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MidWave vs LongWave Infrared Search and Track and Aerosol Scattering Target Acquisition Performance

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MIDWAVE VS LONGWAVE INFRARED SEARCH AND TRACK AND AEROSOL SCATTERING TARGET ACQUISITION PERFORMANCE

by

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A dissertation submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
in CREOL, the College of Optics and Photonics
at the University of Central Florida
Orlando, Florida

Summer Term
2020

Major Professor: Ronald G. Driggers
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ABSTRACT

The decision on whether to use a mid wave infrared (MWIR) or long wave infrared (LWIR) sensor for a given task can be a formidable verdict. The scope entails facts about the observable source, the atmospheric interactions, and the sensor parameters within the hardware device. Even when all the individual metrics are known, the combination ultimately determines whether a MWIR or LWIR sensor is more appropriate. Despite the vast number of variables at play, the reduction of inputs through focused studies can provide essential insight into MWIR and LWIR comparisons. This dissertation focuses on the roles of point source target detection, atmospheric scattering and absorption effects, and target identification has for MWIR vs LWIR performance.

The point source analysis details the Pulse Visibility Factor (PVF) and how it affects the Signal to Noise (SNR) for Infrared Search and Track (IRST) tasks. The PVF is an essential parameter that not only depends upon camera system hardware but also the dynamics of the imaged point source target. The numerical predictions of the PVF show how the hardware transfer function spreads the point source object across the detector array. As a result, it is a critical aspect for MWIR vs LWIR IRST system performance.

Atmospheric effects are another essential study for MWIR and LWIR imaging performance. Given the magnitude of atmospheric variables, the focus here is to reduce the atmospheric conditions with known particulates and concentrations to provide
predictable results. The analysis details how a sparse aerosol medium can absorb and scatter incident light to produce a blur and compromise image quality. Predictions of the aerosol Modulation Transfer Function (MTF) detail the differences in MWIR vs LWIR performance due to aerosols. The MTFs are then added into the Night Vision Integrated Performance Model (NVIPM) to calculate the ability to identify a target at range for typical MWIR and LWIR sensors.
ACKNOWLEDGMENTS

First and foremost, I would like to thank my family and a special thanks to my wife Christy Butrimas. Without her and the rest of my family’s support, this would not be possible.

I would also like to thank my advisor, Dr. Ronald Driggers, for his supervision and support throughout my graduate research. I cannot thank you enough for all the knowledge you have bestowed onto me. I am honored to be considered one of your students.

This dissertation would not be possible without the efforts of my committee members: Dr. Kathleen Richardson, Dr. Sean Pang, and Dr. Gerald Holst. Thank you for the time and efforts throughout my research. A special thanks goes to Dr. Gerald Holst for the many discussions and reviews throughout my research. Your attention to detail was vital and educational.

I would like to thank Dr. Natan Kopeika and Dr. Arkadi Zilberman for their support and review on the research topics. Their guidance and experience was important throughout this research.

I would like to acknowledge my place of employment JRM Technologies. Their support of a flexible schedule amidst tight deadlines allowed me to efficiently manage my time to complete the necessary research within this dissertation.

I extend my thanks to SPIE for publishing large portions of this work as peer-reviewed papers during my studies, and later allowing their use in the present document.
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<td>Contrast Threshold Function</td>
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<tr>
<td>EOIR</td>
<td>Electro Optical InfraRed</td>
</tr>
<tr>
<td>ESF</td>
<td>Edge Spread Function</td>
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<tr>
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<td>Fill Factor</td>
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<tr>
<td>FOV</td>
<td>Field of View</td>
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<td>FPA</td>
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<tr>
<td>ID</td>
<td>Identification</td>
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<td>IRST</td>
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<td>Line Spread Function</td>
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<td>NVIPM</td>
<td>Night Vision Integrated Performance Model</td>
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<td>PID</td>
<td>Probability of Identification</td>
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<tr>
<td>PSF</td>
<td>Point Spread Function</td>
</tr>
<tr>
<td>PVF</td>
<td>Pulse Visibility Factor</td>
</tr>
<tr>
<td>RH</td>
<td>Relative Humidity</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>SAA</td>
<td>Small Angle Approximation</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
<td>----------------------------------</td>
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<tr>
<td>SCR</td>
<td>Signal to Clutter Ratio</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>TTP</td>
<td>Targeting Task Performance</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
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CHAPTER ONE: GENERAL INTRODUCTION

The performance of an infrared search and track (IRST) sensor depends on a large number of variables that are important for determining system performance. One of the variables is the pulse visibility factor, or PVF. The PVF is linearly related to IRST performance metrics, such as signal-to-noise (SNR) or signal-to-clutter ratio (SCR). Maximizing the performance of an IRST through a smart design of the sensor requires understanding and optimizing the PVF. The resulting peak, average, or worst case PVF may cause large variations in the sensor SNR or SCR as the target position varies in the sensor field of view (FOV) and corresponding position on the focal plane. As a result, the characteristics of the PVF are not straightforward.

The definitions and characteristics for the PVF to include ensquared energy (best case PVF), worst case PVF, and average PVF are provided as a function of $F\lambda/d_{CC}$ ($d_{CC}$ is the center to center distance between pixels, i.e. pixel pitch). The metric $F\lambda/d_{CC}$ is a generalized figure of merit that permits broad analysis of the PVF. We show the PVF trends when the target has a finite size but still unresolved on the focal plane (smaller than an instantaneous field of view [IFOV]). The target size is constrained to be no less than 2% of the IFOV but also no greater than 100% to study the effects on the PVF as a function of target size. Finally, we describe the characteristics of the PVF when optical degradations, such as aberrations, are inherent in the sensor transfer function. The results have illustrated that small $F\lambda/d_{CC}$ with large fill factor maximized the PVF at the expense of greater variability. Larger $F\lambda/d_{CC}$ can reduce the PVF variations but result in
a decreased PVF. Finite target sizes and additional optical degradation decrease the PVF compared to diffraction-limited systems.

Another domain of sensor imaging performance that is often not considered is scattering and absorption of light by atmospheric aerosols. Aerosol scattering and absorption has always been difficult to characterize due to many dependencies including particulate composition, size of the particulates, optical parameters of the particulates, wavelength, range to target, aperture size, and spatially anisotropic line of sight effects (e.g., patchy fog). In addition, large path length complexities that can lead to varying aerosol composition, concentration, density distribution, turbulence, and molecular absorption present extreme difficulties to separate out the effects. As a first step to understand aerosol behavior, reducing such variables is essential. In this analysis, a controlled laboratory experiment and numerical study determines the line spread function (LSF) from an approximately uniformly distributed medium comprised of 5μm radius glass spheres. The detection of the scattered and transmitted light produces a blur in the sensor. That blur represents an MTF in the frequency domain. The computed aerosol MTFs exhibit quantifiable dependencies on all the aforementioned variables. This exemplifies the variability of the aerosol MTF and the difficulty to characterize it. In addition, mid wave infrared and long wave infrared aerosol MTFs are predicted from numerical methods simulating light propagating through a uniformly distributed water droplet medium. The water droplets varied as a function of droplet radius and concentration that demonstrates changes in the absorption and scattering of
light from the medium. The changes result in differences for mid wave and long wave imaging performance.

There are numerous metrics that describe atmospheric “resolution”. These include visibility, sky-to-ground ratio, and the ability to discern detail on the 1976 tri-bar target. Atmospheric aerosols between the target and sensor that can contribute to a blur are often overlooked in target acquisition performance. Typically, performance models only consider extinction and turbulence within the prediction processes. In this dissertation, the aerosol MTF is included into range acquisition algorithms to determine how scattering and absorption effects change the target identification predictions. The aerosols are monodisperse water droplets comparable to a tenuous fog or mist. Incorporating the aerosol MTF into the system MTF gives the opportunity to utilize the Night Vision Integrated Performance Model (NVIPM) to predict target identification range with aerosol contributions. The aerosol MTF is a function of range, water droplet composition, wavelength, and aperture size. The analysis focuses on these variables with an emphasis on wavelength dependence to characterize mid wave and long wave performance. Results show that the mid wave systems have a substantial diffraction advantage over long wave systems. When the aerosol MTF is included into NVIPM, mid wave systems suffer more degradation than long wave through scattering and absorption events. Only in the limit of increasing optical depths do the mid wave and long wave performance models begin to converge verifying that the aerosols can be the limiting factor for target identification. Though utilization of the U.S. Army’s NVIPM computer code predicts acquisition range, the aerosol MTFs are applicable to all
imaging systems. This includes such systems as long range thermography, remote sensing, and satellite imaging.

**Publication Details**

Some of the results in this dissertation have been reported in the form of a research paper that is published in a peer-reviewed journal.

Chapter Two

Published in Opt. Eng. 58(7) 073105 (27 July 2019)

[https://doi.org/10.1117/1.OE.58.7.073105](https://doi.org/10.1117/1.OE.58.7.073105)

This content within this chapter detail the PVF and its impact for IRST. The analysis provides insight into point targets, point targets with aberration MTF, and finite size targets. This measures the IRST performance for MWIR and LWIR systems specific to the PVF.
CHAPTER TWO: PULSE VISIBILITY FACTOR (PVF) AND ITS IMPACT ON INFRARED SEARCH AND TRACK (IRST) SYSTEMS

PVF Introduction

IRST sensors are typically used to search, detect, and track aircraft. There are many scenarios of IRST such as 1) distributed aperture sensors on the host platform to detect and track aircraft at short range, 2) the ground-based system that is charged with unmanned aerial system detection and tracking, and 3) the long range detection and tracking of aircraft in the forward sector of the host aircraft. In all of these cases, the mission of the sensor system is to find and track unresolved targets (smaller angle than the sensor instantaneous field of view or IFOV) [1, 2]. IRST sensors with the condition of unresolved targets are very different, in performance terms, from typical target acquisition systems that find and identify resolved targets [3-5]. Low cost, commercially available unmanned aerial vehicles (UAVs) increase the urgency of developing techniques such as IRST to detect, recognize and/or identify such targets.

IRST sensor design over the past few years using a physics-based SNR model have been investigated for calculating the performance of staring IRST sensors. References [6-13] extensively cover the model. SNR is a function of target, atmospheric, and sensor parameters.

It is a basic SNR model, but is extensive in that it includes many important sensor parameters such as dark current, read noise, optics emission temperature as

well as a target model input and MODTRAN atmospheric transmission and emission inputs.

The contrast irradiance is defined as the background to target irradiance at the entrance aperture of the sensor. It is a function of the slant range to the target for a modeled or measured target signature. It can vary spatially and as a function of angular orientation to the sensor but a subpixel target is typically assumed to be constant. Equation 1 is the equation used to calculate the signal from the target contrast irradiance at the sensor aperture.

$$\text{Signal} = \int PVF \frac{E_{\text{CONTRAST}}(\lambda)}{(hc/\lambda)} \frac{\pi D^2}{4} t_{\text{INT}} \tau_{\text{OPTICS}}(\lambda) \tau_{\text{COLDFILTER}}(\lambda) \tau_{\text{ATM}}(\lambda) QE(\lambda) d\lambda$$  \hspace{1cm} (1)

D is the sensor’s entrance pupil diameter in centimeters, t_{\text{INT}} is the integration time in seconds, PVF is the pulse visibility factor (0 ≤ PVF ≤ 1), $E_{\text{CONTRAST}}$ (W / (cm²-μm)) is the target to background contrast irradiance defined at the entrance aperture of the IRST, h is Planck’s constant, c is the speed of light, λ is wavelength, $\tau_{\text{OPTICS}}$ is the transmission of the optics, $\tau_{\text{COLDFILTER}}$ is the transmission of the cold filter, $\tau_{\text{ATM}}$ is the atmospheric transmission, and QE is the quantum efficiency of the detector. The value $hc/\lambda$ has units of energy per photon and the transmission factors are unitless. The QE relates number of incident photons to number of electrons generated within the detector. The left hand side of equation 1 is the number of electrons belonging to the registered signal. A constant contrast irradiance for a given range, constant target intensity, and constant target size are reasonable conditions for unresolved targets that
do not vary significantly in range or aspect to the sensor. The remaining variable in equation 1, the PVF, is the ratio of the target flux integrated by a single detector (the one with the largest impinging integrated flux) to the total target flux on the focal plane.

\[
PVF = \frac{\iiint E_c(x,y) \text{rect}\left(\frac{x}{d_x}, \frac{y}{d_y}\right) dx dy}{\iiint E_c(x,y) dx dy}
\]

In equation 2, \(d_x\) and \(d_y\) are the horizontal and vertical extents respectively, measured in \(\mu\text{m}\), for the active detector region. This analysis assumes square detectors \((d_x = d_y = d)\) as illustrated in figure 1. The rect function models the detector area that integrates the flux from the source target. \(E_c\) is the impulse response on the detector array in units of \(W/cm^2\).

Figure 1 illustrates the concept of the PVF. The grey squares depict the detectors’ active surface and the black spaces between the active areas are detector
dead space. Fill factor describes how much area is covered by active detectors versus focal plane area (ratio that is unitless). The pitch is the distance between the detectors centers so the fill factor would be detector size squared divided by pitch squared \((FF = (d/d_{CC})^2)\). The smaller the fill factor results in smaller flux collected by the detectors. We also characterize PVF as a function of \(F\lambda/d_{CC}\), where \(F\) is f-number, \(\lambda\) is center wavelength, and \(d_{CC}\) is the detector pitch. In the frequency domain, \(F\lambda/d_{CC}\) is the ratio of detector cutoff to optical cutoff. In the spatial domain, \(2.44F\lambda/d_{CC}\) is the ratio of the Airy disk diameter to the detector linear dimension. For a small \(F\lambda/d_{CC}\) (upper left), the optical spot generated by a point target is mostly smaller than a single detector. The upper middle case is a medium \(F\lambda/d_{CC}\) and the upper right is a high \(F\lambda/d_{CC}\). Higher \(F\lambda/d_{CC}\) provides less flux on a detector since the optical spot extends over into adjacent areas. The best case PVF is a target spot (optical spot) centered on the detector and is sometimes called “ensquared energy”. When the target spot is located in the corner between detectors (lower figure), then the PVF is worst case that corresponds to the lowest signal possible. Another important metric is the average PVF that corresponds to an average ratio given the response to a large number of random spot positions on the focal plane. Finally, the variation in PVF is important due to different locations of the target spot on the focal plane. In this chapter, we describe the variation as a standard deviation of the PVF signal variation due to random spot locations.

Included in the temporal noise sources are background noise from the scene (in this case, the dominant noise factor), shot noise of the thermal emission from the lens, shot noise of the background, dark current shot noise, and read noise.
\[ \text{Noise}_{\text{temporal}} = \sqrt{\text{background}^2 + \text{shot}_{\text{lensEmiss}}^2 + \text{shot}_{\text{dark}}^2 + \text{read}^2} \] (3)

The SNR is the ratio of equation 1 to equation 3 is one of the primary metrics for IRST sensor performance. The noise terms in equation 3 can be dependent upon detector element size, which can complicate and reduce the SNR when \( F\lambda/d_{\text{CC}} \) is small.

**PVF and Ensquared Energy as a function of \( F\lambda/d_{\text{CC}} \)**

Simulation code performed PVF analysis that determined the PSF for an unresolved source target propagating through a camera system onto a simulated detector array. For a diffraction-limited system, a point source appears as a sombrero function in the focal plane.

\[ \text{somb}(r) = \left( \frac{2J_1(\pi r/\lambda F)}{\pi r/\lambda F} \right)^2 \] (4)

The independent variables \( F \) is the F number (focal length/aperture diameter) and \( r \) is the spatial coordinate extent on the detector array. \( J_1 \) is the Bessel function of the first kind.

