1996

Computing optimal cocomo effort multiplier values and optimal casebase subsets using monte carlo methods

Robert Joseph Maidhof
rrrw@comcast.net

COMPUTING OPTIMAL COCOMO EFFORT MULTIPLIER VALUES AND OPTIMAL CASEBASE SUBSETS USING MONTE CARLO METHODS

by

ROBERT JOSEPH MAIDHOF
B.S.E.T University of Central Florida, 1984

A thesis submitted in partial fulfillment of the requirements
for the degree of Master of Science
in the Department of Electrical and Computer Engineering
in the College of Engineering
at the University of Central Florida
Orlando, Florida

Fall Term
1996

ABSTRACT

There have been many studies performed and techniques applied to solve the problem of estimating man-month effort for software projects. Despite all the effort expended to solving this problem the results achieved from the various techniques have not been embraced by the software community as very reliable or accurate.

This thesis uses Monte Carlo methods to obtain optimal values for COCOMO effort multipliers which minimize the average of the absolute values of the relative errors (AARE) of man-month estimates for two industry supplied casebases. For example, when using three COCOMO cost drivers (complexity, language experience, application experience) and the COCOMO effort multiplier values, AARE values were 60% for casebase 1 and 53% for casebase 2; using Monte Carlo to obtain optimal effort multiplier values, AARE values were 34% for casebase 1 and 41% for casebase 2.

By repeatedly removing the cases which contributed the greatest Absolute Relative Error, the Monte Carlo method was also used to determine optimal casebase subsets with AARE values of less than 10%. This latter approach identifies casebase cases for which the cost drivers may have been rated incorrectly or cases which are not rated consistently with respect to a subset of cases.

TABLE OF CONTENTS

List Of Figures iv

List Of Tables v

List Of Equations vi

CHAPTER 1. Introduction........ 1

CHAPTER 2. Methods 5
COCOMO 6
Monte Carlo Method 7
Factor Effort Multiplier Values 8
CaseBases 9
Basic Equations 12
AARE Method Flow 16

CHAPTER 3. Monte Carlo Method 20
Results 23
25-Case Casebase Result Comparison 23
16-Case Casebase Result Comparison 27
Case Removal 31

CHAPTER 4. Conclusions 38

Appendix A. Case Removal Data for 16 Case Casebase 42

Appendix B. Case Removal Data for 25-Case Casebase 55

List Of References 68

LIST OF FIGURES

Figure 1. Complexity (CPLX) Adjustment Values....8

Figure 2. Language Experience (LEXP) Adjustment Values....9

Figure 3. Application Experience (AEXP) Adjustment Values....9

Figure 4. Method Flow For Determining AARE....17

Figure 5. Monte Carlo Results For 25-Case Casebase....25

Figure 6. AARE Comparison Among Monte Carlo, Cocomo, and Solver (25 Cases)....26

Figure 7. Monte Carlo Results For 16-Case Casebase....29

Figure 8. AARE Comparison Among Monte Carlo, Cocomo, and Solver (16 Cases)....30

Figure 9. Method Flow For Removing Bad Cases....33

Figure 10. Case Removal, 16-Case Casebase, A=3.2, B=1.05, CPLX, LEXP, AEXP....36

LIST OF TABLES

Table 1. Cocomo Factor Effort Multiplier Values 2

Table 2. Casebase Data - 25 Cases 11

Table 3. Casebase Data - 16 Cases 12

Table 4. Random Numbers Per Iteration 19

Table 5. 25-Case Casebase -Factor Effort Multipliers And Max_Range 21

Table 6. 16-Case Casebase -Factor Effort Multipliers And Max_Range 21

Table 7. AARE Comparison of 1 To 25 Million Iterations, Max_Range = 6.5 28

LIST OF EQUATIONS

Equation 1. Absolute Relative Error 13

Equation 2. Estimated Man-Months For Case i 14

Equation 3. Absolute Relative Error 15

Equation 4. Average Absolute Relative Error 16

Equation 5. Absolute Relative Error 34

CHAPTER 1

INTRODUCTION

The process of estimating the development time and cost for a software project has proved to be an extremely difficult task. In general, the effort expended on development exceeds the estimated effort. Consequently, software projects are subject to cost overruns and very often are delivered after the scheduled completion date. The process of software cost estimation is typically performed at the beginning of a project when there are many variables and unknown factors. Many factors influence the amount of time required to complete a software project. In most cases the first step is to predict the number of lines of code required to complete the development. From this point, various factors are taken into account in an attempt to estimate the total effort for completing the development. This approach raises the questions: What factors are to be included in the process? How much influence will these factors have on the development process?

A standard practice throughout the software industry is to ask an expert who is familiar with the development environment for an effort estimate on the project. This expertise is typically derived from having worked on projects that contain qualities similar to the new project. The estimation process involves using past experiences on software development projects and then adjusting the estimate based on how various factors for the new project are different then the previous projects.

This is the basic concept of the COCOMO model for estimating man-month efforts for software development projects. COCOMO focuses on estimating the influence of various factors, also known as cost drivers, on the development effort. First an estimate of the software size is made which is then converted into man-months effort. This effort is adjusted by the influence of the applicable factors. The model has assigned numerical values for each factor that is used when calculating the man-month effort. The COCOMO model assigns numerical values to factors based on a rating scale consisting of the levels Very Low, Low, Nominal, High, Very High, Extra High. Table 1 presents an example of some COCOMO factors and their associated values. Each level for a specific factor has a pre-assigned value which is used when adjusting the man-month estimate. Consequently the factor values represent their influence on the man-month effort. The rating for a factor is determined by an expert who is familiar with the project.

Factor	Very Low	Low	Average	High	Very High	Extra High
Complexity	0.70	0.85	1.00	1.15	1.30	1.65
Language Experience	1.14	1.07	1.00	0.95		
Application Experience	1.29	1.13	1.00	0.91	0.82	

Table 1. COCOMO Factor Effort Multiplier Values

This paper challenges the process of pre-assigning the numerical values for the factor levels. Using Monte-Carlo analysis optimal factor multipliers were determined for each casebase. An iterative process was used to determine factor values which provided the best Absolute Average Relative Error for each of the casebases. This process eliminates pre-determining which factors have the most influence on the effort adjustment. For example, in COCOMO the application experience factor has a rating range from 1.29 to .82 while the language experience factor has a range from 1.14 to .95. Due to the wider range of the experience factor values it can have a larger influence on the man-month effort calculation. Pre assigning values also determines the degree of difference between the individual levels for a factor. Using the Monte Carlo method these constraints are eliminated. The only constraints applied were a minimum of 0 and maximum limit which was determined empirically by adjusting so no factor values where converging on the upper limit. The only other constraint applied to the random factors was that they logically increase or decrease according to their factor effort multiplier value. For example, on the application experience factor a value for the very low rating (AEXP_VL) would have to be higher than a rating for the low value (AEXP_L) which in turn would be higher than the nominal rating (AEXP_N). Summarizing, this states that AEXP_VL > AEXP_L > AEXP_N> ...). Logically this makes sense because using developers with a lower level of experience will increase the man month effort required to complete a project.

The first goal was to produce factors which would provide the lowest absolute average relative error for a casebase using Monte Carlo methods. This indicates that the factors will produce man-month estimates that are as close to the actual man-months as possible. Using the industry supplied ratings for the cases factors assumes that the cases which were grouped into a casebase were rated in a consistent and predictable manner. Second, a series of iterations were performed in which after each run the case with the highest absolute relative error was removed. This approach was used to show the effect of removing cases which possibly were not classified in a manner consistent with the other cases in the casebase. This process also provided a mechanism for determining if a new case was classified consistently with the cases in the casebase. Finally, a comparison was made between the results achieved using the Monte Carlo methods, COCOMO, and the math tool Solver.

Chapter 1 introduces the problem that this paper attempts to solve. Chapter 2 presents the basic method that was used described in terms of the applicable equations, variables, and the basic flow of the process. After the generic process is described then the specific casebase data used and the results are described in Chapter 3. The last chapter summarizes the data obtained and draws conclusions from the results while also suggesting some future work which could be done to possibly further improve the results.

CHAPTER 2

METHODS

There are many techniques for estimating software development costs. Most of them are a combination of the following techniques [Heemstra, 1992] :

- Estimates made by an expert
- Estimates based on reasoning by analogy
- Estimates based on Price-to-win
- Estimates based on available capacity
- Estimates based on the use of parametric models

This paper uses the technique which is a combination of estimates made by an expert and a parametric model. While the other techniques can also supply reasonable estimates they are not considered in this paper. The process of estimating software development costs is not as simple as estimating a predictably repetitive task such as producing integrated circuits or vacuum cleaners. There are many reasons for this; some of the main ones are:

- Source instructions are not a uniform commodity, nor are they the essence of the desired product.
- Software requires the creativity and cooperation of human beings, whose individual and group behavior is generally hard to predict.
- Software has a much smaller base of relevant quantitative historical experience, and is hard to add to the base by performing small controlled experiments.

One criticism which tends to coincide with the reasons listed above is that " much of what we believe about which approaches are best is based on anecdotes, gut feelings, expert opinions, and flawed research, not on careful, rigorous software-engineering experimentation" [Fenton, 1994].

COCOMO

Perhaps the industries most popular regression-based cost model was completed in 1981 by Barry Boehm of TRW Corporation, and it is called Constructive Cost Model (COCOMO) [Boehm, 1981]. COCOMO is based on data from 63 projects, collected by Boehm during the mid-to-late 1970's. "He clustered the data into three groupings, which he called modes. he then derived two linear equations for each mode in the log-log domain; one equation for estimated effort as a function of software size and one for estimated development time as a function of estimated effort. Boehm and his colleagues identified 15 cost drives as those factors that contributed most to the observed variations in effort and schedule for software projects similar in size and mode. The ranges of cost driver values were derived by expert judgment using the Delphi procedure. Boehm illustrated, by example, how to construct a regression-based cost model; hence the name of the model"[Fairley, 1994].

Monte Carlo Method

The first portion of this paper will focus on the technique of using Monte Carlo methods for determining factor values. The COCOMO model uses predetermined fixed values for each factor effort multiplier value. These fixed values imply that when a factor effort multiplier value is determined that the perception of the difference between the two is the same as was implied by the COCOMO tool. For example, the Complexity factor has a value for Very Low equal to 0.70 while the Low value is 0.85. It is very possible that in reality when the factor effort multiplier values were determined it was assumed that a Very Low value required an effort which was significantly less , maybe 50%, less than a Low value. In that case a Very Low value optimally would be 0.425 when a Low value of .85 is used. The fixed values also restrict how much adjustment one factor can have on the man month estimation versus another factor. For example, the Complexity factor has a value range from 0.85 to 1.15 (Low to High) whereas the language experience factor has a range from 0.95 to 1.07 (Low to High). Looking at the range of these numbers shows that it has been predetermined that the language factor can have a greater percentage of effect on the man month estimate than the language experience factor. The Monte Carlo method eliminates these assumptions and provides a mechanism for determining the optimal factor effort multiplier values.

Factor Effort MultiplierValues

The factor values presented in Figure 1 - Figure 3 are from the COCOMO model [Boehm, 1981]. The Monte Carlo values listed are simply variables which are used to describe the factor effort multiplier values relative to each other. For the Complexity factor the restriction of C1 < C2 < C3 < C4 < C5 would apply when randomly choosing the values. Similar restrictions were applied to the Language Experience factor, L1 < L2 < L3 and the Application experience factors, A1 < A2 < A3.

Factor Effort Multiplier Value	Description	Monte Carlo Value	COCOMO Value
Very Low	Rebuilt system with large segments of existing code	C1	0.70
Low	Entirely new stand alone	C2	0.85
Nominal	Composite system independent subsystems development with considerable overlap	C3	1.00
High	Composite system independent subsystems development in parallel	C4	1.15
Very High	Entirely new must interface with other systems	C5	1.30

Figure 1. Complexity (CPLX) Adjustment Values

Factor Effort Multiplier Value	Description	Monte Carlo Value	COCOMO Value
High	3 years	L1	0.95
Nominal	1 year	L2	1.00
Low	4 months	L3	1.07

Figure 2. Language Experience (LEXP) Adjustment Values

Factor Effort Multiplier Value	Description	Monte Carlo Value	COCOMO Value
High	6 years	A1	0.91
Nominal	3 years	A2	1.00
Low	1 year	A3	1.13

Figure 3. Application Experience (AEXP) Adjustment Values

Casebases

While the COCOMO model allows for up to 6 ratings for each factor, the ratings provided in Figure 1- Figure 3 were the only ratings used with the casebases provided for this paper. Table 2 and Table 3 summarize the casebase data which was supplied from industry sources. Each case within the casebase has been described in terms of how many Lines of Code were required to complete the project, what factor effort multiplier values were assigned to the three factors Complexity (CPLX), Language Experience (LEXP), and Application Experience (AEXP), and the actual number of man months that were required to complete the project. Since the concept is to not use pre-assigned factor effort multiplier values they are labeled simply as variables which will be determined using Monte Carlo methods. For example, case_1 of Table 2 has a complexity factor effort multiplier value of C2 which indicates that the industry source rated the case's complexity as Low (see Figure 1), a Language experience rating of L2 which equates to a rating of Nominal (see Figure 2), and an Application experience of A3 which implies a rating of Low (see Figure 3). This particular case required 25000 Lines of Code and 72 Man-Months of effort to complete the project. The data for each casebase was obtained from an industry source which assumes that the projects were rated in a consistent and predictable manner. The process of identifying cases which are consider erroneous and probably rated in an inconsistent fashion compared to the other cases in the casebase is looked at in a later portion of this paper.

case #	LOCS	CPLX	LEXP	AEXP	Man-Months
case 1	25000	C2	L2	A3	72.0
case 2	1120	C3	L1	A1	7.5
case 3	2581	C3	L2	A2	9.0
case 4	1111	C3	L2	A2	5.4
case 5	740	C3	L3	A1	4.5
case 6	4131	C3	L2	A2	8.5
case 7	1200	C2	L2	A3	3.0
case 8	4900	C3	L1	A1	5.8
case 9	2950	C1	L2	A2	33.0
case 10	5346	C1	L2	A2	22.0
case 11	900	C2	L1	A1	2.5
case 12	1127	C3	L3	A3	4.5
case 13	5800	C2	L2	A2	18.9
case 14	12352	C3	L1	A2	32.5
case 15	300	C2	L3	A3	3.0
case 16	300	C2	L3	A3	1.5
case 17	167	C3	L2	A2	0.7
case 18	4557	C1	L1	A2	9.7
case 19	2892	C1	L2	A3	10.5
case 20	1726	C3	L2	A2	11.0
case 21	17176	C3	L2	A2	47.5
case 22	2956	C1	L2	A3	10.5
case 23	1706	C3	L2	A2	11.0
case 24	12288	C1	L2	A2	153.0
case 25	5166	C1	L2	A2	38.0

Table 2. Casebase Data - 25 Cases

case #	LOCS	CPLX	LEXP	AEXP	man-months
case 1	72000	C4	L2	A2	750.0
case 2	12760	C1	L1	A1	125.0
case 3	7000	C5	L2	A2	550.0
case 4	88000	C5	L3	A1	972.0
case 5	100000	C4	L2	A2	450.0
case 6	55700	C5	L2	A2	495.0
case 7	140000	C5	L1	A1	900.0
case 8	46000	C3	L1	A1	102.0
case 9	129600	C2	L2	A2	427.0
case 10	20000	C1	L2	A2	115.0
case 11	30000	C5	L3	A1	260.0
case 12	5000	C5	L2	A1	72.0
case 13	16700	C5	L2	A1	170.0
case 14	5000	C5	L2	A1	12.0
case 15	40000	C2	L2	A1	80.0
case 16	27000	C5	L2	A1	150.0

Table 3. Casebase Data - 16 Cases

Basic Equations

The Average Absolute Relative Error (AARE) was used as a gauge for determining when the optimum factor values had been reached. The ideal optimum AARE for a casebase would result when the estimated man months for each case in a casebase are equal to the actual man-months required to complete each case. While it is not realistic to estimate the man-months exactly, the goal was to derive the optimal factor values from the casebase data received from the industry sources. The basic process consisted of randomly choosing factor values for each factor within the casebase and calculating the

AARE. This process was repeated for many iterations and the factors which resulted in the lowest value AARE were considered the optimal values for the particular casebase. The process is described in more detail in the following sections.

Before calculating the AARE for the entire casebase the Absolute Relative Error (ARE) for each case within the casebase must be calculated. The Absolute Relative Error (ARE) for an individual case is the estimated man-months minus the actual man-months divided by the actual man-months (Equation 1). The actual man-months value was provided in the casebase data from the industry source, it is the number of man-months that were required to complete the tasks associated with a specific case. The estimated man months is calculated using the Monte Carlo techniques applied to the COCOMO model.

$$ARE_i = \left| \frac{estimated(MM_i) - actual(MM_i)}{actual(MM_i)} \right|$$

Equation 1. AbsoluteRelative Error

ARE_i = Absolute value of Relative Error for i^{th} case in the casebase

$estimated(MM_i)$ = estimated man-months to complete the i^{th} case in the casebase

$actual(MM_i)$ = actual man-months to complete the i^{th} case in the casebase

Calculating the ARE provides a means for gauging how close the tool is estimating the number of man-months required to complete a case compared to how many man-months were actually required to complete the case.

