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A SIMPLIFICATION OF UNDERWATER ACOUSTIC EQUATIONS

Mr. Morris Middleton

ABSTRACT

This Research Report presents some of the equations of underwater acoustics that relate to the signal excess noise received by a transducer. The basic structural equation is developed, as are defining equations for each term in that equation. An analysis is performed utilizing typical values to ascertain if the elements of the structural equation can be simplified. Results delineate that several terms of that equation can be neglected while maintaining a relative high degree of accuracy.

A SIMPLIFICATION OF UNDERWATER ACOUSTIC EQUATIONS

BY

MORRIS G. MIDDLETON

RESEARCH REPORT

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in
Engineering in the Graduate Studies Program of
Florida Technological University, 1973

Orlando, Florida

PREFACE

The simulation techniques employed in the simulation of the operational equipment of the Armed Services are more complex than ever before. The characteristics of the operational equipment and the environment in which it operates can be expressed in elaborate mathematical equations that encompass the most minute detail. With the aid of digital computers these equations can be solved quickly, efficiently, and accurately. However, associated with the accuracy of the solution of these equations is a dollar value. Each term of the equation can be extremely expensive to implement and often is because a mathematician/programmer wants to be mathematically precise and includes terms of equations that contribute little to the final results. Training devices that simulate vehicles in the ocean are a prime candidate for a "purist" to exploit. As a project engineer and supervisor of engineers, the writer has been associated with several "purists" that have spent numerous man-months attempting to obtain an equation and exact solution for a given ocean condition. When the exact equation and solution was obtained and implemented into hardware/software, the improvement was so minute that the operator was unable to detect improvement. The research report affords the writer an opportunity to investigate some of the rigorous equations that are associated with underwater acoustics, which the "purists" delight in exploiting.

The equations of underwater acoustics that relate to the signal excess noise received by a transducer are rigorously described in the acoustical literature. The objective of this paper is to examine some underwater acoustic equations and ascertain if a simplification can be obtained without affecting the overall results. The literature that is available on underwater acoustics would fill a university library and is growing each day. Thus, this paper addresses some of the factors that contribute to the signal excess noise equation and is by no means a complete treatment of the subject.

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CHAPTER I

BACKGROUND

Nature has long made use of acoustic waves for the communication and navigation of her animal species. In these cases, the frequencies are normally within or very close to human audio range (20 to 16,000 Hz), and the functions performed vary from simple detection to the sophisticated high-speed navigation of porpoises and bats.

Leonardo da Vinci in 1490 wrote: "If you cause your ship to stop and place the head of a long tube in the water and place the outer extremity to your ear, you will hear ships at a great distance from you."

This is the earliest recorded use of passive sonar and, as is readily apparent, does not provide any indication of direction, and is very range limited. Yet, even during World War I, a very similar method was widely used by all nations. During World War I, the development by Fressenden of the electrodynamic underwater sound source and the development by Lanzevin of the piezoelectric plate transducer greatly increased the detection range over the previously used underwater bell and stethoscope, and sonar became a useful medium for detection and navigation. Using the new techniques, a submarine could be detected occasionally at a distance up to 1500 meters. However, the war ended before the techniques developed could be put to practical use.

The years following World War I saw a steady, though extremely slow advance in applying underwater sound to practical needs. In the

United States only a handful of men at the Naval Research Laboratory was engaged in underwater sound research. A fairly adequate sonar system had been developed by 1935 and in 1938 quantity production was started to equip the American ships with equipment for both underwater listening and echo ranging.

During the years of World War II a large group of scientists was organized to begin investigation in all phases of underwater acoustics. Most of our present concepts as well as practical applications had their origin during this period. The word "sonar" was coined during this period as a counterpart of the then-glamorous word "radar" and came into use later only after having been dignified as an acronym for SOund NAVigation and Ranging.

CHAPTER II

THEORY

The seas and oceans of the world have been used by man since the beginning of time. However, man had only limited information about this most common, yet complex part of our world until the twentieth century. The ocean has many phenomena and effects peculiar to underwater sound that produce a variety of quantitative effects. These diverse effects can be conveniently and logically grouped together in a number of quantities that are referred to as sonar parameters, which, in turn, are related by the sonar equations. These equations are the working relationships that tie together the effects of the medium, the target and the detection equipment.

The sonar equations are founded on a basic equality between the desired and undesired portion of the received signal. Of the total acoustic field at the receiver, the desired portion is called the signal and the undesired portion is called the background. If the sonar set is passive, the background noise is the sound of the ocean, its numerous biological and man-made objects. However, if the sonar set is active, the background noise has the same parameters as for passive sonar plus the reverberations caused by its own echo ranging. To utilize a sonar system for detection, classification, torpedo homing, fish finding, etc., a certain signal to background noise ratio is required. If the signal level is slowly increased in a constant

background, detection can occur when the signal level equals the level of the background which just masks it. Thus, it is customary to equate the signal level which exactly equals the minimum detection signal level of the system with a detection probability of 50 percent. The difference between the received signal and the minimum detectable signal level is considered to be positive or negative signal excess and delineated by N_{SE} .

During research for this paper, a variety of methods were reviewed to ascertain a standard equation for the computation of signal excess. Each document reviewed presented a slight variation of the others. A general expression can be produced by putting in logarithmic (db) form all factors which either detract from or enhance signal detection. Thus, the broadest possible way to describe N_{SE} is:

$$N_{SE} = \text{Received Signal Level} - \text{Minimum Detection Signal Level}$$

The factors that either contribute or detract from signal excess is presented in the following equation:

$$N_{SE} = I_o - PL + T_s - N_{TOT} + N_{DI} - N_{RD}$$

where

I_o = Source level (ownship transmitter for active sonar; target noise for passive sonar)

PL = Propagation loss

T_s = Target strength

N_{TOT} = Total noise

N_{DI} = Directivity index

N_{RD} = Recognitional differential

Numerous volumes of text have been written on each of the above components of the signal excess equation. References 3, 4, and 5

present excellent descriptions of each component. In the following subsections a brief description of each component is presented.