To gain understanding on how the PVF varies required analysis of three positions cases for unresolved point targets. The three cases are the center pixel location, the corner location, and averaged random positions within the extents of a detector’s pixel pitch (independent of fill factor). This provides insight into the boundary extremes as well as the ensemble average to account for randomized motion of the
target on the detector. The target location at the center is the PSF position at the geometric center of the pixel. The target location at the corner covers 4 pixels at the half pixel pitch distance from the geometrical center independent of the fill factor as illustrated in the last image of figure 1. The average PVF is a collection of randomized locations of the target on the pixel then averaged to produce a PVF curve.

The Matlab code developed to model equation 4 as a function of $F\lambda/d_{CC}$ then positioned the PSF onto a square detector array to compute the PVF for each of the 3 positional cases. The positioning logic was constrained such that the PSF only varied within a single pixel pitch. This is because beyond the corner or a pixel, the main lobe of the PSF effectively moves to a new detector location that is exactly the same as simply confining the positioning within the bounds of the center to corner of a single detector element.

Figure 2a illustrates the PVF and ensquared energy as a function of $F\lambda/d_{CC}$ for the center position (best case), corner position (worst case) and then the average of random positions within the center to corner extents. The average consisted of randomly positioned (Monte Carlo) PSF locations on a single detector element. The PVF is computed by integrating the PSF over a single detector element and then dividing the total integrated PSF. This gives a normalized metric that is the PVF for a detector element of interest. The integration does account for the location of the lobe (e.g. the location of the main center peak) as well as fill factor in order to determine the correct PVF. The integration of the imaged point is consistent with the radiometry of a
source intensity (W/sr). The PVF is only applicable for unresolved sources, that are no larger than the size of a pixel, imaged on a detector.

Figures 2b and 2c portray the histogram of the randomized PVF data for different $Fλ/d_{CC}$ with a fill factor of one. The abscissa is the PVF value in bin size segments and the ordinate is the number of occurrences that a random PVF is within a given bin and its corresponding size. The red bar marks where the average is located among all the random PVF data. The most important detail that depicted in these histograms is that the distribution of the random PVFs is not Gaussian. This is expected since the PVF will never exceed the center (best) or the corner (worst) case. These boundaries shape the distribution of the standard deviation as a function of $Fλ/d_{CC}$. 
Figure 2: (a) PVF of point source through a diffraction limited system with a fill factor of 1. (b) Random PVFs for $F\lambda/d_{CC} = 0.6$, FF=1 and (c) random PVFs for $F\lambda/d_{CC} = 1$, FF=1

The size of the PSF approaches to zero with respect to the detector size when $F\lambda/d_{CC}$ decreases toward zero and all the energy from the unresolved target is centralized on a single point (within the limits of diffraction). As a result, the PVF is confined into a single detector element resulting in the PVF approaching unity for the
case when positioned at the center. For the PSF positioned at the corner between 4 pixels (depicted in the bottom image of figure 1), the PVF approaches 0.25 as $F\lambda/d_{CC}$ tends to zero since the energy is equally distributed among 4 detector elements. In addition to the PVF trends shown in figure 2a, the bars on the green average plot reveal the standard deviation from the average for the randomly positioned PSFs and computed PVF. These bars represent the mathematically computed standard deviation from the calculated randomized PVFs above and below the average PVF. The total standard deviation, illustrated in the orange plot in figure 2a, is the sum of the above and below deviations from the randomized PVF dataset added in quadrature since the randomized positions are independent of each other.

As $F\lambda/d_{CC}$ increases, the PVF for the center position case reduces but a plateau is observed due to the first zero ring from equation 4 crossing over to other detector elements. As a result, the same fringe energy is outside the detector and the same amount from the central lobe of the airy disk is still within the detector. This results in a leveling off the PVF since the zero does not contribute to the PVF as the zero ring moves radially outward. Only when the main first lobe begins to extend past the detector element does the PVF continue to decrease as energy spreads to other detector elements thereby decreasing the PVF. The corner case has similar behavior except that the PVF upper limit is 0.25. A slight drop is observed just beyond $F\lambda/d_{CC} = 0.5$ followed by a plateau. This is due to the first zero ring of the airy disk not contributing as it extends beyond the 4 detectors covered in the corner case.
The randomly positioned PSF shows that the average PVF tends toward the center case than the corner case. This is because even a slight shift from the corner case towards the center gives a substantial increase in the energy on the detector being analyzed thereby increasing the PVF. This is the reason why the average tends upward towards the center position case rather than the corner case as $F\lambda/d_{CC}$ approaches zero. Also, the standard deviation plots as a function of $F\lambda/d_{CC}$ illustrate that there is more variance in the smaller PSF size (smaller $F\lambda/d_{CC}$) than the larger except in the extreme case where $F\lambda/d_{CC}$ is near zero. This illustrates small and large PSFs have less deviation from the average than the case when $F\lambda/d_{CC}$ is approximately 0.5. The small standard deviation ($\sigma$) for large $F\lambda/d_{CC}$ can be explained by the fact that the PSF is quite large in this case and all the PVFs converge leading to smaller deviations from the average. The extreme case of very small $F\lambda/d_{CC}$ results in a small $\sigma$ because this is the case of a very small point source and most of the energy is confined within the pixel. Only when the PSF is exactly on the corner or edge does the PVF become significantly smaller and approach 0.25 or 0.5 respectively. The result is that the standard deviation is less for very small $F\lambda/d_{CC}$.

In addition to the trends identified with the standard deviation, another fact is that it has non-zero skewness. This is observed in figure 2a in the $\sigma$ bars on the average plot and the histogram plots of figures 2b and 2c. For $F\lambda/d_{CC} < 1$, it can be seen that the integral over the histogram has a negative skew. This is because there are larger differences in the PVF for random positions that are closer towards corners and edges when compared to cases that are near the center. As $F\lambda/d_{CC}$ grows beyond 1.5, the
standard deviation in the average is much smaller and all the different positions converge since this is the case where the PSF is considerably large and is less sensitive to different positions.

Figure 3a and 3b reveal the same analysis as Figure 2a but with the fill factors of 0.75 and 0.5. A fill factor of 0.75 still relates to existing systems and a fill factor of 0.5 was analyzed for completeness. The PVF does include the flux falling on the dead space produced by the non-unity fill factor. Figure 3c and 3d illustrate the same histogram details as figures 2b and 2c but with a fill factor 0.75. The abscissa is the PVF value in bin size segments and the ordinate is the number of occurrences that a random PVF is within a given bin and its corresponding size.
Figure 3: (a) PVF of a point source through a diffraction limited system with a fill factor of 0.75. (b) PVF of point source through a diffraction limited system with a fill factor of 0.5. (c) Random PVFs for $F\lambda/d_{cc} = 0.6$, FF=0.75 and (d) random PVFs for $F\lambda/d_{cc} = 1$, FF=0.75
Figure 3a and 3b illustrate the consequence of the inactive vs active regions of the detector (fill factor < 1) and that the PVF is smaller overall for smaller fill factors. In essence, if the flux is outside the integrating regions of the detector, then no energy contributes to the PVF. Only when the energy distribution expands outward for larger $F\lambda/d_{cc}$ does the PVF continue to decrease. Another consequence of smaller fill factor is that there is a greater chance for a single frame to miss the central lobe of the target (e.g. zero integrated flux effects). The average no longer approaches unity and the corner case PVF approaches zero as $F\lambda/d_{cc}$ tends to zero. It is not surprising that the average PVF approaches the numerical value of the fill factor for $F\lambda/d_{cc}$ equal to zero. When $F\lambda/d_{cc}$ is near zero, the PSF approaches a point image on the detector and the randomized locations averaged together numerically approach the value of the fill factor. In this limit, the average PVF with the small point image is mathematically equivalent to a normalized Monte Carlo integration of the detector. Only the active area of the detector contributes to the summation and the integrated result converges to the value of the fill factor.

Note that the standard deviation of the average PVF curve is greater as the fill factor decreases as the histograms of figures 3c and 3d display. This is due to greater variations in the integrated flux for the PVF since some random locations are outside the active region of the detector element. The smaller fill factor cases also show that the rate of descent of the standard deviation becomes less steep. Again, this is due to the PSF having more flux over the inactive region leading to more deviation from the average PVF for larger $F\lambda/d_{cc}$ and smaller fill factors. The more gradual slope in
standard deviation curves for smaller fill factors means the PVF exhibited greater variations for larger $F\lambda/d_{cc}$. This is a direct consequence of the larger inactive regions on the detector and why the standard deviation has a sharper descent for a fill factor of 0.75 than for 0.5 for $F\lambda/d_{cc} < 1$. When $F\lambda/d_{cc} > 1$, the PSF begins to become significantly large enough where the standard deviation in the PVF are small for all fill factors resulting in the similar trends. The PVF in this range is less sensitive to fill factor differences. Figures 4a-4c illustrates the center, corner, and average PVF as a function of $F\lambda/d_{cc}$ for fill factors of 1, 0.75, 0.5.
Figure 4: (a) PVF for center location for different fill factors and (b) PVF for corner location for different fill factors and (c) PVF for average of random locations for different fill factors and standard deviation $\sigma$

Figure 4a depicts the best case where the PSF is at the center of the pixel for decreasing fill factor. The PVF with the smaller fill factor is smaller for increasing $F\lambda/d_{CC}$ due to the reduced active detector area. Figure 4b demonstrates the worst case where the target PSF is at the corner of 4 detector elements. This case shows a low PVF where the maximum is 0.25 at a fill factor of one. A fill factor less than one results in
even smaller PVF due to the smaller active region of the detector element integrating the incident flux. The PVF is observed to approach zero when $F\lambda/d_{cc}$ is so small that the PSF is confined within the inactive region for the fill factor $< 1$. In figure 4c, the average of randomized locations on the detector with different fill factors is plotted as a function of $F\lambda/d_{cc}$. Just as in figure 4a and 4b, the PVF is smaller with decreasing fill factor. This is due to the random positions are increasingly located in the inactive regions resulting in less flux on the pixel resulting in a smaller PVF. Additionally, the PVF approaches the numeric value of the fill factor as $F\lambda/d_{cc}$ trends to zero because the averaged PVF acts just like a normalized Monte Carlo integration. The dotted lines in figure 4c are the standard deviations for each of the PVF plots as a function of both $F\lambda/d_{cc}$ and fill factor. These lines show less deviation with larger fill factor. Smaller fill factor results in more zero integrated flux effects leading to larger standard deviations overall.

IRST applications are directly dependent upon the PVF (as shown in equation 1) which means the best case scenario is the target located on the center with a small $F\lambda/d_{cc}$. Despite this being the best case, it is difficult to keep a small target size (small $F\lambda/d_{cc}$) on the center of a pixel. It is simply not practical. The case of when the target is at the corner is the worst especially when the fill factor is less than one. Zero integrated flux effects can result in the target having a zero PVF resulting in no signal for IRST. The most realistic case is the average of randomized PSF locations. Figure 4c clearly demonstrates that a larger fill factor (ideally one) is the best realistic case for improving search and track applications. Not only is this the best overall average, the standard
deviation is smaller for increasing fill factor giving better stability on the frame by frame collection for IRST processing.

Binning or summing small detector elements to create an effective larger element is an important issue to consider. There is also another technique called matched filter processing that combines detector elements in a weighted fashion that matches the blur spot. These approaches are analyzed with the same technique provided in this chapter and the results of these techniques are provided in reference 9. For high $F\lambda/d_{cc}$ cases, binning and matched filtering can significantly enhance SNR.

**Finite Target Sizes**

The finite target can be any complex shape but for the sake of simplicity, a square is analyzed. In the previous section, the independent variable $F\lambda/d_{cc}$ generalized the PVF analysis. In this section, finite target sizes on the detector have a similar generalization. The target IFOV divided by the detector IFOV (tgt IFOV / det IFOV) can characterize the range in a single unitless IFOV ratio for more generalized analysis. One key assumption here is that the target intensity is constant. Different target sizes are in reference to the angular size differences as a function of range, the physical size of the target is constant. In addition to the IFOV ratio, $F\lambda/d_{cc}$ is changed along with fill factor to view the PVF trends as a function of different variables. The square target does not extend beyond the size of a pixel thereby keeping the designation that it is considered an unresolved target and the rules of integration to compute the PVF are the same as the point source case.
The square is Fourier transformed into the spatial frequency domain, multiplied by the diffraction MTF, and then inverse Fourier transformed back so that the target appears as it would on a detector array. The imaged target is then integrated to determine the PVF on a pixel. Figure 5 illustrates the PVF of a square target as a function of the IFOV ratio and fill factor for $F\lambda/d_{CC} = 0.6$. 
Figure 5: (a) Square target for $F\lambda/d_{CC} = 0.6$ and fill factor 1, (b) square target for $F\lambda/d_{CC} = 0.6$ and fill factor of 0.75, and (c) square target for $F\lambda/d_{CC} = 0.6$ and fill factor of 0.5
The center case is when the target center is located at the detector center. Figures 5a-5c details that the PVF does not approach one when the target IFOV to detector IFOV ratio is less than unity (unresolved target). Even though the target is very small compared to the pixel, the IFOV ratio is much less than one, the diffraction blur by the optics truncates the high spatial frequencies of the small target image sufficiently that the flux is distributed among other detector elements. The diffraction blur for an $\lambda d_{CC}$ of 0.6 is enough to make the blur spot of the target large enough to cover more than one detector hence the reason why the PVF does not approach one. The PVF for the small target IFOV ratio does correspond to the PVF in figures 2a, 3a, and 3b at the discrete value of $\lambda d_{CC}$ of 0.6 as anticipated. For larger target IFOV ratios, the PVF does not correlate to the unresolved point source because the target size is larger and diffraction effects result in greater spreading of the target’s flux. This reduces the flux on the detector causing a smaller PVF. Figures 5b and 5c illustrate that for a fill factor less than one, the PVF decreases. A decreasing fill factor means less active area on the detector for flux integration resulting in a reduced PVF independent of IFOV ratio.

The corner case shows more trends that are interesting as a function of fill factor. When the fill factor is one, the corner case PVF remains near 0.25 for unresolved targets. The flux is integrated across 4 detector elements and diffraction effects spread the flux out further resulting in a PVF that is just less than 0.25. This demonstrates that a target flux is collected across many pixels. Only when $\lambda d_{CC}$ is near zero are diffraction effects minimized and the PVF approaches 0.25 exactly. Since $\lambda d_{CC}$ is 0.6, the PVF is less than the ideal case of 0.25. The corner case also indicates that for fill
factors less than one, the PVF trends towards zero at smaller IFOV ratios. Only when the relative target size becomes large enough to have energy on the active region of the detector does the PVF begin to increase (Figure 5b). This transition occurs for larger IFOV ratios for smaller fill factors (Figure 5c).

The average case illustrates that there is more variation for smaller fill factors. This is an expected trend since random positioning of the target in relation to the pixel under analysis shows greater variances for smaller target size (smaller IFOV ratio). Zero integrated flux effects are stronger with decreasing fill factor leading to more variation in the PVF for random positions. The standard deviation plots portray this trend despite the differences being small due to diffraction effects spreading the flux out on the image plane. The average plots demonstrate that the small target IFOV ratios do not converge to the numerical value of the fill factor seen before in the unresolved point source PVF plots in figure 4c. The fact that $F\lambda/d_{CC}$ is 0.6 and not zero is the reason why the PVF is less than the fill factor value despite the target size being much smaller than the detector size.

