Failure Mode Identifications Of Rc Beams Externally Strengthened With

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FAILURE MODE IDENTIFICATIONS OF RC BEAMS EXTERNALLY STRENGTHENED WITH CARBON FIBER REINFORCED COMPOSITE

by

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A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the Department of Civil and Environmental Engineering in the College of Engineering and Computer Science at the University of Central Florida Orlando, Florida

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ABSTRACT

The application of carbon reinforced-fiber polymers (CFRP) to structures is a new development that is still under intense research. However, the rehabilitation or retrofit of damage reinforced concrete members by the external bonding of CFRP is becoming increasingly popular in the construction industry. The objective of the tests presented in this thesis is to study different CFRP designs on the reinforced concrete beams and compare their failure modes. The main goal is to determine the CFRP design on the reinforced concrete beams that result in a progressive and gradual failure mode with enough warning before final failure. Different CFRP designs are investigated and compared with theoretical predictions. A retrofitting concept is also employed in this research. The retrofitting concept is the idea of strengthening cracked structures. The strengthening of the beams performed in the lab is carried out under sustained loads and on previously cracking the beams to simulate the realistic case that is usually faced in practice on the field. The RC beams are strengthened in flexure to double their flexural capacity by applying the adequate amounts of CFRP to the tension face of the beams.

Due to the CFRP strengthening and increasing the strength capacity of the beams, different CFRP anchorage methods are employed to the beams for additional shear reinforcement to ensure flexural failure.

The different CFRP anchorage methods will also be observed for their effectiveness during the debonding and propagation mechanism as well as evaluated for their progressive failure mode.
To my sister, Christabel Elsa Maade O’Riordan-Adjah for her continuous support in spirit (RIP).
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LIST OF ABBREVIATIONS

\(a_1, a_2, \text{ and } a_3\) = Parameters, derived from the equation of bending moment;

\(l\) = External virtual unit load acting on the beam in the direction of \(\delta\);

\(a\) = Depth of equivalent rectangular stress block = \(c^*\beta\);

\(A_{\text{cfrp}}\) = Area of composite [thickness (t) x width of composite (Wa = Wf)];

\(\text{AdhesiveC}\) = Center of gravity of the adhesive;

\(\text{AdhesiveF}\) = Adhesive force;

\(\text{AdhesiveM}\) = Adhesive moment;

\(A_f\) = Area of FRP;

\(A_s\) = Cross sectional area of tension reinforcement bars;

\(A_{sp}\) = Cross sectional area of compression reinforcement bars;

\(b\) = Beam width;

\(b_a\) = Width of adhesive layer;

\(b_f\) = FRP plate width;

\(b_p\) = Width of FRP plate;

\(b_w\) = Beam width

\(c\) = Depth of neutral axis for cracked section;

\(c\) = The distance from the neutral axis to the extreme compression fiber of the concrete;

\(C_c\) = The compressive force of concrete;

\(\text{CompSteelC}\) = Center of gravity of the compression steel;
CompSteelF = Compression steel force;

CompSteelM = Compression steel moment;

ConcreteC = Center of gravity of the concrete section;

ConcreteF = Concrete force;

ConcreteM = Concrete moment;

Cs = Compressive force of the reinforcing steel;

Cw = Composite Width (width of adhesive and fiber);

df = Effective depth of FRP (distance from the centroid of the CFRP to the extreme compression fiber of the concrete);

dp = Distance from the centroid of the compressive reinforcing steel to the extreme compression fiber of the concrete;

ds = Distance from the centroid of the tension reinforcing steel to the extreme compression fiber of the concrete;

Ea = Adhesive modulus of elasticity;

ec = Ultimate strain of concrete;

Ec = Modulus of elasticity of concrete;

ec = Concrete strain;

Ec = Modulus of elasticity of concrete;

Ecc = Modulus of elasticity of carbon fiber;

ecc = Concrete strain at peak of confined concrete;

Ecf = Modulus of elasticity composite;
Ecf = The modulus of elasticity of the CFRP;

E_{EL} = Elastic energy, energy released at failure;

ef = Ultimate strain of composite;

E_f = Modulus of elasticity of the FRP strip;

EI = Flexural rigidity (always positive);

E_m = Modulus of matrix;

E_p = Elastic modulus of the FRP plate;

E_p = Modulus of elasticity of the composite plate;

Es = Modulus of elasticity of steel reinforcement;

E_{TOT} = Total energy calculated as the area under the moment-deflection curve;

ey = Yield strain of reinforcing steel;

fc = Concrete stress;

fcp = Concrete strength at peak for confined concrete;

Fcfrp = Stress of the CFRP;

fcp = Concrete strength at peak for unconfined concrete;

fcp = Compressive strength of concrete after 28 days;

fct = Splitting tensile strength of concrete;

f_{cm} = Tensile strength of concrete;

F_{ds} = Design strength of FRP,

FiberC = Center of gravity of the fiber;

FiberF = Fiber force;
FiberM = Fiber moment;

fp = Average confining stress = zero for unconfined concrete;

fs = Stress in the tension reinforcing steel;

fsp = Stress in the compressive reinforcing steel.

f_t = Concrete tensile strength;

f_{tu} = Tensile strength of concrete under biaxial stresses;

fy = Yield strength of steel;

G_a = Shear modulus of the adhesive layer;

h = Beam height;

h = Overall depth of section;

i = i^{th} layer;

I = Moment of inertia of cross-sectional area, computed about the neutral axis;

I_c = Moment of inertia of concrete beam;

I_{cs} = Second moment of area of strengthened concrete equivalent cracked section;

I_{eff} = Effective moment of inertia of the transformed section;

I_f = Cracked transformed moment of inertia in terms of FRP;

I_p = Moment of inertia for FRP plate;

I_T = Moment of inertia of the strengthened beam based on concrete;

I_{xx} = Moment of inertia with respect to the parallel centroidal axis.

K_n = Normal stiffness per unit area of epoxy;

l_{t,\text{max}} = Limiting cut-off point;
$L_o =$ Distance between the cut-off point and the support of the beam;

$L_o =$ Distance of FRP curtailment from support;

$L_o =$ Unbonded length of the strip;

$M =$ Internal moment in the beam;

$m =$ Internal virtual moment in the beam;

$M_a =$ Adjusted bending moment;

$M_o =$ Bending moment in the concrete beam at the cutoff point due to external load;

$n =$ Number of layers

$P_1, P_2, P_3 =$ Loads corresponding to intersecting slopes,

$P_u =$ Predicted concentrated anchorage failure load;

$q =$ External distributed load applied on concrete beam;

$Q =$ First moment;

$q =$ Horizontal shear flow;

$S_1, S_2, S_3 =$ Tangential lines to Load-Deflection curve;

$t =$ Thickness of fiber;

$t_a =$ Thickness of the adhesive layer;

$TenSteelC =$ Center of gravity of the tension steel;

$TenSteelF =$ Tension steel force;

$TenSteelM =$ Tension steel moment;

$t_f =$ Thickness of FRP;

$t_p =$ Thickness of the composite plate;
Ts = The tension force of the reinforcing steel;

V = Shear force;

V = Ultimate shear force at the FRP end;

V_c = Shear force in the concrete beam;

V_f = Fiber volume fraction;

V_m = Matrix volume fraction;

V_o = Shear force at curtailment location;

V_p = Shear force in the plate beam;

V_v = Volume fraction of voids;

W = Weight of carbon fiber in grams per square millimeter (g/mm^2);

W_a = Width of Adhesive;

W_f = Width of fiber;

x = Depth of neutral axis of strengthened section;

x = Neutral axis;

x_1 = bottom layer of fiber;

x_2 = top layer of fiber;

x_3 = top layer of adhesive or bottom layer of concrete;

y_bar = Distance of the strip to the neutral axis of the transformed section;

y_p = Distance between the composite plate and the neutral axis;

α = Factor for the stirrup type effect;

α_f = Shear force factor;
\( \alpha_f \) = Short term modular ratio of FRP to concrete;

\( \beta \) = Coefficient used in normal stress definition;

\( b \) = Failure mode factor;

\( \beta \) = Mean stress factor (0.85);

\( \delta \) = External displacement of the point caused by the real loads acting on the beam;

\( \Delta \) = Maximum anticipated displacement or displacement at failure,

\( \Delta_y \) = Displacement at yielding which for most reinforced concrete and masonry;

\( \gamma \) = Reinforcement factor;

\( \gamma \) = Specific gravity of carbon fiber (g/cm\(^3\));

\( \sigma_1 \) = Principle stresses in one direction;

\( \sigma_2 \) = Principle stresses in two directions;

\( \sigma_x \) = Longitudinal normal stress in FRP;

\( \sigma_{z_{\text{max}}} \) = Maximum normal interface stress at FRP curtailment location (peeling stress);

\( \tau \) = Elastic shear interface stress at plate curtailment location;

\( \tau_{\text{max}} \) = Maximum shear interface stress at plate curtailment location;

\( \xi_f \) = Peeling stress factor;

\( \psi_f \) = Moment factor;
CHAPTER 1: INTRODUCTION AND LITERATURE REVIEW

1.1 Background

Methods of strengthening existing reinforced concrete structures is a research area in civil engineering. Besides the high cost of replacing existing concrete structures, strengthening the structures also restores and enhances the load-bearing capacity to reduce deflection at service loading, or to limit the width and distribution of cracks in concrete.

The reinforced concrete structures that mostly require external strengthening are buildings and bridges. For many years, steel plate bonding has been used for such purposes. Unfortunately, using steel plate has not been the best solution due to a few flaws, which include its heavy weight and corrosion in the adhesion zone. This flaw in steel led to the research and use of fiber-reinforced polymers (FRP).

The non-corrosive, non-magnetic, non-conductive, and resistant to chemicals as well as a high strength-to-weight ratio are the characteristics of FRP that makes it a better strengthening material over steel.

Among different types of FRP materials, carbon fiber reinforced polymers (CFRP) seem to be the most commonly used in this area due to its strength, stiffness, durability and fatigue characteristics. CFRP materials are also known to perform better at elevated temperatures and possess better damping characteristics as well as have the best resistance to chemical corrosion compared to other FRP.

Although external strengthening using CFRP has been a success in terms of
retrofitting and increasing the strength of reinforced concrete structures, there is an issue with its failure mode. The failure mode is sudden and does not provide adequate warning. Improved methods of CFRP application on reinforced concrete structures are required for a better, progressive and predictable failure mode.

1.2 Literature Review

The following sections give the historical background, previous works in CFRP used for strengthening, analytical models and research on CFRP rehabilitation and retrofitting.

1.2.1 Historical Background and Previous Work in CFRP Used for Strengthening RC Structures

Composite materials combine two or more distinct phases to produce a material, which has properties better than either of the base materials. Fiber composites are a two-phase material in which one phase reinforces the other. High strength fibers are used as the primary means of carrying load and a matrix material binds the fibers into a cohesive structural unit. The combination of fibers and resin produces a whole material with strength and stiffness approximately half that of the fibers.

Humphreys (2003) reports on the historical background of FRP and some examples of its use on retrofitted structures. He states that the benefits of natural fiber composite materials, which include wood, bone, muscle tissue and grass, have been exploited for centuries. It is believed and evidence shows that the Egyptians used the natural fiber composite papyrus to make boats sails and ropes as early as 4000BC. Straw has been used to reinforce bricks for over 2000 years and this method is still used today.
Synthetic fiber composites originated in the late 19th century when the first man-made polymer, phenol-formaldehyde, was reinforced with linen fiber to make Bakelite, commonly used in early electrical equipment.

In 1936, DuPont invented the first room temperature curing resin, saturated polyester, which was released in 1942. The first epoxy resin system was produced in 1938 and Ciba introduced Araldite epoxy resin system in 1942. During this time reinforcing fibers were undergoing rapid development and in 1941 Owens-Corning began production of the world’s first woven glass fabric.

Lots of money and resources were assigned by the military to develop and research composite components in the 1950’s. This was at the time when composites tooling technology was developing and the mechanical properties of composites were improving at a fast rate. Thinner and stronger laminates were being produced and the advantages of composites began to include high strength to weight ratios and high stiffness to weight ratios, resistance to fatigue and the ability to retain these properties in extreme environments.

The end of the “Cold War” in the late 1980’s produced excess fiber composites resources in need of new applications. This resulted in the development of reinforcing fiber such as carbon and aramid, which offer improved strength and stiffness and better impact performance over glass. Resins have also undergone considerable development, most significantly the introduction of different polyester, vinyl ester and epoxy formulations. These new resins offer improved performance over the original high
temperature curing phenolic resin in areas such as mechanical properties, chemical
resistance and bonding.

Humphrey also added that the most common fiber composite materials used for
rehabilitation and retrofit projects are epoxy resins and carbon or glass fibers. The epoxy
resin system is provided in two parts, which must be combined in exact amounts to
ensure correct properties of the hardened resin. Once mixed the resin has around 30
minutes of working time (depending on the constituents) before it begins to harden.
Carbon fibers or glass fibers are often used in these projects in the form of strips of either
dry cloth or pre-made laminates which can be bonded to the structure after the damaged
area has been prepared.

According to Meier (1987), rehabilitation of beams and slabs using fiber
reinforced polymer started about 15 years ago with a research performed at the Swiss
Federal Laboratories for Materials Testing and Research. Since then, several theoretical
and experimental studies have been done across the world to study the behavior of
reinforced concrete structural elements strengthened with epoxy bonded CFRP plates.

Composites are particularly useful in rehabilitation and retrofit projects due to
their strength-to-weight and stiffness-to-weight ratios, chemical resistance,
manufacturing versatility and superior adhesion. Composites in a nutshell when applied
to a surface layer either protect and/or improve on the response of the element. The
materials are usually bonded externally to the structure in the form of tows, fabrics,
plates, strips and jackets. To date these materials have been used effectively in the repair,
strengthening and seismic retrofit of existing structures and concrete members such as beams, columns, bridge superstructure and substructure and shear walls.

Humprey mentioned that the first recorded commercial uses of composites as a method of repair for traditional structures were in Japan in the late 1980’s and in Switzerland in 1991. Since that time thousands of composite repairs have been undertaken on structures around the world. Structures that have been repaired using composite materials include bridges, parking garages, pre-cast pre-stressed curved concrete roof structures and wooden railway sleepers (crossties) amongst others.

The following are examples of fiber composites and its application on the rehabilitation and retrofitting of buildings and infrastructure:

- The reinforced concrete Bennetts Creek Bridge crossing on New York State route 248 constructed in 1926, which carried around 300 vehicles per day, had undergone significant deterioration from de-icing salts and its capacity had been accordingly reduced to 10 tonne. An on-site detour was constructed to allow passage of light traffic while the bridge was undergoing reconstruction. An FRP structure was chosen by New York State Department of Transport, Federal Highways Association, Hardcore Composites and a number of consulting engineers primarily due to its resistance to icing salts, light weight and ability to allow the bridge to be made in two lightweight modular components which could be transported to site inexpensively and assembled rapidly on site. New bridge abutments were constructed while the deck was being designed and constructed in
the factory. The new bridge was delivered to site and installed within six hours. Additional work, including railings and approaches were completed within six months. The estimated cost for the project was around US$400,000 compared to an estimated US$1.45M required for the permanent upgrade. However, the cost estimate for construction of the FRP bridge deck eliminated a number of costs such as cost of in-house engineers and researchers, overhead charges and manufacturers’ profit.

The simple span T-beam bridge structure with integral deck approximately 40 ft long which consists of 26 parallel beams spaced at 4.5 ft centers was built in 1932. The five lanes bridge carries State Route 378 over Wyantskill Creek, New York State with approximately 30000 vehicles per day. Inspection of the bridge showed that it was deteriorating and needed immediate attention. Due to the importance of the bridge authorities decided to rehabilitate the structure rather than reconstruct a new bridge or post load restrictions. Externally bonded steel plates have been used in applications like this since the early sixties and are a proven method of improvement with little imposition to bridge traffic. However steel plates can be difficult to install and can suffer from corrosion if exposed to the environment. FRP laminates were chosen over traditional steel plates due to their versatility, ease of installation and excellent durability. Subsequent analysis at service live load showed that after the installation of FRP plates stresses in the main reinforcing steel were moderately reduced, concrete stresses were
moderately increased and transverse live load distribution to the beams was slightly increased. This project also showed a total rehabilitation cost of US$300,000 compared to a replacement cost of approximately US$1.2M.

In Italy, a set of Precast Prestressed Concrete (PC) shells suffered deterioration due to the thermal effects of a chimney located near one of the roofs PC elements. PC shells, which have thin profiles and produce poor fire performance as well as provide little cover to the reinforcing steel, have been used since the 1960’s. The damaged roof section required the need for strengthening or replacement of the shell. The option of replacement of the shell externally was eliminated due to the size of the roof element, its location in the building (unable to be accessed externally by crane) and the potential disruption to plant production. Similarly, access from the inside via scaffolding or trussed support structures was impossible due to the location of machinery. A number of options were considered to strengthen the roof panel in place. Some of the options included externally retrofitting steel prestressing cables, strengthening by bonded steel plates and bonded CFRP laminates. Bonded CFRP laminates were chosen as the most suitable option as they could be installed with minimal interruption to plant production, their pliable nature allows them to follow compound curves and they have been shown to provide an adequate and durable method of strengthening. An investigation of the effectiveness of the repair was undertaken and consisted of load deflection measurements taken before and after the repair. It was found that
the roof structure deflected at mid span approximately 16% less after the CFRP laminates were installed. It was also concluded that the repair method corrected the loss of flexure / shear stiffness. This was the first commercial application of CFRP flexible sheets for repair in Italy.

- The Houghton Highway is a dual carriageway bridge in Australia joining Queensland’s capital Brisbane with the northern shire of Redcliffe. The structure consists of an integral concrete overlay tied to prestressed concrete T-beams. In 1991 routine inspection identified deterioration of the prestressed concrete piles. The concrete and steel degradation was too advanced for concrete repair alone and a strengthening system was required to reinstate the columns original capacity and to cover the existing cracks. Due to the octagonal shape of the prestressed columns and proximity of the columns to seawater, the traditional method using steel jackets was eliminated. Concrete was considered but avoided due to the tendency for cracks to migrate through the concrete encasement. Externally bonded fiber composite materials were identified as potential candidates as they can be applied on site in a pliable form, which can easily follow the shape of the column. Fiber composites had also been demonstrated to conceal repaired areas without reflecting previous cracks as well as offer adequate re-strengthening and protect the concrete piles. In total 500 piles were repaired and the rehabilitation project was completed in the year 2000.

- Fiber-reinforced polymer (FRP) composite materials have the potential to
revolutionize the repair of sign structures with cracked secondary support members. The Federal Highway Administration (FHWA) has researched the use of FRP for more than 20 years, and FRP has been used on a variety of bridges and other highway structures. Using FRPs to repair cracked overhead sign structures represents one of the latest applications of these strong and durable materials in maintaining the Nation's aging highway infrastructure. FRPs can provide structural integrity to overhead sign supports and prevent them from failing.

The Federal Highway Administration (FHWA) manages a Federal-aid bridge program with an inventory of 600,000 bridges, each of which being greater than 20 feet in length. The average age of a bridge on the U.S. Interstate System is 45 years old. The Federal Government spent $4 Billion per year through apportionments during the 1998-2003 Transportation Equity Act of 21st Century legislation (TEA-21). The states and local agencies spent about the same amount from their combined matching shares and other tax revenues, thus doubling the annual spending to $7-$8 Billion for bridge improvement. The highway infrastructure continues to face numerous challenges, i.e., increasing growth demands and heavier trucks as well as trying to preserve aging and rapidly deteriorating highway bridges. FHWA’s strategy for the upcoming years is to stay ahead of the bridge deterioration curve by focusing on the use of emerging high performance structural materials and innovative quality designs for more durable and reliable structures. The TEA-21 legislation launched an important initiative and established the Innovative Bridge Research and Construction (IBRC) Program, which
provided $108 million over six years to advance high performance materials in bridge applications. The IBRC Program was one of the largest Federal Government funded initiatives in the world; it was crafted to seek new and innovative material technologies for building more durable and effective bridges as well as extending the service life of the continually aging bridge inventory. Through this pursuit and among many other emerging new materials, the fiber reinforced polymer (FRP) composite technology has been demonstrated with great success for bridge applications. FHWA has been developing research in FRP composites materials over the past 25 years. The development of the advanced FRP composite technology from the aerospace stealth aircraft and commercial industries is an engineer's dream for innovative structural design and application. It has been found that the characteristics of a composites element or system can be tailored and designed to meet any desired specifications. The highly corrosion and fatigue resistance composites materials are making inroads into the civil infrastructure industry. These outstanding composites are among the leading materials in structural engineering applications today. In the six-year period, the IBRC program funded 246 proposals of high performance materials and concepts in bridge design and construction. Of these applications, 127 are constructed with FRP composite materials. Some of the applications have been or are being demonstrated consistently in several states to capture the performance of the FRP composites under variable environments and to spread the wealth of knowledge gained.
1.2.2 Experimental and Analytical Studies of FRP Bonding

A number of issues related to the structural behaviors of FRP strengthened RC structures need to be studied. The most important issue is the mechanism of the bond between the FRP sheets and concrete. This is important because of the significance of the bond and its role in transferring the stress from the concrete structures to the externally bonded FRP sheets. A better understanding on the interfacial bond is therefore necessary for achieving a safe and appropriate design of FRP sheet strengthened RC structures.

RC beams strengthened with FRP sheets or plates commonly failed by ripping of concrete layer between the plate and the longitudinal reinforced bars or debonding of the plate because of the plate-end effect. These failure modes are due to the tension force transferred to the concrete beam from the FRP plate by the adhesive layer and the concrete cover. Since the plate ends are free and the beam stiffness is discontinuous, stress-concentration in the adhesive layer or in the interface between concrete and the tension reinforcing steel at the plate-end region results in debonding or ripping-off failure of the strengthened beam.

The mechanical property of unidirectional carbon fibers, their resin to form the composite and their effect on concrete was studied and explained by Neubauer and Rostasy (1996). The adhesives, mainly epoxy resins, with tensile strength of 4350 psi exceed the tensile strength of concrete by more than a factor of 10, and blends with quartz filler. Consequently, they exhibit low shrinkage and creep, as well as high temperature and chemical resistance. The mechanical properties of the CFRP plates in the
longitudinal direction are mostly exclusively governed by the fibers. Their stress-strain behavior is the same as that of the fiber i.e. linear-elastic. The tensile strength of the matrix, which is 8-13 ksi, is much higher than that of concrete, which is a key factor in the transfer of bond stresses even though the matrix contribution to the strength of the plate is negligible. The high ultimate strain of 3 to 5% of the matrix ensures the composite action of the fibers over the entire range of possible plate tensile stresses. Durability tests performed by Toutanji and Gomez (1997) to determine the potential use of FRP sheets as strengthening materials in harsh environments and the ductility of concrete beams externally bonded with FRP tow sheets to the tension face showed that specimens subjected to wet/dry environmental conditions and those kept at room temperature exhibited significant improvement in load capacity when FRP sheets were bonded to the tension face of the concrete beams. Beams bonded with CFRP and epoxy type II (polyoxypropylenediamine hardener/epoxy resin) exhibited the highest load capacity; under these conditions i.e. room temperature or wet/dry.

A survey by Bonacci (1996) gives a statistical background of beams strengthened for flexure with CFRP. Results from his survey showed that 67% of CFRP used for beam flexure strengthening failed by debonding or anchorage. In only 22% of the tests surveyed, rupture of the FRP was achieved, with the rest of the beams failing in shear or compression. Consequently, Bonacci concludes that CFRP debonding at values of about half of its ultimate strain is not unusual. This is due to the weakness in the concrete substrate rather than in the epoxy. Finally, Bonacci suggests a few methods to efficiently
use the CFRP, above all more research on anchorage, development length and bond stress distribution. This should lead to a research on the use of clamps, anchor bolts, U-shaped straps, or wraps near the FRP plate ends and staggered cutoff of multilayer laminates.

Smith and Teng (2001) reviewed existing models of debonding of the FRP plate end, either by separation of the concrete cover or interfacial debonding of the FRP plate from concrete. Their results showed that the models developed for steel plated concrete beams gave better predictions than those developed for FRP-plated concrete beams.

However, Aprile and Spacone (2001) showed that RC beams strengthened with elasto-plastic steel plates or elastic-brittle carbon plates revealed different behaviors. The steel plate yields before the internal reinforcement does, however that was not observed with the carbon plates. Moreover, bond stress distribution in the shear span is different for steel plates than for carbon plates.

An experimental program by Shehata et al (2001) aimed at studying the behavior of reinforced concrete beams strengthened either in shear, flexure, or both by applying externally glued CFRP laminates revealed the following results. Two modes of failure for beams strengthened in flexure and shear were detected. The first mode of failure was by debonding of the CFRP laminate when the deformation attained a value of 5%. The second mode was separation between the concrete cover that bonded the CFRP laminate when the shear stress in the concrete cover approached its strength value.

An experimental program presented by Takahashi and Sato (2003) on the behavior of RC beams strengthened using continuous CFRP sheets showed that the
bonded CFRP sheets was effective with U-jackets and a buffer layer in upgrading the strength and stiffness of the RC beam. The reinforced concrete beams were externally reinforced with epoxy-bonded CFRP sheets and tested to failure using a symmetrical two-point concentrated static loading system.

Although RC beams strengthened externally by bonded FRP have an increased ultimate capacity compared to the original beam, the peeling off of FRP from the beam occurs mostly before the beam achieves its ultimate anticipated capacity.

The American Concrete Society (2000) recommends limiting the longitudinal shear stress between the FRP plate and the concrete substrate to 116 psi in order to prevent premature peeling failure. This value was based on the bonding of steel plates. They further suggest that the longitudinal shear stress be checked at the plate ends, where the shear force acting on the strengthened portion of the member will be at its greatest, and at the location in the span where the steel reinforcement first yields.

Swamy and Mukhopadhyaya (1999) caution designers to be aware of the fact that debonding of the FRP plate usually starts where there is significant shearing displacement across diagonal or transverse cracks.

FRP plate bonding and its debonding characteristic has been modeled and predicted in many ways. Arduini and Nanni (1997) presented experimental and analytical results for beams precracked and subsequently strengthened with CFRP sheets. For the analytical results, a previous model presented by Arduini et al (1993) was modified to include the effects of precracking, unloading, repairing, and the final loading cycle. The
model took into account the nonlinear properties of concrete in compression, the tensile strength of concrete and the concrete-adhesive interface properties. Results from this study show that precracked RC flexural members can be strengthened which have close results to strengthened virgin specimens. The analytical model could also be used to predict the load-deflection behavior of precracked members.

Saadatmanesh and Malek (1998) developed an analytical model to calculate the shear and normal (peeling) stress concentrations at the cut-off point or around flexural cracks. Saadamanesh and Malek also provided guidelines for strengthening of the simply supported RC beams with FRP plates based on a previous model by Malek (1997) and Malek et al (1998). The model was a closed form solution for the maximum shear stress at the plate end assuming linear elastic behavior of the materials, no slip and complete composite action between the plate and concrete. The model was based on the interfacial shear and normal stresses in the concrete beam at the cut-off point, which normally leads to premature local failure in the concrete beam and separation of the plate. The maximum shear stress at the plate is given by the following equations:

\[ \tau_{\text{max}} = t_p (b_1 \sqrt{A} + b_2) \]  

Equation 1

\[ A = \frac{G_s}{t_a t_p E_p} \]

\[ b_1 = \frac{y_p a_i E_p}{I_T E_c} \]

\[ b_2 = \frac{y_p E_p}{I_T E_c} (2a_i L_o + a_2) \]
\[ b_3 = E_p \left[ \frac{y_p}{I_c E_c} \left( a_1 L_o^2 + a_2 L_o + a_3 \right) + 2b_l \frac{t_a t_p}{G_a} \right] \]

\[ M(x_o) = a_1 x_o^2 + a_2 x_o + a_3 \]

Where:

- \( G_a \) = Shear modulus of the adhesive layer;
- \( t_a \) = Thickness of the adhesive layer;
- \( t_p \) = Thickness of the composite plate;
- \( E_p \) = Modulus of elasticity of the composite plate;
- \( y_p \) = Distance between the composite plate and the neutral axis;
- \( E_c \) = Modulus of elasticity of concrete;
- \( L_o \) = Distance between the cut-off point and the support of the beam;
- \( I_T \) = Moment of inertia of the strengthened beam based on concrete;

and \( a_1, a_2, \) and \( a_3 \) are parameters, derived from the equation of bending moment, \([M(x_o)]\) assumed to be quadratic.

The maximum normal (peeling) stress is expressed by (Malek and Saadatmanesh 1996):

\[ f_{n,\text{max}} = \frac{K_n}{2\beta^3} \left( \frac{V_p}{E_p I_p} - \frac{V_c + \beta M_o}{E_c I_c} \right) + \frac{q E_p I_p}{b_p E_c I_c} \]

Equation 2

Where:

- \( K_n \) = Normal stiffness per unit area of epoxy;}
\( V_p, V_c = \) Shear force in the plate beam or concrete beam;

\( E_p, E_c = \) Elastic modulus of the FRP plate or concrete;

\( M_o = \) Bending moment in the concrete beam at the cutoff point due to external load;

\( I_p, I_c = \) Moment of inertia for FRP or concrete beam;

\( \beta = \) Coefficient used in normal stress definition;

\( b_p = \) Width of FRP plate;

\( q = \) External distributed load applied on concrete beam.

