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Operation and Calibration Procedures for a Small Four-component Strain Gage Balance

Gary Allen Rasponi
University of Central Florida

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OPERATION AND CALIBRATION PROCEDURES
FOR A SMALL FOUR-COMPONENT STRAIN
GAGE BALANCE

BY
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B.S.E., Florida Technological University, 1972

RESEARCH REPORT
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Orlando, Florida
1974
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Also, a note of thanks to Mr. Joseph Haibach who assisted in the fabrication of the calibration assembly.
ABSTRACT

The Florida Technological University four-component strain-gage balance is an internally mounted, half-inch diameter balance capable of measuring four components of load: front and rear normal force, axial force, and rolling moment. Measurement of these components is accomplished by means of sixteen strain gages that are mounted on the balance and wired into four full bridge circuits. When the balance is subjected to a load, the strain gages, through a small resistance change, indicate the strain a balance element is undergoing.

This report presents a description of the balance and its support equipment, and outlines specific calibration procedures necessary to their successful implementation. These calibration procedures take two forms: that of calibrating the readout equipment, and that of calibrating the balance itself. Also contained in this report is a method of reducing calibration data into a set of parameters applicable to the balance.

To aid in the calibration of the balance, a calibration assembly was designed and built. Two calibration models were designed to facilitate the incremental loading of the balance and the interpretation of the readout data.
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CHAPTER ONE

INTRODUCTION

In 1971, Florida Technological University purchased a small four-component strain gage balance from Kenney Engineering Corporation as an accessory to a four-inch supersonic wind tunnel. This balance enables force and moment measurements to be made on a scaled down model that is subjected to an air loading of supersonic velocity in the wind tunnel. The information in this research report has been gathered for the purpose of providing a reference source and guide for experimentation with this balance.

At the time of purchase, the only information available concerning this balance was a users manual. This manual was deficient in several areas, most noticeably through the lack of final calibration constants. The users manual was also obscure on the calibration procedure itself, and since the balance had been received in an un-calibrated state, clarification was in order.

Several main areas of investigation are embodied in this report.

1. The description and use of a calibration assembly fabricated for this balance is presented.

2. How the strain gages detect and transmit force loadings is explained.
3. A trace of the balance readout signal is made from the strain gages to the display panel.

4. Systematic calibration procedures for both the balance and the readout equipment are established.

It was originally intended that this report also include some initial calibration values for this balance. However, due to an unfortunate accident, the balance was extensively damaged, preventing the completion of this part of the research. The damage occurred while the author was in the midst of calibrating the readout equipment. It is important to note that the remaining development was partially theoretical and no experimental evidence was available to completely substantiate the conclusions. In light of this situation, the tone of the research was shifted to become that of a guide to an experimenter who would attempt calibration of the balance after it had been repaired.
CHAPTER TWO

THE BALANCE

General
The balance is internally mounted, designed to fit within the hollowed out cavity of a model as illustrated in Figure 1. There are four strain-gage bridges located on the balance. These bridges measure the model normal force, rolling moment, and axial force. The purpose of such measurements is to obtain an estimate of loadings that will prevail on the full size projectile in flight, both for structural integrity and for performance reasons. Figure 2 depicts a model under an air loading with the resulting forces and moments of interest indicated.

Forces and moments acting on a projectile in flight or on a model in a wind tunnel are divided into two broad categories of static and dynamic loadings. Static loadings are those resulting from the motion of air over the model having a fixed alignment with respect to the relative wind. Referring to Figure 2, this says that the model is not rolling about or accelerating along its axes. Dynamic loads are those resulting from variations with time of orientation and linear accelerations. The FTU balance has been designed to measure static loads only.
Figure 1 Illustration of model mounting. Figure shows internal balance with hollowed out cavity of model.
Figure 2 Model under air loading with resulting forces and moments of interest indicated. Arrows indicate positives.
The balance assembly consists of three sections as illustrated in Figure 3. The front normal force bridge is located on the forward section, axial force and rolling moment bridges are located on the center section, and the rear normal force bridge is located on the aft section.

Figure 4 is a photograph of the front portion of the center section of the balance. The variable cross-section beam in the center of the photograph provides the mounting base for the axial force strain gage bridge. A close look at the beam will reveal some of the structural damage that had occurred. This is substantiated by the high resistance readings obtained from the gages within the bridge.

2.1 The Calibration Assembly

From physical considerations, it was decided that the balance could not be calibrated in the wind tunnel mounting. Investigation was begun to find an alternative method of support for calibration. The outcome of the investigation pointed to an assembly that could be fabricated such that all calibration procedures could take place apart from the wind tunnel.

The calibration assembly had to satisfy several design considerations.

