High Pressure Hydraulic Supply System Model

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HIGH PRESSURE HYDRAULIC
SUPPLY SYSTEM MODEL

By

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B.S., University of Kentucky, 1967

RESEARCH REPORT

Submitted in partial fulfillment of the
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ABSTRACT

A mathematical model is derived to provide quasi-steady state predictions of the performance of a high pressure hydraulic supply system, using equations which govern the physical processes as opposed to equations which match input-output characteristics. Model equations are developed to describe the operation of the power source, control valves, energy source, gas side of the system, hydraulic accumulator, the motorpump, and hydraulic side of the system.

The accuracy of the model is then checked by inserting known parameters from a previously developed control system and comparing model predictions with performance data from this system.
ACKNOWLEDGEMENTS

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List of Symbols

- a: Gas generator constant coefficient, lbsm/s/psi
- A_g: Area of gas side of accumulator piston, sq in.
- A_o: Area of oil side of accumulator piston, sq in.
- A_p_g: Area of piston in warm gas motor, sq in.
- A_r: Ratio of area of gas side of accumulator piston to area of oil side
- D_p: Pump volumetric displacement, cu. in/rad
- I: Inertia of rotating mass in pump, in/lb/s^2
- K_fl: Flow limiter flow constant, cu. in/s/psid
- K_m: Volumetric displacement of motor, cu. in/rad
- K_prv: Relief valve flow constant, cu. in/s/psid
- K_rv: Regulator valve flow constant, lbsm/s/psid
- M_{ap}: Accumulator piston mass, lbsm
- n: Gas generator burn rate exponent
- P_dp: Pump differential pressure, psid
- P_g: Gas generator pressure, psi
- P_{g}': Rate of change of P_g, psi/s
- P_{rv}: Regulator valve cracking pressure, psi
- P_{rvd}: Relief valve differential pressure, psi
- P_s: System differential pressure, psi
- Q_{a}: Rate of oil flow from accumulator, cu. in/s
- Q_{p}: Pump flow, cu. in/s
- R: Gas constant = (1545 x 12)/molecular weight of gas, in lbsf/lbsm - °R
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$T_M$</td>
<td>Motor torque due to gas pressure, in/lbsf</td>
</tr>
<tr>
<td>$V_{AI}$</td>
<td>Instantaneous volume of oil in accumulator, cu. in.</td>
</tr>
<tr>
<td>$V_{FA}$</td>
<td>Volume of oil in accumulator when full, cu. in.</td>
</tr>
<tr>
<td>$V_g$</td>
<td>Gas system volume, cu. in.</td>
</tr>
<tr>
<td>$V_{sg}$</td>
<td>Volumetric burn rate of gas generator, cu. in/s</td>
</tr>
<tr>
<td>$V_{gi}$</td>
<td>Initial gas system volume, cu. in.</td>
</tr>
<tr>
<td>$W_g$</td>
<td>Mass in gas side of system, lbsm</td>
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<tr>
<td>$W_{gg}$</td>
<td>Gas generator mass flowrate, lbsm/s</td>
</tr>
<tr>
<td>$W_M$</td>
<td>Motor gas mass flowrate, lbsm/s</td>
</tr>
<tr>
<td>$W_{RV}$</td>
<td>Regulator valve flowrate, lbsm/s</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle of inclination of bent-axis motorpump, degrees</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>Angular velocity of motorpump, rad/s</td>
</tr>
<tr>
<td>$\rho_M$</td>
<td>Density of gas in motor side of motorpump, lbsm/cu. in.</td>
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I. INTRODUCTION

High pressure (1000-4000 psi) hydraulic control systems have been used extensively throughout industry to provide accurate positional control. In this report, a general mathematical model will be generated for a hydraulic supply system which contains typical component parts (i.e., actuator(s), pressure supply, control valves, hydraulic accumulator, hydraulic power source). The actuator in this model is used only as a means of generating a hydraulic demand on the system, and is not modeled as a dynamic second-order device. Developing a dynamic simulation of an actuator could be the subject of a research paper in itself.

The model developed herein will be based on equations that govern the physical processes as opposed to equations which match input-output characteristics. The model should be general enough to be readily adaptable to study and/or analysis of similar control systems by modifying the general equations to incorporate the characteristics of the particular system being studied.

As with any model of a physical process, simplifying assumptions will be made where appropriate. The number of assumptions made depends on the degree of complexity and detail desired in the model. Since the intent of this report is to present a relatively simple model which is easy to use and understand, the following assumptions, which should have little effect on the model of the
overall system, will be made:

1 **Constant Temperature** - Although it is recognized that the temperature of a hydraulic control system may vary by several degrees during operation, the temperatures considered in this report (<200°F, 93.3°C) are not severe enough to significantly affect the structural properties of the materials used - typically stainless steel and aluminum. However, the thermal expansion of the hydraulic fluid will be allowed for in sizing reservoirs for quiescent storage in order to preclude the occurrence of damaging pressures which could result from temperature cycling during non-operating periods. For extremely precise models, it might be desirable to include thermal effects during system operation.

2 **Rigid Walls** - It is assumed that the volumetric expansion which can occur in tubing, accumulators, etc. upon application of pressure is insignificant in its effect on the system model.

