Two-Phase Flow Pressure Drop Across Thick Restrictions of Annular Geometries

Summer 1982

Saeed Ghandeharioun

University of Central Florida
TWO-PHASE FLOW PRESSURE DROP ACROSS THICK RESTRICTIONS OF ANNULAR GEOMETRIES

BY

SAEED GHANDEHARIOUN
B.S.M.E., University of Miami, 1979

RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the Master of Science in Engineering in the Graduate Studies Program of the College of Engineering University of Central Florida Orlando, Florida

Summer Term
1982
ABSTRACT

This paper presents the methods of predicting the steady-state two-phase flow (steam and water) pressure drop across the restrictions of annular geometries formed when tubes extend through circular holes in tube support plates.

Two approaches are discussed and a detailed sample calculation of the one selected is presented. The major areas of discussion are the orientation of tubes-to-tube support plate holes, and the thickness of tube support plate.

Finally, the conclusion gives a comparison of the methods and recommendations for future investigations.
ACKNOWLEDGEMENTS

The author wishes to express special thanks to Dr. E. R. Hosler, the academic and research report advisor, for his guidance and support throughout the course of this study.

Thanks are also extended to the other members of my committee, Dr. F. S. Gunnerson, for his helpful advice and donation of his time and his books, and Dr. R. G. Denning, for serving on my committee.

Finally, my sincere appreciation and special thanks is given to Miss Dian Brandstetter for her complete cooperation and expert accomplishment of typing this report.
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS ........................................ iii
LIST OF TABLES ........................................... v
LIST OF FIGURES .......................................... vi
NOMENCLATURE .............................................. vii
SUBSCRIPTS ................................................ ix

Chapter
1. PRESSURE DROP OF TWO-PHASE FLOW ACROSS ANNULAR ORIFICES ...................... 4

2. PRESSURE DROP OF TWO-PHASE FLOW ACROSS SHORT AND LONG LENGTH RESTRICTIONS .... 18

3. CONCLUSIONS AND RECOMMENDATIONS ..................................... 23

APPENDIX 1 ............................................. 26
APPENDIX 2 ............................................. 41
APPENDIX 3 ............................................. 45
REFERENCES CITED ...................................... 48
BIBLIOGRAPHY ............................................ 49
LIST OF TABLES

1. Results of calculations for saturated single-phase pressure drop for various flow conditions .......................... 34

2. Values of $\ddot{R}$ as a function of $\ddot{Z}$ .................. 46
LIST OF FIGURES

1. Annular gap between tube and tube support plate .................................. 2
2. Kinetic term for viscous flow in annular orifice ........................................... 13
3. Friction factor for annuli of fine clearance and for parallel plates .............. 14
4. Annular orifice coefficient versus Reynolds number for Sharp-Edge orifice .... 15
5. Summary of concentric-orifice coefficients .................................................. 16
6. Summary of tangent-orifice coefficients ..................................................... 17
7. Short and long length restrictions ............................................................... 19
8. Configuration of two-phase flow channel and its tube support plate for the numerical example ................................................................. 27
9. Variation of quality with height in a uniformly heated channel .................. 29
10. Pressure drop versus mixture quality across the support plate for both concentric and tangent orifices ......................................................... 35
11. Pressure drop versus mixture quality across the support plate for various slip ratios (concentric orifices) ..................................................... 36
12. Pressure drop versus mixture quality across the support plate for various slip ratios (tangent orifices) ..................................................... 37
13. Pressure drop versus mixture quality across the support plate for various flow rates (concentric orifices) ............................................. 38
14. Pressure drop versus mixture quality across the support plate for various flow rates (tangent orifices) ............................................. 39
15. Void fraction versus mixture quality for the annular gap between the tube and the support plate at the system pressure of 1000 psia ................. 47
**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>cross-sectional area, ft²</td>
</tr>
<tr>
<td>α</td>
<td>void fraction</td>
</tr>
<tr>
<td>C</td>
<td>overall annular orifice coefficient</td>
</tr>
<tr>
<td>C&lt;sub&gt;c&lt;/sub&gt;</td>
<td>coefficient of stream contraction in an orifice</td>
</tr>
<tr>
<td>&lt;C&gt;</td>
<td>area vena contracta/area of restriction</td>
</tr>
<tr>
<td>D</td>
<td>outside diameter of annular orifice, ft</td>
</tr>
<tr>
<td>d</td>
<td>inside diameter of annular orifice or outside diameter, ft</td>
</tr>
<tr>
<td>Fr</td>
<td>Froude number, ( \frac{U^2}{gD_h} )</td>
</tr>
<tr>
<td>f&lt;sub&gt;P&lt;/sub&gt;</td>
<td>friction factor for flow between parallel plates of infinite, Fig. 3</td>
</tr>
<tr>
<td>G</td>
<td>mass flux, lbm/hr-ft²</td>
</tr>
<tr>
<td>H</td>
<td>height, ft</td>
</tr>
<tr>
<td>h</td>
<td>fluid enthalpy, BTU/lbm</td>
</tr>
<tr>
<td>K&lt;sup&gt;′&lt;/sup&gt;</td>
<td>single-phase loss coefficient</td>
</tr>
<tr>
<td>K</td>
<td>kinetic term for viscous flow in annular orifice, Fig. 2</td>
</tr>
<tr>
<td>L</td>
<td>restriction (orifice) length or thickness of tube support plate, ft</td>
</tr>
<tr>
<td>m</td>
<td>mass flow rate, lbm/s</td>
</tr>
<tr>
<td>ΔP</td>
<td>total pressure drop across restriction or orifice, psi</td>
</tr>
<tr>
<td>Q</td>
<td>volume flow rate, ft³/s</td>
</tr>
</tbody>
</table>
q_s  sensible heat added per pound mass of incoming coolant, BTU/lbm
q_T  total heat added in channel per pound of mass of incoming coolant, BTU/lbm
R    outer radius of annular orifice, ft
r    inner radius of annular orifice or outer radius of tube, ft
Re   Reynolds number, \( \frac{D_hG}{u} \)
S    slip ratio
U    volumetric flux (superficial velocity), ft/s
V    fluid specific volume, ft^3/lbm
X    flow quality
Z    annulus length-to-width ratio
u    fluid absolute viscosity, lbm/hr-ft
\( \rho \) fluid density, lbm/ft^3
\( \phi^2 \) two-phase multiplier
\( \sigma \) restriction(s) flow area/channel flow area
D_h  hydraulic diameter (for annular gap; \( D_h = D - d \)), ft
SUBSCRIPTS

