The Effects Of Electrode Geometry On Current Pulse Caused By Electrical Discharge Over An Ultra-fast Laser Filament

Matthew Bubelnik
University of Central Florida

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THE EFFECTS OF ELECTRODE GEOMETRY ON CURRENT PULSE CAUSED BY ELECTRICAL DISCHARGE OVER AN ULTRA-FAST LASER FILAMENT

by

MATTHEW BUBELNIK
Bachelors of Science, Stony Brook University, 2003

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the College of Optics and Photonics at the University of Central Florida Orlando, Florida

Spring Term
2005
ABSTRACT

The time-resolved electrical conductivity of a short-pulse generated plasma filament in air was studied. Close-coupled metal electrodes were used to discharge the stored energy of a high-voltage capacitor and the resulting microsecond-scale electrical discharge was measured using fast current sensors. Significant differences in the time dependence of the current were seen with the two electrode geometries used. Using sharp-tipped electrodes additional peaks in the time-resolved conductivity were seen, relative to the single peak seen with spherical electrodes. We attribute these additional features to secondary electron collisional ionization brought about by field enhancement at the tips. Additional discrepancies in the currents measured leaving the high-voltage electrode and that returning to ground were also observed. Implications for potential laser-induced discharge applications will be discussed.
I would like to dedicate this paper to all the people that made it possible to attended the College of Optics and Photonics at the University of Central Florida with out your love, and support none of this would be possible. Also, like to dedicate this thesis to the memory of my late roommate and friend James Weslee Keefe (1981-2005) Git-R-Done.
ACKNOWLEDGMENTS

There are so many people that I would like to deep heart felt thanks. Without their support, this thesis could not have been completed. I would like to thank DARPA, Lockheed Martin, and the US Taxpayers for providing the funds that supplied resources that were used in the experiments that were presented in this thesis. My advisor, Prof. Craig Siders, whose infinite faith in my abilities has helped me achieve my very best. Without him, the funds for this research could never been obtained, thanks for the paychecks. In addition, I would like to thank my laboratory partner, Mr. Matthew K Fisher, keeping the laser in peak conditioned. His watchful eye has kept me safe while operating the high voltage power supply and not to mention the long days and weekends slaving over the laser. Matthew also helped in the editing process in the grammar correction to this document. Dr. Shimkaveg, thank you for your design of the solenoid safety switch, “winky,” which saved countless hours that would have been wasted in waiting for the capacitor to fully discharge. My mom and dad also need to be thanked for the support and encouragement during my entire lifetime. Lastly, Very special thanks for those not mention above but no thanks should be given to Charley, Frances, Ivan, and Jeanne, because I hate hurricanes.
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CHAPTER 1: INTRODUCTION

1.1 The Femtosecond Laser and Filamentation

A femtosecond is $10^{-15}$ seconds. In that timescale, light will travel about 1/30 of the width of a human hair. These short pulses can achieve peak powers on the order of a terawatt, by using modest amount of energy (10-100 mJ). With high peak power, the laser experiences nonlinear effects in air [1-4]. This is where the index of refraction of air is proportional to the intensity of the beam [5, 6]. Since, the laser beam typically has Gaussian like intensity profile; the central portion of the beam sees a higher index than the edges. This produces an effect like that of a positive lens. In atmospheric air, this occurs for powers on the order of a GW. As the beam focuses down, the intensity increases to the point where multi-photon ionization begins (approximately $10^{14}$ W/cm$^2$ for molecular oxygen, which has the lowest ionization potential, 12.1-eV, of all the major constituencies of air). This plasma will cause the beam to diverge due to the lower index of refraction of the plasma, given by [7]

$$\Delta n_{\text{plasma}} = -\frac{n_e}{2n_{\text{crit}}}, \quad (1.1)$$

where,

$$n_{\text{crit}} = \frac{\pi}{r_c \lambda^2} \quad (1.2)$$
\( \lambda \) is the vacuum wave length of the laser and \( r_e = 2.8179 \times 10^{-15} \) m, is the classical electron radius. This plasma acts like an equally strong negative lens on the beam, as compared to the self-focusing lens [7]. If the laser has enough peak power, the self focusing and defocusing will come in to a dynamic balance, with approximately 1 part in 1000 of the air in the filament ionized, until the laser pulse loses enough energy in the ionization process so that the peak power is less than the critical power and the beam will diverge. This section of the beam, where self focusing and divergence due to the plasma cancel one another out, is called the filament [7]. These filaments can be from a few meters to greater than 100 meters in length. Since the filament consists of a weakly ionized gas/plasma, it can be used to conduct electricity and induce electrical discharges. Typical resistivity on the order of 1 \( \Omega \) cm, this is about 10 times less than the resistivity of seawater.

**1.2 Motivation Behind the Work**

One application of using laser filaments is to induce multiple discharges over long distances [8, 9]. The standard police TASERs use compressed air to launch two metal wires toward the target, then discharge a current through the target and stun it. The biggest drawback with these TASERs is that they get at most two shots and both electrodes must hit the target. These wires have a limited range and once used it takes time to reload. One day, high power femtosecond lasers will be shrunk down into a man portable size and the laser-induced plasma can replace the wires allowing for longer range and multiple shots [9].
The ability to control lighting strikes with such a filament is another such application. By propagating a filament into a thunder-cloud, one can predetermine the time and place of a lighting strike, thus helping to prevent loss of life and property or even cause a lighting strike that otherwise would not occur naturally [10, 11].

1.3 Overview

The purpose of this thesis is to discuss a method optimally apply an electric field to a plasma filament in order to measure the filament’s conductivity. This thesis investigates differences seen in the time-resolved discharge current when using two different electrode geometries. Two such designs are sharp electrodes and larger spherical electrodes. The sharp points will enhance the electric field around the tips, while the larger spherical electrodes allow for a more uniform electric field in the gap. The difference in the current that was allowed to pass in-between the electrodes was observed from this signal generated by Pearson coils; which were around both the hot and ground leads of the capacitor.
CHAPTER 2: BASIC THEORY OF FILAMENTATION

2.1 Femtosecond Pulse Generation

Ultra-short pulses are generated via a process called modelocking [12]. Modelocking is the process where short pulses are created by constructively interfering many longitudinal modes that are locked in phase. There are many ways to lock these phases together; active, passive, and self-modelocking. In active modelocking, a modulator is placed into the cavity so only photons with phases that match the window of the modulator will see gain and lase. In passive modelocking, a saturable absorber is used that only allows high intense light to pass through, the modes that are in phase. The laser used in these experiments is a Kerr lens modelocked, titanium doped sapphire laser. Modelocking happens through the non-linear process known as Kerr effect within the laser rod. Only the modes that add constructively will have enough intensity to focus within the gain media. This lensing will give the beam a smaller waist. If an aperture were to be placed in the focal plane of the Kerr lens that was large enough for the high intense beam but small enough to block the CW mode, then all the energy stored within the gain media will be forced all into the same mode so lasing can occur [12].

Most oscillators do not have enough peak intensities to induce strong non-linear optical effects. Therefore, these ultra-short pulses need to be amplified. The peak power of these femtosecond pulses will either saturate or destroy a second amplification stage unless the pulse was stretched out in time. This can be achieved by adding chirp to the pulse. The peak power is lowered so that as it passes though the gain media, it does not induce damage to nonlinear effects with in the gain media that would be detrimental to
the amplification system. After the pulse is amplified, it may be recompressed by adding the opposite chip that was used to stretch the pulse. This process is known as chirped pulse amplification [12].