Source Level (I_0)

The reference power level in active sonar is equivalent to a level one yard from a hypothetical point source and is expressed by the equation

$$I_0 = 71.5 + D + \log P$$

where

D = Transmitting directivity index

P = Radiated power output

This equation assumes we have a nondirectional projector in a homogeneous absorption-free media. Although this situation is never reached in the real world of operational sonars, the above equation is used as a standard throughout most texts. To obtain the constant and ascertain where the other terms come from the derivation of this equation is as follows.

The intensity (I) of the sound emitted by the projector, at a large distance r , is related to the rms pressure (P_r) in dynes per square centimeter by the plane wave expression

$$I = \frac{P_r^2}{\gamma c} \times 10^{-7} \text{ watts/cm}^2$$

when

γ = Density g/cm^3

c = Velocity of sound cm/sec

using typical values

$$\gamma = 1 \text{ gr/cm}^3$$

$$c = 1.5 \times 10^5 \text{ cm/sec or } 4,920 \text{ ft/sec}$$

and converting to yards

$$I = 5.58 \times 10^{-9} P_r^2 \text{ watts/yd}$$

For a nondirectional projector, the intensity corresponds to a radial power output of

$$P = 4 \pi r^2 I = 70.08 \times 10^{-9} P_r^2$$

at a distance of 1 yard, the Power is

$$P = 70.08 \times 10^{-9} P_1^2 \text{ watts}$$

where P_1 is the rms pressure at 1 yard in dynes per square centimeter.

If we convert to db

$$\log P = \log 70.08 + \log 10^{-9} + \log P_1^2$$

and let

$$10 \log P_1^2 = \text{Source level } (I_0)$$

then

$$10 \log P = 10 \log 70 + 10 \log 10^{-9} + I_0$$

$$10 \log P = 10 (\log 7 - 8) + I_0$$

$$10 \log P = -71.55 + I_0$$

$$I_0 = 71.5 + 10 \log P$$

If we now add the transmitting directivity index, we have the original equation

$$I_0 = 71.5 + D + 10 \log P$$

This energy is transmitted from the source projector through the ocean to a target by surface ducting, convergence zone or bottom bounce or a combination of any or all three.

The near surface propagation paths for sound are extremely dependent on the near surface water temperature. If the temperature

is constant or increases with depth, the sound velocity profile is positive and the sound rays are bent concaved upward. A surface layer is defined as that vertical portion of the ocean from the surface to that greatest depth at which maximum temperature is found. When sound rays are trapped within the layer and bounce off the surface the method of transmission is defined as surface ducting. The existence of the convergence zone propagation path is controlled solely by environmental and physical conditions. The sound energy that leaves the surface layer is bent downward over that portion of the profile where the velocity increases with depth. If the sound velocity at a given depth equals the sound velocity at the layer depth, the sound ray will become horizontal at different ranges and results in their physical concentration at the surface, thus giving a convergence zone. In bottom bounce propagation, acoustic energy is reflected off the ocean bottom. In this mode of transmission all sound rays that leave the source at angles greater than the bottom grazing ray strike the bottom. These rays are reflected off the bottom and form a detection annulus at the surface. Figure 1 depicts each method.

Propagation Loss (P_L)

In traveling through the sea, an underwater sound signal becomes delayed, distorted and weakened. The propagation loss may be considered to be the sum of energy loss due to spreading and attenuation. Spreading loss is a geometrical effect representing the regular weakening of a sound signal as it is spread outward from the source. Attenuation loss includes the effect of absorption, scattering, variation in temperature, and leakage out of the sound channel.

