Optimizing The Performance Of As-manufactured Grazing Incidence X-ray Telescopes Using Mosaic Detector Arrays

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OPTIMIZING THE PERFORMANCE OF AS-MANUFACTURED GRAZING INCIDENCE X-RAY TELESCOPES USING MOSAIC DETECTOR ARRAYS

by

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ABSTRACT

The field of X-ray astronomy is only forty (43) years old, and grazing incidence X-ray telescopes have only been conceived and designed for a little over fifty (50) years. The Wolter Type I design is particularly well suited for stellar astronomical telescopes (very small field-of-view). The first orbiting X-ray observatory, HEAO-1 was launched in 1977, a mere twenty-eight (28) years ago. Since that time large nested Wolter Type I X-ray telescopes have been designed, build, and launched by the European Space Agency (ROSAT) and NASA (the Chandra Observatory). Several smaller grazing incidence telescopes have been launched for making solar observations (SOHO, HESP, SXI). These grazing incidence designs tend to suffer from severe aberrations and at these very short wavelengths scattering effects from residual optical fabrication errors are another major source of image degradation. The fabrication of precision optical surfaces for grazing incidence X-ray telescopes thus poses a great technological challenge. Both the residual “figure” errors and the residual microroughness or “finish” of the manufactured mirrors must be precisely measured, and the image degradation due to these fabrication errors must be accurately modeled in order to predict the final optical performance of the as-manufactured telescope. The fabrication process thus consists of a series of polishing and testing cycles with the predictions from the metrology data of each cycle indicating the strategy for the next polishing cycle. Most commercially available optical design and analysis software analyzes the image degradation effects of diffraction and aberrations, but does not adequately model the image degradation effects of surface scatter or the effects of state-of-the-art mosaic detectors.
The work presented in this dissertation is in support of the Solar X-ray Imager (SXI) program. We have developed a rigorous procedure by which to analyze detector effects in systems which exhibit severe field-dependent aberrations (conventional transfer function analysis is not applicable). Furthermore, we developed a technique to balance detector effects with geometrical aberrations, *during the design process*, for wide-field applications. We then included these detector effects in a complete systems engineering analysis (including the effects of diffraction, geometrical aberrations, surface scatter effects, the mirror manufacturer error budget tree, and detector effects) of image quality for the five SXI telescopes being fabricated for NOAA’s next generation GOES weather satellites. In addition we have re-optimized the remaining optical design parameters after the grazing incidence SXI mirrors have been imperfectly fabricated. This ability depends critically upon the adoption of an image quality criterion, or merit function, appropriate for the specific application. In particular, we discuss in detail how the focal plane position can be adjusted to optimize the optical performance of the telescope to best compensate for optical figure and/or finish errors resulting from the optical fabrication process. Our systems engineering analysis was then used to predict the increase in performance achieved by the re-optimization procedure. The image quality predictions are also compared with real X-ray test data from the SXI program to experimentally validate our system engineering analysis capability.
Dedicated to my family:

My parents Nelly and Ivan

My sister Julia
ACKNOWLEDGMENTS

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Through the years I learned many useful things, but the ones that are the most precious are the lessons of life. I had a long journey through time and space, coming from a distant country in the Eastern Europe, Bulgaria, to the US and achieving a goal of “reaching the stars”. But nothing can be compared to the stars that my parents “reached” – growing two girls throughout the long, poor and dark period of the communist regime, giving them an education in exchange of many sacrifices and restrictions and finally enjoying the success through the achieved goals of their children. I am extremely thankful to my best friends – my mom, dad and my sister, to their advices, encouragement and support in every step of my life. But most of all I am grateful to their unconditional love, which made me a better person and kept me going throughout the very difficult moments in my country and as a foreigner in the country that I call “the big land”- US.

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<td>Surface Autocovariance</td>
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<tr>
<td>ADPSF</td>
<td>Aperture Diffraction Point Spread Function</td>
</tr>
<tr>
<td>ALEXIS</td>
<td>Array of Low Energy X-ray Imaging Sensors</td>
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<tr>
<td>APSF</td>
<td>Aerial Point Spread Function</td>
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<tr>
<td>ASF</td>
<td>Angle Spread Function</td>
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<td>AUDPSF</td>
<td>Average Unregistered Detected Point Spread Function</td>
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<td>AXAF</td>
<td>Advanced X-ray Astrophysical Facility</td>
</tr>
<tr>
<td>B-K</td>
<td>Beckmann-Kirchoff</td>
</tr>
<tr>
<td>BRDF</td>
<td>Bi-directional Reflectance Distribution Function</td>
</tr>
<tr>
<td>CREOL</td>
<td>Center of Research and Education in Optics and Lasers</td>
</tr>
<tr>
<td>CTS</td>
<td>Circularity Test Stand</td>
</tr>
<tr>
<td>DPSF</td>
<td>Detected Point Spread Function</td>
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<tr>
<td>ESA</td>
<td>European Space Agency</td>
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<tr>
<td>EXOSAT</td>
<td>European X-ray Observatory Satellite</td>
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<tr>
<td>FOV</td>
<td>Field-of-view</td>
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<tr>
<td>GOES</td>
<td>Geostationary Operational Environmental Satellite</td>
</tr>
<tr>
<td>GPSF</td>
<td>Geometrical Point Spread Function</td>
</tr>
<tr>
<td>GSD</td>
<td>Geometric Spectral Density</td>
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<tr>
<td>GSFC</td>
<td>Goddard Space Flight Center</td>
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<tr>
<td>HEAO</td>
<td>High-Energy Astronomy Observatory</td>
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<tr>
<td>HPR</td>
<td>Half Power Radius</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>HPR$_{twa}$</td>
<td>Half Power Radius Field Weighted Average</td>
</tr>
<tr>
<td>HT#17</td>
<td>Harvey-Thompson Number Seventeen</td>
</tr>
<tr>
<td>MTF</td>
<td>Modulation Transfer Function</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
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<tr>
<td>NOAA</td>
<td>National Oceanic and Atmospheric Administration</td>
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<tr>
<td>NSRE</td>
<td>Number of Spatial Resolution Elements</td>
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<td>OFOV</td>
<td>Operational field-of-view</td>
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<tr>
<td>OSAC</td>
<td>Optical Surface Analysis Code</td>
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<td>OTF</td>
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<tr>
<td>PMI</td>
<td>Phase Measuring Interferometer</td>
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<td>PSD</td>
<td>Power Spectral Density</td>
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<tr>
<td>PSF</td>
<td>Point Spread Function</td>
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<tr>
<td>RDPSF</td>
<td>Registered Detected Point Spread Function</td>
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<tr>
<td>Rms</td>
<td>Root-mean-square</td>
</tr>
<tr>
<td>ROSAT</td>
<td>Roentgen Satellite</td>
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<tr>
<td>R-R</td>
<td>Rayleigh-Rice</td>
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<tr>
<td>RSEPSF</td>
<td>Residual Surface Error Point Spread Function</td>
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<tr>
<td>RXTE</td>
<td>Rossi X-ray Timing Explorer</td>
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<tr>
<td>SAS</td>
<td>Small Astronomy Satellite</td>
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<tr>
<td>Sco X-1</td>
<td>Constellation Scorpios</td>
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<tr>
<td>SN*</td>
<td>Serial Number* denotes the sequence of manufacturing (i.e. 002, 003, etc.)</td>
</tr>
<tr>
<td>SSPSF</td>
<td>Surface Scatter Point Spread Function</td>
</tr>
<tr>
<td>STF</td>
<td>Surface Transfer Function</td>
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</table>
SXI  Solar X-ray Imager
UDPSF  Unregistered Detected Point Spread Function
WS  Wolter-Swarzschild
1.0 INTRODUCTION

Electromagnetic radiation can take on many forms such as radio waves, microwaves, infrared, visible, ultraviolet, X-ray and gamma radiation. Due to the dual nature of electromagnetic radiation, we can talk of either wave or particle (photon) properties. The wavelength and photon energy are reciprocally related \( E = h \nu = \frac{hc}{\lambda} \). There are two basic types of sources of electromagnetic radiation; (i) stimulated or spontaneous emission of radiation at discrete wavelengths corresponding to atomic or molecular energy levels, and (ii) thermal radiation, produced by accelerating charges in material bodies under the conditions of thermal equilibrium in the absence of other external energy sources. The photoelectric effect provides convincing evidence that photons of light can transfer energy to electrons. Is the inverse process also possible? In other words, can all or part of the kinetic energy of a moving electron be converted into a photon? Indeed, the inverse photoelectric effect not only does occur, but it had been discovered (though not at all understood) prior to the theoretical work of Plank and Einstein.

1.1 X-rays

In 1895 a German physicist, Wilhelm Roentgen, made the classic observation that a highly penetrating radiation of unknown nature is produced when fast electrons impinge upon matter. These X-rays (so called because of their unknown nature) were soon found to travel in straight lines even through electric and magnetic fields, to mysteriously pass through opaque materials, to cause phosphorescent substances to glow, and to expose photographic plates.
It was soon learned that the faster the impinging electrons, the more penetrating the resulting X-rays. Also, the greater the number of electrons, the greater the intensity of the X-ray beam. The wave nature of X-rays was established in 1906 by Barkla, who was able to exhibit their polarization. Figure 1-1(a) is a diagram of a classic X-ray tube. A cathode supplies electrons by thermionic emission when heated by an adjacent filament through which an electric current is passed. The high potential difference, \( V \), maintained between the cathode and a metallic target accelerates the electrons. The face of the target is at an angle relative to the electron beam, and the X-rays that emerge from the target pass through the side of the tube.

Classical electromagnetic theory predicts the production of continuous broadband \textit{bremsstrahlung} (braking radiation) by electrons being de-accelerated when impinging upon a target. Figure 1-1(b) shows the X-ray spectra of a tungsten and molybdenum target at a 35 Kv accelerating potential.

![Diagram of a classical X-ray tube and X-ray spectra of tungsten and molybdenum.](image)

The curves in Figure 1-1(b) exhibit two distinct features not accountable in terms of electromagnetic theory: (i) the intense spikes in the X-ray spectrum for the molybdenum target is
a non-classical effect that is characteristic of the electron structures of the target atoms after having been disturbed by the bombarding electrons, and (ii) the *bremsstrahlung* X-rays produced at a given accelerating potential $V$ vary in wavelength, but none has a wavelength shorter than a certain value $\lambda_{\text{min}}$. Furthermore, for a particular $V$, $\lambda_{\text{min}}$ is the same for both the tungsten and the molybdenum targets.

The second observation is readily understood in terms of the quantum theory of radiation. Most of the electrons incident upon the target lose their kinetic energy gradually in numerous collisions, their energy going into heat (thus X-ray tube targets are usually made of high melting-point metals utilizing an efficient means of cooling). A few electrons, though, lose most or all of their energy in single collisions with target atoms; this is the energy that is evolved as X-rays. X-ray production, then, except for the spikes mentioned in observation (i) above, represents an inverse photoelectric effect. Instead of photon energy being transferred into electron kinetic energy, electron kinetic energy is being transferred into photon energy.

The speed or kinetic energy of the (atomic or molecular) particles making up a medium thus sets a limit on the wavelength (or energy) of the radiation emitted. The speed of the particles is also a measure of temperature. Very low temperatures (hundreds of degrees below zero Celsius) produce long wavelength radio waves and microwaves as shown in Figure 1-2. Cool bodies at ambient temperatures (about 30 degrees Celsius) produce infrared radiation, and very high temperatures (millions of degrees Celsius) produce X-rays. Note that visible light - the only radiation perceived by the human eye – consists of wavelengths a million times shorter than the typical radio wavelengths, and the wavelengths of X-rays range from hundreds to thousands of times shorter than those of visible wavelengths.
X-rays have the ability to knock electrons loose from atoms. Over the years these exceptional properties have made X-rays useful in many fields, such as medicine and research into the nature of the atom. Also, when the photons collide with electrons with high energy, a process called Compton Scattering can be observed, where the photons change from low energy to high energy photons. This type of scattering is significant around black holes, where matter is dense and has been heated to many millions of degrees. X-ray astronomy can reveal these hot spots in the universe - regions where particles have been energized or raised to high temperatures by gigantic explosions or intense gravitational fields. Along with the collapsed worlds of neutron stars and black holes, X-ray astronomers and solar physicists are paying more and more attention to our star, the Sun, a giant thermonuclear fusion reactor (five orders of magnitude more massive than Earth) hovering just 23,000 Earth radii away in the vacuum of space.

1.2 Overview of X-ray Astronomy

The study of objects that emit X-ray, gamma-ray, and ultraviolet radiation became possible with the advent of the space age. Because the Earth's atmosphere has the ability to block high-energy radiation, such radiation is possible to be observed only if the detectors are sent into space (above the atmosphere).
In 1962, the science of X-ray astronomy was born with the flight of a small Aerobee rocket launched from White Sands, New Mexico. A team of scientists sent three Geiger counters to investigate whether celestial sources other than the Sun also emitted X-rays. The instruments recorded an unexpected source of X-rays located in the constellation Scorpius, later dubbed Sco X-1. During the next 8 years, instruments launched on rockets and balloons detected several dozen bright X-ray sources in the Milky Way Galaxy and a few sources in other galaxies. Due to the growing excitement over X-ray astronomy, in 1970 NASA launched the first Small Astronomy Satellite (SAS-1). SAS-1's task was to perform the first survey of the X-ray sky from which a catalog of X-ray sources could be developed. Several hundred X-ray sources were discovered: including binary star systems - systems in which two stars travel together; supernova remnants - the remains of stars that have exploded; the nearby Andromeda Galaxy - a galaxy similar to the Milky Way; and several galaxy clusters - large gravitationally-bound groupings of galaxies.

Among the instruments used for studying X-ray sources was a small X-ray telescope aboard NASA's Copernicus satellite, two of NASA's Orbiting Solar Observatory satellites, the Defense Department's Vela 5-A, the Astronomical Netherlands Satellite, the British Ariel 5, and NASA's Small Astronomy Satellite (SAS-3). Numerous discoveries were made due to the ambitious rocket and balloon experiments: binary X-ray pulsars - a neutron star orbiting a normal companion and creating an X-ray emission that appears to wink on and off; X-ray bursters - compact objects that suddenly increase in intensity and then fade; X-ray emission from active stars; active galaxies where the central regions emit huge amounts of X-rays like the so-called "radio" galaxies, known for producing strong radio waves; quasars, radiating up to a thousand times as much energy as the Milky Way Galaxy from an area no larger than the solar system.
These early experiments detected the presence of an isotropic X-ray background radiation arriving from all directions, the origin of which was a subject of intense speculation. Many of the observed sources, due to their X-ray faintness, distance, etc. remained unidentified with any known astronomical objects.

The first large orbiting X-ray observatory, HEAO-1, was launched in 1977 by NASA. It was one in a series of three High-Energy Astronomy Observatory satellites. HEAO-1 weighted 3.5 tons and carried into orbit four experiments that surveyed the sky and detected sources of X-ray and gamma-ray emission but had no capability of producing images of emitting objects. The number of cataloged X-ray sources reached to approximately 1,500. HEAO-1 operated until early 1979 and some of the accomplishments achieved were: the first precise measurement of the energy spectrum of the diffuse X-ray background radiation, implying a possible origin in universal hot plasma; a very large bubble of hot gas in the constellation Cygnus stretching across more than 1,000 light years of space and containing the mass of several hundred thousand Suns; a new black hole candidate; and the discovery that the class of objects known as active galactic nuclei are powerful sources of X-rays.

Scientists were studying X-ray sources mainly by determining their positions, measuring their X-ray spectrum, and monitoring changes in their X-ray brightness over time. With the launch of the second High Energy Astronomy Observatory in 1978, HEAO-2 (known as the Einstein Observatory), it became possible instead of simply locating the positions of the cosmic X-ray sources to produce images of these sources. The Einstein Observatory was the first imaging X-ray telescope to be deployed in Earth orbit. With it, astronomers obtained X-ray images of such extended optical objects as supernova remnants, normal galaxies, clusters of
galaxies, and active galactic nuclei. Along with the revealing that all classes of objects known to
the classical optical astronomy were also sources of X-rays the Einstein Observatory discovered
that all stars, from the coolest to the very hottest, emit significant amounts of X-rays. Now
astronomers recognize that a significant part of the radiation emitted by virtually every type of
object in the cosmos is X-rays.¹

Riccardo Giacconi was involved in much of the early X-ray astronomy work. He led the
team in the construction and operation of the Einstein Observatory and was the first director of
the Space Telescope Science Institute from 1981 to 1993. In 2002 he was awarded the Nobel
Prize in Physics for his pioneering contributions to astrophysics, which have led to the discovery
of cosmic X-ray sources.

During the 1980s the European, Russian, and Japanese space agencies continued to launch
successful X-ray astronomy missions, such as the European X-ray Observatory Satellite
(EXOSAT), Granat, the Kvant module (of the Mir space station), Tenma, and Ginga. These
missions were more modest in scale than the HEAO program in the 1970s and were directed
toward in-depth studies of known phenomena.

In 1990, ROSAT [Roentgen Satellite], a joint project of Germany, the United States, and
Great Britain, was launched. Operational until 1999, it was instrumental in the discovery of
X-ray emissions from comets and conducted an all-sky survey in the X-ray region of the
spectrum. Five other satellites launched in the 1990s are still operational. ALEXIS [Array of
Low Energy X-ray Imaging Sensors] was launched in 1993; a minisatellite containing six
coffee-can-sized wide-angle, ultrasoft-X-ray telescopes, it provided the data for a unique sky
map for studying celestial flashes of soft X rays. Also launched in 1993, the Advanced Satellite
for Cosmology and Astrophysics is a joint Japanese-American project; containing four X-ray
telescopes, its primary purpose is the X-ray spectroscopy of such astrophysical entities as quasars
(RXTE) to study the variations in the emission of such X-ray sources as black-hole candidates,
active galactic nuclei, white dwarf stars, neutron stars, and other high-energy sources. The RXTE
played a key role in the discovery in 1996 of a “pulsing burster” located near the center of the
Milky Way. Unlike other X-ray sources, this one burst, oscillated, and flickered simultaneously,
with bursts lasting from 6 to 100 seconds. Before it burned out, the unexplained object was the
brightest source of X rays and gamma rays in the sky, radiating more energy in 10 seconds than
the sun does in 24 hours. BeppoSAX, a joint Italian-Dutch satellite, was launched in 1996.
When on Dec. 14, 1997, for 1 or 2 seconds the most energetic burst of gamma radiation ever
detected was recorded by the Compton Gamma Ray Observatory, BeppoSAX recorded the X-ray
afterglow of the burst, thereby providing a relatively accurate location for the source. The
Chandra X-ray Observatory was deployed from NASA’s space shuttle and boosted into a high
earth orbit in 1999; it focuses on such objects as black holes, quasars, and high-temperature
gases throughout the X-ray portion of the electromagnetic spectrum. Also launched in 1999 was
X-ray Multimirror Mission, an ESA satellite that carries an optical-ultraviolet telescope together
with three parallel mounted X-ray telescopes, allowing it to simultaneously observe phenomena
in two regions of the spectrum.

1.3 The SXI Mission

When the National Oceanic and Atmospheric Administration (NOAA) started a project for
providing a continuous and reliable stream of environmental information for supporting “space
weather” forecasting, an appropriate X-ray imaging system had to be designed and constructed. This system will be used for severe storm tracking and meteorological research and will provide X-ray images of the solar corona. The main mission of the so called Solar X-ray Imager (SXI) is going to be solar imaging for better prediction of space weather. Space weather is a term used to describe the dynamic environment of energetic particles, the solar wind streams, and the coronal mass ejections emanating from the sun. For this purpose a hyperboloid-hyperboloid design\textsuperscript{2,3} was developed by J. Harvey and P. Thompson which trades on-axis resolution for improved image quality over a wider field than a defocused Wolter Type I. The design was determined by constraining the first order optical design parameters at those of the baseline Wolter Type I and varying the vertex radii of curvature of the two hyperboloids, their conic constants and their vertex to vertex separation. A whole family of telescope designs was thus developed, where each member of the family was optimized at a different operational field-of-view (OFOV). One member of the family, designed as HT\#17, was chosen for the SXI program since it is well suited for the GOES missions, which require imaging the entire solar disk.

1.4 Motivation and Goals for this Research

Astronomical telescopes have traditionally been designed to optimize the on-axis optical performance of the aerial image. An aplanatic design may be used to obtain diffraction-limited performance over a small useable field-of-view (FOV). For near diffraction-limited visible or infrared stellar telescopes, that has historically been an adequate strategy. For telescopes requiring a larger FOV, attempts to correct astigmatism and field curvature, in addition to spherical aberration and comma, have occasionally been made. Furthermore, once the telescope design has been optimized, the design has frequently been submitted to an optical shop for
manufacture on a best effort basis. In such situations, the customer is stuck with the resulting optical performance without an opportunity to balance optical fabrication errors with other available design (or assembly and alignment) parameters.

For X-ray telescopes utilizing mosaic detector arrays, detector effects and surface scatter effects dominate geometrical aberrations at small field angles. For wide-field applications, there is thus little merit in an aplanatic design (why use a precious design variable to correct coma if detector effects are going to dominate coma for small field angles, and field curvature or astigmatism is going to dominate coma at large field angles). For the SXI program, we want to optimize the field-weighted-average detected image quality over a predetermined operational field-of-view (OFOV). It is thus the goal of this research to develop and utilize sophisticated image analysis software to: (1) optimize the initial SXI telescope design to balance the effects of geometrical aberrations, assumed optical fabrication errors, surface scatter effects, and detector effects, then (2) to re-optimize any remaining optical design parameters based upon measured metrology data after the mirror manufacturing process has been completed. Only by exercising this extensive analysis capability we will realize the best final image quality possible with current optical design, fabrication, and testing technology.

1.5 Dissertation Content

This dissertation contains eight chapters including the appendices and references. Chapter 1 (this chapter) starts with the discovery of X-rays and a review of their nature and behavior that makes them potentially very valuable to astronomers studying the universe in which we live. A very brief historical overview of the very young scientific discipline of X-ray astronomy is presented and the Solar X-ray Imager mission is described. This chapter also states the
motivation and goals of this dissertation, and provides a brief executive summary of the following chapters.

Chapter 2 provides a historical background of grazing incidence X-ray telescopes and discusses the classical Wolter Type I and the Wolter-Schwarzschild designs in some detail. An appropriate image quality criterion for wide-field applications (such as high energy solar physics) is suggested and discussed. The Harvey-Thompson Hyperboloid-Hyperboloid X-ray telescope design developed in response to this image quality criterion is then discussed and geometrical performance comparisons of these three grazing incidence X-ray telescope types are presented.

Most imaging systems today include a mosaic detector array in the focal plane. Optical designers of astronomical telescopes typically produce a design that yields a superb on-axis aerial image in the focal plane, and detector effects are included only in the analysis of the final system performance. When used with a mosaic detector array in the focal plane, detector effects eliminate the advantage of an aplanatic design even at small field angles. For wide fields-of-view, the focal plane is frequently despaced to balance field curvature with defocus thus obtaining better overall performance. In Chapter 3 we demonstrate that including detector effects in the design process results in a different optimal (non-aplanatic) design for each OFOV that is even superior to an optimally despaced aplanatic design.

Chapter 4 describes the complete system engineering analysis of image quality that was performed for each of the SXI telescope mirrors (four flight models and a spare) manufactured by Goodrich Optical Systems. This analysis includes all of the effects that contribute to image degradation: geometrical aberrations, diffraction effects, scattering effects, various miscellaneous
residual errors in the mirror manufacturer’s error budget tree, and finally the detector effects discussed in Chapter 3. This systems engineering analysis capability required the use of a specialized MATLAB software package developed specifically for the SXI program. Comprehensive image quality predictions are presented for each pair of the “as manufactured” SXI mirrors.

Chapter 5 compares the performance predictions for the 5 different “as-manufactured” SXI telescopes with each other and with predictions for the design to which they were fabricated. The variation between the predictions for the five “as-manufactured” SXI telescopes is also summarized as this represents the consistency of current state-of-the-art grazing incidence X-ray telescope fabrication technology.

Chapter 6 summarizes the dissertation results and states conclusions concerning state-of-the-art optical fabrication, testing, and modeling capabilities of grazing incidence X-ray telescopes for wide-angle imaging applications.
2.0 FUNDAMENTALS OF GRAZING INCIDENCE X-RAY TELESCOPE DESIGNS

A brief historical background of grazing incidence X-ray telescopes will first be presented. The optical design parameters of three specific grazing incidence X-ray telescope designs will then be discussed in some detail, and their geometrical performance will be compared for the specific 1st-order parameters of the Solar X-ray Imager (SXI) currently being built for the National Oceanic and Atmospheric Administration (NOAA). SXI is expected to become a standard subsystem aboard the next generation of Geostationary Operational Environmental Satellites (GOES weather satellites). Much of the material in this chapter is taken directly from Reference 2.

2.1 Historical Background

The two-mirror grazing incidence X-ray imaging systems described by Wolter in 1952 (Types I, II, and III) were axially symmetric, confocal, and followed the principles of on-axis stigmatic imaging laid down over 300 years earlier by Newton, Gregory, and Cassegrain. In a second paper published that same year, Wolter attempted to formulate completely aplanatic versions of his designs (the Wolter-Schwarzschild designs). Although his aim was to create an X-ray microscope, Wolter unwittingly became the father of modern X-ray astronomy eleven years later when the first Wolter Type I X-ray telescope was launched into space in 1963.

