Analysis of Unstable-Resonator and Optical-Train Laser Alignment Systems by the Use of Matrix Methods of Gaussian Optics

1975

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ANALYSIS OF UNSTABLE-RESONATOR AND OPTICAL-TRAIN LASER ALIGNMENT SYSTEMS BY THE USE OF MATRIX METHODS OF GAUSSIAN OPTICS

BY

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B.S., California State University, San Diego, 1972

RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the degree of Master of Science in the Graduate Studies Program of Florida Technological University

Orlando, Florida
1975
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A high-power gasdynamic-laser example system is described, and its optical elements are represented in the ABCD-matrix formalism of Gaussian optics. Propagation of Gaussian beams, image formation, focusing, and steering by transverse refractive-index gradients are modeled with the ABCD matrix technique. The effects of small translations and rotations of optical elements are represented as matrices and vectors in the ABCD-matrix formalism, and are analyzed as component misalignments with single and multiple beam reflections. Example calculations show that alignment beams making multiple round trips inside an unstable resonator can easily provide satisfactory resonator alignment and designation of the aim point of an output optical train.

Dr. R. L. Phillips
Director of Research Report
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A high-power gasdynamic-laser example system is described, and its optical elements are represented in the ABCD-matrix formalism of Gaussian optics. Propagation of Gaussian beams, image formation, focusing, and steering by transverse refractive-index gradients are modeled with the ABCD matrix technique. The effects of small translations and rotations of optical elements are represented as matrices and vectors in the ABCD-matrix formalism, and are analyzed as component misalignments with single and multiple beam reflections. Example calculations show that alignment beams making multiple round trips inside an unstable resonator can easily provide satisfactory resonator alignment and designation of the aim point of an output optical train.
INTRODUCTION

The development of lasers and optical systems for long-distance beam propagation has evolved from modest beginnings into high-power lasers, amplifiers, and beam-expanding telescopes. With the advent of the unstable resonator and aerodynamic window, design of a high-power gasdynamic or chemical laser, with large mode volume, good transverse mode discrimination, all-reflecting optics, and no solid windows, was made possible. Coupling this laser to an output optical train, which expands the beam, focuses, and directs it, makes possible long-distance propagation. For example, a 1 m diameter nearly diffraction-limited beam of 10.6 μm radiation can propagate over 70 km without appreciable spreading in beam diameter.

With a nominal beam diameter of 1 m at a distance of 70 km from the laser system, the beam must be aligned to within ± 7 μrad to hit a target. Krupke and Sooy (1969) have shown that similar angular tolerances are present in the alignment requirements of unstable resonators. Thus, the required performance of any alignment technique is to fix the angular orientation of certain optical components to within errors in the microradian range. An analytical technique which can predict this performance is the topic of this report.

To perform this analysis, the following approach will be
used. The basic laser system will be described. Then, the mathematical basis of the ABCD matrix technique will be presented, and applied to simple optical elements. Special matrix techniques pertinent to alignment systems will be discussed. Finally, general applications to the basic laser system will be presented.
CHAPTER I

AN EXAMPLE LASER SYSTEM

System Description

The laser system, shown in figure 1, consists of three subsystems: the laser, the output optical train, and the alignment group. This division into subsystems is an artificial one, because the subsystem boundaries are determined only by the point where the alignment group joins the rest of the system, and this will vary from system to system.

The basic functions of each subsystem are as follows:

The laser provides the high-power output beam, with steering optics to direct it to the output optical train. The output optical train takes the high-power beam, expands it, sets its focus, and directs it to the target. The alignment group measures the angular orientation and position of various optical components in both the laser and the output optical train, making possible their correct alignment.

Subsystem Description

Laser Subsystem

The laser subsystem is shown in figure 2. It consists of resonator convex mirror M-1, resonator concave mirror M-2,
Fig. 1. High-power laser system--subsystem block diagram
Fig. 2. Laser subsystem
resonator hole-coupling mirror M-3, beam-turning mirror M-4, aerodynamic window W-1, apertures A-1 and A-2, the flowing gain medium, and (not shown) associated mirror mounts, shutters, and hardware.

The example resonator is a positive-branch confocal unstable resonator, with an output approximately described by the bare-resonator, fundamental geometric mode of a uniform-intensity annulus with a plane wavefront (Krupke and Sooy 1969). The laser output propagates along a line joining the centers of curvature of mirrors M-1 and M-2. The basic alignment requirement for the laser is to rotate M-1 and M-2 to the position that this line of propagation coincides with the common centerline of A-1, A-2, and the hole in M-3. Then minor rotations of M-3 and M-4 are used to direct the laser output into the output optical train.

Output Optical Train

The output optical train is shown in figure 3. It consists of beam-clipping mirror M-5, convex mirror M-6, concave mirror M-7, beam-directing mirror M-8, power meter P-1, aperture A-3, and associated mirror mounts and hardware. Mirrors M-6 and M-7 form a Cassegrain telescope; M-6 translates to focus it. M-5 is used to remove that central portion of the beam which would not pass through the Cassegrain telescope, but instead return to the laser. The beam sample it clips out is used for power measurements in power meter P-1. Aperture A-3 insures that what passes through
Fig. 3. Output optical train
it will not spill over M-6, and that the backside of M-7 is protected from beam damage. Beam-directing mirror M-8 points the focused Cassegrain-telescope output at the target.

Alignment Group

The alignment group is shown in figure 4. It consists of alignment laser with focusing collimator L-1, spatial-filter aperture A-6, concave mirror M-9, steering mirror M-10, apertures A-4 and A-5, pellicle beamsplitter S-1, a shutter, associated mirror and aperture mounts, and hardware. Laser L-1 provides the alignment beam, which passes through the spatial filter A-6 to mirror M-9. M-9 collimates the beam and sends it to M-10 which directs the beam through aperture A-5 to the pellicle. The pellicle beamsplitter is a thin membrane, about 5 \( \mu \)m thick, and coated for a 50:50 beamsplit. To insure that the alignment beam hits the pellicle in the same place for each alignment, mirrors M-8 and M-9 are simultaneously adjusted so that the beam passes through apertures A-4 and A-5. The pellicle directs the input alignment beam to the laser, and allows a portion of the return alignment beam from the laser to pass through it to the output optical train, and finally, to the target.

Basic Alignment Procedure

The basic alignment technique is to center the alignment beam on all mirrors and apertures by rotating and translating the appropriate mirrors. By insuring that the return alignment beam hits the same spot on the pellicle as the input alignment
Alignment group
beam, along with having both a uniform, doughnut-shaped intensity pattern for the return beam at the pellicle and a uniform intensity pattern for the return beam at A-6, the propagation axis for the high-power beam is made the same as the propagation axis of the alignment beam.
CHAPTER II

ABCD MATRIX LAW FOR GAUSSIAN OPTICS.

General Form

Consider some general system shown in figure 5. The system \( \mathbf{H} \) has some input column vector \( \hat{\mathbf{r}}_1 \):

\[
\hat{\mathbf{r}}_1 = \begin{bmatrix} r_1 \\ r_1' \end{bmatrix}.
\]  

(1)

The system \( \mathbf{H} \) has some output column vector \( \hat{\mathbf{r}}_2 \):

\[
\hat{\mathbf{r}}_2 = \begin{bmatrix} r_2 \\ r_2' \end{bmatrix}.
\]  

(2)

In the ABCD matrix formalism, the matrix \( \mathbf{H} \) represents a linear system, with the system output \( \hat{\mathbf{r}}_2 \) related to the system input \( \hat{\mathbf{r}}_1 \) by

\[
\hat{\mathbf{r}}_2 = \mathbf{H} \hat{\mathbf{r}}_1.
\]  

(3)

\[
\begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_1 \\ r_1' \end{bmatrix}
\]  

(4)

\[
r_2 = A r_1 + B r_1',
\]  

(5)

\[
r_2' = C r_1 + D r_1'.
\]  

(6)

Consider some composite system shown in figure 6. With
Fig. 5. General matrix block diagram

Fig. 6. Composite matrix system block diagram

Fig. 7. Equivalent composite matrix block diagram
\( H_k \) defined as \( \tilde{H} \) in equations (3) and (4), a general transfer function matrix \( H_k \), which relates the input \( \tilde{r}_1 \) to the output \( \tilde{r}_k \), can be found from the following expression:

\[
H_t = H_{k-1} \cdots H_2 H_1 .
\]  

Equation (7) is depicted by figures 6 and 7, which are equivalent.

**Application to Rays and Spherical Waves In Optical Systems**

As described concisely by Yariv (1971, chap. 2), both paraxial rays and spherical waves can be handled with the ABCD matrix method. Shown in figure 8 is some input ray defined by \( \tilde{r}_1 \) of equation (1), some optical system with matrix \( H \), and some output ray defined by \( \tilde{r}_2 \) of equation (2). Associated with \( H \) is some input coordinate system \((x_1, y_1, z_1)\) and some output coordinate system \((x_2, y_2, z_2)\). The input vector \( \tilde{r}_1 \) is located at the \( z_1 = 0 \) plane and is defined in terms of \((x_1, y_1, z_1)\) as

\[
\begin{align*}
\tilde{r}_1 & = x_1 \\
\tilde{r}_1' & = dx_1/dz_1 .
\end{align*}
\]

The output vector \( \tilde{r}_2 \) is located at the \( z_2 = 0 \) plane and is defined in terms of \((x_2, y_2, z_2)\) as

\[
\begin{align*}
\tilde{r}_2 & = x_2 \\
\tilde{r}_2' & = dx_2/dz_2 .
\end{align*}
\]

with \( \tilde{r}_2 \) related to \( \tilde{r}_1 \) by equation (3). \( H \) can represent propagation along a distance, propagation through optical elements, combinations of both, etc.
Fig. 8. Matrix system for optical rays
SPHERICAL WAVE

\[ r = x \quad r' = \frac{dx}{dz} = \tan \theta \]

PARAXIAL APPROXIMATION:

\[ r' = \theta = \frac{x}{R} \]

Fig. 9. Spherical wave and equivalent paraxial ray
The geometrical relationship between a spherical wave and one of its associated paraxial rays is shown in figure 9. The input wave's radius of curvature can be expressed as

$$R_1 = r_1/r_1'$$  \hspace{1cm} (12)

with a corresponding expression for the radius of curvature $R_2$ of the output wave. The relationship between the radii of curvature of the input and output waves is expressed by

$$R_2 = \frac{AR_1 + B}{CR_1 + D}$$  \hspace{1cm} (13)

with the optical system matrix $H$ again represented by

$$H = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

### Application to Gaussian Beams

As described by Yariv (1971, chaps. 3 and 4), Sinclair and Bell (1969, chap. 4), and Kogelnik and Li (1966), the output modes of a stable laser can be approximately described by Gaussian beam parameters. The complex parameter $q(z)$ is defined in terms of the beam radius of curvature $R(z)$ and beam width $w(z)$ by

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\gamma' w^2(z)}$$  \hspace{1cm} (15)

where $z$ is the coordinate in the direction of beam propagation, $\lambda$ is the wavelength, and $j$ is $\sqrt{-1}$. Because $q(z)$ transforms in the same way as the radius of curvature of spherical waves, it is called the complex radius of the Gaussian beam, and the relationship between some input $q_1$ and some output $q_2$ can be found...
from the system's ABCD matrix:

\[ q_2 = \frac{A q_1 + B}{C q_1 + D} \]  \hspace{1cm} (16)

The Gaussian beam can be handled with the same ABCD matrices that are used for paraxial rays and spherical waves.

**Application to Imaging Systems**

*Image Formation*

Klein (1970, chap. 3) has shown that the requirement of image formation is a value of zero for the B term in the ABCD matrix relating the object plane to the image plane. In this case, the image lateral magnification will be given by A. If B \( \neq 0 \), the location \( z_2 = d \) in the output coordinate system \((x_2, y_2, z_2)\) where the image is located can be found by finding

\[ H_1 = D H \]  \hspace{1cm} (17)

where \( D \) is given by (Yariv 1971, p. 20)

\[ D = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \]  \hspace{1cm} (18)

The \( D \) matrix represents propagation along a length \( d \). With the elements of \( H \) given by equation (14) for some general case, \( H_1 \) can be written as

\[ H_1 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} A+dC & B+dD \\ C & D \end{bmatrix} \]  \hspace{1cm} (19)

The image-formation requirement can be specified as

\[ B_1 = 0 \]  \hspace{1cm} (20)
yielding a value of \( d \) given by

\[
d = -\frac{B}{D}.
\]

(21)

The value of the lateral image magnification is

\[
A_1 = A - \frac{BC}{D}.
\]

(22)

Focusing

Consider some input spherical wave with radius of curvature \( R_1 \) and representative paraxial ray \( \mathbf{r}_1 \) defined by equations (1) and (12), some system matrix \( \mathbf{H} \) defined by equation (14) and some output spherical wave with radius of curvature \( R_2 \) and representative paraxial ray \( \mathbf{r}_2 \) defined similarly to \( \mathbf{r}_1 \). The requirement that the output spherical wave be focused is that

\[
\mathbf{r}_2 = 0
\]

(23)

for all input rays \( \mathbf{r}_1 \) satisfying equation (12). This implies that

\[
A + \frac{B}{R_1} = 0.
\]

(24)

If the requirement of equation (24) is not met, the location of the point of focus \( z_2 = d \) in the output coordinate system

\((x_2, y_2, z_2)\)

can be found by forming

\[
\mathbf{H}_f = \mathbf{D} \mathbf{H},
\]

(25)

where \( \mathbf{D} \) is defined by equation (13) and the vector \( \mathbf{r}_3 \) is given by

\[
\mathbf{r}_3 = \mathbf{H}_f \mathbf{r}_1.
\]

(26)

The focusing requirement of equations (23) and (24), when applied to vector \( \mathbf{r}_3 \), gives

\[
d = -\frac{A R_1 + B}{C R_1 + D}.
\]

(27)
Equations (13) and (27) have similar forms; an interpretation of this is that the output spherical wave must be propagated a distance equal to its radius of curvature for it to reach a focus.

