An Investigation of a Variable Geometry Diffuser for FTU's Four Inch Supersonic Wind Tunnel

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AN INVESTIGATION OF A VARIABLE GEOMETRY DIFFUSER FOR FTU'S FOUR INCH SUPersonic WIND TUNNEL

BY

WILLIAM ROBERT FREED
B.S.M.E., Michigan Technological University, 1970

Research Report

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering: Mechanical Engineering in the Graduate Studies Program of the College of Engineering of Florida Technological University

Orlando, Florida
1977
INVESTIGATION OF A VARIABLE GEOMETRY DIFFUSER
FOR FTU'S FOUR INCH SUPERSONIC WIND TUNNEL

BY

WILLIAM R. FREED

ABSTRACT

The primary object of the investigation reported in this paper was to obtain information that would aid in the design of a more efficient diffuser for FTU's tunnel, and thus increase the run time.

Presently FTU's four inch supersonic wind tunnel uses a constant area, normal shock, diffuser to recover the fluid pressure after the test section. Also, FTU's tunnel is of the intermittent blowdown type, which provides only a relatively short test time before the storage pressure decreases to a limiting value at which flow in the test section ceases to be supersonic. The use of a constant area diffuser and normal shock pressure recovery has the disadvantage of always entailing a large loss in stagnation pressure. These losses increase as the test section Mach number increases. Since a diffuser employing a system of oblique shocks should have a better pressure recovery than one with a single normal shock, efforts were made to improve FTU's wind tunnel along these lines. Variable area diffusers whose throats can be closed after flow has been established were of interest in this report because of their higher pressure recovery.
The maximum run time of FTU's wind tunnel is limited by the overall operating pressure ratio required to maintain supersonic flow in the test section area. If one can reduce the losses in the tunnel, the operating pressure ratio can be reduced. The reduction in operation pressure can result in an increase in run time. In FTU's tunnel, the majority of losses occurs in the second throat area or the supersonic diffuser. Tunnel run time improvement may be required to conduct heat transfer studies or to conduct force, moment and pressure tests.

The results of the one-dimensional analyses of a variable geometry supersonic diffuser are very promising in that they show a longer run time can be obtained for FTU's tunnel. By using a variable geometry diffuser, an intermittent blowdown wind tunnel run time can be increased two to three times that of a constant area diffuser at high Mach numbers. At the design Mach number of 4.0, the theoretical run time can be increased 321 percent over the run time of a constant area diffuser.

The references cited made it possible to geometrically design a relatively simple, yet efficient contractable wall (convergent-constant area-divergent) type diffuser. Three flat plates were chosen to form the side walls of the adjustable diffuser. The length of the plates were a compromise between mechanical construction requirements and the need to keep the wall convergent angle relatively small for the Mach number range of FTU's tunnel and to minimize energy losses. The first adjustable diffuser plate has an overall length of 14.5 inches.
The angle of convergent for design was chosen to be 7 degrees at the design Mach number of 4.0. The second diffuser plate that forms the constant area passage has an overall length of 12 inches. The third diffuser plate that forms the divergent section has an overall length of 13.5 inches.
ACKNOWLEDGMENT

The author wishes to express his appreciation to Dr. M. Varney and Dr. D. B. Wall whose comments and suggestions contributed to the successful completion of this research report. Special thanks to Mr. James K. Beck for his direct aid and helpful suggestions on the manuscript.

Also, a note of appreciation to K. V. Davis, P. D. Dillard, S. I. Jester, and L. A. Krzywicki, who did the preliminary typing of this report. Special thanks to Mrs. L. A. Krzywicki who did the final typing of this report.

Finally, the author takes this opportunity to express his appreciation to The Boeing Company for their financial support under the Graduate Study Program.
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\( P_f \) Final stagnation pressure in storage tank, psia.
\( P_i \) Initial stagnation pressure in storage tank, psia.
\( P_o \) Settling chamber stagnation pressure, psia.
\( \Delta P_o \) Frictional losses in pressure regulator and duct work, psia.
\( P_{t1} \) Stagnation pressure upstream of shock wave, psia.
\( P_{t2} \) Stagnation pressure downstream of shock wave, psia.
\( T_i \) Initial stagnation temperature of storage tank, degrees Rankine (°R).
\( t_p \) Wind tunnel run time, seconds.
\( V \) Storage tank volume, cubic feet.
\( x \) Characteristic axial length of diffuser, inches.

### GREEK

- \( \phi \) Diffuser divergent half angle, degrees.
- \( \Delta \) Diffuser contraction half (turning) angle, degrees.
- \( \theta \) Shock wave angle, degrees.
- \( n \) Diffuser efficiency, dimensionless.
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NOMENCLATURE

This list of nomenclature is a partial listing of symbols used in this research report. Symbols that are only used infrequently are defined locally and do not appear in this list.

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<th>MEANING</th>
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<td>Nozzle throat area, square inches.</td>
</tr>
<tr>
<td>$A_2$</td>
<td>Diffuser throat area, square inches.</td>
</tr>
<tr>
<td>$A_T$</td>
<td>Test section area, square inches.</td>
</tr>
<tr>
<td>$D$</td>
<td>Hydraulic diameter, inches.</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter of air storage tank, feet.</td>
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<tr>
<td>$e$</td>
<td>Base of natural (Napierian) logarithms, 2.71828...</td>
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<tr>
<td>$f$</td>
<td>Friction coefficient, dimensionless.</td>
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<td>$H$</td>
<td>Height of test section area, inches.</td>
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<td>$h_T$</td>
<td>Second throat height, inches.</td>
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<tr>
<td>$k$</td>
<td>Polytropic exponent, ratio of specific heats, dimensionless.</td>
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<td>$L$</td>
<td>Length of constant area second throat, inches.</td>
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<td>Second diffuser plate length, inches.</td>
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<td>$L_{p_3}$</td>
<td>Third diffuser plate length, inches.</td>
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<td>$\ell$</td>
<td>Length of storage tank, feet.</td>
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<tr>
<td>$M$</td>
<td>Mach number, dimensionless.</td>
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<td>$M_D$</td>
<td>Design Mach number, dimensionless.</td>
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<td>$M_T$</td>
<td>Test section Mach number, dimensionless.</td>
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<td>$P_e$</td>
<td>Diffuser exit pressure, psia.</td>
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CHAPTER I

DESCRIPTION OF FTU'S FACILITY

General

FTU's four-inch supersonic wind tunnel is a conventional blowdown, single-pass type capable of generating flow Mach numbers in the range of 1.5 to 5.0. The tunnel facility, consisting of the pressurization system, flow circuit, associated instrumentation, and model hardware was designed and fabricated by Kenney Engineering Corporation [1]. The facility is designed for semi-automatic operation requiring only the opening of the manual gate valve, adjusting the operating pressure and flow-on timer, and depressing the start button. Further convenience has been incorporated into the design through the addition of a remote tunnel controller which the operator may utilize as far as 25 feet from the main console. A diagrammatic layout of FTU's intermittent blowdown tunnel is shown in Figure 1. The major components such as the air storage tank, isolation gate valve, main control valve, conical subsonic diffuser and settling chamber, variable nozzle, test section, supersonic diffuser, subsonic diffusers, and flow silencer are shown. Additional information on the installation is supplied in the following paragraphs and in References [1,2].
Fig. 1. Diagrammatic Layout of FTU's Intermittent Blowdown Tunnel
Pressurization System

The 329 cubic foot air storage tank is supplied by a 50 horsepower two-stage compressor regulated to a maximum pressure of 250 pounds per square inch. Following compression, the air flows through an aftercooler and separator for removal of the condensed water. It is then processed through an oil-vapor filter and finally through a silica gel dryer to remove the excess water vapor. The stored air has a design dew point in the range of -20 to -40 degrees Fahrenheit. The dryer has the capacity for an eight-hour pumping period, after which the silica gel must be regenerated by the internally mounted electric strip heaters. The tunnel stagnation pressure is automatically controlled by a feedback system between the stilling chamber and the main four-inch double-ported control valve.

Tunnel Circuit

The circuit is made up of components typical to a single-pass blowdown wind tunnel. A conical subsonic diffuser reduces the air velocity to the screened stilling chamber. The air then passes to the nozzle and test section assembly which is unique to the Kenney-engineered facility [1]. The test section Mach number is predicated on the test section to nozzle throat area ratio. The throat height is varied through a smoothly operating mechanism and the flexible upper and lower walls assume a gradually decreasing curvature. This design yields an unlimited selection of test section Mach numbers for the range from 1.5 to 5.0. The flexible nozzle can be removed and replaced with precisely contoured nozzle blocks, if desired. The test section
windows, of 1-1/4 inch plate glass give a 26 square inch viewing area.