During the past fifty years, the original Wolter designs have been studied in detail. In 1957, A. K. Head presented a closed-form solution of two aplanatic grazing incidence mirror
surfaces working between finite foci.\textsuperscript{9} The equations analyzed by Wolter are a special case of the Head equations where one focus is at infinity. In 1969 Mangus and Underwood described the process of determining the 1st-order optical design properties of a Wolter Type I X-ray telescope, and reported the results of both laboratory and rocket flight tests of a prototype instrument for the wavelength region of 6-100Å.\textsuperscript{10} Similarly, in 1970, Mangus reported upon the optical design of a Wolter Type II grazing incidence telescope for far and extreme ultraviolet (100-900Å) solar and astrophysical observations.\textsuperscript{11} In 1972-73, Van Speybroeck and Chase took advantage of computerized ray tracing algorithms to ‘empirically’ and parametrically determine the effects of varying design parameters upon the imaging performance of the Wolter\textsuperscript{12} and Wolter-Schwarzschild\textsuperscript{13} Type I telescopes. Their findings were extremely useful but lacked the identification and interpretation of conventional aberrations (i.e. coma, astigmatism, etc.). In 1976, Chase presented a modified version of the Head equations to describe an aplanatic grazing incidence microscope with improved geometrical performance for possible application in the field of controlled fusion research.\textsuperscript{14} In 1977, Werner\textsuperscript{15} attempted the computational optimization of a Wolter Type I telescope by relaxing the surface shape constraint to that of a generalized axial polynomial. This resulted in almost flat imaging response across the field of view but simultaneously sacrificed the possibility of diffraction limited performance. Also in 1977, Winkler and Korsch\textsuperscript{16} published an apparently decisive and thorough formulation of two-mirror grazing incidence aberration theory. Their results showed, however (due to their limited precision) that any classical Wolter type telescope was \textit{already} aplanatic. This is clearly not true as evidenced by Wolter’s second paper\textsuperscript{7}. In 1979 a paper by Cash \textit{et al} \textsuperscript{17} concluded that standard, near normal incidence aberration theory could be \textit{exactly} applied to grazing incidence optical elements. Korsch\textsuperscript{18} discussed a first order coma term \textit{not} present in normal aberration
theory for a single mirror. Nariai\textsuperscript{19} stated quite decisively in 1987 that “it is not possible to use ordinary aberration theory because the expansion of aberrations in series of powers on the height of the object and on the radius of the pupil does not converge, etc.” And in 1988 Nariai\textsuperscript{20} showed analytically that all aberrations in his expansion must be integrated over the entire annular pupil, and those aberration coefficients in grazing incidence systems are apparently themselves a function of pupil coordinates.

From the above historical review, it is clear that considerable confusion and several contradictions are present in the early attempts to understand and formulate an aberration theory for grazing incidence X-ray imaging systems. However, during 1985-88, Saha performed an extensive analysis of the aberrations of all Wolter types of grazing incidence telescopes as well as all combinations of normal incidence paraboloid-hyperboloid and paraboloid-ellipsoid telescopes.\textsuperscript{21-24} Saha’s theory has been shown to predict image degradation (geometrical rms image size) that agrees well with real ray trace data for both Wolter Type I and Wolter Type II grazing incidence telescopes.\textsuperscript{25}

2.2 The Classical Wolter Type I Design

A Wolter Type I grazing incidence X-ray telescope made up of a paraboloid and hyperboloid is illustrated in Figure 2-1.\textsuperscript{5,10,12} The equation for a paraboloid with its vertex at $z_p$ is given by

$$r_p^2 = 2 R_p (z - z_p)$$

where $R_p$ is the paraboloid vertex radius of curvature and $r_p$ is the radius of the paraboloid at the axial position $z$. The equation for a hyperboloid centered at $z_h$ is given by
\[
\frac{(z-z_h)^2}{a^2} - \frac{r_h^2}{b^2} = 1
\]  

(2-2)

where \(a\) and \(b\) are the semi-major and semi-minor axes of the hyperboloid. The eccentricity of the hyperboloid is determined by \(a\) and \(b\)

\[
\varepsilon = \sqrt{\frac{b^2}{a^2} + 1}
\]

(2-3)

Figure 2-1. Wolter Type I grazing incidence telescope configuration.

The separation of the two hyperboloid foci is given by \(2a\varepsilon\). If we superpose the rear hyperboloid focus with the paraboloid focus, the front hyperboloid focus becomes the system focus and \(f_j = z_f - z_j\) is the nominal focal length (as measured from the mirror joint) of the telescope. If the origin of our coordinate system an arbitrary distance \(z_i\) in front of the front edge of the paraboloid mirror, then

\[
z_p = z_j + f_j + 2\varepsilon a + R_p/2, \quad \text{and} \quad z_h = z_j + f_j + a\varepsilon
\]

(2-4)

The optical prescription of a classical Wolter Type I X-ray telescope can thus be completely defined by the three independent parameters \(R_p, a\) and \(b\) (or \(R_p, a\) and \(\varepsilon\)). An optimized
(maximized effective collecting area) Wolter Type I telescope can be obtained if we require the grazing angles of reflection from the paraboloid and the hyperboloid to be equal near their point of intersection. This constraint reduces the number of independent parameters defining the optical prescription to two.

For our purposes it is more convenient to choose the telescope radius at the intersection of the paraboloid and the hyperboloid, $r_j$, and the nominal focal length of the telescope, $f_j$, as the parameters defining the optical prescription. The grazing angle at the joint is then given by

$$\alpha = \frac{1}{4} \arctan \left( \frac{r_j}{f_j} \right).$$  \hspace{1cm} (2-5)$$

The actual focal length, as measured from the system principle/nodal point, is slightly larger than the nominal focal length

$$f = f_j + \frac{r_j^2}{2 f_j},$$ \hspace{1cm} (2-6)$$

and the plate scale is the reciprocal of this focal length, expressed in arc sec per micrometers.

In addition to the telescope radius, $r_j$, and the nominal focal length, $f_j$, the remaining optical design parameters include the length of the paraboloid mirror, $L_p$, the length of the hyperboloid mirror, $L_h$, and the width of the gap between the two mirror elements. From these input parameters, the actual dimensions of the mirror elements can be calculated as well as the obscuration ratio of the collecting aperture which determines both the geometrical collecting area and the diffraction-limited image characteristics.

A specific optical design must be used in order to obtain quantitative performance predictions; hence, the baseline design for the NOAA SXI telescope has been chosen for this
discussion. The SXI baseline design system parameters, optical prescription, and other significant (and perhaps redundant) quantities are presented in Table 2-1.

Table 2-1. Various SXI optical and geometrical parameters (all lengths in mm).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Telescope System</th>
<th>Paraboloid</th>
<th>Hyperboloid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint Focal Length (f) =</td>
<td>655</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Radius at Joint (r) =</td>
<td>80</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Optic Length (L, or L_s) =</td>
<td>-</td>
<td>47.5</td>
<td>47.5</td>
</tr>
<tr>
<td>Gap About Joint (g) =</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Grazing Angle at Joint (θ) =</td>
<td>1.74088713 (degrees)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Nodal Focal Length (f_n) =</td>
<td>659.68549618</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Plate Scale (m) =</td>
<td>0.31257666 (arc-sec/μm)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Vertex Radius (R_v) =</td>
<td>-</td>
<td>-2.43145733</td>
<td>-2.44046251</td>
</tr>
<tr>
<td>Semi-Major Axis (a) =</td>
<td>-</td>
<td>-</td>
<td>328.71572867</td>
</tr>
<tr>
<td>Semi-Minor Axis (b) =</td>
<td>-</td>
<td>-</td>
<td>28.3248367</td>
</tr>
<tr>
<td>Eccentricity (ε) =</td>
<td>-</td>
<td>1.00000000</td>
<td>1.00370526</td>
</tr>
<tr>
<td>Conic Constant (-ε^2) =</td>
<td>-</td>
<td>-1.00000000</td>
<td>-1.00742425</td>
</tr>
<tr>
<td>Separation of Foci (2aε) =</td>
<td>-</td>
<td>-659.86714092</td>
<td>-659.86714092</td>
</tr>
<tr>
<td>Inner Radius (r_min or r_max) =</td>
<td>-</td>
<td>80.07594899</td>
<td>75.4143272</td>
</tr>
<tr>
<td>Midplane Radius (r_mid or r_out)</td>
<td>-</td>
<td>80.79388287</td>
<td>77.5969410</td>
</tr>
<tr>
<td>Outer Radius (r_out or r_max) =</td>
<td>-</td>
<td>81.05495111</td>
<td>79.77145135</td>
</tr>
<tr>
<td>Front Position (z_front) =</td>
<td>-</td>
<td>0.00000000</td>
<td>52.50000000</td>
</tr>
<tr>
<td>Midplane Position (z_mid or z_out)</td>
<td>-</td>
<td>23.75000000</td>
<td>76.25000000</td>
</tr>
<tr>
<td>Rear Position (z_r or z_max) =</td>
<td>-</td>
<td>47.50000000</td>
<td>100.00000000</td>
</tr>
<tr>
<td>Linear Obscuration Ratio (Θ) =</td>
<td>0.98246071</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Geometrical Collecting Area (A) =</td>
<td>725.67160688</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Location of Focus (z_f) =</td>
<td>-</td>
<td>1374.86741092</td>
<td>705.00000000</td>
</tr>
</tbody>
</table>

We made extensive but careful use of the commercially available optical design and analysis code, ZEMAX, to obtain quantitative geometrical optical performance predictions in terms of rms image radius. Although ZEMAX exhibits several inherent difficulties associated with grazing incidence systems, the basic ray tracing features were found to be consistent with results computed by the optical surface analysis code (OSAC) which was developed specifically for the analysis of grazing incidence X-ray telescopes. OSAC does not have the optimization capability necessary for developing new designs. Some of the above difficulties include the following: 1.) default ray patterns that are not ideally suited for these extremely large obscuration ratios, 2.) reliable aberration coefficients are not always provided for these extremely large obscuration ratios, and 3.) the length and relative positions of the grazing incidence mirror segments are difficult to maintain during optimization of design parameters.
The ZEMAX parameters of the SXI baseline design are presented in Table 2-2 according to the standard “lens editor” format for the program. Note that the conic constants listed are equal to \((-\varepsilon^2)\), where \(\varepsilon\) is the eccentricity of the conic surface. The value of the conic constant for the primary mirror (surface #5) is thus unity. This design gives a classical Wolter Type I X-ray telescope. Also shown in Table 2-2 is an optimal spherical focal surface (surface #8) which has a radius of curvature of about \(-35\)mm. This allows us to evaluate the geometrical performance on both a plane and a curved focal surface. The presence of surface numbers 1 through 4 and 7 is simply to provide reference planes from which to track the relative positions of surface limits and rays within the layout.

Table 2-2. ZEMAX lens editor values for SXI baseline design.

<table>
<thead>
<tr>
<th>ZEMAX “Lens Editor” Values for SXI Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface</td>
</tr>
<tr>
<td>OBJ</td>
</tr>
<tr>
<td>1*</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>STO*</td>
</tr>
<tr>
<td>6*</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>IMA</td>
</tr>
</tbody>
</table>

The classical Wolter Type I X-ray telescope design produces an ideal on-axis geometrical point image (zero spherical aberration); however, field curvature is a dominant limiting factor determining the off-axis performance of grazing incidence X-ray telescopes if a flat focal surface must be used. The focal plane of such systems is frequently despaced to improve the off-axis performance; although, this results in a degraded (defocused) on-axis image.

Geometrical optical performance from ray trace data is conveniently expressed in terms of rms image radius expressed in arc sec. This quantity is plotted as a function of field angle for
several different positions of the focal plane in Figure 2-2. The minus sign associated with the focal shift indicates a displacement toward the telescope mirrors. Also shown for comparison is the performance curve that would be achieved with a curved detector conforming to the optimally curved focal surface.

Figure 2-2. Geometrical performance of a classical Wolter Type I X-ray telescope design for the SXI telescope system parameters.

Note that the curve for the best focal surface in Figure 2-2 appears to have a linear and a quadratic component. This is consistent with findings of Van Speybroeck and Chase.\textsuperscript{13} For small field angles, the linear component dominates and will be associated with a conventional coma-like aberration.\textsuperscript{28,29} Similarly, the quadratic component of the curve will be associated with a conventional astigmatism-like aberration.\textsuperscript{28,29} The curve corresponding to the Gaussian image plane ($dz = 0$) is designated as a Classical Wolter Type I and also appears to consist primarily of a linear and a quadratic component. The linear component is the same as for the
best focal surface as evidenced by the slope at small field angles. However, the quadratic component is significantly larger since it contains a contribution from both astigmatism-like aberrations and field curvature.

In accordance with Saha’s rigorous aberration theory, the image degradation (as indicated by rms image size) that is linear with field angle is attributed to coma, and the degradation that is quadratic with field angle is due to a combination of field curvature and astigmatism. Similarly, we will consider any on-axis image degradation to be caused by a combination of defocus and spherical aberration. Since the pupil dependence was integrated out of these performance curves when calculating the rms image size, we cannot distinguish between various orders of aberrations. For example, the linear component of these curves represents all orders of linear coma. Likewise, the quadratic component includes third-order field curvature and astigmatism as well as all higher-order aberration terms that have a quadratic dependence on field angle; this includes the fifth-order aberration usually referred to as oblique spherical aberration. In fact, some seventh-order terms with a quadratic field dependence can contribute as much to the image degradation as the third-order coma term. There are also, no doubt, cubic and higher-order contributions to the curves in Figure 2-2; however, they do not appear to play a significant role for field angles less than 21 arc min.

Despacing the focal plane of the classical Wolter Type I grazing incidence telescope clearly balances field curvature with defocus, thus improving the wide-field performance at the expense of the small-field performance. There are no additional design variables available for further correcting or balancing aberrations. This classical Wolter Type I design produces a stigmatic image on-axis in the Gaussian focal plane and has thus been used in virtually every X-ray stellar
telescope built in the last forty years, including the Einstein Observatory,\textsuperscript{30} ROSAT,\textsuperscript{31} and AXAF (the Chandra Observatory).\textsuperscript{32}

2.3 The Wolter-Schwarzschild Design

The Wolter-Swarzschild (WS) grazing incidence telescope design consists of two coaxial, non-conic, aspheric mirror surfaces of revolution that strictly satisfy the Abbe sine condition and therefore eliminates all orders of coma.\textsuperscript{6,12,22} Figure 2-3 compares the geometrical performance of the unique WS design with the SXI 1st-order optical system parameters to that of the classical Wolter Type I design.

![Figure 2-3. Comparison of the geometrical performance of a classical Wolter and a WS design for the SXI telescope system parameters. Also displayed is the percent reduction in rms image radius in going from a classical Wolter Type I design to the WS design.](image)

Figure 2-3 also illustrates the percent reduction in the geometrical rms image radius vs. field angle of the WS design relative to the classical Wolter Type I design. The small-field resolution of a WS telescope is clearly intrinsically superior to that of a classical Wolter Type I telescope;
however, this increase in performance falls off rapidly with increasing field angle. Furthermore, when other system errors are considered (scattering effects, detector effects, etc.) the improvement in system performance may be reduced to a negligible value.33

As with the Wolter Type I design, the focal plane of the Wolter-Swarzschild telescope can also be despaced for wide-field applications. Figure 2-4 illustrates a family of geometrical performance curves for various amounts of focal plane despase. Again, these curves represent the balancing of field curvature with defocus, thus improving the wide-field performance at the expense of the small-field performance. For the WS design we used a ray trace program developed specifically for this purpose. The code is based upon general surface equations derived for grazing incidence telescopes.22 All two-mirror telescopes can be analyzed with the code; however, it has no optimization capability.

![Figure 2-4. Geometrical performance of a Wolter-Schwarzschild telescope design (SXI 1st-order parameters) for several different axial positions of the focal plane.](image)
It is readily shown that the best focal surface is nearly the same for the equivalent Wolter Type I and WS designs, indicating that the field curvature aberration is nearly identical. This is expected from the well-known Petzval theorem.34

2.4 Image Quality Criteria for Wide-field Imaging Applications

On-axis resolution (perhaps expressed as a fractional encircled energy) is an appropriate image quality criterion for a stellar telescope that is going to be accurately pointed at the object of interest. However, the image quality criterion for a wide-field imaging application should be expressed in terms of some field-weighted-average “resolution” over the pre-determined operational field-of-view (OFOV). This is certainly the case for the SXI telescope operating in a staring mode, recording and transmitting full solar disc images of solar flare activity for study by NOAA scientists and solar physicists. For the purposes of this study, we therefore follow Burrows, et al in adopting the field-weighted-average geometrical rms image radius as the relevant image quality criterion for wide-field imaging applications35

$$\sigma_{f.w.a} = \frac{1}{A_T} \int_{\theta=0}^{\theta'} \sigma_{rms}(\theta) 2\pi\theta d\theta$$

(2-7)

Here $A_T = \pi \theta'^2$ where $\theta'$ is the angular radius of the OFOV. The total number of spatial resolution elements in the OFOV is closely related to the above merit function, and can be approximated as the number of these area-weighted-average resolution elements in the OFOV

$$N = \# of \ Res. \ Ele. = 2 \int_{\theta=0}^{\theta'} \frac{\theta}{\sigma_{f.w.a}} d\theta$$

(2-8)

Since sunspots or solar flares have an equal probability of appearing anywhere on the solar disc, the total information content of a given snapshot of the solar disc is maximized if we
minimize the field-weighted-average resolution element as degraded by all error sources. In particular, if the SXI telescope is not going to be routinely pointed to the particular feature of interest, this image quality criterion is vastly superior to one that maximizes the on-axis image quality.

### 2.5 The Harvey-Thompson Hyperboloid-Hyperboloid Design

For wide-field imaging applications, there is little merit in an optical design exhibiting stigmatic imaging on-axis; Harvey and Thompson therefore departed from the classical Wolter Type I design in favor of a hyperboloid-hyperboloid design that provides additional design variables. The hyperboloid-hyperboloid grazing incidence telescope design consists of a primary mirror and a secondary mirror described by the following equations

\[
\frac{(z - z_{cp})^2}{a_p^2} - \frac{r_p^2}{b_p^2} = 1 \quad \frac{(z - z_{cs})^2}{a_s^2} - \frac{r_s^2}{b_s^2} = 1
\] (2-9)

The primary mirror hyperboloid and the secondary mirror hyperboloid are centered at \( z_{cp} \) and \( z_{cs} \) respectively and the constants \( a \) and \( b \) are the semi-major and semi-minor axes of the hyperboloid. The eccentricity of a hyperboloid is determined by \( a \) and \( b \)

\[
\varepsilon = \sqrt{\frac{b^2}{a^2} + 1}
\] (2-10)

The separation of the two hyperboloid foci is given by \( 2ae \). We refer to the intersection of these two hyperboloids as the mirror *joint*, i.e., when \( z = z_j \) then \( r_p = r_s \equiv r_j \). We can therefore rearrange the above equations for \( r_j^2 \) and equate the resulting expressions
\[ b_p^2 \left( \frac{(z_j - z_{cp})^2}{a_p^2} - 1 \right) = b_s^2 \left( \frac{(z_j - z_{cs})^2}{a_s^2} - 1 \right) \]  

(2-11)

or, using Equation

\[ (\varepsilon_p^2 - 1)(z_j - z_{cp})^2 - b_p^2 = (\varepsilon_s^2 - 1)(z_j - z_{cs})^2 - b_s^2 \]  

(2-12)

Dividing by the quantity \((\varepsilon_s^2 - 1)\), we obtain

\[ \frac{(\varepsilon_p^2 - 1)}{(\varepsilon_s^2 - 1)}(z_j - z_{cp})^2 - \frac{b_p^2}{(\varepsilon_s^2 - 1)} = (z_j - z_{cs})^2 - a_s^2 \]  

(2-13)

If \(S_{cc}\) is the directional distance from the center of the primary mirror hyperboloid to the center of the secondary mirror hyperboloid, we have \(z_{cp} = z_{cs} - S_{cc}\). Note also that we have not yet specified the origin of the coordinate system. For the purpose of solving this quadratic equation, let us set \(z_{cs} = 0\), then \(z_j = S_{csj}\) ≡ distance from \(z_{cs}\) to \(z_j\)

\[ \frac{(\varepsilon_p^2 - 1)}{(\varepsilon_s^2 - 1)}(S_{csj} - S_{cc})^2 - \frac{(\varepsilon_p^2 - 1)}{(\varepsilon_s^2 - 1)} d_p^2 = S_{csj}^2 - a_s^2 \]  

(2-14)

This quadratic equation for \(S_{csj}\) can now be written as

\[ A S_{csj}^2 + B S_{csj} + C = 0 \]

where

\[ K = \frac{(\varepsilon_p^2 - 1)}{(\varepsilon_s^2 - 1)}, \quad A = 1 - K, \quad B = -2 K S_{cc}, \quad \text{and} \quad C = K(a_p^2 - S_{cc}^2) - a_s^2. \]  

(2-16)

A schematic diagram of the resulting hyperboloid-hyperboloid grazing incidence X-ray telescope is shown in Figure 2-5. As in Figure 1-1 for the classical Wolter Type I design we have chosen the origin of the coordinate system to be an arbitrary distance \(z_1\) in front of the front edge of the paraboloid mirror. Note that the front focus of the primary mirror does \(\textit{not}\) coincide with
the rear focus of the secondary mirror as is the case with the classical Wolter Type I design. In Figure 2-5 this \textit{confocal delta} is indicated as the quantity $\Delta_{p}$.

Similarly, the system focal plane does \textit{not} lie at the front focus of the secondary mirror. This displacement is indicated as $\Delta f$.

![Figure 2-5. Hyperboloid-hyperboloid grazing incidence X-ray telescope design.](image)

The optimization capability of the ZEMAX ray trace code was used to balance on-axis geometrical performance for off-axis performance. The classical Wolter Type I SXI baseline design was used as a starting point and the $1^{st}$-order optical design parameters were constrained, but the vertex radii of curvature of the two mirrors, their conic constants, and the vertex-to-vertex separation were allowed to vary.

A family of hyperboloid-hyperboloid grazing incidence X-ray telescope designs was thus developed where each member of the family provides optimum performance for a different OFOV. The resulting performance curves are illustrated in Figure 2-6. Nariai discussed a similar hyperboloid-hyperboloid design while attempting to design a coma-free grazing
incidence X-ray telescope.\textsuperscript{19,20} The curves in Figure 2-6 are designated by the field angle, $\theta_B$, at which the rms image radius was minimized.

Figure 2-6. Geometrical performance of a new family of optimized hyperboloid-hyperboloid designs.

Note that the locus of minima for this family of curves is a straight line with non-zero slope on this plot of rms image radius vs. field angle. We thus interpret this empirical ray trace data as indicating that the shaded area represents an uncorrectable linear coma-like aberration. This is consistent with Nariai’s conclusion that coma can be minimized, but not eliminated with a hyperboloid-hyperboloid grazing incidence X-ray telescope design.\textsuperscript{20} The non-zero on-axis values of rms image radius clearly represent some combination of defocus and spherical aberration. We also know that these grazing incidence telescopes suffer from severe field curvature, astigmatism, and oblique spherical aberration.\textsuperscript{15,19-24} Since the $5^{th}$-order oblique spherical aberration has the same field dependence as $3^{rd}$-order astigmatism and the same pupil dependence as $3^{rd}$-order spherical aberration,\textsuperscript{29} it is reasonable to interpret each of the above
designs as having balanced defocus, field curvature, 3rd-order spherical aberration, 3rd-order astigmatism, and oblique spherical aberration, leaving only linear coma at the unique field angle, $\theta_b$. Optimum use has thus been made of the five independent design variables.

The design designated as $\theta_b = 12.1$ yields geometrical image sizes that are nearly the same on-axis and at a field angle of 16.5 arc min, slightly above the solar limb, and is thus similar to the design chosen for the SXI mission. The hyperboloid-hyperboloid design chosen for the SXI mission is completely defined by the parameters listed in Table 2-3 and Table 2-4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value (mm)</th>
</tr>
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despaced classical Wolter Type I design and an optimally despaced Wolter-Schwarzschild X-ray telescope design. This new insight and understanding was central to the decision by NOAA to adopt the Harvey-Thompson design for the SXI program, and also a major factor in the way that the “as-manufactured” SXI mirrors are modeled in Chapter 4 of this dissertation. Hence, the results of the analysis presented in Reference 36 are included in detail throughout the remainder of this chapter.

In order to determine the optimum amount of focal plane despace for wide-field applications using a Wolter Type I design, we used Eq. (2-7) to integrate each of the curves in Figure 2-2 (the ray trace analysis was extended to a field angle of 30 arc min, and performed at additional despace values). Figure 2-7 illustrates the resulting plots of field-weighted-average geometrical rms image radius as a function of OFOV. For a particular OFOV (21 arc min for example), we can plot the y-intercepts vs. the focal plane despace and obtain a curve from which we can determine the optimum focal plane despace for that OFOV as shown in Figure 2-8.

![Figure 2-7. Field-weighted-average rms image radius vs. OFOV for the classical Wolter Type I design for different values of focal plane despace.](image-url)
For a particular OFOV (21 arc min for example), we can plot the y-intercepts vs. the focal plane despace and obtain a curve from which we can determine the optimum focal plane despace for that OFOV as shown in Figure 2-8.

![Figure 2-8](image.png)

Figure 2-8. Field-weighted-average rms image radius vs. focal plane despace for a Wolter Type I design with an OFOV of 21 arc min.

A despace of 0.11 mm producing a field-weighted-average rms image radius of $\sigma_{fwa} = 5.03$ arc sec is clearly optimum for an OFOV of 21 arc min. Note that this is down from $\sigma_{fwa} = 6.50$ arc sec for the Gaussian focal plane.

This procedure has been repeated for the Wolter-Schwarzschild optical design. Figure 2-9 illustrates the resulting plots of field-weighted-average geometrical rms image radius as a function of OFOV. We again plot the y-intercepts vs. the focal plane despace for a 21 arc min OFOV, and obtain a curve from which we can determine the optimum focal plane despace for that OFOV as shown in Figure 2-10.
In this case a despace of 0.12 mm producing a field-weighted-average rms image radius of $\sigma_{\text{fwa}} = 3.85$ arc sec is optimum for an OFOV of 21 arc min. Note that this is down from $\sigma_{\text{fwa}} = 5.85$ arc sec for the Gaussian focal plane.

Figure 2-10. Field-weighted-average rms image radius vs. focal plane despace for a Wolter-Schwarzschild design with an OFOV of 21 arc min.
Finally, we can integrate each of the curves in Figure 2-6 to obtain the field-weighted-average rms image radius as a function of OFOV for a family of optimal hyperboloid-hyperboloid designs (see Figure 2-11).

![Figure 2-11. Field-weighted-average geometrical rms image radius vs. OFOV for the new family of hyperboloid-hyperboloid designs.](image)

Plotting the y-intercepts of the curves in Figure 2-11 at a particular OFOV (again 21 arc min) vs. the field angle, \( \theta_b \), at which the aberrations are balanced, and drawing a smooth curve through the data points, we obtain a curve from which we can determine at what field angle, \( \theta_{\theta} \), to balance the aberrations in order to obtain the optimum design for that particular OFOV. We see from Figure 2-12 that a \( \theta_b \) of about 14.18 arc min yielding \( \sigma_{\text{fwa}} = 3.69 \) arc sec is optimum for an OFOV of 21 arc min. If one of our existing designs does not fall very near the minimum of this curve, we can readily go to ZEMAX and obtain a new design optimized for that OFOV.
Figure 2-12. Field-weighted-average geometrical rms image radius vs. $\theta_B$ for hyperboloid-hyperboloid designs with an OFOV of 21 arc min.