2.2 Filamentation

Filamentation is the balance of nonlinear self-focusing and plasma defocusing [7]. This phenomenon occurs when the beam has a power greater than the critical power, given by [14, 15]

\[
P_{\text{crit}} = \frac{\pi (0.16 \lambda_0)^2}{8 n_0 n_2}.
\]

(2.1)

Where \( n_2 \) is the nonlinear index of refraction, given by [5]

\[
n_2 = \frac{96 \pi 10^7}{c} \chi^{(3)} \frac{N}{n_0} \frac{cm^2}{W}.
\]

(2.2)

The power within the filament is clamped on the order of \( P_{\text{crit}} \). For 800 nm light in air \( n_2 = 5.6 \times 10^{-19} \) which gives a critical power is 3 GW. If the power is greater than \( P_{\text{crit}} \), then there is the possibility that multiple filaments can form simultaneously. The intensity of the filament is also clamped at the ionization intensity of air, \( 4 \times 10^{14} \text{ W/cm} \) [16]. This will
also lead to a minimum size to the filament. The minimum radius of a filament is given by

\[ \pi r^2 = \frac{P_{\text{crit}}}{I_{\text{peak}}} \Rightarrow r = \sqrt{\frac{P_{\text{crit}}}{\pi I_{\text{peak}}}} \approx 50 \mu m , \]  

(2.3)

This value has been observed experimentally.

2.3 Resistivity of a Plasma Filament

2.3.1 Calculations of the Resistivity the Filament

Since the filament is a long channel of ionized media, this channel could be used to carry current along its path. The density of electrons is calculated from knowing that filaments need the change in the index of the due to self focusing and defocusing must be equal. This is given by

\[ n_e I = \Delta n_{Kerr} = -\Delta n_{\text{plasma}} = \frac{n_e}{2n_{\text{crit}}} \]  

(2.5)

Given the parameter, from the previous section and \( r_e \) is \( 2.8 \times 10^{-13} \) cm, we calculate the electrons density to be, \( 5 \times 10^{16} \) cm\(^{-3}\). Considering the number density of air is \( 2.5 \times 10^{19} \) this yields a ratio for a weakly ionized gas, of about \( 10^{-3} \). Using this information a Drude approximation for the resistivity, given as [17]
\[
\frac{1}{\sigma} = \rho = \frac{v_m}{r_e c^2 n_e} \approx 0.71 \Omega \text{cm} \tag{2.8}
\]

2.3.2 Results From Literature

Plasma filaments have been previously measured to have a resistivity of 1.2 \(\Omega\text{cm}\) [18]. According to Ladouceur et al, the resistivity ranges from 0.28 \(\Omega\text{cm}\) to 50 \(\Omega\text{cm}\) over a 150 ns window [19]. These values depend linearly with the collision frequency. The conductivity of a plasma filament can be calculated by knowing the electron density. According to Raizer for this given electron density of the weakly ionized gas, he would estimate the resistivity to be 0.35 \(\Omega\text{cm}\) [13]. This value is good agreement with the value calculated above.

2.3.3 Mathematical Model

Using a model discussed by Tzortakis, the results from the experiments can be simulated [20, 21]. The three coupled differential equations given by [20]:

\[
\frac{\partial n_e}{\partial t} = m_e - \gamma n_e - \beta_{ep} n_e n_p
\]
\[
\frac{\partial n_p}{\partial t} = \gamma n_e - \beta_{ep} n_e n_p - \beta_{np} n_p n_p
\]
\[
\frac{\partial n_n}{\partial t} = \gamma n_e - \beta_{np} n_p n_p \tag{2.9}
\]
where $\gamma$ is the impact ionization due to an external field in air at atmospheric conditions is given by [21]:

$$\gamma = \left( \frac{N}{N_0} \right) \frac{5.7 \times 10^8 \left( 3.34 \times 10^{-7} E \left( \frac{N}{N_0} \right) \right)^5}{1 + 0.3 \left( 3.34 \times 10^{-7} E \left( \frac{N}{N_0} \right)^{2.5} \right)} \quad (2.10)$$

$\eta$ is the attachment rate of electrons to diatomic oxygen in atmospheric conditions given by [21]:

$$\eta = 1.22 \times 10^8 \left( \frac{N}{N_0} \right) \exp \left[ \frac{-42.4}{E_0} \right] + 10^8 \left( \frac{N}{N_0} \right)^2 \frac{0.62 + 800(E_0)^2}{1 + 10^3(E_0)^2 [E_0 (1 + 0.03E_0^2)]^{1/3}} \quad (2.11)$$

where,

$$E_0 = \frac{3.34 \times 10^{-5} E}{(N/N_0)} \quad (2.12)$$

$N$ is the current number density of air, $N_0 = 2.688 \times 10^{25}$ m$^{-3}$, $\beta_{ep} = 2.2 \times 10^{-13}$ m$^3$/s is the electron ion recombination rate, and $\beta_{np} = 2.2 \times 10^{-13}$ m$^3$/s is ion-ion recombination rate [20, 21]. These differential equations can be solved using Mathematica 5.1 (code can be
found in the appendix). In addition, the discharge from the RCL circuit is incorporated into the model via the second order differential equation [22]

\[ L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0 \]  

(2.13)

The mean free path, in air at one atm, is about 0.4 µm [13]. The electron on average needs to gain 66 eV for electron collisional ionization [13]. In order for this to happen, if an electric field of 1000 kV/cm, the electron needs to travel 66 µm. This would mean that the electron would have 165 collisions before it reaches the required energy for ionization. Thus, Ohm’s law applies to this situation.

The parameters of the model are the same for the experimental setup discussed in Chapter 3.2. This simulation accounts for two cases constant spatial dependence (1D) and a gaussian dependence (2D), of charge carriers. Figure 2.1, Multimedia 2.1 and Multimedia 2.2 are generated from the output of the model. Figure 2.1 a) shows the time dependence of the charge carriers with time, while Multimedia 2.1 shows the time and spatial dependence of the charge carrier densities. The electron density drops quickly due to the both recombination and attachment. In the 2D model, the spatial distribution widens thus allowing a greater conductivity than the 1D model. Multimedia 2.2 (2D) shows the conductivity’s dependence with time. Figure 2.1 b) shows the resistivity (inverse of the conductivity) dependence with time. One can note that the resistivity starts out low (dominated by electrons) and then tend towards large resistivity when the
number density of electrons diminishes (dominated by ions). These results for the conductivity can be integrated and give the results for resistance per unit length and the total resistance of a 2mm gap shown in Figure 2.1 c) and d). Figure 2.1 d) shows the results for both models. From both of these models, one can solve Equation 2.13 and can give the expected current pulse generated from the models as shown in Figure 2.1 e) and f). The special distribution of the charge carries increases the temporal duration of the resulting current pulse.

Figure 2.2 shows a simulation of what the data taken should look like as one change the electric field between the electrodes with the 1 D model. This model suggests that for low electric fields, ≤17.5kV/cm, that the relationship between the current and the electric field can be approximated by a linear function. This linear region is where Ohms law with a constant conductivity applies [22],

\[ \bar{J} = \sigma \cdot \bar{E} \]  

(2.14)

where \( \bar{J} \) is the current density, \( \sigma \) is the electrical conductivity and \( \bar{E} \) is the electric field.

Figure 2.3 shows the electron collisional time, attachment time and 2mm electron drift time as a function of electric field. These are plots of the inverse of the equations 2.10, 2.11 and 2 mm divided by the electron mobility times the field. The collisional and attachment times shown in Figure 2.3 will equal one another at approximately 20 kV/cm, in agreement with the breakdown threshold of air at STP and Figure 2.2.
Multimedia 2.1 Movie showing the spatial and temporal dependence of the charge carrier densities electrons (red) positive ions (blue) negative ions (green) from the 2D in log time. Note the blue line state off covering the red line.

Multimedia 2.2 Movie showing the spatial and temporal dependence of the conductivity of the filament, from the 2D model in log time.
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Figure 2.2 Total charge plotted as a function of electric field for 1D (red) and 2D (blue) models.

Figure 2.3 Collision time (red) and attachment time (green) Drift time 2mm (black) as a function of electric field. Note that they cross at about 20 kV/cm.
2.5 Simulated Conduction Over Salty Water Jet

According to the *Handbook of Chemistry and Physics 58th Ed.*, the resistivity of sea water ranges from 128 Ωcm to 13 Ωcm, depending on salinity [23]. Most common seawater has a solute concentration of about 35 g/l and has a conductivity of about 25 Ωcm [23], approximately 20 times greater than the filament.