B  boiling region
e  exit of channel
f  saturated liquid
g  saturated vapor
H  homogeneous flow
i  inlet of channel
N  non-boiling region
o  orifice
S  separated flow
T  total
TP two-phase
TSP tube support plate
INTRODUCTION

In shell and tube type heat exchangers, support plates are spaced periodically along the tube bundles to maintain the proper geometric arrangement among the tubes. When the shell side flow is parallel to the tube axis, the flow must pass through the annular shaped clearance between the outside diameter of the tubes and the hole in the support plate (Figure 1), increasing the shell-side pressure drop. Since the support plates contribute a major portion of the pressure drop, predicting shell-side pressure drop for two-phase flow is an important design consideration.

A review of literature indicates that there has been previous work to

(a) predict single-phase pressure drop in complicated geometries (such as support plates) and
(b) predict two-phase pressure drop in simple geometries (such as tubes and channels).

However, there has been little or no work to predict two-phase pressure drop in complicated geometries.

This work is to provide one or more rational approaches of predicting two-phase pressure drop by
Fig. 1. Annular gap between tube and tube support plate

A. CONCENTRIC  

B. TANGENT
tying together the previous work in (a) and (b) to be able to predict shell-side pressure drop with two-phase flow.
CHAPTER 1

PRESSURE DROP OF TWO-PHASE FLOW ACROSS ANNULAR ORIFICES

In general, orifices are used to measure flow rates. However, many flow restrictions, such as clearance between a tube and a tube support plate may be analyzed by treating them as orifices. It is the intent here to evaluate the pressure drop for two-phase flow across annular orifices.

A review of literature indicates that in two-phase systems, it has been observed experimentally that for a given mass flux, the pressure drop can be much greater than for a corresponding single-phase system (1). The classical approach, which has been taken to predict two-phase pressure losses, is to multiply the equivalent saturated single-phase pressure loss by a multiplier, $\phi^2$, which is a function of (at least) flow quality and system pressure. Thus

$$ \Delta P_{TP} = \Delta P_f \cdot \phi^2 (X, p, \ldots) $$

or

$$ \Delta P_{TP} = \frac{k}{2g_c \rho_f} \cdot \phi^2 (X, p, \ldots) \quad (1-1) $$
where $\hat{\mathcal{K}}$ is the single-phase loss coefficient, which will be discussed in more detail for various geometries of annular gaps between a tube and a tube support plate later in this chapter.

Since the pressure drop in two-phase flow is closely related to the flow pattern, two principle types of flow models will appear in the analysis of two-phase pressure drop in this chapter. They are homogeneous flow model which regards the two-phase to flow as a single-phase possessing mean fluid properties, and the separated flow model which considers the phases to be artificially segregated into two streams; one of liquid and one of vapor.

Before presenting the commonly accepted expressions for multiplier, $\phi^2$, in homogeneous flow model and separated flow model, it is necessary to make the following assumptions:

1. One-dimensional flow
2. Steady-state flow
3. Adiabatic flow across the tube support plate, so that the quality, $X$, is constant.
4. Pressure drop across support plate is small compared to the total pressure, so that the densities, $\rho_f$ and $\rho_g$, do not change.
5. Flow properties are in terms of cross-sectional averages taken across the annular gap flow cross section.
6. The parameters affecting voids; i.e.; quality, pressure, and mass velocity are nearly the same upstream and downstream of the tube support plate, so that void fraction, $\alpha$, is nearly constant, and is given by (4)

$$\alpha = \frac{1}{1 + \left(\frac{1 - \bar{X}}{\bar{X}}\right)(\frac{V_f}{V_g})S}$$

(1-2)

where $S$ is the slip ratio, and it is defined as the ratio of the average velocity of the vapor phase to that of the liquid. For homogeneous flow the slip ratio is equal to 1.0, and for nonhomogeneous flow is greater than 1.0 due to the fact that the vapor, because its buoyancy, has a tendency to slip past the liquid.

Therefore, in order to find the commonly accepted expressions for $\phi^2$ based on the foregoing assumptions for homogeneous and separated flow models, it is suggested (1) that the expected behavior of $\phi^2$ is to be examined. For this purpose, for two-phase flow it can be written

$$\Delta P_{TP} = k \frac{G^2}{2G_{c} \rho_{TP}}$$

(1-3)

in which $\rho_{TP}$ is the appropriate two-phase density. For the case of saturated homogeneous two-phase flow ($\rho_{TP} = \rho_{H}$), equation (1-3) can be written

$$(\Delta P_{TP})_H = k \frac{G^2}{2G_{c} \rho_{f}} \frac{\rho_f}{\rho_H}$$

(1-4)

where $\rho_{H}$ is homogeneous density as
Thus, by comparing equations (1-5), (1-4), and (1-1), it is found that

\[ \phi_H^2 = (1 + \frac{V_f \xi}{V_f} x) \] (1-6)

or

\[ (\Delta P_{TP})_H = \frac{3}{2} \frac{G^2}{\rho_f} \left(1 + \frac{V_f \xi}{V_f} x\right) \] (1-7)

which is the expression for prediction of pressure drop in two-phase homogeneous flow.