Arya and Farmer (2001) suggested semi-empirical methods of design for dealing with end plate and debonding failures in members strengthened externally with FRP due to local FRP separation. Arya and Farmer suggest that end plate separation can be avoided by:

- Extending the FRP beyond the point at which it is theoretically no longer required (anchorage length);
- Limiting the longitudinal shear stress between the FRP and the substrate to 0.11 ksi.

Neubauer and Rostasy (1997) proposed a model to be used to estimate the anchorage length based on the maximum ultimate bond force that can be developed by the FRP at cut-off point where there is a discontinuity in the bonded surface. The maximum ultimate bond force, which can also be used to estimate the theoretical cut-off point of the FRP, is
given by.

\[ T_{k,\text{max}} = 0.5k_b b f \sqrt{E_f t f f_{\text{cm}}} \quad \text{Equation 3} \]

\[ l_{t,\text{max}} = 0.7 \frac{E_f t f}{f_{\text{cm}}} \geq 20\text{in} \]

Where:

\( l_{t,\text{max}} = \) Limiting cut-off point. Extending the FRP the greater of \( l_{t,\text{max}} \) and 20 in., past this point, obtains the actual cut-off point;

\[ k_b = 1.06 \left( \frac{2 - \frac{b_f}{b}}{1 + \frac{b_f}{400}} \right) \geq 1.0; \]

\( b_f = \) Plate (in.)

\( b_w = \) Beam width

\( t_f = \) Plate thickness

\( E_f = \) Elastic modulus of the FRP

\( f_{\text{cm}} = \) Tensile strength of concrete

The longitudinal shear stress was calculated using the following:

\[ \tau = \frac{V A_f \alpha_f (h - x)}{I_{cs} b_a} \quad \text{Equation 4} \]

Where:

\( V = \) Ultimate shear force at the FRP end
\( \alpha_f \) = Short term modular ratio of FRP to concrete

\( A_f \) = Area of FRP

\( x \) = Depth of neutral axis of strengthened section

\( h \) = Overall depth of section

\( I_{cs} \) = Second moment of area of strengthened concrete equivalent cracked sect.

\( b_a \) = Width of adhesive layer.

For the debonding criteria, the conditions were proposed to limit the strain in the concrete at the interface with FRP. For steel reinforcement the ultimate strain should not exceed 5 \( \varepsilon_y \). The strain in the concrete at the FRP interface should not exceed 0.8\% when the applied loading is uniformly distributed or 0.6\% if combined high shear forces and bending moments are present, such as where the applied loads are concentrated at a point and at hogging regions close to supports.

El-Mihilmy et al (2001), present from experimental results that delamination of the concrete cover (anchorage failure) due to stress concentrations that develop at the plate curtailments can occur before the beam achieves its ultimate flexural strength. They also presented closed-form expressions for estimating the anchorage failure load for FRP-strengthened beams subjected to either concentrated or uniform loads. Their proposed method for calculating shear and normal stresses at the plate curtailments was adopted from Roberts’ (1989) analytical model. El-Mihilmy et al modified Roberts’ model to account for the non-linearity that exist at the concrete-adhesive interface. The modified
expressions are as follows:

\[ \alpha_f = 0.28 \sqrt{\frac{E_\alpha t_f}{E_f}} \]  

Equation 5

\[ \xi_f = 1.3\sqrt{\alpha_f} \]

\[ \psi_f = 1.35 - 12.5 \frac{L_o}{L}, \frac{L_o}{L} \leq 0.1 \]

\[ M_a = \frac{P}{2} L_o \psi_f^2 \]

\[ \tau = \frac{V_o t_f (d_f - c)}{I_f} \]

\[ \sigma_x = \frac{M_a}{I_f} (d_f - c) \]

\[ \tau_{\max} = \tau + \alpha_f \sigma_x \]

\[ \sigma_{z\max} = 1.3 \sqrt{\alpha_f \tau_{\max}} \]

\[ \sigma_{1,2} = \frac{\sigma_{z\max}}{2} \pm \sqrt{\left( \frac{\sigma_{z\max}}{2} \right)^2 + \tau_{\max}^2} \]

\[ f_t = k \sqrt{f_{cp}} \]

\[ f_{tu} = f_t \left( 1 + \frac{\sigma_2}{f_{cp}} \right) \]

\[ P_u = \frac{3.8 f_t I_f}{(\xi_f + 2)(d_f - c) t_f + \psi_f^2 \alpha_f L_o} \]

Where:
c = Depth of neutral axis for cracked section;

d_f = Effective depth of FRP;

E_a = Adhesive modulus of elasticity;

E_f = Modulus of elasticity of FRP;

fcp = Concrete compressive strength;

f_t = Concrete tensile strength;

f_{tu} = Tensile strength of concrete under biaxial stresses;

I_f = Cracked transformed moment of inertia in terms of FRP;

L_o = Distance of FRP curtailment from support;

M_a = Adjusted bending moment;

P_u = Predicted concentrated anchorage failure load;

t_a = Thickness of adhesive;

V_o = Shear force at curtailment location;

\alpha_f = Shear force factor;

\xi_f = Peeling stress factor;

\sigma_1 = Principle stresses in one direction;

\sigma_2 = Principle stresses in two directions;

\sigma_z = Longitudinal normal stress in FRP;

\sigma_{z\max} = Maximum normal interface stress at FRP curtailment location (peeling stress);
\[ \tau = \text{Elastic shear interface stress at plate curtailment location}; \]

\[ \tau_{\text{max}} = \text{Maximum shear interface stress at plate curtailment location}; \]

\[ \psi_f = \text{Moment factor}. \]

A model proposed by Rizkalla and Hassan (2003) is used to predict the load at which the CFRP would debond. The closed form analytical solutions were proposed to predict the interfacial stresses for near surface mounted FRP strips and externally bonded FRP sheets. Debonding of the strips is assumed to occur as a result of high stress concentration at cutoff point. Using a simple supported beam subjected to a concentrated load \( P \) at mid span, the shear stress at the strip cutoff \( \tau \) is expressed in terms of the effective moment of inertia \( I_{\text{eff}} \) and the thickness of the CFRP strip \( t_f \) as follows:

Shear stress at cutoff by Rizkalla and Hassan (2003):

\[
\tau = \frac{t_f}{2} \left[ \frac{N * P * L_o * y_{\text{bar}}}{2I_{\text{eff}}} * \omega + \frac{N * P * y_{\text{bar}}}{2I_{\text{eff}}} \right]
\]

Equation 6

\[ \tau_{\text{max}} = \frac{f_{\text{cp}} * f_{\text{ct}}}{f_{\text{cp}} + f_{\text{ct}}} \]

Where:

\[ \omega = \sqrt{\frac{2 * G_a}{t_a * t_f * E_f}} \]

\[ N = \frac{E_f}{E_{cc}} \]
\( E_t \) = Modulus of elasticity of the FRP strip;
\( E_{cc} \) = Modulus of elasticity of concrete;
\( G_a \) = Shear modulus of the adhesive;
\( t_a \) = Thickness of the adhesive;
\( L_o \) = Unbonded length of the strip;
\( y_{bar} \) = Distance of the strip to the neutral axis of the transformed section;
\( I_{eff} \) = Effective moment of inertia of the transformed section;
\( f_{cp} \) = Compressive strength of concrete after 28 days;
\( f_{ct} \) = Splitting tensile strength of concrete.

This model predicts that debonding will occur when the shear stress reaches a maximum value, which depends on the concrete properties. Equating the shear stress equations of \( \tau \) and \( \tau_{\text{max}} \), debonding loads for the CFRP can be determined for a simply supported beam subjected to a concentrated load at mid span.

1.2.3 Anchorage

The term anchorage used in this paper refers to the additional CFRP strips on the sides of the RC beam to increase the shear strengthening to insure flexural failure and provide a perfect bond anchored specimen.

In the study by Sharif et al (1994) to investigate the strengthening of initially loaded RC beams using FRP plates, 0.12 in. FRP plates were glue to the beam sides in
the shear span in addition to the FRP plates on the tension face. In the same study, “I-jackets” a special fiberglass plate which wraps around the sides and the bottom to the beam were also used for additional shear strengthening. The wrapping and side plates were used in an attempt to eliminate the diagonal tension cracks and force the beams to fail in flexure.

Results of Sharif et al, show that the additional FRP plates and “I-jackets” bonded to the sides of the beams, allowed the beams to develop their full flexural strength, resulting in concrete crushing in the constant moment region and produced the highest ductility index. Sharif et al (1994) also concluded that “I-jacket” FRP plates provided the best anchorage system to eliminate plate separation and diagonal tension failure and developed the flexural strength of the repaired beams.

Sagawa et al, presented anchoring methods of carbon fiber sheet (CFS) for strengthening of RC beams where CFS for anchoring wrapped the RC beams in a U-shape. A U-shaped anchor consisted of two L-shaped pieces of CFS that were bonded together on the bottom of the RC beam. Cutting parallelograms from the CFS formed the L-shapes. After CFS for flexural strengthening was bonded to the tension face of the beam, anchoring CFS was inclined 45 degrees to the longitudinal axis of the beam. The 45 degrees anchoring method was compared to the anchoring of the CFS oriented perpendicular (90 degrees) to the beam axis. Three RC beams with one layer of CFS for flexural strengthening but varying U-shaped anchor methods were tested. In the first U-shaped anchor beam, the end of the flexural strengthening CFS was strengthened by CFS
inclined at 45 degree to the axis this was referred to as U1-45-1. The second, U1-45-2, had CFS inclined at 45 degree bonded at two places. The first 45-degree anchor is similar to the previous 45-degree anchor and the second, is at a distance of 6 in. from the first. The last, U1-90, had CFS anchored perpendicular (90 degrees) to the axis. Sagawa et al, observed the following:

- In the U1-45-1, the effect of the anchoring CFS became apparent after the debonding area reached the anchoring CFS and the load increased linearly.
- The entire length of CFS for flexural strengthening broke in fragments at the ultimate stage (U1-45-1).
- In U1-45-2, the flexural strengthening CFS broke at the center of span and the debonding area of the CFS only extended to the inside of the anchoring CFS.
- For the U1-90, the bottom of the anchoring CFS slipped from the required position at the same time as the debonding area of the flexural strengthening CFS reached the anchoring CFS.
- The deflection increased at virtually constant load and the flexural strengthening CFS ruptured at the anchoring area as well for the U1-90.

In conclusion, “U-shaped Anchoring Method” could limit the propagation of debonding of CFS but the effectiveness of anchoring depends upon the fiber direction and the bonding area of the anchoring CFS.

In Zhang et al. (2003) experiment to determine crack widths in RC beams externally bonded with CFRP sheets, 2 in. wide CFRP stripes bonded at 45 degree at both sides of
the beam was used in addition to the three and four layers of CFRP sheets applied to the tension surface of each RC beam. The 45 degree shear reinforcement according to Zhang et al., were added due to the FRP strengthening which increases the strength capacity of the beams and consequently, the demand of extra shear reinforcement. The extra shear reinforcement was to insure flexural failure and to prevent shear failure of the concrete beams.

The strength and ductility of concrete beams reinforced with carbon fiber-reinforced polymer plates and steel were studied by Duthinh and Starnes. In their experiments, two beam specimens with carbon FRP plates covering the tensions face of the beams were anchored with carbon fiber fabric wraps. The first specimen had six layers of wrap placed diagonally at each end of the beam to anchor the plates. For the second specimen, two layers of wrap were used diagonally at one end and transversely at the other. After the four-point loading test, the following observations were made:

- The first specimen with the diagonal wrap at each end of the beam failed due to concrete crushing.
- Prior to concrete crushing, wide 45° shear cracks were observed but there was neither anchorage failure nor debonding.
- Wide flexure-shear cracks, which extended vertically above the load point at one end were observed in the second specimen. The transverse wrap at the other end ruptured at one edge of the beam, causing the FRP plate to debond abruptly.
- There was no evidence of concrete crushing in the second specimen.
Duthinh and Starnes concluded that wrapping with FRP fabric combined with adhesion, is effective in anchoring the procured FRP plates and increases the anchorage capacity above that expected for adhesive bond only. Moreover, if proper anchorage is provided the effective strain limit (or stress level) currently proposed informally for FRP reinforcement by ACI 440 is close to being achievable for the procured carbon FRP plates use in this experiment.

Duthinh and Starnes also recommend that a proper design procedure for external strengthening with FRP plates should take into account enhancement of anchorage by mechanical clamping, wrapping with FRP fabric or other means.

Pornpongsaroj and Pimanmas (2003) report an experimental program conducted to examine the effect of end wrapping on the peeling characteristics of FRP-strengthened beams. In their experiment, three different wrapping techniques were considered, U-, L- and X-wrappings. For the U-wrapped strengthened beam, end peeling was prevented but shear-flexural peeling was observed. In the L- and X-wrapped beams no critical peeling was observed but the beams failed in flexural concrete crushing mode.

1.2.4 Ductility

Ductility is an important characteristic of any structural element. It is a desirable structural property because it allows stress redistribution and provides warning of impending failure. In structural design like steel reinforced concrete beams, the beams are normally under reinforced to enable failure to be initiated by yielding of the steel reinforcement. A ductile failure is the deformation of the beam, followed by concrete
crushing and ultimate failure at no excess loss of load capacity and finally the yielding of the steel. A ductile failure is achieved by designing the tensile reinforcement ratio to be substantially below the balance ratio, which is the ratio at which steel yielding and concrete crushing occur simultaneously. ACI 318 requires the reinforcement limit of 0.75 of the reinforcement ratio which results in a net tensile strain at nominal strength of 0.00376. ACI however, has a limit of 0.004 for the tensile strain at nominal moment which is slightly more conservative. The reinforcement ratio therefore provides a measure for ductility and the ductility corresponding to the maximum allowable steel reinforcement ratio provides a measure of the minimum acceptable ductility.

Ductility allows structures to be capable of sustaining high proportions of their initial strength when a major earthquake imposes large deformations in order to minimize major damage and to ensure their survival with moderate resistance with respect to lateral forces.

Ductility is defined by Paulay and Priestley (1992) as the ability of structures and their components or of the materials used to offer resistance in the inelastic domain to withstand large deformations which may exceed their elastic limit. Park and Paulay also emphasis the importance of safety as a major factor in structural design by warning that any type of brittle failure should be avoided since this could limit warning time and cause lives to be endangered. They conclude by advising that structures with a ductile behavior will be able to experience large deflections while still holding near ultimate loads.

Since strengthening using CFRP is a fairly new innovation, understanding the
effect of this material on the ductility of a reinforced concrete beam is important. Different methods have been proposed for the determination of ductility. Ductility has generally been measured by a ratio called a ductility index. The ductility index is usually expressed as a ratio of rotation, curvature or deflection at failure to the corresponding property at yield. Other methods for determining ductility have also been developed and one such method is the *Energy Method* by Naaman and Jeong (1995), which is discussed below.

The proposed ductility index by Naaman and Jeong was based on experimental testing of prestressed concrete beams with FRP tendons. Their ductility index is expressed as a ratio of the total energy of the beam to the elastic energy released at failure. This method is applicable to beams with steel reinforcement, FRP reinforcement or a combination of both. Naaman and Jeong further explain the relation between steel reinforcement and ductility by stating that, since reinforced concrete structures usually behave in a ductile manner if an appropriate amount of steel reinforcement is added, ductility can be achieved by the inelastic deformation of the steel before failure. During this period, the concrete beam consumes much of the energy causing the elastic energy released at failure to be reduced. However, this is not the same case for FRP reinforced beams, since FRP rarely attains inelastic deformation. This results in a huge amount of elastic strain energy building up and released at failure, which exceeds that of steel reinforcement. The ductility index was developed based on the difference in elastic energy.
\[ \mu = \frac{1}{2} \left( \frac{E_{\text{TOT}}}{E_{\text{EL}}} + 1 \right) \]  

Equation 7

Where:

\[ E_{\text{TOT}} = \text{Total energy calculated as the area under the moment-deflection curve up to the failure load.} \]

\[ E_{\text{EL}} = \text{Elastic energy, energy released at failure and can be found by investigating unloading tests. The elastic energy released at failure can also be estimated by using the area of a triangle formed at the failure load with a weighted average slope of the two initial straight lines of the moment-deflection curve.} \]

Several modifications have been made to Naaman and Jeong’s energy method to calculate energy ductility of FRP strengthened beams. Although, the energy method was originally used to calculate the ductility for concrete beams with internal FRP reinforcement, it has been very helpful in determining ductility in cases where the yielding point is difficult to establish due to lack of steel strain measurements.

Grace et al. (1998) in their study on the behavior andductility of simple and continuous FRP reinforced beams modified Naaman and Jeong’s energy method for measuring ductility. Based on Naaman and Jeong’s energy method, Grace et al, proceeded to firstly determine the point that separates the elastic energy from the inelastic energy and secondly use these energies to express the ductility index. In their modification to determine the magnitudes of the elastic and inelastic energies, the following parameters
were considered:

- Modulus of elasticity and failure strength of the reinforcement.
- Type of reinforcing bars and stirrups.
- Failure mode.
- Concrete softening at compressive flexural failure.

The final modified equation to determine the slope of the line separating the elastic energy from the inelastic energy as shown in Figure 1 and taking into account all the above parameters is given as:

\[
S = \alpha \beta \gamma \frac{E_f}{E_s} \times \frac{f_y}{f_{ys}} \frac{P_1S_1 + (P_2 - P_1)S_2 + (P_3 - P_2)S_3}{P_3} \quad \text{Equation 8}
\]

Where:

\( \alpha = \) Factor for the stirrup type effect,

- Steel = 1.0
- GFRP = 0.95
- CFRP = 0.98

\( \beta = \) Effect of failure mode,

- Compressive flexure = 1.0
- Flexural shear = 0.95
- Shear = 0.98

\( \gamma = \) Factor for the type of reinforcement,

- Steel = 1.0
- GFRP = 4.0
- CFRP = 2.1

$E_t$ = FRP modulus of elasticity,

$E_s$ = Steel modulus of elasticity,

$F_y$ = Steel yield strength,

$F_{ds}$ = Design strength of FRP,

$P_1$, $P_2$, $P_3$ = Loads corresponding to intersecting slopes,

$S_1$, $S_2$, $S_3$ = Tangential lines to Load-Deflection curve.

---

In conclusion, Grace et al. proposed a better measure of ductility, which is the “energy ratio”. The “energy ratio” is defined as the ratio of the inelastic energy to the total energy.
The “energy ratio” in relation to ductility was interpreted as follows:

- When the “energy ratio” is greater than 75%, the beam will exhibit a ductile failure.
- When the “energy ratio” is between 70 and 74% the beam is considered semi ductile.
- When the “energy ratio” is below 69%, the beam exhibits a brittle failure.

Orozco and Maji (2004) in their study to determine the energy released in fiber-reinforced plastic reinforced concrete beams also employed the methodology of Naaman and Jeong (1995). In the study, a set of 30 concrete beams reinforced with carbon/epoxy FRP and four reinforced with comparable size steel rebars were subjected to static bending tests. An analytical evaluation of the fracture energy in these experiments showed that there is ductility due to large fraction of the total strain energy that is absorbed in the concrete because of the formation of distributed cracking. In summary,

Orozco and Maji observed that even though the ductility of the 30 reinforced beams with variations in reinforcement ratio, overwrapping configurations, addition of stirrups and addition of fibers was less than that of comparable steel reinforced beams, energy dissipation by concrete cracking ensures good ductility in FRP reinforced beams.

1.3 Objective

The objective of the tests presented in this thesis is to study differences in failure modes CFRP strengthened reinforced concrete beams. The main goal is to identify the designs that result in a progressive and gradual failure mode with enough warning before
final failure. In order to achieve this goal, the change in strength and ductility of the beams as the number and orientation of the transverse CFRP anchors change are investigated. The increase in strength is targeted at approximately twice the strength when fully anchored of the control beam (reinforced concrete beam without CFRP) while the ductility and failure modes are investigated with different anchoring orientations (90, 60 and 45 degrees). Six reinforced concrete beams were constructed and tested. With the exception of the control beam, each specimen was applied with the same number of layers of CFRP for flexural strengthening. Four of the flexurally strengthened specimen had additional lateral anchorage strips. The additional lateral anchorage strips varied for each of the four specimens. The variations include a descending order of 90° lateral anchorage strips from the support to the mid span for one of the specimen and an ascending order of 90° lateral anchorage strips from the support to the mid span for another specimen. One of the last two specimens has a 45° lateral anchorage strips and the last specimen has a combination of 45, 60 and 90° lateral anchorage strips.
CHAPTER 2: MATERIAL PROPERTIES

2.1 Concrete

Rinker Materials in Orlando, Florida produced the concrete used for the research. Two cubic yards of concrete was requested which was enough for a 9x12.5 in slump-cone, twelve 6x12 in cylinders, two 24x12x11 in. supports and six 132x11x6 in. beams. The concrete which arrived in a ready-mix truck was placed in the formwork, compacted and leveled immediately after the slump-cone test. The test cylinders and support formwork were also filled.

2.1.1 Specifications of Concrete

- 5000 psi (compressive strength of concrete) at 28 days;
- 3/8 aggregate;
- No admixtures;
- 4 in. slump.

2.1.2 Cylinder Test

Compressive strength test by ACI code specifications was performed on the standard 6x12 in. concrete cylinders after 7, 14 and 28 days respectively, using the Universal Testing System to determine if the concrete met the specification (5000 psi).

Three concrete cylinders were tested each time using the Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens provided by ASTM. In summary, the test method consisted of applying a compressive axial load to the concrete
cylinders at a rate within an approximate range until failure occurs. The compressive strengths of the concrete cylinders are calculated by dividing the maximum load attained during the test by the cross-sectional area of the concrete cylinders. The graphs presented in Figure 2,

Figure 3 and Figure 4 show the maximum compressive load applied to each cylinder before failure and the cross-head displacements. A summary of the average loads and compressive strengths of the 7, 14 and 28 days test are also presented in

Figure 2: Graph of 7 Days Cylinder Test.
Figure 3: Graph of 14 Days Cylinder Test.

Figure 4: Graph of 28 Days Cylinder Test.
Table 1: Summary of Compressive Cylinder Test.

<table>
<thead>
<tr>
<th>Days After Casting</th>
<th>7</th>
<th>14</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Load (lbs)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder 1</td>
<td>130530</td>
<td>157930</td>
<td>179920</td>
</tr>
<tr>
<td>Cylinder 2</td>
<td>132780</td>
<td>156200</td>
<td>187660</td>
</tr>
<tr>
<td>Cylinder 3</td>
<td>134630</td>
<td>160680</td>
<td>185530</td>
</tr>
<tr>
<td>Average Maximum Load (lbs)</td>
<td>132647</td>
<td>158270</td>
<td>184370</td>
</tr>
<tr>
<td>Compressive Strength (psi)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder 1</td>
<td>4612</td>
<td>5581</td>
<td>6358</td>
</tr>
<tr>
<td>Cylinder 2</td>
<td>4692</td>
<td>5519</td>
<td>6631</td>
</tr>
<tr>
<td>Cylinder 3</td>
<td>4757</td>
<td>5678</td>
<td>6556</td>
</tr>
<tr>
<td>Average Compressive Strength (psi)</td>
<td>4687</td>
<td>5593</td>
<td>6515</td>
</tr>
</tbody>
</table>

2.2 Steel

Steel members, which include compression and tension reinforcement bars, stirrups, tie-wires and chairs, were provided by Hugh Supply Inc. in Orlando Florida.

2.2.1 Compression and Tension Reinforcement

Each beam constructed consisted of three 10 ft #5 longitudinal bars, with a 0.625 in. diameter and two 10 ft #3 longitudinal bars, with a 0.375 in. diameter. All the longitudinal bars were Grade 60 i.e. with nominal yield strength of 60000 psi.

2.2.2 Stirrups

Each beam contained thirty-two #3 Grade 60 stirrups (diameter .375 in.) at 4 in. spacing with the exception of the mid span, which was a foot wide.
2.2.3 Tie-wires and Chairs

Steel tie-wires were used to tie the stirrups to the longitudinal bars and the 1.5 in. chairs to the longitudinal bars. The cross-section and the side view in Figure 5, show the positions of the longitudinal bars and the 1.5 in. steel chairs respectively.
Figure 5: Positions of Longitudinal Bars, Stirrups and Steel Chairs.
2.3 Carbon Fiber Reinforced Polymer (CFRP)

CFRP laminate reinforcing consists of bonding the CFRP strip with the concrete structure using a high-strength epoxy resin as the adhesive. The CFRP strips are unidirectional; the fibers are oriented only in the longitudinal direction. Correspondingly, the strip strength in this direction is proportional to the fiber strength i.e. the strength of a unidirectional CFRP has a high strength in the longitudinal direction compared to the other directions (latitudinal, diagonal etc). There are three main types of carbon fibers, which are high strength, high modulus and ultra-high modulus with elastic moduli of \(3.3 \times 10^4\), \(5.4 \times 10^4\) and \(5.1 \times 10^4\) to \(7.5 \times 10^4\) ksi respectively. Tensile strengths of the carbon fibers range between 406 to 740, 261 and 145 to 254 ksi for high strength, high modulus and ultra-high modulus respectively. It should be noted that the fibers with higher strengths have lower stiffness (modulus of elasticity). A composite consists of two or more materials combined to produce a product that exceeds their individual properties. A fiber reinforced polymer is a combination of high strength fibers and a matrix. The fiber is the strength of the composite and the matrix is the product that holds the fibers together and acts as a load transfer median. For this research, 635-epoxy resin, which acts as the matrix, will be applied to the unidirectional carbon fiber to form the composite. U.S. Composites in West Palm Beach, Florida produced the 635-epoxy resin, hardener and unidirectional carbon fiber.

2.3.1 Undirectional Carbon Fiber

The unidirectional carbon fiber reinforcing from U.S. Composite is used to improve
tensile strength and stiffness in one direction while adding minimum thickness and weight. Fiber bundles are held in place by a polyester fill thread for easy handling and wet out. The unidirectional carbon tape used for this research has the following properties:

- Weight: 11 ounce per square yard;
- Weave: Continuous unidirectional rovings;
- Tow Size: 12K (12000 filaments in a tow);
- Nominal Thickness: 0.021 in.;
- Width: 13 in.;
- Fiber Modulus of Elasticity: $3.3 \times 10^4$ ksi;
- Tensile strength: $3.6 \times 10^2$ ksi.

A section of the 13 in. width unidirectional carbon fiber to be cut into 4 in. width strips is shown in Figure 6.
2.3.2 635 Epoxy Resin

The 635-epoxy resin used in this research from U.S. Composite is also common for lamination with fiberglass, carbon fiber, Kevlar or any type of reinforcement. The syrup-like consistency generates fast wet-out and easy application. In general, epoxy resins are made by reacting epichlorohydrin with bis-phenol A, which is linear polymer that cross-links, forming thermosetting resins by the reaction with amine-type compounds.

Some of the properties of the 635-epoxy are:
- Barcol Hardness: 34 (Barcol hardness is used to determine the hardness of both reinforced and non-reinforced rigid plastics. The specimen is placed under the indentor of the Barcol hardness tester, and a uniform pressure is applied to the specimen until the dial indication reaches a maximum. The depth of the penetration is converted into absolute Barcol numbers);
- Heat Distortion temperature: 160°F;
- Flexural Strength: 17000 psi;
- Flexural Modulus: 4.5 x 10^5 psi;
- Tensile Strength: 9000 psi;
- Tensile Elongation: 2.2%;
- Mixing ratio: 3:1 Epoxy to hardener. (30 ounces of resin would require 10 ounces of hardener in order to catalyze).

According to Cheremisinoff and Cheremisinoff (1995) epoxy resins in general have:

- Low polymerization shrinkages;
- Excellent mechanical strength;

  a. Although viscous liquids in their thermoplastic state, when cured, they are up to seven times more durable and tougher than cured phenolic resins. They are cured by means of a curing agent. No volatile by-products are generated during the curing process;
b. They have high adhesive strengths;

c. The pure epoxy resins (without agents) have an almost indefinite shelf life. They are chemically stable up to about 400° F.

- Good electrical properties;
- Chemical resistant;

a. High-temperature cured epoxies show much better performance than the room- temperature-cured epoxies;

b. High-temperature cured epoxies are less resistant than the polyesters to the wide range of corrosive, chemicals, but show better resistance on the alkaline side.