The smallest nozzle on hand was a dye jet nozzle from an old Coherent Dye laser, model number 740. This nozzle produces a stream of water of about 2.1 mm wide by 0.38 mm thin. In section 2.3, it was estimated that the laser filament had a diameter of 100 μm. This makes the cross-sectional area 101 times greater in the stream of salty water. Since the resistivity is a factor of 20.8 times greater in the filament, the resistance in the stream is 1/5 of the filament, using:

\[ \rho = \frac{R \sigma \text{area}}{L} \]  

(2.9)

The same current can be passed in the water jet as in the filament of equal length, but with 1/63.7 of the voltage.

The nozzle is made from a metallic conductive substance so this would be a good point to couple current in to the stream with out deflecting the water jet. A dye pump was used to pump salty water (~35 g of Morton non-iodized table salt to 1 liter of water). A Simpson 270 multimeter was used to measure the resistance of the stream over varying distances. One probe was attached to the nozzle and the other probe was the target of the stream. Figure 2.8 shows the results of the simulation. Figure 2.8 shows that for this
salinity, the resistance per unit length is 4.408 kΩ/cm. By using Equation 2.9 we can find the resistivity of this sea water solution to be 33.5 Ωcm. The discrepancy between this value and that of the list value of seawater could be attributed to the use of table salt only. Ocean water has other solutes that are not found in the table salt attributing to its lower resistivity [23]. With the table salt solution, gives a resistance of about 1/3 than that of the filament.

Even though using a stream of water is probably more cost affective than using an ultra-fast laser system to produce the conductive media, the water jet has several disadvantages. There is a limited range as to how far the stream may be launched. Gravity has a significant effect and gives the water jet a ballistic trajectory. There is a chance that the target will get wet to the point where the current will conduct over the target’s surface and not through the target. Even though the water jet is a multiple shot system, there is still need to reload with more salty water when the reservoir is low [9].

Figure 2.4 Graph of the resistance of salt-water jet. Blue points show raw data and red line is a least square fit. The fitting results are displayed; the slope is in units of kΩ/cm.
CHAPTER 3 : EXPERIMENTAL SET UP

3.1 Laser Parameters

In these experiments we are using femtosecond laser pulses generated from: a Spectra-Physics Tsunami modelocked laser, which is pumped by a Spectra-Physics Millennia Pro Vs J, a diode pumped CW frequency doubled Nd:YVO$_4$. An acousto-optic programmable dispersive filter, Dazzler from Fast Light, shapes these pulses. The pulses are then sent through a chirp pulse amplifier, Spectra-Physics TSA, which is pumped by a frequency, doubled Coherent Evolution Nd: YLF at 1 kHz and a Spectra-Physics Quanta-Ray ND: YAG at 10 Hz. The chirp of the outgoing pulse is controlled by adjusting the spacing of the compressor gratings. The chirp of the pulses was set so that only one filament was passing though the electrodes ~2-3 m away. For the sharp electrodes, the filament crossed at the tips and with the larger spherical electrodes, the filament passed through a small hole through the center of the sphere.

3.2 High Voltage Circuit

3.2.1 Design Layout

The following figure 3.1 depicts the set up of the circuit used in the experiments. Element “a” is a Glassman High Voltage Inc. series PK 30KV high voltage power supply. Element “b” is a high voltage rated 32 MΩ resistor. Element C is a 0.5 µF capacitor rated for 25 kV. Element d) is a safety switch that will discharge the capacitor, 2 metal connectors that are separated about 4 cm in air. The magnet force of the solenoid
susends one of the connectors. When current is applied to the solenoid, the switch is open.

Figure 3.1 Schematic of high voltage circuit a) is a Glassman High Voltage Inc. series PK 30KV’ high voltage power supply. b) is a high voltage 32 MΩ resistor. c) a 0.5 µF capacitor rated for 25KV. d) Safety switch followed by a 10 e) Position of electrode f) laser filament coming form hot electrode to ground electrode. g) Front Pearson coil model number 6585 h) Back Pearson coil model 2100

Element e) are the electrodes in the circuit. Two pointy electrodes, whose diameter are as small as 300µm and have a base of 3.18mm, and two ball electrodes whose diameter is 3cm with a hole drilled through the center with a diameter of 3.7mm. Element f) is the laser filament crossing over or through the electrodes which causes the capacitor to discharge. Elements g and h are Pearson coils that are used to measure the current leaving and/or entering the electrode from the capacitor. Both coils have a sensitivity of 1 V/A. The back coil, model 2100, has a slower rise time, 20ns, than the front coil, model 6585, which has a rise time of, 1.5ns. When the coils were reversed, consistent results were found.
3.2.2 Short Circuit Analysis

In order to characterize the circuit, the gap where the filament would propagate
the gap between the electrodes was shorted via a total break down between the points.
Instead of using the Pearson coils stated above, they were replaced with a Low sensitivity
(1mV/A) Pearson coil, model 4997. Figure 3.2 show the oscilloscope trace and a fit to
the data of the spark discharge. The result of the fit yielded, L 411 nH, R to be 60 mΩ,
and C to be 0.5 µF.

Figure 3.2 Oscilloscope trace of a spark discharge over the electrode gap. Peak current
found to be 14 kA with a ~375 kHz oscillation.
CHAPTER 4: DISCHARGE OVER POINT ELECTRODES

4.1 Enhanced Electric Field Caused by Sharp Electrodes

There is an electric field enhancement around sharp tip conductors [24, 25, 27]. This is explained by the charge distribution on the surface of a sharp conductor. The more charge present in a small area will translate into a higher electric field. There are two ways to describe the charge density at a tip of a conductor.

The first way is to assume that the point is made out of two conducting spheres that are separated by some distance, not so close that one sphere will polarize the other [24]. Then connect those two spheres with a conducting wire so that each sphere will have the same potential when voltage is applied. In the case of a point, one of the spheres is smaller than the other, with the radius of curvature as the tip of the point. Since the potential of these two spheres is equal and the potential of a sphere with a charge is [24]

\[ V = \frac{kQ_1}{r_1} = \frac{kQ_2}{r_2}. \]  

(4.1)

So, the field right outside the spheres is [24]

\[ E_n = \frac{kQ_n}{r_n^2} = \frac{V}{r_n}. \]  

(4.2)
The ratio of the electric field given by the second small sphere to the first large sphere gives the amount of electric field enhancement to be [24]

\[ \frac{E_2}{E_1} = \frac{r_1}{r_2}. \]  

(4.3)

In the approximation of a tip where \( r_2 \gg r_1 \), the field at the tip becomes [23]

\[ E_{\text{tip}} \approx E_o \left( \frac{a}{r_{\text{tip}}} \right)^\frac{1}{2}, \]  

(4.4)

where \( E_o \) would be the field of a sphere with the larger radius, \( a \) is the length of the tip and \( r_{\text{tip}} \) is the radius of the tip [23].

Given the parameters of the points used in these experiments: \( r_{\text{base}} = 1.59\text{mm}, \) \( r_{\text{tip}} = 0.15\text{mm}, \) and the length of the tip, \( a = 2.34\text{mm} \), the ratio is calculated to be 3.94 times the field generated by the base.