The appropriate two-phase density for separated flow is not as well defined (1). It has been suggested (Chisholm, 1973) that the momentum density should be used. Thus, equation (1-3) becomes

\[ (\Delta P_{TP})_S = \frac{3}{2} \frac{G^2}{\rho_f} \left(\frac{\rho_f}{\rho}\right) \] (1-8)

where \( \rho \) is the momentum density, and is given by

\[ \rho = \frac{1}{\rho_f(1-\alpha)} + \frac{\rho_g \alpha}{\rho_f(1-\alpha)} \] (1-9)

Thus, by comparing equations (1-9), (1-8), and (1-1), it is found that

\[ \phi_S^2 = \frac{(1 - X)^2}{1 - \alpha} + \frac{V_f \xi x^2}{V_f} \alpha \] (1-10)

or

\[ (\Delta P_{TP})_S = \frac{3}{2} \frac{G^2}{\rho_f} \left(\frac{(1 - X)^2}{1 - \alpha} + \frac{V_f \xi x^2}{V_f} \alpha\right) \] (1-11)
which is the expression for prediction of pressure drop in two-phase separated flow.

For the clearance between a tube and a tube support plate which may be regarded as an annular orifice, the single-phase loss coefficient can be defined as

\[
\frac{K}{\rho} = \frac{1}{C^2}
\]  

(1-12)

where \( C \) is called the overall annular orifice coefficient.

The experimental study by Bell and Bergelin (2) on the single-phase flow (water and oil) through various single annular orifices indicates that the annular orifice coefficient, \( C \), is a function of the annular orifice dimensions, the annular orifice Reynolds number, and the orientation of annular orifice; i.e., concentric or tangent position of tube to tube support plate hole (Figure 1). The equations defining orifice length-to-width ratio, and orifice Reynolds number are given by

\[
Z = \frac{2L}{D - d}
\]  

(1-13)

\[
Re_o = \frac{(D - d)G_o}{\mu}
\]  

(1-14)

where \( G_o \) is mass flux through an annular orifice.

It should be noted that for the two-phase flow through an annular orifice, the orifice coefficient may be estimated at the Reynolds number of the entire flow rate considered in the state of saturated liquid \((Re_o = Re_{of})\). Consequently, the results of Bell and
Bergelin's analysis (2) for prediction of the overall annular orifice coefficient may be used.

Bell and Bergelin have developed suitable equations expressing the overall annular orifice coefficient for various geometries of thick annular orifices with single-phase flow, with the designation of the flow ranges as viscous, turbulent, or transition, based on the following Reynolds-number ranges:

1. The viscous-flow range refers to Reynolds numbers less than 40, where the predominant effect is energy loss by the viscous shear in the fluid, and the kinetic effects are confined to the fluid as it flows into the orifice.

2. The turbulent-flow range refers to Reynolds numbers above 4000, where the predominant effects are the kinetic-energy losses associated with stream acceleration, contraction, limited expansion, and turbulent friction. It should be noted that the stream expands from the vena contracta to the full area of the annulus with a partial recovery of the kinetic energy as pressure. The expansion from the vena contracta begins a finite distance downstream from the orifice entrance, and no pressure recovery will be obtained in an orifice whose thickness is less than this distance (Figure 7).
3. The transition-flow range refers to Reynolds numbers between 40 and 4000, where both kinetic and viscous phenomena are important.

The equations of the overall annular thick orifice coefficients are given by

(A) Viscous Range, Concentric Orifice.

\[
\frac{1}{C^2} = \frac{64}{Re} + \frac{48}{Re} \frac{Z}{k} + \frac{K}{Re} \tag{1-15}
\]

where \( K \) is taken from Figure 2.

(B) Viscous Range, Tangent Orifice.

\[
\frac{1}{C^2} = \frac{128}{Re} + \frac{96}{5} \frac{Z}{Re} + \frac{K}{Re} \tag{1-16}
\]

where \( K \) is taken from Figure 2.

(C) Transition Range, Concentric Orifice.

\[
\frac{1}{C^2} = \frac{1}{C_c^2} - \left[ 2 \sqrt{\left( \frac{1}{C_c^2} - \frac{64}{Re} \right)} - 2 \right] F + 2f_p Z \tag{1-17}
\]

where \( F = 0 \), for \( Z > 1.15 \), and

\[
F = 1 - e^{-0.95(Z-1.15)} \text{, for } Z > 1.15, \text{ where } C_c \text{ and } f_p \text{ are taken from Figures 4 and 3 at the appropriate Reynolds number.}
\]

(D) Turbulent Range, Concentric Orifice.

\[
\frac{1}{C^2} = \frac{1}{C_c^2} - \left[ \frac{2}{C_c^2} - 2 \right] F + 2f_p \cdot Z \tag{1-18}
\]

where \( F = 0 \), for \( Z < 1.15 \), and

\[
F = 1 - e^{-0.95(Z-1.15)} \text{, for } Z > 1.15.
\]

\( C_c \) and \( f_p \) are taken from Figs. 4 and 3, respectively.
(E) Turbulent Range, Tangent Orifice.

\[
C = \frac{2}{A_0} \left\{ L^2 \left\langle \left[ \frac{3}{2} \left( \frac{1}{Z} - \frac{1}{C_c^2} - \left( \frac{2}{C_c} - 2 \right) \frac{f_p}{Z} \right) \right] \right. \\
\left. + \left( \frac{2f_p}{Z \left[ \frac{1}{C_c^2} - \left( \frac{2}{C_c} - 2 \right) \right]^{3/2}} \frac{1}{C_c^2} - \left( \frac{2}{C_c} - 2 \right) \frac{f_p}{Z} \right) \right\rangle \right. \\
\left. \left[ \frac{\pi}{2} + \sin^{-1}\left( \frac{1}{C_c^2} - \left( \frac{2}{C_c} - 2 \right) - \frac{Zf_p}{Z} \right) \right] \right. \\
\left. \left. + \left( \frac{1}{C_c^2} - \left( \frac{2}{C_c} - 2 \right) \frac{f_p}{Z} \right) \right\rangle \right. \\
\left. \left[ \frac{3}{2} \left( \frac{1}{Z} - \frac{1}{C_c^2} - \left( \frac{2}{C_c} - 2 \right) \right) \right] \right. \\
\left. \left. + \left[ \frac{1}{C_c^2} - \left( \frac{2}{C_c} - 2 \right) \frac{f_p}{Z} \right] \right\rangle \right. \\
\left. \frac{\pi}{2} + \sin^{-1}\left( \frac{1}{C_c^2} - \left( \frac{2}{C_c} - 2 \right) - \frac{Zf_p}{Z} \right) \right. \\
\left. + \frac{1}{C_c^2} - \left( \frac{2}{C_c} - 2 \right) \frac{f_p}{Z} \right\} \\
\right\} \tag{1-19}
\]