The requirement that an optical system bring a Gaussian beam into focus is for the output beam parameter $q_2$ of equations (15) and (16) be imaginary. Perhaps an easier approach to finding the Gaussian focus is to use equation (16) to find $q_2$, use equation (15) to find $R_2$ and $w_2$, and then use equation (28), below.

Sinclair and Bell (1969, p. 94) have shown that if $R$ and $w$ are known, then the Gaussian beam has propagated a distance $z$ from its focus ($z = 0$), where

$$z = \frac{R}{1 + (\lambda R/\gamma w^2)^2}.$$  

(28)

By propagating the Gaussian output beam a distance of $-z$, the beam is brought to a focus.

Individual Optical Components

Yariv (1971, p. 20), Klein (1970, chap. 3), and Kogelnik and Li (1966, p. 1313) give matrix representations for various optical elements. The following optical elements and their matrices will be used later in this paper:

As presented in equation (18), the matrix $D$ for propagation a distance $d$ is as follows:

$$D = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}.$$  

(29)

Propagation through a thin lens of focal length $f$ can be
represented by the matrix \( \mathbf{L} \), as follows:

\[
L = \begin{bmatrix}
1 & 0 \\
-1/f & 1
\end{bmatrix}.
\]  

(30)

A spherical mirror with radius of curvature \( R \) can be represented by the matrix \( \mathbf{M} \), as follows:

\[
M = \begin{bmatrix}
1 & 0 \\
-2/R & 1
\end{bmatrix}.
\]  

(31)

A confocal telescope, with input at lens (or mirror) number one, output at lens (or mirror) number two, and lens (or mirror) separation equal to the sum of the signed focal lengths, can be represented by the matrix \( \mathbf{T} \), as follows:

\[
T = \begin{bmatrix}
-f_2/f_1 & f_1+f_2 \\
0 & -f_1/f_2
\end{bmatrix}.
\]  

(32)

Note that the input vector is at the plane of the first lens, and the output vector is at the plane of the second lens.
CHAPTER III

SPECIAL MATRIX APPLICATIONS

Static Misalignment of Individual Components

Optical components are considered rigid bodies for the purpose of misalignment analysis, and as such have three translational degrees of freedom and three rotational degrees of freedom. Each optical element will have associated with it an input coordinate system \((x_1, y_1, z_1)\) in rectangular Cartesian coordinates and an output coordinate system \((x_2, y_2, z_2)\) in rectangular Cartesian coordinates. For the purposes of the following analysis, two special cases will be considered: For a simple lens, the input and output coordinate systems are identical; for a mirror, the \(x\) axes are identical, while the input and output \(y\) axes are antiparallel, as are the \(z\) axes; and both coordinate systems have a common origin. Deviations from these special cases are treated in the Appendix. For both cases, the positive \(z\) axis is considered the axis of propagation. The fixed coordinate system \((X, Y, Z)\), in which the optical element's translation and rotation are measured, is identical to the input coordinate system prior to translation or rotation.

Because of the two-dimensional nature of the ABCD matrix technique, two matrices are required to handle a three-dimensional
situation. The $H_x$ matrix relates the input ray vector $\vec{r}_{x,1}$, a function of $x_1$ and $z_1$, to the output ray vector $\vec{r}_{x,2}$, a function of $x_2$ and $z_2$. The $H_y$ matrix relates the input ray vector $\vec{r}_{y,1}$, a function of $y_1$ and $z_1$, to the output ray vector $\vec{r}_{y,2}$, a function of $y_2$ and $z_2$.

Translation of the optical element in the $X$ direction will require the use of the $H_x$ matrix. Translation in the $Y$ direction will require the use of the $H_y$ matrix. Translation in the $Z$ direction will be equivalent for both $H_x$ and $H_y$.

Rotation about an axis parallel to the $Y$ axis is handled using the $H_x$ matrix. Rotation about an axis parallel to the $X$ axis is handled using the $H_y$ matrix. Rotation about the $Z$ axis itself has no effect for optical elements with rotational symmetry about the $Z$ axis; for other cases, consult the Appendix.

General translations and rotations can be separated into parts requiring the $H_x$ and $H_y$ matrices individually. For translations, the vectorial nature of the displacement is used, and vector components in the $X$, $Y$, and $Z$ directions are taken. Rotations, in general, cannot be treated as vectors, for unless they have parallel axies of rotation, they do not commute. However, infinitesimal rotations are vectorial in nature; Goldstein (1959, pp. 124-32) has shown that they are pseudovectors or axial vectors. The first-order approximation used by Goldstein in defining the infinitesimal rotation is the paraxial approximation. Thus, the rotations which can be handled with the ABCD
matrix technique must be infinitesimal, and as such, can be treated as vectors. To accomplish this, the vector associated with a rotation has a magnitude equal to the angle of rotation and a direction parallel to the axis of rotation, with sense according to the right-hand rule. The components of this vector in the X and Y directions are used with the $H_y$ and $H_x$ matrices individually.

Because general translations and infinitesimal rotations of optical elements can be broken up into separate components, the following techniques are given for the X-Z plane only. In applying the techniques to the Y-Z plane, the asymmetry associated with the infinitesimal-rotation pseudovector requires that a positive rotation is pointed in the -X direction.

Component Translation Parallel To the Optical Axis

Individual component translation parallel to the optical axis can be handled with the $D$ matrix of equation (29). For a simple lens translated a distance $d$ in the $+Z$ direction, the new output vector $\vec{r}_2$ can be written in terms of the lens matrix $L$ of equation (30), the $D$ matrix and its inverse $D^{-1}$, and the input vector $\vec{r}_1$, by

$$\vec{r}_2 = D^{-1} L D \vec{r}_1 . \tag{33}$$

Forming the inverse matrix $D^{-1}$ is the same as forming the matrix to propagate a distance $-d$.

For a mirror, the form of the output vector is changed
from that of a lens because the mirror reverses the direction of
propagation upon reflection. For a translation of the mirror
a distance $d$ along the $+Z$ axis, the new output vector is
\[ \vec{r}_2 = D M D \vec{r}_1 , \] (34)
with $D$ defined by equation (29) and $M$ defined by equation (31).

Equation (33) can be used for any optical element whose $z_1$ and $z_2$ axes are parallel. Equation (34) can be used for
any optical element whose $z_1$ and $z_2$ axes are antiparallel.

Component Translation Perpendicular
To the Optical Axis

The translation of an optical element perpendicular to the
optical axis is treated by the addition of a translation vector
$\vec{t}_1$ to the input vector, and a translation vector $\vec{t}_2$ to the
output vector. For a simple lens or mirror, with general matrix
$H$, translated a distance $d$ in the $+X$ direction, the new output
vector $\vec{r}_2$ can be written in terms of the input vector $\vec{r}_1$, as
follows:
\[ \vec{r}_2 = H (\vec{r}_1 + \vec{t}_1) + \vec{t}_2 . \] (35)
In the case of a simple lens or mirror, the input $x_1$ axis and
the output $x_2$ axis are equivalent. Thus $\vec{t}_1$ and $\vec{t}_2$ can be written
as follows:
\[ \vec{t}_1 = \begin{bmatrix} -d \\ 0 \end{bmatrix} \] (36)
\[
\hat{t}_2 = -\hat{t}_1 = \begin{bmatrix} d \\ 0 \end{bmatrix}.
\tag{37}
\]

Using the values of \(t_1\) and \(t_2\) from equations (36) and (37) in equation (35) gives
\[
\hat{r}_2 = H \hat{r}_1 + (H - I) \hat{t}_1,
\tag{38}
\]
where \(I\) is the identity matrix. For a plane mirror, \(H = I\), and the translation (parallel to the mirror's surface) has no effect on the output vector.

Small Rotations of Components About An Axis Perpendicular to the Direction of Propagation

For an input ray to see a pure rotation of an optical element, the axis of rotation must pass through the point of intersection of that ray and the input \(z_1 = 0\) plane. Otherwise, the rotation is accompanied with a translation; see the Appendix for details. For the purposes of this analysis, the axis of rotation is the \(+Y\) axis, and the displacement in the \(Z\) direction, resulting from the rotation, which the input ray will see, is neglected. This assumption is reasonable if the range of values of \(x_1\) is small compared to the separation between various optical elements.

Because, in the paraxial approximation, the \(r_1'\) component of the \(\hat{r}_1\) input vector is the angle between the input ray and the \(z_1\) axis, small rotations of the \(z_1\) axis about the \(Y\) axis (equivalent to the \(y_1\) axis) add directly to the \(r_1'\) component. Thus, for a thin lens or mirror, the vector \(\hat{t}_1\) is added to the
input vector \( \vec{r}_1 \), and a corresponding rotation vector \( \vec{\theta}_2 \) is added to the output vector, to form the new output vector \( \vec{r}_2 \), where

\[
\vec{r}_2 = H (\vec{r}_1 + \vec{\theta}_1) + \vec{\theta}_2
\]

\[
\vec{\theta}_1 = \begin{bmatrix} 0 \\ \theta \end{bmatrix}
\]

For a thin lens, with identical input and output coordinate systems, the vector \( \vec{\theta}_2 \) represents a rotation in the opposite sense to \( \vec{\theta}_1 \), and is given by

\[
\vec{\theta}_2 = -\vec{\theta}_1 = \begin{bmatrix} 0 \\ -\theta \end{bmatrix}
\]

For a thin lens, equation (39) becomes

\[
\vec{r}_2 = L \vec{r}_1 + (L - I) \vec{\theta}_1
\]

Using the elements of \( L \) from equation (30),

\[
\vec{r}_2 = L \vec{r}_1 + \begin{bmatrix} 0 & 0 \\ -1/f & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \theta \end{bmatrix}
\]

\[
\vec{r}_2 = L \vec{r}_1
\]

Equation (44) indicates that small rotations about the Y axis do not effect propagation through a thin lens.

Because, for a mirror, the output \( y_2 \) axis is antiparallel to the input \( y_1 \) axis, the rotation vector \( \vec{\theta}_2 \) for a mirror is opposite in sign for that of a thin lens. Thus, a new output

\[
\vec{r}_2 = L \vec{r}_1
\]
vector \( \vec{r}_2 \) for a mirror with matrix \( \hat{M} \) rotated through an angle \( \theta \) about the Y axis can be defined in terms of the input vector \( \vec{r}_1 \) and the rotation vector \( \vec{\theta}_1 \) by

\[
\vec{r}_2 = \hat{M} (\vec{r}_1 + \vec{\theta}_1) + \vec{\theta}_1,
\]
(45)

\[
\vec{r}_2 = \hat{M} \vec{r}_1 + (\hat{M} + \mathbb{I}) \vec{\theta}_1.
\]
(46)

Using the elements of \( \hat{M} \) from equation (31),

\[
\vec{r}_2 = \hat{M} \vec{r}_1 + \begin{bmatrix} 2 & 0 \\ -2/R & 2 \end{bmatrix} \begin{bmatrix} 0 \\ \theta \end{bmatrix}.
\]
(47)

By defining the vector

\[
\hat{\phi} = \begin{bmatrix} 0 \\ 2\theta \end{bmatrix},
\]
(48)

equation (47) can be written as

\[
\vec{r}_2 = \hat{M} \vec{r}_1 + \hat{\phi} = \hat{M} (\vec{r}_1 + \hat{\phi}).
\]
(49)

The rotation vector \( \hat{\phi} \) can be added to the input vector before the mirror matrix operation is performed, or it can be added afterward; this is permitted because the product \( \hat{M} \hat{\phi} \) equals the vector \( \hat{\phi} \). The nature of the rotation vector \( \hat{\phi} \) indicates that output ray is rotated through an angle equal to twice the angle of mirror rotation.

