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Lesson Study As A Catalyst For Including Concepts And Procedures In Planning And Implementing Fraction-Based Mathematics Lessons: An Elementary School Case Study

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LESSON STUDY AS A CATALYST FOR INCLUDING CONCEPTS AND PROCEDURES IN PLANNING AND IMPLEMENTING FRACTION-BASED MATHEMATICS LESSONS: AN ELEMENTARY SCHOOL CASE STUDY

by

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ABSTRACT

This qualitative case study investigated how lesson study influenced teachers' pedagogical strategies and content knowledge by leveraging the interplay between conceptual understanding and procedural fluency, with conceptual knowledge forming the foundation for procedural fluency in the planning and implementation of fraction-based mathematics tasks. The study aimed to address two research questions: (1) How does the engagement of teachers in the creation and analysis of fraction-based mathematics tasks in lesson study influence their instructional decision-making processes? and (2) How do teachers construct pedagogical strategies that integrate procedural and conceptual knowledge through lesson study? The sample size consisted of seven elementary school teachers who worked collaboratively to modify fraction tasks. Triangulated data from observations and interviews revealed two key findings. First, the study's findings highlight the transformative power of collaborative professional growth among teachers and their subsequent shift towards engaging in more conceptual, student-centered mathematics instruction, addressing the first research question. Second, the findings suggest that lesson study can be a valuable tool for enhancing teachers' pedagogical content knowledge and improving fraction instruction when utilized consistently, thus answering the second research question. Implications for future teacher education are discussed, emphasizing the importance of sustained, collaborative learning opportunities to support conceptual understanding in mathematics education.

This is dedicated to my family. This would not happen without each of you by my side.

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Abbreviation	Meaning
CoP	Communities of Practice
FLDOE	Florida Department of Education
NAEP	National Assessment of Educational Progress
NCTM	National Council of Teachers of Mathematics
PSSM	Principles and Standards for School Mathematics
PLC	Professional Learning Community

CHAPTER 1: INTRODUCTION

The United States is a global superpower renowned for its military and economic prowess. As a leader on the world stage, the United States recognizes the crucial role of mathematics education in preparing its students for the challenges and opportunities of the 21st century. The nation continuously invests in mathematics education reform policies and modernizing instructional practices to ensure that American students are well-equipped to thrive in an increasingly complex and interconnected world (U.S. Department of Education, 2019).

As Wiburg and Brown (2006) note, "to graduate students capable of living and working in a rapidly changing world with a global economy, it is necessary to fundamentally change how teachers instruct, and students learn" (p. 20). The United States has a proud history of rising to challenges and embracing change. By harnessing this spirit of innovation, the nation has the potential to positively transform mathematics education and empower its students to reach new heights of success.

One promising approach to achieve this transformation is the implementation of high-quality mathematics lessons and modes of instruction that align conceptual and procedural knowledge in an iterative format developed through teacher communities. The Japanese-inspired lesson study is a collaborative and inquiry-based approach to instructional improvement. It involves an iterative process of studying, planning, teaching, and reflecting on standards, instructional delivery, and assessments (Fernandez & Yoshida, 2004).

"Improving something as complex and culturally embedded as teaching requires the efforts of all players, including students, parents, and politicians. But teachers must be the primary driving force behind change. They are the best positioned to understand the problems that students face and generate possible solutions" (Stigler & Hiebert, 1999, p. 135). Thus, lesson study deviates from the norms of the existing practice of professional learning initiatives as it combines the educators' position to effect immediate change with research-informed knowledge.

This study, conducted at ABC Elementary in Aspen County (pseudonyms), utilized Robert Yin's (2013) case study design to explore the real-world applications of lesson study in third-grade classrooms. This research provided valuable insights into how lesson study can transform teaching practices and improve student learning outcomes. It focused on creating and executing mathematics lessons that effectively combine conceptual and procedural aspects of teaching fractions. By evaluating the impact of lesson study on teachers' pedagogical and content knowledge, the study identified effective strategies for enhancing mathematics instruction, particularly in the area of fractions (NAEP, 2022).

Statement of Problem

United States students have unfinished learning with fractions, which can hinder their overall mathematical progress and potentially limit their qualifications for future technological and engineering roles (Fennell, 2007; Schoenfeld, 2002). Proficiency in fractions is essential for building robust mathematical skills and provides a solid foundation for comprehending abstract concepts in subjects such as algebra (National Mathematics Advisory Panel [NMAP], 2008). Many instructional tasks tend to favor specific procedures or steps that should be followed to facilitate learning (Hiebert, 1984). This reliance on procedural dependency can exacerbate the confusion surrounding fractions.

Fractions are uniquely challenging to students because of their rampant mathematical complexity (Bruce et al., 2023). Many students tend to see fractions as meaningless symbols or view the numerator and denominator as separate numbers rather than understanding them as part of a unified whole. In addition, fractions have "multiple situationally dependent meanings"

(Bruce et al., 2023, p. 4), which involve both procedural and conceptual components. For example, fractions can present part-whole relationships or magnitude, as in the case of a pizza divided into eight equal slices; if you consumed three slices, it means that you ate $\frac{3}{8}$ of the whole pizza. Fractions can be used to express measurements by comparing the length of an object to the total length of the measuring tool. For example, if you measure an object that is three inches long using a 12-inch ruler, the object's length can be represented as the fraction $\frac{3}{12}$.

Fractions can also represent the result of mathematical operations. For example, when dividing 2 by 3, it can be represented by the fraction $\frac{2}{3}$. In terms of multiplication, fractions are used to scale quantities. For example, if a recipe serves five people, and you want to double it to serve ten people, you would multiply all the ingredients by the fraction $\frac{2}{1}$, as 2 represents the desired scale. Thus, grasping fractions requires "complex mental processes such as spatial reasoning and proportional reasoning, which are crucial for various content areas like probability, measurement, and geometry" (Bruce et al., 2023, p.4). Fractions also serve as a foundation for understanding algebra, which is a gateway to higher mathematics (NMAP, 2008).

In addition, the challenges students encounter when learning about fractions tend to become exacerbated because of a lack of exposure to the various models necessary to teach fractions (Tuba, 2017). How fractions are presented in mathematics textbooks is problematic as many tasks rely heavily on the area model because students are intuitively familiar with partitioning (as most of them have had exposure to sharing items) (Cathcart et al., 2013). However, this can deeply limit how students perceive fractions, as it is not the only way to represent them.

The linear model, for instance, demonstrates the idea that there is always another fraction between any two numbers (Cathcart et al., 2013). This is a crucial concept that is often overlooked in fraction instruction. Interestingly, research shows that many teachers also have emerging understanding in interpreting and making meaning of the models for fractions (Tuba, 2017). The third-grade mathematics curriculum at ABC Elementary includes tasks that often involve reinforcing skills and fluency. The tasks often have a singular correct answer and follow prescribed methods. While the task aims to develop students' skills in using mathematical procedures efficiently and accurately, it can limit opportunities for students to explore or develop critical thinking since they are required to follow specific procedures.

The *Principles and Standards for School Mathematics* (PSSM) emphasizes the "importance of students learning mathematics with understanding and actively constructing new knowledge based on their experiences and prior knowledge" (National Council of Teachers of Mathematics [NCTM], 2000, p. 2). The challenges related to students' understanding and acquisition of fractions are often magnified when the procedural curriculum does not align with the recommended instructional approaches outlined by NCTM.

Furthermore, the role of teachers in the learning process is crucial, as their understanding of the subject directly influences their instructional practices (Ellis, 2004). If teachers have unfinished learning of fractions, they may unknowingly transmit these to their students (Charalambous & Pitta-Pantazi, 2006). This can unwittingly lead to teachers experiencing difficulties in explaining complex fraction concepts or modeling appropriate problem-solving techniques (Özpınar & Arslan, 2021). It can also unintentionally limit students' exploration of alternative solutions or hinder their ability to construct new problem-solving approaches (Wiburg & Brown, 2006).

Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations recommends that "mathematics instruction should be consistent with research-informed and equitable teaching practices" (NCTM, 2020, p. 25). *Catalyzing Change* recommends disrupting the cycle of rote teaching and learning of fractions and instead prioritizing "teaching fraction concepts and operations conceptually" (NCTM, 2020, p. 100). These educational reform recommendations in *Catalyzing Change* (NCTM, 2020) advocate for a shift in teachers' pedagogy. It encourages educators to view teaching as engaging students in ways they can comprehend and to view learning as a collaborative process of knowledge co-construction. Therefore, the curriculum and the instructional methods teachers employ must be critically reassessed and modified to align with the transformative principles advocated by *Catalyzing Change* (NCTM, 2020).

As educational stakeholders, we have an exciting opportunity to enhance our current instructional systems and drive transformative changes that will positively impact the learning trajectory of mathematics. As Wiburg and Brown (2006) note, "to graduate students capable of living and working in a rapidly changing world with a global economy, it is necessary to fundamentally change how teachers instruct, and students learn" (p. 20).

Therefore, leveraging the synergistic relationship between conceptual understanding and procedural through transformative approaches such as lesson study holds promise for enhancing mathematics instruction, specifically in fractions. By examining the impact of lesson study on teachers' pedagogy and their ability to promote a comprehensive understanding of fractions through well-developed mathematical lessons, this study aimed to contribute valuable insights and recommendations for improving mathematics instruction to empower teachers on their mathematical journey (Bruce et al., 2023; Özpınar & Arslan, 2021; NCTM, 2020).

Organizational Context

ABC Elementary is in an urban setting, and its entire student population qualifies for the free and reduced lunch program (FLDOE, 2022). In Aspen County, the Department of Education employs a grading system to assess public K-12 schools annually, using *A* through *F* letter grades. The grade is determined by calculating the points earned for various achievement components, such as learning gains and learning gains of the lowest 25% of students. Based on the results of the 2023 standardized tests, ABC Elementary received a *C* grade. The standardized test scores indicated that only 43% of third-grade students were proficient in mathematics (FLDOE, 2022).

ABC Elementary has approximately 550 students, 54 teachers, and 15 support staff. Approximately 70% of the staff and 90% of the student population identify as Black (FDOE, 2022). Approximately 65% of students are non-native English speakers. Table 1 reveals an approximation of the ethnic composition of ABC Elementary students.

Table 1 ABC Elementary Ethnic Diversity Breakdown

Ethnicity	Breakdown
White	3%
Hispanic	7%
Black (Haitians, West Indians, Africans)	90%

The administration strongly recommends that teachers utilize the district's prescribed curriculum to create curricular continuity across grade levels. Due to the structure of the prescribed lessons, which typically include an average of ten PowerPoint slides saturated with various tasks and strategies, teachers tend to focus more on procedural-type tasks. The excessive focus on procedural knowledge is further intensified by the time constraints imposed by high-stakes testing. "High-stakes testing affects the nature of instruction" (Schoenfeld, 2002, p. 21) as teachers resort to procedural means to maximize coverage of all mathematics topics. This leads to classroom environments where students are expected to memorize formulas, rhymes, and algorithms without grasping the underlying concepts (Boaler, 1998). For example, teachers use rhymes and mnemonics such as “Keep it, Change it, Flip it” to help students remember the procedure for dividing any number by a fraction, which involves keeping the first number or fraction, changing the division sign to multiplication, and flipping the second fraction or finding the reciprocal.

However, the administration at ABC Elementary is committed to promoting transformative practices that can positively impact students’ learning outcomes. They recognize the need to address the school’s challenges and, therefore, support the implementation of lesson study to accomplish this goal. By embracing the collaborative and research-informed approach of lesson study, the administration hoped to change the trajectory of students’ learning outcomes and create a more effective and equitable learning environment. Therefore, *Catalyzing Change* (NCTM, 2020) was an important component of the framework to further enhance the educational practices and outcomes within the school.

Conceptual Framework

This study incorporated *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations* (NCTM, 2020) as the overarching conceptual framework to guide the research. I also included Hiebert’s seminal Site Theory (1984, further developed in 1986), which links procedural and conceptual knowledge, and Wenger’s (1998) Communities of Practice (CoP) theory to explore the dynamics of learning in voluntary settings. Drawing upon these frameworks, this study examined how educators collaborated within professional networks to foster shared understandings about the intricate relationship between procedural and conceptual knowledge by embedding both knowledge types in mathematics learning (NCTM, 2020).

Catalyzing Change recommends that "early childhood settings and elementary schools should build a strong foundation of deep mathematical understanding, emphasize reasoning and sense-making, and ensure the highest quality mathematics education for each and every child" (NCTM, 2020, p. 69). These concepts resonate throughout NCTM's *Taking Action* series under the phrase 'ambitious teaching,' which positions students as capable of understanding mathematical concepts and transferring these ideas to new situations. It involves presenting high-quality mathematics tasks that encourage students' thinking and problem-solving abilities while fostering and valuing their capabilities (Huinker & Bill, 2017).

Furthermore, NCTM's *Taking Action* series recommends building procedural fluency from a foundation of conceptual understanding (NCTM, 2020). Thus, conceptual knowledge and procedural knowledge are intricately intertwined as their development occurs simultaneously and reciprocally. As students engage in mathematical tasks, their conceptual understanding of the underlying principles and relationships informs their procedural fluency, while their growing procedural skills reinforce and deepen their conceptual grasp. This iterative process allows for continuous refinement and strengthening of both types of knowledge.

Catalyzing Change (NCTM, 2020) contributes to the conceptual framework because it advocates for implementing interrelated practices and processes to ensure students' success in mathematics. In the context of this study, three of the practices contributed to the framework:

- Broaden the purposes of learning mathematics.
- Implement equitable mathematics instruction.
- Develop deep mathematical understanding (NCTM, 2020).

While Hiebert's (1984) work served as a precursor to the synthesis of ideas housed within *Catalyzing Change* (NCTM, 2020), it aligned with interrelated practices, processes, and ambitious teaching. Hiebert's seminal research provided the content-specific framework for the study. Together, *Catalyzing Change* (NCTM, 2020) and Hiebert's (1984) Site Theory framework emphasize the importance of deep mathematical understanding and the link between procedural and conceptual knowledge.

Hiebert (1984) emphasized three distinct sites to present high-quality mathematics tasks to link conceptual and procedural knowledge. In Site 1, mathematics symbols are assigned meanings through symbol interpretation (Hiebert, 1984). For instance, when dealing with fractions, students are presented with symbols like $\frac{1}{6}$ which are then linked to visual representations to which students can understand and relate. This approach offers students a more concrete and intuitive understanding of an abstract mathematical concept. Understanding the concept of the fraction symbol and its representation of a specific quantity or relationship allows them to develop a holistic understanding of a part-whole relationship, which corresponds to *Catalyzing Change*, 'broadening the purposes of learning mathematics' (NCTM, 2020, p. 12). Thus, understanding the meaning behind mathematical symbols will more likely engage students in meaningful learning rather than procedural dependency. This positions them to "develop deep mathematical understanding as confident and capable learners" (NCTM, 2020, p. 11).

In Hiebert's (1984) procedural execution (Site 2), pictures or models were utilized to illustrate how rules make sense by offering visual representations that assisted students in connecting abstract mathematical concepts with tangible examples (Hiebert, 1984). In terms of fractions, envision dividing one representation of a circular cookie into two halves and another representation of a circular cookie into four equal parts. Placing these pieces on top of each other can reveal that two-fourths and one-half are equivalent. Through hands-on exploration of the concrete models, it becomes evident that two-fourths and one-half constitute a whole.

