Computer Analysis of the Flow of a Dissociating Gas Through a Porous Matrix

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COMPUTER ANALYSIS OF THE FLOW OF A DISSOCIATING GAS THROUGH A POROUS MATRIX

BY

DAVID P. LIPPY
B.S.A.A.E., University of Illinois, 1970

RESEARCH REPORT

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The person who stimulated my interest in porous matrix cooling was Archie Ossin. The formulation of the computer program was undertaken at his suggestion, and he has provided comments and references which have been very useful in completing the project. His help is greatly appreciated.

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Special thanks are given to Mrs. Jeannie Pfann who deciphered my equations and typed the final copy of the report.
ABSTRACT

A computer model has been developed to analyze the flow of a dissociating gas through a porous metal matrix. The program predicts the transient temperature distributions through the coolant gas and matrix along with the pressure distribution and mass flow rate.

The differential equations used in developing the program are documented in the literature or represent logical extensions of documented equations. The derivation of the finite difference equations is presented.

Comparisons of experimental data with computer predictions are shown and indicate that the predictions fall within the experimental error in the data.

A source listing of the computer program is contained in the Appendix.
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LIST OF VARIABLES

A = empirical heat transfer coefficient
B = empirical heat transfer coefficient
Co_e = concentration of oxygen at boundary layer edge
c_p = boundary layer specific heat at constant pressure
c_p_C = coolant gas specific heat at constant pressure
c_p_m = matrix specific heat at constant pressure
E = coolant activation energy
G_h = blockage coefficient
h_v = volumetric heat transfer coefficient
H_r = recovery enthalpy
H_w = boundary layer enthalpy at wall temperature T_mL
k_b = thermal conductivity of matrix base metal
k_c = thermal conductivity of coolant gas
k_m = thermal conductivity of matrix
L = thickness of matrix
M = exponent in viscosity equation
M_ND = molecular weight of undissociated gas
M_D = molecular weight of dissociated gas
m = coolant gas mass flow rate
P = pressure
P_e = boundary layer pressure
P_s = coolant gas supply pressure
- $q_{net}$ = net heat flux to matrix downstream surface
- $q_{cond_{in}}$ = heat conduction into a node
- $q_{cond_{out}}$ = heat conduction out of a node
- $q_{conv}$ = convective heat flux
- $q_{cw}$ = cold wall heat flux
- $q_{stored}$ = heat stored in a node
- $R$ = coolant gas constant
- $R_0$ = isothermal dissociation rate
- $t$ = time
- $T$ = temperature at time $t$
- $T'$ = temperature at time $t + \Delta t$
- $T^*$ = viscosity reference temperature
- $T_c$ = coolant gas temperature
- $T_i$ = initial temperature
- $T_m$ = matrix temperature
- $T_o$ = radiation sink temperature
- $u$ = coolant gas flow velocity
- $x$ = distance from upstream surface of matrix
- $a$ = amount of dissociation
- $\beta$ = inertial resistance coefficient
- $\Gamma$ = matrix permeability
- $\Delta H$ = heat of dissociation
- $\Delta H_{re}$ = heat of reaction with oxygen
- $\Delta t$ = time step
- $\Delta x$ = distance between nodes
\( \varepsilon \) = matrix surface emissivity

\( \xi \) = molecular weight exponent in blockage term

\( \eta \) = correction for type of coolant in blockage term

\( \eta_{ND} \) = blockage correction for no dissociation

\( \mu \) = viscosity

\( \mu^* \) = reference viscosity

\( \rho_c \) = coolant gas density

\( \rho_b \) = matrix base metal density

\( \rho_m \) = matrix density

\( \sigma \) = Boltzmann's constant

\( \phi \) = porosity

**Subscripts**

\( e \) = property evaluated at boundary layer edge

\( i \) = initial value

\( L \) = property evaluated at downstream surface of matrix

\( n \) = property evaluated at node \( n \)

\( N \) = property evaluated at downstream surface node

\( s \) = coolant gas supply reservoir condition

\( 1 \) = coolant gas supply reservoir condition

\( 2 \) = property evaluated at upstream surface of matrix
I. INTRODUCTION

As peak velocities for advanced antiballistic missiles and reentry vehicles have increased, protecting critical areas such as nose tips and air vane leading edges has become increasingly more important and difficult. These surfaces must withstand not only the extreme aerodynamic heating environment, but also the particle impact environments encountered during flight. These particle environments may result from traversing a rain cloud or a nuclear surface debris cloud. Nuclear surface debris clouds are created by nuclear blasts occurring close enough to the earth's surface so that dust and debris are swept up into the familiar mushroom cloud. Penetrating surfaces such as air vane leading edges may also experience particle environments due to ablation products being swept off the missile forebody.

Refractory metals such as tungsten have become strong candidates for advanced nose tips and leading edges. Refractory metals have high melting points and high strength over a wide range of temperatures. Their high temperature capability allows designers to take advantage of the hot wall heat blockage effect while maintaining material strength for particle erosion resistance. The major problem associated with using refractory metals is that they oxidize rapidly when exposed to the high temperatures generated in hypersonic flight. The oxidation problem can be greatly reduced by forming a porous matrix from the refractory metal and then forcing a liquid or gas
through the matrix. The transpiring liquid or gas enters the boundary layer and reacts with the boundary layer oxygen. By forcing enough transpirant through the matrix, the amount of oxygen diffusing to the metal surface can in theory be driven to zero, thus preventing surface oxidation of the metal matrix. The transpirant serves a second purpose in that the metal is cooled as the transpirant passes through the matrix. By careful selection of the coolant and matrix material, it is possible to obtain a zero recession nose tip which is highly desirable from a missile controllability standpoint.

One of the most promising candidates for cooling a refractory metal matrix such as tungsten is ammonia. Ammonia has several inherent properties which make it superior to other coolants:

1. It is relatively cheap and readily available.
2. It has a high specific heat which allows it to absorb large amounts of heat while passing through the matrix.
3. It has a high heat of formation. Hence, when the ammonia dissociates at high temperatures, it will absorb large amounts of energy.
4. The dissociation rate is high at the temperatures of interest.
5. The tungsten matrix serves as a catalyst for the dissociation reaction.
6. Only gaseous products are formed from the dissociation reaction which means that there will be no clogging of the matrix. There is also no erosion of the matrix walls resulting from the dissociation reaction.

7. The dissociation products, nitrogen and hydrogen, have low molecular weights for effective boundary layer blockage and readily react with the boundary layer oxygen.

8. The dissociation reaction rate increases with temperature, and thus the hot spots of the matrix receive the most cooling from the dissociation reaction.

Collectively, the properties listed above result in a coolant system which can be packaged in a small volume, since very little ammonia is needed to cool the areas of interest. This in turn minimizes the center of gravity shift as the coolant is expended. Since the weight and size of the missile control system are directly affected by the missile center of gravity shift during flight, the reduction in this variable resulting from the use of an ammonia coolant system leads to a smaller missile control system.

To effectively design a porous matrix thermal protection system, it is necessary to be able to predict the performance of the matrix and coolant when exposed to the flight environment. In the past, several researchers have developed analytical models to predict the performance of such systems. Most previous models were developed to analyze steady state conditions with non-dissociating coolants. Recently, steady state models for dissociating coolants have become available, but little work has been attempted to develop transient
models. Some transient models do exist for non-dissociating coolants, but none have been developed that will analyze a reactive coolant and the thermal environment applicable to advanced missile components. Such a model is necessary to properly analyze the highly transient environments experienced by advanced antiballistic missiles and reentry vehicles.

