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Properties of a Dynamic Measure of Industrial Concentration

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PROPERTIES OF A DYNAMIC MEASURE OF INDUSTRIAL CONCENTRATION

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PREFACE

Chapter I of this paper presents a review of several commonly used concentration measures. Chapter II focuses on the Grossack Model of Permanent Industrial Concentration and explains it in depth. Chapter III and Appendix A explain the mathematical methodology and statistical tests used to evaluate the Grossack Model Indices. Chapter IV and V present the results and conclusions of the statistical tests. Appendix B contains information on, and a source listing of, the computer program used to perform the statistical tests and Print Tables 3-26.

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ABSTRACT

Several commonly used concentration measures are defined and discussed. The Grossack Model of Permanent Industrial Concentration and its three concentration indices are explained. Monte Carlo Techniques are used to generate synthetic data which is then used to compute theoretical values of the Grossack Model Indices. Values of other commonly used concentration indices are computed for comparison purposes. The Grossack Model Indices are then subjected to a variety of statistical tests. The results of these tests are presented and evaluated and a conclusion is drawn.

I. REVIEW OF COMMON CONCENTRATION INDICES

The concept of concentration is concerned with the relative distribution of sizes of firms within and between industries and is, generally speaking, closely linked to the level of competitive activity in an industry. If Industry A consists of one firm with a 50% market share and two firms each with 25% market shares, and Industry B consists of one 60% firm and two 20% firms, then the question arises, "Which industry is more concentrated?" To this end, various mathematical measures or concentration indices have been developed by econometricians in order to provide an answer to questions of this type.

The simplest and easiest to understand of the concentration indices is the concentration ratio which is merely the sum of the market shares of the k largest firms in an industry, where the firm's rank and market share are determined according to some given criteria such as assets, sales, output, profits, employment, and so on. That is,

$$CR_k = \sum_{i=1}^{k} X_i \tag{1-1}$$

where X_i is the market share of the i'th largest firm of the industry in question. The most commonly used concentration ratios are those for four, eight, and twenty firms (CR_4, CR_8, CR_{20}). Concentration ratios are computed and made available by the Bureau of the Census at regular intervals.

Concentration ratios have the disadvantage that they do not reflect monopolistic tendencies within an industry. As can be seen

from (1-1), if a very large firm absorbs a very small firm, the resultant change in CR_k will be very small. The limitations of CR_k as a concentration index were explained by Marshall Hall and Nicolaus Tideman [1]. Hall and Tideman identified six properties a "good" concentration index should possess:

(1) The index should be unambiguous and one dimensional. It should be obvious to the user of the statistic whether one industry is more, less, or equally concentrated than another.

(2) Concentration is independent of the actual size of the industry. Rather, concentration should be a function of the relative shares of the firms in the industry. M. A. Adelman [2, p. 100] detailed another difficulty relating to the actual size of an industry when he stated that "n, the number of firms, is inherently unknowable and is subject to large arbitary variation." Thus, if the number of firms in an industry is unknown, the total size of the industry is also an unknown.

(3) The concentration index should change if there is a change in the relative rankings of firms within the industry. It should increase if there is a shift of market share from a small firm to a large firm and it should decrease for a shift in the opposite direction.

(4) If each firm in an industry is split into n equal sized firms, the concentration index should be reduced by a factor of $\frac{1}{n}$.

(5) If an industry consists of n equal sized firms, the concentration index should be a decreasing function of n.

(6) A concentration index should be scaled to range from 0 to 1.

The concentration ratio, CR_k, does not satisfy properties 2, 3, and 4, because, first of all, CR_k deals only with k firms at a time, ignoring the rest, and secondly, because it weights each firm equally.

Another commonly used index, developed by O. C. Herfindahl in 1950 [3], satisfies all six of the above properties. In the

Herfindahl Index each firm is weighted [Equation (1-2)] by its relative share of the total market. Thus, the relative sizes of firms are more important than the absolute number of firms. The formula for the Herfindahl Index is:

$$H = \sum_{i=1}^{n} X_i^2 \tag{1-2}$$

where X_i is as in (1-1) and n is the total number of firms in an industry. Hall and Tideman [1] criticize the Herfindahl Index on the grounds that it deemphasizes the importance of the total number of firms, and they feel that the number of firms in an industry is indicative of entry requirements for that industry. They argue that a large number of firms in an industry indicates easy entry, while a small number of firms indicates that entry is difficult. However, it should be pointed out that, in reality, the ease with which a firm enters an industry is determined more by such factors as capital requirements, management abilities and experience, and the potential ability of the new firm to succeed. The number of firms in an industry is important only inasmuch as it reflects these factors. Similarly, the idea that a large number of firms in an industry indicates easy entry is not necessarily true. Consider the computer industry, where competition is fierce, despite the fact that one firm (IBM) has an overwhelmingly large market share. The cost of entering the industry is extremely high, yet there is a large number of firms in the industry.

However, the ability of firms in a given industry to prevent other firms from entering the industry does affect the ease with which new firms can enter that industry. Hall and Tideman [1] developed a concentration index which weighted each firm by its rank in the industry, thereby emphasizing the absolute number of firms in an industry:

$$HLT = 1/\left[2\sum_{i=1}^{n} iX_i - 1\right] \quad (1\text{-}3)$$

The Hall-Tideman Index, of course, satisfies all six properties.

Michael O. Finkelstein and Richard M. Friedburg [4] developed another concentration measure, called the entropy measure because it resembles the entropy measure of the kinetic theory of gases in Physics. Finkelstein and Friedburg conjectured that if the degree of competitive activity of k equal sized firms is f(k), then the contribution of each firm is $\frac{1}{k}$ f(k). They then assume that a few large firms constitute a greater competitive threat than a greater number of proportionately smaller firms. Based on this assumption, they concluded [4, p. 691] that "The contribution each firm makes to the total degree of competitive activity will be equal to the contribution that firm would make in a market otherwise identical but composed solely of firms equal to it in size." Denoting the entropy measure of concentration by C, it follows that:

$$f(C) = \frac{1}{n_1} f(n_1) + \frac{1}{n_2} f(n_2) + \ldots + \frac{1}{n_k} f(n_k) \quad (1\text{-}4)$$

where $(1/n_i)$ is the market share of the i'th firm. If all firms are of equal size, then

$$f(C) = f(k) \quad (1\text{-}5)$$

$$C = k. \quad (1\text{-}6)$$

Finkelstein and Friedburg chose a simple log function for f because it has the property of increasing without bound at a diminishing rate (that is, it assumes a slow decrease in competitive activity as the number of firms in an industry declines). This gives

$$\log C = \frac{1}{n_1} \log n_1 + \frac{1}{n_2} \log n_2 + \ldots + \frac{1}{n_k} \log n_k \quad (1\text{-}7)$$

$$C = n_1^{1/n_1}\, n_2^{1/n_2} \ldots n_k^{1/n_k} = \prod_{i=1}^{k} n_i^{1/n_i} \quad (1\text{-}8)$$

In terms of market shares, this becomes:

$$C = \prod_{i=1}^{k} X_i^{-X_i} \tag{1-9}$$

The entropy measure is a kind of "Numbers-Equivalent." Finkelstein and Friedburg state [4, p. 691] that: "We say that two markets are equivalent if they have the same total competitive activity. Given any market we seek to determine an equivalent equal-firm market. The entropy measure, which we denote by C, is the number of firms in this equivalent market."

M. A. Adelman [2] commented on using the inverse of the Herfindahl Index as a numbers-equivalent. He expressed the Herfindahl Index in terms of the mean and variance of the sample of firms, as follows:

$$n\sigma^2 = H-(1/n) \tag{1-10}$$

Where H is the Herfindahl Index for a particular sample, n the number of firms in the sample, and σ^2 the variance of the sample. If the industry in question consists of equal sized firms, the variance is zero and (1-10) reduces to:

$$N = 1/H \tag{1-11}$$

Where N is a numbers-equivalent, the number of equal sized firms necessary in an industry to generate the specific index of concentration.

Janos Horvath [5] came up with another concentration index called the "Comprehensive Concentration Index," which incorporates both relative and absolute market shares. The index is defined as:

$$CCI = X_1 + \sum_{j=2}^{n} (X_j)^2 \ (2 - X_j) \tag{1-12}$$

Where X_1 is the net market share of the largest firm in the industry and X_j is the net market share of the j'th largest firm in the industry. Hence, the largest firm is given a weight of unity. The remaining

firms are weighted according to the market share they do not have. The CCI Index differs from the H Index in that the CCI treats the largest firm independently and adds the weight factor of $(2-X_j)$ for the other firms.

The concentration indices discussed thus far have two things in common: First: they are all static measures, which cannot indicate the degree of monopoly in an industry since the monopoly power of large firms depends on their ability to inhibit the entry and growth of small firms. However, this can only be measured over time, and static measures of concentration cannot incorporate the time dimension. Second: the indices presented concern themselves only with observed market shares. Irvin M. Grossack [6] proposed three "permanent" concentration indices based on a dynamic model which divides the observed market shares into permanent and transitory components. Professor Grossack then develops estimates of his indices based on observed market shares from two time periods.

II. THE GROSSACK MODEL AND INDICES

Irvin M. Grossack [6] developed three dynamic measures of concentration which attempt to measure the ability of large firms in an industry to inhibit the entry and growth of small firms. Since this power must be measured over time, only a dynamic measure can incorporate this power. Professor Grossack divides the variables which determine a large firm's power into two components: a permanent and a transitory component. The author then goes on to point out that it is possible to isolate the permanent and transitory components in some "base" year only with respect to some "reference" year.

Professor Grossack develops his model with the following definitions, assumptions, and restrictions. Designating x_{i0} and x_{it} as deviations of the i'th firm's market shares from the mean (for years 0 and t), the first assumption is that the total deviations from the mean are equal to the sum of the permanent and transitory deviations from their means. That is,

$$x_{i0} = x_{i0p} + x_{i0s} \qquad (2\text{-}1)$$

$$x_{it} = x_{itp} + x_{its}$$

where the p and s subscripts denote the permanent and transitory components. The next assumption is that the permanent components in years 0 and t are perfectly correlated, and, correspondingly, the transitory components in years 0 and t are perfectly uncorrelated. In other words,

$$r(x_{i0p}, x_{itp}) = 1 \qquad (2\text{-}2)$$

$$r(x_{i0s}, x_{its}) = 0 \tag{2-3}$$

The author then assumes that the permanent and transitory components in any particular year are perfectly unrelated. This gives us

$$r(x_{i0p}, x_{i0s}) = 0 \tag{2-4}$$

$$r(x_{itp}, x_{its}) = 0 \tag{2-5}$$

which implies that

$$r(x_{i0p}, x_{its}) = 0 \tag{2-6}$$

$$r(x_{itp}, x_{i0s}) = 0 \tag{2-7}$$

The last two assumptions [(2-6) and (2-7)] are especially unrealistic. If a large firm were to gain a transitory advantage it would most likely attempt to make it a permanent one. Similarly, if a small firm gained an advantage, the larger firm would attempt to take it from them adding it to their store of permanent components. Thus, the permanent components of year t would probably not be perfectly unrelated to the transitory components of year 0, as assumption (2-7) suggests. Also, since even "permanent" advantages can and do change over time, it is unlikely that the permanent components of year 0 are perfectly unrelated to the transitory components of year t, as (2-6) implies.

Grossack defines a Herfindahl Index as derived by Adelman and with the sum of squared deviations term modified as the 'true' index of permanent concentration.

$$HT\ (0) = \sum_{i=1}^{n} x_{i0p}^{2} + \frac{1}{n} \tag{2-8}$$

where HT (0) is the true index for year 0. The corresponding index for year t would be:

$$HT\ (t) = \sum_{i=1}^{n} x_{itp}^{2} + \frac{1}{n} \tag{2-9}$$

Since it is impossible to directly measure x_{i0p} and x_{itp}, Professor Grossack derives approximations for HT based on two different assumptions. The first assumption is a proportionality assumption which states that the ratios of the variances of the permanent components to the observed data variances are the same for both years. Based on this assumption, the author derives

$$\Sigma x^2_{i0p} = r\Sigma x^2_{i0} \tag{2-10}$$

where r is the correlation coefficient of the observed data (x_{i0} on x_{it}). Substituting (2-10) into (2-8) gives

$$HT = HP = r\sum_{i=1}^{n} x^2_{i0} + \frac{1}{n} \tag{2-11}$$

as an index of permanent industrial concentration, where HP refers to the value of HT under the proportionality assumption. The second assumption is an equality assumption which states that the permanent advantages have the same absolute impact on the permanent components of every firm's market shares in both years. The author derives

$$\Sigma x^2_{i0p} = b\ \Sigma x^2_{i0} \tag{2-12}$$

where b is the regression coefficient of the observed data between the two years. Substituting (2-12) into (2-8) gives

$$HT = HE = b\sum_{i=1}^{n} x^2_{i0} + \frac{1}{n} \tag{2-13}$$

as another index of permanent industrial concentration. HE refers to the value of HT under the equality assumption.

Despite the fact that the model's assumptions are very restrictive and somewhat unrealistic, the Grossack Model obviously goes to the heart of the problem of monopoly power and represents an important theoretical advance. The way of the future appears to

be in the direction of a dynamic statistic as a measure of industrial concentration. There is an urgent need, especially in the anti-trust courts, for a statistic suitable for measuring monopoly power and concentration. Professor Grossack does relax some of the assumptions of the model. However, he asserts that relaxing the assumptions does little to change the concentration indices (HP and HE). Such an assertion needs to be tested empirically before any claim can be made.

This research is an attempt to subject HP and HE to statistical testing when assumptions (2-6) and (2-7) are relaxed and to draw a conclusion based on the results. This will be done when x_{i0} and x_{it} are computed by two different methods. The first method, referred to as the addition method, is to compute x_{i0} and x_{it} according to equation (2-1). The second method is referred to as the multiplication method, wherein x_{i0} and x_{it} are computed by

$$x_{i0} = (x_{i0p})(x_{i0s}) \qquad (2\text{-}14)$$

$$x_{it} = (x_{itp})(x_{its})$$

Monte Carlo Techniques are used to generate synthetic values of x_{i0p}, x_{i0s}, x_{its}, and x_{its} for certain specific values of λ, where either $\lambda = r(x_{i0p}, x_{its})$ or $\lambda = r(x_{itp}, x_{i0s})$. The values of λ used are ± 1, ± 0.9, ± 0.7, ± 0.5, ± 0.3, 0. Values for x_{i0} and x_{it} are then obtained by both the addition and multiplication methods. Values of HT, HP, and HE are then computed and subjected to appropriate statistical tests.

III. MATHEMATICS

In order to test the properties of the Grossack Model Indices when assumptions (2-6) and (2-7) are violated, sample values for x_{i0p}, x_{i0s}, x_{itp}, and x_{its} with specified correlations must be generated. The most effective way to generate the required values is to use Monte Carlo techniques to generate synthetic values for X_{i0p}, X_{i0s}, X_{itp}, and X_{its} with specified correlations where

$$\left.\begin{aligned} x_{i0p} &= X_{i0p} - \bar{X}_{i0p} \\ x_{i0s} &= X_{i0s} - \bar{X}_{i0s} \\ x_{itp} &= X_{itp} - \bar{X}_{itp} \\ x_{its} &= X_{its} - \bar{X}_{its} \end{aligned}\right\} \quad i = 1, 2, \ldots, n \qquad (3\text{-}1)$$

and X_{i0p}, X_{i0s}, X_{itp}, X_{its} are the permanent and transitory components of a firm's market share. Since the X's (X_{i0p}, X_{i0s}, X_{itp}, X_{its}) represent market shares, they are subject to the following restrictions, imposed by the structure of the market:

$$0 \leq X_{i0p} \leq 1 \qquad 0 \leq X_{itp} \leq 1 \qquad (3\text{-}2)$$

$$0 \leq X_{i0s} \leq 1 \qquad 0 \leq X_{its} \leq 1$$

$$\sum_{i=1}^{n} X_{i0} = \sum_{i=1}^{n} (X_{i0p} + X_{i0s}) = 1 \qquad (3\text{-}3)$$

$$\sum_{i=1}^{n} X_{it} = \sum_{i=1}^{n} (X_{itp} + X_{its}) = 1$$

Since variables generated by Monte Carlo techniques will not automatically conform to restrictions (3-2) and (3-3) above, it is

necessary to transform or normalize the variables so that they meet the specifications.* After the variables are normalized, they are converted to deviations from means using (3-1) and are then used to compute the indices of the Grossack model. The indices are then subjected to a variety of statistical tests.

The first step in generating the required variables is to compute M, the covariance matrix of the X^a vector where

$$X^a = \begin{bmatrix} X_{i0p} \\ X_{i0s} \\ X_{itp} \\ X_{its} . \end{bmatrix} \tag{3-4}$$

Arbitrary values for the variances are chosen based on data from the steel and tobacco industries. The values used for the variances are given in Table 1. The resultant covariances are then computed using the given values of the variances and correlations:

$$\sigma_{ab} = \lambda_{ab} \sigma_a \sigma_b \tag{3-5}$$

where $\lambda_{ab} = r_{ab} = r(a, b)$ = correlation between a and b. This is done for the following correlations:**

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* The data normalization procedure is explained in detail in Appendix A.

** Note that: $r(X_a, X_b) = r(x_a, x_b)$

where $x_{ai} = X_{ai} - \bar{X}_a$

$x_{bi} = X_{bi} - \bar{X}_b$.

For proof, refer to Appendix A.

Correlation	Equivalent Correlation	Grossack Model Value	Value used in Computing M	
$r(X_{i0s}, X_{i0p})$	$r(x_{i0s}, x_{i0p})$	0	(A) 0	(B) λ
$r(X_{itp}, X_{i0p})$	$r(x_{itp}, x_{i0p})$	1	1	1
$r(X_{its}, X_{i0p})$	$r(x_{its}, x_{i0p})$	0	λ	0
$r(X_{itp}, X_{i0s})$	$r(x_{itp}, x_{i0s})$	0	0	λ
$r(X_{its}, X_{i0s})$	$r(x_{its}, x_{i0s})$	0	0	0
$r(X_{its}, X_{itp})$	$r(x_{its}, x_{itp})$	0	λ	0

where λ is a scalar which assumes the values $\lambda = 0, \pm 0.3, \pm 0.5, \pm 0.7, \pm 0.9, \pm 1$. Only one set of correlation values is used at a time (A or B above) to compute M. It should be noted that the effect of using value set (A) above instead of the Grossack Model values is to violate assumption (2-6), while (B) violates assumption (2-7).

Once the M matrix is computed, the next step is to compute the lower-triangular factorization of the M matrix: $M = PP^T$ where P is lower triangular. As M is symmetric, P^T is computed by the square root method of Cholesky [7].

The elements of $Q = P^T$ (nxn) are given by:

$$q_{ii} = (m_{ii})^{1/2} \tag{3-6}$$

$$q_{1j} = m_{1j}/q_{11} \text{ for } j = 2, \dots, n \tag{3-7}$$

$$q_{ii} = (m_{ii} - \sum_{k=1}^{i-1} q_{ki}^2)^{1/2} \quad \text{for } j = 2, \dots, n \tag{3-8}$$

$$q_{ij} = \frac{1}{q_{ii}} (m_{ij} - \sum_{k=1}^{i-1} q_{ki}\, q_{kj}) \text{ for } j = i+1, \dots, n \text{ and } i = 2, \dots, n. \tag{3-9}$$

Since M is a covariance matrix, M is positive definite. Thus, except when M is ill-conditioned, the q_{ii} are always real numbers. It is possible for the Cholesky method to "fail" (that is, one or more q_{ii} and q_{ij} are complex numbers) if M is ill-conditioned or is not positive definite.

The next step is to compute the X matrix using the method of Neiswanger and Yancey [8]:

$$X = Q^T S = (P^T)^T S = PS \tag{3-10}$$

where S is a matrix of random normal deviates with mean 0 and variance of 1. This produces values for X_{i0p}, X_{i0s}, X_{itp}, and X_{its} based on correlations set (A) or (B). These values are then normalized to conform to the restrictions of equations (3-2) and (3-3). These restrictions are inherent to the system. For instance, no firm can have a negative market share, nor can a firm have a market share greater than 100%, hence (3-2). Similarly, the sum of the market share of all the firms in the market should equal the entire market, or 1, hence (3-3). The normalization procedure and its effects are explained in Appendix A. It should be noted that the normalization procedure preserves correlations but does not preserve variances or covariances.