Consequently, when students encounter the operation $\frac{1}{2} + \frac{2}{4}$, they holistically understand why the result is one whole. Utilizing tangible manipulatives or models corresponds with the call of *Catalyzing Change* to "implement equitable mathematics instruction" (NCTM, 2020, p. 45). These tools promote equitable learning by offering multiple visual representations of fractions, such as area, linear, and set models. Therefore, Heibert's (1984) Site 2 and *Catalyzing Change* (NCTM, 2020) promote visual support for students, enhancing their understanding of the procedural steps and reinforcing the underlying logic behind applied mathematical rules.

The solution evaluation (Site 3) (Hiebert, 1984) involves assessing the reasonableness of answers in mathematical tasks by critically examining students' steps, strategies, and outcomes. By delving into the nuances of students' processes, this evaluative process helps identify early conceptions or errors in their problem-solving approaches. Educators then provide targeted support and guidance to improve students' understanding and mathematical reasoning skills. This positions the students as active doers and sense-makers of mathematics, aligning with the recommendation of *Catalyzing Change* to "develop deep mathematical understanding" (NCTM, 2020, p. 11).

Combining the *Catalyzing Change* (NCTM, 2020) and Heibert's (1984) Site Theory frameworks shows that conceptual understanding forms the foundation upon which procedural fluency is built. Both types of knowledge are essential and thus work in tandem to foster deep mathematical understanding. Conceptual knowledge involves comprehending the underlying principles, relationships, and meanings behind mathematical ideas, while procedural knowledge encompasses the skills and steps required to solve problems efficiently and accurately (Hiebert, 1986; NCTM, 2020). As students develop conceptual understanding, they are better equipped to make sense of the procedures they apply. This enables them to adapt their problem-solving strategies to novel situations.

Furthermore, as students practice and refine their procedural skills, they simultaneously reinforce and deepen their conceptual understanding by connecting abstract ideas to concrete applications. This interplay between conceptual and procedural knowledge aligns with the recommendations of *Catalyzing Change*, which advocates for "developing deep mathematical understanding" (NCTM, 2020, p. 11) by engaging students in meaningful learning experiences that emphasize both types of knowledge. By intentionally aligning conceptual and procedural knowledge in mathematics instruction, educators can cultivate students' ability to make sense of mathematical ideas and apply their understanding flexibly across various contexts.

To facilitate this caliber of learning, teachers in ABC Elementary purposefully engaged in "ongoing, critical conversations to improve the mathematics learning experiences and outcomes of each and every child" (NCTM, 2020, p. 125). This was accomplished through another component of the conceptual framework, Wenger's (1998) Communities of Practice (CoP).

Communities of Practice

CoP has proliferated since ancient times as people have long recognized the value of coming together in informal social structures to address shared challenges and explore common issues (Wenger et al., 2002). CoP theory (explained further in Chapter 2) explores how learning occurs as individuals naturally gravitate toward others who share similar interests, goals, or concerns. Thus, CoP provided a valuable framework for understanding how professionals engage in collaborative and informal networks to develop shared understandings and engage in work-relevant knowledge building (Hara, 2010). The definition of CoP may vary among different scholarly communities; therefore, this study adopts the following definition: "Communities of practice are collaborative, informal networks that support professional practitioners in their efforts to develop shared understandings and engage in work-relevant knowledge building" (Hara, 2010, p. 12). Thus, at the core of CoP is the concept of a shared professional identity, emphasizing the importance of collective engagement and mutual learning (Wenger, 1998).

Lesson study (Wiburg & Brown, 2006) (examined in greater detail in Chapter 2) can operationalize CoP, as teachers can engage in deep discussions, exchange ideas, and collectively problem-solve. Within the CoP framework, lesson study fostered the development of a shared professional identity among the participating teachers. It encouraged sharing tacit knowledge and expertise as teachers collectively analyzed student's thinking, instructional strategies, and assessment methods. *Catalyzing Change* calls for sufficient instructional time for teachers to engage in extended mathematical inquiries and discussions. Teachers should have the support of administrators and the instructional time to "go deep with mathematics" (NCTM, 2020, p. 14). Therefore, since the *Catalyzing Change* (NCTM, 2020) framework emphasized the importance of professional collaboration and ongoing critical conversations to enhance mathematics learning experiences, it aligned perfectly with CoP's focus on collaborative, informal networks for shared knowledge building.

The intersection of *Catalyzing Change*, Hiebert's (1984) Site Theory, and Wenger's (1998) CoP theory culminated in a robust conceptual framework for exploring how informal networks within education can cultivate shared knowledge about the intricate relationship between conceptual and procedural knowledge in the learning of fractions in mathematics. Figure 1 summarizes the conceptual framework utilized in this study.

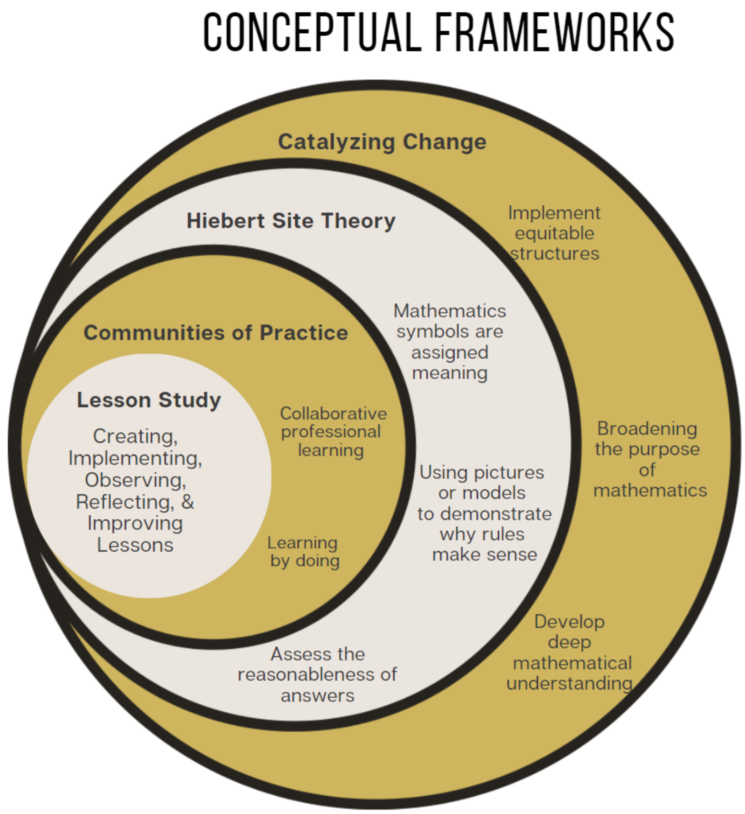

Figure 1 Conceptual Framework

<u>Significance of Study</u>

In today's world, there is a pressing demand for students with strong technical capabilities, as many future occupations necessitate some mathematical expertise. Specifically, fractions hold a crucial educational significance as they serve as a foundation for more complex mathematical subjects (Schoenfeld, 2002). Consequently, it is logical to assume, given the position of the NCTM, that aspects of conceptual and procedural learning should be embedded within mathematics tasks.

Therefore, this study aimed to enhance teachers' pedagogical practices and content knowledge through lesson study centered on co-creating mathematics lessons containing conceptual and procedural knowledge. The research findings can inform educational practices and professional learning initiatives, helping teachers improve their instructional strategies and make informed decisions that promote students' mathematical understanding.

Purpose of Study

This study aimed to investigate how educators co-construct mathematical tasks and implement pedagogical strategies that effectively align procedural and conceptual knowledge. Additionally, the study aimed to explore how the engagement of teachers in the analysis of mathematics tasks contributed to their instructional decision-making, specifically in the context of fractions. The following research questions guided this study:

- How does the engagement of teachers in the creation and analysis of fraction-based mathematics tasks in lesson study influence their instructional decision-making processes?
- How do teachers construct pedagogical strategies that integrate procedural and conceptual knowledge through lesson study?