**First-Order Approximation to Transverse Refractive-Index Gradients**

Gradients in the index of refraction, perpendicular to the direction of propagation, can be handled with the ABCD matrix.
technique, if the following first-order approximation is considered: The optical path along which the transverse gradient exists is broken up into smaller paths along which the gradient is constant. In this case, the effect of the gradient on a plane wave is shown schematically in figure 10. The plane wave, propagating a distance $\Delta s$, is rotated through an angle $\Delta \theta$, which is found from the geometry of figure 10, using the small-angle approximation:

$$\Delta \theta = \frac{\Delta s - \Delta s(1 - \Delta n/n)}{\Delta x}, \quad (50)$$

$$\frac{\Delta \theta}{\Delta s} = \frac{\Delta n}{\Delta x n}, \quad (51)$$

where the $\Delta$ operator signifies a small change, and $n$ is the index of refraction. Taking the limit as $\Delta s \to 0$,

$$\frac{d\theta}{ds} = \frac{dn}{dx n}. \quad (52)$$

With the refractive-index gradient a constant, integrating equation (52) yields

$$\theta_2 - \theta_1 = \frac{dn}{dx n} s. \quad (53)$$

In the paraxial approximation,

$$\theta = \frac{dx}{ds}. \quad (54)$$

Substituting equation (54) into equation (52) and integrating twice with respect to $s$ yields

$$x_2 - x_1 = \frac{dn}{dx} \frac{s^2}{2n} + \theta_1 s. \quad (55)$$

Equations (53) and (55) can be put in vector matrix notation, by observing that $x_1$ and $\theta_1$ are the two components of the $r_1$
\[ \lambda_2 = \frac{\lambda_0}{n + \Delta n} \approx \frac{\lambda_0}{n} \left[ 1 - \frac{\Delta n}{n} \right] \]

\( \lambda_0 \) is the free-space wavelength.

\[ \Delta s = k \lambda_1, \]

\[ \lambda_1 = \frac{\lambda_0}{n} \]

Fig. 10. Transverse refractive-index gradient
input vector, and \( x_2 \) and \( \theta_2 \) are the two components of the \( \mathbf{r}_2 \) output vector:

\[
\begin{bmatrix}
  x_2 \\
  \theta_2
\end{bmatrix} =
\begin{bmatrix}
  1 & 6 \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  \theta_1
\end{bmatrix} +
\begin{bmatrix}
  \frac{dn}{dx} \frac{s^2}{2n} \\
  \frac{dn}{dx} \frac{s}{n}
\end{bmatrix}.
\tag{56}
\]

Equation (56) can be written as

\[
\mathbf{r}_2 = \mathbf{S} \mathbf{r}_1 + \mathbf{N},
\tag{57}
\]

where \( \mathbf{S} \) is a form of the \( \mathbf{D} \) matrix of equation (29), and the vector \( \mathbf{N} \) is defined by

\[
\mathbf{N} =
\begin{bmatrix}
  \frac{dn}{dx} \frac{s^2}{2n} \\
  \frac{dn}{dx} \frac{s}{n}
\end{bmatrix}.
\tag{58}
\]

Thus the effect of a constant, transverse refractive-index gradient \( \frac{dn}{dx} \) over a path length \( s \) is accounted for by adding the vector \( \mathbf{N} \) to the output vector for propagation over that length \( s \).

**Single-Component Rotation with Multiple-Pass Beams**

Two mirrors facing each other, as in a Fabry-Perot interferometer, a stable resonator, or an unstable resonator, can have multiple reflections from the reflecting surfaces. When one of these surfaces is misaligned by a small rotation about an axis perpendicular to the direction of propagation, this rotation steers the beam each time it reflects from the rotated surface.

Figure 11 shows the geometry used for the matrix analysis.
Fig. 11. Multiple-pass geometry
Equation (31) defines the matrix \( M_1 \) for the rotated mirror and the matrix \( M_2 \) for the unrotated mirror. \( M_1 \) has been rotated through a small angle \( \theta \). The input vector for \( M_1 \), prior to the \( i \)-th reflection from its surface, is \( \vec{r}_{1,i} \), \( i = 1, 2, 3, \ldots \). The output vector for \( M_1 \), after the \( i \)-th reflection from its surface, is \( \vec{r}_{2,i} \), \( i = 1, 2, 3, \ldots \). The two mirrors are separated by a distance \( d \), which has a propagation matrix \( D \) defined by equation (29).

The rotation vector \( \hat{\theta} \) is associated with the rotation angle \( \theta \); its definition and use are given in equations (48) and (49).

After the first reflection from \( M_1 \),

\[
\vec{r}_{2,1} = M_1 \vec{r}_{1,1} + \hat{\theta} .
\]  

(59)

After the ray propagates the distance \( d \) to mirror \( M_2 \), is reflected, and propagates back to \( M_1 \), it becomes the next input, as follows:

\[
\vec{r}_{1,2} = D M_2 D \vec{r}_{2,1} .
\]  

(60)

After reflecting a second time from \( M_1 \),

\[
\vec{r}_{2,2} = M_1 D M_2 D \vec{r}_{2,1} + \hat{\theta} .
\]  

(61)

Defining

\[
E = M_1 D M_2 D ,
\]  

(62)

\[
\vec{r}_{2,2} = E \vec{r}_{2,1} + \hat{\theta} .
\]  

(63)

Extending equation (63) for another round trip,

\[
\vec{r}_{2,3} = E (E \vec{r}_{2,1} + \hat{\theta}) + \hat{\theta} .
\]  

(64)

This can be generalized, for \( i \) reflections from \( M_1 \), to
\[ \overrightarrow{r}_{2,1} = F^{i-1} \overrightarrow{r}_{2,1} + (F^{i-2} \overrightarrow{\phi} + \ldots + \overrightarrow{I} \overrightarrow{\phi}) , \quad (65) \]

where the notation \( F^{i-1} \) indicates the matrix multiplication of \( i-1 \) \( F \) matrices. Substituting the value of \( \overrightarrow{r}_{2,1} \) from equation (59) into equation (65),

\[ \overrightarrow{r}_{2,1} = F^{i-1} M_1 \overrightarrow{r}_{1,1} + (F^{i-1} + \ldots + F + I) \overrightarrow{\phi} . \quad (66) \]

For the purposes of rotational analysis, the assumption is made that the input vector \( \overrightarrow{r}_{1,1} \) is zero. Thus, the final output vector is

\[ \overrightarrow{r}_1 = (F^{i-1} + \ldots + F + I) \overrightarrow{\phi} , \quad (67) \]

where the input beam hits the rotated mirror \( i \) times.

DeRusso, Roy, and Close (1965, pp. 281-6) discuss the Cayley-Hamilton technique, which can be used to reduce the matrix polynomial of equation (67), as follows: Since \( F \) is a 2 x 2 matrix, the matrix polynomial \( P(F) \), which is a function of powers of \( F \), can be written as

\[ P(F) = \alpha I + \beta F , \quad (68) \]

where \( I \) is the identity matrix and \( \alpha \) and \( \beta \) are constants.

The eigenvalues of \( F \), \( \gamma_1 \) and \( \gamma_2 \), must satisfy the corresponding scalar equation:

\[ P(\gamma) = \alpha + \beta \gamma . \quad (69) \]

The eigenvalues \( \gamma_1 \) and \( \gamma_2 \) are solutions of

\[ |\gamma I - F| = 0 , \quad (70) \]

where \( | \) \( | \) denotes the determinant of the matrix. With

\[ F = \begin{bmatrix} A & B \\ C & D \end{bmatrix} , \quad (71) \]
and with $B, C \neq 0$,

\[ \gamma_1 = \frac{A + D}{2} + \sqrt{\frac{(A + D)^2}{4} - (AD - BC)} \quad , \quad (72) \]

\[ \gamma_2 = \frac{A + D}{2} - \sqrt{\frac{(A + D)^2}{4} - (AD - BC)} \quad . \quad (73) \]

Since the matrices which form the product matrix $F$ are all unimodular, the determinant of $F$ is one:

\[ AD - BC = 1 . \quad (74) \]

Thus equations (72) and (73) reduce to

\[ \gamma_1 = \frac{A + D}{2} + \sqrt{\frac{(A + D)^2}{4} - 1} \quad , \quad (75) \]

\[ \gamma_2 = \frac{A + D}{2} - \sqrt{\frac{(A + D)^2}{4} - 1} \quad . \quad (76) \]

The particular form of $P(F)$ from equation (67) is

\[ P(F) = F^{i-1} + \ldots + F + I . \quad (77) \]

This is a geometric series, and thus $P(\gamma)$ can be written

\[ P(\gamma) = \frac{\gamma^i - 1}{\gamma - 1} . \quad (78) \]

Inserting $P(\gamma)$ and the individual eigenvalues of $F$ into equation (69) gives the following equations:

\[ \frac{\gamma_1^i - 1}{\gamma_1 - 1} = \alpha_1 \gamma_1 + \beta_1 \gamma_1 \quad , \quad (79) \]

\[ \frac{\gamma_2^i - 1}{\gamma_2 - 1} = \alpha_1 \gamma_2 + \beta_1 \gamma_2 \quad . \quad (80) \]

These equations can be solved for $\alpha_1$ and $\beta_1$:

\[ \alpha_1 = \frac{\gamma_2 \left( \frac{\gamma_1^i - 1}{\gamma_1 - 1} \right) - \gamma_1 \left( \frac{\gamma_2^i - 1}{\gamma_2 - 1} \right)}{\gamma_2 - \delta_1} \quad , \quad (81) \]
\[ \beta_1 = \frac{\gamma_2^i - 1}{\gamma_2 - 1} - \frac{\gamma_1^i - 1}{\gamma_1 - 1}. \]

Equation (67) can be written in reduced form as
\[ \bar{r}_1 = (\alpha_1 I + \beta_1 F) \bar{\Phi}. \]

Using specific eigenfunctions of \( F \), further simplifications of equation (83) are possible; the particular case of the positive-branch confocal unstable resonator will be treated later.
CHAPTER IV

APPLICATION OF MATRIX-ANALYSIS TECHNIQUES
TO THE EXAMPLE LASER SYSTEM

The example laser system, described in chapter I, can be analyzed in two parts: high-power laser performance, and alignment system performance. Both parts have many common characteristics.

High-Power Laser Performance

Static Alignment Requirements

The basic alignment requirements for the optical elements of the high-power laser system are to orient the laser mirrors for proper laser output, and to orient the output-train mirrors so that the output beam stays on those mirrors and arrives at the designated target. It is beyond the scope of this paper to establish proper performance factors for these requirements. Instead, this paper describes analysis techniques which determine how well the alignment requirements are met.

The basic technique to determine performance is to propagate an initially-aligned ray (the zero vector) through the optical train, insert appropriate element misalignments, and observe the ray at various planes where beam location is critical.
The unstable-resonator output ray will be determined with the geometric-optics method of Krupke and Sooy (1969).

**Unstable-resonator mirror alignment**

Siegman and Sziklas (1974) have shown that the predicted output for a typical high-power gasdynamic laser has a complicated phase and amplitude distribution. Their wave-optics model demonstrates some of the detailed characteristics of a real laser's output which is not present in the geometric-optics model, and the wave-optics approach should be used when the effects neglected by the geometric model (diffraction, flowing saturable gain medium, etc.) are significant. However, the geometric-optics model used by Krupke and Sooy (1969) has shown good agreement with their experimental results, and will be used for this analysis.

Shown in figure 12 is a misaligned unstable resonator, with the output ray passing through the center of curvature of the convex and concave mirrors. Mirror designations are the same as in chapter I, with the following additions: \( R_c \) is the radius of curvature of the concave mirror \( M-2 \). \( R_v \) is the radius of curvature of the convex mirror \( M-1 \); it is a negative number. The separation between mirrors \( M-1 \) and \( M-2 \) is called \( d_{1-2} \). The separation between mirrors \( M-1 \) and \( M-3 \) is called \( d_{1-3} \). A small rotation of mirror \( M-1 \) is termed \( \theta_v \). A small rotation of mirror \( M-2 \) is called \( \theta_c \). The directions of the rotations are negative as shown in figure 12, which is consistent with previous notation.