As shown in Equation 2 the estimated man-months for a specific case i is a product of the basic COCOMO model multiplied by the factor effort multiplier values for each factor which has been determined to be included in the calculation . In this paper the three factors Complexity (CPLX), Language Experience (LEXP), and Application Experience (AEXP) have been used in calculating the man-month effort. Once again this is primarily a function of the data that was received from the industry sources for the casebases used.

$$estimated(MM)_i = A(L_i)^B (Factor\_1_i)(Factor\_2_i)(Factor\_n_i)$$

Equation 2. Estimated Man-Months for Case i

A,B = parameters for COCOMO organic or embedded models

L_i = thousands of Lines of code required to complete the i th case

$(Factor\_1_i)\ldots(Factor\_n_i)$ = effort multiplier values for each factor

Substituting the three factors CPLX, LEXP, and AEXP into Equation 2 to generate the estimated man months and then substituting Equation 2 into Equation 1 results in the absolute relative error for an individual case. Equation 3 contains the associated factors used for calculating the absolute relative error for an individual case within a casebase.

$$ARE_i = \left| \frac{estimated(MM_i) - actual(MM_i)}{actual(MM_i)} \right| = \left| \frac{A(L_i)^B (CPLX_i)(LEXP_i)(AEXP_i) - M_i}{M_i} \right|$$

Equation 3. Absolute Relative Error

i = case number within a casebase

M_i = actual man-months for the i th case

A,B = parameters for COCOMO organic or embedded models

L_i = thousands of lines of code required for the i th case

CPLX $_i$ = complexity factor effort multiplier values for the i th case

LEXP $_i$ = language experience factor effort multiplier values for the i th case

AEXP $_i$ = application experience factor effort multiplier values for the i th case

After the ARE was calculated for each case the AARE could then be derived. As Equation 4 shows, calculating the Average Absolute Relative Error (AARE) is the summation of the ARE for each case in the casebase divided by the total number of cases.

$$\text{AARE} = \frac{1}{N}\sum_{i=1}^{N}\left|ARE_i\right|$$

Equation 4. Average Absolute Relative Error

AARE = Average Absolute Relative Error for the entire casebase

i = case number

ARE_i = Absolute Relative Error for ith case in the casebase (Equation 3)

N = number of cases in the casebase

i = case number

AARE Method Flow

The iterative process of generating random numbers, calculating the ARE for a case, and then calculating the AARE for the entire casebase is summarized in the C language pseudocode of Figure 4.

```
for (NUMBER_RUNS)
{
        /* Generate the required random numbers to be used for factor values */
        for (each FACTOR)
        {
                for (each FACTOR_RATING of the current FACTOR)
                {
                        get a random number between 0 and 1 and scale it to MAX_RANGE.
                        sort the random number in ascending order.
                }
        }

        /* Calculate the ARE for each case using the randomly generated factor values */
        for (each CASE in the CASEBASE)
        {
                calculate the Absolute Relative Error (ARE) for the current CASE. (Equation 3)
                Add the current CASE's ARE  to the Total Absolute Error for this RUN.
        }

        /* Calculate the AARE and save it off and the associated factor values if it is the lowest */
        calculate the Absolute Average Relative Error (AARE) for this RUN.(Equation 4)
        If (this is the lowest AARE for this NUM_RUNS)
                save off the AARE and FACTOR values as optimal for this run.
}
```

Figure 4. Method Flow for Determining AARE

The process of generating a set of random numbers to be used as the effort multiplier values for each factor associated with the casebase is a very simple process. Following the pseudocode of Figure 4, when the 16-case casebase was used it required that 5 random numbers be generated for the CPLX factor effort multiplier values (C1-C5), 3 random numbers for the LEXP factor effort multiplier values (L1-L3), and 2 random numbers for the AEXP factor effort multiplier values (A1-A2). After generating the required factor effort multiplier values for each factor the ARE was calculated for each case in the casebase. The sum of the individual case's AREs was then summed together and divided by the number of cases in the casebase to obtain the AARE. This process was repeated for a the required number of runs and the factor effort multiplier values which resulted in the lowest AARE was determined to be the optimal factor values for the casebase.

The Number of Runs was determined to be 1 million. This number of runs provided results which would produce a minimum AARE which would change less than 5% as the number of runs was increased . This number was determined by running many series of runs with varying numbers from 100,000 up to 5 million on a series of cases.

A random number generator was used which generated numbers between 0 and 1. This number was then scaled by the value of MAX_RANGE. The value for MAX_RANGE was usually 2.5 as it was determined that most optimum factor values were less than 2.5. This determination came from a combination of the results obtained from the Solver math tool and empirical data which indicated that the factor values were not converging on the upper limit. For the exceptional cases where the optimum factor values were determined to be greater than 2.5 the MAX_RANGE was adjusted accordingly. The optimum values which were used a guideline for determining the MAX_RANGE values were taken from the work performed using the Solver math tool. The MAX_RANGE numbers for the various iterations run are summarized in the Results portion of this paper.

The casebases contained in this study contained cases which had the following factors associated with them:

- Complexity (CPLX)
- Language Experience (LEXP)
- Application Experience (AEXP)

Each factor has a rating associated with it. There are 6 distinct ratings for each factor. The possible ratings are Very Low (VL), Low (L), Nominal (N), High (H), Very High (VH), and Extra High (EH). Table 2 and Table 3 summarize the factor effort multiplier values that were supplied from the industry source for each casebase.

Table 4 lists the number of rating values associated with each factor which determined the specific quantity of random numbers that would need to be generated.

	16-case Casebase	25-case Casebase
Factor	# Factor Values (i.e., Random #'s)	# Factor Values (i.e., Random #'s)
CPLX	5	3
LEXP	3	3
AEXP	2	3
Total Random #'s per Run	10	9

Table 4. Random Numbers per Iteration

The basic process of generating random numbers for factor values in an iterative process was a simple procedure which was accomplished using the C programming language. The concept is adaptable in the manner that the number of factors could vary among casebases but can easily be handled by the process described in Figure 4.

CHAPTER 3

MONTE CARLO METHOD

The use of the Monte Carlo methods for determining factor effort multiplier values can only be verified as a useful technique if there are existing casebases from industry sources. "In general, data collection is not common in the software community. It is labor and time-intensive and requires an attitude not only focused on the constructive part but also on the analytical part of software engineering. Furthermore, data collection, usable for software cost estimation, is limited to a relatively small number of software development organizations. Only a few organizations realize large software projects each year" [Heemstra, 1992]. Two casebases were used for this paper to determine if the process of using Monte Carlo methods would produce acceptable results when determining factor values for man-month estimation of software development. A comparison of the results between the Monte Carlo methods, COCOMO, and the Solver math tool provides a means for analyzing the various techniques.

The two casebases used for the analysis each provided ratings for the three factors CPLX, LEXP, and AEXP. Consequently, it was determined that the Monte Carlo techniques would be applied to each possible combination of factors. Table 5 and Table 6 show that the same combinations of factors were run for each casebase.

	A= 3.2, B=1.05	A=2.8, B=1.2		
Factor Effort Multipliers	MAX_RANGE	MAX_RANGE	Qty. Random Numbers (10^6 iterations)	Time (mins)
CPLX	0 - 2.5	0 - 2.5	5×10^6	16
LEXP	0 - 2.5	0 - 2.5	3×10^6	14
AEXP	0 - 2.5	0 - 2.5	2×10^6	12
CPLX and LEXP	0 - 2.5	0 - 2.5	8×10^6	19
CPLX and AEXP	0 - 2.5	0 - 2.5	7×10^6	17
LEXP and AEXP	0 - 2.5	0 - 2.5	5×10^6	16
CPLX and LEXP and AEXP	0 - 2.5	0 - 2.5	10×10^6	45

Table 5. 25-Case Casebase - Factor Effort Multipliers and MAX_RANGE

	A= 3.2, B=1.05	A=2.8, B=1.2		
Factor Effort Multipliers	MAX_RANGE	MAX_RANGE	Qty. Random Numbers (10^6 iterations)	Time (mins)
CPLX	0 - 2.5	0 - 2.5	3×10^6	19
LEXP	0 - 2.5	0 - 2.5	3×10^6	19
AEXP	0 - 2.5	0 - 2.5	3×10^6	19
CPLX and LEXP	0 - 2.5	0 - 2.5	6×10^6	23
CPLX and AEXP	0 - 6.5	0 - 4.5	6×10^6	23
LEXP and AEXP	0 - 3.5	0 - 2.5	6×10^6	23
CPLX and LEXP and AEXP	0 - 6.5	0 - 2.5	9×10^6	26

Table 6. 16-Case Casebase - Factor Effort Multilpiers and MAX_RANGE

It was decided to run all combinations to see if there were any substantial differences in the results that occurred as a function of a specific factor. Since the industry supplied casebases did not specify the details on the development mode for the cases, each combination was also run using both the COCOMO Organic model (A=3.2, B=1.05) and the Embedded model (A=2.8, B=1.2). The organic mode is characterized as being a stable development environment, less innovative, and relatively small in size. The embedded mode consists of developing within tight constraints, innovative, complex, and having high volatility of requirements.

Also listed in Table 5 and Table 6 are the quantity of random numbers for each combination of factors and the time required to complete the 1 million iterations for the specific combination. In choosing random numbers it becomes very important that the random number generator not start repeating itself before the required quantity of random numbers is generated. The largest quantity of random numbers to be generated at any one time was 10 million. A 32-bit random number generator was used for this paper which means that there was no problem with getting into a repeating sequence. As expected, the more random numbers that had to be processed the longer the time was required to complete the run. Certainly the times are a direct function of the specific computer on which the code was running. These results were obtained on a 486 66 MHz machine with 8 megabytes of RAM.

Results

This section will discuss the results obtained using the Monte Carlo methods. Figure 5 shows the results that were obtained for the 25-case casebase using the Monte Carlo Methods. The optimal AARE was determined for each combination of factors for both the embedded and organic modes. The factor values which resulted in the optimal AARE are the result of performing 1 million iterations of choosing random numbers. A random number was chosen for each factor value and the ARE calculated for each case and then consequently the AARE for the entire casebase. Also included are the factor values which resulted in the optimal AARE for each combination of factors.

25-Case Casebase Result Comparison

Figure 6 provides a summary of the AARE results for the 25-case casebase. The data presents a comparison of the optimal AARE Monte Carlo results versus the COCOMO and Solver math tool results. The percent difference and absolute difference between the Monte Carlo results versus the COCOMO and Solver math tool results are provided for both the organic and embedded modes. The average AARE for all the factor combinations is also provided.

The Monte Carlo and Solver results tracked each other very closely. The AARE percent difference for the organic mode was typically less than 1%, the average percent difference was 1.066%. The factor combination of CPLX, LEXP, and AEXP resulted in a

6% difference, but this is still only an absolute difference of 0.021229. The AARE percent difference between the Monte Carlo results and COCOMO were considerably higher. The percent difference ranged from a low of 5.951885% to a high of 21.381900%. Across all factor combinations this resulted in an average AARE percent difference of 13.89%. For each individual factor combination the Monte Carlo methods produced factor values which resulted in a lower AARE than COCOMO.

In the embedded mode the results tracked very similarly to the organic mode. The average percent difference between the Monte Carlo and Solver results was only 2.995910% while the percent difference to COCOMO was 15.885390%. Once again, in all factor combinations the Monte Carlo derived factor values resulted in a lower AARE than the COCOMO factor values.

	Average Relative Error (ARE)														
	A=3.2, B=1.05								A=2.8, B=1.2						
	CPLX	LEXP	AEXP	CPLX, LEXP	CPLX,AEXP	LEXP,AEXP	CPLX, LEXP, AEXP		CPLX	LEXP	AEXP	CPLX, LEXP	CPLX,AEXP	LEXP,AEXP	CPLX,LEXP,AEXP
case 1	0.110771	0.356358	0.374161	0.333241	0.377839	0.313999	0.479519		0.441548	0.590735	0.839605	0.571799	0.817020	0.615293	0.599968
case 2	0.584860	0.704807	0.835463	0.684986	0.825128	0.697001	0.688441		0.664494	0.759324	0.866432	0.764486	0.867222	0.858016	0.867779
case 3	0.168788	0.000011	0.107843	0.017339	0.012760	0.040103	0.040552		0.238618	0.165732	0.449102	0.152621	0.404903	0.192372	0.140826
case 4	0.428281	0.312178	0.386362	0.300260	0.320963	0.339770	0.284294		0.538510	0.494332	0.666088	0.486385	0.639299	0.510479	0.479236
case 5	0.552227	0.137071	0.822530	0.118621	0.811382	0.172481	0.299038		0.659932	0.026159	0.864616	0.004077	0.865416	0.440557	0.524446
case 6	0.442168	0.735037	0.547909	0.765101	0.712879	0.665438	0.805376		0.417578	0.553280	0.025689	0.577692	0.107979	0.503680	0.599652
case 7	0.099358	0.342421	0.360041	0.319542	0.363682	0.300498	0.464317		0.095248	0.001614	0.154583	0.013499	0.140408	0.013799	0.004181
case 8	1.528453	0.797903	0.002127	0.918624	0.065072	0.845447	0.897584		1.549805	0.829110	0.015102	0.789883	0.009099	0.079060	0.004863
case 9	0.744189	0.686188	0.720034	0.693605	0.728740	0.698777	0.699319		0.760677	0.732899	0.823623	0.745206	0.814622	0.741428	0.758100
case 10	0.283647	0.121225	0.216003	0.141995	0.240384	0.156476	0.157996		0.267303	0.182262	0.460016	0.219937	0.432460	0.208374	0.259415
case 11	0.024708	0.296113	0.607663	0.274228	0.626605	0.277499	0.273099		0.231251	0.444629	0.691785	0.471318	0.698919	0.672366	0.717393
case 12	0.303559	0.342152	0.151131	0.370848	0.049506	0.323282	0.263851		0.436627	0.613312	0.286120	0.649895	0.282447	0.645225	0.499562
case 13	0.087448	0.114313	0.005868	0.095322	0.014906	0.069614	0.134498		0.048771	0.049673	0.306861	0.037178	0.264229	0.016155	0.001294
case 14	0.191291	0.152911	0.278637	0.096033	0.414909	0.114250	0.032639		0.380070	0.010003	0.001450	0.031234	0.078663	0.011604	0.018186
case 15	0.743566	0.498409	0.682759	0.504993	0.681910	0.505461	0.537851		0.828582	0.505636	0.781247	0.508175	0.783933	0.495857	0.574378
case 16	0.487132	0.003183	0.365518	0.009986	0.363820	0.010921	0.075702		0.657163	0.011273	0.562495	0.016350	0.567866	0.008285	0.148756
case 17	0.396981	0.274522	0.352767	0.261951	0.283787	0.303624	0.245111		0.633679	0.598612	0.734949	0.592304	0.713684	0.611429	0.586629
case 18	0.373918	0.003839	0.503653	0.020245	0.456894	0.041625	0.047532		0.372004	0.002480	0.011141	0.078696	0.062742	0.000859	0.125710
case 19	0.212613	0.034086	0.021407	0.056915	0.040579	0.064251	0.008425		0.265552	0.180307	0.052067	0.218073	0.072943	0.167653	0.217260
case 20	0.554265	0.463747	0.521583	0.454455	0.470595	0.485258	0.442007		0.615624	0.578829	0.721885	0.572210	0.699572	0.592278	0.566255
case 21	0.152262	0.386259	0.236747	0.410279	0.368555	0.330650	0.442458		0.402531	0.536792	0.014802	0.560945	0.096219	0.487719	0.582672
case 22	0.194307	0.011629	0.001344	0.034989	0.018273	0.042495	0.014629		0.246006	0.158492	0.026838	0.197263	0.048269	0.145501	0.196428
case 23	0.559686	0.470269	0.527402	0.461090	0.477034	0.491519	0.448794		0.620963	0.584678	0.725748	0.578151	0.703744	0.597941	0.572279
case 24	0.753178	0.697216	0.729872	0.704372	0.738272	0.709361	0.709885		0.713979	0.680781	0.789208	0.695489	0.778451	0.690975	0.710900
case 25	0.599919	0.509207	0.562140	0.520807	0.575757	0.528895	0.529743		0.592888	0.545636	0.699966	0.566570	0.684655	0.560145	0.588505
Optimal AARE	0.423103	0.338042	0.396839	0.342793	0.401609	0.341148	0.360906		0.507176	0.393463	0.462857	0.403977	0.465391	0.394682	0.413787
Factors	Factor Values														
C1	0.847168			1.492622	1.050973		0.755983		0.770112			1.387149	1.060333		0.335000
C2	0.851079			1.502700	1.074843		0.803288		0.778889			1.436835	1.070898		0.352852
C3	0.863827			1.555244	1.200308		0.820983		0.784407			1.477000	1.089788		0.380944
L1		0.614240		0.421467		0.279225	0.772409			0.562697		0.372802		0.289976	0.660876
L2		1.039249		0.679801		0.433684	1.190770			0.859497		0.591067		0.429478	1.048263
L3		1.664731		1.093284		0.707014	1.611156			2.246282		1.555327		1.127234	2.345063
A1			0.342369		0.303151	2.257977	1.022332				0.312280		0.284857	1.144770	1.227894
A2			0.927164		0.854762	2.300202	1.106157				0.567558		0.562580	1.937356	2.216608
A3			1.052890		0.982198	2.321491	1.185133				0.993965		0.916765	2.032156	2.337194

Figure 5. Monte Carlo Results for 25-Case Casebase

	Optimal AARE								
	A=3.2, B=1.05								
	CPLX	LEXP	AEXP	CPLX, LEXP	CPLX,AEXP	LEXP,AEXP	CPLX, LEXP, AEXP	Average	Median
Solver	0.423174	0.339678	0.398310	0.339677	0.398691	0.339677	0.339677	0.368412	0.339678
Cocomo	0.449908	0.418695	0.431842	0.436023	0.442827	0.413217	0.428770	0.431612	0.431842
Monte Carlo	0.423103	0.338042	0.396839	0.342793	0.401609	0.341148	0.360906	0.372063	0.360906
Difference : Monte Carlo - Solver	-0.000071	-0.001636	-0.001471	0.003116	0.002918	0.001471	0.021229	0.003651	0.001471
%Diff = MOnte Carlo -vs- Solver	-0.016778	-0.481633	-0.369310	0.917342	0.731895	0.433058	6.249761	1.066334	0.433058
Difference = Monte Carlo - COCOMO	-0.026805	-0.080653	-0.035003	-0.093230	-0.041218	-0.072069	-0.067864	-0.059549	-0.067864
%Diff = Monte Carlo -vs- COCOMO	-5.957885	-19.262948	-8.105511	-21.381900	-9.307924	-17.440957	-15.827600	-13.897818	-15.827600
	Optimal AARE								
	A=2.8, B=1.2								
	CPLX	LEXP	AEXP	CPLX, LEXP	CPLX,AEXP	LEXP,AEXP	CPLX,LEXP,AEXP	Average	Median
Solver	0.506355	0.394257	0.477932	0.395673	0.398691	0.394261	0.393223	0.422913	0.395673
Cocomo	0.547219	0.518081	0.541982	0.532386	0.442827	0.526041	0.525323	0.519123	0.526041
Monte Carlo	0.507176	0.393463	0.462857	0.403977	0.465391	0.394682	0.413787	0.434476	0.413787
Difference : Monte Carlo - Solver	0.000821	-0.000794	-0.015075	0.008304	0.066700	0.000421	0.020564	0.011563	0.000821
%Diff = Monte Carlo -vs- Solver	0.162139	-0.201391	-3.154214	2.098703	16.729748	0.106782	5.229603	2.995910	0.162139
Difference = Monte Carlo - COCOMO	-0.040043	-0.124618	-0.079125	-0.128409	0.022564	-0.131359	-0.111536	-0.084647	-0.111536
%Diff = Monte Carlo -vs- COCOMO	-7.317546	-24.053768	-14.599193	-24.119530	5.095444	-24.971247	-21.231890	-15.885390	-21.231890

Figure 6. AARE Comparison Among Monte Carlo, COCOMO and Solver (25 cases)

16-Case Casebase Result Comparison

The AARE results obtained for the 16-case casebase are presented in Figure 7. The comparison of AARE results presented in Figure 8 indicate that the results varied more using the 16-case casebase compared to the 25-case casebase. In general, the Monte Carlo tracked the results from the Solver math tool. In all cases the Monte Carlo methods produced factor values which resulted in an AARE that was lower than the AARE from the COCOMO values. In the organic and embedded modes the Monte Carlo average AARE was 42% to 43%, while the Solver tool was extremely close with values from 41% to 42%. These results were considerably lower than the COCOMO results, which were 51% in the organic mode and 63% in the embedded mode.