This approach is only accurate under the assumption that the two spheres are completely isolated. They need to be far apart compared to both of their radii. A model of a tapered conical approach is needed [25]. A series of touching spheres will simulate a conical point with radii decreasing by a factor \( k \), where \( k > 1 \). Given an angle of taper, \( \beta \), \( k \) can be found by [25]
\[ \beta = \sin^{-1}\left[ \frac{1-k}{1+k} \right]. \]  \hfill (4.5)

According to Fricker, for small values of \( k \) yields a charge density at the tip of \([24]\)

\[ \sigma_{\text{tip}} = \frac{1}{2} \sigma_{\text{base}} \left( \frac{r_{\text{tip}}}{r_{\text{base}}} \right)^{-\gamma}, \]  \hfill (4.6)

Where \( \gamma \) is given by

\[ \gamma = -\frac{\ln(2)}{\ln(k)}. \]  \hfill (4.7)

Since \( 0 < k < 1 \), \( \gamma \) will always be a positive number so the charge at the tip of the electrode will be proportional to the inverse of the tip radius to the \( \gamma \) power \([24]\).

If given the parameters stated before, the angle of the tip can be found by using trigonometry, \( \beta = 33.45^\circ \). By using Equation 4.5, the value of \( k \) is determined to be 0.289 with corresponding \( \gamma \) of 0.559. The ratio of the charge density at tip to the base is therefore 1.86. The ratio of the charge densities should be equal to the ratio of the electric fields. This estimation is slightly smaller than the simpler one discussed above.

Let there be a point \( p \) that is on axis of the tip. The electric field of this tip can be modeled as a series of rings with changing radii. The electric field of a thin ring is given by \([27]\)
To simplify this problem, the electrode is separated into three sections, as shown in Figure 4.1. These sections are: I) a semi-sphere with radius \( r \) centered at zero, II) a conical section that has radii \( r \) through \( R \) over the length over the tip, \( L \), starting at \( -L \) to zero, and III) a semi-sphere with radius \( R \) centered at \( -L \). Assume the charge density of the tip be field enhancement times \( \sigma \), for simplicity let’s take the charge density distribution over the conical section to be a quadric function since \( 1/\gamma \) is close to 2, this density will vary from \( 3.461\sigma \) to \( 1\sigma \). The last section will have a charge density of \( \sigma \), where \( \sigma \) is the charge density of a sphere with radius \( R \), and potential \( V \).

\[
E_z(z) = \frac{Q}{4\pi \varepsilon_o} \frac{z}{(z^2 + r^2)^{3/2}},
\]  

(4.8)

Figure 4.1 Depiction of a point electrode. Angle “a” is the angle of the radius with the axis of integration.
To find the field of a 1/2 sphere, the rate of change of the radius is found to be the radius of the sphere times the sine of the angle made by the point, the center, and a point on the sphere. Then the small segment of charge is given by

\[ dQ = \frac{Q}{SA} (R \, d\theta \, (2\, \pi R \, \sin(\theta))) . \]  

(4.9)

When equation is integrated from 0 – \( \pi/2 \) gives the field of the left half of a circle and from \( \pi/2 - \pi \) gives the other half. For the field given by sections I and III we have the equation,

\[
E_z(z) = \int_0^{\pi/2} 4\pi R^2 \sigma \frac{\sigma_{enh} ((z - r\cos(\theta))\sin(\theta))}{8\pi \varepsilon_0 \left( (z - r\cos(\theta))^2 + (r\sin(\theta))^2 \right)^{1/2}} \, da + \\
\int_{\pi/2}^{\pi} 4\pi R^2 \sigma \frac{(z + L - R\cos(\theta))\sin(\theta)}{8\pi \varepsilon_0 \left( (z + L - R\cos(\theta))^2 + (R\sin(\theta))^2 \right)^{3/2}} \, da ,
\]

(4.10)

where \( \sigma_{enh} \) is the enhancement factor found in equation 4.6.

In order to add in the electric field due to section II with a varying charge density, the assumption is made since that the field disruption is like that of a quadratic, since \( \gamma \) is close to 0.5 on can This leads \( dQ(z) \) to be,
\[ dQ(x) = \sigma \left\{ 2\pi \int_{-L}^{0} \left( \frac{R - r}{L} - r \right) \sqrt{1 + \frac{R - r}{L} \cdot dz} - \left( \frac{1 - \sigma_{ehn}}{L^2} \right) (x - L)^2 + 1 \right\}, \quad (4.11) \]

where the first term, on the right hand side, is the charge density at the base of the tip, the second term is the surface area of the conical section, and the last term is the approximation of the charge distribution on the surface of the cone. The integral for the electric field is given by

\[ E_z(z) = \int_{z}^{z_i} \frac{Q(z)}{4\pi\varepsilon_0} \frac{(x - z)}{((x - z)^2 + r^2)^{3/2}}. \quad (4.12) \]

These three sections were integrated and added using Mathematica 5.1 (code located in appendix). The following figure 4.2 is generated assuming \( \frac{\sigma}{4\pi\varepsilon_0} = 1 \).

Figure 4.2 shows the electric field of the proposed summation of thin rings, as discussed above. The actual electrode is a conductor so the field within it should be zero. This is not the case due to the simple fact that the horizontal charge distribution as a function of \( z \) is not correct, but it should give a slight over-estimation of the peak field because in order to cancel out the field in the middle the charge distribution needs to be shifted away from the tip.
Figure 4.2 Axial Electric field of a tip conductor, horizontal axis is distance from the tip on the tip axis in meters. The vertical axis is the electric field in V/m when \( \frac{\sigma}{4\pi\varepsilon_0} = 1 \).

Blue lines signify where the separate sections of integration occurred. The electrode is in the same position in figure 4.1 where the small semi circle is center at zero. Top figure shows axial electric field with the enhancement factor given in the first method [23]. Bottom figure shows the enhancement factor given by the latter [24].

Since the electric field is dependent on voltage, the arbitrary units can be easily converted to V/cm, knowing that the field shown in the Figure 4.2 is directly proportion
to \( \frac{\sigma}{4\pi\varepsilon_0} \). In the approximation made above the charge density, \( \sigma \), is that of a sphere with a radius equal to that of the base, equation 4.1. To find the field at any point on the graph, \( E_{arb} \), needs to be multiplied by

\[
E = E_{arb} \frac{V}{4\pi R}.
\] (4.13)

A more exact approach can be found in a paper written by Atten et al, which gives the radial dependence of the electric field for a point and a plane [26]. Their results provide the field distribution shown in Figure 4.3. This is compared to a picture taken of a point-plane setup taken under corona discharge conditions shown in Figure 4.4. Figures 4.3 with 4.4 shows that the plasma follows the curve present in [26]. When a filament is introduced, the plasma formed by the discharge follows the path of the filament. This shift in the electric field induced by the filament could lead to a spark discharge into the plane as shown in Multimedia 4.1.
Figure 4.3 Field lines for a cylinder (2mm) with a cone that is capped with a sphere (100µm) at 15 kV with an infinite plane at 30mm from point. The right figure is a closed look of the figure on the left [26].

Figure 4.4 Image of corona discharge from a point to a plane without the presence of a filament.
Multimedia 4.1 Video file showing spark and arc discharge through the filament in a point-plane configuration. The spark is traveling along the filament path. The arc discharge is shifting and collecting at the spot on the plane where the filament intersects. Please click above.

4.2 Discharge Induced by Laser Filament Over the Gap

Total breakdown was observed only with the presence of the plasma filament at 25 kV at a 3 cm gap. At this separation and voltage, no breakdown was observed without the filament. This indicates that the laser plasma is inducing the discharge over the gap.

Pearson coil signals were observed over various gap lengths. Secondary features in the time resolved current made it difficult to measure peak current; the area of the signal under an 80 ns window was measured to find total charge that was passed through the first electrode, which is shown in Figure 4.3.

Figure 4.3 shows as the distance in-between the electrodes increases, the amount of charge that was carried through the first Pearson coil plateaus as the bias voltage was increased. In order to make the several data series into one series, the amount of charge was plotted against a relative electrical field. By assuming that the scaling of the electric
field is the same as the scaling of the field between two plates separated by a distance, d, with a potential difference, V, the field in-between the plates is given by \( |E| = \frac{V}{d} \) [27]. Figure 4.6 was generated when this scaling factor is used. From the slope of the figure and a filament radius of 50 µm, the resistivity of the filament can be calculated to be 54.07 Ωcm. However, the value of the \( R^2 \) is too low, and the value should not be trusted further investigation is required.