By repeating this procedure for different OFOV’s, and constructing plots similar to that depicted in Figure 2-12 for each of those OFOV’s, we obtain a plot of the minimum field-weighted-average rms image radius (characterizing the optimum hyperboloid-hyperboloid design for that OFOV) versus the OFOV. Figure 2-13 summarizes all of this data, and shows a detailed and meaningful comparison of the geometrical performance of the classical Wolter Type I design, the Wolter-Schwarzchild design, and the new hyperboloid-hyperboloid design for wide-field X-ray imaging applications. Each point on the curves represents the optimal despace value (or the optimal hyperboloid-hyperboloid design) for that OFOV. Note that the optimally despaced Wolter-Schwarzschild design and the optimum hyperboloid-hyperboloid design always significantly outperform the optimally despaced classical Wolter Type I design. However, the optimally despaced Wolter-Schwarzschild design only outperforms the optimum hyperboloid - hyperboloid design for OFOV’s less than approximately 18 arc min.
Figure 2-13. Comparison of the field-weighted-average rms image radius versus OFOV for three different types of X-ray telescopes for wide-field imaging applications.

For OFOV’s greater than approximately 18 arc min, the *optimum hyperboloid-hyperboloid design* outperforms the *optimally despaced Wolter-Schwarzschild design*. Recall that all of this data is specifically for grazing incidence X-ray telescopes with the 1st-order properties outlined in Table 2-1.

It should be emphasized here that the above analysis is restricted to the *geometrical performance* (image quality degraded only by aberrations due to residual optical design errors) of the three grazing incidence X-ray telescope design types. A complete systems engineering analysis including image degradation by diffraction effects, geometrical aberrations, surface scattering effects (from residual optical fabrication errors), detector effects, alignment errors, metrology errors, and other potential error sources for the SXI telescope will be reported in the Chapter 4 for each of the “as-manufactured” SXI mirrors.
3.0 INCLUDING DETECTOR EFFECTS IN THE DESIGN OF GRAZING INCIDENCE X-RAY TELESCOPES

Most imaging systems today include a mosaic detector array in the focal plane. Optical designers of astronomical telescopes typically produce a design that yields a superb aerial image in the focal plane, detector effects are then treated in some ad-hoc manner. However, detector effects are not usually included in the optical design process. It was shown in the last chapter that, due to the large off-axis aberrations of grazing incidence X-ray telescopes, an optimal family of non-aplanatic hyperboloid-hyperboloid designs exists where each member of the family is optimum for a given operational field-of-view (OFOV). Each of these optimum designs was a unique balance of small-field and large field aberrations, resulting in a maximum number of spatial resolution elements for that OFOV.

When a mosaic detector array is placed in the focal plane of these grazing incidence X-ray telescopes, detector effects eliminate the advantage of an aplanatic design even at small field angles; i.e., detector effects dominate the image degradation at small field angles and aberrations (field curvature, astigmatism, and oblique spherical aberration) dominate the image degradation at large field angles. Again, balancing the small-field performance with the large-field performance is frequently achieved by despacing the focal plane to obtain better overall performance. In this chapter it will be demonstrated that including detector effects in the design process results in a different optimal (non-aplanatic) design for each OFOV that is superior to merely optimally despacing a particular design that may have yielded a superior aerial image.
3.1 Detection with Mosaic Detector Arrays

The modulation transfer function (MTF) is widely used in the initial specification and design of many imaging systems, as well as in the subsequent detailed analysis of the images they produce. However, implicit in this is the mathematical assumption that the imaging system is both linear and shift-invariant, i.e., that the location (and strength) of a point source can be chosen arbitrarily.

When a single detector is scanned over an aerial image, the detected image (in the scan direction) can be modeled by the convolution of the aerial image with the detector; or conversely, one can multiply the MTF of the imaging system by the detector MTF. However, these line-scan devices all employ a discrete sampling interval in the direction perpendicular to the scan direction, and the MTF approach to system performance analysis is not directly applicable to these scanning techniques or imaging systems utilizing staring mosaic detector arrays. The sampling causes these systems to exhibit a particular kind of local shift variance which causes the appearance of the reconstructed image to vary with the location of the aerial PSF relative to the sampling (i.e., pixel) grid.\textsuperscript{36,37}

For example, in an imaging system utilizing a staring mosaic detector array, the aerial image is sampled (averaging over each detector pixel) to produce a detected point spread function (DPSF). An interpolation scheme can then be used to reconstruct a smooth DPSF; however, the detailed characteristics of the DPSF varies substantially with the registration (or lack thereof) of the aerial PSF on a given detector pixel. In other words, the imaging process using a staring mosaic detector array is not a shift-invariant process. This detector registration (or alignment) process must therefore be discussed in some detail.
Assuming a Gaussian aerial PSF slightly larger than a detector pixel, Figure 3-1 illustrates the resulting DPSF and reconstructed DPSF for the following three situations: (i) when the aerial PSF is precisely “registered” at the center of a detector pixel, (ii) when the aerial PSF is positioned on the boundary between two detector pixels, and (iii) when the aerial PSF is positioned at a point where four detector pixels meet.

If the detector array is not “registered” we get substantially different quantitative results for various characteristics of the reconstructed DPSF. For example, the half power radius (HPR) of the reconstructed DPSF can increase by more than 40% over the registered value. For an application where the telescope is being operated as a staring telescope recording fine detail in an extended image (random location of aerial PSF on pixel), the “average unregistered” detected point spread function (AUDPSF) is given by the convolution of the registered detected point spread function (RDPSF) by the unit cell of the sampling grid. The calculation of both the reconstructed registered DPSF and the reconstructed average unregistered DPSF is thus illustrated in Figure 3-2. Since the aerial PSF is represented as a dense numerical array, the averaging over the individual pixels is referred to as a “binning” operation. Care is taken to
precisely “register” the sampling detector grid by positioning it so as to maximize the signal produced by a given pixel. We then use a cubic interpolation technique to reconstruct the “registered” \textit{DPSF}. Finally, we convolve by the unit cell of the sampling grid to produce the average unregistered \textit{DPSF}.

If detector effects other than the finite pixel size are present (such as charge spreading from pixel to pixel), these effects should be included in the \textit{DPSF} before performing the cubic interpolation to model the reconstructed \textit{DPSF}.

![Figure 3-2. A graphical illustration of the numerical computation technique for modeling both the reconstructed “registered” \textit{DPSF} (RDPSF) and the reconstructed “average unregistered” \textit{DPSF} (AUDPSF) is indicated.](image)

In testing the SXI mirrors at the NASA/MSFC X-ray Test Facility, the centroid (or peak) of the aerial \textit{PSF} can be precisely registered at the center of a detector pixel; hence, comparing the test results to the predicted \textit{RDPSF} is appropriate. However, in operational use, the SXI telescope will be staring at the full solar disc with numerous features of interest at arbitrary positions on the surface of the sun. It will be impossible to “register” this extended image on the detector array. Predictions of the operational performance of the SXI telescope are thus best modeled as the \textit{AUDPSF}.
3.2 Image Quality Criterion for Applications using Mosaic Detectors

In Section 2.4 a brief discussion of a field-weighted-average image quality criterion was discussed. Only residual geometrical optical design (ray) errors were being considered; hence, the field-weighted-average geometrical rms image radius was adopted as the image quality criterion. In modeling the combination of geometrical aberration and detector effects we will now choose the field-weighted-average half power radius \( HPR_{fwa} \) of the \( RDPSF \) and the \( AUDPSF \) as meaningful measures of a spatial resolution element in the images detected by the SXI telescope.

Similar to Eq.(2-7), this \( HPR_{fwa} \) is determined by a two-dimensional integration over a circular \( OFOV \) of the field-dependent half power radius, \( HPR(\theta) \) of either the \( RDPSF \) or the \( AUDPSF \):

\[
HPR_{fwa} = \frac{1}{A_T} \int_{\theta=0}^{\theta=\pi} HPR(\theta) 2\pi \theta d\theta
\]

(3-1)

where

\[
A_T = \pi (OFOV)^2
\]

(3-2)

Again minimizing the \( HPR_{fwa} \) over a given OFOV will provide the maximum number of spatial resolution elements over that OFOV, thus optimizing the information content of the detected image.

\[
N_{eff} = \# \ of \ average \ resol. \ elem. = \frac{1}{2} \int_{\theta=0}^{\theta=\pi} \frac{\theta}{HPR^2(\theta)} d\theta
\]

(3-3)

Since the SXI telescope is not going to be routinely pointed at a particular feature of interest, this image quality criterion is vastly superior to one which maximizes the on-axis image quality.
3.3 Balancing Geometrical Aberrations with Detector Effects

An aplanatic optical design (corrected for spherical aberration and coma) will produce an excellent aerial image of an on-axis point source. However, grazing incidence X-ray telescopes exhibit very severe off-axis aberrations. Figure 3-3 illustrates the HPR of the geometrical PSF vs. field angle in the paraxial focal plane for a near-aplanatic hyperboloid-hyperboloid grazing incidence X-ray telescope design with the SXI 1st-order design parameters. Included in the same graph is the HPR vs. field angle of the RDPSF and the AUDPSF where we have assumed the 15.8 μm (5.0 arc sec) detector pixels to be used in the SXI instrument. Note that the image quality is clearly “detector-limited” for small field angles and “aberration-limited” for large field angles.

Figure 3-3. Comparison of the HPR of the aerial PSF, the RDPSF, and the AUDPSF in the paraxial focal plane of a near-aplanatic grazing incidence X-ray telescope design.

It is common practice to despace the operational focal plane of a wide-field imaging system to balance field curvature with defocus, thus improving wide-field image quality (of the aerial
image) at the expense of small-field image quality. However, when detector effects are included, there is virtually no small-field image degradation due to this despacing operation until the resulting defocused on-axis geometrical PSF exceeds the detector pixel size. A despace of 55 μm is allowed in the SXI design before the defocused geometrical PSF (annulus) completely fills a detector pixel.

Figure 3-4 illustrates the HPR of the geometrical PSF vs. field angle for the despaced near-aplanatic hyperboloid-hyperboloid grazing incidence X-ray telescope design, and again compares it with the HPR vs. field angle of the RDPSF and the AUDPSF. Note that the geometrical performance (no detector effects) is improved substantially for large field angles at the expense of degraded small-field performance. And indeed, when detector effects are included, the wide-angle performance is improved substantially with virtually no additional degradation at small field angles.

![Graph](image)

Figure 3-4. Comparison of the HPR of the aerial PSF, the RDPSF, and the AUDPSF of a near-aplanatic grazing incidence X-ray telescope with a despaced focal plane.
We will now determine precisely what value of despace will minimize the field-weighted-average $HPR$ defined by Eq.(3-1). Using the near-aplanatic grazing incidence X-ray telescope design defined by the Zemax Lens Editor values listed in Table 3-1, we performed extensive ray trace analyses and calculated the $HPR$ of the $AUDPSF$ for a variety of field angles and despace values as illustrated in Figure 3-5 (a). We then used Eq.(3-1) and performed a 2-dimensional integration of this data to produce the $HPR_{fwa}$ vs. $OFOV$ curves illustrated in Figure 3-5 (b). Note that the on-axis ordinate values of these two sets of curves are the same, but the off-axis values of the curves in Figure 3-5(b) are substantially reduced due to the averaging process. Finally, to find the optimum despace for a given $OFOV$, we plotted the $HPR_{fwa}$ at that $OFOV$ as a function of despace. This curve is shown in Figure 3-5(c) for an $OFOV$ of 21 arc min. From the curve in Figure 3-5(c) it is obvious that the optimum focal plane despace for a 21 arc min $OFOV$ is about 124 μm. This value of despace produces an $HPR_{fwa}$ of about 5.0 arc sec, down from 6.5 arc sec for the paraxial focal plane. This procedure has been repeated for different $OFOV$’s to obtain a plot of the optimally despaced $HPR_{fwa}$ as a function of the $OFOV$. In Figure 3-5(d), this curve is compared to the performance of the system when the detector is despaced by 55 μm and when the mosaic detector array is positioned in the paraxial focal plane.

Table 3-1. ZEMAX lens editor values for SXI baseline design.

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Figure 3-5. (a) Illustration of the $HPR$ vs. field angle of the $AUDPSF$ for a variety of focal plane despace values, (b) $HPR_{fwa}$ vs. $OFOV$ for the same data, (c) $HPR_{fwa}$ vs. focal plane despace for $OFOV = 21$ arc sec, and (d) Additional improvement in image quality when the focal plane despace is optimized for each $OFOV$.

It is clear from Figure 3-5(d) that there is considerable improvement in the field-averaged image quality produced by a given telescope design if the focal plane position is optimized for a particular $OFOV$. And the optimum amount of despace clearly depends upon the size of the detector pixels. The obvious question that arises is whether we can achieve even better performance (field-averaged image quality) if we include the detector effects in the optical design process that determines the telescope mirror prescription. This may mean that we get an inferior aerial image in the focal plane (produced by the telescope alone) even though the final system performance (including detector effects) is improved.
3.4 Including the Detector Effects in the Optical Design Process

As previously discussed in Chapter 2, a family of optimal grazing incidence hyperboloid-hyperboloid X-ray telescope designs was developed, where each member of the family is the optimum design for a different $OFOV$. Figure 2-6 illustrated the geometrical rms image radius vs. field angle for this optimal family of hyperboloid-hyperboloid grazing incidence X-ray telescope designs with the SXI 1st-order design parameters. The different optical designs represented by the curves in Figure 2-6 are designated by the field angle, $\theta_B$, at which the rms image radius was minimized.

We have included detector effects in the optical design process determining the telescope mirror prescriptions through the procedure similar to that described in the previous section. Figure 3-6(a) illustrates the $HPR$ of the $AUDPSF$ for a variety of optimal hyperboloid-hyperboloid optical designs vs. field angle. We again used Eq.(3-1) and performed a 2-dimensional integration of this data to produce the $HPR_{fwa}$ vs. $OFOV$ curves illustrated in Figure 3-6(b). To find the optimum optical design for a given $OFOV$, we again plotted the $HPR_{fwa}$ at that $OFOV$ as a function of the parameter $\theta_B$ which defines the different members of the family of optimal optical designs. This curve is shown in Figure 3-6(c) for an $OFOV$ of 21 arc min. From the curve in Figure 3-6(c) we see that the optimum optical design for a 21 arc min $OFOV$ is designated by $\theta_B = 13.5$ arc min. This procedure has been repeated for different $OFOV$'s to obtain a plot of the optimal hyperboloid-hyperboloid grazing incidence X-ray telescope designs as a function of the $OFOV$. This curve is illustrated in Figure 3-6 (d) and compared to the performance of the system when an aplanatic optical design is used with the detector optimally despaced for each $OFOV$, when an aplanatic optical design is despaced by
55 µm, and when the mosaic detector array is positioned in the paraxial focal plane of an aplanatic optical design.

Figure 3-6. (a) Illustration of the HPR vs. field angle of the AUDPSF for a variety of optimal optical designs, (b) $HPR_{fw}$ vs. $OFOV$ for the same data, (c) $HPR_{fw}$ vs. optical design parameter $\theta_B$ for an $OFOV = 21$ arc sec, and (d) Illustration of additional improvement in image quality when the optical design is optimized for each $OFOV$.

Minimizing the $HPR_{fw}$ over a given $OFOV$ will maximize the number of resolution elements, $N$, over that $OFOV$. And, of course, increasing the number of angular resolution elements over the $OFOV$ increases the amount of information in the image. Figure 3-7 illustrates the number of angular resolution elements as a function of the $OFOV$ for the AUDPSF for four situations: (a) an aplanatic grazing incidence X-ray telescope design with the SXI 1st-order design parameters having a mosaic detector array in the paraxial focal plane, (b) the same
aplanatic grazing incidence X-ray telescope design with the mosaic detector array despaced until the defocused geometrical PSF just fills a detector pixel, (c) the same optical design with the mosaic detector array optimally despaced for each OFOV, and finally (d) having the optimal hyperboloid-hyperboloid optical design for each OFOV.

From the curves in Figure 3-7 we conclude that: (a) for \(OFOV < 9\) arc min, the detector effects are so dominant that all four situations provide the same result; i.e., there is no penalty in performance for using the classical Wolter Type I design (no advantage to the aplanatic design); (b) for \(9 < OFOV < 21\) arc min, despacing the detector until the geometrical PSF just fills a detector significantly improves the system performance; (c) for \(OFOV > 21\) arc min, optimally despacing (by more than 55 \(\mu m\)) the detector for each \(OFOV\) yields even more improvement in
wide-field performance with no loss in small-field performance. However, for $OFOV > 13$ arc min, further substantial improvement in optical performance can be obtained by balancing detector effects with geometrical aberrations. This requires a different optimum (non-aplanatic) hyperboloid-hyperboloid optical design for each $OFOV$. More specifically, we see that for a 30 arc min $OFOV$, the optimum optical design yields a 50.4% increase in the number of angular resolution elements over that obtained with an aplanatic design with the mosaic detector array located in the paraxial focal plane, and a 10.9% improvement over the aplanatic design with an optimally despaced detector array. The improvement clearly decreases with decreasing $OFOV$. For example, for a 21 arc min $OFOV$ there is approximately a 41% increase in performance over the aplanatic design with the detector in the paraxial focal plane and a 9.5 % increase over the aplanatic design with an optimally despaced detector array.
4.0 PERFORMING A SYSTEMS ENGINEERING ANALYSIS OF IMAGE QUALITY

A complete systems engineering analysis of the SXI X-ray telescope performance requires that we look at the effects of aperture diffraction, geometrical aberrations, surface scattering, and all other potential error sources such as assembly and alignment errors and metrology errors that appear in the mirror manufacturer’s error budget tree\textsuperscript{40}.

A linear systems approach (multiplying MTFs or convolving PSFs) to performing a complete systems engineering analysis of image quality has been used in the SXI program. Figure 4-1 illustrates the preliminary error budget tree constructed at the beginning of the program.

Figure 4-1. The SXI error budget tree, indicating the usual practice of considering detector effects only after the imaging system has been designed to produce the best possible aerial image.
A tops-down process of assigning initial error budget allocations is usually done from past experience or preliminary analysis. As the program goes forward the error budget tree becomes a living document in which the initial allocations are replaced with actual achieved values as the system is designed, and components are fabricated, assembled, and aligned. If requirements based on this detailed bottoms-up re-allocation of errors are achieved with margin to spare in some areas, further re-allocations can be made to relax other requirements (thus reducing cost and schedule) while maintaining the top-level image quality requirement. Note the symbolic notation in Figure 4-1 indicating that the individual PSF’s from diffraction effects, geometrical aberrations, surface scattering phenomena, and the composite effects of all the errors in the mirror manufacturer’s error budget tree will be convolved to obtain the aerial image in the focal plane. This process is perhaps illustrated more clearly in Figure 4-2.

Figure 4-2. Illustration of the individual image degradation mechanisms and their resulting convolution (aerial image or systems PSF) of the SXI telescope for a field angle of 15 arc min and a wavelength of 44.7Å.
Most commercially-available optical design and analysis software packages are based upon geometrical ray-trace analyses. They therefore do an excellent job of modeling the effects of geometrical aberrations. Most of those codes also provide an adequate diffraction analysis for optical systems with clear apertures or for modest obscuration ratios; however, they fail to provide accurate diffraction analysis for the very large obscuration ratios inherent to grazing incidence X-ray telescopes. Also none of the commercially-available codes adequately model the image degradation effects of surface scatter phenomena or detector effects. We have therefore developed our own MATLAB code to perform the complete systems engineering analysis of image quality presented in this chapter. We will now discuss the calculation of each of the individual PSF’s making up the aerial image before proceeding to show their combined effect upon the image quality of the SXI telescopes.

### 4.1 Diffraction Effects of Highly-obscured Annular Apertures

The diffraction-limited imaging performance of annular apertures has been discussed in detail by Tschunko.\textsuperscript{41} The irradiance distribution of an aberration-free image formed by an annular aperture with a linear obscuration ratio of $\varepsilon$ is given by the expression

$$I(x) = \frac{1}{(1-\varepsilon^2)^2} \left[ 2 \frac{J_1(x)}{x} - \varepsilon^2 2 \frac{J_1(\varepsilon x)}{\varepsilon x} \right]^2 \tag{4-1}$$

where $x$ is a normalized (dimensionless) radius in the focal plane,

$$x = \frac{\pi r}{\lambda f / D} \tag{4-2}$$

The diffraction pattern of an annular aperture changes significantly with the obscuration ratio. For $\varepsilon < 0.6$ the majority of the energy is concentrated in the central lobe of the diffraction
pattern.\textsuperscript{42} For large obscuration ratios, the cross terms in the above squared modulus represent a dominant interference effect that produces an irradiance distribution made up of ring groups as illustrated in Figure 4-3.

![Image of annular aperture with obscuration ratio \( \varepsilon = 0.8 \) and its diffraction pattern.]

Figure 4-3. Annular aperture with obscuration ratio \( \varepsilon = 0.8 \) and its diffraction pattern.

Tschunko has calculated the normalized irradiance versus the normalized radius in the Fraunhofer diffraction pattern of a variety of annular apertures with obscuration ratios ranging from \( \varepsilon = 0 \) to \( \varepsilon = 0.99 \) and displayed the results in a log-log scale as shown in Figure 4-4\textsuperscript{41}

![Image of normalized irradiance versus normalized image radius for different obscuration ratios ranging from \( \varepsilon = 0 \) to \( \varepsilon = 0.99 \).]

Figure 4-4. Normalized Irradiance versus normalized image radius for different obscuration ratios ranging from \( \varepsilon = 0 \) to \( \varepsilon = 0.99 \).
Tschunko also shows that, for $\varepsilon > 0.8$, the number of rings in each ring group does not vary with obscuration ratio and is given by

$$n = \frac{2}{1 - \varepsilon}.$$  \hspace{1cm} (4-3)

Additional insights from Tschunko’s work, in terms of fractional encircled energy for different obscuration ratios can be gained from the Figure 4-5 below. Note that 90% of the energy is contained within the first ring group and 95% within the second ring group, independent of the obscuration ratio. This is compared to 84% of the energy in the central lobe of the Airy pattern produced by an unobscured circular aperture. The central ring group clearly replaces the Airy disc as the meaningful image size. The central lobe itself contains a very small fraction of the energy and in no way represents a meaningful image size or resolution.\(^{41}\)

![Fractional Encircled Energy](image)

**Figure 4-5.** Fractional Encircled Energy for different obscuration ratios.

For a Wolter Type I X-ray telescope with an obscuration ratio $\varepsilon = 0.98$, there are 100 rings per ring group. Harvey\(^{42}\) pointed out that the angular radius of the image is thus given by
\[ \theta_0 = \frac{2\lambda}{D(1 - \varepsilon)}, \]  

which is almost two orders of magnitude larger than the Airy disc of an un-obscured circular aperture of the same diameter. Thus, diffraction effects in X-ray telescopes are not necessarily negligible, since the effects of the high obscuration ratios inherent in grazing incidence optics offsets the effects of the very short X-ray wavelengths.

This diffraction-limited PSF (or ADPSF) must be convolved with the geometrical PSF, which must in turn be convolved with the scattering function and the cumulative effects of all the other miscellaneous error sources. However, the diffraction rings described by Eq.(4-1) and illustrated in Figure 4-4 requires high sampling density and associated computational problems are encountered when performing these convolutions numerically. Therefore, the following semi-empirical expression for the diffraction-limited irradiance distribution in the focal plane was developed:

\[ I(x) = I_0 \left[ \left( \frac{x}{0.643} \right)^3 + 1 \right]^{-1/3} \sin \left[ \frac{x(1 - \varepsilon)/2}{x(1 - \varepsilon)/2} \right] \]  

This formula is an approximation to the actual behavior of the diffraction-limited PSF of highly obscured annular apertures. It does not model the high-spatial-frequency diffraction rings, but does accurately describe the envelope of the diffraction rings as illustrated in Figure 4-6. Since the high-spatial-frequency diffraction rings would be smoothed out upon convolution with the other functions anyway (the geometrical PSF, surface scatter PSF, and the residual miscellaneous error PSF), the above semi-empirical equation provides sufficient accuracy for our calculations.
Figure 4-6. The ADPSF approximation for Grazing Incidence telescope

Figure 4-7 illustrates the full two-dimensional aperture diffraction PSF that is created in our MATLAB program to model the diffraction effects for the grazing incidence SXI telescopes.

Figure 4-7. The two-dimensional Aperture Diffraction PSF for grazing incidence X-ray telescopes as simulated in our MatLab code.
4.2 Geometrical Performance of the HT#17 Optical Design

In Chapter 2.0 a family of optimal hyperboloid-hyperboloid grazing incidence X-ray telescope designs was discussed, where each member of the family was optimized for a different operational field-of-view (OFOV). The design characterized by $\theta_B = 12.1$ in Figure 2-6 yields geometrical image sizes that are nearly the same on-axis and at a field angle of 16.5 arc min, slightly above the solar limb, and is thus similar to the design chosen for the SXI mission. The specific hyperboloid-hyperboloid design chosen for the SXI mission was optimized for an 18 arc-min OFOV, and was designated as HT#17. Its surface data summary, as simulated with ZEMAX is shown in Table 4-1, and the 1st-order optical system design parameters that were constrained during the ZEMAX optimization process are listed in Table 4-2.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Type</th>
<th>Radius</th>
<th>Thickness</th>
<th>Glass</th>
<th>Semi-Diameter</th>
<th>Conic</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBJ</td>
<td>Standard</td>
<td>Infinity</td>
<td>Infinity</td>
<td>-</td>
<td>Infinity</td>
<td>0</td>
</tr>
<tr>
<td>1*</td>
<td>Standard</td>
<td>Infinity</td>
<td>75</td>
<td>-</td>
<td>81.96441520</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Standard</td>
<td>Infinity</td>
<td>50</td>
<td>-</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Standard</td>
<td>Infinity</td>
<td>50</td>
<td>-</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Standard</td>
<td>Infinity</td>
<td>1298.38079460</td>
<td>-</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>STO*</td>
<td>Standard</td>
<td>-2.31585269</td>
<td>-697.51745069</td>
<td>Mirror</td>
<td>81.50626000</td>
<td>-1.00008509</td>
</tr>
<tr>
<td>6*</td>
<td>Alternate</td>
<td>-2.52172061</td>
<td>4.136653895</td>
<td>Mirror</td>
<td>80</td>
<td>-1.00735892</td>
</tr>
<tr>
<td>7</td>
<td>Standard</td>
<td>Infinity</td>
<td>0</td>
<td>-</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>IMA</td>
<td>Alternate</td>
<td>Infinity</td>
<td>-</td>
<td>-</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4-2. Input Optical System Parameters for SXI telescopes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_j$</td>
<td>655.0 mm</td>
<td>Nominal focal length (from joint)</td>
</tr>
<tr>
<td>$r_j$</td>
<td>80.0 mm</td>
<td>Mirror joint radius</td>
</tr>
<tr>
<td>$L_p$</td>
<td>47.5 mm</td>
<td>Length of primary mirror</td>
</tr>
<tr>
<td>$L_s$</td>
<td>47.5 mm</td>
<td>Length of secondary mirror</td>
</tr>
<tr>
<td>gap</td>
<td>5.0 mm</td>
<td>Width of gap between primary and secondary mirrors.</td>
</tr>
</tbody>
</table>
The geometrical performance of the HT#17 grazing incidence X-ray telescope design, as determined from real ray-trace data using ZEMAX, is illustrated in Figure 4-8. The geometrical rms image radius in arc-sec is plotted versus field angle for several different axial focal planes. Note that the optimum focal plane for an 18 arc-min radius OFOV (as determined by the technique discussed in Chapter 2.0) lies 107 μm inside of the best axial focus.