Once the variables are normalized, the net market shares are computed for times 0 and t:

$$\begin{aligned} X_{i0} &= X_{i0p} + X_{i0s} \\ X_{it} &= X_{itp} + X_{its}. \end{aligned} \tag{3-11}$$

The values of X_{i0} and X_{it} are then sorted into descending order and the following commonly used concentration indices are computed for both X_{i0} and X_{it}:

$$CR_4 = \sum_{i=1}^{4} X_i \qquad \text{Concentration Ratio} \tag{3-12}$$

$$CR_8 = \sum_{i=1}^{8} X_i \qquad (3\text{-}13)$$

$$HF = \sum_{i=1}^{n} X_i^2 \qquad \text{Herfindahl Index} \qquad (3\text{-}14)$$

$$HLT = 1/(2\sum_{i=1}^{n} iX_i - 1) \qquad \text{Hall-Tideman Index} \qquad (3\text{-}15)$$

$$C = \prod_{i=1}^{n} (X_i)^{-X_i} \qquad \text{Entropy} \qquad (3\text{-}16)$$

$$CCI = X_i + \sum_{i=1}^{n} (X_i)^2 (2 - X_j) \qquad \text{Comprehensive Concentration Index} \qquad (3\text{-}17)$$

$$N = 1/H \qquad \text{Numbers-Equivalent} \qquad (3\text{-}18)$$

Mean values of each of the seven indices above for both times 0 and t are computed for each separate M matrix. These indices are presented in Tables 19-26.

Once the X's have been generated and normalized, the X's are converted to deviations from means (x's) using (3-1) to give a matrix of the x values: [x]. The covariance matrix of the [x] matrix is computed and a mean value is derived. The mean resultant variances of the normalized [x] matrices are given in Table 2.

Net deviations from mean market share are then computed by both of the following methods:

$$x_{i0} = x_{i0p} + x_{i0s} \qquad \text{Addition Method} \qquad (3\text{-}19)$$

$$x_{it} = x_{itp} + x_{its}$$

and

$$x_{i0} = (x_{i0p})(x_{i0s}) \qquad \text{Multiplication Method} \qquad (3\text{-}20)$$

$$x_{it} = (x_{itp})(x_{its}).$$

x_{i0} and x_{it} are computed by both the addition and multiplication methods using the same values for x_{i0p}, x_{i0s}, x_{itp}, x_{its}. The

resultant values of x_{i0} and x_{it} are then used to compute the Grossack Model Indices and are subjected to statistical tests.

The next step is to regress x_{it} on x_{i0}. The regression coefficient ($\bar{b}$), coefficient of correlation (r), and standard error of b (sb) are computed. The Grossack Model Indices are then computed as follows:

$$HT_0 = \sum_{i=1}^{n} x_{i0p}^2 + \frac{1}{n} \qquad (3\text{-}21)$$

True Permanent Herfindahl Index

$$HT_t = \sum_{i=1}^{n} x_{itp}^2 + \frac{1}{n} \qquad (3\text{-}22)$$

$$HP = r \sum_{i=1}^{n} x_{i0}^2 + \frac{1}{n} \qquad \text{Proportionality Model} \qquad (3\text{-}23)$$

$$HE = b \sum_{i=1}^{n} x_{i0}^2 + \frac{1}{n} \qquad \text{Equality Model.} \qquad (3\text{-}24)$$

The standard errors associated with HP and HE are also computed.

For each different value of λ (and resultant covariance matrix M), 50 values each of HP, HE, and associated standard errors of HP, HE are derived for both addition and multiplication methods. The mean values of HP, HE, standard errors, standard deviation, and root mean square errors of HP, HE are computed for both addition and multiplication methods. Mean HT_0, HT_t, HF_0, and HF_t are also computed for each value of λ. In addition, the estimated standard errors of each HP, HE are analyzed in the following ways:

(1) The number of times the estimated parameter values are greater than 2 standard errors ($HP > 2sHP$, $HE > 2sHE$).

(2) The number of times the absolute values of the estimated parameters minus the true parameters are greater than 2 standard errors ($|HP - HT| \geqslant 2sHP$, $|HE - HT| \geqslant 2sHE$). The above statistics are summarized and presented in Tables 3-18.

IV. THE ANALYSIS

Table 1 yields the variances used to generate the covariance matrix (M Matrix) used to produce the initial X Matrix, and Table 2 lists the mean resultant variances derived from the normallized [x] matrices. Note how the initial and resultant variances differ. This is a direct result of the scale factors used in the normalization procedure. The scale factors range from a low of approximately 10.6 to a high of 71.2 over the four matrices.

Tables 3-18 contain the summary statistical data for indices HP, HE, and HT. Tables 3 and 4 contain this data for Matrix I when $\lambda = r\,(x_{its}, x_{i0p})$. Tables 5 and 6 contain similar data for Matrix I with $\lambda = (x_{itp}, x_{i0s})$. The same pattern holds for Matrix II (Tables 7-10), Matrix III (Tables 11-14), and Matrix IV (Tables 15-18).

Tables 3, 4, 7, 8, 11, 12, 15, and 16 give the summary statistical data when $\lambda = r(x_{its}, x_{i0p})$ in Matrices I-IV. Examination of this data (for the addition method of computation) reveals trends for certain of the statistics. For the Standard Herfindahl Index computed at times 0 and t [HF (0) and HF (T) in the Tables], the trends were erratic. No trends were evident for HF (0) in any matrix, whereas HF (T) decreases as λ decreases ($\lambda \to -1$) in Matrices II-IV. HF (T) displays no trend in Matrix I. For the Permanent Herfindahl Index for times 0 and t [HT (0), HT (T) in the tables], no trends are evident in any matrix. However, HT (T) >HT (0) for all values of λ in all four matrices.

There are no trends for mean HP in Matrices I-III. In Matrix IV, though, mean HP tends to increase as λ approaches zero. Mean HE tends to decrease as λ decreases for all four matrices. No trends are evident for the standard deviation of HP, HE except for Matrix III, where the standard deviation of HE decreases as λ decreases. The trends for mean standard error were quite erratic. The mean standard error for HP: (a) increases as λ approaches zero in Matrices I and II, (b) displays no trends in Matrix III, and, (c) decreases as λ decreases in Matrix IV. In Matrix IV, the trend is somewhat irregular. The mean standard error of HE displays similar behavior except for Matrix II, where there is no trend. Note the small magnitude of the standard errors in relationship to the standard deviations and mean values of HP, HE. The root mean square error (RMSE) of HP, HE display no trends in Matrices I and II; whereas, in Matrices III and IV the trend is for RMSE to decrease as λ decreases, so long as $\lambda \leqslant 0$. For $\lambda > 0$, there is no trend. The values for mean HP, HE are consistently greater than two standard errors (50 out of 50-100%).

To simplify the analysis, let N_1 equal the number of times $|HP - HT|$ is greater than two standard errors. Let N_2 be the corresponding number for HE. Then it can be seen from the tables that N_1, N_2 decrease as λ approaches zero, in all matrices. Also note that N_1, N_2 are quite "large" even when $\lambda = 0$ (the minimum value of N_1, N_2 throughout all of the given charts is 33 = 66%).

Tables 5, 6, 9, 10, 13, 14, 17, and 18 give the summary statistical data for Matrices I-IV when $\lambda = r(x_{itp}, x_{i0s})$. Examination of these tables reveals certain trends. The standard Herfindahl Index computed for time 0 [HF (0)] tends to decrease as λ decreases in Matrices I and III, but HF (0) exhibits no trends in Matrices II and IV. HF (T) displays no trends in any of the four

matrices. The permanent Herfindahl Index - HT (0), HT (T) have no trends in any of the matrices. However, HT (T) >HT (0) consistently in all of the matrices. The mean values of HP, HE tend to decrease as λ decreases in Matrices I-IV except for HE in Matrix I, where no trend is evident. No trends are evident for the standard deviation of HP, HE in Matrices I and II. However, in Matrices III and IV, the standard deviations of HP, HE tend to decrease as λ decreases. The mean standard errors of HP and HE tend to increase as λ approaches zero in Matrices III and IV. No trends are evident for mean standard error in Matrices I and II. The standard errors are "small" in comparison with the standard deviations, as was observed when $\lambda = r(x_{its}, x_{i0p})$. The root mean square errors for HP and HE tend to decrease as λ approaches zero in Matrices I-IV except for HE in Matrix IV, where the trend is irregular.

For every value of λ in all four matrices, the values for mean HP and mean HE are consistently greater than two standard errors. N_1 and N_2 (where N_1 and N_2 are defined as before) tend to decrease as λ approaches zero in all matrices, as before. Similarly, N_1 and N_2 are both quite "large" when $\lambda = 0$ (minimum value of N_1, N_2 is 72%).

Examination of Tables 3-18 for the multiplication method reveals highly unusual results. What trends are evident are consistent in all four matrices, regardless of whether $\lambda = r(x_{its}, x_{i0p})$ or $\lambda = r(x_{itp}, x_{i0s})$.

In every table, mean HP = mean HE = 0.05 = 1/20. In other words, mean HP and mean HE are constant throughout the tables. This is even more noteworthy when one remembers that there are 20 firms in each sample.

The behavior of the root mean square error statistics is

somewhat similar. No trends are evident for the root mean square error of HP and HE, but RMSE HP = RMSE HE in every case.

Standard deviations and mean standard errors display no trends. However, it should be noted that both the standard deviations and mean standard errors are of extremely small magnitude (10^{-6} and 10^{-8}). Mean HP and HE are always greater than two standard errors. When N_1 and N_2 are as defined before, it can be seen that N_1 and N_2 are constantly equal to 50 (100%). In other words, $|HP - HT|$ and $|HE - HT|$ are greater than two standard errors in every case.

Tables 19-26 contain mean values of certain commonly used concentration indices computed for Matrices I-IV with various values of λ [$\lambda = r(x_{its}, x_{i0p})$ or $\lambda = r(x_{itp}, x_{i0s})$]. The values of the indices given are expected values computed from a set of 50 for each time point (0 and t) and each value of λ. The indices are fairly consistent and no major fluctuations are to be seen. Note the apparent correspondence between the entropy measure and the numbers-equivalent index.

TABLE 1.--VARIANCES USED IN GENERATING INITIAL X MATRICES

MATRIX	XIOP	XIOS	XITP	XITS
I	0.181584E-02	0.216776E-03	0.218776E-02	0.243084E-03
II	0.218776E-02	0.243084E-03	0.256500E-02	0.285000E-03
III	0.112099E-01	0.747325E-02	0.747327E-02	0.498217E-02
IV	0.747327E-02	0.498217E-02	0.560493E-02	0.373663E-02

TABLE 2.--MEAN RESULTANT VARIANCES (COMPUTED FROM NORMALIZED X MATRICES)

MATRIX	XIOP	XIOS	XITP	XITS
I	0.157156E-03	0.204794E-04	0.188422E-03	0.216044E-04
II	0.163817E-03	0.194922E-04	0.190378E-03	0.227200E-04
III	0.157363E-03	0.108229E-03	0.105109E-03	0.750839E-04
IV	0.158101E-03	0.107637E-03	0.118605E-03	0.830382E-04

TABLE 3.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONNEGATIVE CORRELATIONS BETWEEN XITS AND XIOP, ADDITION AND MULTIPLICATION METHODS (MULT. SHOWN IN PARENTHESES), MATRIX I.

LAMBDA=	1.0	0.9	0.7	0.5	0.3	0.0
MEAN- HF(0)	0.5344E-01	0.5368E-01	0.5349E-01	0.5348E-01	0.5384E-01	0.5317E-01
MEAN- HF(T)	0.5632E-01	0.5677E-01	0.5563E-01	0.5553E-01	0.5523E-01	0.5381E-01
MEAN- HT(0)	0.5307E-01	0.5331E-01	0.5296E-01	0.5314E-01	0.5319E-01	0.5284E-01
MEAN- HT(T)	0.5355E-01	0.5400E-01	0.5360E-01	0.5376E-01	0.5385E-01	0.5339E-01
MEAN- HP	0.5326E-01 (0.5000E-01)	0.5349E-01 (0.5000E-01)	0.5322E-01 (0.5000E-01)	0.5319E-01 (0.5000E-01)	0.5342E-01 (0.5000E-01)	0.5283E-01 (0.5000E-01)
MEAN- HE	0.5440E-01 (0.5000E-01)	0.5472E-01 (0.5000E-01)	0.5408E-01 (0.5000E-01)	0.5399E-01 (0.5000E-01)	0.5397E-01 (0.5000E-01)	0.5309E-01 (0.5000E-01)
S.D.- HP	0.1154E-02 (0.4664E-06)	0.1565E-02 (0.4792E-06)	0.1261E-02 (0.4643E-06)	0.1698E-02 (0.4620E-06)	0.1662E-02 (0.4474E-06)	0.1173E-02 (0.4781E-06)
S.D.- HE	0.1427E-02 (0.4697E-06)	0.2033E-02 (0.4637E-06)	0.1502E-02 (0.4656E-06)	0.2105E-02 (0.4630E-06)	0.1834E-02 (0.4483E-06)	0.1361E-02 (0.4776E-06)
MEAN OF S.E.- HP	0.2628E-04 (0.4842E-08)	0.2744E-04 (0.5190E-08)	0.3626E-04 (0.4211E-08)	0.3882E-04 (0.5057E-08)	0.5015E-04 (0.5918E-08)	0.3886E-04 (0.2901E-08)
MEAN OF S.E.- HE	0.3608E-04 (0.8546E-08)	0.3760E-04 (0.1087E-07)	0.4607E-04 (0.6072E-08)	0.4938E-04 (0.6975E-08)	0.5867E-04 (0.7215E-08)	0.4280E-04 (0.3865E-08)
RMSE- HP	0.5797E-03 (0.3236E-02)	0.5250E-03 (0.3592E-02)	0.6046E-03 (0.3152E-02)	0.5675E-03 (0.3571E-02)	0.7272E-03 (0.3459E-02)	0.3873E-03 (0.3058E-02)
RMSE- HE	0.1436E-02 (0.3236E-02)	0.1581E-02 (0.3592E-02)	0.1254E-02 (0.3152E-02)	0.1054E-02 (0.3571E-02)	0.1076E-02 (0.3459E-02)	0.4566E-03 (0.3058E-02)
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
|HP-HT| > 2*S.E.	48 (50)	42 (50)	45 (50)	47 (50)	45 (50)	39 (50)
|HE-HT| > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	47 (50)	44 (50)

TABLE 4.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONPOSITIVE CORRELATIONS BETWEEN XITS AND XIOP, ADDITION AND MULTIPLICATION METHODS (MULT. SHOWN IN PARENTHESES), MATRIX I.

LAMBDA=	0.0	-0.3	-0.5	-0.7	-0.9	-1.0
MEAN- HF(0) MEAN- HF(T)	0.5317E-01 0.5381E-01	0.5372E-01 0.5350E-01	0.5378E-01 0.5316E-01	0.5354E-01 0.5253E-01	0.5345E-01 0.5193E-01	0.5334E-01 0.5159E-01
MEAN- HT(0) MEAN- HT(T)	0.5284E-01 0.5339E-01	0.5320E-01 0.5388E-01	0.5336E-01 0.5411E-01	0.5320E-01 0.5386E-01	0.5306E-01 0.5374E-01	0.5291E-01 0.5357E-01
MEAN- HP	0.5283E-01 (0.5000E-01)	0.5327E-01 (0.5000E-01)	0.5334E-01 (0.5000E-01)	0.5317E-01 (0.5000E-01)	0.5319E-01 (0.5000E-01)	0.5314E-01 (0.5000E-01)
MEAN- HE	0.5309E-01 (0.5000E-01)	0.5316E-01 (0.5000E-01)	0.5303E-01 (0.5000E-01)	0.5267E-01 (0.5000E-01)	0.5238E-01 (0.5000E-01)	0.5216E-01 (0.5000E-01)
S.D.- HP	0.1173E-02 (0.4781E-06)	0.1393E-02 (0.4689E-06)	0.1754E-02 (0.4705E-06)	0.1557E-02 (0.4678E-06)	0.1550E-02 (0.4658E-06)	0.1304E-02 (0.4631E-06)
S.D.- HE	0.1361E-02 (0.4776E-06)	0.1215E-02 (0.4652E-06)	0.1629E-02 (0.4726E-06)	0.1323E-02 (0.4710E-06)	0.1182E-02 (0.4658E-06)	0.8394E-03 (0.4655E-06)
MEAN OF S.E.- HP	0.3886E-04 (0.2901E-08)	0.4433E-04 (0.4082E-08)	0.4134E-04 (0.5852E-08)	0.3193E-04 (0.5009E-08)	0.2052E-04 (0.5478E-08)	0.1454E-04 (0.4925E-08)
MEAN OF S.E.- HE	0.4280E-04 (0.3865E-08)	0.4296E-04 (0.5665E-08)	0.3832E-04 (0.8255E-08)	0.2677E-04 (0.7877E-08)	0.1537E-04 (0.9939E-08)	0.1005E-04 (0.9397E-08)
RMSE- HP	0.3873E-03 (0.3058E-02)	0.5532E-03 (0.3406E-02)	0.5907E-03 (0.3784E-02)	0.4575E-03 (0.3544E-02)	0.5172E-03 (0.3351E-02)	0.6184E-03 (0.3087E-02)
RMSE- HE	0.4566E-03 (0.3058E-02)	0.4078E-03 (0.3406E-02)	0.5222E-03 (0.3784E-02)	0.6652E-03 (0.3544E-02)	0.7611E-03 (0.3351E-02)	0.8169E-03 (0.3087E-02)
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
|HP-HT| > 2*S.E.	39 (50)	45 (50)	42 (50)	47 (50)	46 (50)	48 (50)
|HE-HT| > 2*S.E.	44 (50)	41 (50)	44 (50)	50 (50)	50 (50)	50 (50)

TABLE 5.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONNEGATIVE CORRELATIONS BETWEEN XITP AND XIOS,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX I.

LAMBDA=	1.0	0.9	0.7	0.5	0.3	0.0
MEAN- HF(0)	**	0.5509E-01	0.5515E-01	0.5486E-01	0.5394E-01	0.5306E-01
MEAN- HF(T)	**	0.5394E-01	0.5427E-01	0.5456E-01	0.5397E-01	0.5384E-01
MEAN- HT(0)	**	0.5296E-01	0.5318E-01	0.5331E-01	0.5296E-01	0.5280E-01
MEAN- HT(T)	**	0.5350E-01	0.5385E-01	0.5399E-01	0.5352E-01	0.5335E-01
MEAN- HP	**	0.5478E-01	0.5477E-01	0.5446E-01	0.5350E-01	0.5272E-01
	**	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)
MEAN- HE	**	0.5419E-01	0.5432E-01	0.5431E-01	0.5349E-01	0.5303E-01
	**	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)
S.D.- HP	**	0.1872E-02	0.2012E-02	0.2434E-02	0.1364E-02	0.1106E-02
	**	(0.4789E-06)	(0.4554E-06)	(0.4639E-06)	(0.4733E-06)	(0.4823E-06)
S.D.- HE	**	0.1620E-02	0.1876E-02	0.2352E-02	0.1210E-02	0.1207E-02
	**	(0.4744E-06)	(0.4548E-06)	(0.4698E-06)	(0.4725E-06)	(0.4810E-06)
MEAN OF S.E.- HP	**	0.2956E-04	0.3749E-04	0.3989E-04	0.4550E-04	0.3930E-04
	**	(0.4498E-08)	(0.4891E-08)	(0.4989E-08)	(0.3900E-08)	(0.3134E-08)
MEAN OF S.E.- HE	**	0.2601E-04	0.3426E-04	0.3829E-04	0.4636E-04	0.4405E-04
	**	(0.3818E-08)	(0.4570E-08)	(0.5318E-08)	(0.4677E-08)	(0.3713E-08)
RMSE- HP	**	0.1969E-02	0.1770E-02	0.1447E-02	0.8564E-03	0.6016E-03
	**	(0.3170E-02)	(0.3431E-02)	(0.3717E-02)	(0.3121E-02)	(0.2998E-02)
RMSE- HE	**	0.1370E-02	0.1309E-02	0.1275E-02	0.7201E-03	0.5642E-03
	**	(0.3170E-02)	(0.3431E-02)	(0.3717E-02)	(0.3121E-02)	(0.2998E-02)
HP > 2*S.E.	**	50	50	50	50	50
	**	(50)	(50)	(50)	(50)	(50)
HE > 2*S.E.	**	50	50	50	50	50
	**	(50)	(50)	(50)	(50)	(50)
|HP-HT| > 2*S.E.	**	50	50	50	43	45
	**	(50)	(50)	(50)	(50)	(50)
|HE-HT| > 2*S.E.	**	50	50	50	47	44
	**	(50)	(50)	(50)	(50)	(50)

**-INDICATES SUBROUTINE MFSD UNABLE TO COMPUTE A SOLUTION

TABLE 6.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONPOSITIVE CORRELATIONS BETWEEN XITP AND XIOS,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX I.