The computer model described in the following chapters has been developed to treat the transient one-dimensional heating of a porous matrix which is being cooled by a dissociating gas. It is assumed that the downstream matrix surface is experiencing laminar stagnation point heating. The program utilizes a finite difference solution to the differential equations describing the problem. Since matrix surface temperatures at the convective boundary are high, the effects of surface reradiation have been incorporated. Boundary layer heat blockage is determined from a linear model, and rate limited dissociation of the coolant is based on a state-of-the-art model described by Gorton. The analytical model is based on the assumption that the gas mass flow rate and pressure distribution can be determined from steady state continuity and momentum equations, while the temperatures of the matrix and gas are determined from transient energy equations. A one-dimensional geometry as shown in Figure 1 has been chosen for analysis.

Chapter II describes the differential equations for the flow of a reactive gas through a porous matrix. Chapter III develops the finite difference equations which correspond to the differential equations of Chapter II. The computer model is described in Chapter
Fig. 1. Problem Geometry
IV along with data to validate the model. Some notes on application of the model to real world problems are also contained in Chapter IV.

A source listing of the computer program and a sample output are contained in the Appendix.
II. DIFFERENTIAL EQUATIONS FOR FLOW THROUGH A POROUS MATRIX

The one-dimensional steady state differential continuity equation for the flow of a gas through a porous matrix may be written as:

\[ \frac{dm}{dx} = 0 \]  \hspace{1cm} \text{II.1}

where

\( m \) = coolant gas mass flow rate.

Integrating this expression gives:

\[ m = \text{constant}. \]  \hspace{1cm} \text{II.2}

The pressure distribution within the porous matrix is given by the following modified Darcy's equation as given by Koh:\(^4\):

\[ -\frac{dp}{dx} = \frac{\mu m}{\Gamma \rho_c} + \beta \rho_c \left[ \frac{m}{\rho_c} \right]^2 \]  \hspace{1cm} \text{II.3}

where

\( \mu \) = coolant gas viscosity
\( \rho_c \) = coolant gas density
\( \Gamma \) = matrix permeability
\( \beta \) = inertial resistance coefficient.

The first term on the righthand side of equation II.3 is due to viscous forces, while the second term is due to inertia forces. The matrix permeability and inertial resistance coefficient are properties of the porous matrix and are determined experimentally\(^5\).
With the exception of a change in sign convention, the transient one-dimensional energy equation for the metal matrix is given by Curry and Cox as:

$$\rho_m c_{pm} \frac{dT_m}{dt} = \frac{\partial}{\partial x} \left[ k_m \frac{\partial T_m}{\partial x} \right] - h' (T_m - T_c) \quad \text{II.4}$$

where

- $\rho_m$ = matrix density
- $c_{pm}$ = matrix specific heat
- $T_m$ = matrix temperature
- $T_c$ = coolant gas temperature
- $k_m$ = matrix conductivity
- $h'$ = volumetric heat transfer coefficient.

The thermal conductivity of the matrix, $k_m$, is defined in terms of the porosity of the matrix and the thermal conductivity of the metal from which the matrix is constructed. Koh gives the following expression for matrix conductivity:

$$k_m = k_b \left[ \frac{1 - \phi}{1 + 10\phi^2} \right] \quad \text{II.5}$$

where

- $k_b$ = base metal thermal conductivity
- $\phi$ = matrix porosity.

The matrix density is expressed as:

$$\rho_m = \rho_b (1 - \phi) \quad \text{II.6}$$

The specific heat of the matrix is equal to the specific heat of the base metal when used in equation II.4. The volumetric heat transfer coefficient is a function of the mass flow rate, the coolant gas specific heat at constant pressure, the matrix thermal conductivity,
and two empirical constants. The volumetric heat transfer co-efficient will be discussed in more detail in Chapter III.

Curry and Cox give the transient one-dimensional coolant gas energy equation as:

\[ \rho_c c_p \left( \frac{\partial T_c}{\partial t} + u \frac{\partial T_c}{\partial x} \right) - \left[ \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} \right] = \]

\[ \frac{\partial}{\partial x} \left[ k_c \frac{\partial T_c}{\partial x} \right] + \frac{h_c}{\phi} (T_m - T_c) \]

where

- \( \rho_c \): coolant gas density
- \( c_p \): coolant gas specific heat at constant pressure
- \( u \): coolant gas flow velocity
- \( k_c \): coolant gas thermal conductivity.

In this form the coolant gas energy equation will not account for the energy transfer associated with a reactive gas. Therefore, the equation must be generalized by adding a term to account for coolant gas dissociation. This term is based on the recent work of Gorton and results in the following energy equation:

\[ \rho_c c_p \left( \frac{\partial T_c}{\partial t} + u \frac{\partial T_c}{\partial x} \right) - \left[ \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} \right] = \]

\[ \frac{\partial}{\partial x} \left[ k_c \frac{\partial T_c}{\partial x} \right] + \frac{h_c}{\phi} (T_m - T_c) - \]

\[ \Delta H R_0 e^{-\frac{E}{RT_m}} \]

where

- \( \Delta H \): heat of dissociation of coolant gas
- \( R_0 \): isothermal reaction rate
- \( E \): activation energy
- \( R \): coolant gas constant.
To solve equations II.1 and II.3 for the coolant gas flow rate and pressure distribution, it is necessary to specify two boundary conditions. The first boundary condition states that the coolant gas supply pressure is imposed on the upstream surface of the matrix.

\[ P(0, t) = P_s(t) \]  

The second boundary condition states that the pressure imposed on the downstream surface of the matrix is equal to the pressure generated by the inviscid flow field external to the matrix, i.e., the boundary layer pressure.

\[ P(L, t) = P_e(t) \]  

To solve the coolant and matrix energy equations, two boundary conditions and one initial condition are needed for each equation. Assuming that the matrix and coolant are initially at some uniform ambient temperature, the initial conditions for the matrix energy equation and coolant energy equation are respectively:

\[ T_m(x, 0) = T_i \]  

and

\[ T_c(x, 0) = T_i \]  

The boundary condition for the matrix energy equation at \( x = L \) is given by Gorton and Spurlock \(^{11}\) as:

\[ -k_m \frac{\partial T_m}{\partial x} = q_{\text{net}} \]  

where

\[ q_{\text{net}} = \text{net heat flux to the surface.} \]
This boundary condition assumes that heat conduction through the coolant gas is negligible in comparison to the heat conduction through the metal matrix.

The net heat flux is given by Radovich\textsuperscript{12} as:

\[
q_{\text{net}} = \frac{q_{\text{cw}}}{H_r} \left[ H_r - (H_w - \Delta H_{\text{re}} C_{O_e}) \right] C_h - \sigma e (T_{mL}^4 - T_o^4)
\]

where

- \(q_{\text{cw}}\) = cold wall heat flux
- \(H_r\) = recovery enthalpy
- \(H_w\) = boundary layer enthalpy at wall temperature \(T_{mL}\)
- \(\Delta H_{\text{re}}\) = heat of reaction
- \(C_{O_e}\) = concentration of oxygen at boundary layer edge
- \(C_h\) = blockage coefficient
- \(\sigma\) = Boltzmann constant
- \(\varepsilon\) = emissivity of metal matrix
- \(T_o\) = radiation sink temperature.

In this expression the cold wall heat flux is the heat flux that would be obtained at the surface of the matrix if there were no coolant flow and the temperature of the surface were at some reference temperature, usually room temperature or absolute zero. The heat of reaction is the heat released when coolant gas burns with the oxygen in the boundary layer. It has been assumed that this reaction will be diffusion limited, i.e., all of the oxygen entering the boundary layer will be consumed.

The blockage coefficient in equation II.14 indicates the ratio
of the actual amount of convective surface heating to the amount of surface heating that would exist with no coolant gas flow. The addition of coolant gas into the boundary layer inhibits the transfer of heat to the surface. For a laminar stagnation point, $G_h$ is given by Ossin\textsuperscript{13} as:

$$G_h = \frac{1}{1 + \eta \frac{m \dot{H}_r}{q_{cw}}}$$

where

- $m = \text{coolant mass flow rate}$
- $\eta = \text{correction for type of coolant}$.