3 **Constant Bulk Modulus** - Although the bulk modulus in reality varies with temperature, pressure, and the amount of entrained air, it is assumed that for the conditions considered herein, the effect is negligible.
II. MODEL DESCRIPTION

The system considered in this model operates in the following manner:

The hydraulic servoactuators are powered by a closed, recirculating hydraulic system with the energy being supplied by a gas-driven motor pump. An accumulator provides the extra hydraulic power capacity to meet the transient flow requirements of the system. Gas power to drive the system is derived from the products of combustion of a solid propellant gas generator. (Although this is a typical aerospace system, the model should be readily adaptable to include equations describing the power source for the particular system to be modeled.) Gas pressure is maintained by pressure relief valves. Transient pressure peaks in the hydraulic system which are due to the accumulator reaching its full position and the pump speed changing are limited by the hydraulic relief valve.

Fluid is supplied to the pump from a reservoir, which also contains a spring to maintain positive pressure during quiescent storage. Even though this will not enter into the model equations, it was mentioned as being typical for systems similar in nature to the system which will be used to check the model developed herein.
III. DERIVATION OF MODEL EQUATIONS

A. POWER SOURCE

As mentioned earlier, the power source included in this model is a solid-propellant warm gas generator. In order to use the model with a different power source, a new set of equations describing the power source characteristics would have to be developed at this point. The equation which describes the output (i.e., mass flowrate) of the gas generator as a function of gas pressure is derived in the following manner:

The rate of propellant consumption ($\dot{W}_{gg}$) is related to the linear burning rate ($r_b$) for the propellant. It can be assumed that the linear burning rate is given by Baumeister [1]

$$r_b = c P_g^n$$  \hspace{1cm} (1)

where

$c$ and $n$ are experimentally determined constants.

If $\rho_p$ represents the specific weight of the solid propellant (lbs/cu. ft.) and $A_p$ the area of the burning surface (assumed constant), then the propellant consumption rate can be expressed as follows:

$$\dot{W}_{gg} = a P_g^n$$  \hspace{1cm} (2)

where

$a = \rho_p A_p C.$
The values of $a$ and $n$ depend on the composition of propellant as well as the propellant geometry involved.

B. PRESSURE REGULATOR VALVE

The pressure regulator valve is normally designed such that no flow occurs until the system pressure meets or exceeds some predetermined level, hereinafter called the cracking pressure ($P_{RV}$). Thus the operation of the valve is illustrated in Figure 1, and can be described as follows:

$$\dot{W}_{RV} = K_{DRV} (P_g - P_{RV})$$

$$K_{DRV} = K_{RV} \quad P_g > P_{RV}$$

$$K_{DRV} = 0 \quad P_g < P_{RV}$$

$K_{RV}$ is a constant which is dependent on the design of the particular valve used and must be determined for each system to which this model is applied.

C. ENERGY SOURCE

Flow of gas through the motor is proportional to the specific weight of the gas and the volumetric flowrate. The volumetric flowrate is equal to the motor angular velocity (rad/s) multiplied times the motor pump geometric constant (cu. in./rad), which can vary depending on the particular unit being used. Thus

$$\dot{W}_M = \frac{P_g}{RT} K_M \dot{\theta} = \rho_M K_M \dot{\theta} \left( \dot{\theta} = \frac{Q_p}{D_p} \right)$$

The geometric constant $K_M$ is determined based on the number of pistons in the pump, the piston stroke per gas intake cycle,
piston area, and pump revolution during intake cycle. This constant is usually available from the supplier of the unit used.

D. GAS PRESSURE FUNCTION

The conservation of mass equation for the gas side of the system can then be expressed as follows:

\[ \dot{W}_{gg} = \dot{W}_{RV} + \dot{W}_M + \frac{d}{dt} W_g \]  

(5)

\( \dot{W}_{gg}, \dot{W}_{RV}, \) and \( \dot{W}_M \) were developed in equations 2, 3, and 4 respectively. The last term in equation (5) represents the rate of change of mass contained within the gas side of the system. This can be expressed as follows:

\[ \frac{d}{dt} W_g = \frac{d}{dt} \frac{P_g V_g}{RT} \]  

(6)

Having assumed the system operates with essentially a constant temperature, this can be expressed as

\[ \frac{V_g}{RT} \frac{d}{dt} p_g + \frac{P_g}{RT} \frac{d}{dt} V_g = \frac{\dot{P}_g V_g}{RT} + \frac{P_g \dot{V}_g}{RT} \]  

(7)

For this model, the variation in the gas system volume (\( \dot{V}_g \)) is created by an increasing gas generator volume due to propellant consumption, and a transient variation introduced as hydraulic accumulator piston motion occurs due to transient hydraulic demands which exceed the instantaneous capability of the motor-pump. Gas system volume can then be expressed as

\[ V_g = V_{gi} + \dot{V}_{gg} t + A_R (V_{AF} - V_{AI}) \]  

(8)
Since $V_i$, $V_{gg}$, $A_R$, and $V_{AF}$ are constants, equation (8) can be differentiated to yield

$$\dot{V}_g = \dot{V}_{gg} + A_R \frac{d}{dt} (-V_{AI})$$

(9)

But $d/dt (-V_{AI})$ is simply the rate of flow of oil out of the hydraulic accumulator. Equation (9) then becomes

$$\dot{V}_g = \dot{V}_{gg} + A_R Q_A$$

(10)

Combining equations 2, 3, 4, 7, and 10 and solving for $\dot{P}_g$ results in:

$$\dot{P}_g = \frac{RT}{V_g} \left[ a P^n - K_{DRV} (P_g - P_{RV}) \right] - \frac{P_g}{V_g} \left[ \frac{K_{OP}}{D_p} + \dot{V}_g \right]$$

(11)

This expresses the time variation in system pressure as a function of the performance of the power source, the pressure regulator valve(s), and the energy source.