The overall difference with the unresolved point source and the finite target size discussed here is the dependence on $F\lambda/d_{CC}$ and the effect it makes on the PVF. Even though the target can be very small on a detector (IFOV ratio $<< 1$), the PVF does not approach the numeric value of the fill factor like the unresolved point source PVF demonstrated unless $F\lambda/d_{CC}$ is near zero. The constant $F\lambda/d_{CC}$ adds significant blur to the system thereby reducing the PVF as a result. For IRST applications, it might seem that a smaller $F\lambda/d_{CC}$ is better since smaller target sizes show increasing trends in the
PVF. The consequence of this choice is the zero integrated flux effects. The small targets on the detector with little blur results in significant zero integrated flux effects resulting in greater standard deviations in the PVF. The frame to frame differences in the PVF can be unacceptable so a tradeoff is at hand where less variation and zero integrated flux effects can be attained but at the expense of PVF. Such blinking effects can be acceptable for human consumption but can be difficult to process for IRST imaging processing algorithms. The optical flow as a function of angular velocity of the sensor can cause varying periodicity of the blinking effects that can disrupt or confuse software processing algorithms.

Now consider the same case portrayed in figures 6a-6c but with a larger $F\lambda/d_{CC}$. Figures detail the same square target on a detector element but with $F\lambda/d_{CC} = 1.6$. 
Figure 6: (a) Square target for $F\lambda/d_{CC} = 1.6$ and fill factor 1, (b) square target for $F\lambda/d_{CC} = 1.6$ and fill factor of 0.75, and (c) square target for $F\lambda/d_{CC} = 1.6$ and fill factor of 0.5.
Figures 6a-6c depict essentially the same details as the cases in figures 5a-5c. The main difference is the PVF is smaller for all cases and fill factors when compared to the Fλ/d_{CC} = 0.6 case. The Fλ/d_{CC} = 1.6 shows that the standard deviation is reduced but at the expense of a lower PVF. The larger Fλ/d_{CC} corresponds to better sampled system or a system with more diffraction blur. The system benefit is less variations in the PVF for random locations (e.g. less zero integrated flux effects). Though they are still measurable variations in the small target size (small target IFOV ratio) it reduces as Fλ/d_{CC} increases.

Figure 7 below illustrates the PVF for a finite target size and discrete target to detector IFOV ratios. In addition, the PVF is a function of Fλ/d_{CC} at a fill factor of 0.75.

![Graph of PVF vs. Fλ/d_{CC} for different IFOV ratios](image)

Figure 7: Average PVF for square target and fill factor of 0.75 with discrete target IFOV ratios. IFOV ratio of 0.01 corresponds to a point source that corresponds to the average PVF in figure 3a.
Figure 7 simply illustrates the average PVF trends are as a function of $F\lambda/d_{CC}$ rather than the target IFOV ratio shown in figures 5a-5c and 6a-6c. The PVF here shows monotonic decreasing trends as a function of $F\lambda/d_{CC}$ but also increases for smaller target IFOV ratios. The smaller the ratio, the smaller the target size is on the detector which result in more of the target flux is concentrated on the detector (e.g. a point source). The smaller $F\lambda/d_{CC}$ means less diffraction blur which leads to the result that the PVF increases for the smaller target size. The case for the IFOV ratio of 0.01 and $F\lambda/d_{CC}$ near zero is considered a point source with little diffraction applied. The average of randomized locations in this case yields the numeric value of the fill factor of 0.75. Even though the targets are of finite size, the target image is still within a single pixel. The target is still treated as a point source ($W/sr$) that is integrated over the detector IFOV.

**Realistic MTF**

The analysis in the previous sections only considered diffraction blur by the optics. A more realistic system MTF would need to consider all MTFs up to the detector where the flux is integrated. Such MTFs include atmospheric, defocus, aberrations, motion jitter, etc. To maintain a realistic MTF that is tractable, the aberration MTF is the additional MTF of choice to add into the system MTF. The proceeding analysis is the same as the PVF and Ensquared Energy section above but now includes the aberration MTF. This section details the PVF of an unresolved source as a function of $F\lambda/d_{CC}$ but
now with aberrations added into the system MTF. Equation 5 details the aberration MTF added into the system MTF for realistic MTF analysis of PVF on the detector [14].

\[
MTF(f)_{ABERRATION} \approx 1 - \left( \frac{W}{A} \right)^2 \left[ 1 - 4 \left( \frac{f}{f_o} - 0.5 \right)^2 \right]
\] (5)

\(W\) is the RMS wavefront error measuring in fractions of waves and \(A = 0.18\). The wavefront peak to peak error was set to \(\lambda/4\) resulting in a RMS wavefront error of 0.072 since the RMS is the peak to peak divided by 3.5 (RMS = PP/3.5). The cutoff frequency is \(f_o\) and is identical to the diffraction optical cutoff frequency.

To analyze the PVF, the same point source intensity from equation 4 was processed to add the aberration MTF for additional degradation of the image of the source object. The object is Fourier transformed into the spatial frequency domain, multiplied by the total system MTF, and then inverse Fourier transformed back. This process accurately represents the imaged object integrated on the detector array.

Figure 8 describes the PVF as a function of \(F\lambda/d_{CC}\) for fill factors of 1, 0.75, and 0.5 for a realistic MTF due to aberrations.
Figure 8: (a) PVF of point source with realistic MTF with a fill factor of 1, (b) PVF of point source with realistic MTF with a fill factor of 0.75, and (c) PVF of point source with realistic MTF with a fill factor of 0.5
Figure 8 shows the same trends seen in figures 2a, 3a, and 3b. The center cases in figures 8a-8c illustrate that the PVF approaches unity as $F\lambda/d_{CC}$ approaches zero. However, the PVF decreases much faster with increasing $F\lambda/d_{CC}$ due to the increased blur from the smaller system MTF. Additionally, the center cases of figures 8a-8c portray a smaller plateau for the center PVF case compared to the diffraction-limited case of figure 2a. The aberration MTF sufficiently blurs the PSF such that the PSF rings for a diffraction limited system are less apparent. There is also distinct change in slope for the PVF in the center case as $F\lambda/d_{CC}$ increases beyond 0.5. This is the case where the central lobe blur spot from the target intensity starts to occupy adjacent detector elements. This coupled with the additional MTF for realistic blur results in a change of PVF slope for larger $F\lambda/d_{CC}$. This transition region occurs for smaller $F\lambda/d_{CC}$ as the fill factor decreases. This is expected because the detector element is fully occupied by the main lobe of the PSF for smaller $F\lambda/d_{CC}$ when the fill factor is less than one.

The corner cases in figures 8a-8c depict the same trends as in the diffraction-limited case demonstrated in figures 2a, and 3a-3b. The main difference is that the PVF is smaller overall due to greater spreading of the flux across detector elements with the additional MTF applied. The PVF approaches 0.25 as $F\lambda/d_{CC}$ approaches zero for a fill factor of one because the flux is always spread evenly across 4 detector elements. As the fill factor decreases, the PVF approaches zero for small $F\lambda/d_{CC}$ since the flux is incident upon inactive regions of the detector. The result is that the PVF decreases with decreasing fill factor and is less for all $F\lambda/d_{CC}$ due to more blur applied by additional MTFs in the system.
The average cases show that there is more variation for smaller fill factors. This is an expected trend since random positioning of the target in relation to the analyzed pixel demonstrates greater variances for smaller and smaller $F\lambda/d_{CC}$. The essential point here is that with the additional MTF applied; the PVF is less than what is observed in the diffraction-limited case. The average PVF has less variation but at the expense of it being lower as $F\lambda/d_{CC}$ increases.

The standard deviation trends show that with increasing fill factor that is still less than one, more variations are observed. This is due to the zero integrated flux effects coupled with additional blurring from the non-diffraction limited system. The flux falls onto the inactive regions of the detector resulting in significant differences in the PVF. A result of a non-diffraction limited system MTF is that the standard deviations are less at the consequence of a smaller PVF. The standard deviation also decreases as a function of $F\lambda/d_{CC}$. The more flux spread across detector elements results in the average PVF to be less sensitive to variations in the randomized location of the target on the detector. Even if the imaged point source is incident on an inactive region of the detector, the additional blur spreads out the flux sufficiently where the PVF does not approach zero. In addition, the deviations in the randomized PVFs exhibit little to no skewness unlike the diffraction-limited cases. The diffraction-limited system displayed significant asymmetry where the variations below the average were more than above the average. When additional MTFs that degrade the system performance are present, the skewness in randomized PVFs is significantly reduced.
Figure 9 details the average and standard deviations of the PVF as a function of $F\lambda/d_{CC}$ for different fill factors.

Figure 9 and figure 4c detail the same analysis with the only difference is that figure 9 has the aberration MTF applied to the system. The most notable difference is how quickly the PVF decreases as a function of $F\lambda/d_{CC}$ because of the degraded system with aberrations. In addition, the greater blur produces smaller variations in the PVF for random target locations. The increased blur reduces variations in the PVF at the expense of a producing a smaller PVF since the flux is spread out more on the detector array.
**PVF Conclusions**

PVF is a primary contributor to IRST performance (equation 1). Given the dependence on IRST performance on PVF, it is vital to understand how the PVF behaves under different conditions. The number of independent variables and permutations in analyzing the PVF for IRST can be near endless; our analysis focused on a select few to study PVF trends. The investigation here covers PVF trends as a function of $F\lambda/d_{CC}$, target size, fill factor, location of point on focal plane, and non-diffraction limited system. The choice of independent variables was not arbitrary. All of the figures are a function of $F\lambda/d_{CC}$ or target size to detector IFOV ratio. These are generalizations that are unitless metrics that provide a broader insight to the PVF in a single plot. The PVF generalizations were for the ensquared energy (best case PVF) where the spot was at the center of the detector element, the average PVF from randomly located spots, the worst case PVF at the corner, and the standard deviation of the PVF.

The PVF for an unresolved point source in a diffraction-limited system was best for the largest fill factor with the smallest $F\lambda/d_{CC}$. Despite this conclusion, the standard deviation shows greater variations in the PVF for smaller $F\lambda/d_{CC}$ because of zero integrated flux effects. These effects can lead to a loss of the target (PVF near zero) on the focal plane for randomized locations. A tradeoff between zero integrated flux effects with smaller $F\lambda/d_{CC}$ and less variation with larger $F\lambda/d_{CC}$ directly affect the PVF. The unresolved point source PVF was further analyzed in a non-diffraction limited system by including an aberration MTF into the system. The effect from the aberration wavefront
error was a reduction in the PVF. In addition to the unresolved point source analysis, the PVF for a finite target size was examined in a diffraction-limited system. The target assumptions are that it has a constant intensity and the physical size is constant. In the limit of a small target angular size, the PVF agreed with the unresolved point source results. Overall, the larger the target angular size, the smaller the PVF. Understanding the trends of the PVF is important to gauge the performance of IRST applications.

For observer in the loop, studies have shown $F\lambda/d_{CC}$ values approaching 2 provide the best range performance for resolved target acquisition sensors. These smaller detectors create new opportunities in sensor design trades and enhanced performance. However, our results suggest that IRST applications require $F\lambda/d_{CC}$ near 1.0. Values of $F\lambda/d_{CC}$ too small introduce more variations in the PVF especially for smaller fill factors. This does not give frame to frame stability in the PVF required for IRST image processing algorithms. For less than unity fill factors, spatial variance increases. This variance can be minimized by defocusing but this leads to reduced PVF. Larger $F\lambda/d_{CC}$ simply has too low of a PVF. With $F\lambda/d_{CC}$ about 1.1, the peak PVF is approximately 0.6 which is a good compromise that has been a historic benchmark in IRST design. Factoring this into a nominal detector pitch of 10μm and an average wavelength of 4μm for MWIR and 10μm for LWIR, the F number would need to be 2.75 for MWIR and 1.1 for LWIR systems. The larger F number for the MWIR system is a simpler optics design than the 1.1 F number LWIR system. This shows that when only analyzing the PVF, the MWIR system adheres to traditional F number designs better.
than LWIR. In addition to the PVF, noise that is dependent upon detector element size can further complicate and reduce the SNR especially for small $F\lambda/d_{cc}$.

This analysis provides insight on PVF behavior and is useful in IRST design. However, the PVF alone cannot optimize IRST system performance. For example, a smaller aperture may provide small changes in the PVF but will reduce SNR significantly as shown in equation 1. Also, the contrast irradiance can improve significantly with smaller ranges to the target whereas the PVF can exhibit small changes in comparison. This shows that optimizing the whole IRST system encompasses more than just the PVF but the details here provide PVF optimizations as a subset.
CHAPTER THREE: EFFECTS OF AEROSOL MTF ON IMAGE QUALITY

MTF Introduction

In order to understand the details of sensor resolution performance, the fundamentals on MTF system analysis needs to be understood. The most effective way to analyze any imaging system is by measuring how well sinusoidal objects are transferred through an imaging system. In other words, the system transfer function provides the ability to fully characterize imaging performance by measuring the imaged modulation as a function of spatial frequency. Figure 10a depicts a sinusoid that has a non-unity modulation at a given period (spatial frequency). Figure 10b shows how an ideal square “bar” target of unity contrast will be transformed by a non-ideal imaging system. This leads to an MTF function that results in a loss of contrast by blurring. One other significant note about the MTF is that it is the Fourier Transform of the Point Spread Function (PSF). The governing equation for the MTF is also shown below.

\[
MTF(f) = \frac{I_{max}(f) - I_{min}(f)}{I_{max}(f) + I_{min}(f)}
\]  

(6)
Imaging systems do not have infinite spatial frequency bandwidth which is to say that a sensor and anything between the object and the sensor causing image degradation (such as the atmosphere or obscurants like smoke) will limit how well object details can be imaged. As the spatial frequency of the object increases, the bar patterns are closer together and the system transfer function blur in the image plane will eventually reach a point where there is no modulation from black to white (i.e.
computation of equation 6 produces a value of 0). This limit is called the cutoff frequency and defines the spatial frequency limit that can be sufficiently reconstructed for viewing. Any object of spatial frequency above the cutoff will not be resolved. As mentioned before, each stage in the image pipeline (figure 11) has an MTF but the overall system frequency limit is based upon which component of the imaging system pipeline has the worst transfer function.

![Figure 11: Common sources of transfer functions in imaging pipeline](image)

Long range imaging coupled with well-designed large format focal plane arrays (FPAs) has shown that the atmosphere can quickly become a significant transfer function in the imaging pipeline. More specifically, the limitations can be predominantly from light-matter interactions. To illustrate this point more clearly, consider the moon at night in a clear and hazy atmosphere. The clear night conditions can show a sharp transition from the moon disk edge to the dark background. The hazy night can show a
blurry edge transition from the moon boundary. This demonstrates how the atmosphere can add a significant blurring transfer function to the imaging pipeline. The atmosphere has scattered the light propagating through the atmosphere such that a blur can be observed in the image of the moon. In fact, the blur can be sufficiently large enough where the overall system performance of the sensor, in essence all the transfer functions throughout the imaging pipeline in figure 11, is limited by the atmospheric transfer function.

**Aerosol MTF Introduction**

Image quality analysis requires examination of not just the camera hardware, such as the optics, detector, and electronics, but also atmospheric effects such as turbulence, transmission, and scattering. Typically, only turbulence and transmission are considered. However, the effects of scattering and absorption can be significant and therefore cannot be overlooked. Observable scattering effects are not only dependent on the scattering medium but also on the camera aperture and location. In this chapter, the focus is on scattering from larger sized particles of radius \( r > 1 \mu m \) and its effect on imaging performance.