LAMBDA=	0.0	-0.3	-0.5	-0.7	-0.9	-1.0
MEAN- HF(0) MEAN- HF(T)	0.5306E-01 0.5384E-01	0.5303E-01 0.5449E-01	0.5253E-01 0.5435E-01	0.5209E-01 0.5421E-01	0.5182E-01 0.5460E-01	** **
MEAN- HT(0) MEAN- HT(T)	0.5280E-01 0.5335E-01	0.5327E-01 0.5387E-01	0.5339E-01 0.5402E-01	0.5313E-01 0.5376E-01	0.5360E-01 0.5419E-01	** **
MEAN- HP	0.5272E-01 (0.5000E-01)	0.5269E-01 (0.5000E-01)	0.5221E-01 (0.5000E-01)	0.5185E-01 (0.5000E-01)	0.5168E-01 (0.5000E-01)	** **
MEAN- HE	0.5303E-01 (0.5000E-01)	0.5326E-01 (0.5000E-01)	0.5289E-01 (0.5000E-01)	0.5261E-01 (0.5000E-01)	0.5266E-01 (0.5000E-01)	** **
S.D.- HP	0.1106E-02 (0.4823E-06)	0.1076E-02 (0.4785E-06)	0.1027E-02 (0.4709E-06)	0.9033E-03 (0.4833E-06)	0.8809E-03 (0.4379E-06)	** **
S.D.- HE	0.1207E-02 (0.4810E-06)	0.1386E-02 (0.4779E-06)	0.1155E-02 (0.4693E-06)	0.1172E-02 (0.4891E-06)	0.1274E-02 (0.4345E-06)	** **
MEAN OF S.E.- HP	0.3930E-04 (0.3134E-08)	0.4239E-04 (0.4822E-08)	0.4190E-04 (0.5465E-08)	0.3467E-04 (0.4984E-08)	0.2377E-04 (0.7008E-08)	** **
MEAN OF S.E.- HE	0.4405E-04 (0.3713E-08)	0.5204E-04 (0.5564E-08)	0.5529E-04 (0.5312E-08)	0.4917E-04 (0.5592E-08)	0.3791E-04 (0.5935E-08)	** **
RMSE- HP	0.6016E-03 (0.2998E-02)	0.8069E-03 (0.3542E-02)	0.1304E-02 (0.3612E-02)	0.1439E-02 (0.3413E-02)	0.2100E-02 (0.3971E-02)	** **
RMSE- HE	0.5642E-03 (0.2998E-02)	0.4610E-03 (0.3542E-02)	0.6862E-03 (0.3612E-02)	0.7200E-03 (0.3413E-02)	0.1089E-02 (0.3971E-02)	** **
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	** **
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	** **
|HP-HT| > 2*S.E.	45 (50)	47 (50)	50 (50)	50 (50)	50 (50)	** **
|HE-HT| > 2*S.E.	44 (50)	40 (50)	43 (50)	46 (50)	50 (50)	** **

**-INDICATES SUBROUTINE MFSD UNABLE TO COMPUTE A SOLUTION

TABLE 7.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONNEGATIVE CORRELATIONS BETWEEN XITS AND XIOP, ADDITION AND MULTIPLICATION METHODS (MULT. SHOWN IN PARENTHESES), MATRIX II.

LAMBDA=	1.0	0.9	0.7	0.5	0.3	0.0
MEAN- HF(O)	0.5357E-01	0.5369E-01	0.5410E-01	0.5364E-01	0.5370E-01	0.5346E-01
MEAN- HF(T)	0.5658E-01	0.5667E-01	0.5661E-01	0.5558E-01	0.5520E-01	0.5406E-01
MEAN- HT(O)	0.5319E-01	0.5329E-01	0.5365E-01	0.5324E-01	0.5326E-01	0.5309E-01
MEAN- HT(T)	0.5370E-01	0.5386E-01	0.5413E-01	0.5379E-01	0.5378E-01	0.5362E-01
MEAN- HP	0.5338E-01 (0.5000E-01)	0.5348E-01 (0.5000E-01)	0.5379E-01 (0.5000E-01)	0.5332E-01 (0.5000E-01)	0.5336E-01 (0.5000E-01)	0.5311E-01 (0.5000E-01)
MEAN- HE	0.5457E-01 (0.5000E-01)	0.5467E-01 (0.5000E-01)	0.5480E-01 (0.5000E-01)	0.5409E-01 (0.5000E-01)	0.5396E-01 (0.5000E-01)	0.5334E-01 (0.5000E-01)
S.D.- HP	0.1355E-02 (0.4643E-06)	0.1396E-02 (0.4485E-06)	0.2101E-02 (0.4777E-06)	0.1350E-02 (0.4738E-06)	0.1482E-02 (0.4803E-06)	0.1347E-02 (0.4764E-06)
S.D.- HE	0.1654E-02 (0.4515E-06)	0.1676E-02 (0.4505E-06)	0.2486E-02 (0.4478E-06)	0.1561E-02 (0.4723E-06)	0.1678E-02 (0.4611E-06)	0.1354E-02 (0.4556E-06)
MEAN OF S.E.- HP	0.2774E-04 (0.5595E-08)	0.3030E-04 (0.5580E-08)	0.4054E-04 (0.6354E-08)	0.4142E-04 (0.4617E-08)	0.4194E-04 (0.4645E-08)	0.4028E-04 (0.3733E-08)
MEAN OF S.E.- HE	0.3798E-04 (0.1027E-07)	0.4113E-04 (0.1036E-07)	0.5170E-04 (0.1143E-07)	0.5204E-04 (0.6372E-08)	0.4988E-04 (0.6566E-08)	0.4434E-04 (0.4676E-08)
RMSE- HP	0.5082E-03 (0.3380E-02)	0.5780E-03 (0.3483E-02)	0.5978E-03 (0.4149E-02)	0.4967E-03 (0.3447E-02)	0.6154E-03 (0.3523E-02)	0.5982E-03 (0.3348E-02)
RMSE- HE	0.1506E-02 (0.3380E-02)	0.1505E-02 (0.3483E-02)	0.1303E-02 (0.4149E-02)	0.1011E-02 (0.3447E-02)	0.9286E-03 (0.3523E-02)	0.5614E-03 (0.3348E-02)
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
|HP-HT| > 2*S.E.	46 (50)	45 (50)	45 (50)	45 (50)	45 (50)	44 (50)
|HE-HT| > 2*S.E.	50 (50)	50 (50)	50 (50)	47 (50)	46 (50)	42 (50)

TABLE 8.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONPOSITIVE CORRELATIONS BETWEEN XITS AND XIOP, ADDITION AND MULTIPLICATION METHODS (MULT. SHOWN IN PARENTHESES), MATRIX II.

LAMBDA=	0.0	-0.3	-0.5	-0.7	-0.9	-1.0
MEAN- HF(0)	0.5346E-01	0.5367E-01	0.5375E-01	0.5377E-01	0.5349E-01	0.5360E-01
MEAN- HF(T)	0.5406E-01	0.5345E-01	0.5305E-01	0.5272E-01	0.5188E-01	0.5165E-01
MEAN- HT(0)	0.5309E-01	0.5330E-01	0.5325E-01	0.5342E-01	0.5317E-01	0.5317E-01
MEAN- HT(T)	0.5362E-01	0.5380E-01	0.5382E-01	0.5405E-01	0.5368E-01	0.5372E-01
MEAN- HP	0.5311E-01	0.5325E-01	0.5339E-01	0.5336E-01	0.5325E-01	0.5343E-01
	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)
MEAN- HE	0.5334E-01	0.5312E-01	0.5305E-01	0.5284E-01	0.5238E-01	0.5232E-01
	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)
S.D.- HP	0.1347E-02	0.1762E-02	0.1746E-02	0.1476E-02	0.1469E-02	0.1785E-02
	(0.4764E-06)	(0.4535E-06)	(0.4735E-06)	(0.4558E-06)	(0.4937E-06)	(0.4771E-06)
S.D.- HE	0.1364E-02	0.1454E-02	0.1551E-02	0.1239E-02	0.1026E-02	0.1096E-02
	(0.4556E-06)	(0.4550E-06)	(0.4743E-06)	(0.4712E-06)	(0.4687E-06)	(0.4805E-06)
MEAN OF S.E.- HP	0.4028E-04	0.4116E-04	0.3316E-04	0.3522E-04	0.1862E-04	0.1239E-04
	(0.3733E-08)	(0.5023E-08)	(0.4363E-08)	(0.5291E-08)	(0.5096E-08)	(0.5131E-08)
MEAN OF S.E.- HE	0.4434E-04	0.4021E-04	0.2999E-04	0.2988E-04	0.1364E-04	0.8460E-05
	(0.4676E-08)	(0.6047E-08)	(0.6242E-08)	(0.7504E-08)	(0.9690E-08)	(0.9861E-08)
RMSE- HP	0.5982E-03	0.6098E-03	0.4995E-03	0.6581E-03	0.4512E-03	0.7271E-03
	(0.3348E-02)	(0.3643E-02)	(0.3579E-02)	(0.3697E-02)	(0.3439E-02)	(0.3466E-02)
RMSE- HE	0.5614E-03	0.3734E-03	0.5220E-03	0.7587E-03	0.8910E-03	0.9461E-03
	(0.3348E-02)	(0.3643E-02)	(0.3579E-02)	(0.3697E-02)	(0.3439E-02)	(0.3466E-02)
HP > 2*S.E.	50	50	50	50	50	50
	(50)	(50)	(50)	(50)	(50)	(50)
HE > 2*S.E.	50	50	50	50	50	50
	(50)	(50)	(50)	(50)	(50)	(50)
|HP-HT| > 2*S.E.	44	43	43	44	46	48
	(50)	(50)	(50)	(50)	(50)	(50)
|HE-HT| > 2*S.E.	42	41	40	49	50	50
	(50)	(50)	(50)	(50)	(50)	(50)

TABLE 9.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONNEGATIVE CORRELATIONS BETWEEN XITP AND XIOS,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX II.

LAMBDA=	1.0	0.9	0.7	0.5	0.3	0.0
MEAN- HF(0)	**	0.5518E-01	0.5486E-01	0.5459E-01	0.5469E-01	0.5391E-01
MEAN- HF(T)	**	0.5420E-01	0.5421E-01	0.5417E-01	0.5480E-01	0.5443E-01
MEAN- HT(0)	**	0.5303E-01	0.5311E-01	0.5320E-01	0.5357E-01	0.5342E-01
MEAN- HT(T)	**	0.5357E-01	0.5369E-01	0.5378E-01	0.5422E-01	0.5393E-01
MEAN- HP	**	0.5490E-01	0.5449E-01	0.5420E-01	0.5425E-01	0.5351E-01
	**	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)
MEAN- HE	**	0.5439E-01	0.5416E-01	0.5399E-01	0.5427E-01	0.5371E-01
	**	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)
S.D.- HP	**	0.1751E-02	0.1429E-02	0.1701E-02	0.1723E-02	0.1711E-02
	**	(0.4669E-06)	(0.4641E-06)	(0.4753E-06)	(0.4628E-06)	(0.4565E-06)
S.D.- HE	**	0.1673E-02	0.1439E-02	0.1704E-02	0.1816E-02	0.1677E-02
	**	(0.4784E-06)	(0.4648E-06)	(0.4728E-06)	(0.4649E-06)	(0.4543E-06)
MEAN OF S.E.- HP	**	0.2721E-04	0.3603E-04	0.3807E-04	0.4614E-04	0.4430E-04
	**	(0.4441E-08)	(0.4297E-08)	(0.4086E-08)	(0.5448E-08)	(0.4638E-08)
MEAN OF S.E.- HE	**	0.2466E-04	0.3397E-04	0.3608E-04	0.4711E-04	0.4727E-04
	**	(0.3655E-08)	(0.4672E-08)	(0.4350E-08)	(0.6681E-08)	(0.5623E-08)
RMSE- HP	**	0.2003E-02	0.1505E-02	0.1192E-02	0.9195E-03	0.6428E-03
	**	(0.3212E-02)	(0.3263E-02)	(0.3429E-02)	(0.3855E-02)	(0.3711E-02)
RMSE- HE	**	0.1521E-02	0.1192E-02	0.1042E-02	0.9340E-03	0.4966E-03
	**	(0.3212E-02)	(0.3263E-02)	(0.3429E-02)	(0.3855E-02)	(0.3711E-02)
HP > 2*S.E.	**	50	50	50	50	50
	**	(50)	(50)	(50)	(50)	(50)
HE > 2*S.E.	**	50	50	50	50	50
	**	(50)	(50)	(50)	(50)	(50)
|HP-HT| > 2*S.E.	**	50	50	49	44	46
	**	(50)	(50)	(50)	(50)	(50)
|HE-HT| > 2*S.E.	**	50	50	48	45	43
	**	(50)	(50)	(50)	(50)	(50)

**-INDICATES SUBROUTINE MFSD UNABLE TO COMPUTE A SOLUTION

TABLE 10.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONPOSITIVE CORRELATIONS BETWEEN XITP AND XIOS,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX II.

LAMBDA=	0.0	-0.3	-0.5	-0.7	-0.9	-1.0
MEAN- HF(0) MEAN- HF(T)	0.5391E-01 0.5443E-01	0.5314E-01 0.5435E-01	0.5218E-01 0.5359E-01	0.5225E-01 0.5439E-01	0.5180E-01 0.5421E-01	** **
MEAN- HT(0) MEAN- HT(T)	0.5342E-01 0.5393E-01	0.5332E-01 0.5384E-01	0.5280E-01 0.5325E-01	0.5348E-01 0.5393E-01	0.5356E-01 0.5399E-01	** **
MEAN- HP	0.5351E-01 (0.5000E-01)	0.5277E-01 (0.5000E-01)	0.5196E-01 (0.5000E-01)	0.5205E-01 (0.5000E-01)	0.5165E-01 (0.5000E-01)	** **
MEAN- HE	0.5371E-01 (0.5000E-01)	0.5325E-01 (0.5000E-01)	0.5251E-01 (0.5000E-01)	0.5284E-01 (0.5000E-01)	0.5252E-01 (0.5000E-01)	** **
S.D.- HP	0.1711E-02 (0.4565E-06)	0.1250E-02 (0.4736E-06)	0.8769E-03 (0.4775E-06)	0.1160E-02 (0.4652E-06)	0.7993E-03 (0.4980E-06)	** **
S.D.- HE	0.1677E-02 (0.4543E-06)	0.1435E-02 (0.4720E-06)	0.1112E-02 (0.4779E-06)	0.1413E-02 (0.4558E-06)	0.1031E-02 (0.5017E-06)	** **
MEAN OF S.E.- HP	0.4430E-04 (0.4688E-08)	0.4374E-04 (0.4535E-08)	0.2905E-04 (0.2884E-08)	0.2946E-04 (0.5341E-08)	0.2202E-04 (0.6836E-08)	** **
MEAN OF S.E.- HE	0.4727E-04 (0.5623E-08)	0.5145E-04 (0.5419E-08)	0.3728E-04 (0.2985E-08)	0.4163E-04 (0.4865E-08)	0.3328E-04 (0.5413E-08)	** **
RMSE- HP	0.6428E-03 (0.3711E-02)	0.7601E-03 (0.3597E-02)	0.9272E-03 (0.3007E-02)	0.1613E-02 (0.3860E-02)	0.2140E-02 (0.3955E-02)	** **
RMSE- HE	0.4966E-03 (0.3711E-02)	0.4088E-03 (0.3597E-02)	0.4510E-03 (0.3007E-02)	0.8371E-03 (0.3860E-02)	0.1287E-02 (0.3955E-02)	** **
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	** **
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	** **
|HP-HT| > 2*S.E.	46 (50)	46 (50)	50 (50)	50 (50)	50 (50)	** **
|HE-HT| > 2*S.E.	43 (50)	41 (50)	41 (50)	48 (50)	50 (50)	** **

**-INDICATES SUBROUTINE MFSD UNABLE TO COMPUTE A SOLUTION

TABLE 11.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONNEGATIVE CORRELATIONS BETWEEN XITS AND XIOP,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX III.

LAMBDA=	1.0	0.9	0.7	0.5	0.3	0.0
MEAN- HF(0)	0.5513E-01	0.5532E-01	0.5548E-01	0.5549E-01	0.5521E-01	0.5558E-01
MEAN- HF(T)	0.5666E-01	0.5682E-01	0.5572E-01	0.5551E-01	0.5447E-01	0.5381E-01
MEAN- HT(0)	0.5303E-01	0.5317E-01	0.5307E-01	0.5336E-01	0.5309E-01	0.5334E-01
MEAN- HT(T)	0.5202E-01	0.5215E-01	0.5207E-01	0.5217E-01	0.5207E-01	0.5218E-01
MEAN- HP	0.5389E-01	0.5404E-01	0.5412E-01	0.5389E-01	0.5356E-01	0.5333E-01
	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)
MEAN- HE	0.5440E-01	0.5450E-01	0.5414E-01	0.5383E-01	0.5324E-01	0.5270E-01
	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)	(0.5000E-01)
S.D.- HP	0.1698E-02	0.1618E-02	0.1922E-02	0.2413E-02	0.1562E-02	0.1477E-02
	(0.4532E-06)	(0.4359E-06)	(0.4439E-06)	(0.4682E-06)	(0.4852E-06)	(0.4182E-06)
S.D.- HE	0.1823E-02	0.1598E-02	0.1691E-02	0.2052E-02	0.1273E-02	0.1182E-02
	(0.4828E-06)	(0.4542E-06)	(0.4617E-06)	(0.4632E-06)	(0.4868E-06)	(0.4512E-06)
MEAN OF S.E.- HP	0.1322E-03	0.1393E-03	0.1308E-03	0.1467E-03	0.1376E-03	0.1566E-03
	(0.1601E-07)	(0.1712E-07)	(0.1272E-07)	(0.1377E-07)	(0.9901E-08)	(0.1156E-07)
MEAN OF S.E.- HE	0.1516E-03	0.1623E-03	0.1366E-03	0.1501E-03	0.1301E-03	0.1318E-03
	(0.1754E-07)	(0.1776E-07)	(0.1160E-07)	(0.1137E-07)	(0.7343E-08)	(0.7910E-08)
RMSE- HP	0.1400E-02	0.1571E-02	0.1689E-02	0.1558E-02	0.1209E-02	0.1422E-02
	(0.3202E-02)	(0.3363E-02)	(0.3249E-02)	(0.3604E-02)	(0.3261E-02)	(0.3510E-02)
RMSE- HE	0.1738E-02	0.1681E-02	0.1464E-02	0.1118E-02	0.7653E-03	0.1191E-02
	(0.3202E-02)	(0.3363E-02)	(0.3249E-02)	(0.3604E-02)	(0.3261E-02)	(0.3510E-02)
HP > 2*S.E.	50	50	50	50	50	50
	(50)	(50)	(50)	(50)	(50)	(50)
HE > 2*S.E.	50	50	50	50	50	50
	(50)	(50)	(50)	(50)	(50)	(50)
|HP-HT| > 2*S.E.	43	41	44	40	39	40
	(50)	(50)	(50)	(50)	(50)	(50)
|HE-HT| > 2*S.E.	44	42	43	32	34	38
	(50)	(50)	(50)	(50)	(50)	(50)

TABLE 12.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONPOSITIVE CORRELATIONS BETWEEN XITS AND XIOP,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX III.

LAMBDA=	0.0	-0.3	-0.5	-0.7	-0.9	-1.0
MEAN- HF(0)	0.5558E-01	0.5518E-01	0.5476E-01	0.5504E-01	0.5525E-01	0.5527E-01
MEAN- HF(T)	0.5381E-01	0.5248E-01	0.5179E-01	0.5113E-01	0.5043E-01	0.5007E-01
MEAN- HT(0)	0.5334E-01	0.5304E-01	0.5292E-01	0.5309E-01	0.5300E-01	0.5311E-01
MEAN- HT(T)	0.5218E-01	0.5206E-01	0.5193E-01	0.5207E-01	0.5205E-01	0.5210E-01
MEAN- HP	0.5333E-01 (0.5000E-01)	0.5255E-01 (0.5000E-01)	0.5216E-01 (0.5000E-01)	0.5209E-01 (0.5000E-01)	0.5235E-01 (0.5000E-01)	0.5416E-01 (0.5000E-01)
MEAN- HE	0.5270E-01 (0.5000E-01)	0.5174E-01 (0.5000E-01)	0.5127E-01 (0.5000E-01)	0.5097E-01 (0.5000E-01)	0.5066E-01 (0.5000E-01)	0.5047E-01 (0.5000E-01)
S.D.- HP	0.1477E-02 (0.4182E-06)	0.1225E-02 (0.4524E-06)	0.1495E-02 (0.5020E-06)	0.1480E-02 (0.4261E-06)	0.1378E-02 (0.4826E-06)	0.1654E-02 (0.4318E-06)
S.D.- HE	0.1182E-02 (0.4512E-06)	0.8583E-03 (0.4461E-06)	0.8158E-03 (0.4909E-06)	0.6615E-03 (0.4679E-06)	0.3957E-03 (0.4646E-06)	0.1651E-03 (0.4439E-06)
MEAN OF S.E.- HP	0.1566E-03 (0.1166E-07)	0.1447E-03 (0.1168E-07)	0.1222E-03 (0.1118E-07)	0.1019E-03 (0.1434E-07)	0.6263E-04 (0.1488E-07)	0.1256E-04 (0.1595E-07)
MEAN OF S.E.- HE	0.1318E-03 (0.7910E-08)	0.1016E-03 (0.9177E-08)	0.7745E-04 (0.9103E-08)	0.4975E-04 (0.1462E-07)	0.1825E-04 (0.1490E-07)	0.1507E-05 (0.1880E-07)
RMSE- HP	0.1422E-02 (0.3510E-02)	0.8691E-03 (0.3157E-02)	0.1509E-02 (0.3072E-02)	0.1581E-02 (0.3262E-02)	0.1254E-02 (0.3205E-02)	0.1625E-02 (0.3266E-02)
RMSE- HE	0.1191E-02 (0.3510E-02)	0.1426E-02 (0.3157E-02)	0.1864E-02 (0.3072E-02)	0.2287E-02 (0.3262E-02)	0.2515E-02 (0.3205E-02)	0.2774E-02 (0.3266E-02)
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
|HP-HT| > 2*S.E.	40 (50)	33 (50)	41 (50)	47 (50)	47 (50)	50 (50)
|HE-HT| > 2*S.E.	38 (50)	49 (50)	50 (50)	50 (50)	50 (50)	50 (50)

TABLE 13.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONNEGATIVE CORRELATIONS BETWEEN XITP AND XIOS,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX III.