for \( Z > 9 \), and \( Re > 10,000 \). \( C_c \) and \( f_p \) are taken from Figures 4 and 3, respectively.
The experimental analysis of Bell and Bergelin for the flow through annular openings are presented for wide range of flow rates and orifice dimensions in the Figures 5 and 6. From these curves, it was concluded that for high Reynolds number \((\text{Re}_o > 5000)\) in thick orifices with a fixed mass flow rate, the pressure recovery begins at a Z-value of about 1.0 up to a Z-value of 6.0. For longer orifice channels the wall friction causes an increase in the pressure drop until the channel length give Z-numbers in the range of 10 to 100 where the overall annular orifice coefficients drop to about 0.65, the value for shapred-edge orifice (Figure 4). At higher Z-values the friction resistance lowers the value of overall annular orifice coefficient still more.

A numerical example is presented in Appendix 1, which has been solved based on the foregoing discussion of predicting the two-phase flow pressure drop across the annular orifice.
Fig. 2. Kinetic term for viscous flow in annular orifice

Fig. 3. Friction factor for annuli of fine clearance and for parallel plates.

Reynolds number, $Re = \frac{(D-d)G}{\mu}$

**Fig. 4.** Annular orifice coefficient versus Reynolds' number for Sharp-Edge Orifice

Fig. 5. Summary of concentric-orifice coefficients

Fig. 6. Summary of tangent-orifice coefficients

CHAPTER 2

PRESSURE DROP OF TWO-PHASE FLOW ACROSS SHORT AND LONG LENGTH RESTRICTIONS

This chapter primarily deals with the Janssen's prediction of steady state two-phase flow pressure drop across short, and long length restrictions, based on a one-dimensional momentum balance.

A review of literature indicates that Janssen (3) obtained two equations (2-1 and 2-2) for prediction of pressure drop across restrictions of circular and rectangular geometries depend on whether the vena contracta occurs inside or outside of the restriction (Fig. 7). Thus, for short length restriction where the vena contracta is outside of the restriction, the equation of pressure drop is given by

$$\Delta P_{TP}^{\text{short}}(\text{restriction}) = \frac{G^2}{2g_0 \rho_f} \left( \frac{1}{\sigma^2} \frac{1}{\alpha^2_3} \right) \left[ \frac{V_k}{V_f} \right] \left[ X^2 \alpha_3 \right]

\left\{ \frac{1}{\alpha_3} - \frac{\sigma^2 \alpha_5}{\alpha^2_5} \right\} + (1 - X)^2 (1 - \alpha_3) \left\{ \frac{1}{(1 - \alpha_3)^2} \right\}

- \frac{\sigma^2 \alpha_5^5}{(1 - \alpha_5)^2} \right\} - 2\sigma \left\{ \frac{V_k}{V_f} \right\}

X^2 \left( \frac{1}{\alpha_3} - \frac{\sigma \alpha_5}{\alpha^5_5} \right) + (1 - X)^2

\left( \frac{1}{1 - \alpha_3} - \frac{\sigma \alpha_5}{1 - \alpha_5} \right) \right]\]

(2-1)
Fig. 7. Short and long length restrictions

where

\[ G_1 = \text{mass flux in channel} \]

\[ \hat{\alpha}_3 = \frac{\alpha_3 + \alpha_5}{2} \]

and the \( \alpha \)'s are void fractions at different locations as shown in Figure 7 with the assumption of slip flow at all locations.

In long length restrictions where the vena contracta occurs within the restriction, the equation of pressure drop is given by

\[
\Delta P_{TP} \left( \text{long} \right) = \frac{G_1^2}{2g_c^0 \sigma_f} \left[ 1 - \frac{V}{V_f} X^2 \right] \left[ \frac{\sigma^2}{\sigma_1^2} \left\{ \frac{V}{V_f} X^2 \hat{\alpha}_1 \right. \right.
\]

\[
\left. \left( \frac{1}{\sigma^2 \alpha^2_3} - \frac{1}{\sigma^2 \alpha^2_4} \right) + (1 - X)^2 (1 - \hat{\alpha}_1) \right. \}
\]

\[
\left. \frac{1}{\sigma^2 (1 - \alpha_3)^2} - \frac{1}{(1 - \alpha_4)^2} \right\} - \frac{2}{\sigma^2} \left\{ \frac{V}{V_f} X^2 \right. \left. \frac{\sigma}{\sigma_4} + \frac{\sigma}{\sigma_5} \right. \}
\]

\[
\left. \left( \frac{1}{\sigma \alpha_3} - \frac{1}{\sigma \alpha_4} + \frac{\sigma}{\sigma_4} - \frac{\sigma^2}{\sigma^2_4} \right) + (1 - X)^2 \right. \}
\]

\[
\frac{1}{\sigma (1 - \alpha_3)^2} - \frac{1}{1 - \alpha_4} + \frac{\sigma}{1 - \alpha_4} - \frac{\sigma^2}{1 - \alpha_5} \}
\]

\[
\left\{ \frac{\sigma^2 (1 - \alpha_4)^2}{\sigma^2 (1 - \alpha_1)^2} - \frac{1}{(1 - \alpha_1)^2} \right\} \right. \}
\]

(2.2)

where

\[ \hat{\alpha}_1 = \frac{\alpha_3 + \alpha_4}{2} \]

\[ \hat{\alpha}_2 = \frac{\alpha_1 + \alpha_4}{2} \]
and the $\alpha$'s are void fractions at different locations as shown in Figure 7.