Figure 4-8. Geometrical rms image radius versus field angle for several different axial focal planes.

Approximately 10,000 rays are traced, only about 350 of which pass through the highly obscured annular aperture, for each data point in the above family of curves. To adequately model the two-dimensional geometrical PSF (GPSF) depicted in Figure 4-2, many more rays are required. We therefore trace approximately 1,000,000 with ZEMAX, of which about 35,000 pass thru the highly obscured annular aperture and reach the focal plane to produce a conventional spot diagram as illustrated in Figure 4-9.
This ray intercept data is then imported into a MATLAB code that creates a numerical grid and performs a binning operation upon the data. The number of rays in each bin to is then calculated to obtain the two-dimensional ray density function, or GPSF, shown in Figure 4-10.

This GPSF can then be numerically convolved with the other PSF’s illustrated in Figure 4-1 to achieve a systems engineering analysis of image quality.
4.3 Image Degradation due to Surface Scatter Phenomena

When light is reflected from an imperfect optical surface, the reflected radiation consists of both specularly reflected radiation and diffusely reflected, or scattered, radiation as illustrated schematically in Figure 4-11. The light scattered from optical surface irregularities degrades optical performance in several different ways: a.) it reduces optical throughput since some of the scattered radiation will not even reach the focal plane, b.) the wide-angle scatter will produce a veiling glare which reduces image contrast or signal-to-noise ratio, and c.) the small-angle scatter will decrease resolution by broadening the image core.

![Figure 4-11](image)

Figure 4-11. Residual optical fabrication errors result in diffusely reflected, or scattered, radiation which, at X-ray wavelengths, can dominate both diffraction effects and geometrical aberrations in the degradation of image quality.

The surface height distribution function and the surface autocovariance function shown in Figure 4-12 are the relevant statistical parameters associated with scattered light behavior from a random rough surface. The rms roughness determines how much light is scattered out of the specular beam and the surface autocovariance function determines the angular distribution of the scattered light. Since the amount of scattered light is determined by the surface roughness relative to the wavelength, X-ray imaging systems require super-smooth optical surfaces.
The surface autocovariance (ACV) function and the surface power spectral density (PSD) function are related by the Fourier transform operation as illustrated in Figure 4-13. Note that the peak value of the ACV function is the surface variance (square of the rms surface roughness), and, from the autocorrelation theorem of Fourier transform theory, the area (volume) under the surface PSD function is equal to the surface variance.
Surface scatter phenomena is really a diffraction process where the diffracting “aperture” is a random phase function rather than a binary amplitude function. Two approaches that are commonly used in the scatter community to perform the necessary diffraction calculations are the Rayleigh-Rice (R-R) scatter theory$^{43}$ and the Beckmann-Kirchoff (B-K) scatter theory.$^{44}$ The classical vector diffraction theory of R-R agrees well with experimental observations for scatter from “smooth” surfaces for all angles of incidence, and scattered angles. This agreement between theory and experiment weakens when the surface-height deviations have amplitudes much more than $\frac{\lambda}{100}$, where $\lambda$ is the wavelength of the scattered radiation. On the other hand, the classical scalar diffraction theory of B-K holds for rougher surfaces, where surface errors have amplitudes on the order of $\frac{\lambda}{10}$. However, this is true only if a small angle of incidence is used and if the scatter occurs at small angles. Thus, although the B-K theory can treat surfaces that are much rougher than R-R can handle, the B-K theory has small-angle assumptions inherent in its derivation. These assumptions raise questions about the B-K theory’s ability to accurately handle wide-angle scattering and large angles of incidence. Other useful references dealing with surface scatter for general optical applications include *Introduction to Surface Roughness and Scattering* by J.M. Bannet and L. Mattsson$^{45}$ *Optical Scattering, Measurement and Analysis* by John C. Stover$^{46}$ and a various papers published over the years by Eugene Church$^{47-51}$ Aschenbach provides a nice discussion of the effects of surface scatter phenomena upon the specific application of X-ray telescopes.$^{52}$

In 1976 Harvey and Shack formulated a surface scatter theory in a linear systems format resulting in a surface transfer function (STF) that relates scattering behavior to surface topography.$^{53,54}$ Insight into the scattering process was inferred by considering the nature of the surface transfer function, and its Fourier transform, the angle spread function (ASF) shown in
Figure 4-14. The constant component of the transfer function transforms into a delta function (specularly reflected radiation) and the bell-shaped component transforms into a bell-shaped scattering function as illustrated on the right side of Figure 4-14. Note that this ASF is scattered \textit{radiance}, not irradiance or intensity. This was consistent with the fact that the bi-directional reflectance distribution function (BRDF) was defined as reflected \textit{radiance} divided by incident irradiance.\textsuperscript{55}

Figure 4-14. The surface transfer function and the associated angle spread function.

For a broad class of scattering surfaces, including optical surfaces polished with conventional techniques on ordinary materials, the ASF exhibited shift-invariant behavior in direction cosine space with respect to incident angle as illustrated in Figure 4-15. This led to a modest following among the radiometric community of BRDF curves plotted in the Harvey-Shack $\beta - \beta_0$ format.

Bob Breault made extensive use of this format in building a catalog of BRDF data from various materials and surfaces for use in his APART baffle design program.\textsuperscript{56} Today the ASAP, Trace-Pro, and ZEMAX codes all use some form of the Harvey-Shack surface scatter theory.\textsuperscript{57-59}
During the 1980’s the STF was modified to include the extremely large incident angles inherent to grazing incidence Wolter Type I X-ray telescopes. This still required no explicit smooth surface approximation, and was successfully used to model image degradation due to residual optical fabrication errors in grazing incidence X-ray telescopes.60-62

For the purpose of including surface scatter effects in our systems engineering analysis of image quality for the SXI program, we exercised the computer program EEGRAZPC which was developed by the Perkin-Elmer Corporation for NASA/GSFC for use on the Advanced X-ray Astrophysical Facility (later re-named the Chandra Observatory)63

The EEGRAZPC program calculates the one-dimensional surface scatter profile produced by a single “barrel stave” of a grazing incidence X-ray telescope as shown in Figure 4-16. An assumed (or measured) surface autocovariance function is provided as input for the EEGRAZPC program. This one-dimensional scatter profile was then input into our MATLAB program which converted it into a two-dimensional surface scatter point spread function (SSPSF) by integrating
azimuthally around the annular aperture and adding up the associated one-dimensional scatter functions as schematically illustrated in Figure 4-16(c).

Figure 4-16. (a) Grazing incidence X-ray telescope, (b) one-dimensional scattered irradiance distribution in the focal plane due to a single barrel stave, or azimuthal element of the telescope, and (c) the final two-dimensional scatter function obtained by adding up all the one-dimensional scatter functions while azimuthally integrating around the annular aperture of the telescope.

Information concerning the relevant surface statistics (residual optical fabrication errors) must be provided as input to the EEGRAZPC code in the form of either an average axial surface ACV function or an average axial surface PSD function. The surface PSD is a convenient specification for residual optical fabrication errors, and becomes a requirement for the mirror manufacturer. The two-dimensional SSPSF (see Figure 4-17) can then be numerically convolved with the other PSF’s illustrated in Figure 4-2 to obtain the aerial image.

Figure 4-17. Surface scatter PSF produced from the 1-D data provided by EEGRAZPC.
4.4 Miscellaneous Errors in the Mirror Manufacturer’s Error Budget Tree

Refer back to the error budget tree in Figure 4-1, and the schematic illustration of our technical approach to performing a complete systems engineering analysis of image quality for the SXI telescopes provided in Figure 4-2. From Figure 4-1 we see that the fourth contributor to the aerial image (labeled RSEPSF) is really the composite effect of a variety of miscellaneous error sources from the mirror manufacturer’s error budget tree that affect the image core. There are contributions from optical fabrication errors, assembly and alignment errors, environmental errors, and an original reserve allocation included by the program manager of the subcontract for manufacturing the SXI telescope mirrors. Each of these categories was further broken down into individual errors contributing to the final image degradation. The reserve allocation was eventually re-distributed to other error sources as experience was gained from manufacturing and testing the five SXI mirrors.

These miscellaneous error sources associated with the manufacture of the SXI mirrors all contribute to the width of the final image core, as opposed to the “wings” of the aerial image produced by the scatter effects discussed in the previous section.\(^6^4\) However, like the scattering effects discussed in the last section, a given azimuthal element of the SXI mirror produces a one-dimensional broadening of the image core (due to the extreme grazing angle). And since there are many different contributing error sources, we can invoke the central limit theorem of Fourier transform theory and assume that the composite effect of the various error sources is to form a one-dimensional Gaussian intensity for the image core.\(^6^3\)-\(^6^5\) This one-dimensional image core (produced by a differential azimuthal element of the SXI telescope) is then converted into a two-dimensional residual surface error point spread function (RSEPSF) by integrating
azimuthally around the annular aperture. This was shown by Glenn to be equivalent to dividing the one-dimensional distribution by $2\pi r$. The resulting two-dimensional RSEPSF (see Figure 4-18) can then be numerically convolved with the other PSF’s illustrated in Figure 4-2 to obtain the aerial image.

![Residual surface error point spread function (RSEPSF)](image)

Figure 4-18. Residual surface error point spread function (RSEPSF) produced from the mirror manufacturer’s error budget tree.

### 4.5 Modeling the Aerial Image of the HT#17 Optical Design

The system PSF, or aerial image, produced by a grazing incidence X-ray telescope includes image degradation due to the effects of aperture diffraction, geometrical aberrations, surface scatter phenomena, and all of the miscellaneous residual error sources included in the mirror manufacturer’s error budget tree. A MATLAB code, called PSFGraz, has been developed by the Optical Design and Image Analysis Laboratory at CREOL specifically for the SXI program. The PSFGraze executive program, or driver program, executes a series of MatLab modules (each containing sub-modules and those likewise consisting of lower level or standardized sub-modules) which ultimately calculates the aerial point spread function (APSF) or system PSF for a given grazing incidence telescope configuration. The software calculates an APSF for a
particular wavelength, obscuration ratio, focal length, surface scatter statistics, field-angle, etc.

Figure 4-19 is a flow chart of the PSFGraz MatLab Code.

![Flow Chart](image)

Figure 4-19. Flow Chart for the PSFGraz MatLab Code.

The four functions discussed in the previous sections of this chapter are each calculated in a particular module of the PSFGraz Code. The heart of the PSFGraz Code is the convolution engine that numerically convolves those four functions. Once the user has obtained a particular APSF, operating conditions (inputs) can be altered to account for various field positions and wavelengths. PSFGraze must be run again for any change in these operational parameters.
Recall that the ADPSF depends upon the obscuration ratio of the grazing incidence mirrors and the wavelength (or X-ray energy) of the incident radiation. Likewise, the geometrical ray intercept data obtained by using the ZEMAX ray trace code must be input into the PSFGraz Code to obtain the GPSF. The GPSF is a strong function of field angle due to the severe field-dependent aberrations of grazing incidence X-ray telescopes. Similarly, the EEGRAZPC code is used to obtain the one-dimensional surface scatter profile, which is then input into the PSFGraz Code to obtain the SSPSF. The SSPSF depends upon the grazing angle of the X-ray mirrors, the wavelength of the incident radiation, and the surface roughness (ACV or PSD) of the mirrors. The contractual requirement for the surface PSD that was imposed upon the mirror manufacturer is illustrated in Figure 4-20.

![Figure 4-20](image-url)
The form of the required PSD is given by the following equation, and the values of the constants involved are included in Figure 4-20.

\[
PSD(f) = \sqrt{\pi} \ell_1 \sigma_1^2 e^{-(\sigma_1 f)^2} + \sqrt{\pi} \ell_2 \sigma_2^2 e^{-(\sigma_2 f)^2} + \frac{2\ell_3 \sigma_3^2}{1 + (2\pi \ell_3 f)^2}.
\] (4-6)

An example of the PSFGraz output is the aerial image illustrated below in Figure 4-21. This particular example is for the HT#17 grazing incidence X-ray telescope design operating at a field angle of 15 arc min, at a wavelength of 44.7 Å, with the required surface PSD illustrated in Figure 4-20.

Figure 4-21. Illustration of the system PSF (or aerial image) of the HT#17 X-ray telescope design operating at a field angle of 15 arc min and a wavelength of 44.7 Å.

The contractual requirement imposed upon the mirror manufacturer was expressed in terms of the fractional encircled energy (at best axial focus) for an on-axis point source and one at a field angle of 20 arc min. The PSFGraz Code thus provided fractional encircled energy curves as illustrated in Figure 4-22 and Figure 4-23.
Figure 4-22. On-axis fractional encircled energy at best axial focus for the HT#17 X-ray telescope design for a wavelength of 13.3 Å.

Figure 4-23. Fractional encircled energy at best axial focus for the HT#17 X-ray telescope design at a field angle of 20 arc min and a wavelength of 13.3 Å.
By plotting the fractional encircled energy curves for the ADPSF, GPSF, SSPSF, and the RSEPSF, as well as the aerial image; we can readily see which image degradation mechanism dominates for a given set of conditions. For example, note that for small field angles, scatter effects dominate geometrical aberrations; however, for large field angles, geometrical aberrations dominate scatter effects.

A comparison of the contractual requirements (and goals) upon fractional encircled energy at best axial focus, and the image quality predictions made with the PSFGraz Code are shown in Table 4-3 below. Note that our image quality predictions indicate the requirement will be satisfied in all cases, and the goals will all be satisfied except for on-axis aerial image for a wavelength of 44.7 Å.

Table 4-3. Performance predictions for HT#17 compared to requirements and goals.

<table>
<thead>
<tr>
<th>Fractional Encircled Energy</th>
<th>On-axis (θ = 0.0) (5.0 arc sec Dia. Circle)</th>
<th>Off-axis (θ = 20 arc min) (20.0 arc sec Dia. Circle)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>λ = 44.7 Å</td>
<td>λ = 13.3 Å</td>
</tr>
<tr>
<td>Req’mt</td>
<td>0.55</td>
<td>0.41</td>
</tr>
<tr>
<td>Goal</td>
<td>0.60</td>
<td>0.45</td>
</tr>
<tr>
<td>CREOL Prediction</td>
<td>0.56</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Although the contractual requirements were express in terms of fractional encircled energy at best axial focus (a carry-over from government contracts on stellar X-ray telescopes), the SXI is a wide-field staring telescope and the HT#17 optical design has been optimized for an 18 arc min operational field of view. The operational focal plane position for that optimum design is located 0.107 mm inside of best axial focus. By performing the ray trace analyses at the operational
focal plane for various field angles, performing the diffraction and scattering analysis at two different wavelengths (13.3 Å and 44.7 Å), executing the necessary numerical convolutions to obtain the various aerial images, then calculating the half power radius (HPR) for each one, we have produced Figure 4-24 which nicely summarizes the expected aerial image quality of the HT#17 optical design.

![Figure 4-24. HPR versus field angle for the aerial image of the HT#17 X-ray telescope design.](image)

### 4.6 Systems Engineering Analysis of the “as-manufactured” SXI Telescopes

As each of the SXI telescope mirrors (four flight models and a spare) were fabricated and tested by Goodrich Optical Systems, the mirror metrology data was used to make image quality predictions to assure compliance with the top-level image quality requirements. Understanding the hyperboloid-hyperboloid grazing incidence X-ray telescope design in detail is necessary in order to model the “as-manufactured” telescopes with the ZEMAX ray trace code.
An optical layout of the hyperboloid-hyperboloid grazing incidence X-ray telescope was shown in Figure 2-5, but is duplicated here as Figure 4-25. The origin of the coordinate system is located at the front of the front edge of the primary mirror. Recall that the front focus of the primary mirror does not coincide with the rear focus of the secondary mirror as is the case with the classical Wolter Type I design. This \textit{confocal delta} is indicated as the quantity $\Delta_{ps}$ in Figure 2-5. Similarly, the system focal plane does not lie at the front focus of the secondary mirror. This displacement is indicated as $\Delta_f$. The five independent optical design parameters that define the optical prescription are listed in Table 4-4. Several additional optical design parameters and the axial locations of all relevant geometrical features are listed in Table 4-5.

![Optical layout of hyperboloid-hyperboloid grazing incidence X-ray telescope design.](image)

**Table 4-4. Input ZEMAX Optical Design Parameters for the HT#17 Optical Design**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{vp}$</td>
<td>-2.3158526900 mm</td>
<td>Primary mirror vertex radius of curvature</td>
</tr>
<tr>
<td>$k_p$</td>
<td>-1.0000850860</td>
<td>Primary mirror conic constant.</td>
</tr>
<tr>
<td>$R_{vs}$</td>
<td>-2.5217206144 mm</td>
<td>Secondary mirror vertex radius of curvature</td>
</tr>
<tr>
<td>$k_s$</td>
<td>-1.0073589211</td>
<td>Secondary mirror conic constant.</td>
</tr>
<tr>
<td>$S_{vv}$</td>
<td>-697.5174506900 mm</td>
<td>Vertex to vertex separation</td>
</tr>
</tbody>
</table>
Table 4-5. Other Redundant but Useful Optical Design Parameters

\[ \varepsilon_p = \sqrt{-k_p} \]  
Primary mirror eccentricity

\[ \varepsilon_s = \sqrt{-k_s} \]  
Secondary mirror eccentricity

\[ a_p = -\frac{R_{vp}}{\varepsilon_p^2 - 1} \]  
Primary mirror hyperboloid constant

\[ b_p = -\frac{R_{vp}}{\sqrt{\varepsilon_p^2 - 1}} \]  
Primary mirror hyperboloid constant

\[ a_s = -\frac{R_{vs}}{\varepsilon_s^2 - 1} \]  
Secondary Mirror hyperboloid constant

\[ b_s = -\frac{R_{vs}}{\sqrt{\varepsilon_s^2 - 1}} \]  
Secondary Mirror hyperboloid constant

\[ z_j = 0 \]  
Origin of coordinate system.

\[ z_j = z_1 + L_p + \frac{gap}{2} \]  
Axial position of joint (mm)

\[ z_f = z_j + f_j \]  
Axial position of design focal plane (mm)

\[ z_{cp} = z_j + a_p \sqrt{1 + (r_j / b_p)^2} \]  
Center; primary hyperboloid (mm)

\[ z_{pv1} = z_{cp} - a_p \]  
Axial position of pri. mirror vertex (mm)

\[ z_{pf1} = z_{cp} - a_p \varepsilon_p \]  
Axial position of pri. mirror front focus (mm)

\[ z_{sv1} = z_{pv1} + S_{vv} \]  
Axial position of sec. mirror vertex (mm)

\[ z_{sf1} = z_{sv1} - a_s (\varepsilon_s - 1) \]  
Axial position of sec. mirror front focus (mm)

\[ z_{sf2} = z_{sf1} + 2a_s \varepsilon_s \]  
Axial position of sec. hyp back focus (mm)

\[ z_{cs} = z_{sv1} + a_s \]  
Axial position of center of sec. hyp. (mm)

\[ z_{sv2} = z_{sv1} + 2a_s \]  
Axial position of sec. hyp. back vertex (mm)

\[ \Delta f = z_f - z_{sf1} \]  
Delta Focus (mm)

\[ \Delta_{ps} = z_{sf2} - z_{pf1} \]  
Confocal Delta (mm)
The primary and the secondary mirror surface profiles of the hyperboloid-hyperboloid optical design are given by the following equations

\[ r_p = \frac{b_p}{a_p} \sqrt{(z-z_{cp})^2 - a_p^2} \quad \text{(primary mirror)} \]  \hspace{1cm} (4-7)

\[ r_s = \frac{b_s}{a_s} \sqrt{(z-z_{cs})^2 - a_s^2} \quad \text{(secondary mirror).} \]  \hspace{1cm} (4-8)

The mirror profiles of the HT#17 optical design are illustrated in Figure 4-26. The SXI telescope 1st-order system parameters are also listed, as are the defining parameters for the specific hyperboloid-hyperboloid design, and the five (redundant) optical prescription parameters required by the ZEMAX ray trace code.

Figure 4-26. HT#17 design mirror profiles, with the 1st-order system parameters, the specific hyperboloid-hyperboloid design parameters, and ZEMAX optical prescription listed.
Note that the design length of the primary and the secondary mirror is 47.5 mm with a 5.0 mm gap at the joint (100 mm total length), and the radius at the mirror joint is 80 mm. The nominal focal length is 655 mm, with the joint focal length just slightly greater than that. The actual focal length (measured from the principle/nodal point) is 659.9036 mm.

The optical fabrication process consists of a series of fabrication/testing cycles. The mirror metrology data is obtained from three different instruments: (1) the WEGU instrument measures absolute diameter at the forward and aft ends of both the primary and the secondary mirrors, (2) the circularity test stand (CST) measures radial “runout” at the forward and aft ends of both the primary and the secondary mirrors, and (3) the Zygo axial interferometer measures the axial figure at sixteen different azimuthal positions. These three sets of data are combined to form a surface error map for each optical element. At the end of each testing cycle, the metrology data is used to form the strategy for the next fabrication cycle. As the process nears the end of this series of cycles, the mirror manufacturer would provide us with the fabrication errors obtained from the metrology data so we could make “as-manufactured” image quality predictions. The fabrication errors were provided to us in the form of three “figure” error coefficients and “mid” and “high” spatial frequency surface PSD plots.

The three “figure” error coefficients consisted of: (1) an average radius error, (2) a delta radius error, and (3) an average axial sag for both the primary mirror and the secondary mirror. Since these errors are departures from the design mirror profile, the fabricated mirror profile could readily be calculated. From the metrology data (average radius) at the front, rear, and center of each mirror element, Eq(4-7) and Eq.(4-8) could be used to form three equations with three unknowns which could then be solved for the three hyperbolic constants \(a\), \(b\), and \(z_c\) for a
hyperboloid fit to the “as-manufactured” mirror profile. The equations in Table 4-5 could then be used to calculate the ZEMAX design parameters so detailed ray trace analyses of the “as-manufactured” telescope could be performed.

The first SXI mirror to be fabricated (Serial Number 1, or SN001) was broken in a mishap in the Goodrich optical fabrication shop. The second SXI mirror (SN002) to be fabricated was thus intended to be a fully functional mirror; however, it would be installed in the Engineering Model of the SXI Telescope that would undergo the vibration and thermal cycling tests required of all flight hardware. The SN002 mirror would then become a spare mirror, in case something happened to one of the four flight models (SN003, SN004, SN005, and SN006). Figure 4-27 is a photograph of the SN002 mirror being fabricated in the Goodrich Optical Shop. Note that the primary and secondary mirrors are an integral unit, separated by a groove or “gap” on a single mirror substrate. Figure 4-28 is a photograph of the Lockheed Martin SXI Telescope Engineering model into which the SN002 mirror was installed for the vibration and thermal cycling tests required for all flight instruments.

![Image](image-url) Figure 4-27. The SXI SN002 grazing incidence X-ray telescope mirror being fabricated in the Goodrich Optical Shop.
A complete systems engineering analysis of the predicted image quality for each of these five grazing incidence X-ray telescope mirrors was performed and will be documented in the remainder of this chapter.

### 4.6.1 SXI Engineering Model ("As-manufactured" SN002 SXI Telescope)

The mirror profiles for SN002, including the fabrication errors, are shown in Figure 4-29. The fabrication figure error coefficients are tabulated along with the graph of the mirror profile. The *average radius error* of the primary mirror and the secondary mirror are designated as $r_{p,\text{bar}}$ and $r_{s,\text{bar}}$ respectively. The *delta radius error* of the primary mirror and the secondary mirror are designated as $\delta r_p$ and $\delta r_s$ respectively. And the *average axial sag* of the primary mirror and the secondary mirror are designated as $a_p^2$ and $a_s^2$ respectively.

The measured mirror dimensions at the front, midpoint, and rear of both the primary and secondary mirrors are also tabulated in Figure 4-29 along with their respective axial locations.
This is the data that is used to calculate the hyperboloid-hyperboloid fit to the actual “as-manufactured” surface profiles as explained in the last section. And finally, the ZEMAX optical prescription parameters for the hyperboloid-hyperboloid fit to the “as-manufactured” mirror profiles are included in Figure 4-29.

![Figure 4-29. “As-manufactured” SN002 mirror profile, with the fabrication error coefficients, mirror dimensions, and optical prescription of the hyperboloid-hyperboloid fit.](image)

The primary and secondary mirrors were polished and figured into a single glass cylindrical substrate. After the figuring and polishing was completed, the substrate was end-cut to eliminate any undesirable rolled-down edges. A 5.0 mm groove, referred to as the *gap*, separating the primary mirror profile from the secondary mirror profile was included in the mirror design. This groove was also routinely expanded at the end of the fabrication process to eliminate undesirable edge effects at the rear of the primary mirror and at the front of the secondary mirror. A wire-frame layout of the hyperboloid-hyperboloid fit to the “as-manufactured” SN002 SXI
telescope mirror is illustrated in Figure 4-30, as simulated with ZEMAX. Note the “as-manufactured” end-cut dimensions shown at the bottom of the illustration.

Figure 4-30. Wire-frame layout of the hyperboloid-hyperboloid fit to the “as-manufactured” SN002 SXI telescope mirror, as simulated with ZEMAX.