Definition of Terms

The study employed the following terms:

Pedagogical Strategies:

The methodology and specific techniques used by educators to facilitate meaningful classroom interactions, build on students' prior learning, and help them develop relevant skills. Effective pedagogical strategies include facilitating discovery-based experiences, allowing for multiple pathways to arrive at correct answers, and promoting student discourse (Hiebert & Lefevre, 1986; NCTM, 2020).

Conceptual Knowledge:

The "knowledge about facts, [generalizations,] and principles" (Baroody et al., 2007, p. 107). It can encompass an "explicit or implicit understanding of the principles that govern a domain and the interrelations between pieces of knowledge in a domain" (Rittle-Johnson & Alibali, 1999, p. 175). Conceptual knowledge includes concepts such as problem-solving, reasoning, and communication. It utilizes students' prior knowledge and encourages them to think deeply about mathematical concepts.

Procedural Knowledge:

The knowledge of the procedures, algorithms, and rules necessary to carry out mathematical computations and problem-solving (Star, 2005). This includes having familiarity with individual symbols and syntactic conventions and understanding the sequential or algorithmic steps to solve a problem (Hiebert & Lefevre, 1986). It includes facility and fluency with arithmetic operations.

Mathematics Tasks:

"A set of problems that address the same mathematical idea" (Boston et al., 2019, p. 9).

Lesson Study:

An iterative process of collaborative teacher inquiry, where educators engage in a systematic cycle of observing, reflecting, revising, and reteaching a lesson (Akiba & Wilkinson, 2015). It involves a group of teachers working together to design and refine instructional practices to improve students' learning outcomes.

Lesson:

A curated collection of mathematical tasks, deliberate instructional strategies, and tailored student experiences designed with a specific educational goal in mind.

CHAPTER 2: LITERATURE REVIEW

Internationally and nationally, students' acquisition of fraction-based knowledge has been fraught with difficulties (Fennell, 2007; Hiebert, 1984; Pitsolantis & Osana, 2013; Siegler et al., 2013). These difficulties pose great challenges for students as they strive to develop their deep understanding and proficient application of this abstract topic. Research from global and national studies reveals that traditional teaching has strongly emphasized a procedural approach, leaving conceptual understanding underdeveloped, which may explain the challenges encountered in this domain (Rittle-Johnson & Alibali, 1999).

Mathematics educational theorists suggest that an integrated approach involving conceptual and procedural elements may significantly enrich students' mathematical comprehension and learning outcomes in fraction acquisition (Hiebert & Lefevre, 1986; Rittle-Johnson, 2019; Star, 2005). Therefore, this research draws upon the frameworks of *Catalyzing Change* (NCTM, 2020) and Hiebert's (1984) theories to form the robust structure for designing mathematics lessons that integrate procedural and conceptual knowledge. Additionally, the CoP framework, which emphasizes the value of shared learning and collaboration, is operationalized through lesson study, which serves as the central mechanism for the development of these mathematics lessons (Wenger, 1998; Wiburg & Brown, 2006).

Because educators play a crucial role in facilitating student learning, my aim is to explore how the lesson study can aid in this endeavor. Therefore, the following research questions guides this literature review:

- How does the engagement of teachers in the creation and analysis of fraction-based mathematics tasks in lesson study influence their instructional decision-making processes?

- How do teachers construct pedagogical strategies that integrate procedural and conceptual knowledge through lesson study?

The potential effects of understanding these approaches on contemporized mathematics lessons necessitate further exploration and review. Therefore, this literature review seeks to map the existing literature in this area, identifying key concepts, theories, and gaps in the literature.

The review commences by providing an overview of the global perspective on mathematics instruction, which reveals a dominance of procedural knowledge. It then delves into the classifications of procedural and conceptual knowledge and explores the empirical separability of these two forms of knowledge. The literature review also explores the sequencing of procedural and conceptual knowledge acquisition, aiming to determine the optimal order in which these two types of knowledge should be introduced and developed in mathematics education.

Furthermore, the literature review also delves into mathematics lessons, identifying the conceptual and pedagogical elements that should be embedded within them. It then examines students' difficulties with fraction acquisition and the progression of fraction benchmarks. The literature review continues with a discussion of CoP as well as the historical development of lesson study, tracing its origins and evolution over time. The review seeks to explore how lesson study contributes to teachers' content knowledge and identifies challenges with its implementation.

The literature review concludes with a rationale for catalyzing changes in mathematics instruction. By synthesizing existing research, this literature review aims to enhance understanding of the interplay between procedural and conceptual knowledge in mathematics through lesson study.

I utilized the following keywords and controlled vocabulary to search relevant databases, including Google Scholar, APA PsycInfo, ProQuest, and EBSCOhost, to find literature that addressed the research questions. The keywords used were *educators, pedagogical strategies, procedural knowledge, conceptual knowledge, procedural and conceptual knowledge, lesson study, fractions, teachers' content knowledge for fractions, teachers' professional development, communities of practice, engagement, analysis, and mathematics tasks.* The journals provided valuable insights and research findings related to the research questions.

My inclusion criteria were:

- Empirical studies (both qualitative and quantitative) and literature reviews that discussed or analyzed fraction-based mathematical tasks.
- Studies conducted in an educational context, such as elementary or middle schools.
- Studies specifically focused on tasks designed to teach fractions or examined the relationship between the tasks and students' conceptual understanding or procedural learning outcomes between 2003-2023.
- Studies that discussed teachers' content knowledge for fractions, teachers' professional development, or lesson study.

My exclusion criteria were:

- Studies that did not have a clear research methodology, opinion pieces, and editorial comments.
- Studies that discussed mathematical tasks in general, without a specific focus on fraction-based tasks.

Studies published in languages that I could not translate.

Global Perspective on Mathematics Instruction

Studies conducted internationally highlight a shared commonality: schools that prioritize traditional, procedural instruction hinder students' ability to develop mathematical proficiency, leading to diminished enjoyment of the subject (Boaler, 1998; Chirove & Ogbonnaya, 2021; Özpınar & Arslan, 2021). A three-year ethnographic study comparing the teaching approaches of two schools in the United Kingdom underscores this assertion (Boaler, 1998). The first school utilized a firmly entrenched traditional, procedural approach, with students primarily working individually with textbooks during mathematics classes. Despite an intense focus on exams, the students developed inert knowledge that proved challenging to apply beyond textbook questions. These students struggled to interpret unfamiliar problems and to transfer learned procedures to real-life situations. The overemphasis on procedural knowledge significantly inhibited students' capacity to generalize and apply knowledge to novel situations (Boaler, 1998).

Conversely, students at the second school enjoyed a conceptual approach to mathematics instruction. They were encouraged to work collaboratively on open-ended, authentic projects. They were afforded the opportunity to think about and use mathematics in novel situations. As a result, they believed mathematics involved active and flexible thought and could adapt methods to fit new situations (Boaler, 1998).

An intriguing aspect of this research was the inclusion of firsthand accounts from students, allowing their perspectives to be heard and shedding light on how different teaching methods influenced their learning and application of mathematical concepts. Students at the first school expressed disinterest and a lack of enjoyment in mathematics compared to their counterparts at the second school. These findings underscore the significance of Boaler's research in highlighting the need for teachers to reconsider traditional procedural teaching methods, which can hinder students' retention capabilities and diminish their enjoyment of mathematics.

A study of eleventh graders in South Africa presented more nuanced findings. The study revealed that certain concepts in algebra required procedural instruction as a solid grasp of representations such as symbols, equations, and tables was necessary (Chirove & Ogbonnaya, 2021). Like Boaler's (1998) findings, students with limited procedural knowledge faced challenges in their conceptual understanding. However, a lack of conceptual knowledge proved to be a significant obstacle for students attempting to solve algorithmic tasks. Interestingly, students who demonstrated a strong command of procedural and conceptual knowledge performed better in algebra than those with proficiency in only one type of knowledge.