1. The assembly had to be inexpensive, both in material and design. No intricate design would be considered, and no expensive material chosen.
Figure 3  Four Component Strain-Gage Balance. Photograph shows balance broken down into three sections. From top to bottom, they are forward section, center section, and aft section.
Figure 1: BRODWINER APPEARANCE SHOWING THE POSITION OF THE CAPILLARIES AND VESSELS RESISTANT TO THE PRESSURE OF THE CONTACT OR EXTERNAL FORCES."
Figure 4  Photograph enlargement showing the portion of the center section of the balance that contains the axial force measurement channel.
2. The assembly had to be structurally sound such that the imposition of maximum loads would not cause any deflections.

3. The assembly had to provide good accuracy. The tolerances found in the wind tunnel mounting had to be at least equaled in the calibration assembly.

4. The assembly had to include a provision for the application of positive and negative loadings.

5. The assembly had to include a provision for leveling the balance under load.

With these considerations in mind, a calibration assembly was designed and built. Figure 5 is a photograph of the resultant design. The balance can be mounted in the assembly as it would be in the supersonic wind tunnel. Axial force loadings may be accomplished with the assembly tipped over on its back, putting the balance in a vertical plane.
Figure 5  Calibration assembly for the four component strain-gage balance. Assembly shown in position for normal force and rolling moment loadings.
3.1 Signal Detection

The balance is built to measure four force loading components. They are front and rear normal force, rolling moment, and axial force. In addition, proper combination of the normal force data yields the pitching moment about any pre-selected reference point. The balance has been designed to measure the following maximum loads:

- Front normal force: ± 25 pounds
- Rear normal force: ± 25 pounds
- Rolling moment: ± 25 inch-pounds
- Axial force: ± 20 pounds

And from geometric considerations:

- Pitching moment: ± 75 inch-pounds

Each force measuring channel contains four strain-gages wired into a bridge circuit as illustrated in Figure 6. Each of these four independent bridges has four conductors leading out of the balance. These conductors are color coded such that input power to the bridge is red
Supply Voltage ($E_0$)

Red wire

Black wire

R denotes resistance in ohms.

Figure 6 The electrical arrangement of four strain gages in a bridge circuit.
and black, with red as plus input. The output from the bridge is green and white, with white as plus output. Each strain-gage in a bridge has a resistance of 350 ohms. (4)

The signal voltage from a bridge circuit can be calculated as follows, referring to Figure 6. The current flowing through gages 1 and 3, and through 2 and 4 is:

\[ I_{1-3} = \frac{E_0}{(R_1 + R_3)} \]  (3.10)

\[ I_{2-4} = \frac{E_0}{(R_2 + R_4)} \]  (3.11)

The voltage drop across gages 1 and 2 is:

\[ \Delta E_1 = I_{1-3} R_1 = E_0 \left[ \frac{R_1}{(R_1 + R_3)} \right] \]  (3.12)

\[ \Delta E_2 = I_{2-4} R_2 = E_0 \left[ \frac{R_2}{(R_2 + R_4)} \right] \]  (3.13)

The signal voltage, \( E_s \), is equal to:

\[ E_s = (E_0 - \Delta E_1) - (E_0 - \Delta E_2) \]  (3.14)

which reduces to

\[ E_s = - \Delta E_1 + \Delta E_2 \]  (3.15)

Substituting equation (3.12) and (3.13) into (3.15);
\[ E_s = -E_0 \left[ \frac{R_1}{R_1 + R_3} \right] + E_0 \left[ \frac{R_2}{R_2 + R_4} \right] \]
\[ = E_0 \left[ \frac{R_2}{R_2 + R_4} - \frac{R_1}{R_1 + R_3} \right] \]
\[ = \frac{E_0 (R_2 R_1 + R_2 R_3 - R_1 R_2 - R_1 R_4)}{(R_1 + R_3)(R_2 + R_4)} \] \hspace{1cm} (3.16)

or finally

\[ E_s = \frac{E_0 (R_2 R_3 - R_1 R_4)}{(R_1 + R_3)(R_2 + R_4)} \] \hspace{1cm} (3.17)

Therefore, from equation (3.17); if a zero load condition exists on the balance, all resistances will be equal and the signal voltage, \( E_s \), will be equal to zero. As the balance is loaded, however, the gage resistances change and the output signal will take on a finite value which is transmitted to the readout panel.

To get a correlation between the actual load impressed on the balance and the signal voltage produced, a special calibration model was designed. A detailed description of this model, as well as the calibration procedure itself, is treated in another chapter of this report.