Figure 4-31 illustrates the departure of the “as-manufactured” mirror profiles from the HT#17 optical design profiles in the region of the mirror joint. The expanded ordinate and abscissa of this graph shows the radial and axial shift of the mirror joint for the “as-manufactured” telescope from its design location. This is actually a virtual joint, as it occurs in the gap between the primary and secondary mirror surfaces. However, there is a mathematical joint with a specific radius and axial location. The joint radius, joint axial location, and the axial location of the telescope focal plane are also listed on the figure.
The departure between the hyperboloid-hyperboloid fit and the actual “as-manufactured” mirror surface is shown of Figure 4-32. The straight line in the middle of the figure represents the “as-manufactured” hyperboloid-hyperboloid mirror profile. The curve on the left shows the deviation from the “as-manufactured” mirror profile of the hyperboloid-hyperboloid fit for the primary mirror. Likewise, the curve on the right shows the deviation from the “as-manufactured” mirror profile of the hyperboloid-hyperboloid fit for the secondary mirror. The front edge, rear edge, and midpoint of the “as-manufactured” mirrors are a perfect fit, as these are the points that were used to calculate the hyperboloid parameters. Note that the greatest departure of the hyperboloid-hyperboloid fit from the actual mirror surface is less than half a nanometer. However, it is mirror slope errors illustrated in Figure 4-33 that produce reflected ray deviations that might introduce errors in our geometrical ray trace analysis.
Figure 4-32. Surface error due to the Hyperboloid-Hyperboloid fit for SN002.

Figure 4-33. Local slope error due to the Hyperboloid-Hyperboloid fit for SN002.

The local slope errors of the primary and secondary mirrors can cause the corresponding ray errors to either add or subtract. We thus calculated the contribution to the rms image size
assuming that the ray deviations from the two mirrors were correlated, and also assuming that they were uncorrelated. For all five of the SXI mirrors, both of these numbers were shown to be negligible compared to the geometrical image size as determined by exhaustive ray tracing. This justifies our approximating the “as-manufactured” mirror surfaces with hyperboloids which can be easily modeled with ZEMAX.

Although somewhat redundant with Figure 4-29, we then had our MATLAB code calculate and print out the summary graph and tabulated parameters illustrated in Figure 4-34. This included the “as-manufactured” system parameters, and the necessary ZEMAX input parameters in precisely the format accepted by the ZEMAX code.

![Figure 4-34. Summary chart illustrating mirror profiles, telescope system parameters, and ZEMAX input parameters for the “as-manufactured” SN002 SXI telescope.](image-url)
Using the optimization capability of ZEMAX, the best axial focus of the “as-manufactured” SN002 SXI telescope was then determined. The optimum focal position for an operational field of view (OFOV) of 18 arc-min was then found by the method described in Chapter 2. The geometrical rms image size versus field angle for a variety despaced focal planes was determined by exhaustive ray tracing. This data is illustrated in Figure 4-35. The minus sign in front of each despase value indicates that we are moving the focal plane from best axial focus towards the telescope mirrors.

![Figure 4-35. Geometrical rms image radius vs. field angle for a variety of different focal plane positions.](image)

Continuing to use the technique described in Chapter 2, the field-weighted-average rms image radius versus operational field-of-view is calculated for each of the above focal positions. This family of curves is shown in Figure 4-36.
Figure 4-36. Field weighted average rms image radius vs. operational filed of view for SN002.

For an OFOV = 18 arc min, we now plot the y-intercepts vs. the focal plane despace and obtain a curve from which we can determine the optimum focal plane position as shown in Figure 4-37.

Figure 4-37. The optimum focal plane position for the “as-manufactured” SN002 SXI telescope.
The aerial point spread function (APSF) for the “as-manufactured” SN002 SXI telescope can now be modeled as described in Section 4.5. The obscuration ratio and the effective focal length used in the calculations of the ADPSF are taken directly from the “as-manufactured” mirror properties simulated with ZEMAX. Also from ZEMAX we obtain the ray intercept data with the focal plane and import it into the PSFGraz MATLAB code that creates the two-dimensional geometrical point spread function (GPSF).

On order to calculate the surface scatter point spread function (SSPSF), we need to provide the measured surface power spectral density (PSD) function as input into our version of the EEGRAZPC code, and this surface PSD had to be expressed in terms of a sum of Gaussian and Lorentzian functions. Figure 4-38 shows the one-dimensional, one-sided surface PSD that was presented to the mirror manufacturer as a requirement (and a goal) for the SXI mirrors. Also shown in Figure 4-38 is the “as-manufactured” surface PSD for SN002 SXI telescope mirrors as obtained from the metrology data from the two instruments used to evaluate surface quality during the optical fabrication process. The phase measuring interferometer (PMI) provides the surface PSD from a spatial frequency of 0.01 mm\(^{-1}\) to 10 mm\(^{-1}\). The WYKO instrument provides the PSD from a spatial frequency of 1.0 mm\(^{-1}\) to 1000 mm\(^{-1}\). By combining the two sets of data, we synthesize a surface PSD over the entire spatial frequency range from 0.01 mm\(^{-1}\) to 1000 mm\(^{-1}\). We then used a MATLAB fitting program to fit this synthesized surface PSF with a combination of Gaussian and Lorentzian functions that adequately describe the surface characteristics. For the “as-manufactured SN002 SXI mirrors, we used the four Gaussian functions and one Lorentzian function shown in Eq. (4-9)

\[
PSD(f) = \sqrt{\pi}\ell_1\sigma_1^2 e^{-\left(\pi\ell_1 f\right)^2} + \sqrt{\pi}\ell_2\sigma_2^2 e^{-\left(\pi\ell_2 f\right)^2} + \sqrt{\pi}\ell_3\sigma_3^2 e^{-\left(\pi\ell_3 f\right)^2} + \sqrt{\pi}\ell_4\sigma_4^2 e^{-\left(\pi\ell_4 f\right)^2} + \frac{2\ell_5\sigma_5^2}{1 + (2\pi\ell_5 f)^2}. \tag{4-9}
\]
Figure 4-38 shows this fit (the green line) to the measured Surface PSD data, and its comparison to the required and goal PSD’s. The values of the parameters used for fitting Eq(4-9) to the metrology data are also tabulated in Figure 4-38.

\[
\begin{align*}
\sigma_1 &= 150 \text{ A}, \quad \ell_1 = 8.0 \text{ mm} \\
\sigma_2 &= 48 \text{ A}, \quad \ell_2 = 2.5 \text{ mm} \\
\sigma_3 &= 12 \text{ A}, \quad \ell_3 = 0.8 \text{ mm} \\
\sigma_4 &= 7.0 \text{ A}, \quad \ell_4 = 0.11 \text{ mm} \\
\sigma_5 &= 7.0 \text{ A}, \quad \ell_5 = 0.05 \text{ mm}
\end{align*}
\]

Figure 4-38. Comparison of the measured surface PSD for the “as-manufactured” SN002 SXI mirrors to the requirement and the goal imposed upon the mirror manufacturer.

The miscellaneous residual surface error point spread function (RSEPSF) for the “as-manufactured” SN002 SXI telescope is calculated from the final error allocations in the mirror manufacturer’s error budget tree. We mentioned earlier that the preliminary error budget tree becomes a *living document* throughout the fabrication and assembly and alignment process, with initial error budget allocations (requirements) being changed to reflect actual achievements at each step in the process. Figure 4-39 is the final error budget tree for the “as-manufactured”
SN002 SXI telescope. Note that the “reserve” allocations in the preliminary error budget tree shown in Figure 4-1 have been re-allocated to other error sources.

![Error Budget Tree](image)

Figure 4-39. The final error budget tree for the “as-manufactured” SN002 SXI telescope indicates that the reserve allocations have been reallocated to other error sources.

As described earlier, the values of the various error sources are sum-root-squared to obtain the top-level rms image core diameter of 3.481 arc sec. This one-dimensional Gaussian image core is converted into a two-dimensional RSEPSF by the PSFGraz Code. Finally the ADPSF, GPSF, SSPSF and the RSEPSF are numerically convolved by the PSFGraz Code to obtain the predicted aerial point spread function (APSF) for the “as-manufactured” SN002 SXI telescope.

The contractual requirement for the mirror manufacturer was expressed in terms of the fractional encircled energy of the aerial image at the focal plane exhibiting best axial focus for an on-axis object point and an object point at a 20 arc min field angle. Figure 4-40 to Figure 4-43 thus illustrate our image quality predictions under those conditions for a wavelength of 44.7 Å and a wavelength of 13.3 Å.
Figure 4-40. Fractional encircled energy predictions for the “as-manufactured” SN002 telescope at best axial focus for a wavelength 44.7Å and a field angle of zero.

Figure 4-41. Fractional encircled energy predictions for the “as-manufactured” SN002 telescope at best axial focus for a wavelength 13.3Å and a field angle of zero.
Figure 4-42. Fractional encircled energy predictions for the “as-manufactured” SN002 SXI telescope at best axial focus for a wavelength 44.7 Å and a field angle of 20 arc min.

Figure 4-43. Fractional encircled energy predictions for the “as-manufactured” SN002 SXI telescope at best axial focus for a wavelength 13.3 Å and a field angle of 20 arc min.
Note from the previous figures that surface scatter effects dominate geometrical aberrations at small field angles, and geometrical aberrations dominate surface scatter effects at large field angles. It is also evident that surface scatter effects are more severe for shorter wavelengths.

Table 4-6 compares our performance predictions for the “as-manufactured” SN002 SXI telescope with the contractual requirements and goals. The performance predictions of the mirror manufacturer, Goodrich Optical Systems, are also included in the comparison.

Table 4-6. Comparison of fractional encircled energy predictions for the “as-manufactured” SN002 SXI telescope with program requirements and goals

<table>
<thead>
<tr>
<th></th>
<th>Fractional Encircled Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>On-axis (θ = 0.0)</td>
</tr>
<tr>
<td></td>
<td>(5.0 arc sec Dia. Circle)</td>
</tr>
<tr>
<td></td>
<td>λ = 44.7 Å</td>
</tr>
<tr>
<td>Req’mt</td>
<td>0.55</td>
</tr>
<tr>
<td>Goal</td>
<td>0.60</td>
</tr>
<tr>
<td>CREOL Prediction</td>
<td>0.60</td>
</tr>
<tr>
<td>Goodrich Prediction</td>
<td>0.58</td>
</tr>
</tbody>
</table>

We see from Table 4-6 that our predictions meet or exceed all requirements and goals. Our predictions also agree very well with those of Goodrich Optical systems, except for the 13.3 Å, on-axis case.

Although the contractual image quality requirements were expressed in terms of the fractional encircled energy at best axial focus, a more meaningful image quality criterion for the SXI mission is some field-weighted-average measure of resolution. We have chosen the
field-weighted-average half power radius ($\text{HPR}_{\text{fwa}}$) as an appropriate image quality criterion, and the HT#17 design was optimized to minimize this quantity for an 18 arc min OFOV. Figure 4-44 illustrates the predicted HPR of the aerial image for the “as-manufactured” SN002 SXI telescope for two different wavelength over the entire field-of-view of interest at the optimal focal plane position ($dz = -0.098 \text{ mm}$) for an 18 arc-min OFOV. A 3-D isometric plot of the aerial image at various field angles is also illustrated.

Figure 4-44. Predicted half power radius of the aerial image vs. field angle for the “as-manufactured” SN002 SXI telescope at two different wavelengths.

If we consider the HPR of the aerial image to define a spatial resolution element, then the number of spatial resolution elements in a given OFOV is given by

$$N = 2\pi \int_{\theta=0}^{\text{OFOV}} \frac{\theta}{\pi \text{HPR}^2(\theta)} d\theta$$

(4-10)
Minimizing the $HPR_{fwa}$ over a given OFOV clearly maximizes the number of spatial resolution elements in that OFOV. This is equivalent to maximizing the amount of information in the OFOV. Figure 4-45 illustrates the predicted number of spatial resolution elements in the aerial image as a function of OFOV for the “as-manufactured” SN002 SXI telescope for the two wavelengths of interest.

![Figure 4-45](image)

Figure 4-45. Number of predicted spatial resolution elements vs. OFOV for the aerial image of the “as-manufactured” SN002 SXI telescope at two different wavelengths.

Following the procedure described in Chapter 3 for calculating image degradation due to detector effects, the half power radius for the average UDPSF for the “as-manufactured” SN002 SXI telescope was determined as a function of field angle for wavelengths of 13.3Å and 44.7Å. These predictions are shown in Figure 4-46. Again, 3-D isometric plots of the average UDPSF are displayed at a variety of different field angles. Figure 4-47 illustrates the number of spatial resolution elements calculated from Eq.(4-10) for the average UDPSF.
Figure 4-46. Predicted half power radius of the average UDPSF vs. field angle for the “as-manufactured” SN002 SXI telescope at two different wavelengths.

Figure 4-47. Number of predicted spatial resolution elements vs. OFOV when detector effects are included with the “as-manufactured” SN002 SXI telescope at two different wavelengths.
Figure 4-48 summarizes the results of our systems engineering analysis of image quality for the “as-manufactured” SN002 SXI telescope. It illustrates the predicted half power radius (HPR) of the point spread function (PSF) as a function of field angle as we progressively include more error sources in the analysis. The lower curve illustrates the predicted image quality based solely upon geometrical analysis (ray tracing). The predicted aerial image (middle curves) includes diffraction effects, geometrical aberrations, surface scatter effects, and all of the error sources in the mirror manufacturer’s error budget tree. Of course, the diffraction effects and surface scatter effects vary with wavelength. And finally, the top two curves include all of the above errors, plus a rigorous analysis of image degradation due to detector effects has been included. This comparison shows how truly inadequate the simple geometrical analysis is for calculating image quality for many applications.

![Graph showing comparison of image quality predictions for the “as-manufactured” SN002 SXI telescope based upon: 1) geometrical analysis only, 2) physical optics analysis including all error sources affecting the aerial image (in particular surface scatter phenomena), and 3) including a rigorous analysis of mosaic detector array effects.]
4.6.2 “As-manufactured” SN003 SXI Telescope

The mirror profiles for SN003, including the fabrication errors, are shown in Figure 4-49. The fabrication figure error coefficients are again tabulated along with the graph of the mirror profile, as are the measured mirror dimensions at the front, midpoint, and rear of both the primary and secondary mirrors along with their respective axial locations. This data that is used to calculate the hyperboloid-hyperboloid fit to the actual “as-manufactured” surface profiles as explained in Section 4.5. And finally, the ZEMAX optical prescription parameters for the hyperboloid-hyperboloid fit to the “as-manufactured” mirror profiles are included in Figure 4-49.

![Figure 4-49. “As-manufactured” SN003 mirror profile, with the fabrication error coefficients, mirror dimensions, and optical prescription of the hyperboloid-hyperboloid fit.](image)

The primary and secondary mirrors were polished and figured into a single glass cylindrical substrate. After the figuring and polishing was completed, the substrate was end-cut to eliminate any undesirable rolled-down edges. A 5.0 mm groove, referred to as the gap, separating the
primary mirror profile from the secondary mirror profile was included in the mirror design. This
groove was also routinely expanded at the end of the fabrication process to eliminate undesirable
edge effects at the rear of the primary mirror and at the front of the secondary mirror. A
wire-frame layout of the hyperboloid-hyperboloid fit to the “as-manufactured” SN003 SXI
telescope mirror is illustrated in Figure 4-50, as simulated with ZEMAX. Note the
“as-manufactured” end-cut dimensions shown at the bottom of the illustration.

![Figure 4-50](image)

Figure 4-50. Wire-frame layout of the hyperboloid-hyperboloid fit to the “as-manufactured” SN003 SXI telescope mirror, as simulated with ZEMAX.

Figure 4-51 illustrates the departure of the “as-manufactured” mirror profiles from the
HT#17 optical design profiles in the region of the mirror joint. The expanded ordinate and
abscissa of this graph shows the radial and axial shift of the mirror joint for the
“as-manufactured” telescope from its design location. This is actually a virtual joint, as it occurs
in the gap between the primary and secondary mirror surfaces. However, there is a mathematical
joint with a specific radius and axial location. The joint radius, joint axial location, and the axial
location of the telescope focal plane are also listed on the figure.
Figure 4-51. Comparison of the HT#17 design and the SN003 mirror profile near the mirror joint.

The departure between the hyperboloid-hyperboloid fit and the actual “as-manufactured” mirror surface is shown of Figure 4-52. The straight line in the middle of the figure represents the “as-manufactured” hyperboloid-hyperboloid mirror profile. The curve on the left shows the deviation from the “as-manufactured” mirror profile of the hyperboloid-hyperboloid fit for the primary mirror. Likewise, the curve on the right shows the deviation from the “as-manufactured” mirror profile of the hyperboloid-hyperboloid fit for the secondary mirror. The front edge, rear edge, and midpoint of the “as-manufactured” mirrors are a perfect fit, as these are the points that were used to calculate the hyperboloid parameters. Note that the greatest departure of the hyperboloid-hyperboloid fit from the actual mirror surface is approximately six hundredths of a nanometer. However, it is mirror slope errors illustrated in Figure 4-53 that produce reflected ray deviations that might introduce errors in our geometrical ray trace analysis.
Figure 4-52. Surface error due to the Hyperboloid-Hyperboloid fit for SN002.

Figure 4-53. Local slope error due to the Hyperboloid-Hyperboloid fit for SN002.
The local slope errors of the primary and secondary mirrors can cause the corresponding ray errors to either add or subtract. We thus calculated the contribution to the rms image size assuming that the ray deviations from the two mirrors were correlated, and also assuming that they were uncorrelated. For all five of the SXI mirrors, both of these numbers were shown to be negligible compared to the geometrical image size as determined by exhaustive ray tracing. This justifies our approximating the “as-manufactured” mirror surfaces with hyperboloids which can be easily modeled with ZEMAX.

Although somewhat redundant with Figure 4-49, we then had our MATLAB code calculate and print out the summary graph and tabulated parameters illustrated in Figure 4-54. This included the “as-manufactured system parameters, and the necessary ZEMAX input parameters in precisely the format accepted by the ZEMAX code.

![Figure 4-54. Summary chart illustrating mirror profiles, telescope system parameters, and ZEMAX input parameters for the “as-manufactured” SN003 SXI telescope.](image-url)
Using the optimization capability of ZEMAX, the best axial focus of the “as-manufactured” SN003 SXI telescope was then determined. The optimum focal position for an operational field of view (OFOV) of 18 arc-min was then found by the method described in Chapter 2. The geometrical rms image size versus field angle for a variety despaced focal planes was determined by exhaustive ray tracing. This data is illustrated in Figure 4-55. The minus sign in front of each despase value indicates that we are moving the focal plane from best axial focus towards the telescope mirrors.

![Figure 4-55. Geometrical rms image radius vs. field angle for a variety of different focal plane positions.](image)

Continuing to use the technique described in Chapter 2, the field-weighted-average rms image radius versus operational field-of-view is calculated for each of the above focal positions. This family of curves is shown in Figure 4-56.
Figure 4-56. Field weighted average rms image radius vs. operational filed of view for SN003.

For an OFOV = 18 arc min, we now plot the y-intercepts vs. the focal plane despace and obtain a curve from which we can determine the optimum focal plane position as shown in Figure 4-57.

Figure 4-57. The optimum focal plane position for the “as-manufactured” SN003 SXI telescope.
The aerial point spread function (APSF) for the “as-manufactured” SN003 SXI telescope can now be modeled as described in Section 4.5. The obscuration ratio and the effective focal length used in the calculations of the ADPSF are taken directly from the “as-manufactured” mirror properties simulated with ZEMAX. Also from ZEMAX we obtain the ray intercept data with the focal plane and import it into the PSFGrasz MATLAB code that creates the two-dimensional geometrical point spread function (GPSF).

On order to calculate the surface scatter point spread function (SSPSF), we need to provide the measured surface power spectral density (PSD) function as input into our version of the EEGRAZPC code, and this surface PSD had to be expressed in terms of a sum of Gaussian and Lorentzian functions. Figure 4-58 shows the one-dimensional, one-sided surface PSD that was presented to the mirror manufacturer as a requirement (and a goal) for the SXI mirrors. Also shown in Figure 4-58 is the “as-manufactured” surface PSD for SN003 SXI telescope mirrors as obtained from the metrology data from the two instruments used to evaluate surface quality during the optical fabrication process. The phase measuring interferometer (PMI) provides the surface PSD from a spatial frequency of 0.01 mm\(^{-1}\) to 10 mm\(^{-1}\). The WYKO instrument provides the PSD from a spatial frequency of 1.0 mm\(^{-1}\) to 1000 mm\(^{-1}\). By combining the two sets of data, we synthesize a surface PSD over the entire spatial frequency range from 0.01 mm\(^{-1}\) to 1000 mm\(^{-1}\). We then used a MATLAB fitting program to fit this synthesized surface PSF with a combination of Gaussian and Lorentzian functions that adequately describe the surface characteristics. For the “as-manufactured SN003 SXI mirrors, we used three Gaussian functions and two Lorentzian functions as shown in Eq. (4-11)

\[
PSD(f) = \sqrt{\pi} \ell_1 \sigma_1^2 e^{-\left(\frac{\pi f}{\ell_1}\right)^2} + \frac{2\ell_2 \sigma_2^2}{1 + \left(2\pi \ell_2 f\right)^2} + \sqrt{\pi} \ell_3 \sigma_3^2 e^{-\left(\frac{\pi f}{\ell_3}\right)^2} + \frac{2\ell_4 \sigma_4^2}{1 + \left(2\pi \ell_4 f\right)^2} + \sqrt{\pi} \ell_5 \sigma_5^2 e^{-\left(\frac{\pi f}{\ell_5}\right)^2} \quad (4-11)
\]
Figure 4-58 shows this fit (the green line) to the measured Surface PSD data, and its comparison to the required and goal PSD’s. The values of the parameters used for fitting Eq(4-11) to the metrology data are also tabulated in Figure 4-58.

\[
\begin{align*}
\sigma_1 &= 125 \text{ A}, \quad \ell_1 = 4.1 \text{ mm} \\
\sigma_2 &= 23 \text{ A}, \quad \ell_2 = 1.2 \text{ mm} \\
\sigma_3 &= 2.7 \text{ A}, \quad \ell_3 = 0.02 \text{ mm} \\
\sigma_4 &= 3.7 \text{ A}, \quad \ell_4 = 0.003 \text{ mm} \\
\sigma_5 &= 1.6 \text{ A}, \quad \ell_5 = 0.0008 \text{ mm}
\end{align*}
\]

Figure 4-58. Comparison of the measured surface PSD for the “as-manufactured” SN003 SXI mirrors to the requirement and the goal imposed upon the mirror manufacturer.

The miscellaneous residual surface error point spread function (RSEPSF) for the “as-manufactured” SN003 SXI telescope is calculated from the final error allocations in the mirror manufacturer’s error budget tree. We mentioned earlier that the preliminary error budget tree becomes a *living document* throughout the fabrication and assembly and alignment process, with initial error budget allocations (requirements) being changed to reflect actual achievements at each step in the process. Figure 4-59 is the final error budget tree for the “as-manufactured”
SN003 SXI telescope. Note that the “reserve” allocations in the preliminary error budget tree shown in Figure 4-1 have been re-allocated to other error sources.

![Error Budget Tree](image)

Figure 4-59. The final error budget tree for the “as-manufactured” SN003 SXI telescope indicates that the reserve allocations have been reallocated to other error sources.

As described earlier, the values of the various error sources are sum-root-squared to obtain the top-level rms image core diameter of 2.697 arc sec. This one-dimensional Gaussian image core is converted into a two-dimensional RSEPSF by the PSFGras Code. Finally the ADPSF, GPSF, SSPSF and the RSEPSF are numerically convolved by the PSFGras Code to obtain the predicted aerial point spread function (APSF) for the “as-manufactured” SN003 SXI telescope.

The contractual requirement for the mirror manufacturer was expressed in terms of the fractional encircled energy of the aerial image at the focal plane exhibiting best axial focus for an on-axis object point and an object point at a 20 arc min field angle. Figure 4-60 to Figure 4-63 thus illustrate our image quality predictions under those conditions for a wavelength of 44.7 Å and a wavelength of 13.3 Å.
Figure 4-60. Fractional encircled energy predictions for the “as-manufactured” SN003 telescope at best axial focus for a wavelength 44.7Å and a field angle of zero.

FEcE = 0.57 in a 5 arc sec diameter circle.

Figure 4-61. Fractional encircled energy predictions for the “as-manufactured” SN003 telescope at best axial focus for a wavelength 13.3Å and a field angle of zero.

FEcE = 0.46 in a 5 arc sec diameter circle.
Figure 4-62. Fractional encircled energy predictions for the “as-manufactured” SN003 SXI telescope at best axial focus for a wavelength 44.7 Å and a field angle of 20 arc min. Geo. Aberrations Dominate Scatter Effects For Large Field Angles

\[ FEcE = 0.42 \text{ in a } 20 \text{ arc sec diameter circle.} \]

\[ \lambda = 44.7\,\text{Å} \quad \theta = 20 \text{ arc min} \]

Figure 4-63. Fractional encircled energy predictions for the “as-manufactured” SN003 SXI telescope at best axial focus for a wavelength 13.3 Å and a field angle of 20 arc min. Geo. Aberrations Dominate Scatter Effects For Large Field Angles

\[ FEcE = 0.38 \text{ in a } 20 \text{ arc sec diameter circle.} \]

\[ \lambda = 13.3\,\text{Å} \quad \theta = 20 \text{ arc min} \]
Note from the previous figures that surface scatter effects dominate geometrical aberrations at small field angles, and geometrical aberrations dominate surface scatter effects at large field angles. It is also evident that surface scatter effects are more severe for shorter wavelengths.

Table 4-7 compares our performance predictions for the “as-manufactured” SN003 SXI telescope with the contractual requirements and goals. The performance predictions of the mirror manufacturer, Goodrich Optical Systems, are also included in the comparison.

Table 4-7. Comparison of fractional encircled energy predictions for the “as-manufactured” SN003 SXI telescope with program requirements and goals

<table>
<thead>
<tr>
<th></th>
<th>Fractional Encircled Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>On-axis ($\theta = 0.0$) ($5.0$ arc sec Dia. Circle)</td>
</tr>
<tr>
<td>$\lambda = 44.7$ Å</td>
<td>$\lambda = 13.3$ Å</td>
</tr>
<tr>
<td>Req'mt</td>
<td>0.55</td>
</tr>
<tr>
<td>Goal</td>
<td>0.60</td>
</tr>
<tr>
<td>CREOL Prediction</td>
<td>0.57</td>
</tr>
<tr>
<td>Goodrich Prediction</td>
<td>0.62</td>
</tr>
</tbody>
</table>

We see from Table 4-7 that our predictions reach and even exceed the requirements for all the cases except for 13 Å off-axis case. We also meet the goal for the 44 Å off-axis case and we exceed the goal for 13 Å, on-axis case. In all cases our predictions are somewhat lower than those of Goodrich.

Although the contractual image quality requirements were expressed in terms of the fractional encircled energy at best axial focus, a more meaningful image quality criterion for the
SXI mission is some field-weighted-average measure of resolution. We have chosen the field-weighted-average half power radius ($\text{HPR}_{\text{fwa}}$) as an appropriate image quality criterion, and the HT#17 design was optimized to minimize this quantity for an 18 arc min OFOV. Figure 4-64 illustrates the predicted HPR of the aerial image for the “as-manufactured” SN003 SXI telescope for two different wavelength over the entire field-of-view of interest at the optimal focal plane position ($dz = -0.102 \text{ mm}$) for an 18 arc-min OFOV. A 3-D isometric plot of the aerial image at various field angles is also illustrated.