E. ACCUMULATOR FLOW

An expression will now be developed for the flow from the hydraulic accumulator. It will be assumed that relatively low flow velocities are produced in the accumulator. This permits the assumption that the fluid in the accumulator is at hydraulic system pressure. The net force on the accumulator piston, neglecting any frictional effects (which under dynamic conditions should be minimal in a well-designed system), becomes

$$P_g A_p - P_s A_o = \text{net force}$$

(12)
The flow from the accumulator can be expressed as

\[ \frac{dx}{dt} A_0 \]

where \( \frac{dx}{dt} \) is the accumulator piston velocity.

Thus the rate of change of accumulator flow can be expressed as

\[ Q_A = A_0 \frac{d^2 x}{dt^2} \]

\[ Q_A = A_0 \frac{F}{M_{AP}} = A_0 \frac{P_A - P_{A0}}{M_{AP}} \quad (13) \]

This applies only when the instantaneous accumulator volume \( V_{AI} \) lies between 0 (accumulator empty) and \( V_{FA} \).

The instantaneous volume is determined by

\[ V_{AI} = V_{FA} + \int_0^+ Q_A \, dt \quad (14) \]

\[ Q_A = 0 \quad 0 \geq V_{AI} \text{ or } V_{AI} \geq V_{FA}. \]

F. EQUATION DESCRIBING MOTOR PUMP TORQUE

Due to the transient response characteristics of the motor-pump, an equation must be developed to express the net torque acting on the rotating mechanism. Applying the conservation of energy principle to the hydraulic side of the motorpump yields

\[ \left[ \frac{P_2}{\rho_2} - \frac{P_1}{\rho_1} \right] + \frac{V_2^2 - V_1^2}{2} + (U_2 - U_1) \right] \frac{W}{g} = \dot{H} + T_P \theta \quad (15) \]
where $P_i$, $\rho_i$, $V_i$, and $U_i$; are the pressure, density, velocity, and intrinsic internal energy at point $i$ ($i = 1$ at entrance and $i = 2$ at exit), $W$ is the weight flow through the pump, $\dot{H}$ is the rate of heat flow to the control volume, and $T_p$ is the torque applied to the pump shaft. Under the assumption that the pumping process is a uniform, constant temperature, adiabatic process (i.e., $U_2 = U_1$) and noting that $V_1 = 0$, $V_2^2 < \frac{P_2}{\rho_2}$, $\rho_1 \sim \rho_2$ (incompressible fluid), and $P_2 - P_1 = P_{DP}$, equation (15) reduces to

$$\frac{P_{DP}}{\rho} \frac{W}{g} = T_p$$  \hspace{1cm} (16)

However, mass flow through pump divided by density is equal to pump volumetric displacement per radian multiplied by pump rad/s, i.e.,

$$\frac{W}{g \rho} = D_p \theta$$  \hspace{1cm} (17)

Therefore equation (16) reduces to

$$T_p = \frac{P_{DP}}{D_p}$$  \hspace{1cm} (18)

which is a standard equation used in determining torque output of hydraulic pumps.

Deriving a comparable expression for the gas side of the pump (i.e., the motor) is somewhat more difficult. Since the flowing medium in the motor is a warm gas, the constant temperature and density assumptions made in the pump analysis are not valid for the motor.
Therefore, in order to develop an expression for the motor torque, the motorpump cross-section and timing diagram in Figures 2 and 3, respectively will be used. From the motor timing diagram it can be seen that the motor pistons are exposed to gas system pressure for approximately 93 percent of a power stroke (considering the power stroke as being inlet open to exhaust open). It is therefore assumed that the useful work extracted from the warm gas is done at nominal gas system pressure \( P_g \).

Therefore, the force acting on each piston is just

\[ F = A_{pg} P_g \]  \hspace{1cm} (19)

The component of this force normal to the shaft, which produces the useful torque, can be derived by considering the geometry depicted in the cross section of Figure 2. The motorpump considered in this model is a bent-axis unit. The angle \( \alpha \) depicted in this figure is the angle between the pump axis and the motor axis. This normal force can be expressed as

\[ F_N = P_g A_{pg} \sin \alpha \]  \hspace{1cm} (20)

where

\[ F_N \] is the normal force. The torque generated by \( F_N \) is

\[ F_N \frac{d_M}{2} \sin \beta \]  \hspace{1cm} (21)

where

\[ d_M \] is the working diameter of the motor (see cross-sectional view) and \( \beta \) is the angle of rotation from top dead center, as defined in Figure 3.
The motorpump of this model contains nine (9) pistons on both the gas and oil sides. Therefore, since the pistons are 40 degrees apart (360°/9 pistons), there are typically three pistons being subjected to system pressure, with one possibly being in the expansion region. It will be assumed that since the expansion region is such a small portion of a total power stroke and the average pressure there is probably approximately one-half total system pressure, the torque generated in this region can be neglected. Thus, the total torque acting on the motor shaft due to the hot gas is expressed as

\[ T_M = \frac{P_g A_{PE} d_M}{g} \sin \alpha \left[ \sin \beta + \sin (\beta + 40^\circ) + \sin (\beta + 80^\circ) \right] \]  

(22)

From the timing diagram, it can be seen that \( \beta \) varies from 8 degrees to approximately 125 degrees during a power stroke.

For pumps of the type included in this mode, the theoretical torque typically oscillates at a relatively high frequency. The particular pump model considered in checking this simulation displayed 150 Hz oscillation at approximately 1000 rpm. Therefore, it is assumed that the torque is constant at its mean value, i.e.,

\[ T_M = \frac{P_g A_{PE} d_M}{2g} \sin \alpha \int_{8}^{45} \left[ \sin \beta + \sin (\beta + 40^\circ) + \sin (\beta + 80^\circ) \right] d\beta \]  

(23)

where

\( K \) is a constant which converts the distance between adjacent pistons to radians \( (K = 57.3/40 = 1.433 \text{ rad}^{-1}) \). The
theoretical torque can therefore be expressed as

\[ T_M = P_g D_g \]  \hspace{1cm} (24)

where

\[ D_g = 1.15 A_{pg} d_M \sin \alpha \]  \hspace{1cm} (25)

(For the unit considered in the model checkout, this value becomes 0.123 cubic inches.)