On the basis of his own steam-water data, Janssen suggested that the void fractions be estimated by assuming slip flow everywhere except at the vena contracta.

In view of the foregoing equations, it can be seen that the vena contracta ratio is the only major factor to be depended on the geometry of the restriction. Consequently, it would be possible to apply the Janssen's equations to any form of restrictions; i.e., the annular gap between a tube and a tube support plate, providing that the proper value of vena contracta ratio is used.

Since Janssen did not address the question of when a restriction may be considered short or long, the experimental study by Harshe, Hussain, and Weisman (3) with freon and its vapor on both single and multiple hole circular cross-sectional area restrictions suggests that equation (2-1) may be used for short length restriction, the ratio of restriction length to restriction diameter is less than or equal to 2.0, and equation (2-2) for long length restrictions when the ratio is greater than 2.0. Moreover, the study recommends that in short length restrictions (0.5 < $L/D$ < 2.0) and long length restriction (with void fraction greater than 50%), the void fraction at the vena contracta be based on partial mixing at the
vena contracta which depends on void fraction and geometry of restriction. It is also suggested that for multiple hole restrictions the ratio of restriction length to restriction diameter should be determined using diameter of a single hole.

However, it should be noted that the validity of foregoing limitations on the ratio of restriction length to restriction diameter remains to be tested for this case study where there is an annular gap, not a circular hole between the tube and the tube support plate. The restriction diameter should be taken as hydraulic diameter of the annular gap.

When there is significant vaporization across the restriction (large difference between the inlet and the outlet void fraction), it is suggested by Harshe, Hussain, and Weisman (3) that the void fraction at the vena contracta be based on the exit quality of restriction. Furthermore, the study recommends the Hugmark correlation (Appendix 2) for obtaining the relationship between void fraction and quality for slip flow condition.
CHAPTER 3

CONCLUSIONS AND RECOMMENDATIONS

The goal of this paper was to present the methods of predicting two-phase flow pressure drop across the tube support plate due to the existence of the small annular gaps between tubes and tube support plate. This was achieved by considering two approaches:

1. The first method considers annular clearances between the tubes and the tube support plate as annular orifices, assuming that the entire two-phase flow is considered in the state of saturated liquid. The approach is found to be easy to use based on the available data and equations for finding the overall annular orifice coefficients, and simple calculation of two-phase multiplier. However, the method has lack of data in the area of the annular orifice coefficient for the cases of multiple annular orifices, and various eccentric orifices other than tangent.

2. The second method deals with the Janssen's
prediction of the two-phase flow pressure drop across short and long length restrictions. Since the Janssen's approach deals with the nature of the two-phase flow at every location throughout the restrictions, the method should be more accurate. However, there are many difficulties associated with this method; e.g., accurate prediction of the void fractions at various axial locations and the appropriate values of the vena contracta ratios.

Since the Janssen's equations of two-phase pressure drops require the extensive measurements of void fractions at various axial locations, it would be difficult to apply them for design purposes of large scale heat exchangers. However, the first method is easier to use since it does not require any measurements of flow conditions throughout the shell and should provide adequate accuracy for design purposes.

Based on the findings of this study, it is recommended that future investigations are needed to

1. Examine the effect of geometrical parameters; i.e., number of tubes and pitch-to-diameter ratio of tubes, on loss coefficient.
2. Experimentally confirm Janssen's method on restrictions with annular cross-section area of flow.
APPENDIX 1

NUMERICAL EXAMPLE OF CALCULATING THE TWO-PHASE FLOW PRESSURE DROP ACROSS ANNULAR ORIFICES

A 3-ft.-high two-phase flow channel is in the shape of a cylindrical shell with the inside diameter of 2.5 inches. The channel contains seven heating tubes, which pass through a 3/4 inch thick tube support plate at the middle of the channel (Fig. 8). Assuming the channel receives heat uniformly, and operates at a pressure of 1000 psia, an exit quality of 10 percent, and inlet water temperature of 520°F with flow rate of 6 GPM. Compute the pressure drop across the tube support plate for the following two-phase flow models:

(a) homogeneous flow model
(b) separated flow model with the slip ratio of 2.

Solution:

The thermodynamic and physical properties of saturated steam and water are given by

At 520°F: \( h_i = 511.9 \text{ BTU/lbm} \)
\[ V_f = 0.0209 \text{ ft}^3/\text{lbm} \]
Fig. 8. Configuration of two-phase flow channel and its tube support plate for the numerical example
At 1000 psia: \( h_f = 542.4 \text{ BTU/lbm} \)
\( h_{fg} = 649.4 \text{ BTU/lbm} \)
\( V_f = 0.0216 \text{ ft}^3/\text{lbm} \)
\( V_g = 0.4456 \text{ ft}^3/\text{lbm} \)
\( V_{fg} = 0.4240 \text{ ft}^3/\text{lbm} \)
\( \mu_f = 0.233 \text{ lbm/hr.-ft.} \)

First, it is necessary to determine if the tube support is in the two-phase flow region of the channel. For this purpose, we calculate the non-boiling height for the uniformly heated channel (Fig. 9). Thus

\[
\frac{H_N}{H} = \frac{q_s}{q_T} = \frac{h_f - h_1}{(h_f + X_e h_{fg}) - h_1}
\]

\[
= \frac{542.4 - 511.9}{(542.4 + 0.1 \times 649.4) - 511.9}
\]

\[
= 0.32
\]