![Average Charge Vs. Bias Voltage Front Coil](image)

Figure 4.5 Average charge vs. bias voltage of front coil.
Figure 4.6 Electric field vs. Charge data taken from Fig 4.5. Data fitted to line the slope is related to the conductivity of the filament. Slope is in units of (nC cm) / kV.

Other experiments were conducted where an object was placed in-between the two electrodes or the grounded electrode was rotated by 90 degrees, so the filament would not contact it. Both of these experiments, shown in Figure 4.7, show current moving through the front and back coils. The currents seen here are filament dependent. With the laser chirp adjusted to produce no filaments at or near the electrode gap, no current was observed at any bias voltage from 0-25 kV. All of these curves are insignificant when compared to the blue curve, a long distance discharge, except for the yellow one, in which the electrodes are perpendicular to the path of the filament without a dielectric between them. There is a possibility that a filament stimulated corona discharge between the hot and ground leads exist due to the enhanced electric field generated by the tips. This could also explain why there seems to be a threshold potential
(figure 4.5) independent of the gap spacing (after 7 cm) and why the $R^2$ value of the linear fit is so low. In addition, the data taken here does not match the model presented in Chapter 2.4.

![Figure 4.7 Comparison of front and back coils so several IV curves](image)

### 4.3 Secondary Features of the Time Resolved Current

The shapes of the time resolved current pulses were surprising. There was not just one decaying pulse as the model suggested in Chapter 2. The expected pulse was observed. Occasionally, with occurrence frequency increasing as voltage was increased at short gap spacing, another current spike was observed that was slightly delayed with respect to the predicted pulse. Figure 4.8 and Multimedia 4.2 show an example of a pulse with this extra feature with ranging voltages. When the bias voltage was set to 27 kV multiple secondary features were observed, shown in Figure 4.9. The delay between the secondary features is equal to that of delay of the first poles to the second. This can be some sort of periodic or echoing effect.
The delays of these secondary features ranged from tens of ns to one µs after the ignition pulse. The secondary features were observed on both the front and back coils, even though the front signal is only shown in the figures. These oscilloscope traces are triggered off the laser-timing signal. In a few instances, it appears that there is no delay as shown in Figure 4.10.

These features were observed for short gap spacing. Shown in Figure 4.10, the gap spacing ranged from 3.2 cm to 5.4 cm with a constant bias voltage of 21 kV. From these figures, the amplitude dependence of these secondary features on distance between the electrodes and bias voltage can be inferred. These spikes were also observed under negative bias at the same short distances. Figure 4.11 and Multimedia 4.3 show the typical current trace for a -20 kV bias with gap spacings of 3.2 m and 6 cm.

These secondary features of the time resolve current are dependent on the voltage applied and the gap spacing. Their position in time with respect to the initial current pulse corresponds to voltage and gap distance, the higher the voltage and shorter the gaps, the shorter the delay. This can lead to the conclusion that there is an electric field dependence on the position of the spikes. These features are not present in the model shown in Chapter 2 nor have they been seen in the current literature. The only time scale that is on this magnitude is electron collisions. These secondary features were not observed at high voltages and short gaps with the use of larger spherical electrodes discussed in the next chapter.
Figure 4.8 Oscilloscope traces of odd behavior of current pulse at high bias voltages. All taken form the Front Coil at gap spacing of 3.2 cm. Top Left) bias voltage 25 kV, Top Right) 17 kV, Bottom Left) 17 kV, Bottom Right) 15 kV
Multimedia 4.2 Gif file showing the oscilloscope trace as bias voltages changes from a large to small value (repeats twice). Yellow curve front coil and red curve back coil. Please click above.

Figure 4.9 Oscilloscope trace of multiple secondary features.
Figure 4.10  Example of secondary pulse at zero delay for gap spacing of 3.2 cm and bias voltage of 21 kV Front Coil

Figure 4.11 Current spikes for a bias of 21 kV left front coil in yellow and in purple shows an integration window for figure 4.5 Top Left) gap spacing of 3.2 cm, Top Right) 4.0 cm, Bottom) 5.4 cm.
Figure 4.12 Current spikes for negative bias of -20 kV front coil in yellow and rear coil in purple. Taken on a lower bandwidth setting than the others Left) gap spacing of 3.2mm, Right) 6cm

Multimedia 4.3 Gif file showing oscilloscope traces as negative bias voltages changes from large value to small (repeats twice). Please click above.
Figure 4.13 show the plot of the delay of the secondary feature with respect to the initial current pulse. The data displayed in this graph shows taken with both positive (Black) and negative biases (red). The data was taken with three different gap separations (3.3 cm, 3.6 cm, and 4 cm) at 1 kV intervals from (25 ~ 19 kV), where these secondary features were observed. This shows a decreasing trend in the delay with respect to the applied field (bias voltage over gap spacing). Figure 4.14 shows this data plotted on top of Figure 2.3. This data has a similar slope to that of the electron ionization collisional time. If a factor of 1.7 is accounted for an electric field enhancement due the sharp tips, it will shift the data on top of the collisional time, as shown by the lighter points.

![Positive and Negative Bias vs. Delay](image)

Figure 4.13 The dependence of delay of secondary features with respect to the initial current pulse for both positive (black) and negative (red) bias.
Figure 4.14 Comparing various time scales (collisional time [red], drift time for 2mm[Black], Attachment time [Green] ) with data from Figure 4.13 (Darker points) and data From Figure 4.13 shifted by a factor by a factor of 1.7(Lighter Points).
CHAPTER 5: DISCHARGE OVER SPHERICAL ELECTRODES

5.1 Electric Field Caused by Spherical Electrode With Cylindrical Hole

In order to tackle this problem, the electrode needs to be broken into two parts, a cylinder with a constant radius with length equal to the diameter of the sphere, and a second cylinder with a constantly changing radius, and their results added by the principle of superposition. The assumption is made that all the surface charge is uniformly distributed over the entire surface of the electrode and there are no sharp points. This is reasonable approximation neglecting the edge at the hole. This is a third order approximation, where zero order would be a point, first order would be a sphere, and second order would be a hollow sphere with a hole. The rate of change of radius would be the same as half the length of a chord on a circle that is perpendicular the filament path see Figure 5.1.

![Figure 5.1 Sketch of a cross sectional of electrode with cylindrical hole](image)

Figure 5.1 Sketch of a cross sectional of electrode with cylindrical hole
As in Chapter 4.1, the on-axis electric field will be found by integrating over a series of thin rings, given by equation 4.8. To find the electric field at a point \( p \) on the \( z \)-axis, the principle of superposition is used to sum all the thin rings comprising the cylinder. Taking the origin at the center and assuming that \( r \ll R \), the function is integrated along \( z \) from \( z + R \) to \( z - R \). This would give the following equation (solved using Mathematica 5.1. the code is located in the appendix),

\[
E_z(z) = \int_{z-R}^{z+R} \frac{Q}{4\pi\varepsilon_0} \frac{z'}{(z'^2 + r^2)^{3/2}} dz' = \frac{\sigma 2rR\pi}{4\pi\varepsilon_0} \frac{1}{2R\sqrt{R^2 + r^2 - 2Rz + z^2}} - \frac{1}{\sqrt{2R\sqrt{R^2 + r^2 + 2Rz + z^2}}} , \tag{5.2}
\]

where \( R \) is the radius of the sphere (length of the cylinder) and \( \sigma \) is the charge density of the cylinder.

Equation 5.2 is plotted versus the variable \( z \), using \( R \) and \( r \) given from the parameters in Chapter 3 and \( \frac{\sigma}{4\pi\varepsilon_0} = 1 \) and shown in Figure 5.2.
Figure 5.2 Inner cylinder electric field plot of Equation 5.2. The blue lines indicate the edges of the cylinder. The horizontal axis is the distance in meters and the vertical axis is the electric field where \( \frac{\sigma}{4\pi\varepsilon_0} = 1 \). R and r are given from Chapter 3.