LAMBDA=	1.0	0.9	0.7	0.5	0.3	0.0
MEAN- HF(0)	0.6163E-01	0.6007E-01	0.5891E-01	0.5808E-01	0.5699E-01	0.5524E-01
MEAN- HF(T)	0.5351E-01	0.5372E-01	0.5370E-01	0.5360E-01	0.5334E-01	0.5385E-01
MEAN- HT(0)	0.5353E-01	0.5326E-01	0.5324E-01	0.5311E-01	0.5316E-01	0.5325E-01
MEAN- HT(T)	0.5230E-01	0.5223E-01	0.5214E-01	0.5214E-01	0.5205E-01	0.5217E-01
MEAN- HP	0.5879E-01 (0.5000E-01)	0.5758E-01 (0.5000E-01)	0.5622E-01 (0.5000E-01)	0.5553E-01 (0.5000E-01)	0.5443E-01 (0.5000E-01)	0.5315E-01 (0.5000E-01)
MEAN- HE	0.5478E-01 (0.5000E-01)	0.5456E-01 (0.5000E-01)	0.5396E-01 (0.5000E-01)	0.5364E-01 (0.5000E-01)	0.5301E-01 (0.5000E-01)	0.5267E-01 (0.5000E-01)
S.D.- HP	0.2990E-02 (0.6075E-06)	0.2922E-02 (0.4364E-06)	0.2602E-02 (0.6970E-06)	0.2222E-02 (0.4473E-06)	0.1546E-02 (0.4855E-06)	0.1598E-02 (0.4451E-05)
S.D.- HE	0.1607E-02 (0.4876E-06)	0.1826E-02 (0.4622E-06)	0.1491E-02 (0.4602E-06)	0.1466E-02 (0.4724E-06)	0.1071E-02 (0.4698E-06)	0.1270E-02 (0.4651E-06)
MEAN OF S.E.- HP	0.1478E-03 (0.1920E-07)	0.1416E-03 (0.1825E-07)	0.1554E-03 (0.1421E-07)	0.1491E-03 (0.1369E-07)	0.1525E-03 (0.1190E-07)	0.1481E-03 (0.1214E-07)
MEAN OF S.E.- HE	0.8207E-04 (0.8009E-08)	0.8679E-04 (0.8966E-08)	0.1012E-03 (0.7721E-08)	0.1011E-03 (0.8418E-08)	0.1077E-03 (0.9094E-08)	0.1277E-03 (0.8935E-08)
RMSE- HP	0.5616E-02 (0.3723E-02)	0.4732E-02 (0.3438E-02)	0.3437E-02 (0.3471E-02)	0.2919E-02 (0.3238E-02)	0.1781E-02 (0.3314E-02)	0.1223E-02 (0.3436E-02)
RMSE- HE	0.1534E-02 (0.3723E-02)	0.1680E-02 (0.3438E-02)	0.1190E-02 (0.3471E-02)	0.1188E-02 (0.3238E-02)	0.9349E-03 (0.3314E-02)	0.1178E-02 (0.3436E-02)
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
|HP-HT| > 2*S.E.	50 (50)	50 (50)	49 (50)	46 (50)	46 (50)	40 (50)
|HE-HT| > 2*S.E.	49 (50)	49 (50)	43 (50)	37 (50)	41 (50)	46 (50)

TABLE 14.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONPOSITIVE CORRELATIONS BETWEEN XITP AND XIOS,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX III.

LAMBDA=	0.0	-0.3	-0.5	-0.7	-0.9	-1.0
MEAN- HF(0) MEAN- HF(T)	0.5524E-01 0.5385E-01	0.5408E-01 0.5361E-01	0.5277E-01 0.5303E-01	0.5153E-01 0.5356E-01	0.5067E-01 0.5344E-01	0.5011E-01 0.5401E-01
MEAN- HT(0) MEAN- HT(T)	0.5325E-01 0.5217E-01	0.5324E-01 0.5211E-01	0.5278E-01 0.5186E-01	0.5318E-01 0.5216E-01	0.5304E-01 0.5207E-01	0.5322E-01 0.5215E-01
MEAN- HP	0.5315E-01 (0.5000E-01)	0.5207E-01 (0.5000E-01)	0.5125E-01 (0.5000E-01)	0.5072E-01 (0.5000E-01)	0.5032E-01 (0.5000E-01)	0.5008E-01 (0.5000E-01)
MEAN- HE	0.5267E-01 (0.5000E-01)	0.5192E-01 (0.5000E-01)	0.5130E-01 (0.5000E-01)	0.5106E-01 (0.5000E-01)	0.5071E-01 (0.5000E-01)	0.5049E-01 (0.5000E-01)
S.D.- HP	0.1598E-02 (0.4451E-06)	0.1461E-02 (0.4547E-06)	0.8327E-03 (0.4860E-06)	0.4664E-03 (0.4355E-06)	0.2166E-03 (0.4565E-06)	0.2509E-04 (0.4595E-06)
S.D.- HE	0.1270E-02 (0.4651E-06)	0.1259E-02 (0.4552E-06)	0.7836E-03 (0.4980E-06)	0.6078E-03 (0.4438E-06)	0.4469E-03 (0.3909E-06)	0.1666E-03 (0.4618E-06)
MEAN OF S.E.- HP	0.1481E-03 (0.1214E-07)	0.1503E-03 (0.1192E-07)	0.1189E-03 (0.1061E-07)	0.9606E-04 (0.1543E-07)	0.6143E-04 (0.1395E-07)	0.1556E-04 (0.2031E-07)
MEAN OF S.E.- HE	0.1277E-03 (0.8935E-08)	0.1438E-03 (0.8167E-08)	0.1265E-03 (0.6867E-08)	0.1490E-03 (0.8291E-08)	0.1423E-03 (0.6833E-08)	0.9629E-04 (0.1065E-07)
RMSE- HP	0.1223E-02 (0.3436E-02)	0.1483E-02 (0.3466E-02)	0.1674E-02 (0.2895E-02)	0.2632E-02 (0.3365E-02)	0.2885E-02 (0.3221E-02)	0.3249E-02 (0.3333E-02)
RMSE- HE	0.1178E-02 (0.3436E-02)	0.1531E-02 (0.3466E-02)	0.1628E-02 (0.2895E-02)	0.2280E-02 (0.3365E-02)	0.2492E-02 (0.3222E-02)	0.2828E-02 (0.3333E-02)
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
|HP-HT| > 2*S.E.	40 (50)	43 (50)	50 (50)	50 (50)	50 (50)	50 (50)
|HE-HT| > 2*S.E.	46 (50)	46 (50)	50 (50)	50 (50)	50 (50)	50 (50)

TABLE 15.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONNEGATIVE CORRELATIONS BETWEEN XITS AND XIOP,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX IV.

LAMBDA=	1.0	0.9	0.7	0.5	0.3	0.0
MEAN- HF(0)	**	0.5564E-01	0.5572E-01	0.5536E-01	0.5529E-01	0.5551E-01
MEAN- HF(T)	**	0.5757E-01	0.5679E-01	0.5612E-01	0.5558E-01	0.5409E-01
MEAN- HT(0)	**	0.5315E-01	0.5322E-01	0.5314E-01	0.5318E-01	0.5328E-01
MEAN- HT(T)	**	0.5239E-01	0.5237E-01	0.5234E-01	0.5239E-01	0.5252E-01
MEAN- HP	** **	0.5426E-01 (0.5000E-01)	0.5427E-01 (0.5000E-01)	0.5355E-01 (0.5000E-01)	0.5363E-01 (0.5000E-01)	0.5340E-01 (0.5000E-01)
MEAN- HE	** **	0.5487E-01 (0.5000E-01)	0.5452E-01 (0.5000E-01)	0.5376E-01 (0.5000E-01)	0.5367E-01 (0.5000E-01)	0.5291E-01 (0.5000E-01)
S.D.- HP	** **	0.1611E-02 (0.4777E-06)	0.2086E-02 (0.5069E-06)	0.1595E-02 (0.4495E-06)	0.2123E-02 (0.4923E-06)	0.1326E-02 (0.4473E-06)
S.D.- HE	** **	0.1656E-02 (0.4578E-06)	0.1993E-02 (0.5406E-06)	0.1711E-02 (0.4539E-06)	0.1981E-02 (0.4731E-06)	0.1164E-02 (0.4692E-06)
MEAN OF S.E.- HP	** **	0.1518E-03 (0.2054E-07)	0.1491E-03 (0.1530E-07)	0.1721E-03 (0.1616E-07)	0.1515E-03 (0.1411E-07)	0.1537E-03 (0.1218E-07)
MEAN OF S.E.- HE	** **	0.1784E-03 (0.2274E-07)	0.1706E-03 (0.1536E-07)	0.1846E-03 (0.1641E-07)	0.1578E-03 (0.1316E-07)	0.1333E-03 (0.9899E-08)
RMSE- HP	** **	0.1634E-02 (0.3288E-02)	0.2001E-02 (0.3493E-02)	0.1101E-02 (0.3328E-02)	0.1345E-02 (0.3478E-02)	0.1190E-02 (0.3441E-02)
RMSE- HE	** **	0.1991E-02 (0.3288E-02)	0.1845E-02 (0.3493E-02)	0.1240E-02 (0.3328E-02)	0.1105E-02 (0.3478E-02)	0.1085E-02 (0.3441E-02)
HP > 2*S.E.	** **	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
HE > 2*S.E.	** **	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
|HP-HT| > 2*S.E.	** **	42 (50)	46 (50)	33 (50)	43 (50)	42 (50)
|HE-HT| > 2*S.E.	** **	45 (50)	46 (50)	34 (50)	38 (50)	43 (50)

**-INDICATES SUBROUTINE MFSD UNABLE TO COMPUTE A SOLUTION

TABLE 16.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONPOSITIVE CORRELATIONS BETWEEN XITS AND XIOP,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX IV.

LAMBDA=	0.0	-0.3	-0.5	-0.7	-0.9	-1.0
MEAN- HF(0)	0.5551E-01	0.5546E-01	0.5531E-01	0.5512E-01	0.5487E-01	**
MEAN- HF(T)	0.5409E-01	0.5305E-01	0.5208E-01	0.5128E-01	0.5042E-01	**
MEAN- HT(0)	0.5328E-01	0.5328E-01	0.5321E-01	0.5299E-01	0.5287E-01	**
MEAN- HT(T)	0.5252E-01	0.5250E-01	0.5243E-01	0.5231E-01	0.5213E-01	**
MEAN- HP	0.5340E-01 (0.5000E-01)	0.5313E-01 (0.5000E-01)	0.5265E-01 (0.5000E-01)	0.5252E-01 (0.5000E-01)	0.5217E-01 (0.5000E-01)	** **
MEAN- HE	0.5291E-01 (0.5000E-01)	0.5228E-01 (0.5000E-01)	0.5165E-01 (0.5000E-01)	0.5123E-01 (0.5000E-01)	0.5062E-01 (0.5000E-01)	** **
S.D.- HP	0.1326E-02 (0.4473E-06)	0.1598E-02 (0.4760E-06)	0.1303E-02 (0.4969E-06)	0.1341E-02 (0.4803E-06)	0.1155E-02 (0.4886E-06)	** **
S.D.- HE	0.1164E-02 (0.4692E-06)	0.1084E-02 (0.4708E-06)	0.8095E-03 (0.4845E-06)	0.6153E-03 (0.4804E-06)	0.3312E-03 (0.4698E-06)	** **
MEAN OF S.E.- HP	0.1537E-03 (0.1216E-07)	0.1427E-03 (0.1409E-07)	0.1285E-03 (0.1436E-07)	0.1030E-03 (0.1479E-07)	0.6062E-04 (0.1382E-07)	** **
MEAN OF S.E.- HE	0.1333E-03 (0.9899E-08)	0.1087E-03 (0.1225E-07)	0.8145E-04 (0.1493E-07)	0.5276E-04 (0.1721E-07)	0.1819E-04 (0.1676E-07)	** **
RMSE- HP	0.1190E-02 (0.3441E-02)	0.1188E-02 (0.3432E-02)	0.1149E-02 (0.3427E-02)	0.1134E-02 (0.3178E-02)	0.1071E-02 (0.2995E-02)	** **
RMSE- HE	0.1085E-02 (0.3441E-02)	0.1300E-02 (0.3432E-02)	0.1818E-02 (0.3427E-02)	0.1942E-02 (0.3178E-02)	0.2340E-02 (0.2995E-02)	** **
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	** **
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	** **
|HP-HT| > 2*S.E.	42 (50)	45 (50)	36 (50)	49 (50)	47 (50)	** **
|HE-HT| > 2*S.E.	43 (50)	43 (50)	50 (50)	49 (50)	50 (50)	** **

**-INDICATES SUBROUTINE MFSD UNABLE TO COMPUTE A SOLUTION

TABLE 17.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONNEGATIVE CORRELATIONS BETWEEN XITP AND XIOS,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX IV.

LAMBDA=	1.0	0.9	0.7	0.5	0.3	0.0
MEAN- HF(O)	0.6105E-01	0.6063E-01	0.5865E-01	0.5812E-01	0.5684E-01	0.5548E-01
MEAN- HF(T)	0.5410E-01	0.5439E-01	0.5372E-01	0.5412E-01	0.5414E-01	0.5393E-01
MEAN- HT(O)	0.5335E-01	0.5334E-01	0.5307E-01	0.5330E-01	0.5312E-01	0.5323E-01
MEAN- HT(T)	0.5245E-01	0.5246E-01	0.5229E-01	0.5241E-01	0.5235E-01	0.5240E-01
MEAN- HP	0.5856E-01 (0.5000E-01)	0.5809E-01 (0.5000E-01)	0.5603E-01 (0.5000E-01)	0.5571E-01 (0.5000E-01)	0.5438E-01 (0.5000E-01)	0.5336E-01 (0.5000E-01)
MEAN- HE	0.5518E-01 (0.5000E-01)	0.5508E-01 (0.5000E-01)	0.5392E-01 (0.5000E-01)	0.5399E-01 (0.5000E-01)	0.5332E-01 (0.5000E-01)	0.5278E-01 (0.5000E-01)
S.D.- HP	0.3254E-02 (0.7607E-06)	0.4063E-02 (0.7853E-06)	0.2489E-02 (0.4863E-06)	0.2226E-02 (0.4698E-06)	0.1972E-02 (0.4527E-06)	0.1697E-02 (0.4805E-06)
S.D.- HE	0.2077E-02 (0.4989E-06)	0.2492E-02 (0.4995E-06)	0.1638E-02 (0.4448E-06)	0.1380E-02 (0.4421E-06)	0.1529E-02 (0.4332E-06)	0.1228E-02 (0.4788E-06)
MEAN OF S.E.- HP	0.1440E-03 (0.1969E-07)	0.1560E-03 (0.2048E-07)	0.1537E-03 (0.1434E-07)	0.1559E-03 (0.1433E-07)	0.1611E-03 (0.1269E-07)	0.1521E-03 (0.1292E-07)
MEAN OF S.E.- HE	0.8800E-04 (0.9264E-08)	0.1042E-03 (0.1060E-07)	0.1015E-03 (0.8597E-08)	0.1137E-03 (0.1004E-07)	0.1278E-03 (0.1127E-07)	0.1320E-03 (0.1091E-07)
RMSE- HP	0.5648E-02 (0.3535E-02)	0.5562E-02 (0.3571E-02)	0.3415E-02 (0.3230E-02)	0.2893E-02 (0.3461E-02)	0.1981E-02 (0.3243E-02)	0.1178E-02 (0.3466E-02)
RMSE- HE	0.2203E-02 (0.3535E-02)	0.2286E-02 (0.3571E-02)	0.1301E-02 (0.3230E-02)	0.1107E-02 (0.3461E-02)	0.1165E-02 (0.3243E-02)	0.1153E-02 (0.3466E-02)
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
|HP-HT| > 2*S.E.	50 (50)	50 (50)	49 (50)	46 (50)	44 (50)	36 (50)
|HE-HT| > 2*S.E.	50 (50)	46 (50)	40 (50)	44 (50)	36 (50)	40 (50)

TABLE 18.--COMPARING THE VALUES OF HP AND HE WITH DIFFERENT NONPOSITIVE CORRELATIONS BETWEEN XITP AND XIOS,ADDITION AND MULTIPLICATION METHODS(MULT. SHOWN IN PARENTHESES), MATRIX IV.

LAMBDA=	0.0	-0.3	-0.5	-0.7	-0.9	-1.0
MEAN- HF(0)	0.5548E-01	0.5424E-01	0.5268E-01	0.5162E-01	0.5058E-01	0.5011E-01
MEAN- HF(T)	0.5393E-01	0.5440E-01	0.5371E-01	0.5368E-01	0.5385E-01	0.5392E-01
MEAN- HT(0)	0.5323E-01	0.5370E-01	0.5302E-01	0.5279E-01	0.5286E-01	0.5314E-01
MEAN- HT(T)	0.5240E-01	0.5273E-01	0.5225E-01	0.5214E-01	0.5218E-01	0.5238E-01
MEAN- HP	0.5336E-01 (0.5000E-01)	0.5219E-01 (0.5000E-01)	0.5135E-01 (0.5000E-01)	0.5065E-01 (0.5000E-01)	0.5025E-01 (0.5000E-01)	0.5008E-01 (0.5000E-01)
MEAN- HE	0.5278E-01 (0.5000E-01)	0.5214E-01 (0.5000E-01)	0.5156E-01 (0.5000E-01)	0.5095E-01 (0.5000E-01)	0.5063E-01 (0.5000E-01)	0.5049E-01 (0.5000E-01)
S.D.- HP	0.1697E-02 (0.4805E-06)	0.1531E-02 (0.4464E-06)	0.7125E-03 (0.4818E-06)	0.6559E-03 (0.5184E-06)	0.1432E-03 (0.4505E-06)	0.2642E-04 (0.5012E-06)
S.D.- HE	0.1228E-02 (0.4788E-06)	0.1291E-02 (0.4680E-06)	0.7374E-03 (0.4682E-06)	0.8904E-03 (0.4365E-06)	0.3309E-03 (0.4493E-06)	0.1505E-03 (0.4403E-06)
MEAN OF S.E.- HP	0.1521E-03 (0.1292E-07)	0.1633E-03 (0.1792E-07)	0.1233E-03 (0.1317E-07)	0.1045E-03 (0.1435E-07)	0.6444E-04 (0.1531E-07)	0.1344E-04 (0.1642E-07)
MEAN OF S.E.- HE	0.1320E-03 (0.1091E-07)	0.1712E-03 (0.1467E-07)	0.1484E-03 (0.9050E-08)	0.1608E-03 (0.9601E-08)	0.1698E-03 (0.8518E-08)	0.8391E-04 (0.7909E-08)
RMSE- HP	0.1178E-02 (0.3466E-02)	0.1864E-02 (0.3962E-02)	0.1876E-02 (0.3134E-02)	0.2332E-02 (0.3037E-02)	0.2720E-02 (0.2979E-02)	0.3176E-02 (0.3262E-02)
RMSE- HE	0.1153E-02 (0.3466E-02)	0.1892E-02 (0.3962E-02)	0.1657E-02 (0.3134E-02)	0.2027E-02 (0.3037E-02)	0.2337E-02 (0.2979E-02)	0.2759E-02 (0.3262E-02)
HP > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
HE > 2*S.E.	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)	50 (50)
|HP-HT| > 2*S.E.	36 (50)	47 (50)	47 (50)	50 (50)	50 (50)	50 (50)
|HE-HT| > 2*S.E.	40 (50)	45 (50)	48 (50)	49 (50)	50 (50)	50 (50)

TABLE 19.--COMPARING THE MEAN VALUES OF VARIOUS CONCENTRATION INDICES COMPUTED WITH DIFFERENT CORRELATIONS(LAMBDA) BETWEEN XITS AND XIOP, MATRIX I.