A remotely-driven model support strut allows additional flexibility in tunnel operation. A linkage has been provided which permits the model to be driven either in an angular mode or translation mode in the vertical plane. Effectively, this permits the model angle of attack to be varied through ±20 degrees, maximum, at the most aft center of rotation to any intermediate plus and minus (including 0) range at a more forward center of rotation (∞ for pure translation). The strut has a total movement of ±3 inches; therefore, extreme care must be exercised when changing the model's center of rotation. Limit switches have been installed to prevent driving the model into the top or bottom of the test section.

The second throat is a simple four inch square 40 inch long duct having no wall adjustments. It is followed by a 40 inch long transition section to an 8-1/2 inch round section which enters an extension fitted to the large silencer. The diffuser section is caster-mounted to allow access to the model and strut.

Instrumentation

The supplied instrumentation is contained in the tunnel console. The information displayed includes:
1. Storage tank pressure: 0 - 300 psig.
2. Tunnel stagnation pressure: 0 - 300 psig.
3. Tunnel stagnation temperature: -75 to +225°F.
4. Six data channels from test section: -30 in. Hg to +30 psig.
5. One data channel from test section: -30 in. Hg to +100 psig.
6. Tunnel preset pressure.

7. Strut position and angle of attack.

8. Strain gage readout for normal forces, axial force, and rolling moment.

The test-section/model pressures above may be read out from eight-inch Bourdon-type gages which may be sealed off during tunnel operating to facilitate data recording after the run is completed. Tunnel stagnation temperature is required to define the Reynolds number, which can vary as a result of expansion from the storage tank pressure to the running pressure.

The half-inch diameter four-component strain gage balance has been designed to the following maximum load conditions:

1. Front normal force ±25 lb.
2. Rear normal force ±25 lb.
3. Axial force ±20 lb.
4. Rolling moment ±20 lb.-in.

The loads are read-out by meter indicators or, for better reading accuracy, by a null-balance digital system.

A viewing screen and a schlieren optical system is available for flow visualization. It is a standard single-pass system utilizing six-inch parabolic mirrors of 48-inch focal length and a 1000 watt BH6 mercury vapor light source.
CHAPTER II

PURPOSE OF INVESTIGATION

FTU's wind tunnel provides only a relatively short test time before the stagnation pressure decreases to a limiting value at which flow in the test section ceases to be supersonic. The use of a constant area diffuser and normal shock pressure recovery has the disadvantage of always entailing a large loss in stagnation pressure. These losses increase as the test section Mach number increases. Since a diffuser employing a system of oblique shocks should have a better pressure recovery than one with a single normal shock, efforts were made to improve FTU's wind tunnel along these lines. Variable area diffusers whose throats can be closed after flow has been established were of interest in this report because of their higher pressure recovery.

The maximum run time of FTU's wind tunnel is limited by the overall operating pressure ratio required to maintain supersonic flow in the test section area. If one can reduce the losses in the tunnel, the operating pressure ratio can be reduced. The reduction in operation pressure can result in an increase in run time. In FTU's tunnel, the majority of losses occurs in the second throat area or the supersonic diffuser. Tunnel run time improvement may be required to conduct heat transfer studies or to conduct force, moment and pressure tests. For example, heat transfer studies may be required to determine
the heat transfer rates at the leading edge of a wing on an airplane model. Another example would be the increase in run time required to measure the forces and rolling moment due to the deflection of a control surface on a model.

This research report will be concerned with the use of a variable geometry supersonic diffuser at the test section exit to recover the fluid pressure and thus provide for a longer run time. Also, the information in this research report has been gathered for the purpose of providing a preliminary description of a variable geometry supersonic diffuser which can be adapted to FTU's tunnel and provide background knowledge of supersonic diffusers which can be used in future investigations.

The aerodynamic design of supersonic diffusers for wind tunnels has been discussed in numerous studies cited in the references. For optimum pressure recovery, it has been found that the diffuser should have a throat with a cross section less than that of the test section. Satisfactory throat areas have been defined in terms of the operating Mach number. However, the overall diffuser configuration has not been defined. Studies have been made in the attempt to optimize such configuration variables as the angle of convergence between the test section and the diffuser throat, the length of the diffuser throat and the angle of divergence after the second throat. Included angles of convergence from quite small up to thirty degrees or more have been used, as have second throat lengths of zero to more than ten test section lengths. The test section length is defined here to be some
characteristic dimension of the test section such as its height, width, diameter or hydraulic diameter. The results of studies cited in the references have not been conclusive and the tunnel designer must choose a configuration that he can reasonably expect to work on the basis of previous diffuser studies and hope it works well in his tunnel.
CHAPTER III

SUPersonic Diffusers Background

Simple Divergent Diffuser

The simplest, and also the least effective, is a simple divergent passage starting at the end of the test section, with a normal shock standing just behind the test section. The air is compressed to subsonic speed in the shock, and further slowed down in the conventional subsonic diffuser behind it. The pressure ratio required then is the ratio of total pressures across a normal shock wave at the test section Mach number. In practice, the simple diffuser does not give the expected pressure recovery. Observations by MacDonald [3], and Neumann and Lustwerk [4] of shock compression in divergent ducts have shown separation of the stream to exist. This is due to the interaction of shock wave and boundary layer. The above interaction produces a flow that is different from the normal shock model, which neglects viscous effects, and one that usually results in lower pressure recovery or higher stagnation pressure losses.

Constant Area Diffuser (Existing on FTU's Tunnel)

A long constant area duct ahead of a subsonic diffuser more nearly realizes the normal shock recovery than the simple diffuser type. Such a duct, provided it is long enough, gives nearly the same recompression as a normal shock, even though the mechanism is quite
different [5]. The compression occurs through a system of shocks interacting with the thickened boundary layer. Recovery through such a dissipative system is not the most efficient recovery, but it is often the most practical. Its virtue is its stability with respect to variations of inlet conditions. It is possible to design more efficient diffusers (fixed and variable geometry) for specific conditions, but these may perform quite badly at off-design points.

**Fixed Convergent-Divergent Geometry Diffuser**

It would appear to be possible to use a convergent-divergent nozzle in reverse (that is, supersonic-subsonic) as a completely isentropic diffuser. For this, the diffuser throat (second throat) would have to match the nozzle throat exactly, with due allowances for the boundary layer. This could perhaps be done when the flow is established, but one must first consider the starting process.

As the tunnel starts, the velocity in the nozzle will increase until sonic velocity \( (M = 1.0) \) is reached at the narrowest part of the tunnel circuit, that is, at the nozzle throat (area \( A_1 \)). Then, as the overall tunnel pressure ratio increases, a normal shock wave will form and will move from the nozzle throat into the diverging portion of the nozzle. The supersonic region behind the nozzle throat will terminate in a normal shock wave, which will reduce the total pressure of the air. At any other point in the circuit where the Mach number is sonic, particularly at the diffuser throat, the mass flow must be the same value as at the nozzle throat, since the mass flow and total tempera-
ture are unchanged by passing through the shock. This means that the diffuser throat area must be greater than the nozzle throat area, since the total-pressure is reduced through the shock. The worst condition (that is, the greatest loss in total pressure) occurs when the normal shock is standing at the largest section where the flow is supersonic, which is the test section. Then, for starting, the minimum diffuser throat area \( (A_2) \) is given by the following relationship:

\[
A_2 = A_1 \left( \frac{P_{t1}}{P_{t2}} \right) \tag{1}
\]

where \( (P_{t1}/P_{t2}) \) is the total pressure ratio across a normal shock at the Mach number in the test section. Thus the diffuser throat area for starting must be greater than the nozzle throat in the ratio of the total pressures on the two sides of a normal shock at the operating Mach number of the test section. This relation also gives the minimum pressure ratio which will start the tunnel, that is, the ratio of total pressure \( (P_0) \) upstream of the nozzle to the total pressure \( (P_e) \) at the diffuser exit must be greater than that through a normal shock in the test section.

During starting, a normal shock will not remain stably in one position in a convergent passage because of changing pressure gradient, and as soon as the pressure ratio exceeds by a small amount the value required for starting, the shock moves from the test section, passes
through the second throat, and takes up a position in the diffuser where the area is approximately equal to the test section area.

Once a tunnel has been started, for best (most efficient) operating, the shock should be at the diffuser throat, as this is the point of minimum supersonic Mach number downstream of the test section. To achieve this, the pressure ratio can be reduced.

Knowing the diffuser throat area \( A_2 \) from the starting conditions, one can find the Mach number \( M_2 \) in the second throat, and hence the corresponding total pressure ratio from the normal shock relations. Thus there is a considerable gain in efficiency compared with a simple divergent diffuser at high test section Mach numbers.