To find the electric field caused by the outer surface, the ring-like approach is used but the changing radius of the ring is accounted for. The radius of the ring is given by, \( R \sin(a) \), where \( a \) is the angle formed by the point on the sphere where the ring is located, the origin, and the point \( p \), where the field is to be evaluated, Figure 5.1. The distance from the point to the ring will be the distance from the origin to point \( p \) less the distance from the origin to the ring, \( R \cos(a) \). The small element of the charge on the ring will be equal to

\[
dQ = \frac{Q}{SA} (Rd) (2 \pi R \sin(a)) ,
\]

where, \( SA \) is the surface area of the sphere for \( r \ll R \). It can be shown that
\[ SA = 4\pi R^2 \left(1 - \frac{r^2}{R^2}\right), \quad (5.4) \]

which leads to,

\[ dQ = \frac{Q}{2} \left(1 - \frac{r^2}{R^2}\right)^{3/2} \sin(a) \, da. \quad (5.3) \]

To find the electric field, Equation (4.8) needs to be integrated with these new substitutions with respect to angle \( a \). This integration (solved using Mathematica 5.1) yields the following equation,

\[
E_z(z) = \int \frac{Q}{8\pi\epsilon_0 \left(1 - \frac{r^2}{R^2}\right)^{3/2}} \frac{(z - R\cos(a))\sin(a)}{(z - R\cos(a))^2 + (R\sin(a))^2)^{1/2}} \, da = \\
= \frac{Q}{4\pi\epsilon_0 \left(1 - \frac{r^2}{R^2}\right)^{3/2}} \frac{R - z\cos(a)}{z^2 \sqrt{R^2 + z^2 - 2Rz\cos(a)}} + C, \quad (5.6)
\]

where \( C \) is the constant of integration.

The limits of integration will range from \( a = \sin^{-1}(r/R) \) to \( a = \pi - \sin^{-1}(r/R) \). In the special case where the limits of integration were from 0 to \( \pi \) (i.e. \( r = 0 \)) we will get the field for a full sphere.
\[ E_z(z) = \begin{cases} 
-\frac{4\pi R^2 \sigma}{4\pi \varepsilon_o} \frac{1}{z^2} & z \leq -R \\
0 & -R < z < R \\
+\frac{4\pi R^2 \sigma}{4\pi \varepsilon_o} \frac{1}{z^2} & z \geq R 
\end{cases} \]

(5.7)

Figure 5.3 is a plot over all \( z \), of Equation 5.7, when \( \frac{\sigma}{4\pi \varepsilon_o} = 1 \). If the parameters from Chapter 3 were applied to Equation 5.6 and was plotted over \( z \) where \( \frac{\sigma}{4\pi \varepsilon_o} = 1 \), Figure 5.4 is obtained.

To find the field by both the inner cylinder to the outer partial sphere the two fields are superimposed together, as shown in Figure 5.6.

Figure 5.3 Electric field of full sphere. It is a plot of Equation 5.6. The blue lines indicate the edges of the sphere. The horizontal axis is the \( z \)-axis is in meters and the vertical axis is the electric field where \( \frac{\sigma}{4\pi \varepsilon_o} = 1 \). \( R \) is given from section 3.2.1 and \( r = 0 \).

The dashed line shows the field for a point.
Figure 5.4 Electric field of hollow sphere with small hole on axis. It is a plot of Equation 5.6. The blue lines indicate the edges of the sphere. The horizontal axis is the z-axis in meters and the vertical axis is the electric field where \( \frac{\sigma}{4\pi\varepsilon_0} = 1 \). R and r is given from Chapter 3. The Dashed line shows the field for a point.

Figure 5.5 Electric field of a hollow sphere of radius R with a hole and cylindrical tube with the radius r. The tube has the same charge density as the sphere. It is a plot of Equation 5.6 plus Equation 5.2. The blue lines indicate the edges of the sphere. The horizontal axis is the z-axis in meters and the vertical axis is the electric field where \( \frac{\sigma}{4\pi\varepsilon_0} = 1 \). R and r are given from Chapter 3.

From these figures, it is noted that the field is reduced by about \( \frac{1}{4} \) from that of a full sphere. In addition, the field effects caused by the inner cylinder are about one
hundredth of that of the outer sphere. From the comparison in Figure 5.5 and Figure 4.2 when compared to a particular voltage, the value of $\sigma$ also will scale with the surface area. The electric field of the larger spheres is approximately a factor of 20-40 less than field for the shape electrodes, depending on the chosen enhancement factor.

5.2 Discharge Induced by a Laser Filament Over the Gap

Breakdown over a 1.08 cm gap was observed at 15.3 kV when the filament was going through the spherical electrodes. This breakdown was not observed without the filament until 20 kV. This shows that the breakdown is reduced with the presence of the plasma.

The spherical electrodes were used to measure the conductivity of the filament. A single filament was shot through the middle of the holes drilled into the electrodes. The signals from the Pearson coils were measured in two ways. With the absence of the secondary features, a large window could be used to measure the average conductivity of the gap. The peak height was used to find the maximum conductivity. The bias voltage was varied from 0-25 kV in increments of 5 kV. The gap between the electrodes was also varied. The results of these experiments are shown in Figure 5.6-9. These results have a striking resemblance to the model that was shown in Chapter 2, Figure 2.5.

The front coil signal continuously increases with voltage and does not plateau as seen in Figure 4.5 at long distances and voltages greater than 12.5 kV. When comparing Figure 5.6 with that of Figure 5.7 they show that the current is making its way to the rear electrode even at longer distances.
With this electrode configuration, there were no other secondary current pulses observed as in Figure 4.8. This is most likely because the field was not close enough to that of breakdown.

Figure 5.6 Average front coil peak current measurements with varying electrode gap distance.
Figure 5.7 Average back coil peak current measurements with varying electrode gap distance

Figure 5.8 Front coil peak current measurements with varying electrode gap distance.
Figure 5.9 Back coil peak current measurements with varying electrode gap distance.

As in the previous chapter, it would make better sense to look at how the current/charge varies with electric field. By using the same scaling of the field between the two electrodes, $|E| = \frac{V}{d}$, the Figures 5.10-5.14 are produced. These graphs are similar to the model shown in Chapter 2 Figure 2.6. This set of data is more linear than that of Figure 4.6. The resistivity of the filament can be found from the inverse of these slopes (ohms law [equation 2.8]). It should be noted that the linear fits to the data on the front coils have low values for $R^2$. The values for the rear coil produce a better least squares fit than the front coil.

If the fits were forced through the origin, the minimum resistivity as measured from the front coil is 3.0 $\Omega$cm and for the back coil, we measure 2.94 $\Omega$cm. These numbers are within good agreement. If there were some offset and the y-intercept was
allowed to vary, slightly greater values of the resistivity, 4.74 \( \Omega \text{cm} \) for the front coil and 3.11 \( \Omega \text{cm} \) for the back coil, are obtained. The square values are larger than what is shown in Figures 5.12 and 13. The \( R^2 \) is 0.80 for the front coil and 0.97 for the back coil.

Since the plasma channel created by the filament has a lifetime, the current decays. This decay of the time resolved current is faster than the other case of the pointy electrodes, 5-25 ns. The average resistance over a 250 ns window is found to be 60 \( \Omega \text{cm} \) on the front coil and 40 \( \Omega \text{cm} \) on the rear coil, forcing these fits through the origin. If the y-intercept was allowed to vary, the resistivity is found to be 49.6 \( \Omega \text{cm} \) for the front coil and 52 \( \Omega \text{cm} \) for the rear coil with \( R^2 \) values of 0.77 and 0.95 respectively.