LAMBDA	TIME	CR4	CR8	HERFINDAHL	HALL-TIDEMAN	C(ENTROPY)	CCI	NUMB-EQUIV
1.0	0	0.2716	0.4996	0.5344E-01	0.5842E-01	0.1931E 02	0.1686E 00	19
1.0	T	0.2971	0.5349	0.5632E-01	0.6226E-01	0.1871E 02	0.1800E 00	18
0.9	0	0.2751	0.5034	0.5368E-01	0.5876E-01	0.1927E 02	0.1690E 00	19
0.9	T	0.3032	0.5410	0.5677E-01	0.6269E-01	0.1866E 02	0.1821E 00	18
0.7	0	0.2729	0.5023	0.5349E-01	0.5858E-01	0.1930E 02	0.1683E 00	19
0.7	T	0.2937	0.5305	0.5563E-01	0.6147E-01	0.1887E 02	0.1771E 00	18
0.5	0	0.2717	0.4993	0.5348E-01	0.5844E-01	0.1930E 02	0.1683E 00	19
0.5	T	0.2897	0.5255	0.5553E-01	0.6118E-01	0.1890E 02	0.1767E 00	18
0.3	0	0.2748	0.5034	0.5384E-01	0.5892E-01	0.1923E 02	0.1698E 00	19
0.3	T	0.2892	0.5206	0.5523E-01	0.6079E-01	0.1895E 02	0.1753E 00	18
0.0	0	0.2657	0.4951	0.5317E-01	0.5804E-01	0.1936E 02	0.1662E 00	19
0.0	T	0.2735	0.5045	0.5381E-01	0.5898E-01	0.1923E 02	0.1684E 00	19
-0.3	0	0.2746	0.5038	0.5372E-01	0.5887E-01	0.1926E 02	0.1691E 00	19
-0.3	T	0.2734	0.5018	0.5350E-01	0.5857E-01	0.1930E 02	0.1684E 00	19
-0.5	0	0.2754	0.5050	0.5378E-01	0.5890E-01	0.1925E 02	0.1693E 00	19
-0.5	T	0.2693	0.4944	0.5316E-01	0.5791E-01	0.1938E 02	0.1669E 00	19
-0.7	0	0.2729	0.5022	0.5354E-01	0.5860E-01	0.1929E 02	0.1679E 00	19
-0.7	T	0.2607	0.4841	0.5253E-01	0.5704E-01	0.1949E 02	0.1637E 00	19
-0.9	0	0.2710	0.4996	0.5345E-01	0.5848E-01	0.1930E 02	0.1673E 00	19
-0.9	T	0.2539	0.4745	0.5193E-01	0.5605E-01	0.1962E 02	0.1604E 00	19
-1.0	0	0.2704	0.4990	0.5334E-01	0.5834E-01	0.1933E 02	0.1673E 00	19
-1.0	T	0.2487	0.4678	0.5159E-01	0.5545E-01	0.1968E 02	0.1588E 00	19

TABLE 20.--COMPARING THE MEAN VALUES OF VARIOUS CONCENTRATION INDICES COMPUTED WITH DIFFERENT CORRELATIONS(LAMBDA) BETWEEN XITP AND XIOS, MATRIX I.

LAMBDA	TIME	CR4	CR8	HERFINDAHL	HALL-TIDEMAN	C(ENTROPY)	CCI	NUMB-EQUIV
1.0	0	**	**	**	**	**	**	**
1.0	T	**	**	**	**	**	**	**
0.9	0	0.2883	0.5221	0.5509E-01	0.6068E-01	0.1899E 02	0.1759E 00	18
0.9	T	0.2786	0.5080	0.5394E-01	0.5919E-01	0.1921E 02	0.1707E 00	19
0.7	0	0.2883	0.5227	0.5515E-01	0.6077E-01	0.1897E 02	0.1746E 00	18
0.7	T	0.2802	0.5115	0.5427E-01	0.5961E-01	0.1914E 02	0.1713E 00	19
0.5	0	0.2853	0.5187	0.5486E-01	0.6040E-01	0.1903E 02	0.1739E 00	18
0.5	T	0.2821	0.5146	0.5456E-01	0.6000E-01	0.1909E 02	0.1722E 00	18
0.3	0	0.2771	0.5059	0.5394E-01	0.5912E-01	0.1921E 02	0.1703E 00	19
0.3	T	0.2777	0.5076	0.5397E-01	0.5922E-01	0.1920E 02	0.1710E 00	18
0.0	0	0.2675	0.4942	0.5306E-01	0.5792E-01	0.1938E 02	0.1657E 00	19
0.0	T	0.2754	0.5058	0.5384E-01	0.5907E-01	0.1922E 02	0.1691E 00	19
-0.3	0	0.2685	0.4935	0.5303E-01	0.5787E-01	0.1939E 02	0.1665E 00	19
-0.3	T	0.2812	0.5138	0.5449E-01	0.5988E-01	0.1910E 02	0.1724E 00	18
-0.5	0	0.2614	0.4848	0.5253E-01	0.5704E-01	0.1949E 02	0.1636E 00	19
-0.5	T	0.2829	0.5128	0.5435E-01	0.5976E-01	0.1913E 02	0.1723E 00	18
-0.7	0	0.2548	0.4769	0.5209E-01	0.5630E-01	0.1958E 02	0.1613E 00	19
-0.7	T	0.2784	0.5104	0.5421E-01	0.5956E-01	0.1915E 02	0.1707E 00	18
-0.9	0	0.2529	0.4713	0.5182E-01	0.5581E-01	0.1964E 02	0.1605E 00	19
-0.9	T	0.2831	0.5145	0.5460E-01	0.6003E-01	0.1908E 02	0.1732E 00	18
-1.0	0	**	**	**	**	**	**	**
-1.0	T	**	**	**	**	**	**	**

**-INDICATES SUBROUTINE MFSD UNABLE TO COMPUTE A SOLUTION

TABLE 21.--COMPARING THE MEAN VALUES OF VARIOUS CONCENTRATION INDICES COMPUTED WITH DIFFERENT CORRELATIONS(LAMBDA) BETWEEN XITS AND XIOP, MATRIX II.

LAMBDA	TIME	CR4	CR8	HERFINDAHL	HALL-TIDEMAN	C(ENTROPY)	CCI	NUMB-EQUIV
1.0	0	0.2725	0.5025	0.5357E-01	0.5868E-01	0.1928E 02	0.1685E 00	19
1.0	T	0.3002	0.5401	0.5658E-01	0.6268E-01	0.1868E 02	0.1804E 00	18
0.9	0	0.2760	0.5052	0.5369E-01	0.5887E-01	0.1927E 02	0.1698E 00	19
0.9	T	0.3017	0.5413	0.5667E-01	0.6276E-01	0.1866E 02	0.1811E 00	18
0.7	0	0.2799	0.5088	0.5410E-01	0.5923E-01	0.1920E 02	0.1712E 00	19
0.7	T	0.3025	0.5379	0.5661E-01	0.6243E-01	0.1871E 02	0.1818E 00	18
0.5	0	0.2731	0.5028	0.5364E-01	0.5876E-01	0.1927E 02	0.1681E 00	19
0.5	T	0.2903	0.5287	0.5558E-01	0.6139E-01	0.1887E 02	0.1760E 00	18
0.3	0	0.2750	0.5047	0.5370E-01	0.5883E-01	0.1926E 02	0.1692E 00	19
0.3	T	0.2887	0.5233	0.5520E-01	0.6087E-01	0.1895E 02	0.1756E 00	18
0.0	0	0.2708	0.4994	0.5346E-01	0.5845E-01	0.1930E 02	0.1674E 00	19
0.0	T	0.2766	0.5073	0.5406E-01	0.5925E-01	0.1918E 02	0.1704E 00	19
-0.3	0	0.2742	0.5025	0.5367E-01	0.5870E-01	0.1927E 02	0.1689E 00	19
-0.3	T	0.2728	0.5002	0.5345E-01	0.5849E-01	0.1931E 02	0.1686E 00	19
-0.5	0	0.2747	0.5020	0.5375E-01	0.5876E-01	0.1925E 02	0.1697E 00	19
-0.5	T	0.2675	0.4922	0.5305E-01	0.5779E-01	0.1939E 02	0.1666E 00	19
-0.7	0	0.2757	0.5060	0.5377E-01	0.5896E-01	0.1925E 02	0.1694E 00	19
-0.7	T	0.2645	0.4894	0.5272E-01	0.5738E-01	0.1946E 02	0.1646E 00	19
-0.9	0	0.2710	0.5003	0.5349E-01	0.5853E-01	0.1930E 02	0.1674E 00	19
-0.9	T	0.2514	0.4736	0.5188E-01	0.5597E-01	0.1962E 02	0.1594E 00	19
-1.0	0	0.2735	0.5025	0.5360E-01	0.5865E-01	0.1928E 02	0.1685E 00	19
-1.0	T	0.2507	0.4698	0.5165E-01	0.5554E-01	0.1967E 02	0.1593E 00	19

TABLE 22.--COMPARING THE MEAN VALUES OF VARIOUS CONCENTRATION INDICES COMPUTED WITH DIFFERENT CORRELATIONS(LAMBDA) BETWEEN XITP AND XIOS, MATRIX II.

LAMBDA	TIME	CR4	CR8	HERFINDAHL	HALL-TIDEMAN	C(ENTROPY)	CCI	NUMB-EQUIV
1.0	0	**	**	**	**	**	**	**
1.0	T	**	**	**	**	**	**	**
0.9	0	0.2891	0.5233	0.5518E-01	0.6084E-01	0.1896E 02	0.1754E 00	18
0.9	T	0.2795	0.5103	0.5420E-01	0.5951E-01	0.1916E 02	0.1715E 00	18
0.7	0	0.2882	0.5208	0.5486E-01	0.6050E-01	0.1903E 02	0.1740E 00	18
0.7	T	0.2808	0.5113	0.5421E-01	0.5955E-01	0.1915E 02	0.1712E 00	18
0.5	0	0.2829	0.5146	0.5459E-01	0.6003E-01	0.1908E 02	0.1739E 00	18
0.5	T	0.2806	0.5104	0.5417E-01	0.5946E-01	0.1917E 02	0.1716E 00	18
0.3	0	0.2854	0.5181	0.5469E-01	0.6016E-01	0.1907E 02	0.1738E 00	18
0.3	T	0.2852	0.5185	0.5480E-01	0.6030E-01	0.1904E 02	0.1738E 00	18
0.0	0	0.2766	0.5054	0.5391E-01	0.5909E-01	0.1921E 02	0.1701E 00	19
0.0	T	0.2808	0.5139	0.5443E-01	0.5984E-01	0.1911E 02	0.1719E 00	18
-0.3	0	0.2681	0.4954	0.5314E-01	0.5799E-01	0.1937E 02	0.1665E 00	19
-0.3	T	0.2807	0.5114	0.5435E-01	0.5965E-01	0.1913E 02	0.1723E 00	18
-0.5	0	0.2569	0.4789	0.5218E-01	0.5646E-01	0.1956E 02	0.1615E 00	19
-0.5	T	0.2726	0.5006	0.5359E-01	0.5863E-01	0.1928E 02	0.1679E 00	19
-0.7	0	0.2563	0.4801	0.5225E-01	0.5654E-01	0.1955E 02	0.1616E 00	19
-0.7	T	0.2799	0.5127	0.5439E-01	0.5980E-01	0.1911E 02	0.1713E 00	18
-0.9	0	0.2530	0.4725	0.5180E-01	0.5580E-01	0.1964E 02	0.1597E 00	19
-0.9	T	0.2800	0.5116	0.5421E-01	0.5956E-01	0.1916E 02	0.1716E 00	18
-1.0	0	**	**	**	**	**	**	**
-1.0	T	**	**	**	**	**	**	**

**-INDICATES SUBROUTINE MFSD UNABLE TO COMPUTE A SOLUTION

TABLE 23.--COMPARING THE MEAN VALUES OF VARIOUS CONCENTRATION INDICES COMPUTED WITH DIFFERENT CORRELATIONS(LAMBDA) BETWEEN XITS AND XIOP, MATRIX III.

LAMBDA	TIME	CR4	CR8	HERFINDAHL	HALL-TIDEMAN	C(ENTROPY)	CCI	NUMB-EQUIV
1.0	0	0.2860	0.5224	0.5513E-01	0.6078E-01	0.1894E 02	0.1742E 00	18
1.0	T	0.2938	0.5398	0.5666E-01	0.6272E-01	0.1863E 02	0.1796E 00	18
0.9	0	0.2890	0.5264	0.5532E-01	0.6109E-01	0.1892E 02	0.1756E 00	18
0.9	T	0.3025	0.5433	0.5682E-01	0.6293E-01	0.1861E 02	0.1805E 00	18
0.7	0	0.2900	0.5253	0.5548E-01	0.6116E-01	0.1887E 02	0.1770E 00	18
0.7	T	0.2932	0.5286	0.5572E-01	0.6158E-01	0.1883E 02	0.1774E 00	18
0.5	0	0.2897	0.5252	0.5549E-01	0.6110E-01	0.1889E 02	0.1768E 00	18
0.5	T	0.2904	0.5277	0.5551E-01	0.6127E-01	0.1889E 02	0.1764E 00	18
0.3	0	0.2868	0.5220	0.5521E-01	0.6080E-01	0.1892E 02	0.1746E 00	18
0.3	T	0.2807	0.5137	0.5447E-01	0.5994E-01	0.1908E 02	0.1717E 00	18
0.0	0	0.2924	0.5283	0.5558E-01	0.6139E-01	0.1885E 02	0.1771E 00	18
0.0	T	0.2755	0.5049	0.5381E-01	0.5894E-01	0.1923E 02	0.1698E 00	19
-0.3	0	0.2862	0.5219	0.5519E-01	0.6084E-01	0.1892E 02	0.1746E 00	18
-0.3	T	0.2603	0.4842	0.5248E-01	0.5694E-01	0.1950E 02	0.1635E 00	19
-0.5	0	0.2841	0.5174	0.5476E-01	0.6030E-01	0.1902E 02	0.1737E 00	18
-0.5	T	0.2500	0.4703	0.5179E-01	0.5575E-01	0.1964E 02	0.1597E 00	19
-0.7	0	0.2869	0.5197	0.5504E-01	0.6060E-01	0.1897E 02	0.1749E 00	18
-0.7	T	0.2408	0.4570	0.5113E-01	0.5450E-01	0.1977E 02	0.1553E 00	20
-0.9	0	0.2864	0.5233	0.5525E-01	0.6092E-01	0.1892E 02	0.1748E 00	18
-0.9	T	0.2243	0.4343	0.5043E-01	0.5265E-01	0.1991E 02	0.1502E 00	20
-1.0	0	0.2902	0.5247	0.5527E-01	0.6097E-01	0.1893E 02	0.1764E 00	18
-1.0	T	0.2105	0.4143	0.5007E-01	0.5105E-01	0.1999E 02	0.1458E 00	20

TABLE 24.--COMPARING THE MEAN VALUES OF VARIOUS CONCENTRATION INDICES COMPUTED WITH DIFFERENT CORRELATIONS(LAMBDA) BETWEEN XITP AND XIOS, MATRIX III.

LAMBDA	TIME	CR4	CR8	HERFINDAHL	HALL-TIDEMAN	C(ENTROPY)	CCI	NUMB-EQUIV
1.0	0	0.3314	0.5860	0.6163E-01	0.6844E-01	0.1756E 02	0.1963E 00	16
1.0	T	0.2724	0.5006	0.5351E-01	0.5854E-01	0.1929E 02	0.1684E 00	19
0.9	0	0.3207	0.5715	0.6007E-01	0.6661E-01	0.1788E 02	0.1920E 00	17
0.9	T	0.2730	0.5023	0.5372E-01	0.5882E-01	0.1923E 02	0.1690E 00	19
0.7	0	0.3166	0.5602	0.5891E-01	0.6518E-01	0.1817E 02	0.1899E 00	17
0.7	T	0.2732	0.5032	0.5370E-01	0.5880E-01	0.1925E 02	0.1692E 00	19
0.5	0	0.3125	0.5560	0.5808E-01	0.6444E-01	0.1836E 02	0.1861E 00	17
0.5	T	0.2723	0.5010	0.5360E-01	0.5865E-01	0.1926E 02	0.1688E 00	19
0.3	0	0.3025	0.5455	0.5699E-01	0.6321E-01	0.1857E 02	0.1811E 00	18
0.3	T	0.2680	0.4977	0.5334E-01	0.5832E-01	0.1931E 02	0.1662E 00	19
0.0	0	0.2877	0.5223	0.5524E-01	0.6085E-01	0.1892E 02	0.1754E 00	18
0.0	T	0.2773	0.5050	0.5385E-01	0.5900E-01	0.1922E 02	0.1699E 00	19
-0.3	0	0.2775	0.5070	0.5408E-01	0.5922E-01	0.1918E 02	0.1707E 00	19
-0.3	T	0.2726	0.5015	0.5361E-01	0.5863E-01	0.1927E 02	0.1686E 00	19
-0.5	0	0.2630	0.4895	0.5277E-01	0.5746E-01	0.1944E 02	0.1642E 00	19
-0.5	T	0.2677	0.4937	0.5303E-01	0.5788E-01	0.1938E 02	0.1659E 00	19
-0.7	0	0.2475	0.4657	0.5153E-01	0.5527E-01	0.1969E 02	0.1586E 00	19
-0.7	T	0.2737	0.5006	0.5356E-01	0.5861E-01	0.1928E 02	0.1687E 00	19
-0.9	0	0.2314	0.4444	0.5067E-01	0.5340E-01	0.1987E 02	0.1526E 00	20
-0.9	T	0.2719	0.5011	0.5344E-01	0.5851E-01	0.1931E 02	0.1675E 00	19
-1.0	0	0.2127	0.4178	0.5011E-01	0.5132E-01	0.1998E 02	0.1463E 00	20
-1.0	T	0.2775	0.5058	0.5401E-01	0.5923E-01	0.1918E 02	0.1708E 00	19

TABLE 25.--COMPARING THE MEAN VALUES OF VARIOUS CONCENTRATION INDICES COMPUTED WITH DIFFERENT CORRELATIONS(LAMBDA) BETWEEN XITS AND XIOP, MATRIX IV.

LAMBDA	TIME	CR4	CR8	HERFINDAHL	HALL-TIDEMAN	C(ENTROPY)	CCI	NUMB-EQUIV
1.0	0	**	**	**	**	**	**	**
1.0	T	**	**	**	**	**	**	**
0.9	0	0.2933	0.5260	0.5564E-01	0.6135E-01	0.1884E 02	0.1777E 00	18
0.9	T	0.3073	0.5508	0.5757E-01	0.6384E-01	0.1845E 02	0.1837E 00	17
0.7	0	0.2936	0.5283	0.5572E-01	0.6143E-01	0.1884E 02	0.1787E 00	18
0.7	T	0.3039	0.5399	0.5679E-01	0.6273E-01	0.1864E 02	0.1815E 00	18
0.5	0	0.2921	0.5246	0.5536E-01	0.6103E-01	0.1892E 02	0.1771E 00	18
0.5	T	0.2984	0.5337	0.5612E-01	0.6197E-01	0.1879E 02	0.1806E 00	18
0.3	0	0.2880	0.5222	0.5529E-01	0.6081E-01	0.1894E 02	0.1759E 00	18
0.3	T	0.2907	0.5265	0.5558E-01	0.6123E-01	0.1887E 02	0.1761E 00	18
0.0	0	0.2897	0.5272	0.5551E-01	0.6130E-01	0.1887E 02	0.1765E 00	18
0.0	T	0.2735	0.5093	0.5409E-01	0.5940E-01	0.1917E 02	0.1708E 00	19
-0.3	0	0.2899	0.5249	0.5546E-01	0.6118E-01	0.1886E 02	0.1752E 00	18
-0.3	T	0.2675	0.4922	0.5305E-01	0.5783E-01	0.1939E 02	0.1665E 00	19
-0.5	0	0.2907	0.5246	0.5531E-01	0.6100E-01	0.1893E 02	0.1759E 00	18
-0.5	T	0.2547	0.4769	0.5208E-01	0.5629E-01	0.1957E 02	0.1609E 00	19
-0.7	0	0.2860	0.5205	0.5512E-01	0.6072E-01	0.1894E 02	0.1742E 00	18
-0.7	T	0.2435	0.4611	0.5128E-01	0.5481E-01	0.1974E 02	0.1571E 00	20
-0.9	0	0.2836	0.5191	0.5487E-01	0.6047E-01	0.1900E 02	0.1731E 00	18
-0.9	T	0.2246	0.4342	0.5042E-01	0.5261E-01	0.1992E 02	0.1500E 00	20
-1.0	0	**	**	**	**	**	**	**
-1.0	T	**	**	**	**	**	**	**

**-INDICATES SUBROUTINE MFSD UNABLE TO COMPUTE A SOLUTION

TABLE 26.--COMPARING THE MEAN VALUES OF VARIOUS CONCENTRATION INDICES COMPUTED WITH DIFFERENT CORRELATIONS(LAMBDA) BETWEEN XITP AND XIOS, MATRIX IV.