In practice, the shock is maintained just downstream of the throat, for reasons of stability, and the second throat made a little larger than the minimum to allow for uncertainty in boundary layer calculations, effects of model wakes, and so forth, so that the gain in efficiency is not quite so great. Fixed convergent-divergent diffusers are mainly used with fixed geometry nozzles and are generally not used with a variable geometry nozzle.

**Variable Geometry Diffuser**

If the size of the second throat can be varied when the tunnel is running, it is possible after the starting shock has passed through to reduce the second throat size nearly to that of the first throat, with allowances for boundary layers, model wakes, and so forth. In this case, with the shock brought up close behind the second throat,
the operating pressure ratio can be reduced further and the efficiency improved over a fixed convergent-divergent geometry diffuser, though at the cost of considerable mechanical complication.
CHAPTER IV

RUN TIME IMPROVEMENT

Run Time

The derivation of the relation with which run time may be computed employs a number of basic principles given in the references cited and will not be derived in this research report. The following equation was adapted from Pope [6] and may be used to estimate the run time of a blowdown tunnel for runs at constant stagnation pressures.

\[ t_p = \frac{0.0706}{k+1} \frac{V}{A_T \sqrt{T_i (A_1/A_T)}} \frac{P_i}{P_0} \left[ 1 - \left( \frac{P_f}{P_i} \right)^{\frac{k+1}{2k}} \right] \]  

Equation (2) gives the run time in seconds for runs at constant stagnation pressure \( P_0 \) when a storage tank of volume \( V \), cubic feet, at initial stagnation pressure \( P_i \) and initial stagnation temperature \( T_i \), degrees Rankine (R), is blowdown to an end pressure \( P_f \) through a test section of \( A_T \) square feet. From Equation (2), the maximum run time obviously occurs when the final pressure \( P_f \) is a minimum.

It has been determined [7] that the variable geometry diffuser gives the minimum final pressure and will be the main objective of this report. By using a variable geometry diffuser, the set stagnation pressure \( P_0 \) required to maintain supersonic flow in the test section can be reduced after the tunnel has started. By reducing the stagna-
tion pressure, the run time can be increased as shown in Equation (2). Additional discussion of the other parameters in Equation (2) is supplied in the following paragraphs.

The run time could also be increased by increasing the capacity of the air storage tank and by increasing the initial pressure of the storage tank. The storage tank volume and initial stagnation pressure is limited by the existing facility and will not be investigated in this research report. The initial temperature of the fluid in the air storage tank is relatively fixed by the aftercooler and ambient temperature and will not be investigated in this report.

The polytropic exponent \(k\) of expansion process in the storage tank is a function of the rate at which the air is used, the total amount used, and the shape of the storage tank. It is somewhere between an isentropic and an isothermal process. From preliminary data reported by Pope [7], it appears that the polytropic exponent may be estimated to be 1.2 for a cylindrical storage tank with a length to diameter ratio \((L/d)\) of 3.0.

**Establishing the Minimum Allowable Operating Pressure**

As a first step in the design of a wind tunnel diffuser, the assumption can be made that no boundary layer is present, and a simple one-dimensional analysis can then be undertaken. If a normal shock during the starting process and running condition is assumed to be inevitable, the pressure ratio at which a wind tunnel will start and run can be computed. These computed values, subject to the
limitations imposed by a one-dimensional analysis that neglects friction, are presented in Figure 2 for the range of Mach numbers from 1.5 to 5.0.

Curve (a) of Figure 2 represents the recommended operating pressure ratio of the tunnel by Kenney Engineering Corporation [1]. The theoretical pressure ratios necessary for starting the tunnel were determined for the case where a normal shock wave is located in the region of the test section, and is shown in Figure 2 as curve (b). This curve also represents the pressure ratio required if there is no contraction downstream of the test section, (that is, no "second throat"). Curve (b) was established from the normal shock pressure ratio \( \frac{P_{t1}}{P_{t2}} \) from NACA Report 1135 [8] at the test section Mach number \( M_T \). Curve (c) of Figure 2 is the theoretical pressure ratio required for running with an optimum fixed contraction in the diffuser. For this case, the second throat is large enough to swallow the shock and hence not as small as the optimum would be after the tunnel is started. Curve (c) was established by determining the Mach number \( M_2 \) in the second throat after the tunnel has started (that is, isentropic flow between the two throats) and finding the corresponding normal shock pressure ratio \( \frac{P_{t1}}{P_{t2}} \) at this Mach number. Curve (d) of Figure 2 is the optimum running pressure ratio of a variable geometry diffuser (that is, one which can be reduced in area after the shock has passed). Curve (d) was adapted from Equation (2) of Diggins and Lange [9] and represents the optimum pressure recoveries faired at various facilities for the range of Mach numbers from 1.75 to 6.5. See
Fig. 2. Operating pressure ratio, $P_o/P_e$, as a function of test section Mach number, $M_T$. 
Appendix A for detailed method of determining theoretical pressure ratios.

The results of the one-dimensional analysis may be summarized as follows:

1. For starting conditions, the minimum pressure across a wind tunnel (stilling chamber through diffuser) must be equal to the loss in stagnation pressure across a normal shock at the test section Mach number.

2. For operating conditions, the minimum pressure across a wind tunnel must be equal to the loss in stagnation pressure across a normal shock at the Mach number in the diffuser throat.

These conclusions must be accepted with reservations since it has been assumed that no boundary layer is present and that the only possible transition from supersonic to subsonic flow during the starting operation is a normal shock. The assumption of no boundary layer, that is, no wall friction, ignores irreversibilities which are actually present, whereas the assumption of a normal shock may assume greater irreversibilities than are actually necessary. A more realistic model might have to take into account the possibility of oblique shocks, the role of boundary layer development and shock wave interaction with the boundary layer. The tunnel in any case must be started by the ratios given by curve (b) of Figure 2, but then the stagnation pressure may be reduced, if a variable geometry diffuser is used, to the values obtained from curve (d) of Figure 2.

Figure 2 shows that by using a variable geometry diffuser, an
intermittent blow down tunnel can be run with a smaller (closer to one) overall pressure ratio. The smaller overall pressure ratio for running results in a longer run time before the flow breaks down and becomes subsonic in the test section.

**Diffuser Efficiency**

The most common definition of diffuser efficiency \( n \) as defined by Shapiro [5] is based on the stagnation pressure ratio \( (P_{t2}/P_{t1}) \) for a given diffuser inlet Mach number \( (M_1) \) and is given by Equation (3).

\[
n = \frac{\left(1 + \frac{k-1}{2} M_1^2\right) \left(P_{t2}/P_{t1}\right)^{\frac{k-1}{k}}}{\frac{k-1}{2} M_1^2} - 1
\]

Equation (3) is based on the assumption that the velocity leaving the diffuser is negligible, state 1 (subscript 1) is the actual state entering the diffuser and state 2 (subscript 2) is the actual state leaving the diffuser. For this report, it is also assumed there is no loss in stagnation pressure between the settling chamber and supersonic diffuser inlet, the Mach number \( (M_1) \) at entrance to the diffuser is the same as the test section Mach number \( (M_T) \), flow is uniform across the cross sectional area of the tunnel, and there is full subsonic recovery in the subsonic diffuser. For this case, the diffuser efficiency also becomes the overall wind tunnel efficiency.
With the aid of Equation (3), the efficiencies for constant area (normal shock) diffuser, fixed geometry diffuser and variable geometry diffuser are shown in Figure 3 for the range of test section Mach numbers from 1.5 to 5.0. The normal shock pressure recovery, assuming full subsonic recovery, is a convenient reference, or standard, for comparing the performance of actual supersonic diffusers and wind tunnels. It can be seen from Figure 3, for a diffuser inlet Mach number of 4.0 that there is a theoretical improvement of 15.8 percent in diffuser efficiency if a variable geometry diffuser is used instead of a constant area, normal shock diffuser. It can also be seen from Figure 3, that the diffuser efficiency increases with decreasing Mach number. The increase in efficiency is mainly due to the normal shock occurring at a lower Mach number in the second throat. See Appendix B for detailed method of determining the theoretical diffuser efficiencies. If the diffuser is more efficient, the wind tunnel can run for a longer time before the flow breaks down and becomes subsonic in the test section.

Second Throat Area

For fixed convergent-divergent geometry diffusers, sizing of the second throat to allow the normal shock to pass through during the starting process is accomplished as follows. A method adapted from Shapiro [5] for finding the theoretical area of a second throat (A2) uses the ideal that the area of a second throat at a Mach number of 1.0 is greater than that of the nozzle throat area (A1) upstream of the
Fig. 3. Diffuser efficiency, $n$, as a function of test section Mach number, $M_T$. 