![Graph showing predicted half power radius of the aerial image vs. field angle for the “as-manufactured” SN003 SXI telescope at two different wavelengths.]

Again we consider the HPR of the aerial image to define a spatial resolution element, and then the number of spatial resolution elements in a given OFOV is given by

$$N = 2\pi \int_{\theta=0}^{\theta_{\text{OFOV}}} \frac{\theta}{\pi \text{HPR}^2(\theta)} d\theta$$

(4-12)
Minimizing the $HPR_{fwa}$ over a given OFOV clearly maximizes the number of spatial resolution elements (N) in that OFOV. This is equivalent to maximizing the amount of information in the OFOV. Figure 4-65 illustrates the predicted N as a function of OFOV for the “as-manufactured” SN003 SXI telescope for the two wavelengths of interest.

![Figure 4-65. Number of predicted spatial resolution elements vs. OFOV for the aerial image of the “as-manufactured” SN003 SXI telescope at two different wavelengths.](image)

Following the procedure described in Chapter 3 for calculating image degradation due to detector effects, the half power radius for the average UDPSF for the “as-manufactured” SN003 SXI telescope was determined as a function of field angle for wavelengths of 13.3Å and 44.7Å. These predictions are shown in Figure 4-66. Again, 3-D isometric plots of the average UDPSF are displayed at a variety of different field angles. Figure 4-67 illustrates the number of spatial resolution elements calculated from Eq.(4-12) for the average UDPSF.
Figure 4-66. Predicted half power radius of the average UDPSF vs. field angle for the "as-manufactured" SN003 SXI telescope at two different wavelengths.

Figure 4-67. Number of predicted spatial resolution elements vs. OFOV when detector effects are included with the "as-manufactured" SN003 SXI telescope at two different wavelengths.
Figure 4-68 summarizes the results of our systems engineering analysis of image quality for the “as-manufactured” SN003 SXI telescope. It illustrates the predicted half power radius (HPR) of the point spread function (PSF) as a function of field angle as we progressively include more error sources in the analysis. The lower curve illustrates the predicted image quality based solely upon geometrical analysis (ray tracing). The predicted aerial image (middle curves) includes diffraction effects, geometrical aberrations, surface scatter effects, and all of the error sources in the mirror manufacturer’s error budget tree. Of course, the diffraction effects and surface scatter effects vary with wavelength. And finally, the top two curves include all of the above errors, plus a rigorous analysis of image degradation due to detector effects has been included. This comparison shows how truly inadequate the simple geometrical analysis is for calculating image quality for many applications.

Figure 4-68 Comparison of image quality predictions for the “as-manufactured” SN003 SXI telescope based upon: 1) geometrical analysis only, 2) physical optics analysis including all error sources affecting the aerial image (in particular surface scatter phenomena), and 3) including a rigorous analysis of mosaic detector array effects.
4.6.3 “As-manufactured” SN004 SXI Telescope

The mirror profiles for SN004, including the fabrication errors, are shown in Figure 4-69. The fabrication figure error coefficients are again tabulated along with the graph of the mirror profile, as are the measured mirror dimensions at the front, midpoint, and rear of both the primary and secondary mirrors along with their respective axial locations. This data that is used to calculate the hyperboloid-hyperboloid fit to the actual “as-manufactured” surface profiles as explained in Section 4.5. And finally, the ZEMAX optical prescription parameters for the hyperboloid-hyperboloid fit to the “as-manufactured” mirror profiles are included in Figure 4-69.

The primary and secondary mirrors were polished and figured into a single glass cylindrical substrate. After the figuring and polishing was completed, the substrate was end-cut to eliminate any undesirable rolled-down edges. A 5.0 mm groove, referred to as the gap, separating the
primary mirror profile from the secondary mirror profile was included in the mirror design. This groove was also routinely expanded at the end of the fabrication process to eliminate undesirable edge effects at the rear of the primary mirror and at the front of the secondary mirror. A wire-frame layout of the hyperboloid-hyperboloid fit to the “as-manufactured” SN004 SXI telescope mirror is illustrated in Figure 4-70, as simulated with ZEMAX. Note the “as-manufactured” end-cut dimensions shown at the bottom of the illustration.

![Figure 4-70](image)

Figure 4-70. Wire-frame layout of the hyperboloid-hyperboloid fit to the “as-manufactured” SN004 SXI telescope mirror, as simulated with ZEMAX.

Figure 4-71 illustrates the departure of the “as-manufactured” mirror profiles from the HT#17 optical design profiles in the region of the mirror joint. The expanded ordinate and abscissa of this graph shows the radial and axial shift of the mirror joint for the “as-manufactured” telescope from its design location. This is actually a virtual joint, as it occurs in the gap between the primary and secondary mirror surfaces. However, there is a mathematical joint with a specific radius and axial location. The joint radius, joint axial location, and the axial location of the telescope focal plane are also listed on the figure.
Figure 4-71. Comparison of the HT#17 design and the SN004 mirror profile near the mirror joint.

The departure between the hyperboloid-hyperboloid fit and the actual “as-manufactured” mirror surface is shown of Figure 4-72. The straight line in the middle of the figure represents the “as-manufactured” hyperboloid-hyperboloid mirror profile. The curve on the left shows the deviation from the “as-manufactured” mirror profile of the hyperboloid-hyperboloid fit for the primary mirror. Likewise, the curve on the right shows the deviation from the “as-manufactured” mirror profile of the hyperboloid-hyperboloid fit for the secondary mirror. The front edge, rear edge, and midpoint of the “as-manufactured” mirrors are a perfect fit, as these are the points that were used to calculate the hyperboloid parameters. Note that the greatest departure of the hyperboloid-hyperboloid fit from the actual mirror surface is less than 0.15 nanometers. However, it is mirror slope errors illustrated in Figure 4-73 that produce reflected ray deviations that might introduce errors in our geometrical ray trace analysis.
Figure 4-72. Surface error due to the Hyperboloid-Hyperboloid fit for SN004.

Figure 4-73. Local slope error due to the Hyperboloid-Hyperboloid fit for SN004.
The local slope errors of the primary and secondary mirrors can cause the corresponding ray errors to either add or subtract. We thus calculated the contribution to the rms image size assuming that the ray deviations from the two mirrors were correlated, and also assuming that they were uncorrelated. For all five of the SXI mirrors, both of these numbers were shown to be negligible compared to the geometrical image size as determined by exhaustive ray tracing. This justifies our approximating the “as-manufactured” mirror surfaces with hyperboloids which can be easily modeled with ZEMAX.

Although somewhat redundant with Figure 4-69, we then had our MATLAB code calculate and print out the summary graph and tabulated parameters illustrated in Figure 4-74. This included the “as-manufactured system parameters, and the necessary ZEMAX input parameters in precisely the format accepted by the ZEMAX code.
Using the optimization capability of ZEMAX, the best axial focus of the “as-manufactured” SN004 SXI telescope was then determined. The optimum focal position for an operational field of view (OFOV) of 18 arc-min was then found by the method described in Chapter 2. The geometrical rms image size versus field angle for a variety despaced focal planes was determined by exhaustive ray tracing. This data is illustrated in Figure 4-75. The minus sign in front of each despase value indicates that we are moving the focal plane from best axial focus towards the telescope mirrors.

![Figure 4-75. Geometrical rms image radius vs. field angle for a variety of different focal plane positions.](image)

Continuing to use the technique described in Chapter 2, the field-weighted-average rms image radius versus operational field-of-view is calculated for each of the above focal positions. This family of curves is shown in Figure 4-76.
For an OFOV = 18 arc min, we now plot the y-intercepts vs. the focal plane despace and obtain a curve from which we can determine the optimum focal plane position as shown in Figure 4-77.

Figure 4-77. The optimum focal plane position for the “as-manufactured” SN004 SXI telescope.
The aerial point spread function (APSF) for the “as-manufactured” SN004 SXI telescope can now be modeled as described in Section 4.5. The obscuration ratio and the effective focal length used in the calculations of the ADPSF are taken directly from the “as-manufactured” mirror properties simulated with ZEMAX. Also from ZEMAX we obtain the ray intercept data with the focal plane and import it into the PSFGratz MATLAB code that creates the two-dimensional geometrical point spread function (GPSF).

On order to calculate the surface scatter point spread function (SSPSF), we need to provide the measured surface power spectral density (PSD) function as input into our version of the EEGRAZPC code, and this surface PSD had to be expressed in terms of a sum of Gaussian and Lorentzian functions. Figure 4-78 shows the one-dimensional, one-sided surface PSD that was presented to the mirror manufacturer as a requirement (and a goal) for the SXI mirrors. Also shown in Figure 4-78 is the “as-manufactured” surface PSD for SN004 SXI telescope mirrors as obtained from the metrology data from the two instruments used to evaluate surface quality during the optical fabrication process. The phase measuring interferometer (PMI) provides the surface PSD from a spatial frequency of 0.01 mm\(^{-1}\) to 10 mm\(^{-1}\). The WYKO instrument provides the PSD from a spatial frequency of 1.0 mm\(^{-1}\) to 1000 mm\(^{-1}\). By combining the two sets of data, we synthesize a surface PSD over the entire spatial frequency range from 0.01 mm\(^{-1}\) to 1000 mm\(^{-1}\). We then used a MATLAB fitting program to fit this synthesized surface PSF with a combination of Gaussian and Lorentzian functions that adequately describe the surface characteristics. For the “as-manufactured SN004 SXI mirrors, we used four Gaussian functions and one Lorentzian functions as shown in Eq. (4-13)

\[
PSD(f) = \sqrt{\pi} \ell \sigma^2 e^{-\pi f^2} + \frac{2 \ell^2 \sigma^2}{1 + (2\pi \ell f)^2} + \sqrt{\pi} \ell \sigma^2 e^{-(\pi f)^2} + \sqrt{\pi} \ell \sigma^2 e^{-(\pi f)^2} + \sqrt{\pi} \ell \sigma^2 e^{-(\pi f)^2} \quad (4-13)
\]
Figure 4-78 shows this fit (the green line) to the measured Surface PSD data, and its comparison to the required and goal PSD’s. The values of the parameters used for fitting Eq(4-13) to the metrology data are also tabulated in Figure 4-78.

![Figure 4-78. Comparison of the measured surface PSD for the “as-manufactured” SN004 SXI mirrors to the requirement and the goal imposed upon the mirror manufacturer.](image)

The miscellaneous residual surface error point spread function (RSEPSF) for the “as-manufactured” SN004 SXI telescope is calculated from the final error allocations in the mirror manufacturer’s error budget tree. We mentioned earlier that the preliminary error budget tree becomes a *living document* throughout the fabrication and assembly and alignment process, with initial error budget allocations (requirements) being changed to reflect actual achievements at each step in the process. Figure 4-79 is the final error budget tree for the “as-manufactured”
SN004 SXI telescope. Note that the “reserve” allocations in the preliminary error budget tree shown in Figure 4-1 have been re-allocated to other error sources.

As described earlier, the values of the various error sources are sum-root-squared to obtain the top-level rms image core diameter of 2.894 arc sec. This one-dimensional Gaussian image core is converted into a two-dimensional RSEPSF by the PSFGras Code. Finally the ADPSF, GPSF, SSPSF and the RSEPSF are numerically convolved by the PSFGras Code to obtain the predicted aerial point spread function (APSF) for the “as-manufactured” SN004 SXI telescope.

The contractual requirement for the mirror manufacturer was expressed in terms of the fractional encircled energy of the aerial image at the focal plane exhibiting best axial focus for an on-axis object point and an object point at a 20 arc min field angle. Figure 4-80 to Figure 4-83 thus illustrate our image quality predictions under those conditions for a wavelength of 44.7 Å and a wavelength of 13.3 Å.
Figure 4-80. Fractional encircled energy predictions for the “as-manufactured” SN004 telescope at best axial focus for a wavelength 44.7 Å and a field angle of zero.

FE\textsubscript{cE} = 0.56 in a 5 arc sec \textit{diameter} circle.

Figure 4-81. Fractional encircled energy predictions for the “as-manufactured” SN004 telescope at best axial focus for a wavelength 13.3 Å and a field angle of zero.

FE\textsubscript{cE} = 0.45 in a 5 arc sec \textit{diameter} circle.
Figure 4-82. Fractional encircled energy predictions for the “as-manufactured” SN004 SXI telescope at best axial focus for a wavelength 44.7 Å and a field angle of 20 arc min.

Figure 4-83. Fractional encircled energy predictions for the “as-manufactured” SN004 SXI telescope at best axial focus for a wavelength 13.3 Å and a field angle of 20 arc min.
Note from the previous figures that surface scatter effects dominate geometrical aberrations at small field angles, and geometrical aberrations dominate surface scatter effects at large field angles. It is also evident that surface scatter effects are more severe for shorter wavelengths.

Table 4-8 compares our performance predictions for the “as-manufactured” SN004 SXI telescope with the contractual requirements and goals. The performance predictions of the mirror manufacturer, Goodrich Optical Systems, are also included in the comparison.

Table 4-8. Comparison of fractional encircled energy predictions for the “as-manufactured” SN004 SXI telescope with program requirements and goals

<table>
<thead>
<tr>
<th></th>
<th>Fractional Encircled Energy</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>On-axis (θ = 0.0)</td>
</tr>
<tr>
<td></td>
<td>(5.0 arc sec Dia. Circle)</td>
</tr>
<tr>
<td>λ = 44.7 Å</td>
<td>Req’mt: 0.55</td>
</tr>
<tr>
<td></td>
<td>Goal: 0.60</td>
</tr>
<tr>
<td></td>
<td>CREOL Prediction: 0.56</td>
</tr>
<tr>
<td></td>
<td>Goodrich Prediction: 0.61</td>
</tr>
</tbody>
</table>

We see from Table 4-8 that for the 44 Å and 13 Å on-axis case our predictions exceed the requirements but not the goal. We also don’t agree well with Goodrich’s predictions for these cases. For 44 Å, off-axis case we meet and exceed the requirements and the goal, and agree well with Goodrich’s predictions. For 13 Å, off-axis case we meet the requirements but not the goal and agree precisely with Goodrich’s predictions.

Although the contractual image quality requirements were expressed in terms of the fractional encircled energy at best axial focus, a more meaningful image quality criterion for the
SXI mission is some field-weighted-average measure of resolution. We have chosen the field-weighted-average half power radius ($HPR_{fwa}$) as an appropriate image quality criterion, and the HT#17 design was optimized to minimize this quantity for an 18 arc min OFOV. Figure 4-84 illustrates the predicted HPR of the aerial image for the “as-manufactured” SN004 SXI telescope for two different wavelength over the entire field-of-view of interest at the optimal focal plane position ($dz = -0.097$ mm) for an 18 arc-min OFOV. A 3-D isometric plot of the aerial image at various field angles is also illustrated.

![Figure 4-84. Predicted half power radius of the aerial image vs. field angle for the “as-manufactured” SN004 SXI telescope at two different wavelengths.](image)

Again we consider the HPR of the aerial image to define a spatial resolution element, and then the number of spatial resolution elements in a given OFOV is given by

$$N = 2 \pi \int_{0}^{OFOV} \frac{\theta}{\pi HPR^2(\theta)} d\theta$$  \hspace{1cm} (4-14)
Minimizing the $HPR_{fwa}$ over a given OFOV clearly maximizes the number of spatial resolution elements ($N$) in that OFOV. This is equivalent to maximizing the amount of information in the OFOV. Figure 4-85 illustrates the predicted $N$ in the aerial image as a function of OFOV for the “as-manufactured” SN004 SXI telescope for the two wavelengths of interest.

Figure 4-85. Number of predicted spatial resolution elements vs. OFOV for the aerial image of the “as-manufactured” SN004 SXI telescope at two different wavelengths.

Following the procedure described in Chapter 3 for calculating image degradation due to detector effects, the half power radius for the average UDP SF for the “as-manufactured” SN004 SXI telescope was determined as a function of field angle for wavelengths of 13.3Å and 44.7Å. These predictions are shown in Figure 4-86. Again, 3-D isometric plots of the average UDP SF are displayed at a variety of different field angles. Figure 4-87 illustrates the number of spatial resolution elements calculated from Eq.(4-14) for the average UDP SF.
Figure 4-86. Predicted half power radius of the average UDPSF vs. field angle for the “as-manufactured” SN004 SXI telescope at two different wavelengths.

Figure 4-87. Number of predicted spatial resolution elements vs. OFOV when detector effects are included with the “as-manufactured” SN004 SXI telescope at two different wavelengths.
Figure 4-88 summarizes the results of our systems engineering analysis of image quality for the “as-manufactured” SN004 SXI telescope. It illustrates the predicted half power radius (HPR) of the point spread function (PSF) as a function of field angle as we progressively include more error sources in the analysis. The lower curve illustrates the predicted image quality based solely upon geometrical analysis (ray tracing). The predicted aerial image (middle curves) includes diffraction effects, geometrical aberrations, surface scatter effects, and all of the error sources in the mirror manufacturer’s error budget tree. Of course, the diffraction effects and surface scatter effects vary with wavelength. And finally, the top two curves include all of the above errors, plus a rigorous analysis of image degradation due to detector effects has been included. This comparison shows how truly inadequate the simple geometrical analysis is for calculating image quality for many applications.

Figure 4-88. Comparison of image quality predictions for the “as-manufactured” SN004 SXI telescope based upon: 1) geometrical analysis only, 2) physical optics analysis including all error sources affecting the aerial image (in particular surface scatter phenomena), and 3) including a rigorous analysis of mosaic detector array effects.
4.6.4 “As-manufactured” SN005 SXI Telescope

The mirror profiles for SN005, including the fabrication errors, are shown in Figure 4-89. The fabrication figure error coefficients are again tabulated along with the graph of the mirror profile, as are the measured mirror dimensions at the front, midpoint, and rear of both the primary and secondary mirrors along with their respective axial locations. This data that is used to calculate the hyperboloid-hyperboloid fit to the actual “as-manufactured” surface profiles as explained in Section 4.5. And finally, the ZEMAX optical prescription parameters for the hyperboloid-hyperboloid fit to the “as-manufactured” mirror profiles are included in Figure 4-89.

![Figure 4-89. “As-manufactured” SN005 mirror profile, with the fabrication error coefficients, mirror dimensions, and optical prescription of the hyperboloid-hyperboloid fit.](image)

The primary and secondary mirrors were polished and figured into a single glass cylindrical substrate. After the figuring and polishing was completed, the substrate was end-cut to eliminate any undesirable rolled-down edges. A 5.0 mm groove, referred to as the gap, separating the
primary mirror profile from the secondary mirror profile was included in the mirror design. This
groove was also routinely expanded at the end of the fabrication process to eliminate undesirable
dge effects at the rear of the primary mirror and at the front of the secondary mirror. A
wire-frame layout of the hyperboloid-hyperboloid fit to the “as-manufactured” SN005 SXI
telescope mirror is illustrated in Figure 4-90, as simulated with ZEMAX. Note the
“as-manufactured” end-cut dimensions shown at the bottom of the illustration.

![Figure 4-90](image_url)

**Figure 4-90.** Wire-frame layout of the hyperboloid-hyperboloid fit to the “as-manufactured” SN005 SXI telescope mirror, as simulated with ZEMAX.

Figure 4-91 illustrates the departure of the “as-manufactured” mirror profiles from the
HT#17 optical design profiles in the region of the mirror joint. The expanded ordinate and
abscissa of this graph shows the radial and axial shift of the mirror joint for the
“as-manufactured” telescope from its design location. This is actually a virtual joint, as it occurs
in the gap between the primary and secondary mirror surfaces. However, there is a mathematical
joint with a specific radius and axial location. The joint radius, joint axial location, and the axial
location of the telescope focal plane are also listed on the figure.
The departure between the hyperboloid-hyperboloid fit and the actual “as-manufactured” mirror surface is shown in Figure 4-92. The straight line in the middle of the figure represents the “as-manufactured” hyperboloid-hyperboloid mirror profile. The curve on the left shows the deviation from the “as-manufactured” mirror profile of the hyperboloid-hyperboloid fit for the primary mirror. Likewise, the curve on the right shows the deviation from the “as-manufactured” mirror profile of the hyperboloid-hyperboloid fit for the secondary mirror. The front edge, rear edge and midpoint of the “as-manufactured” mirrors are a perfect fit, as these are the points that were used to calculate the hyperboloid parameters. Note that the greatest departure of the hyperboloid-hyperboloid fit from the actual mirror surface is approximately three-tenths of a nanometer. However, it is mirror slope errors illustrated in Figure 4-93 that produce reflected ray deviations that might introduce errors in our geometrical ray trace analysis.
Figure 4-92. Surface error due to the Hyperboloid-Hyperboloid fit for SN005.

Figure 4-93. Local slope error due to the Hyperboloid-Hyperboloid fit for SN005.
The local slope errors of the primary and secondary mirrors can cause the corresponding ray errors to either add or subtract. We thus calculated the contribution to the rms image size assuming that the ray deviations from the two mirrors were correlated, and also assuming that they were uncorrelated. For all five of the SXI mirrors, both of these numbers were shown to be negligible compared to the geometrical image size as determined by exhaustive ray tracing. This justifies our approximating the “as-manufactured” mirror surfaces with hyperboloids which can be easily modeled with ZEMAX.

Although somewhat redundant with Figure 4-89, we then had our MATLAB code calculate and print out the summary graph and tabulated parameters illustrated in Figure 4-94. This included the “as-manufactured system parameters, and the necessary ZEMAX input parameters in precisely the format accepted by the ZEMAX code.

![Figure 4-94. Summary chart illustrating mirror profiles, telescope system parameters, and ZEMAX input parameters for the “as-manufactured” SN005 SXI telescope.](image)
Using the optimization capability of ZEMAX, the best axial focus of the “as-manufactured” SN005 SXI telescope was then determined. The optimum focal position for an operational field of view (OFOV) of 18 arc-min was then found by the method described in Chapter 2. The geometrical rms image size versus field angle for a variety despaced focal planes was determined by exhaustive ray tracing. This data is illustrated in Figure 4-95. The minus sign in front of each despac value indicates that we are moving the focal plane from best axial focus towards the telescope mirrors.

![Figure 4-95. Geometrical rms image radius vs. field angle for a variety of different focal plane positions.](image)

Continuing to use the technique described in Chapter 2, the field-weighted-average rms image radius versus operational field-of-view is calculated for each of the above focal positions. This family of curves is shown in Figure 4-96.
Figure 4-96. Field weighted average rms image radius vs. operational field of view for SN005.

For an OFOV = 18 arc min, we now plot the y-intercepts vs. the focal plane despace and obtain a curve from which we can determine the optimum focal plane position as shown in Figure 4-97.

Figure 4-97. The optimum focal plane position for the “as-manufactured” SN005 SXI telescope.
The aerial point spread function (APSF) for the “as-manufactured” SN005 SXI telescope can now be modeled as described in Section 4.5. The obscuration ratio and the effective focal length used in the calculations of the ADPSF are taken directly from the “as-manufactured” mirror properties simulated with ZEMAX. Also from ZEMAX we obtain the ray intercept data with the focal plane and import it into the PSFGraz MATLAB code that creates the two-dimensional geometrical point spread function (GPSF).

On order to calculate the surface scatter point spread function (SSPSF), we need to provide the measured surface power spectral density (PSD) function as input into our version of the EEGRAZPC code, and this surface PSD had to be expressed in terms of a sum of Gaussian and Lorentzian functions. Figure 4-98 shows the one-dimensional, one-sided surface PSD that was presented to the mirror manufacturer as a requirement (and a goal) for the SXI mirrors. Also shown in Figure 4-98 is the “as-manufactured” surface PSD for SN005 SXI telescope mirrors as obtained from the metrology data from the two instruments used to evaluate surface quality during the optical fabrication process. The phase measuring interferometer (PMI) provides the surface PSD from a spatial frequency of 0.01 mm\(^{-1}\) to 10 mm\(^{-1}\). The WYKO instrument provides the PSD from a spatial frequency of 1.0 mm\(^{-1}\) to 1000 mm\(^{-1}\). By combining the two sets of data, we synthesize a surface PSD over the entire spatial frequency range from 0.01 mm\(^{-1}\) to 1000 mm\(^{-1}\). We then used a MATLAB fitting program to fit this synthesized surface PSF with a combination of Gaussian and Lorentzian functions that adequately describe the surface characteristics. For the “as-manufactured SN005 SXI mirrors, we used three Gaussian functions and two Lorentzian functions as shown in Eq. (4-15)

\[
PSD(f) = \sqrt{\pi\ell_1}\sigma_1^2 e^{-(\pi f\ell_1)^2} + \sqrt{\pi\ell_2}\sigma_2^2 e^{-(\pi f\ell_2)^2} + \frac{2\ell_3\sigma_3^2}{1+(2\pi\ell_3 f)^2} + \sqrt{\pi\ell_4}\sigma_4^2 e^{-(\pi f\ell_4)^2} + \frac{2\ell_5\sigma_5^2}{1+(2\pi\ell_5 f)^2} \tag{4-15}
\]
Figure 4-98 shows this fit (the green line) to the measured Surface PSD data, and its comparison to the required and goal PSD’s. The values of the parameters used for fitting Eq(4-15) to the metrology data are also tabulated in Figure 4-98.

![Figure 4-98](image)

\[
\begin{align*}
\sigma_1 &= 140 \text{ Å}, \quad \ell_1 = 6.0 \text{ mm} \\
\sigma_2 &= 40 \text{ Å}, \quad \ell_2 = 3.7 \text{ mm} \\
\sigma_3 &= 24 \text{ Å}, \quad \ell_3 = 1.5 \text{ mm} \\
\sigma_4 &= 6.3 \text{ Å}, \quad \ell_4 = 0.01 \text{ mm} \\
\sigma_5 &= 5.3 \text{ Å}, \quad \ell_5 = 0.0015 \text{ mm}
\end{align*}
\]

Figure 4-98. Comparison of the measured surface PSD for the “as-manufactured” SN005 SXI mirrors to the requirement and the goal imposed upon the mirror manufacturer.

The miscellaneous residual surface error point spread function (RSEPSF) for the “as-manufactured” SN005 SXI telescope is calculated from the final error allocations in the mirror manufacturer’s error budget tree. We mentioned earlier that the preliminary error budget tree becomes a *living document* throughout the fabrication and assembly and alignment process, with initial error budget allocations (requirements) being changed to reflect actual achievements at each step in the process. Figure 4-99 is the final error budget tree for the “as-manufactured”
SN005 SXI telescope. Note that the “reserve” allocations in the preliminary error budget tree shown in Figure 4-1 have been re-allocated to other error sources.