3.2 **Signal Transmission**

Once the signal voltage has been generated in the strain-gage bridge,
it is transmitted to the readout panel. As it reaches the panel, the signal is amplified before being registered on a panel meter. The panel meters, one for each force measuring channel, are located in a display housing (see Figure 7). Starting at the upper left and going clockwise, the first meter reads front normal force, the second reads rear normal force, the third is axial force, and finally, rolling moment.

Each meter has a plus to minus 500 microamp movement with an accompanying set of adjustments. To the left of each meter, there are four trim potentiometers used to adjust and align the amplifiers which boost the signal voltage, $E_s$. A specific step-by-step meter calibration is presented in Chapter Five.

Power for the readout panel and balance originates from a plug-in amplifier supply board located within the readout panel housing. There are two power sources on the supply board. One of these sources is a nine-volt supply that provides power to the strain-gage bridges. The other power supply provides plus and minus fifteen volts, direct current, to the amplifiers. (4)
Figure 7. The model shows the ten forces exerting on the second layer.
Figure 7 The display panel showing the four force measuring channels.
CHAPTER FOUR

DATA REDUCTION

Most small balances have interactions between force components. (3) These interactions can be interpreted as the effect of one component on another, as the balance is subjected to a loading. This effect must be evaluated before model testing can be attempted. For example, as a normal force is impressed on the balance, the axial force channel may indicate a load, even though there is no axial force being directly applied. As the balance is loaded further, this interaction may vary in a linear or non-linear manner.

4.1 Linear Interactions

To illustrate how an interaction can vary linearly, consider the following situation. Figure 8 represents a balance that is capable of measuring front and rear normal forces. Station 1 is the front normal force gage location, while station 2 is the rear normal force gage location. A calibration shell is attached to the balance forward of station 1, as indicated. A normal force $F$ is applied to the shell at one of three loading notches machined into the calibration shell at known locations. The notches are spaced at 1.5 inches with one notch coinciding with station 1, and another notch coinciding with station 2.
Balance with calibration shell attached. Front and rear normal force gage stations are located.
With $F$ applied at station 1, a moment diagram, Figure 9a, can be constructed. The moment at station 1 is zero, and the moment at station 2 is $(3F)$ inch-pounds. The slope of the moment curve is linear and equal to $F$ inch-pounds per inch.

Now suppose $F$ is applied to station 2. Again, a moment diagram, Figure 9b, can be constructed. The moment at station 1 is $(-3F)$ inch-pounds, and the moment at station 2 is zero. Once again the slope of the moment curve is linear and equal to $F$ inch-pounds per inch. It can be concluded that no matter where the normal force $F$ is applied, the slope of the moment curve is constant for a given value of $F$.

The figure below is a plot of the above data. The moments at stations 1 and 2 are plotted versus distance along the balance. For graphical clarity, let $F$ be equal to 20 pounds.
Figure 9a  Moment diagram for F applied at station 1

Figure 9b  Moment diagram for F applied at station 2
These data can be presented in tabular form.

<table>
<thead>
<tr>
<th>Applied Load</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>F applied at 1</td>
<td>F applied at 2</td>
</tr>
<tr>
<td>Moment at 1</td>
<td>0 inch-lbs.</td>
</tr>
<tr>
<td>Moment at 2</td>
<td>-60 inch-lbs.</td>
</tr>
</tbody>
</table>

The moments generate a signal voltage which is displayed on a panel meter calibrated to read in pounds. The above moment table would be transformed into the following table of meter readings.

<table>
<thead>
<tr>
<th>Meter Readings</th>
<th>Applied Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>F applied at 1</td>
<td>F applied at 2</td>
</tr>
<tr>
<td>Front normal (F_{1ind})</td>
<td>0 lbs.</td>
</tr>
<tr>
<td>Rear normal (F_{2ind})</td>
<td>-20 lbs.</td>
</tr>
</tbody>
</table>

Note that in general, the meter readings ($F_{1ind}$) and ($F_{2ind}$) do not represent the true forces applied to the balance. A close look at the above table reveals that if a force $F$ is applied directly over station 1, no bending moment is produced there and hence, $F_{1ind}$ equals zero. However, it is desired that the front panel
meter indicate the presence of a front normal force. To accomplish this, the signal from the rear normal force channel \( F_{2\text{ind}} \) is connected to the front normal panel meter, and the front normal signal \( F_{1\text{ind}} \) is connected to the rear normal panel meter. In this way, a force \( F \) applied directly over station 1 will show up as a front normal force \( F_{1\text{ind}} \) on the display panel. Rewiring to eliminate negatives, the above table then becomes:

<table>
<thead>
<tr>
<th>Meter Readings</th>
<th>Applied Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front normal ( F_{1\text{ind}} )</td>
<td>20 lbs.</td>
</tr>
<tr>
<td>Rear normal ( F_{2\text{ind}} )</td>
<td>0 lbs.</td>
</tr>
</tbody>
</table>

Note how the indicated (panel readout) loadings have been interchanged because of the wiring changes discussed above.