Therefore, the non-boiling height is

\( H_N = 0.32 \times H = 0.32 \times 36" = 11.52 \text{ in.} \)

and the boiling height is

\( H_B = H - H_N = 36 - 11.52 = 24.48 \text{ in.} \)

Since, \( H_{TSP} > H_N \), the two-phase flow condition exists at the tube support plate.
\[ H = \text{height of channel} \]
\[ H_{\text{TSP}} = \text{height of tube support plate} \]
\[ H_N = \text{non-boiling height} \]
\[ H_B = \text{boiling height} \]
\[ X = \text{quality at exit} \]
\[ X_{\text{TSP}} = \text{quality at tube support plate} \]

Fig. 9. Variation of quality with height in a uniformly heated channel

Assuming the existence of the tube support plate does not change the linear behavior of quality with height, the quality at the tube support plate can be obtained from

\[ X_{\text{TSP}} = \frac{H_{\text{STP}} - H_N}{H_B} X_e \]
\[ = \frac{18" - 11.52"}{24.48"} = 2.65\% \]

Under the steady state condition, the mass flow rate remains constant throughout the channel \( \dot{m}_T = \dot{m}_H = \dot{m}_B \). Therefore

\[ \rho_f = \frac{1}{V_f} = 47.847 \text{ lbm/ft}^3 \]
\[ @ 520^\circ F \]

And

\[ \dot{m}_T = 6 \frac{\text{gal}}{\text{min}} \times \frac{2.23 \times 10^{-3} \text{ ft}^3/\text{s}}{1 \frac{\text{gal}}{\text{min}}} \times 47.847 \frac{\text{lbm}}{\text{ft}^3} \]
\[ \dot{m}_o = \frac{1}{7} \dot{m}_T = \frac{1}{7} \times 0.640 = 0.0914 \text{ lbm/s} \]

Since the cross-sectional areas of the annular gaps between the tubes and the tube support plate are the same, we assume each gap carries the same mass flow rate under the steady state condition regardless of their annular geometries. Thus

\[ \dot{m}_o = \frac{1}{7} \dot{m}_T = \frac{1}{7} \times 0.640 = 0.0914 \text{ lbm/s} \]

and mass flux is

\[ G_o = \frac{\dot{m}_o}{A_o} = \frac{0.0914}{\frac{\pi}{4}(0.53^2 - 0.50^2)} = 542.33 \text{ lbm/ft}^2\text{-s} \]

Regarding the annular gaps between the tubes and the tube support plate as annular orifices, the annular orifice Reynolds number can be obtained from equation (1-14):

\[
R_{e_o} = Re_{of} = \frac{(D - d)G_o}{\mu_f} = \frac{(0.53 - 0.50)\text{ft} \times (542.33)\text{lbm/ft}^2\text{-s}}{0.233 \text{ lbm/hr-ft}} \times (3600) \text{ s/hr} = 20,948
\]

Since \( R_{e_o} > 4000 \), the flow is turbulent.

Also, \( Z = \frac{2L}{D-d} = \frac{2(3/4)^n}{0.53 - 0.50} = 50 \)

To obtain the two-phase flow pressure drop across the tube support plate, we examine the following cases:
**CASE I**

Assume the tubes are in the concentric position inside the support plate holes. Therefore, from equation (1-18)

\[
\dot{K} = \frac{1}{\rho^2} = \frac{1}{C_c^2} - \left[ \frac{2}{C_c} - 2 \right] F + 2f_p Z
\]

where

\[
Z = 50
\]
\[
F = 1 - e^{-0.95(50-1.15)} = 1
\]
\[
f_p = 0.0073 \quad \text{(from Fig. 3)}
\]
\[
C_c = 0.65 \quad \text{(from Fig. 4)}
\]

resulting \(\dot{K} = \frac{1}{C^2} = 2.020\)

Thus, the pressure drops in two-phase flow are

(a) Homogeneous flow model

(Eq. 1-7): \((\Delta P_{TP})_H = \dot{K} \frac{G^2}{2g_c^0 f} \left(1 + \frac{V_f g}{V_f} X\right)\)

where \(X = X_{TSP} = 2.65\%

Thus \n
\[
(\Delta P_{TP})_H = 2.020 \frac{2(54.23)^2}{2(32.2)(46.296)144} \left[1 + \frac{4240}{0.0216(0.0265)}\right] = 2.104 \text{ psi}
\]

(b) Separated flow model \((S = 2)\)

(Eq. 1-11): \((\Delta P_{TP})_S = \dot{K} \frac{G^2}{2g_c^0 f} \left[\frac{(1 - X)^2}{1 - \alpha} + \frac{V_f}{V_g} X^2\right]\)

where

\[
\alpha = \alpha_{TSP} = \frac{1}{1 + \left(\frac{1 - X_{TSP}}{X_{TSP}}\right) S \frac{V_f}{V_g}}
\]
\[
\frac{1}{1 + \left( \frac{1 - 0.0265}{0.0265} \right)^2 \left( \frac{0.0216}{0.4456} \right)} = 21.92\% 
\]

Therefore,

\[
(\Delta P_{TP})_S = 2.020 \cdot \frac{(542.33)^2}{2(32.2)(46.296)144} \left[ \frac{(1 - 0.0265)^2}{1 - 0.2192} \right] + \left( \frac{0.4456}{0.0216} \right)^2 = 1.771 \text{ psi}
\]

**CASE II**

Assume the tubes are in the tangent position inside the support plate holes. Therefore, from equation (1-19) with

\[
Z = 50 \\
f_p = 0.0073 \text{ (from Fig. 3)} \\
C_c = 0.65 \text{ (from Fig. 4)} \\
A_o = \frac{\pi}{4}(0.53^2 - 0.50^2) \cdot \frac{1}{144} = 1.68 \times 10^{-4} \text{ ft}^2 \\
L = 3/4 \text{ in.} = 0.0625 \text{ ft.} \\
R = 0.0221 \text{ ft.} \\
r = 0.0208 \text{ ft.}
\]

resulting \( C = 0.752 \)

or \( K = \frac{1}{C^2} = 1.768 \)