$\eta$ is computed from the following expression:

$$\eta = \eta_{ND} \left\{ 1 + \alpha \left[ \left( \frac{M_{ND}}{M_D} \right)^{\xi} - 1 \right] \right\}$$

where

- $\eta_{ND} = \text{correction for undissociated coolant gas}$
- $M_{ND} = \text{molecular weight of undissociated gas}$
- $M_D = \text{molecular weight of dissociated gas}$
- $\xi = \text{empirical molecular weight exponent}$
- $\alpha = \text{amount of dissociated gas}$.

According to Gorton\textsuperscript{14}, the amount of dissociation is a function of the matrix temperature and can be expressed as:

$$\alpha = e^{-\frac{E}{RT_{mL}}}$$

where

- $E = \text{activation energy}$
- $R = \text{coolant gas constant}$.
The ratio $E/R$ is determined experimentally and has a value of $9500^\circ R$ for ammonia\textsuperscript{15}.

The second boundary condition for the matrix energy equation results from assuming that the heat transfer from the upstream surface of the metal matrix to the upstream coolant is negligible. The upstream metal matrix surface can then be treated as adiabatic, and the boundary condition is mathematically expressed as:

$$\frac{\partial T_m}{\partial x} \bigg|_{x=0} = 0$$ \hspace{1cm} \text{(II.18)}

The boundary condition for the coolant gas at $x = L$ is given by Curry and Cox\textsuperscript{16} as:

$$\frac{\partial^2 T_c}{\partial x^2} \bigg|_{x=L} = 0$$ \hspace{1cm} \text{(II.19)}

By assuming a large coolant gas supply reservoir and negligible heat transfer by means of conduction upstream in the coolant gas at $x = 0$, the boundary condition for the coolant gas energy equation at $x = 0$ is:

$$T_c \bigg|_{x=0} = T_c \bigg|_{x=\infty} = T_{cS}$$ \hspace{1cm} \text{(II.20)}

The set of coupled differential equations, boundary conditions, and initial conditions presented in this chapter describe the flow of a dissociating gas through a porous metal matrix. Due to the complexity of the equations, they cannot be readily solved to obtain an exact closed form solution. Therefore, an approximate method of solution must be found. Such a method is the method of finite differences\textsuperscript{17}. The finite difference solution to the differential equations is given
in the following chapter.
III. FINITE DIFFERENCE EQUATIONS FOR FLOW THROUGH A POROUS MATRIX

A forward finite difference procedure, or Euler method of solution, has been chosen to solve the equations given in Chapter II. This procedure works on the premise that the temperature at a given position in the body at time $t + \Delta t$ may be determined from equation III.1.

$$T' = T + \frac{dT}{dt} \Delta t$$  \hspace{2cm} \text{III.1}

In equation III.1, $T'$ is the temperature at time $t + \Delta t$, while $T$ is the temperature at time $t$. $\frac{dT}{dt}$ is evaluated at time $t$, and $\Delta t$ is the time interval. In the limit as $\Delta t \to 0$, the solution obtained from equation III.1 becomes exact, but excellent approximate solutions may be obtained for finite values of $\Delta t$. The problem then becomes one of determining $\frac{dT}{dt}$ from the known values of temperature at time $t$.

The first step in obtaining the finite difference equations is to divide the geometry given in Figure 1 into a number of equally spaced nodes as shown in Figure 2. At each node an energy balance can be written for the coolant gas and for the metal matrix. Figure 3 shows a typical internal node and the energy terms corresponding to the metal matrix energy equation II.4. The energy balance for this node can be written as:

$$\dot{q}_{\text{stored}} = \dot{q}_{\text{cond in}} - \dot{q}_{\text{cond out}} - \dot{q}_{\text{conv}}$$  \hspace{2cm} \text{III.2}
Fig. 2. Finite Difference Geometry
In this equation \( \dot{q}_{\text{stored}} \) is the amount of energy stored in the node \( n \) per unit time. \( \dot{q}_{\text{condin}} - \dot{q}_{\text{condout}} \) represents the net amount of energy conducted into the node per unit time, and \( \dot{q}_{\text{conv}} \) represents the amount of energy convected to the coolant gas per unit time.

From the geometry shown in Figure 3, the stored energy term, \( \dot{q}_{\text{stored}} \), may be written as:

\[
\dot{q}_{\text{stored}} = \rho_{mn} c_{pmn} \Delta x \frac{T_{mn} - T_{mn+1}}{\Delta t}
\]

where

- \( \Delta x \) = distance between nodes
- \( T_{mn}' \) = matrix temperature at node \( n \) at time \( t + \Delta t \)
- \( T_{mn} \) = matrix temperature at node \( n \) at time \( t \).

Similarly the terms \( \dot{q}_{\text{condin}} \) and \( \dot{q}_{\text{condout}} \) may be written as:

\[
\dot{q}_{\text{condin}} = \frac{k_{mn-1} + k_{mn}}{2} \frac{T_{mn-1} - T_{mn}}{\Delta x}
\]

\[
\dot{q}_{\text{condout}} = \frac{k_{mn} + k_{mn+1}}{2} \frac{T_{mn} - T_{mn+1}}{\Delta x}
\]

The net heat flux due to conduction is then:

\[
\dot{q}_{\text{cond}} = \frac{1}{2\Delta x} \left[ (k_{mn-1} + k_{mn}) T_{mn-1} - (k_{mn-1} + 2k_{mn} + k_{mn+1}) T_{mn} + (k_{mn} + k_{mn+1}) T_{mn+1} \right]
\]

Finally, the convective heat flux term, \( \dot{q}_{\text{conv}} \), may be written as:

\[
\dot{q}_{\text{conv}} = h_{n} \Delta x (T_{mn} - T_{cn})
\]

Substituting equations III.3, III.6, and III.7 into equation III.2 results in:
Fig. 3. Matrix Energy Equation Terms
\[ \rho_n c_{p_m} \Delta x \left[ \frac{T_n^m - T_m^n}{\Delta t} \right] = \frac{1}{2\Delta x} \left[ (k_{m-1} + k_m) T_{m-1}^n - (k_{m-1} + 2k_m + k_{m+1}) T_m^n + (k_m + k_{m+1}) T_{m+1}^n \right] \]

\[ - h_n^* \Delta x (T_m^n - T_c^n) \]

Solving for the new matrix temperature at time \( t + \Delta t \):

\[ T_{n}^m = \left[ 1 - \frac{\Delta t (k_{m-1} + 2k_m + k_{m+1})}{2\rho_m c_{p_m} (\Delta x)^2} - \frac{h_n^* \Delta t}{\rho_m c_{p_m}} \right] T_m^n + \frac{\Delta t (k_m + k_{m-1})}{2\rho_m c_{p_m} (\Delta x)^2} T_{m-1}^n + \frac{\Delta t (k_m + k_{m+1})}{2\rho_m c_{p_m} (\Delta x)^2} T_{m+1}^n + \frac{h_n^* \Delta t}{\rho_m c_{p_m}} T_c^n \]

All terms on the right-hand side of equation III.9 are evaluated for the known temperature at time \( t \) to obtain the new temperature \( T_m^n \) at time \( t + \Delta t \).