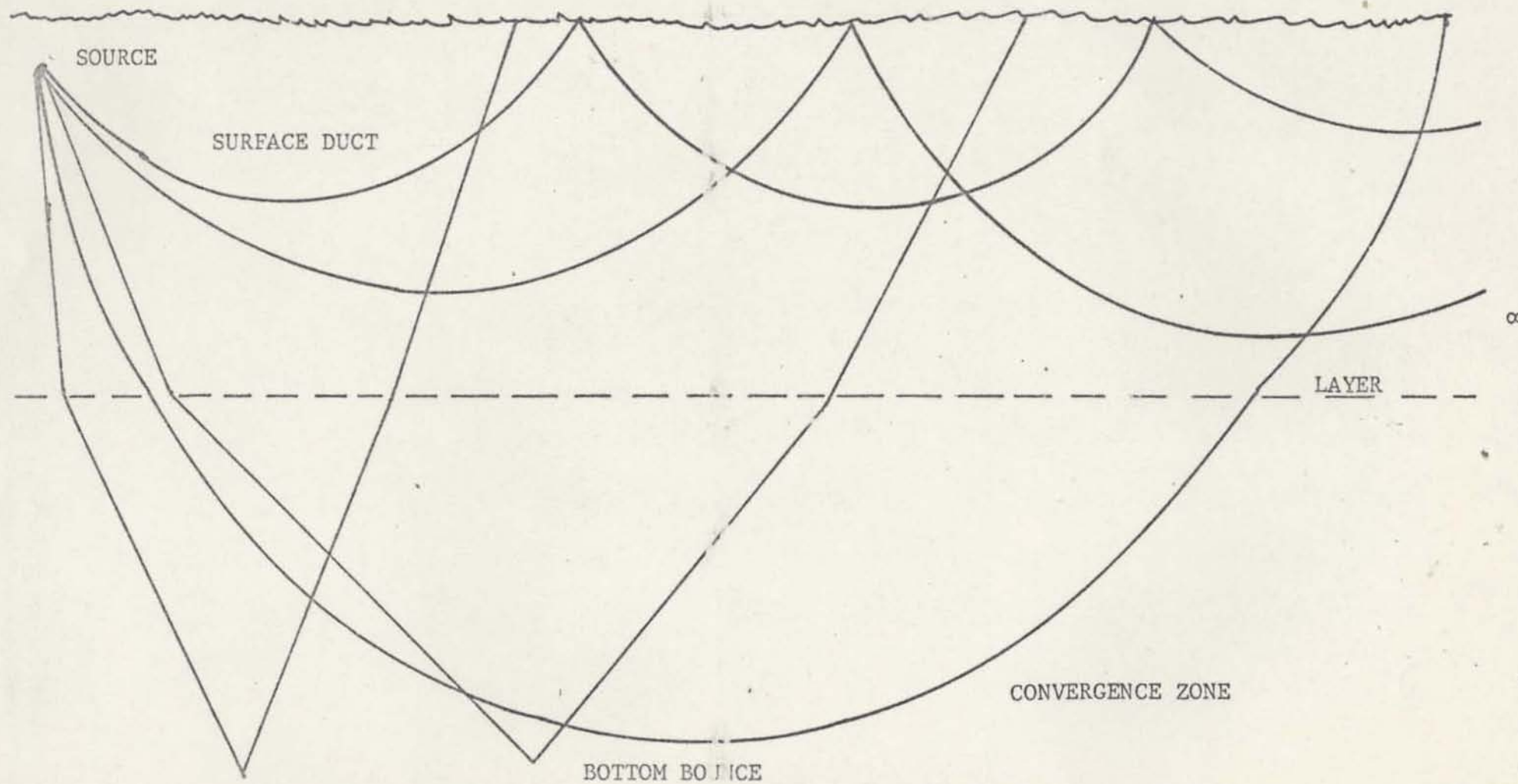


Figure 1.--Methods of Transmission

Attenuation may not be constant and cannot be accurately predicted from theoretical considerations. A simplified equation that is used as a working rule that contains spreading loss and attenuation loss but does not include specific propagation conditions is

$$PL = 20 \log r + (\alpha + \alpha_L)r \times 10^{-3}$$

$20 \log r$ = Spherical spreading

α = Absorption coefficient

$$\alpha = A \frac{S f_t f^2}{f_t^2 + f^2} + B \frac{f^2}{f_t}$$

α_L = Leakage coefficient that varies with frequency (0 - 12db)

where

$$A = \text{Constant} = 1.86 \times 10^{-2}$$

$$B = \text{Constant} = 2.68 \times 10^{-2}$$

S = Salinity (Parts/thousand)

$$f_t = 21.9 \times 10^6 - 1320/(T + 273)$$

f = Frequency in kilohertz

T = Temperature in degrees Centigrade

The above equation considers only surface duct transmission.

When convergence zone or bottom bounce mode of transmission is used for detection, the effects of pressure must be taken into consideration.

Measured and theoretical data agree that the formula

$$\alpha = \alpha_0 (1 - 1.93 \times 10^{-5} d)$$

where α_0 is the value of absorption at zero depth and d is depth in feet, the absorption of sound in sea water decreases by about 2 percent for every increase of 1,000 feet in depth. Thus, a ray trace of the bundle of rays in the convergence zone mode would have a propagation loss of

$$P_L = 20 \log r + (\alpha + \alpha_L)r \times 10^{-3} (1 - 1.93 \times 10^{-5} d)$$

As stated previously, the above equation is a working equation that is used for the temperate zone and deep water (depth greater than 100 fathoms). The Arctic region of the world produces unique propagation effects, thus requiring the use of different propagation equations. The Arctic region ice causes a combination of upward and downward refraction from the rough surface underneath the ice and produces a number of peculiarities. The most pronounced peculiarities are the rapid attenuation of high and low frequencies similar to band-pass filtering, low frequencies traveling faster than the high frequencies and the best propagation occurring in the octave of 15 to 30 Hz. The propagation loss in shallow water depends upon many natural variables of the sea surface, water medium and bottom type. Because of its sensitivity to these variables, the transmission loss in shallow water is only approximately predictable in the absence of specific knowledge of variables, especially the sound velocity and density structure of the bottom. The fluctuation of sound velocity is due to the existence of random inhomogeneities in the body of the sea and to the fact that these inhomogeneities are in motion relative to the source and receiver. For rough prediction purposes, tables of the data plus three different equations based upon range are used for shallow water propagation loss computations. These tables are based upon some 100,000 measurements in shallow water in the frequency range of 0.1 to 10 kHz and are used as a standard by companies and agencies of the government.

Target Strength (T_S)

The term "target strength" refers to the echo return by an underwater target. The target strength of many geometric shapes and forms have been found theoretically, in most cases for applications to radar. However, to obtain the exact target strength of an object of any complexity, it is best to utilize measured data of the target in its environment. Urlick (reference 5) gives a list of a number of mathematical forms for which the target strength has been determined. However, these idealized expressions should be taken only as crude approximations for targets of complex internal construction for which penetration and scattering are suspected to occur. Yet these equations are often useful for predicting target strength for which no measured data exists.

The simplest target to analyze is a sphere. The target strength does not depend on the direction of the incident sound or the direction in which the reflected sound is measured. For this reason, spheres are convenient targets and frequently serve as experimental targets in echo-ranging measurements. Unfortunately, very few objects encountered in every day experience are perfect spheres (mines and sonobuoys being the exception). The object chosen to analyze for this paper is a finite cylinder which closely approximates a submarine. In real life the submarine target strengths are perhaps most noteworthy for their variability. Not only do individual echoes vary greatly from echo to echo on a single submarine, but average values of echoes from submarine to submarine, as measured by different workers at different times, are vastly different. The foremost items that influence target strength are aspect, range and pulse duration. Thus, the equation for target

strength of a finite cylinder with a variable direction of incidence is given by

$$T_S = 10 \log \left[\frac{AL^2}{2\lambda \left(\frac{\sin B}{B} \right)^2} \right] \cos^2 \theta$$

where

A = Cylinder radius

L = Length of cylinder

λ = Wavelength

B = KL sin θ

K = 2π / wavelength

θ = Angle with the normal

Noise Total (N_{TOT})

Noise is defined as any undesired sound or an erratic, intermittent, or statistically random oscillation. In audioacoustics three terms of noise are used: random noise, white noise, and ambient noise. Random noise is defined as an oscillation whose instantaneous magnitude is not specified for any given instant of time. The instantaneous magnitudes of a random noise are specified only by probability distribution functions giving the function of the total time that the magnitude, or some sequence of magnitudes, lies within a specified range. White noise is used to describe a noise of a uniform distribution of energy as a function of frequency in the audible frequency range. Ambient noise is the noise that exists in the medium because of uncontrolled sources. Horton (reference 3) goes into great detail concerning the various types of noise that are distinguishable in the

ocean and contribute to the total noise spectrum. A brief abstract of some of the various types noise is:

1. Thermal Noise - Thermal agitation of water molecules, accompanied by a release of acoustic energy. Lower energy level than other noise, thus regarded as lower bound in determining minimum detectable signal.