Scattering and absorption effects are dependent upon wavelength, concentration and size of the scatterers, complex refractive index of the scatterers, range through the scattering medium, aperture size, and line of sight variations in the medium. Scattering regimes can be divided into three distinct regions: Rayleigh, Mie, and Geometric (figure 12). The Mie regime is the focus in this analysis.
Rayleigh

Rayleigh scattering occurs when the particle size is much smaller than the wavelength of light \((r \ll \lambda)\), where \(r\) is the radius of the particulate. Light matter interactions govern this process resulting in dipole radiation that leads to broad scattering angles and is heavily dependent upon wavelength \((1/\lambda^4)\).

Mie

Mie theory \((r \sim \lambda)\), is the domain that is widely used for aerosol sized particles in the visible through infrared imaging realms. It is a process that redirects light according to the principles of diffraction. A rigorous analytical solution for Mie theory exists if the particle is spherical in shape \([15-19]\). Particulate sizes approximately equal to the wavelength predominately scatter light in the forward direction, which produces a blur in the image plane. This blur is characterized by an MTF. The MTF cutoff spatial frequency that characterizes imaging performance is \(2r/\lambda\) \([17]\).
**Geometric**

Geometric scattering ($r \gg \lambda$) is a complex scattering regime that is difficult to analyze due to all the different interactions that occur. Light can refract, diffract, reflect, absorb, as well as reflect multiple times (i.e. bounce around), and transmit through the particle. Large particles are not common and typically settle out.

The classical form for the aerosol MTF is a 2 part function that is a Gaussian function for spatial frequencies below the cutoff and a constant beyond the cutoff. The cutoff frequency ($f_c = 2r/\lambda$) defines the extent of the scattering angle based upon diffraction theory. The variable $r$ is the particulate radius of the scatterer. The basic form of the MTF is shown in equation 7a and 7b below that account for scattering and absorption [14, 20-21].

\[
MTF_{AEROSOL}(z, f) = \exp \left( -z \gamma \left( \frac{f}{f_c} \right)^2 - \alpha z \right) \quad \text{for} \quad f < f_c
\] (7a)

\[
MTF_{AEROSOL}(z, f) = \exp(-z \gamma - \alpha z) \quad \text{for} \quad f > f_c
\] (7b)

In these equations, $\gamma$ is the scattering coefficient, $\alpha$ is the absorption coefficient, $z$ is the range from the source object to the sensor, and $f_c$ is the cutoff frequency as mentioned above. The MTF corresponds to the irradiance distribution of scattered light that is proportional to a Gaussian function in the spatial frequency domain. This is the classical form of the small angle approximation (SAA) for analytically modeling the aerosol MTF. The Fourier transform of the irradiance distribution in the spatial domain gives the Gaussian form MTF found in equation 7a. Figure 13 details the plot of equations 7a and 7b.
Figure 13: Aerosol scattering MTF functional form

Figure 13 shows that scattered light modulation drops as a Gaussian function until the cutoff frequency. Beyond the cutoff, there is no change in modulation because the angular extent of the phase function has been reached. This is the maximum angle of diffraction scattering theory resulting in an asymptote of the MTF. The size of the scatterer and the wavelength determine the spatial frequency cutoff ($2r/\lambda$). The concentration of the scatterers determines the amplitude of the asymptote. The overall effect is that the aerosol MTF shifts horizontally with the scatterer size and shifts vertically with the concentration of the medium (i.e. scattering and absorption coefficients) [14, 20-21].

All of these details pertaining to the MTF exclude realistic limitations imposed by the sensor. The imaging performance in equation 7a and 7b above is at the aperture of the sensor and assumes the size of the aperture is infinite. It does not fully account for
what is recorded on the image plane. Real systems have 3 finite constraints that affect the MTF which include FOV, dynamic range, and bandwidth of the sensor [17, 20, and 24-26].

The scattering angles for EOIR (ElectroOptical InfraRed) wavelengths and micron size particulates are larger than what can be fully captured by a sensor’s FOV. One example of this is simply viewing a full moon with the naked eye. The scattered light covers such a large angular distribution (approximately 0.5 radian) that the eye cannot capture it all within its FOV. One would need to turn their head to see the full extent of the 0.5 radian scattered light distribution. The smaller FOV of the sensor manifests as a smaller measured blur, which translates into a wider MTF. This means that a narrow FOV system will measure the cutoff frequency to be larger than what it actually is at the pupil. Figure 14 illustrates this phenomenon.

![Diagram showing sensor finite FOV limits on aerosol blur](image.png)

Figure 14: Sensor finite FOV limits on aerosol blur

The dynamic range of the sensor is another limitation on how well the blur can be measured. The scattered intensities can decay sufficiently away from the source leading
to a condition where the levels are below the background noise or dynamic range of the sensor. This will result in a smaller measured blur that is a wider MTF in the frequency domain. Just as in the case for the FOV, the finite dynamic range will result in larger measured cutoff frequency than what is at the entrance pupil. Figure 15 illustrates this concept of a smaller measured scattered light as a function of the sensor’s dynamic range.

![Intensity I](image)

Figure 15: Dynamic range limit on scattered light distribution

In addition to the dynamic range and FOV limiting the observable blur within a sensor, the spatial frequency bandwidth is another sensor hardware limitation to consider. This can be the optics, detector, and even the electronics sub systems producing an intrinsic blur that can be described through an MTF. The spatial frequency where the sensor system MTF is zero defines the bandwidth of the sensor. A sensor system with a low bandwidth can impose a substantial blur of the unscattered light.
source. This blur by the sensor can mask the scattering blur by the atmosphere thereby impeding the observable scattering blur contributions.

Instrumentation can be a significant factor when trying to quantify the aerosol MTF. As history has shown [17, 24-26], poorly performing sensors did not have the dynamic range or bandwidth to observe that the aerosol blur can be significant especially in IR bands. Now with the advent of more advanced sensors with very high resolution, the seemingly subtleties of the aerosol MTF are now at the forefront of imaging performance.

Atmospheric aerosol blur is difficult to characterize due to many variables [20-35] and other phenomena, such as turbulence. The size and concentration distributions along with any spatially anisotropic densities throughout the line of sight path (e.g. patchy fog) pose a significant issue for making broader conclusions on scattering effects. The research in references [23-25] detail how the aerosol MTF can be complicated due to limits of the imaging hardware. A number of open atmosphere aerosol MTF measurements and models show that the variability in the atmosphere and the source target can change the outcome of the aerosol MTF [20-22, 26-27, 30-31, 34]. Also, efforts to restore imagery affected by the aerosol MTF have shown that the more accurate the predicted or measured aerosol MTF is, the better the restoration effort [28-29, 35]. Assumptions about the source, aerosol medium, and the imaging hardware can produce results that are seemingly contradictory [32, 33]. However, within the assumptions and constraints of the analysis, seemingly contradictory conclusions are correct in their own right.
Given the aforementioned complexities behind characterization of the aerosol MTF, it is important to reduce the number of independent variables while maintaining a meaningful test environment. This involves controlling the medium and the surroundings within a laboratory environment for greater deterministic behavior and more stable results in measurements. In this experiment, simplifications to the medium are organized by the introduction of monodisperse micron sized glass spheres at a known concentration. In addition, the source target is a tilted edge target that is collimated. The collimating lens is required for precise resolution measurements to ensure the source is imaged as if it were from infinity. Measurements of tilted edge images through the medium gauge the aerosol MTF performance. These assumptions and procedures are key differences compared to other research [36-41].

A numerical prediction model provides a parallel view to supplement the experimental study. The same conditions and assumptions from the experiment apply to the numerical predictions. The transmitted and scattered irradiances are calculated from a directional light source propagating though a monodisperse glass sphere medium suspended in water. The aerosol MTF is derived from the Fourier Transform of the total irradiance distribution collected at the aperture. The purpose of the predictions is not only to compare to the measured results but also to consider conditions outside the constraints of experimentation. This includes differences in range, wavelength, and aperture size.

Mie Theory is the mathematical model of choice to model the aerosol MTF. It can precisely handle scattering regimes ranging from \( r \ll \lambda \) to \( r \sim \lambda \). Its fundamental
dependence on the complex refractive index allows determining both the absorption and scattering contributions to determine how the incident light interacts with the particulate. In addition, it is a theory that utilizes wave optics phenomena thereby generating absorption and angular dependent scattering contributions.

Numerical MWIR and LWIR aerosol MTFs predictions are also discussed for scattering and absorption caused by water droplets. Water droplets provide a meaningful setting since it is an important constituent in infrared imaging and abundant in the atmosphere. Droplet size and concentration computations vary as a function of temperature and relative humidity. The variations affect the scattered and absorbed light, which affect the predicted MTFs. In addition, aerosol MTF trends as a function of aperture size are examined. Other factors, such as dynamic range, within the imaging hardware must be considered when analyzing performance at the image plane [23-25]. However, we restrict the numerical analysis to the aperture plane.

**Mie Theory and Aerosol MTF**

Mie theory models light matter scattering and absorption interactions of plane waves by a homogeneous sphere. The solution is an infinite series of spherical multipole partial waves. The $r\sim\lambda$ regime produces the strongest forward scattering of light compared to the other scattering regimes and is the focus throughout this chapter. Strong forward scattering presents the best conditions for observing a blur. Mie theory is rigorous, but simplified analytical solutions are found which assume spherical scatterers and single particle constituent composition. Development of the solutions into numerical
algorithms can predict the scattered and transmitted irradiances. A full treatment of Mie theory can be found in references [15-19].

The Mie theory phase function and scattering function predict the scattering weight as a function of angle from the incident propagation direction. Both equations assume a single particle type. The assumption of homogeneity simplifies both experimental measurements as well as numerical simulations of the scattering and transmission events

\[
P(\theta, \lambda) = \frac{|S_1(\theta, \lambda)|^2 + |S_2(\theta, \lambda)|^2}{2\pi k^2 r^2 Q_{SCAT}(\lambda)}, \tag{8a}
\]
\[
F(\theta, \lambda) = \pi k^2 r^2 Q_{SCAT}(\lambda) P(\theta, \lambda). \tag{8b}
\]

The variable P is the normalized phase function with circular azimuthal symmetry and F is the scattering function for an isotropic medium. The integral of P(\theta) over the angle 2\pi\sin(\theta)d\theta is unity. The angle \(\theta\) is the angular deviation from the initial propagation direction and ranges from (0 \(\leq\) \(\theta\) \(\leq\) \(\pi\)). S_1 and S_2 are the scattering amplitudes computed from Mie theory and are unitless values. The variable k is the wave number (k = 2\pi/\lambda), r is the particulate radius, and Q_{SCAT} is the scattering efficiency [15] that is unitless. The normalized phase function in equation 8a gives the angular dependence upon the scattered irradiance distribution on the observation plane. The normalized phase function is more widely used in radiative transfer theory but the scattered irradiance is related to the scattering function from equation 8b. The scattered irradiance in units of [W/cm^2] is
The variable $I_o$ is the incident irradiance, and $R$ is the distance from the scattering particle to the observation plane. In this analysis, the observation plane is the aperture plane of the imaging device. The angle $\theta$ is the same as defined in equation 8 but projected out to the aperture plane to determine the scattered irradiance distribution at aperture. An angle $\theta \approx 0$ represents infinitesimal aperture contributions whereas an angle of $\theta = \pi/2$ signifies infinitely large aperture contributions. Note that the limit for an infinite aperture is $\pi/2$. Angles greater than $\pi/2$ for $\theta$ represent backscattering and not applicable for the forward scattering focus.

The transmitted irradiance is also part of the observed irradiance distribution in the aperture plane. The main difference is that the transmitted irradiance is unperturbed from the incident propagation direction. By definition, it is neither scattered nor absorbed. Equation 10 details the transmitted irradiance definition.

$$I_{TRANSMITTED}(\lambda) = I_o \exp(-\tau(\lambda)).$$  \hspace{1cm} (10)

The optical depth, $\tau$, is a spectral unitless measure which takes on the form

$$\tau(\lambda) = \gamma(\lambda)z = (\alpha(\lambda) + \beta(\lambda))z,$$ \hspace{1cm} (11a)

$$\tau(\lambda) = \pi r^2 Q_{EXT}(\lambda)Nd.$$ \hspace{1cm} (11b)

The variables $\gamma$, $\alpha$, and $\beta$ are the spectral extinction, absorption, and scattering coefficients respectively in units of [m$^{-1}$]. The variable $\beta$ represents the light scattered
out of the line of sight. The variable \( z \) is the length in meters \([m]\) light propagates through the medium. Equation 11b is the optical depth derived from Mie theory and depends on the particulate radius \( r \) \([m]\) and \( N \) is the number of scatters per unit volume \([m^{-3}]\). The variable \( Q_{\text{EXT}} \) is the extinction efficiency \([15]\) and is a unitless variable.

The total irradiance distribution that incorporates both absorption and scattering irradiances is determined by combining equation 9 and 10 with an additional term to account for background noise.

\[
I_{\text{OBSERVED}}(\theta, \lambda) = \exp(-\tau(\lambda)) I_0 \delta(\theta) + I_{\text{SCAT}}(\theta, \lambda) + I_{\text{BACKGROUND}}
\]  \hspace{1cm} (12)

The first term depicts the irradiance that is neither absorbed nor scattered. The delta function is added to the transmitted irradiance to indicate that the source is assumed to be directional (i.e. collimated). This assumption is not uncommon given that distant objects, such as sunlight or distant vehicles, are examples of directional sources. In addition, resolution measurements require the source to be collimated to avoid issues such as depth of focus. The second term \( I_{\text{SCAT}} \) accounts for the light scattered into the line of sight. The \( \theta \) dependence shows that the observed irradiance will increase for increasing angles accepted by the aperture plane. The third term is the background, or path, irradiance contributions. It is typically independent of angle and can be omitted if the background contributions are negligible.

Equation 12 presents a representation of the observable scattered and transmitted irradiance at the aperture plane. Imposing additional assumptions on these equations demonstrate how they conform to other publications. In the limit of the small
angle approximation [14, 17, 20-21], the scattering functions of 8a and 8b can be approximated by a Gaussian function of $\theta$. Substituting this for $I_{\text{SCAT}}$, equation 12 agrees with equations found in other references [17-18, 24-25]. In the limit of an infinitesimal aperture, the scattering contribution is approximately zero and approaches Beer's law of extinction for the observed irradiance (equation 10). At these boundary conditions, the aperture captures almost no scattered light.

Observation of the transmitted irradiance is the on-axis incident light propagation axis, and the scattered irradiance is any angle away from the optic axis. The aperture size will limit the angular extent of the collected scattered irradiance. This limitation on the arrival angle ($\theta$ in equation 12) not only depends on the size, but also on how far the scattering occurred from the sensor. Equation 12 constitutes all the scattering, absorption, and even background contributions within the aforementioned assumptions. Determining the aerosol MTF involves taking the Fourier Transform of equation 12.

The next section depicts the experimental setup and measurements of the aerosol MTF and provides numerical predictions for the scattering blur from 10$\mu$m sized glass spheres. This includes details pertaining to anisotropic scattering medium trends [36-42]. The context of anisotropic scattering simply refers to the positional dependence of the scattering medium along the path from source to camera. In essence, the optical depth is not constant as the light propagates through its path. The calculated scattering MTFs characterize the effects on image quality as a function of distance from the scattering medium to the camera system. The rest of the chapter focuses on numerical analysis in MWIR and LWIR bands. The scattering medium is an isotropic distribution of
water droplets. The water droplets vary in size and concentration to gauge MWIR and LWIR scattering MTF performance.