Thus, the pressure drop in two-phase flow are

(a) Homogeneous flow model from equation (1-7)

\[
(\Delta P_{TP})_H = 1.768 \cdot \frac{(542.33)^2}{2(32.2)(46.296)144} \left[ \frac{(1 - 0.0265)^2}{1 + \frac{0.424}{0.0216} \cdot 0.0265} \right] = 1.841 \text{ psi}
\]
(b) Separated flow model from equation (1-11)

$$\left(\Delta P_{TP}\right)_S = 1.768 \frac{(542.33)^2}{2(32.2)(46.296)144} \left[\frac{(1 - .0265)^2}{1 - .2192} + .4456 \left(\frac{.0265}{.0216}ight)^2\right]$$

$$= 1.550 \text{ psi}$$

The foregoing procedure have been applied to the following flow conditions:

- Flow rate = 6 to 8 GPM
- Exit quality = 0.05 to 0.25 percent
- Slip ratio = 1 to 3

which the results are presented in Table 1 and the graphs of pressure drop across tube support plate versus the quality at the tube support plate.
# Table 1

## Results of Calculations for Saturated Single-Phase Pressure Drop for Various Flow Conditions

<table>
<thead>
<tr>
<th>Flow Rate (GPM)</th>
<th>$m_T$</th>
<th>$C_0$</th>
<th>$Re_0$</th>
<th>$f_p$</th>
<th>$\frac{1}{C^2}$</th>
<th>$\Delta P_{fP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.0914</td>
<td>542.33</td>
<td>20948</td>
<td>0.0073</td>
<td>0.0071</td>
<td>1.768</td>
</tr>
<tr>
<td>7</td>
<td>0.1067</td>
<td>633.11</td>
<td>24455</td>
<td>0.0071</td>
<td>0.0070</td>
<td>1.757</td>
</tr>
<tr>
<td>8</td>
<td>0.1220</td>
<td>723.89</td>
<td>27961</td>
<td>0.0070</td>
<td>0.0070</td>
<td>1.756</td>
</tr>
</tbody>
</table>

**Tangent Orifice**

**Concentric Orifice**
FLOW RATE = 6 GPM
SLIP RATIO = 2

Fig. 10. Pressure drop versus mixture quality across the support plate for both concentric and tangent orifices.
CONCENTRIC ORIFICES
FLOW RATE = 6 GPM

Fig. 11. Pressure drop versus mixture quality across the support plate for various slip ratios.
TANGENT ORIFICES
FLOW RATE = 6 GPM

Fig. 12. Pressure drop versus mixture quality across the support plate for various slip ratios.
CONCENTRIC ORIFICES
SLIP RATIO = 2

Fig. 13. Pressure drop versus mixture quality across the support plate for various flow rates.
TANGENT ORIFICES
SLIP RATIO = 2

Fig. 14. Pressure drop versus mixture quality across the support plate for various flow rates
DISCUSSION

From these curves (Figure 10 through Figure 14), it can be seen that:

1. The pressure drop across the support plate increases linearly as the quality across the support plate increases. This is found to be true for the following cases:
   (i) both homogeneous and separated flow models
   (ii) both concentric and tangent orifices
   (iii) for all flow rates.
2. The pressure drop in homogeneous flow models are found to be much greater than the separated flow models.
3. It is found that the pressure drop across the concentric annular gaps is greater than the tangent annular gaps.
APPENDIX 2

SAMPLE CALCULATION OF THE TWO-PHASE (AIR-WATER) FLOW PRESSURE DROP ACROSS ANNULAR ORIFICES

This Appendix gives the sample calculation of two-phase flow (air-water) pressure drop across the tube support plate similar to Appendix (1), and comparison with the experimental result from the University of Central Florida two-phase flow apparatus (Fig. 8), when a mixture of 6 GPM of water with 20 ft³/hr of air at room temperature of 78°F and the atmospheric condition flows in the channel.

Solution:

The physical properties of air and water are given by

For air: \( \rho_g = 0.0737 \text{ lbm/ft}^3 \)
\[ V_g = 13.568 \text{ ft}^3/\text{lbm} \]

For water: \( \rho_f = 62.32 \text{ lbm/ft}^3 \)
\[ V_f = 0.016 \text{ ft}^3/\text{lbm} \]
\[ u_f = 2.08 \text{ lbm/hr-ft} \]
Therefore, the mass flow rate of air and water can be found

\[ \dot{m}_{\text{air}} = \dot{m}_{g} = 20 \ \frac{\text{ft}^3}{\text{hr}} \times \frac{0.0737 \ \text{lbm}}{\text{ft}^3} \times \frac{1 \ \text{hr}}{3600 \ \text{s}} = 0.00041 \ \text{lbm/s} \]

\[ \dot{m}_{\text{water}} = \dot{m}_{f} = 6 \ \frac{\text{gal}}{\text{min}} \times \frac{2.23 \times 10^{-3} \ \text{ft}^3}{\text{s}} \times \frac{1 \ \text{gal}}{1 \ \text{min}} \times 62.32 \ \frac{\text{lbm}}{\text{ft}^3} = 0.83384 \ \text{lbm/s} \]

Consequently, the total mass flow rate is given by

\[ \dot{m}_{T} = \dot{m}_{g} + \dot{m}_{f} = 0.00041 + 0.83384 = 0.83425 \ \text{lbm/s} \]

Under the steady-state condition, we assume the total mass flow rate remains constant throughout the channel and each of the annular gaps between the tubes and the support plate holes carries the same mass flow rate. Thus

\[ \dot{m}_{o} = \frac{1}{7} \dot{m}_{T} = \frac{1}{7} \times 0.83425 = 0.11918 \ \text{lbm/s} \]

and mass flux is

\[ G_{o} = \frac{\dot{m}_{o}}{A_{o}} = \frac{0.11918}{\frac{\pi}{4}(0.53^2 - 0.50^2) \frac{1}{144}} = 707.16 \ \text{lbm/ft}^2\cdot\text{s} \]