Suppose it is desired to know what percent of the applied load shows up as a front normal force \( F_{1\text{ind}} \) and rear normal force \( F_{2\text{ind}} \). This is accomplished by dividing the indicated load by the applied load. The above table then reduces to the following:
Indicated Load/Applied Load

<table>
<thead>
<tr>
<th></th>
<th>Applied Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F applied at 1</td>
</tr>
<tr>
<td>$F_{1\text{ind}} \ (\text{lb/lb})$</td>
<td>1</td>
</tr>
<tr>
<td>$F_{2\text{ind}} \ (\text{lb/lb})$</td>
<td>0</td>
</tr>
</tbody>
</table>

In other words

\begin{align*}
F_{1\text{ind}} &= (1)F_1 + (0)F_2 \quad (4.10) \\
F_{2\text{ind}} &= (0)F_1 + (1)F_2 \quad (4.11)
\end{align*}

There are no interactions between channels (i.e. the effect of an applied load at station 1 on $F_{2\text{ind}}$ is zero). This is the ideal case. In reality, interactions do exist. To illustrate the effect of these interactions, assume that the following is a table of meter readings for this balance.

<table>
<thead>
<tr>
<th>Meter Readings</th>
<th>Applied Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F applied at 1</td>
</tr>
<tr>
<td>Front normal ($F_{1\text{ind}}$)</td>
<td>19 lbs.</td>
</tr>
<tr>
<td>Rear normal ($F_{2\text{ind}}$)</td>
<td>2 lbs.</td>
</tr>
</tbody>
</table>
In other words, for this example, the application of a 20 pound load at station 1 (front gage) results in a panel readout of $F_{1\text{ind}} = 19$ lbs., and $F_{2\text{ind}} = 2$ lbs. Then if the 20 pound load changed its position to that of station 2, the readouts would become $F_{1\text{ind}} = 3$ lbs., and $F_{2\text{ind}} = 19$ lbs. Now, if as before, the indicated load is divided by the applied load, the following table would result.

<table>
<thead>
<tr>
<th>Indicated Load/Applied Load</th>
<th>Applied Load</th>
<th>20 lbs. at 1</th>
<th>20 lbs. at 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{1\text{ind}}$ (lb/lb)</td>
<td>.95</td>
<td>.15</td>
<td></td>
</tr>
<tr>
<td>$F_{2\text{ind}}$ (lb/lb)</td>
<td>.10</td>
<td>.95</td>
<td></td>
</tr>
</tbody>
</table>

or

$$F_{1\text{ind}} = (.95)F_1 + (.15)F_2 \quad (4.12)$$

$$F_{2\text{ind}} = (.10)F_1 + (.95)F_2 \quad (4.13)$$

The above procedure can be extended to the general case where the interactions are not known. The following equation can then be written.

$$F_{1\text{ind}} = A_{11}F_1 + A_{12}F_2 \quad (4.14)$$

$$F_{2\text{ind}} = A_{21}F_1 + A_{22}F_2 \quad (4.15)$$
where

\[ F_{1\text{ind}} = \text{indicated front normal force, lbs.} \]
\[ F_{2\text{ind}} = \text{indicated rear normal force, lbs.} \]
\[ F_1 = \text{true front normal force, lbs.} \]
\[ F_2 = \text{true rear normal force, lbs.} \]
\[ A_{11} = \text{the effect of an applied load at station 1 on } F_{1\text{ind}}. \]
\[ A_{12} = \text{the effect of an applied load at station 2 on } F_{1\text{ind}}. \]
\[ A_{21} = \text{the effect of an applied load at station 1 on } F_{2\text{ind}}. \]
\[ A_{22} = \text{the effect of an applied load at station 2 on } F_{2\text{ind}}. \]

Solve equations (4.14) and (4.15) for \( F_1 \) and \( F_2 \), respectively.

\[ F_1 = \frac{F_{1\text{ind}} - A_{12}F_2}{A_{11}} \]  
(4.16)

\[ F_2 = \frac{F_{2\text{ind}} - A_{21}F_1}{A_{22}} \]  
(4.17)

Solve equations (4.16) and (4.17) simultaneously.

\[ F_1 - \frac{A_{12}A_{21}F_1}{A_{22}A_{11}} = \frac{A_{22}F_{1\text{ind}} - A_{12}F_{2\text{ind}}}{A_{11}A_{22}} \]
\[ F_2 - \frac{(A_{12}A_{21}F_2)}{A_{11}A_{22}} = \frac{(A_{11}F_{2\text{ind}} - A_{21}F_{1\text{ind}})}{A_{11}A_{22}} \]