Finite difference equations for the matrix temperatures at the matrix boundaries, nodes 2 and N, were derived in a manner similar to that described above for the internal nodes. The resulting equations are:
\[
T_{m2}^* = \left[ 1 - \frac{\left(k_{m2} + k_{m3}\right) \Delta t}{\rho_{m2} c_{pm2} (\Delta x)^2} - \frac{h^*_2 \Delta t}{\rho_{m2} c_{pm2}} \right] T_m + \frac{\left(k_{m2} + k_{m3}\right) \Delta t}{\rho_{m2} c_{pm2} (\Delta x)^2} T_m3 + \frac{h^*_2 \Delta t}{\rho_{m2} c_{pm2}} T_c \quad \text{III.10}
\]

for node 2 and
\[
T_{mN}^* = \left[ 1 - \frac{\left(k_{mN-1} + k_{mN}\right) \Delta t}{\rho_{mN} c_{pmN} (\Delta x)^2} - \frac{h^*_N \Delta t}{\rho_{mN} c_{pmN}} \right] T_m + \frac{2\Delta t \sigma_e T_m^3}{\rho_{mN} c_{pmN} \Delta x} - \frac{2\Delta t q_{cw} c_p G_h}{\rho_{mN} c_{pmN} \Delta x H_r} \right] T_{mN} + \frac{\left(k_{mN-1} + k_{mN}\right) \Delta t}{\rho_{mN} c_{pmN} \Delta t} T_{mN-1} + \frac{h^*_N \Delta t}{\rho_{mN} c_{pmN}} T_c \quad \text{III.11}
\]

for node N. In this expression \(c_p\) is the specific heat at constant pressure of the boundary layer.

An energy balance can also be written at each node for the coolant gas. These coolant gas energy equations are then solved for the new coolant gas temperature using the same procedures as described previously for the metal matrix temperatures. The resulting equations are:
\[ T_c' = \left[ 1 - \frac{(k_{c2} + k_{c3}) \Delta t}{\rho_{c2} c_{p_{c2}} (\Delta x)^2} \right] T_{c2} - \frac{h \Delta t}{\rho_{c2} c_{p_{c2}} \phi} - \frac{2 \Delta t u_2}{\Delta x} \] 

\[ T_{c1} = \left[ \frac{(k_{c2} + k_{c3}) \Delta t}{\rho_{c2} c_{p_{c2}} (\Delta x)^2} \right] T_{c2} + \left[ \frac{2 \Delta t \rho_{c1} c_{p_{c1}} u_1}{\rho_{c2} c_{p_{c2}} \Delta x} \right] T_{c1} \]

\[ + \frac{p' - p_2}{\rho_{c2} c_{p_{c2}}} - \left[ \frac{\Delta H \Delta t R_0}{\rho_{c2} c_{p_{c2}}} \right] e^{-\frac{E}{RT_{m2}}} \]

III.12

for node 2

\[ T_{c_n} = \left[ 1 - \frac{(k_{c_{n-1}} + 2k_{c_n} + k_{c_{n+1}}) \Delta t}{2 \rho_{c_n} c_{p_{c_n}} (\Delta x)^2} \right] T_{c_n} + \left[ \frac{\Delta t u_n}{\Delta x} \right] T_{c_n} + \left[ \frac{(k_{c_n} + k_{c_{n+1}}) \Delta t}{2 \rho_{c_n} c_{p_{c_n}} (\Delta x)^2} \right] + \left[ \frac{\rho_{c_{n-1}} c_{p_{c_{n-1}}} u_{n-1} \Delta t}{\rho_{c_n} c_{p_{c_n}} \Delta t} \right] \]

\[ T_{c_{n-1}} + \left[ \frac{(k_{c_n} + k_{c_{n+1}}) \Delta t}{2 \rho_{c_n} c_{p_{c_n}} (\Delta x)^2} \right] T_{c_{n+1}} + \left[ \frac{h_n \Delta t}{\rho_{c_n} c_{p_{c_n}} \phi} \right] \]

III.13

for nodes 2 < n < N, and
for node N. By boundary condition II.20, the coolant gas temperature at node 1 is defined as:

$$T_{c1} = T_{cs}$$  \text{III.15}

The above equations can be used to calculate metal matrix and coolant gas temperatures as a function of time provided the time step is small enough to prevent instabilities. For example, to ensure that the solution to the internal matrix energy equation III.9 is stable, all terms must be equal to or greater than zero. Therefore:

$$1 - \left( \frac{k_{m_{n-1}} + k_{c_{N}}}{{\rho}c_{N}c_{p_{c_N}}(\Delta x)^2} \right) \frac{\Delta t}{2} \left( \frac{h_{N}}{\rho_{c_N}c_{p_{c_N}}} \right) \frac{\Delta t}{\Delta x} + \frac{2\Delta t}{\Delta x} \frac{u_{N}}{c_{p_{c_N}}} > 0$$  \text{III.16}
Solving for the time step, $\Delta t$:

$$
\Delta t \leq \frac{\rho_{m_n} c_{Pm_n}}{(k_{m_n-1} + 2k_{m_n} + k_{m_n+1})} + \frac{h_n}{2(\Delta x)^2} \quad \text{III.17}
$$

By using a time step less than or equal to the value given by the righthand side of equation III.17, a stable solution will always result.

Expressions similar to equation III.17 exist for each of the matrix and coolant temperature equations previously developed. These equations are:

$$
\Delta t \leq \frac{\rho_{m_2} c_{Pm_2}}{(k_{m_2} + k_{m_3})} + h_2 \quad \text{III.18}
$$

for equation III.10,

$$
\Delta t \leq \frac{\rho_{m_N} c_{Pm_N}}{(k_{m_N-1} + k_{m_N})} + h_N + \frac{2\sigma_{T_m} T_{m_N}^3}{\Delta x} + \frac{2q_{cw} c_p g_h}{\Delta x H_r} \quad \text{III.19}
$$

for equation III.11,

$$
\Delta t \leq \frac{1}{\rho_{c_2} c_{Pc_2}} + \frac{h_2}{\rho_{c_2} c_{Pc_2}} + \frac{2U_2}{\Delta x} \quad \text{III.20}
$$
for equation III.12,

\[
\Delta t \leq \frac{1}{2\rho c_n c_{pcn} (\Delta x)^2} \left( \frac{h_n^\prime}{\rho c_n c_{pcn} \phi} + \frac{u_n}{\Delta x} \right)
\]

III.21

for equation III.13, and

\[
\Delta t \leq \frac{1}{\rho c_n c_{pcn} (\Delta x)^2} \left( \frac{h_n^\prime}{\rho c_n c_{pcn} \phi} + \frac{2u_n}{\Delta x} \right)
\]

III.22

for equation III.14. Equation III.15 is not a function of time; therefore, no stability criterion exists for this equation.

The volumetric heat transfer coefficient \( h_n^\prime \), appearing in the above equations is defined by Ossin and Cawthon as:

\[
h_n^\prime = \frac{mc_{pcn}}{\Delta x} \left[ A \left( \frac{\Delta x m_{pcn}}{k_{mn}} \right) + B \left( \frac{\Delta x m_{pcn}}{k_{mn}} \right)^2 \right]
\]

III.23

where A and B are experimentally determined constants which are functions of the coolant gas, matrix material, and matrix porosity.

The coolant mass flow rate and pressure distribution are determined by integrating equation II.3. Assuming that the coolant gas obeys the perfect gas law, equation II.3 becomes:
\[- \frac{dp}{dx} = \frac{m\mu}{\Gamma} \left[ \frac{RT_c}{p} \right] + \beta m^2 \left[ \frac{RT_c}{p} \right] \]  

III.24

Integrating from \( x = 0 \) to \( x_n \):

\[ \quad p_n^2 - p_0^2 = \frac{mR}{\Gamma} \int_0^{x_n} \mu T_c \, dx + \beta m^2 R \int_0^{x_n} T_c \, dx \]  

III.25

If it is assumed that the viscosity, \( \mu \), is a function of temperature to some power:

\[ \mu = \mu^* \left[ \frac{T_c}{T_c^*} \right]^M \]  

III.26

where

\[ \mu^* = \text{reference viscosity at temperature } T_c^* \]

then equation III.25 becomes

\[ \quad p_n^2 - p_0^2 = \frac{2}{\Gamma} \cdot mR \mu^* T_c^* \left[ \frac{T_c^*}{T_c} \right]^M \int_0^{x_n} T_c \, dx + 2 \beta m^2 R \int_0^{x_n} T_c \, dx \]  