Using equations (18) and (23), which express the theoretical torque available from the pump and motor respectively, the net theoretical torque available is

\[ T_{NT} = D_g P_g - D_p P_{DP} \]  \hspace{1cm} (26)

However, to determine the torque actually available to drive the motorpump, the torque losses encountered (i.e., frictional losses) in operation must be subtracted from the total net theoretical torque available.

In order to derive an expression for the sum of all the losses encountered, the steady-state operating characteristics of a motorpump typical of the unit considered in this model are used. An example of the operating curve is shown in Figure 4. Letting \( T_L \) be the sum of all the losses, an expression for steady-state operation would be

\[ T_{NT} - T_L = D_g P_g - D_p P_{DP} - T_L = 0 \]  \hspace{1cm} (27)

or

\[ T_L = D_g P_g - D_p P_{DP}. \]  \hspace{1cm} (28)
Given a constant supply pressure, the functional relationship between pump differential pressure ($P_{DP}$) and demand flow from the pump is as shown in Figure 4. Thus a functional relationship between $T_L$ and pump flow can be developed, using this figure and equation (27). The typical shape of this relationship is shown in Figure 5. Since torque is equal to the product of inertia of the rotating mass and the angular acceleration as stated by Sears and Zemansky [2], and angular acceleration is the rate of change of pump flow divided by the pump displacement per radian, the relationship depicted in the referenced figure can be developed as follows:

**Steady State**

$$D_g P_g - D_p P_{DP} - T_L = 0$$

**Transient**

$$D_g P_g - D_p P_{DP} - T_L (Q_p) = I \frac{d^2 \phi}{dt^2}$$  \hspace{1cm} (29)

where

$T_L (Q_p)$ means $T_L$ is a function of $Q_p$ and $I$ is inertia of rotating mass in pump.

But

$$\frac{d^2 \phi}{dt^2} = \frac{dQ_p}{dt}/D_p$$

therefore

$$\frac{dQ_p}{dt} = \frac{D_p}{I} [D_g P_g - D_p P_{DP} - T_L (Q_p)].$$  \hspace{1cm} (30)

The pressure drop across the pump ($P_{DP}$) is equal to system differential pressure ($P_S$) plus the pressure drop across the flow limiter ($P_{FL}$). $P_{FL}$ can be expressed as
\[ P_{FL} = K'_{FL} (Q_p - Q_{PL}) \]  \hspace{1cm} (31)

where

\[ Q_{PL} = \text{pump flow limit} \]

\[ K'_{FL} = K_{FL} \quad Q_p > Q_{PL} \]

\[ K'_{FL} = 0 \quad Q_p < Q_{PL} \]

\[ K_{FL} = \text{flow limiter constant} \sim \text{cu. in/s/psi.} \]

The pump flow expression [equation (30)] can then be written as follows:

\[ \frac{dQ_p}{dt} = \frac{D_p}{I} [D_g P_g - D_p (P_S + P_{FL}) - T_L (Q_p)] \]

\[ = \frac{D_p}{I} [D_g P_g - D_p P_S - D_p K'_{FL} (Q_p - Q_{PL}) - T_L (Q_p)] \]  \hspace{1cm} (32)

G. SYSTEM PRESSURE FUNCTION

All that is lacking in the model equations is an expression of system pressure in terms of known constants and variables. This will be developed using a property of hydraulic fluids known as bulk modulus, which expresses the unit change in volume produced by a unit change in pressure and is defined by Harrison and Bollinger [3] as

\[ \beta = \frac{\Delta P}{\Delta v/v} \]  \hspace{1cm} (33)

where

\[ \beta = \text{bulk modulus} \]

\[ P = \text{pressure} \]

\[ v = \text{volume}. \]
Equation (33) can also be written [1]

\[ \beta = \rho \frac{\Delta P}{\Delta \rho} \]

or

\[ \frac{1}{\rho} \Delta \rho = \frac{1}{\beta} \Delta P. \]

(34)

Introducing a \( \Delta t \) term on each side of equation (34) and taking the limit as \( \Delta t \) approaches zero yields

\[ \frac{1}{\rho} \frac{dp}{dt} = \frac{1}{\beta} \frac{dP}{dt} \]

(35)

By definition, \( \rho = M/V \) where \( M \) is mass of fluid contained in volume \( V \). Therefore,

\[ \frac{d\rho}{dt} = \left( V \frac{dM}{dt} - M \frac{dV}{dt} \right) / V^2 \]

(36)

However, \( M = \rho V \) and \( dM/dt = \rho Q \), therefore

\[ \frac{d\rho}{dt} = \frac{1}{V^2} \left[ V (\rho Q) - \rho V \frac{dV}{dt} \right] \]

\[ \frac{d\rho}{dt} = \frac{\rho}{V} \left( Q - \frac{dV}{dt} \right) \]

(37)

where

\( Q \) is the volumetric flowrate.

For this model, \( dV/dt \) is just the opposite of the accumulator flow (i.e., positive accumulator flow decreases overall hydraulic system volume as accumulator piston displaces hydraulic fluid).