2.3.3 Composite

The composite, which is a combination of the carbon fiber fabric and a proportional mix of the 635-epoxy resin and its hardener, could reach high tensile strengths. The following computation is used to determine the modulus of elasticity of the composite, assuming that the materials follow a one-dimensional Hooke’s law and that strains in the composite, fiber and matrix are equal (all moduli are in one direction).

\[ E_c = E_f V_f + E_m V_m \]  
Equation 9

\[ V_f = \frac{W}{t * \gamma} \]  
Equation 10

\[ V_f + V_m + V_v = 1 \]  
Equation 11
Where:

\[ E_c = \text{Modulus of the composite} \]
\[ E_f = \text{Modulus of carbon fiber} \]
\[ E_m = \text{Modulus of matrix} \]
\[ V_f = \text{Fiber volume fraction, it is the ratio between fiber cross-sectional area and composite controlling the property fiber reinforced composite material. The higher the fiber volume fraction, the higher the modulus, strength and many other properties of the composite.} \]
\[ V_m = \text{Matrix volume fraction} \]
\[ V_v = \text{Volume fraction of voids.} \]
\[ W = \text{Weight of carbon fiber} \]
\[ t = \text{Nominal thickness of carbon fiber} \]
\[ \gamma = \text{Specific gravity of carbon fiber} \]

A typical carbon/epoxy composite assuming 40% fibers (\( V_f \)) and 60% resin (\( V_m \)) by volume will have the following properties:

- Density: \( 5.7 \times 10^{-2} \text{ lb/in}^3 \);
- Tensile Strength: \( 3.3 \times 10^5 \text{ psi} \);
- Modulus of Elasticity: \( 1.58 \times 10^7 \text{ psi} \);

**2.4 Elastomeric Bearing Con-Slide Bearing Pad**

Elastomeric Bearing Con-Slide bearing pad was used between the support and the
test specimen to prevent excessive movement of the specimen during loading as well as between the steel loading fixture attached to the testing machine at a spacing of 11 in. which will produce a two point load. Figure 7 shows the dimensions and placement of the elastomeric pad. Properties of the Elastomeric Bearing Con-Slide bearing pad are:

- Durometer: 90 +/- 5 (Shore A);
- Compressive Strength: 18 ksi max;
- Maximum Design Pressure: 1.5 ksi;
- Thickness: 1 in.;
- Width: 2 in.;
- Length: 24 in.;
2.5 Hydro-Stone Gypsum Cement

Hydro-stone gypsum cement is hard and strong with high water absorption resistance. The cement was used at the base of the support to stabilize and level the support during testing. Properties of the cement are as follows:

- Use consistency (parts of water by weight per 100 parts plaster): 32;
- 1 hour compressive strength: 4000 psi;
- Dry compressive strength: 10,000 psi;
- Maximum setting expansion: 0.24%;
- Wet density: 119 lb/ft$^3$;
- Dry density: 108 lb/ft$^3$;
- Set time (Machine Mix): 17 to 20 min.

The 24x12 in. base area was elevated ½ in. and the wet cement poured underneath the support and 1 ½ in. to the sides at 2 in. spacing from the sides as shown in Figure 8.
2.6 Hook-System

Two carrying hooks with a total carrying capacity of a thousand pounds were designed to move the six hundred pounds beams for testing. One hook is placed at each end of each beam. Hook dimensions were as follows:

- ½-13x3 Forged eye bolt;
- ½-13 thread rod low carbon;
- ½-13 hex finish nut zinc;
- ½ USS flat washer zinc;
- ½-13x1 ¼ Hex coupling nut zinc.

The various parts of the hook-system as listed above were assembled as follows:

1. The forged eyebolt was threaded half way into the hex coupling;
2. Two hex finish nuts were then attached onto the lower end of the 3 in. rod with a washer in between;

3. The upper end of the threaded rod was then screwed into the remaining portion section of the hex coupling.

4. The hook-system was then suspended in the steel cage by 2x4 wood before pouring the concrete to form the beams. The ¼ hex coupling nut and every member below it was buried in the concrete making it easier to screw and unscrew the eyebolt when the concrete cured.

The hook system assembly as well as its position above and below the concrete surface level is shown in Figure 9.

Figure 9: Hook-System Assembly.
CHAPTER 3: OUTLINE OF EXPERIMENTS PROGRAM

The purpose of these experiments is to evaluate the different modes failures of the RC beams strengthened after initial flexural cracks (retrofitting). Six beams will be tested in this experiment. Different CFRP application methods were employed in this experiment to determine the best strengthening method as well as a better failure mode (progressive and not abrupt). The beams, which were simply supported under four-point bending, were initially cracked using a minimum load (approximately 20% of the ultimate strength) before CFRP application. A minimum amount of five layers of CFRP which was predicted to double the strength of the control beam (RC beam without composite) was used on the tension face of five of the beams with four of the specimens having transverse anchorage as well. The sixth beam was a control specimen. The specimens with transverse anchorage were however designed with the minimum layer of flexural strengthening so debonding or peeling at the plate end and mid span will occur and propagate through the entire span.

3.1 Designs and Analysis of Specimens

The RC beams for this research were designed and analyzed to meet the steel, top and bottom concrete cover requirements per ACI Building Code and Commentary (ACI 318-02/318R-02). The design criteria were to ensure prevention of shear failure; hence enough shear reinforcement was provided, consisting of #3 bars (stirrups). This will allow for the RC beam to fail in flexure, which will enhance the study to determine the CFRP application on the reinforced concrete beam that results in a progressive and
gradual failure mode with enough warning before final failure due to flexure.

The final analysis showed that the following dimensions will qualify for the RC beam for this study:

\[ f_{cp} = 5000 \text{ psi} \] – Compressive strength of concrete;  
\[ f_y = 60000 \text{ psi} \] – Yield stress of steel;  
\[ h = 11 \text{ in.} \] – Beam height;  
\[ d_p = 1.688 \text{ in.} \] – Compression depth;  
\[ b = 6 \text{ in.} \] – Beam width;  
\[ A_s = 3 \text{ #5 bars} \] – Cross sectional area of tension reinforcement bars;  
\[ A_{sp} = 2 \text{ #3 bars} \] – Cross sectional area of compression reinforcement bars;  
\[ d_s = 9.188 \text{ in.} \] – Structural depth to tension steel reinforcement measured from the center of the three #5 tension reinforcement.

### 3.2 Construction of Specimens

Six beam specimens were constructed, each with a cross-section of 11x 6 in. and a length of 132 in. The beams were reinforced with both compression and tension reinforcement. Longitudinal bars (two #3 on top compression, three #5 on bottom tension) will run 129 in. end-to-end. #3 vertical stirrups for shear reinforcement were in closed loops with 4 in. spacing from the center with the longitudinal compression bars acting as “stirrup-support bars” through the beam span. Figure 10, shows the cross section and longitudinal view of the RC beam with a clear span of 12 in. at the mid span.
Figure 10: RC Beam Reinforcement.

* Value theoretically calculated.

** Measured from the center of compression bars to center of all three tensile bars
3.3 Experimental Test Set-up and Flexural Test of Specimens

The two-stage experimental test set-up was performed using the MTS 243.45 Actuator shown in Figure 11. The two stages were:

- Pre-cracking of all six RC beams
- Flexural test of all the beams, five RC beam specimens strengthened with CFRP and one without any CFRP as a control were performed using a “static cyclic loading” method. The “static cyclic loading” was applied in a stepwise manner by loading and releasing from 25%, 50%, and 75% and until failure of the predicted ultimate moment of the beams under four-point loading. Three cycles were performed for each loading and releasing stage.

The specifications for the Actuator used in this research are as follows:

- Rod Diameter: 4.5 in.;
- Stroke: +/- 10 in.;
- Piston Area:
  - Tension: 34.36 in²;
  - Compression: 50.26 in²;
- Force Rating
  - Tension: 100 kip;
  - Compression: 146 kip.
Figure 11: Loading Frame and the MTS 243.45 Actuator.

The experimental set-up in relation to specimen positioning, placement and area occupied are shown in Figure 12, Figure 13 and Figure 14. All specimens were tested in a four-point bending configuration, which is also shown in Figure 15.
Figure 12: Front View of Test Set-Up.
Figure 13: Side View of Test Set-Up.
Figure 14: Top View of Test Set-Up.
Figure 15: Load Configuration.
3.4 Methods and Applications of CFRP to RC Beams

Applications of the CFRP to the five pre-cracked RC beam specimens were done. The crack moment for the RC beam was estimated to be approximately 13% of the calculated ultimate moment. Cracking of the RC beams was performed using the MTS 243.45 Actuator as mentioned in the earlier section.

The cracked beams were then turned over with their cracked tension face up, resting on the 6x6 in. wooden block at the midpoint. This allowed the cracks to be exposed and widened by the self-weight of the beam. The CFRP layers were chosen to optimize the performance of the composite beams and to compare the different anchorage modes. Analytical models and test experiments discussed in Chapter 4 helped to decide on the number of layers to apply. The application of the CFRP for external strengthening then followed with recommendations by the manufacturer. See Appendix (Construction and Test Set-up) for details on manufacturer’s recommendations. The wet lay-up procedure was used. In the wet lay-up procedure, the dry carbon fabric was saturated by applying the mix (epoxy and hardener) and bonded to the RC beam. The following steps were taken for preparation and application of the CFRP.

a. Surface preparation – The concrete surface was sanded and the sharp edges ground to a 0.6 in. radius. It was then cleaned with acetone to ensure a good bond.

b. Epoxy mixture – Mixing ratio was 3:1 epoxy to hardener. 30 ounces of resin would require 10 ounces of hardener in order to catalyze. Mixing was done in single portions for 20 ounces of resin plus 6.7 ounces of hardener. Each portion
was mixed thoroughly in the 21/2-quart bucket for 3 minutes. *Note: Product requires a mixture of 3 parts resin to 1 part hardener. This can be measured by either weight or by volume.*

c. Application - The bonding surface of five 4x120 in. CFRP fabric (strips) were cleaned with acetone. Acetone was also applied to the reverse sides of the fabrics to weaken the adhesive of the self-release-tape (SRT) and removed gently so the fiber do not come apart. The five strips were for the first layer of the five beams. The mixture (adhesive) was applied to the concrete surface of all five beams and allowed to dry for 15 minutes.

- The CFRP strips were then stretched and placed on the concrete surface with the adhesive for all five beams.
- The strips were then pressed down with the fiberglass roller to keep the CFRP strips tight and wrinkle-free.
- A thick layer of the saturating mixture is then applied over the CFRP strips.
- The paint roller is used to remove any trapped air pockets and to work the saturating mixture in to the fabric.
- After 30 minutes an additional layer of saturating mixture was applied and the above procedure was repeated to bond the additional five layers of CFRP strips. This was recommended by the manufacturer.
- The transverse strips were applied in the same way as the 4x120 in. strips but this time, the strips were placed on the side of the beams very carefully so they fall within the markings.
- For the transverse strips, that required multiply layers, the same procedure for, “multiply layers” used previously was applied.
- The concrete beams strengthened with CFRP strips were allowed to cure for three days at room temperature.

Details of the designs that were used to externally strengthen the RC beam after the cracked beams were cleaned and the sharp edges rounded off to a radius of 0.6 in. are discussed in Section 3.4 of this document. Figure 16, shows a summary of the design methods used.
Design 1 (Flexural Strengthened Beam - Tension face only)

Design 1-5 (Tension Face)

5 layers (1/2"x4") composite strip

Design 2 (90 deg. Transverse + Tension face)

DESIGN 2
(Side View of 90° Transverse)
beam specimen composite strip bottom strip

Design 3 (45 deg. Transverse + Tension face)

DESIGN 3
(Side View of 45° Transverse)
beam specimen composite strip bottom strip

Design 4 (45,60,90 deg. Transverse + Tension face)

DESIGN 4
(Side View of 45°, 60°, 90° Transverse)
beam specimen composite strip bottom strip

Design 5 (90 deg. Transverse + Tension face)

DESIGN 5
(Side View of 90° Transverse)
beam specimen composite strip bottom strip

Figure 16: Summary of Design Methods.
3.4.1 Design 1

Five strips of 4x120 in. CFRP were applied to the tension face for flexural strengthening. This allowed the clear span of the RC beam to be covered, leaving 1 in. gaps on both sides to the beam width as shown in Figure 17. The distance between the cut-off point (the end of the CFRP) and the support of the beam was 3 in. Debonding was expected at the plate ends as predicted by analytical models described in Chapter 4.
Figure 17: Tension Face with CFRP as will Apply to All Designs.
3.4.2 Design 2

In addition to the five strips of 4x120 in. of CFRP applied to the tension face for flexural strengthening as shown in Figure 17, 90° transverse strips were used to anchor the tension strips.

As shown in Figure 18, the flexural CFRP strips were strengthened at the plate ends with two layers of 26x4 strips and a 36 in. space provided in the mid span between transverse anchors so intermediate debonding will initiate in the mid span and propagate to the ends.
Figure 18: Side and Bottom View of Design 2 Anchorage Strip Alignment
3.4.3 Design 3

In addition to the five strips of 4x120 in. of CFRP applied to the tension face for flexural strengthening as shown in Figure 17, 45° transverse strips were used to anchor the tension strips as shown in Figure 19.

It should be noted that unlike the 90° transverse strips where the strips were continuous the 45° transverse strips are discontinuous. The 45° transverse strips end at the edge of the beam and overlaps with another 45° transverse starting at the edge. This application method is to avoid folds and wrinkles of the CFRP at the corners and makes it easier to apply.

The 45° transverse strips are expected to provide the highest anchorage strength due to its 45° inclination which provides a strong tension resistance during loading. The other design methods will provide shear or lower tension resistance which is not as effect as the tension resistance provided by the 45° transverse strips. The 60° transverse strips will be the closest in terms of anchorage strength but has a higher inclination hence will be likely subjected to both tension and shear force.
Figure 19: Side and Bottom View of Design 3 Anchorage Strip Alignment.
3.4.4 Design 4

In addition to the five strips of 4x120 in. of CFRP applied to the tension face for flexural strengthening as shown in Figure 17, 45°, 60° and 90° transverse strips were used to anchor the tension strips as shown in Figure 20.

The angle change in this design specimen is to create a sequence in anchoring strength. As discussed in 3.4.3 Design 3, the higher the inclination of the transverse strips the lower its resistance to applied load in tension. Example, since the 90° transverse strips have no inclination, it is likely for the strips to be subjected to shear forces while the 60° transverse strips are likely to be subjected to a combination of shear and tension forces.

With the resistance in tension providing a higher strength, the 45° transverse is more likely to provide a greater resistance in tension during loading. The sequential anchoring strength is hence provided by the combination of 45°, 60° and 90° transverse strips which are intended to allow debonding to be initiated in the mid span and propagate to the ends where the anchoring strength is greatest during loading.
Figure 20: Side and Bottom View of Design 4 Anchorage Strip Alignment.
3.4.5 Design 5

Design 5 as shown in Figure 21, is the reverse of Design 2 i.e. the height of the 90° transverse strips are in ascending order from the support to the mid span whereas in Design 2 it was in a descending order.

Debonding was expected at the plate ends since the first and the shortest 90° transverse strip was placed 14 in. from the CFRP cut-off points allowing 14 in. of the five layered flexural CFRP unanchored on both ends of the beam. Additionally, the shortest 90° transverse strips presumed to the weakest of the transverse strips were the first to be encountered if debonding occurred from the end plates and will allow propagation to the mid span.
Figure 21: Side and Bottom View of Design 5 Anchorage Strip Alignment.
3.5 Verification of Test Procedures, Calibration and Instrumentation

The testing procedures and data acquisition system were verified by a trial test, which was performed with a 4x4x80 in. wood beam, used as a test specimen. The test allowed for calibration, which included, loading rate, displacement, control mode, relative ending level, and maximum load. A checklist of test procedures was developed and is presented in Appendix.

The composite and control beams were instrumented to tract the displacement behavior throughout testing.

Six linear potentiometers (6 in. range) were placed at the supports and mid span and two (1.5 in. range) at quarter spans to measure deflections as shown in Figure 22. There were two potentiometers at each support and two at the mid span (left and right). The 1.5 in. Range sliding resistor potentiometer were place quarter spans on each side to the mid span potentiometers in the in the middle of the beam. The support deflections were subtracted from the mid span deflection. The subtraction was done so the mid span deflections were corrected for support deflections.
Figure 22: Elevation of Potentiometer Alignment.
Figure 23: Support Potentiometers.

Figure 24: Mid Span and Quarter Span Potentiometers.
3.6 Data and Records

Data and records from the test were collected and stored by two main systems. The load and displacement of the MTS Series 243 Hydraulic Actuator is collected by the Actuator computer system while the Data Acquisition System saves and collects the real time data.

See Appendix A for details on the data acquisition system.

3.7 Experimental Problems

The load steel plate for the four-point loading had to be carefully positioned, aligned and placed. This took lots of time. Since the quarter span (1.5 in. range) linear potentiometers were installed under the specimen, they were removed after the 75% static cyclic loading stage to prevent instrument damage.
CHAPTER 4: ANALYTICAL STUDY

The purpose of this analytical study is to develop a model that predicts the flexural behavior of RC beams strengthened with CFRP. The model is developed for two conditions. Bonded, assuming a perfect bond between the concrete and CFRP for flexural strengthening, and unbonded, assuming no bonding of the CFRP within the mid span but anchored at the ends. Since four of the six testing specimens will be fully anchored using CFRP strips the perfect bonded model can be used to predict the nominal capacity and failure modes of the anchored specimens.

A shear flow analysis will also be investigated to predict the debonding conditions and to establish the minimum number of layers of CFRP for flexural strengthening to double the strength of the control beam as well as enable debonding. Shear flow analysis will include methods suggested in Chapter 1 and a derived model.

Ductility, which is an important property for safe structural design, will also be considered. The ductility for the model control RC beam will be analyzed and compared with the analysis of the model RC beam strengthened with CFRP assuming perfect bonding. The Conventional Method by Paulay and Priestley (1992) and the Energy Method by Naaman and Jeong (1995) will used for the ductility analysis.
4.1 Ultimate Capacity Prediction

In this thesis, since the ACI code does not provide methods of predicting the bending capacity of RC beams externally reinforced with carbon fiber reinforced plastics (CFRP) composites, a method will be developed to take into account the additional reinforcement of the composite.

Dimensions and properties of the specimens are as follows:

\( b = 6 \text{ in.} \) - Beam width;

\( h = 11 \text{ in.} \) - Beam height;

\( d_s = 9.188 \text{ in.} \) - Structural depth to tension steel reinforcement measured from the center of the three #5 tension reinforcement.

\( d_f = 11.02 \text{ in.} \) - Structural depth to CFRP reinforcement;

\( d_p = 1.688 \text{ in.} \) - Distance from extreme compression to centroid of compression reinforcement;

\( t = 0.02 \text{ in.} \) - Nominal thickness of a layer of carbon fabric;

\( E_s = 29 \text{ Mpsi} \) - Modulus of elasticity of steel reinforcement;

\( E_c = 285 \text{ Mpsi} \) - Modulus of elasticity of concrete;

\( E_{cc} = 15.87 \text{ Mpsi} \) - Modulus of elasticity of composite;

\( E_f = 34 \text{ Mpsi} \) - Modulus of elasticity of carbon fiber;

\( A_{sp} = 0.22 \text{ in}^2 \) - Cross sectional area of compression reinforcement bars;

\( A_s = 0.93 \text{ in}^2 \) - Cross sectional area of tension reinforcement bars;

\( A_{cfp} = 0.12 \text{ in}^2 \) - Area of composite;
\[ e_y = \frac{f_y}{E_s} \] - Yield strain of reinforcing steel;

\[ e_c = 0.003 \] - Ultimate strain of concrete;

\[ e_f = 0.01 \] - Ultimate strain of composite;

\[ f_y = 66 \text{ kips} \] - Yield stress of steel;

\[ f_{cp} = 6.5 \text{ kips} \] - Compressive strength of concrete;

\[ W_a = \text{Width of Adhesive}; \]

\[ W_f = \text{Width of fiber}. \]

The following sections show the assumptions as well as methods used to predict the ultimate capacity of the model specimen assuming:

1. Perfect Bonding – The entire length of the specimen is bonded.
2. Unbonded – The specimen is only bonded at the ends.

### 4.1.1 Strain Compatibility Method

A “strain compatibility” method was used to predict the nominal strength of the specimens to estimate load levels for the tests.

In addition to “strain compatibility”, force equilibrium and the following assumptions were employed:

1. Concrete tensile strength is ignored;
2. Linear strain distribution through the cross-section;
3. Small flexural deformations;
4. No shear deformations;
5. Perfect bond between materials;
6. Unbonded within the beam span but anchored at the ends for the unbonded case.

7. Stress-strain curve for concrete is approximated by Mander (1988);

8. Stress-strain curve for the reinforcing steel is approximated as elastic-plastic;

9. The maximum usable concrete compressive strain at crushing of the concrete will be taken as 0.003 according to ACI 318-02 10.2.3.

**4.1.1.1 Bonded Nominal Capacity of Beams**

A perfectly bonded assumption is made between the composite and the RC beam neglecting shear deformations. The concrete is assumed to be nonlinear in compression and to exhibit a post-cracking tension-stiffening behavior in tension. The steel reinforcement will be modeled as elastic-plastic and the composite as linear elastic as shown in the section equilibrium and compatibility relation in Figure 25.
The analysis was based on the internal forces, strains and stresses in the cross-section of the composite beam. Using similar triangles, the corresponding strains for the CFRP (\(e_f\)), tensile reinforcing steel (\(e_s\)) and compressive reinforcing steel (\(e_{sp}\)) are calculated as in Equation 12, 13 & 14.

\[
e_f = 0.003 \left( \frac{df - c}{c} \right) \quad \text{Equation 12}
\]

\[
e_s = 0.003 \left( \frac{ds - c}{c} \right) \quad \text{Equation 13}
\]

\[
e_{sp} = 0.003 \left( \frac{c - dp}{c} \right) \quad \text{Equation 14}
\]
Where:

-\( d_f \) = The distance from the centroid of the CFRP to the extreme compression fiber of the concrete;

-\( c \) = The distance from the neutral axis to the extreme compression fiber of the concrete;

-\( d_s \) = The distance from the centroid of the tension reinforcing steel to the extreme compression fiber of the concrete;

-\( d_p \) = The distance from the centroid of the compressive reinforcing steel to the extreme compression fiber of the concrete.

The following are equations for the stresses of the CFRP and reinforcing steel, and are obtained from their stress-strain behavior:

-\( F_f = E_{cf} \cdot e_f \) \hspace{1cm} \text{Equation 15}

-If \( e_s < e_y \), \( f_s = E_s \cdot e_s \) otherwise \( f_s = f_y \) \hspace{1cm} \text{Equation 16}

-If \( e_{sp} < e_y \), \( f_{sp} = E_s \cdot e_{sp} \) otherwise \( f_{sp} = f_y \) \hspace{1cm} \text{Equation 17}

Where:

-\( F_f \) = The stress in the CFRP;

-\( E_{cf} \) = The modulus of elasticity of the CFRP;

-\( e_y \) = The yield strain of the reinforcing steel;

-\( f_s \) = The stress in the tension reinforcing steel;

-\( E_s \) = The modulus of elasticity of the tension reinforcing steel;
\( f_y \) = The yielding stress of the reinforcing steel;
\( f_{sp} \) = The stress in the compressive reinforcing steel.

Each of the corresponding internal forces can be determined by multiplying the stress by their cross-sectional areas. The forces are as follows:

\[
\begin{align*}
T_f &= F_f \times A_f \quad \text{Equation 18} \\
T_s &= f_s \times A_s \quad \text{Equation 19} \\
C_s &= f_{sp} \times A_{sp} \quad \text{Equation 20} \\
C_c &= \beta \times f_{cp} \times b \times c \quad \text{Equation 21}
\end{align*}
\]

Where:

\( T_f \) = The tension force of the CFRP;
\( A_f \) = The cross-sectional area of the CFRP;
\( T_s \) = The tension force of the reinforcing steel;
\( A_s \) = The cross-sectional area of the tension reinforcing steel;
\( C_s \) = The compressive force of the reinforcing steel;
\( A_{sp} \) = The cross-sectional area of the compressive reinforcing steel;
\( C_c \) = The compressive force of concrete;
\( \beta \) = Mean stress factor (0.85);
\( f_{cp} \) = Compressive stress of concrete taken from compression cylinder test;
\( b \) = Width of beam.

The depth of the neutral axis from the extreme compression fiber (c) was obtained
from the equilibrium of the internal forces of the beam. The total compressive forces are equal to the total tensile forces,

\[ C_c + C_s = T_s + T_f \]  

Equation 22

Equations 18 to 21 into equation 22 results;

\[ \beta f_{cp} b c + f_{sp} A_{sp} = f_s A_s + F_f A_f \]  

Equation 23

Note: If no CFRP is used (control beam), \( T_f \) is zero.

The neutral axis depth can be calculated using the quadratic equation (MathCAD).

With this parameter known, the internal nominal moment (\( M \)) is obtained by taking the sum of the moments about the middle of the cross section:

\[ M_n = \frac{h}{2} - a + C_s \left( \frac{h}{2} - d_s \right) + T_s \left( d_s - \frac{h}{2} \right) + T_f \left( df - \frac{h}{2} \right) \]  

Equation 24

Where:

\[ a = \text{Depth of equivalent rectangular stress block} = c^*\beta; \]

Computations of resisting moments for the control beam and RC beams with 1, 3 and 5 layers of CFRP are presented in Appendix B. Results were compared with ultimate moments generated from moment-curvature programs.

4.1.1.2 Unbonded Nominal Capacity of Beam

For the unbonded condition, since there is no longer compatibility between the RC cross-section and the composite, analyzes will solely be based on the moment
deflection study using the iteration method as discussed in Section 4.2.1 by assuming constant CFRP strains.

4.2 Moment Deflection Study

4.2.1 Iteration Method

An iteration method was developed to calculate the mid span deflection for both the bonded and unbonded conditions for the RC beam externally strengthened with CFRP. The iteration was accomplished by a MathCAD program and presented in the Appendix B. The program contains two major parts. The first is to create moment-curvature (M-φ) curves while the second is to use the M-φ curve to calculate the moment-deflection (M-δ) curve by integration.

4.2.1.1 Creating Moment-Curvature Curves

The M-φ relation was developed based on the relation of concrete and the force and moment equilibrium of the section.
The compressive behavior of concrete follows the Mander model [Mander 1988], which is a nonlinear function between the stress and the strain for confined and unconfined concrete. In this thesis, the concrete was modeled as unconfined. Mander’s equation is as follows:

\[ f_c = \frac{f_{ccp} \times x \times r}{r - 1 + x'} \quad \text{Equation 25} \]

\[ f_{ccp} = f_{cp} \left( 2.254 \sqrt{1 + \frac{7.94 f_p}{f_{cp}}} - \frac{2 f_p}{f_{cp}} - 1.254 \right) \]

\[ x = \frac{e_c}{e_{cc}} \]

\[ e_{cc} = 0.002 \left[ 1 + 5 \left( \frac{f_{ccp}}{f_{cp}} - 1 \right) \right] \]
\[ r = \frac{E_c}{E_c - E_{sec}} \]

\[ E_c = 57000\sqrt{f_{cp}} \quad \text{(psi)} \]

\[ E_{sec} = \frac{f_{ccp}}{e_{cc}} \]

Where:

\[ f_c = \text{Concrete stress}; \]
\[ e_c = \text{Concrete strain}; \]
\[ f_{ccp} = \text{Concrete strength at peak for confined concrete}; \]
\[ f_{cp} = \text{Concrete strength at peak for unconfined concrete}; \]
\[ f_p = \text{The average confining stress} = \text{zero for unconfined concrete}; \]
\[ e_{cc} = \text{Concrete strain at peak of confined concrete}. \]

According to the plane section assumptions, relation between curvature (\(\phi\)) and strain for concrete (\(e_c\)), composite (\(e_f\)), tension (\(e_s\)) and compression (\(e_{sp}\)) steel can be found using a program described by its flow chart shown in Figure 27. Strain hardening, which allows concrete strength to increase over a long-term, was not considered in the moment curvature analysis.
Start \rightarrow \text{Assume } f = 0.00001 \rightarrow F = F + 0.00001

Try c = ? \rightarrow c = c + 0.01

ecf = i * c/n * F \rightarrow ec = c * F

Concrete Failure End

N \rightarrow ec < 0.004 \rightarrow Y

esp = (c-dp) * F \rightarrow es = (ds-c) * F \rightarrow ef = (df-c) * F

frp \leftarrow Ecf*ef \text{ if } ef < 0.01, \quad frp \leftarrow 0 \text{ if } ef > 0.01, \quad frp \leftarrow 0 \text{ if } ef < 0

fs \leftarrow Es*es \text{ if } |es| < ey, \quad fs \leftarrow Es*ey \text{ if } |es| > ey

Y \rightarrow T = As*fs + Af*frp

N \rightarrow fc = f*c/n*b*fc

Ci = i*c/n*b*fc \rightarrow Ct = \sum_{i} Ci

C = Ct + Cs

C = T \rightarrow Y

M = \sum_{i} Ci \left[ \left( \frac{h}{2} \right) - \left( c - \left( i - \frac{1}{2} \right) * \frac{c}{n} \right) + (Ts + Cs) \right] * \left( \frac{h}{2} - dp \right) + Tf * \left( \frac{h}{2} \right)

OUTPUT M - F

Figure 27: Flow Chart of M-φ Curve for Bonded Condition.
4.2.1.2 Creating Moment-Displacement Curves

The (M-δ) behavior was predicted using Principles of Virtual Work. To compute the deflection (δ), virtual unit load acting in the direction of “δ” is placed on the beam at that point and the internal virtual moment “m” is determined by the method of sections at any location “x” from the left support as shown in Figure 28.