III.27

The two integrals in the last expression are approximated by the trapezoidal rule as:

\[ \int_0^{x_n} T_c \, dx = \frac{\Delta x}{2} \left[ T_{c_n} + \sum_{k=3}^{n-1} 2T_{c_k} + T_{c_2} \right] \]  

III.28

and

\[ \int_0^{x_n} T_c^{M+1} \, dx = \frac{\Delta x}{2} \left[ T_{c_n}^{M+1} + \sum_{k=3}^{n-1} 2T_{c_k}^{M+1} + T_{c_2}^{M+1} \right] \]  

III.29
Substituting equations III.28 and III.29 into equation III.27 gives:

\[
\begin{align*}
\frac{p_2^2 - p_n^2}{mR} &= \left( \frac{\mu^{*}T_c*_{c}^{-M}}{\Gamma} \right) \left[ T_{c_n} + \sum_{k=3}^{n-1} 2T_{c_k} + T_{c_2} \right] \\
&\quad + \frac{M+1}{T_{c_2}} + \delta m \left[ T_{c_n} + \sum_{k=3}^{n-1} 2T_{c_k} + T_{c_2} \right]
\end{align*}
\]

This results in equation III.30:

\[
\text{III.30}
\]

Since \( P_2 \) and \( P_N \) are known from the boundary conditions, they can be substituted into equation III.30 which can then be solved for \( m \).

\[
m = \frac{-BB + \sqrt{(BB)^2 - 4(AA)(CC)}}{2(AA)}
\]

\[
BB = \frac{R\Delta x\mu^{*}T_c*_{c}^{-M}}{\Gamma} \left[ T_{c_n} + \sum_{k=3}^{n-1} 2T_{c_k} + T_{c_2} \right]
\]

\[
AA = R\Delta x\delta \left[ T_{c_n} + \sum_{k=3}^{n-1} 2T_{c_k} + T_{c_2} \right]
\]

\[
CC = P_2^2 - p_N^2
\]

This results in equation III.31:

\[
\text{III.31}
\]

Once the coolant mass flow rate is known, the pressure distribution can be found from equation III.30 since \( P_n \) is the only unknown in the equation.
The equations presented in this chapter can now be used to obtain transient solutions to the problem of flow through a porous matrix. The equations can be solved by hand calculations; however, the time steps required for stability are small, and the time required to generate the solution to any practical problem would be unreasonably long. By incorporating the finite difference equations in a digital computer program, problem solutions can be generated quickly and efficiently. Such a computer program is described in the next chapter.
IV. COMPUTER MODEL AND APPLICATIONS

Computer Model

The computer model, as it currently exists, has several limitations which are worth noting. The program is restricted to problems where the initial coolant gas temperature is equal to the initial matrix temperature. The temperature is also considered constant through the system at time zero. The coolant gas supply pressure is maintained at its initial value throughout the run, while the external pressure at $x = L$ is allowed to vary with time. A simplified flow chart of the program is shown in Figure 4.

The first section of the program consists of statements for inputting data. The input data consist of metal matrix properties, coolant gas properties, boundary and initial conditions, matrix dimensions, and program control constants. The gas and matrix thermo-physical properties are input in table form as functions of temperature. The convective heating parameters at $x = L$ are input as functions of time. Data are extracted from table inputs by means of a linear interpolation subroutine.

After all the necessary data have been input to the program, the program initializes the matrix and coolant gas temperatures at each node. The initial mass flow rate and initial pressure distribution are determined. Initial values of volumetric heat transfer coefficient, blockage correction, gas density, and gas velocity are also computed.
Fig. 4. Program Flow Chart
at this time. The program then prints the initial values of cold wall heat flux, recovery enthalpy, external and reservoir pressure, reservoir temperature, and mass flow rate. The initial pressure at each node is also printed.

Following the initial printout, material properties are determined for the coolant gas and matrix at each node. Using equations developed in Chapter III, the maximum time step for each node is determined. The smallest of these time steps is chosen as the time step to be used so that stability is achieved at each node. If this time step would cause the program to pass a time designated for output, the time step is reduced so that temperatures will be computed at the desired print time.

Having determined the proper time step, the external flow parameters are determined at the new time from the input data. The mass flow rate and pressure distribution can then be calculated at the new time. With these, the volumetric heat transfer coefficients, coolant densities, and coolant velocities are computed for each node. The blockage correction is also computed at this time.

All the parameters needed to solve the matrix and coolant gas energy equations for new temperatures are now available. Matrix temperatures are calculated first followed by the coolant temperatures. If the time is equal to a print time, the program will print the values of time, cold wall heat flux, recovery enthalpy, external and reservoir pressures, reservoir temperature, and mass flow rate. The matrix and coolant gas temperature distributions and the
pressure distribution are printed as functions of distance from the upstream matrix surface.

The program will next compute the coolant gas and matrix properties that are functions of temperature for use in the next loop through the program. It then returns to compute the next time step or stops if the problem stop time has been reached.

A source listing and sample program output are contained in the Appendix to this report.

Model Validity

As with any new computer model, there is always some doubt as to whether or not the model is valid. In cases where no experimental data exist, the model's validity may be assessed based on its formulation. If it is based on equations that have been rigorously derived and validated by past investigators, then the confidence level in the new model is usually high. The differential equations described in Chapter II have been developed in such a manner. The basic equations are, for the most part, identical to those used by several past investigators. The new terms which have been added to generalize the equations are based on experimentally verified, state-of-the-art work published by individuals recognized in the field of transpiration cooling. Similarly, the finite difference equations have been derived using standard techniques which are well documented in the literature\textsuperscript{19}. These facts all lend credibility to the model.
The ultimate test of any analytical solution to a physical problem is the comparison of the analytical solution to experimental data. Fortunately, a small sample of experimental data is available for comparison with computer predictions. The experiments of interest are reported by Kinnaird, Cawthon, and Joyner. They involved the exposure of both cooled and passive porous tungsten specimens to the environment created by the Avco 10 megawatt air arc facility. Both nitrogen and ammonia were used as coolant gases.

Pressure probe and calorimeter measurements indicated that the model stagnation point heat flux in the 10 megawatt arc was 3500 BTU/ft²–sec. The total enthalpy of the arc jet was 4000 BTU/lb, and the model stagnation pressure was 11 atmospheres. All models were configured to give essentially one-dimensional coolant flow and heat transfer.

The test specimens employed in the 10 megawatt tests were constructed of 70 percent dense sintered tungsten. Pretest calibrations indicated a matrix permeability of $1.55 \times 10^{-13}$ ft² and an inertial resistance coefficient of $1.93 \times 10^8$ ft⁻¹. The empirical heat transfer coefficients were obtained from the work of Ossin and Cawthon.

Figure 5 shows the computer prediction for the nitrogen test along with the experimental data. Surface temperature was the only measured data obtained during the test of interest to this study. Surface temperatures were determined by means of an optical pyrometer positioned to view the model stagnation point. Due to the inherent errors of optical pyrometry and the errors associated with the data...
Fig. 5. Nitrogen Test Temperatures
acquisition, a large experimental error of approximately $\pm 10$ percent exists at each data point. This $\pm 10$ percent also includes the errors resulting from the fact that the emissivity of the surface as a function of temperature is not accurately known. With these observations in mind, the agreement between the experimental data and the computer simulation is acceptable since the prediction falls within the experimental error bands.

Figure 6 shows the correlation between the ammonia test data and the computer model prediction. Again, the agreement is acceptable when the errors associated with the test data are considered. In addition to the errors described for the nitrogen test, the ammonia test specimen holder had a defective seal which ruptured at about 2.3 seconds\(^2\). This effectively stopped the coolant flow through the matrix and precipitated a thermal stress failure at 2.5 seconds. The loss of coolant flow as the seal began to leak is evident from the sharp rise in temperature beginning at 2.1 seconds. Prior to this time, the model prediction is acceptable.

Based on the general agreement of the model predictions with the sparse experimental data available, it is concluded that the model should be considered valid until such time as additional experimental data may prove it otherwise.