Using equations (35) and (37), the time variation in system pressure can be expressed as follows:

\[ \frac{dP}{dt} = \frac{\beta}{V} \left( Q - \frac{dV}{dt} \right) \]

(38)
where Q is comprised of pump flow \((Q_p)\), pressure relief valve flow 
\((Q_{RV})\), and demand flow \((Q_D)\). For this model, \(Q_D\) is the flow
required to drive the control actuators. Thus equation (38) can
be rewritten as

\[
\frac{dP}{dt} = \frac{\dot{Q}}{V} (Q_p + Q_A - Q_{RV} - Q_D)
\]  
(39)

where \(Q_{RV} = K'_{PRV} (P_S - P_{RV})\)

\[
K'_{PRV} = K_{PRV} \quad P_S \geq P_{RV}
\]

\[
K'_{PRV} = 0 \quad P_S < P_{RV}
\]

\(K_{PRV}\) is a relief valve flow constant which must be determined for
each system studied, where applicable) and

\[V = V_I + V_A\]

where

\[V_I = \text{hydraulic system volume with full accumulator}\]

\[V_A = \text{accumulator volume.}\]

This now completes the series of equations required to generate a
computerized model of a hydraulic control system. The equations
are summarized below, and a summary of the applicable constants
and their values as used in the checkout program are also pres-
ented. Prior to using this model for any system, the rationale
behind the equations derivations should be checked to verify
consistency with the system being considered.

H. SUMMARY OF EQUATIONS

(For notation, refer to equations in body of report by
number in parentheses.)
Power Source

\[ \dot{W}_{gg} = a P^n_g \]  \hspace{1cm} (2)

Pressure Regulator Valve

\[ \dot{W}_{RV} = K_{DRV} (P_g - P_{RV}) \]  \hspace{1cm} (3)

\[ K_{DRV} = K_{RV} \quad P_g \geq P_{RV} \]

\[ K_{DRV} = 0 \quad P_g < P_{RV} \]

Energy Source

\[ \dot{W}_M = \rho_M K_M \dot{\theta} \]  \hspace{1cm} (4)

\[ \dot{\theta} = \frac{Q_P}{D_P} \]

Gas Pressure

\[ \dot{P}_g = \frac{RT}{V_g} \left[ a \ P^n_g - \ K_{DRV} (P_g - P_{RV}) \right] - \frac{P_g}{V_g} \left[ \frac{K_M Q_P}{D_P} + V_g \right] \]  \hspace{1cm} (11)

\[ \dot{V}_g = \frac{V_g}{p_g} + A_R \ Q_A \]

Accumulator Flow

\[ A_Q = \frac{A_O}{M_{AP}} (P_g \ A_g - P_s \ A_0) \]  \hspace{1cm} (13)

\[ Q_A = 0 \quad 0 \geq V_{AI} \ or \ V_{AI} \geq V_{FA} \]

\[ V_{AI} = V_{FA} + \int_0^{+} Q_A \ dt \]  \hspace{1cm} (14)

Motorpump Flow

\[ \frac{dQ_P}{dt} = \frac{D_P}{I} \left[ D_g P_g - D_P P_s - D_P K'_{FL} (Q_P - Q_{PL}) - T_L (Q_P) \right] \]  \hspace{1cm} (32)
System Pressure Function

\[ \frac{dP}{dt} = \frac{6}{V} (Q_P + Q_A - Q_{RV} - Q_D) \]  (39)

[Note: As can be seen in the glossary of terms included in the model listing in Appendix I, various terms are included in the model which are not discussed in the body of the report. These are terms which were used to facilitate programming the specific model and are not considered germane to the basic descriptive equations for a general model. This was done in an attempt to present a simplified approach which could be used for various applications, and to avoid prejudicing the model toward a particular system.]
IV. - MODEL CHECKOUT/CONCLUSIONS

In order to verify that the mathematical model developed in this report provides a viable means of making at least a preliminary assessment of hydraulic supply system performance, a checkout run was made using input parameters from an existing system. Typical printout data from the checkout runs are shown in the appendix.

The input data were obtained from nominal design values from the components in the selected system. Since a tolerance exists on each of these nominal values, the data obtained are not presented as exact solutions for the various output functions. It is not considered likely that a single system would be manufactured with each component meeting the nominal specified design value. Rather, these data are presented to indicate the trend the output data display.

For each condition checked, the program was structured to iterate for 500 milliseconds, which was sufficient time for the system studied to achieve steady-state operating condition. In each case the last three iterations were printed to verify that a steady state solution had been achieved, which was indicated by the identical values being printed in each output position.

The output included as Appendix II is presented as being typical data for the system which was picked for model verification.
The nominal operating pressures varied from 1900-2100 psi on the gas side of the system and from 2800 to approximately 3700 psi on the oil side in actual system operation; model predictions compare favorably with these values. Also included are data which indicate that hydraulic system pressure decayed to zero. This indicates the capabilities of the modeled system have been exceeded (i.e., hydraulic demand was too high for system to maintain pressure). These data were generated as a self-check by inserting input values which were known to exceed the system design limits, thereby verifying model capability to predict anomalies in system operation.

Based on the data generated when design parameters of an existing hydraulic control system were input into the model and the model was exercised using various demand levels, it is concluded that the basic equations used to describe the performance of the various system components are correct and can be used as the core around which a specific model can be developed. As was done in the checkout runs, additional equations which are not germane to the basic model may be developed for particular components and combined with the basic equations to form a complete system model.