Analysis of the geometry of figure 12, using the small-angle
Fig. 12. Unstable resonator misalignment geometry
approximation, gives the value of the ray vector \( \vec{r}_a \) at the center of curvature of the convex mirror \( M-1 \) as

\[
\vec{r}_a = \left[ \begin{array}{c} R_v \Theta_v \\ R_c \Theta_c - R_v \Theta_v \\ R_c + R_v - d_{1-2} \end{array} \right]. 
\] (84)

This vector is then propagated backward to the plane passing through the center of the hole-coupling mirror \( M-3 \), and the resulting vector \( \vec{r}_b \) is the input to mirror \( M-3 \):

\[
\vec{r}_b = \left[ \begin{array}{cc} 1 & R_v - d_{1-3} \\ 0 & 1 \end{array} \right] \vec{r}_a, 
\] (85)

\[
\vec{r}_b = \left[ \begin{array}{c} R_v \Theta_v + (R_v - d_{1-3})(R_c \Theta_c - R_v \Theta_v) \\ R_c + R_v - d_{1-2} \\ R_c \Theta_c - R_v \Theta_v \\ R_c + R_v - d_{1-2} \end{array} \right]. 
\] (86)

If a small translation of \( M-1 \) in the \( x \) direction is designated \( x_v \), and a small translation of \( M-2 \) in the \( x \) direction is designated \( x_c \), the effect of these translations can be introduced into equation (86) by adding \( x_v/R_v \) to each term \( \Theta_v \) and by adding \( x_c/R_c \) to each term \( \Theta_c \). Rotation of \( M-1 \) or \( M-2 \) about the \( z \) axis, and translation of \( M-1 \) or \( M-2 \) along the \( z \) axis will not change the output direction, as long as the resonator is aligned. The vector \( \vec{r}_b \) can now be propagated through the rest of the optical train, to determine where the misaligned beam will travel.
Because mirrors M-3 and M-4 are oriented at 45° with respect to the axis of beam propagation, they require the general techniques of the Appendix to completely handle the effects of their misalignments. However, a few general observations can be made. Because the mirrors are plane, their rotation can be handled with the techniques of equation (49), with the axes of rotation in the plane of each mirror. Translation of each mirror in the plane of the mirror will have no effect, while translation of each mirror in a direction normal to its surface will produce the combined effect of translation along the axis of propagation, and translation perpendicular to the axis of propagation, as handled by equations (34) and (38).

**Aerodynamic-window steering**

Non-uniformity in the effective index of refraction of the flowing medium in aerodynamic windows can cause steering of laser beams passing through them. Because of the short length of interaction between beam and window, this steering can be treated as pure beam rotation, as handled with equations (48) and (49), with the $M$ matrix being the identity matrix $I$ of a plane mirror.

**Output-optical-train alignment**

Each mirror in the output optical train can be misaligned using techniques expressed by equations (34), (38), and (49), or in the Appendix. A comparison of the misalignment results can be an aid to troubleshooting. For instance, if the laser
system's output were to miss the target, it may be possible to rule out rotation of a resonator mirror as a possible cause, because the amount of rotation required would cause the beam to miss the Cassegrain telescope's entrance aperture completely. Because of the angular demagnification of the Cassegrain telescope, the tightest angular tolerances for the mirrors of the output optical train would be on M-7 and M-8. A knowledge of misalignment effects in the output optical train can be used to establish a performance tradeoff between mirror-mount rigidity and beam-pointing accuracy at the target.

Cassegrain-Telescope Focusing

Determination of the Cassegrain-telescope mirror separation required to focus the beam on target can be determined in two ways. A plane-wave output from the confocal unstable resonator is assumed. Then the spherical-wave propagation techniques of equations (10-13) and (23-27), or the Gaussian-beam propagation techniques of equations (15), (16), and (28) can be used with the associated matrices of the optical train to propagate to the focus. The mirror separation in the Cassegrain telescope can be varied analytically or numerically to find the relationship between mirror separation and point of focus.

Temperature-Gradient Beam Steering

Propagation of a laser beam over long distances close to the ground becomes a function of the interaction between the heated earth and the air above it. The variation of the surface
terrain composition (grass, trees, dirt, water, hummocks, etc.) and the variation in local heating from the sun (clouds, shade, etc.) can cause the air temperature near the surface to vary appreciably. Because the sun can provide a fairly constant energy source, a fairly-constant temperature gradient can be maintained in the air near the earth's surface.

To make an estimate of this effect on beam propagation, a constant downward temperature gradient is assumed along the entire beam path. The relationship between this gradient and the gradient in the index of refraction is shown below.

The beam-propagation coordinate system has the z axis for the direction of propagation, with the x axis in the vertical upward direction. The beam starts at the origin, initially pointed along the z axis. Using this notation, the results of propagation are given by equations (57) and (58), where S is the length of the beam path, and the refractive-index gradient is found from the following derivation.

Born and Wolf (1970, p. 88) indicate that the approximate relationship between n, the index of refraction of air, and the density of air, is

$$n^2 - 1 = K \rho$$  \hspace{1cm} (87)

where K is a constant. Defining

$$n - 1 = \Delta n$$  \hspace{1cm} (88)

with $\Delta n$ small for air. Using this definition in equation (87),

$$\Delta n (2 + \Delta n) = K \rho.$$  \hspace{1cm} (89)
Retaining only first-order terms in $\Delta n$, 

$$\Delta n = \frac{K_1 p}{2}.$$  \hspace{1cm} (90)

Using the ideal gas law

$$\frac{p}{T} = K_1,$$  \hspace{1cm} (91)

with $K_1$ a constant and the pressure $p$ also a constant along the horizontal beam path, equation (90) can be written as a function of the temperature $T$:

$$\Delta n = \eta \frac{T_s}{T},$$  \hspace{1cm} (92)

where $\eta$ is the resulting lumped constant of proportionality, and $T_s$ is the standard temperature for which $\eta$ is determined.

Writing equation (92) in terms of equation (88) and differentiating with respect to $x$,

$$\frac{dn}{dx} = - \frac{\eta T_s}{T^2} \frac{dT}{dx}.$$  \hspace{1cm} (93)

If the temperature along the beam path is approximately equal to the standard temperature $T_s$, then equations (91) and (93) reduce to

$$\Delta n = \eta,$$  \hspace{1cm} (94)

$$\frac{dn}{dx} = - \frac{\Delta n}{T} \frac{dT}{dx}.$$  \hspace{1cm} (95)

Born and Wolf (1970, p. 88) give a typical value of $2.929 \times 10^{-4}$ for $\Delta n$ in air at typical conditions ($14.5^\circ C$, 1 atm). Using this, equation (95) can be written as follows:
\[
\frac{dn}{dx} = -1.018 \times 10^{-6} \frac{dT}{dx} \tag{96}
\]

As a numerical example, consider a 1\textdegree C variation in temperature over a height of 1 m, with a path length of 1 km. With the air hotter at the bottom of the beam, equations (57) and (58) indicate that the temperature gradient has shifted the beam 0.5 m up at the target, and pointed it up at an angle of $10^{-3}\text{rad}$. This steering effect produces a displacement at the target that is quadratic in the distance to the target, and a beam rotation at the target that is linear in the distance to the target.

Temperature-gradient beam steering can also play a role in the performance of the laser alignment system. An alignment beam sent to the target will experience the same steering as the high-power laser beam (ignoring high-power effects). In addition, containment ducts and the chamber enclosing the gain medium, often partially evacuated during high-power operation, may be at typical atmospheric conditions during alignment, and thus sustain significant temperature gradients due to heating (from the sun, etc.) during alignment. Even when materials such as INVAR are used to minimize differential thermal-growth problems, the air itself can become the limiting factor in precision alignments. As a typical example, a 10 cm diameter beam propagating over a 10 m path would require that the air-temperature difference across the path be less than 0.01\textdegree C if the beam steering angle has to be less than 1 \mu rad.
Alignment-System Performance

The Alignment Hypotheses

The fundamental alignment hypothesis is that when the alignment system has "perfectly" aligned the unstable resonator, the resonator will still be "perfectly" aligned for the high-power beam. An additional hypothesis is that the return alignment beam will follow the same path as the high-power beam, from the unstable resonator to the target. A measure of alignment system performance is how well these two hypotheses are met.

Basic Misalignment Analysis

The techniques used to analyze the effects of misalignments of mirrors in the output optical train on the propagation of the high-power beam through that train can also be used to analyze the alignment beam propagation. A primary difference between alignment-beam propagation and high-power beam propagation is that the alignment beam may experience multiple reflections from misaligned elements. Multiple reflections can be handled either by applying the appropriate single-reflection equation to the beam each time it reflects from the misaligned surface, or by using a general expression such as equation (83).

Multiple Passes Inside the Unstable Resonator

To determine the effect of the unstable resonator on an alignment beam which propagated through it, it is necessary to know how many times the beam reflects from each mirror of the
resonator. This can be determined experimentally, by observing the intensity pattern the beam makes as it hits the various mirrors. It can be determined by repeated numerical solution to the Fresnel diffraction equation. Two alternative analytical techniques are discussed here.

**Geometric-optics propagation**

Certain unstable resonators can be approximately analyzed by using geometric optics for alignment beam propagation. These are the nonconfocal resonators, and are characterized by a non-zero value for the C element in their round-trip matrix of equation (62). Confocal resonators with collimated inputs must be excluded because geometric optics predicts an infinite number of round trips inside the cavity would be required for the beam to leave the cavity. A noncollimated input beam can be propagated through a confocal unstable resonator without getting trapped.

The input alignment beam is represented by two vectors, \( \mathbf{u} \) and \( \mathbf{v} \), of the form given in equation (1). They represent spherical waves, and the relationship between their first and second components is expressed in equation (12). In forming the second component of each, the same radius of curvature \( R \) for the input alignment beam is used. For their first components, the value of \( u \) is taken as the radius of the input beam's outer edge (in cross-section), while the value of \( v \) is the radius of the input beam's inner edge, which if formed by the hole in
the hole-coupling mirror. The two vectors are

\[
\begin{align*}
\vec{u} &= \begin{bmatrix} u \\ u' \end{bmatrix} = \begin{bmatrix} b_0 \\ b_0/R \end{bmatrix}, \\
\vec{v} &= \begin{bmatrix} v \\ v' \end{bmatrix} = \begin{bmatrix} b_1 \\ b_1/R \end{bmatrix},
\end{align*}
\tag{97}
\tag{98}
\]

where \( b_0 \) is the input beam cross-sectional outer radius, \( b_1 \) is the input beam cross-sectional inner radius, and \( R \) is the input beam radius of curvature.

These two vectors are propagated through the resonator, and each time they arrive at the hole-coupler mirror M-3, the current values of \( v \) and \( u \) are compared respectively with \( b_0 \), the effective outer radius of M-3 (as determined by the limiting aperture A-1), and \( b_1 \), the radius of the hole in M-3. When the value of \( u \) grows larger than \( b_1 \), output coupling of the return alignment beam takes place. However, if \( v \) is less than \( b_1 \), there is still a portion of the beam that will continue to propagate inside the resonator. To represent the continuing portion, the current value of \( u \) is replaced by \( b_1 \), and the current value of \( u' \) is multiplied by \( b_1/u \). This process continues until the current value of \( v \) becomes greater than \( b_1 \). An additional refinement to note in this algorithm is that the aperture A-2 may clip the alignment beam during its passes through the cavity. This can be taken into account by modifying \( \vec{u} \) appropriately whenever clipping (the current value of \( u \) being greater than the aperture radius)
occurs. Also, it is possible for the beam to grow too big for the resonator, even with no beam coupling out. This occurs when the current value of $v$ exceeds $b_0$ or the radius of aperture A-2.

**Gaussian-beam propagation**

A second analytical technique for determining the number of round trips the alignment beam makes inside the resonator uses Gaussian-beam propagation, as described in equations (15) and (16). The input beam outer and inner cross-sectional radii $b_0$ and $b_1$ are assumed to be certain fractions of the input Gaussian beam-width parameter $w$. The geometric-optics assumption is made that the beam edge remains distinct as the Gaussian beam propagates, indicating that the ratios $b_0/w$ and $b_1/w$ remain constant during propagation, changing only when the beam passes through a limiting aperture. Thus, at each aperture and at the hole coupling mirror, the inner and outer beam radii are calculated from the Gaussian beam parameters, and the same output-coupling criterion for the geometric-optics ray vectors is used for the Gaussian beam.

It is the nature of the unstable resonator that there exists no Gaussian-beam eigenfunction for it. Consequently, the Gaussian beam must diffract out of the resonator in a finite number of passes. Thus, this Gaussian-beam propagation technique can be used for confocal unstable resonators with collimated inputs.
Questions concerning the regions of validity for these two approximate analytical techniques have to be handled on an individual basis. Even if the results of a more-exact wave optics (Fresnel diffraction) approach were used, the results might not agree with experiment. Such factors as distortion of the input wavefront, atmospheric turbulence, and coherence length of the laser beam can cause variation in experimental observation. These analytical techniques are an approximation to the experimental situation, and as such, have utility.

Resonator mirror rotational sensitivity

Once the number of round trips inside the resonator has been determined, equations (75-83) can be used to determine resonator mirror rotational sensitivity. However, the expressions for the eigenvalues of the $F$ matrix, given in equations (75) and (76), must be modified for the case of the confocal resonator. The modification is required because the $C$ element of the $F$ matrix is zero, and so the confocal eigenvalues are

$$\gamma_1 = A, \quad (99)$$

$$\gamma_2 = D. \quad (100)$$

By noting that the focal length of a spherical mirror is half of its radius of curvature, the matrix $F$ can be formed from the product of the confocal telescope matrix $T$ from equation (32) and the $D$ matrix from equation (29), with the $d$ value equal to the sum of the focal lengths. The resulting $F$ matrix is expressed
as follows:

\[
E = \begin{bmatrix}
- R_1/R_2 & d(1 - R_2/R_1) \\
0 & - R_2/R_1
\end{bmatrix}, \quad (101)
\]

where \( R_1 \) is the signed radius of curvature of the mirror being rotated, \( R_2 \) is the signed radius of curvature of the other resonator mirror, and \( d \) is the mirror separation.

