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Identifying non-stationarities in random EM fields: are speckles really disturbing?

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Abstract: In dealing with random EM fields, ensemble averaging is an ubiquitous procedure. However, we demonstrate that spatial non-stationarities such as enhanced backscattering can be identified even from one single realization (snapshot) of the wave interaction with a random medium. Fourth-order correlations between field components at two different spatial points are shown to provide the necessary information.

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References and links

1. J. W. Goodman, "Some fundamental properties of speckle," J. Opt. Soc. Am. **66**, 1145–1150 (1976).
2. J. W. Goodman, *Statistical Optics* (John Wiley & Sons, Inc., New York, 1985).
3. G. Korn and T. Korn, *Mathematical Handbook for Scientists and Engineers* (McGraw-Hill, New York, 1961).
4. Y. N. Barabanenkov, Y. A. Kravtsov, V. D. Ozrin, and A. I. Saichev, "Enhanced backscattering in optics," Prog. Opt. **29**, 65–197 (1991).
5. P. Rochon and D. Bissonnette, "Lensless imaging due to back-scattering," Nature **348**, 708–710 (1990).
6. J. W. Goodman, *Introduction to Fourier Optics* (Roberts & Company, Englewood, 2005), 3rd ed.
7. S. Huard, *Polarization of Light* (John Wiley & Sons, New York, 1997).
8. J. Ellis and A. Dogariu, "Differentiation of globally unpolarized complex random fields," J. Opt. Soc. Am. A **21**, 988–993 (2004).
9. J. Ellis and A. Dogariu, "Complex degree of mutual polarization," Opt. Lett. **29**, 536–538 (2004).
10. M. Nieto-Vesperinas and J. A. Sanchez-Gil, "Enhanced long-range correlations of coherent waves reflected from disordered media," Phys. Rev. B **46**, 3112–3115 (1992).
11. L. Tsang and A. Ishimaru, "Theory of backscattering enhancement of random discrete isotropic scatterers based on the summation of all ladder and cyclical terms," J. Opt. Soc. Am. A **2**, 1331–1338 (1985).
12. C. Schwartz and A. Dogariu, "Enhanced backscattering of vortex waves from volume scattering media," Opt. Commun. **263**, 135–140 (2006). Schwartz, Chaim Dogariu, Aristide.
13. I. Bereznyy and A. Dogariu, "Time-resolved mueller matrix imaging polarimetry," Opt. Express **12**, 4635–4649 (2004).
14. G. Popescu and A. Dogariu, "Optical path-length spectroscopy of wave propagation in random media," Opt. Lett. **24**, 442–444 (1999).
15. Y. N. Barabanenkov, Y. A. Kravtsov, V. D. Ozrin, and A. I. Saichev, "Enhanced backscattering - the universal wave phenomenon," Proc. IEEE **79**, 1367–1370 (1991).
16. K. M. Yoo, G. C. Tang, and R. R. Alfano, "Coherent backscattering of light from biological tissues," Appl. Opt. **29**, 3237 (1990).
17. A. Derode, V. Mamou, F. Padilla, F. Jenson, and P. Laugier, "Dynamic coherent backscattering in a heterogeneous absorbing medium: Application to human trabecular bone characterization," Appl. Phys. Lett. **87**, 114101 (2005).
18. P. Sheng, *Scattering and localization of classical waves in random media* (World Scientific, Singapore, 1990).

1. Introduction

Optical waves interacting with randomly inhomogeneous media give rise to complicated electromagnetic fields. Such a field can be considered as the superposition of a large number of

elementary waves with random phases and states of polarization. When the initial source of radiation is coherent, the superposition of these waves produces a random interference, i. e. a speckle pattern [1]. It is commonly assumed that the sources of these elementary waves are uncorrelated and sufficiently separated to produce a uniform phase distribution. As a result, in one point of the speckle pattern, the field components are described by circular Gaussian statistics [2]. Additionally, this random interference is usually considered to be both spatially stationary and spatially ergodic. That is to say that the ensemble averaged quantities do not depend on position, and that the spatial statistics are equivalent to the ensemble statistics at a point [3]. Of course, if the properties of the randomly inhomogeneous medium vary in time then the speckle pattern is dynamic and similar assumptions are usually made about the temporal stationarity and ergodicity. For the purposes of this paper, we will restrict ourselves to considering only the spatial properties of random fields.

There is little to say about the wave-matter interaction when the resulting random field obeys Gaussian statistics. However, there are instances in which the assumptions of Gaussianity, stationarity, and ergodicity are not fully satisfied. For instance, specific spatial correlations between sources can produce non-stationary statistics in a speckle field. In this context, one intriguing situation is that of the weak localization of waves in reflection [4]. When a plane wave is incident upon a random medium, the probabilities of any given scattering path and its time reversed pair are equal. In the exact backscattering direction, all such pairs interfere constructively. The location of this maximal interference is independent of a particular path or realization, leading to a "coherent" effect known as enhanced backscattering (EBS). Because of this phenomenon, the intensity distribution is a random process that is spatially non-stationary; its mean value varies with the spatial location.