![Figure 28: Real and Virtual Loads as it applies to the Principle of Virtual Work.](image)

When the real loads act on the beam, the mid point is deflected by “δ” as shown in part A of Figure 28. Assuming that the real loads cause linear elastic material response, the element “dx” will deform by;
\[ \text{d} \alpha = (M/EI) \text{d}x. \quad \text{Equation 26} \]

The *external virtual work* done by the unit load presented in part B of Figure 28 is \(1*\delta\), and the *internal virtual work* done by the moment “\(m\)” is;

\[ m \text{d} \alpha = m(M/EI) \text{d}x. \quad \text{Equation 27} \]

Integration is required to sum the effects on all the elements “\( \text{d}x \)” along the beam hence

\[ 1*\delta = \int_0^l \frac{mM}{EI} \text{d}x, \quad (\text{external virtual work} = \text{internal virtual work}) \quad \text{Equation 28} \]

Where;

1 = External virtual unit load acting on the beam in the direction of \(\delta\);

\(m\) = Internal virtual moment in the beam, expressed as a function of \(x\) and caused by the external virtual unit load;

\(\delta\) = External displacement of the point caused by the real loads acting on the beam;

\(M\) = Internal moment in the beam, expressed as a function of \(x\) and caused by the real loads;

\(E\) = Modulus of elasticity of the material;

\(I\) = Moment of inertia of cross-sectional area, computed about the neutral axis;

\(EI\) = Flexural rigidity (always positive).

From the M-\(\phi\) relation developed and the virtual work equation:
\[ 1 \delta = \int_{0}^{L} \frac{mM}{EI} \, dx \quad \text{Equation 29} \]

The M-\( \delta \) relation can be established using drawings presented in Figure 29.

Figure 29: Moment Diagram of Beam with Real and Virtual loads

A=Four-point loaded beam  
B=Moment diagram for four-point loaded beam  
C=Unit load beam  
D=Moment diagram for unit loaded beam

To determine the mid span deflection of the beam, the following were taken into account.
- Four point bending analysis parts A & B of Figure 29.
- Unit load bending analysis parts C & D of Figure 29.
- Half of the beam was used for analysis since beam is symmetric resulting in a distance “x” from the left support to be less or equal to L/2.
- The half beam analysis resulted in unit load moment \( m = \frac{1}{2}x \) (using method of sections).
- The moment-curvature relation \( \phi = \frac{M}{EI} \) was also employed.

The above accounts, transforms the Virtual Work Principle equation, Equation 29 to;

\[
\sum_{j=1}^{\frac{1}{2}} \phi_j + \phi_{j-1} \frac{1}{2} x \Delta x = \int_0^\frac{1}{2} \phi x \Delta x
\]

Equation 30

A numerical summation was then used to determine the midpoint deflection by dividing the beam into “n” segments and using the trapezoidal solution. For every segment in the beam, the curvature and its distance from the left support was computed as follows;

\[
\phi = \frac{\phi_i + \phi_{i-1}}{2}
\]

Equation 31

\[
x = (i - \frac{1}{2})\Delta x
\]

Equation 32

Therefore;

\[
\sum_{j=1}^{\frac{1}{2}} \phi_j + \phi_{j-1} \frac{1}{2} (i - \frac{1}{2})\Delta x \Delta x
\]

Equation 33

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Where;

\[ dx = \Delta x; \]

\[ \Delta x = \frac{L}{n}. \]

Example:

Consider the four point bending beam divided into 20 segments shown in Figure 30. The figure shows the different segments and their corresponding moment-curvature values. Only one load point is showed in this figure since the other loaded point will be on the other half of beam.

Figure 30: Half Beam Divided into “n” Segments for M-φ Computation.
For the above section, the displacement (deflection) will be calculated as follows;

\[
\delta_{\text{midspan}} = \sum_{i=1}^{10} \frac{\phi_i + \phi_{i-1}}{2} (i - \frac{1}{2})(\frac{L}{n})^2
\]

\[
= \left[ \frac{\phi_1 + \phi_0}{2} + \frac{\phi_2 + \phi_1}{2} \right] + \cdots + \frac{\phi_{10} + \phi_9}{2} \left( \frac{L}{20} \right)
\]

This approach can be used directly for a bonded RC beam and composite. That is for every applied load, the applied moment can be computed. The moment for each segment was calculated and the corresponding curvatures calculated by interpolation using the \( M-\phi \) curve constructed.

When all the curvatures are computed for half the beam, the mid span displacement is calculated using Equation 33.

Unlike the bonded case, calculating the displacement of unbonded beams is not easy since the force in the composite can not be directly calculated as in the bonded beams. The moment curvature for the unbonded case is computed assuming constant CFRP strains.

**4.3 Prediction of Maximum Deflection**

Based upon the derivation and formulation to compute the deflection at mid span, the maximum deflections for the control and composite beam (RC beam externally strengthened with CFRP) were predicted at concrete failure and debonding.

**4.3.1 Control Beam**

For the control beam, the maximum moment from the moment-curvature
developed using the iteration method for the RC beam without composite was first compared with the manual computation maximum moment using principles discussed and derived in Section 4.1.1.1 assuming that there was no CFRP therefore $T_f$ the tension force in CFRP was zero. The results were different by 1%. The moment-curvature data was then used to compute the moment-displacements and to predict the maximum deflection of the control beam. Manual computations, moment-curvature and moment-displacement computations are present in Appendix B. The predicted maximum deflection, load and moment values will also be compared with experimental results in Chapter 5.

4.3.2 Composite Beam

For the composite beams, the derived principles in Section 4.1.1.1 were employed for the manual computations. However, predictions were made for composite beams with 1, 3, and 5 layers of CFRP assuming perfect bonding (full anchorage). Manual computations, compared with the iteration method in Section 4.2.1 were within a percentages difference of 2-3%. Manual computations, moment-curvature and moment-displacement computations are also present in Appendix B for the composite beams with 1, 3, and 5 layers of CFRP. The predicted maximum deflection, load and moment values for the composite beam with 5 layers of CFRP will be compared with experimental results in Chapter 5.

4.4 Interface Shear and Stress Analysis

The number of layers to use for flexural strengthening was determined from the
moment deflection study, which was targeted at twice the strength of the control beam assuming full anchorage. With this criterion, the prediction of debonding loads and end plate conditions are necessary due to design methods presented in Chapter 3 which shows some of the specimen that are not fully anchored. As discussed in Chapter 3, design methods are targeted to follow certain failure and debonding patterns to aid in the study and investigation of CFRP strengthened RC beams. These predictions were achieved by an interface shear and stress analysis.

4.4.1 Shear Flow Analysis

The horizontal shear flow was analyzed assuming an uncracked section and using the equation;

\[ q = \frac{VQ}{I} \]  

Equation 34

Where;

- \( q \) = Shear flow which is the horizontal shear force per unit distance along the longitudinal axis of the beam;
- \( V \) = Shear force;
- \( Q \) = First moment;
- \( I \) = Moment of inertia.

Since different materials make up the cross section (concrete, steel, adhesive and carbon fiber), the modulus of elasticity of each material is taking into account.
The moment of inertia is first computed using the parallel axis theorem, which requires that the neutral axis of the cross section shown in Figure 31 have to be determined. The neutral axis of the cross section was first computed by determining the center of gravity of each component.

The neutral axis for each component in the cross section as shown in Figure 31 was calculated with the section above the concrete interface assumed to be negative.

Figure 31: Cross Section with “x” and “y” axes.
The following equations were derived using the cross section:

\[
\text{ConcreteC} = \frac{-h}{2} \quad \text{- Center of gravity of the concrete section; \quad Equation 35}
\]

\[
\text{TenSteelC} = -dp \quad \text{- Center of gravity of the tension steel; \quad Equation 36}
\]

\[
\text{CompSteelC} = -h + dp \quad \text{- Center of gravity of the compression steel; \quad Equation 37}
\]

\[
\text{AdhesiveC} = \frac{ta}{2} (2*i - 1) + t*(i - 1) \quad \text{- Center of gravity of the adhesive; \quad Equation 38}
\]

\[
\text{FiberC} = ta*i + \frac{t}{2} (2*i - 1) \quad \text{- Center of gravity of the fiber; \quad Equation 39}
\]

\[
x = \frac{\text{ConcreteM} + \text{CompSteelM} + \text{TenSteelM} + \text{AdhesiveM} + \text{FiberM}}{\text{ConcreteF} + \text{TenSteelF} + \text{CompSteelF} + \text{AdhesiveF} + \text{FiberF}} \quad \text{Equation 40}
\]

\[
\text{ConcreteM} = \text{Ecc*b*h*ConcreteC} \quad \text{- Concrete moment; \quad Equation 41}
\]

\[
\text{CompSteelM} = (Es-Ecc)*As*\text{CompSteelC} \quad \text{- Compression steel moment; \quad Equation 42}
\]

\[
\text{TenSteelM} = (Es-Ecc)*As*\text{TenSteelC} \quad \text{- Tension steel moment; \quad Equation 43}
\]

\[
\text{AdhesiveM} = \sum \text{AdhesiveC*Cw*ta*Ea} \quad \text{- Adhesive moment; \quad Equation 44}
\]

\[
\text{FiberM} = - \sum \text{FiberC*Cw*t*Ef} \quad \text{- Fiber moment; \quad Equation 45}
\]

\[
\text{ConcreteF} = \text{Ecc*b*h} \quad \text{- Concrete force; \quad Equation 46}
\]

\[
\text{TenSteelF} = (Es-Ecc)*As \quad \text{- Tension steel force; \quad Equation 47}
\]
CompSteelF = (Es-Ecc)*Asp - Compression steel force;  Equation 48

AdhesiveF = n*Ea*ta*Cw - Adhesive force;  Equation 49

FiberF = n*Ef*t*Cw - Fiber force;  Equation 50

Where;

x = Neutral axis;

E_{cc} = Modulus of Elasticity of concrete;

E_s = Modulus of Elasticity of steel;

E_f = Modulus of Elasticity of carbon fiber;

E_a = Modulus of Elasticity of adhesive;

A_s = Area of tension steel;

A_{sp} = Area of compression steel;

h = Beam height;

C_w = Composite Width (width of adhesive and fiber)

d_p = Cover

\( t_a \) = Thickness of adhesive

\( t \) = Thickness of fiber

n = Number of layers

i = \( i^{th} \) layer
From the parallel axis theorem, which states that, the moment of inertia of an area with respect to any axis in its plane is equal to the moment of inertia with respect to a parallel centroidal axis plus the product of the area and the square of the distance between the two axis, the inertia was computed. From the cross section above, using the axis a-a, and applying the parallel axis theorem results in the equation;

\[ I_{aa} = I_{xx} + \sum Ad^2 \]  
Equation 51

Taking the Modulus of Elasticity of each component into account,

\[ EI_{aa} = EI_{xx} + E \sum Ad^2 \]  
Equation 52

\[ EI_{xx} = \frac{Ecc}{12} * b * h^3 \]  
Equation 53

\[ \sum Ad^2 = \text{ConcreteAd}^2 + \text{TenSteelAd}^2 + \text{CompSteelAd}^2 + \text{AdhesiveAd}^2 + \text{FiberAd}^2 \]  
Equation 54

\[ \text{ConcreteAd}^2 = E_{cc} * b * h * (x - \text{ConcreteC})^2 \]  
Equation 55

\[ \text{TenSteelAd}^2 = (E_s - E_{cc}) * A_s * (x - \text{TenSteelC})^2 \]  
Equation 56

\[ \text{CompSteelAd}^2 = (E_s - E_{cc}) * A_{sp} * (\text{CompSteelC} - x)^2 \]  
Equation 57

\[ \text{AdhesiveAd}^2 = E_a * C_w * t_a * \sum (\text{AdhesiveC} - x)^2 \]  
Equation 58

\[ \text{FiberAd}^2 = E_f * C_w * t * \sum (\text{FiberC} - x)^2 \]  
Equation 59
Where;

\[ \sum A_d^2 = \text{The sum of the area of each component multiplied by square of their distance from the neutral axis}; \]

\[ I_{xx} = \text{Moment of inertia with respect to the parallel centroidal axis}. \]

In calculating the first moment \( Q \), it is assumed that the adhesive between the concrete face and the fiber will transmit the horizontal shear forces that act between the concrete and the fiber. At the fiber section, the horizontal shear force (per unit distance along the axis of the beam) is the shear flow along the contact surface \( a-a \). The shear flow is calculated by taking \( Q \) as the first moment of the cross-sectional area below the contact surface \( a-a \) (adhesive and fiber). Therefore, \( Q \) is the first moment of the adhesive and fiber calculated with respect to the neutral axis. After calculating the shear flow, the amount of the adhesive and fiber needed to resist the shear force can be determined due to the relationship between the shear stresses and the thickness of the member.

Additionally, the correlation between the thickness level of the composite and debonding can be further investigated from the shear flow analysis i.e. the greater the composite thickness the lesser the interface strain and hence the increase in interface stress which leads to debonding at low loads. The results from the shear flow analysis can therefore be used to determine the number of layers needed for flexural strengthening targeting twice the strength of the control beam. Hence the thickness of CFRP can then be determined by the shear flow demand.
The first moment $Q$ for the adhesive and fiber were computed using the first moment equation as follows.

$$Q = \int y \, dA$$  \hspace{1cm} \text{Equation 60}

$$EQ = \int Ey \, dA \text{ (for the different component with different modulus)}$$  \hspace{1cm} \text{Equation 61}

$$dA = Cw \cdot dy$$  \hspace{1cm} \text{Equation 62}

Therefore;

$$EQ = E \cdot Cw \int y \, dy$$  \hspace{1cm} \text{Equation 63}

$$EQ = E \cdot Cw \cdot \frac{1}{2} \left[ y_1^2 - y_2^2 \right]$$  \hspace{1cm} \text{Equation 64}

For the fiber,

$$y_1 = x_1 - x$$  \hspace{1cm} \text{Equation 65}

$$y_2 = x_2 - x$$  \hspace{1cm} \text{Equation 66}

$$E = E_c$$  \hspace{1cm} \text{Equation 67}

For the adhesive,

$$y_1 = x_2 - x$$  \hspace{1cm} \text{Equation 68}

$$y_2 = x_3 - x$$  \hspace{1cm} \text{Equation 69}
E = E_a 

Equation 70

Therefore;

\[ EQ = E_c \cdot C_w \cdot \frac{1}{2} \left[ (x_1 - x)^2 - (x_2 - x)^2 \right] + E_a \cdot C_w \cdot \frac{1}{2} \left[ (x_2 - x)^2 - (x_3 - x)^2 \right] \]

Equation 71

Where:

\( x_1 \) = bottom layer of fiber;

\( x_2 \) = top layer of fiber;

\( x_3 \) = top layer of adhesive or bottom layer of concrete.
4.4.2 Analytical Models

Debonding at the plate ends and mid span before concrete failure (crushing) is expected depending on the design specimen in this research so the debonded CFRP will propagate from the plate ends to the mid span or vice versa. The reason for this mode of failure is to aid in the study, investigation and behavior of the transverse anchorage and improve the failing mode of CFRP. The four main debonding criteria investigated were:

1. Debonding due to the interface shear stress between concrete and adhesive exceeding the shear strength of the concrete-adhesive interface.

2. Debonding due to the interface shear stress between the FRP and the adhesive exceeding the strength of the FRP-adhesive interface.

3. Debonding initiated by the development of a flexural crack in the maximum bending moment region, where the debonding begins at one of the flexural cracks and propagates towards the support.

4. Debonding due to local shear-tension failure where a crack initiates around one of the plate ends at the level of the tension steel reinforcement and propagates horizontally toward the mid span of the concentrated load resulting in delamination of the concrete cover.

The following analytical models were used for the debonding predictions:

1. Aryaa and Farmer (2001)

2. Roberts (1989)

The analytical model by Aryaa and Farmer (2001) presented in Chapter 1, for an uncracked RC beam strengthened with CFRP was used to predict interface stress and shear flow between the concrete and composite interface. The analysis, which was done using five layers of CFRP, was compared with the shear flow analysis presented above. There was a 20% discrepancy between the two methods with Aryaa and Farmer having a higher shear flow.

Since the RC beams for this research were initially cracked before applying the CFRP, Roberts’s (1989) analytical model which took into account initial cracking of the beams presented in Chapter 1 was used to predict the debonding load and end plate conditions for a cracked section externally strengthened with CFRP. It was very important to estimate these conditions (debonding load and end plate stresses) because of the design methods employed. Debonding at both mid spans and end plates were incorporated in the design methods to investigate the failure modes hence the importance the debonding loads. Using Roberts, the minimum layers of CFRP to use for external strengthening to approximately double the strength of the pure RC beam and also for debonding to occur at strength was predicted.

Malek et al (1998) closed form solution to predict the maximum shear stress at the plate end assuming linear elastic behavior of the materials, no slip and complete composite action between the plate and concrete gave a large estimate, which had to be further reviewed. The model was based on the interfacial shear and normal stresses in the
concrete beam at the cut-off point, which normally leads to premature local failure in the concrete beam and separation of the plate. The equations for the maximum shear stress at the plate are presented in Chapter 1.

Computations of the shear flow analysis, Aryaa and Farmer, Roberts and Malek et al are presented in the Appendix B.

4.5 Ductility

Ductility, as presented in Chapter 1, is very important since it gives us an idea of the structures ability to sustain inelastic deformations without significant loss in resistance. Two methods were used to predict the ductility of the control beam (pure RC beam) and the composite beam (RC beam externally strengthened with 5 layers of CFRP). The methods were;

- Conventional Method - Displacement Ductility by Paulay and Priestley (1992)

The predictions were done using data from the predicted moment-displacements program for the pure RC beam and the composite beam assuming perfect bonding.

4.5.1 Displacement Ductility

Ductility has generally been measured by a ratio called a ductility index or factor ($\mu$). The ductility index is usually expressed as a ratio of rotation ($\theta$), curvature ($\varphi$), or displacement ($\Delta$) at failure to the corresponding property at yield. For this study, displacement will be used as the primary measurement of ductility.

Displacement ductility is the most convenient quantity to evaluate a structure’s
capacity to develop ductility. Paulay and Priestley presented the displacement ductility as:

\[ \mu_A = \frac{\Delta}{\Delta_y} > 1 \]  

Equation 72

Where;

\[ \Delta = \text{Maximum anticipated displacement at failure}, \]
\[ \Delta_y = \text{Displacement at yielding which for most reinforced concrete and masonry structures, is assumed to occur simultaneously with the yield curvature } \phi_y. \]

From the moment-curvature and moment-displacement analysis for the control and composite beams, the ductility for both specimens was predicted. The yielding displacements, which were hard to predict from the moment-displacement plot and data, were derived from the moment-curvature analysis. The estimated yielding loads for the control and composite beam were derived from the yielding moments. The yielding loads were then used to predict the yielding displacements.

The control and composite beams had a ductility index greater than one showing that they were both ductile according to the Conventional Paulay and Priestley Method. The control beam was more ductile than the composite beam with their estimated ductility index of 2.20 and 1.75 respectively. The disparity between the composite beam and the control with the composite beam having a higher ductility than the control beam could be the perfect bonding assumption made for the composite beam.
4.5.2 Energy Method

The energy method by Naaman and Jeong (1995) as presented in Chapter 1 was also used to predict the ductility of the control and composite beams. As stated in Chapter 1, the advantages of this method as stated below will improve the ductility estimation.

- The energy method, which was originally used to calculate the ductility for concrete beams with internal FRP reinforcement, has been very helpful in determining ductility in cases where the yielding point is difficult to establish due to lack of steel strain measurements.

- The elastic energy can be determined without having unloading data. This is done based on the experimental load-deflection curve and is the area of the triangle formed at the failure load by the line having the weighted average slope of the two initial straight lines of the curve.

The initial straight lines (slopes) drawn on the curve to approximate the stiffness and the loads at the intersection of these lines are estimated at the initial cracking, approximate yielding and failure points. The parallelism of the initial slope was checked with the estimated unloading slope. In the case where the initial slope seem to be parallel with the unloading slope; the structure possibly has not yielded.

The total energy absorbed in failure of the specimen was obtained by calculating the area under the load-displacement curve. MathCAD was used to calculate the area under the load-displacement curve using the Simpson’s integration method.

The ductility index and the energy ratio using the “energy method” for the
predicted control and the composite beams were 3.717, 4.223 and 84.64%, 86.57% respectively. The “energy” method shows that the predicted composite ductility assuming perfect bonding is greater than the control beam ductility. The energy ratios were above 75% which implies both specimens were ductile.

**4.6 Examples**

The results of the programs give us an idea of the following.

- M-φ relation for the pure concrete beam (control)
- M-φ relation for the specimen assuming perfect bonding concrete beam
- M-δ relation for the specimen assuming perfect bonding
- M-δ relation for the control beam
- M-φ relation for the specimen assuming unbonded concrete beam

![Figure 32: Prediction of M-φ Curve for Control Beam and Bonded 5 Layer Composite.](image-url)
Figure 33: Prediction of M-δ Curve Control Beam and Bonded 5 Layer Composite.
Figure 34: Prediction of M-\(\phi\) Curve for Unbonded Beam.
CHAPTER 5: TEST RESULTS AND DISCUSSION

The test results of the six beams described in Chapter 3 are presented in the following chapter. Their behavior from initial loading and throughout the static cyclic test to failure is described using recorded data, observed crack patterns and mode of failure. The behavior of each beam is also compared to that predicted by the model presented in Chapter 4.

5.1 Load Deflection

For each beam, the load and deflection results of the tests are presented. The results presented are from the initial loading to the response of the static cyclic loading behavior of each beam. As stated above, the predicted behavior in terms of capacity and stiffness of each beam is compared to the experimental results.

5.1.1 Initial Loading

An average initial flexural load for about 32% of the calculated ultimate moment of the control RC beam was applied to all six beams in preparation for the CFRP application. The load was applied until the first flexural cracks were observed. The cracking load level was intended to simulate a reasonable service condition and allow the formation of three to four cracks in the region of constant moment. The strengthening that will be applied to the cracked beams will be referred to as “retrofitting”. When retrofitting existing RC structures, cracks at the bottom of the RC members are expected hence the cracking of the beams before CFRP application as a representation of retrofitting in the field. The cracking load versus displacement curves for the six beams
as well as a table for maximum cracking loads and displacements taken from the actuator (MTS) are shown in Figure 35 and
Table 2 respectively.

Figure 35: Initial Cracking Load versus Displacement Plot for all Beams.
Table 2: Summary of Initial Cracking Loads Versus Maximum Displacement.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Max. Cracking Load (lb)</th>
<th>Max. Cracking Disp. (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>7013</td>
<td>0.46</td>
</tr>
<tr>
<td>Design 1</td>
<td>4684</td>
<td>0.29</td>
</tr>
<tr>
<td>Design 2</td>
<td>5387</td>
<td>0.34</td>
</tr>
<tr>
<td>Design 3</td>
<td>5925</td>
<td>0.26</td>
</tr>
<tr>
<td>Design 4</td>
<td>6966</td>
<td>0.44</td>
</tr>
<tr>
<td>Design 5</td>
<td>5194</td>
<td>0.31</td>
</tr>
</tbody>
</table>

5.1.2 Quasi Static Cyclic Loading: Load Versus Deflection Response

A summary of the test plots for all the design specimens and the control beam are shown in Figure 36. A detailed description of the testing procedure and observations are also presented in this section.
Figure 36: Summary of Design Specimens and Plot Results.
For a gradual and systematic loading to aid in observing the behavior of the beams during loading, each beam was statically loaded in 25, 50 and 75% increments of the predicted ultimate capacity of the perfect bonded beam with the exception of the control beam in which the ultimate capacity of the control beam was used. Every incremental stage was repeated three times hence “quasi static cyclic loading.” After the third cycle at 75% ultimate were completed, quasi static monotonic loading was applied until failure, which was observed to be concrete crushing or debonding. The predicted ultimate load for both the control beam and the 5 layer composite beam assuming perfect bonding were 18.2 and 33.6 kips respectively as shown in Figure 33.

5.1.2.1 Control

The control beam as shown in Figure 37 was tested as a baseline to be compared with the CFRP strengthened beams. Quasi static cyclic loading, in increments of 25, 50 and 75% of the predicted ultimate baseline capacity was used in loading the specimen. Every incremental stage was repeated three times resulting in a three cycle pattern. After each loading stage (the end of the third cycle of each increment), the beam was inspected for cracks and any possible signs of failure.
The load-deflection plot for the actual beam and the load-deflection plot obtained from the theoretical model are shown in Figure 38.
The beam appeared to display linear behavior to the cracking load. Large increase in deflection was noticed after what seem to be the yielding of the steel reinforcement with very little increase in load, which was applied in 0.1 in. displacement increments. This behavior continued until failure was caused by the crushing of the concrete at the top of beam as shown in Figure 39. The test was terminated once crushing of the concrete was observed.

Figure 38: Theoretical and Experimental Load Displacement for Control Beam.
The theoretical model prediction of the test beam behavior was close to the experimental results. The test results at the maximum load capacity and corresponding deflections are compared and presented in Table 3.
Table 3: Tested and Theoretical Properties of the Control Beam.

<table>
<thead>
<tr>
<th></th>
<th>Max. Load (kips)</th>
<th>Max. Deflection (in.)</th>
<th>Mode of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tested</td>
<td>21.0</td>
<td>1.58</td>
<td>Concrete Crushing</td>
</tr>
<tr>
<td>Theoretical</td>
<td>18.2</td>
<td>1.21</td>
<td>Concrete Crushing</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>13</td>
<td>23</td>
<td></td>
</tr>
</tbody>
</table>

The significant difference between the actual data and theoretical prediction was the maximum deflection at concrete crushing. The model under predicted this test value by 23%. As shown in Figure 38, the theoretical model predicted a wider plastic range before concrete crushing on the load-displacement plot. The model did not account for strain-hardening hence resulting in a plastic range assuming steel is ideal after yielding.

5.1.2.2 Design 1

The beam specimen strengthened with five layers of CFRP on the tension face was tested using the same loading procedure as was used for the control beam. That is static cyclic loading, in increments of 25, 50 and 75% of the predicted ultimate capacity as well as the three cycles for each incremental stage. After each loading stage (the end of the third cycle of each increment), the beam was inspected for cracks and any possible failure signs.

The load-deflection plot for the actual beam and the load-deflection plot obtained from the theoretical model are shown in Figure 40.
The beam appeared to have linear behavior until possibly when it reached the cracking moment. After this point, the linear behavior continued but the stiffness was reduced. Cracking sounds were heard when loading was about 76% of the predicted ultimate capacity which could be an indication of the CFRP debonding from the concrete. Plate being suddenly separated from the RC beam rather than by the ultimate flexural capacity of the section is known as a debonding failure. Those that initiate at or near a plate end and then propagated from the plate end are referred to as plate end debonding. The load was held when the cracking sound was heard and the beam inspected. There was no visible failure or debonding at this point. Loading was then continued with very small displacement increments (0.1 in.). The loading was continued until a loud explosive
sound was heard. The load was once again held and the beam inspected. After inspection, it was observed that the CFRP had debonded (plate end debonding) on end of the beam and propagated close to the mid span as shown in Figure 41. There was a 30% drop from the load capacity attained by the beam after debonding.

Figure 41: Design 1 Beam at Failure after Testing.

The theoretical model prediction of the test beam behavior was close to the experimental results. The test results at the maximum nominal capacity and corresponding deflections are compared and presented in
Table 4.
Table 4: Tested and Theoretical Properties of Design 1 Beam.

<table>
<thead>
<tr>
<th>Tested</th>
<th>Max. Load (kips)</th>
<th>Max. Deflection (in.)</th>
<th>Mode of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>24</td>
<td>0.9</td>
<td>Debonding of CFRP from Concrete Interface.</td>
</tr>
<tr>
<td>Theoretical</td>
<td>33.6</td>
<td>1.38</td>
<td>Debonding</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>28</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

The significant difference between the actual data and theoretical prediction was the maximum deflection at debonding. The model over predicted this test value by 35%. As shown in Figure 40, the theoretical model predicted a wider plastic range before debonding and did not take into account strain hardening. Consequently, the theoretical model assumes a perfect bonding which is not the case in the Design 1 beam, which had no anchorage.

5.1.2.3 Design 2

In addition to the beam specimen strengthened with five layers of CFRP on the tension face, 90° transverse anchorage of lengths 4, 6, 8 and 10 in. respectively were used to anchor the tension five layers CFRP as shown in Figure 42. See Chapter 3, Design 2 for details on anchorage. This specimen was tested using the same loading procedure as used for the control beam. That is static cyclic loading, in increments of 25, 50 and 75% of the predicted ultimate capacity as well as the three cycles for each incremental stage. After each loading stage (the end of the third cycle of each increment), the beam was
inspected for cracks and any possible failure signs.

Figure 42: Design 2 Beam Test Set-up.