![Electric field vs Charge Over 250ns Window Front Coil](image)

\[ y = 0.4951x \]
\[ R^2 = 0.6778 \]

**Figure 5.10** Plot of current vs. field using data from Figure 5.6
Electric field vs Charge Over 250ns Window Back Coil

$y = 0.3233x$

$R^2 = 0.9174$

Figure 5.11  Plot of current vs. field using data from Figure 5.7

Electric field vs Peak current in Front Coil

$y = 16.56x + 23.167$

$R^2 = 0.795$

Figure 5.12  Plot of current vs. field using data from Figure 5.8
This data shown above has similar trends to that of the 1D model presented in Chapter 2. The actual values of the amount of charge passing through the filament are underestimated in that model. This can be easily compensated by a factor of 2-4 in the total conductivity or the recombination time. In addition, some constants used in the model are estimates taken from the literature and could be off by a factor less than ten. Figure 5.14 shows the data taken from Figures 5.10 and 5.11 and in comparison to the model with and without being adjusted. Figure also included data taken at one cm gap between the electrodes.

However, the 1D model generates current pulses that are shorter than what was observed. When the spatial dependence of the charge carriers is accounted for, in the 2D model, the current pulses are obtained approximately to what was observed in the laboratory. Figure 5.15 shows the comparison of the 2D model with the data used different initial electron densities. It should be noted that the model uses a lower initial
electron density. This 2D model also tends to increase in slope at the larger values of field, same as what the data should suggest.

Figure 5.14 shows a comparison of data shown in Figure 5.10 (red), Figure 5.11 (blue), data taken at one cm electrode gap spacing Front coil (Green) back coil (Black). The model shown is discussed in Chapter 2 (black line).
Experiments were performed like that of Figure 4.7 (90 degrees between the electrodes and the beam propagation). In this configuration, there was no significant current in the electrodes on the order of a few pC. The currents seen here are filament dependent. Even with the laser chirp adjusted to produce no filament at or near the electrode gap, no significant current was observed at any bias voltage from 0-25 kV. This small signal could be attributed to the charge moving in the circuit to compensate for the presence of the filament.
CHAPTER 6: DISCUSSION

The results of the spherical electrodes discussed in Chapter 5 gave results that not only matched the models presented in Chapter 2, but also agreed with the values found within the literature [17, 18]. The 2D model gives more equivalent current pulse shapes, using the same initial charge carrier densities, compared to the 1D model, was observed and trends with the data better than the 1D model.

The secondary features to the time resolved current make for an interesting story. They are caused by a delayed decrease in the resistance of the gap. The 2D model, at fields near breakdown, shows an increase in conductivity caused by the increase of cross sectional area. (The 2D model does not turn off after the increase in the conductivity; further study is needed.) The experiments suggest the secondary features are attributed to an increase in the electron density charge. This will lower the resistance of the gap and allow more charge to pass. The time scales of the delay of these peaks significantly changes with the electric field, from ten’s ns to approximately a µs. The only time scale that also changes that significantly with the electric field is the electron collision time, shown in Figure 2.3 / Figure 4.14. When the field between the two electrodes is high this electron collisional time is short enough that there are a significant number of electrons remaining to ionize the air molecules thus producing a peak. This hypothesis could also explain the echoing effect see in Figure 4.9. At even higher fields these electron collisional time is so short that other ionization collisions can occur. At the point where the collisional time is equal to the attachment time is when avalanches can form, leading to the development of break down of the gap. The secondary features are a warning that
avalanches are soon to follow, as some times seen in the laboratory to the detrement to the equipment.

These secondary features to the time resolved current should have been seen with the spherical electrodes, but the field, 10 kV at 1 cm, was never strong enough to reduce the electron collisional time to observe the phenomena, near break down which was observed at 15 kV, while the sharp electrodes were only at few kV away from breakdown. More careful study with the spherical electrodes at higher fields is needed.

While using the sharp electrodes, the current dependence on electric field was erratic at low values, as shown in Figure 4.6. At high voltages and long distances, the current is finding other ways to get to ground, thus giving misleading results. This can explain the threshold in Figure 4.5. More experiments should be conducted with better confidence to find the cause of the voltage dependant threshold.
CHAPTER 7: CONCLUSIONS

The geometry of the electrodes plays a pivotal role in measuring the conductivity of a filament. The secondary features of the time resolved current are caused by the high fields generated by the sharp electrodes. The underlying physics of the secondary features of the time resolved current are consistent with electron collisions, but further research is required. There are no such descriptions of the secondary features to time resolved current that have been found in the current literature.

These secondary features could produce misleading results in the resistivity of the filament if one were to measure over long time scales. The current dependence on electric field becomes erratic at low values, as shown in Figure 4.6. If the resistivity of the filament needs to be measured, then a uniform electric field needs to be used. With the uniform field caused by the spherical electrodes, measurements are in close agreement with published values [18, 19] and with the model presented in Chapter 2.

It has also been demonstrated that the filament can lower the voltage at which breakdown occurs and controlled the destination of the current. This can have many useful applications ranging from making better lighting protection to an improved tool for law enforcement. The high resistance of the plasma filament will make such technology tough to be implemented over long distances.
The following contain the Mathematica® 5.1 sheet used to do calculations for this Thesis. The following is the code used to model the experiment from Chapter 2 and 5

Discharge Modeling for PhASER Project

- Pearson Triggered

```mathematica
(* Begin code *)

(* End code *)
```

- Phototube Triggered

```mathematica
(* Begin code *)

(* End code *)
```

2.27 Data

- Data taken in the study

```mathematica
(* Begin code *)

(* End code *)
```
Plots of the Data

Modeling Section

• A Single Modeling Run

$\text{Model:}$

\[ y = a + bx + \epsilon \]

\[ \epsilon \sim N(0, \sigma^2) \]

\[ y = x \\
\]
Modeling live cross-section

The reaction model is a two-section model directly corection. Under such an agreement, the HS model with happy configurations can be modelled with the knowledge and is as follows:

\[
\text{Model: Corection} \quad \text{Corection} \quad \text{Corection}
\]

\[
\text{Model: HS Model} \quad \text{HS Model} \quad \text{HS Model}
\]

\[
\text{Model: Knowledge} \quad \text{Knowledge} \quad \text{Knowledge}
\]

\[
\text{Model: Corection} \quad \text{Corection} \quad \text{Corection}
\]

\[
\text{Model: HS Model} \quad \text{HS Model} \quad \text{HS Model}
\]

\[
\text{Model: Knowledge} \quad \text{Knowledge} \quad \text{Knowledge}
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\[
\text{Model: Corection} \quad \text{Corection} \quad \text{Corection}
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\[
\text{Model: HS Model} \quad \text{HS Model} \quad \text{HS Model}
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\[
\text{Model: Knowledge} \quad \text{Knowledge} \quad \text{Knowledge}
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\text{Model: Corection} \quad \text{Corection} \quad \text{Corection}
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\text{Model: HS Model} \quad \text{HS Model} \quad \text{HS Model}
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\text{Model: Knowledge} \quad \text{Knowledge} \quad \text{Knowledge}
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\text{Model: Corection} \quad \text{Corection} \quad \text{Corection}
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\text{Model: HS Model} \quad \text{HS Model} \quad \text{HS Model}
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\text{Model: Knowledge} \quad \text{Knowledge} \quad \text{Knowledge}
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\text{Model: Corection} \quad \text{Corection} \quad \text{Corection}
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\text{Model: HS Model} \quad \text{HS Model} \quad \text{HS Model}
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\text{Model: Knowledge} \quad \text{Knowledge} \quad \text{Knowledge}
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\text{Model: Knowledge} \quad \text{Knowledge} \quad \text{Knowledge}
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\[
\text{Model: HS Model} \quad \text{HS Model} \quad \text{HS Model}
\]
• Models of Spatial Distribution of Sensitivity and Objectivity