2. Cavitation Noise - Pockets are formed when acoustic pressure exceeds static pressure. When the pressure equalizes, the cavities collapse and acoustical energy is released. This is the major component of ships sound.

3. Ambient Noise - A catchall term for general water noise when the individual noise sources are not easily identifiable. This noise is greatest near the shore and in shallow water because of the surf and movement of the sand and shells. In open sea or deep water this noise is of extremely low level.

4. Water Noise - Rainfall and the noise caused by water impacting on the ship's hull make up the major portion of this category. The magnitude and frequency of water noise is independent of depth to about 300 feet.

5. Marine Life Noise - Fish, shrimp and other marine life as well as birds, beasts and insects are included in this category. Fish noise is the limiting interference to the operation of sonar equipment in many locations of the world.

6. Ship Traffic Noise - General ship noise, not associated with a specific vessel, or having directional characteristics relative to the listening point.

7. Industrial Noise (in harbor or channel) - From factories, dredging operation, trains, and various machinery. This noise usually lacks directional variation because of transmission through the bottom rather than through the water.

8. Ship Sounds - Noise produced by own ship during the monitor cycle. This noise is usually low in frequency and when combined with sea life is generally the limiting factor in detection.

9. Reverberation - Reverberation is the backscattering of the transmitted energy. Reverberation is divided into three separate types: (1) volume reverberation which is assumed to be caused by scattering in the volume of the ocean by entrapped gas bubbles, dust and small marine organisms, (2) surface reverberation, caused by the scattering at the surface; and (3) bottom reverberation, which results from scattering at the bottom. Numerous investigations have been made to identify the precise sources and mechanisms that cause the various reverberation phenomena. However, the problem is still largely unresolved. Abners (reference 1) list nine possible causes for reverberation and disagrees with Horton (reference 3) as to the importance of convection cells, the microthermal structure and velocity microstructure of the ocean.

The sources of noise as described above can be divided into four categories: ambient (N_{AMB}), own ship (N_{OS}), volume reverberation (R_V), and surface reverberation (R_S). Bottom reverberation equations are identical with surface-reverberation equations with the exception of the grazing angle (angle of acoustic rays that strike ocean bottom tangential and are reflected upward) correction factor and variation of bottom type. The grazing angle correction factor is obtained from the equation

$$\theta = \sin^{-1}(d/r)$$

where

d = depth of the bottom

r = range

Since the bottom reverberation is most predominant in shallow water and less predominant in deep water; this term will be neglected in this analysis to be consistent with the previous equation for deep water.

The combination of all noise into a single equation was accomplished by Lockheed (reference 7) and is represented by the equation:

$$N_{TOT} = 10 \log (10^{0.1 N_{AMB}} + 10^{0.1 N_{OS}} + 10^{0.1 R_V} + 10^{0.1 R_S})$$

$$N_{AMB} = -55 - 17 \log f_{OS} + 30 \log (1 + 1.28S - .039S^2)$$

where

f_{OS} = Own ship frequency (an input representative of receiving spectrum)

S = Sea state

The own ship noise (N_{OS}) is a term derived empirically for each class of ships and submarines. This data is usually depicted in the form of a graph of noise versus speed of the vessel. The noise generated by own ship has numerous origins. The predominant causes of N_{OS} are propellers, machinery, cavitation and wave slap against the hull. Own ship noise is usually linear until a critical speed is reached and thereafter is exponential. For the purpose of this paper a numerical value will be chosen for a particular speed and a particular class of ship.

$$R_V = I_O - 2 P_L + 10 \log m_V - \frac{N_{DI}}{2} + 10 \log \gamma + 20 \log R + 55.9$$

$$R_S = I_O - 2 P_L + 10 \log m_S - \frac{N_{DI}}{2} + 10 \log \gamma + 20 \log R + 25.1$$

where

R_V = Volume reverberation level

R_S = Interface reverberation level

I_O = Effective radiated power

P_L = Propagation loss

m_V = Volume scattering coefficient

m_S = Area scattering coefficient

τ = Pulse length in milliseconds

R = Target range

Directivity Index (N_{DI})

The directivity factor for a transducer is defined as

$$DF = \frac{1}{\frac{1}{4} \int_0^{4\pi} [f(\phi)]^2 d\Omega}$$

In the above equation $f(\phi)$ is the ratio of the voltage output of a hydrophone for a signal incident at an angle ϕ with the acoustic axis to the voltage output when $\phi = 0$. The directivity factor may also be defined as the ratio of the response measured at a remote point in a free field on the principal axis to the average response measured on the surface of a sphere passing through the remote point, the center of which is at the transducer. Since the function $f(\phi)$ cannot normally be determined in practice, the directivity of a transducer cannot be determined by applying the above equation directly. Consequently, in general, the directivity factor must be determined by a process of integrating measured directivity patterns. Most transducers are designed so that the minor lobes are suppressed well below the level of the major lobe, the directivity index can be determined sufficiently accurate from various charts such as those presented by Albers

(reference 2). The directivity index of a transducer is the directivity factor expressed in decibels. It is ten times the logarithm to the base ten of the directivity factor.