The load-deflection plot for the actual beam and the load-deflection plot obtained from the theoretical model are shown in Figure 43.
The beam appeared to have linear behavior until possibly where it appeared that the cracking moment was reached. The linearity continued to approximately 80% of the predicted ultimate capacity. After this point, the linear behavior continued but the stiffness was reduced. Cracking sounds were heard when loading was about 90% of the predicted ultimate capacity which could be an indication of the CFRP debonding from the concrete. Unlike the plate end debonding observed from Design 1, the cracking sound seems to be coming from the mid span. This type of debonding is initiated at an intermediate flexural or flexural-shear crack and then propagates from such a crack towards a plate end.

The load was then held when the cracking sound was heard and the mid span of
the beam carefully inspected. The five layers CFRP at the mid span seem to have debonded slightly (a little gap between the CFRP and concrete) but with the load still on the specimen and deflected it was difficult to draw any conclusions at this point. Other parts of the beam including the transverse anchorages remained intact with no signs of debonding or shearing. Loading was then continued with very small displacement increments (0.1 in.) but the beam continued to deflect more with these small increments.

The loading was continued until a loud explosive sound was heard. The load was once again held and the beam inspected. After inspection, it was observed that about one inch of the 4 in. CFRP 90° transverse anchorage closest to the mid span on one side of the beam had sheared transversely from the top to the bottom of the beam. The specimen had reached 95% of the predicted ultimate capacity just before the shearing. There was a 10% drop from the load capacity attained by the beam with a visible but small debonding of the five layers CFRP at the mid span. The rest of the beam still remained intact.

Loading was then continued at a lower displacement increment (0.05 in.). Another explosive sound not quiet as loud as the previous one was heard when the load reached 91% of the predicted ultimate capacity. After a close observation, it was noticed that another in of the same 4 in. transverse anchorage stripped in the same manner. The second shear in the 4 in. transverse anchorage occurred simultaneously with concrete crushing at the top fiber of the beam (compression face). The test was terminated once the crushing began. A final examination of the specimen showed that debonding occurred in the mid span and propagated to one side of the beam. The propagation reached the first
anchorage (4 in. transverse) and sheared it as shown in Figure 44. Although the propagation did not seem to go beyond the 4 in. transverse, there were signs of slightly debonding between the 4 and 6 in. transverse and indications of shear in the 6 and 8 in. transverse respectively. Knocking on the five layers CFRP between the 4 and 6 in. transverse made a hollow sound, which was an indication of debonding. The remaining sections sounded solid implying there was no debonding. The 10 in. end anchorages had no signs of shear or stress. The load drop and load increase during the two shearing incidence of the 4 in. transverse produced a two-step progression before concrete crushing. The two-step progression before concrete crushing is a warning indication.

![Shearing of CFRP](image)

Figure 44: Design 2 Beam at failure after testing.

The theoretical model prediction of the test beam behavior different from the experimental results. The test results at the maximum load capacity and corresponding deflections are compared and presented in Table 5.
Table 5: Tested and Theoretical Properties of Design 2 Beam.

<table>
<thead>
<tr>
<th>Max. Load (kips)</th>
<th>Max. Deflection (in.)</th>
<th>Mode of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tested</td>
<td>26.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Theoretical</td>
<td>33.6</td>
<td>1.38</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>21</td>
<td>1.4</td>
</tr>
</tbody>
</table>

The significant difference between the actual data and theoretical prediction was the maximum load at debonding and concrete crushing. The model over predicted this test value by 21%. On the other hand, tested Design 2 beam exhibited some progression after debonding before concrete crushing. The maximum deflection predicted was however close to the tested maximum deflection due to the provision of some anchorage.

5.1.2.4 Design 3

In addition to the beam specimen strengthened with five layers of CFRP on the tension face, 45° transverse anchorage of the same lengths was used to anchor the tension five layers CFRP as shown in Figure 45. See Chapter 3, Design 3 for details on anchorage. This specimen was tested using the same loading procedure as used for the control beam. That is static cyclic loading, in increments of 25, 50 and 75% of the predicted ultimate capacity as well as the three cycles for each incremental stage. After each loading stage (the end of the third cycle of each increment), the beam was inspected for cracks and any possible failure signs.
Figure 45: Design 3 Beam Test Set-up.

The load-deflection plot for the actual beam and the load-deflection plot obtained from the theoretical model are shown in Figure 46.
The beam had a linear behavior until possibly where it appeared that the cracking moment was reached. The linearity continued to approximately 100% of the predicted ultimate capacity. After this point, the linear behavior continued but the stiffness was reduced. Cracking sounds were heard when loading was about 5% above the predicted ultimate capacity which could be an indication of the CFRP debonding from the concrete. The cracking sound seemed to be coming from the mid span like the previous specimen (Design 2). The load was held when the cracking sound was heard and the mid span of the beam carefully inspected. The five layers CFRP at the mid span did not show any signs of debonding. Additional loading in increments of 0.1 in. displacements produced a series of continues cracking sound on about three quarters into the third 0.1 in.
displacement increment. The loading was halted again but with the load still on the specimen for inspection. A little gap between the CFRP and concrete was noticed. An indication of debonding initiated at an intermediate flexural or flexural-shear crack, which is likely to propagate towards the plate end.

All transverse anchorages remained intact with no signs of debonding or shearing. Loading was continued with displacement increments (0.1 in.) and the beam continued to deflect more with these small increments. The loading was continued until the loudest explosive sound among the entire test was heard. The load was held and the beam inspected. After inspection, it was observed that the two 45° transverse anchorages from the mid span and on the same side of the beam had sheared and concrete crushing simultaneously as shown in Figure 47. The 45° transverse anchorages sheared evenly and diagonally in the middle from the top of the anchorage to the bottom of the beam.

Although the end anchorages seemed to be unaffected, further inspection showed minute shearing indications. Tapping on the five layers CFRP also revealed indications of debonding from the mid span to the beginning of the end anchorage on the side of the beam where the shearing occurred. On the other side of the beam, there were absolutely no signs of debonding on the five layers CFRP or shearing on the anchorages. The specimen had reached 14% beyond the predicted ultimate capacity just before the shearing. There was however a 20% drop from the load capacity attained by the beam but proceeded to concrete crushing with no warning unlike the two step progression observed in Design 2.
Figure 47: Design 3 Beam at Failure after Testing.

The theoretical model prediction of the test beam behavior was close to the experimental results. The test results at the maximum nominal capacity and corresponding deflections are compared and presented in Table 6.
Table 6: Tested and Theoretical Properties of Design 3 Beam.

<table>
<thead>
<tr>
<th></th>
<th>Max. Load (kips)</th>
<th>Max. Deflection (in.)</th>
<th>Mode of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tested</td>
<td>32.2</td>
<td>1.58</td>
<td>Debonding and concrete crushing Simultaneously.</td>
</tr>
<tr>
<td>Theoretical</td>
<td>33.6</td>
<td>1.38</td>
<td>Debonding</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>4</td>
<td>12.6</td>
<td></td>
</tr>
</tbody>
</table>

The main difference between the actual data and theoretical prediction was the maximum deflection at debonding and concrete crushing, which happened simultaneously in this test. The model also slightly over predicted the maximum load test value by 4%. The behavior for this specimen was however very similar to the model. The elastic and plastic regions seem to parallel each other. The tested specimen (Design 3) also showed a high strength due to the provision of anchorage.

5.1.2.5 Design 4

In addition to the beam specimen strengthened with five layers of CFRP on the tension face, 45°, 60°, and 90° transverse combination anchorage of varying lengths was used to anchor the tension five layers CFRP as shown in Figure 48. See Chapter 3, Design 4 for details on anchorage. This specimen was tested using the same loading procedure as used for the control beam. That is static cyclic loading, in increments of 25, 50 and 75% of the predicted ultimate capacity as well as the three cycles for each incremental stage. After each loading stage (the end of the third cycle of each increment), the beam was inspected for cracks and any possible failure signs.
Figure 48: Design 4 Beam Test Set-up.

The load-deflection plot for the actual beam and the load-deflection plot obtained from the theoretical model are shown in Figure 49.
The beam had a linear behavior until possibly where it appeared that the cracking moment was reached. The linearity continued to approximately 75% of the predicted ultimate capacity. After this point, the linear behavior discontinued but the stiffness was increased. Cracking sounds were heard when loading reached 90% of the predicted ultimate capacity. The displacement control of 0.1 in. increments was used for loading at this point. Cracking sounds were still heard after every 0.1 in. increment until the fifth 0.1 in. increment when a loud sound was heard at a maximum load of 94% of the predicted ultimate capacity. This was followed by a 5% load drop, which was caused by failure in the 90° transverse anchorage close to the mid span. Upon examination of the failed anchorage with the load still held, it was noted the anchorage shear into two in the
transverse direction and debonding from the concrete. Loading was continued the beam regained load to the initial 94% of the predicted ultimate capacity when a sudden explosive sound was heard. The load was held for inspection but it dropped to approximately 30% of the predicted ultimate capacity. The inspection showed the 45° and 60° anchorage on the same side as the failed 90° anchorage were sheared and debonded from the beam as shown in Figure 50. The five layers CFRP had also debonded which was propagated from the mid span. Additional loading increased the load to about 8% from dropping load when it suddenly dropped to the predicted ultimate capacity of the control beam followed by concrete crushing.

The anchorages were able to prevent the five layers CFRP from debonding however, when the anchorages failed the second time, the load drop was large enough to cause the beam to fall to a low load. The debonding of the five layers CFRP no longer contributed to the load carrying capacity of the beam and therefore the beam behaved similarly to the control beam.

Even though the first load drop was very small, it provide a progression until the second load which was huge. The two-step progression provides for warning however unlike Design 2 where the progression was within 20%, this specimen was within 30%.
Figure 50: Design 4 Beam at Failure after Testing.

The test results at the maximum load capacity and corresponding deflections are compared and presented in Table 7.

Table 7: Tested and Theoretical Properties of Design 4 Beam.

<table>
<thead>
<tr>
<th></th>
<th>Max. Load (kips)</th>
<th>Max. Deflection (in.)</th>
<th>Mode of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tested</td>
<td>27.5</td>
<td>1.6</td>
<td>Debonding and Concrete Crushing</td>
</tr>
<tr>
<td>Theoretical</td>
<td>33.6</td>
<td>1.38</td>
<td>Debonding</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>18</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>
5.1.2.6 Design 5

In addition to the beam specimen strengthened with five layers of CFRP on the tension face, 90° transverse anchorage of lengths 10, 8, 6 and 4 in. respectively were used to anchor the tension five layers CFRP as shown in Figure 51. This is similar to Design 2 but it is the reverse and there more spacing from the cut-off point of the five layers CFRP to the first anchorage. This design method was to encourage plate end debonding. See Chapter 3, Design 5 for details on anchorage. This specimen was tested using the same loading procedure as used for the control beam. That is static cyclic loading, in increments of 25, 50 and 75% of the predicted ultimate capacity as well as the three cycles for each incremental stage. After each loading stage (the end of the third cycle of each increment), the beam was inspected for cracks and any possible failure signs.

Figure 51: Design 5 Beam Test Set-up.
The load-deflection plot for the actual beam and the load-deflection plot obtained from the theoretical model are shown in Figure 52.

![Theoretical and Experimental Load Displacement for Design 5 Beam.](image)

The beam appeared to have linear behavior until possibly where it appeared that steel was yielding. The linearity continued to approximately 85% of the predicted ultimate capacity. After this point, the linear behavior continued but the stiffness was reduced. Cracking sounds were heard when loading was about 90% of the predicted ultimate capacity which could be an indication of the CFRP debonding from the concrete. Plate end debonding was observed immediately following the cracking sound from one end of the beam. The load was then held when the cracking sound was heard and the
whole beam mainly the mid span and the ends close to the support of the beam carefully inspected. The five layers CFRP at one end of the beam was debonding and moving close the first 4 in. anchorage. Other parts of the beam including the transverse anchorages remained intact with no signs of debonding or shearing. Loading was then continued with very small displacement increments (0.1 in.) but the beam continued to deflect more with these small increments. The loading was continued until a loud explosive sound was heard. The load was once again held and the beam inspected. After inspection, it was observed that the end plate, which started debonding, had reached the first anchorage and sheared it as shown in Figure 53. The beam had reached its maximum capacity of approximately 96% of the predicted ultimate capacity. There was a load drop to 80% of the predicted ultimate capacity but the beam regained strength to about 85% of the predicted ultimate capacity before dropping again to 80% of the predicted ultimate capacity. The beam regained strength again to about 83% of the predicted ultimate capacity when it finally dropped to capacity of the control beam followed by concrete crushing. This sequence produced a three-stage progression. After careful inspection of the crushed beam, it was observed that even though the anchorages remained intact on the side, the ones on the side of the beam were debonding occurred had sheared slightly underneath the beam on the tension and five layers CFRP had debonded all the way to the mid span. It was possible that in addition to the plate end debonding, debonding could also have been initiated at an intermediate flexural or flexural-shear crack in the mid span and then propagated to the closest anchorage from the mid span. The rest of the
anchorage still remained intact. The three-step progression before concrete crushing is a warning indication.

Figure 53: Design 5 Beam at Failure after Testing.

The test results at the maximum nominal capacity and corresponding deflections are compared and presented in Table 8.
Table 8: Tested and Theoretical Properties of Design 5 Beam.

<table>
<thead>
<tr>
<th></th>
<th>Max. Load (kips)</th>
<th>Max. Deflection (in.)</th>
<th>Mode of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tested</td>
<td>27</td>
<td>1.5</td>
<td>Progressive debonding followed by concrete crushing.</td>
</tr>
<tr>
<td>Theoretical</td>
<td>28.5</td>
<td>1.9</td>
<td>Debonding</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>5</td>
<td>21</td>
<td></td>
</tr>
</tbody>
</table>

5.1.3 Summary of Loading Capacity and Displacements

A summary of the tested and theoretical properties for the control and CFRP strengthened beams are shown in Figure 54 and Table 9 respectively.
Figure 54: Plots of Theoretical and Experimental curves for Control and Design Beams.
Table 9: Summary of Tested and Theoretical Properties

<table>
<thead>
<tr>
<th>Test Beams</th>
<th>Control</th>
<th>Design 1</th>
<th>Design 2</th>
<th>Design 3</th>
<th>Design 4</th>
<th>Design 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tested Max. Load (kips)</td>
<td>21.0</td>
<td>24.0</td>
<td>26.4</td>
<td>32.2</td>
<td>27.5</td>
<td>27.0</td>
</tr>
<tr>
<td>Theoretical Max. Load (kips)</td>
<td>18.2</td>
<td>33.6</td>
<td>33.6</td>
<td>33.6</td>
<td>33.6</td>
<td>33.6</td>
</tr>
<tr>
<td>% Difference</td>
<td>13</td>
<td>29</td>
<td>21</td>
<td>4</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>Tested Max. Deflection (in.)</td>
<td>1.6</td>
<td>0.9</td>
<td>1.4</td>
<td>1.6</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td>Theoretical Max. Deflection (in.)</td>
<td>1.21</td>
<td>1.35</td>
<td>1.35</td>
<td>1.35</td>
<td>1.35</td>
<td>1.35</td>
</tr>
<tr>
<td>% Difference</td>
<td>24</td>
<td>33</td>
<td>4</td>
<td>16</td>
<td>16</td>
<td>10</td>
</tr>
</tbody>
</table>
5.2 Crack Configuration

The crack patterns were observed at the 25, 50 and 75% stages while the beams were still loaded at the end of each third cycle for each stage. The crack patterns were carefully marked from the beginning and end of the crack as shown in Figure 55. Assigning numbers at the end of the cracks signifying the incremental stages differentiated the cracks.

5.2.1 Control

New cracks were not observed at the end of the 25% cycle. The existing cracks at the mid span were from the cracks developed during the initial loading. At the end of the 50% cycle, small cracks began to develop vertically in constant moment region and the existing ones were extended. The cracks were similar in width, but displayed no distinct spacing pattern.

At the end of the 75% cycle, additional smaller cracks were developed with the tension area and the existing cracks continued to grow in both width and length propagating and migrating towards the load at approximately 45-degree angles. The cracks continued after loading was increasing beyond the 75% load which reached about 80% of the beam depth until failure occurred.
5.2.2 Design 1

The first new cracks for the specimen (not including existing cracks from initial loading) were observed at the 50% incremental loading stage at the end of the cycle with the load still on the beam. At the end of the third cycle of the 75% incremental stage, more small vertical cracks began to form in the constant moment region. No cracks were observed at the supports at any point during the testing. Crack patterns were again observed after the debonding of the CFRP. More flexural cracks formed concentrating within the mid span (constant moment region) and traveling up towards the load. Unlike the control beam, the cracks were smaller in width and evenly spaced. With the exception of a few larger cracks that formed in the constant moment region, most of the cracks,
traveled approximately 60% of the beam depth before debonding. The larger cracks reached about 75% of the beam depth before debonding. Figure 56 shows cracks observed at the end of the 75% cycle which are marked with the number “7” and at debonding which are marked “D”.

![Figure 56: Crack Configuration of Design 1 Beam.](image)

### 5.2.3 Design 2

New cracks for the specimen (not including existing cracks from initial loading) were observed at the 50% incremental loading stage at the end of the cycle with the load still on the beam. These cracks close to micro cracks were developing and concentrating in the mid span (constant moment region). At the end of the third cycle of the 75% incremental stage, visible vertical cracks began to form in the constant moment region. No cracks were observed at the supports but very few and little cracks between anchorages. Crack patterns were again observed after the debonding of the five layers.
CFRP, shearing of the anchorage and crushing of concrete. More flexural cracks formed concentrating within the mid span (constant moment region) and extended up towards the load. New cracks were however not developed between the anchorages but rather the existing ones progressed slightly. The mid span cracks were slender and evenly spaced unlike the ones in the control beam. The cracks within the anchorages only extended about 20% of the beam depth while those in the constant moment region extended approximately 80% of the beam depth. Figure 57 shows crack pattern at the 50% and 75% cycles marked with the number “5” and “7” respectively.

Figure 57: Crack Configuration for Design 2 Beam.
5.2.4 Design 3

Fewer cracks were observed for this specimen. The least among all the beams tested. A couple of cracks were developed only at the constant moment region at the 50% stage. There were hardly any noticeable cracks beyond the first 45° transverse anchorages on both sides of the beam including the supports, even at 75% stage. In the mid span however, although there were very few cracks, they were wide, long and evenly spaced.

The crack patterns observed after the debonding of the five layers CFRP, shearing of the anchorage and crushing of concrete were extensions from the previous cracks. New cracks were not formed after the 75% stage marked “7” as shown in Figure 58. The cracks in the constant moment region extended approximately 90% of the beam depth almost coinciding with the concrete crushing zone on the top of the beam.

Figure 58: Crack Configuration for Design 3 Beam.
5.2.5 Design 4

Cracks were developed in almost every section of the beam with the exception of the supports at the 75% stage. Very fewer cracks were observed at the 50% stage. At the 75% stage, the cracks, which were wide spread, were concentrated in the mid span and between the anchorages which are marked “7” as shown in Figure 59. In the mid span however, a couple of the cracks were very close to the 90° anchorages with some appearing to be coming from underneath the anchorage. New cracks were also developed after concrete crushing and the existing ones extended to approximately 85% of the beam depth.

Figure 59: Crack Configuration for Design 4 Beam.
5.2.6 Design 5

Cracks were developed in almost every section of the beam with the exception of the supports at the 50% stage. At the 75% stage, new cracks were observed between the anchorages along the beam as shown in Figure 60 and marked with the number “7”. Most of the cracks only extended to the about 30% of the beam depth with the longest cracks in the constant moment region only reaching approximately 60% of the beam depth.

Figure 60: Crack configuration for Design 5 Beam.
**5.3 Evaluation of Test Results**

The maximum strength of each tested beam and the percent increase in strength with respect to the control beam is shown in Table 10.

Table 10: Tested Maximum Load Comparison of Strengthened Beams to Control Beam.

<table>
<thead>
<tr>
<th>Tested Beams</th>
<th>Maximum Load (kips)</th>
<th>Percent Increase w/r to Control Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>21</td>
<td>N/A</td>
</tr>
<tr>
<td>Design 1</td>
<td>24</td>
<td>13</td>
</tr>
<tr>
<td>Design 2</td>
<td>27.7</td>
<td>20</td>
</tr>
<tr>
<td>Design 3</td>
<td>32.2</td>
<td>35</td>
</tr>
<tr>
<td>Design 4</td>
<td>27.5</td>
<td>24</td>
</tr>
<tr>
<td>Design 5</td>
<td>27.1</td>
<td>22</td>
</tr>
</tbody>
</table>

The Design 1 beam has the lowest increase in strength with respect to the control beam among all the design beams. This is an indication of the major role played by anchoring the beams. With all the design beams having the same number of layers (5 layers) used for flexural strengthening, the low increase in strength of the Design 1 could only be attributed to it not being anchored.

Design 3 attained the highest increase in strength with its 45° transverse anchorage system. The increase in strength after Design 3 is Design 2 (descending 90° transverse anchorage from the support) followed by Design 4 (45, 60, 90° transverse anchorage from the support).
anchorage) and Design 5 (ascending 90° transverse anchorage from the support) respectively.

The strength of all the design beams is above that of the control beam, which shows that the RC beam can be externally strengthened. Consequently, anchoring the strengthened beams also increases the strengthened beams even more. Anchoring allows the externally strengthened beams to approach a perfectly bonded condition. Investigating the anchoring systems of all the design beams also shows some closeness in the numbers, which can be further, evaluated for a better or improved anchoring system.

Design 2, Design 4 and Design 5 attained strengths of 27.7, 27.5 and 27.1 kips respectively. This implies that either these design function similarly or there is excess (non effect) anchorage in the system.

Since the purpose of this research is to study the failure modes of the design beams in addition to external strengthening to increase the strength of the beams, the plots of the design beams will be reviewed and analyzed. From the plots presented in Figure 54, the failure modes of all the design beams can be compared. Design 1 with the lowest increase in strength among all the design beams, has the least displacement and an abrupt failure mode. Design 3 even though has the highest increase in strength and a high displacement also has an abrupt failure mode. Design 2, 4, and 5 have some progression but their drop in load to failure has to be investigated. Table 11 shows the progression of design beams 2, 4 and 5 respectively. In the table, Load 1 represents the highest load attained by the design beam before the first drop and Load 2
represents the second highest load attained by the design after the first drop. For Design 2 and 4, the second “drop load” is also the load at failure representing the one step progression while in Design 5 the third “drop load” is the failure load representing a two step progression.

Table 11: Summary of Progression Design Beams.

<table>
<thead>
<tr>
<th>Test Beam</th>
<th>Design 2</th>
<th>Design 4</th>
<th>Design 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load 1(kips)</td>
<td>27.7</td>
<td>27.5</td>
<td>27.1</td>
</tr>
<tr>
<td>Deflection (in.)</td>
<td>1.07</td>
<td>1.25</td>
<td>0.99</td>
</tr>
<tr>
<td>Load Drop (kips)</td>
<td>24.4</td>
<td>19.9</td>
<td>22.1</td>
</tr>
<tr>
<td>Deflection (in.)</td>
<td>1.11</td>
<td>1.36</td>
<td>1.03</td>
</tr>
<tr>
<td>% Drop</td>
<td>12</td>
<td>28</td>
<td>18</td>
</tr>
<tr>
<td>Load 2(kips)</td>
<td>26.4</td>
<td>21.4</td>
<td>22.9</td>
</tr>
<tr>
<td>Deflection (in.)</td>
<td>1.31</td>
<td>1.56</td>
<td>1.29</td>
</tr>
<tr>
<td>Load Drop (kips)</td>
<td>22.3</td>
<td>17</td>
<td>21.6</td>
</tr>
<tr>
<td>Deflection (in.)</td>
<td>1.36</td>
<td>1.64</td>
<td>1.3</td>
</tr>
<tr>
<td>% Drop</td>
<td>16</td>
<td>21</td>
<td>6</td>
</tr>
<tr>
<td>Load 3(kips)</td>
<td>N/A</td>
<td>N/A</td>
<td>22.6</td>
</tr>
<tr>
<td>Deflection (in.)</td>
<td>N/A</td>
<td>N/A</td>
<td>1.4</td>
</tr>
<tr>
<td>Load Drop (kips)</td>
<td>N/A</td>
<td>N/A</td>
<td>16.6</td>
</tr>
<tr>
<td>Deflection (in.)</td>
<td>N/A</td>
<td>N/A</td>
<td>1.5</td>
</tr>
<tr>
<td>% Drop</td>
<td>N/A</td>
<td>N/A</td>
<td>27</td>
</tr>
</tbody>
</table>
From the progression table, Design 4 has the highest load drop percentages of 21 and 28 respectively. Design 2 and Design 5 both 90° transverse anchorages but varying in their ascending or descending order from the support have load drop percentages of 12 and 16 for Design 4 and 6 and 18 for Design 5.

Design 4 therefore with the highest strength among the three progression beams cannot be reliable due to its large drop load percentage. Design 5 even though with a larger drop load of 27% compared to Design 2 whose largest drop load is 16% has a better progression failure mode. It should also be noted that in addition to the Design 5 beam having a two step progression, it also has the lowest percentage drop load of 6%.

### 5.4 Ductility

The ductility of the test specimens is analyzed using the energy method and conventional method as discussed in Chapter 4. Table 12, Table 13 and Table 14 show a comparison of tested ductility index of the strengthened beams to the control beam for the conventional and energy method, a comparison of the conventional and energy method values and the “energy ratios” of all the beams. The “energy ratio” as defined previously, is the ratio of the inelastic energy to total energy. The “energy ratio” category and interpretation are as follows:

- If the energy ratio is greater than 75%, the beam will exhibit a ductile failure.
- If the energy ratio is between 70 and 74% the beam is semi-ductile.
- If the energy ratio is below 69% the beam is brittle.

Computation of ductility and analysis using MathCAD are presented in Appendix B. The
ductility computation for all specimen were estimated at their maximum failure loads.

Table 12: Ductility Index and Energy Ratio using Energy Method.

<table>
<thead>
<tr>
<th>Test Beams</th>
<th>Ductility Index</th>
<th>Percent Decrease w/r to Control Beam</th>
<th>Energy Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>2.704</td>
<td>N/A</td>
<td>77.32</td>
</tr>
<tr>
<td>Design 1</td>
<td>1.264</td>
<td>53.25</td>
<td>34.52</td>
</tr>
<tr>
<td>Design 2</td>
<td>1.288</td>
<td>52.37</td>
<td>43.71</td>
</tr>
<tr>
<td>Design 3</td>
<td>1.619</td>
<td>40.13</td>
<td>55.32</td>
</tr>
<tr>
<td>Design 4</td>
<td>1.813</td>
<td>32.95</td>
<td>61.91</td>
</tr>
<tr>
<td>Design 5</td>
<td>1.321</td>
<td>51.15</td>
<td>39.08</td>
</tr>
</tbody>
</table>

Table 13: Ductility Index using the Conventional Method.

<table>
<thead>
<tr>
<th>Test Beams</th>
<th>Ductility Index</th>
<th>Percent Decrease w/r to Control Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>3.850</td>
<td>N/A</td>
</tr>
<tr>
<td>Design 1</td>
<td>2.100</td>
<td>45.45</td>
</tr>
<tr>
<td>Design 2</td>
<td>2.403</td>
<td>37.58</td>
</tr>
<tr>
<td>Design 3</td>
<td>2.569</td>
<td>33.27</td>
</tr>
<tr>
<td>Design 4</td>
<td>2.632</td>
<td>31.64</td>
</tr>
<tr>
<td>Design 5</td>
<td>2.231</td>
<td>42.05</td>
</tr>
</tbody>
</table>

160
Table 14: Comparison of Conventional and Energy Method for Ductility Index.

<table>
<thead>
<tr>
<th>Test Beams</th>
<th>Ductility Index Conventional Method</th>
<th>Ductility Index Energy Method</th>
<th>Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>2.704</td>
<td>3.850</td>
<td>29.8</td>
</tr>
<tr>
<td>Design 1</td>
<td>1.264</td>
<td>2.100</td>
<td>39.8</td>
</tr>
<tr>
<td>Design 2</td>
<td>1.288</td>
<td>2.403</td>
<td>46.4</td>
</tr>
<tr>
<td>Design 3</td>
<td>1.619</td>
<td>2.569</td>
<td>36.9</td>
</tr>
<tr>
<td>Design 4</td>
<td>1.813</td>
<td>2.632</td>
<td>31.1</td>
</tr>
<tr>
<td>Design 5</td>
<td>1.321</td>
<td>2.231</td>
<td>40.8</td>
</tr>
</tbody>
</table>

The results from the ductility analyze as presented above show the reduction in ductility of the composite beams with respect to the control beam. The ductility indexes acquired using the energy method were smaller for the entire test beam than the ductility indexes using the conventional method. The ductility indexes for the control beam only differed by 30% (the lowest in percentage difference) showing that both methods agree on ductility measurements for typical steel reinforced concrete beams to some extent. The order of ductility indexes reduction with respect to the control beam for both methods was also the same i.e. Control, Design 4, Design 3, Design 5, Design 2 and Design 1. Although, the difference between the ductility indexes for the strengthened beams varied by as much as 46% for the Design 2 beam, there is a reasonable agreement for the rest of the beams which were equal or less than 40%. The high difference in percentage (46%) could be attributed to the conventional ductility measurement method, which overestimates the actual ductility of a CFRP strengthened reinforced concrete beam due
to the oversimplification of this measurement method. Recall also that the yielding loads used to determine the displacement yield points were estimates from the theoretical yielding loads.