As described earlier, the values of the various error sources are sum-root-squared to obtain the top-level rms image core diameter of 2.910 arc sec. This one-dimensional Gaussian image core is converted into a two-dimensional RSEPSF by the PSFGraz Code. Finally the ADPSF, GPSF, SSPSF and the RSEPSF are numerically convolved by the PSFGraz Code to obtain the predicted aerial point spread function (APSF) for the “as-manufactured” SN005 SXI telescope.

The contractual requirement for the mirror manufacturer was expressed in terms of the fractional encircled energy of the aerial image at the focal plane exhibiting best axial focus for an on-axis object point and an object point at a 20 arc min field angle. Figure 4-100 to Figure 4-103 thus illustrate our image quality predictions under those conditions for a wavelength of 44.7 Å and a wavelength of 13.3 Å.
Figure 4-100. Fractional encircled energy predictions for the “as-manufactured” SN005 telescope at best axial focus for a wavelength 44.7Å and a field angle of zero.


Figure 4-101. Fractional encircled energy predictions for the “as-manufactured” SN005 telescope at best axial focus for a wavelength 13.3Å and a field angle of zero.
Figure 4-102. Fractional encircled energy predictions for the “as-manufactured” SN005 SXI telescope at best axial focus for a wavelength 44.7 Å and a field angle of 20 arc min.

Figure 4-103. Fractional encircled energy predictions for the “as-manufactured” SN005 SXI telescope at best axial focus for a wavelength 13.3 Å and a field angle of 20 arc min.
Note from the previous figures that surface scatter effects dominate geometrical aberrations at small field angles, and geometrical aberrations dominate surface scatter effects at large field angles. It is also evident that surface scatter effects are more severe for shorter wavelengths.

Table 4-9 compares our performance predictions for the “as-manufactured” SN005 SXI telescope with the contractual requirements and goals. The performance predictions of the mirror manufacturer, Goodrich Optical Systems, are also included in the comparison.

<table>
<thead>
<tr>
<th>Fractional Encircled Energy</th>
<th>On-axis ($\theta = 0.0$) (5.0 arc sec Dia. Circle)</th>
<th>Off-axis ($\theta = 20$ arc min) (20.0 arc sec Dia. Circle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 44.7 \text{ Å}$</td>
<td>0.55</td>
<td>0.41</td>
</tr>
<tr>
<td>$\lambda = 13.3 \text{ Å}$</td>
<td>0.41</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Table 4-9. Comparison of fractional encircled energy predictions for the “as-manufactured” SN005 SXI telescope with program requirements and goals

We see from Table 4-9 that in all the cases our predictions reach or exceed the requirements. We also reach or exceed the goals in all cases except for $44 \text{ Å}$, on-axis and $13 \text{ Å}$, off-axis cases. Our predictions are slightly below than Goodrich’s predictions for the $44 \text{ Å}$ and $13 \text{ Å}$, on-axis case, but they are higher that Goodrich’s predictions for $44 \text{ Å}$ and $13 \text{ Å}$, off-axis cases.

Although the contractual image quality requirements were expressed in terms of the fractional encircled energy at best axial focus, a more meaningful image quality criterion for the SXI
mission is some field-weighted-average measure of resolution. We have chosen the field-weighted-average half power radius (HPR\textsubscript{fwa}) as an appropriate image quality criterion, and the HT#17 design was optimized to minimize this quantity for an 18 arc min OFOV. Figure 4-104 illustrates the predicted HPR of the aerial image for the “as-manufactured” SN005 SXI telescope for two different wavelength over the entire field-of-view of interest at the optimal focal plane position (dz = -0.113 mm) for an 18 arc-min OFOV. A 3-D isometric plot of the aerial image at various field angles is also illustrated.

![Figure 4-104. Predicted half power radius of the aerial image vs. field angle for the “as-manufactured” SN005 SXI telescope at two different wavelengths.](image)

Again we consider the HPR of the aerial image to define a spatial resolution element, then the number of spatial resolution elements in a given OFOV is given by

\[
N = 2\pi \int_{\theta=0}^{\theta_{\text{OFOV}}} \frac{\theta}{\pi \text{HPR}^2(\theta)} d\theta
\]  

(4-16)
Minimizing the $HPR_{fwa}$ over a given OFOV clearly maximizes the number of spatial resolution elements (N) in that OFOV. This is equivalent to maximizing the amount of information in the OFOV. Figure 4-105 illustrates the predicted N in the aerial image as a function of OFOV for the “as-manufactured” SN005 SXI telescope for the two wavelengths of interest.

Following the procedure described in Chapter 3 for calculating image degradation due to detector effects, the half power radius for the average UDPSF for the “as-manufactured” SN005 SXI telescope was determined as a function of field angle for wavelengths of 13.3Å and 44.7Å. These predictions are shown in Figure 4-106. Again, 3-D isometric plots of the average UDPSF are displayed at a variety of different field angles. Figure 4-106 illustrates the number of spatial resolution elements calculated from Eq.(4-16) for the average UDPSF.
Figure 4-106. Predicted half power radius of the average UDPSF vs. field angle for the “as-manufactured” SN005 SXI telescope at two different wavelengths.

Figure 4-107. Number of predicted spatial resolution elements vs. OFOV when detector effects are included with the “as-manufactured” SN005 SXI telescope at two different wavelengths.
Figure 4-108 summarizes the results of our systems engineering analysis of image quality for the “as-manufactured” SN005 SXI telescope. It illustrates the predicted half power radius (HPR) of the point spread function (PSF) as a function of field angle as we progressively include more error sources in the analysis. The lower curve illustrates the predicted image quality based solely upon geometrical analysis (ray tracing). The predicted aerial image (middle curves) includes diffraction effects, geometrical aberrations, surface scatter effects, and all of the error sources in the mirror manufacturer’s error budget tree. Of course, the diffraction effects and surface scatter effects vary with wavelength. And finally, the top two curves include all of the above errors, plus a rigorous analysis of image degradation due to detector effects has been included. This comparison shows how truly inadequate the simple geometrical analysis is for calculating image quality for many applications.

Figure 4-108. Comparison of image quality predictions for the “as-manufactured” SN005 SXI telescope based upon; 1) geometrical analysis only, 2) physical optics analysis including all error sources affecting the aerial image (in particular surface scatter phenomena), and 3) including a rigorous analysis of mosaic detector array effects.
4.6.5 “As-manufactured” SN006 SXI Telescope

The mirror profiles for SN006, including the fabrication errors, are shown in Figure 4-109. The fabrication figure error coefficients are again tabulated along with the graph of the mirror profile, as are the measured mirror dimensions at the front, midpoint, and rear of both the primary and secondary mirrors along with their respective axial locations. This data that is used to calculate the hyperboloid-hyperboloid fit to the actual “as-manufactured” surface profiles as explained in Section 4.5. And finally, the ZEMAX optical prescription parameters for the hyperboloid-hyperboloid fit to the “as-manufactured” mirror profiles are included in Figure 4-109.

![Figure 4-109. “As-manufactured” SN006 mirror profile, with the fabrication error coefficients, mirror dimensions, and optical prescription of the hyperboloid-hyperboloid fit.](image)

The primary and secondary mirrors were polished and figured into a single glass cylindrical substrate. After the figuring and polishing was completed, the substrate was end-cut to eliminate any undesirable rolled-down edges. A 5.0 mm groove, referred to as the gap, separating the
primary mirror profile from the secondary mirror profile was included in the mirror design. This
groove was also routinely expanded at the end of the fabrication process to eliminate undesirable
edge effects at the rear of the primary mirror and at the front of the secondary mirror. A
wire-frame layout of the hyperboloid-hyperboloid fit to the “as-manufactured” SN006 SXI
telescope mirror is illustrated in Figure 4-110, as simulated with ZEMAX. Note the
“as-manufactured” end-cut dimensions shown at the bottom of the illustration.

![Wire-frame layout of the hyperboloid-hyperboloid fit to the “as-manufactured” SN006 SXI telescope mirror, as simulated with ZEMAX.](image)

Figure 4-111 illustrates the departure of the “as-manufactured” mirror profiles from the
HT#17 optical design profiles in the region of the mirror joint. The expanded ordinate and
abscissa of this graph shows the radial and axial shift of the mirror joint for the
“as-manufactured” telescope from its design location. This is actually a virtual joint, as it occurs
in the gap between the primary and secondary mirror surfaces. However, there is a mathematical
joint with a specific radius and axial location. The joint radius, joint axial location, and the axial
location of the telescope focal plane are also listed on the figure.
Figure 4-111. Comparison of the HT#17 design and the SN006 mirror profile near the mirror joint.

The departure between the hyperboloid-hyperboloid fit and the actual “as-manufactured” mirror surface is shown of Figure 4-112. The straight line in the middle of the figure represents the “as-manufactured” hyperboloid-hyperboloid mirror profile. The curve on the left shows the deviation from the “as-manufactured” mirror profile of the hyperboloid-hyperboloid fit for the primary mirror. Likewise, the curve on the right shows the deviation from the “as-manufactured” mirror profile of the hyperboloid-hyperboloid fit for the secondary mirror. The front edge, rear edge, and midpoint of the “as-manufactured” mirrors are a perfect fit, as these are the points that were used to calculate the hyperboloid parameters. Note that the greatest departure of the hyperboloid-hyperboloid fit from the actual mirror surface is approximately a tenth of a nanometer. However, it is mirror slope errors illustrated in Figure 4-113 that produce reflected ray deviations that might introduce errors in our geometrical ray trace analysis.
Figure 4-112. Surface error due to the Hyperboloid-Hyperboloid fit for SN006.

Figure 4-113. Local slope error due to the Hyperboloid-Hyperboloid fit for SN006.
The local slope errors of the primary and secondary mirrors can cause the corresponding ray errors to either add or subtract. We thus calculated the contribution to the rms image size assuming that the ray deviations from the two mirrors were correlated, and also assuming that they were uncorrelated. For all five of the SXI mirrors, both of these numbers were shown to be negligible compared to the geometrical image size as determined by exhaustive ray tracing. This justifies our approximating the “as-manufactured” mirror surfaces with hyperboloids which can be easily modeled with ZEMAX.

Although somewhat redundant with Figure 4-109, we then had our MATLAB code calculate and print out the summary graph and tabulated parameters illustrated in Figure 4-114. This included the “as-manufactured system parameters, and the necessary ZEMAX input parameters in precisely the format accepted by the ZEMAX code.

Figure 4-114. Summary chart illustrating mirror profiles, telescope system parameters, and ZEMAX input parameters for the “as-manufactured” SN006 SXI telescope.
Using the optimization capability of ZEMAX, the best axial focus of the “as-manufactured” SN006 SXI telescope was then determined. The optimum focal position for an operational field of view (OFOV) of 18 arc-min was then found by the method described in Chapter 2. The geometrical rms image size versus field angle for a variety despaced focal planes was determined by exhaustive ray tracing. This data is illustrated in Figure 4-115. The minus sign in front of each despase value indicates that we are moving the focal plane from best axial focus towards the telescope mirrors.

Continuing to use the technique described in Chapter 2, the field-weighted-average rms image radius versus operational field-of-view is calculated for each of the above focal positions. This family of curves is shown in Figure 4-116.
Figure 4-116. Field weighted average rms image radius vs. operational filed of view for SN006.

For an OFOV = 18 arc min, we now plot the y-intercepts vs. the focal plane despace and obtain a curve from which we can determine the optimum focal plane position as shown in Figure 4-117.

Figure 4-117. The optimum focal plane position for the “as-manufactured” SN006 SXI telescope.
The aerial point spread function (APSF) for the “as-manufactured” SN006 SXI telescope can now be modeled as described in Section 4.5. The obscuration ratio and the effective focal length used in the calculations of the ADPSF are taken directly from the “as-manufactured” mirror properties simulated with ZEMAX. Also from ZEMAX we obtain the ray intercept data with the focal plane and import it into the PSF.Graz MATLAB code that creates the two-dimensional geometrical point spread function (GPSF).

On order to calculate the surface scatter point spread function (SSPSF), we need to provide the measured surface power spectral density (PSD) function as input into our version of the EEGRAZPC code, and this surface PSD had to be expressed in terms of a sum of Gaussian and Lorentzian functions. Figure 4-118 shows the one-dimensional, one-sided surface PSD that was presented to the mirror manufacturer as a requirement (and a goal) for the SXI mirrors. Also shown in Figure 4-118 is the “as-manufactured” surface PSD for SN006 SXI telescope mirrors as obtained from the metrology data from the two instruments used to evaluate surface quality during the optical fabrication process. The phase measuring interferometer (PMI) provides the surface PSD from a spatial frequency of 0.01 mm\(^{-1}\) to 10 mm\(^{-1}\). The WYKO instrument provides the PSD from a spatial frequency of 1.0 mm\(^{-1}\) to 1000 mm\(^{-1}\). By combining the two sets of data, we synthesize a surface PSD over the entire spatial frequency range from 0.01 mm\(^{-1}\) to 1000 mm\(^{-1}\). We then used a MATLAB fitting program to fit this synthesized surface PSF with a combination of Gaussian and Lorentzian functions that adequately describe the surface characteristics. For the “as-manufactured SN006 SXI mirrors, we used three Gaussian functions and two Lorentzian functions as shown in Eq. (4-17)

\[
PSD(f) = \sqrt{\pi} \ell_1 \sigma_{\ell_1}^2 e^{-(\pi f \sigma_{\ell_1})^2} + \frac{2\ell_2 \sigma_{\ell_2}^2}{1 + (2\pi \ell_2 f)^2} + \frac{2\ell_3 \sigma_{\ell_3}^2}{1 + (2\pi \ell_3 f)^2} + \sqrt{\pi} \ell_4 \sigma_{\ell_4}^2 e^{-(\pi f \sigma_{\ell_4})^2} + \sqrt{\pi} \ell_5 \sigma_{\ell_5}^2 e^{-(\pi f \sigma_{\ell_5})^2} \quad (4-17)
\]
Figure 4-118 shows this fit (the green line) to the measured Surface PSD data, and its comparison to the required and goal PSD’s. The values of the parameters used for fitting Eq(4-17) to the metrology data are also tabulated in Figure 4-118.

![Figure 4-118](image)

The miscellaneous residual surface error point spread function (RSEPSF) for the “as-manufactured” SN006 SXI mirrors is calculated from the final error allocations in the mirror manufacturer’s error budget tree. We mentioned earlier that the preliminary error budget tree becomes a *living document* throughout the fabrication and assembly and alignment process, with initial error budget allocations (requirements) being changed to reflect actual achievements at each step in the process. Figure 4-119 is the final error budget tree for the “as-manufactured”
SN006 SXI telescope. Note that the “reserve” allocations in the preliminary error budget tree shown in Figure 4-1 have been re-allocated to other error sources.

As described earlier, the values of the various error sources are sum-root-squared to obtain the top-level rms image core diameter of 2.460 arc sec. This one-dimensional Gaussian image core is converted into a two-dimensional RSEPSF by the PSFGraz Code. Finally the ADPSF, GPSF, SSPSF and the RSEPSF are numerically convolved by the PSFGraz Code to obtain the predicted aerial point spread function (APSF) for the “as-manufactured” SN006 SXI telescope.

The contractual requirement for the mirror manufacturer was expressed in terms of the fractional encircled energy of the aerial image at the focal plane exhibiting best axial focus for an on-axis object point and an object point at a 20 arc min field angle. Figure 4-120 to Figure 4-123 thus illustrate our image quality predictions under those conditions for a wavelength of 44.7 Å and a wavelength of 13.3 Å.
Figure 4-120. Fractional encircled energy predictions for the “as-manufactured” SN006 telescope at best axial focus for a wavelength 44.7Å and a field angle of zero.

Scatter Effects Dominate Geo. Aberrations
For Small Field Angles

SN006

FEcE = 0.65 in a 5 arc sec
diameter circle.

\[ \lambda = 44.7\text{Å} \]
\[ \theta = 0.0 \text{ arc min} \]

Figure 4-121. Fractional encircled energy predictions for the “as-manufactured” SN006 telescope at best axial focus for a wavelength 13.3Å and a field angle of zero.

Scatter Dominates Geo. Aberrations for Small Field Angles

SN006

FEcE = 0.55 in a 5 arc sec
diameter circle.

\[ \lambda = 13.3\text{Å} \]
\[ \theta = 0.0 \text{ arc min} \]
Fractional Encircled Energy

Radius of Circle (arc sec)

SN006

Geo. Aberrations Dominate Scatter Effects For Large Field Angles

FEcE = 0.41 in a 20 arc sec diameter circle.

λ = 44.7 Å
θ = 20 arc min

Figure 4-122. Fractional encircled energy predictions for the “as-manufactured” SN006 SXI telescope at best axial focus for a wavelength 44.7 Å and a field angle of 20 arc min.

Fractional Encircled Energy

Radius of Circle (arc sec)

SN006

λ = 13.3 Å
θ = 20 arc min

FEcE = 0.39 in a 20 arc sec diameter circle.

Figure 4-123. Fractional encircled energy predictions for the “as-manufactured” SN006 SXI telescope at best axial focus for a wavelength 13.3 Å and a field angle of 20 arc min.
Note from the previous figures that surface scatter effects dominate geometrical aberrations at small field angles, and geometrical aberrations dominate surface scatter effects at large field angles. It is also evident that surface scatter effects are more severe for shorter wavelengths.

Table 4-10 compares our performance predictions for the “as-manufactured” SN006 SXI telescope with the contractual requirements and goals. The performance predictions of the mirror manufacturer, Goodrich Optical Systems, are also included in the comparison.

Table 4-10. Comparison of fractional encircled energy predictions for the “as-manufactured” SN006 SXI telescope with program requirements and goals

<table>
<thead>
<tr>
<th></th>
<th>Fractional Encircled Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>On-axis ($\theta = 0.0$)</td>
</tr>
<tr>
<td></td>
<td>(5.0 arc sec Dia. Circle)</td>
</tr>
<tr>
<td></td>
<td>$\lambda = 44.7 , \text{Å}$</td>
</tr>
<tr>
<td></td>
<td>$\lambda = 13.3 , \text{Å}$</td>
</tr>
<tr>
<td>Req’mt</td>
<td>0.55</td>
</tr>
<tr>
<td>Goal</td>
<td>0.60</td>
</tr>
<tr>
<td>CREOL Prediction</td>
<td>0.65</td>
</tr>
<tr>
<td>Goodrich Prediction</td>
<td>0.61</td>
</tr>
</tbody>
</table>

We see from Table 4-10 that our predictions meet or exceed the requirements except for the 13.3 Å off-axis case. We also exceed the goal except for 44.7 Å and 13.3 Å off-axis cases. For all cases our predictions also agree well with those of Goodrich Optical systems.

Although the contractual image quality requirements were expressed in terms of the fractional encircled energy at best axial focus, a more meaningful image quality criterion for the SXI mission is some field-weighted-average measure of resolution. We have chosen the
field-weighted-average half power radius (HPR$_{fwa}$) as an appropriate image quality criterion, and the HT#17 design was optimized to minimize this quantity for an 18 arc min OFOV. Figure 4-124 illustrates the predicted HPR of the aerial image for the “as-manufactured” SN006 SXI telescope for two different wavelength over the entire field-of-view of interest at the optimal focal plane position (dz = -0.108 mm) for an 18 arc-min OFOV. A 3-D isometric plot of the aerial image at various field angles is also illustrated.

![Figure 4-124. Predicted half power radius of the aerial image vs. field angle for the “as-manufactured” SN006 SXI telescope at two different wavelengths.](image)

Again we consider the HPR of the aerial image to define a spatial resolution element, then the number of spatial resolution elements in a given OFOV is given by

$$ N = 2\pi \int_{\theta=0}^{\theta=\theta_{OFOV}} \frac{\theta}{\pi \text{HPR}^2(\theta)} d\theta $$

(4-18)
Minimizing the $HPR_{fwa}$ over a given OFOV clearly maximizes the number of spatial resolution elements in that OFOV. This is equivalent to maximizing the amount of information in the OFOV. Figure 4-125 illustrates the predicted number of spatial resolution elements in the aerial image as a function of OFOV for the “as-manufactured” SN006 SXI telescope for the two wavelengths of interest.

Following the procedure described in Chapter 3 for calculating image degradation due to detector effects, the half power radius for the average UDPSF for the “as-manufactured” SN006 SXI telescope was determined as a function of field angle for wavelengths of 13.3Å and 44.7Å. These predictions are shown in Figure 4-126. Again, 3-D isometric plots of the average UDPSF are displayed at a variety of different field angles. Figure 4-127 illustrates the number of spatial resolution elements calculated from Eq.(4-18) for the average UDPSF.
Figure 4-126. Predicted half power radius of the average UDPSF vs. field angle for the “as-manufactured” SN006 SXI telescope at two different wavelengths.

Figure 4-127. Number of predicted spatial resolution elements vs. OFOV when detector effects are included with the “as-manufactured” SN006 SXI telescope at two different wavelengths.
Figure 4-128 summarizes the results of our systems engineering analysis of image quality for the “as-manufactured” SN006 SXI telescope. It illustrates the predicted half power radius (HPR) of the point spread function (PSF) as a function of field angle as we progressively include more error sources in the analysis. The lower curve illustrates the predicted image quality based solely upon geometrical analysis (ray tracing). The predicted aerial image (middle curves) includes diffraction effects, geometrical aberrations, surface scatter effects, and all of the error sources in the mirror manufacturer’s error budget tree. Of course, the diffraction effects and surface scatter effects vary with wavelength. And finally, the top two curves include all of the above errors, plus a rigorous analysis of image degradation due to detector effects has been included. This comparison shows how truly inadequate the simple geometrical analysis is for calculating image quality for many applications.

Figure 4-128. Comparison of image quality predictions for the “as-manufactured” SN006 SXI telescope based upon; 1) geometrical analysis only, 2) physical optics analysis including all error sources affecting the aerial image (in particular surface scatter phenomena), and 3) including a rigorous analysis of mosaic detector array effects.
5.0 PERFORMANCE PREDICTION COMPARISONS

This chapter shows a detailed comparison of our performance predictions for the five “as-manufactured” SXI telescopes, and also includes the performance predictions for the H-T#17 grazing incidence X-ray telescope design in the comparison. We start by comparing the Goodrich metrology data and the resulting system parameters for the five “as-manufactured” telescopes. We then compare the performance predictions at the best axial focus to the contractural requirements and goals. We then proceed to compare the performance predictions for the various telescopes at their respective optimal focal planes. This is done for the geometrical performance (ray trace analysis), the aerial image in the optimal focal plane, and the average unregistered detected point spread function (AUDPSF). The total number of spatial resolution elements in an 18 arc min OFOV is then calculated and compared for all of the “as-manufactured” telescopes and the H-T#17 design for both the aerial image and the detected image at two different wavelengths. And finally, our performance predictions are compared to a limited amount of actual X-ray test data provided by Lockheed Martin.

5.1 Comparison of Optical Fab Errors and Resulting System Parameters

The Goodrich metrology data from the “as-manufactured” SXI mirrors was summarized in terms of deterministic low spatial frequency “figure” errors and random mid and high spatial frequency “finish” errors. The deterministic figure errors were expressed in terms of the Average Radius Error for the primary and the secondary mirrors, $\bar{R}_p$ and $\bar{R}_s$, the Delta Radius Error for the primary and the secondary mirrors, $\Delta R_p$ and $\Delta R_s$, and the Average Axial Sag Error
for the primary and the secondary mirrors, $a_p^2$ and $a_s^2$. The random finish errors were measured as “mid” and “high” spatial frequency surface PSD’s and presented for each of the telescopes in the previous chapter. These optical fabrication figure errors are tabulated in Table 5-1 along with the distance from the mirror joint to the best axial focus and the axial position of the front ($z_{p1}$) and back ($z_{p3}$) of the primary mirror, and the front ($z_{s1}$) and back ($z_{s3}$) of the secondary mirror. These axial positions of the final end-cut mirrors are measured from the front of the design primary mirror; i.e., 50 mm in front of the design joint.

<table>
<thead>
<tr>
<th></th>
<th>SN002</th>
<th>SN003</th>
<th>SN004</th>
<th>SN005</th>
<th>SN006</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{r}_p$</td>
<td>10.38 um</td>
<td>-50.90 um</td>
<td>-30.09 um</td>
<td>-158.51 um</td>
<td>-43.68 um</td>
</tr>
<tr>
<td>$\bar{r}_s$</td>
<td>-13.53 um</td>
<td>-51.10 um</td>
<td>-40.54 um</td>
<td>-151.46 um</td>
<td>-38.56 um</td>
</tr>
<tr>
<td>$\Delta r_p$</td>
<td>-1.02 um</td>
<td>0.42 um</td>
<td>0.81 um</td>
<td>1.17 um</td>
<td>0.31 um</td>
</tr>
<tr>
<td>$\Delta r_s$</td>
<td>0.38 um</td>
<td>-0.04 um</td>
<td>-0.03 um</td>
<td>-0.99 um</td>
<td>0.05 um</td>
</tr>
<tr>
<td>$a_p^2$</td>
<td>0.0372 um</td>
<td>0.0315 um</td>
<td>0.0272 um</td>
<td>0.0491 um</td>
<td>0.0491 um</td>
</tr>
<tr>
<td>$a_s^2$</td>
<td>0.0420 um</td>
<td>0.0018 um</td>
<td>-0.0191 um</td>
<td>0.0412 um</td>
<td>-0.0041 um</td>
</tr>
<tr>
<td>$f_{jb}$</td>
<td>654.8061 mm</td>
<td>655.3412 mm</td>
<td>655.0379 mm</td>
<td>655.8205 mm</td>
<td>655.4013 mm</td>
</tr>
<tr>
<td>$z_{p1}$</td>
<td>-0.7863 mm</td>
<td>8.286 mm</td>
<td>6.75 mm</td>
<td>0.043 mm</td>
<td>4.3635 mm</td>
</tr>
<tr>
<td>$z_{p3}$</td>
<td>44.6851 mm</td>
<td>36.268 mm</td>
<td>43.75 mm</td>
<td>39.513 mm</td>
<td>37.3285 mm</td>
</tr>
<tr>
<td>$z_{s1}$</td>
<td>50.8611 mm</td>
<td>63.437 mm</td>
<td>60.45 mm</td>
<td>60.1665 mm</td>
<td>62.147 mm</td>
</tr>
<tr>
<td>$z_{s3}$</td>
<td>97.5901 mm</td>
<td>89.219 mm</td>
<td>92.05 mm</td>
<td>96.1815 mm</td>
<td>93.015 mm</td>
</tr>
</tbody>
</table>

Table 5-2 shows the “as-manufactured” system parameters for all of the SXI telescopes in terms of radial distance from the optical axis to the mirror joint, $r_j$, the axial distance from the beginning of the coordinate system to the joint, $z_j$, and the minimum and maximum average radius of the primary mirror, $r_{pmin}$ and $r_{pmax}$. The obscuration ratio, $\varepsilon$, is calculated from $r_{pmin}$ and $r_{pmax}$. The average grazing angle, $\alpha$; the effective focal length, $f_{eff}$; the distance from the joint to
the operational focal plane, $f_j$; and the distance from the joint to the best axial focus, $f_{jb}$ are all tabulated. Also included in Table 5-2 is the error of the hyperboloid-hyperboloid fit to the as-manufactured mirror surface profile, and the one-dimensional width of the image core due to the miscellaneous errors in the mirror manufacturer’s error budget tree. All of these parameters are either inputs to, or outputs from the MATLAB codes and ZEMAX files that are used for our systems engineering analysis of image quality.