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = \frac{1}{\rho c^2} \nabla \cdot (\kappa \nabla T)
\]

\[
\nabla \cdot \left[ \begin{array}{ccc}
\rho & 0 & 0 \\
0 & \rho & 0 \\
0 & 0 & \rho
\end{array} \right] \nabla u + \nabla \cdot \left[ \begin{array}{ccc}
\tau_{x,1} & 0 & 0 \\
0 & \tau_{y,1} & 0 \\
0 & 0 & \tau_{z,1}
\end{array} \right]
\]

\[
\tau_{x,1} = \frac{1}{2} \left( \frac{7}{3} \frac{2000}{10} \right) (x \text{ in units of } 1000)
\]

\[
\tau_{y,1} = 3.9 \quad (y \text{ in units of } 1000)
\]

\[
\tau_{z,1} = 0.6 \quad (z \text{ in units of } 1000)
\]

\[
T = 3.2 \quad \text{(total energy density per cell)}
\]

\[
\text{max} = 7.3 \times 10^2 \text{ (peak cell)}
\]

\[
\text{max} = 2.34 \times 10^2 \text{ (max density per cell)}
\]

\[
\sigma = 0.34 \times 10^{-3} \text{ (total energy density per cell)}
\]

\[
\text{max} = 3.2 \times 10^2 \text{ (max density per cell)}
\]

\[
\text{max} = 2.34 \times 10^{-3} \text{ (total energy density per cell)}
\]

\[
\rho = 2.34 \times 10^{-3} \text{ (total energy density per cell)}
\]

\[
T = 3.2 \times 10^2 \text{ (total energy density per cell)}
\]

\[
\text{max} = 2.34 \times 10^{-3} \text{ (total energy density per cell)}
\]

\[
\text{max} = 3.2 \times 10^2 \text{ (total energy density per cell)}
\]

\[
\text{max} = 2.34 \times 10^{-3} \text{ (total energy density per cell)}
\]

\[
\rho = 2.34 \times 10^{-3} \text{ (total energy density per cell)}
\]

\[
\sigma = 0.34 \times 10^{-3} \text{ (total energy density per cell)}
\]
**Collision Time**

\[
\text{collision}[	ext{field}_1] = \frac{\max \left(5.7 \times 10^{-2}, 1.0 \times 10^{-3}\right)}{\max / \text{sec}}
\]

attachmentplot = LogPlot[attachment[x], \((x, 500, 50000), \text{Frame} = \text{True}, \text{GridLines} = \text{True}, \text{FrameLabel} = \{"Log", "\text{attachment}[x]"}, \text{"Collision Time \{ms\}"}, \{, \}, \text{Flaestyle} = \{\text{RedColor}[0, 1, 0]\}];

**Attachment Time**

attachmenttime\[\text{field}_1\] = attachment[x], \((x, 500, 50000), \text{Frame} = \text{True}, \text{GridLines} = \text{True}, \text{FrameLabel} = \{"Log\text{attachment}[x]"}, \text{"Attachment Time \{ms\}"}, \{, \}, \text{Flaestyle} = \{\text{RedColor}[0, 1, 0]\}, \text{Flaestyle} = \{\text{BlueColor}[0, 1, 0]\}, \text{Flaestyle} = \{\text{BlackColor}[0, 1, 0]\};

Show[attachmentplot, attachmenttimeplot, attachmenttime\[\text{field}_2\] = attachment[x], \((x, 500, 50000), \text{Frame} = \text{True}, \text{GridLines} = \text{Automatic}, \text{FrameLabel} = \{"Log\text{attachment}[x]"}, \text{"Attachment Time \{ms\}"}, \{, \}, \text{Flaestyle} = \{\text{RedColor}[0, 1, 0]\}, \text{RedColor}[0, 1, 0]\};

FindRoot[attachment\[\text{field}_1\] = attachment[x], \((x, 50000)\); 

**Dissociative Recombination Time**

\[
\text{dissociation} \left(\frac{\text{sec}}{\text{cm}^2}\right)
\]

Dissociationplot = LogPlot[\(\frac{10^5}{\text{dissociation}}\), \((x, 500, 50000), \text{Frame} = \text{True}, \text{GridLines} = \text{Automatic}, \text{FrameLabel} = \{"Log\text{dissociation}[x]"}, \text{"Dissociative Recombination Time \{ms\}"}, \{, \}, \text{Flaestyle} = \{\text{RedColor}[0, 1, 0]\}];

**Mutual Neutralization Time**

\[
\text{mutualneutralization} \left(\frac{\text{sec}}{\text{cm}^2}\right)
\]

Mutualneutralizationplot = LogPlot[\(\frac{10^5}{\text{mutualneutralization}}\), \((x, 500, 50000), \text{Frame} = \text{True}, \text{GridLines} = \text{Automatic}, \text{FrameLabel} = \{"Log\text{mutualneutralization}[x]"}, \text{"Mutual Neutralization Time \{ms\}"}, \{, \}, \text{Flaestyle} = \{\text{RedColor}[0, 1, 0]\}];

LogPlot[\(\frac{10^5}{\text{mutualneutralization}}\), \((x, 500, 50000), \text{Frame} = \text{True}, \text{GridLines} = \text{Automatic}, \text{FrameLabel} = \{"Log\text{mutualneutralization}[x]"}, \text{"Mutual Neutralization Time \{ms\}"}, \text{"Using Longear's method", \{, \}, \text{Flaestyle} = \{\text{RedColor}[0, 1, 0]\}];
This next code is for the approximation of the electric field near a point electrode on axis shown in chapter 4.

\[
\begin{align*}
R &= 0.03147/2 \\
L &= 0.023368 \\
k &= 0.2 \\
r &= 0.00150 \\
Enh &= 1.86 \\
Etip[z_] &= \\
&\int_{0}^{\pi/4} \left( \left( \int_{L}^{R} \left( 2\pi a \left( \frac{R-r}{L} a + r \right) \sqrt{1 + \frac{R-r}{L} a} \right) \right) da \right) \\
&\times \frac{1 - Enh}{L^2} (r + L)^2 + 1 \right) dx + \\
&\int_{0}^{\pi/2} \left( 2\pi r^2 \left( ((z + L) - (R \cos[\theta])) \sin[\theta] \right) \\
&\times ((z - (R \cos[\theta]))^2 + (r \sin[\theta]^2)^2)^{3/2} \right) d\theta \\
&+ \int_{0}^{\pi/2} \left( 2\pi r^2 \left( Enh \left( ((z) - (r \cos[\theta])) \sin[\theta] \right) \\
&\times ((z) - (r \cos[\theta]))^2 + (r \sin[\theta]^2)^2)^{3/2} \right) d\theta \\
\end{align*}
\]

\text{EtipF}[z_] := \begin{cases} 
\text{Etip}[z] & z <- (R + L) \\
\text{Etip}[z] & -(R + L) \leq z \leq r \\
0 & z > r 
\end{cases}

\text{Plot}[\text{Etip}[z], \{z, -1.4 (L + R), 10 r\}, \text{PlotRange} \rightarrow \text{All}, \\
\text{PlotRange} \rightarrow \text{All}, \text{Frame} \rightarrow \text{True}, \\
\text{FrameLabel} \rightarrow \\
\{"z (m)", \text{"Electric Field where } \sigma/4\pi\varepsilon_0=1 \text{ (arbs)}\}, \\
\text{"Electric field of tip where } \text{Enh}=1.86", \text{""}, \\
\text{GridLines} \rightarrow \{(r, 0, -L, -(L + R)), \{(\})\}]

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This sheet was used to approximate the electric field of a spherical electrode with a cylindrical hole on axis of the center of the hole.

**Electric Field of a Spherical Conductor**

\[
E_{\text{field}}(x, y, z) = \frac{1}{4\pi\epsilon_0} \int \frac{q}{r^2} \, dx
\]

\[
E_{\text{field}}(x, y, z) = \left( \frac{x}{(x^2 + y^2 + z^2)^{3/2}} \right) dx
\]

**Hollow Sphere with Hole**

**Sum of Cylinder and Partial Sphere**
LIST OF REFERENCES