If the parameter \( m \) is defined as the ratio of the absolute values of the radii of curvature of the concave and convex mirrors, respectively,

\[
m = \frac{|R_c|}{|R_v|}, \quad (102)
\]

then \( E \) can be defined for the convex mirror as

\[
E_v = \begin{bmatrix}
1/m & d(1 + 1/m) \\
0 & m
\end{bmatrix}, \quad (103)
\]

with a corresponding expression for the concave mirror,

\[
E_c = \begin{bmatrix}
m & d(1 + m) \\
0 & 1/m
\end{bmatrix}. \quad (104)
\]

By using the values of the \( A \) and \( D \) elements of \( E_v \) and \( E_c \) as the appropriate eigenvalues (from equations (99) and (100)) for equations (81) and (82), equation (83) can be evaluated, giving the rotational sensitivities for the confocal unstable resonator's convex and concave mirrors in the form of an output ray vector. This vector is propagated to the pellicle and other significant planes in the alignment group and output optical train, where its location is used as an alignment.
Alignment Sensitivities

To evaluate alignment sensitivities for an optical component anywhere in the laser system, it is necessary to define what alignment criterion is used. If the alignment criterion specifies that the return alignment beam shall hit the pellicle no more than 1 cm (for example) from where the input alignment beam hit the pellicle, then the alignment sensitivity for each component would be measured in terms of shift of the return beam, relative to the input beam, at the pellicle. Alignment sensitivity can be defined in terms of misalignment of single optical components or groups of optical components. It can also be calculated in terms of steering of the high-power beam, in the following manner.

A minor misalignment is assumed for a particular component. The standard alignment procedure is followed, and the return beam at, for instance, the pellicle, is brought into "perfect" alignment. To do this, some other optical element had to be misaligned to compensate for the particular element's original misalignment. The combined effect can result in a misalignment of the high-power beam, even though the alignment system indicates "perfect" alignment. Thus an alignment sensitivity can be defined in terms of error in high-power beam pointing.

These general alignment sensitivities all relate to the question of how well the path of the alignment beam duplicates the path of the high-power beam. Such items as aerodynamic-
window steering (present for the high-power beam but absent during alignment), temperature-gradient beam steering (present in different amounts during high-power operation and alignment), and component deviation from "perfect" alignment (producing different results for the high-power beam and the alignment beam) cause the two beam paths to differ in amounts predicted by the analysis techniques of this paper.

**Example Calculations**

**Calculation of the Rotational Sensitivities of the Cassegrain System Components**

An analysis of the rotational sensitivities of components in the Cassegrain telescope system illustrates the operation of the ABCD matrix technique. The component values are chosen to be somewhat representative of present and future optical systems. However, integral values are chosen wherever possible, to ease computation.

Figure 13 shows the details of the Cassegrain system, consisting of mirrors M-6, M-7, and M-8. Mirror M-6 is a convex spherical mirror with a radius of curvature $R_6$ of -1.0 m. Mirror M-7 is a concave spherical mirror with a radius of curvature $R_7$ of 10.0 m. Mirror M-8 is plane. The mirror diameters are sized to accept a 0.1 m diameter beam at the input to the Cassegrain telescope and produce a 1.0 m diameter output beam. The separation between M-6 and M-7 has been increased from its confocal separation of 4.5 m by translating M-6. This increased
Fig. 13. Cassegrain system detail
separation of 4.525 m allows the collimated input beam to come to a geometric focus at the target 1.0 km from M-8.

To obtain the rotational sensitivity coefficient for each component, an initially-aligned ray represented by the zero vector will be assumed as an input to the rotated component. Because the optical components under consideration are mirrors, the effect of rotation is to add to the output ray leaving the mirror under consideration the rotation vector $\phi$. However, since the input vector was the zero vector, the output vector for the component before rotation is also the zero vector. Thus, the effect of the component rotation is represented by the rotation vector $\phi$ applied to the composite matrix which represents propagation from the component under consideration.

Two rotational sensitivity coefficients can be defined for each component in the Cassegrain system. The coefficient $T_1$ is defined as the ratio of a unit displacement of the beam location at the target to the amount of rotation required to produce that displacement, and has units of m/rad. The second coefficient $T_2$ represents the ratio of a unit beam rotation at the target to the amount of component rotation needed to produce that rotation, and has the units of rad/rad.

M-8 rotational sensitivity coefficients

To find the sensitivity coefficients $T_{1,8}$ and $T_{2,8}$, it is necessary to find the matrix which propagates a ray from M-8 to
the target. These two sensitivity coefficients can be found
from the following vector equation for the ray vector $\vec{r}$ at target:

$$\vec{r} = \begin{bmatrix} r \\ r' \end{bmatrix} = \begin{bmatrix} T_{1,8} \\ T_{2,8} \end{bmatrix} \theta = \begin{bmatrix} 1 \frac{m}{m} \\ 0 \frac{rad}{m} \end{bmatrix} \begin{bmatrix} 10^3 \frac{m}{rad} \\ 1 \frac{rad}{rad} \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix}.$$  \( \theta \) \( \text{(105)} \)

Dividing by $\theta$,

$$\vec{r}_8 = \begin{bmatrix} T_{1,8} \\ T_{2,8} \end{bmatrix} = H_3 \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$  \( \text{(106)} \)

where $H_3$ is the propagation matrix from M-8 to the target.

Carrying out the matrix operation gives

$$\vec{r}_8 = \begin{bmatrix} 2 \times 10^3 \frac{m}{rad} \\ 2 \frac{rad}{rad} \end{bmatrix}.$$  \( \text{(107)} \)

In a similar manner, the effect of rotation of M-7 can be found from

$$\vec{T}_7 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1006 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$  \( \text{(108-a)} \)

$$\vec{T}_7 = \begin{bmatrix} 2012 \frac{m}{rad} \\ 2 \frac{rad}{rad} \end{bmatrix}.$$  \( \text{(108-b)} \)

The rotation-coefficient vector $T_6$ for mirror M-6 is given by

$$\vec{T}_6 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1006 \\ 100.100 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{4.525}{2} \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$  \( \text{(110)} \)

$$\vec{T}_6 = \begin{bmatrix} 200.2 \\ -0.2 \end{bmatrix} \begin{bmatrix} 100.100 \\ 0.095 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 200.19 \frac{m}{rad} \\ 0.19 \frac{rad}{rad} \end{bmatrix}.$$  \( \text{(111)} \)
Rotational Misalignment Sensitivities of Elements with Multiple Reflections

Multiple reflections of the alignment beam inside the unstable resonator

Resonator description

The resonator being analyzed is a positive-branch confocal unstable resonator. Numerical values have been chosen to be representative of typical existing resonators, but with integral values to ease computation. The resonator in figure 2. Convex mirror M-1 has a radius of curvature of -333 cm, and an effective diameter, as determined by the hole in mirror M-3, of 4.543 cm. The distance between M-1 and concave mirror M-2 is 200 cm. Mirror M-2 has a radius of curvature of 733 cm, and has an effective diameter, as determined by aperture A-2, of 10 cm. Using these values for the radii of curvature of M-1 and M-2 gives the value of the geometric magnification m, defined by equation (102), as

\[ m = 2.2012. \] (111)

The separation between hole-coupling mirror M-3 and convex mirror M-1 is 7 cm, while the separation between mirrors M-3 and M-4 is 25 cm. The distance between beam-turning mirror M-4 and aerodynamic window W-1 is 150 cm.

Gaussian-beam propagation inside the resonator

The alignment beam to be analyzed is collimated at the
surface of mirror M-3, with a beam width at M-3 of 5 cm. The effective diameter of M-3, as determined by aperture A-1, is 10 cm. Consequently, the truncated outer edge of the input alignment beam is exactly one times the beamwaist, while the truncated edge of the hole in the middle is 0.4543 times the beamwaist.

To ease the calculations, a simplifying assumption is made: The separation between mirrors M-1 and M-3 can be neglected. This is justified for two reasons; the distance involved is only 3.5% of the curved-mirror separation, and this distance is always made as small as possible for positive-branch unstable resonators, in order to reduce high-power loading on the back (uncooled) side of the hole-coupler mirror.

The ABCD matrix required for Gaussian-beam propagation is one that takes the beam from the hole-coupling mirror into the resonator, bounces back and forth between the concave and convex mirrors an arbitrary number of times, and ends up back at the hole-coupling mirror, ready to leave the resonator. This composite matrix can be written as a product of two matrices, an input matrix which propagates the beam from M-3 to M-2, through M-2, and from M-2 to M-1, and a round-trip matrix which represents \( p \) round trips of the sequence for propagation through M-1, from M-1 to M-2, through M-2, and back to the M-1/M-3 common location. Propagation through the round-trip section zero times is equivalent to raising the matrix to the zeroth power, and yields the
identity matrix. In terms of the basic matrices $M_\psi$ (the convex mirror $M-1$), $M_c$ (the concave mirror $M-2$), and $D_{1-2}$ (the 200 cm separation between $M-2$ and the $M-1/M-3$ combination), the composite matrix $G_p$ for the $p$th round trip inside the resonator can be written as

$$G_p = (D_{1-2} M_c D_{1-2} M_\psi)^p (D_{1-2} M_c D_{1-2}) .$$

(112)

By use of the Cayley-Hamilton technique (DeRusso, Roy, and Close, 1965, pp. 281-6), the round-trip portion of the $G_p$ matrix can be evaluated. Combining this result with the other portion of the $G_p$ matrix, and writing the result in terms of the parameter $m$, as defined in equation (102), and $R_c$, the radius of curvature of concave mirror $M-2$, gives

$$G_p = \begin{bmatrix}
-m^{-p-1} & \frac{R_c}{2} (m^p - m^{-p-2}) \\
-2m^{-p} & m^{-p-1}
\end{bmatrix} .$$

(113)

Having found $A$, $B$, $C$, and $D$ for the $G_p$ matrix, the new value of the complex beam parameter $q$ can be determined by applying equation (16). By identifying the real and imaginary parts of the reciprocal of $q$, and noting how they correspond to the radius of curvature and the beam width, as shown in equation (15), the beam size and curvature when it returns to the hole-coupling mirror can be determined. The geometric-optics approximation that the intensity distribution is still an illuminated annulus with an outer radius equal to the beam width and the inner radius is still $0.4543$ times the beam width will determine if any significant amount of radiation couples out of the cavity after
the \( ith \) round trip. A summary of the results of these calculations for from zero to twelve round trips is presented in table 1. The results show an output coupling after both 10 and 11 round trips inside the cavity. With any fewer round trips, the beam is still too small to get out, while with more round trips, the beam is too big. With the more-realistic assumption that diffraction has blurred the edges of the output beam, filling in the hole in the middle, output coupling would still occur for the twelfth round trip. However, the \textit{beam width} for each round trip is increasing by a factor approximately equal to the magnification \( m \) for each round trip after the sixth, and with the approximation that the watts per square centimeter in the output beam varies inversely with the square of the \textit{beam width}, the twelfth round-trip output will only be \( 4\% \) as intense as the output from the tenth round trip, and thus can be ignored.