EBS is typically observed as the result of an ensemble average over many realizations of the interaction between a coherent wave and a multiple scattering medium. This average is usually performed by changing the medium's configuration but it can also be achieved through wavelength diversity [5]. Of course, the presence of an enhancement in the backscattering direction is masked by the random intensity distribution that constitutes the speckle pattern. The existence of this spatial non-stationarity however should be present in each realization of the interaction between a coherent wave and a random medium and one can ask the question: is this information retrievable without having access to the ensemble average? To answer it, we will inspect such random field distributions by examining their statistical properties beyond the second-order field correlations. We will show that through an appropriate statistical analysis, the presence and location of the spatial non-stationarity associated with enhanced backscattering can be identified even in a single realization of the random electromagnetic field.

2. Theory

Given a spatial non-stationarity, it follows that the field distribution is non-ergodic. Specifically, as the ensemble average depends on location, it is not possible for the spatial average to equal the ensemble average at every point. However, in practice it may be possible to treat the field as *locally, spatially ergodic*. That is to say that the spatial average over some region about a given point may recover the ensemble average value at that point. However, the concept of local spatial ergodicity raises a number of issues regarding the length scales of the field distribution, the field measurement, and its characterization. For instance, the region of spatial averaging must be sufficiently large to provide reasonable statistics, and yet not so large as to wash out the spatial non-stationarity. We will discuss these aspects in the context of different methods for characterizing such field distributions.

Based on the assumption of local ergodicity over certain spatial domains, we will examine two different methods that may be capable of discerning ensemble-like information from one

single realization of the random field. The simplest approach to duplicate the ensemble intensity average is to take a moving spatial intensity average. The effective intensity $\bar{I}$ at a point r calculated for a spatial subdomain A can be defined as

$$\bar{I}_A(r) = \frac{1}{\|A\|} \int_A I(r+r_0) dr_0, \quad (1)$$

where

$$I(r) = E_x^*(r)E_x(r) + E_y^*(r)E_y(r). \quad (2)$$

As the phenomenon of interference between time reversed paths is polarization dependent, another possibility would be to examine polarimetric quantities [6]. Specifically, the constructive interference relies on polarization similarity, that can be gauged by the degree of polarization estimated over a spatial subdomain A [7]. This effective degree of polarization $\bar{P}$ can be defined as

$$\bar{P}_A(r) = \frac{\sqrt{\int_A S_1^2 dr + \int_A S_2^2 dr + \int_A S_3^2 dr}}{\int_A I dr}. \quad (3)$$

where

$$\begin{aligned} S_1(r) &= E_x^*(r)E_x(r) - E_y^*(r)E_y(r) \\ S_2(r) &= E_x^*(r)E_y(r) - E_y^*(r)E_x(r) \\ S_3(r) &= i(E_x^*(r)E_y(r) - E_y^*(r)E_x(r)) \end{aligned} \quad (4)$$

It is interesting to note that, unlike the effective intensity, this quantity inherently involves fourth-order field correlations, and as such, can be expected to be more sensitive to fluctuations in the field distribution [8].

Both parameters defined in Eqs. (1) and (3) can be used to encode the spatial distribution of a random field (a speckle-like image) by producing an average over a subdomain A and then associating that value with the corresponding location in the initial field. However, the choice for the size of such a subdomain is arbitrary and most importantly, introduces an artificial length scale. In other words, one may find an appropriate size of the subdomain for which the non-stationarity can be revealed but the size of such subdomain is not known *a priori*; moreover, this choice may depend on characteristic length scales of the specific problem. These length scales are the physical extent of the non-stationarity and the overall size of the available data, i.e. the largest scale length in the random field. In fact, the existence of an appropriate size of the subdomain is inherently tied to the existence of a spatial non-stationarity.

To avoid having to find an optimum size of the subdomain that would reveal a specific non-stationarity, we examine another possible measure that is based on the recently introduced complex degree of mutual polarization (CDMP) of a random electromagnetic field [9]. The magnitude of the CDMP measures the polarization similarity between two points, r_1 and r_2 , in a random electromagnetic field. Under the assumption of a fully correlated and locally fully polarized field, the CDMP simplifies to

$$V^2(r_1, r_2) = \frac{(E_x^*(r_1)E_x(r_2) + E_y^*(r_1)E_y(r_2))^2}{(E_x^*(r_1)E_x(r_1) + E_y^*(r_1)E_y(r_1))(E_x^*(r_2)E_x(r_2) + E_y^*(r_2)E_y(r_2))} \quad (5)$$

This two-point measure does not rely upon the choice of an artificial subdomain, and as such, it can provide a direct measure of the length scale associated with a certain physical process. We emphasize that, as in the case of the effective degree of polarization, the CDMP parameter

contains fourth-order correlations between field components. However, in this case field components at different spatial locations are involved as opposed to the measure defined in Eq. (3). Thus, one expects even higher sensitivity to specific characteristics of a random field.