When comparing the Monte Carlo and Solver results, there were a few factor combinations which did not track each other as closely as the others. These were exceptions to the results obtained for the other combinations and to all combinations in the 25-case casebase. In the organic mode the two combinations of CPLX, AEXP and CPLX, LEXP, AEXP produced percent differences of 27.58% and 47.15%. In the embedded mode the factor combination of CPLX, AEXP produced a percent difference of 27.50%. Since all three of these combinations had been run with a MAX_RANGE of greater than 2.5, it was decided to increase the number of iterations from 1 million to 25 million to see if this was possibly contributing to the source of error. The results shown in Table 7 clearly show that increasing the number of iterations to 25 million resulted in an

AARE which was 8.5% lower than the value at 1 million iterations. The initial evaluation for determining the number of iterations to run was performed with a MAX_RANGE of 2.5. For the few instances where the MAX_RANGE was increased above 2.5 it appears that the number of iterations should have also been increased.

Factors	MODE	AARE @ 1 million iterations	AARE @ 2 million iterations	AARE @ 15 million iterations	AARE @ 25 million iterations	% difference from 1 to 25 million iterations
CPLX, LEXP, AEXP	Organic (A=3.2, B=1.05)	0.408934	0.402379	0.381042	0.374122	8.5 %

Table 7. AARE Comparison of 1 to 25 million iterations, MAX_RANGE = 6.5

	Average Relative Error (ARE)														
	A=3.2, B=1.05								A=2.8, B=1.2						
	CPLX	LEXP	AEXP	CPLX, LEXP	CPLX,AEXP	LEXP,AEXP	CPLX, LEXP, AEXP		CPLX	LEXP	AEXP	CPLX, LEXP	CPLX,AEXP	LEXP,AEXP	CPLX,LEXP,AEXP
case 1	0.551796	0.692666	0.574847	0.569800	0.515687	0.581554	0.307479		0.580888	0.606979	0.595668	0.578477	0.597011	0.591948	0.571457
case 2	0.790113	0.786239	0.743524	0.769791	0.814234	0.783003	0.889533		0.837975	0.824809	0.705595	0.861825	0.871543	0.821516	0.862268
case 3	0.930683	0.963737	0.949835	0.933222	0.878145	0.950627	0.912722		0.946405	0.967309	0.966368	0.944241	0.922700	0.966058	0.942251
case 4	0.440386	0.165948	0.749473	0.157300	0.419941	0.063755	0.086631		0.367465	0.007120	0.615809	0.003661	0.326989	0.022981	0.008442
case 5	0.054691	0.276796	0.000448	0.012325	0.139661	0.015334	0.629606		0.036049	0.028448	0.000488	0.042009	0.003809	0.008708	0.059361
case 6	0.320185	0.644356	0.508017	0.345087	0.195076	0.515778	0.144035		0.282548	0.562378	0.549784	0.253586	0.034780	0.545641	0.226944
case 7	0.015898	0.632813	0.559439	0.248441	0.020055	0.627254	0.486872		0.192568	0.568964	0.275655	0.018023	0.268880	0.560863	0.096857
case 8	0.012837	0.006910	0.208119	0.111335	0.008051	0.022154	0.004961		0.076694	0.000256	0.680907	0.015825	0.097273	0.019057	0.032789
case 9	0.287945	0.000646	0.384250	0.025494	0.116275	0.362414	0.023832		0.160156	0.397580	0.437800	0.001053	0.047800	0.451030	0.045114
case 10	0.634291	0.477776	0.277578	0.494009	0.451044	0.288974	0.535203		0.697998	0.448918	0.433058	0.674593	0.675435	0.427842	0.634736
case 11	0.324147	0.007295	0.697435	0.017739	0.299455	0.130715	0.312338		0.349954	0.020366	0.605173	0.023920	0.308358	0.004066	0.019007
case 12	0.628093	0.805437	0.833505	0.641716	0.614506	0.834476	0.703038		0.726597	0.833233	0.833940	0.715560	0.709102	0.839311	0.772046
case 13	0.441206	0.707668	0.749840	0.461675	0.420791	0.751298	0.553811		0.507753	0.699746	0.701018	0.487882	0.476255	0.710689	0.589583
case 14	1.231442	0.167375	0.001032	1.149702	1.312965	0.006855	0.781775		0.640419	0.000601	0.003638	0.706639	0.745388	0.035868	0.367722
case 15	0.106061	0.554341	0.330109	0.513735	0.022388	0.322356	0.008548		0.093660	0.819954	0.812243	0.303590	0.006571	0.753621	0.053108
case 16	0.048791	0.451327	0.530479	0.010373	0.087107	0.533216	0.162555		0.007073	0.394347	0.396913	0.033009	0.056464	0.416422	0.172134
Optimal AARE	0.426160	0.458833	0.506121	0.403859	0.394711	0.424360	0.408934		0.406513	0.448813	0.538379	0.353993	0.384272	0.448476	0.340864
Factors	Factor Values														
C1	0.565721			1.162532	1.788038		2.144640		0.340654			0.474705	0.854045		0.477071
C2	0.574852			1.168483	1.897524		2.465449		0.373545			0.575830	1.087169		0.538244
C3	0.579554			1.191419	2.059830		4.101087		0.396483			0.592198	1.277733		0.626557
C4	1.178115			1.679491	2.680473		5.429621		0.662880			0.862222	1.486867		0.784820
C5	1.544182			2.209461	5.715790		5.799465		1.019093			1.371180	3.428814		1.271462
L1		0.576163		0.533746		0.412020	0.935618			0.368336		0.611981		0.530633	0.505060
L2		0.807836		0.673294		0.484142	1.432784			0.621613		0.773226		0.846952	0.556048
L3		2.301456		1.052437		1.819896	3.484229			1.599653		1.170689		2.225846	1.045449
A1			0.691296		0.280031	1.419557	0.148387				0.618980		0.316233	0.707191	1.201824
A2			1.117525		0.474927	2.271846	0.233989				0.639503		0.428672	0.762011	1.553158

Figure 7. Monte Carlo Results for 16-Case Casebase

	Optimal AARE								
	A=3.2, B=1.05								
	CPLX	LEXP	AEXP	CPLX, LEXP	CPLX,AEXP	LEXP,AEXP	CPLX, LEXP, AEXP	Average	Median
Solver	0.422244	0.502870	0.506027	0.402476	0.309366	0.460776	0.277902	0.411666	0.422244
Cocomo	0.501787	0.550925	0.542219	0.498709	0.493536	0.539404	0.490735	0.516759	0.501787
Monte Carlo	0.426160	0.458833	0.506121	0.403859	0.394711	0.424360	0.408934	0.431854	0.424360
Difference = Monte Carlo - Solver	0.003916	-0.044037	0.000094	0.001383	0.085345	-0.036416	0.131032	0.020188	0.001383
%Diff = Monte Carlo -vs- Solver	-0.927426	8.757134	-0.018576	-0.343623	-27.587065	7.903189	-47.150434	-8.480972	-0.343623
Difference = Monte Carlo - COCOMO	-0.075627	-0.092092	-0.036098	-0.094850	-0.098825	-0.115044	-0.081801	-0.084905	-0.092092
%Diff : Monte Carlo -vs- COCOMO	-15.071534	-16.715887	-6.657458	-19.019107	-20.023869	-21.327984	-16.669078	-16.497845	-16.715887
	Optimal AARE								
	A=2.8, B=1.2								
	CPLX	LEXP	AEXP	CPLX, LEXP	CPLX,AEXP	LEXP,AEXP	CPLX,LEXP,AEXP	Average	Median
Solver	0.397368	0.502376	0.538263	0.375785	0.301374	0.502376	0.340446	0.422570	0.397368
Cocomo	0.657261	0.644702	0.628290	0.643955	0.617104	0.617818	0.604995	0.630589	0.628290
Monte Carlo	0.406513	0.448813	0.538379	0.353993	0.384272	0.448476	0.340864	0.417330	0.406513
Difference = Monte Carlo - Solver	0.009145	-0.053563	0.000116	-0.021792	0.082898	-0.053900	0.000418	-0.005240	0.000116
%Diff : Monte Carlo -vs- Solver	-2.301393	10.661934	-0.021551	5.799061	-27.506686	10.729016	-0.122780	-0.394628	-0.021551
Difference = Monte Carlo - COCOMO	-0.250748	-0.195889	-0.089911	-0.289962	-0.232832	-0.169342	-0.264131	-0.213259	-0.232832
%Diff = Monte Carlo -vs- COCOMO	-38.150446	-30.384426	-14.310430	-45.028302	-37.729783	-27.409690	-43.658377	-33.810208	-37.729783

Figure 8. AARE Comparison Among Monte Carlo, COCOMO and Solver (16 cases)

Case Removal

The Monte Carlo and Solver results tracked each other for most every factor combination for each casebase and consistently outperformed the COCOMO factor values for calculating AARE. But there were a few instances where the tools did not track each other as might have been expected. While there are many variables which could contribute to the discrepancies in results between the tools this section of the paper investigates the process of removing 'bad' cases and determining if a new case will fit into an existing casebase.

A casebase consists of a collection of data from a set of completed projects. The data depends a great deal on the judgment of the expert and their ability to consistently rate the factors associated with a project. The process becomes a difficult exercise because the expert is required to remember facts and circumstances related to previous projects. As a result of this there is a very good possibility that cases may not be rated in a consistent manner. This results in what can be classified as bad cases within a casebase. It is also not possible to use cases from different organizations due to the many variations in the local expertise and work environment.

In an attempt to identify bad cases in this papers casebases some additional processing was added to the procedure of determining the optimal AARE. The method for determining a bad case is shown in Figure 9. The process consisted of once again iteratively generating random numbers for the factor values, calculating the AARE, and then saving off the factor values for the optimal AARE. After the optimal AARE was determined with all cases in the casebase present the individual case with the highest ARE was removed from the casebase. At this point the procedure was repeated but there was one less case in the casebase then the previous set of iterations. This procedure continued until the STOP_CRITERIA was met. The STOP_CRITERIA indicates to the program when it no longer has to search for factors which will result in a lower AARE and no more cases will be removed. The process of removing one case at a time was continued until the stopping criteria was met. The STOPPING_CRITERIA was that all remaining cases in the casebase had to have an AARE of less than 10%.

```
while (STOP_CRITERIA has NOT been reached)
{
   for (NUMBER_RUNS)
   {
         /* Generate the required random numbers to be used for factor values */
         for (each FACTOR)
         {
                  for (each FACTOR_RATING of the current FACTOR)
                  {
                           get a random number between 0 and 1 and scale it to MAX_RANGE.
                           sort the random number in ascending order.
                  }
         }

         /* Calculate the ARE for each case using the randomly generated factor values */
         for (each CASE in the CASEBASE)
         {
                  calculate the Absolute Relative Error (ARE) for the current CASE. (Equation 3)
                  Add the current CASE's ARE  to the Total Absolute Error for this RUN.
         }

         /* Calculate the AARE and save it off and the associated factor values if it is the lowest */
         calculate the Absolute Average Relative Error (AARE) for this RUN.
         If (this is the lowest AARE for this NUM_RUNS)
                  save off the AARE and FACTOR values as optimal for this run.
   }
   If (the STOP_CRITERIA has NOT been reached)
         remove case from the casebase with the highest ARE from the most recent  RUN.
   Else
         terminate the process because the STOP_CRITERIA  has been met.
}
```

Figure 9. Method Flow for Removing Bad Cases

Similar to when the data was obtained when the determining the optimal AARE a variety of factor combinations were run. Runs were made for both the organic and embedded modes with each single factor, all three factors, and also varying the A and B variables with each single factor and all three factors. Equation 5 represents the basic equation with all the variables for the Absolute Relative Error for a case.

$$ARE_i = \left| \frac{estimated(MM_i) - actual(MM_i)}{actual(MM_i)} \right| = \left| \frac{A(L_i)^B (CPLX_i)(LEXP_i)(AEXP_i) - M_i}{M_i} \right|$$

Equation 5. Absolute Relative Error

The following is a summary of the factor combinations run for the process which consisted of removing cases until the stopping criteria was met. These combinations were run for both the 16 and 25-case casebases. The complete set of data obtained when running these combinations is presented in Appendix A.

1. A=3.2, B=1.05 (Embedded)
   - ratings for only CPLX are variables
   - ratings for only LEXP are variables
   - ratings for only AEXP are variables
   - ratings for only CPLX, LEXP and AEXP are variables
2. A=2.8, B=1.2 (Organic)
   - ratings for only CPLX are variables
   - ratings for only LEXP are variables
   - ratings for only AEXP are variables
   - ratings for only CPLX, LEXP and AEXP are variables

3.

- A, B and ratings for only CPLX are variables
- A, B and ratings for only LEXP are variables
- A,B and ratings for only AEXP are variables
- A,B and ratings for only CPLX, LEXP, and AEXP are variables

To better illustrate the process which was used for removing cases until the stopping criteria was met consider the data presented in Figure 10 which consists of keeping the A and B variables constant in the organic mode (A=3.2, B=1.05) and varying the factors CPLX, LEXP, and AEXP. The first series of 1 million iterations, with all 16 cases, resulted in an optimal AARE of 0.408934. During this series of iterations case number three (3) had the highest ARE of 0.912722. Therefore, at the completion of the first series case 3 was removed from the casebase and a second series of 1 million iterations was run. At the end of the second series, the new casebase of 15 cases (i.e., case 3 removed) now had an AARE of 0.369662, and case two (2) now had the highest ARE of 0.898823. This sequence was repeated until all the remaining cases had an ARE of less than 10%. For this particular set of sequences, on the fourteenth series all three remaining cases had an ARE of less than 10%.

	A=3.2, B=1.05													
	Average Relative Error (ARE)													
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14
case 1	.307479	.350631	.310700	.308289	.186879	.126832	.293985	.115881	.204070	*				
case 2	.889533	.898823	*											
case 3	.912722	*												
case 4	.086631	.231770	.111914	.044150	.122032	.095932	.204961	.060568	.063007	.096830	.128850	.306365	*	
case 5	.629606	.528064	.622025	.627701	*									
case 6	.144035	.048119	.081604	.061877	.178006	.080644	.058474	.079354	.086163	.295763	.000615	.025268	.018062	.018235
case 7	.486872	.254610	.217402	.129281	.081104	.139256	.344695	.207176	.035102	.163767	.053209	.017031	.001544	.000843
case 8	.004961	.132416	.042589	.010896	.011451	.108830	.147355	.326673	*					
case 9	.023832	.163294	.053837	.158136	.133712	.268203	.007433	.046126	.047325	.040492	.198716	.007523	.137513	*
case 10	.535203	.641153	.454886	.597983	.800611	*								
case 11	.312338	.072199	.072552	.154392	.060332	.091854	.039822	.134564	.131618	.324655	*			
case 12	.703038	.686272	.619183	.543344	.568986	.480034	*							
case 13	.553811	.528621	.427818	.313870	.352397	.218746	.466653	*						
case 14	.781775	.882367	1.284904	*										
case 15	.008548	.011323	.053524	.163573	.100028	.000218	.078446	.102430	.053350	.215509	.002290	.205685	.004530	.002850
case 16	.162555	.115276	.073919	.287786	.215475	.466323	.001029	.008727	.084972	.029698	.237420	*		
Optimal AARE	.408934	.369662	.316204	.261637	.234251	.188807	.164285	.120167	.088201	.166673	.103517	.112375	.040412	.007309
Factors	Factor Values													
C1	2.144640	1.186360	1.697248	1.595208	.527736	.862513	1.415148	.314892	1.331935	.744141	.402421	.504055	.175162	.120223
C2	2.465449	2.007115	1.712408	1.743379	1.196616	1.263784	2.205898	1.389433	2.169951	.785574	1.295022	1.513573	1.361476	1.722950
C3	4.101087	2.159647	2.206088	1.745865	1.796108	2.466702	3.703541	2.699347	2.994298	1.139391	3.111623	2.156700	2.333880	2.020319
C4	5.429621	3.647911	3.646796	4.663845	3.656933	4.909636	5.033302	4.193018	5.369241	1.638023	3.492093	3.559678	3.783523	5.582765
C5	5.799465	5.088128	4.945005	5.466055	4.578302	5.250845	5.800512	4.423592	6.331812	2.752569	4.550116	4.120010	4.521645	6.009422
L1	.935618	.218087	2.564693	3.679713	1.686327	.634887	1.317029	.905479	.597170	1.956276	2.742599	2.095124	2.893033	.097981
L2	1.432784	.242888	3.302322	3.937376	2.093017	1.014853	1.887774	1.081002	.630066	2.129910	3.363386	2.239356	3.201787	.113612
L3	3.484229	.395266	5.118034	5.477132	2.833384	1.172667	2.809904	1.886768	1.019774	4.512254	4.437647	2.512798	6.338513	2.272899
A1	.148387	1.054030	.096827	.088099	.186758	.405141	.134598	.310586	.400418	.243678	.119049	.184878	.119767	2.662661
A2	.233989	1.926433	.150449	.099012	.279240	2.136757	.195309	.512708	.618427	.502034	.148517	.239978	.159732	3.387646

Figure 10. Case Removal, 16-Case Casebase, A=3.2, B=1.05, CPLX, LEXP, AEXP

It is interesting to note how the removal of just three cases, in this situation cases 2,3, and 14, resulted in a 36% lower AARE for the casebase. The AARE dropped from 0.408934 to 0.261637. This is a significant drop in the AARE and it is possible to now consider cases 2, 3, and 14 as having been improperly characterized by the expert and classified as bad cases for this casebase. Certainly many criteria can be applied for determining what constitutes a bad case. A user may indicate that the AARE must be below 20% before no more cases are removed. For this particular instance that would mean that five cases (2, 3, 14, 5,6) would need to be removed. Or possibly that all remaining cases must have an ARE which is less than 40%. This criteria would need to be identified on a case by case basis. The stopping criteria that all cases must have an ARE of less than 10% is probably not a very realistic criteria because that indicates that the software man-month estimate would be within 10% of the actual man-months required to complete the projects. Unfortunately software tools as a whole are no where near that accurate. This criteria was used to fully illustrate how the AARE changes as a function of removing cases from a casebase.