(a) Variable Geometry Diffuser 
(b) Fixed Geometry Diffuser 
(c) Constant Area Diffuser 

Diffuser Efficiency, $n$

Mach Number, $M_T$
shock by the ratio of the stagnation pressures at the two throats.

\[ \frac{A_2}{A_1} = \frac{P_{t1}}{P_{t2}} \]  

(4)

Instead of this ratio, it is often convenient to deal with the ratio of test section area \( (A_T) \) to diffuser throat area \( (A_2) \), that is \( (A_T/A_2) \). This is called the diffuser contraction ratio. Thus, in terms of the test section area, the minimum permissible second throat area ratio for starting is given by Equation (5).

\[ \frac{A_2}{A_T} = \frac{A_2}{A_1} \frac{A_1}{A_T} = \frac{P_{t1}}{P_{t2}} \frac{A_1}{A_T} \]  

(5)

It should be noted that the second throat area that permits starting does very little supersonic diffusing after the tunnel has started. What can be done now in principle if a variable geometry diffuser is used, is to reduce the area of the second throat, after the flow has started, until the second throat area \( (A_2) \) equals the nozzle throat area \( (A_1) \), for optimum pressure recovery. The flow is then the ideal one, with supersonic test section and isentropic diffuser.

In practice, it is not possible to reduce the second throat area \( (A_2) \) to the ideal value of the nozzle throat area \( (A_1) \). However, some contraction after starting is possible, up to some value at which the boundary layer effects prevent sufficient mass flow for maintaining a
supersonic test section. Pope [7] suggests that the second throat area found should be increased (30% or more) to allow for the boundary layer and the wake of a model. See Appendix C for detailed method of determining the second throat area \(A_2\).

The results of these calculations for the range of Mach numbers from 1.5 to 5.0 are shown in Figure 4. Figure 4 shows the variation of the theoretical second throat area with test section Mach number of a variable geometry diffuser for starting and running conditions. The curve for variable geometry diffuser (start), also represents the optimum diffuser area ratio for a fixed geometry diffuser. Thus, using a variable geometry diffuser, an intermittent tunnel can be run with a smaller (closer to one) overall pressure ratio as shown in Figure 2 and can run for a longer time.

Calculating Run Times

There are two types of runs which are generally employed with blowdown wind tunnels: (1) constant pressure and (2) constant mass. Constant mass runs may be accomplished with a heater or thermal mass which holds the stagnation temperature constant. FTU's blowdown tunnel has a semi-automatic control valve \((P_0\) valve) which maintains a constant pressure in the stilling chamber while the available pressure in the storage tank is decreasing.

The following equation was adapted from Pope [6] and may be used to estimate the run time of a blowdown tunnel for runs at constant stagnation pressure.
Fig. 4. Theoretical diffuser area ratio, $A_2/A_T$, as a function of test section Mach number, $M_T$. 
\[ t_p = \frac{0.0706}{k+1} \frac{V}{A_T \sqrt{T_{i1}} (A_1/A_T)} \frac{p_i}{p_0} \left[ 1 - \left( \frac{p_f}{p_i} \right)^{\frac{k+1}{2k}} \right] \]  

The maximum run time obviously occurs when the final pressure \((p_f)\) is a minimum. It has been determined that the variable geometry diffuser gives the minimum final pressure. The actual run does not continue until the tank pressure drops to the stagnation pressure \((p_0)\), but rather stops when the pressure reaches some higher value due to the losses in the duct work and in the regulator. Pope [7] states that the losses for duct work and regulator varies from about \(0.1p_0\) for very-small-mass runs (hypersonic tunnels) to somewhere around \(1.0p_0\) for high-mass runs. For the purpose of this report, friction losses \((\Delta p_0)\), are assumed to vary linearly from \(0.9p_0\) to \(0.2p_0\) for the range of Mach numbers from 1.5 to 5.0.

The polytropic exponent \((k)\) of expansion process in the storage tank is a function of the rate at which the air is used, the total amount used, and the shape of the storage tank. From preliminary data reported by Pope [7], it appears that the polytropic exponent \((k)\) may be estimated to be 1.2 for a cylindrical storage tank with a length to diameter ratio \((\ell/d)\) of 3.0, as compared to \((k)\) equals 1.4 for the isentropic expansion process. For the purpose of this report, the polytropic exponent \((k)\) will be taken as 1.2 for use in Equation (2).

Using the above relationships and the values of operating pressure ratio developed previously in Appendix A and the nozzle area ratio \((A_T/A_1)\), the estimated run time may now be determined for the
different diffuser configurations. See Appendix D for details of calculating run times. Figure 5 shows the maximum obtainable run time, $t_p$, as a function of test section Mach number, $M_T$, for the cases of a variable geometry diffuser (run) and constant area diffuser (existing) for the range of Mach numbers from 1.5 to 5.0. Figure 5 shows that by using a variable geometry diffuser, an intermittent blowdown wind tunnel run time can be increased 2 to 3 times that of a constant area diffuser at high Mach numbers. Above a Mach number of about 4.0 the maximum capability of the tunnel is approached and there is a rapid decrease in run time. Preliminary calculations indicated that the overall starting pressure ratio of the wind tunnel can not be obtained by the existing air compressor at a Mach number of 5.0. Figure 5 also indicates the upper range of Mach numbers obtainable is about 4.5.
Fig. 5. Run time, $t_p$, as a function of test section Mach number, $M_T$. 

Variable Geometry Diffuser (Run)

Constant Area Diffuser (Existing)
CHAPTER V

PRELIMINARY DIFFUSER DESIGN

The Diffuser Problem

The more efficiently the diffuser function, that is, the lower
the storage tank pressure can lower before breakdown of the flow occurs,
the longer will be the blowdown times in an intermittent tunnel. In
undergoing the deceleration from supersonic velocities to rest, the
air in the wind tunnel must pass through a shock system. Since the
entropy gain through a normal shock increases with the Mach number at
which the shock occurs, every effort should be made in the diffuser
to produce this normal shock at the lowest obtainable Mach number.
This allows lower pressures to occur on the upstream side of the
normal shock which in-turn mean lower stilling chamber pressures.
In order to do this it is necessary to produce in the diffuser oblique
shocks which are reflected and terminate in a normal shock at a lower
Mach number. It has been found that if the proper combination of
oblique shocks and normal shock is produced, then the entropy gain
through this whole shock system will actually be less than through a
single normal shock occurring at the Mach number of the test section,
as is the case of the existing FTU's wind tunnel. To find the best
such combination, while using a simple converging-diverging flat-
surface diffuser, is the aim of this research report. Practically
speaking, finding this "proper combination" means finding the best location of the diffuser throat from the test section exit in conjunction with the best throat opening for the range of Mach numbers from 1.5 to 5.0.

**Diffuser Entrance Shape**

Investigations have been conducted to determine the best inlet angle for the convergent portion of the diffuser. Lukasiewicz [10] found contraction angles of the order of 5 degrees are best at Mach numbers from 7 to 10. Neumann and Lustwerk [4] used included entrance angles between 10 and 30 degrees in their investigation. They found the best diffuser efficiency was obtained with the smallest entrance angle tested. A smaller inlet angle might have shown a slight additional improvement; however, tests at smaller angles were not attempted because an increase in length of the test apparatus would have been necessary, and the result increase in friction area might have offset the gain obtained by the smaller contraction angle.

Based upon the results reported above and other investigations cited in this report, contraction angle of 7 degrees was selected for design purposes at a design Mach number of 4.0. The 7 degree contraction angle was chosen because it allowed contraction of the diffuser at the design Mach number to the minimum theoretical second throat height. Other factors considered were the overall length requirement of the diffuser, which imposes restrictions upon the constant area passage after convergence and the length of the divergent portion. The overall length requirement was to remain within the existing 40
inch constant area second throat diffuser envelope and the divergent angle was not to exceed an angle of approximately 7 degrees. The losses in the diffuser increase with increasing Mach number. A variable geometry diffuser should be designed to have a maximum efficiency at the highest Mach number or at some other critical point. From Figure 5, it can be seen that the maximum run time of FTU's tunnel would be near the Mach number of 4.0 if a variable geometry diffuser is used. Thus a design Mach number of 4.0 was chosen because it was near the critical point of FTU's tunnel.

**Diffuser Throat Area Sizing**

Minimum pressure ratios and minimum area ratios to start and run the tunnel are shown in Figures 2 and 4 respectively as predicted by a one-dimensional analysis. A reduction of settling chamber pressure (Po) can then be made once the tunnel has started. This reduction increases the viscous effects at the diffuser throat. If the supply pressure is lowered, the minimum throat opening should be increased to accommodate the thicker boundary layer.