\textbf{Rotation of resonator mirrors}

Now that the number of round trips inside the resonator has been established, the effect of rotation of the resonator convex and concave mirrors can be determined, using equations (80-83) and (99-104). For the convex mirror, the number of round trips equals the number of reflections from its surface, while for the concave mirror, the number of reflections from its surface is one more than the number of round trips. Because of the nature of the \( F \) matrix of equation (62), used to
<table>
<thead>
<tr>
<th>Number of Round Trips</th>
<th>Round-Trip Matrix Coefficients</th>
<th>Radius of Curvature (m)</th>
<th>Beam Width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A ( \times 10^{-5} )</td>
<td>B</td>
<td>C ( \times 10^{-5} )</td>
</tr>
<tr>
<td>0</td>
<td>45.129.7</td>
<td>2.90859</td>
<td>-27.285.1</td>
</tr>
<tr>
<td>1</td>
<td>20.638.6</td>
<td>7.72377</td>
<td>-12.395.6</td>
</tr>
<tr>
<td>2</td>
<td>9.376.07</td>
<td>17.4143</td>
<td>-5.631.27</td>
</tr>
<tr>
<td>3</td>
<td>4.259.52</td>
<td>39.0180</td>
<td>-2.558.27</td>
</tr>
<tr>
<td>4</td>
<td>1.935.09</td>
<td>86.0102</td>
<td>-1.162.22</td>
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<tr>
<td>5</td>
<td>879.107</td>
<td>189.382</td>
<td>-527.992</td>
</tr>
<tr>
<td>6</td>
<td>399.376</td>
<td>416.894</td>
<td>-239.866</td>
</tr>
<tr>
<td>7</td>
<td>181.435</td>
<td>917.679</td>
<td>-108.970</td>
</tr>
<tr>
<td>8</td>
<td>82.4256</td>
<td>2.020.00</td>
<td>-49.5049</td>
</tr>
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<td>9</td>
<td>37.4458</td>
<td>4.446.43</td>
<td>-22.4899</td>
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<tr>
<td>10</td>
<td>17.0115</td>
<td>9.787.49</td>
<td>-10.2172</td>
</tr>
<tr>
<td>11</td>
<td>7.72828</td>
<td>21.544.2</td>
<td>-4.64161</td>
</tr>
<tr>
<td>12</td>
<td>3.51094</td>
<td>47.423.2</td>
<td>-2.10867</td>
</tr>
</tbody>
</table>
describe the round-trip matrix for mirror rotation, the resultant matrix multiplier of the \( \hat{\mathbf{r}} \) vector in equation (83) represents the propagation matrix for the rotation vector through the last reflection from the mirror surface being rotated. Thus, in the case of the concave resonator mirror M-2, the \( \hat{r}_1 \) vector of equation (83) is the misalignment vector for the \((P-1)\)th round trip, located just after reflection from M-2, while for resonator convex mirror M-1, \( \hat{r}_1 \) is the misalignment vector for the \(P\)th round trip, at a location just after reflection from M-1. To bring the misalignment vector for the convex mirror up to the hole-coupling mirror M-3 for propagation outside the resonator, it must be propagated the distance \(d\) to M-2, through M-2, and then back the distance \(d\) to M-3 (M-1/M-3 separation is assumed negligible). The misalignment vector for M-2 needs only to be propagated the distance \(d\) to hole-coupling mirror M-3.

Table 2 lists the calculated parameters for convex mirror M-1, while Table 3 lists the calculated rotational parameters for concave mirror M-2. Because of the large number of round trips inside what is effectively a magnifying telescope, the misalignment ray vector has its displacement component greatly magnified and its angular component greatly demagnified.

Definition of misalignment criteria

Because of the large number of round trips the alignment beam makes inside the resonator, observation of a sharply-defined shadow region for the beam exiting the resonator, and correlating
### TABLE 2

**CONVEX MIRROR M-1 MISALIGNMENT PARAMETERS**

<table>
<thead>
<tr>
<th>Number of Round Trips</th>
<th>Matrix for Propagation of Misalignment Vector to Hole-Coupling Mirror M-3</th>
<th>Misalignment Vector at M-3 ($x \theta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>10</td>
<td>0.839188</td>
<td>8,143.65</td>
</tr>
<tr>
<td>11</td>
<td>0.832359</td>
<td>17,931.1</td>
</tr>
</tbody>
</table>

### TABLE 3

**CONCAVE MIRROR M-2 MISALIGNMENT PARAMETERS**

<table>
<thead>
<tr>
<th>Number of Round Trips</th>
<th>Matrix for Propagation of Misalignment Vector to Hole-Coupling Mirror M-3</th>
<th>Misalignment Vector at M-3 ($x \theta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>10</td>
<td>4,892.91</td>
<td>17,929.5</td>
</tr>
<tr>
<td>11</td>
<td>10,771.6</td>
<td>39,474.8</td>
</tr>
</tbody>
</table>
Fig. 14. Misaligned Unstable Resonator
the movement of this shadow with the rotation of resonator mirrors, may not be possible. However, because of the extreme magnification of the displacement of the beam centerline, a small rotation of a curved resonator mirror can cause the output beam to miss entirely the 10 cm diameter output aperture A-1, which restricts the diameter of the beam leaving the resonator after reflection from mirror M-3. Consequently, a readily-observable critical alignment angle \( \theta_{cr} \) can be defined as that angle of rotation of the resonator mirror in question which causes the associated misalignment vector, located at the hole-coupling mirror M-3, to have a displacement component equal to the sum of the radius of the output aperture A-1 and the beam width of the output alignment beam. With a rotation at the critical angle, all of the output beam inside its Gaussian beam width will miss the output aperture, as shown in figure 14. Thus, a significant reduction of intensity of the output alignment beam will be observed with a mirror rotation at its associated critical angle. Using the beam widths given in table 1, and the mirror-rotation data from tables 2 and 3, the values for the critical angles can be established. For convex mirror M-1, the critical angle for the tenth round trip is

\[
\theta_{cr,10} = 5.49071 \mu \text{rad}. \tag{114}
\]

For the eleventh round trip, the critical angle for M-1 is

\[
\theta_{cr,11} = 3.81436 \mu \text{rad}. \tag{115}
\]
For the tenth round trip, the critical angle for M-2 is
\[ \theta_{cr,10} = 2.49390 \text{ \mu rad} . \] (116)

For the eleventh round trip, the critical angle for M-2 is
\[ \theta_{cr,11} = 1.73265 \text{ \mu rad} . \] (117)

A misalignment vector representing the maximum misalignment which can occur without an appreciable reduction in output intensity can be found by inserting the appropriate values of \( \theta_{cr} \) in the expressions given for the misalignment vector in tables 2 and 3.

Alignment-beam misalignment due to elements with two reflections

Mirrors M-3 and M-4 have the alignment beam reflecting from them twice, and thus require multiple-pass considerations to calculate their rotational sensitivities. An application of equation (67) shows that the misalignment matrix for either of these mirrors is just the sum of the identity matrix \( \mathbb{I} \) and the round-trip matrix which represents propagation of the alignment beam between the first reflection and the second reflection. For hole-coupling mirror M-3, the round-trip matrix is equal to the matrix previously calculated (see table 1) for Gaussian beam propagation. For beam-turning mirror M-4, the round-trip matrix can be formed by taking the round-trip matrix for M-3 and both premultiplying it and postmultiplying it by the propagation matrix for the M-3/M-4 separation. The sum of this
round-trip matrix and the identity matrix $I$ is a matrix which propagates $M-4$'s misalignment vector back to $M-4$. Since all other misalignment vectors associated with that portion of the alignment beam which passes through the resonator have been propagated to a reference location at $M-3$, it would be convenient to calculate $M-4$'s misalignment vector, referenced at the $M-3$ location. Then a critical angle $\theta_{cr}$ can be calculated for $M-4$, based on the same definition used for the critical angles presented in equations (114-17). However, the alignment beam, when it arrives at $M-3$ the second time, has been reflected only once from $M-4$. Thus, $M-4$'s critical angle is determined by a matrix which is not the same as its misalignment matrix. This is an example of what can occur in general if a beam-limiting aperture is located somewhere in the middle of the round-trip portion of an optical system with multiple reflections.

The misalignment parameters for mirror $M-3$ are presented in Table 4. Table 5 shows the misalignment parameters for mirror $M-4$. $M-4$'s parameters have been calculated based on a 30 cm separation between $M-3$ and $M-4$. The initial misalignment matrix for $M-4$ was calculated at $M-4$, and then multiplied by a propagation matrix for the $M-3/M-4$ separation distance, but with the sign of the distance ($B$ component) reversed to indicate propagation from $M-4$ back to $M-3$. The misalignment vector was calculated from the misalignment matrix referenced to the $M-3$ location. However, the critical angle was calculated from the value of the
### Table 4

**HOLE-COUPLING MIRROR M-3 MISALIGNMENT PARAMETERS**

| Number of Round Trips | Matrix for Propagation of Misalignment Vector to Hole-Coupling Mirror M-3 | Misalignment Vector at M-3 (x | $\theta_{cr3}$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
<td>$B$</td>
<td>$C$</td>
</tr>
<tr>
<td>10</td>
<td>1.00017</td>
<td>9,787.49</td>
<td>-1.02172 x10^{-4}</td>
</tr>
<tr>
<td>11</td>
<td>1.00008</td>
<td>21,544.2</td>
<td>-4.64161 x10^{-5}</td>
</tr>
</tbody>
</table>

### Table 5

**BEAM-TURNING MIRROR M-4 MISALIGNMENT PARAMETERS, REFERENCED TO M-3**

| Number of Round Trips | Matrix for Propagation of Misalignment Vector to Hole-Coupling Mirror M-3 | Misalignment Vector at M-3 (x | $\theta_{cr4}$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
<td>$B$</td>
<td>$C$</td>
</tr>
<tr>
<td>10</td>
<td>1.00017</td>
<td>9787.19</td>
<td>-1.02172 x10^{-4}</td>
</tr>
<tr>
<td>11</td>
<td>1.00008</td>
<td>21543.94</td>
<td>-4.64161 x10^{-5}</td>
</tr>
</tbody>
</table>
misalignment vector prior to its second reflection from M-4.

If the misalignment vector's value presented in table 4 were used in lieu of its actual value when encountering the limiting aperture at M-3, the calculation of the critical angle would be too large by an amount equal to \(3.1 \times 10^{-5}\) times the critical angle for 10 round trips, and \(1.4 \times 10^{-5}\) times the critical angle for 11 round trips.

An analysis can be made for each element in the alignment group depicted in figure 4, examining the rotational sensitivity of each optical element, and calculating a \(\theta_{cr}\) and a misalignment vector referenced to M-3 for each. However, the value of the misalignment vectors from these calculations, when evaluated at their associated critical angles, should not be significantly different from results for M-4. This is because the large number of round trips which the alignment beam makes inside the resonator causes the resonator to permit an output alignment beam for only a narrow range of output alignment-beam vectors.

A Comparison Between High-Power Beam Pointing and Alignment-Beam Pointing

**Unstable-resonator alignment**

Any alignment system for an unstable resonator is required to precisely position the optical axis of the resonator. Failure to do so can cause damage to the nozzles, sidewalls, and apertures surrounding the desired optical beam path. For the alignment system under present consideration, a worst-case evaluation can be
made in the following manner. Mirrors M-1 and M-2 are considered, in turn, to be rotated at their critical angles. The vector, located at M-3, which designates the optical axis of the resonator with rotated mirrors is given by equation (86); this vector is evaluated at the appropriate critical angles, and the displacement component of this vector is the amount by which the misaligned resonator's optical axis is displaced from its desired position. The displacement of the optical axis at the other end of the resonator (at M-2) is found by propagating the vector for the optical axis back to M-3.

Inserting the appropriate values for M-1 in equation (86), and taking the tenth round-trip value of the critical angle \( \theta_{cr,1,10} \) from equation (114) allows calculation of the optical-axis vector for the worst case, as follows:

\[
\vec{r}_{b,1,10} = \begin{bmatrix} -8.991 \\ 1.665 \end{bmatrix} \quad \theta_{cr,1,10} = \begin{bmatrix} -4.93670 \times 10^{-5} \text{ m} \\ 9.14203 \times 10^{-6} \text{ rad} \end{bmatrix} \quad (118)
\]

Propagating this vector back to M-2 increases its displacement component to 6.70111 \( \times 10^{-5} \) m. Likewise, with M-2 rotated at its largest critical angle, the optical-axis vector is

\[
\vec{r}_{b,2,10} = \begin{bmatrix} -12.461 \\ 3.665 \end{bmatrix} \quad \theta_{cr,2,10} = \begin{bmatrix} -3.10765 \times 10^{-5} \text{ m} \\ 9.10401 \times 10^{-6} \text{ rad} \end{bmatrix} \quad (119)
\]

Propagating this vector back to M-2 increases its displacement component to 4.86472 \( \times 10^{-5} \) m. All of these values are significantly smaller than a nominal 0.5 mm accuracy with which an alignment beam can be positioned on the center of an aperture.
with simple visual observation, and thus, for a visual alignment technique, the alignment system under discussion is completely adequate for positioning the optical axis at the desired location in the gain medium.

**Aerodynamic-window steering**

Because flow of the aerodynamic window is not present during alignment, but is present during high-power operation, any steering caused by the flow will make the high-power beam strike a down-range target at a point different from where the alignment beam hits. To calculate the magnitude of this effect, the propagation matrix from the aerodynamic window to the target is needed. With an assumed zero displacement and a nominal 50 μrad angular steering due to the window, the window is equivalent to a mirror rotated 25 μrad. Now with a separation of 8.0 m between aerodynamic window W-1 of figure 2 and mirror M-7 of figures 3 and 13, the propagation matrix from W-1 to the target can be found by a continuation of the process used to generate the matrix multiplier of equation (110). The beam pointing at the target due to aerodynamic-window steering is expressed by the vector

\[
\begin{align*}
\hat{r}_{W-1} &= \begin{bmatrix} 2 \times 10^{-6} & 100.100 \\ -0.01 & -0.030120 \end{bmatrix} \begin{bmatrix} 0 \\ 50 \times 10^{-6} \end{bmatrix}, \\
\hat{r}_{W-1} &= \begin{bmatrix} 5.0050 \times 10^{-3} m \\ -1.5060 \times 10^{-6} \text{ rad} \end{bmatrix}.
\end{align*}
\]
The A term of the matrix multiplier in equation (120) is very small because the value of the separation between M-6 and M-7 in the Cassegrain telescope was chosen to make it zero. When the separation value of 4.52497502 m was inserted in the expression for the A term of the matrix, $2 \times 10^{-6}$ was the result. However, since the aerodynamic window is assumed to produce no beam displacement, the A term of the multiplier matrix has no effect on the beam displacement at the target.