**Measured aerosol MTF from suspended particulates in the Visible Band**

A light source back illuminates translucent plastic sheet partially painted black to provide a tilted edge target for edge spread function (ESF) measurements. A tilted edge target is used rather than a pinhole to provide the ability to take multiple samples of the edge target to improve the sampling density [43]. This is also detailed in ISO 12233. It removes aliasing artifacts that can come from systems that are under sampled. The light emanating from the edge target is incident upon a collimating lens located one focal length away. Light that is outside the physical extents of the lens is blocked to prevent undesired light from entering the medium. This also limits the amount of divergent light from the edge target. The diameter of the lens is 50.8mm and the focal length is 75cm.

A transparent flat clear glass fish tank filled with water provides the medium for the glass sphere scatterers. The optical power of the tank is zero making it a well suited container for the scatterers. The spheres are obtained from Cospheric LLC and have a radius of 5μm. The density of the glass spheres provides sufficient single scattering and absorption interactions for MTF measurements from the target ESF through the medium. This is achieved by ensuring the optical depth is small which also minimizes multiple scattering effects. Multiple scattering effects are more significant with optical depths much greater than 1. The fish tank contents are agitated to deliver an
approximate homogeneous medium. After the particulates are agitated, an approximate settling time of 2-3 seconds ensures turbulence from the water is minimal while maintaining sufficient particulate suspension. Table 1 details the medium metrics for the experimental setup.

Table 1: Medium parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass sphere radius, r [μm]</td>
<td>5</td>
</tr>
<tr>
<td>Glass sphere density [# of particles/cm³]</td>
<td>15000</td>
</tr>
<tr>
<td>Length of medium, z [cm]</td>
<td>10.2</td>
</tr>
<tr>
<td>Glass Refractive Index (n + ik) at λ=0.5μm</td>
<td>1.5185 + 7.235x10⁻⁹ [44]</td>
</tr>
<tr>
<td>Water Index (n + ik) at λ=0.5μm</td>
<td>1.335 + 1x10⁻⁹ [44]</td>
</tr>
<tr>
<td>Optical depth [unitless]</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The experiment requires the use of two fish tanks to determine the aerosol MTF from the particulates. One contains the micron size glass spheres suspended in water and another that was filled with water. Linear system theory shows that the MTF computations from the imagery the camera collects is a combination of sub system MTFs

\[
MTF_{SYSTEM+AEROSOL} = MTF_{TANK} \cdot MTF_{WATER} \cdot MTF_{AEROSOL} \cdot MTF_{CAMERA}, \quad (13a)
\]

\[
MTF_{SYSTEM} = MTF_{TANK} \cdot MTF_{WATER} \cdot MTF_{CAMERA}. \quad (13b)
\]

Calculation of the aerosol MTF is performed by dividing equation 13a by equation 13b. In this experiment, the aerosol MTF does include the effect of light transmitting
from water to glass to air. The angles of the scattered light arriving at the camera aperture slightly increase from the water to glass to air interface. This slight increase in the blur causes the MTF calculation to scale down horizontally (i.e. $MTF(x)$ scales down horizontally $MTF(ax)$).

A Sony alpha 6000 camera (table 2) captures images of the collimated tilted edge at 14 bit depth resolution. The configuration of the camera is set to capture 3 color channels at 14 bit resolution. Only the green channel data is used for ESF measurements. The mean wavelength from the green channel data is 0.5μm. A fitted error function determines the ESF from the tilted edge target imaged onto the camera. Observations show that different functions, such as the Fermi-Dirac distribution, is an acceptable function to use as well [45] but the error function is mathematically convenient. After measuring the ESF, differentiating the fitted function gives a Line Spread Function (LSF). The LSF is Fourier transformed and normalized to provide an MTF of the system.

Table 2: Camera Specification for collecting imagery for MTF measurements

<table>
<thead>
<tr>
<th>Pixel Array Size</th>
<th>6000x4000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bit Depth Resolution</td>
<td>14</td>
</tr>
<tr>
<td>Horizontal IFOV [mrad]</td>
<td>0.0196</td>
</tr>
<tr>
<td>Aperture Size [mm]</td>
<td>12.5</td>
</tr>
<tr>
<td>Focal Length [mm]</td>
<td>200</td>
</tr>
<tr>
<td>Integration time [s]</td>
<td>1</td>
</tr>
</tbody>
</table>
In an effort to better model real scattering and absorption performance, consideration of the separation distance from the medium to the camera is one aspect of the experimental design (figure 16). This simulates a variable line of sight medium (e.g., spatially anisotropic medium or patchy fog). Successive measurements at different separation distances are conducted to characterize the scattering phenomena known as the shower curtain effect [36-42] and absorption effects [23]. In the shower curtain effect, light received from a more distant object involves smaller scattering angles and thus reduces blur than light received from a closer object.

Figure 16: Spatially anisotropic scattering along path from the medium to camera. The medium has thickness d and the variable separation distance from medium to camera is comprised of air. An enclosure ensures no light outside the extents of the lens passes through to the medium.

The separation distances are 1m, 0.75m, 0.5m, and 0.2m. The minimum distance for the medium never falls below the focal length of the camera (200mm). This ensures the light entering the aperture of the camera will hit the detector. A total of 8
ESF measurements are made at each separation distance; 4 measurements with scatterers present and 4 without scatterers present. Each ESF measurement is a composition of 2 sub ESF measurements that are combined to define a single ESF at half the sampling rate to reduce sampling errors [43]. MTFs calculations from each final ESF permit solving for the aerosol MTF by dividing the MTF with scatterers (equation 13a) by the MTF without scatterers (equation 13b). Averaging the MTFs for each range to gives an overall aerosol MTF for the 4 different separation distances.

A single average MTF is the final result from the 4 aerosol MTFs at a given separation distance. Figure 17 depicts the Edge Spread Function observations with and without scatterers and the average aerosol MTF measurements for the different ranges.

![Figure 17: (a) 50mrad view of ESF without scatterers, (b) 50mrad view of ESF with scatterers, (c) ESF plots of (a) and (b), (d) Average aerosol MTF, obtained by dividing](image-url)
the measured MTFs with and without scatterers. Error bars show 1 standard deviation in average. The arrows point to the experimental MTF cutoff location.

Figure 17a and 17b depict the lit target area with a black background defining the edge spread functions with and without the glass sphere scatterers in the medium. The field of view of the ESF in figures 17a and 17b is 50mrad. Figure 17c shows the ESF with and without the glass spheres. The ESF for image B demonstrates the effects of extinction and scattering. Extinction shows the loss of contrast with the smaller modulation and scattering into the line of sight shows a wider spreading of the ESF. Figure 17d shows the MTF computations at 4 different separation distances (or ranges). Since the MTFs calculations are from dividing equation 13a by 13b, the plane that the MTF relates to is the aperture of the camera system.

One essential detail in Figure 17b is that the image has significant background irradiance in comparison to figure 17a. The ESF of image B in figure 17c shows the result of the constant background offset. The background irradiance is in reference to the minimum amount of light observed throughout the image. This is considered to be a constant offset for all pixels ($I_{BACKGROUND}$ in equation 12). Despite the enclosure permitting the desired target light to pass through the lens, oblique light rays are present and produce background contributions (glare). This background irradiance offset cannot be neglected when computing the LSF. Computing the LSF involves taking the derivative of the ESF, which removes any DC offset. The constant background, or pedestal, is divided out. It is necessary to add the background irradiance back into the LSF. Measurement of the average minimum signal from the collected images and
adding it back into the LSF is an essential step for modeling the background of the imaged target. This is equivalent to determining $I_{\text{BACKGROUND}}$ of equation 12. The constant background in the LSF will produce a delta function response in the MTF at zero spatial frequency. The amplitude of the delta function response in figure 17d is in accordance with the strength of the background irradiance. The larger background results in a greater delta function response in the MTF.

The transition from a varying spatial frequency dependent MTF to one that is weak to no spatial frequency dependence defines the cutoff frequency. This changeover in the experimental MTF delineates where the scattering effects are no longer perceivable, and no spatial frequency dependence on scattered irradiance can be observed (arrows in figure 17d). The 0.2m range case does not have an arrow since it is not obvious where the cutoff is but it is estimated to be near 0.5 cyc/mrad. The cutoff frequency is significantly higher than the theoretical prediction of $2r/\lambda$. This is a consequence of the aperture limiting the observable blur for a given separation distance. The truncated blur in the spatial domain results in a shift to a higher spatial frequency compared to theoretical predictions. Sadot and Kopeika [8] detail the same phenomenology with hardware constraints as demonstrated here. In addition, the near constant MTF response beyond the cutoff frequency corresponds to the unobstructed light from the target (i.e. neither scattered nor absorbed). This corresponds to the first term in equation 12. Ideally, the MTF should be independent of spatial frequencies beyond the cutoff frequency. The second term in equation 12, $I_{\text{SCAT}}$, corresponds to the aerosol MTF below the cutoff frequency.
The aerosol MTF also demonstrates the effect of the separation distance between the medium and the camera (i.e. spatially anisotropic line of sight scattering effects). At larger distances from the medium, the observable blur at the aperture has a weak angular dependence. The finite aperture size cannot capture the scattered light at larger angles. This corresponds to an MTF with larger amplitude and greater cutoff frequency. Equation 12 shows that in the limit of small angles, the observable irradiance is mostly transmitted irradiance. Conversely, as the distance between the medium and the camera decreases, the camera collects a larger scattering angular spread. The decrease in range increases the scattered irradiance (equation 9) and the greater angular extent shows a greater scattering irradiance contribution (equation 12). This diminishes the observable transmitted irradiance from the unabated edge response. Figure 17d shows a significant MTF degradation response at shorter distances to the medium. The shower curtain effect is the reason for the performance reduction [36-42].

The error bars in figure 17d denote the standard deviation in the MTFs computations from the ESF measurements. The sources of these variances are difficult to isolate individually since all the components are mixed together. Multiple scattering is minimized by controlling the concentration of the medium but could never be truly eliminated. Other sources of variance include the back lit target not being ideally uniform, imperfections in the fish tank surfaces, measurement error, error when dividing the MTF with scatterers to the one without scatterers, oblique light and reflections from the container walls producing a glare, and scatterers that are not truly homogeneously distributed within the medium. Such consequences can account for the MTFs in figure
17d not demonstrating a clear knee transition and high spatial frequency regions not showing a constant response as expected.

**Predicted aerosol MTF from suspended particulates in the Visible Band**

The numerical model applies Mie theory and in the same configuration as the experimental setup (tables 1 and 2). The basis behind using Mie theory is that it is more accurate than the small angle approximation and relies on fundamental parameters such as the scatterer concentration, size, and complex refractive index. The separation distances from the aperture and the medium are 100m, 1m, 0.75m, 0.5m, and 0.2m. The 100m separation provides insight into the case of large separation distance similar to remote sensing applications viewing through fog from afar. The numerical method predicts the irradiance distribution across the aperture as a function of the separation distance. Equation 14 details the numerical prediction model.

\[
I_{OBSERVED}(\theta) = I_o \exp \left( - \sum_{i=1}^{K} \tau_{i-1} \right) \delta(\theta) + \\
\sum_{n=1}^{K} \left[ \left( I_o \exp \left( - \sum_{i=1}^{n} \tau_{i-1} \right) F_n(\min(\theta, \theta_{max})) \right) \right] / k^2 R_n^2 + I_{BACKGROUND}
\]

The model uses K equally spaced segments of plane parallel slabs to determine the Mie theory based diffusely scattered and collimated transmitted light as it propagates through the medium. Equation 14 is similar to methods described by Ishimaru [17] and is a discretization of equation 12. The variables noted here are the same as in equations 9, 10, and 12. The first term is the transmitted term with the delta
function to maintain the directional source as dictated by the collimated lens configuration in the previous section. The second term has the summation index n to denote a segment within the K slab segments of the medium. The variable $\tau_n$ is the per slab optical depth with the initial condition that $\tau_0$ is 0 and the incident irradiance is $I_0$.

The summation of the optical depth in the second term establishes how much irradiance is incident to each layer. The variable $\theta_{max}$ is the maximum angle subtended from the scatterer to the sensor aperture for each slab segment. This ensures the scattered light beyond this threshold angle does not contribute to the observable irradiance. The background irradiance is added to provide a constant noise to contribute to the observable irradiance.

The Fourier transform of equation 14 determines the aerosol MTF at the aperture of the system. The first term, the transmitted term, is a delta function in the spatial domain resulting in a constant response in the spatial frequency domain. If the transmission is not 0, this gives the constant high spatial frequency response in the MTF. The second term is the scattering term. The angular distribution of the total scattering contributions gives the varying spatial frequency response in the MTF. The last term is a constant background offset to the observable irradiance. The MTF response is a delta function at zero spatial frequency.

Since the experimental measurements involve the glass beads suspended in water, the numerical model accounts for the Mie theory of particulates within water. Additionally, the scattered and transmitted irradiance distribution at the end of the medium accounts for refraction from the water to glass to air interface. Figure 18
portrays the light source, a medium with depth (d), and the separation distance from the aperture plane. The number of sub division slabs is set to 10. Additional slab segments are not necessary since the medium is not of significant length, and the solution shows good convergence with a relative error ~2%.

Figure 18: Numerical configuration for predicted spatially anisotropic scattering and absorption effects from the medium to the camera with thickness d. The separation distance varies to model the shower curtain effect.

Not only did Mie theory predict the scattering properties, but it also predicts the absorption properties allowing calculation of both scattered and transmitted irradiances. Once the scattered and transmitted irradiance distributions are computed at the aperture plane, the result is Fourier transformed and normalized to provide the aerosol MTF (figure 19).
Figure 19: Mie Theory aerosol MTFs for different camera to medium separation distances (aka range). Arrows denote predicted cutoff frequency

Figure 19 values generally fit within the error bars of figure 17d. The cutoff frequency shows trends of shifting towards lower spatial frequencies with decreasing separation distance. The MTFs also demonstrates greater spatial frequency dependence below the cutoff frequency for decreasing separation distance. All of these details are consistent with the shower curtain effect.

At large separation distances, the blur is almost undetectable at the aperture. In the limit of large separation distance, only the transmitted light and a narrow angular component of the scattered light reach the aperture. The MTF appears as a transmission loss over nearly all spatial frequencies (equation 10). As the separation distance decreases, the MTF amplitude reduces. This trend agrees with the MTF measurements of figure 17d and the shower curtain effect.
The measurements of figure 17d show that adding the background irradiance offset back to the LSF affects the delta function response at zero spatial frequency. The same background offset measurement for the different ranges is added into the numerically predicted results for the same corresponding ranges. The measured background light is normalized in order to scale it properly since the simulations assume normalized source irradiance. This provides the background that generates the delta function response shown in figure 19. The smaller the separation distance, the greater the background illumination since the aperture collects more light. Since there is no background measurement for the 100 meter range case, sampling the edge of the predicted spread function provides the background irradiance. The edge is the largest acceptance angle into the aperture from the scattering location. It is the aperture radius divided by the range from the scatterer to the camera.