This research underscores the importance of nurturing both types of knowledge to enhance students' mathematical proficiency and problem-solving abilities. The findings suggest that a balanced approach to teaching, integrating both procedural and conceptual knowledge, is crucial for promoting deeper understanding and more effective problem-solving in algebra (Boaler, 1998).

Examining the instructional practices of teachers in different countries revealed a shared commonality. Eisenhart et al. (1993) investigated teaching styles in southern America, Hussein (2022) studied teaching styles in Kurdistan, Iraq, and Özpınar and Arslan (2021) examined instructional methods in Turkey. While the teachers in these studies recognized the importance of conceptual learning, they often reverted to procedural teaching methods. School testing demands undermined the available instructional time, and the teachers resorted to procedural means to maximize coverage of all mathematics topics. Tensions between procedural and conceptual knowledge were further exacerbated because teachers' evaluations were tied to their student's learning outcomes. Therefore, there was an unconscious but present institutional accountability for using procedural knowledge.

Even though the American, Iraqi, and Turkish teachers were aware of the importance of teaching mathematics conceptually, their pedagogical practices were heavily influenced by procedural knowledge. The reliance on procedural knowledge across different countries suggests an international pattern for this instructional approach, highlighting a crucial area of focus for future pedagogical development.

As this commonality transcends geographical boundaries, it emphasizes a universal trend of elevated tensions between procedural and conceptual knowledge. While procedural knowledge is essential for certain concepts, an excessive emphasis on procedural approaches can impede students' conceptual understanding and ability to apply mathematical concepts in different contexts.

Striking a balance between procedural and conceptual teaching methods is crucial for fostering effective mathematics learning (Byrnes & Wasik, 1991; Eisenhart et al., 1993; Pitsolantis & Osana, 2013; Rittle-Johnson, 2019; Star, 2005). When there is an imbalance between these two knowledge types, the process of learning mathematics can become overwhelming instead of stimulating, potentially hindering students' intellectual growth and overall development in the subject. Thus, a thorough understanding of how these two types of knowledge is categorized, defined, and differentiated in mathematics education is warranted.

Classifications of Procedural and Conceptual Knowledge

Global studies revealed a predominant focus on procedural knowledge in mathematics instruction, often characterizing it as rote memorization and superficial. In the field of educational psychology, conceptual and procedural knowledge is seen as a *type* of knowledge. In contrast, some mathematics scholars define conceptual knowledge and procedural knowledge as the *quality* of knowledge (Star, 2005). Both types of knowledge are interrelated aspects of mathematical understanding that can vary in terms of their quality or depth (Rittle-Johnson, 2019).

The term conceptual refers to the interlinking of concepts and is associated with deep understanding, flexibility, critical judgment, and evaluation (Hiebert, 1986; Star, 2005). Traditionally, conceptual knowledge was regarded as the knowledge of principles, concepts, and definitions (Rittle-Johnson, 2019; Star, 2005). The rich interconnections between concepts could be implicit or explicit and may or may not be verbalized (Rittle-Johnson & Alibali, 1999). Historically, conceptual knowledge was often regarded as profound, comprehensive, and represented the depth of knowledge, while procedural knowledge was often perceived as superficial (Rittle-Johnson et al., 2015; Star, 2005).

Conversely, the term procedural is associated with rote memorization and inflexibility (Hiebert, 1986; Star, 2005). Star (2005) rejected the oversimplification of procedural knowledge in mathematics education, arguing that its association with rote memorization, superficial understanding, and limited depth occurred because procedural knowledge was automated through extensive practice, reinforcing the misconception that it was only a product of memorization. He challenged this perception and argued for a redefinition of procedural knowledge that goes beyond superficial memorization, suggesting that it can encompass deep learning and understanding.

This idea was further supported by researchers Baroody, Feil, and Johnson (2007), who agreed that conceptual knowledge does not necessarily require rich interconnections and can even be disjointed, particularly among novices. A more contemporary perspective suggests that the level of interconnectedness intensifies as expertise in the subject develops, challenging the notion that the richness of connections is an inherent characteristic of conceptual knowledge. An example of this can be seen when students demonstrate conceptual knowledge by stating a definition or principle, but this knowledge may be superficial if the student cannot apply it to solve problems or make connections to other concepts (Baroody et al., 2007; Star, 2005).

Thus, modern definitions acknowledge that procedures are not always automatized but require “conscious selection, reflection, and sequencing of steps” (Rittle-Johnson, 2019, p. 126). Therefore, procedural knowledge has the potential to reach depths of understanding previously underestimated as it employs constructs such as skills, strategies, productions, and interiorized actions (Byrnes & Wasik, 1991). In tasks where the procedures are heuristic, a student’s choice can reflect deep and sophisticated thinking (Star, 2005). The selection and implementation of procedural knowledge necessitate careful consideration, and the verbalization of the problem-solving process can ensure that procedural knowledge is deep. Procedures that are interconnected or embedded with other procedures require thoughtful consideration; this careful application of precise steps alludes to the fact that procedural knowledge can be deep (Baroody et al., 2007).

To substantiate their argument, Star and Stylianides (2013) utilized a fraction task to show how a student could propose novel methods for determining a fraction between two given fractions. Depending on the perspective taken, the task could assess conceptual knowledge, procedural knowledge, neither, or potentially both. Thus, in a departure from traditional views, conceptual knowledge can sometimes appear superficial, while procedural knowledge can possess depth (Schneider & Stern, 2005).

While reform mathematics policies emphasize the integration of conceptual knowledge, there is a lack of consensus in defining, operationalizing, and measuring it, leading to its underutilization in elementary classrooms (Byrnes & Wasik, 1991). In their research, Crooks and Alibali (2014) examined eighty-two articles focused on equivalence, cardinality, and inversion in mathematics. They identified five different types of conceptual knowledge: connection knowledge, general principle knowledge, knowledge of principles underlying procedures, category knowledge/symbol knowledge, and domain structure knowledge. These forms of knowledge can be explicit and verbalizable or more implicit. To address the varied definitions of conceptual knowledge, the authors proposed consolidating it into two branches: general principle knowledge, which involves understanding mathematical ideas independent of specific problems or procedures, and knowledge of principles, which involves connecting concepts to specific procedures.

Skemp (1976) differentiated between instrumental understanding, which is similar to procedural knowledge, and relational understanding, which is more closely related to conceptual knowledge. He argued for the importance of fostering relational understanding in mathematics education. Therefore, a growing number of mathematics researchers are advocating for a broader awareness within the scholarly communities regarding the specific usage and purpose of the terms conceptual and procedural knowledge (Crooks & Alibali, 2014). They advocate for a call to action to attend to the precision of these terminologies, as this can encourage educators or mathematics researchers to develop specific lessons and activities designed to target each concept type. Empirically separating both kinds of knowledge can allow for a nuanced exploration, possibly leading to a more effective integration of procedural and conceptual knowledge in mathematics education.

Empirical Separability of Procedural and Conceptual Knowledge – Two Views

According to Star, while procedural and conceptual knowledge are mutually interdependent, they can be empirically separated (Star, 2005). To accomplish this, measures were developed to test if conceptual and procedural knowledge could be evaluated independently and with sufficient validity (Schneider & Stern, 2005). Four different measures before and after an intervention and a three-step data analysis strategy involving confirmatory factor analyses (CFA) were used to evaluate convergent and divergent validity for pretest and posttest data. Structural equation modeling (SEM) was employed to test different hypotheses concerning the causal relations between the two types of knowledge (Schneider & Stern, 2005).

Results from these experiments suggested that conceptual knowledge could be independently measured with high convergent validity for pretest and posttest data. Procedural knowledge could only be independently measured for posttest but not pretest data. A likely explanation could be that students lacked the necessary procedural knowledge at the pretest. Students' conceptual pretest knowledge can strongly influence their conceptual and procedural posttest knowledge; thus, the results favor the concept-view, especially if students are relatively new to a mathematical domain.