<table>
<thead>
<tr>
<th>Table 5-2. Relevant “as-manufactured” X-ray telescope system parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HT#17</strong></td>
</tr>
<tr>
<td>$r_j$ (mm)</td>
</tr>
<tr>
<td>$z_j$ (mm)</td>
</tr>
<tr>
<td>$r_{pmin}$ (mm)</td>
</tr>
<tr>
<td>$r_{pmax}$ (mm)</td>
</tr>
<tr>
<td>$\varepsilon$ (deg)</td>
</tr>
<tr>
<td>$\alpha$ (deg)</td>
</tr>
<tr>
<td>$f_{eff}$ (mm)</td>
</tr>
<tr>
<td>$f_j$ (mm)</td>
</tr>
<tr>
<td>$f_{jb}$ (mm)</td>
</tr>
<tr>
<td>$\delta z$ (mm)</td>
</tr>
<tr>
<td>fit error</td>
</tr>
<tr>
<td>RSEPSF 1-D Width</td>
</tr>
</tbody>
</table>

5.2 Comparison of Performance Predictions at Best Axial Focus

We have discussed at length (in the previous chapters) the reasons for not operating these wide-field telescopes at the best axial focus, and have taken great pains to calculate the optimum focal plane position for each of the “as-manufactured” SXI telescopes. However, for historical
reasons, and because X-ray testing of the image quality can more easily be performed at best axial focus; the contractual image quality requirements were expressed in terms of fractional encircled energy (FEE) at best axial focus. In particular, there is a contractual requirement for the FEE for two wavelengths, 44.7 Å and 13.3 Å, and for two different field angles, $\theta = 0$ and $\theta = 20.0$ arc min.

Comparison of geometrical ray trace data for each of the “as-manufactured” SXI telescope mirrors at best axial focus is shown on Figure 5-1. The ray trace data for the HT#17 design is also included in the comparison. This family of curves gives some indication of the variation in the manufacturing figure errors between the various telescopes. Note that the variation in the rms image size relative to the mean image size is quite large at the small field angles compared to that at the large field angles. It is also interesting to note that the H-T#17 design has the largest rms image size for field angles less than 5 arc min.

Figure 5-1. Comparison of the geometrical rms image radius vs. field angle at best axial focus for H-T#17, SN002, SN003, SN004, SN005 and SN006.
Our on-axis FEE performance predictions for the H-T#17 design and the “as-manufactured” mirror sets SN002, SN003, SN004, SN005 and SN006 are summarized in Table 5-3 for easy comparison with the contractual requirements and goals. The predictions of Goodrich are also shown for comparison. The actual FEE curves were shown for each of the telescopes in the previous chapter.

Table 5-3. Comparison of the predicted Fractional Encircled Energy at best axial focus.

<table>
<thead>
<tr>
<th></th>
<th>Fractional Encircled Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>On-axis (θ = 0.0)</td>
</tr>
<tr>
<td></td>
<td>(5.0 arc sec Dia. Circle)</td>
</tr>
<tr>
<td>λ = 44.7 Å</td>
<td>0.55</td>
</tr>
<tr>
<td>Req’mt</td>
<td>SXI Mirrors</td>
</tr>
<tr>
<td>Goal</td>
<td>0.60</td>
</tr>
<tr>
<td>CREOL HT#17</td>
<td>0.56</td>
</tr>
<tr>
<td>Goodrich SN002</td>
<td>0.60</td>
</tr>
<tr>
<td>CREOL SN002</td>
<td>0.58</td>
</tr>
<tr>
<td>Goodrich SN003</td>
<td>0.62</td>
</tr>
<tr>
<td>CREOL SN004</td>
<td>0.56</td>
</tr>
<tr>
<td>Goodrich SN004</td>
<td>0.61</td>
</tr>
<tr>
<td>CREOL SN005</td>
<td>0.58</td>
</tr>
<tr>
<td>Goodrich SN005</td>
<td>0.60</td>
</tr>
<tr>
<td>CREOL SN006</td>
<td>0.65</td>
</tr>
<tr>
<td>Goodrich SN006</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>Off-axis (θ = 20 arc min)</td>
</tr>
<tr>
<td>(20.0 arc sec Dia. Circle)</td>
<td>0.41</td>
</tr>
<tr>
<td>λ = 44.7 Å</td>
<td>0.41</td>
</tr>
<tr>
<td>λ = 13.3 Å</td>
<td>0.40</td>
</tr>
<tr>
<td>CREOL SN002</td>
<td>0.47</td>
</tr>
<tr>
<td>Goodrich SN002</td>
<td>0.48</td>
</tr>
<tr>
<td>CREOL SN003</td>
<td>0.46</td>
</tr>
<tr>
<td>Goodrich SN003</td>
<td>0.50</td>
</tr>
<tr>
<td>CREOL SN004</td>
<td>0.45</td>
</tr>
<tr>
<td>Goodrich SN004</td>
<td>0.52</td>
</tr>
<tr>
<td>CREOL SN005</td>
<td>0.45</td>
</tr>
<tr>
<td>Goodrich SN005</td>
<td>0.49</td>
</tr>
<tr>
<td>CREOL SN006</td>
<td>0.55</td>
</tr>
<tr>
<td>Goodrich SN006</td>
<td>0.53</td>
</tr>
</tbody>
</table>

There are five different “as-manufactured” telescopes with four sets of requirements and goals for each telescope. Our predictions meet or exceed the contractual requirement in all cases except for the off-axis 13.3Å case for SN003 (FEE = 0.38 instead of 0.40) and SN006 (FEE = 0.39 instead of 0.40). Our predictions meet or exceed even the contractual goal for 12 of the 20 cases for the “as-manufactured” telescopes. As can be seen from the table, our
predictions also agree well with the Goodrich’s predictions. In fact, if we calculate the per cent error from the mean of the two predictions, and average over the 20 cases, we get a 2.9% mean deviation from the mean.

5.3 Comparison of Performance Predictions at Operational Focal Plane

In Chapter 4 we performed extensive analysis to find the focal plane position, for each as-manufactured SXI telescope, that minimizes the field-weighted-average half power radius ($HPR_{fwa}$) of the aerial point spread function ($APSF$) over an OFOV of 18 arc min radius. Minimizing the $HPR_{fwa}$ maximizes the number of spatial resolution elements over the desired $OFOV$, thus maximizing the amount of information contained in the image. Figure 5-2 shows the sensitivity of this information content to the focal plane position for the SN004 SXI telescope.

![Figure 5-2. Number of Spatial Resolution Elements vs. Focal Plane Despace for SN004](image)
The number of spatial resolution elements in an 18 arc min radius OFOV at the midpoint of the spectral range of interest is indicated on Figure 5-2 for the best axial focus, a focal plane despace of 0.055 mm, and the optimum focal plane position determined to be $dz = -0.097$ mm. Recall from Chapter 3 that $dz = -0.055$ mm was the focal plane position where the geometrical PSF just filled a detector pixel. It is significant that placing the detector in the optimum focal plane position will more than double the amount of information that would be obtained at best axial focus, and will result in almost a 20% increase in the amount of information that would be obtained at a despace of 0.055 mm.

5.3.1 Predictions of the Geometrical Performance

Comparison of the geometrical ray trace data for each of the “as-manufactured” SXI telescope at their respective optimum focal planes for an OFOV of 18 arc min radius is shown in Figure 5-3. The ray trace data for the HT#17 design is also included in the comparison.

Figure 5-3. Comparison of the geometrical rms image radius vs. field angle for OFOV = 18 arc-min for H-T#17, SN002, SN003, SN004, SN005 and SN006.
The average rms image radius has a value for approximately 4.2 arc sec on axis and goes up to approximately 9 arc sec at a 21 arc min filed angle. This is a significant reduction in image size at the larger field angles from those shown in Figure 5-1 for best axial focus. The variation in the geometrical performance predictions for the different SXI telescopes is an indication of the degree of uniformity with which the mirror manufacturer was able to produce multiple telescopes to the same design; i.e., a measure of their telescope to telescope optical figure errors.

The discrete geometrical spot diagrams from which the rms image radius is calculated by the ZEMAX ray trace code are converted to a ray intercept density function (GPSF) for use in the PSF Graz MatLab code. Figure 5-4 illustrates the HPR of the GPSF for each of the “as-manufactured” SXI telescope mirrors at their respective optimum focal planes for an OFOV of 18 arc min radius. Note that the performance curves did not change significantly from those indicated in Figure 5-3; however, the values at 21 arc min dropped to 7.6 arc sec on average.

Figure 5-4. Comparison of the HPR of GPSF vs. field angle for OFOV = 18 arc-min for H-T#17, SN002, SN003, SN004, SN005 and SN006.
5.3.2 Predictions of the Aerial Image

In Figure 5-5 and Figure 5-6 we compare the HPR of the aerial image (including all system errors except for detector effects) for each of the “as-manufactured” SXI telescope mirrors for a wavelength of 44.7 Å and 13.3 Å respectively.

Figure 5-5. Comparison of the HPR of the APSF vs. field angle for OFOV = 18 arc-min for H-T#17, SN002, SN003, SN004, SN005 and SN006 for a wavelength of 44.7 Å.

Figure 5-6. Comparison of the HPR of the APSF vs. field angle for OFOV = 18 arc-min for H-T#17, SN002, SN003, SN004, SN005 and SN006 for a wavelength of 13.3 Å.
As expected the \(HPR\) of the aerial image is somewhat larger than for the \(GPSF\) due to the degradation effects of diffraction effects, surface scatter effects, and all miscellaneous residual errors in the mirror manufacturer’s error budget tree. Diffraction effects degrade images more for longer wavelengths and surface scatter effects degrade images more for short wavelengths; however, the scatter effects dominate, resulting in the \(HPR\) being larger for the 13.3 Å radiation.

We also note that the H-T#17 design outperforms all of the as-manufactured SXI telescopes over the entire field-of-view. SN005 exhibits the poorest performance predictions of all the as-manufactured SXI telescopes.

### 5.3.3 Predictions of the Detected Image

Finally, in Figure 5-7 and Figure 5-8 we compare the \(HPR\) of the average unregistered detected \(PSF\) (AUDPSF) for each of the “as-manufactured” SXI telescope mirrors for a wavelength of 44.7 Å and 13.3 Å respectively.

![Figure 5-7](image)

Figure 5-7. Comparison of the HPR of the AUDPSF vs. field angle for OFOV = 18 arc-min for H-T#17, SN002, SN003, SN004, SN005 and SN006 for wavelength of 44.7 Å.
Figure 5-8. Comparison of the HPR of the AUDPSF vs. field angle for OFOV = 18 arc-min for H-T#17, SN002, SN003, SN004, SN005 and SN006 for wavelength of 44.7 Å.

For the comparison of the HPR of the AUDPSF we used the procedure described in Chapter 3 and as can be noted the performance predictions for each SXI telescope has become even worse compared to the performance predictions for the HPR of the aerial image. However, the performance for the individual mirrors compared to each other did not change. This behavior is expected since the detector effects only broaden the aerial image PSF according to the procedure described in Chapter 3.

5.3.4 Summary and Uniformity of System Performance

Recalling Eq.(3-3), we calculate and compare the number of spatial resolution elements in an 18 arc min OFOV for all as-manufactured SXI Mirrors. This comparison is illustrated in Figure 5-9 in the form of a bar graph. A spatial (angular) resolution element has been defined as a circle whose radius is equal to the HPR of the corresponding PSF. The total number of detector pixels in an 18 arc min radius OFOV is indicated on the graph as a reference.
It should be noted that, for staring mosaic detector arrays, the size of a \textit{detected} resolution element at any given field angle is either the size of the corresponding aerial \textit{PSF} or the size of the detector pixel, \textit{whichever is largest}. It is this fact that invalidates the presumed merits of an aplanatic telescope design, even at small field angles, when used with a mosaic detector array.

Comparison of the percent reduction (from the HT#17 design) in the number of spatial (angular) resolution elements in an 18 arc min radius OFOV for each of the “as-manufactured” SXI telescope mirrors is shown on Figure 5-10. Again we see that SN004 and SN006 have the best predicted performance of the as-manufactured SXI telescopes and SN005 has the worst predicted performance. In Figure 5-11, we present the mean percent reduction (from the HT#17 design) in the number of spatial resolution elements in an 18 arc min radius \textit{OFOV}, and the mean deviation from the mean percent reduction.
Figure 5-10. Per cent reduction in the number of resolution elements (from that predicted for the H-T#17 SXI design) in an 18 arc min \(OFOV\) due to optical fabrication errors.

Figure 5-11. Mean percent reduction in the number of resolution elements (from that predicted for the H-T#17 SXI design) in an 18 arc min \(OFOV\) due to optical fabrication errors, and the mean deviation from the mean for the five SXI telescopes.
Figure 5-11 indicates that the mean reduction in the number of predicted spatial resolution elements (of the five as-manufactured SXI telescopes from that predicted for the H-T#17 optical design) is about 24% over the SXI spectral range for the aerial image and about 15% for the detected image. The mean deviation from the mean is about 4.5% and 3% respectively. This last chart is Goodrich’s Report Card on the optical fabrication of the five SXI mirrors, and represents quite good uniformity in the optical fabrication of these very challenging grazing incidence X-ray telescopes.

**5.4 Comparison of Performance Predictions with X-ray Test Data**

The X-ray imaging performance of SN002, the SXI Engineering Model, was tested by Lockheed Martin personnel at the X-ray calibration facility (XRCF) at NASA/MSFC. The SXI engineering model mirror assembly was mounted on a Five Axis Mount (FAM) and used with the X-ray Detector Assembly (XDA), both of which were developed under the AXAF program. The XDA consisted of a pinhole array, two calibrated Flow Proportional Counters (FPCs), and a laboratory CCD detector and associated low-noise camera. Several different FPCs were used a Beam Normalization Detectors as well. This experimental set-up is illustrated in Figure 5-12.67

![Figure 5-12. Schematic of the XRCF setup used in the SXI Acceptance Test Measurements.](image)
In order to assess the image quality of SN002, the fractional encircled energy of the mirror assembly was measured at two discrete incident photon energies with an FPC behind various sized pinholes in the array. Additional properties of the mirror assembly were measured with the CCD in the focal plane. Figure 5-13 are photographs taken in the X-ray Calibration Facility during the SXI SN002 testing.

![Figure 5-13. Photographs of the X-ray Calibration Facility during SN002 testing.](image)

On-axis fractional encircled energy measurements were made at two different discrete wavelengths at best axial focus of the SN002 mirror during the mirror acceptance tests at the XRCF. The experimental data is tabulated in Table 5-4.

<table>
<thead>
<tr>
<th>Pinhole Diameter (microns)</th>
<th>Encircled Energy (%) Cu-L (.93 keV, 13.3 Å)</th>
<th>Encircled Energy (%) C-K (.277 keV, 44.7 Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35000</td>
<td>100. ± 1.1</td>
<td>100. ± 2.4</td>
</tr>
<tr>
<td>1000</td>
<td>93.6 ± 1.4</td>
<td>96.8 ± 2.3</td>
</tr>
<tr>
<td>300</td>
<td>90.7 ± 1.4</td>
<td>95.1 ± 2.3</td>
</tr>
<tr>
<td>200</td>
<td>88.1 ± 1.4</td>
<td>91.5 ± 2.2</td>
</tr>
<tr>
<td>100</td>
<td>81.2 ± 2.3</td>
<td>90.0 ± 2.2</td>
</tr>
<tr>
<td>70</td>
<td>80.6 ± 1.3</td>
<td>87.1 ± 2.1</td>
</tr>
<tr>
<td>50</td>
<td>76.0 ± 1.2</td>
<td>83.5 ± 2.0</td>
</tr>
<tr>
<td>40</td>
<td>74.7 ± 1.2</td>
<td>82.4 ± 2.0</td>
</tr>
<tr>
<td>30</td>
<td>69.8 ± 1.1</td>
<td>77.3 ± 1.9</td>
</tr>
<tr>
<td>20</td>
<td>58.4 ± 1.0</td>
<td>68.1 ± 1.6</td>
</tr>
<tr>
<td>15</td>
<td>47.8 ± 0.8</td>
<td>53.6 ± 1.3</td>
</tr>
<tr>
<td>10</td>
<td>35.5 ± 0.7</td>
<td>39.0 ± 0.9</td>
</tr>
</tbody>
</table>
Similar experimental data for the off-axis fractional encircled energy measurements are provided in Table 5-5. These tables were directly taken from a Lockheed Martin technical report. They were also published in Proc. SPIE 4138.67

Table 5-5. Encircled Energy in the Plane of Best Focus and 20 arc min Off-Axis Pinhole Diameter (microns)

<table>
<thead>
<tr>
<th>Pinhole Diameter (microns)</th>
<th>Encircled Energy (%) Cu-L (.93 keV, 13.3 Å)</th>
<th>Encircled Energy (%) C-K (.277 keV, 44.7 Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35000</td>
<td>100. ± 1.3</td>
<td>100. ± 2.1</td>
</tr>
<tr>
<td>1000</td>
<td>95.4 ± 1.3</td>
<td>99.2 ± 2.6</td>
</tr>
<tr>
<td>300</td>
<td>90.0 ± 1.3</td>
<td>95.6 ± 2.8</td>
</tr>
<tr>
<td>200</td>
<td>88.7 ± 1.3</td>
<td>89.7 ± 2.7</td>
</tr>
<tr>
<td>150</td>
<td>84.5 ± 1.2</td>
<td>91.7 ± 2.7</td>
</tr>
<tr>
<td>100</td>
<td>69.0 ± 1.1</td>
<td>72.5 ± 2.2</td>
</tr>
<tr>
<td>70</td>
<td>50.5 ± 0.8</td>
<td>56.2 ± 1.8</td>
</tr>
<tr>
<td>50</td>
<td>27.4 ± 0.5</td>
<td>30.0 ± 1.0</td>
</tr>
<tr>
<td>40</td>
<td>26.3 ± 0.4</td>
<td>26.9 ± 0.7</td>
</tr>
</tbody>
</table>

A comparison of our on-axis fractional encircled energy predictions for SN002 with the experimental X-ray test data is shown in Figure 5-14.

Figure 5-14. Comparison of on-axis fractional encircled energy predictions for SN002 with experimental measurements taken at the NASA/MSFC X-ray Calibration Facility.
The above agreement between the predicted fractional encircled energy and the experimental measurements are quite good for the small circle sizes (less than 3 arc sec radius), but the experimental measurements are more than ten percent lower than the predictions for circle radii greater than 5 arc sec. This is due to the rather large increase in measured encircled energy as the circles were progressively increased to quite large values (see Table 5-4). If this experimental data were normalized to unity for the 200 μm diameter pinhole rather than the 35000 μm diameter pinhole we obtain a much better agreement in the comparison curves as illustrated in Figure 5-15. This re-normalization of the experimental data is not necessarily justified; however, it is plausible that there was some X-ray scattering (or stray radiation) produced by the experimental setup, which made the wings of the on-axis point spread function to appear larger than expected. A disproportionate fraction of the measured encircled energy would thus be at large angular radii.

Figure 5-15. Comparison of on-axis fractional encircled energy predictions for SN002 with re-normalized experimental data (not necessarily justified).
It should be noted that for both sets of curves, the contractual requirement for on-axis fractional encircled energy in a five arc sec diameter circle is satisfied.

A comparison of the off-axis (20 arc min field angle) fractional encircled energy predictions in the plane of best axial focus for the SXI SN002 Engineering Model mirror with experimental measurements made during the mirror acceptance tests at the XRCF are shown in Figure 5-16.

![Graph showing comparison of off-axis fractional encircled energy predictions for SN002 with experimental measurements taken at the NASA/MSFC X-ray Calibration Facility.](image)

Figure 5-16. Comparison of off-axis fractional encircled energy predictions for SN002 with experimental measurements taken at the NASA/MSFC X-ray Calibration Facility.

Once again we have good agreement between our performance predictions of fraction encircled energy and the experimental measurements for the smaller circle sizes (less than 12 arc sec radius), with larger departures as the size of the circle increases. And again, both the predictions and the experimental measurements indicate that the the contractual fractional encircled energy requirements for both the 44.7 Å and the 13.3 Å wavelength will be satisfied.
As part of the X-ray acceptance tests, the Lockheed Martin personnel also scanned the SN002 aerial image in both the x and y directions with a small pinhole for a variety of different field angles at a focal plane 30 μm inside of best axial focus (dz = -0.030 mm). They then constructed the plot of rms image diameter as a function of field angle illustrated in Figure 5-17. Although it is difficult to correlate these measurements quantitatively with our performance predictions of HPR for the SXI SN002 aerial image, we see the same general behavior as that illustrated in Figure 5-5 and Figure 5-6.

Figure 5-17. Illustration of experimental measurements of rms image diameter as a function of field angle for the SXI SN002 mirror. This curve has the same general shape as our predictions of the HPR versus field angle.
6.0 SUMMARY AND CONCLUSIONS

A scientist performs experiments in order to test theoretical models. An optical systems engineer develops analytical and numerical models to avoid performing a series of time-consuming and expensive trial and error experiments. Space-based X-ray astronomy experiments are particularly time-consuming and expensive to perform. This dissertation was devoted to exhaustive numerical modeling to \textit{predict}, and \textit{optimize}, the optical performance of five separate as-manufactured grazing incidence X-ray telescopes. These Solar X-ray Imager (SXI) telescopes were built by Lockheed Martin’s Solar and Astrophysics Laboratory (LMSAL) in Palo Alto, CA for use as add-on instruments on NOAA’s next-generation GOES weather satellites. Goodrich Optical Systems in Danbury, CT fabricated the grazing incidence X-ray mirrors to an optical design optimized for the SXI Mission by CREOL’s Optical Design and Image Analysis Laboratory. The optimization of the H-T#17 optical design was reported in a previous dissertation by Patrick Thompson in 2000.

The numerical modeling described in this dissertation is unique in that it is a complete \textit{systems engineering analysis} of image quality including geometrical aberrations, diffraction effects, surface scatter effects, and all of the the other miscellaneous errors appearing in the mirror manufacturer’s error budget tree. These error sources all degrade the aerial image produced by a grazing incidence X-ray telescope. That aerial image is then sampled by a mosaic detector array whose signals are transmitted to earth for further analysis. Since commercially available optical design and analysis software typically model only the geometrical aberrations and diffraction effects, many of the numerical models (in the form of MATLAB codes) needed
for this task were developed by CREOL’s Optical Design and Image Analysis Laboratory during the course of this project.

Perhaps the most significant contribution of this dissertation is the detailed analysis of the detector effects upon the image quality of a *staring* wide-field grazing incidence X-ray telescope utilizing a mosaic detector array. Grazing incidence X-ray telescopes suffer from severe field-dependent aberrations. The aerial image produced in the focal plane is therefore non-isoplanatic, or *shift-variant*, with respect to field angle. Furthermore, staring mosaic detector arrays are inherently *locally shift-variant* due to pixel registration issues. The conventional transfer function approach for analyzing detector effects is thus not applicable to the SXI mission. Chapter three of this dissertation describes in detail a rigorous analysis of detector effects upon the systems performance of the SXI telescope. Since detector effects limit the resolution at small field angles and geometrical aberrations limit the resolution at large field angles, we demonstrate that the optical design can be optimized to balance detector effects with geometrical aberrations. This process obviously depends upon choosing an appropriate image quality criterion for this wide-field application. Although most modern imaging instruments utilize mosaic detector arrays, including detector effects in the optical design process for astronomical telescopes has certainly not yet become a routine practice.

The second major contribution of this dissertation is the development of a detailed technique (involving exhaustive numerical modeling) for re-optimizing the telescope design to partially compensate for image degradation due to optical fabrication errors in the form of surface figure errors. For the SXI telescope, once the mirrors are fabricated, the only remaining design variable is the focal plane position. Thus, as reported in Chapter four of this dissertation, using the mirror
metrology data from Goodrich Optical Systems, we have determined a different unique optimum focal plane position for each of the five as-manufactured SXI telescopes. This is again not yet routine practice among telescope manufacturers. In fact, Lockheed Martin is, at the time of this writing, planning to despace the focal plane from the best axial focus by a fixed amount for all five of the SXI telescopes.

Of course our main role in the SXI program was to model the predicted performance of each of the five SXI telescopes during the optical fabrication process (using the Goodrich optical metrology data) to provide the prime contractor, Lockheed Martin, with the assurance that the contractual requirements on image quality would be satisfied. Since the optical fabrication process consists of a dozen or more polishing/testing cycles where the metrology data from each testing cycle defines the strategy for the next polishing cycle, these image quality predictions had a large impact upon the cost and schedule of the program as they were used to determine when to stop the optical fabrication process for each SXI telescope. These image quality predictions were somewhat redundant with work done by the subcontractor, Goodrich Optical Systems, who was manufacturing the SXI telescope mirrors; however, we served as an independent check upon their predictions, with no vested interest in the outcome. Our fractional encircled energy predictions at best axial focus presented in Chapter four agreed well with similar predictions made by Goodrich, giving Lockheed Martin a high level of confidence that the contractual requirements would be satisfied.

Finally, with the exhaustive image quality predictions, based upon real metrology data for five different SXI telescopes fabricated to the same H-T#17 optical design, we have a significant data base to assess the current state of the art in grazing incidence X-ray telescope fabrication
technology. These results, reported in Chapter five, should be of interest to the high energy astrophysics community, and will be published in the near future.

And lastly, the first of these SXI telescopes is scheduled to be launched with the GOES-N weather satellite on a Delta II rocket on May 4, 2005. After the GOES-N satellite is operational, we will collaborate with Lockheed Martin personnel to publish a comparison of our image quality predictions with real on-orbit observations.
1. http://chandra.harvard.edu