**MWIR and LWIR aerosol MTF predictions from water droplets**

MWIR (3-5μm) and LWIR (8-12μm) experimental measurements with glass spheres suspended in water cannot be performed due to water absorption in infrared bands. The correlation between the experimental (measured aerosol MTF from suspended particulates) and the numerical analysis (predicted aerosol MTF from suspended particulates) suggests that the same numerical approach (equation 14) can be used to characterize MWIR and LWIR performance. Equation 14 accounts for both absorption and scattering effects in the first and second terms of the equation. The advantage of the numerical method is that it permits investigation of any particulate
type. In this section, the medium consists of water droplets suspended in air. A path length of 3km is selected with the camera aperture near the edge of the isotropic medium. The diameter of the aperture plane is set to 51mm (~2in) with a MWIR and LWIR wavelength set to 4μm and 10μm respectively. The complex refractive index of water is obtained at each respective central wavelength [46]. Figure 18 illustrates the numerical simulation configuration except the length of the medium is 3km and the separation distance is no less than 200mm.

The droplet radius and concentration calculations are derived from empirical formulas that compute the parts per million by volume (ppmv) of water vapor as a function of temperature and relative humidity (RH) [47]. The ppmv provides a simple method to scale the concentration and radius of the water droplets to simulate a medium from a known initial supposition. This relationship between the ppmv and the water droplets is an initial assumption of 4.2x10^5 droplets per cubic meter for the concentration and a 5μm radius at 70% RH and 302K. The radii and concentrations are consistent with a tenuous fog [48-49]. The concentration is small but the larger path length compensates to give a sufficient optical depth. As the RH and temperature changes, the concentration scales with the calculated ppmv. The new volume of water droplets determines the radius of a single droplet assuming spherical symmetry (table 3). The medium consists of only monodisperse droplets with scattering and absorption properties. These assumptions are consistent with natural phenomena like the corona [15]. Although water vapor absorption can be significant, it is not considered here.
Table 3: Water droplet radius and concentration for different relative humidity and temperature

<table>
<thead>
<tr>
<th>Test Case</th>
<th>Temperature (K)</th>
<th>RH (%)</th>
<th>Radius (μm)</th>
<th>Concentration (#/m³)</th>
<th>Optical Depth MW/LW</th>
<th>Corresponding Figure number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>290</td>
<td>40</td>
<td>7.45</td>
<td>1.29x10^5</td>
<td>0.439/0.638</td>
<td>20 and 22</td>
</tr>
<tr>
<td>2</td>
<td>302</td>
<td>40</td>
<td>6.02</td>
<td>2.4x10^5</td>
<td>0.604/0.802</td>
<td>20 and 23</td>
</tr>
<tr>
<td>3</td>
<td>302</td>
<td>90</td>
<td>4.17</td>
<td>7.26x10^5</td>
<td>0.833/0.955</td>
<td>20 and 24</td>
</tr>
</tbody>
</table>

The true composition of water droplet radius size and concentrations are more complicated than what is presented in table 3. However, sizes and concentrations derivations form empirical formulas is an improvement over arbitrary size and concentration selections. The focus here is not on droplet size and concentration predictions but on imaging performance with trends dependent upon droplet size and concentration.

The 3km path through the medium is broken up into 6000 slabs for more accurate results (K = 6000 in equation 14). The water droplet medium is isotropic throughout the complete 3km path. The Fourier transform of equation 14 gives the aerosol MTF for the cases in table 3. Figure 20 illustrates the MWIR and LWIR aerosol MTF predictions for size and concentrations of cases 1-3 in table 3.
The aerosol MTFs demonstrate a low spatial frequency response that quickly flattens for increasing frequencies. The most notable trend in figure 20 is the improvement in the MTF in the MWIR when compared to LWIR. The MWIR case has a higher cutoff frequency in comparison to the LWIR case. This agrees with the theory that the MTF cutoff frequency is proportional to droplet radius and inversely proportional to the wavelength ($2r/\lambda$). This leads to the fact that LWIR blur, in the spatial domain, is larger in angular extents than the MWIR blur. MWIR is slightly more sensitive to changes in water droplet size since the average wavelength is smaller. The spectral differences of the blur are due to the scattering function of equation 8b.

Also, there is less absorption for MWIR in contrast to LWIR. Water has a smaller imaginary component for the refractive index in MWIR than LWIR [46]. The LWIR MTF amplitudes display less sensitivity to concentration changes. This absorption dominance
and diminished scattering account for the LWIR MTF observations. The MWIR MTF amplitude is more sensitive to changes in concentration. This is traced back to the properties of water having greater scattering in the MWIR due to the larger real component for the refractive index [46]. Conversely, case 3 shows that MWIR performance trends towards LWIR. This suggests that MWIR and LWIR performance shows fewer differences in their respective MTFs under those conditions.

The MTFs do not have the delta function response illustrated in the previous sections. Given that a constant background, or path radiance, is assumed to be well below the signal (large SNR) from the simulated source, the background can be neglected ($I_{\text{BACKGROUND}} = 0$ in equation 12 and 14). This assumption is not uncommon since it maintains unperturbed MTF trends. The result is the MTFs approach unity as the spatial frequency approaches zero.

_Aerosol MTF applied to test images_

Many conclusions can be drawn just from the MTFs alone in figure 20. Some aspects of imaging performance can be difficult to quantify because the MTF trends may not be sufficient in exposing all aspects of imaging performance. Therefore, applying the MTFs to a test image that contains a vast spectrum of spatial frequencies is necessary. This allows in depth insight on image performance that is otherwise difficult to ascertain with the MTF plots alone. The first step involves making 2D separable MTFs from the 1D MTFs in figure 20. Then, multiply the 2D MTFs to the Fourier transformed test image in the spatial frequency domain. The modified Fourier
spectrum image is then inverse Fourier transformed back to the spatial domain. Figure 21 shows the USAF 1951 resolution test image and Figures 22-24 show the image modifications with the MTFs from figure 20 to the test image.

![USAF-1951 Test Image with line plot for bar pattern group -1 element 6](image)

Figure 21: Test Image with line plot for bar pattern group -1 element 6
Figure 22: Case 1 from Table 3. (a) MWIR aerosol MTF applied, (b) LWIR aerosol MTF applied, (c) line plot of bar pattern in group -1 element 6.
Figure 23: Case 2 from Table 3. (a) MWIR aerosol MTF applied, (b) LWIR aerosol MTF applied, (c) line plot of bar pattern in group -1 element 6.
Figure 24: Case 3 from Table 3. (a) MWIR aerosol MTF applied, (b) LWIR aerosol MTF applied, (c) line plot of bar pattern in group -1 element 6.

Figures 22a-24a shows that the MWIR MTFs have greater sensitivity to water droplet concentration due to greater scattering into the line of sight. The overall MWIR image quality is better than LWIR at lower concentrations but shows greater degradation than the LWIR imagery as the concentration increases. Figures 22b-24b illustrates that the LWIR MTFs are less sensitive to changes in concentration due to the reduction of scattering into the line of sight and greater absorption. The LWIR imagery shows degradation for increases in concentration but not as strongly as the MWIR imagery. Figures 22b-24b also details that the LWIR set of images looks more washed
out. This is consistent with the larger angular blur (narrower MTF) and greater absorption that manifests as an overall loss of contrast in the image for the LWIR case. Only in case 3 (figure 24a and 24b) does MWIR and LWIR show similar performance.

Figure 22c-24c show line plots of the bar pattern in group -1 on element 6. The sinusoidal shape on all the plots is a consequence of the MTFs attenuating the higher harmonics of the 3 bar pattern more than the fundamental frequency. Despite the subtle differences in the observable blur in all the figures, the modulation is clearly evident from the line plots. The LWIR modulation is 0.23 whereas the MWIR modulation is considerably improved at 0.35. Figure 23c demonstrates similar trends as figure 22c with the LWIR modulation at 0.21 vs the MWIR modulation at 0.29. Figure 24c shows that the line plots are nearly identical confirming that MWIR and LWIR aerosol MTFs show comparable performance in this test case. This substantiates the aforementioned details about the MWIR and LWIR aerosol MTFs.

**Aerosol MTF for small aperture plane**

The size of aperture plane can be a variable of concern for the aerosol MTF in addition to diverse medium characteristics. In this section, the same MWIR and LWIR numerical analysis is performed but for smaller and larger apertures. Figure 25 illustrates the MWIR and LWIR aerosol MTFs from water droplet cases 2-3 from table 3 and for an aperture size of 12mm.
Figure 25: MTFs for cases 2-3 in table 3 and an aperture size of 12mm. (a) MWIR and (b) LWIR.

The scattering and absorption dependent trends in figure 25 are the same as in figure 20. This is expected since the medium is the same. However, the smaller aperture produces MTFs that are larger overall and have a greater cutoff frequency. The smaller aperture results in smaller scattering angles arriving at the aperture (into the line of sight). The constraint on the angular extent produces an effective blur that has a weaker angular dependence for both MWIR and LWIR cases (equations 12 and 14 approach equation 10). Under this constraint, the blur predictions for MWIR and LWIR are similar. The resultant MTFs show comparable trends both in amplitude and in cutoff frequency.

Figure 26 details the aerosol MWIR and LWIR MTFs at 302K for an aperture size of 127mm.
Figure 26: MTFs for cases 2-3 in table 3 and an aperture size of 127mm. (a) MWIR and (b) LWIR

Figure 26 portrays trends that are different from what figure 25 illustrates. The interesting trends of figure 26 are the lower amplitude and cutoff frequency shifts to the left. This is a byproduct of the larger aperture and its ability to collect a larger angular distribution of scattered light (equation 12 and 14). The scattered distributions are no longer similar between MWIR and LWIR, unlike the MTFs of figure 25. The MTFs demonstrate similar performance for high concentration and lower droplet radius, but this is expected since the medium does not change. The larger aperture uncovers greater scattering dependence in MWIR and weaker dependence in the LWIR. The MWIR MTF cutoff frequency shows a more noticeable difference for the 2 cases than LWIR due its inverse dependence on wavelength for a given droplet size. The differences are subtle, but it is clear that the smaller aperture does minimize scattering effects for both MWIR and LWIR.
Figure 27: MTF dependence on aperture for case 2 outlined in table 3. (a) MWIR and (b) LWIR.

Figure 27 summarizes the significance of the aperture size. In the limit of a large aperture, the MWIR and LWIR MTFs detail little spatial frequency dependence. The truncation of the blur is minimal. Conversely, the small aperture truncates the blur significantly and produces MTFs that are similar for MWIR and LWIR. The mid-size aperture shows the most difference comparing MWIR to LWIR.

**Aerosol MTF Discussion**

Though the experimentation here is a simplification, it demonstrates the effects on imaging performance. Particulates, like atmospheric aerosols, can adversely affect image quality if sufficient forward scattering exists. In addition, a finite aperture constrains the observable scattered light. This results in different aerosol MTF
performance as a function of the separation distance between the medium and the camera.

Collimating the incident light is an important step to ensure all light entering the medium has a common direction. If the propagation direction is not collimated, the scattered irradiance predictions require accounting for the different incident angles. Variances in the propagation direction of the incident light leads to variances in the scattered light distribution on the aperture plane. Such variances complicate the experimental results as well as any attempts at analyzing with numerical predictions.

The numerical model not only shows that there is general agreement with the experimental results but also that application of Mie theory can be an acceptable methodology for aerosol MTF predictions. As long as all the material refractive indices, concentrations, and sizes of the particulates are known for the medium, Mie theory determines the spatial distribution of light. The solution can yield sufficient results for predicting aerosol MTFs when $t\sim\lambda$.

The Mie theory MWIR and LWIR aerosol MTF trends provide insight into imaging performance for water droplet based atmospheric conditions. The MTF predictions have low cutoff frequencies that illustrate an overall loss of contrast in the image. This is mostly true for LWIR but both MWIR and LWIR predictions show that not only can the MTFs produce a noticeable blur, it also varies as a function of concentration and droplet radius. The droplet conditions are maintained to be inline with tenuous fog environments. These conditions did lead to imagery comparisons showing marginal but observable differences. The most insightful aspects are the roles of both scattering and
absorption within the droplet medium for MWIR and LWIR as well as the dependence on aperture size.

The aperture size variations in figure 27 show similar trends observable in figures 17c and 19 with the spatially anisotropic medium. The conclusion in the anisotropic medium case is that the scattering events far from the camera led to narrow scattered light angles collected by the finite aperture. This results in MTF improvements with increasing separation distance and degraded MTFs for decreasing separation distance. Figure 27 shows the same trends as a function of aperture size. The blur is a function of the distance to the medium and the size of the aperture. Though the trends point to having a smaller aperture to improve aerosol MTF performance, it requires balancing with other MTFs that define the system.

**Aerosol MTF Conclusions**

Characterization of the aerosol MTF is a difficult task due to the dependence on many different variables. One method is to reduce the variables to understand fundamental concepts without loss of generality behind the scattering process and its effects on imaging performance. The experiment and simulations described here assume that there is a single particulate type with known concentration, size, and complex refractive index. The medium contains an isotropic distribution of scatterers. The MTF calculations from measurements and predictions by Mie theory demonstrate trends that agree with known phenomena and provide insight into how scattering affects image quality.
The experimental procedure is defined to determine the transmitted and scattered light incident upon the finite size aperture of the camera system. Collimated light from a back lit tilted edge propagates through a tank filled with water provides the conditions to measure a baseline MTF without scatterers present. Glass spheres with a radius of 5μm are dispersed in the water and the MTF measurement repeats again with the scatterers present. Experimental results show that the MTF is nearly constant at high spatial frequencies, which accounts for transmitted light that is neither scattered nor absorbed. The MTF also varies as a function of separation distance between the medium and the camera demonstrating the shower curtain effect. The numerical predictions derived from Mie theory show general agreement with the experimental measurement results. The trends show the shower curtain effect, absorption and scattering effects, and even background light are all part of the MTF predictions as well as the observations in the MTF measurements. Results show that scatterers closest to the camera with significant background irradiance significantly degrade imaging performance.

Mie theory predictions and analysis are also performed on MWIR and LWIR bands. The medium consists of water droplets that provide a close resemblance to real atmospheric conditions. Though the medium is only water based, it is an adequate characterization of atmospheric conditions where water droplets are dominant. The predictions involve variances in droplet concentration and radius to cause changes in the water droplets for absorption and scattering MTF analysis. Results show that the LWIR MTF is less sensitive to change in the concentration due to the lower scattering
and greater absorption. Conversely, MWIR shows better performance despite the greater scattering dependence with droplet concentration. Only in the limit of high concentration and small droplet radius do the MWIR and LWIR performances show similar image quality. There is no clear advantage of MWIR vs LWIR under such conditions for the aerosol MTF.

For a smaller aperture size, the angular dependence of the scattered light projected onto the aperture is weak. The MWIR and LWIR MTFs are similar under this condition. Conversely, a larger aperture shows greater variation in the MWIR MTF than the LWIR MTF. The conclusions about MWIR verses LWIR aerosol MTF detail that a smaller aperture exhibits the best performance. However, this can be in conflict with other aspects of the system design and must consider the complete system for a full analysis.

This study considers air-borne water droplets only. The total atmospheric transmission must include the molecular absorption, which is dominated by water vapor absorption in the MWIR and LWIR spectral regions. The effect of absorption on the aerosol MTF is detailed elsewhere [23].
CHAPTER FOUR: EFFECTS OF AEROSOL MTF ON TARGET IDENTIFICATION

Aerosol MTF for Target Identification Introduction

One of the difficulties in analyzing aerosol MTF performance and its impacts on imaging tasks is the domain in which the aerosol MTF is determined. Previous research shows varying conclusions on the importance on the aerosol blur [14-15, 17-18, 20-25, 28-29, 32-36, 39, 42, 50]. Despite the differences in results, the conclusions align within the constraints of the experiments.