Furthermore, Barzel et al. (2013) adapted Anderson and Krathwohl's (2001) two-dimensional taxonomy, which structures knowledge types according to cognitive processes: explicit verbalization, application, and visual representation to determine if mathematics tasks can be specifically geared either toward conceptual or procedural fraction knowledge.

The taxonomy revealed that the developing separate measures for conceptual and procedural knowledge allowed for a more accurate assessment of students' understanding and skills in each domain. This can assist teachers with identifying students' unique struggles and strengths within this domain. Furthermore, educators can more efficiently design interventions and instructional strategies that specifically target one type of knowledge or the other, which can lead to improved learning outcomes.

However, Baroody et al. (2007) suggest that deep procedural knowledge, including comprehension of a task and algorithmic sequence, cannot exist independent of conceptual knowledge. Similarly, deep conceptual knowledge relies upon the interconnection of knowing the tools for applying and extending mathematical ideas. This shows that deep procedural and conceptual knowledge cannot be separated psychologically as they rely on each other. Baroody et al. (2007) suggest that students need a deep understanding of the underlying concepts to develop their procedural skills. Likewise, it is only effective to teach conceptual understanding by developing procedural skills in tandem. Both deep procedural and conceptual knowledge are necessary for a comprehensive understanding of mathematics. The contribution of their research centers on the emphasis of developing deep procedural and deep conceptual knowledge in mathematics education. This point of view contrasts with Star (2005), who proposed that deep, flexible procedural knowledge can be achieved with or without conceptual knowledge.

Thus, while superficial procedural and conceptual knowledge may exist in isolation, deep procedural knowledge cannot exist without deep conceptual knowledge, and vice versa. This iterative view emphasizes the simultaneous development of conceptual and procedural knowledge, each informing the progress of the other (Baroody et al., 2007; Hurrell, 2021; Rittle-Johnson et al., 2015). This perspective is the foundation of the conceptual framework underpinning this study. Nonetheless, it's important to note that this is one of many viewpoints embraced by mathematics communities; the bi-directional perspective, explored further below, has also garnered favor within scholarly circles.

Balancing Conceptual and Procedural Knowledge in Mathematics

Research has shown that students' learning is influenced by their conceptual knowledge (abstract understanding of principles and relationships) and their procedural knowledge (efficient problem-solving); thus, both procedural and conceptual knowledge are necessary requirements in mathematics education. However, the optimal ordering of this knowledge is a topic for debate.

Rittle-Johnson and Alibali (1999) conducted an empirical study to examine the optimal ordering of procedural and conceptual knowledge. Sixty fourth graders received either conceptual-only instruction or procedural-only instruction in a clinical setting. The results showed that gains in conceptual knowledge led to gains in procedural knowledge. Students who developed their conceptual understanding first tended to have more sophisticated procedural knowledge as they could generate new procedures that could be applied and transferred to novel contexts. However, gains in procedural knowledge also influenced the gains in conceptual knowledge.

In a later study, Rittle-Johnson, Fyfe, and Loehr (2016) evaluated the effects of conceptual-only instruction or conceptual-with-procedural instruction. Their study emphasized the importance and timing of conceptual instruction and the sequence of presenting procedural tasks on a sample of 180 children from thirteen classrooms across U.S. public schools.

The results revealed that students who received two iterations of conceptual instruction gained greater conceptual and procedural knowledge than those who received conceptual and procedural instruction. This study challenged conventional thinking as it showed that providing direct conceptual instruction within a lesson, rather than a comparable amount of time spent on procedural instruction after the conceptual instruction, can promote a more robust understanding of mathematical concepts. However, the reality of modern education is influenced by high-stakes tests that focus primarily on procedures, necessitating the inclusion of procedural tasks in instruction. The authors further acknowledged that it is not enough to present random procedural tasks simultaneously with conceptual tasks; instead, procedural tasks should be presented sequentially; since gains in one can lead to gains in the other, the ordering of procedural tasks matters. The study supports the idea that teaching conceptual and procedural knowledge in tandem, rather than in isolation, is an effective approach to improving students' mathematics learning outcomes (Rittle-Johnson et al., 2016).

These findings have contributed to developing the bidirectional approach in mathematics education (Rittle-Johnson, 2019). The bidirectional approach posits that knowledge acquisition is continuous rather than categorical, allowing for both types of knowledge to develop simultaneously.

Data collected from schools in Pittsburgh, Michigan, and Massachusetts, which rigorously implemented reform mathematics instructions with fidelity, revealed that the reform curricula improved student learning and reduced disparities among different groups of students (Schoenfeld, 2002). Therefore, reform curricula that emphasize conceptual and procedural elements in mathematics tasks have been shown to positively influence the trajectory of student learning. To change the trajectory of student learning, teachers need to alter the types of tasks and their dissemination methods- from strictly procedural to a combination of both conceptual and procedural (Rittle-Johnson et al., 2015).

The importance of developing conceptual knowledge prior to procedural knowledge in mathematics education has been supported by several studies. Rittle-Johnson and Alibali's (1999) study found that students who developed their conceptual understanding first tended to have more sophisticated procedural knowledge, as they could generate new procedures and transfer them to novel contexts. Similarly, Rittle-Johnson, Fyfe, and Loehr's (2016) study revealed that students who received two iterations of conceptual instruction gained greater conceptual and procedural knowledge compared to those who received conceptual and procedural instruction simultaneously. Furthermore, research by Siegler (2002) emphasizes the importance of conceptual understanding in the development of procedural fluency. Hiebert's Site Theory (1984) also supported the notion that conceptual knowledge should precede procedural knowledge as it provides a meaningful context for procedural skills.

However, it is essential to note that these findings still might have possible limitations. For instance, the study conducted by Rittle-Johnson and Alibali (1999) was primarily confined to a laboratory setting, potentially influencing the outcomes. Moreover, their research was primarily focused on fractions, leaving the applicability of their findings to other domains within mathematics in question. Nonetheless, these investigations contribute greatly to the ongoing discourse on the interplay between procedural and conceptual knowledge in mathematics learning and instruction.

Modern literature reveals that procedural and conceptual knowledge are crucial, indispensable components of mathematics learning (Hurrell, 2021), and instruction should target each type of knowledge (Baroody et al., 2007; Hurrell, 2021; Rittle-Johnson et al., 2015; Schneider & Stern, 2005). As students engage in mathematical tasks, their conceptual understanding of the underlying principles and relationships informs their procedural fluency, while their growing procedural skills reinforce and deepen their conceptual grasp. This iterative process allows for continuous refinement and strengthening of both types of knowledge. The synergistic relationship between conceptual understanding and procedural knowledge can be leveraged through transformative approaches such as lesson study to enhance mathematics instruction, specifically in fractions.

Mathematics Lessons

A curriculum reflects an institution's educational goals and objectives and encompasses all the planned learning experiences and instructional strategies. It is a roadmap that guides teachers in designing and delivering instruction (Prideaux, 2003). Within the curriculum lies tasks, which are problems, or "set of problems that address the same mathematical idea" (Boston et al., 2019, p. 9). Within the context of this study, I use the term 'lesson' to refer to the collection of tasks, planned instructional strategies, and tailored student experiences designed with a specific fraction-based educational goal. Thus, the lessons will include both mathematics tasks as well as pedagogical strategies designed to incorporate both conceptual and procedural elements.

Tasks

The type and caliber of tasks presented to students influence their understanding and perception of mathematics (Stein & Smith, 1998). Tasks that consist of rote memorization lead to surface-level engagement. In contrast, tasks that utilize students' prior knowledge and challenge them to think conceptually can offer richer and more in-depth cognitive experiences for students (Hiebert, 1986; Stein & Smith, 1998).