CHAPTER 4

CONCLUSIONS

The process of estimating the man-month effort required for developing software is not a simple task. There have been many studies performed and techniques applied to solve this problem. Even with all the effort that has been expended to solving this problem the results achieved from the various techniques have not been embraced by the software community as a reliable and accurate solution.

Goodman [Goodman, 1992],condensed all the problems associated with software cost estimation rather plainly when he wrote "one conclusion seems to have been drawn by every major research project that has looked at this problem [software cost estimation], and this seems to be borne out by every industrial organization the author has spoken to ... put simply, the conclusion and the starting premise is that no one technique, metric or tool is ideal or universally accepted. Goodman felt that based on this conclusion, one should adapt a strategy employing several estimation techniques that could attack the problem from many angles as opposed to just one.

The COCOMO regression-based model presents an equation for estimating the number of man-months required to develop a software product. This model has been accepted as a good starting point for estimating man-months of effort but can only produce results which many times are considered marginal or unacceptable. It was the intent of this thesis to take the basic COCOMO model and modify its associated assumptions in the selection of factor values to achieve an improvement in the man-month estimates for a casebase compared to both the COCOMO model and the Solver math tool. Then take the modified process a step further and attempt to identify whether a new case would fit into the existing casebase and if all cases in the casebase have been classified in a consistent manner.

This does not imply that COCOMO is the 'best' method for determining man-month estimates for all software products. For many instances there are probably a combination of methods which can generate equal if not better results. The basis behind this paper was to provide a technique using Monte Carlo methods to improve the results achieved when using the COCOMO model.

This thesis accomplished the goal of providing a method for determining optimal effort multiplier values for a casebase. Using the Monte Carlo method on two industry supplied casebases, optimal effort multiplier values were determined which consistently yielded lower AARE results than those achieved with COCOMO effort multiplier values. The fact that the results using the Monte Carlo methods consistently tracked the results obtained using the Solver math tool also indicates that the optimal values were determined with the available data.

The Monte Carlo techniques were used on casebases which contained data obtained form industry sources. The effort multiplier values were supplied for the cost drivers Complexity (CPLX), Application Experience (AEXP), and Language Experience (LEXP). Since it was decided to use data from real world industry projects as opposed to fabricated projects the supplied data is considered a constraint. The author believes that the three factors used in the project are very relevant for estimating man-month effort but there are most certainly other factors which could additionally influence the final estimates. The model provided in this paper is generic in nature and allows for additional factors to be included when determining optimal factor values.

The second objective of this paper was to provide a mechanism for determining if a new case can be added to an existing casebase and if all cases within a casebase have been consistently rated. The process of applying factor effort multiplier values will vary from person to person. For example, classifying a project as having a 'Very Low' complexity is subject to an individuals experience and capabilities. For this reason it becomes important that cases which are grouped into a casebase are rated in a consistent and predictable manner.

The process of determining which case is providing the greatest AARE, removing it from the casebase, and then recalculating the AARE indicates how well a case fits into the casebase. The data clearly demonstrates that as cases are removed from the casebase the AARE can be lowered.

Certainly there is the opportunity to continue to improve on the basic process of estimating man-month effort using the process presented in this paper. One item which was identified in the paper is to determine how many iterations should be run when the maximum range for the random generator is increased above 2.5. As the number of iterations is increased the processing time also increases, so at some point it can become an extremely time consuming effort to calculate the optimal effort adjustment factors.

A second item would be to investigate how much effect additional factors could have on the AARE. This paper used three factors because that was what was provided by the industry sources. The basic model allows for additional factors to be added, so if a casebase with the appropriate data was obtained perhaps the AARE could be brought down to even lower levels.

A third option might be to investigate the use of neural networks combined with the process of using Monte Carlo methods for determining the optimal factor values and how well a case fits into an existing casebase. The capability of neural nets to learn from their inputs and adjust their outputs accordingly might provide a mechanism for generating man-month estimates without using a pre-defined formula.

In closing, this paper was intended to present an enhancement to an existing method for determining man-month estimates for software development projects. Perhaps this incremental improvement will result in further studies which will continue to improve the overall goal of achieving more accurate estimates.

Appendix A

Case Removal Data for 16-Case Casebase

	A=3.2, B=1.05											
	Average Relative Error (ARE)											
	Series 1	2	3	4	5	6	7	8	9	10	11	12
case 1	.551796	.570020	.565945	.561603	.574036	*						
case 2	.790113	.812519	.811211	*								
case 3	.930683	.897149	*									
case 4	.440386	.169655	.175814	.193009	.182071	.172956	.169979	.144377	.143958	.145775	.169992	*
case 5	.054691	.011808	.021396	.031613	.002357	.004064	.000747	.007388	.010078	.007657	.002726	.035902
case 6	.320185	.008698	.001215	.019673	.006386	.004687	.008303	.039404	.039914	.037706	.008288	.013216
case 7	.015898	.460193	.449361	.419123	.438357	.454387	.459622	*				
case 8	.012837	.036291	.006398	.013089	.015993	.013164	.037368	.004965	.016480	.005224	.004704	.001482
case 9	.287945	.331557	.360301	.350539	.353301	.358176	.348080	.346384	.339450	*		
case 10	.634291	.673332	.671052	.679751	*							
case 11	.324147	.002818	.004621	.025387	.012177	.001169	.002426	.033346	.033852	.031657	.002411	.007310
case 12	.628093	.448171	.452265	.463692	.456423	.450365	.448387	.431373	*			
case 13	.441206	.170872	.177023	.194192	.183270	.174168	.171196	.145632	.145213	.147028	*	
case 14	1.231442	*										
case 15	.106061	.038317	.006332	.008831	.004541	.003031	.012651	.015285	.026056	.000048	.000977	.010820
case 16	.048791	.556177	.544633	.512408	.532906	.549990	*					
Optimal AARE	.426160	.345838	.303398	.267147	.230152	.198742	.165876	.129795	.094375	.053585	.031516	.013746
Factors	Factor Values											
C1	.565721	.505328	.508854	.495398	.332517	.031403	.345572	.116950	.193215	.505181	.003901	.362711
C2	.574852	.539644	.516438	.524319	.522089	.518154	.526304	.527673	.533271	.519754	.520237	.514105
C3	.579554	.592975	.575870	.579699	.581360	.564677	.593591	.575050	.562779	.575198	.569517	.571361
C4	1.178115	1.130214	1.140924	1.152337	1.119657	1.112485	1.117858	1.108771	1.128282	1.108471	1.120069	1.076920
C5	1.544182	2.291228	2.274231	2.226785	2.256966	2.282119	2.290333	2.360977	2.362135	2.357120	2.290297	2.301491

Figure A-1. 16 Cases, CPLX, A=3.2, B=1.05

	A=3.2, B=1.05													
	Average Relative Error (ARE)													
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14
case 1	.692666	.694006	.693006	.693296	.694032	*								
case 2	.786239	.785605	.786492	*										
case 3	.963737	*												
case 4	.165948	.166913	.170153	.170417	.163253	.164014	.165667	.160557	.150854	.158073	.173540	.160449	*	
case 5	.276796	.279951	.277596	.278279	.280012	.280750	.278486	.281092	.275729	.278693	.277863	*		
case 6	.644356	.645907	.644749	.645085	.645937	.646300	*							
case 7	.632813	.631724	.633248	.631629	.633866	.632197	.634378	*						
case 8	.006910	.009898	.005719	.010156	.004023	.008599	.002619	.000384	.002159	.003618	.000136	.000239	.000113	.002400
case 9	.000646	.003719	.000460	.001405	.003803	.004824	.001692	.005297	.002123	.001978	.000829	.127719	.137572	*
case 10	.477776	.480054	.478353	.478847	.480098	.480631	.478996	.480878	.477005	*				
case 11	.007295	.006129	.002217	.001897	.010550	.009630	.007634	.013805	.025524	.016806	.001874	.013935	.001045	.000850
case 12	.805437	.806286	*											
case 13	.707668	.708943	.707991	.708267	*									
case 14	.167375	.162283	.166085	.164982	.162185	.160993	.164647	.160442	.169098	.164314	.165654	.017621	.006127	.010246
case 15	.554341	.547561	.552622	.551154	.547429	.545843	.550708	.545109	*					
case 16	.451327	.453720	.451933	.452451	.453766	.454326	.452609	.454585	.450517	.452766	*			
Optimal AARE	.458833	.425513	.397902	.368297	.339913	.308010	.273744	.233572	.194126	.153749	.103316	.006835	.036214	.004499
												.005286		
Factors	Factor Values													
L1	.576163	.577872	.575481	.578020	.574511	.577129	.573708	.572428	.570973	.570139	.572131	.572072	.003737	.570835
L2	.807836	.804312	.806943	.806180	.804244	.803420	.805948	.803038	.406760	.805717	.806645	.704205	.002189	.699101
L3	2.301456	2.298793	2.289854	2.289124	2.308894	2.306792	2.302231	2.316331	.430244	2.323187	2.280508	2.316629	.000640	2.286730

Figure A-2. 16 Cases, LEXP, A=3.2, B=1.05

	A=3.2, B=1.05													
	Average Relative Error (ARE)													
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14
case 1	.575154	.575126	.575154	.574950	.575116	.575026	.574681	.574961	.574817	*				
case 2	.743142	.743259	.743391	.743259	.787629	*								
case 3	.949872	*												
case 4	.749100	.749214	.749344	.749215	*									
case 5	.000273	.000207	.000274	.000206	.000185	.000027	.000839	.000181	.000519	.000360	.000919	.000040	.000448	.000117
case 6	.508372	.508340	.508372	.508136	.508329	.508224	.507825	.508149	.507982	.508414	*			
case 7	.558783	.558984	.559211	.558985	.635200	.634916	.635195	*						
case 8	.209918	.209367	.208743	.209363	.000364	.001144	.000378	.000521	.001167	.000307	.000325	.000572	.000364	.090826
case 9	.383252	.383343	.383251	.383915	.383374	.383667	.384790	.383880	.384348	.383132	.382358	*		
case 10	.278099	.278052	.278099	.277753	.278035	.277883	.277296	.277771	.277527	.278162	.278566	.277873	*	
case 11	.696985	.697123	.697279	.697124	.749466	.749271	*							
case 12	.833258	.833333	*											
case 13	.749468	.749582	.749711	*										
case 14	.000455		.000516	.000003	.172820	.172175	.172809	.172691	.174087	.172868	.173391	.173595	.172820	*
case 15	.332089	.331483	.330796	.331479	.101376	.102235	.101391	.101548	.099690	.101313	.100617	.100345	.101376	.000977
case 16	.529780	.529994	.530237	.529996	.611220	.610917	.611215	.611160	*					
Optimal AARE	.506125	.476494	.451027	.428030	.400260	.365044	.326642	.292318	.252517	.206365	.156029	.110485	.068752	.030640
Factors	Factor Values													
A1	.692325	.692010	.691653	.692008	.572417	.572864	.572425	.572507	.571541	.572384	.572022	.571881	.572417	.520237
A2	1.116719	1.116793	1.116719	1.117254	1.116817	1.117054	1.117961	1.117226	1.117604	1.116622	1.115998	1.117069	1.116524	1.117155

Figure A-3. 16 Cases, AEXP, A=3.2, B=1.05

	A=3.2, B=1.05													
	Average Relative Error (ARE)													
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14
case 1	.307479	.350631	.310700	.308289	.186879	.126832	.293985	.115881	.204070	*				
case 2	.889533	.898823	*											
case 3	.912722	*												
case 4	.086631	.231770	.111914	.044150	.122032	.095932	.204961	.060568	.063007	.096830	.128850	.306365	*	
case 5	.629606	.528064	.622025	.627701	*									
case 6	.144035	.048119	.081604	.061877	.178006	.080644	.058474	.079354	.086163	.295763	.000615	.025268	.018062	.018235
case 7	.486872	.254610	.217402	.129281	.081104	.139256	.344695	.207176	.035102	.163767	.053209	.017031	.001544	.000843
case 8	.004961	.132416	.042589	.010896	.011451	.108830	.147355	.326673	*					
case 9	.023832	.163294	.053837	.158136	.133712	.268203	.007433	.046126	.047325	.040492	.198716	.007523	.137513	*
case 10	.535203	.641153	.454886	.597983	.800611	*								
case 11	.312338	.072199	.072552	.154392	.060332	.091854	.039822	.134564	.131618	.324655	*			
case 12	.703038	.686272	.619183	.543344	.568986	.480034	*							
case 13	.553811	.528621	.427818	.313870	.352397	.218746	.466653	*						
case 14	.781775	.882367	1.284904	*										
case 15	.008548	.011323	.053524	.163573	.100028	.000218	.078446	.102430	.053350	.215509	.002290	.205685	.004530	.002850
case 16	.162555	.115276	.073919	.287786	.215475	.466323	.001029	.008727	.084972	.029698	.237420	*		
Optimal AARE	.408934	.369662	.316204	.261637	.234251	.188807	.164285	.120167	.088201	.166673	.103517	.112375	.040412	.007309
Factors	Factor Values													
C1	2.144640	1.186360	1.697248	1.595208	.527736	.862513	1.415148	.314892	1.331935	.744141	.402421	.504055	.175162	.120223
C2	2.465449	2.007115	1.712408	1.743379	1.196616	1.263784	2.205898	1.389433	2.169951	.785574	1.295022	1.513573	1.361476	1.722950
C3	4.101087	2.159647	2.206088	1.745865	1.796108	2.466702	3.703541	2.699347	2.994298	1.139391	3.111623	2.156700	2.333880	2.020319
C4	5.429621	3.647911	3.646796	4.663845	3.656933	4.909636	5.033302	4.193018	5.369241	1.638023	3.492093	3.559678	3.783523	5.582765
C5	5.799465	5.088128	4.945005	5.466055	4.578302	5.250845	5.800512	4.423592	6.331812	2.752569	4.550116	4.120010	4.521645	6.009422
L1	.935618	.218087	2.564693	3.679713	1.686327	.634887	1.317029	.905479	.597170	1.956276	2.742599	2.095124	2.893033	.097981
L2	1.432784	.242888	3.302322	3.937376	2.093017	1.014853	1.887774	1.081002	.630066	2.129910	3.363386	2.239356	3.201787	.113612
L3	3.484229	.395266	5.118034	5.477132	2.833384	1.172667	2.809904	1.886768	1.019774	4.512254	4.437647	2.512798	6.338513	2.272899
A1	.148387	1.054030	.096827	.088099	.186758	.405141	.134598	.310586	.400418	.243678	.119049	.184878	.119767	2.662661
A2	.233989	1.926433	.150449	.099012	.279240	2.136757	.195309	.512708	.618427	.502034	.148517	.239978	.159732	3.387646

Figure A-4. 16 Cases, CPLX, LEXP, AEXP, A=3.2, B=1.05

	A=2.8, B=1.2												
	Average Relative Error (ARE)												
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13
case 1	.580188	.594089	.603408	.572856	.598925	.593990	.593985	*					
case 2	.828730	.834504	*										
case 3	.943909	*											
case 4	.338014	.366366	.369136	.374188	.360948	.117043	.080372	.083352	.083556	.082146	.102144	*	
case 5	.037778	.003416	.019622	.055902	.008541	.003661	.003673	.002329	.018787	.003392	.006284	.001910	.000877
case 6	.249143	.281302	.284444	.290174	.275157	.001493	.043086	.039706	.039475	.041074	.018392	.028962	.002195
case 7	.248094	.194639	.189416	.179892	.204853	.664708	*						
case 8	.011444	.053131	.009740	.004064	.015290	.002554	.000234	.008190	.007124	.001948	.001019	.004402	.012585
case 9	.182630	.184478	.215089	.216380	.229623	.207113	.198523	.220646	.218810	.224005	*		
case 10	.680766	.691529	.721244	.694333	*								
case 11	.319688	.348825	.351672	.356863	.343257	.092600	.054914	.057977	.058186	.056737	.077288	.067711	*
case 12	.713867	.726122	.727320	*									
case 13	.484834	.506899	.509054	.512985	.502682	.312872	.284335	.286654	.286813	*			
case 14	.716796	.643267	.636083	.622983	.657317	*							
case 15	.064395	.061988	.022125	.020445	.003200	.032512	.043698	.014889	.017280	.010515	.006160	.011434	.019823
case 16	.039157	.005349	.009698	.017627	.003155	.386028	.443592	.438914	*				
Optimal AARE	.402465	.366394	.333432	.301438	.266913	.219507	.174641	.128073	.091254	.059974	.035214	.022884	.008870
Factors	Factor Values												
C1	.360092	.347952	.314433	.344788	.167373	.236950	.336042	.314891	.203307	.295545	.303809	.277156	.305161
C2	.363549	.362727	.349111	.348538	.342647	.352659	.356480	.346640	.347457	.345146	.339450	.337649	.348325
C3	.372456	.387806	.371828	.369738	.373872	.367301	.368328	.365226	.370865	.367524	.368617	.369862	.372876
C4	.663986	.642001	.627261	.675582	.634351	.642157	.642165	.638325	.627795	.641985	.643836	.638593	.640376
C5	1.066542	1.020863	1.016400	1.008261	1.029591	1.422554	1.481635	1.476833	1.476505	1.478776	1.446558	1.461572	1.423552