The energy ratio analysis using the energy method and the criteria to determine the ductility of the beams showed that the Control beam was ductile and the rest of the test specimens were brittle (Design 1, 2, 3, 4 and 5).
CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The following conclusions were drawn from the experimental and theoretical analysis of the research.

6.1.1 Flexural Load Capacity

This research confirmed that the load capacity of a reinforced concrete beam can be increased by external strengthening. The load capacity of the steel reinforced concrete beam increased as much as 13 to 35% depending on the anchorage system in addition to the five layers of CFRP. None the less, the load capacity of the Design 1 beam without any anchorage system was 13% more than the Control beam, proving that the reinforced concrete beams can be externally strengthened to increase their load capacity. The analysis provided upper and lower bounds not capable of capturing debonding behavior. The upper and lower bounds were estimated with a parallel study of a baseline specimen (control beam) and a perfectly bonded specimen.
6.1.2 Anchorage

The anchorage system proved its ability to increase the loading capacity as well as improve the failure mechanism i.e. controlling the failure modes of the anchored specimens by providing some progression. The difference in the loading capacity of the externally strengthened beam without anchorage (Design 1) to the rest of the externally strengthened beams with anchorage ranged from 12.5 to 34%. This proved that anchorage is needed in external strengthening of CFRP beams in order for the specimen to approach a perfect bond or maximize its strength during loading.

The anchorage also played an important role in the failure mechanism by limiting the debonding propagation as well as providing a gradual failure mechanism as was revealed in the two-step progression failure mode in the Design 5 specimen. The beam specimen reaching its maximum strength followed by a drop but progressively, regaining its strength is referred to as a step progression failure. In the case of the Design 5 specimen, this scenario occurs twice hence the “two-step” progression.

Although in most cases, the anchorages seem to be too strong to allow a smooth propagation, this was balanced by an increased in load capacity i.e. the stronger the anchorage, the lesser the progression but the higher the specimen strength.

6.1.3 Failure Modes

The failure modes for all the design beams were unique and different in terms of their CFRP (both flexural and shear strengthening components) debonding mechanisms.
Design 1 beam with flexural strengthening only to debond at only one end and there was no concrete crushing.

Design 2 beam with flexural strengthening as well as 90° anchorage was designed to debond at the mid span and propagate to the ends of the beam but instead, the debonding propagated to one side of the beam and sheared the anchorage. However, the debonding was initiated from the mid span. The Design 2 beam also had a one step progression before failure as shown in Figure 54.

The Design 3 beam with 45° anchorage initiated debonding from the anchorage close to the mid span but there was no gradual progression. The debonding propagated to one side of the beam causing the first two anchorages to debond followed by concrete crushing. The flexural layers seem to be hollow when it was tapped after the test. This is a sign of flexural debonding.

The Design 4 beam with a combination of 45, 60 and 90° anchorages was designed to debond from the mid span and propagate to the ends but debonded at the mid span propagated to one side of the beam, sheared a section of the first anchorage (90°) and was followed by concrete crushing.

The Design 5 beam, which was the closest prediction in terms of a gradual and progressive failure, was designed to debond from the ends and propagate to the mid span. Even though the propagation from only one end to the beam was initiated it did not propagate all the way through to the mid span. A close investigation of the beam specimen however showed that there was debonding in the mid span. There was a two
step progression as explained in Section 6.1.2 with very little load drop.

### 6.1.4 Ductility

External strengthening while increasing strength reduced the ductility of the reinforced concrete beam. The ductility index of the reinforced concrete beam decreased as much as 31 to 53%. The design beams with a better mode failure (progression failure) had a better ductility in terms of reduction with respect to the Control beam. Design 4, 3, 5, 2 and 1 were reduced in ductility by 33, 40, 51, 52 and 53% respectively with respect to the Control Beam.

The estimated area under the curves for the specimens showed that the energy absorbing capacity was enhanced. It was observed from load-capacity and energy absorbing ability of the specimens that the strengths of the specimen had no direct relation with the anchorage methods i.e. the specimen with the highest energy absorbing capacity did not have the highest failure load.

### 6.2 Recommendations

The following recommendations are based on the experimental testing of the research:

1. More specimens need to be tested for a more accurately characterization of the effect of anchorage on flexural beam strengthening.

2. The anchorage methods used in this research has to be further investigated. The anchorage areas that could be studied will include but not limited to the following:
   a. Number of anchorage to be used on each specimen.
   b. Layers of anchorage (single or double layers).
3. Although this research targeted the CFRP to double the strength of the control beam as well as to provide a progressive failure further research on varying the layers could be studied.

4. Further research could include:
   a. Varying the layers of several specimens with the same reinforcement, dimension and anchorage method. This will determine the effectiveness of the anchorage procedures (method).
   b. Varying the anchorage lengths and number on different specimens with same the number of flexural strengthening layer.
Design of formwork

Design of formwork complied and followed requirements as stated in the Building Code Requirements for Structural Concrete (ACI 318-02) and Commentary (ACI 318R-02) Chapter 6.

- 6.1.1: Forms shall result in a final structure that conforms of shapes, lines and dimensions of the members as required by the design drawings and specifications.
- 6.1.2: Forms shall be substantial and sufficiently tight to maintain position and shape.
- 6.1.3: Forms shall be properly braced or tied together to maintain position and shape.
- 6.1.4: Forms and their supports shall be designed so as not to damage previously placed structure.
- 6.1.5: Design of formwork shall include consideration of the following factors:
  a. Rate and method of placing concrete;
  b. Construction loads, including vertical, horizontal, and impact loads.

Form Work

- Held 6 (132x11x 6) in beams.
- ¾ in plywood were used for the base, sides and dividers.
- 2x4 in standard wood were used for reinforcing the sides and diagonals.
- Wood screws and nails were used for fastening.
- Corners, small openings and cracks of formwork were sealed with adhesive (Polyurethane Premium Construction Adhesive).

- Dimensions of form work

  a. Base plywood = 70x161.5 in.
  b. Lateral Sides (2) plywood = 11x133.5 in.
  c. Front & Back plywood = 11x39.75 in.
  d. Dividers (5) plywood = 11x132 in.
  e. Four longitudinal (2x4) in, two for side1, two for side2 = 141.5 in.
  f. Four longitudinal (2x4) in, two for front, two for back = 49.25 in.
  g. Fourteen (2x4) in diagonals (6 in height), seven for side1, seven for side 2.
  h. Six (2x4) in diagonals (6 in height) three for front, three for back.

Formwork had sufficient support with all the side bracings using the longitudinal (2x4) in wood and also tied with diagonals to maintain its position and shape (ACI 6.1.3).

The formwork for the two supports for beams testing were of the following dimensions,
materials and construction method:

a. Height = 11 in.

b. Length = 24 in.

c. Width = 12 in.

d. 6 in diagonals @ height 3.5 in from the top on each side of the two support to stay clear during beam bending.

e. 4 in width on the top.

f. Total of four #4 longitudinal rebars of length 21 in and seven #3 stirrups are used for each support. Stirrups were of the same dimensions used for the beams. Three stirrups with 9 in spacing along the four #4 longitudinal bars (two #4 in compression and two #4 in tension). Two stirrups were placed on the top of the cage in compression and two stirrups on the bottom in tension.

g. Two PVC pipes approximately 14 in long ran through the sides of the formwork of the supports. The pipes were 6 in from the ground and 14 in apart. This is for transporting and moving the supports.
Dobies/Chairs

Dobies and chairs were used for elevation from the bottom of formwork and on the sides of the cage to hold it in place and provide the necessary cover needed.

The 1.5 in chairs from Hughes Supplies were attached to the bottom of the steel cage to elevate it to provide the necessary cover below in the tension area. The 1.5 in chairs were tied on the two #5 longitudinal bars @ 1ft spacing.

½ in dobies were made in the Structures lab by build a platform with half inch dividers and concrete mix.

Placement

Formwork

Formwork was designed and constructed based on the ACI code stated in Chapter 6. The floor, on which the complete form was placed, was lined with plastic to prevent leakage of mortal from damaging the floor and the form was substantial and sufficiently tight to
reduce any leakages (ACI 6.1.2).

With the formwork in position and preparing for pouring of concrete, the ACI 5.7 – Preparation of Equipment and Place of deposit was followed.

5.7.1 – Preparation before concrete placement shall include the following:

a. All equipment for mixing and transporting concrete shall be clean. (See Pouring of Concrete).

b. All debris and ice shall be removed from spaces to be occupied by concrete; (Formwork was swept and well cleaned).

c. Forms shall be properly coated. (Formwork was well coated with vegetable oil).

d. Masonry filler units that will be in contact with concrete shall be well drenched.

e. Reinforcement shall be thoroughly clean of ice or other deleterious coatings.

Steel Cage

Steel cage with dobies attached to the sides to provide the necessary cover and chairs attached to the base to provide the necessary elevation (cover) is placed in the coated formwork. See chapters 2 & 3 for details on steel cage dimensions.
Hooks

Hooks were placed 10 in from the end of the beams (two hooks per beam). The hooks were placed in a two stacked (2x4) in wood 46.5 in long in order to have the coupling flashed with the concrete beam when the concrete is poured. The (2x4) in wood stacked is to hold the hooks in place during pouring. The wood stacks were drilled to the formwork to keep the wood stack and the hooks in place.

Supports

The formworks for the supports were turned upside down with the diagonal at the bottom and the wider base on top, which made it easier during pouring of concrete. The formwork was well coated and 0.5 in dobies placed on the diagonal sides before placing the reinforcement. The ¾ in diameter 26 in long PVC pipes ran through one side of the
support formwork, through the reinforcement steel cage through the other hole on the 
other side of the formwork. Two PVC pipes were used for each support. The PVC pipes 
provided adequate room to pass a #4 bar through, which were used for moving the 
supports from one location to the other. Tie wires were used to tie the reinforcement steel 
to the PVC pipe to hold them in place.

See Dobies and Chairs.

**Concrete Pouring**

Rinker poured concrete upon request. Concrete required for the beams (six 132x11x6) in, 
supports (two 24x11x12) in, a slump cone and fifteen cylinders (12x6) in was estimated 
to be approximately 1.4 cubic yards. 

Rinker was requested to provide two cubic yards of 5000 psi concrete, 3/8 aggregate and 
slump of 4 in. 

On the day prior to pouring, formwork was inspected, 1 in dobies tied to the top cage and 
four 46x1 ½x ¾ in wood strips used to hold the dobies in case concrete causes the steel 
cage to float during pouring.

The following items were ready for pouring; 

- Wheel barrow
- Trowels
- Hard Hat / Goggles / Gloves
- Vibrator
- Shovel
- 15 cylinders
- Slump Cone
- Digital Camera
- Drill
- Buckets

On December 9, 2003 @ 10a.m., the concrete arrived for pouring. The slump cone test was performed and the result was approximately 1.5 in slump. The concrete was accepted and test cylinders (15) filled and compacted while the formwork for both the beam and the support were also filled using the vibrator.

The following guidelines were specifically followed. ACI code Chapter 5.

- 5.10.1: Concrete shall be deposited as nearly as practical in its final position to avoid segregation due to rehandling or flowing.
5.10.2: Concreting shall be carried on at such a rate that concrete is at all times plastic and flows readily into spaces between reinforcement.

5.10.3: Concrete that has partially hardened or been contaminated by foreign materials shall not be deposited in the structure.

5.10.4: Retempered concrete or concrete that has been remixed after initial set shall not be used unless approved by the engineer.

5.10.5: After concreting is started, it shall be carried on as a continuous operation until placing of a panel or section, as defined by its boundaries or predetermined joints, is completed except as permitted or prohibited by 6.4.

5.10.6: Top surfaces of vertically formed lifts shall be generally level.

5.10.7: When construction joints are required, joints shall be made in accordance with 6.4.

5.10.8: All concrete shall be thoroughly consolidated by suitable means during placement and shall be thoroughly worked around reinforcement and embedded fixtures and into corners of forms.

Curing

5.11.1: Concrete (other than high-early-strength) shall be maintained above 50 F and in a moist condition for at least the first 7 days after placement, except when in accordance with 5.11.3.

From text “Reinforced Concrete” by Edward G. Nawy gives the following guidelines for
methods of good curing conditions, which will be followed;

1. Continuously sprinkling with water.
2. Ponding with water.
3. Covering the concrete with wet burlap, plastic film, or waterproof curing paper.
4. Using liquid membrane-forming curing compounds to retain the original moisture in the wet concrete.

Notes;

1. The first 7 days (early age) are very critical. That is when a large proportion of autogenously shrinkage takes place.
2. The benefits of internal curing by virtue of water entrainment go far beyond the improvements in long-term strength gain.
3. A significant reduction in permeability is achieved by the major increase in the length of curing time available, hence a resulting high performance of the finished product.

After 3 hours of pouring the concrete, it was covered with plastic wrap. It remained covered until 7 days when the first cylinder test was performed.

**Cylinder Test**

ACI code;

5.1.2 – Requirements for fcp (compressive strength of concrete) shall be based on tests of
cylinders made and tested as prescribed in 5.6.3.

5.1.3 – Unless otherwise specified, fcp shall be based on 28-day tests. If other than 28 days, test age for fcp shall be as indicated in design drawings or specifications.

5.6.3.1 – Samples for strength tests shall be taken in accordance with “Method of Sampling Freshly Mixed Concrete” (ASTM C 172).

5.6.3.2 – Cylinders for strength tests shall be molded and laboratory-cured in accordance with “Practice for Making and Curing Concrete Test Specimens in the Field” (ASTM C 31) and tested in accordance with “Test Method for Compressive Strength of Cylindrical Concrete Specimens” (ASTM C 39).

See Chapter 1 for results of Cylinder Test.

The formwork was removed after the seventh day when the strength of the concrete from
the test cylinder results exceeded 70% of the 5000 psi.

Test Set-Up.

1. The testing frame, which held the actuator was, moved 12 in forward to create enough testing space for the 132x11x6 in beams. See Chapter 3 for details on experimental set-up.

2. Specimen Pre-cracking
   
   a. The cracking moment for the virgin beam was first estimated (moment crack - Mcr).
   
   b. The six beam specimens were placed on two 6x6 in wooden block supports and positioned for loading using the Actuator. *Note the value of estimate moment crack.*
   
   c. The loading rates and displacements were set based on the estimated cracking moment and beams were loaded until first flexural cracks were
observed.

d. The cracked specimens were flipped with the tension face up but pivoted at midpoint with one and only one of the 6 x 6 wooden blocks so the cracks are widened by the self-weight of the beam.

3. Support set-up

a. The supports were positioned [10.5 ft. (126 in.) clear span]

b. A plastic sheet was laid down on the floor where the supports will be positioned and four, ½ in square wooden blocks were used to elevate each support before hydro stone cement was poured. The elevators were to ensure that the hydro stone flowed underneath the support and covered all gaps. The hydro stone will stabilize the support as well as level it during testing and prevent the support from excessive movement. See Chapter 1 for details on support and hydro stone.

c. Formwork with 2 in. clearance was set before hydro stone was poured. Form dimensions (28x16x4) in.

d. Elastomeric pads (12x2x1) were then placed on the support and sealed along the edges with polyurethane Premium Construction Adhesive.
4. Preparing Specimens for composite application.
   a. Surface preparation- Concrete surface sanded, dusted and cleaned of all
dust particles. Edges of beam ground and rounded off to a radius of 0.6 in
to smoothen sharp corners to prevent composite breaks or tear.
   b. Beams were well marked with pencil for composite placement. Tension
and transverse (beams which require transverse application). See beam
designs for details on transverse application.
5. Cutting CFRP strips

a. Lengths of 120 in of CFRP were carefully rolled off the 50 yards long and 13 in width CFRP reel.

b. The 120 in point were marked with a yellow chalk liner and a blue 2 in wide Safe Release Tape (SRT) was placed horizontally parallel to the chalk mark.

c. The CFRP was then carefully cut along the line with a scissors so there was an inch of SRT on each side of the CFRP after it has been cut.

d. The SRT held the CFRP ends in place and prevented the loose ends from coming apart. The SRT was however removed before application of adhesive. The same procedure will be followed for the rest of the required lengths and widths. All strips were 4 in wide.
CFRP Strips cutting alignment

2" SRT 4" 2" SRT

4" 4" 4"

13" width CFRP Strip

4" Yellow markings
The dimensions of the cut CFRP strips were as follows:

**Design 1**
- Five strips of 4x120 in. for flexural strengthening tension face application.

**Design 2 (90 degrees transverse)**
- Five strips of 4x120 in. for flexural strengthening tension face application.
- 4(10+6+10) strips = 4 (26x4) in strips. Transverse
- 2(8+6+8) strips = 2 (22x4) in strips. Transverse
- 2(6+6+6) strips = 2 (18x4) in strips. Transverse
- 2(4+6+4) strips = 2 (14 x 4 in) strips. Transverse

**Design 3 (45 degrees transverse)**
- Five strips of 4x120 in. for flexural strengthening tension face application.
- 4 (11.4+6+11.4) strips = 4 (28.8 x 4 in) strips. Transverse
- 2 (15.3+6+15.3) strips = 2 (36.6 x 4 in) strips. Transverse

**Design 4 (45, 60, 90 degrees mix transverse)**
- Five strips of 4x120 in. for flexural strengthening tension face application.
- 2 (15.6+6+15.6) 45 deg. Strips = 2 (37.2 x 4 in) strips.
- 2 (11.3+6+11.3) 60 deg. Strips = 2 (28.6 x 4 in) strips.
- 2 (6+6+6) 90 deg. Strips = 2 (18 x 4 in) strips.

**Design 5 (90 deg. Transverse – Reverse Design 2)**

- Five strips of 4 x 120 in. for flexural strengthening tension face application.
- 4(10+6+10) strips = 4 (26x4) in strips. Transverse
- 2(8+6+8) strips = 2 (22x4) in strips. Transverse
- 2(6+6+6) strips = 2 (18x4) in strips. Transverse
- 2(4+6+4) strips = 2 (14 x 4 in) strips. Transverse

*See Chapter 3 for details of design methods.*

6. Detailed Resin Mixing & Application (Mixing 635 Thin Epoxy Resin & Epoxy Hardener)

a. Mixing/Application Materials

i. Two 21/2 quart buckets
ii. 635 Thin Epoxy Resin
iii. Epoxy Hardener
iv. Mixing sticks (2)
v. Foam brushes (2)
vi. Plastic squeegee
vii. Brushes
viii. Towels
ix. Fiber glass rollers
x. Measuring cup

b. Working Gear

i. Boots

ii. Coveralls

iii. Face mask

iv. Gloves

v. Goggles

c. Mixing

- 30 ounces of resin would require 10 ounces of hardener in order to catalyze. The product should be mixed thoroughly with a clean paint stick for approximately 2 to 3 minutes, during which you must scrape the side of the container while mixing. You should look for lines or separations in the mixture that represent uncombined resin, mix until all separations have disappeared.

*Note: Product requires a mixture of 3 parts resin to 1 part hardener. This can be measured by either weight or by volume.*

- Mixing was done in single portions for 20 ounces of resin + 6.7 ounces of hardener. Each portion was mixed thoroughly in the 21/2-quart bucket for 3 minutes.

d. Application

- The bonding surface of five 4x120 in CFRP fabric (strips) was
cleaned with acetone. Acetone was also applied to the reverse sides of the fabrics to weaken the adhesive of the SRT and removed gently. The five strips were for the first layer of the five beams.

- The mixture (adhesive) was applied to the concrete surface of all five beams and allowed to dry for 15 minutes.

- The CFRP strips were then stretched and placed on the concrete surface with the adhesive for all five beams.

- The strips were then pressed down with the fiberglass roller to keep the CFRP strips tight and wrinkle free.

- A thick layer of the saturating mixture is then applied over the CFRP strips.

- The paint roller is used to remove any trapped air pockets and to work the saturating mixture in to the fabric.

- After 30 minutes an additional layer of saturating mixture was applied and the above procedure was repeated to bond the additional five layers of CFRP strips.

- The five concrete beams with five layers of CFRP strips were allowed 24 hours before applying the transverse strips.

- The transverse strips were applied in the same way as the 4x120 in strips but this time, the strips were placed on the side of the
beams very carefully so they fall within the markings.

- For the transverse strips, which required double layers, the same procedure for, multiply layers used previously was applied.
- The concrete beams strengthened with CFRP strips were allowed to cure for three days at room temperature.

7. Instrumentation

a. Four EA-06-250BG strain gages were placed on the flexural side (bottom) and one EA-06-20CBW-120 strain gage on the top of each design beam (Design 1-5). The control beam had only one EA-06-250BG strain gage on the bottom and one EA-06-20CBW-120 on top. The strain gages for design beams (Design 1-5) spaced evenly and at the same positions for each beam to measure and compare the strain profile of the beam at mid span before testing.
Properties of Strain gage EA-06-250BG:

I. Gage Type – EA-06-250BG (Series EA Strain Gages)

II. Resistance in Ohms – 120 +/- 0.15%

III. Gage Factor @ 75°F – 2.095 +/- 0.5%

IV. Temperature Range – Cryogenic to approx. +400°F for static
measurements to +500°F for dynamic strain.

V. Strain limits – 30,000 to 50,000 microstrain (3% to 5%), tension or compression.

VI. Fatigue life- Over $10^7$ cycles @ +/-1400 microstrain; over $10^6$ cycles @ +/-1500 microstrain of approx. 2800 microstrain unidirectional (tension or compression). Longer gage lengths and lower resistance gages show greater endurance.

VII. Cement – Compatible with Certified M-Bond 200 for fast installation.

- Properties of Strain gage EA-06-20CBW-120:

I. Gage Type – EA-06-20CBW-120 (Series EA Strain Gages)

II. Resistance in Ohms – 120 +/- 0.4% @ 75°F.

III. Gage Factor @ 75°F – 2.055 +/- 0.5%

IV. Temperature Range – -100°F to +350°F for continuous use in static measurements; -320°F to +400°F for special or short-term exposure.

V. Strain limits – Approximately 5% for gage lengths 1/8 in and larger and approximately 3% for gage lengths under 1/8 in.

VI. Fatigue life- $10^8$ cycles @ +/-1200 µin/in (micro strain); $10^6$ cycles @ +/-1500 µin/in (micro strain); $10^5$ cycles @ +/-1800
μin/in (micro strain); $10^6$ cycles @ +/-2800 μin/in (micro strain) for unidirectional (tension or compression) loading.

Longer gage lengths and lower resistance gages show greater endurance.

VII. Cement – Compatible with Certified M-Bond 200 for fast installation. Super Glue.

- Installation
  
i. For design beams (Design 1-5), with their tension face up, the strain gage positions were marked with SRT and adhesive applied to the marked section to level and smoothen the sections before strain gage application.

ii. Super glue was first tested and used for the strain gages installation.

iii. Beam surface where strain gage will be placed was marked, thoroughly degreased with a solvent and dried.

iv. Gage was removed carefully from the acetate envelop with tweezers and place on a chemically clean glass plate with the bond side of the gage down.

v. A 4 in piece of M-M No. PCT-2 cellophane tape was placed over
the gage and terminal. The gage was carefully centered on the tape and the tape lifted at a shallow angle, bringing the gage up with the tape.

vi. The gage/tape assembly was positioned so that the triangle alignment marks on the gage were over the layout lines on the specimen.

vii. The gage end of the tape assembly was lifted at a shallow angle to the specimen surface (about 30°) until the gage and terminal are free of the specimen surface. Lifting was continued until the tape was free from the specimen approx. ½ in beyond the terminal. The loose end of the tape was tucked under and pressed to the specimen surface so that the gage and terminal lied flat, with the bonding surface exposed.

viii. The M-Bond 200 catalyst was applied to the bond surface of the gage and terminal. Very little catalyst is needed and should be applied in a thin, uniform coat. Lift the brush-cap out of the catalyst bottle and wipe the brush approximately 10 strokes against the lip of the bottle to wring out most of the catalyst. Allow catalyst to dry at least one minute under normal ambient conditions of +75°F (+24°C) and 30%-65% relative humidity.

ix. The tucked-under tape end of the assembly was lifted and held in
the same position. Two drops of the Super glue adhesive were applied at the fold formed by the junction of the tape and specimen surface. The adhesive application was approx. ½ in outside the actual gage installation area. This insured that local polymerization taking place when the adhesive came in contact with the specimen surface did not cause unevenness in the gage glue-line.

x. The tape was immediately rotated to approx. a 30° angle so that the gage is bridged over the installation area. While holding the tape slightly taut, a single wiping stroke was slowly and firmly made over the gage and tape assembly with a piece to gauze bringing the gage back down over the alignment marks on the specimen. Pressure was applied when wiping over the gage.

xi. Pressure was applied for a minute with thumb to the gage and terminal area and waited 2 minutes before removing tape.

xii. The gage and terminal strip were solidly bonded in place and the tape was pulled back, peeling slowly off the surface.

b. Eight potentiometers were used for the experimental testing to measure deflections. Two 1.5 in Range sliding resistors at each support, two 1.5 in Range sliding resistors at quarter spans and two 6 in Range sliding resistors at the mid span.

- Properties of the potentiometers
i. 1.5 in Range sliding resistor

ii. +/- 10 volts

iii. Material

iv. Research

- Installation

i. The two support potentiometers (left and right) were assembled on a aluminum frame and hot glued on the supports while the mid span potentiometers (left and right) were positioned on L-angles and hot glued to the sides of the beam. The quarter span potentiometers were held on the steel frame and placed underneath the beam.

ii. See Chapter 3 for potentiometer placement and arrangement.

8. Data Collection:

Data Acquisition

Data System Name: National Instruments

Title: Labview (FRP Test)

Loading: MTS Station Manager

Data Collection:
The data acquisition and collecting system were assembled as presented in the drawing above. A separate acquisition system for dynamic loading test was set for the accelerometer sensors.

Accelerometer sensors: 3 (one data acquisition system)

Three beams for accelerometer test (see Testing Order and Conditions below).

Pre-Loading Procedure

Constant loading rate was set at 0.1 in/min i.e. [Displacement controlled @ 0.1 in/min]

Displacement Limit: Relative- 4 in (max.)

Loading Condition: Upper limit loading controls were used (to enable ramp back to zero)

Check Actuator:
- Actuator was cycled prior to setting beams into place.
  - Is the system responding (Check interlocks).
  - Hose (ensure hoses are not kinked).
  - Water pump (ensure cooling pump is on)

**Set-Up**

**Step 1:** Beam placement and positioning on supports – Beams were carefully lifted by eyebolts at the ends of the beam and place on the elastomeric pads attached to the support. The elastomeric pads and beams were marked to aid with alignment. Beam were checked to ensure beam stability so it will not rock or tip over.

**Step 2:** Beam Prep1 – With the beam in position, potentiometers (2 @ each supports, quarter spans and 2 @ mid span) were carefully placed making sure the quarter span potentiometer tips were in direct contact with the glass slides attached to the beam for a smooth contact. The potentiometers were checked for reading by applying a small load (actuator).

**Step 3:** Beam Prep 2 - Strain gages were then wired from the beam to the Channels.

*Note, there were four strain gages on the bottom (tension face) and one on top (compression face) for all specimens except the control beam with only one on each face.*

**Step 4:** Quick Check:

- Connections were secure
- Connecting wires were loose and free from all tension
- Wires were not caught anywhere (under beam)
- Potentiometer tips were making contact
- Elastomeric Pads on actuator testing head were in place but not making any contact with the beam.
- Actuator was ready.

**Step 5: Loading**

- Check all data acquisition systems
  - Potentiometers
  - Strain Gages
  - Load-Displacement
- Check set conditions
  - Label-Title each beam specimen
  - Max. Load
  - Max. Displacement
  - Displacement Rate
- Check Safety
  - Personnel have safety clear goggles, gloves and hardhat on.
  - Position (ensure testing area is clear of personnel or position at right locations).
  - Ensure all exterior doors are closed.
- Ready
First accelerometer test (Not required for all specimen. See “Conditions” for each specimen).

Actuator slowly lowered onto beam (no contact initially until acquisition system is ready).

Load to 10% of total load of the beam specimen and check readings on acquisition system to verify response.

Load beam to 25% of total load.

- Reduce load to approximately 500lbs.
- Increase again to 25%.
- Repeat two more cycles.
- At the end of the 3rd cycle, perform accelerometer test on required specimen. Increase load to 50% of the total load.

Note: Visible gaps between actuator and beam are required after every 3rd cycle for accelerometer test (lift actuator clear off beam specimen).

- Reduce load to 500lbs to complete the first cycle for 50%.
- Increase load again to 50% and reduce to 500lbs two more cycles.
- At end of the 3rd cycle for 50%, perform accelerometer test on required specimen.
- Increase load to 75% of the total load of the specimen and
repeat three cycles by reducing load to approximately 500lbs and increasing to 75% of the ultimate capacity.