A schematic of the system used in checking the model developed herein is included as Figure 6.
V. MODEL LIMITATIONS

As was indicated earlier, this particular model was developed for a hydraulic system which might typically be encountered in an aerospace/missile system application. In order to be used for other (i.e., commercial) applications, the derivations of the model equations must be reviewed in light of the system being considered. As an example, the model does not consider hydraulic line length, since this is not typically a driving design constraint for the relatively short tubing lengths used in missile applications. However, in commercial/industrial control systems, hydraulic line length can become a significant factor in system design. Relatively long operating times can result in elevated hydraulic fluid temperatures in industrial systems, whereas in the short durations of typical missile systems, temperature effects are miniscule, and were neglected in the model equations.

Therefore, although it is felt the model is a viable tool for predicting control system performance, the basis for the model equations must be considered prior to applying the model.
Figure 1. Typical Operating Characteristic of Pressure Regulator Valve
Figure 2. Cross-Section of Warm Gas Motorpump
Figure 3. Hot Gas Motor Timing Diagram
Figure 4. Steady State Operating Characteristics of a Typical Pump
Figure 5. Typical Friction Torque versus Pump Flow Characteristic
Figure 6. Schematic of System Used to Check Model
APPENDIX I

Listing of Model Program
THE FOLLOWING IS A GLOSSARY OF TERMS USED IN THIS PROGRAM.

A
AD = AREA OF GAS SIDE OF ACCUMULATOR PISTON " SQ. IN.
ARM = DISPLACEMENT PER RADIUS OF MOTOR " CU. IN. / RAD
ARK = REGULATOR VALVE FLOW CONSTANT " LBS/SEC/PSI
AO = AREA OF OIL SIDE OF ACCUMULATOR PISTON " SQ. IN.
APA = ACTUATOR PISTON AREA " SQ. IN.
APM = ACCUMULATOR PISTON MASS " LBS/IN/SEC/SEC
AR = AREA RATIO (GAS/OIL) OF ACCUMULATOR PISTON

B
BMOD = BULK MODULUS OF OIL IN SYSTEM " PSI
BBR = BURN RATE OF GAS GENERATOR

D
DP = DISPLACEMENT OF PUMP " CU. IN./RAD.
DPBIT = VARIATION IN SYSTEM PRESSURE WITH TIME(DP/DT) " PSI/SEC
DPR = PRESSURE ABOVE REGULATOR VALVE CRACKING PRESSURE " PSI
DPRES = VARIATION IN RESERVOIR PRESSURE WITH TIME(DP/DT) " PSI/SEC
DPRV = PRESSURE ABOVE RELIEF VALVE CRACKING PRESSURE " PSI
DSPCT = CHANGE IN SYSTEM PRESSURE WITH TIME " PSI/SEC
DQAD = RATE OF CHANGE OF ACCUMULATOR FLOW " CU. IN./SEC/SEC
DQFL = FLOW ExCEEDING FLOW LIMITER LIMIT " CU. IN./SEC
DQPD = RATE OF CHANGE OF PUMP FLOW " CU. IN./SEC/SEC
DQTIME = TIME DURATION OF MOTION " SEC
DVDM = RATE OF CHANGE OF GAS VOLUME " CU. IN./SEC
DVNT = RATE OF CHANGE OF RESERVOIR VOLUME " CU. IN./SEC
DXYD = RESERVOIR PISTON VELOCITY " IN./SEC

E
ELM = EQUIVALENT LINEAR MOTION " IN.

F
F = NET FORCE ON ACCUMULATOR PISTON " LBS
FDS = TOTAL DAMPING AND FRICTION FORCE " LBS
FD = ACCUMULATOR PISTON DAMPING
FF = TOTAL FORWARD FORCE ON ACCUMULATOR PISTON " LBS
FG = FORCE ON ACCUMULATOR PISTON DUE TO GAS PRESSURE " LBS
FLC = FLOW LIMITER FLOW CONSTANT " PSI/CU. IN./SEC
FLDF = FLOW LIMITER DIFFERENTIAL PRESSURE " PSI
FRES = NET FORCE ON RESERVOIR PISTON " LBS
FRG = TOTAL FORWARD FORCE ON RESERVOIR PISTON " LBS
FRF = COULOMB FRICTION FORCE ON ACCUMULATOR PISTON
FRG = FORCE ON ACCUMULATOR PISTON DUE TO OIL " LBS
FSP = RESERVOIR PISTON FORCE DUE TO SPRING RATE " LBS