Figure A-5. 16 Cases, CPLX, A=2.8, B=1.2

	A=2.8, B=1.2											
	Average Relative Error (ARE)											
	Series 1	2	3	4	5	6	7	8	9	10	11	12
case 1	.606979	.605176	.607764	.607760	.595361	.595155	*					
case 2	.824809	.823838	.825363	*								
case 3	.967309	*										
case 4	.007120	.008495	.012290	.018937	.000380	.017820	.013296	.000916	.028355	.001476	.028262	.019214
case 5	.028448	.023993	.030389	.030380	.000271	.000779	.025620	.000766	.014743	.025620	.003769	.010861
case 6	.562378	.560371	.563252	.563248	.549441	.549213	.561104	.549218	*			
case 7	.568964	.566577	.570328	.568790	.567048	.569768	.570008	*				
case 8	.000256	.005796	.002908	.000660	.004703	.001608	.002165	.000548	.002570	.000624	.003376	.000900
case 9	.397580	.403989	.394789	.394801	.438893	.439624	.401648	.439605	.417295	.401648	*	
case 10	.448918	.446390	.450018	.450013	.432627	.432339	.447313	.432346	.441144	*		
case 11	.020366	.018953	.015053	.008221	.027293	.009370	.014019	.028624	.001457	.029200	.001362	.007937
case 12	.833233	.832468	*									
case 13	.699746	.698369	.700345	.700343	.690870	*						
case 14	.000601	.005190	.001397	.001388	.030180	.030703	.003514	.030690	.014717	.003514	.033782	.018715
case 15	.819954	.828301	.816319	.816335	*							
case 16	.394347	.391570	.395557	.395552	.376444	.376127	.392584	.376135	.385803	.392584	.374263	*
Optimal AARE	.448813	.414632	.384698	.350495	.309459	.274773	.243127	.206539	.163260	.122095	.074136	.011525
Factors	Factor Values											
L1	.368336	.370376	.367171	.368485	.369973	.367649	.367444	.368040	.367295	.368012	.366998	.368573
L2	.621613	.624464	.620372	.620377	.639989	.640314	.623423	.640305	.630382	.623423	.642226	.632866
L3	1.599653	1.597437	1.591323	1.580613	1.610512	1.582415	1.589702	1.612599	1.565441	1.613502	1.565590	1.580167

Figure A-6. 16 Cases, LEXP, A=2.8, B=1.2

	A=2.8, B=1.2												
	Average Relative Error (ARE)												
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13
case 1	.595701	.595503	.595391	.595447	.459893	.286821	.102357	.102190	.051729	.051673	.051892	.052005	.051934
case 2	.705087	.704493	.705512	.695833	.593854	.512592	.512077	.324677	.287065	.286683	.286886	*	
case 3	.966371	*											
case 4	.615146	.614370	.615700	.603070	.469990	.363945	.363272	.118721	.069638	.069139	.069404	.069453	.069416
case 5	.000569	.000081	.000195	.000058	.335149	.762984	*						
case 6	.549820	.549600	.549475	.549537	.398600	.205888	.000490	.000304	.055884	.055945	.055701	.055576	.055655
case 7	.274405	.272943	.275450	.251637	.000732	.199203	.200472	.661543	*				
case 8	.683807	.687200	.681383	.736642	*								
case 9	.437684	.438386	.438784	.438587	.920615	*							
case 10	.433104	.432827	.432670	.432748	.242678	.000002	.258648	.258882	.329637	.329715	*		
case 11	.604492	.603695	.605061	.592082	.455318	.346337	.345646	.094324	.043883	.043370	.043642	.043692	.043655
case 12	.833653	.833318	*										
case 13	.700502	.699898	.700933	.691104	.587540	.505015	.504491	.314178	.275981	.275593	.275799	.275837	*
case 14	.001919	.000092	.003356	.029399	.374528	.649545	.651291	*					
case 15	.815370	.819028	.812757	*									
case 16	.395873	.394655	.396742	.376916	.168014	.001549	.000492	.383394	.460442	*			
Optimal AARE	.538344	.509739	.486672	.461005	.417243	.348535	.293924	.250913	.196782	.158874	.130554	.099313	.055165
Factors	Factor Values												
A1	.620048	.621297	.619155	.639504	.853912	1.024763	1.025848	1.419850	1.498929	1.499732	1.499305	1.499227	1.499286
A2	.639451	.639763	.639940	.639852	.854248	1.127984	1.419737	1.420001	1.499812	1.499900	1.499554	1.499375	1.499488

Figure A-7. 16 Cases, AEXP, A=2.8, B=1.2

	A=2.8, B=1.2								
	Average Relative Error (ARE)								
	Series 1	2	3	4	5	6	7	8	9
case 1	.571457	.611462	.521957	.568944	.502624	.592286	*		
case 2	.862268	.862621	*						
case 3	.942251	*							
case 4	.008442	.089497	.042815	.011222	.053986	.079605	.017809	.011765	.011765
case 5	.059361	.039531	.181727	.065573	.229517	.007871	.057956	.000343	.000343
case 6	.226944	.114822	.093362	.249545	.043819	.161742	.005501	.017694	.017694
case 7	.096857	.004788	.017008	.127134	.030038	.032704	.060514	.088923	.088923
case 8	.032789	.060000	.004936	.035902	.150437	.044354	.106473	.151893	*
case 9	.045114	.023256	.012184	.020816	.025084	.106898	.097791	.028316	.028316
case 10	.634736	.625033	.744939	.682296	*				
case 11	.019007	.119657	.071684	.039216	.083164	.054125	.045985	.039773	.039773
case 12	.772046	.748250	.760510	*					
case 13	.589583	.546739	.568812	.622671	.506173	.511398	.553611	*	
case 14	.367722	.510499	.436941	.257453	.645685	*			
case 15	.053108	.005519	.086336	.025825	.004820	.011964	.064761	.018562	.018562
case 16	.172134	.085712	.130236	.238878	.003886	.014426	.099575	.034065	.034065
Optimal AARE	.340864	.296493	.262389	.226575	.189936	.147034	.110998	.043482	.029930
Factor	Factor Values								
C1	.477071	.653406	.372266	.624857	.093611	.250975	.352735	.170870	.170870
C2	.538244	.703096	.582518	.791672	.822373	.637445	.776370	.593800	.593800
C3	.626557	.883022	.702569	.950544	.870704	.888969	.920556	.726667	.726667
C4	.784820	.949346	.978311	1.188752	1.418908	1.034802	.958365	.830947	.830947
C5	1.271462	1.942398	1.666328	1.858657	2.449775	1.910715	2.246109	1.811497	1.811497
L1	.505060	.479147	.693838	.533776	.503343	1.196761	1.010416	.296265	.296265
L2	.556048	.523652	.712691	.559025	.584636	1.464547	1.163117	.403867	.403867
L3	1.045449	.979531	1.341342	1.165886	.971054	2.146945	2.063832	.620301	.620301
A1	1.201824	.922568	.751686	.751831	.713829	.361482	.353745	1.450668	1.450668
A2	1.553158	1.236147	1.084409	1.025923	.948308	.425499	.540718	1.907179	1.907179

Figure A-8. 16 Cases, CPLX, LEXP, AEXP, A=2.8, B=1.2

	A= variable, B= variable										
	Average Relative Error (ARE)										
	Series 1	2	3	4	5	6	7	8	9	10	11
case 1	.633412	.610969	.614023	.628411	.621881	.561699	*				
case 2	.869940	.863518	*								
case 3	.950983	*									
case 4	.078238	.321807	.203440	.148625	.193927	.029319	.026458	.055986	.095364	.050517	.050517
case 5	.037320	.002254	.008466	.010654	.011011	.201197	.039230	.082795	.018632	.076509	.076509
case 6	.038887	.273761	.163544	.129136	.180333	.011339	.007081	.020905	.109590	*	
case 7	.892888	.355541	.624087	.782706	.698003	*					
case 8	.022788	.054710	.017493	.044646	.041203	.012381	.099727	.024481	.012855	.013893	.013893
case 9	.027659	.095275	.063674	.006977	.031721	.038141	.067407	.033246	.083678	.134707	*
case 10	.736678	.730816	.857341	*							
case 11	.222909	.391260	.317202	.313908	.359384	.245471	.250105	.254810	*		
case 12	.764965	.795627	.787688	.807504	*						
case 13	.471684	.571773	.531570	.544866	.578237	.524620	.528407	*			
case 14	.410208	.226236	.273871	.154979	.053745	.084868	.072160	.005696	.035894	.068250	.068250
case 15	.019881	.016223	.000105	.008509	.014600	.065351	.042541	.107614	.019901	.007882	.007882
case 16	.164192	.082407	.024594	.023333	.045797	.115021	.107806	.097126	.015549	.001282	.001282
Optimal AARE	.396414	.359478	.320507	.277250	.235820	.171764	.124092	.075851	.048933	.050434	.036389
Factors	Factor Values										
C1	.383009	.487882	.202303	.142904	.094113	.268839	.134966	.193496	.038216	.009397	.009397
C2	.395394	.511152	.382085	.279507	.224028	.337396	.318468	.327197	.207148	.337250	.337250
C3	.416647	.561995	.395267	.281337	.239727	.377095	.377272	.367251	.215399	.354394	.354394
C4	.590644	.841558	.618426	.424750	.335973	.609203	.484244	.457421	.303547	.431091	.431091
C5	1.457988	1.457191	1.256835	.947382	.695438	1.368367	1.254218	1.386674	.706273	1.165497	1.165497
A	1.251177	1.195542	1.344111	1.474138	1.794466	.832036	.892608	.719047	1.441540	.806030	.806030
B	1.384025	1.325771	1.368576	1.425944	1.438864	1.513961	1.517099	1.549303	1.510088	1.538862	1.538862

Figure A-9. 16 Cases, CPLX, A=variable, B=variable

	A= variable, B= variable											
	Average Relative Error (ARE)											
	Series 1	2	3	4	5	6	7	8	9	10	11	12
case 1	.644558	.618805	.624838	.653661	.637297	.583815	*					
case 2	.799667	.809354	.811514	*								
case 3	.967400	*										
case 4	.046480	.006385	.059235	.017840	.036688	.028278	.005976	.010163	.071799	.017427	.005679	.004502
case 5	.133364	.062922	.081986	.155940	.114748	.030871	.012063	.004529	.034476	.096050	.147995	.015132
case 6	.599940	.573692	.578928	.610048	.592095	.537308	.553424	*				
case 7	.554201	.549631	.569420	.545616	.591684	.539044	.531505	.561153	*			
case 8	.083933	.065055	.034252	.106501	.010662	.062475	.094508	.027139	.009039	.015315	.009634	.013821
case 9	.233180	.342069	.310001	.200627	.260692	.485250	.418929	.429134	.375703	.294549	.207775	*
case 10	.474114	.453751	.452665	.486678	.465520	.420965	.434157	.428317	.418879	*		
case 11	.025140	.039945	.000417	.057496	.032182	.007891	.028194	.046756	.007523	.028635	.085957	.012109
case 12	.831335	.830760	*									
case 13	.711300	.701467	.700117	.718127	*							
case 14	.011992	.015442	.037406	.010298	.024074	.042530	.036029	.049133	.110435	.019802	.039449	.051516
case 15	.686999	.782892	.769179	.645125	.718296	*						
case 16	.429265	.402709	.404032	.443130	.419392	.362464	.379250	.373155	.368357	.423657	*	
Optimal AARE	.452054	.416992	.388142	.357776	.325278	.281899	.249403	.214387	.174527	.127919	.082748	.019416
Factors	Factor Values											
L1	.346901	.320978	.331054	.337008	.321623	.298964	.326474	.283929	.319989	.328934	.331379	.308694
L2	.497853	.497193	.522987	.461941	.515315	.501621	.514417	.480977	.547440	.515673	.507684	.538337
L3	1.371958	1.324646	1.344242	1.346066	1.405059	1.191587	1.302334	1.227821	1.318999	1.408361	1.481199	1.334909
A	3.782663	3.651000	3.626839	3.995776	3.679508	3.579974	3.537449	3.841488	3.743920	3.399205	3.900761	2.908532
B	1.158075	1.183026	1.169021	1.156699	1.161208	1.206080	1.193968	1.192309	1.163156	1.182803	1.143456	1.232503

Figure A-10. 16 Cases, LEXP, A=variable, B=variable

	A= variable, B= variable												
	Average Relative Error (ARE)												
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13
case 1	.552821	.560362	.556491	.546271	.568694	.557072	.558603	.552456	.564664	*			
case 2	.761892	.761466	.757618	.762992	.763710	*							
case 3	.932772	*											
case 4	.809714	.802816	.807155	*									
case 5	.016953	.005573	.007848	.037433	.014210	.011962	.005134	.019927	.004328	.019148	.004156	.001402	.005726
case 6	.468536	.479842	.472586	.463020	.489407	.475489	.475951	.468975	.484908	.489089	*		
case 7	.681141	.666884	.677199	.669876	.671784	.670434	.680902	*					
case 8	.018371	.005721	.003691	.002105	.006598	.001206	.022033	.041812	.003457	.021864	.020165	.014576	.021488
case 9	.369669	.360507	.356598	.403216	.332966	.367939	.355167	.375958	.347048	.322978	.354623	*	
case 10	.131933	.165507	.136519	.137558	.178968	.155572	.147571	.138321	.173477	.170301	.151415	.155878	*
case 11	.742989	.738645	.738889	.740558	.741599	.740066	*						
case 12	.829619	.832093	*										
case 13	.774162	.772690	.770251	.774210	*								
case 14	.022286	.007444	.042876	.002022	.000066	.008122	.009401	.016206	.001340	.005999	.006395	.008615	.002380
case 15	.096565	.120730	.113321	.112179	.107350	.113543	.091859	.069422	.110532	.091804	.093607	.099604	.091621
case 16	.596777	.590718	.590246	.593667	.595248	.592796	.599143	.607746	*				
Optimal AARE	.487887	.458067	.430806	.403470	.372550	.335837	.294576	.254536	.211219	.160169	.105060	.056015	.030304
Factors	Factor Values												
A1	.678831	.658825	.696799	.689355	.706311	.648919	.689266	.660884	.701134	.663368	.690838	.662091	.668111
A2	1.488260	1.375189	1.494330	1.497413	1.465775	1.376219	1.494480	1.481316	1.462649	1.401598	1.491027	1.415253	1.442439
A	3.942121	3.891584	3.932328	3.705703	3.616390	3.975462	3.808708	3.856756	3.625947	3.933926	3.775312	3.937955	3.848967
B	.946052	.963569	.943756	.962480	.961327	.960151	.950084	.952456	.963384	.951683	.952288	.953858	.955625

Figure A-11. 16 Cases, AEXP, A=variable, B=variable

	A= variable, B= variable											
	Average Relative Error (ARE)											
	Series 1	2	3	4	5	6	7	8	9	10	11	12
case 1	.385411	.580129	.569764	.495205	.593919	.522357	*					
case 2	.903324	.909508	*									
case 3	.936789	*										
case 4	.036321	.029495	.078475	.133576	.060634	.108934	.073998	.073998	.056161	.014226	.103685	*
case 5	.543597	.098833	.113771	.300954	.063228	.298037	.000909	.000909	.012507	.100263	.036757	.036757
case 6	.064553	.089875	.130238	.012545	.123238	.108995	.215485	*				
case 7	.031702	.034006	.097876	.115754	.036223	.005140	.032895	.032895	.191621	*		
case 8	.101741	.105284	.170193	.069383	.079575	.101190	.111448	.111448	.077681	.064710	.022167	.022167
case 9	.100165	.200373	.144267	.010648	.041477	.104892	.007837	.007837	.086932	.000083	.000246	.000246
case 10	.690798	.686907	.675096	.722218	*							
case 11	.011003	.172579	.047158	.016310	.097085	.164369	.068212	.068212	.151384	.058058	.006351	.006351
case 12	.748083	.789600	.823913	*								
case 13	.519199	.532970	.624444	.483113	.610182	*						
case 14	.511505	.262402	.056521	.478193	.051938	.016221	.048613	.048613	.022503	.027958	.084346	.084346
case 15	.074748	.175005	.058405	.037893	.116661	.024681	.068192	.068192	.019043	.115431	*	
case 16	.007366	.023997	.189568	.108144	.144726	.000267	.172383	.172383	*			
Optimal AARE	.354144	.312731	.269978	.229534	.168240	.132280	.079997	.064943	.077229	.054390	.042259	.029973
Factors	Factor Values											
C1	.300297	.352866	.396433	.182534	.285980	.384120	.170503	.170503	.226107	.155883	.465377	.465377
C2	.384911	.385664	.423432	.202248	.470338	.419691	.436945	.436945	.457175	.377441	.586358	.586358
C3	.563355	.489114	.536363	.349967	.681723	.739916	.585199	.585199	.681206	.472197	.759099	.759099
C4	.786670	.531230	.614865	.395357	.722771	.764317	.649041	.649041	.734529	.500534	.899831	.899831
C5	1.088766	1.081275	1.157309	.735777	1.465858	1.378823	1.240589	1.240589	1.032684	.917415	1.415081	1.415081
L1	.580429	.757562	.614564	.727866	.654357	.595803	.563055	.563055	.462461	.639553	.512673	.512673
L2	.773856	1.199051	.835571	1.069821	.865167	1.156389	.749808	.749808	.614950	.711170	.629899	.629899
L3	1.197799	1.453100	1.478541	1.478802	1.369647	1.431472	1.268266	1.268266	1.078451	1.469311	1.208813	1.208813
A1	1.167902	1.164322	.396757	.816591	.774289	.210179	.460449	.460449	1.104606	.651655	.639240	.639240
A2	1.470796	1.263009	.532302	.942008	.967776	.210282	.538999	.538999	1.113763	.858960	.709509	.709509
A	2.471866	1.100099	3.820817	3.261549	1.406074	3.316473	3.305835	3.305835	1.984753	3.418103	2.768010	2.768010
B	1.248358	1.373593	1.340466	1.326848	1.374978	1.488322	1.357363	1.357363	1.324199	1.294081	1.311166	1.311166