- At the end of the 3rd cycle for the 75% ultimate capacity, perform accelerometer on required specimen.
- Specimen will then be loaded to failure.

**Step 6: Observations:**

- Detailed pictures of before and after loading.
- Detailed pictures crack configurations.
  - Supports
  - Mid span
- Check all data acquisition system
  - Does data look realistic
  - Graphs-Plots
  - Save Data

**Step 7: Prep for next test:**

- Disconnect all strain gages wires carefully.
- Move potentiometers temporarily from testing area.
- Move tested specimen carefully and gently using eyebolts from testing area (tested specimen may have weak spots).
- Clear and clean testing area of all debris or concrete pieces.
- Clean elastomeric pad well.
Step 8: Repeat Steps 1 through 7 for new test.

Specimen Testing Order and Conditions

Control Beam 1.

Ultimate capacity: 18kips.

10% of Ultimate capacity: 1.8kips

25% of Ultimate capacity: 4.5kips.

50% of Ultimate capacity: 9kips.

75% of Ultimate capacity: 13.5kips

Limit for testing: 25kips.

Conditions:

- Accelerometer test required.

- Failure mode:
  
  o Concrete crushing.

2. Design 1 (Flexural Strengthened Beam – Tension face only)

Ultimate capacity: 30kips.

10% of Ultimate capacity: 3.0kips.
25% of Ultimate capacity: 7.5kips.

50% of Ultimate capacity: 15kips.

75% of Ultimate capacity: 22.5kips

Limit for testing: 35kips.

Conditions:

- No Accelerometer test required.
- Failure mode:
  - Debonding.

3. Design 2 (90 deg. transverse + Tension face)

Ultimate capacity: 30kips.

10% of Ultimate capacity: 3.0kips.

25% of Ultimate capacity: 7.5kips.

50% of Ultimate capacity: 15kips.

75% of Ultimate capacity: 22.5kips

Limit for testing: 35kips.
Conditions:

- Accelerometer test required.
- Failure mode:
  - Debonding.

4. Design 5 (90 deg. Transverse + Tension face)

Ultimate capacity: 30kips.
10% of Ultimate capacity: 3.0kips.
25% of Ultimate capacity: 7.5kips.
50% of Ultimate capacity: 15kips.
75% of Ultimate capacity: 22.5kips
Limit for testing: 35kips.
Conditions:
- Accelerometer test required.
- Failure mode:
5. Design 3 (45 deg. Transverse + Tension face)

- Debonding.

Ultimate capacity: 30kips.

10% of Ultimate capacity: 3.0kips.

25% of Ultimate capacity: 7.5kips.

50% of Ultimate capacity: 15kips.

75% of Ultimate capacity: 22.5kips

Limit for testing: 35kips.

Conditions:

- No Accelerometer test required.
- Failure mode:
  - Debonding.

6. Design 4 (45, 60, 90 deg. Transverse + Tension face)
Ultimate capacity: 30kips.

10% of Ultimate capacity: 3.0kips.

25% of Ultimate capacity: 7.5kips.

50% of Ultimate capacity: 15kips.

75% of Ultimate capacity: 22.5kips

Limit for testing: 35kips.

Conditions:

- No Accelerometer test required.
- Failure mode:
  - Debonding.

After testing was complete for all beams, the specimens were reviewed for additional debonding effects.

**Miscellaneous**

The information herein is general information designed to assist customers in determining whether our products are suitable for their applications. Our products are intended for sale to industrial and commercial customers. We require customers to inspect and test our products before use and to satisfy themselves as to contents and
suitability for their specific applications.

We warrant that our products will meet our written specifications. **Nothing herein shall constitute any other warranty express or implied, including any warranty of merchantability or fitness for a particular purpose**, nor is any protection from any law or patent to be inferred. All patent rights are reserved. The exclusive remedy for all proven claims is limited to replacement of our materials and in no event shall we be liable for special, incidental or consequential damages.

January 2001

**EPOTUF® 37-127**

Product Code: 37127-00

**Liquid Epoxy Resin**

**DESCRIPTION**

EPOTUF® 37-127 is a low viscosity 100% reactive diluted liquid epoxy resin based on Bisphenol-A and containing EPOTUF® 37-058 (C12 – C14 glycidyl ether).

**APPLICATIONS**

- Adhesives
- Grouts and coatings
- Wet lay-up laminating
• Potting and encapsulation

• Flooring

FEATURES

• Low viscosity and good color

• Excellent toughness

• Excellent flexibility

PROPERTIES

Viscosity at 25°C, cps 600

Color, Gardner 1 max.

Pounds per Gallon, Solution 9.2

Epoxide Equivalent Weight, on Solids 197

STORAGE

EPOTUF® 37-127, as with most liquid epoxies, may crystallize during extended storage or when stored at low temperatures. Resin that has crystallized can be remelted by holding it at 130°F to 150°F until all the crystals have melted. Warm storage (130°F to 150°F) is recommended. Remelting of crystallized resin has no effect on performance.

*Read the EPOTUF® 37-127 Material Safety Data Sheet before handling, storing, or using this product.*
APPENDIX B: MATHCAD COMPUTATIONS
Constants

**Numerical constants**

Mpsi = 10^6 psi  \( \text{kips} = 10^3 \text{ psi} \)

**Initialization**

\( \text{ORIGIN} = 1 \)

**Dimensions and Properties of Beam**

- **Height** \( h = 11 \text{ in} \)
- **Width** \( b = 6 \text{ in} \)
- **Structural Depth to tension steel reinforcement** \( ds = 9.1875 \text{ in} \)
- **Structural Depth to FRP reinforcement** \( df = 11.02 \text{ in} \)
- **Compression Depth** \( dp = 1.6875 \text{ in} \)
- **Total horizontal cover (5.5+5.5)** \( hc = 1 \text{ in} \)
- **Total vertical cover (1.125+1.125)** \( Vc = 2.25 \text{ in} \)
- **Structural Depth to FRP reinforcement** \( df = 11.02 \text{ in} \)
- **Dist. from support to first point of applied load** \( X = 57 \text{ in} \)

**Materials**

**Steel**

- **Yield Stress** \( f_y = 66000 \text{ psi} \)
- **Modulus of Elasticity** \( E_s = 290 \text{ Mpsi} \)
- **Yield strain of reinforcing steel** \( e_y = \frac{f_y}{E_s} \)

**Concrete**

- **Compressive Strength** \( f_{cp} = 6.5 \text{ kips} \)

**Properties of Unidirectional Carbon Tape**:

- **Weight** \( W_t = 11 \frac{oz}{yd} \)
- **Weight** \( W = 3.729 \times 10^{-4} \frac{g}{mm^2} \)
- **Tow size** \( T_S = 12000 \text{ ft} \)
- **Thickness** \( t = 0.508 \text{ mm} \)
- **Tension** \( T = 0.02 \text{ in} \)
- **Gravity of carbon fiber** \( \gamma = 1.6 \frac{g}{cm^3} \)
**FRP**

Modulus of Elasticity of fiber \( E_f = 54 \text{ Mpsi} \)

Width of FRP \( w_f = 4 \text{ in} \)

Thickness of fiber \( t_f = 0.1 \text{ in} \)

Area of Fiber \( A_f = w_f t_f \)

**Adhesive**

Modulus of Elasticity of adhesive \( E_a = 500 \text{ kpsi} \)

Width of adhesive \( w_a = 4 \text{ in} \)

Thickness of adhesive \( t_a = 0.01 \text{ in} \)

Area of Adhesive \( A_a = w_a t_a \)

Shear Modulus of Adhesive \( G_a = 0.51 \times 10^6 \frac{\text{lb}}{\text{in}^2} \)

**Composite**

Modulus of Elasticity of Composite \( E_{cf} = 15.87 \text{ Mpsi} \)

**Reinforcement**

Area of compression reinforcement \( A_{sp} = 0.22 \text{ in}^2 \)

Diameter of compression bar \( d_c = 0.375 \text{ in} \)

Area of tension reinforcement \( A_{st} = 0.93 \text{ in}^2 \)

Diameter of tension bar \( d_t = 0.625 \text{ in} \)

For concrete compressive strength \( f_{cp} = 6000 - 12000 \text{ psi} \) (see Navy page 53)

Density of concrete \( w_c = 155 \)

Modulus of Elasticity \( E_c = \left[ \left( 40000 + \sqrt{f_{cp} \text{ psi}} + 1.0 \times 10^6 \text{ psi} \right) \frac{w_c}{145} \right]^{1.2} \)

**Reinforcement**

Compression steel \( A_{sp} = 0.22 \text{ in}^2 \)

Tension steel \( A_{st} = 0.93 \text{ in}^2 \)

Composite \( A_p = 0 \text{ in}^2 \)

Diameter of stirrups (\#3) \( d_b = 0.375 \text{ in} \)

Diameter of compression bar \( d_c = 0.375 \text{ in} \)

Diameter of tension bar \( d_t = 0.625 \text{ in} \)
Calculation of \( f_c \) using Mander’s Model for Concrete (see Priestley Chap. 3 page 95):

\[
h_x := b - d_b - b_c
\]
\[
h_y := h - d_b - V_c
\]
\[
N_y := 0
\]
\[
N_z := 0
\]
\[
A_b := A_{sp}
\]
\[
s := 4 \text{ in}
\]
\[
\rho_y := \frac{N_y}{A_b}
\]
\[
\rho_z := \frac{N_z}{A_b}
\]

Typical value for rectangular sections

\[K_e = 6.75\]

The compression strength of confined concrete is directly related to the effective confining stress \( f_{LP} \) that can be developed at yield of the transverse reinforcement. For rectangular sections:

In the Y-direction

\[FL_{py} := K_e \rho_y f_y\]

In the Z-direction

\[FL_{pz} := K_e \rho_z f_y\]

\[FL_p := \begin{cases} FL_{pz} & \text{if } FL_{pz} \geq FL_{py} \\ FL_{py} & \text{otherwise} \end{cases}\]

Concrete strength at peak for confined concrete

\[
f_{cp} := \frac{f_{cp} \left( 2.254 \sqrt{1 + \frac{7.94 f_{LP}}{f_{cp}}} - 2 \frac{f_{LP}}{f_{cp}} - 1.254 \right)}{2.254}\]

Concrete Strain at peak of confined concrete

\[
decc := 0.002 \left[ 1 + 5 \left( \frac{f_{cp}}{f_{cp}} - 1 \right) \right]\]

\[
E_{sec} := \frac{f_{cp}}{ecc}\]

\[
T := \frac{E_c}{E_c - E_{sec}}\]
Hand Calculation of Ultimate Moment for RC Beam without Composite

MOMENT DESIGN

Check of Stresses:

Ratio of nonstressed tension reinforcement
\[ \rho := \frac{A_s}{b \cdot ds} \]

Ratio of nonstressed compression reinforcement
\[ \rho' := \frac{A_{sp}}{b \cdot ds} \]

\[ \beta_1 := 0.85 - 0.05 \left( \frac{f_{cp} - 4000 \text{ psi}}{1000 \text{ psi}} \right) \quad \text{(for } 4000 \text{ psi} < f_{cp} = 8000 \text{ psi)} \]

\[ \Delta p := \frac{0.85 \beta_1 f_{cp} \cdot \Delta p}{f_y \cdot ds} \left( \frac{87000 \text{ psi}}{87000 \text{ psi} - f_y} \right) \]

Since \( \rho - \rho' \geq \Delta p \),

\[ f_{sp} := 87000 \left[ 1 - \frac{0.85 f_{cp} \beta_1 \Delta p}{(\rho - \rho') f_y \cdot ds} \right] \text{ psi} \]

Since \( f_{sp} < f_y \)

\[ a := \frac{A_{sp} f_{sp} f_{sp}}{0.85 f_{cp} \cdot b} \]

Neutral Axis Depth

\[ c := \frac{a}{\beta_1} \]

Strain in tension reinforcement

\( \epsilon_1 > 0.005 \) (Tension Steel yields, use \( f_y \))

Calculation of Nominal Moment resistance without Composite:

\[ M_n := (A_{sp} f_{cp} - A_{sp} \cdot f_{sp}) \left( ds - \frac{a}{2} \right) + A_{sp} f_{sp} (ds - \Delta p) \]

Calculation of ultimate load resisted by beam:

( Dist. from support to 1st applied load position)

\[ x := \left( \frac{1}{2} - 0.5 \cdot \frac{t}{l} \right) \]

Ultimate Load

\[ P := 2 \frac{M_n}{x} \]
Hand Calculation of Ultimate Moment for RC Beam with Composite. Perfect Bonding is assumed.

**Analysis for 6 Layers**

**Effective Depth of Cross Section**

Number of layers of CFRP: \( n = 5 \)

Total FRP thickness: \( t_f = n \cdot t_f + \frac{t_f}{2} (2n - 1) \)

Effective depth: \( d_f = h + t_f \)

**Calculation of Fiber Volume Fraction (V_f):**

Fiber Volume Fraction is a parameter controlling the properties of a fiber-reinforced composite material. The higher the fiber volume fraction, the higher the modulus, strength, and many other properties of the composite.

\[
V_f = \frac{W}{t_f} \quad \text{Flexural Modulus (4.5 - 5.0) \( 10^6 \) psi} \quad FM = 0.51 \cdot 10^{-6} \quad \frac{\text{lb}}{\text{in}^2}
\]

**Calculation of Modulus of the Composite:**

Assume that the materials follow one-dimensional Hooke’s law and that strains in the composite, fiber, and matrix are equal; the equation below can be derived (all moduli in one direction):

\[
E_{cf} = E_f V_f + E_m V_m
\]

where \( E_{cf} \) = Modulus of the composite

\( E_f \) = Modulus of fiber

\( E_m \) = Modulus of matrix

\( V_f \) = Fiber volume fraction

\( V_m \) = Matrix volume fraction

\( 2E_{f\text{val}} = E_f \cdot V_f + E_m \cdot V_m \)

\( V_f + V_m + V_v = 1 \)

where \( V_v \) = volume fraction of voids.

**Calculate “c” (distance from extreme compression fiber to neutral axis) using Whitney Block and Equilibrium. Assume Perfect Bonding:**

Using similar triangles with max. strain @ 0.003

Using mean stress factor \( \beta = 0.85 \)

(The mean stress factor converts the actual stress-strain relationship of concrete into a rectangular stress-strain equivalent)
Assume concrete compressive strain @ extreme compression fiber

\[ ec = 0.003 \]

Carbon fiber strain

\[ \varepsilon_{\text{CFRP}} = \frac{d_f - C}{C} \]

Tensile reinforcing steel strain

\[ \varepsilon_s = \frac{d_s - C}{C} \]

Compressive reinforcing steel strain

\[ \varepsilon_p = \frac{C - d_p}{C} \]

Assume tensile steel has yielded

\[ f_s = f_y \]

Using the equilibrium equation:

\[ C_s + C_c - T_f - T_s = 0 \]

where

\[ T_s = A_s f_y \quad T_f = E_{\text{CFRP}} f_{\text{CFRP}} \]

\[ C_c = 0.85 f_{\text{t}} c_{\text{t}} b' a' \quad C_s = E_s \varepsilon_p A_s p \]

\[ A_s f_y = \left[ \frac{ec}{C} \left( d_f - C \right) \right] E_{\text{CFRP}} - \left[ (k_{p} b' c_{p}) \varepsilon_s \left( d_s - C \right) \right] + E_s \left( C - d_p \right) - 0.003 A_s \]

solve for \( C \)

Carbon fiber strain

\[ \varepsilon_{\text{CFRP}} = \frac{d_f - C}{C} \]

Tensile reinforcing steel strain

\[ \varepsilon_s = \frac{d_s - C}{C} \]

Compressive reinforcing steel strain

\[ \varepsilon_p = \frac{C - d_p}{C} \]

Check strains:

Stress in the tension reinforcing steel

\[ f_s = \begin{cases} (E_s \varepsilon_s) & \text{if } \varepsilon_s < \varepsilon_y \\ f_y & \text{otherwise} \end{cases} \]

\( f_s = f_y \) implies tensile steel has yielded.

Stress in the compressive reinforcing steel

\[ f_p = \begin{cases} (E_s \varepsilon_p) & \text{if } \varepsilon_p < \varepsilon_y \\ f_y & \text{otherwise} \end{cases} \]

\( f_p \) is not equal to \( f_y \) implies compression steel has not yielded.
Calculation of Nominal Moment resistance:
Assuming tensile steel yields, internal forces will be as follows

Tension steel bars: \( T_s = A_s \cdot f_s \)
Tension CFRP: \( T_f = F_{cfrp} \cdot A_{df} \)
Compression in concrete: \( C_c = 0.85 \cdot f_{cp} \cdot b \cdot c \)
Compression steel bars: \( C_s = f_{sp} \cdot A_{sp} \)

Equilibrium Check: \( C_c + C_s - (T_s + T_f) = 0 \text{ lb} \)

Taking moments about midpoint:
Depth of equivalent rectangular stress block \( a = C \beta \)

\( M_n = C_s \left( \frac{h}{2} - 2a \right) + C_c \left( \frac{h}{2} - a \right) + T_s \left( ds - \frac{h}{2} \right) + T_f \left( df - \frac{h}{2} \right) \)

Calculation of ultimate load resisted by beam:
(Dist. from support to 1st applied load position) \( x = \left( \frac{1}{2} - 5 \text{ ft} \right) \)

\( P = 2 \frac{M_n}{x} \)
Moment Curvature Control Beam

Incremental Variables

Compression depth "c" incremental variable \( \delta_c \approx 0.01 \text{in} \)

Number of layers in the compression zone \( N_{\text{strips}} = 20 \)

Program

\[
\begin{align*}
\text{for } & j = 1 \text{ to } N_{\text{strips}} \\
& a_j \leftarrow j \\
& \text{return } a
\end{align*}
\]

\[
\begin{align*}
\phi & \leftarrow 0.00000000000005 \\
\phi_{\text{max}} & \leftarrow 0.004 \\
\delta & \leftarrow 0.000034 \\
\text{count} & \leftarrow 1 \\
\text{while } & \phi_{\text{current}} \leq \phi_{\text{max}} \\
& \phi_{\text{current}} + 1 \leftarrow \phi_{\text{current}} + \delta \\
& \text{count} \leftarrow \text{count} + 1 \\
& \text{return } \phi
\end{align*}
\]
\( \text{set: } M_1 \leftarrow 0 \text{ lb}-\text{in} \)

\begin{align*}
\text{for } j & \in \{1, \ldots, \text{length}(\phi)\} \\
C_j & \leftarrow 0.01 \text{ lbf} \\
T_j & \leftarrow 0 \text{ lbf} \\
e_j & \leftarrow \frac{\text{dr}}{2} \\
c_{\text{count}} & \leftarrow 1 \\
\text{while } \left| C_j - T_j \right| \geq 0.01 \cdot \left| C_j \right| \wedge \text{count} < 10000 \\
e_j & \leftarrow e_j - \delta_c \\
e_j & \leftarrow e_j + \frac{\phi_j}{\text{in}} \\
e_j & < 0.006 \\
e_{\text{ef}} & \leftarrow \frac{c_j}{N_{\text{strips}}} \cdot \frac{\phi_j}{\text{in}} \\
\text{for } k & \in \{1, \ldots, \text{length}(x)\} \\
FC_k & \leftarrow \left[ \frac{c_j}{x} \right]^{(i-1) \cdot \left( \frac{s_k}{c_j} \right)} \\
C_{p,k} & \leftarrow \frac{e_j}{N_{\text{strips}}} \cdot b \cdot FC_k \\
C_{p,k,j} & \leftarrow C_{p,k} \left[ \frac{h_j}{2} \right] \cdot \left( e_j - \frac{i_k - 1}{2} \right) \frac{c_j}{N_{\text{strips}}} \\
C_p & \leftarrow \sum C_{p,j} \\
C_{j,y} & \leftarrow \sum C_{p,j} \\
es_{p,j} & \leftarrow \frac{(c_j - \text{dp})}{\text{in}} \cdot \phi_j \\
es_j & \leftarrow \frac{(d_s - c_j)}{\text{in}} \cdot \phi_j
\end{align*}
for $j \in \text{ContinueFromtestAbove}$
while ContinueFromtestAbove

$\mathbf{f}_j \leftarrow \mathbf{f}_j - \mathbf{e}_j \mathbf{y}_j$ if $|\mathbf{e}_j| < \mathbf{e}_j$
$\mathbf{f}_j \leftarrow \mathbf{f}_j - \mathbf{e}_j \mathbf{y}_j$ if $|\mathbf{e}_j| > \mathbf{e}_j$
$\mathbf{f}_p_j \leftarrow \mathbf{f}_p_j - \mathbf{f}_j \mathbf{A}_p$

$\mathbf{T}_j \leftarrow \mathbf{T}_j + \mathbf{C}_j$

count $\leftarrow$ count + 1

difference $j \leftarrow C_j - T_j$

$M_j \leftarrow C_j + \left[ \mathbf{T}_j \left( \mathbf{d}_j - \frac{\mathbf{b}}{2} \right) \right] \left[ \mathbf{C}_j \left( \frac{\mathbf{b}}{2} - \mathbf{d}_j \right) \right]$

return $\left[ \left( \begin{array}{ccc} \mathbf{c} & -\mathbf{M} & \mathbf{T} \\ \mathbf{T} & \mathbf{I} & \mathbf{e} \\ \mathbf{e} & \mathbf{e} & \mathbf{C}_j \end{array} \right) \right]^{T}$
Moment Curvature Assuming Perfect Bonding

Analysis for 5 Layers

test :=
\[ M_1 \leftarrow 0 \text{ lbf-in} \]
for \( j \in 1 \ldots \text{length}(y) \)
\[ C_j \leftarrow 0.01 \text{ lbf} \]
\[ T_j \leftarrow 0 \text{ lbf} \]
\[ c_j \leftarrow \frac{df}{2} \]
\[ \text{count} \leftarrow 1 \]
while \[ |C_j - T_j| \geq 0.01 \cdot |C_j| \land \text{count} < 10000 \]
\[ c_j \leftarrow c_j - \delta_c \]
\[ \delta_c \leftarrow 0.006 \]
\[ c_j \leftarrow c_j \]
\[ \text{count} \leftarrow \text{count} + 1 \]
\[ e_{cf} \leftarrow \frac{c_j}{N_{\text{strips}}} \cdot \phi_j \]
\[ x \leftarrow \frac{e_{cf}}{e_{c}} \]
for \( k \in 1 \ldots \text{length}(x) \)
\[ F_{C_k} \leftarrow \frac{e_{cf} \cdot x_k \cdot r}{r - 1 + (x_k)^2} \]
\[ C_{P_k} \leftarrow \frac{c_j}{N_{\text{strips}}} \cdot b \cdot F_{C_k} \]
\[ C_{py_k} \leftarrow C_{P_k} \left[ \frac{b}{2} \right] - \left[ c_j - \left( k - \frac{1}{2} \right) \cdot \frac{c_j}{N_{\text{strips}}} \right] \]
\[ C \leftarrow \sum C_p \]
\[ C_j \leftarrow \sum C_{py} \]
\[ e_{py_j} \leftarrow \frac{(c_j - dp)}{\text{in}} \cdot \phi_j \]
Moment Curvature For Varying Composite Layers Assuming Perfect Bonding (1,3,5 Layers)

**Thickness Levels (Layer 1,3,5) t=0.02 for one thickness**

\[
t = \begin{align*}
& t_1 = 0.02 \text{ in} \\
& t_\text{max} = 3.08 \text{ in} \\
& \delta = 0.04 \text{ in} \\
& \text{count} = 1 \\
\end{align*}
\]

while \( t_{\text{count}} \leq t_{\text{max}} \)

\[
\begin{align*}
& t_{\text{count}} + 1 = t_{\text{count}} + \delta \\
& \text{count} = \text{count} + 1
\end{align*}
\]

return \( t \)

**Note:** The effective depth of the whole cross section varies with the thickness of the FRP and adhesive hence the effective depth for a layered system is \((h+t = d)\).

\[
test(t) = \begin{align*}
M_1 & \leftarrow 0 \text{ lb-ft in} \\

c_1 & \leftarrow 0 \text{ lb-ft} \\
&T_1 = 0 \text{ lb-ft} \\
&c_j = \frac{h}{2} \\
&\text{count} = 1 \\
\end{align*}
\]

while \( \left| C_j - T_j \right| \geq 0.01 \left| G_j \right| \land \text{count} < 10000 \)

\[
\begin{align*}
c_j & \leftarrow c_j = \delta_c \\
&c_c = c_j - \frac{\phi_j}{\ln} \\
&\text{if } \zeta_j < 0.004 \\
&\sigma_c = \frac{c_j}{N_{\text{strips}}} \\
&x_e = \frac{e}{\text{ef}} \\
\end{align*}
\]
for \( j \in \text{ContinueFrom} \)

while \( \text{ContinueFrom} \)

for \( k \in 1 \ldots \text{length}(\cdot) \)

\[
F_{ck} = \frac{\text{fcP} \cdot x_k \cdot r}{\left[ r - 1 + \{x_k\} \right]}
\]

\[
C_{pk} = \frac{c_j}{N_{\text{strip}s}} \cdot F_{ck}
\]

\[
\text{Cpy}_{pk} = C_{pk} \left[ \frac{b}{2} \right] \left[ c_j - \left( \frac{b}{2} \right) \right] \left[ c_j - \left( \frac{b}{2} \right) \right] \left[ c_j - \left( \frac{b}{2} \right) \right]
\]

\[
C = \sum C_P
\]

\[
C_j = \sum \text{Cpy}
\]

\[
\text{exp}_j = \frac{(c_j - dp)}{\Phi_j}
\]

\[
\text{es}_j = \frac{(ds - c_j)}{\Phi_j}
\]

\[
\text{ef}_j = \frac{(h + d - e_j)}{\Phi_j}
\]

\[
\text{frp}_j = \text{Es} \cdot \text{ef}_j \quad \text{if} \quad \text{ef}_j \leq 0.01
\]

\[
\text{frp}_j = 0 \quad \text{psi} \quad \text{if} \quad \text{ef}_j > 0.01
\]

\[
\text{fs}_j = \text{Es} \cdot \text{es}_j \quad \text{if} \quad \text{es}_j \leq ey
\]

\[
\text{fs}_j = \text{Es} \cdot ey \quad \text{if} \quad \text{es}_j > ey
\]

\[
\text{fap}_j = ty \quad \text{if} \quad \left| \text{exp}_j \right| \geq ey
\]

\[
\text{fap}_j = \text{exp}_j \cdot \text{Es} \quad \text{if} \quad \left| \text{exp}_j \right| < ey
\]

\[
\text{frp}_j = 0 \quad \text{psi} \quad \text{if} \quad \text{ef}_j < 0
\]
test := ContinueFromTestAbove
for j \in \text{ContinueFromTestAbove}
  while \text{ContinueFromTestAbove}
    Cs := \text{IsPt}_j \text{Asp}
    A_T := \text{Wt}_t
    T_f := \text{sp}_j A_T
    T_s := \text{fs}_j A_s
    C_{ij} := Cs + C
    Cs := \text{sp}_j \text{Asp}
    T_T := \text{sp}_j A_T
    T_s := \text{fs}_j A_s
    C_{ij} := Cs + C
    T_j := T_s + T_f
    count := count + 1
    \text{difference}_j := C_{ij} - T_j
    \begin{aligned}
    \left[ \left[ M_j := C_{ij} + \left[ T_s \left( \frac{ds - \frac{h}{2}}{2} \right) + C_s \left( \frac{h}{2} - dp \right) \right] + T_f \left( \frac{df}{2} \right) \right] \right] \end{aligned}
  \end{aligned}
return M^T

M := for \ w \in 1.. \text{length}(t)
  M_w := test(t_w)
return M

M := for \ i \in 1.. \text{length}(M)
  M_i := M_i^T
return M

M_{\text{matrix}} := temp := M_1
for \ i \in 2.. \text{rows}(M)
  temp := augment(temp, M_i)
return temp
Moment-Displacement (\( \phi \)) Computation

Analysis for Control Beam

No. of segments (strips) beam is divided

\[ N_{\text{strips}} = 42 \]

Data from moment-curvature computation are used for interpolation

Interpolation data assembly system

\[ \text{calc} := \text{augment} \left( \phi, \frac{M}{\text{lb-m}} \right) \]

\[ \text{calc} := \text{esort} \left( \text{calc}, 2 \right) \]

Applied load (\( P \)) computation - variable (Randomly selected)

\[ P_1 := \begin{align*} & P_1 \leftarrow 0.001 \text{ lb-f} \\ & \text{for } r \in 2 \ldots 35 \\ & P_r \leftarrow 500 \text{ lb-f} \cdot r \\ & \text{return } P \end{align*} \]

Program

\[ i := \begin{align*} & \text{for } j \in 1 \ldots N_{\text{strips}} \\ & y_j \leftarrow j \\ & \text{return } a \end{align*} \]

\[ g := \begin{align*} & \text{for } t \in 1 \ldots N_{\text{strips}} \\ & a_t \leftarrow t \\ & \text{return } a \end{align*} \]

Computation of Max. moment for each applied load

\[ M_{\text{max}} := \begin{align*} & \text{for } k \in 1 \ldots \text{length}(P) \\ & \frac{P_k \cdot X}{2} \\ & M_{\text{max}} \leftarrow \frac{P_k \cdot X}{2} \\ & \text{return } M_{\text{max}} \end{align*} \]
Computation of "m" moments for load (P) per "Nstrips":

\[
M_{vec} := \begin{cases} 
\text{for } t \in 1.. \text{length}(P) \\
\text{for } k \in 1.. N\text{strips} \\
M_{vec,k,t} \leftarrow P_t \left( \frac{2}{L_{\text{strips}}} \right) \left( \frac{1}{N\text{strips}} \right) \text{ if } \left( \frac{L}{N\text{strips}} \right) \leq 57-\text{in} \\
M_{vec,k,t} \leftarrow P_t \left( \frac{57-\text{in}}{L_{\text{strips}}} \right) \left( \frac{L}{N\text{strips}} \right) \text{ if } \left( \frac{L}{N\text{strips}} \right) \leq 69-\text{in} \\
M_{vec,k,t} \leftarrow P_t \left( \frac{2}{L_{\text{strips}}} \right) \left( \frac{L}{N\text{strips}} \right) \left( \frac{P_t}{2} \right) \left( \frac{L}{24} \right) \left( \frac{L}{N\text{strips}} \right) \text{ if } \left( \frac{L}{N\text{strips}} \right) > 69-\text{in} \\
\end{cases}
\]

return \( M_{vec} \)

Calculation & Interpolation of curvature using previous curvature values and computed moments

\[
\Phi_{vec} := \begin{cases} 
\text{for } i \in 1.. \text{cols}(M_{vec}) \\
\Phi_{vec,i} \leftarrow \text{interp}(\text{calc}(\Phi_i \text{-in}, \text{calc}(\Omega_i \text{-in}), M_{vec}) \\
\end{cases}
\]

return \( \Phi_{vec} \)

Computation of Displacement from curvature computation using Trapezoidal rule for the curvatures:

\[
D := \begin{cases} 
\text{for } t \in 1.. 40 \\
\text{for } y \in 2.. N\text{strips} \left( \frac{1}{2} \right) \\
\alpha_y \leftarrow \left[ \frac{\Phi_{vec,y.t} + \Phi_{vec,y-1,t}}{2} \right] \left( y - \frac{1}{2} \right) \left( \frac{L}{N\text{strips}} \right)^2 \\
\end{cases}
\]

\[ c_t \leftarrow \sum \alpha \]
Moment-Displacement (c) Computation

Analysis for 5 Layers Composite Beam

See Moment-Displacement for Control Beam.
Note: interpolation is done using moment-curvature value from 5 Layers moment-curvature computation.