As a point of interest, the matrix and coolant temperature distributions for the ammonia test are shown at 2.0 seconds into the run in Figure 7. The pressure distribution at this time is shown in Figure 8. To determine the effect of the ammonia dissociation on temperature and pressure, the program was rerun for
Fig. 6. Ammonia Test Temperatures
Figure 7. Ammonia Test Temperatures at 2.0 Seconds
Fig. 8. Ammonia Test Pressures at 2.0 Seconds
the ammonia test conditions but with the heat of dissociation set to zero. The results of this run are also shown in Figure 7. Note that while the temperatures for no dissociation are higher, they are not substantially higher. The reason for this is that the flow rate is high enough so that the dissociation reaction does not have enough time to occur to the extent where temperatures would be substantially lowered. The temperature difference between the dissociated and non-dissociated runs would be much more evident if the flow rate was lower and the dwell time of the ammonia in the matrix was longer. This has been verified in the experiments of Gorton.

Since the change in temperature was small when the heat of dissociation was set to zero, the change in the pressure distribution was also small since the pressure distribution is proportional to the temperature distribution. For this reason, the zero dissociation pressure distribution is not shown in Figure 8.

Model Applications

The computer program described above may now be used to predict the transient thermal response of a porous matrix with some degree of confidence. It will be particularly useful in predicting the thermal response of transpiration cooled nose tips which experience laminar stagnation point heating. It can be used to perform trade studies where both inert and reactive gases are candidate coolants. With slight modifications, it can be used to predict the thermal response of transpiration cooled missile control surfaces and rocket nozzles.
The program should be found extremely useful in designing further tests of transpiration cooled specimens. By performing parametric studies with coolant and coolant supply pressure as variables, the experimenter can now design his test conditions to obtain the desired temperature response in any convective heating facility, and the program will give results which are more meaningful than those obtained from past steady state models.

Thermal stress analysis of porous cooled matrices will now be enhanced since the maximum thermal stress usually occurs before steady state conditions are reached.

In general, the design of transpiration cooled matrices will be made easier and more reliable by implementation of this computer program.
V. CONCLUSIONS AND RECOMMENDATIONS

The objectives established in Chapter I have been met in that a computer program has been developed to perform transient thermal analysis of a convectively heated porous metal matrix being cooled by a transpiring reactive gas. The model has been verified by comparison with the past work of other authors and by comparison with experimental data. The program is now developed to the point where it can be used with some degree of confidence to predict the performance of gas-cooled porous matrices being used for flight applications. The program should also be very useful in designing future ground test programs.

Since the computer program is new, there are many areas in which it could be improved and generalized. The most significant of these are included here as recommendations for future work:

1. The transient continuity and momentum equations should be incorporated in the program to determine their effect on the final solution.

2. A variable supply pressure and temperature capability should be provided for.

3. Coolant gas properties are currently taken as independent of the amount of dissociation. A scheme for determining dissociation dependent properties should be incorporated.
4. The program should be generalized to permit the analysis of liquid as well as gaseous coolants. Such two-phase models are documented in the literature\textsuperscript{24}.

5. Provisions for analyzing off-stagnation point turbulent flow should be added.

6. The initial conditions for the matrix and coolant energy equations should be generalized to permit nonuniform initial temperature distributions.
APPENDIX

SOURCE LISTING AND SAMPLE OUTPUT
ONE-DIMENSIONAL TRANSPERSION COOLING COMPUTER PROGRAM

DIMENSION P(I),PP(I),TM(I),TMP(I),TC(I),TCP(I),TITLE(10)
DIMENSION TW(I),CM(I),TWZ(I),EM(I),CP(I),TW3(I)
DIMENSION T5(I),CK(I),CP(I),TIM(15),QCW(15),HR(15),PLEX(15)
DIMENSION XM(11),CPM(I),XK(11),CPC(I),HPH(11),IC(11),HMOD(11)
SIGMA=4.7543E-13

INPUT SECTION

READ(*1) TITLE
FORMAT(21A4)
READ(*2) GAMMA,RETA,XL,N
FORMAT(3E15.6,I12)
READ(*4) M,J,K
FORMAT(3I2)
READ(*3) (TW(I),CM(I),I=1,M)
FORMAT(4E15.6)
READ(*3) (TW(I),CM(I),I=1,J)
READ(*3) (TW(I),CP(I),I=1,K)
READ(*3) A,R,PHI,H0
READ(*5) MM,KK
FORMAT(3I2)
READ(*3) (TW(I),CK(I),I=1,MM)
READ(*3) (TWS(I),CP(I),I=1,MM)
READ(*5) TINT,P(I),COE,HR,E,H,VISCR,REFT,VEXP
READ(*3) CPBL,TS,P,R,WMU,WMU,VEXP,DIS,ETA
READ(*4) MM
READ(*3) (TIM(I),HCW(I),HR(I),PLEX(I),I=1,MM)
READ(*3) TIMF,TIMP,TIMC,KMUM
NN=NN+1
TIME=0.0

COMPUTE INITIAL PRESSURE DISTRIBUTION AND MASS FLOW RATE

DO 10 I=1,NN
TM(I)=TINT
TM(I)=TINT
TCP(I)=TINT
TC(I)=TINT
CONTINUE
P(N+1)=PLEX(1)
VISCR=VISCR*(TINT/REFT)**VEXP
C=(P(I)**2,-P(N+1)**2)/2.6767E+05
AA=H*TINT*X*X*ETA
Hd=H*TINT*X*X*VISCI/GAMMA
XM(0)=(-BH+SQR(H9**2,-4.*AA*C))/(2.*AA)
DX=X/X/(N-1)**P2)
DO 11 I=3,N
P(I)=SQR(P(I-1)**2,-2.*XM*UT**R*TINT*DX*(VISCI/GAMMA+XMDOT*RETA)*I)
11 CONTINUE
DO 12 I=1,NN
PP(I)=P(I)
12 CONTINUE
Q=QCW(1)

HHEC=HM(I)
TIME=0.0
CALL CLINE(KK,TC(I),TW(1),CP(I),CP(I))
CALL CLINE(M,TINT,TW(1),CM(1),XKM(1))
AD=DX*XMDOT*CPC(1)/XKM(1)
HPR(I)=XMDOT*CPC(1)/DX*(AD+H*AH**2)
DO 70 I=1,NN
HPR(I)=HPR(1)
70 CONTINUE
ALFA=EXP(-ER/TINT)
ETA1=ETA*(1+ALFA*((WMU/WMD)*WEXP-1))
GH=(1/(1+ETA1*XMDOT*HREC/W))
DO 71 I=1,NN
RHOC(I)=P(I)/(R*TINT)*144*
UC(I)=XMDOT/(RHOC(I)*PHI)
71 CONTINUE
GO TO 1000
C
C DETERMINE TIME STEP
C
1001 D1=RHUM*CPM(2)/((XKM(2)+XKM(3))/DX**2+HPR(2))
DO 30 J=3,N
D1N=RHUM*CPM(I)/((XKM(I-1)+2*XKM(I)+XKM(I+1))/(2*DX**2)+HPR(I))
IF(D1NLT.DT) DT=D1N
30 CONTINUE
D1N=RHUM*CPM(N+1)/((XKM(N)+XKM(N+1))/DX**2+HPR(N+1)+2*SIGMA*EMM1
1+STM(N+1)*3/6.12+2*CPBL*GH/(DX*HREC))
IF(D1NLT.DT) DT=D1N
DO 31 J=1,N
D1N=1/((XKC(I-1)+2*XKC(I)+XKC(I+1))/(2*RHOC(I)*CPC(I)*DX**2)+HPR(I)/PHI*RHOC(I)*CPC(I)+UC(I)/DX))
IF(D1NLT.DT) DT=D1N
31 CONTINUE
D1N=1/((XKC(N+1)+XKC(N))/RHOC(N+1)*CPC(N+1)*DX**2)+HPR(N+1)/(PH
1+RHC(N+1)*CPC(N+1)+2*UC(N+1)/DX)
IF(D1NLT.DT) DT=D1N
IF(TIMC+DT,DT) GO TO 60
IF(TIME+DT,GT,PRN1) DT=PRN1-TIME
TIME=TIME+DT
GO TO 61
60 WHITE(*,63) DT
63 FORMAT(2OH TIME STEP TOO SMALL,E15.6)
GO TO 9000
61 CALL CLINE(MMM,TIME,TM(1),WCW(1),0)
CALL CLINE(MMM,TIME,TM(1),HR(I),HREC)
CALL CLINE(MMM,TIME,TM(1),PL(I),P(N+1))
C
C COMPUTE NEW MASS FLOW RATE
C
C C=-(P(2)**2,-P(N+1)**2)*6.67E+05
BB=R*DX*VISC*HFT**(-VEXP)/GAMMA
SUMT=TC(N+1)**(VEXP+1)+TC(2)**(VEXP+1)
DO 32 I=3,N
SUM=SUM+2*TC(I)**(VEXP+1)
32 CONTINUE
H3=RH*SUMT
SUM1=TC(N+1)+TC(2)
DO 33 I=3,N
SUM1=SUM1+2*TC(I)
33 CONTINUE
AA=R*DX*BETA*SUMT1
C COMPUTE NEW PRESSURE DISTRIBUTION