Figure A-12. 16 Cases, CPLX, LEXP, AEXP, A=variable, B=variable

Appendix B

Case Removal Data for 25-Case Casebase

	A= 3.2, B=1.05																
	Average Relative Error (ARE)																
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
case 1	.137623	.210311	.218372	.206001	.148483	.139748	.146164	.151765	.140651	.131380	.130431	.206700	.212990	.194275	.189500	.192412	.139952
case 2	.576553	.497838	.404042	.409752	.405717	.404365	.310798	.282018	.280346	.285536	.280088	.280044	*				
case 3	.152155	.005452	.193253	.181822	.189901	.192608	.379951	.437576	.440924	.430533	*						
case 4	.416840	.308436	.179264	.187126	.181569	.179707	.050850	.011215	.008912	.016059	.008558	.008496	.008091	.010978	.048049	.052208	.049431
case 5	.543267	.458364	.357196	.363354	.359002	.357544	.256622	.225580	.223776	.229374	.223499	.223451	.223133	.225394	.254428	*	
case 6	.471027	.744478	*														
case 7	.125934	.197875	.205854	.193609	.136683	.128037	.134387	.139930	.128930	.119755	.118816	.194301	.200526	.182004	.177278	.180160	.128239
case 8	1.579049	*															
case 9	.736888	.720030	.719074	.720997	.734786	*											
case 10	.263201	.215993	.213316	.218701	.257315	.263773	.258160	.254502	.264725	.267121	.273024	.219604	.215367	.226310	.229433	.227739	*
case 11	.001131	.062691	.069769	.058907	.008405	.000734	.006368	.011286	.001527	.006613	.007446	.059520	.065043	.048611	.044419	.046975	.000914
case 12	.289622	.157570	.000218	.009796	.003027	.000758	.156209	.204490	.207296	.198590	.207728	.207802	.208296	.204780	.159621	.154556	.157937
case 13	.065388	.005671	.000952	.009212	.056465	.063642	.058371	.053770	.062900	.070517	.071296	.008638	.003470	.018845	.022768	.020376	.063474
case 14	.215129	.441011	.710167	.693784	.705362	.709242	*										
case 15	.737367	.720586	.718725	.721581	*												
case 16	.474733	.441172	.437449	.443162	.469719	.473752	.470790	.468204	.473335	*							
case 17	.384914	.270575	.134331	.142624	.136763	.134799	.001113	.042918	.045347	.037808	.045721	.045785	.046213	.043168	.004067	.000319	.002609
case 18	.413131	.503673	.508808	.498480	.424420	.412035	.422800	.429815	.410209	.405613	.394292	*					
case 19	.190140	.138251	.135307	.141227	.183670	.190768	.184598	.180578	.191814	.194448	.200936	.142220	.137562	.149590	.153023	.151161	.189327
case 20	.545345	.460829	.360121	.366251	.361919	.360467	.260005	.229104	.227308	.232881	.227032	.226984	.226668	.228919	*		
case 21	.175320	.393801	.654140	.638293	.649493	.653246	.912948	*									
case 22	.171311	.118215	.115204	.121261	.164691	.171954	.165641	.161527	.173025	.175720	.182359	.122277	.117511	.129819	.133332	.131426	.170479
case 23	.550875	.467387	.367904	.373959	.369680	.368246	.269006	.238480	.236706	.242211	.236434	.236386	.236074	*			
case 24	.746134	.729868	.728945	*													
case 25	.588500	.562135	.560639	.563647	.585213	.588819	.585685	.583642	*								
Optimal AARE	.422063	.368009	.347524	.330161	.311061	.289712	.264761	.228133	.206925	.190260	.173844	.155872	.146227	.138558	.128720	.115733	.100263
Factros	Factor Values																
C1	.871347	.927176	.930342	.923974	.878308	.870671	.877309	.881634	.869545	.866711	.859731	.922905	.927917	.914976	.911282	.913285	.872222
C2	.871654	.927347	.933524	.924045	.879975	.873281	.878197	.882489	.873973	.866870	.866143	.924580	.929400	.915061	.911402	.913633	.873438
C3	.881113	1.044904	1.240074	1.228195	1.236591	1.239404	1.434098	1.493984	1.497464	1.486664	1.497999	1.498091	1.498704	1.494343	1.438330	1.432047	1.436242

Figure B-1. 25 Cases, CPLX, A=3.2, B=1.05

	A= 3.2, B=1.05																
	Average Relative Error (ARE)																
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
case 1	.356358	.356746	.360930	.364899	.369170	.365215	.357327	.349684	.354026	.336483	.349756	*					
case 2	.704807	.648548	.652033	.652479	.650049	*											
case 3	.000011	.000297	.003382	.006308	.009457	.006541	.000725	.004910	.001709	.014643	.004857	.012839	.032018	.035512	.012552	.011175	.012167
case 4	.312178	.311982	.309860	.307847	.305681	.307687	.311687	.315563	.313361	.322257	.315526	.303355	.290163	.287760	*		
case 5	.137071	.137501	.138323	.140778	.138979	.141336	.145812	.141793	.153258	.139789	.146121	.142564	.138785	.134092	.141986	.136454	.130545
case 6	.735037	.735534	*														
case 7	.342421	.342806	.346947	.350875	.355102	.351188	.343381	.335816	.340113	.322751	.335887	.359641	*				
case 8	.797903	*															
case 9	.686188	.686099	.685131	.684212	*												
case 10	.121225	.120974	.118263	.115692	.112924	.115487	.120597	.125549	.122736	.134102	.125503	.109953	.093098	.090028	.110205	.111415	.110543
case 11	.296113	.161963	.170273	.171338	.165542	.169251	.171128	.170966	.173555	.171808	.174768	.173790	.168848	.169461	.175217	.171849	.161451
case 12	.342152	.341484	.340205	.336387	.339184	.335519	.328557	.334808	.316976	.337925	.328077	.333609	.339487	*			
case 13	.114313	.114632	.118070	.121330	.124840	.121590	.115110	.108830	.112397	.097985	.108889	.128607	.149979	.153872	.128288	.126754	.127859
case 14	.152911	.008531	.001470	.002751	.004224	.000239	.002499	.002304	.005419	.003316	.006878	.005702	.000245	.000492	.007419	.003366	.009147
case 15	.498409	.498658	.499136	.500563	.499518	.500887	.503489	*									
case 16	.003183	.002684	.001728	.001126	.000965	.001774	.006979	.002306	.015635	.000024	.007337	.003202	.001191	.006646	.002530	.003900	.010770
case 17	.274522	.274315	.272077	.269954	.267669	.269785	.274004	.278092	.275770	.285153	.278054	.265216	.251302	.248767	.265424	*	
case 18	.003839	.186013	.174253	.172746	.180948	.175700	.173043	.173272	.169609	.172082	.167893	.169277	.176270	.175403	.167257	.172023	*
case 19	.034086	.033809	.030829	.028003	.024961	.027778	.033395	.038838	.035746	.048239	.038787	.021695	.003169	.000205	.021972	.023302	.022343
case 20	.463747	.463593	.461939	.460370	.458681	.460245	.463364	.466385	.464669	*							
case 21	.386259	.386655	.390932	.394988	.399354	.395311	.387249	.379438	.383875	.365945	*						
case 22	.011629	.011346	.008297	.005405	.002292	.005175	.010922	.016492	.013328	.026112	.016440	.001050	.020006	.023459	.000767	.000594	.000386
case 23	.470269	.470118	.468484	.466934	.465265	.466810	.469891	.472876	*								
case 24	.697216	.697129	.696195	*													
case 25	.509207	.509067	.507553	.506116	.504571	.506002	*										
Optimal AARE	.338042	.312520	.293752	.275505	.256161	.236176	.222061	.206551	.191305	.173663	.160318	.145036	.128043	.110475	.093965	.076083	.065024
Factors	Factor Values																
L1	.614240	.731305	.724053	.723124	.728182	.724945	.723307	.723449	.721190	.722714	.720131	.720985	.725297	.724762	.719739	.722678	.731752
L2	1.039249	1.039547	1.042753	1.045794	1.049067	1.046036	1.039992	1.034136	1.037463	1.024021	1.034191	1.052581	1.072513	1.076144	1.052283	1.050852	1.051883
L3	1.664731	1.663903	1.662316	1.657581	1.661050	1.656505	1.647868	1.655622	1.633504	1.659489	1.647273	1.654135	1.661425	1.670478	1.655251	1.665921	1.677321

Figure B-2. 25 Cases, LEXP, A=3.2, B=1.05

	A= 3.2, B=1.05																	
	Average Relative Error (ARE)																	
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
case 1	.372990	.368447	.376950	.375784	.372207	.365768	.363703	.367547	.353770	.374038	.368944	.369029	.363002	.349344	*			
case 2	.835185	*																
case 3	.101179	.100875	.106972	.116590	.258846	.272461	.277053	.276664	.278043	.279655	.288017	.297554	.304175	.302519	.301786	.300068	*	
case 4	.381778	.381569	.385763	.392378	.490224	.499588	.502747	.502479	.503428	.504537	*							
case 5	.822230	.822938	*															
case 6	.559472	.559999	.549420	.532732	.285915	.262294	.254327	.255002	.252608	.249812	.235304	.218758	.207270	.210142	.211415	.214395	.206281	.039616
case 7	.358882	.354386	.362802	.361648	.358108	.351734	.349691	.353495	.339860	.359920	.354878	.354962	.348997	.335479	.367788	*		
case 8	.003822	.000179	.003797	.000361	.001445	.002691	.011329	.000707	.004813	.006141	.000446	.000652	.002951	.006134	.007192	.006355	.002163	.000098
case 9	.717943	.717847	.719760	.722779	*													
case 10	.210147	.209880	.215238	.223690	.348700	.360664	.364699	.364357	.365570	.366986	.374334	.382715	*					
case 11	.606999	.608565	.607009	.608637	.609061	.609549	.604060	.608219	.606611	*								
case 12	.151855	.154661	.149408	.150128	.152338	.156316	.157591	.155217	.163727	.151207	.154354	.154301	.158024	.166461	.146296	.130538	.136942	.132845
case 13	.001558	.001896	.004897	.015615	.174131	.189302	.194419	.193985	.195523	.197319	.206636	.217263	.224641	.222796	.221979	.220065	.225276	*
case 14	.288188	.288623	.279885	.266100	.062219	.042707	.036126	.036684	.034706	.032396	.020413	.006744	.002745	.000373	.000679	.003140	.003562	.141234
case 15	.683030	.684078	.682115	.682384	.683210	*												
case 16	.366059	.368157	.364230	.364769	.366420	.369394	.370347	.368572	.374933	.365575	.367927	.367888	.370671	*				
case 17	.347932	.347712	.352135	.359113	.462315	.472192	.475523	.475241	.476242	.477411	.483478	*						
case 18	.514885	.515397	.505121	.488910	.249150	.226204	.218465	.219121	.216795	.214079	.199986	.183912	.172753	.175543	.176779	.179674	.171792	.009893
case 19	.022241	.025477	.019421	.020251	.022798	.027384	.028855	.026117	.035928	.021495	.025123	.025062	.029354	.039080	.015833	.002333	.005050	.000327
case 20	.518009	.517846	.521116	.526274	.602558	.609859	.612321	.612113	*									
case 21	.245985	.246407	.237955	.224621	.027420	.008547	.002181	.002721	.000808	.001426	.013017	.026238	.035417	.033121	.032105	.029724	.036207	.169368
case 22	.000491	.002820	.003377	.002527	.000079	.004772	.006276	.003475	.013514	.001255	.002457	.002395	.006787	.016740	.007048	.025637	.018082	.022915
case 23	.523872	.523711	.526941	.532036	.607392	.614604	.617037	*										
case 24	.727854	.727762	.729608	*														
case 25	.558870	.558720	.561713	.566433	.636251	.642933	*											
AvgAbsErr	.396858	.378665	.359375	.342444	.322419	.304448	.286671	.267873	.248052	.225203	.206354	.186248	.171291	.154811	.135355	.111193	.089484	.064537
Factors	Factor Values																	
A1	.342948	.341582	.342940	.341519	.341149	.340723	.345513	.341884	.343287	.343741	.341795	.341865	.340635	.339547	.344100	.343814	.340904	.341676
A2	.934089	.934405	.928069	.918073	.770235	.756087	.751315	.751719	.750285	.748610	.739920	.730009	.723128	.724849	.725611	.727396	.722536	.622707
A3	1.051993	1.048512	1.055027	1.054134	1.051394	1.046459	1.044877	1.047822	1.037267	1.052796	1.048893	1.048958	1.044340	1.033875	1.058888	1.078433	1.070490	1.075571

Figure B-3. 25 Cases, AEXP, A=3.2, B=1.05

	A= 3.2, B=1.05																
	Average Relative Error (ARE)																
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
case 1	.278072	.412532	.412532	.057709	.926344	.492257	.492257	.172150	.172150	.215453	.305735	.590423	.209682	.209682	.209682	.209682	.202730
case 2	.998029	*															
case 3	.081061	.456373	.456373	.253279	.390080	.034190	.034190	.142052	.142052	.197642	.414959	.243129	.057109	.057109	.057109	.057109	.418516
case 4	.367941	.001714	.001714	.486395	.043883	.288669	.288669	.409891	.409891	.448127	.026771	.479413	.351466	.351466	.351466	.351466	.600047
case 5	.997150	.001208	.001208	.948655	.645456	.022191	.022191	.302910	.302910	.201975	.414979	.064110	.191188	.191188	.191188	.191188	.389515
case 6	.594375	1.526834	*														
case 7	.264940	.398018	.398018	.046841	.906551	.476924	.476924	.160107	.160107	.202964	.292319	.574082	.217802	.217802	.217802	.217802	.210922
case 8	.987995	1.007799	1.007799	.878481	.159578	.397708	.397708	.178925	.178925	.253737	.360126	.442863	.359647	.359647	.359647	.359647	.483723
case 9	.788657	.810962	.810962	.901066	.717347	.827741	.827741	*									
case 10	.408171	.470631	.470631	.722953	.208480	.517618	.517618	.646467	.646467	*							
case 11	.995512	.533524	.533524	.976387	*												
case 12	.013458	1.120436	1.120436	*													
case 13	.022137	.036952	.036952	.587024	.301197	.007076	.007076	.110655	.110655	.117260	.049878	.311295	.542970	*			
case 14	.172203	.068990	.068990	.262820	.811997	.078746	.078746	.224782	.224782	.056281	.027246	.196010	.032063	.032063	.032063	.032063	.378798
case 15	.647911	.529731	.529731	.598689	.036467	.154132	.154132	.430372	.430372	.073592	.629131	.003256	.067909	.067909	.067909	.067909	.083917
case 16	.295821	.059462	.059462	.197379	.927065	*											
case 17	.333338	.056555	.056555	.458276	.008461	.249726	.249726	.377584	.377584	.417913	.026510	.450913	.315961	.315961	.315961	.315961	.578151
case 18	.010277	.480019	.480019	.633986	.380725	.424961	.424961	.572516	.572516	.513200	.414531	.272259	.530133	.530133	*		
case 19	.301498	.298862	.298862	.359248	.058149	.345345	.345345	.579238	.579238	.656433	*						
case 20	.507222	.219024	.219024	.599573	.254573	.445418	.445418	.539927	.539927	.569737	.241232	.594130	.494377	.494377	.494377	*	
case 21	.273873	1.018888	1.018888	.035137	.926990	.433640	.433640	.189326	.189326	.112264	.961478	*					
case 22	.285258	.282561	.282561	.344351	.082751	.330124	.330124	.569455	.569455	.648446	.234762	.092385	.453494	.453494	.453494	.453494	*
case 23	.513215	.228523	.228523	.604443	.263640	.452164	.452164	.545523	.545523	.574971	.250461	.599067	*				
case 24	.796084	.817604	.817604	.904542	.727279	.833794	*										
case 25	.669465	.704349	.704349	.845270	.557938	.730591	.730591	.802553	*								
Optimal AARE	.464146	.480898	.435422	.531932	.444522	.377151	.353117	.386357	.361875	.328750	.310008	.350952	.294139	.273403	.250063	.225632	.371813
Factors	Factor Values																
C1	1.739957	.351818	.351818	.379396	1.207626	.974406	.974406	.855789	.855789	.789670	.715231	1.565069	.667080	.667080	.667080	.667080	.796675
C2	2.267203	.504751	.504751	.445999	1.565610	1.581740	1.581740	1.697764	1.697764	1.989470	.889305	1.660413	.702961	.702961	.702961	.702961	1.543154
C3	2.374115	.850559	.850559	.898606	1.863730	1.835788	1.835788	1.825035	1.825035	2.015002	1.475767	2.033329	1.616035	1.616035	1.616035	1.616035	2.224516
L1	1.146042	.530644	.530644	.825918	.927794	.180894	.180894	.157555	.157555	.334406	.577357	.256348	.326230	.326230	.326230	.326230	.284965
L2	1.287627	1.036116	1.036116	1.199022	1.020093	.291039	.291039	.249905	.249905	.364057	1.139776	.345865	.455455	.455455	.455455	.455455	.382298
L3	1.536512	1.494186	1.494186	1.970562	2.210146	.714594	.714594	.526055	.526055	1.201936	1.402277	.938915	2.326750	2.326750	2.326750	2.326750	1.902744
A1	.001507	1.519793	1.519793	.055938	.166049	1.437942	1.437942	1.400733	1.400733	.635666	.545367	.945714	.414969	.414969	.414969	.414969	.278246
A2	.312399	1.717414	1.717414	.720239	.759857	2.011599	2.011599	1.954924	1.954924	1.136682	.874222	1.118465	1.331315	1.331315	1.331315	1.331315	.710584
A3	.335444	2.069466	2.069466	1.515483	.924178	2.483717	2.483717	2.116781	2.116781	1.285812	.987030	2.121950	1.891347	1.891347	1.891347	1.891347	1.035474