Further

\[ F_1(A_{11}A_{22} - A_{12}A_{21}) = A_{22}F_{1\text{ind}} - A_{12}F_{2\text{ind}} \quad (4.18) \]

\[ F_2(A_{11}A_{22} - A_{12}A_{21}) = A_{11}F_{2\text{ind}} - A_{21}F_{1\text{ind}} \quad (4.19) \]

And finally

\[ F_1 = \frac{(A_{22}F_{1\text{ind}} - A_{12}F_{2\text{ind}})}{(A_{11}A_{22} - A_{12}A_{21})} \quad (4.20) \]

\[ F_2 = \frac{(A_{11}F_{2\text{ind}} - A_{21}F_{1\text{ind}})}{(A_{11}A_{22} - A_{12}A_{21})} \quad (4.21) \]

Using the values for the calibration constants in the previous example, the following can be shown.

\[ A_{11} = .95 \]

\[ A_{12} = .15 \]

\[ A_{21} = .10 \]

\[ A_{22} = .95 \]
Equations (4.20) and (4.21) then become:

\[ F_1 = (0.95F_{1\text{ind}} - 0.15F_{2\text{ind}})/(0.95)^2 - (0.10)(0.15) \]
\[ F_2 = (0.95F_{2\text{ind}} - 0.10F_{1\text{ind}})/(0.95)^2 - (0.10)(0.15) \]

To illustrate the use of these equations, consider the following example. Suppose that during an actual test, the loading on an airplane model made the front normal force panel meter indicate 10 pounds, and the rear normal force panel meter indicate 15 pounds. Using the above equations:

\[ F_1 = 0.95(10) - 0.15(15) / (0.8975) \]
\[ F_2 = 0.95(15) - 0.10(10) / (0.8975) \]
\[ F_1 = 8.08 \text{ lbs.} \]
\[ F_2 = 14.75 \text{ lbs.} \]

This procedure may be extended to a balance capable of measuring four components. In this case, there would be four equations to solve simultaneously, instead of only two. These equations would be similar to equations (4.14) and (4.15) described previously.
\[ F_{1\text{ind}} = A_{11} F_1 + A_{12} F_2 + A_{13} X + A_{14} R \]  \hspace{1cm} (4.22)

\[ F_{2\text{ind}} = A_{21} F_1 + A_{22} F_2 + A_{23} X + A_{24} R \]  \hspace{1cm} (4.23)

\[ X_{\text{ind}} = A_{31} F_1 + A_{32} F_2 + A_{33} X + A_{34} R \]  \hspace{1cm} (4.24)

\[ R_{\text{ind}} = A_{41} F_1 + A_{42} F_2 + A_{43} X + A_{44} R \]  \hspace{1cm} (4.25)

where

\( F_{1\text{ind}} \) = indicated front normal force, pounds.

\( F_{2\text{ind}} \) = indicated rear normal force, pounds.

\( F_1 \) = true front normal force, pounds.

\( F_2 \) = true rear normal force, pounds.

\( X_{\text{ind}} \) = indicated axial force, pounds.

\( R_{\text{ind}} \) = indicated rolling moment, inch-pounds.

\( X \) = true axial force, pounds.

\( R \) = true rolling moment, inch-pounds.

\( A_{ij} \) = calibration constants.

Equations (4.22) through (4.25) can be solved simultaneously and put into the form of equations (4.20) and (4.21). To determine the calibration constants, a table of loading data similar to the one
presented earlier is needed.

Suppose that the following is the result of applying a variable load $F$ directly over station 1.

<table>
<thead>
<tr>
<th>Meter Readings</th>
<th>Load Applied</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F$ applied at station 1</td>
</tr>
<tr>
<td>$F_{1\text{ind}}$ (lbs.)</td>
<td>4.9</td>
</tr>
<tr>
<td>$F_{2\text{ind}}$ (lbs.)</td>
<td>.10</td>
</tr>
<tr>
<td>$X_{\text{ind}}$ (lbs.)</td>
<td>.05</td>
</tr>
<tr>
<td>$R_{\text{ind}}$ (inch-pounds)</td>
<td>.02</td>
</tr>
<tr>
<td>Applied Load (lbs.)</td>
<td>5</td>
</tr>
</tbody>
</table>

Divide the indicated load by the applied load.