Sadot et al show the presence and importance of the aerosol blur and its dependence on weather conditions as a function of time through a series of measurements [29]. In addition, analysis shows that the SAA theory provides a mathematical model to compare with measured results for the aerosol MTF [17-18, 23-25, 35]. The results show that the theory and the open atmosphere measurements of the aerosol MTF are an important component of the total system MTF. The assumptions include small angle forward scattering with single scattering events. The most notable domain suppositions include particulate absorption [23] and camera hardware constraints (FOV, dynamic range, and spatial frequency resolution) [23-25, 35]. The outcomes show that the classical description of the aerosol MTF [14, 20-21] changes resulting in shifts and scales to the MTF. The most notable change is the horizontal scaling that broadens the MTF as a function of instrumentation constraints [24]. With all the assumptions known, the aerosol MTF measurement or prediction can be used to effectively restore images through image processing techniques [28-29, 35]. The better
the understanding on measurement and prediction conditions, the more effective the image restoration process can be.

The dependence on experimental design details is further illustrated through analysis of the shower curtain effect [17, 36, 39, 42, 50]. The results show that the aerosol blur effects can dramatically fluctuate depending upon anisotropic assumptions with the aerosols. The closer they are to the camera, the lower the aerosol MTF is. Greater scattering angles are observable under this condition. Conversely, concentration of aerosols at the source generates diminishing scattering angles thereby improving the overall aerosol MTF. This is important since it states that the line of sight from sensor to target is not symmetric. The MTF is not the same for an air to ground condition verses a ground to air.

Eismann and LeMasters [42] conclude that the scattering blur is merely a radiometric effect that did not have significant spatial frequency dependence in the MTF. The numerical predictions did not account for the camera system limitations such as the FOV or aperture size limiting the observable blur. Other experiments show that the instrumentation effects can significantly affect the cutoff frequency of the aerosol MTF [24-25, 35]. In addition, the aerosol MTF measurement from satellite imagery [42] satisfies the condition for the shower curtain effect where the bulk of the scattering and absorption occurs in the far field from the sensor. This result produces a blur that has a weak angular dependence across the observation plane. The MTF in this case behaves like an overall loss of contrast. The most important conclusion here is that the results are correct within the experimental conditions.
A final example about the fluctuating (various and sometimes appear to be contradictory) conclusions on the significance of the aerosol MTF is the work from Bissonnette [32]. He concludes that the aerosol blur is observable under large optical depths with heavy fog and rain conditions. Kopeika and Sadot show that key differences within the results lay within the instrumentation details used to measure the aerosol MTF [33]. The optical cutoff of the sensor and the assumption on the shape of the phase function accounts for the key differences in the results. The conclusions by Bissonnette that clear and light hazy conditions do not produce an observable aerosol MTF are correct only within the constraints of the experimental conditions. The publications from both signify the critical aspect that the aerosol MTF can differ significantly depending upon the assumptions made within the analysis.

This brief review on different literature sources shows that the characteristics of the aerosol MTF are dependent upon the instrumentation and the assumptions about the experiment or prediction. In this study, the aerosol MTF is incorporated into the system MTF to perform acquisition range analysis. Therefore, it is important to identify the assumptions pertaining to the aerosols and sensor parameters to avoid similar occurrences. The assumptions include the light from the target is collimated, the aerosols are a monodisperse composition of spherical water droplets, and no multiple scattering (i.e. single scattering only). Though the monodisperse aerosols are a simplification, it does represent realistic environmental conditions that are worthy for investigation. Such conditions can produce corona effects [15]. Optical depths for such conditions are predominately single scattering events. In addition, target identification
sensors will have a narrow field of view with a finite aperture size and objects at large ranges leading to an approximately collimated target emittance. These details show that such real world conditions are in alignment with the assumptions here. These assumptions are used to build a set of aerosol MTFs that vary as a function of range for 4 water droplet compositions, wavelength (MWIR and LWIR), and aperture size. The aerosol MTFs are incorporated into NVIPM to predict target identification ranges.

The theory section outlines the concepts behind the aerosol MTF and the target task performance (TTP) algorithm used within NVIPM. The aerosol MTF theories cover the SAA model as well as Mie theory. The TTP procedure details how NVIPM utilizes the input parameters ranging from the sensor details to the identification criteria. The next section portrays the setup and procedure in NVIPM to create range ID plots for the 4 different droplet compositions, wavelength, and aperture size. The discussion section details the analysis of the range data from NVIPM. This includes facts about the results and the trends for MWIR vs LWIR configurations. The conclusion summarizes all the experiment and emphasizes the important aspects of the observable trends for MWIR and LWIR sensor systems.

**Aerosol MTF and Target Identification Theory**

Numerous authors use the SAA model for aerosol MTF analysis [17-18, 23, 35]. This chapter uses the model by Sadot and Kopeika [23].

\[
MTF_{AEROSOL}(z, f) = \exp\left(-\gamma z \left(\frac{f}{f_c}\right)^2\right) \exp\left[-\alpha z \left\{\exp\left(-\gamma z \left(1 - \left(\frac{f}{f_c}\right)^2\right)\right) - \exp(-\gamma z)\right\}\right] \quad \text{for} \quad f \leq f_c. \tag{15a}
\]
\[ MTF_{\text{AEROSOL}}(z, f) = \exp(-\gamma z) \exp[-\alpha z \{1 - \exp(-\gamma z)] \quad \text{for} \ f > f_c. \quad (15b) \]

The variables \( \gamma \) and \( \alpha \) are the scattering and absorption coefficients respectively. These variables represent the strength of scattering and absorption as light propagates through the aerosols. The most important aspect of the scattering coefficient is that it represents what is scattered into the sensor. The scattering strength is dependent upon the aerosols and what is observable within the sensor constraints. The spatial frequency is \( f \) and the cutoff spatial frequency is \( f_c \). The cutoff frequency marks the location in frequency space where the aerosol MTF no longer displays any spatial frequency dependence. Its definition is proportional to the aerosol radius divided by the wavelength (\( 2r/\lambda \)). The variable \( z \) provides the range dependence for the aerosol MTF. Figure 28 shows a normalized aerosol MTF from the SAA model.

Figure 28: Example of SAA aerosol MTF with \( \gamma \) and \( \alpha \) equal to 0.15km\(^{-1}\), \( z \) is 3km, and \( f_c \) is 1cyc/mrad
The SAA model is a simplification that is analytic. Another method separate from the SAA model is to use the numerical Mie theory technique to predict the aerosol blur in the spatial domain. This method is described in chapter 3.

In order to model target identification, the NVIPM target task performance must be understood. The algorithm depends upon the contrast of the target, the contrast threshold function (CTF), the TTP metric, and the V number to determine probability of identification (PID). A more rigorous derivation of the TTP model and PID are in references [14, 21, 51].

\[
\text{CTF}_{SYS}(f) = \frac{\text{CTF}(f)}{\text{MTF}_{SYS}(f)} \tag{16a}
\]

\[
\text{MTF}_{SYS}(f) = \text{MTF}_{AEROSOL}(f) \cdot \text{MTF}_{CAMERA}(f) \tag{16b}
\]

\[
\text{TTP} = \int_{\xi_1}^{\xi_2} \frac{C}{\sqrt{\text{CTF}_{SYS}(f)}} df \tag{16c}
\]

\[
V = \left( \frac{\sqrt{A_{tgt}}}{z} \right) \text{TTP} \tag{16d}
\]

\[
\text{PID} = \frac{\left( \frac{V}{V_{50}} \right)^E}{1 + \left( \frac{V}{V_{50}} \right)^E} \quad \text{where} \; E = 1.51 + 0.24 \frac{V}{V_{50}} \tag{16e}
\]

The most important aspect of equation 16a is the inclusion of the system MTF to define the system CTF. This is the location where the aerosol MTF is included into the TTP model. The system CTF is a unitless measure. Equation 16b illustrates linear system theory where the aerosol MTF simply multiplies to the other MTFs to define the
total system MTF. Equation 16c details the TTP where C is the target contrast and the integration is over all spatial frequencies from the lower cuton to the upper cutoff [14, 21]. The TTP has units of cycles/mrad. Equation 16d is the V number that characterizes how easy it is to find a target. It is a unitless metric that is dependent upon the TTP, characteristic size (the root of the target area $\sqrt{A_{tgt}}$), and the range to the target z. The final expression (equation 16e) details the computation of the probability of the task at hand. In this case, the task is identification and the variable $V_{50}$ is the V number for 50% probability of identification. It is set to 13. In the context of NVIPM, PID is predicted as a function of range.

An array of ranges serve as the independent variables to determine the corresponding V numbers from equation 16d and subsequently the PID value with equation 16e. This is the process that NVIPM utilizes to determine PID vs range.

**NVIPM Probability of Identification Predictions**

Measuring the aerosol MTF impacts on target identification requires running NVIPM permutations with the aerosol MTF as a function of range, droplet concentration and size, wavelength, and aperture size. The wavelength permutation is only 2 wavelength bands. The MWIR case is 4μm and the LWIR case is 10μm. The aperture size evaluations consider a small aperture case of 3in and a large aperture case of 6in. The water droplet size and concentration combinations are listed in table 4 below. This is the same data tabulated in table 3 in chapter 3.
Table 4: Water droplet radius and concentration

<table>
<thead>
<tr>
<th>Test Case</th>
<th>Radius (µm)</th>
<th>Concentration (#/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>7.45</td>
<td>1.29x10⁵</td>
</tr>
<tr>
<td>3</td>
<td>6.02</td>
<td>2.4x10⁵</td>
</tr>
<tr>
<td>4</td>
<td>4.17</td>
<td>7.26x10⁵</td>
</tr>
</tbody>
</table>

A Mie theory numerical solution is used to generate the irradiance distribution according to equation 14 and applying the Fourier transform produces the aerosol MTF. An in depth analysis and discussion into this process is covered in chapter 3. The scattered and transmitted irradiance is computed in a sequential fashion through the aerosol segments. The initial irradiance is collimated and normally incident to the first segment. Each segment seeds the next with the transmitted irradiance from the previous segment thereby computing a downstream (or waterfall) transmitted and scattered irradiance distributions. The number of segments is set to 12000 to provide sufficient spatial division for all ranges. The irradiance propagates to the aperture plane from each segment and summed up. This determines the total observable irradiance at the aperture. The Fourier transform of the total irradiance gives the aerosol MTF prediction. In the infrared region, Modtran assumes only single scattering is present. All radiation scattered out of the FOV contributes to extinction. It does not include radiation scattered into the system, which is the basis of the analysis here.
The size and concentrations in table 4 are commensurate with a tenuous fog [48-49]. In addition, Mie theory requires the wavelength and complex refractive index for the water droplets [46]. Aerosol MTF calculations for wavelength, aperture size, and range variables are performed with the droplet compositions to provide a sufficient testbed for comparing MWIR and LWIR performance. The NVIPM configuration performs PID predictions in 1000 meter range increments with the aerosol MTF prediction at each range increment included. The range extends from 0 to 12000 meters to satisfy range dependent PID data. The arrangement for the water droplet cases is a single NVIPM xml file for each droplet configuration.

NVIPM requires many inputs into the model to perform range prediction analysis. Table 5 details the common values shared among the MWIR and LWIR sensor configuration within NVIPM.

Table 5: Common NVIPM parameters for MWIR and LWIR sensors

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MWIR Value</th>
<th>LWIR Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pixel Pitch [μm]</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Background Temp [K]</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Pixel Array Size</td>
<td>1024x1024</td>
<td>1024x1024</td>
</tr>
<tr>
<td>Beer’s Law Transmission [km⁻¹]</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Fill Factor [unitless]</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Target Temp [K]</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Well Fill [%]</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Target ΔT Variation [K]</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Integration time [ms]</td>
<td>16.67</td>
<td>16.67</td>
</tr>
<tr>
<td>Target Characteristic Size [m]</td>
<td>3.11</td>
<td>3.11</td>
</tr>
<tr>
<td>Optics Transmission</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>TTP Metric (V₅₀) [unitless]</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Optics Temperature [K]</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>TTP Target Contrast [unitless]</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>F/λ/d [unitless]</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
One aspect to point out about the input data common for all sensors is the transmission. This transmission value of 0.85 km$^{-1}$ characterizes the molecular transmission effect and is independent of the aerosol MTF. It is intentional to have the transmission specified the same across MWIR and LWIR sensors so that the PID trends are almost entirely due to the aerosol MTF variations. This applies to all cases of NVIPM runs. Another common setup is all noise sources are set to 0 throughout NVIPM for all test cases but photoelectron shot noise is present. This ensures the system is resolution limited. Table 6 details the parameters that are not common with the MWIR and LWIR configurations.

Table 6: Non common NVIPM parameters for MWIR and LWIR sensors

<table>
<thead>
<tr>
<th></th>
<th>MWIR 6in aperture</th>
<th>MWIR 3in aperture</th>
<th>LWIR 6in aperture</th>
<th>LWIR 3in aperture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength [μm]</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Well Capacity [Me]$^-$</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>F number [unitless]</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Focal Length [in]</td>
<td>30</td>
<td>15</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>FOV [mrad]</td>
<td>13.44</td>
<td>26.88</td>
<td>33.6</td>
<td>67.2</td>
</tr>
<tr>
<td>System cutoff [cyc/mrad]</td>
<td>38.1</td>
<td>19.05</td>
<td>15.24</td>
<td>7.62</td>
</tr>
</tbody>
</table>

Each dataset (water droplet case) contains the 4 sensor combinations in table 6. Each sensor configuration adds the aerosol MTFs as generic MTF arrays (i.e. range
loop) before each optics definition for all range permutations. This satisfies the range, wavelength, and aperture dependence of aerosol MTFs incorporated into NVIPM to generate range predictions. This process applies to all the 4 datasets thereby covering the water droplet combinations necessary for the range acquisition analysis.

The data in table 6 ensures that $F\lambda/d$ never falls below 2 guaranteeing the sensor is well sampled for both wavebands. The consequence is the focal length is different for each sensor configuration. This means the FOV is different for each sensor but this is the tradeoff for ensuring no sampling artifacts are part of the sensor system.

In order to analyze the impacts on PID from the aerosol MTF, a baseline needs to be established. Test case 1 in table 4 shows the case with no aerosol MTF contributions. Figure 29 illustrates the PID results for MWIR and LWIR sensors with 3” and 6” apertures for test case 1.

![Figure 29: PID predictions with no aerosol MTF](image)
The MWIR and LWIR acquisition ranges for PID in figure 29 are baselines to quantify the impacts of scattering and absorption in association with the aerosol MTF. This reaffirms the known facts about MWIR having better long range identification predictions. Previous research shows that when $F\lambda/d = 2$, a transparent atmosphere range is proportional to $D/\lambda$. While Figure 29 contain $\tau = 0.85$/km, at PID = 0.5 the curves follow $D/\lambda$ dependency [14].