Using the definition in Eq. (5), one can calculate the CDMP spatial decay length $L(r)$ by evaluating in each point r the decay of $|V^2(r, r + \delta r)|$ for increasing values of δr averaged azimuthally. For identically polarized fields, $|V^2(r, r + \delta r)|$ is unity, while for a uniformly random distribution of states of polarization $|V^2(r, r + \delta r)|$ averages to one half. After evaluating the CDMP decay length for each point, these values can be used to generate a completely new spatial representation where each point is encoded in its corresponding value of the CDMP decay length.

All three of these approaches, the effective intensity $\bar{I}$, the effective degree of polarization $\bar{P}$, and the CDMP decay length $L(r)$ will be used to examine speckle fields that, upon ensemble averaging, manifest coherent backscattering. A single realization of the speckle pattern can be written as

$$I(\mathbf{k}_i, \mathbf{k}_f) = I_0 + \sum_{l,m} |A_{lm}|^2 \cos(\mathbf{k}_i + \mathbf{k}_f) \cdot (\mathbf{r}_l - \mathbf{r}_m) + F(\mathbf{k}_i, \mathbf{k}_f), \quad (6)$$

where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the wave vectors of the incident and scattered plane wave and A_{lm} represents the complex amplitude of the wave having $\mathbf{r}_l$ and $\mathbf{r}_m$ as the ending points of a multiple scattering trajectory inside the random medium [10, 11]. The second term in Eq. 6 represents the non-stationary component that upon ensemble averaging leads to a cone of enhanced intensity. This intensity enhancement around the backscattering direction has a width on the order of λ/l^* , where l^* is the so-called transport mean free path. The third term $F(\mathbf{k}_i, \mathbf{k}_f)$ represents the speckle fluctuations and, in the case of a Gaussian random field, averages to zero.

3. Experiment

To test the different methods for locating the presence of non-stationarity, a typical EBS experiment was conducted. The setup, built around a continuous-wave laser operating at 488nm and a cooled CCD array, was described earlier [12]. In addition, a full polarimetric measurement was performed in each pixel of the resolved speckle using a rotating quarter-wave plate and subsequent Fourier analysis [13]. In the experiments, a 2mm beam was incident on the sample and produced in the plane of the CCD a speckled field with an average size of the speckle of about $64\mu\text{m}$. The sample was mounted on a spin plate that allowed observing single realizations of the scattered field as well as the corresponding ensemble average.

The scattering media used were different diffusing materials exhibiting minimal absorption. A large range of transport mean free paths, l^* , was covered using different solid samples: (A) Suba IVTM polishing pad (Rodel), (B) Spectralon[®](Labsphere), (C) DuraporeTM-HVLP filter paper (Millipore), and (D) compressed TiO₂ powder (DuPont). The scattering strengths of these samples are very different. From the widths of the corresponding EBS cones the estimated values of l^* were $40\mu\text{m}$, $20\mu\text{m}$, and $7\mu\text{m}$ for samples A, B, and C respectively. For the TiO₂ sample a scattering mean free path of approximately $1\mu\text{m}$ was determined using optical path-length spectroscopy [14].

4. Results

The results of applying the analysis methods are summarized in Fig. 1. The first two rows illustrate a typical realization of the random distribution of backscattered intensity and the corresponding result of the ensemble average, respectively. The familiar appearance of a speckle field (first row) can be observed for all samples, and as can be seen, no sample specific information is practically available in these intensity distributions.

Fig. 1. (color online) Images corresponding to samples A, B, C, and D as described in the text. (i) single realization speckle intensity image, (ii) ensemble average, (iii) image encoded in the calculated effective intensity, (iv) image encoded in the calculated effective degree of polarization, (v) image encoded in the calculated polarization decay.

As a result of the ensemble average on the other hand, the extent of the enhanced intensity may provide means to discriminate between the different structures as can be seen in the second row. However, this ability is restricted by the spatial resolution and the extent of the accessible field (experimentally limited by the pixel size, numerical aperture, and number of pixels available). For instance, in the case of sample D, the ensemble average appears as an almost constant background. It is evident that in this case, one cannot conclude that the absence of a region of enhanced intensity is due to the peak being either too large or too narrow or simply because the recording is performed away from the backscattering direction.