C

C COEFA=XMDOt*R*DX
COEFB=VISCR*REFT**(-VEXP)/GAMMA
TSUM=0.
TSUM1=U..
DO 34 I=3,N
TSUM=TSUM+TC(I)**(VEXP+1)+TC(I-1)**(VEXP+1)
TSUM1=TSUM1+TC(I)*TC(I-1)
P(I)=SORT(P(2)**2.-COEFA*(COEFB*TSUM+BETA*XMDOt*TSUM1)*1.4988E-06)
CONTINUE

C

C CALCULATE HEAT TRANSFER COEFFICIENT

C DO 35 J=2,NN
A3=DX*XMDOt*CPC(I)/XKM(I)
HPR(I)=XMDOt*CPC(I)/UX*(A*B+B*A*2)
CONTINUE

C

C COMPUTE BOUNDARY LAYER BLOCKAGE TERM

C ALFA=EXP(-ER/TM(N+1))
FTA1=ETA*(1.*ALFA*((WMU/WMD)**WEXP-1))
G=-((1./((1.+ETA*XMDOt*HREC/U))
CONTINUE

C

C COMPUTE FLUID DENSITY AND VELOCITY

C DO 73 I=1,NN
RMOC(I)=M(I)/(H*TC(I))**144*
UC(I)=XMDOt/(RMOC(I)*H)
CONTINUE

C

C SOLVE SOLID ENERGY EQUATION

C TM(2)=(1.-XKM(2)+XKM(3))*DT/(RHOM*CPTM(2)*DX**2.)-HPR(2)*DT/(RHM*CPTM(2))
2PR(2)*DT/(RHM*CPTM(2)*DX**2.)*TMP(3)+H
2PR(2)*DT/(RHM*CPTM(2)))*TCP(2)
DO 36 J=3,N
X=DT*(XKM(I-1)+2.*XKM(I)+XKM(I+1))/2.*RHM*CPTM(I)*DX**2.
Y=HPR(I)*DT/(RHM*CPTM(I))
TM(I)=(1.-X-Y)*TMP(I)+DT*(XKM(I)+XKM(I-1))/2.*RHM*CPTM(I)*DX**2.)*TMP(I-1)+Y*
TCP(I)
CONTINUE

C

C SOLVE FLUID ENERGY EQUATION

C X=(XKC(2)+XKC(3))*DT/(RHOUC(2)*CPC(2)*DX**2.)
Y=HPR(2)*DT/(RHOUC(2)+CPC(2)*H)
TC(I)=(1.-X-Y-2.*DT*UC(2)/DX)*TCP(2)+XTCP(3)
1*TMP(2)+2.*DT*HUC(2)*CPC(1)*UC1(1)/RHOUC(2)*CPC(2)*DX)*TCP(1)+
2P(2)*HPR(2)/((RHOUC(2)*CPC(2))**1.8509-HDIS*DT*RHO/(RHOUC(2)*CPC(2)))*E
3X*TMP(2))
DO 40 I=3,N
X=(XKC(I-1)+2.*XKC(I)+XKC(I+1)) *UT/(2.*RHOC(I) *CPC(I) *DX**2.)
Y=PHI(I) *DT/(PHI*I *RHOC(I) *CPC(I))
V=(XKC(I)+XKC(I-1)) *UT/(2.*RHOC(I) *CPC(I) *DX**2.)
VV=NERU(I) *CPC(I-1) *DT/(RHOC(I)*CPC(I))
VVV=(XKC(I+1)+XKC(I+2)) *UT/(2.*RHOC(I) *CPC(I) *DX**2.)
TC(I)=[1.+X-Y-OT*UC(I)/DX)*TCP(I)+(V+VV)*TCP(I-1)+VVV*TCP(I+1)]/Y**4
1MP(I)=H015*S*RT*(RHOC(I)*CPC(I)) *EXP(-ER/TMP(I))^2*(I-P(I))/RHOC(I)*CPC(I)
DO 40 1=J,N
V=<XKC(I)+XKC(I-1)) *UT/(2.*RHOC(I) *CPC(I) *DX)**2.)
VV=RHOC(I-1) *CPC(I-1) *UT/(RHOC(I)*CPC(I))
VVV=(XKC(I)+XKC(I+1)) *UT/(2.*RHOC(I) *CPC(I) *DX)**2.)
TC(N+1)=<1.-X-Y-2.OT*UC(N+1)/OX)*TCP(N)+X*TCP(N)+Y*TMP(N+1)-V+
1P(N+1)-P(N+1))/RHO(N+1) *CPC(N+1) *UX(N+1) /RHOC(N+1) *CPC(N+1) *DX)**2.*18509+V*TCP(N)
IF(TIME.EQ.TPRNT) GO TO 1000
IF(TIME.EQ.TMPF) GO TO 1000
GO TO 1002
C
PRIN SECTION
1000 WRITE(b,54)
54 FORMAT(1HI)
WRITE(b,1) TITLE
WRITE(b,55)
55 FORMAT(/5X,4HTIME,12X,3HQCW,11X,4HHREC,8X,12HEXT PRESSURE,3X,12HI
INT PRESSURE,4X,8HRES TEMP/5X,5H(SEC),6X,13H(HTU/FT2-SEC),4X,8(HTU
/LB),9X,5H(Psi),10X,5H(Psi),8X,7H(DEG R/)//)
WRITE(b,50) TIME*D,HREC,P(N+1),P(1),TC(1)
50 FORMAT(6EL1.6)
WRITE(b,56)
56 FORMAT(/5X,4HMDOT/2X,10H(LB/FT2-S)///)
WRITE(b,50) XMDOT
WRITE(b,53)
53 FORMAT(/5X,4X,HMATIX TEMP,5X,10HFLUID TEMP,5X,8HPRESSU
1RE,2X,5H(FT),9X,7H(DEG R),8X,7H(DFG R),9X,5H(Psi)//)
X=0.0
DO 51 I=2,N
WRITE(b,52) X,TM(I),TC(I),P(I)
52 FORMAT(4EL1.6)
X=X+DX
51 CONTINUE
TP9HN=TIME+TIMPR
C
Determine properties for next time step
1002 DO 20 I=2,N
CALL CLINE(M,TM(I),TW(I),CM(I),XM(I))
CALL CLINE(K,TM(I),TW2(I),CP(I),CPM(I))
CALL CLINE(MT,TMI,TW3(I),CK(I),XXC(I))
CALL CLINE(KK,TC(I),TW5(I),CPC(I))
20 CONTINUE
CALL CLINE(M,TM(N+1),TW1(I),E[M(I),EMMIS)
DO 74 I=1,N
PP(I)=P(I)
TMP(I)=TM(I)
TCP(I)=TC(I)
74 CONTINUE
IF(TIME.LT.TIMF) GO TO 1001
9000 STOP
END
### Sample Ammonia Run