Stein and Smith (1998) proposed that mathematics tasks go through three distinct phases: (1) the task as it appears in the curriculum, (2) the task as it is presented by the teacher, and (3) the task as it is engaged with by the students. They also highlight potential misalignments that can occur at different stages of the task implementation process. Specifically, how a task is initially presented in the curriculum materials, how the teacher ultimately sets it up in the classroom, and how students interpret and engage with it during the learning process.

For example, a task that started with a high cognitive demand might, during its implementation, be reduced to a more routine or simplified version, thereby changing the quality of cognitive engagement. Figure 3, taken from Stein and Smith's (1998) work, represents the misalignment with the shape of a triangle, rather than the intended outcome of a rectangle.

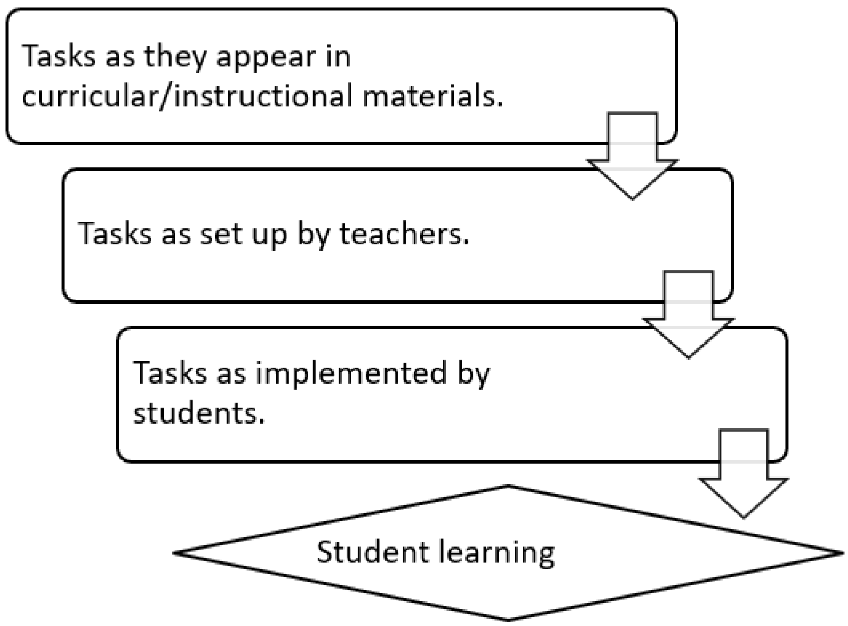

Figure 2 The Mathematics Task Framework

While consistently using high-quality tasks can enhance students' mathematical outcomes, the caliber of these tasks must be evaluated at all levels to ensure that the cognitive demands remain intact (Boston et al., 2019). Based on the ideas of Stein and Smith, Boston (2019) crafted an Instructional Quality Assessment (IQA) Potential of the Task Rubric (Boston et al., 2019) to assess elements of ambitious mathematics instruction, such as the level of instructional tasks, task implementation, opportunities for mathematical discourse, and teachers' expectations. According to the IQA Potential of the Task Rubric a high-quality task will have a Level 4 rating. It will require students to:

- Solve a genuine problem.
- Develop an explanation for why formulas or procedures work.
- Identify patterns.
- Justify generalizations.
- Make conjectures and explicit connections between representations.

The conceptually rich mathematics task is the cumulation of deep mathematical understanding, which calls on students to draw upon diverse strategies, utilize various tools, and access their prior experiences for exploration (Boston et al., 2019; NCTM, 2020). In addition, the tasks should be designed with a flexible structure that allows students to engage with them through multiple entry points, as this ensures access and equity. Similarly, solutions to the tasks should be expressed through various representations (Boston et al., 2019; NCTM, 2020).

High-quality tasks should contribute to developing procedural fluency among students. This fluency goes beyond rote memorization; it involves a deep understanding of procedures and applying them flexibly in various situations (Boston et al., 2019; NCTM, 2020). This flexibility is crucial as it empowers students to adapt their mathematical skills to different contexts, promoting a more comprehensive and adaptable mathematical competence.

The distinction between Level 3 and Level 4 tasks is rooted in the student's capacity to provide explanations, generalizations, or explicit evidence of their thinking (Boston et al., 2019). This intentional act of prompting students to communicate their academic reasoning purposefully is at the heart of enhanced pedagogical strategies.

Pedagogical Strategies

Enhanced pedagogical strategies have the potential to unleash students' conceptual thinking in a myriad of ways. One such pedagogical skill is providing opportunities for students to articulate their thought processes. This helps students solidify their understanding while enabling educators to assess the depth of their knowledge. Another pedagogical strategy occurs when educators help students form connections between isolated segments of information by enabling students to transfer previously acquired academic skills to new and unfamiliar contexts (NCTM, 2020).

Furthermore, pedagogical strategies that are cognizant of students' needs and resonate with their daily lives can also make mathematics instruction applicable and more meaningful to the learners (Boston et al., 2019). This approach demonstrates the practical applicability of fractions in everyday situations and can lead to a more engaging learning environment.

Another critical facet of pedagogical strategies includes crafting and utilizing purposeful questions to guide and scaffold students' cognitive processes. These questions serve as navigational tools, fostering a deeper understanding of the task's underlying concepts and encouraging students to explore different dimensions of their thinking. Thus, using purposeful, higher-order thinking questions during fraction acquisition can encourage students to build their critical reasoning skills while deepening their knowledge base (Huinker & Bill, 2017).

Formulating these types of questions ensures that students can reason quantitively and abstractly and that they construct explanations and justifications while examining the reasoning of others (Huinker & Bill, 2017). These actions can help students to analyze, synthesize, and evaluate fractions from multiple perspectives, thus cultivating a more profound comprehension beyond mere memorization.

Fraction Acquisition in Students and Teachers

Student difficulties with fraction acquisition have persisted over time (Fennell, 2007; Namkung & Fuchs, 2019; Pitsolantis & Osana, 2013). According to the National Assessment of

Educational Progress (NAEP) data, in 2017, 32% of fourth-grade students were able to correctly determine whether given fractions were greater than, less than, or equal to the benchmark fraction $\frac{1}{2}$. Likewise, in 2009, only 25% of fourth-grade students successfully identified the fraction closest to $\frac{1}{2}$ (NAEP, 2022). Difficulties arise in fraction acquisition because many concepts within this domain contradict what students understand about natural numbers (Fazio & Siegler, 2022). For example, with natural numbers, multiplication yields a greater result, while division produces a lesser one. However, this pattern is not true for fractions as multiplication does not always make the answer greater, and division does not always make it lesser. Thus, this pervasively challenging skill demands closer inspection to comprehend the root causes of students' confusion.

One significant factor linked to students' difficulties with fractions is known as whole-number bias (Namkung & Fuchs, 2019; Roesslein & Codding, 2018). This bias occurs when students misconstrue the fraction's magnitude (ratio of numerator to denominator) and instead view the numerator and denominator as independent whole numbers. This misconception leads to a domino effect when performing arithmetic functions such as adding or subtracting fractions; for example, adding $\frac{1}{6}$ and $\frac{5}{6}$ and getting a result of $\frac{6}{12}$ (Namkung & Fuchs, 2019). Similarly, students may believe that $\frac{1}{5}$ is greater than $\frac{1}{4}$ since five is greater than four.

Another factor that adds to students' difficulties within this domain lies in the comparison between fractions and whole numbers (Namkung & Fuchs, 2019; Pitsolantis & Osana, 2013). While whole numbers follow a linear and logical order, fractions possess a unique characteristic: "there is infinite density of fractions in every segment of the number line" (Namkung & Fuchs, 2019, p. 37). This means that between any two fractions, there exists an infinite number of other fractions. Students often rely on their deeply ingrained counting strategies to identify a greater number, as each number in a counting sequence is greater than its predecessor. However, this skill is not applicable when it comes to ordering fractions. As a result, students commonly struggle with the misapplication of whole-number properties when attempting to determine the value of a fraction, leading to errors in their understanding of fraction concepts.