Target aim-point designation with alignment laser beam

Focus at target

Angular misalignment of mirrors M-6, M-7, and M-8 will produce the same beam steering at target for both the alignment beam and the high-power beam. As previously shown in equation (121), the aerodynamic window will produce a permanent offset of 5 mm at target between the alignment beam and the high-power beam. The mirrors M-1, M-2, M-3, and M-4, which experience multiple reflections of the alignment beam, will also cause an offset at target between the aim point of the alignment beam and the aim point of the high-power beam. This is because of the different form of the misalignment vector, referenced to M-3, for the two beams. For the high-power beam, the misalignment vector for M-1 and M-2 angular misalignment is given by equation (86), while equation (48) gives the misalignment vector for M-3, and when used to represent misalignment of M-4, it can be
propagated back to M-3 for reference. On the other hand, the misalignment vectors for the alignment beam with multiple reflections take the forms presented in tables 2-5.

To evaluate the target aim-point designation, the matrix representing propagation from M-3 to the target is needed. This can be found using the matrix multiplier of equation (120), and calculating the required matrix by premultiplying the matrix of equation (120) by the propagation matrix which represents the beam path between M-3 and the aerodynamic window W-1. With the M-4/W-1 separation of 3.0 m, and the M-3/M-4 separation of 30 cm, the propagation matrix from M-3 to the target is

$$H_3 = \begin{bmatrix} 2 \times 10^{-6} & 100.100 \\ -0.01 & -0.03012 \end{bmatrix} . \quad (122)$$

As in the case of equation (120), the value of the A term of the $H_3$ matrix is nonzero only because of roundoff errors in its calculation. Taking a zero value for the A term of $H_3$, and calculating beam displacement at target (focused case) for rotations of mirrors M-1, M-2, M-3, and M-4, for the largest critical angles (tenth round trip) gives the displacement values shown in table 6. An examination of the signs of the components of the vectors in equations (118) and (119) might suggest that the displacement of the high-power beam at target is in the opposite direction to the displacement of the alignment beam. However, because of the small (ideally zero) value of the A term of the $H_3$ matrix, the displacement at the target is dominated by the angular component of the $r_b$ vectors of equations (118-19), rather than by
<table>
<thead>
<tr>
<th>Mirror</th>
<th>Displacement (mm)</th>
<th>Angle of Rotation ($\theta_{cr}$ in rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Power</td>
<td>Alignment Beam</td>
</tr>
<tr>
<td>M-1</td>
<td>0.91512</td>
<td>0.91478</td>
</tr>
<tr>
<td>M-2</td>
<td>0.91131</td>
<td>0.91478</td>
</tr>
<tr>
<td>M-3</td>
<td>0.91462</td>
<td>0.91478</td>
</tr>
<tr>
<td>M-4</td>
<td>0.91462</td>
<td>0.91475</td>
</tr>
</tbody>
</table>

**TABLE 6**

**DISPLACEMENT AT TARGET DUE TO MISALIGNMENT (FOCUSED CONDITION)**
the displacement component. Thus, the displacements of both the alignment beam at the target and the high-power beam at the target, for a focused condition, follow one another, and the maximum discrepancy is $3.5 \times 10^{-6}$ m, for M-2 rotation. Since the spot size of a TEM$_{00}$ Gaussian beam with diameter of 1.0 m at M-8 would focus to a spot nominally 2 cm in diameter, any large errors in assuming that the alignment beam points at the same location as the high-power beam is not caused by the different way in which the misaligned mirrors affect the two beams.

Focus at infinity

If the separation of mirrors M-6 and M-7 in the Cassegrain telescope is set to 4.5 m, instead of 4.525 m for focus on target, the telescope itself becomes focused on infinity. This changes the form of its ABCD matrix, giving a non-zero value for the A term. The propagation matrix from M-3 to the target, with this defocused condition, is

$$H_3' = \begin{bmatrix} 10 & 263.1 \\ 0 & 0.1 \end{bmatrix}.$$  \hspace{1cm} (123)

Because of the non-zero value of the A term of $H_3'$, the value of the displacement component of the misalignment vector propagated to the target becomes significant. For the alignment beam, this displacement component grows rapidly with small angular misalignment. However, because of the limiting effect of aperture A-1 associated with hole-coupling mirror M-3, the maximum
value the displacement component can assume is 5.0 cm. Thus, assuming the same critical angles as used in table 6, but with the associated displacements based on those critical angles the lesser of the calculated values and the 5.0 cm limit, the displacements at target using the propagation matrix \( H^3 \) are calculated and presented in table 7. Note that the effect of the 5.0 cm displacement component completely dominates the displacement of the alignment beam at target. Thus, although misalignment at the critical angle is satisfactory for maintaining the same displacement at target for both the alignment and high-power beams at focus, it is not satisfactory for the defocused cases, and a tighter specification of alignment, which will keep the alignment beam centered on the optical axis, is needed to reduce the error of aim-point designation by the alignment beam.
<table>
<thead>
<tr>
<th>Mirror</th>
<th>Displacement (mm)</th>
<th>Angle of Rotation ($\theta_{cr}$ in $\mu$rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Power</td>
<td>Alignment Beam</td>
</tr>
<tr>
<td>M-1</td>
<td>1.9116</td>
<td>502.40</td>
</tr>
<tr>
<td>M-2</td>
<td>2.0845</td>
<td>502.40</td>
</tr>
<tr>
<td>M-3</td>
<td>2.4040</td>
<td>502.40</td>
</tr>
<tr>
<td>M-4</td>
<td>2.4013</td>
<td>502.40</td>
</tr>
</tbody>
</table>
CONCLUSION

The ABCD matrix technique has been shown to be an effective analytical tool for modelling alignment systems for high-power lasers, and predicting their performance. Use of this analysis technique on an example system has shown that a laser alignment system with a visual alignment criteria can be entirely adequate to perform successful alignment of an unstable resonator, and to designate the aimpoint at a target. Specific expressions for the effect of mirror rotational misalignment (or corresponding mirror translations) show that an alignment beam which makes many reflections from the resonator mirrors becomes highly sensitive to misalignment, and easily meets the typical micro-radian alignment requirements for unstable resonators.
APPENDIX

ABCD MATRIX CONSIDERATIONS FOR THREE-DIMENSIONAL OPTICAL SYSTEMS

The ABCD matrix technique of Gaussian optics applies to an optical system in which all the optical elements lie on a common axis of ray propagation, designated the Z axis. The technique is two dimensional, in that one other axis, designated the X axis, is used to represent a ray's displacement from the axis of propagation. The technique is also confined to paraxial rays. Skewed rays and rays that make large angles with respect to the optical axis are not allowed. As has been shown in chapter III, this two-dimensional technique can be generalized to certain types of three-dimensional paraxial rays by considering separately the ABCD matrices associated with the two orthogonal X and Y axes representing displacements from the Z axis of propagation. In this treatment, displacements and small angles are handled as vectors, and their vector components are handled separately.

The ABCD matrix technique can be extended to rays propagating in arbitrary directions in three-dimensional space, with the following provision. A generalized Z axis is defined, which traces a ray along a three-dimensional path joining the various optical elements. These elements are oriented so that this
Z-axis ray enters each element at the origin of its input coordinate system, travels through the optical element to the origin of its output coordinate system, and then travels on to the origin of the input coordinate system of the next optical element, etc. A paraxial ray now is defined as one which is paraxial with respect to this generalized Z axis, at all points along the axis. Equivalently, a ray is considered paraxial if it is almost parallel to the generalized Z axis, and the sine and tangent of the angle between it and this Z axis can be approximated by the angle itself.

Equivalent Radius of Curvature for Tilted Spherical Mirrors

The equivalent radius of curvature of a tilted spherical mirror can be determined by examining the ray-tracing results of Klein (1970, pp. 62-69), for the general case of nonparaxial rays. By examining his expressions for the direction cosines which eventually become terms in his ABCD matrix paraxial approximation, the equivalent radius of curvature \( R_{eq} \) can be written as follows:

\[
R_{eq} = \frac{R}{\cos \psi}, \tag{124}
\]

where \( \psi \) is the angle of incidence, which is approximated for the generalized paraxial rays by the angle between the optical element's surface normal at the origin of its input coordinate system and the generalized Z axis.
General Composite Three-Dimensional Systems

Consider some general optical system with an input coordinate system \((x_1, y_1, z_1)\) and some output coordinate system \((x_2, y_2, z_2)\), with some input ray vector

\[
\mathbf{r}_{x,1} = \begin{bmatrix} x_1 \\ dx_1/dz_1 \end{bmatrix}
\]

(125)

transformed to some output ray vector

\[
\mathbf{r}_{x,2} = \begin{bmatrix} x_2 \\ dx_2/dz_2 \end{bmatrix}
\]

(126)

by the ABCD matrix equation

\[
\mathbf{r}_{x,2} = H_x \mathbf{r}_{x,1}.
\]

(127)

A corresponding set of equations can be written for the \(y\) direction:

\[
\mathbf{r}_{y,1} = \begin{bmatrix} y_1 \\ dy_1/dz_1 \end{bmatrix},
\]

(128)

\[
\mathbf{r}_{y,2} = \begin{bmatrix} y_2 \\ dy_2/dz_2 \end{bmatrix},
\]

(129)

\[
\mathbf{r}_{y,2} = H_y \mathbf{r}_{y,1}.
\]

(130)

Now suppose that the whole optical system is translated and rotated by amounts small enough to preserve the generalized paraxial approximation. The displaced input and output coordinate systems are \((X_1, Y_1, Z_1)\) and \((X_2, Y_2, Z_2)\), respectively.

To account for this displacement, the following approach is taken.
First, the two input vectors are transformed into the displaced input coordinate system \((X_1, Y_1, Z_1)\). In general, the transformation will produce new input vectors which do not lie in the \(Z_1 = 0\) plane, and so, they must be propagated to that plane, using the \(\mathbf{D}\) matrix of equation (29). Then the input vectors are operated on by their respective \(\mathbf{H}\) matrices, bringing them to the \(Z_2 = 0\) plane in the displaced output coordinate system \((X_2, Y_2, Z_2)\). Next, these output vectors are transformed into the undisplaced output coordinate system \((x_2, y_2, z_2)\). Finally, they are propagated, again by the \(\mathbf{D}\) matrix, to the \(z_2 = 0\) plane in the output coordinate system \((x_2, y_2, z_2)\). A list of the general coordinate transformations is given in Hodgman (1959, p. 417).
LIST OF SYMBOLS

A: the "A" component of an ABCD matrix; see p. 11.
A-1: an aperture in the laser subsystem; see p. 5.
A-2: an aperture in the laser subsystem; see p. 5.
A-3: an aperture in the output optical train; see p. 7.
A-4: an aperture in the alignment group; see p. 9.
A-5: an aperture in the alignment group; see p. 9.
A-6: an aperture in the alignment group; see p. 9.
A_1: the "A" component of the $H_1$ matrix; see p. 17.

b_1: the input alignment beam's cross-sectional inner radius at hole-coupling mirror M-3; see p. 47.

b_0: the input alignment beam's cross-sectional outer radius at hole-coupling mirror M-3; see p. 47.

B: the "B" component of an ABCD matrix; see p. 11.
B_1: the "B" component of the $H_1$ matrix; see p. 17.
C: the "C" component of an ABCD matrix; see p. 11.
C_1: the "C" component of the $H_1$ matrix; see p. 17.

D: the "D" component of an ABCD matrix; see p. 11.

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$D_1$: the "D" component of the $H_1$ matrix; see p. 17.

$D_1$: the ABCD matrix representing propagation a distance $d$ in free space; see p. 17.

$D_1$: the ABCD matrix representing propagation between the $M-1/M-3$ mirror combination and mirror $M-2$, equal to 200 cm.; see p. 58.

$f$: the focal length of a lens or spherical mirror; see p. 20.

$f_1$: the focal length of the input lens (or mirror) of the confocal telescope described by the $I$ matrix; see p. 20.

$f_2$: the focal length of the output lens (or mirror) of the confocal telescope described by the $I$ matrix; see p. 20.

$E$: the ABCD matrix representing one round trip inside a two-mirror resonator; see p. 32.

$E_p$: the ABCD matrix representing the $p$th round trip of an alignment beam inside the unstable resonator; see p. 58.

$H$: a general ABCD matrix; see p. 11.

$H_f$: the ABCD matrix which brings a given spherical wave to a focus; see p. 18.

$H_1$: the ABCD matrix which describes propagation from an object plane to its corresponding image plane; see p. 17.