<table>
<thead>
<tr>
<th>TIME (SEC)</th>
<th>QCW (BTU/FT²-SEC)</th>
<th>HREC (BTU/LB)</th>
<th>EXT PRESSURE (PSI)</th>
<th>INT PRESSURE (PSI)</th>
<th>RES TEMP (DEG H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.350000E+04</td>
<td>0.400000E+04</td>
<td>0.170000E+03</td>
<td>0.150000E+04</td>
<td>0.600000E+03</td>
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</table>

**MDOT (LB/FT²-SEC)**

<table>
<thead>
<tr>
<th>X (FT)</th>
<th>MATRIX TEMP (DEG H)</th>
<th>FLUID TEMP (DEG R)</th>
<th>PRESSURE (PSI)</th>
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<tbody>
<tr>
<td>0.0</td>
<td>0.600000E+03</td>
<td>0.6000E+03</td>
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<tr>
<td>0.520750E-02</td>
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<td>0.6000E+03</td>
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<tr>
<td>0.104150E-01</td>
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<td>0.6000E+03</td>
<td>0.150000E+04</td>
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<td>0.150000E+04</td>
</tr>
<tr>
<td>0.208300E-01</td>
<td>0.600000E+03</td>
<td>0.6000E+03</td>
<td>0.150000E+04</td>
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### SAMPLE AMMONIA RUN

<table>
<thead>
<tr>
<th>TIME (SEC)</th>
<th>QCw (BTU/FT(^2)-SEC)</th>
<th>MREC (BTU/Lb)</th>
<th>EXT PRESSURE (PSI)</th>
<th>INT PRESSURE (PSI)</th>
<th>RES TEMP (DEG W)</th>
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<tbody>
<tr>
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<td>0.350000E+04</td>
<td>0.400000E+04</td>
<td>0.172000E+03</td>
<td>0.150000E+04</td>
<td>0.600000E+03</td>
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<td>MDOT (LB/FT(^2)-S)</td>
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<td></td>
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<tr>
<td>0.160737E+01</td>
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<table>
<thead>
<tr>
<th>X (FT)</th>
<th>MATRIX TEMP (DEG W)</th>
<th>FLUID TEMP (DEG R)</th>
<th>PRESSURE (PSI)</th>
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<tbody>
<tr>
<td>0.0</td>
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<td>0.60506E+03</td>
<td>0.13160E+04</td>
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<td>0.74784E+03</td>
<td>0.62001E+03</td>
<td>0.10983E+04</td>
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<td>0.156225E-01</td>
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<td>0.67036E+03</td>
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<td>0.208300E-01</td>
<td>0.18739E+04</td>
<td>0.75295E+03</td>
<td>0.17200E+03</td>
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### SAMPLE AMMONIA RUN

<table>
<thead>
<tr>
<th>TIME (SEC)</th>
<th>QCw (BTU/FT(^2)-SEC)</th>
<th>MREC (BTU/Lb)</th>
<th>EXT PRESSURE (PSI)</th>
<th>INT PRESSURE (PSI)</th>
<th>RES TEMP (DEG W)</th>
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<tbody>
<tr>
<td>0.100000E+00</td>
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<td>0.400000E+04</td>
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<td>0.600000E+03</td>
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<td>MDOT (LR/FT(^2)-S)</td>
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<table>
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<tr>
<th>X (FT)</th>
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<th>FLUID TEMP (DEG R)</th>
<th>PRESSURE (PSI)</th>
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<tbody>
<tr>
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<td>0.150000E+04</td>
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<td>0.520750E-02</td>
<td>0.70314E+03</td>
<td>0.61376E+03</td>
<td>0.13228E+04</td>
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<td>0.104150E-01</td>
<td>0.87147E+03</td>
<td>0.64103E+03</td>
<td>0.11103E+04</td>
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<td>0.208300E-01</td>
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<td>0.81300E+03</td>
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<td>TIME (SEC)</td>
<td>QCW (BTU/FT²-SEC)</td>
<td>HREC (BTU/LB)</td>
<td>EXT PRESSURE (PSI)</td>
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<tr>
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<td>------------------</td>
<td>-------------</td>
<td>------------------</td>
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<tr>
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MDOT (LB/FT²-S)

0.153320E+01

<table>
<thead>
<tr>
<th>X (FT)</th>
<th>MATRIX TEMP (DEG H)</th>
<th>FLUID TEMP (DEG R)</th>
<th>PRESSURE (PSI)</th>
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<tbody>
<tr>
<td>0.1</td>
<td>0.720853E+03</td>
<td>0.607485E+03</td>
<td>0.150000E+04</td>
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<tr>
<td>0.2</td>
<td>0.781250E+03</td>
<td>0.525197E+03</td>
<td>0.132809E+04</td>
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<td>0.3</td>
<td>0.991588E+03</td>
<td>0.664134E+03</td>
<td>0.111455E+04</td>
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<td>0.4</td>
<td>0.143361E+04</td>
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<td>0.024377E+03</td>
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<tr>
<td>0.5</td>
<td>0.232356E+04</td>
<td>0.860610E+03</td>
<td>0.172000E+03</td>
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</table>

SAMPLE AMMONIA RUN

<table>
<thead>
<tr>
<th>TIME (SEC)</th>
<th>QCW (BTU/FT²-SEC)</th>
<th>HREC (BTU/LA)</th>
<th>EXT PRESSURE (PSI)</th>
<th>INT PRESSURE (PSI)</th>
<th>RES TEMP (DEG H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250000E+00</td>
<td>0.350000E+04</td>
<td>0.400000E+04</td>
<td>0.172000E+03</td>
<td>0.150000E+04</td>
<td>0.600000E+03</td>
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</tbody>
</table>

MDOT (LB/FT²-S)

0.150011E+01

<table>
<thead>
<tr>
<th>X (FT)</th>
<th>MATRIX TEMP (DEG H)</th>
<th>FLUID TEMP (DEG R)</th>
<th>PRESSURE (PSI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.795553E+03</td>
<td>0.612047E+03</td>
<td>0.150000E+04</td>
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<td>0.2</td>
<td>0.866852E+03</td>
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<td>0.132845E+04</td>
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<tr>
<td>0.3</td>
<td>0.110428E+04</td>
<td>0.687703E+03</td>
<td>0.112663E+04</td>
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<tr>
<td>0.4</td>
<td>0.156225E+04</td>
<td>0.791864E+03</td>
<td>0.083551E+03</td>
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<td>0.5</td>
<td>0.208330E+04</td>
<td>0.913741E+03</td>
<td>0.172000E+03</td>
</tr>
</tbody>
</table>
FOOTNOTES


5 Archie Ossin and Don M. Cawthon, Evaluation of the CONAP Concept for Advanced ABM Nose Tips, OR 12,840 (Orlando, Fl.: Martin Marietta Aerospace, 1974).


8 Ossin and Cawthon, CONAP Concept.

9 Curry and Cox, "Heat and Mass Transfer."

10 Gorton, "Effect of Reactive Coolants."


14  Gorton, "Ammonia as a Reactive Transpiration Coolant."

15  Ibid.

16  Curry and Cox, "Heat and Mass Transfer."


18  Ossin and Cawthon, CONAP Concept.


20  Kinnaird, Cawthon, and Joyner, Jr., "Active Oxidation Protection."

21  Ossin and Cawthon, CONAP Concept.

22  Kinnaird, Cawthon, and Joyner, Jr., "Active Oxidation Protection."

23  Gorton, "Ammonia as a Reactive Transpiration Coolant."

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