N_{DI} of a projector provides a convenient means for computing the index level of an outgoing signal in terms of the total acoustic energy radiated. Since the N_{DI} is an empirically determined number which differs for each sonar or class of sonar, a typical numerical value will be chosen to be used for this paper.

Recognition Differential (N_{RD})

The separation of a signal from its background depends upon the time-frequency characteristics of the signal, the signal-to-noise ratio, the degree of correlation of the noise, the receiving bandwidth, the method of processing and the skill of the sonar operator. For a given set of conditions, the difference between signal level and interference level which corresponds to a detection probability of 50 percent is designated as recognition differential. Because there is no specification concerning false alarm, the term N_{RD} is quantitatively almost meaningless and is not used in recent publications. The term has been given a new name by current psychoacoustic literature, such as Urlick (reference 5), as being "detection index" having the equation of

$$d^{\frac{1}{2}} = \frac{M(s+n) - M_n}{\sigma}$$

where

$M(s+n)$ - mean signal-plus-noise amplitude

M_n = Mean noise amplitude

σ = Variance

However, numerous texts and other recent literature, such as Lockheed (reference 7), depend entirely on the recognition differential given by the formula

$$M = L_{50} - L_n$$

where

L_{50} = Signal level for a 50 percent probability of recognition

L_n = Noise level

Figure 2 depicts the graphical representation for recognition differential versus observational probability. The scale for recognition differential is from minus five to plus five db and, as expected, a recognition differential of zero is depicted for an observation probability of fifty percent.

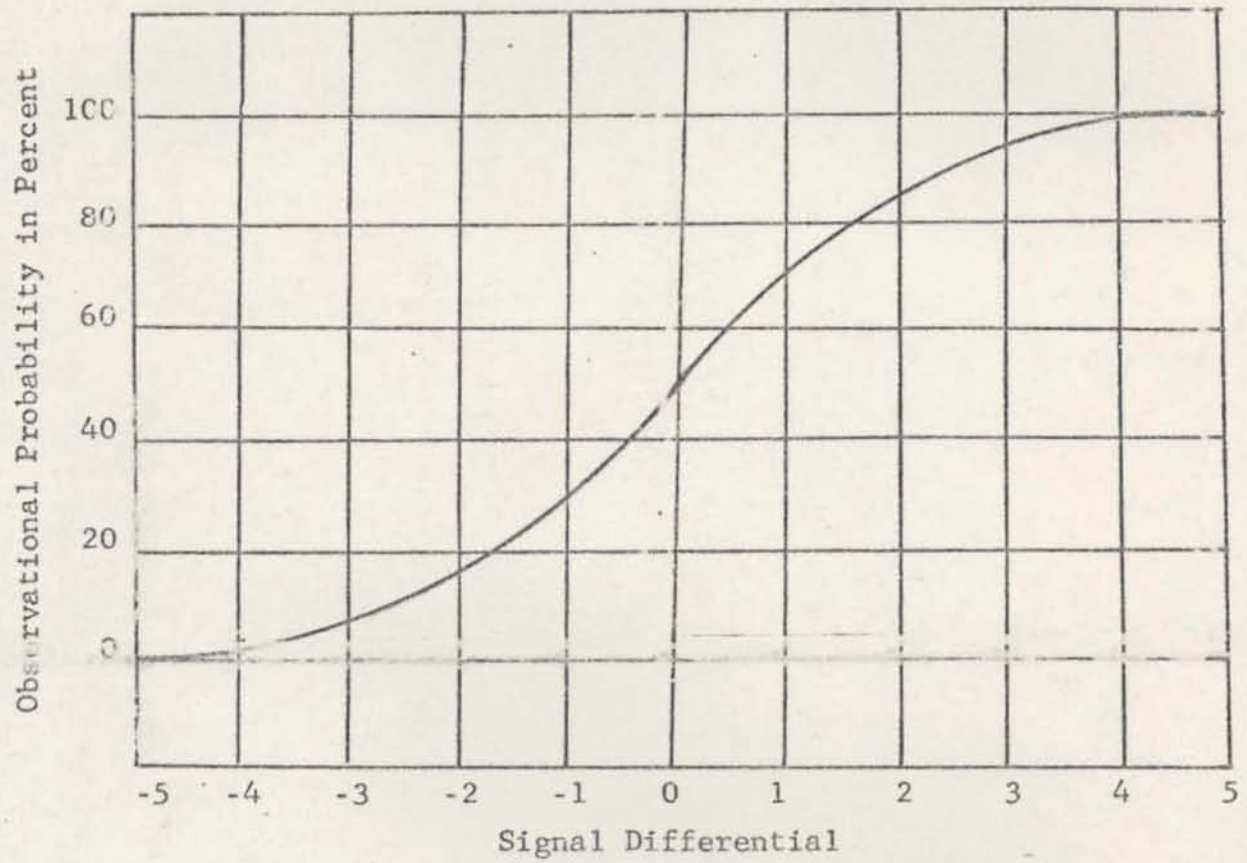


Figure 2.--Recognitional Differential
versus Observational Probability

CHAPTER III

ANALYSIS

As stated in the introduction of this paper, it is the intent to ascertain if the rigorous sonar equation can be simplified. To accomplish this, each equation or set of equations will be analyzed by using typical numerical values. Thus, each term of the equation can be analyzed as to the overall contribution it makes. From page 4

$$N_{SE} = I_O - P_L + T_S - N_{TOT} + N_{DI} - N_{RD}$$

Source Level (I_O)

The source level equation only contains two variables, power and directivity index. If we choose a beamwidth of 30 degrees at the 10 db downpoint on the transducer radiation pattern and use a power rating of a typical high-powered sonar, we have

$$I_O = 71.6 + D + 10 \log P$$

where

$$D = 20 \text{ db}$$

$$P = 140 \text{ db}$$

$$\begin{aligned} I_O &= 71.6 + 20 + 10 \log 6000 \\ &= 71.6 + 20 + 10 (4.778) \\ &= 139.38 \end{aligned}$$

Charts presented by Albers (reference 2) delineate that there is substantial loss in db (20 db) for directivity index between the 5 and 30 degree beamwidth at 10 db downpoint but relative little change in

db level (7 db) for beamwidths between 30 and 90 degrees. The above example is based upon 30 degree beamwidth at 10 db downpoint. Thus, substantial variation in beamwidth can be achieved without substantial change in db for directivity index. If the power is taken as one-half the above example, the result is only a 3 db loss. Thus, it is readily apparent that although the values can vary over a considerable range each term contributes significantly and none of the terms can be simplified or left out without a sacrifice to the entire equation.