The second throat height (ht) was determined from the second throat area (A2) with 30 percent allowances [7] for boundary layer effects, model wake effects and other effects that might have been overlooked in the preliminary design. ht is obtained from the following relationship:

\[
h_t = \frac{1.3A_2}{4} = \frac{1.3A_T}{4(A_T/A_l)}
\]  

(6)
For the design Mach number, $M_D = 4.0$, Equation (6) becomes

$$h_t = \frac{(1.3)(16)}{(4)(10.72)} = 0.4851 \text{ inches}$$

**Second Throat Location**

Optimizing a variable geometry diffuser to find the length of the first diffuser plate involves many variables. A special configuration would be needed for every Mach and Reynolds number to obtain the best performance. However, for practical reasons, a diffuser with a fixed first plate length is desirable. Its shape should be selected to be reasonably efficient in the range of Mach numbers for FTU's tunnel and have a maximum efficiency at either the highest Mach number or at some other critical point dictated by other requirements.

Using a turning angle of $7$ degrees at the design Mach number of $M_D = 4.0$, the length of the first diffuser plate can now be determined. From geometry considerations as shown in Figure 6, the location of the constant area second throat may be determined as outlined below:
From geometry considerations. \[ \tan \delta = \frac{y}{x} \] (7)

or \[ x = \frac{y}{\tan \delta} = \frac{(H - h_t)/2}{\tan \delta} = \frac{(4 - 0.4851)/2}{\tan 7^\circ} \] (8)

\[ x = 14.313 \text{ inches} \]

The length of the first diffuser plate is given by \[ L_{p_1} = \frac{y}{\sin \delta} = \frac{(H - h_t)/2}{\sin \delta} = \frac{(4 - 0.4851)/2}{\sin 7^\circ} \] (9)

\[ L_{p_1} = 14.421 \text{ inches} \]
Thus, the second throat area should be located approximately 14.5 inches from the exit of the test section area. This makes allowances for the reed cover plate at diffuser entrance when the diffuser is in the open position for starting and running at lower Mach numbers.

**Second Throat Length Requirement**

It has been found from various studies [4,10] reviewed for this research report that to increase the operating efficiency of supersonic diffusers, the second throat area should be elongated from 3H to 10H, where H is the height of the test section area. This permits the separation region caused by the normal shock wave boundary layer interaction to again fill the passageway before the flow is introduced into the subsonic diffuser. In the case for this diffuser, the minimum area for starting is greater than the minimum value computed from a one-dimensional analysis which does not take into account friction in the elongated throat.

A constant area passage of second plate length, \( l_{p2} = 12 \) inches, was chosen for the following reasons: (1) to stay within the existing 40 inch second throat diffuser envelope and (2) not to exceed a divergent angle of approximately 7 degrees for subsonic pressure recovery downstream of the second throat area.

**Third Plate Length Requirement**

The existing constant area diffuser length is 40 inches. Allowance for the first diffuser plate in the open position is 14.5 inches. The second throat constant area passage length allowance is 3H or 12
inches. The third diffuser plate length \( l_{p3} \) is then given by the following relationship at the design Mach number of 4.0.

\[
l_{p3} = 0.40 - l_{p1} - 3H = 13.5 \text{ inches}
\]  

Thus, the divergent angle \( \phi \) downstream of the constant area passage is given by

\[
\sin \phi = \frac{(H-ht)/2}{l_{p3}} = 0.1302
\]

or

\[
\phi = \arcsin 0.1302 = 7.5 \text{ degrees}
\]

**Proposed Diffuser Description**

Three flat-plates are used to form the side walls of the adjustable diffuser. The length of the plates were a compromise between mechanical construction requirements and the need to keep the entrance convergent angle \( \phi \) relatively small for the Mach number range of FTU's tunnel and to minimize energy losses. The first adjustable diffuser plate has an overall length \( l_{p1} \) of 14.5 inches. The angle of convergent \( \phi \) for design was chosen to be 7 degrees at the design Mach number \( M_D \) of 4.0. The second diffuser plate that forms the constant area passage of the second throat area has an overall length \( l_{p2} \) of 12 inches. The third diffuser plate that forms the divergent section has an overall length \( l_{p3} \) of 13.5 inches.

Figure 7 illustrates the proposed model and operation of the
Fig. 7. Variable Geometry Diffuser.
adjustable diffuser. The four hinge joints at the throat plate, to which the piston rod is attached, should be covered with a bonded rubber pad approximately 1/2 inch thick. This makes for a smooth, leakproof hinge for any diffuser setting. The upstream sliding hinge point (diffuser entrance) and the downstream sliding hinge point (diffuser exit) should be free to move longitudinally to allow the necessary freedom of motion for the adjustment. A stainless steel reed cover plate (1/32 inch thick) should be used to cover the gap at these link points. The center plate should be supported and moved by an electrical/pneumatic operated piston rod or jack. Sliding seals of silicon rubber should be used along the diffuser sidewalls. The seals can be placed in a channel of rectangular cross section. These seals may not be absolutely leakproof, but this requirement should not exist in this region of the tunnel. A near sealed compartment is formed by the adjustable plates and the outside tunnel walls which keeps the air behind these plates at near test section pressure. The diffuser should be electrical/pneumatic operated during the run for adjustment to set position. The diffuser throat opening should be recorded by a mechanical indicator to within 0.005 inches. The variable diffuser fixed steel sidewalls should have pressure taps along the center line for measurements.

Proposed Diffuser Operation

Depending on the Mach number involved, the diffuser should be set at an open position for starting, and then immediately after flow has been established, the diffuser should be closed to the running position
for given Mach number. Opening and closing should be automatic. The minimum and maximum positions should be preset on dials or limit switches. To be compatible with the existing control system circuit, microswitches, electric/pneumatic solenoid valves, relays, and so forth, should be designed where the operation of the diffuser position can be controlled by simply pressing a single start/stop button or switch. Care should be taken to provide for a fool-proof control system.

**Analysis of Proposed Diffuser**

Figure 8 shows the proposed oblique shock diffuser model, the upper boundary of which is the streamline of symmetry (center line). The oblique shock originating at the bend (point A) is reflected from angle ($\delta$) as the original shock AB. Because of the deceleration through each shock, a point is reached, depending on the initial Mach number and turning angle beyond which regular reflection is impossible. The last shock which should be in the second throat area, is a normal shock and reduces the entire stream to subsonic speeds.

The potential advantage of such a diffuser is that the supersonic deceleration occurs across several oblique shocks of small turning angle. Preliminary calculations indicate that the overall pressure ratio of 1.11 is obtained at a design Mach number of 4.0 as compared to a pressure ratio of 7.21 across a normal shock. This indicates a smaller overall pressure ratio of the oblique shock diffuser can result in a potential longer run time before the flow breaks down and becomes subsonic in the test section. See Appendix E for detailed analysis of
Fig. 8. Diffuser model for oblique shock analysis.
proposed diffuser.

It should be noted that in practice the inviscid flow pattern of Figure 8 is seriously modified [11,12] by the interaction between the shock waves and the boundary layers on the walls.
CHAPTER VI

CONCLUSION OF INVESTIGATION

In the case of a variable geometry supersonic diffuser, the overall operating pressure ratio can be reduced, the efficiency improved and the run time increased over that of a constant area diffuser.

The information reviewed in the cited references makes it possible to geometrically design a relatively simple, yet efficient variable geometry diffuser for FTU's four-inch supersonic wind tunnel. A detailed design analysis of a variable geometry diffuser was not undertaken in this research report. This is because the variation of pressure and shearing stress along the solid walls should be included in the analysis. Such a calculation would require knowledge of turbulent boundary layer characteristics in converging and diverging channels and an understanding of shock-wave and boundary-layer interaction.

For optimum pressure recovery, it has been found that a variable geometry diffuser is more efficient than a constant area diffuser. At the design Mach number of 4.0, the overall pressure ratio \((P_o/P_e)\) of an oblique shock diffuser (convergent-constant area-divergent) was 1.11 as compared to a normal shock pressure ratio of 7.21. In the one-dimensional analysis there was an increase of 15.8 percent in diffuser efficiency with a variable geometry diffuser as compared to a constant area diffuser at the design Mach number of 4.0.
The results of the one-dimensional analysis of a variable geometry supersonic diffuser is very promising in that it shows a longer run time can be obtained for FTU's intermittent blowdown wind tunnel. Figure 5 shows that by using a variable geometry diffuser, an intermittent blowdown wind tunnel run time can be increased two to three times that of a constant area diffuser at high Mach numbers. At the Design Mach number of 4.0, the theoretical run time can be increased 321 percent over the run time of a constant area diffuser.