Considering the annular gaps between the tubes and the tube support plate holes as annular orifices, the annular orifice Reynolds number can be found from equation (1-14):

\[ \text{Re}_{o} = \text{Re}_{f} = \frac{(D - d) \ G_{o}}{\mu_{f}} \]

\[ = \frac{(0.53 - 0.50) \text{ft}(707.16) \text{lbm/ft}^2\cdot\text{s}(3600)\text{hr/s}}{2.08 \ \text{lbm/hr-ft}} \]
\[ \approx 3060 \]

Since \( 40 < Re_o < 4000 \), the flow is transitional.

Assuming the tubes are in the concentric position inside the support plate holes, from equation (1-17) can be found

\[ K = \frac{1}{C^2} = \frac{1}{C_{c}^2} \left[ 2 \sqrt{\frac{1}{C_{c}^2} - \frac{64}{Re_o}} - 2 \right] F + 2f_p Z \]

where

\[ Z = 50 \]
\[ F = 1 \]
\[ f_p = 0.0135 \text{ (from Fig. 3)} \]
\[ C_c = 0.65 \text{ (from Fig. 4)} \]

resulting \[ \frac{K}{K} = \frac{1}{C^2} = 2.654 \]

Since there is no state change from water to air, no relationship can be found between the specific volumes of air and water for defining \( V_{fg} \) in the equation (1-7) of pressure drop in two-phase homogeneous flow. Therefore, the equation (1-11) of pressure drop in separated flow with slip ratio equal to 1.0 is being used instead

\[ Eq. (1-11): \Delta P_{TP} = \frac{\dot{m}_K}{2g_c \rho_f} \left[ \frac{(1 - X)^2}{1 - \alpha} + \frac{V_g}{V_f} \frac{X^2}{\alpha} \right] \]

where

\[ X = X_{TSP} = \frac{\dot{m}_K}{\dot{m}_T} = \frac{0.00041}{0.83425} = 0.00049 \]

and quality is assumed to remain constant throughout the channel under the steady-state condition.
\[ \alpha = \alpha_H = \alpha_{TSP} = \frac{1}{1 + \left(\frac{1 - X_{TSP}}{X_{TSP}}\right)\frac{V_p}{V_g}} \]

\[ = \frac{1}{1 + \left(\frac{1 - 0.00049}{0.00049}\right)\frac{0.01605}{13.568}} \]

\[ = 29.30\% \]

Thus, the predicted value of two-phase pressure drop can be found

\[
(\Delta P_{TP})_H = 2.654 \frac{(707.16)^2}{2(32.2)(62.32)144} \left[ \frac{(1 - 0.00049)^2}{1 - 0.293} + \frac{(13.568)(0.00049)^2}{0.01605} \frac{0.293}{0.293} \right] \]

\[ = 3.246 \text{ psi} \]

From the comparison between the predicted value (3.246 psi) and the experimental value (4.729 psi) of two-phase pressure drop, it can be seen that the predicted value underestimated the experimental value by 45.7%. The difference might be caused by:

1. The assumptions which are stated in Chapter 1.
2. The positions of pressure taps before and after the support plate.

Also, it should be noted that the two-phase multiplier, $\phi$, are derived for two-phase flow with one component (liquid and its vapor), not two components (air and water).

Since the two-phase pressure drop for separated flow model is less than the homogeneous flow model, the calculation of pressure drop for the separated flow model will not be presented.
APPENDIX 3

THE HUGHMARK CORRELATION (4)

In a two-phase flow, the relationship between quality and void fraction is given by Hughmark 1962 as

\[ \frac{1}{X} = 1 - \frac{V_g}{V_f} \left( 1 - \frac{\mathcal{R}}{\alpha} \right) \]

where \( \mathcal{R} \) is related to a parameter \( \mathcal{Z} \) (Table 2) which is defined as follows:

\[ \mathcal{Z} = (Re)^{1/6} (Fr)^{1/8} (1 - \alpha)^{-1/4} \]

or

\[ \mathcal{Z} = \left( \frac{D_h G}{(1 - \alpha) \mu_f + \alpha \mu_g} \right)^{1/6} \left( \frac{U^2}{g D_h} \right)^{1/8} (1 - \alpha)^{-1/4} \]

where \( U \) is volumetric flux, and is given by

\[ U = \frac{Q}{A} \]

Also, Figure (15) is provided for comparison between the Hughmark Correlation and equation (1-2) for void fractions versus mixture qualities.
## TABLE 2

VALUES OF $\overset{*}{R}$ AS A FUNCTION OF $\overset{*}{Z}$

<table>
<thead>
<tr>
<th>$\overset{*}{Z}$</th>
<th>1.3</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overset{*}{R}$</td>
<td>0.185</td>
<td>0.225</td>
<td>0.325</td>
<td>0.49</td>
<td>0.605</td>
<td>0.675</td>
<td>0.72</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\overset{*}{Z}$</th>
<th>8.0</th>
<th>10.0</th>
<th>15.0</th>
<th>20.0</th>
<th>40.0</th>
<th>70.0</th>
<th>130.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overset{*}{R}$</td>
<td>0.767</td>
<td>0.78</td>
<td>0.808</td>
<td>0.83</td>
<td>0.88</td>
<td>0.93</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Fig. 15. Void fraction versus mixture quality for the annular gap between the tube and the support plate (Appendix 1) at the system pressure of 1000 psia.

HUGHMARK CORRELATION for slip flow with $G_0 = 542.33$ lbm/ft$^2$-s

Equation (1-2) for $S = 2$
REFERENCES CITED


BIBLIOGRAPHY