In the third row of Fig. 1, we present the results of calculating the effective intensity $\bar{I}$ over a region (square box) containing 61×61 pixels that was scanned across the entire speckle image shown in the first row. For each location, the value of $\bar{I}$ was attributed to the central pixel of the box. This method basically performs the subset average of intensity including many points

instead of the ensemble average at the central point of this domain. Since the average intensity in the EBS region is higher than the background, one could also expect a similar effect in the effective intensity image. However, as can be seen in Fig. 1, no such increase of intensity can be observed for samples A and D, but the reasons are quite different. In the case A, the non-stationarity cannot be resolved due to the size of the averaging box while in the case D, the process is simply stationary over the limited field available. A certain increased intensity is prevalent in the results for samples B and C where an intensity enhancement concentrated towards the center of the image may indicate the presence of a spatial non-stationarity.

In a completely similar manner, we have evaluated the effective degree of polarization for a circular area with a radius of 31 pixels and the results are shown in the fourth row of Fig. 1. Because the interference effects leading to the enhanced scattering rely upon polarization similarity, it is expected that regions with a higher degree of polarization should indicate the presence of EBS. This is indeed seen in our results where the $\bar{P}$ values around 0.5 clearly indicate the polarization similarity. It is interesting to note the increase in the $\bar{P}$ values from sample A to D and the existence of non-stationarities similar to the ones in the ensemble averages shown in the second row.

Of course, for a specific sample, both $\bar{I}$ and $\bar{P}$ images could have been optimized in order to illustrate the presence and identify the location of the enhancement peak. However, that would necessitate *a priori* knowledge about the extent of the non-stationarity in order to select an appropriate size of the averaging box as the size of the box is sample specific. This requirement can be avoided by using a higher-order polarimetric measure as demonstrated in the last row of Fig. 1 where the images are encoded in the decay length of CDMP evaluated as described before. As can be seen, now there is an even stronger progression from left to right. As the size of the enhancement increases, the number of points having longer polarization decay lengths rises. Note that there is no additional image processing involved and that the color coding indicates the actual decay length of CDMP measured in pixels. Samples B, C, and D all show strong spatial polarization correlations with increasing values of the polarization decay lengths. In the case A, there is simply not enough pixel resolution to evaluate a two-point characteristic such as CDMP.

Perhaps the most interesting observation concerns the compressed TiO₂ powder (sample D) which has a very small l^* leading to a large EBS cone. As pointed out before, in this case the ensemble average image cannot confirm the presence of a non-stationarity in the random distribution of intensity. Because of the limited angular resolution of the optical system, one cannot identify the presence of a region with enhanced intensity. However, the existence of the coherent enhancement is clearly visible in only one realization of the random speckle pattern when the higher-order two-point correlations of the field are examined. The high values of the CDMP decay length clearly indicate the existence of polarization similarities that are specific to EBS. Detailed features of the wave-matter interaction are therefore prevalent in one single realization of the emerging random field but their characterization requires access to higher-order statistics of the field.

5. Conclusions

We have investigated three different methods to detect the presence and locate the non-stationary feature from enhanced backscattering in a single realization of the random field. The use of an effective intensity over certain subdomains merely provides a limited spatial intensity average to be substituted for an ensemble average and it cannot reliably detect the presence of spatial non-stationarities.

Having access to polarimetric information across the spatial extent of the field allows building fourth-order joint statistical parameters. Specific features such as the presence of non-

stationary points can now be identified in a random field. Furthermore, when using mutual polarization measures such as the CDMP decay length these features can be found without any prior knowledge of the spatial size of the non-stationarity. We have shown that fourth-order field correlations evaluated at pairs of points in a random electromagnetic field can reveal properties that until now were inferred only through an ensemble average.

The ability to identify a spatial non-stationarity using only one single realization of a random field opens up new avenues for using multiple light scattering techniques. Over the years, EBS has become a standard tool for determining the scale length of wave-matter interactions in random, complex media such as heterogeneous photonic structures, biological tissues, and others [15, 16, 17]. EBS has also been proposed as a mechanism for sensing objects embedded in highly scattering media [12]. In addition, the unique feature of self-referencing a direction of propagation may have applications in remote sensing. Without the need for an ensemble average, this coherent phenomenon may become relevant for the study of ultrafast and transient phenomena or in situations that require a large number of realizations such as the search for Anderson localization [18]. On the other hand, very slow processes that are essentially stationary in time may be characterized for one single realization of the scattering medium.

Techniques using multi-order field correlations as a means of detecting spatial non-stationarities are not limited to solely polarimetric phenomena. As such, it should be possible to generalize our method to field correlations of arbitrary order, with a corresponding increase in the number of compared spatial locations. Furthermore, as we are relying upon field correlations, these analysis techniques are not necessarily specific to electromagnetic phenomena, and may find application in the measurement and characterization of any random field or process.

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