The next detail to investigate is adding in the aerosol MTF from the SAA model (equation 15a and 15b) to gain insight on scattering only and absorption only cases. A characteristic value of 0.2km$^{-1}$ is assumed for the absorption coefficient. The scattering coefficient assumption is 0.2km$^{-1}$ for the 3in aperture size and 0.3km$^{-1}$ for the 6in aperture size. The reason for this is the scattering coefficient in equations 15a and 15b characterize what is scattered into the aperture. As a result, the change from 0.2km$^{-1}$ to 0.3km$^{-1}$ are a characteristic values to help illustrate how an increase in aperture size can affect the aerosol MTF and range acquisition predictions. The cutoff frequencies are set 0.5, 0.3, 0.20, and 0.12 cyc/mrad for 3in MWIR, 6in MWIR, 3in LWIR, and 6in LWIR respectively. These are all characteristic values that align with the Mie theory aerosol MTF predictions. The cutoff frequencies decrease as the aperture size increases. This reflects the point that an aperture size increase allows for greater observable scattering blur. Additionally, the cutoff is larger for MWIR since it is inversely proportional to wavelength [23-24]. Figure 30 details the scattering and absorption only PID for the SAA model. The absorption only case has the scattering coefficient set to zero and the absorption coefficient set to its characteristic value. The scattering only case has the
absorption coefficient set to zero and the scattering coefficient set to its characteristic values.

Figure 30: SAA aerosol MTF for PID range predictions, (a) absorption only case $\alpha=0.2\text{km}^{-1}$ and $\gamma=0$, (b) scattering only case $\alpha=0$, $\gamma=0.2\text{km}^{-1}$ for the small aperture, and $\gamma=0.3\text{km}^{-1}$ for the large aperture.  

Figure 30a shows that absorption is significant for PID predictions. Figure 30a is similar to figure 29 in the respect that MWIR outperforms LWIR. The additional absorption compresses the range predictions but the trends are similar to figure 29. The spatial frequency independence of absorption is responsible for the compression of the range predictions. Figure 30b portrays the range predictions for a scattering only case. The maximum ranges for a non-zero probability are greater than absorption. This shows that absorption is a stronger effect in comparison to scattering. The interesting trend in figure 30b is the cross over between the different aperture sizes. The larger aperture results in a greater reduction in range performance. More scattered light is collected with the larger aperture.
The water droplet compositions for test cases 2, 3, and 4 in table 4 provide the aerosol details that generate the aerosol MTFs. Figures 31, 32, and 33 illustrate the range performance predictions for MWIR and LWIR as well as 3in and 6in aperture sizes in these 3 water droplet conditions.

Figure 31: PID range predictions with aerosol MTF for test case 2 in table 4

Figure 31 shows the how absorption and scattering of the aerosol MTF influence range performance predictions. The same trends with MWIR out performing LWIR are observable here just as in figure 29. This shows that the diffraction advantage of MWIR is superior to LWIR even with the droplet conditions identified in table 4 for case 2. The absorption effects are significant resulting in dramatic differences seen in PID ranges compared to the data in figure 29. The larger aperture MWIR case shows that the greater observable scattered irradiance reduces the identification performance at long
ranges. The result is a cross over in PID between the larger MWIR and smaller MWIR aperture configurations. This effect is not readily observable in LWIR given that absorption by water droplets is greater in this band.

Figure 32: PID range predictions with aerosol MTF for test case 3 in table 4

Figure 32 shows a greater overall reduction in the range performance predictions in comparison to figure 31. This is expected since the concentration of droplets is greater in this condition. The greater absorption is mostly responsible for this result. The increase in scattering for the MWIR band shows that the larger aperture has a greater reduction in range performance. The diffraction advantage with aperture and wavelength are now showing less difference demonstrating the degradation impacts by the aerosol MTF.
Figure 33: PID range predictions with aerosol MTF for test case 4 in table 4

Figure 33 details the case with the largest concentration of water droplets. Absorption is clearly a significant effect under this condition. Range performances show that the PID data is converging for all cases (aperture size and wavelength). Only in the MWIR large aperture case, where scattering has the largest effect, does the PID curve show noticeable trends that include more than just absorption.

**Aerosol MTF Target Identification Discussion**

Figures 29-33 detail how the aerosol MTF can affect range performance for suspended water droplets. This is just a few cases among a much larger set of cases that can exist with arbitrary atmospheric conditions. Despite this fact, the choice of a monodisperse aerosols with spherically symmetric water droplets distributed in an
isotropic manner does approximate real world cases. Such conditions arise giving the corona effect that is observable around the sun and moon through thin layers of light mist or fog [15]. Molecular extinction is also part of the configuration with a broad band Beer’s law of $0.85 \text{km}^{-1}$.

The previous section details the raw data for the range predictions. The next step is to analyze the comparison of the MWIR and LWIR data with different aperture sizes and droplet conditions. Figure 34 shows the range predictions for a 50% probability of identification under the test conditions in table 4.

![Figure 34: Range predictions for 50% probability of identification](image)

Figure 34 shows case 1 has the best range performance predictions. This is the case of NVIPM predictions with no aerosol MTF defined in the system. The diffraction
advantage of MWIR systems is clearly evident in this case. Case 2 shows that absorption drastically affects range performance. Also, the large aperture MWIR and LWIR circumstances have greater reduction due to more scattering contributions collected by the sensor. The scattering effect is considerably greater for MWIR whereas absorption effects are greater in LWIR. This is a consequence of the complex refractive index of water [46]. Greater scattering is proportional to the larger real component of the refractive index whereas absorption is proportional to the complex component.

Cases 3 and 4 show the same trends as case 1. The reduction in MWIR range predictions is greater than LWIR as the concentration of droplets increases. The increasing concentrations intensify the absorption and scattering effects with the aerosol MTF. Case 4 demonstrates that range performances are showing trends towards convergence. The increase in concentration of the water droplets significantly reduces the advantages of smaller wavelengths and a larger aperture. This leads to the state that MWIR shows no substantial advantage over LWIR.

Another point of analysis to consider is the 50% probability of identification range ratio of MWIR to LWIR. Figure 35 portrays the relative ranges of MWIR to LWIR for the 2 aperture and 4 water droplet compositions.
Figure 35: MWIR to LWIR relative ranges for 50% probability of identification for test cases in table 4

Figure 35 signifies how the MWIR to LWIR range performance degrades with the water droplet test cases. The relative ranges show that the MWIR does outperform LWIR but the aerosol MTF degrades MWIR performance more than LWIR as the concentration of droplets increases. The smaller aperture relative range is always showing a larger ratio than the larger aperture since the MWIR large aperture circumstance collects the most scattered light. In other words, the TTP metric (equation 16c) for the integration of the MTF has a smaller integrated area for the larger aperture MWIR case that reduce range predictions. In addition to the concentration trends, the size of the droplets also affects the range predictions. The larger droplet size can increase forward scattering and affect the large aperture case more than the small aperture. This is evident in case 2 of figure 35. Cases 3 and 4 have smaller droplet
sizes but the concentration increases. A monotonic decrease in range ratio shows that MWIR performance is degrading faster than LWIR. This illustrates that increasing concentrations will reduce range performance to a point where MWIR will show little to no advantage over LWIR.

The overall trends show that MWIR does outperform LWIR in target identification for a well sampled, narrow FOV system. This is clearly evident when only molecular Beer's law of transmission is present. When the aerosol MTF is incorporated into the system MTF, MWIR shows more variability in the range performance compared to LWIR. The greater scattering in MWIR bands and the aperture dependence gives rise to greater variability in MWIR range predictions. Absorption is nearly constant for all spatial frequencies and the predominance of absorption in LWIR shows trends that have less variability.

The results in this analysis are within the assumptions made earlier. Previous research shows the importance behind the assumptions and how they can affect the results. In addition to these results here, the works by others reaffirms that the conclusions are dependent upon the sensor specification and the aerosol composition. Though the focus is on the aerosol MTF exclusively, it is important in this context of this chapter since the shape of the system MTF will alter the results on PID. The conclusion by Eisman and LeMasters [42] that the aerosol MTF is more of a radiometric effect is similar to figure 30a. The constant MTF response for all spatial frequencies simply reduces range predictions while keeping the trends largely unchanged. Kopeika and Sadot [33] conclude that the aerosol MTF is measurable in smaller optical depths agree
with the PID curves in figure 31 and 32. The instrumentation constraints affect the aerosol MTF prediction thereby showing new trends in the PID range predictions. Though this is in contrast to the findings of Bissonette [32], it demonstrates how the assumptions change the dynamic of the results. The results in this study show that the aerosol MTF effect is dependent upon the aerosols and system hardware. The FOV, aperture size, optical depth, concentration, and particulate size will change the amount of detected radiation thereby affecting PID performance.

**Aerosol MTF Target Identification Conclusion**

The aerosol MTF and its importance on imaging performance are difficult to quantify. Research shows that the most important factors are the assumptions within the experiment. The fact that there is a lack of convergence on previous research is predominantly due to the constraints and scope of analysis. In this chapter, the focus is on monodisperse, isotropic, water droplet aerosols to induce scattering and absorption along the propagation path. The sensor configurations assume a well sampled system (\( F\lambda/d = 2 \)) with a narrow field of view for 4 sensor configurations and no noise sources other than shot noise. The 4 configurations are MWIR and LWIR sensors each with a 3in and 6in aperture size. These are the assumptions for the PID predictions incorporated into NVIPM.

The aerosol MTF predictions are for 4 different radii and concentrations as a function of range, wavelength, and aperture size of the sensor. Mie theory predicts the scattered and transmitted irradiance at the aperture of the sensor. The solution is a
sequential method where the scattered and transmitted irradiance predictions propagate to the sensor aperture determining the total irradiance distribution. The total irradiance is Fourier transformed to compute the aerosol MTF. This aerosol MTF is incorporated into NVIPM as a function of range to give PID predictions. This is done for MWIR and LWIR central wavelengths as well as for the 3in and 6in aperture specifications.

The results show that absorption is a significant effect. The water droplet aerosol attenuates both MWIR and LWIR bands significantly thereby reducing PID ranges. Absorption is more apparent in LWIR bands specifically for water, given its complex refractive index properties, and the PID data reflects these trends. LWIR bands generally scatter less causing the range predictions to follow similar trends for all water droplet permutations. On the other hand, scattering effects are more apparent in MWIR bands. The greater scattering effects in the MWIR configurations show that the aperture size and droplet composition affect the PID range performance. This leads to the conclusion that MWIR systems do exhibit greater degradation with increasing water droplets. However, the diffraction advantage with the smaller wavelength in MWIR systems is not overcome by the aerosol MTF. Only when the concentration and size are sufficiently high do MWIR and LWIR bands show converging range performance. In this limit, MWIR no longer has a clear advantage over LWIR.
CHAPTER FIVE: GENERAL CONCLUSIONS

Imaging performance of a sensor is dependent upon many variables. They pose a challenge for both designing and analyzing a sensor that is set to solve established requirements. In the domain of IR imaging, the choice of sensors predominately comes down to either a MWIR or a LWIR system. The motivation behind this dissertation is to study a focused set of conditions to help identify advantages and disadvantages of MWIR and LWIR imaging systems. The focus includes PVF analysis for IRST performance, MWIR and LWIR aerosol MTF for a monodisperse medium, and aerosol MTF impacts for MWIR and LWIR systems concerning targeting performance.

The variables of focus for the PVF analysis includes $F\lambda/d_{CC}$, fill factor, relative location of object on focal plane, target size, and non-diffraction limited systems. The conclusions show that the average PVF is optimum for a fill factor equal to 1, $F\lambda/d_{CC}$ is 1.1, and a diffraction limited system. The $F\lambda/d_{CC}$ figure of merit permits additional conclusions for a nominal detector size of 10μm. This requires the F number to be 2.75 for MWIR and 1.1 for LWIR systems. In the context of analyzing just the PVF impacts, the MWIR system follows more traditional system design than the LWIR system for the optimum PVF. Despite these conclusions, full IRST optimization requires all the variables in equation 1 must be considered. The PVF was the focus for this MWIR and LWIR study.

The other focus for MWIR and LWIR comparisons is the aerosol MTF. Aerosols can be very complex leading to seemingly endless variables to research. Reducing the variables is essential to understand fundamental aerosol effects on resolution. As a
result, the attention assumes a monodisperse medium comprised of spherical water droplets. The droplet distribution is isotropic but varies in concentration and size resulting in a medium that is commensurate with a tenuous fog or haze. In addition, the size of the aperture changes to analyze the impacts on MWIR and LWIR bands. The results show that the MWIR aerosol MTF demonstrates greater variability as a function of changes to concentration and particulate size. The greater scattering effects in MWIR account for this difference in comparison to LWIR. In addition, changes in aperture show a significant impact on both MWIR and LWIR aerosol MTFs. Overall, the MWIR aerosol MTF shows better performance than the LWIR with the exception for large concentrations and large aperture sizes. The results show similar performance suggesting no clear advantage in these limits.

The last focus for MWIR and LWIR comparison is to factor in the aerosol MTF for target identification. This includes an array of aerosol MTFs as a function of range, concentration, and size for a monodisperse water droplet medium. The aerosol MTF is added into NVIPM to perform range predictions for target identification. The conclusions show that MWIR range performance is superior to LWIR performance. Despite the advantage, increasing concentration and larger aperture sizes permit more scattering effects for MWIR systems thereby reducing identification ranges more rapidly than LWIR systems. Only in the limit of increasing concentrations and aperture size does the MWIR performance approach that of the LWIR performance. Utilization of NVIPM incorporating aerosol MTFs demonstrate a measure of resolution that acquisition range decreases as aerosol scattering and absorption increases. However, the MTFs are
applicable to all imaging systems. For the infrared region, this includes long range thermography, remote sensing, and satellite imaging.

**Future Research**

As chapters 3 and 4 demonstrate, the aerosol MTF is a complicated metric even with the aforementioned assumptions in place. Future research that incorporates generic aerosol constituents with different material properties, size, and concentration distributions permits a broader scope of analysis. These additions allow analysis into scattering and absorption effects for aerosols such as smoke, soot, sea spray, and other particulates at various concentrations and sizes. As a result, a more generic analysis of the aerosol MTF can be performed while maintaining deterministic results.

Typically, aerosol MTF predictions and measurements only account for the light from the target being imaged. Any background illumination, such as the sun, can vary significantly as a function of angle with respect to the viewing direction. The spatial distribution of the background light can alter the aerosol MTF such that it is not symmetric across the aperture or field of view. This is an important area for future work since solar illumination is always present and more accurately accounts for real world imaging situations.

In addition to improvements to the numerical model, aerosol MTF measurements can be performed with duel band sensors. This is a future research opportunity where real world atmospheric measurements can be made to analyze MWIR vs LWIR aerosol MTF performance as well as compare to numerical predictions.
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   https://doi.org/10.1117/1.601130


28. Yitzhak Yitzhaky, Itai Dror, Norman S. Kopeika, "Restoration of atmospherically blurred images according to weather-predicted atmospheric modulation transfer functions," Optical Engineering 36(11), (1 November 1997). http://dx.doi.org/10.1117/1.601526


https://doi.org/10.1117/1.OE.54.3.033101


42. Michael T. Eismann and Daniel A. LeMaster, "Aerosol modulation transfer function model for passive long-range imaging over a nonuniform atmospheric path," Optical Engineering 52(4), 046201 (10 April 2013).
   http://dx.doi.org/10.1117/1.OE.52.4.046201


50. **Luc R. Bissonnette** "Imaging through fog and rain," *Optical Engineering* **31**(5), (1 May 1992). [https://doi.org/10.1117/12.56145](https://doi.org/10.1117/12.56145)


56. John D. O’Conner, Ronald G. Driggers, Richard H. Vollmerhausen, Nicole Devitt, Jeff Olson, "Fifty-percent probability of identification ($N_{50}$) comparison for targets in the visible and infrared spectral bands," Optical Engineering 42(10), (1 October 2003). http://dx.doi.org/10.1117/1.1607331