<table>
<thead>
<tr>
<th>Indicated Load/Applied Load</th>
<th>Applied Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F$ applied at station 1</td>
</tr>
<tr>
<td>$F_{1\text{ind}}$ (lb/lb)</td>
<td>.98</td>
</tr>
<tr>
<td>$F_{2\text{ind}}$ (lb/lb)</td>
<td>.02</td>
</tr>
<tr>
<td>$X_{\text{ind}}$ (lb/lb)</td>
<td>.01</td>
</tr>
<tr>
<td>$R_{\text{ind}}$ (inch-lb/lb)</td>
<td>.004</td>
</tr>
</tbody>
</table>
In equation form, the above table becomes:

\[
\begin{align*}
F_{1\text{ind}} &= 0.98F_1 & (4.26) \\
F_{2\text{ind}} &= 0.02F_1 & (4.27) \\
X_{\text{ind}} &= 0.01F_1 & (4.28) \\
R_{\text{ind}} &= 0.004F_1 & (4.29)
\end{align*}
\]

If equations (4.26) through (4.29) are compared with equations (4.22) through (4.25), the following can be shown.

\[
\begin{align*}
A_{11} &= 0.98 \\
A_{21} &= 0.02 \\
A_{31} &= 0.01 \\
A_{41} &= 0.004
\end{align*}
\]

From this then it follows that if \( F \) is applied directly over station 2, the constants \( A_{12}, A_{22}, A_{32}, \) and \( A_{42} \) can be evaluated. Likewise, if the balance is subjected to an axial load, constants \( A_{13}, A_{23}, A_{33}, \) and \( A_{43} \) can be evaluated. A rolling moment loading would reveal constants \( A_{14}, A_{24}, A_{34}, \) and \( A_{44} \). Equations (4.22) through (4.25) could then be rewritten and solved by using one of several mathematical techniques (i.e. a computer program utilizing the Gauss-Seidel method).
method or the Jacobi method of solution).

These calibration constants can be obtained in another manner. A graph can be drawn of the effect of the applied variable load on one of the four force components. To illustrate this, the following is a graph of the effect of the applied variable load at station 1 on $F_{2\text{ind}}$.

The slope of the above curve is:

$$\text{slope} = \frac{.20}{10} = .02$$
Recall from the previous method of determining calibration constants that the effect of the applied variable load at station 1 on $F_{2\text{ind}}$ was equal to .02, and was constant. The slope of the above curve is .02 and is constant. Therefore the slope of the above curve must equal the calibration constant $A_{21}$ found in equation (4.23). Graphing the other effects in a similar manner will reveal the remaining constants in equations (4.22) through (4.25).

4.2 Non-linear Interactions

The preceding section covers those instances where the interactions are linear, but occasionally an interaction has a definite second order effect. In this case, a linear calibration constant no longer suffices. Typically, such an interaction can be represented as

$$Y = Ax + Bx^2 = (A + Bx)x$$

This can best be illustrated by considering the following graph which shows the effect of an applied load (variable) at station 1 on $F_{2\text{indicated}}$: 

![Graph showing non-linear interaction](image-url)
The equation of the above curve can be represented as:

\[ F_{2\text{ind}} = (0.14 - 0.004F)F \]  \hspace{1cm} \text{(4.30)}

where

\[ F = \text{applied variable load, pounds.} \]

Recalling some values from a previous example, the table of loading data would look like the following.

<table>
<thead>
<tr>
<th>Meter Readings</th>
<th>Load Applied</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F applied at 1</td>
</tr>
<tr>
<td>( F_{1\text{ind}} ) (lbs.)</td>
<td>.95F</td>
</tr>
<tr>
<td>( F_{2\text{ind}} ) (lbs.)</td>
<td>((0.14 - 0.004F)F)</td>
</tr>
</tbody>
</table>

Divide the indicated load by the applied load.

<table>
<thead>
<tr>
<th>Load Indicated/Load Applied</th>
<th>Load Applied</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F applied at 1</td>
</tr>
<tr>
<td>( F_{1\text{ind}} ) (lb/lb)</td>
<td>.95</td>
</tr>
<tr>
<td>( F_{2\text{ind}} ) (lb/lb)</td>
<td>((0.14 - 0.004F))</td>
</tr>
</tbody>
</table>
Putting this data into equation form (similar to equations 4.20 and 4.21, with the appropriate constant change):

\[
F_1 = \frac{(.95F_{1\text{ind}} - .15F_{2\text{ind}})}{(.95)^2 - .15(.14 - .004F)} \quad (4.31)
\]

\[
F_2 = \frac{(.95F_{2\text{ind}} - (.14 - .004F)F_{1\text{ind}})}{(.95)^2 - .15(.14 - .004F)} \quad (4.32)
\]

Notice that there are four variables in equation (4.31); \(F_1, F_{1\text{ind}}, F_{2\text{ind}},\) and \(F\). There are also four variables in equation (4.32); \(F_2, F_{1\text{ind}}, F_{2\text{ind}},\) and \(F\). \(F_1\) and \(F_2\) are the desired quantities since they represent the true front and rear normal forces. The quantity \(F\) must be determined before the solution of these equations is attempted.