P
PACC = PUMP ACCELERATION " RAD/SEC/SEC
PCC = PRESSURE DROP ACROSS CHECK VALVE " PSI
PFP = PUMP DIFFERENTIAL PRESSURE " PSI
PFE = PRESSURE DROP ACROSS FILTER ELEMENT " PSI
PG = SYSTEM GAS PRESSURE " PSI
PJJ = INHERENT INERTIA OF PUMP " IN-LBS-SEC
PMR = RESERVOIR PISTON MASS " LBS
PREG = REGULATOR VALVE CRACKING PRESSURE " PSI
PRE = PRESSURE IN RESERVOIR " PSI
PS = PUMP SPEED " RPM
PFR = PUMP FLOW " CU. IN./SEC
PSM = PUMP SPEED IN RPM/IN
PSD = HYDRAULIC RELIEF CRACKING PRESSURE " PSI
PSS = SYSTEM DIFFERENTIAL PRESSURE " PSI
G  QA = ACCUMULATOR FLOW "CU.IN./SEC"
G  QAL = PREVIOUS VALUE OF QA
G  QD  = DEMAND FLOW "CU.IN.
G  QFL = FLOW LIMITER LIMIT "CU.IN./SEC"
G  QNET= NET SYSTEM FLOW "CU.IN./SEC"
G  QP  = PUMP FLOW "CU.IN./SEC"
G  QPMP = PUMP FLOW IN GPH
G  QRLV= RELIEF VALVE FLOW "CU.IN./SEC"
G  QSVL = SERVOVALVE LEAKAGE FLOW "CU.IN./SEC"
G  QSUM= SUMMATION OF RESERVOIR FLOWS "CU.IN./SEC"
G  QRES = RESERVOIR RAMPING (ASSUMED=0.0 IN MODEL)
G  RGA= GAS CONSTANT (1545/MOL.HT.)
G  RPA= RESERVOIR PISTON AREA "SQ.IN.
G  RFRA= RESERVOIR PISTON AREA ON RETURN SIDE "SQ.IN.
G  RSG= RESERVOIR SPRING SPRING RATE "LBS/IN"
G  RTG= GAS CONSTANT X TEMPERATURE OF GAS
G  RUCF= RELIEF VALVE FLOW CONSTANT "CU.IN./SEC/PSI
G  RSF= SPRING FORCE ON PISTON "LBS"
G  RTM= TORQUE CONSTANT OF MOTOR "CU.IN.
G  RTF= TORQUE DUE TO FLOW LIMITER "IN-LBS"
G  RTFR= CONSTANT FRICITION TORQUE IN SYSTEM "IN-LBS"
G  RTO= TORQUE DUE TO GAS SUPPLY "IN-LBS"
G  RTGO= TORQUE DUE TO GAS TEMPERATURE "DEGREES R"
G  RTN= NET TORQUE "IN-LBS"
G  RTN= TOTAL NUMBER OF ACTUATORS
G  RTP= TOTAL BACK TORQUE ON PUMP "IN-LBS"
G  RTS= BACK TORQUE DUE TO SYSTEM OIL PRESSURE "IN-LBS"
G  RVAC= ACCUMULATOR OIL VOLUME "CU.IN.
G  RVOL= GAS VOLUME IN SYSTEM "CU.IN.
G  RVOLL = SYSTEM OIL VOLUME WITH ACCUMULATOR EMPTY "CU.IN.
G  RVRS= RESERVOIR VOLUME "CU.IN.
G  RIV= INITIAL RESERVOIR VOLUME "CU.IN.
G  RVYS= TOTAL SYSTEM OIL VOLUME "CU.IN.
G  RDOT= RTGX(GAS GENERATOR FLOW - PRESSURE REGULATOR FLOW)
G  WRAT= A CHECK CALCULATION ON REQUIRED GAS FLOW
G  WRG= REQUIRED GAS GENERATOR FLOW RATE "LISH / SEC"
G  WPG= PRESSURE REGULATOR FLOW "LISH / SEC"
G  X = RESERVOIR PISTON LINEAR MOTION "IN.
G  XMAX= MAXIMUM X
G  XMIN= MINIMUM X
G  ZD= ACCUMULATOR PISTON RAMPING CONSTANT
G  THIS CONCLUDES THE LIST OF VARIABLES. THE PROGRAM FOLLOWS.
01060  10  READ(5,*)RT,ELM,TNA,VACC,UG,PG,FS
01070  READ(5,*)VOIL,VRES,USYS,ZD,PR,PRU
01080  READ(5,*)AG,AKM,AKRV,AD,AFA,BMDL,BR,DP,RXDT
01090  READ(5,*)TLC,FRT,FCU,MT,F,J,MR,PRES,DA,AF
01100  READ(5,*)W1,QL,RESD,KGAS,AFPA,AFPA,DRK,ACF
01110  READ(5,*)TCM,TGG,WFG,X,XMAX,XMIN,APR,DURFT
01115  WRITE(6,60)
01120  A=AD/AD
01121  J=0
01122  DD=50  I=1,250
01123  I=I+1
01124  QSL1 = QSL + 0.15
01125  I = 0
01126  DD 50  I = 1,250
01127  I = I + 1
01130  G0 = TNA*AFPA*ELM/DT+QSL
01140  RTG = KGAS*TG0
01150  DVDT = QAO*APR
01160  VN = DVDT#INT4VG
01170  DPR = PG-PR
01180  WPR = AKRV*APR
01190  IF(DPR.LT.0.0) WPR = 0.0
01200  WDOT1 = RTG#WFG-WFR
01210  WDOT2 = DVDT#PFG((AKRV*WPR)/DP)
01220  DPDT = (WDOT1-WDOT2)/VG
01230  FG = DPDT#INT4FG
01240  FG = AD#FG
01250  FG = AQ#FG
01260  FS = AD#FS
01270  FF = FO#FS
01280  FFB#FO#FRT
01290  F = FF#FEG
01300  DGADT = AD#FG/AD
01310  QA = DGADT*RTGA
01320  IF(ABS(FRS),LT,ABS(FF)) GO TO 20
01330  IF(AG#GAL,GT,0.0) GO TO 20
01340  G0 = 0.0
01350  G0 = 0.0
01360  20 CONTINUE
01370  QA = DA
01380  IF(QA.EQ.0.0) FRT = 0.0
01390  IF(QADT.EQ.0.0) FRT = 0.0
01400  21 CONTINUE
01410  VSYS = (-1.0*QA#G0) VSYS
01420  IF(VSYS GE,VOIL,AND,USYS LE,VOIL + VACC) GO TO 25
01440  IF(VSYS LE,VOIL) VSYS = VOIL
01450  IF(VSYS GE,VOIL + VACC) VSYS = VOIL + VACC
01460  QA = 0.0
01460  20 CONTINUE
01470  QA = 0.0
01480  25 CONTINUE
01490  DFS = FS-PRU
01500  ORV = ORV*DFS
01510  IF(DFS,LT,0.0) GRV = 0.0
01520  A = DFS#DFS-GRV
01530  A = DFS+DFS-GRV
01540  GR = DFS+DFS-GRV
01550  25 CONTINUE
01540 DPSD=ONET*MOIL/VSYS
01550 PS=DPSD*0.1*PS
01560 IF(FS.LE.0.)FS=0.
01570 DQFL=OF-DFL
01580 FLDP=FLC*DFL
01590 IF(DQFL.LE.0.)FLDP=0.
01600 FDP=FS*FLDP+PCV*FFE
01610 TP=FRP+DF
01620 TG=FS+DF
01630 TTL=FLD*UF
01640 TG=FG*ICM
01650 OMPM=OF/3.66
01660 TN=TF-TFR
01670 PACC=TN/FJ
01680 DQDF=DF*PACC
01690 OF=OQDF+ODF
01700 IF(OF.GT.0.)GO TO 26
01710 OF=0.0
01720 TN=0.0
01730 26 CONTINUE
01740 DPRES=RHOIL*DVRDT/VRES
01750 PRES=DPRES*DTR+RES
01760 FROI=PFB*RPA*RES*RPA-PRES*RPA
01770 SFDR=X*FGR
01780 FXD=DXDT*RES
01790 RES=FROI-SFDR-FXD
01800 DXDT=DXDT*RES*QT/THR
01810 X=DXDT+TX
01820 IF(X.GT.XMIN)GO TO 30
01830 X=XMIN
01840 DXDT=0.
01850 30 CONTINUE
01860 IF(X.LT.XMAX)GO TO 35
01870 X=XMAX
01880 DXDT=0.
01890 35 CONTINUE
01900 VRES=VRESI-X*RPA
01910 DVRDT=DXDT*RPA
01920 IF(VRES.GT.VRESI)VRES=VRESI
01930 FRFS=OF/DF
01940 FRPH=FRFS*0.5493
01950 DO 90 I=240,250
01960 WRITE(6,70)OA,DF,FRPH,PG,PS
01970 FORMAT(F10.3,F14.3,F10.3,F9.1,F13.1)
01980 50 CONTINUE
01990 WRITE(6,60)
02000 FORMAT(6SN ACCUM. FLOW PUMP FLOW PUMP RPA GAS PRESS. SYS. PR
02010 ICSS.
02020 80 CONTINUE
02030 GO TO 10
02040 END
APPENDIX II