Propagation Loss (P_L)

The propagation loss equation contains three variables for the basic equation. However, the subcomponents of the equation contain four additional variables that must be considered.

$$P_L = 20 \log r + (\alpha + \alpha_L) r \times 10^{-3}$$

$$\alpha = A \frac{3f_T f^2}{f_T^2 + f^2} + B \frac{f^2}{f_T^2}$$

$$\alpha_L = \text{Variable } 0 - 6 \text{ db}$$

$$A = 1.86 \times 10^{-2}$$

$$B = 2.68 \times 10^{-2}$$

$$f_T = 21.9 \times 10^6 = 1520/(T + 273)$$

$$S = 35$$

$$f = 4 \text{ kilohertz}$$

Solving for α and letting the temperature of the water be 60° Fahrenheit, we have

$$\begin{aligned} \alpha = & \frac{(1.86 \times 10^{-2}) (35) (21.9 \times 10^6 - 1520/15 + 273) (4)^2}{(21.9 \times 10^6 - 1520/15 + 273)^2 + (4)^2} \\ & + \frac{(2.68 \times 10^{-2}) (4)^2}{21.9 \times 10^6 - 1520/15 + 273} \end{aligned}$$

$$\begin{aligned}
\alpha &= \frac{(65.1 \times 10^{-2}) (115.7) (16)}{(115.7)^2 + (4)^2} + \frac{(2.68 \times 10^{-2}) (16)}{115.7} \\
&= \frac{120.5 \times 10^1}{1.338 \times 10^4 + 16} + \frac{41.88 \times 10^{-2}}{115.7} \\
&= 90.05 \times 10^{-3} + 3.706 \times 10^{-3} \\
&= 93.756 \times 10^{-3} \text{ or } .093756 \\
&= 9.37 \times 10^{-2} \text{ db/K yard}
\end{aligned}$$

Marsh (reference 6) develops logarithmic equation for α_L , the scattering loss in db per bounce and presents a table of the theoretical sea surface scattering loss versus wave height times frequency. Values for the table are obtained by multiplying the wave height times the transmitter frequency in kilohertz. A wave height of one foot and a transmitting frequency of four kilohertz will result in a 3 db per bounce loss. When α_L is compared to the contribution made by α it is seen from the above calculations that α could be neglected in any surface duct transmitting situation and α contributes very little in the bottom bounce transmission mode. If we include α in the calculation the following propagation loss results

$$\begin{aligned}
P_L &= 20 \log 7.2 \times 10^4 + (9.37 \times 10^{-2} \frac{\text{db}}{\text{K yard}} + \frac{3 \text{ db}}{\text{K yard}}) \\
&\quad (72 \text{ K yards}) (10^{-3}) \\
&= 80 (.85733) + 222.7 \times 10^{-3} \\
&= 68.58 + .2227 \\
&= 68.8027
\end{aligned}$$

It is readily seen that α contributes relative little and can be dropped. The above example considered only one bounce for α_L and convergence zone transmission. A ray trace of surface duct transmission would produce at least ten bounces for the range of 72,000

yards and even with this number of bounces the contribution is still small. The factor that would play a major role in increasing the propagation loss due to α_L would be rough seas, the table values presented by Marsh (reference 6) increase exponentially with higher seas. Thus, unless the seas are moderately high the terms α and α_L can be neglected without degenerating the propagation loss equation.

The small amount that α and α_L contribute to the propagation loss can be reduced further when we take into consideration the effects of pressure on absorption. The equation that includes this effect was stated on page 10 and we have

$$\begin{aligned}
 P_L &= 20 \log r + (\alpha + \alpha_L) (\times 10^{-3}) (1 - 1.93 \times 10^{-5} d) \\
 &\quad (1.93 \times 10^{-5}) (1.5 \times 10^4) \\
 &= 68.58 + .2227 (1 - .2895) \\
 &= 68.58 + .1581 \\
 &= 68.7381
 \end{aligned}$$

From the above calculations it is most obvious that we can neglect all the terms except $20 \log r$. Additional calculations were made with various ranges, frequencies, depths and sea state to insure that the equation did not contribute significantly. The only possible combination that causes the neglected factors to contribute significantly would be a high frequency transmitter and a high sea state. This combination in real life is impractical since the range is reduced by high frequency and sonar is seldom, if ever, operated in a high sea state. Thus, a rule of thumb that has fairly high accuracy for propagation loss is $20 \log r$.

Target Strength (T_S)

The target strength equation contains five variables, utilizing approximate values for the length and radius of a submarine with the acoustic beam striking at an angle of 45° , we have

$$T_S = 10 \log \left[\frac{AL^2}{2 \lambda \left(\frac{\sin B}{B} \right)^2} \right] \cos^2 \theta$$

$$A = 25 \text{ feet}$$

$$L = 425 \text{ feet}$$

$$\lambda = 2 \pi f = 2 \pi (4 \times 10^3)$$

$$B = KL \sin \theta$$

$$K = 2 \pi / \lambda$$

$$B = \frac{2 \pi}{2 \pi (4 \times 10^3)} 425 \sin 45^\circ$$

$$B = 74.34 \times 10^{-3} = .07434$$

$$\frac{\sin 4.26}{.074}^2 = \frac{.0125}{.074}^2 = .0337$$

$$T_S = 10 \log \frac{25 (425)^2}{2 (4 \times 10^3)} \quad (.5)$$

$$= 10 \log 282.3$$

$$= 24.49$$

The above equation can be simplified by equating the term $\left(\frac{\sin B}{B} \right)^2$ to one-tenth when θ is equal to 90° . This simplification does little to change the equation. The controlling factor is the $\cos \theta$ since as the angle changes from zero to 90 degrees the target strength decreases from a maximum value to zero. The orientation of the target is assumed to be broadside (beam aspect) at zero degrees and head (bow aspect) on at 90 degrees. Thus, the theory supports the polar antenna

patterns of various references that delineate maximum target strength at bow aspect.