LAMBDA	TIME	CR4	CR8	HERFINDAHL	HALL-TIDEMAN	C(ENTROPY)	CCI	NUMB-EQUIV
1.0	0	0.3277	0.5807	0.6105E-01	0.6773E-01	0.1768E 02	0.1953E 00	17
1.0	T	0.2798	0.5090	0.5410E-01	0.5935E-01	0.1918E 02	0.1715E 00	19
0.9	0	0.3287	0.5767	0.6063E-01	0.6725E-01	0.1780E 02	0.1942E 00	17
0.9	T	0.2817	0.5134	0.5439E-01	0.5978E-01	0.1912E 02	0.1725E 00	18
0.7	0	0.3138	0.5596	0.5865E-01	0.6504E-01	0.1821E 02	0.1879E 00	17
0.7	T	0.2732	0.5034	0.5372E-01	0.5886E-01	0.1924E 02	0.1690E 00	19
0.5	0	0.3102	0.5553	0.5812E-01	0.6449E-01	0.1831E 02	0.1852E 00	17
0.5	T	0.2782	0.5100	0.5412E-01	0.5947E-01	0.1916E 02	0.1706E 00	19
0.3	0	0.3014	0.5417	0.5684E-01	0.6292E-01	0.1859E 02	0.1816E 00	18
0.3	T	0.2764	0.5064	0.5414E-01	0.5928E-01	0.1915E 02	0.1715E 00	19
0.0	0	0.2917	0.5256	0.5548E-01	0.6120E-01	0.1889E 02	0.1772E 00	18
0.0	T	0.2772	0.5045	0.5393E-01	0.5911E-01	0.1920E 02	0.1708E 00	19
-0.3	0	0.2802	0.5106	0.5424E-01	0.5952E-01	0.1914E 02	0.1721E 00	18
-0.3	T	0.2805	0.5136	0.5440E-01	0.5984E-01	0.1911E 02	0.1721E 00	18
-0.5	0	0.2632	0.4874	0.5268E-01	0.5726E-01	0.1946E 02	0.1646E 00	19
-0.5	T	0.2755	0.5043	0.5371E-01	0.5888E-01	0.1925E 02	0.1688E 00	19
-0.7	0	0.2462	0.4667	0.5162E-01	0.5543E-01	0.1967E 02	0.1579E 00	19
-0.7	T	0.2751	0.5030	0.5368E-01	0.5871E-01	0.1927E 02	0.1698E 00	19
-0.9	0	0.2301	0.4416	0.5054E-01	0.5316E-01	0.1988E 02	0.1516E 00	20
-0.9	T	0.2761	0.5053	0.5385E-01	0.5901E-01	0.1922E 02	0.1701E 00	19
-1.0	0	0.2129	0.4180	0.5011E-01	0.5131E-01	0.1998E 02	0.1463E 00	20
-1.0	T	0.2773	0.5096	0.5392E-01	0.5924E-01	0.1921E 02	0.1695E 00	19

V. CONCLUSIONS

Of the two methods of calculation (addition/multiplication), it appears that the addition method gives the best and most realistic results for HP and HE. The multiplication method seems to give results which are questionable at best.

Since the mean values for HP and HE are always greater than two standard errors, HP and HE are significantly different from zero. The fact that (for the addition method), the number of times $|HP - HT|$, $|HE - HT|$ is greater than two standard errors increases as λ diverges from zero, indicates that violating the model's assumptions regarding λ has a serious effect on indices HP and HE. Based on the data presented in Tables 3-18, it appears that when assumptions (2-6) and (2-7) are violated, the indices HP and HE are not good estimators (that is, $|HP - HE|$, $|HE - HT|$ exceed two standard errors a large number of times).

Furthermore, even when $\lambda = 0$, it appears that HP and HE are not good estimators. This is due to the large number of times that the absolute values of the estimated parameters minus the true parameters ($|HE - HT|$, $|HP - HT|$) exceed two standard errors when $\lambda = 0$. Since this number increases as λ increases absolutely, it appears that HP and HE go from "bad" to "worse" as λ diverges from zero.

One of the reasons for this is the small size of the standard errors. Standard errors tend to be an underestimate of the true

deviation. This, coupled with the limitations due to the methodology, make the indices (HP and HE) look worse than they really are. The small magnitude of the resultant variances coupled with the small sample size result in a sample error which is large relative to the variances. However, the total sample errors are not large enough to be significant or to explain the poor results given by HP and HE.

Despite the fact that, statistically speaking, HP and HE are not good estimators, the tables seem to indicate that they are good "ballpark figures" when $\lambda = 0$. This is especially true when there are a large number of samples and a mean value can be computed. This can be seen in Tables 3-18 where the absolute value of mean HP or HE minus HT ($|\overline{HP} - \overline{HT}|$ or $|\overline{HE} - \overline{HT}|$) decreases as λ approaches zero. In fact, when $\lambda = 0$, the mean value of HP (and HE) is usually very close to the mean value of HT (0).

Since (a) $r(x_{itp}, x_{its}) = 0$ and $r(x_{its}, x_{i0p}) = 0$ are unrealistic assumptions and (b) HP and HE go from "bad" to "worse" when assumption (a) is violated, then it appears that (c) HP and HE (addition method), are not good estimators.

It appears that the multiplication method is less desirable than the addition method. It should be stressed that the fact that the means of HP and HE are always constant and equal to 0.05 is not a result of an error in the computer program which produced Tables 3-18. When mean HP, HE are computed to eight digits it can be seen that they are not exactly equal. A similar statement can be made regarding root mean square error.

The fact that the multiplication method gives nearly identical results for different inputs would imply that the multiplication method is not very sensitive to changes in input. Part of the

reason for this lack of sensitivity may be due to the methodology.

If total market share (X_{i0}, X_{it}) is considered to consist of permanent and transitory components (X_{i0p}, X_{i0s}, X_{itp}, X_{its}), then it is fairly obvious that

$$X_{i0} = X_{i0p} + X_{i0s} \tag{5-1}$$

$$X_{it} = X_{itp} + X_{its}$$

must be true. Converting to deviations from means yields

$$X_{i0} - \overline{X}_{i0} = X_{i0p} + X_{i0s} - (\overline{X_{i0p} + X_{i0s}}) \tag{5-2}$$

$$= (X_{i0p} - \overline{X}_{i0p}) + (X_{i0s} - \overline{X}_{i0s})$$

or

$$x_{i0} = x_{i0p} + x_{i0s} \tag{5-3}$$

$$x_{it} = x_{itp} + x_{its}.$$

Thus, the addition method is straightforward and consistent. Unfortunately, the multiplication method is not so straightforward. If we assume that equation (5-1) is incorrect and that

$$X_{i0} = X_{i0p}X_{i0s} \tag{5-4}$$

$$X_{it} = X_{itp}X_{its}$$

is true then, converting to deviations from means yields

$$X_{i0} - \overline{X}_{i0} = X_{i0p}X_{i0s} - \overline{X_{i0p}X_{i0s}} \tag{5-5}$$

$$X_{it} - \overline{X}_{it} = X_{itp}X_{its} - \overline{X_{itp}X_{its}}.$$

Now if we assume

$$x_{i0} = x_{i0p} \quad x_{i0s} \tag{5-6}$$

$$x_{it} = x_{itp} \quad x_{its}$$

(the multiplication method) then, converting to market shares gives

$$(X_{i0} - \bar{X}_{i0}) = (X_{i0p} - \bar{X}_{i0p})(X_{i0s} - \bar{X}_{i0s}) \tag{5-7}$$

$$= X_{i0p}X_{i0s} - \bar{X}_{i0p}X_{i0s} - \bar{X}_{i0s}X_{i0p} + \bar{X}_{i0p}\bar{X}_{i0s}$$

$$(X_{it} - \bar{X}_{it}) = X_{itp}X_{its} - \bar{X}_{itp}X_{its} - \bar{X}_{its}X_{itp} + \bar{X}_{itp}\bar{X}_{its}$$

Subtracting (5-5) from (5-7) gives

$$0 = \overline{X_{i0p}X_{i0s}} + \bar{X}_{i0p}\bar{X}_{i0s} - \bar{X}_{i0p}X_{i0s} - \bar{X}_{i0s}X_{i0p} \tag{5-8}$$

$$0 = \overline{X_{itp}X_{its}} + \bar{X}_{itp}\bar{X}_{its} - \bar{X}_{itp}X_{its} - \bar{X}_{its}X_{itp}$$

which reduces to

$$\overline{X_{i0p}X_{i0s}} + \bar{X}_{i0p}\bar{X}_{i0s} = \bar{X}_{i0p}X_{i0s} + \bar{X}_{i0s}X_{i0p} \tag{5-9}$$

$$\overline{X_{itp}X_{its}} + \bar{X}_{itp}\bar{X}_{its} = \bar{X}_{itp}X_{its} \quad \bar{X}_{its}X_{itp}$$

changing variables, (5-9) becomes

$$k_{01} = k_{02}X_{i0s} + k_{03}X_{i0p} \tag{5-10}$$

$$k_{t1} = k_{t2}X_{itp} + k_{t3}X_{itp}\,.$$

Equation (5-10) requires that X_{i0s} and X_{i0p} define a straight line and that X_{its} and X_{itp} define another straight line. Equation (5-4) does not give us (5-6) unless (5-10) is true. It is highly unlikely that (5-10) will be satisfied. This is in sharp contrast to the manner in which (5-3) is derived from (5-1). Another difficulty with the multiplication method is that (5-1) is essentially a precondition and cannot be changed. It is impossible to derive (5-6) from (5-1).

Thus, part of the problem with the multiplication model seems to be theoretical: it lacks a consistent theoretical base. In general, the multiplication method is less desirable than the addition method and seems to be of little value.

APPENDIX A

THE DATA NORMALIZATION PROCEDURE AND ITS EFFECTS

The marketplace imposes the following restrictions on the Grossack Model:

$$0 \leqslant X_{i0p} \leqslant 1 \qquad \text{(A-1)}$$

$$0 \leqslant X_{i0s} \leqslant 1$$

$$0 \leqslant X_{itp} \leqslant 1$$

$$0 \leqslant X_{its} \leqslant 1$$

$$\sum_{i=1}^{n} X_{i0} = \sum_{i=1}^{n} (X_{i0p} + X_{i0s}) = 1 \qquad \text{(A-2)}$$

$$\sum_{i=1}^{n} X_{it} = \sum_{i=1}^{n} (X_{itp} + X_{its}) = 1 .$$

These restrictions are a direct result of the structure of the marketplace and must not be violated. No firm can have a negative market share, nor can a firm have more than 100% of the market, hence (A-1). Similarly, the sum total market share for all firms in a particular market must equal 1 (the entire market), hence (A-2).

Now initially

$$X = PS \qquad \text{(A-3)}$$

where X, P, and S are as in (3-10). Clearly, the X matrix, when generated as in (A-3), will not automatically conform to (A-1) and (A-2). Subsequently, the following transformations,

referred to as the normalization procedure, are required to make the X matrix satisfy (A-1) and (A-2).

Given

$$X = \begin{bmatrix} \ldots X_{i0p} \ldots \\ \ldots X_{i0s} \ldots \\ \ldots X_{itp} \ldots \\ \ldots X_{its} \ldots \end{bmatrix}, \tag{A-4}$$

the data is transformed to meet the restrictions as follows:

1. Find $z = \text{MIN}\left\{ X_{i0p}, X_{i0s}, X_{itp}, X_{its} \right\}$ $i = 1, 2, \ldots, n$ (A-5)

Now if $z \leqslant 0$, Set $a = 0$

if $z > 0$, Set $a = | z |$

2. Add a to every element of the X matrix:

$$X' = \begin{bmatrix} \ldots (X_{i0p} + a) \ldots \\ \ldots (X_{i0s} + a) \ldots \\ \ldots (X_{itp} + a) \ldots \\ \ldots (X_{its} + a) \ldots \end{bmatrix} \tag{A-6}$$

This imposes the restriction on the market shares that they be positive or zero (eliminating negative market shares).

3. Compute market share totals:

$$S_0 = \sum_{i=1}^{n} (X'_{i0p} + X'_{i0s}) \tag{A-7}$$

$$S_t = \sum_{i=1}^{n} (X'_{itp} + X'_{its}) \tag{A-8}$$

4. Scale the X' matrix to 1:

$$X^* = \begin{bmatrix} \ldots(X'_{i0p}/S_0)\ldots \\ \ldots(X'_{i0s}/S_0)\ldots \\ \ldots(X'_{itp}/S_t)\ldots \\ \ldots(X'_{its}/S_t)\ldots \end{bmatrix} \qquad \text{(A-9)}$$

(S_0 and S_t are referred to as scale factors). The data in the X^* matrix meets restrictions (A-1) and (A-2).

5. Compute row means:

$$\mu_1 = \frac{1}{n} \sum_{i=1}^{n} X^*_{i0p} \qquad \text{(A-10)}$$

$$\mu_2 = \frac{1}{n} \sum_{i=1}^{n} X^*_{i0s}$$

$$\mu_3 = \frac{1}{n} \sum_{i=1}^{n} X^*_{itp}$$

$$\mu_4 = \frac{1}{n} \sum_{i=1}^{n} X^*_{its}$$

6. Compute deviations from row means:

$$[x^*_i] = \begin{bmatrix} \ldots(X^*_{i0p} - \mu_1)\ldots \\ \ldots(X^*_{i0s} - \mu_2)\ldots \\ \ldots(X^*_{itp} - \mu_3)\ldots \\ \ldots(X^*_{its} - \mu_4)\ldots \end{bmatrix}. \qquad \text{(A-11)}$$

The $[x^*_i]$ matrix of (A-11) represents the matrix used by the computer program to compute the Grossack Model Indices.

Since the purpose of this paper is to test the behavior of the Grossack Model Indices when assumptions (2-6) and (2-7) are violated, it is important to note what effect, if any, the normalization procedure will have on the correlations.

Rephrased, the question is, if we generate (unrestricted) values for X_{i0p}, X_{i0s}, X_{itp}, X_{its} with specific correlations using (A-3), will the corresponding correlations of the normalized x_{i0p}, x_{i0s}, x_{itp}, x_{its} be the same? The answer is that the normalization procedure preserves correlations, but does not preserve the variances and covariances of the X matrix of (A-4).

Consider two sets of data, X and Y, where

$$X = \left\{ X_1, X_2, \ldots, X_n \right\} \quad 0 < n < \infty, \text{ n an integer} \tag{A-12}$$

$$Y = \left\{ Y_1, Y_2, \ldots, Y_n \right\}$$

$$r(X, Y) = e, \quad e \neq 0, \quad e \text{ a constant.}$$

Now let

$$X' = \left\{ X_1 + c, X_2 + c, \ldots, X_n + c \right\}$$

$$Y' = \left\{ Y_1 + c, Y_2 + c, \ldots, Y_n + c \right\}$$

where c is a constant, $c \neq 0$. By definition,

$$r(X', Y') = \frac{\sigma_{X'Y'}}{\sigma_{X'} \sigma_{Y'}} \tag{A-13}$$

with

$$\sigma_{X'Y'} = \frac{1}{n} \sum_{i=1}^{n} (X_i + c - \overline{X_i + c})(Y_i + c - \overline{Y_i + c}) \tag{A-14}$$

$$= \frac{1}{n} \sum_{i=1}^{n} (X_i - \overline{X}_i)(Y_i - \overline{Y}_i) = \sigma_{XY}.$$

Similarly,

$$\sigma_{X'} = \sigma_X \tag{A-15}$$

$$\sigma_{Y'} = \sigma_{Y'}$$

which gives

$$r(X', Y') = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = r(X, Y). \tag{A-16}$$

Thus, step 2 will have no effect on the correlations.

Now let

$$X^* = \left\{ X_1/c_2, \ldots, X_n/c_2 \right\}, c_2 \text{ a constant, } c_2 \neq 0 \qquad \text{(A-17)}$$

$$Y^* = \left\{ Y_1/c_3, \ldots, Y_n/c_3 \right\}, c_3 \text{ a constant, } c_3 \neq 0$$

then

$$r(X^*, Y^*) = \frac{\frac{1}{n} \Sigma \frac{1}{c_2} (X_i - \overline{X}) \frac{1}{c_3} (Y_i - \overline{Y})}{\left[\frac{1}{n} \Sigma \frac{(X_i - \overline{X})^2}{c_2} \; \frac{1}{n} \Sigma \frac{(Y_i - \overline{Y})^2}{c_3} \right]^{1/2}} \qquad \text{(A-18)}$$

$$= \frac{\frac{1}{c_2 c_3} \Sigma (X_i - \overline{X})(Y_i - \overline{Y})}{\frac{1}{c_2 c_3} \left[\Sigma (X_i - \overline{X})^2 \Sigma (Y_i - \overline{Y})^2 \right]^{1/2}}$$

$$= \frac{\sigma_{XY}}{\sigma_X \, \sigma_Y} = r\,(X, Y).$$

Hence,

$$r(X, Y) = r(X', Y') = r(X^*, Y^*) \qquad \text{(A-19)}$$

and step 4 will have no effect on the correlations.

Now let x, y represent the X^*, Y^* deviations from means:

$$x = \left\{ x_1, x_2, \ldots, x_n \right\}, \quad x_i = X_i^* - \overline{X}^* \qquad \text{(A-20)}$$

$$y = \left\{ y_1, y_2, \ldots, y_n \right\}, \quad y_i = Y_i^* - \overline{Y}^*.$$

Thus,

$$r\,(x, y) = \frac{\Sigma (x_i - \overline{x})(y_i - \overline{y})}{\left[\Sigma (x_i - \overline{x})^2 \Sigma (y_i - \overline{y})^2 \right]^{1/2}}. \qquad \text{(A-21)}$$

But, x, y are deviations from means, hence,

$$\Sigma\, x_i = 0 \qquad\qquad \Sigma\, y_i = 0 \tag{A-22}$$

$$\bar{x} = \frac{1}{n}\ \Sigma\, x_i = 0 \qquad \bar{y} = \frac{1}{n}\ \Sigma\, y_i = 0.$$

Then,

$$r(x, y) = \frac{\Sigma x_i y_i}{\left[\Sigma x_i^{\,2}\ \ \Sigma y_i^{\,2}\right]^{1/2}} \tag{A-23}$$

$$= \frac{\Sigma(X_i^* - \bar{X}^*)\,(Y_i^* - \bar{Y}^*)}{\left[\Sigma(X_i^* - \bar{X}^*)^2\ \Sigma(Y_i^* - \bar{Y}^*)^2\right]^{1/2}}$$

$$= \ r(X^*, Y^*)\ = r(X, Y).$$

Thus, step 6 has no effect on the correlations. Combining (A-24) and (A-19) gives.

$$r(X, Y) = r(X', Y') = r(X^*, Y^*) = r(x, y). \tag{A-24}$$

Thus, the normalization procedure and the conversion of the X^* matrix to deviations from means has no effect on the correlations.

Although the normalization procedure has no effect on the correlations, it does affect the variances and covariances. If $X, Y, X', Y', X^*, Y^*, x, y$ are defined as before, then:

$$\sigma_X = \sigma_X{}' \tag{A-25}$$

$$\sigma_Y = \sigma_Y{}'$$

$$\sigma_{XY} = \sigma_{X'Y'}$$

$$\sigma_X{}^* = \sigma_X \,/\, c_2 \tag{A-26}$$

$$\sigma_Y{}^* = \sigma_Y \,/\, c_3$$

$$\sigma_{X^*Y^*} = \sigma_{XY}/c_2\, c_3$$

$$\sigma_x = \sigma_{X^*} \tag{A-27}$$

$$\sigma_y = \sigma_{Y^*}$$

$$\sigma_{xy} = \sigma_{X^* Y^*} .$$

Because the variances and covariances are not preserved in the normalization procedure, the covariance matrix of the resultant [x] matrix is derived and a mean value is computed. The mean resultant x variances derived from each matrix computed with a given set of X variances is presented in Table 2.

APPENDIX B

COMPUTER PROGRAM DOCUMENTATION

Objectives and Performance Requirements

The objective of the FORTRAN Program provided in this Appendix is to generate the data and perform the mathematical analysis of Chapter III in an efficient manner. The program performs the following specific functions:

1. Compute the covariance matrix M using given (input) variances and correlations.
2. Compute the triangular factorization of the M matrix, $M = PP^T$, by the square root method of Cholesky (SUBROUTINE MFSD).
3. Generate a matrix of random normal deviates (SUBROUTINE GAUSS), which is then pre-multiplied by the P matrix to produce a matrix of X values with specific variances and covariances.
4. Normalize and convert the X's to market shares. Compute gross market shares (X_{i0}, X_{it}).
5. Compute expected values for the following concentration indices: CR_4, CR_8, Herfindahl, Hall-Tideman, C (Entropy), CCI (Comprehensive Concentration Index), and N (Numbers-Equivalent). Save the computed values of the indices for print.
6. Convert the X's to deviations from (row) means: x_{i0p}, x_{i0s}, x_{itp}, x_{its}. Compute and print mean covariance matrix of normalized [x] matrix.