Figure B-4. 25 Cases, CPLX, LEXP, AEXP, A=3.2, B=1.05

	A= 2.8, B=1.2																		
	Average Relative Error (ARE)																		
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
case 1	.515276	.586143	.598532	.571799	.594314	.533777	.513955	.514611	.497932	.491984	.329604	.078683	.052233	.054069	.040511	.049278	.042672	.043828	.033421
case 2	.649015	.630308	.628514	.636034	.630079	*													
case 3	.203491	.161038	.156966	.174032	.160517	.191179	.201982	.197597	.212767	.216396	.300682	.431914	.448142	.446669	.450741	.448142	.450513	*	
case 4	.517218	.491486	.489018	.499363	.491171	.509756	.516304	.513646	.522841	.525041	.576128	.655671	*						
case 5	.644243	.625282	.623463	.631086	.625049	.638744	*												
case 6	.482979	.562021	.569602	.537827	.562989	.505902	.485788	.493952	.465707	.458951	.302024	.057689	.027476	.030218	.022636	.027476	.023061	.019280	.013926
case 7	.048974	.004496	.003280	.013498	.000633	.037362	.049803	.049391	.059860	.063592	.165506	.322991	.339592	.338439	.346948	.341446	.345592	.344867	.351398
case 8	1.667442	*																	
case 9	.747151	.734103	.732092	*															
case 10	.225892	.185946	.179790	.196001	.180462	.213239	.223769	.223175	.230053	.235079	.318198	.446678	.468586	.458213	*				
case 11	.191933	.154141	.147534	.161791	.149784	.182067	.192638	.192288	.201183	.204354	.290948	.424760	.438865	.437886	.445116	.440441	.443964	.443347	*
case 12	.410636	.379223	.376210	.388838	.378838	.401525	.409519	.406275	.417500	.420185	.482551	.579654	.591661	*					
case 13	.000119	.046643	.054818	.037178	.052035	.012089	.000992	.000558	.011565	.015489	.122638	.288213	.305667	.304455	.313401	.307616	.311976	.311213	.318080
case 14	.443740	.520690	.528071	.497137	.521633	.466057	.446475	.454423	.426926	.420348	.267573	.029703	.000290	.002959	.004422	.000289	.004009	.007689	.012902
case 15	.819814	.811387	*																
case 16	.639628	.622774	.619828	.626186	.620831	.635228	.639943	*											
case 17	.616779	.596354	.594394	.602606	.596103	.610855	.616053	.613943	*										
case 18	.449548	.524349	.535875	.505520	.534618	.473241	.453523	.454637	.441756	.432346	.276702	.036118	.004906	.014517	.000773	.006797	.008723	.011744	.009178
case 19	.224042	.184000	.177830	.194080	.178503	.211359	.221914	.221318	.228213	.233251	.316568	.445356	.467316	.456918	.465103	*			
case 20	.597891	.576459	.574403	.583019	.576196	.591675	.597129	.594915	.602574	.604406	*								
case 21	.467238	.545440	.552941	.521503	.546399	.489918	.470017	.478095	.450150	.443465	.288203	.046462	.016570	.019283	.011782	.016570	.012201	.008461	.003164
case 22	.203391	.162283	.155949	.172631	.156639	.190370	.201206	.200594	.207673	.212844	.298379	.430594	.453139	.442465	.450867	.454178	*		
case 23	.603476	.582341	.580314	.588810	.582082	.597346	.602725	.600542	.608094	*									
case 24	.697813	.682219	.679817	.686145	*														
case 25	.569879	.547683	.544263	.553270	.544636	.562848	.568699	.568369	.572191	.574983	.621167	*							
Optimal AARE	.505504	.454867	.439283	.426289	.413501	.402727	.390128	.376574	.362175	.347045	.330458	.305320	.278034	.250508	.232027	.209223	.182523	.148804	.106010
Factors	Factor Values																		
C1	.813638	.855625	.862094	.845056	.861389	.826937	.815870	.816495	.809265	.803983	.716619	.581578	.558551	.569454	.560871	.557490	.556409	.554713	.556153
C2	.818726	.857016	.863710	.849266	.861431	.828722	.818011	.818366	.809354	.806141	.718404	.582828	.568536	.569529	.562203	.566940	.563370	.563995	.558372
C3	.820597	.864334	.868529	.850946	.864869	.833281	.822151	.826669	.811039	.807301	.720466	.585265	.568547	.570064	.565869	.568547	.566104	.564012	.561049

Figure B-5. 25 Cases, CPLX, A=2.8, B=1.2

	A= 2.8, B=1.2																
	Average Relative Error (ARE)																
	Series 1	2	3	4	5	6	7	8	9	1	11	12	13	14	15	16	17
case 1	.590735	.592112	.608079	.600922	.598080	.596397	.583905	.562639	.515494	.507510	.518046	*					
case 2	.759324	.758369	*														
case 3	.165732	.165010	.156636	.160390	.161880	.162763	.169314	.180467	.205193	.209380	.203854	.192431	.159279	.007437	.007454	.002004	.000694
case 4	.494332	.493894	.488819	.491094	.491997	.492532	.496503	.503263	.518250	.520788	*						
case 5	.026159	.017903	.016355	.011131	.020876	.009204	.008488	.000749	.004110	.006367	.002491	.000757	.000112	.002228	.000358	.000389	.009400
case 6	.553280	.554624	.570216	.563226	.560451	.558808	.546610	.525845	.479810	.472014	.482302	.503571	*				
case 7	.001614	.000750	.009272	.004779	.002996	.001940	.005901	.019248	.048837	.053848	.047235	.033565	.006109	.187821	.187801	.194324	.197553
case 8	.829110	*															
case 9	.732899	.732668	.729987	*													
case 1	.182262	.181554	.173345	.177025	.178486	.179351	.185773	.196705	.220940	.225045	.219628	.208431	.175936	.027103	.027119	.021777	.019132
case 11	.444629	.442424	.438423	.440385	.438217	.440159	.444920	.439046	.437095	.444666	.438753	.440895	.439208	.440515	.438312	*	
case 12	.613312	.626991	.629556	.638209	.622065	*											
case 13	.049673	.050582	.061118	.056395	.054520	.053409	.045166	.031134	.000024	.005244	.001708	.016081	.057793	.248840	.248818	.255676	*
case 14	.010003	.006073	.001060	.002438	.001426	.002035	.010522	.000052	.003427	.010070	.000471	.003347	.000341	.002669	.001257	.002407	.001756
case 15	.505636	.501445	.500659	.498007	.502954	.497029	.496665	.492737	.494443	.495589	.493621	.492741	.492413	.493488	*		
case 16	.011273	.002890	.001318	.003985	.005909	.005942	.006669	.014526	.011114	.008822	.012757	.014518	.015173	.013025	.014923	.014892	.005743
case 17	.598612	.598265	.594236	.596042	.596759	.597184	*										
case 18	.002480	.006459	.013682	.010140	.014053	.010548	.001954	.012556	.016079	.002412	.013086	.009220	.012264	.009906	.013881	.010171	.010831
case 19	.180307	.179598	.171370	.175058	.176523	.177390	.183827	.194785	.219078	.223193	.217764	.206540	.173967	.024778	.024794	.019439	.016788
case 2	.578829	.578464	.574237	.576132	.576884	.577330	.580637	.586268	*								
case 21	.536792	.538122	.553549	.546633	.543888	.542262	.530194	.509649	.464103	.456390	.466568	.487611	.548680	*			
case 22	.158492	.157764	.149317	.153103	.154607	.155497	.162105	.173355	.198295	.202518	.196945	.185422	.151982	.001177	.001160	.006658	.009379
case 23	.584678	.584319	.580150	.582019	.582761	.583200	.586462	*									
case 24	.680781	.680505	.677301	.678737	*												
case 25	.545636	.545243	.540682	.542726	.543538	.544019	.547587	.553661	.567127	*							
Optimal AARE	.393463	.374834	.358233	.341299	.325184	.309350	.294379	.277594	.259025	.240241	.221015	.199652	.171789	.121582	.087807	.052774	.030142
Factors	Factor Values																
L1	.562697	.564930	.568985	.566997	.569193	.567226	.562402	.568353	.570330	.562659	.568650	.566480	.568188	.566865	.569097	.567014	.567384
L2	.859497	.860241	.868868	.865001	.863465	.862556	.379737	.844316	.818843	.814529	.820222	.831991	.866145	1.022579	1.022562	1.028177	1.030956
L3	2.246282	2.265327	2.268898	2.280946	2.258468	2.285391	.387644	2.304893	2.297142	2.291936	2.300876	2.304877	.311907	.353287	2.305797	2.305725	2.284940

Figure B-6. 25 Cases, LEXP, A=2.8, B=1.2

	A= 2.8, B=1.2																
	Average Relative Error (ARE)																
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
case 1	.882743	*															
case 2	.868089	.867219	*														
case 3	.448289	.443514	.448310	.446035	.442089	.445918	.447129	.448455	.449859	.448548	.454946	.449456	.454048	.451408	.452187	*	
case 4	.665596	.662702	.665608	.664230	.661838	.664159	.664893	.665697	.666548	.665753	.669631	.666303	*				
case 5	.866295	.865413	.865014	*													
case 6	.027202	.036092	.027163	.031398	.038745	.031617	.029361	.026892	.024278	.026720	.014807	.025029	.016480	.021396	.019944	.013108	.012738
case 7	.181658	.218358	.218790	.216699	.204880	.215784	.197538	.200111	.185615	.212034	.215144	.200129	.190932	.185286	.187598	.202513	.198470
case 8	.002510	.009122	.012115	.008271	.000419	.006022	.009501	.000163	.003116	.008561	.004225	.001821	.001667	.003020	.000707	.004569	.003698
case 9	.823363	.821835	.823370	.822642	*												
case 10	.459220	.454540	.459240	.457011	.453143	.456896	.458083	.459383	.460759	.459473	.465745	.460364	.464865	.462277	*		
case 11	.695609	.693601	.692692	.693859	.696243	.694542	.693486	.696420	.695424	.693771	.697654	*					
case 12	.269380	.246688	.246421	.247714	.255021	.248280	.259561	.257970	.266933	.250598	.248676	.257959	.263646	.267136	.265707	.256485	.258985
case 13	.305839	.299831	.305865	.303003	.298038	.302856	.304380	.306048	.307815	.306164	.314215	.307307	.313084	.309763	.310743	.315363	*
case 14	.000023	.008677	.000015	.004108	.011260	.004321	.002125	.000279	.002824	.000446	.012044	.002093	.010415	.005630	.007043	.013698	.014058
case 15	.776118	.769164	.769082	.769479	.771718	.769652	*										
case 16	.552235	.538329	.538165	.538957	.543436	.539304	.546218	.545243	.550736	.540725	.539547	.545236	.548721	*			
case 17	.734558	.732260	.734568	.733473	.731575	.733417	.734000	*									
case 18	.012632	.021395	.012594	.016769	.024011	.016984	.014761	.012327	.009750	.012157	.000413	.010489	.002062	.006908	.005477	.001262	.001626
case 19	.029838	.000293	.000648	.001069	.010772	.001820	.016801	.014688	.026589	.004899	.002346	.014673	.022224	.026859	.024961	.012716	.016035
case 20	.721475	.719064	.721485	.720337	.718345	.720278	.720889	.721559	.722267	*							
case 21	.016298	.025094	.016261	.020451	.027719	.020666	.018435	.015992	.013406	.015822	.004036	.014148	.005691	.010554	.009118	.002354	.001989
case 22	.004018	.026915	.027280	.025517	.015555	.024746	.009367	.011536	.000682	.021585	.024206	.011551	.003799	.000960	.000989	.013560	.010153
case 23	.725343	.722966	.725353	.724221	.722257	.724163	.724766	.725426	*								
case 24	.788897	.787070	.788905	.788034	.786525	*											
case 25	.699524	.696923	.699535	.698296	.696147	.698232	.698892	.699614	.700379	.699665	*						
Optimal AARE	.462270	.444461	.426021	.405981	.386178	.365983	.344747	.322656	.299234	.272933	.244509	.211897	.176741	.145933	.116770	.083563	.057528
Factors	Factor Values																
A1	.308406	.310440	.311361	.310179	.307763	.309487	.310557	.307584	.308593	.310268	.306334	.308195	.308147	.306705	.307852	.309040	.308772
A2	.568395	.573314	.568374	.570717	.574782	.570838	.569590	.568224	.566777	.568129	.561537	.567193	.562463	.565182	.564379	.560596	.560392
A3	1.017273	1.048867	1.049240	1.047439	1.037265	1.046651	1.030944	1.033159	1.020680	1.043423	1.046100	1.033175	1.025257	1.020397	1.022387	1.035227	1.031746

Figure B-7. 25 Cases, AEXP, A=2.8, B=1.2

	A= 2.8, B=1.2																
	Average Relative Error (ARE)																
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
case 1	.599968	.510728	.600108	.679945	.708242	*											
case 2	.867779	*															
case 3	.140826	.158866	.126072	.121806	.093843	.093061	.000362	.502767	.682686	*							
case 4	.479236	.490170	.470293	.467707	.450758	.450284	.394098	.089140	.019913	.195660	.107781	.067463	.258732	.178106	.180747	.178235	*
case 5	.524446	.224345	.438353	.173793	.254351	.005592	.036431	.021421	.042134	.015515	.131687	.006018	.005718	.042752	.006391	.007387	.051601
case 6	.599652	.566064	.627121	.635064	.687126	.688582	*										
case 7	.004181	.051828	.004269	.054376	.072136	.191671	.146242	.251053	.268774	.458701	.138963	.314173	.364116	*			
case 8	.004863	.574516	.047488	.181695	.096595	.543387	.154822	.091161	.022176	.168902	.003381	.068106	.082614	.015669	.022612	.117323	.016899
case 9	.758100	.762957	*														
case 10	.259415	.274284	.251812	.258404	.226392	.187672	.221174	.181616	.199268	.015415	.117755	.344238	.105955	.079604	.000177	.154868	.048472
case 11	.717393	.560761	.744134	.674218	.704269	.565466	.718452	.805140	*								
case 12	.499562	.747022	.834317	*													
case 13	.001294	.027646	.027203	.003269	.012648	.058113	.009916	.112068	.089889	.460726	.156080	.066835	.222560	.339002	*		
case 14	.018186	.130114	.063600	.124671	.067707	.087644	.028199	.132254	.065496	.007790	.157120	.021385	.034484	.098679	.173471	.007807	.007722
case 15	.574378	.508145	.502721	.424121	.412161	.387846	.271298	.337469	.328931	.492579	*						
case 16	.148756	.016291	.005441	.151759	.175679	.224309	.457404	.325062	.342137	.014842	.015212	.125231	.167689	.002702	.219561	*	
case 17	.586629	.595309	.579531	.577478	.564025	.563648	.519049	.276980	.190416	.050912	.120669	.152672	.000847	.064847	.062750	.064744	.021914
case 18	.125710	.007295	.171812	.236374	.177751	.155790	.217819	.362996	.476196	.464081	.422959	.607106	*				
case 19	.217260	.254201	.185989	.178632	.136813	.035843	.068412	.029728	.017616	.068644	.083984	.026732	.051314	.022047	.050572	.015986	.072628
case 20	.566255	.575362	.558807	.556653	.542536	.542142	.495344	.241344	.150514	.004134	.077329	.110909	.048398	.018755	.016555	.018648	.026294
case 21	.582672	.549441	.609850	.617708	.669218	.670659	.841418	*									
case 22	.196428	.234352	.164325	.156772	.113840	.010182	.043618	.003905	.008530	.097085	.059605	.000829	.079294	.049248	.078533	.010202	.101175
case 23	.572279	.581260	.564934	.562811	.548890	.548501	.502353	.251881	.162312	.017965	.090144	.123258	.033837	.032383	.030214	.032277	.012040
case 24	.710900	.716704	.707932	.710505	*												
case 25	.588505	.596767	.584281	.587943	.570156	.548642	.567257	.545277	.555085	.435800	.509794	*					
Optimal AARE	.413787	.404768	.385669	.369805	.346911	.327952	.299667	.253404	.213063	.185547	.146164	.145354	.111966	.078649	.076507	.060748	.039861
Factors	Factor Values																
C1	.335000	.719243	1.002768	1.478643	1.863117	1.644355	.654687	1.339110	.500716	.969368	.801065	.679970	1.029204	.721217	.690357	.443906	.766103
C2	.352852	.750747	1.015717	1.558381	1.899931	1.668616	.661361	1.417590	.530941	1.086361	.817762	.753810	1.096409	.817396	1.005277	.732108	1.427278
C3	.380944	.817115	1.148086	1.716309	2.139101	1.799497	.823655	2.410236	1.031374	1.845873	1.626602	1.789969	2.343281	1.492879	1.317963	1.000800	1.604327
L1	.660876	.465087	.345929	.411034	.312911	.419216	.398494	.167960	.302227	.195542	.288206	.321801	.168567	.230361	.167097	.277026	.168716
L2	1.048263	.627445	.585193	.747473	.551277	.755352	.742995	.404066	.865134	.693770	.825120	1.005747	.613369	.900435	.713849	.968529	.616926
L3	2.345063	1.717898	1.529403	2.154781	1.595330	2.047981	2.493053	1.129418	2.415117	1.273767	1.882747	2.272584	1.385606	1.679711	1.233011	1.872748	1.179810
A1	1.227894	1.274572	.737808	.515308	.503998	.629392	1.082387	.829198	.965041	.996257	.654002	.570449	.714478	.880525	1.428476	1.239785	1.155746
A2	2.216608	1.690229	1.340111	.705243	.791664	.687411	1.682870	1.589714	1.942865	1.586972	1.402925	1.007853	1.488559	1.489656	2.133173	2.066097	2.116076
A3	2.337194	1.732861	1.454533	.779242	.881228	.813947	2.008156	1.880264	2.377936	1.666180	1.453154	1.492269	1.746236	1.650234	2.235314	2.399885	2.379699