Noise Total (N_{TOT})

As stated previously, there are numerous types of noise that contribute to the total noise spectrum. Lockheed (reference 7) combines several factors and presents the following equation as the total noise. The basic total noise equation has only four variables but the subequations have many other variables from the salinity of the water to constants that were derived from empirical data.

$$N_{TOT} = 10 \log (10^{.1 N_{AMB}} + 10^{.1 N_{OS}} + 10^{.1 R_V} + 10^{.1 R_S})$$

$$N_{AMB} = -55 - 17 \log f_{os} + 30 \log (1 + 1.28S - .039S^2)$$

$$f_{os} = 5$$

$$S = .3$$

$$\begin{aligned} N_{AMB} &= -55 - 17 (.6989) + 30 \log (4.84 - .35) \\ &= 11.8 + 19.5 \\ &= -47.3 \end{aligned}$$

This calculation agrees with curves presented in the Lockheed report (reference 7) for a moderate shipping lane, sea state one, and speed of vehicle of 11 - 16 knots and in deep water. This calculation assumes an average ambient noise and does not include intermitting noise sources such as porpoises that can create a sound level of 10 to 20 db.

The radiated noise of own ship varies according to class of ship and speed. Numerous tables and charts are available for various class ships at different frequencies. The destroyer was chosen as the platform of the sonar for this paper and from Urlick (reference 5).

$$N_{OS} = 20 \text{ db}$$

The equation for volume reverberations was obtained from Lockheed (reference 7). Several other references give similar type equation and although the source of volume scattering in the sea has not been definitely established, the following equation is considered a good working equation

$$R_V = I_O - 2 P_L + 10 \log M_V - N_{DI} - 10 \log \tau + 20 \log R + 55.9$$

$$I_O = 113.06 \text{ from previous calculation}$$

$$P_L = 75.626 \text{ from previous calculation}$$

$$M_V = 4 \pi s_V$$

$$s_V = \frac{\text{intensity of backscattering}}{\text{intensity of incident sound wave}}$$

$$S_V = -100 \text{ db (reference 1)}$$

$$s_V = 10^{-10}$$

$$s_V = 10^{-10}$$

$$M = 1.255 (10^{-9})$$

$$N_{DI} = 25 \text{ db}$$

$$\tau = \frac{2L}{V}$$

assume target length $L = 425$ feet and aspect angle of 45°

$$L = L \cos 45^\circ$$

$$= 297.5$$

$$\tau = \frac{595}{4920} = .1208 \text{ sec} = 120.8 \text{ millisecond}$$

$$R = 72,000 \text{ yds}$$

$$R_V = 113.06 - 2(75.626) + 10 \log 1.255 \times 10^{-9} - 25 + 10 \log 120.8 + 20 \log 7.2 \times 10^4 + 55.9$$

$$\begin{aligned}
 &= 113.06 - 151.25 - 90.97 - 25 + 20.82 + 97.15 + 55.9 \\
 &= 19.71
 \end{aligned}$$

As can be seen from the above calculations, each factor contributes significantly and the equation should not be simplified. It was surprising to find that R_V for these sets of conditions was positive. R_V is considered to be negative and is treated as such in most literature. In analyzing each of the terms of the equation it is readily apparent that if the power output or the range is decreased, R_V will become negative. Under operational conditions it is highly unlikely that the power output will be reduced but the range of the target will very likely decrease. Additional calculations were made to ascertain at what range the term R_V would become negative. Using the same factors above, at 6,000 yards R_V becomes negative.

The surface reverberation equation, like volume reverberation equation, is varied in different references. Again, the Lockheed report (reference 7) was chosen for this paper.

$$\begin{aligned}
 R_S = I_O - 2 P_L + 10 \log M_S - \frac{N_{DI}}{2} + 10 \log \gamma + 20 \log R \\
 + 25.1
 \end{aligned}$$

$$I_O = 113.06 \text{ from previous calculations}$$

$$P_L = 75.626 \text{ from previous calculations}$$

$$M_S = 2 s_s$$

$$S_s = 10 \log s_s$$

$$S_s = -50 \text{ db (reference 1)}$$

$$s_s = 10^{-4}$$

$$M_S = 6.28 \times 10^{-4}$$

$$N_{DI} = 25 \text{ db}$$

$$\gamma = 120.8 \text{ millisecond}$$

$$R = 72,000 \text{ yards}$$

$$\begin{aligned} R_S &= 113.06 - 2(75.626) + 10 \log 6.28 \times 10^{-4} - \frac{25}{2} + 10 \log \\ &\quad 120.8 + 20 \log 7.2 \times 10^4 + 25.1 \\ &= 113.06 - 151.25 - 47.97 - 12.5 + 20.82 + 97.15 + 25.1 \\ &= 42.41 \end{aligned}$$

Each factor of R_S contributes significantly and the equation should not be simplified. Again the range can be reduced to a point where the term becomes negative. However, this range is less than 1,000 yards.

$$\begin{aligned} N_{TOT} &= 10 \log (10^{0.1 N_{AMB}} + 10^{0.1 N_{OS}} + 10^{0.1 R_V} + 10^{0.1 R_S}) \\ &= 10 \log (10^{-4.73} + 10^2 + 10^{1.97} + 10^{4.24}) \end{aligned}$$

The term $10^{-4.73}$ can be neglected since its contribution will be very small.