7. Compute x_{i0}, x_{it} by the addition and multiplication methods. Regress x_{it} on x_{i0}. Compute b, r and standard errors.

8. For each sample, compute HT, HP, HE and associated standard errors.

9. Repeat steps 3-8 to generate 50 sets of values for HT, HP, HE, and the standard errors.

10. For the set of indices computed in step 9, compute means, root mean square error, mean standard error, and standard deviations. Test the model indices against their standard errors.

11. Save the values computed in step 10 for print.

12. Repeat steps 1-11 for a specified number of matrices.

13. Print the values computed in steps 5 and 10 in an attractive tabular format (Tables 3-26).

Input/Output Requirements

All input to the program is via cards. The following inputs are required:

Initial input:

Card No. 1 - Initial random number for SUBROUTINE GAUSS

2 - Number of samples (of 20) to compute per matrix

3 - Variances to be used in computing M matrix

Input for each matrix (22 sets):

Card No. 1 - J = number of correlations of Grossack Model which are to be changed. ($0 \leqslant J \leqslant 6.$) If J = 0, no additional cards are needed for that particular matrix

2 - Identification number of correlation to be changed and new value of correlation.

The correlation identification number is determined as follows:

ID No.	Correlation	Grossack Model Value
1	$r\ (x_{i0s}, x_{i0p})$	0
2	$r\ (x_{itp}, x_{i0p})$	1
3	$r\ (x_{its}, x_{i0p})$	0
4	$r\ (x_{itp}, x_{i0s})$	0
5	$r\ (x_{its}, x_{i0s})$	0
6	$r\ (x_{its}, x_{itp})$	0

The program uses the Grossack Model correlation values unless the user specifies otherwise. Caution is advised when changing the correlations, since correlations are transitive. That is, if $r(x, y) = 1$ and $r\ (y, z) = c$ (c a constant, $0 \leqslant |c| \leqslant 1$), then $r(x, z) = c$ also.

All output from the program is sent to the line printer.

Subroutines Required

The following subroutines are required:

(1) MFSD - Triangular factorization of a symmetric positive definite matrix

(2) RANDU - Random number generator

(3) GAUSS - Generates random normal deviates

(4) SAVIDX - Computes means of alternate concentration indices

(5) MATCOV - Computes and prints mean covariance matrix of the normalized [x] matrix.

Source listings of SUBROUTINE SAVIDX and SUBROUTINE MATCOV are provided in this appendix. MFSD, RANDU,

and GAUSS are part of the IBM System/360 Scientific Subroutine Package. For detailed information on these routines, including matrix storage requirements for MFSD, please refer to reference [9].

Variable Identification

The following information is provided in order to provide a link between the mathematics section and the program code:

Variable Name	Description
IX	Odd random integer used to initialize subroutine Gauss
NPTS	Number of samples to compute for each matrix
SCORR(K)	Grossack Model correlations
CORR(K)	Input correlations
M(I, J)	Initial covariance matrix
P(I, J)	Triangular factorization of M matrix
PD(K)	Dummy array used to clear P matrix
RDM(I, J)	Random deviate matrix
RD(K)	Dummy array used to compute RDM
A(K)	Vector, used to input/output matrices M and P with subroutine MFSD
XI0(K)	First X_{i0} then x_{i0}
XIT(K)	First X_{it} then x_{it}
XR(I, J)	Dummy array used to represent XI0, XIT
RIDX(I, J)	Concentration indices (CR_4, CR_8, H, Hall-Tideman, C, CCI)
NNN(K)	N = Numbers-Equivalent Index
SX0	ΣX_{i0}
SXT	ΣX_{it}

Variable Name	Description
SGXI0	Σx_{i0}
SGXIT	Σx_{it}
SGXI0P	Σx_{i0p}
SGXITP	Σx_{itp}
XI0XIT	$\Sigma x_{i0} x_{it}$
B	b, least-squares coefficient of x_{it} on x_{i0}
R	$r(x_{it}, x_{i0})$ correlation
STERB	Standard error of b ($\hat{sb}$)
STER(I, J, K)	Standard error HP, HE
HT(I, J)	'True' permanent Herfindahl Index
HP(I, J)	Proportionality model Herfindahl Index
HE(I, J)	Equality model Herfindahl Index
H(I, J, K)	Dummy array used to compute HP, HE
MU(I, J)	Mean HP, HE
MHT(J)	Mean HT
RMSE(I, J)	Root mean square error
STDV(I, J)	Standard deviation of HP, HE
ICTR(I, J, K)	Counter - HP, HE versus HT, STER
IPRINT(N, K)	
PRINT(N, K)	
ALPHA(K)	Used for printing tables
TITLE(I, J)	
LABEL(I, J)	