The actual upper limit run times of FTU's four-inch wind tunnel at Mach numbers of 2.0 and 3.0 were about one third that predicted by the one-dimensional analysis which neglected viscous effects. In the investigations reviewed for this research report, it was found that the inviscid flow pattern is seriously modified by the interaction between the shock waves and the boundary layers on the walls. Also, the model wake and model support structure has an adverse effect on the flow in the supersonic diffuser.
CHAPTER VII

RECOMMENDATIONS

Since FTU's wind tunnel is designed for classroom demonstrations, it is recommended that a variable geometry diffuser not be installed on FTU's tunnel. The existing constant area diffuser is not the most efficient type, but it is the most practical. Its virtue is its stability with respect to variations of inlet conditions. The variable geometry diffuser is more efficient, but it is relatively complex and a costly addition. It may be more practical if the cost of a variable geometry diffuser is employed to increase the capability of the air supply system.

If a variable geometry diffuser of contractable wall type is to be built for FTU's wind tunnel, it is recommended that a more detailed study be undertaken which includes viscous effects. The investigation should also study the effects of model wake and model support structure on the performance of the supersonic diffuser. Separation and boundary layer thickness control should be investigated in the detailed study.

It is also recommended that an experimental research program be undertaken on FTU's tunnel to establish allowable operating conditions. Items to be investigated would be: (1) maximum run time for a given Mach number, (2) minimum allowable starting and operating pressures for a given Mach number, (3) effects of model on run time and (4) effects of model on starting and operating pressures.
APPENDIX A

ANALYSIS OF ALLOWABLE OPERATING PRESSURE

General

All calculations used in this Appendix were based upon a one-dimensional analysis of the flow. The velocity and pressure were assumed to be uniform across any cross section in the wind tunnel. Air was assumed to be a perfect gas with a specific heat ratio \( k \) equal to 1.4. The pressure ratios for the diffuser were calculated by assuming isentropic compression from conditions before the diffuser to the diffuser throat followed by a transverse shock at that point and a reversible subsonic diffuser after the shock. For the theoretical calculations, the exit velocity was assumed to be zero.

Constant Area Diffuser (Existing on FTU's Tunnel)

For a given test section Mach number, the operating pressure ratio \( \frac{P_0}{P_e} \) was obtained by dividing the stilling chamber pressure \( P_0 \) as recommended by Kenney Engineering Corporation \([1]\) by the diffuser exit pressure \( P_e \), which was assumed to be 14.7 psia for this research report. The results of these calculations for the range of Mach numbers from 1.5 to 5.0 are tabulated in column (1) of Table 1 and are shown in Figure 2 as curve (a).

Normal Shock Pressure Recovery Ratio

When the Mach number at which the tunnel is to operate is known, and since FTU's intermittent blowdown tunnel exhausts to atmosphere,
<table>
<thead>
<tr>
<th>Test Section Mach Number, $M_T$</th>
<th>Existing FTU's Diffuser (1)</th>
<th>Constant Area Diffuser &amp; Starting (2)</th>
<th>Fixed Geometry Diffuser (3)</th>
<th>Variable Geometry Diffuser (4)</th>
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</table>

*Based on data collected by Beck for Reference [2].

**No Data Available.

***Estimated, exceeds given range of Mach numbers reported by Diggins and Lange [9].
the tunnel exit pressure is known (14.7 psia), the minimum allowable starting pressure can be determined from the ratio of stagnation pressure across a normal shock wave at the test section Mach number, $M_T$, and is given by

$$\frac{P_{t2}}{P_{t1}} = \left( \frac{k+1}{2} \right)^{k/(k-1)} \left( \frac{2kM_T}{k+1} - \frac{k-1}{k+1} \right)^{1/(k-1)}$$

Equation (12) above was adapted from Shapiro [5] and is tabulated in NACA Report 1135 [8] as a function of Mach numbers for $k = 1.4$. This assumes that the flow is ideal, that is, frictionless flow with no losses except for across the normal shock. The above expression also represents the theoretical pressure ratio required if there is no contraction downstream of the test section (that is, no second throat) as for the case existing on FTU's tunnel. The results of these calculations for the range of Mach numbers from 1.5 to 5.0 are tabulated in Column (2) of Table 1 and are shown in Figure 2 as curve (b).

**Fixed Geometry Convergent-Divergent Diffuser**

Minimum allowable operating pressure is determined by finding the Mach number ($M_2$) in the second throat after the tunnel has started (isentropic flow between the two throats) and then finding the corresponding normal shock pressure ratio ($P_{t1}/P_{t2}$) at this Mach number. The ratio of the second throat area ($A_2$) to test section area ($A_T$), in terms of the Mach number ($M_2$) at the second throat, may be determined
from the following relationship adapted from Ratty [13]:

$$\frac{A_2}{A_T} = \frac{M_T}{M_2} \left( \frac{1 + \frac{k-1}{2} M_2^2}{1 + \frac{k-1}{2} \frac{M_T^2}{M_2^2}} \right)^{(k+1)/2(k-1)}$$ \hspace{1cm} (13)

for \( k = 1.4 \), the above equation becomes

$$\frac{A_2}{A_T} = \frac{M_T}{M_2} \left( \frac{1 + 0.2 M_2^2}{1 + 0.2 \frac{M_T^2}{M_2^2}} \right)^3$$ \hspace{1cm} (14)

rearranging

$$\frac{(1 + 0.2 M_2^2)^3}{M_2} = \frac{A_2}{A_T} \frac{(1 + 0.2 M_T^2)^3}{M_T}$$ \hspace{1cm} (15)

Thus, for a given area ratio \((A_2/A_T)\) and for a given value of test section Mach number \((M_T)\), the second throat Mach number \((M_2)\) may be found by a trial and error solution.

For starting, the ratio of the second throat area \((A_2)\) to test section area \((A_T)\), in terms of the test section Mach number \((M_T)\), can be found for \( k = 1.4 \) by the following equation adapted from Pope [7]:

$$\frac{A_2}{A_T} = \frac{(5 + M_T^2)^{0.5}}{(7 M_T - 1)^{2.5}} \frac{216}{M_T^6}$$ \hspace{1cm} (16)

or from the following equation adapted from Shapiro [5] with the aid of values tabulated in NACA Report 1135 [8] at the test section Mach number, \( M_T \).
Example calculation for test section Mach number, \( M_T = 3.0 \).

\[
\frac{A_2}{A_T} = \frac{P_{t1}}{P_{t2}} \frac{A_1}{A_T} = \frac{1}{0.3283} \frac{1}{4.235} = 0.7192
\]

\[
\frac{A_2}{A_T} \left( \frac{1 + 0.2M_T^2}{M_T} \right)^3 = (0.7192) (7.3173) = 5.2626
\]

try: \( M_2 = 2.66 \)

\[
\frac{(1 + 0.2M_2^2)^3}{M_2} = 5.2460 + 0.0332
\]

try: \( M_2 = 2.65 \)

\[
\frac{(1 + 0.2M_2^2)^3}{M_2} = 5.2460 - 0.0166
\]

Therefore \( M_2 = 2.65 \)

At \( M_2 = 2.65 \) the operating pressure ratio is

\[
\frac{P_0}{P_e} = \frac{P_{t1}}{P_{t2}} = \frac{1}{0.4416} = 2.264
\]
The results of these calculations for the range of Mach numbers from 1.5 to 5.0 are tabulated in Column (3) of Table 1 and are shown in Figure 2 as curve (c). These values compare with values of pressure recovery as reported by Lukasiewicz [10].