In an actual test of a model in the wind tunnel, \(F_{1\text{ind}}\) and \(F_{2\text{ind}}\) would be known since they represent the meter readings. The determination of \(F\) however, is dependent upon the accuracy desired in the answer for \(F_1\) and \(F_2\). When the desired range of accuracy is 5 percent, \(F\) can be set equal to \(F_{1\text{ind}}\); and \(F_1\) and \(F_2\) can be solved for directly. When the desired range of accuracy is 2 percent or less, \(F\) can be set equal to \(F_1\), and equation (4.31) solved for \(F_1\) by means of the quadratic formula. Equation (4.32) can then be solved by substitution. These percentages were arrived at by substituting values into equations (4.31) and (4.32) for \(F_{1\text{ind}}\) and \(F_{2\text{ind}}\), and solving for \(F_1\) and \(F_2\) using both methods discussed above.
Note that F was set equal to \( F_1 \) because the non-linear interaction occurred when the load \( F \) was applied at station 1. Had the non-linear interaction occurred when \( F \) was applied at station 2, then \( F \) would be set equal to \( F_2 \) in equations (4.31) and (4.32).

4.3 Model Normal Force and Pitching Moment

Although mentioned previously, no formal definition has been presented for normal force and pitching moment. The figure below is an illustration of the balance showing the front and rear normal force gage locations and a typical resultant force loading, \( N_F \). A moment reference center location must be chosen such as distance \( D_{ref} \) below. The distance between stations 1 and 2 is represented as \( D \); and the forces \( F_1 \) and \( F_2 \) are the gage loadings resulting from the model normal force.
The normal force, $NF$, can be calculated by taking the difference between the moments at stations 1 and 2 by dividing by the distance between them. In equation form (taking clockwise moments as positive):

$$NF = \frac{F_1 D_{\text{ref}} - F_2 (D - D_{\text{ref}})}{D}$$  \hspace{1cm} (4.33)

Therefore, with $D_{\text{ref}}$ equal to zero, the normal force $NF$ will equal $F_2$; and with $D_{\text{ref}} = D$, the normal force $NF$ will equal $F_1$. Recall that due to a wiring change, $F_2$ will register on the $F_1$ ind panel meter, and $F_1$ will register on the $F_2$ ind panel meter.

In designing the model, it is necessary to locate the resultant normal force between the fore and aft bridges. This is to ensure that the moment capability of the balance is not exceeded. If the resultant normal force is located ahead of or behind both bridges, $F_1$ and $F_2$ will be of the same sign, and if the resultant normal force is located between both bridges, $F_1$ and $F_2$ will be of opposite sign.

The pitching moment ($PM$) about the reference center can be obtained by linear interpolation between the two bridge moments.

$$PM = F_1 D_{\text{ref}} + \frac{D_{\text{ref}}}{D} \left[ F_2 (D - D_{\text{ref}}) - F_1 D_{\text{ref}} \right]$$  \hspace{1cm} (4.34)
In order to use the above equations, the numerical values of $D$ and $D_{ref}$ must be precisely determined. $D_{ref}$ can be located by knowing the model location with respect to the balance, i.e. through careful measurement following assembly.

Recall the discussion of moments induced on the model by the imposition of a force $F$. Figures 9a and 9b were generated by applying a fixed value load ($F$) to the model. Consider the case where the value of $F$ varies between zero and 15 pounds in 5 pound increments. This can be illustrated by the figure below.
Note that changing the value of F changes the slope of the moment curve, and may also cause small variations in the point where the curve crosses the zero axis because of small differences in gage placement. This range of values would produce an average "electrical center" location of the strain-gage bridge; by applying the variable load F at stations 1 and 2, the electrical centers of each of the bridges would be revealed. The longitudinal distance between the electrical centers would equal the quantity D.
CHAPTER FIVE

CALIBRATION PROCEDURE

5.1 Readout Panel

For the following procedure, refer to Figure 10. Once the power has been turned on, the panel should be allowed to warm up for fifteen minutes. After warm up, the zero control (R31) should be set near the center of its range; it is a ten turn potentiometer. Then the meter zero-adjust trim potentiometer (R20) should be set for a zero indication on the panel meter. The digital potentiometer control (R30) can then be set for a 0000 reading. R30 controls the reference voltage supplied to the amplifiers. The null balance switch (S1A-S1B) should now be depressed and held while the null balance zero trim potentiometer (R18) is adjusted for a zero indication on the meter. It should now be possible to adjust R18 and R20 until switching S1A on and off results in no meter offset.