Figure B-8. 25 Cases, CPLX, LEXP, AEXP, A=2.8, B=1.2

	A= variable, B= variable																
	Average Relative Error (ARE)																
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
case 1	.142200	.038237	.019638	.147240	.067406	.115172	.095529	.070812	.008749	.124917	.080036	.090027	.037963	.060298	.088690	.064539	.067765
case 2	.390422	.435530	.437719	.401641	.410120	.391422	.398715	.388149	.373065	.365388	.400281	.365954	*				
case 3	.026193	.002280	.003526	.014511	.023789	.040481	.031027	.055260	.090768	.064944	.029875	.072749	.051690	.061984	.026305	.080379	.060061
case 4	.159098	.221690	.224781	.174630	.186511	.160603	.170685	.156160	.135430	.124537	.172857	.125386	.163061	.147096	.176301	.124298	.148184
case 5	.283392	.352283	.357891	.299065	.316971	.290042	.299567	.289314	.274946	.252785	.301908	.256481	.296763	*			
case 6	.614732	.613636	.631909	.602772	*												
case 7	.595324	.498093	.474190	.545372	.552288	.555556	*										
case 8	1.732360	*															
case 9	.692665	.696145	.697432	*													
case 10	.239354	.221406	.219336	.253953	.235297	.251877	.235294	.229625	.187686	.221957	.214273	.201430	.196609	.202010	.224895	*	
case 11	.502444	.387375	.360689	.451830	.446628	.457202	.472289	.477918	.522702	*							
case 12	.021309	.053922	.057523	.002566	.011469	.019751	.007554	.025302	.050647	.063226	.004940	.062344	.017000	.036239	.000768	.063771	.034908
case 13	.045387	.017200	.015036	.062757	.016099	.041608	.025338	.008837	.041314	.020717	.011225	.003830	.017658	.009934	.021713	.021547	.002841
case 14	.062400	.131786	.159244	.064436	.122309	.108900	.104873	.143236	.201235	.095961	.106727	.121087	.149623	.141298	.104324	.141384	*
case 15	.503690	.570173	.583781	.524886	.540930	.528335	.525284	*									
case 16	.007380	.140347	.167563	.049772	.081860	.056670	.050568	.054200	.036685	.029826	.041898	.025397	.045238	.021504	.054389	.022285	.026545
case 17	.314791	.089471	.061541	.269798	.186843	.267152	.243641	.248835	.254379	.378745	.236196	.352028	.225263	.274605	*		
case 18	.508073	.529333	.530560	.477113	.507296	.478863	.510796	.520306	*								
case 19	.050111	.061955	.066144	.073215	.066243	.076325	.057883	.054949	.009891	.026108	.033045	.006423	.017907	.017977	.046604	.018352	.016287
case 20	.401733	.431838	.431194	.410572	.411845	.397909	.404221	.391924	.374106	.378189	.405314	*					
case 21	.040428	.042110	.071484	.035879	.025967	.007709	.005211	.042504	.099180	.011357	.007490	.014586	.050308	.038975	.005579	.035318	.035844
case 22	.032436	.043279	.047309	.055794	.048107	.058756	.039890	.036754	.009404	.008070	.014539	.012197	.001127	.000818	.028340	.000197	.002526
case 23	.407577	.437771	.437209	.416388	.417839	.403920	.410192	.398068	.380505	.384236	*						
case 24	.779528	.763089	*														
case 25	.572148	.562927	.561940	.580483	.570404	*											
Optimal AARE	.365007	.305912	.287723	.268849	.249820	.235413	.215187	.199564	.179452	.159435	.137374	.122137	.097709	.084395	.070719	.057207	.043884
Factors	Factor Values																
C1	1.198413	1.306761	1.341219	1.410939	1.114158	1.065157	1.411124	1.108983	1.117460	1.440915	1.433239	1.273029	1.102259	1.241879	1.023450	1.378618	1.408694
C2	1.206354	1.316762	1.349591	1.420745	1.146398	1.092826	1.440020	1.141733	1.145292	1.455128	1.443859	1.282911	1.116743	1.258212	1.034006	1.424132	1.415389
C3	1.221328	1.319885	1.370491	1.458405	1.157173	1.134264	1.460459	1.172011	1.166545	1.485717	1.443923	1.297482	1.117904	1.270352	1.042216	1.433419	1.435808
A	3.402541	2.896264	2.774848	2.794283	3.460774	3.650031	2.799711	3.547258	3.647412	2.913172	2.823827	3.329129	3.686312	3.309600	3.895600	3.014963	2.924679
B	.842268	.900650	.912255	.850804	.878805	.860798	.864304	.871267	.881737	.838459	.866089	.848273	.876989	.866127	.866919	.855206	.865491

Figure B-9. 25 Cases, CPLX, A=variable, B=variable

	A= variable, B= variable														
	Average Relative Error (ARE)														
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
case 1	.001369	.010175	.028393	.003280	.003385	.021118	.034076	.068300	.023534	.023904	.033630	.009667	.015229	.015952	.029213
case 2	.637554	.584066	.594974	.582847	.598679	*									
case 3	.006086	.004883	.010054	.011564	.019884	.008665	.014931	.033825	.027287	.002531	.001479	.002244	.012224	.001905	.004225
case 4	.222961	.227538	.227226	.241052	.209706	.244808	.223695	.172393	.193732	.239164	.237647	.229386	*		
case 5	.254467	.206457	.228052	.237253	.242305	.252794	.223402	.276182	.270384	.237045	.239047	*			
case 6	.636257	.638602	.651387	.612817	*										
case 7	.500561	.492366	.493613	.466410	.525701	.460207	.500474	*							
case 8	.802047	*													
case 9	.690030	.690170	.688342	*											
case 10	.200122	.197821	.190392	.210167	.191456	.202433	.186302	.203133	.194529	.198384	.193755	.206753	.192554	*	
case 11	.109366	.020822	.007168	.023510	.012996	.001663	.002659	.009591	.003547	.028821	.008012	.015277	.008753	.055795	.089451
case 12	.094392	.167608	.138519	.122939	.110418	.104400	.145524	.043675	.062470	.126957	.125134	.054136	.020650	.026197	.030431
case 13	.002965	.006309	.016093	.009068	.013509	.001399	.021252	.004267	.008410	.006359	.012367	.005350	.012834	.004951	.008279
case 14	.252320	.130392	.141714	.124986	.179855	.125324	.132525	.256205	.204239	.152491	.114809	.173362	.142938	.216933	*
case 15	.509365	.480398	.497092	.501176	.499599	.515452	.494202	.505008	*						
case 16	.018729	.039204	.005816	.002353	.000802	.030904	.011596	.010017	.023003	.009124	.013860	.054807	.090380	.074201	.061737
case 17	.063545	.046143	.035496	.025191	.089728	.002183	.039625	.227834	.144159	.012578	.010000	.054779	.056697	.072682	.095950
case 18	.008471	.166409	.144799	.172067	.110531	.160730	.156703	.046692	.093927	.126380	.173653	.114984	.150736	.061104	.011740
case 19	.043302	.043838	.038304	.059341	.030612	.055577	.033627	.021669	.025259	.049898	.045818	.051224	.036964	.047780	.046335
case 20	.429798	.431759	.430123	.441366	.421068	.441833	.427442	.403963	.413298	.438037	*				
case 21	.074708	.084854	.102095	.069857	.081410	.092201	.108058	.018814	.058404	.095804	.105219	.065783	.090694	.060862	.049317
case 22	.024001	.024429	.018662	.040218	.011140	.036180	.013883	.002861	.006009	.030416	.026202	.032084	.017437	.028669	.027344
case 23	.435830	.437807	.436225	.447320	.427167	.447843	.433575	.409977	.419376	*					
case 24	.754202	.752345	*												
case 25	.551162	.549957	.545876	.556904	.546238	.552708	*								
Optimal AARE	.292945	.268098	.246540	.225531	.206009	.187921	.168608	.150800	.127739	.111118	.089375	.076417	.065238	.055586	.041275
Factors	Factor Values														
L1	.804483	.747179	.865434	.755399	.762069	.826775	.730393	.786952	.879537	.756961	.818194	.730203	.931083	.914081	.849834
L2	1.250969	1.006530	1.197767	.996889	1.088448	1.110867	1.005471	1.182590	1.273526	1.053655	1.098907	1.018483	1.279906	1.341727	1.306113
L3	1.449203	1.251309	1.451252	1.213111	1.258003	1.335925	1.220176	1.227416	1.380751	1.283439	1.333689	1.145951	1.379659	1.457946	1.413596
A	3.047024	3.762484	3.161170	3.731875	3.563196	3.329121	3.782827	3.448371	3.111885	3.536683	3.396950	3.711620	2.983318	2.841598	2.945303
B	.912496	.918085	.923696	.919447	.908589	.928807	.924002	.869966	.893428	.927291	.929688	.912483	.917077	.907851	.900858

Figure B-10. 25 Cases, LEXP, A=variable, B=variable

	A= variable, B= variable																
	Average Relative Error (ARE)																
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
case 1	.025730	.061170	.019012	.061177	.002872	.008580	.038614	.048367	.068869	.033086	.054989	.011825	.040260	.034975	.008380	.023819	.027716
case 2	.684044	.744474	*														
case 3	.014251	.002582	.041868	.005097	.041668	.024313	.038024	.019824	.020440	.028604	.012091	.015395	.013284	.005417	.088645	.008375	.053801
case 4	.220083	.199984	.245352	.198338	.248988	.225211	.230611	.219570	.178810	.176273	.185972	.228290	.188159	.211842	*		
case 5	.634920	.703507	.746543	*													
case 6	.581750	.601140	.541321	.605559	.545983	.562410	.534312	*									
case 7	.596846	.586666	.581880	.584089	.578984	*											
case 8	.505944	.199901	.056158	.066699	.106905	.019097	.008192	.016052	.064955	.010842	.006058	.005793	.008003	.016778	.044184	.020723	.006075
case 9	.697455	.692703	.705720	.691911	*												
case 10	.232452	.225063	.251035	.222817	.247611	.242685	.257931	.240950	.217009	.207408	.223050	.227079	.219529	*			
case 11	.218731	.366761	.457034	.437565	.436680	.487357	.474144	.466135	.428327	.462678	*						
case 12	.007116	.001330	.002659	.000330	.004858	.038737	.034134	.006868	.024528	.044965	.037721	.010336	.039007	.003400	.021160	.015673	.042086
case 13	.039814	.031369	.062652	.028520	.057908	.052952	.072660	.051027	.022102	.009644	.029596	.032212	.024834	.032685	.101516	.027800	.053079
case 14	.089232	.090458	.067646	.094106	.077909	.070806	.041813	.070422	.092560	.111337	.084717	.107169	.093830	.095757	.030770	.109565	*
case 15	.531065	.527486	.538901	.528602	.543544	.513552	.510234	.526500	*								
case 16	.062129	.054973	.077803	.057205	.087087	.027104	.020467	.053000	.020136	.008226	.008367	.072943	.013152	.063257	.106242	*	
case 17	.126967	.178340	.079425	.179574	.062124	.128813	.139131	.143863	.232777	.223081	.220529	.091744	.206707	.142948	.000311	.109073	.012762
case 18	.511679	.528694	.473816	.532993	.479141	.492559	.464496	.496734	.547062	*							
case 19	.007220	.022271	.011873	.023401	.008482	.019767	.007155	.014460	.009046	.016255	.004325	.006497	.009930	.012637	.021464	.022326	.025533
case 20	.434855	.422864	.451872	.421540	.453075	.439644	.445628	.436350	.410205	.406871	.415183	.438043	*				
case 21	.002602	.004797	.020638	.001293	.009272	.020878	.050052	.022248	.006176	.012837	.013100	.017565	.003302	.002751	.051491	.018706	.019932
case 22	.012175	.003391	.007549	.004531	.011138	.039590	.026542	.004628	.009867	.035783	.023508	.026410	.029323	.006610	.002023	.003051	.005770
case 23	.440647	.428712	.457524	.427405	.458751	.445359	.451228	.442078	.416112	.412852	.421045	*					
case 24	.769687	*															
case 25	.568881	.564580	.579396	.563326	.577560	.574566	*										
Optimal AARE	.320651	.301801	.281638	.260731	.240026	.221699	.202388	.182171	.162881	.137546	.116017	.092236	.068409	.052421	.043290	.035911	.027417
Factors	Factor Values																
L1	.634485	.599119	.444166	.524908	.458987	.445894	.492023	.371243	.421943	.395168	.393117	.431765	.509250	.358473	.444626	.489523	.435663
L2	1.135725	1.360101	1.105682	1.344254	1.094607	1.209486	1.294322	.974656	1.091431	1.089548	1.068033	1.177710	1.386579	.990412	1.170432	1.362515	1.158699
L3	1.206797	1.401015	1.202338	1.379546	1.193318	1.334381	1.431802	1.034822	1.120808	1.137585	1.120668	1.268559	1.460545	1.037573	1.323653	1.416009	1.254236
A	3.378806	2.897188	3.356268	2.937211	3.371770	3.153328	2.928723	3.942885	3.709851	3.725467	3.757801	3.220274	2.885330	3.916258	2.988704	2.811740	3.079865
B	.883882	.873786	.889246	.874319	.895224	.879538	.871045	.876377	.863739	.869533	.864385	.895056	.868975	.881998	.898766	.891899	.917275

Figure B-11. 25 Cases, AEXP, A=variable, B=variable

	A= variable, B= variable															
	Average Relative Error (ARE)															
	Series 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
case 1	.073114	.119943	.055147	.001338	.055046	.022756	.072606	.119420	.134315	.036235	.008216	.058225	.085904	.085904	.082618	.055970
case 2	.654062	.706099	.722038	.659711	.496081	.539785	.563245	*								
case 3	.035214	.040543	.003277	.020993	.013113	.011730	.151574	.045180	.090953	.002387	.066167	.004815	.050117	.050117	.001499	.029066
case 4	.240821	.220210	.241084	.218626	.214608	.211596	.094792	.151779	.164383	.229078	.151789	.229654	.198580	.198580	*	
case 5	.374279	.474604	.526056	.434387	.257408	.342079	.411525	.220745	.416293	*						
case 6	.552834	.721000	*													
case 7	.489617	.509692	.317247	.455869	.439123	.650148	*									
case 8	.664676	.540441	.433271	.719198	*											
case 9	.729656	.701841	.742216	*												
case 10	.311501	.214067	.324956	.352854	.350505	.350578	.257287	.312708	.255807	.306277	.234373	.384008	*			
case 11	.208970	.335973	.413767	.202190	.073700	.089452	.013196	.107094	.002618	.030644	.176459	.003436	.008182	.008182	.148854	.267263
case 12	.187570	.164169	.203759	.220875	.300598	.204761	.189783	.254662	.113551	.228262	.239685	.129035	.089587	.089587	.066571	.024866
case 13	.125156	.010517	.119516	.017995	.128240	.087660	.033112	.110929	.074062	.095183	.009373	.083429	.002927	.002927	.020805	.056542
case 14	.196509	.089532	.002124	.046240	.122046	.035482	.107513	.076397	.091251	.048274	.029762	.040542	.099940	.099940	.158948	.123550
case 15	.492300	.538713	.549275	.484005	.491936	.457107	.498102	.455943	*							
case 16	.015400	.077425	.098549	.031991	.016129	.085786	.003795	.088113	.034458	.013393	.137825	.090852	.097195	.097195	.100424	.137756
case 17	.083628	.002616	.008719	.047716	.084035	.160605	.289013	.297720	.122569	.045214	.240946	.037022	.067866	.067866	.055233	.041875
case 18	.010654	.096871	.096225	.104181	.229716	.148977	.256298	.140311	.241600	.220209	.333008	.065963	.370248	*		
case 19	.082538	.007855	.173273	.224962	.146483	.095023	.032662	.076220	.074997	.055494	.011807	.161079	.003811	.003811	.192516	*
case 20	.448311	.418694	.437009	.423870	.424791	.431169	.341832	.393470	.384145	.433037	.387138	*				
case 21	.011763	.189459	.123028	.113291	.073172	.018470	.182359	.001196	.187328	.080963	.065468	.091325	.152264	.152264	.151682	.026461
case 22	.064482	.012953	.156138	.209139	.129348	.077542	.013598	.058795	.056133	.036334	.007353	.143990	.024380	.024380	.175495	.084656
case 23	.454007	.425083	.443125	.430037	.430847	.436934	.348636	.399475	.390730	.439068	*					
case 24	.792291	*														
case 25	.613367	.559526	.621530	.637000	.635491	*										
Optimal AARE	.316509	.299076	.296145	.275294	.243448	.222882	.203207	.183898	.166776	.143753	.139958	.108812	.096231	.073396	.104968	.084800
Factors	Factor Values															
C1	1.045789	1.304476	1.079158	1.140771	1.110066	1.097907	.746172	1.224543	.754323	.948285	1.296864	1.043128	1.257949	1.257949	.911559	.936899
C2	1.061679	1.305962	1.120312	1.379315	1.188678	1.233938	.829168	1.269365	.867541	.985994	1.367449	1.236962	1.264457	1.264457	1.072921	.955456
C3	1.146460	1.409312	1.298276	1.442243	1.372206	1.291670	.905922	1.418502	.885486	1.093351	1.399498	1.361471	1.342909	1.342909	1.081091	.975899
L1	.921085	.865971	.940318	1.023690	1.113550	.962163	1.264999	.747063	1.029732	1.112746	.881040	1.043743	.745991	.745991	.665059	1.092042
L2	1.234540	1.209548	1.130746	1.174214	1.153871	1.072749	1.471082	.891087	1.208132	1.239282	.997672	1.180832	.841480	.841480	.972135	1.386095
L3	1.445721	1.375980	1.418553	1.496857	1.434466	1.185588	1.479816	1.001056	1.266409	1.422227	1.136252	1.243296	.878115	.878115	1.139097	1.456896
A1	.669143	.651925	.726736	1.001723	.727179	1.135649	.744479	1.219144	1.038572	1.215358	.695963	1.147544	.887109	.887109	1.344881	.914733
A2	.794533	.898481	1.197044	1.454612	.793293	1.265255	.962215	1.264546	1.285617	1.310614	1.110589	1.267326	1.063270	1.063270	1.412978	1.294449
A3	.873247	.969367	1.244441	1.466261	.869235	1.439743	1.034435	1.370665	1.344145	1.496502	1.172856	1.450845	1.139205	1.139205	1.418592	1.311294

Figure B-12. 25 Cases, CPLX, LEXP, AEXP, A=variable, B=variable

LIST OF REFERENCES

[Boehm, 1981]
Boehm, Barry W., 1981. Software Engineering Economics. Englewood Cliffs, NJ: Prentice Hall, Inc.

[Heemstra, 1992]
Heemstra, F. J., "Software Cost Estimation", *Information and Software Technology*, October 1992, pp 627-639.

[Fenton, 1994]
Fenton, Norman and Pfleeger, Shari, "Science and Substance: A Challenge to Software Engineers", *IEEE Software*, July 1994, pp 86-96.

[Goodman, 1992]
Goodman, P. A., "Application of cost-estimation techniques: industrial perspective", *Information and Software Technology*, June 1992, pp 379-382.