$$\begin{aligned} N_{TOT} &= 10 \log (100 + 93.3 + 17380) \\ &= 10 \log (17573.3) = 10 \log 1.7573 \times 10^4 \\ &= 42.44 \end{aligned}$$

If we neglect the contribution for own ship noise and volume reverberation, the noise total is still 42.3 db and it is obvious that the most predominant factor is the surface reverberation. Thus, the equation can be reduced to

$$N_{TOT} = 10 \log 10^{0.1 R_S} = R_S$$

Directivity Index (N_{DI})

$$N_{DI} = 25 \text{ db (typical value)}$$

Recognitional Differential (N_{RD})

$$N_{RD} = 2 \text{ db per Horton (reference 3)}$$

Total Signal Excess Noise

Combining the numerical values obtained from the previous calculations and inserting them into the original signal excess noise equation, we have

$$\begin{aligned} N_{SE} &= I_O - P_L + T_S - N_{TOT} + N_{DI} - N_{RD} \\ &= 139.38 - 68.74 + 24.49 - 42.44 + 25 - 2 \\ &= 75.69 \end{aligned}$$

Comparative Analysis

Neglecting all the terms of the signal excess equation except the source level and the two way propagation loss, a simplified signal excess equation can be obtained. This equation is

$$N_{SE} = I_O - 20 \log R$$

Using the same source level and range as in the above calculations we have

$$\begin{aligned} N_{SE} &= I_O - 20 \log R \\ &= 139.38 - 20 \log 72,000 \\ &= 138.38 - 68.10 \\ &= 70.28 \end{aligned}$$

Thus, by further simplifying the signal excess equation and deleting all but two factors, results in a difference of only 5.31 db, or 14.25 percent error.

CHAPTER IV

CONCLUSION

The intent of this paper was to ascertain if the signal excess noise equation can be simplified without affecting the solution. As can be seen from the previous discussion, a number of terms that make up the detailed equation can be dropped without drastically affecting the overall numerical answer. The basic equation with all its components is presented on the following page along with the simplified equation. Depending upon the level required, the equation can be further reduced by deleting additional terms. The calculations on the previous page lead one to the conclusion that if all terms are neglected except the source level and the propagation loss a high degree of accuracy is obtained since the other terms of the equation cancel each other. Thus, a general working equation is

$$N_{SE} = I_0 - 20 \log R$$

This equation is most general but, if used judiciously, will provide approximately the same numerical results as does the complete equation.

$$N_{SE} = I_O - P_L + T_S - N_{TOT} + N_{DI} - N_{RD}$$

$$N_{SE} = [71.6 + D + 10 \log P] - [20 \log r + A \frac{S f_t f^2}{f_t^2 + f} + B \frac{f^2}{f_t^2} + 3] + 10 \log \left[\frac{AL^2}{2 \lambda \left(\frac{\sin B}{B} \right)^2} \right] \cos^2 \theta$$

$$+ [10 \log 10^{.1(-55 - 17 f_{os} + 30 \log(1 - 1.23s - .039s^2))} + 10^2$$

$$+ 10^{.1(I_O - 2 P_L + 10 \log M_V - N_{DI} - 10 \log \gamma + 20 \log r + 55.9)}$$

$$+ 10^{.1(I_O - 2 P_L + 10 \log M_S - \frac{N_{DI}}{2} + 10 \log \gamma + 20 \log r + 25.1)] + \frac{1}{\frac{1}{4\pi} \int_0^{4\pi} [f(\phi)]^2 d\Omega} - 2$$

CAN BE REDUCED TO

$$N_{SE} = 71.6 + D + 10 \log P - 20 \log r + 10 \log \left(\frac{AL^2}{2 \lambda} \right) \cos^2 \theta$$

$$+ 10 \log 10^{.1(I_O - 2 P_L + 10 \log M_S - \frac{N_{DI}}{2} + 10 \log \gamma + 20 \log r + 25.1)} + 25 - 2$$

LIST OF REFERENCES

Book Reference

1. Albers, V. M. 1961. Underwater Acoustics. P. 120-140.
New York: Plenum Press.
2. Albers, V. M. 1965. Underwater Acoustic Handbook II. P. 290.
University Park, Pennsylvania: Pennsylvania State University Press.
3. Horton, J. H. 1959. Fundamentals of Sonar. P. 57-72.
Annapolis, Maryland: United States Naval Institute.
4. Kerr, D. E., ed. 1951. Propagation of Short Radio Waves.
13: 445-469. New York: McGraw-Hill.
5. Urlick, R. J. 1967. Principles of Underwater Sound for Engineers.
P. 60-257. New York: McGraw-Hill.

Journal Reference

6. Marsh, H. W.; Schulkin, M.; and Kneale, S. G. Scattering of
underwater sound by the sea surface. Journal of the
Acoustical Society of America. Volume 33, Number 3
(March 1961). P. 1498-1514.

Report Reference

7. Lockheed Electronics Company. Final Math Model Report, Device 14A6.
Plainfield, New Jersey. (November 1968).

BIBLIOGRAPHY

Anti-Submarine Warfare Officer. Washington, D.C.: Bureau of Naval Personnel (1967).

Batzer, W. E.; Vent, R. J.; and Davies, I. E. "Acoustic Volume Scattering in the Pacific: Its Prediction and Its Effect on Submarine Detection," USL Symposium Report No. 995, (27 January 1969).

Ocean Acoustics. (Unpublished Proceedings). Washington: The Catholic University of America (1970).

Pedersen, M. A. and Gordon, D. F. "Normal-Mode Theory Applied to Short-Range Propagation in An Underwater Acoustic Surface Duct," Journal of the Acoustical Society of America, Volume 37, Number 1 (January 1968).

Sonar Ray Trace and Intensity Program. Los Angeles, California: Lockheed California Company, 1968.

Surface Channel Propagation Loss. San Diego, California: Naval Underwater Weapon Center Tech Note 93 (1968).

Take, John T., ed. The Physics of Sound in the Sea. Office of Scientific Research and Development. Washington, D.C.: Government Printing Office, 1946.