A full scale signal (load) can now be applied to the balance element being calibrated. Adjust R30 so that the full scale load appears on the digital indicator. Now set the null balance span trim potentiometer (R23) for a full scale reading on the panel meter. It may be necessary to do this several times between zero and full scale to get the proper adjustment.
Figure 10  Illustration of readout for one force measuring channel.
When calibrating the front and rear normal force meters, the above procedure should be repeated, applying a full scale negative loading. The rolling moment can be checked by changing the direction of roll.

After calibration, before any loading is accomplished on the balance, each channel should be adjusted by turning the zero control potentiometer (R31). R31 acts as a fine tuning adjustment for trim potentiometer R18. This adjustment should be made after the model has been attached to the balance.

5.2 Balance Calibration

The calibration assembly for the four component balance includes the balance calibration stand, a calibration model for normal force and rolling moment loadings, and a calibration model for axial force loadings. Full detailed drawings for both the calibration stand and the calibration models can be found in this chapter.

To calibrate for normal force and rolling moment, the proper calibration model (see Figure 11) is slipped over the balance and the alignment pin engaged in one of the balance slots. The set screw is then secured. The model is now in a position such that the front and rear loading notches are over the centerline of the front and rear normal force gages.
Figure 11  Calibration model for normal force and rolling moment measurement.
Before loading, level carefully in all directions. Adjustment bolts are provided to level the balance in the horizontal plane. Figure 12 shows the calibration assembly and the balance in a normal force measurement position.

Loading for front and rear normal forces can be accomplished by using piano wire, a weight hanger, and a set of weights. The piano wire can be looped over the calibration model and placed in a loading notch. The weight hanger can be hung on the piano wire, and the weights applied to the hanger.

Hysteresis may play a big part in the application of a load. Hysteresis is an indication of the ability of a strain gage to possess the same resistance it had before a load is imposed as after the load is removed. If hysteresis data are needed, weights can be applied in step order and removed in step order. Caution should be taken because the removal of one weight before applying the next destroys the hysteresis data.

When it is desired to calibrate for negative normal forces, the balance holder (see Figure 13) can be inverted to calibrate in this direction. This is done by removing the hex nut retaining the balance holder to the stand, inverting the holder and attaching it to the other side of the stand. Neither the calibration model nor the balance has to be removed from the holder. No wires have to be disconnected. Figure 16 is a detailed drawing of the stand to which the balance holder is attached.
Figure 12  Calibration assembly and balance shown in normal force measurement position.
Figure 13   Balance Holder.
When applying the weights, all components should be recorded at each load, up and down. After the addition of each weight, the calibration model should be leveled.

Rolling moment is calibrated by hanging weights on a cross arm that can be attached to the forward end of the normal force calibration model (see Figure 14). Piano wire, weight hangers, and weights can be used to accomplish loadings. The piano wire can be pulled through the roll hooks and the hangers attached. The weights should be initially all equally distributed on the two weight hangers. Roll can then be generated by the transfer of weights from one hanger to the other.

When it is desired to calibrate the axial force component, the calibration model is disconnected from the balance. The cross arm at the front of the model is removed and a separate weight support (see Figure 15) is installed in its place. The calibration model is re-attached to the balance and the whole assembly is turned on its back so that the balance axis is in a vertical plane; the assembly is then leveled. The axial force calibration model has been designed so that the weight hanger will be centered over the balance. The weight hanger can be put into place and weights added.
Figure 14: Calibration assembly and balance shown in rolling moment measurement position.
Figure 15  Weight support used for axial force calibration.
Figure 16  Stand used to support balance holder.
CHAPTER SIX

CONCLUSIONS

An investigation has been conducted to establish operation and calibration procedures for the Florida Technological University four component strain-gage balance. The following results emerged from this investigation.

1. A calibration assembly was designed and built.

2. A detailed method of determining calibration constants was presented.

3. A detailed procedure for calibrating the readout display panel as well as the balance was outlined.

4. The balance and readout equipment operated satisfactorily prior to damage.

After damage, the front normal, rear normal, and rolling moment strain-gage bridges indicated book value resistances. However, the axial force strain-gage bridge resistance reading indicated that extensive damage had occurred. Upon dissassembling the balance, severe structural damage was noted. No further experimentation was attempted due to the nature of the damage. However, from the troubleshooting that followed, insight was gained into the electrical make-up of the readout system. Research continued on a theoretical basis, with the belief that when the balance was repaired, experimentation would confirm theory.
BIBLIOGRAPHY