**Variable Geometry Diffuser**

The following equation was adapted from NAVORD Report 2421 [9] and represents the optimum pressure recoveries faired at various facilities for the range of Mach numbers from 1.75 to 6.5.

\[
\frac{P_o}{P_e} = \frac{0.0865M^2}{1.081} \tag{18}
\]

The results of calculations for the range of Mach numbers from 1.5 to 5.0 are tabulated in Column (4) of Table 1 and are shown in Figure 2 as curve (d). These results agree approximately with the data presented in Figure 5.37 of Shapiro [5] for variable geometry wind tunnel diffusers.
APPENDIX B

ANALYSIS OF DIFFUSER EFFICIENCY

For a given test section Mach number, the diffuser efficiency (or wind tunnel efficiency) may be obtained from Equation (3) and for \( k = 1.4 \) becomes

\[
\eta = \frac{(1 + 0.2M_T^2)(P_{t2}/P_{t1})^{0.286}}{0.2M_T^2} - 1
\]  

(19)

If one substitutes the values of operating pressure ratio obtained by the method in Appendix A into Equation (19), the efficiencies for the various diffuser configurations may be obtained. The results of these calculations for the range of Mach numbers from 1.5 to 5.0 are tabulated in Table 2 and are shown in Figure 3.
## TABLE 2

**DIFFUSER EFFICIENCY**

<table>
<thead>
<tr>
<th>Test Section Mach Number, $M_T$</th>
<th>Diffuser Efficiency, $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant Area Diffuser &amp; Starting</td>
</tr>
<tr>
<td>1.5</td>
<td>0.930</td>
</tr>
<tr>
<td>2.0</td>
<td>0.798</td>
</tr>
<tr>
<td>2.5</td>
<td>0.676</td>
</tr>
<tr>
<td>3.0</td>
<td>0.575</td>
</tr>
<tr>
<td>3.5</td>
<td>0.496</td>
</tr>
<tr>
<td>4.0</td>
<td>0.433</td>
</tr>
<tr>
<td>4.5</td>
<td>0.383</td>
</tr>
<tr>
<td>5.0</td>
<td>0.341</td>
</tr>
</tbody>
</table>
APPENDIX C
SECOND THROAT AREA

The ratio of \((A_2/A_T)\) second throat area to test section area for a fixed geometry and a variable geometry diffuser for starting can readily be determined from Equation (17), with the use of stagnation pressure ratios \((P_{t1}/P_{t2})\) and test section to nozzle area ratios \((A_T/A_1)\), from NACA Report 1135 [8]. The theoretical ratio of second throat area to test section area \((A_2/A_T)\) of a variable geometry diffuser for running is equal to the nozzle area ratio \((A_1/A_T)\). The results of diffuser area ratio calculations are tabulated in Table 3 and are shown in Figure 4.

TABLE 3
DIFFUSER AREA RATIO

<table>
<thead>
<tr>
<th>Test Section Mach Number, (M_T)</th>
<th>Area Ratio, (A_2/A_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Start</td>
</tr>
<tr>
<td>1.5</td>
<td>0.915</td>
</tr>
<tr>
<td>2.0</td>
<td>0.822</td>
</tr>
<tr>
<td>2.5</td>
<td>0.760</td>
</tr>
<tr>
<td>3.0</td>
<td>0.719</td>
</tr>
<tr>
<td>3.5</td>
<td>0.692</td>
</tr>
<tr>
<td>4.0</td>
<td>0.672</td>
</tr>
<tr>
<td>4.5</td>
<td>0.659</td>
</tr>
<tr>
<td>5.0</td>
<td>0.648</td>
</tr>
</tbody>
</table>
APPENDIX D

CALCULATING RUN TIMES

Theoretical Run Times

The following equation was adapted from Pope [6], for runs at constant stagnation pressure, and will not be derived here.

\[ t_p = \frac{0.0706}{k+1} \frac{V}{A_T \sqrt{T_i} (A_1/A_T)} \frac{P_i}{P_o} \left[ 1 - \frac{P_f}{P_i} \right]^{\frac{k+1}{2k}} \]  \hspace{1cm} (2)

where

- \( t_p \) = Wind tunnel run time, seconds.
- \( P_i \) = Initial stagnation pressure in storage tank, psia.
- \( P_f \) = Final stagnation pressure in storage tank, psia.
- \( P_o \) = Settling chamber stagnation pressure, psia. (constant during run)
- \( V \) = Storage tank volume, cubic feet.
- \( A_T \) = Test section area, square feet.
- \( T_i \) = Initial stagnation temperature of storage tank, degrees Rankine (°R).
- \( K \) = Polytropic exponent of expansion process in storage tank, dimensionless.

FTU's blowdown tunnel facility has the following parameters; storage tank volume \( V \) of 329 cubic feet at 250 psig, test section dimensions of 4 inches by 4 inches. It was assumed that the storage tank initial temperature \( T_i \) is 540 degrees Rankine (°R), the initial
pressure \( (P_i) \) in the storage tank is 264.7 psia, polytropic exponent \((k)\) of expansion process in the storage tank is 1.2 and area of test section \((A_T)\) is 16 square inches.

The set stagnation pressure \((P_0)\), constant during run, is established by multiplying the pressure recovery ratio determined in Appendix A by the tunnel exit pressure \((P_e)\) of 14.7 psia for a given diffuser configuration and a given Mach number.

\[
P_0 = \frac{P_0}{P_e} (14.7 \text{ psia})
\]  

(20)

The final pressure \((P_f)\) in the storage tank at which flow ceases to be supersonic is given by

\[
P_f = P_0 + \Delta P_0
\]

(21)

Where \(\Delta P_0\) is the losses in the duct work and regulator. \(\Delta P_0\) is assumed to vary linearly from \(0.9P_0\) to \(0.2P_0\) for the range of Mach numbers from 1.5 to 5.0 as shown below.

<table>
<thead>
<tr>
<th>Test Section Mach Number, (M_T)</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Losses, (\Delta P_0)</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The nozzle area ratio \((A_T/A_1)\) was obtained from NACA Report 1135 [8] at the test section Mach number. Using the above relationships and Equation (2), the estimated run time may now be determined for the
different diffusers configuration. Substituting known values, Equation (2) can be reduced to

\[
\frac{t_p}{P_0} = \frac{1082.4}{(A_T/A_1)P_0} \left[ 1 - \left(\frac{P_f}{P_1}\right)^{0.917} \right]
\]  

Example calculation for constant area diffuser at a test section Mach number, \(M_T = 3.0\). From Appendix A, the stagnation pressure is

\[
P_0 = (3.05)(14.7) = 44.8 \text{ psia}
\]

Then the storage tank final pressure becomes

\[
P_f = (1 + 0.6)(44.8) = 71.7 \text{ psia}
\]

Substituting values into Equation (22)

\[
\frac{t_p}{P_0} = \frac{1082.4(4.235)}{(44.8)} \left[ 1 - \left(\frac{71.7}{264.7}\right)^{0.917} \right] = 71.4 \text{ seconds}
\]

The results of these calculations for the range of Mach numbers from 1.5 to 5.0 are tabulated in Table 4 and are shown in Figure 5.

Example for Mach number, \(M_T = 5.0\) (normal shock)

\[
P_0 = (16.2)(14.7) = 238.14 \text{ psia}
\]
\[
P_f = (1.2)(238.14) = 285.8 \text{ psia} > 264.7 \text{ psia}
\]

Therefore unable to start tunnel.
<table>
<thead>
<tr>
<th>Test Section Mach Number, $M_T$</th>
<th>Run Time, $t_p$, Seconds</th>
<th>Constant Area Diffuser (Existing)</th>
<th>Variable Geometry Diffuser (Run)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>69.2</td>
<td>66.4</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>74.8</td>
<td>80.2</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>76.0</td>
<td>100.7</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>71.4</td>
<td>122.4</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>61.4</td>
<td>140.1</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>45.4</td>
<td>145.8</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>22.1</td>
<td>133.5</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>...*</td>
<td>99.8*</td>
<td></td>
</tr>
</tbody>
</table>

*Unable to start tunnel
Verification of Run Time Equation

On March 16, 1977, two runs were made using FTU's four inch supersonic wind tunnel in an attempt to verify assumptions made and verify equations used in this research report. Run time data at a test section Mach number ($M_T$) of 3.0 and 2.0 was collected and is tabulated in Table 5 below.

<table>
<thead>
<tr>
<th>Test Section Mach Number, $M_T$</th>
<th>Start of Run</th>
<th>End of Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>2.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>69.0</td>
<td>20.0</td>
<td>62.0</td>
</tr>
<tr>
<td>72.5</td>
<td>34.0</td>
<td></td>
</tr>
</tbody>
</table>

The actual run time measured for a test section Mach number of 3.0 was 26 seconds and for a test section Mach number of 2.0 was 28.5 seconds. The data was collected by noting the final conditions at the moment of breakdown of supersonic flow in the test section as noted by visual observation of the flow with the Schlieren system.
Using the data collected by the above method, and the relationship of Equation (2), the estimated run time can now be computed and compared with the actual values measured. Substituting measured values at a test section Mach number of 3.0, the estimated run time is

\[
\ tp = \frac{(0.0706)(329)(4.235)(144)(250.7)}{(2.2)(16) \sqrt{532.5} (83.7)} \left[ 1 - \left( \frac{144.7}{250.7} \right)^{0.917} \right]
\]

\[tp = 26.7 \text{ seconds}\]

Substituting measured values at a test section Mach number of 2.0 into Equation (2), the estimated run time is

\[
\ tp = \frac{(0.0706)(329)(1.688)(144)(260.7)}{(2.2)(16) \sqrt{531} (34.7)} \left[ 1 - \left( \frac{114.7}{260.7} \right)^{0.917} \right]
\]

\[tp = 27.7 \text{ seconds}\]

The above calculations show that at a test section Mach number of 3.0, the estimated run time is within 2.7 percent of the actual measured run time and at a test section Mach number of 2.0, the estimated run time is within 2.9 percent of the actual measured run time. Thus, Equation (2) is a valid relationship for predicting blowdown tunnel run times.
APPENDIX E

ANALYSIS OF PROPOSED DIFFUSER

Locating Shock Wave Reflection Points

Figure 9 shows the geometry relationships for locating the impingement points of the oblique shock waves with respect to diffuser entrance contraction point A.