Typical Printout of Sample Run
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APPENDIX III

Summary of Values of Constants Used in
Checkout Runs
Summary of Values of Constants Used in Checkout

The following list is a repeat of pages vi and vii, except that the values assigned the various constants in making a checkout run on the model developed in this report are delineated. These values were derived using the known operating characteristics of an existing hydraulic control system. Refer to the aforementioned pages for definitions of the symbols.

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<td>$A_g$</td>
<td>16.75 sq in.</td>
</tr>
<tr>
<td>$A_0$</td>
<td>11.76 sq in.</td>
</tr>
<tr>
<td>$A_{pg}$</td>
<td>*</td>
</tr>
<tr>
<td>$A_R$</td>
<td>Calculated in model</td>
</tr>
<tr>
<td>$D_p$</td>
<td>0.066 cu. in/rad</td>
</tr>
<tr>
<td>$I$</td>
<td>0.0076 in. lb/s^2</td>
</tr>
<tr>
<td>$K_{FL}$</td>
<td>819.7 psi/cu. in/s</td>
</tr>
<tr>
<td>$K_M$</td>
<td>0.119 cu. in/rad</td>
</tr>
<tr>
<td>$K_{PRV}$</td>
<td>0.20 cu. in/s/psi</td>
</tr>
<tr>
<td>$K_{RV}$</td>
<td>0.002 lbsm/s/psi</td>
</tr>
<tr>
<td>$M_{AP}$</td>
<td>4.55 lbsm</td>
</tr>
<tr>
<td>$n$</td>
<td>*</td>
</tr>
<tr>
<td>$P_{DP}$</td>
<td>Calculated in model</td>
</tr>
<tr>
<td>$P_g$</td>
<td>2000 psi (initialized)</td>
</tr>
<tr>
<td>$P_{pg}$</td>
<td>Calculated in model</td>
</tr>
<tr>
<td>Symbol</td>
<td>Value/Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------</td>
</tr>
<tr>
<td>$P_{RV}$</td>
<td>3450 psi</td>
</tr>
<tr>
<td>$P_{RVD}$</td>
<td>Calculated in model</td>
</tr>
<tr>
<td>$P_S$</td>
<td>0.0 psi (initialized)</td>
</tr>
<tr>
<td>$Q_A$</td>
<td>0.0 (initialized)</td>
</tr>
<tr>
<td>$Q_P$</td>
<td>0.0 (initialized)</td>
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<tr>
<td>$R$</td>
<td>996.0 in lbsf/lbsm-°R</td>
</tr>
<tr>
<td>$T_M$</td>
<td>Calculated in model</td>
</tr>
<tr>
<td>$V_{AI}$</td>
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<tr>
<td>$V_{FA}$</td>
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<tr>
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<tr>
<td>$\dot{V}_g$</td>
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<tr>
<td>$V_gi$</td>
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<tr>
<td>$W_g$</td>
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<td>$\dot{W}_g$</td>
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</tr>
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<td>$\dot{W}_M$</td>
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</tr>
<tr>
<td>$\dot{W}_{RV}$</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>*</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>*</td>
</tr>
<tr>
<td>$\rho_M$</td>
<td>*</td>
</tr>
</tbody>
</table>

*Symbols marked with an asterisk were not used in the actual model, but were used in the body of the report in deriving equations necessary to the model.*
List of References


Bibliography