Program Source Listing

A source listing of the FORTRAN Program used to generate Tables 3-26 begins on the next page.

```
COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=66,SIZE=0000K,
                   SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
          C
          C*********************************************************************
          C
          C     MONTE CARLO ANALYSIS OF A PERMANENT INDUSTRIAL CONCENTRATION INDEX
          C
          C     **************      FLORIDA TECHNOLOGICAL UNIVERSITY     **************
          C
          C     PROGRAMMER- T.E.BILLINGS                    DATE- MARCH 1976
          C
          C     THIS PROGRAM GENERATES SYNTHETIC MEASURES OF PERMANENT AND
          C     TRANSITORY MARKET SHARES WITH SPECIFIED CORRELATIONS. THE DATA IS
          C     THEN ADJUSTED TO CONFORM TO SPECIFICATIONS AND CONVERTED TO
          C     DEVIATIONS FROM MEAN PERMANENT/TRANSITORY MARKET SHARE.THE DATA IS
          C     THEN USED TO COMPUTE A 'TRUE' HERFINDAHL INDEX OF PERMANENT
          C     INDUSTRIAL CONCENTRATION AND TWO SEPARATE APPROXIMATIONS OF
          C     THE 'TRUE' INDEX. THE THREE INDICES ARE THEN COMPARED AND
          C     ANALYZED. OTHER CONCENTRATION INDICES ARE COMPUTED AND PRINTED FOR
          C     COMPARISON PURPOSES.
          C
          C
          C     SUBROUTINES REQUIRED
          C     MFSD- TRIANGULAR FACTORIZATION OF A POSITIVE DEFINITE MATRIX(SSP)
          C     GAUSS(RANDU)- RANDOM NUMBER GENERATOR(SSP)
          C     SAVIDX- COMPUTES MEANS OF ALTERNATE CONCENTRATION INDICES
          C     MATCOV- COMPUTES AND PRINTS MEAN RESULTANT COV MATRIX OF X'S
          C
          C     REFERENCES
          C     SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE VERSION III PROGRAMMER'S
          C     MANUAL(IBM MANUAL NO. H20-0205-3), PGS. 77,158.
          C
          C*********************************************************************
          C
          C
ISN 0002   1000 FORMAT(I9)
ISN 0003   1010 FORMAT(4(3X,E14.8))
ISN 0004   1020 FORMAT(' INITIAL RANDOM INTEGER FOR SUB. GAUSS=',I9)
ISN 0005   1030 FORMAT(' NUMBER OF PTS TO PROCESS PER LOOP=',I5)
ISN 0006   1040 FORMAT(' K=',I4)
ISN 0007   1050 FORMAT(1X,I1,5X,E14.8)
ISN 0008   1060 FORMAT(' L=',I1,5X,'CORR(L)=',E14.8)
ISN 0009   1070 FORMAT(' WARNING: SEVERE ERROR IN SUBROUTINE MFSD. ',
               *' CALCULATIONS WITH THIS MATRIX DISCONTINUED.')
ISN 0010   1080 FORMAT(' WARNING: TOLERANCE EXCEEDED IN COMPUTATION OF P MATRIX.',
               *'   IER=',I3)
ISN 0011   1090 FORMAT(5A4,2I2)
ISN 0012   1100 FORMAT('1')
ISN 0013   1110 FORMAT(////////////////////)
ISN 0014   1120 FORMAT(24X,'TABLE ',I2,'.--COMPARING THE MEAN VALUES OF VARIOUS ',
               *'CONCENTRATION INDICES COMPUTED WITH')
ISN 0015   1130 FORMAT(35X,'DIFFERENT CORRELATIONS(LAMBDA) BETWEEN ',A4,' AND ',
               *A4,', MATRIX ',A4,//)
ISN 0016   1140 FORMAT(//,10X,'**-INDICATES SUBROUTINE MFSD UNABLE TO COMPUTE A ',
               *'SOLUTION',////)
ISN 0017   1150 FORMAT(7X,11('-----------'))
ISN 0018   1160 FORMAT(T16,'|',T23,'|',T38,'|',T54,'|',T69,'|',T84,'|',T99,'|',
               *T114,'|')
ISN 0019   1170 FORMAT(7X,'LAMBDA  | TIME |      CR4       |      CR8       |',
               *'   HERFINDAHL  | HALL-TIDEMAN |  C(ENTROPY)  |      CCI      |',
               *'   NUMB-EQUIV')
ISN 0020   1180 FORMAT(T8,F4.1,T16,'|',T20,A1,T23,'|',T28,F6.4,T38,'|',T43,F6.4,
               *T54,'|',T56,E11.4,T69,'|',T71,E11.4,T84,'|',T86,E11.4,T99,'|',
               *T101,E11.4,T114,'|',T121,I2)
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```
ISN 0021        1190 FORMAT(//////////)
ISN 0022        1200 FORMAT(T8,F4.1,T16,'|',T20,A1,T23,'|        **        ',
                    *'|        **        ',5('|        **        '))
ISN 0023        1210 FORMAT(18X,'TABLE ',I2,'.--COMPARING THE VALUES OF HP AND HE ',
                    *'WITH DIFFERENT NON',2A4,' CORRELATIONS BETWEEN ',A4)
ISN 0024        1220 FORMAT(29X,'AND ',A4,',ADDITION AND MULTIPLICATION METHODS(MULT.',
                    *' SHOWN IN PARENTHESES), MATRIX ',A4,/)
ISN 0025        1230 FORMAT(10X,120('-'))
ISN 0026        1240 FORMAT(10X,'LAMBDA=',13X,6('|',5X,F4.1,6X))
ISN 0027        1250 FORMAT(30X,6('|',15X))
ISN 0028        1260 FORMAT(10X,4A4,4X,6('|  ',E11.4,2X))
ISN 0029        1270 FORMAT(30X,6('| (',E11.4,') '))
ISN 0030        1280 FORMAT(10X,4A4,4X,6('|',7X,I2,6X))
ISN 0031        1290 FORMAT(30X,6('|',6X,'(',I2,')',5X))
ISN 0032        1300 FORMAT('+',117X,'**')
ISN 0033        1310 FORMAT(10X,4A4,4X,'|',7X,'**',6X,5('|  ',E11.4,2X))
ISN 0034        1320 FORMAT(30X,'|  ',5X,'**',4X,'  ',5('| (',E11.4,') '))
ISN 0035        1330 FORMAT(10X,4A4,4X,'|',7X,'**',6X,5('|',7X,I2,6X))
ISN 0036        1340 FORMAT(30X,'|',6X,' ** ',5X,5('|',6X,'(',I2,')',5X))
               C
ISN 0037             COMMON IERIND(22),TIND(2,7,22),NIND(2,22)
ISN 0038             DIMENSION P(4,4),X(4,20),RDM(4,20),A(10),XIO(20),XIT(20)
ISN 0039             DIMENSION HT(1000,2),HP(1000,2),HE(1000,2),RMSE(2,2),STDV(4,2)
ISN 0040             DIMENSION ICTR(2,2,2),XE(80),XR(20,2),RIDX(2,7),RDX(14),NNN(2)
ISN 0041             DIMENSION RD(80),STER(1000,2,2),CORR(6),SCORR(6),PD(16),SRDV(4)
ISN 0042             DIMENSION SAVCOR(22),PRINT(22,18),IPRINT(22,8),ICTREQ(8)
ISN 0043             DIMENSION H(1000,2,2),TIME(2),ALPHA(5),TITLE(2,2),LABEL(4,16)
ISN 0044             REAL M(4,4),MU(2,2),MSTER(2,2),MEAN(4),MHT(2)
ISN 0045             EQUIVALENCE (HP(1,1),H(1,1,1)),(HE(1,1),H(1,1,2))
ISN 0046             EQUIVALENCE (RD(1),RDM(1,1)),(PD(1),P(1,1))
ISN 0047             EQUIVALENCE (XE(1),X(1,1)),(XIO(1),XR(1,1)),(XIT(1),XR(1,2))
ISN 0048             EQUIVALENCE (RDX(1),RIDX(1,1)),(SRDV(1),STDV(1,1))
ISN 0049             EQUIVALENCE (ICTREQ(1),ICTR(1,1,1))
ISN 0050             DATA SCORR/0.0E0,1.0E0,4*0.0E0/,TIME/'0','T'/
ISN 0051             DATA TITLE/'NEGA','TIVE','POSI','TIVE'/
ISN 0052             DATA LABEL/'MEAN','- HF','(0) ','    ','MEAN','- HF',
                    *'(T) ','    ','MEAN','- HT','(0) ','    ','MEAN','- HT','(T) ',
                    *'    ','MEAN','- HP',2*'    ','MEAN','- HE',2*'    ','S.D.',
                    *'- HP',2*'    ','S.D.','- HE',2*'    ','MEAN',' OF ','S.E.',
                    *'- HP','MEAN',' OF ','S.E.','- HE','RMSE','- HP',2*'    ','RMSE',
                    *'- HE',2*'    ','HP >',' 2*S','.E. ','    ','HE >',' 2*S','.E. ',
                    *'    ','|HP-','HT| ','> 2*','S.E.','|HE-','HT| ','> 2*','S.E.'/
               C
ISN 0053             READ(5,1000) IX
ISN 0054             READ(5,1000) NPTS
ISN 0055             READ(5,1010) M(1,1),M(2,2),M(3,3),M(4,4)
               C
ISN 0056             WRITE(6,1020) IX
ISN 0057             WRITE(6,1030) NPTS
               C
               C     SET UP CONSTANTS FOR LOOP
               C
ISN 0058             CONS=1.0E0/20.0E0
ISN 0059             RNP=1.0E0/FLOAT(NPTS)
ISN 0060             RNP1=1.0E0/FLOAT(NPTS-1)
ISN 0061             DO 10 J=1,22
ISN 0062          10 IERIND(J)=-1
               C
               C     **************
ISN 0063             DO 220 N=1,22
                     **************
               C
               C     COMPUTE M MATRIX USING SPECIFIED CORRELATIONS AND VARIANCES
               C
```

```
ISN 0064          DO 12 K=1,4
ISN 0065          SRDV(K)=SQRT(M(K,K))
ISN 0066       12 CORR(K)=SCORR(K)
ISN 0067          CORR(5)=SCORR(5)
ISN 0068          CORR(6)=SCORR(6)
ISN 0069          READ(5,1000) K
ISN 0070          WRITE(6,1040) K
ISN 0071          IF(K.EQ.0) SAVCOR(N)=0.0E0
ISN 0073          IF(K.EQ.0) GO TO 14
ISN 0075          DO 14 J=1,K
ISN 0076          READ(5,1050) L,CORR(L)
ISN 0077          WRITE(6,1060) L,CORR(L)
ISN 0078          SAVCOR(N)=CORR(L)
ISN 0079       14 CONTINUE
ISN 0080          K=0
ISN 0081          DO 18 I=1,3
ISN 0082          NN=4-I
ISN 0083          DO 16 J=1,NN
ISN 0084          K=K+1
ISN 0085          M(I,J+I)=CORR(K)*SRDV(I)*SRDV(J+I)
ISN 0086       16 M(J+I,I)=M(I,J+I)
ISN 0087       18 CONTINUE
             C
             C    COMPUTE TRIANGULAR P MATRIX
             C
ISN 0088          DO 25 K=1,16
ISN 0089       25 PD(K)=0.0E0
ISN 0090          A(1)=M(1,1)
ISN 0091          A(2)=M(1,2)
ISN 0092          A(3)=M(2,2)
ISN 0093          DO 40 K=1,4
ISN 0094          A(K+6)=M(K,4)
ISN 0095          IF(K-4) 30,40,40
ISN 0096       30 A(K+3)=M(K,3)
ISN 0097       40 CONTINUE
ISN 0098          IER=0
ISN 0099          EPS=1.0E-5
ISN 0100          NN=4
             C
ISN 0101          CALL MFSD(A,NN,EPS,IER)
             C
ISN 0102          IF(IER.EQ.-1) WRITE(6,1070)
ISN 0104          IF(IER.EQ.-1) GO TO 220
ISN 0106          IF(IER.GT.0) WRITE(6,1080) IER
             C
ISN 0108          P(1,1)=A(1)
ISN 0109          P(2,1)=A(2)
ISN 0110          P(3,1)=A(4)
ISN 0111          P(4,1)=A(7)
ISN 0112          P(2,2)=A(3)
ISN 0113          P(3,2)=A(5)
ISN 0114          P(4,2)=A(8)
ISN 0115          P(3,3)=A(6)
ISN 0116          P(4,3)=A(9)
ISN 0117          P(4,4)=A(10)
             C
             C    GENERATE RANDOM NUMBER MATRIX RDM
             C
ISN 0118          AM=0.0E0
ISN 0119          S=1.0E0
             C
             C    ----------------
ISN 0120          DO 130 NP=1,NPTS
             C    ----------------
```

```
                C
ISN 0121              DO 80 K=1,80
ISN 0122              CALL GAUSS(IX,S,AM,V)
ISN 0123           80 RD(K)=V
                C
                C     COMPUTE MATRIX X=P*RDM
                C
ISN 0124              DO 100 I=1,4
ISN 0125              DO 100 J=1,20
ISN 0126              X(I,J)=0.0E0
ISN 0127              DO 90 K=1,4
ISN 0128           90 X(I,J)=X(I,J)+P(I,K)*RDM(K,J)
ISN 0129          100 CONTINUE
                C
                C     NORMALIZE X MATRIX
                C
ISN 0130              Z=XE(1)
ISN 0131              DO 101 J=2,80
ISN 0132              IF(XE(J).LT.Z) Z=XE(J)
ISN 0134          101 CONTINUE
ISN 0135              IF(Z.GT.0.0E0) Z=0.0E0
ISN 0137              Z=ABS(Z)
ISN 0138              DO 102 J=1,80
ISN 0139          102 XE(J)=XE(J)+Z
ISN 0140              SX0=0.0E0
ISN 0141              SXT=0.0E0
ISN 0142              DO 103 J=1,20
ISN 0143              SX0=SX0+X(1,J)+X(2,J)
ISN 0144          103 SXT=SXT+X(3,J)+X(4,J)
ISN 0145              DO 104 K=1,2
ISN 0146              DO 104 J=1,20
ISN 0147              X(K,J)=X(K,J)/SX0
ISN 0148          104 X(K+2,J)=X(K+2,J)/SXT
                C
                C     DATA IN X MATRIX NOW REPRESENTS MARKET SHARES. COMPUTE
                C     GROSS MARKET SHARES AND SORT.
                C
ISN 0149              DO 106 J=1,20
ISN 0150              XI0(J)=X(1,J)+X(2,J)
ISN 0151          106 XIT(J)=X(3,J)+X(4,J)
ISN 0152              DO 108 J=1,19
ISN 0153              J2=J+1
ISN 0154              DO 108 K=J2,20
ISN 0155              IF(XI0(J).GE.XI0(K)) GO TO 107
ISN 0157              Z=XI0(J)
ISN 0158              XI0(J)=XI0(K)
ISN 0159              XI0(K)=Z
ISN 0160          107 IF(XIT(J).GE.XIT(K)) GO TO 108
ISN 0162              Z=XIT(J)
ISN 0163              XIT(J)=XIT(K)
ISN 0164              XIT(K)=Z
ISN 0165          108 CONTINUE
                C
                C     COMPUTE INDICES: CR4,CR8,HF,HLT,CCI,C,N
                C
ISN 0166              DO 109 J=1,14
ISN 0167          109 RDX(J)=0.0E0
ISN 0168              RIDX(1,1)=XI0(1)+XI0(2)+XI0(3)+XI0(4)
ISN 0169              RIDX(2,1)=XIT(1)+XIT(2)+XIT(3)+XIT(4)
ISN 0170              RIDX(1,2)=RIDX(1,1)+XI0(5)+XI0(6)+XI0(7)+XI0(8)
ISN 0171              RIDX(2,2)=RIDX(2,1)+XIT(5)+XIT(6)+XIT(7)+XIT(8)
ISN 0172              RIDX(1,5)=1.0E0
ISN 0173              RIDX(2,5)=1.0E0
ISN 0174              DO 110 K=1,2
```

```
ISN 0175          DO 110 J=1,20
ISN 0176          Z=FLOAT(J)
ISN 0177          RIDX(K,3)=RIDX(K,3)+XR(J,K)**2
ISN 0178          RIDX(K,4)=RIDX(K,4)+2.0E0*Z*XR(J,K)
ISN 0179          RIDX(K,5)=RIDX(K,5)*((1.0E0/XR(J,K))**XR(J,K))
ISN 0180          IF(J.EQ.1) GO TO 110
ISN 0182          RIDX(K,6)=RIDX(K,6)+(2.0E0-XR(J,K))*(XR(J,K)**2)
ISN 0183      110 CONTINUE
ISN 0184          DO 111 K=1,2
ISN 0185          RIDX(K,4)=(1.0E0/(RIDX(K,4)-1.0E0))
ISN 0186          RIDX(K,6)=RIDX(K,6)+XR(1,K)
ISN 0187          RIDX(K,7)=1.0E0/RIDX(K,3)+0.5E0
ISN 0188      111 NNN(K)=RIDX(K,7)
           C
ISN 0189          CALL SAVIDX(N,NP,NPTS,RIDX,NNN,IER)
           C
           C      CONVERT X MATRIX TO DEVIATIONS FROM (ROW) MEANS.
           C
ISN 0190          DO 114 J=1,4
ISN 0191          MEAN(J)=0.0E0
ISN 0192          DO 114 K=1,20
ISN 0193          MEAN(J)=MEAN(J)+X(J,K)
ISN 0194      114 CONTINUE
ISN 0195          DO 115 J=1,4
ISN 0196          MEAN(J)=MEAN(J)/20.0E0
ISN 0197          DO 115 K=1,20
ISN 0198          X(J,K)=X(J,K)-MEAN(J)
ISN 0199      115 CONTINUE
           C
ISN 0200          CALL MATCOV(N,NP,NPTS,X,M)
           C
           C
           C      COMPUTE XI0, XIT. REGRESS XIT ON XI0.
           C
ISN 0201          DO 130 K=1,2
ISN 0202          DO 119 J=1,20
ISN 0203          IF(K.EQ.2) GO TO 118
ISN 0205          XI0(J)=X(1,J)+X(2,J)
ISN 0206          XIT(J)=X(3,J)+X(4,J)
ISN 0207          GO TO 119
ISN 0208      118 XI0(J)=X(1,J)*X(2,J)
ISN 0209          XIT(J)=X(3,J)*X(4,J)
ISN 0210      119 CONTINUE
ISN 0211          SGXI0=0.0E0
ISN 0212          SGXI0P=0.0E0
ISN 0213          SGXIT=0.0E0
ISN 0214          SGXITP=0.0E0
ISN 0215          XI0XIT=0.0E0
ISN 0216          DO 120 J=1,20
ISN 0217          A1=XI0(J)
ISN 0218          A2=XIT(J)
ISN 0219          SGXI0=SGXI0+A1*A1
ISN 0220          SGXI0P=SGXI0P+X(1,J)**2
ISN 0221          SGXIT=SGXIT+A2*A2
ISN 0222          SGXITP=SGXITP+X(3,J)**2
ISN 0223          XI0XIT=XI0XIT+A1*A2
ISN 0224      120 CONTINUE
ISN 0225          B=XI0XIT/SGXI0
ISN 0226          R=B*SQRT(SGXI0/SGXIT)
ISN 0227          STERB=((1.0E0-(R*R))*SGXIT)/(18.0E0*SGXI0)
ISN 0228          IF(K.EQ.2) GO TO 125
ISN 0230          HT(NP,1)=SGXI0P+CONS
ISN 0231          HT(NP,2)=SGXITP+CONS
ISN 0232      125 HP(NP,K)=(R*SGXI0)+CONS
```

```
      HE(NP,K)=(B*SGXI0)+CONS
      SGSPEC=SGXI0*SQRT(SGXI0)
      STER(NP,1,K)=(STERB*SGSPEC)/SQRT(SGXIT)
      STER(NP,2,K)=STERB*SGXI0
C
C     ----------------
  130 CONTINUE
C     ----------------
C
      DO 140 K=1,2
      MHT(K)=0.0E0
      DO 140 L=1,2
      MSTER(L,K)=0.0E0
      MU(L,K)=0.0E0
      RMSE(L,K)=0.0E0
  140 STDV(L,K)=0.0E0
      DO 150 K=1,2
      DO 150 L=1,NPTS
      MHT(K)=MHT(K)+HT(L,K)
      MU(1,K)=MU(1,K)+HP(L,K)
      MU(2,K)=MU(2,K)+HE(L,K)
      MSTER(1,K)=MSTER(1,K)+STER(L,1,K)
      MSTER(2,K)=MSTER(2,K)+STER(L,2,K)
      RMSE(1,K)=RMSE(1,K)+(HP(L,K)-HT(L,1))**2
  150 RMSE(2,K)=RMSE(2,K)+(HE(L,K)-HT(L,1))**2
      DO 160 K=1,2
      MHT(K)=MHT(K)*RNP
      DO 160 L=1,2
      MU(L,K)=MU(L,K)*RNP
      MSTER(L,K)=MSTER(L,K)*RNP
  160 RMSE(L,K)=SQRT(RMSE(L,K)*RNP)
C
C     COMPUTE STANDARD DEVIATION OF DERIVED INDEXES (HP,HE)
C
      DO 180 K=1,2
      DO 170 L=1,NPTS
      STDV(1,K)=STDV(1,K)+(HP(L,K)-MU(1,K))**2
  170 STDV(2,K)=STDV(2,K)+(HE(L,K)-MU(2,K))**2
      DO 180 L=1,2
  180 STDV(L,K)=SQRT(STDV(L,K)*RNP1)
C
      DO 190 K=1,8
  190 ICTREQ(K)=0
      DO 210 I=1,2
      DO 210 J=1,2
      DO 200 L=1,NPTS
      STERJ=STER(L,J,I)*2.0E0
      HLJ=H(L,I,J)
      IF(HLJ.GT.STERJ) ICTR(1,J,I)=ICTR(1,J,I)+1
  200 IF(ABS(HLJ-HT(L,1)).GT.STERJ) ICTR(2,J,I)=ICTR(2,J,I)+1
  210 CONTINUE
C
C     SAVE DATA FOR PRINT
C
      DO 211 J=1,2
      PRINT(N,J)=MHT(J)
      PRINT(N,J+2)=MU(1,J)
      PRINT(N,J+4)=MU(2,J)
      PRINT(N,J+6)=STDV(1,J)
      PRINT(N,J+8)=STDV(2,J)
      PRINT(N,J+10)=MSTER(1,J)
      PRINT(N,J+12)=MSTER(2,J)
      PRINT(N,J+14)=RMSE(1,J)
      PRINT(N,J+16)=RMSE(2,J)
```

```
ISN 0288                IPRINT(N,J)=ICTR(1,1,J)
ISN 0289                IPRINT(N,J+2)=ICTR(1,2,J)
ISN 0290                IPRINT(N,J+4)=ICTR(2,1,J)
ISN 0291          211   IPRINT(N,J+6)=ICTR(2,2,J)
              C
              C         **************
ISN 0292          220   CONTINUE
              C         **************
              C
              C         PRINT OUTPUTS IN TABLE FORM
              C
ISN 0293                READ(5,1090) ALPHA,IT1,IT2
ISN 0294                WRITE(6,1100)
ISN 0295                DO 240 J=1,2
ISN 0296                N=0
ISN 0297                WRITE(6,1110)
ISN 0298                IT1=IT1+1
ISN 0299                WRITE(6,1120)  IT1
ISN 0300                J2=(J-1)*2+1
ISN 0301                WRITE(6,1130)  ALPHA(J2),ALPHA(J2+1),ALPHA(5)
ISN 0302                DO 225 K=1,2
ISN 0303          225   WRITE(6,1150)
ISN 0304                WRITE(6,1160)
ISN 0305                WRITE(6,1170)
ISN 0306                WRITE(6,1160)
ISN 0307                WRITE(6,1150)
ISN 0308                DO 228 I=1,2
ISN 0309          228   WRITE(6,1160)
ISN 0310                J2=(J-1)*11+1
ISN 0311                J3=J*11
ISN 0312                DO 235 K=J2,J3
ISN 0313                DO 230 I=1,2
ISN 0314                IF(IERIND(K).NE.-1) GO TO 229
ISN 0316                N=-1
ISN 0317                WRITE(6,1200)  SAVCOR(K),TIME(I)
ISN 0318                GO TO 230
ISN 0319          229   WRITE(6,1180)  SAVCOR(K),TIME(I),(TIND(I,JJ,K),JJ=1,6),NIND(I,K)
ISN 0320          230   CONTINUE
ISN 0321          235   WRITE(6,1160)
ISN 0322                WRITE(6,1160)
ISN 0323                WRITE(6,1150)
ISN 0324                IF(N.NE.0) WRITE(6,1140)
ISN 0326                WRITE(6,1100)
ISN 0327          240   CONTINUE
              C
ISN 0328                DO 300 MN=1,2
ISN 0329                MM=(MN-1)*2+1
ISN 0330                DO 300 IPG=1,2
ISN 0331                J2=(MN-1)*11+1
ISN 0332                J2=J2+(IPG-1)*5
ISN 0333                J3=J2+5
ISN 0334                JT=J3
ISN 0335                JS=J2
ISN 0336                IT2=IT2+1
ISN 0337                WRITE(6,1190)
ISN 0338                WRITE(6,1210)  IT2,(TITLE(I,IPG),I=1,2),ALPHA(MM)
ISN 0339                WRITE(6,1220)  ALPHA(MM+1),ALPHA(5)
ISN 0340                DO 250 J=1,2
ISN 0341          250   WRITE(6,1230)
ISN 0342                WRITE(6,1250)
ISN 0343                WRITE(6,1240)  (SAVCOR(J),J=J2,J3)
ISN 0344                WRITE(6,1250)
ISN 0345                WRITE(6,1230)
ISN 0346                WRITE(6,1250)
```

```
ISN 0347          IF(IERIND(JT).EQ.-1) J3=J3-1
ISN 0349          IF(IERIND(JS).EQ.-1) J2=J2+1
ISN 0351          DO 262 J=1,2
ISN 0352          IF(IERIND(JS)) 255,260,260
ISN 0353      255 WRITE(6,1310) (LABEL(I,J),I=1,4),(TIND(J,3,K),K=J2,J3)
ISN 0354          GO TO 262
ISN 0355      260 WRITE(6,1260) (LABEL(I,J),I=1,4),(TIND(J,3,K),K=J2,J3)
ISN 0356          IF(IERIND(JT).EQ.-1) WRITE(6,1300)
ISN 0358      262 CONTINUE
ISN 0359          WRITE(6,1250)
ISN 0360          DO 266 J=1,2
ISN 0361          IF(IERIND(JS)) 263,264,264
ISN 0362      263 WRITE(6,1310) (LABEL(I,J+2),I=1,4),(PRINT(N,J),N=J2,J3)
ISN 0363          GO TO 266
ISN 0364      264 WRITE(6,1260) (LABEL(I,J+2),I=1,4),(PRINT(N,J),N=J2,J3)
ISN 0365          IF(IERIND(JT).EQ.-1) WRITE(6,1300)
ISN 0367      266 CONTINUE
ISN 0368          WRITE(6,1250)
ISN 0369          WRITE(6,1230)
ISN 0370          JJ=4
ISN 0371          DO 280 K=2,16,4
ISN 0372      270 WRITE(6,1250)
ISN 0373          IF(IERIND(JS)) 272,274,274
ISN 0374      272 WRITE(6,1310) (LABEL(I,JJ+1),I=1,4),(PRINT(N,K+1),N=J2,J3)
ISN 0375          WRITE(6,1320) (PRINT(N,K+2),N=J2,J3)
ISN 0376          WRITE(6,1250)
ISN 0377          WRITE(6,1310) (LABEL(I,JJ+2),I=1,4),(PRINT(N,K+3),N=J2,J3)
ISN 0378          WRITE(6,1320) (PRINT(N,K+4),N=J2,J3)
ISN 0379          GO TO 276
ISN 0380      274 WRITE(6,1260) (LABEL(I,JJ+1),I=1,4),(PRINT(N,K+1),N=J2,J3)
ISN 0381          IF(IERIND(JT).EQ.-1) WRITE(6,1300)
ISN 0383          WRITE(6,1270) (PRINT(N,K+2),N=J2,J3)
ISN 0384          IF(IERIND(JT).EQ.-1) WRITE(6,1300)
ISN 0386          WRITE(6,1250)
ISN 0387          WRITE(6,1260) (LABEL(I,JJ+2),I=1,4),(PRINT(N,K+3),N=J2,J3)
ISN 0388          IF(IERIND(JT).EQ.-1) WRITE(6,1300)
ISN 0390          WRITE(6,1270) (PRINT(N,K+4),N=J2,J3)
ISN 0391          IF(IERIND(JT).EQ.-1) WRITE(6,1300)
ISN 0393      276 WRITE(6,1250)
ISN 0394          WRITE(6,1230)
ISN 0395          JJ=JJ+2
ISN 0396      280 CONTINUE
ISN 0397          DO 290 J=1,5,4
ISN 0398          WRITE(6,1250)
ISN 0399          IF(IERIND(JS)) 282,284,284
ISN 0400      282 WRITE(6,1330) (LABEL(I,JJ+1),I=1,4),(IPRINT(N,J),N=J2,J3)
ISN 0401          WRITE(6,1340) (IPRINT(N,J+1),N=J2,J3)
ISN 0402          WRITE(6,1250)
ISN 0403          WRITE(6,1330) (LABEL(I,JJ+2),I=1,4),(IPRINT(N,J+2),N=J2,J3)
ISN 0404          WRITE(6,1340) (IPRINT(N,J+3),N=J2,J3)
ISN 0405          GO TO 286
ISN 0406      284 WRITE(6,1280) (LABEL(I,JJ+1),I=1,4),(IPRINT(N,J),N=J2,J3)
ISN 0407          IF(IERIND(JT).EQ.-1) WRITE(6,1300)
ISN 0409          WRITE(6,1290) (IPRINT(N,J+1),N=J2,J3)
ISN 0410          IF(IERIND(JT).EQ.-1) WRITE(6,1300)
ISN 0412          WRITE(6,1250)
ISN 0413          WRITE(6,1280) (LABEL(I,JJ+2),I=1,4),(IPRINT(N,J+2),N=J2,J3)
ISN 0414          IF(IERIND(JT).EQ.-1) WRITE(6,1300)
ISN 0416          WRITE(6,1290) (IPRINT(N,J+3),N=J2,J3)
ISN 0417          IF(IERIND(JT).EQ.-1) WRITE(6,1300)
ISN 0419      286 JJ=JJ+2
ISN 0420          WRITE(6,1250)
ISN 0421      290 WRITE(6,1230)
ISN 0422          IF((IERIND(JT).EQ.-1).OR.(IERIND(JS).EQ.-1)) WRITE(6,1140)
```

```
   ISN 0424          300 WRITE(6,1100)
                C
   ISN 0425              STOP
   ISN 0426              END
*OPTIONS IN EFFECT*        NAME=  MAIN,OPT=00,LINECNT=66,SIZE=0000K,
*OPTIONS IN EFFECT*        SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
*STATISTICS*      SOURCE STATEMENTS =       425 ,PROGRAM SIZE =     60230
*STATISTICS*  NO  DIAGNOSTICS GENERATED
****** END OF COMPILATION ******
*STATISTICS*  NO  DIAGNOSTICS THIS STEP
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LEVEL 21.8 ( JUN 74 )                                         OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=66,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
                    C
                    C
                    C       SUBROUTINE MATCOV COMPUTES AND PRINTS THE MEAN COVARIANCE MATRICES
                    C       OF THE NORMALIZED X MATRICES FOR EACH CORRELATION. MATCOV ALSO
                    C       COMPUTES THE MEAN VARIANCES OF THE NORMALIZED X MATRICES OVER
                    C       THE ENTIRE RUN.
                    C
   ISN 0002                 SUBROUTINE MATCOV(N,NP,NPTS,X,M)
                    C
   ISN 0003            1000 FORMAT(' M MATRIX AND (AVG) DERIVATIVE USED THIS LOOP(NO.',I4,')')
   ISN 0004            1010 FORMAT(4(1X,E14.8),5X,4(1X,E14.8))
   ISN 0005            1020 FORMAT(' MEAN DERIVATIVE VARIANCES THIS RUN=  ',4(E14.8,5X))
                    C
   ISN 0006                 REAL M(4,4),X(4,20),M1(16),M2(4,4),M3(16),M4(4,4),MEAN(4)
   ISN 0007                 DATA MEAN/4*0.0E0/
   ISN 0008                 EQUIVALENCE (M1(1),M2(1,1)),(M3(1),M4(1,1))
                    C
   ISN 0009                 IF(NP.GT.1) GO TO 100
   ISN 0011                 DO 100 J=1,16
   ISN 0012                 M1(J)=0.0E0
   ISN 0013             100 CONTINUE
   ISN 0014                 DO 110 J=1,16
   ISN 0015             110 M3(J)=0.0E0
   ISN 0016                 DO 150 J=1,4
   ISN 0017                 DO 150 K=1,J
   ISN 0018                 DO 140 L=1,20
   ISN 0019             140 M4(J,K)=M4(J,K)+X(J,L)*X(K,L)/20.0E0
   ISN 0020                 M4(K,J)=M4(J,K)
   ISN 0021             150 CONTINUE
   ISN 0022                 DO 160 J=1,16
   ISN 0023             160 M1(J)=M1(J)+M3(J)
   ISN 0024                 IF(NP.NE.NPTS) RETURN
   ISN 0026                 DO 170 J=1,16
   ISN 0027             170 M1(J)=M1(J)/FLOAT(NPTS)
   ISN 0028                 WRITE(6,1000) N
   ISN 0029                 DO 180 K=1,4
   ISN 0030             180 WRITE(6,1010)  (M(K,I),I=1,4),(M2(K,I),I=1,4)
                    C
   ISN 0031                 DO 190 I=1,4
   ISN 0032             190 MEAN(I)=MEAN(I)+M2(I,I)
   ISN 0033                 IF(N.NE.22) RETURN
   ISN 0035                 DO 200 I=1,4
   ISN 0036             200 MEAN(I)=MEAN(I)/22.0E0
   ISN 0037                 WRITE(6,1020)  (MEAN(I),I=1,4)
   ISN 0038                 RETURN
   ISN 0039                 END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=00,LINECNT=66,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF

*STATISTICS*      SOURCE STATEMENTS =        38 ,PROGRAM SIZE =        1892

*STATISTICS*   NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******
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LEVEL 21.8 ( JUN 74 )                                   OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=66,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
              C
              C
              C        SUBROUTINE SAVIDX COMPUTES MEAN VALUES OF SEVERAL DIFFERENT
              C        CONCENTRATION INDICES COMPUTED FOR EACH CORRELATION AND STORES
              C        THE RESULTS IN COMMON FOR PRINTING.
              C
  ISN 0002             SUBROUTINE SAVIDX(N,NP,NPTS,RIDX,NNN,IER)
              C
  ISN 0003             DIMENSION AVEQ(14),AV1(2,7),RIDX(2,7),NNN(2),NAV(2)
  ISN 0004             COMMON IERIND(22),TIND(2,7,22),NIND(2,22)
  ISN 0005             EQUIVALENCE (AVEQ(1),AV1(1,1))
              C
  ISN 0006             IERIND(N)=IER
  ISN 0007             IF(NP.NE.1) GO TO 110
  ISN 0009             DO 100 J=1,14
  ISN 0010         100 AVEQ(J)=0.0E0
  ISN 0011             NAV(1)=0
  ISN 0012             NAV(2)=0
  ISN 0013         110 DO 120 K=1,2
  ISN 0014             NAV(K)=NAV(K)+NNN(K)
  ISN 0015             DO 120 J=1,7
  ISN 0016             AV1(K,J)=AV1(K,J)+RIDX(K,J)
  ISN 0017         120 CONTINUE
  ISN 0018             IF(NP.NE.NPTS) RETURN
  ISN 0020             RNP=FLOAT(NPTS)
  ISN 0021             DO 130 K=1,2
  ISN 0022             Z=FLOAT(NAV(K))
  ISN 0023             Z=Z/RNP+0.5E0
  ISN 0024             NIND(K,N)=Z
  ISN 0025             DO 130 J=1,7
  ISN 0026             TIND(K,J,N)=AV1(K,J)/RNP
  ISN 0027         130 CONTINUE
              C
  ISN 0028             RETURN
  ISN 0029             END
*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=00,LINECNT=66,SIZE=0000K,
*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
*STATISTICS*     SOURCE STATEMENTS =      28 ,PROGRAM SIZE =      1192
*STATISTICS*  NO  DIAGNOSTICS GENERATED
****** END OF COMPILATION ******
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