**Cracking Moment**

Modulus of Rupture

\[ f_r := 7.5 \sqrt{E_p} \text{ psi} \]

Gross section

\[ I_g := \frac{b h^3}{12} \]

Dist. of extreme tension fiber from the center of gravity

\[ Y_t := \frac{h}{2} \]

Cracking Moment

\[ M_{cr} := \frac{I_g f_r}{Y_t} \]
Shear Flow Computation

Analysis for 5 Layers

Uncracked Section

Condition

Number of layers \( (n) \)

\[ n := 5 \]

\[ i := 1 \ldots n \]

Initialization

Center of Gravity of concrete

\[ \text{ConcreteC} := \frac{-h}{2} \]

Center of Gravity of tension steel

\[ \text{TensionC} := -dp \]

Center of Gravity of compression steel

\[ \text{CompressionC} := -h + dp \]

Center of Gravity of Adhesive

\[ \text{AdhesiveC} := \frac{ta}{2} (2i - 1) + tf (i - 1) \]

Center of Gravity of fiber

\[ \text{FiberC} := ta i + tf (2i - 1) \]

Computation of component Moments

Concrete moment

\[ \text{ConcreteM} := Ec \cdot bh \cdot \text{ConcreteC} \]

Compression Steel Moment

\[ \text{CompressionM} := (Es - Ec) \cdot Asp \cdot \text{CompressionC} \]

Tension Steel Moment

\[ \text{TensionM} := (Es - Ec) \cdot As \cdot \text{TensionC} \]

Adhesive Moment

\[ \text{AdhesiveM} := E_{a} \cdot \text{Wa} \cdot \sum \text{AdhesiveC} \]

Fiber Moment

\[ \text{FiberM} := E_{f} \cdot \text{Wf} \cdot \sum \text{FiberC} \]
Computation of component Forces

Concrete Force

\[ \text{ConcreteF} := F_C \cdot b \cdot h \]

Tension Steel Force

\[ \text{Tension SteelF} := (E_s - E_c) \cdot A_s \]

Compression Steel Force

\[ \text{Compression SteelF} := (E_s - E_c) \cdot A_p \]

Adhesive Force

\[ \text{AdhesiveF} := n \cdot E_a \cdot t_a \cdot W_a \]

Fiber Force

\[ \text{FiberF} := n \cdot E_f \cdot t_f \cdot W_f \]

Neutral Axis Computation of the whole system (c)

\[ c := \frac{\sum \text{ConcreteM} + \sum \text{CompressiveM} + \sum \text{TensionM} + \sum \text{AdhesiveM} + \sum \text{FiberM}}{\sum \text{ConcreteF} + \sum \text{TensionF} + \sum \text{CompressionF} + \sum \text{AdhesiveF} + \sum \text{FiberF}} \]

Moment of Inertia Computation using Parallel Axis Theorem

The moment of inertia of an area with respect to any axis in its plane is equal to the moment of inertia with respect to a parallel centroidal axis plus the product of the area and the square of the distance between the two axes.

From the parallel axis law, \( I = I_{xx} + \sum A_d^2 \)

where \( \sum A_d^2 \) is the sum of area times the distance of the different components. The modulus of elasticity of the different components will be taken into consideration.

The moment of inertia \( I_{xx} \) will be about the interface of the concrete and the composite.

Moment of Inertia with respect to the parallel centroidal axis

\[ I_{xx} := \frac{E_c}{12} \cdot b \cdot h^3 \]

Area-square distance between axis for concrete (Modulus-Ec)

\[ \text{ConcreteAdsq} := E_c \cdot b \cdot h \cdot (c - \text{ConcreteC})^2 \]
Area-square distance between axis for tension steel (Modulus-Es-Ecc)

\[ \text{TenSteelAdsq} = (E_s - E_c) \cdot (c - \text{TenSteelC})^2 \]

Area-square distance between axis for compression steel (Modulus-Es-Ecc)

\[ \text{CompSteelAdsq} = (E_s - E_c) \cdot A_{sp} \cdot (\text{CompSteelC} - c)^2 \]

Area-square distance between axis for Adhesive (Modulus-Ea)

\[ \text{AdhesiveAdsq} = E_a \cdot W_{a-ta} \cdot \sum (\text{AdhesiveC} - c)^2 \]

Area-square distance between axis for Fiber (Modulus-Ef)

\[ \text{FiberAdsq} = E_f \cdot W_{f-tf} \cdot c \cdot \sum (\text{FiberC} - c)^2 \]

Therefore:

\[ F_{f,ad}^2 = I_{xx} + \text{ConcreteAdsq} + \text{CompSteelAdsq} + \text{TenSteelAdsq} + \text{AdhesiveAdsq} + \text{FiberAdsq} \]

First moment (Q) computation

\[ n_1 = 3 \cdot (i - 1) \]
\[ n_2 = n - i \]

Bottom layer of fiber

\[ x_{1_1} = \text{FiberC}_{(i-1)} + \frac{tf}{2} \]

Top layer of fiber

\[ x_{2_1} = x_{1_1} + tf \]

Top layer of adhesive or bottom layer of concrete

\[ x_{3_1} = x_{2_1} + ta \]

\[ EQ_{\text{layer}} = Wf \cdot E_f \cdot \frac{1}{2} \left[ \left( x_{1_1} - c \right)^2 - \left( x_{2_1} - c \right)^2 \right] + Wa \cdot E_a \cdot \frac{1}{2} \left[ \left( x_{2_1} - c \right)^2 - \left( x_{3_1} - c \right)^2 \right] \]

Initializer

\[ EQ_0 = 0 \text{-bf in} \]
\[ EQ_i = EQ_{i-1} + EQ_{\text{layer}} \]

Maximum moment = \( M_{max} \)
Maximum load

\[ P_{\text{max1}} = \frac{2 \cdot M_{\text{max}}}{X} \]

Shear

\[ V = P_{\text{max1}} \frac{1}{I} \]

Shear Flow (q) computation

\[ q = \frac{V - EQ}{E I_{\text{ax}}} \]

Shear Stress

\[ \tau = \frac{q}{W Y} \]
Analytical Model by Aryaa and Farmer

Analysis for 5 Layers

Uncracked Section

Dobonding and Plate End Condition

Limiting the longitudinal shear stress between the FRP and the substrate to $6.8 \text{ N/mm}^2 = 0.11 \text{ ksi}$ will prevent end peeling.

Transformations

\[
\eta_f = \frac{E_f}{E_c}, \quad \eta_c = \frac{E_s}{E_c}
\]

Depth of neutral axis of strengthened section

Initialization

\[i = 1 \ldots n\]

Center of Gravity of concrete

\[\text{Concrete} C \Rightarrow -\frac{h}{2}\]

Center of Gravity of tension steel

\[\text{TenSteel} C \Rightarrow -dp\]

Center of Gravity of compression steel

\[\text{CompSteel} C \Rightarrow -h + dp\]

Center of Gravity of Adhesive

\[\text{Adhesive} C = \frac{th}{2} - (2z - 1) + tf(1 - i)\]

Center of Gravity of fiber

\[\text{Fiber} C = ta + \frac{tf}{2} - (2t - 1)\]

See "Shear Flow Analysis" for computation of neutral axis.

Second moment area of strengthened concrete equivalent cracked section

\[I_{ec} = \frac{b - c'}{3} + \alpha_c As - c - d \varepsilon (ds - c)^2 + 0.6 Asp(c - dp)^2 + \alpha_f A_{frp}(df - c)^2\]

Max. moment for 5 layers

\[P_{max} = \frac{2}{X} M_{max}\]
Proposed Methodology using Roberts Analytical Model

Analysis for 5 Layers

Cracked Section

Roberts model evaluates the interface shear and normal stresses that develop at the plate cut-off point in FRP strengthened concrete beams.

Conditions

- Unbonded length of the strip (dist. between the cut-off point and the support): \( L_0 = 3 \) in
- Distance from support to 1st concentrated load: \( x = 57 \) in
- Number of layers (n): \( n = 5 \)
- Max. Moment of n layers (determined from M-curvature): \( M_n = 8.112 \times 10^5 \) lb-ft
- Predicted concentrated Load: \( P_u = \frac{2M_n}{x} \)
- Shear force @ cut-off point: \( V_{uc} = \frac{P_u}{2} \)

Determination of Cracked Section Properties in Terms of FRP

Transformations:

- Modular ratio of concrete: \( N_c = \frac{E_c}{E_f} \)
- Modular ratio of FRP: \( N_f = \frac{E_f}{E_c} \)
- Modular ratio of steel: \( N_s = \frac{E_s}{E_f} \)
Computation of depth of neutral axis for cracked section (c):

The neutral axis of the cracked section occurs at a distance "c" below the top of the section. For an elastic section, the neutral axis occurs at the centroid of the area which is defined as that point where \( \sum A_y \cdot y_{bar} = 0 \) where \( y_{bar} \) is the distance from the centroid axis to the centroid of the \( i^{th} \) area.

Using Roberts method for calculating the constants for the neutral axis:

\[
\begin{align*}
\delta_k &= \frac{N_c - b}{2} \\
B_k &= A_0 N_s + A_f + A_{sp} N_s \\
C_k &= -(A_0 N_s \cdot ds + A_f \cdot df + A_{sp} N_s \cdot dp)
\end{align*}
\]

Therefore

\[
c = \frac{-b_k + \sqrt{b_k^2 - 4 \cdot \delta_k \cdot C_k}}{2 \cdot \delta_k}
\]

Cracked Transformed Moment of Inertia in terms of FRP (I_f):

Concrete-FRP equivalent inertia

\[
C_f = \frac{N_c \cdot b \cdot c^3}{3}
\]

Tension Steel-FRP equivalent inertia

\[
I_f = N_s \cdot A_s \cdot (ds - c)^2
\]

FRP inertia

\[
I_f = A_f \cdot (df - c)^2
\]

Compression Steel-FRP equivalent inertia

\[
Csf = N_s \cdot A_{sp} \cdot (c - dp)^2
\]

Therefore

\[
I_f = C_f + I_f + F_f + Csf
\]
Computation of Roberts Modfication Factors:

Shear Stress factor
\[ \alpha_f := 0.28 \]

Peeling Stress factor
\[ \zeta_f := 1.37 \sqrt{\alpha_f} \]

Moment factor
\[ \psi_f := 1.35 - 12.5 \frac{L_0}{L} \]

Computation of Elastic Shear and Normal Stresses:

Adjusted bending moment
\[ M_a := \frac{P_0}{2} \cdot L_0 \cdot \psi_f \]

Shear stresses
\[ \tau := \frac{V_0}{I_f} \cdot \sigma \cdot (d_f - c) \]

Normal stresses
\[ \sigma_x := \frac{M_a}{I_f} \cdot (d_f - c) \]

Computation of Max. Shear and Normal Stresses:

Max. Shear stress
\[ \tau_{\text{max}} := \tau + \alpha_f \sigma_x \]

Max. Normal stress
\[ \sigma_{\text{x max}} := \zeta_f \tau_{\text{max}} \]
**Computation of Principle stresses:**

Primary tensile principle stress (Principle stress in one direction)

\[
\sigma_1 = \frac{\sigma_{\text{max}}}{2} + \sqrt{\left(\frac{\sigma_{\text{max}}}{2}\right)^2 + \tau_{\text{max}}^2}
\]

Secondary principle stress (either tension or compression, positive if tension - Principle stresses in two directions)

\[
\sigma_2 = \frac{\sigma_{\text{max}}}{2} - \sqrt{\left(\frac{\sigma_{\text{max}}}{2}\right)^2 + \tau_{\text{max}}^2}
\]

**Checking Failure Criteria:**

a. For Tension-Tension ($\sigma_1 = \sigma_2 = \text{Positive}$). Failure occurs if the principle tensile stress $\sigma_1$ is greater than $f_{tu}$ (ultimate tensile strength of concrete) and $f_{tu} = k' \sqrt{f_{cp}}$. The factor "$k'$" is set to 0.53 for normal strength concrete ($f_{cp}<55\text{MPa}$).

For high strength concrete, ACI recommends 0.59.

b. For Tension-Compression ($\sigma_1 = \text{Positive}, \sigma_2 = \text{Negative}$). Failure occurs if the principle tensile stress is greater than $f_{tu} = f_{cp}(1+\frac{\sigma_2}{f_{cp}})$.

\[
k = \begin{cases} 
0.53 & \text{if } f_{cp} < 8000 \frac{\text{lb}}{\text{in}^2} \\
0.59 & \text{otherwise}
\end{cases}
\]

Therefore

\[
f_{tu} = \begin{cases} 
k \sqrt{f_{cp} \frac{\text{lb}}{\text{in}^2}} & \text{if } (\sigma_1 \times \sigma_2) < 0 \\
k \sqrt{f_{cp} \frac{\text{lb}}{\text{in}^2}} \left(1 + \frac{\sigma_2}{f_{cp}}\right) & \text{otherwise}
\end{cases}
\]

Failure:

\[
1 \text{ if } \sigma_1 > f_{tu} \\
0 \text{ otherwise}
\]
Since the ratio of $\sigma_2/f_{cp}$ is very small, the anchorage failure load can be obtained by equating $\sigma_1$ with 0.95*ft ($f_{t}$ = concrete tensile strength). From this equality, the anchorage failure load $P_f$ for a beam subjected to concentrated loads can be approximated by:

$$f_t = \left( k \cdot \frac{f_{cp}}{\sqrt{\text{in}^2}} \right)$$

$$P_f = \frac{3.8 \cdot f_t}{\left( \sigma_c + 2 \right) \left( d_f - c \right) \left( f_t + \psi_1^2 \cdot \alpha_4 \cdot L_0 \right)}$$

Failure2 := 1 if $P_u > P_f$

0 otherwise
Analytical Model by Malek, Saadatmanesh and Ehsani

Analysis for 5 Layers

Analytical models are developed for predicting the shear and normal stresses at the concrete/FRP interface. The following assumptions are made:
1. Linear elastic and isotropic behavior for FRP, epoxy, concrete and steel reinforcement
2. Complete composite action between plate and concrete: perfect bonding (no slip)
3. Linear strain distribution through the full depth of the section.
4. Analysis is done assuming no cracks.

Effective moment of inertia of the transformed section

Transformations

Steel transformation

\[ N_s = \frac{E_s}{E_c} \]

FRP transformation

\[ N_f = \frac{E_f}{E_c} \]

Adhesive transformation

\[ N_a = \frac{E_a}{E_c} \]

Equivalents

Compression steel concrete equivalent

\[ A_{ec} = N_s \cdot A_{sc} \]

\[ b_{ec} := \frac{A_{ec}}{d_c} \]

\[ A_{et} := N_s \cdot A_{tc} \]

\[ b_{et} := \frac{A_{et}}{d_t} \]

Tension steel equivalent

\[ A_{ef} := N_f \cdot A_{tf} \]

\[ b_{frp} := \frac{A_{ef}}{t_f} \]

FRP concrete equivalent

\[ A_{ca} := N_f \cdot A_{ct} \]

\[ b_{ca} := \frac{A_{ca}}{t_a} \]
Equivalent lengths

Compression steel
\[ \text{be} := \begin{cases} \frac{\text{bec} - b}{2} & \text{if } \frac{\text{bec} - b}{2} > 0 \text{ in} \\ 0 \text{ in} & \text{otherwise} \end{cases} \]

Tension steel
\[ \text{bt} := \begin{cases} \frac{\text{bet} - b}{2} & \text{if } \frac{\text{bet} - b}{2} > 0 \text{ in} \\ 0 \text{ in} & \text{otherwise} \end{cases} \]

Adhesive
\[ \text{ba} := \text{bea} \]

FRP
\[ \text{bf} := \text{bfrp} \]

Concrete inertia 1
\[ \text{Cin1} := \left[ \frac{b c^3}{12} + b c \left( \frac{c}{2} \right)^2 \right] \]

Concrete inertia 2
\[ \text{Cin2} := \left[ \frac{b (h - c)^3}{12} + b (h - c) \left( \frac{h - c}{2} \right)^2 \right] \]

Compression steel inertia
\[ \text{CSin} := \frac{2 \text{bc} c^3}{12} + 2 \text{bc} dc (c - dp)^2 \]

Tension Steel inertia
\[ \text{Tsin} := \frac{2 \text{bt} d^3}{12} + 2 \text{bt} dc (d - c)^2 \]

Adhesive inertia
\[ \text{Ain} := \frac{\text{ba} \tau a^3}{12} + \text{ba} \tau a \left( \frac{h - c + \tau a}{2} \right)^2 \]

FRP inertia
\[ \text{Fin} := \frac{\text{bf} s^3}{12} + \text{bf} d^2 \left( \frac{h - c + \tau a}{2} \right)^2 \]

Moment of inertia of transformed section
\[ I_n := \text{Cin1} + \text{Cin2} + \text{CSin} + \text{Tsin} + \text{Ain} + \text{Fin} \]
Therefore for the bending moment at the ultimate load, the expression \( M(x) = P_{\text{max}}(x-x_0) \), where the origin of \( x_0 \) is define at the left support. Therefore, the coefficients of the polynomial are \( a_1 = a_3 = 0 \) and \( a_2 = P_{\text{max}}. \)

**Constants**

\[ a_1 := 0 \quad \text{lb/in} \]
\[ a_2 := P_{\text{max}} \]
\[ a_3 := 0 \quad \text{lb/in} \]

\[ A := \frac{G_a}{t_f \cdot t_a \cdot E_f} \]

\[ b_1 := \frac{\bar{y}_{\text{bar}} \cdot a_1 \cdot E_f}{t_f \cdot E_c} \]

\[ b_2 := \frac{\bar{y}_{\text{bar}} \cdot E_f}{t_f \cdot E_c} \cdot (2a_1L_o + a_2) \]

\[ b_3 := \frac{E_f}{t_f \cdot E_c} \left( \left( \frac{\bar{y}_{\text{bar}}}{t_f \cdot E_c} \right) \left( a_1L_o^2 + a_2L_o + a_3 \right) + \frac{2}{t_f} \cdot \frac{t_a^2}{G_a} \right) \]

\[ C_1 = b_3 \quad C_2 = -b_3 \]

**Shear Stress**

\[ \tau_{\text{max}} := tf \cdot (b_3 \sqrt{A} + b_2) \]

The shear stress multiplied by the width of the FRP can be assumed as a distributed load per unit length (shear flow) along the interface of each of the beams (concrete beam & FRP beam) with the adhesive layer.

**Shear Flow**

\[ q = \tau_{\text{max}} \cdot W_f \]

\[ b_3 \cdot \frac{2}{N^2} \]

**Discrete Point along the length of beam**

\[ m := \text{min} \]
\[ s := 63 \]

\[ \tau_j := \frac{tf}{m} \left( \frac{b_1}{m} \cdot \frac{t_a^2}{N} \cdot \sqrt{A} \cdot m^2 \cdot \cos h \left( \sqrt{A} \cdot m^2 \right) - b_2 \cdot m^2 \cdot \sqrt{A} \cdot m^2 \cdot \sin h \left( \sqrt{A} \cdot m^2 \right) \right) + 2b_1 \frac{m^4}{N} \cdot j + b_2 \frac{m^3}{N} \]
\[ f_p = C_1 \frac{\text{in}^2}{\text{lb}} \sinh \left( \sqrt{\frac{A \text{ in}^2}{s}} \right) + C_2 \frac{\text{in}^3}{\text{lb}} \cosh \left( \sqrt{\frac{A \text{ in}^2}{s}} \right) + b_1 \frac{\text{in}}{\text{lb}} + b_2 \frac{\text{in}^3}{\text{lb}} + b_3 \frac{\text{in}}{\text{lb}} + b_4 \frac{\text{in}^2}{\text{lb}} \]
## Ductility - Energy Method

### Assuming Perfect Bonding for 5 Layers Composite

<table>
<thead>
<tr>
<th>Load (kips)</th>
<th>Displacement (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1.106</td>
</tr>
<tr>
<td>1</td>
<td>3.36</td>
</tr>
<tr>
<td>2</td>
<td>5.04</td>
</tr>
<tr>
<td>3</td>
<td>6.72</td>
</tr>
<tr>
<td>4</td>
<td>8.4</td>
</tr>
<tr>
<td>5</td>
<td>10.08</td>
</tr>
<tr>
<td>6</td>
<td>11.76</td>
</tr>
<tr>
<td>7</td>
<td>13.44</td>
</tr>
<tr>
<td>8</td>
<td>15.12</td>
</tr>
</tbody>
</table>

\[ \text{Layer5Load} = \\]

\[ \text{Layer5Disp} = \\]

\[ \text{DispSort} = \text{sort(Layer5Disp)} \]

\[ \text{LoadSort} = \text{sort(Layer5Load)} \]

\[ \text{AreaLayer5} = \int_{2.46 \times 10^4}^{1.226} \text{interpLinearDispSort,LoadSort,DispSort,LoadSort,} x \text{ } dx \]
### Computing Slopes at the Cracking, Yielding and Failure Points

<table>
<thead>
<tr>
<th>Data Points @ Cracking</th>
<th>Data Points @ Yielding</th>
<th>Data Points @ Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_cy_1 = c$</td>
<td>$S_cy_1 = c$</td>
<td>$S_cy_1 = 31.3%$</td>
</tr>
<tr>
<td>$S_cy_2 = 2.1 \times 10^2$</td>
<td>$S_cy_2 = 22.5%$</td>
<td>$S_cx_1 = 1.05$</td>
</tr>
<tr>
<td>$S_cy_2 = 0.010$</td>
<td>$S_cx_2 = 0.38$</td>
<td>$S_cx_2 = 3.3%$</td>
</tr>
</tbody>
</table>

#### Slope @ Cracking

$$S_c = \frac{S_cy_2 - S_cy_1}{S_cx_2 - S_cx_1}$$

#### Equation of Line @ Cracking

$$y = S_cy_1 - S_cy_2 - S_cx_1 \cdot (x - S_cx_1) = 0 \text{ solve } y = 200 \cdot S_cx_1$$

#### Equation of Line @ Yielding

$$y = S_cy_2 - S_cy_1 - S_cx_2 \cdot (x - S_cx_2) = 0 \text{ solve } y = 40 \cdot S_cx_2$$

#### Equation of Line @ Failure

$$y = S_cy_3 - S_cy_2 - S_c3 \cdot (x - S_c3) = 0 \text{ solve } y = 21.425443786982485 \cdot 9.94082802366863905485$$
Generating Points for Slopes at the Cracking, Yielding and Failure Points

Points for Cracking Slope

\[ \text{Points for Cracking Slope} \]
\[ x_{Sc1} := x_{Sc1} - 0.006 \]
\[ x_{Sc1_{\max}} := 0.1 \]
\[ \delta := 0.02 \]
\[ \text{count} := 1 \]
\[ \text{while } x_{Sc1_{\text{count}}} \leq x_{Sc1_{\max}} \]
\[ x_{Sc1_{\text{count}}} := x_{Sc1_{\text{count}}} + \delta \]
\[ \text{count} := \text{count} + 1 \]
\[ \text{return } x_{Sc1} \]

\[ y_{Sc1} := 200 \times x_{Sc1} \]

Points for Yielding Slope

\[ \text{Points for Yielding Slope} \]
\[ x_{Sc2} := x_{Sc2_{\delta}} - 0.01 \]
\[ x_{Sc2_{\delta}} := 1.3 \]
\[ \delta := 0.02 \]
\[ \text{count} := 1 \]
\[ \text{while } x_{Sc2_{\text{count}}} \leq x_{Sc2_{\delta}} \]
\[ x_{Sc2_{\text{count}}} := x_{Sc2_{\text{count}}} + \delta \]
\[ \text{count} := \text{count} + 1 \]
\[ \text{return } x_{Sc2} \]

\[ y_{Sc2} := 40 \times x_{Sc2} \]

Points for Failure Slope

\[ \text{Points for Failure Slope} \]
\[ x_{Sc3} := x_{Sc3_{\delta}} - 0.6 \]
\[ x_{Sc3_{\delta}} := 1.3 \]
\[ \delta := 0.6 \]
\[ \text{count} := 1 \]
\[ \text{while } x_{Sc3_{\text{count}}} \leq x_{Sc3_{\delta}} \]
\[ x_{Sc3_{\text{count}}} := x_{Sc3_{\text{count}}} + \delta \]
\[ \text{count} := \text{count} + 1 \]
\[ \text{return } x_{Sc3} \]

\[ y_{Sc3} := 21.412544372 \times 9.9408284023 \times x_{Sc3} \]

Computation of Slope Line Separating Elastic & Inelastic Energy

Failure Point

\[ S_x := 122t \quad S_y := 33t \]

Loads from slope intersection

\[ P_1 := 23.5t \quad P_2 := 31.6t \]

Unloading Slope

\[ S = \left[ \frac{P_1 \cdot x_{\text{Scl}} + (P_2 - P_1) \cdot x_{\text{Sc2}}}{P_2} \right] \]

Unloading Equation

\[ y - S \times (x - 50) = 0 \text{ where } y \to -161.30415428390768257 \quad 158.9756560227631094t \]
Generating Points for Unloading Equation

\[
y := \begin{cases} 
    x_1 & \leftarrow 1 \\
    y_{\text{max}} & \leftarrow 1.3 \\
    \delta & \leftarrow 0.02 \\
    \text{count} & \leftarrow 1 \\
    \text{while } y_{\text{count}} \leq y_{\text{max}} & \\
    \text{count} & \leftarrow \text{count} + 1 \\
    \text{return } x 
\end{cases}
\]

\[
\begin{array}{c|c}
\text{count} & 3 \quad 4 \quad 5 \quad 6 \\
\hline
1 & 1.04 & 1.05 & 1.08 \\
2 & 1.11 & 1.12 & 1.14 \\
\end{array}
\]

\[
y := -161304152839768257 \times 15897366022376319948
\]

Elastic Area Dimensions

\[
B_1 = 1.01 \\
B_2 = 1.22t
\]

Elastic Area

\[
E_{el} = \frac{1}{2} (h_2 - B_1) H
\]

Ductility

\[
\mu := \frac{1}{2} \left( \frac{\text{AreaLayer5}}{E_{el}} + 1 \right)
\]
Energy Ratio: Defined as the ratio of the inelastic energy to total energy. It is proposed as the proper measure of ductility. If the energy ratio is greater than 75%, the beam will exhibit a ductile failure.

- **Ductile failure** - greater than 75%
- **Semiductile** - 70 - 74%
- **Brittle failure** - below 69%

**Inelastic Energy**

\[ E_{in} = \text{Area}_{layer5} \times t_e \]

**Energy Ratio**

\[ ER = \frac{E_{in}}{\text{Area}_{layer5}} \times 10^4 \]
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