For a diffuser inlet Mach number $M_D = 4.0$ and a deflection angle $\delta = 7^\circ$, the solution is outlined below. From NACA Report 1135 [8], Chart 2, for $M_D = 4.0$ and $\delta = 7^\circ$, the incident shock wave angle $\theta_i$ is
equal to 19.7°. From Chart 4, for $M_D = 4.0$ and $\delta = 7^\circ$, $M_b$ is equal to 3.47. From the same chart at $M_b = 3.47$ and $\delta = 7^\circ$, $M_c$ is equal to 3.04. From Chart 2, for $M_b = 3.47$ and $\delta = 7^\circ$, the shock wave angle $\theta_b$ is 22°, here, the angle between the flow direction in region b and the reflected wave. From geometrical consideration, the reflected shock wave angle $\theta_{br}$ is given by

$$\theta_{br} = \theta_b - \delta = 22^\circ - 7^\circ = 15^\circ$$

(23)

From geometrical considerations shown in Figure 9,

$$\tan \theta_i = \frac{H/2}{X_1} = \frac{2.0}{X_1}$$

(24)

or

$$X_1 = \frac{2.0}{\tan \theta_i} = \frac{2.0}{\tan 19.7^\circ} = 5.586 \text{ inches}$$

Also from geometry relationships

$$\tan \delta = \frac{Y_1}{X_1 + X_2} = \frac{Y_1}{5.586 + X_2} = 0.1228$$

(25)

and

$$\tan \theta_{br} = \frac{H/2 - Y_1}{X_2} = \frac{2.0 - Y_1}{X_2} = 0.2679$$

(26)

Solving Equations (25) and (26) simultaneously,
Thus point C is located 8.949 inches downstream from the diffuser entrance contraction point A. In a similar manner, the reflection impingement points E and G can be located.

From Chart 2 for \( M_g = 1.77 \) and \( \delta = 7^\circ \), the shock wave angle \( \theta_g \) is equal to 41.6\(^\circ\). From Chart 4 for \( M_g = 1.77 \) and \( \delta = 7^\circ \), \( M_h \) is equal to 1.51. From the same chart for \( M_h = 1.51 \) and \( \delta = 7^\circ \), \( \theta_h \) is equal to 50.6\(^\circ\). From geometrical consideration, the reflected shock wave angle \( \theta_h \) is

\[
\theta_h = \theta_h - \delta = 50.6^\circ - 7^\circ = 43.6^\circ
\]

(27)

From geometrical consideration

\[
\tan \theta_g = \frac{H/2 - Y_1 - Y_2 - Y_3}{X_7} = \frac{0.325}{X_7} = 0.8878
\]

(28)

or

\[
X_7 = \frac{0.325}{0.8878} = 0.366 \text{ inches}
\]

Thus point H is located 0.366 inches downstream from impingement point G or 14.007 inches downstream from diffuser entrance point A. It is now assumed that the reflected shock wave HI interacts with the...
expansion wave from the corner of the constant second throat area as shown in Figure 8. From geometry considerations, then

\[ \tan \theta_{hr} = \frac{ht/2}{x_g} \]  

or

\[ x_g = \frac{ht/2}{\tan \theta_{hr}} = \frac{0.485/2.0}{\tan 43.6^\circ} = 0.254 \text{ inches} \]  

Thus, point I is located 0.254 inches downstream from impingement point H or 14.261 inches downstream from diffuser entrance point A.

In the immediate neighborhood of the oblique shock wave HI and the Prandtl-Meyer expansion wave, there are large variations in pressure and flow direction and slip lines are formed. However, for this report is now assumed that the reflected shock wave HI is exactly cancelled by the expansion wave and the result is uniform one-dimensional flow downstream of the shock wave HI at a Mach number of 1.26.

**Choking Effect Due to Friction**

If the value of 4fL/D between the diffuser second throat area is known, then for each initial Mach number at the diffuser second throat entrance, the final Mach number in the constant second throat area may be easily found from Table B.4 of Shapiro [5]. With an initial Mach number of 1.26 and an assumed friction coefficient (f) of 0.0025 the value of 4fL/D may be determined. First the hydraulic diameter (D) is
defined as four times the flow area divided by the wetted perimeter or

\[ D = \frac{4A}{P} = \frac{(4)(4)(0.4851)}{2(4) + 2(0.485)} = 0.8653 \]  

Thus,

\[ \frac{4fL}{D} = \frac{(4)(0.0025)(12)}{0.8653} = 0.13868 \]

The maximum value of \( 4fL/D \) from Table B.4 of Shapiro at an entrance Mach number of 1.26 is

\[ \left( \frac{4fL}{D} \right)_{\text{max}} = 0.05183 \]  

Since the value of \( 4fL/D \) is over its maximum value, a normal shock will stand in the constant second throat area. It is assumed that the shock is located near the second throat inlet, thus for a normal shock at 1.26 the pressure loss is

\[ \frac{p_t}{p_g} = \frac{p_{t2}}{p_{t1}} = 0.9857 \]  

and the Mach number downstream of the normal shock is 0.8071.

**Stagnation Losses in Diffuser**

Table II of NACA Report 1135 [8], representing the normal shock relations, may be adapted to oblique-shock calculations. An oblique shock with an approach Mach number \( M_1 \) and incident angle \( \alpha \) may be re-
duced to a normal shock with an approach Mach number $M_x = M_1 \sin \theta$.

Then Table II can be used to find other parameters for an oblique shock with an approach Mach number $M_x$ and a shock angle $\theta$. From the geometry considerations, it follows further that Mach number downstream of the shock is given by

$$M_y = \frac{M_2}{\sin (\theta - \delta)}$$

The results of calculations for the diffuser model of Figure 8 are tabulated in Table 6.

**TABLE 6**

**OBLIQUE SHOCK WAVE PRESSURE RATIO**

<table>
<thead>
<tr>
<th>Approach Mach Number, $M_T$</th>
<th>Oblique Shock Wave Angle, $\theta$ (degrees)</th>
<th>Oblique Mach Number, $M_x$</th>
<th>Stagnation Pressure Ratio, $P_{t1}/P_{t2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00</td>
<td>19.7</td>
<td>1.35</td>
<td>0.9697</td>
</tr>
<tr>
<td>3.47</td>
<td>22.0</td>
<td>1.30</td>
<td>0.9794</td>
</tr>
<tr>
<td>3.04</td>
<td>24.6</td>
<td>1.26</td>
<td>0.9857</td>
</tr>
<tr>
<td>2.67</td>
<td>27.8</td>
<td>1.24</td>
<td>0.9984</td>
</tr>
<tr>
<td>2.30</td>
<td>31.4</td>
<td>1.20</td>
<td>0.9928</td>
</tr>
<tr>
<td>2.04</td>
<td>35.6</td>
<td>1.19</td>
<td>0.9937</td>
</tr>
<tr>
<td>1.77</td>
<td>41.6</td>
<td>1.18</td>
<td>0.9946</td>
</tr>
<tr>
<td>1.51</td>
<td>50.6</td>
<td>1.16</td>
<td>0.9961</td>
</tr>
</tbody>
</table>
The overall pressure ratio is given by the following relationship:

\[
\frac{P_e}{P_o} = \frac{P_b}{P_a} \frac{P_c}{P_b} \frac{P_d}{P_c} \frac{P_e}{P_d} \frac{P_f}{P_e} \frac{P_g}{P_f} \frac{P_h}{P_g} \frac{P_i}{P_h} \frac{P_j}{P_i} \frac{P_k}{P_j} \frac{P_l}{P_k} \frac{P_m}{P_l} \frac{P_n}{P_m} = \frac{P_g}{P_a}
\] (34)

or

\[
\frac{P_e}{P_o} = \frac{P_g}{P_a} = 0.8979
\]

Thus the stagnation pressure upstream of the diffuser inlet is given by

\[
P_o = \frac{P_e}{0.8979} = \frac{14.7}{0.8979} = 16.4 \text{ psia}
\] (35)
The bibliography is divided into two groups; the first group consists of those references referred to specifically in the research report and the second group consists of references pertinent to the investigation which were used by the author.

GROUP 1. REFERENCES CITED


GROUP 2. SELECTED BIBLIOGRAPHY