$H_k$: the ABCD matrix representing the $k$th optical element in an optical system; see p. 12.

$H_t$: the ABCD matrix representing a composite optical system composed of individual optical elements and their associated matrices; see p. 12.

$H_x$: the ABCD matrix representing an optical element's propagation characteristics for a ray vector in the X-Z plane of a three-dimensional system; see p. 22.

$H_y$: the ABCD matrix representing an optical element's propagation characteristics for a ray vector in the Y-Z plane of a three-dimensional system; see p. 22.

$H_3$: the ABCD matrix representing propagation from mirror $M-3$ to the target, with the Cassegrain telescope focused at the target; see p. 72.
$H_3$: the ABCD matrix representing propagation from mirror M-3 to the target, with the Cassegrain telescope focused at infinity; see p. 74.

$H_6$: the ABCD matrix representing propagation from mirror M-8 to the target; see p. 55.

1: the number of times a multiple-reflecting beam hits the mirror being rotated in a general two-mirror system; see p. 32.

$2$: the identity matrix; see p. 25.

$i$: $\sqrt{-1}$; see p. 16.

$k$: a general subscript; see p. 12.

$k$: an arbitrary constant; see p. 29.

$k_1$: an arbitrary constant; see p. 43.

$L$: the ABCD matrix representing a thin lens; see p. 20.

$L-1$: the alignment laser, part of the alignment group; see p. 9.

$m$: the absolute value of the ratio of the radii of curvature of the confocal unstable resonator's concave and convex mirrors; see p. 50.

$M$: the ABCD matrix representing a spherical mirror; see p. 20.

$M_1$: the ABCD matrix representing the mirror being rotated in a general two-mirror system with multiple beam reflections; see p. 32.

$M_2$: the ABCD matrix representing the unrotated mirror in a general two-mirror system with multiple beam reflections; see p. 32.

$M_3$: the ABCD matrix representing concave mirror M-2; see p. 58.

$M_4$: the ABCD matrix representing convex mirror M-1; see p. 58.

$M-1$: the unstable-resonator convex mirror; see p. 5.

$M-2$: the unstable-resonator concave mirror; see p. 5.

$M-3$: the unstable-resonator hole-coupling mirror; see p. 5.

$M-4$: the unstable-resonator beam-turning mirror; see p. 5.
M-5: the beam-clipping mirror in the output optical train; see p. 7.

M-6: the convex focusing mirror in the Cassegrain telescope of the output optical train; see p. 7.

M-7: the concave mirror in the Cassegrain telescope of the output optical train; see p. 7.

M-8: the beam-directing mirror in the output optical train; see p. 7.

M-9: the concave collimating mirror in the alignment group; see p. 9.

M-10: the beam-steering mirror in the alignment group; see p. 9.

n: the index of refraction; see p. 28.

N: the ray vector representing the effect of a transverse refractive-index gradient; see p. 30.

p: the number of round trips made by the alignment beam inside the unstable resonator; see p. 57.

P(μ): a scalar polynomial function of the matrix eigenvalue μ which corresponds to the matrix polynomial P(μ); see p. 33.

P(ξ): a matrix polynomial function of the matrix ξ; see p. 33.

P-1: the power meter in the output optical train; see p. 7.

q(z): the complex radius of a Gaussian beam; see p. 16.

q₁: the complex radius of a Gaussian-beam input to an optical system; see p. 17.

q₂: the complex radius of a Gaussian-beam output from an optical system; see p. 17.

r₁: a general ray vector, used to designate the high-power beam at the target; see p. 55

r₁*: the ray vector representing the high-power beam with misaligned resonator mirrors, located at the center of curvature of mirror M-1; see p. 39.

r₂*: the ray vector representing the high-power beam with misaligned resonator mirrors, located at M-3; see p. 39.
\( \mathbf{r}_p \): the ray vector representing the alignment beam after \( p \) round trips inside the resonator; see p. 61.

\( \mathbf{r}_{b1,10} \): the ray vector representing the high-power beam at \( 1,10 \) M-3, with mirror M-1 misaligned by the angle \( \theta_{cr1,10} \); see p. 69.

\( \mathbf{r}_{b2,10} \): the ray vector representing the high-power beam at \( 2,10 \) M-3, with mirror M-2 misaligned by the angle \( \theta_{cr2,10} \); see p. 69.

\( \mathbf{r}_{x,1} \): the ray vector representing some input ray's components in the X-Z plane of a three-dimensional optical system; see p. 22.

\( \mathbf{r}_{x,2} \): the ray vector representing some output ray's components in the X-Z plane of a three-dimensional optical system; see p. 22.

\( \mathbf{r}_{y,1} \): the ray vector representing some input ray's components in the Y-Z plane of a three-dimensional optical system; see p. 22.

\( \mathbf{r}_{y,2} \): the ray vector representing some output ray's components in the Y-Z plane of a three-dimensional optical system; see p. 22.

\( r_1 \): the displacement component of the \( \mathbf{r}_1 \) vector; see p. 11.

\( r_1' \): the angular component of the \( \mathbf{r}_1 \) vector; see p. 11.

\( \mathbf{r}_1 \): the ray vector representing an input to an optical system; see p. 11.

\( r_2 \): the displacement component of the \( \mathbf{r}_2 \) vector; see p. 11.

\( r_2' \): the angular component of the \( \mathbf{r}_2 \) vector; see p. 11.

\( \mathbf{r}_2 \): the ray vector representing an output from an optical system; see p. 11.

\( R \): the radius of curvature of a spherical wave or a spherical mirror; see p. 15 and 20.

\( R_c \): the radius of curvature of concave mirror M-2; see p. 38.

\( R_{eq} \): the equivalent radius of curvature of a tilted spherical mirror; see p. 79.

\( R_v \): the radius of curvature of convex mirror M-1; see p. 38.
\( R(z) \): the (real) radius of curvature of a Gaussian beam; see p. 16.

\( S \): a distance through some medium in which there exists a uniform, constant gradient in the index of refraction, with the gradient's direction transverse to the direction of propagation; see p. 28.

The ABCD matrix representing propagation the distance \( S \) in a medium with a constant, uniform transverse refractive-index gradient; see p. 28.

\( t_1 \): the translation vector added to the input vector of an optical system being translated perpendicular to the direction of propagation; see p. 24.

\( t_2 \): the translation vector added to the output vector of an optical system being translated perpendicular to the direction of propagation; see p. 24.

\( T \): temperature, in °K; see p. 43.

\( T_s \): a standard or reference temperature; see p. 43.

The ABCD matrix representing a confocal telescope; see p. 20.

\( T_1 \): the rotational sensitivity coefficient which specifies the amount of displacement at the target for a unit rotation of some component in the output optical train; see p. 54.

\( T_2 \): the rotational sensitivity coefficient which specifies the amount of beam rotation at the target for a unit rotation of some component in the output optical train; see p. 54.

\( T_{1,8} \): the \( T_1 \) rotational sensitivity coefficient for mirror M-8; see p. 55.

\( T_{2,8} \): the \( T_2 \) rotational sensitivity coefficient for mirror M-8; see p. 55.

\( T_6 \): the rotational-sensitivity-coefficient vector for mirror M-6, composed of the particular \( T_1 \) and \( T_2 \) for M-6; see p. 55.

\( T_7 \): the rotational-sensitivity-coefficient vector for mirror M-7, composed of the particular \( T_1 \) and \( T_2 \) for M-7; see p. 55.

\( T_8 \): the rotational-sensitivity-coefficient vector for mirror M-8, composed of the particular \( T_1 \) and \( T_2 \) for M-8; see p. 55.
u: the displacement component of the ray vector $\mathbf{u}_j$; see p. 47.

$u'$: the angular component of the ray vector $\mathbf{u}_j$; see p. 47.

$\mathbf{u}_j$: the ray vector representing a spherical-wave input alignment beam's outer beam radius; see p. 47.

$\mathbf{v}_j$: the displacement component of the ray vector $\mathbf{v}_j$; see p. 47.

$v'$: the angular component of the ray vector $\mathbf{v}_j$; see p. 47.

$\mathbf{v}_j$: the ray vector representing a spherical-wave input alignment beam's inner beam radius; see p. 47.

$w$: an abbreviated form for $w(z)$; see p. 19.

$w(z)$: the beam width of a Gaussian beam; see p. 16.

$w_2$: the Gaussian beam width associated with $q_2$; see p. 19.

$W-1$: the aerodynamic window in the laser subsystem; see p. 5.

$x_c$: a small translation of concave mirror $M-2$ in a direction perpendicular to the optical axis; see p. 39.

$x_\gamma$: a small translation of convex mirror $M-1$ in a direction perpendicular to the optical axis; see p. 39.

$x_1$: a coordinate in an input coordinate system; see p. 21.

$x_2$: a coordinate in an output coordinate system; see p. 21.

$x$: a coordinate in a fixed, external coordinate system; see p. 21.

$X_1$: a coordinate in a displaced input coordinate system; see p. 80.

$X_2$: a coordinate in a displaced output coordinate system; see p. 80.

$y_1$: a coordinate in an input coordinate system; see p. 21.

$y_2$: a coordinate in an output coordinate system; see p. 21.

$Y_1$: a coordinate in a fixed, external coordinate system; see p. 21.

$Y_1$: a coordinate in a displaced input coordinate system; see p. 80.
\( r_2 \): a coordinate in a displaced output coordinate system; see p. 80.

\( z \): a coordinate in the direction of propagation; see p. 16.

\( z_1 \): a coordinate in an input coordinate system; see p. 21.

\( z_2 \): a coordinate in an output coordinate system; see p. 21.

\( Z \): a coordinate in a fixed, external coordinate system; see p. 21.

\( Z_1 \): a coordinate in a displaced input coordinate system; see p. 80.

\( Z_2 \): a coordinate in a displaced output coordinate system; see p. 80.

\( \alpha \): the scalar multiplier of the identity matrix \( I \) for the first term of the reduced expression for \( \mathbf{F}(\mathbf{F}) \); see p. 33.

\( \alpha_1 \): the value of \( \alpha \) for the \( i \)-th reflection; see p. 34.

\( \beta \): the scalar multiplier of the round-trip matrix \( \mathbf{F} \) for the second term of the reduced expression for \( \mathbf{F}(\mathbf{F}) \); see p. 33.

\( \beta_1 \): the value of \( \beta \) for the \( i \)-th reflection; see p. 34.

\( \gamma \): an eigenvalue of the \( \mathbf{F} \) matrix; see p. 33.

\( \gamma_1 \): a particular eigenvalue of the \( \mathbf{F} \) matrix; see p. 33.

\( \gamma_2 \): a particular eigenvalue of the \( \mathbf{F} \) matrix; see p. 33.

\( \Delta \): a small or infinitesimal change; see p. 28.

\( \gamma \): an arbitrary constant of proportionality; see p. 43.

\( \lambda \): the wavelength of the Gaussian beam; see p. 16.

\( \lambda_0 \): the free-space wavelength of a propagating beam; see p. 29.

\( \lambda_1 \): the wavelength in a medium for the first path of propagation; see p. 29.

\( \lambda_2 \): the wavelength in a medium for the second path of propagation; see p. 29.

\( \pi \): the constant 3.14159265...; see p. 16.
$\rho$: the density of air; see p. 42.

$\vec{\omega}$: the ray vector representing a mirror misalignment; see p. 27.

$\psi$: the angle of incidence at the surface of an optical element for the generalized $Z$ axis in a three-dimensional system; see p. 79.

$\theta$: the angle of rotational misalignment of an optical element; see p. 26.

$\vec{\theta}_1$: the rotational ray vector added to the input vector of a rotated optical system; see p. 26.

$\vec{\theta}_2$: the rotational ray vector added to the output vector of a rotated optical system; see p. 26.

$\Theta_{c_r}$: a critical angle of misalignment for an optical element in the path of an alignment beam making many round trips inside an unstable resonator; see p. 64.

$\Theta_{c_{r1}}$: the critical angle of misalignment for mirror $M-1$, $\Theta_{c_{r1}}$ with the alignment beam making 10 round trips; see p. 64.

$\Theta_{c_{r11}}$: the critical angle of misalignment for mirror $M-1$, $\Theta_{c_{r11}}$ with the alignment beam making 11 round trips; see p. 64.

$\Theta_{c_{r2}}$: the critical angle of misalignment for mirror $M-2$, $\Theta_{c_{r2}}$ with the alignment beam making 10 round trips; see p. 65.

$\Theta_{c_{r21}}$: the critical angle of misalignment for mirror $M-2$, $\Theta_{c_{r21}}$ with the alignment beam making 11 round trips; see p. 65.

$\Theta_{c_{r3}}$: the critical angle of misalignment for mirror $M-3$; see $\Theta_{c_{r3}}$ p. 67.

$\Theta_{c_{r4}}$: the critical angle of misalignment for mirror $M-4$; see $\Theta_{c_{r4}}$ p. 67.
LIST OF REFERENCES