Moreover, introducing symbolic notation like the division sign in fractions too early can exacerbate students' fragile understanding. They may perceive the notation as representing two separate numbers rather than understanding the relationship between the two quantities (Bruce et al., 2023). Moreover, educators tend to instruct their students in the same manner they were taught themselves (Sigler, 2017). If teachers have a superficial understanding of the underlying rationale behind certain procedures (such as 'invert and multiply'), they will be ill-equipped to convincingly explain to students why the procedures work. Consequently, students are more likely to learn by rote memorization (Sigler, 2017). Misconceptions and mistakes can easily happen as students lack a robust understanding of the underlying concepts.

Teachers' professional learning, supported through mediums such as lesson study, has the potential to dramatically alter the trajectory of students' fraction acquisition. Lesson study can help teachers identify and prioritize effective teaching strategies, develop appropriate scaffolding support, create questions that promote students' higher order thinking, and include aspects of their lived experiences within the context of the mathematics tasks (Wiburg & Brown, 2006).

Furthermore, teachers can better facilitate meaningful connections between concepts and practical applications by identifying strategies that resonate with students' cognitive processes and learning styles (Pitsolantis & Osana, 2013; Roesslein & Codding, 2018). One way they can do this is through scaffolding. This refers to providing instructional support to students exactly when they need it during the learning process. Educational psychologist Vygotsky calls this the ‘zone of proximal development (ZPD).’ The ZPD represents tasks that learners can perform with the guidance of a ‘knowledgeable other’ but cannot yet perform independently (Lourenço, 2012). This concept is particularly relevant to the ideas presented in the conceptual frameworks, as it emphasizes the importance of providing appropriate support and scaffolding to students during the learning process. Therefore, crafting just-in-time scaffolds tailored to students’ current abilities encourages them to develop grit and perseverance because they are engaging in cognitively demanding tasks of their own volition and relying on the teachers’ questions as a bridge of support (Dixon, 2020).

Educators can benefit from a structured approach to addressing these challenges. Therefore, understanding the progression of fraction benchmarks or standards is of paramount importance as this provides teachers with a roadmap for planning effective instruction.

Progression of Fraction Standards (Pre- K to 3rd Grade)

The state in which Aspen County schools are located recently introduced new benchmark standards in mathematics (Citation withheld to preserve confidentiality). Students in Pre-K typically have a basic understanding of fractions, as they frequently engage in dividing objects into equal parts (Neagoy, 2017). Consequently, when they start kindergarten, they often possess foundational proportional reasoning skills, such as recognizing that dividing a whole into two equal parts results in halves (Neagoy, 2017, p. 23).

According to the mathematics standards, in kindergarten, students are tasked with naming and identifying two—and three-dimensional shapes like triangles, circles, rectangles, cubes, and spheres. Their proficiency progresses as they start to create composite shapes, such as combining two triangles to make a rectangle (Citation withheld to preserve confidentiality).

Following the vertical alignment of the standards, in Grade 1, students develop an understanding of fractions by partitioning circles and rectangles into halves and fourths. They learn specific vocabulary words as they use manipulatives to build their conceptual understanding.

In Grade 2, students expand their understanding of fractions as they partition shapes into thirds. This skill is further enhanced as students partition rectangles into two, three, or four equal-sized parts to show that equal-sized parts of the same whole may have different shapes. For example, a square cake can be cut into two equal-sized rectangular pieces or two equal-sized triangular pieces. Here, students' mathematical language develops more extensively as they compose and decompose shapes.

In Grade 3, students start to make sense of the meaning of fractions and subsequently start representing fractions as symbols. They develop their conceptual understanding using manipulatives or visual models. Instruction involves understanding that a "unit fraction is a part of a whole, part of a set, a point on a number line, a visual model or a fractional notation" (Citation withheld to reserve the confidentiality of the institution).

In third grade, students also learn that fractions can represent numbers greater than one, and that whole numbers can be written as fractions. They begin to understand that adding unit fractions can produce more than one whole; for example, $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$ which is equivalent to 1 whole and $\frac{1}{3}$. Students are also tasked with representing fractions using numerals, words, and standard notation. At this point in instruction, a connection needs to circle back to the student's familiarity with whole numbers; just like how the number four is built from four units of one, the fraction $\frac{3}{4}$ can be comprised of three $\frac{1}{4}$ units.

The insights gleaned from exploring the vertical alignment of the fraction benchmarks (Pre-K to 3rd Grade) provide valuable insights into students' expected developmental progression. By having a thorough grasp of the critical content and skills students must possess at each grade level, educators can more effectively design and sequence tasks that foster a deep, connected understanding of fractions, ensuring a balanced development of both conceptual and procedural knowledge.

Overview of Communities of Practice

Teaching is not an isolated activity; it thrives on collaboration and shared knowledge, which are the core components permeating this profession. As pioneered by Lave and Wenger (1991), the concept of CoP can offer transformative benefits to teachers by fostering professional development through collaboration. Cultivating these collaborative opportunities can enrich instructional practice and students' education experiences (Fernandez & Yoshida, 2004).

Wenger (1998) identified several key actions necessary for a CoP to thrive. He emphasized that a CoP should be free to develop and grow based on its own needs, interests, and shared experiences. The members should create opportunities where diverse viewpoints and opinions are valued, as this promotes a sense of collaboration, inclusivity, and shared understanding.

In the context of ABC Elementary, CoPs manifest daily as educators engage in informal discussions to reflect on a lesson's implementation, provide targeted pedagogical support to each other, and share expertise in low-stakes settings (Wenger et al., 2002). These collaborative interactions typically existed along friendship lines or geographic locations. In addition, a CoP should accommodate different levels of participation, acknowledging that not all members may be equally active (Wiburg & Brown, 2006).

This is evident at schools where experienced teachers spend time mentoring and coaching pre-service teachers to bolster their skills. Both public and private spaces within the community should be recognized. Public spaces are where knowledge and information are shared openly with the entire community and can include spaces such as the common planning or faculty room. Private spaces refer to individual or small group settings where members can work independently or have more focused conversations, such as the team's planning pod or the individual classroom (Akiba et al., 2019).

Furthermore, a CoP should strive to create an environment where members feel valued, supported, and rewarded for their contributions. A healthy balance of familiarity and excitement should be cultivated to promote active participation and sustain members' interests. CoP should have a consistent and predictable schedule for their activities as this supports the continuous growth and development of the community (Akiba et al., 2019; Fernandez & Yoshida, 2004; Wiburg & Brown, 2006).

A crucial distinction between CoP and their closest counterpart, professional learning communities (PLCs), is that a CoP cannot be imposed upon an existing system but is cultivated over time through shared experiences and mutual engagement. While a PLC is intentionally created within an organization, a CoP emerges naturally as professionals come together to collaborate, share knowledge, and develop a shared identity (Wenger, 1998).

Furthermore, within the PLC at ABC Elementary, the focus lies on holding educators accountable for their students' data and performance. Whereas in a CoP, the focus lies on building educators' knowledge, competence, and innovation (Wenger et al., 2002). Additionally, within a PLC, an expert usually disseminates information by talking to teachers, whereas in a COP, its members work collaboratively to develop knowledge and then execute that knowledge.

In this sense, CoP members take on dual roles: they are both learners and practitioners, working together to develop and implement new ideas and strategies (Wenger, 1998).

A thriving CoP is guided by three essential characteristics: domain, community, and practice (Wiburg & Brown, 2006). At ABC Elementary, the domain represents the teaching profession and, more specifically, the teaching of mathematics strategies related to fractions. The second characteristic is the community, which embodies a group of dedicated educators at ABC Elementary who unite to enhance students' understanding of fractions by developing effective instructional strategies for teaching this concept. The third characteristic is practice, which encompasses the shared activities in which community members engage. This includes joint activities, discussions, and lesson planning sessions to improve teaching skills and share best practices for teaching fractions. Community members develop a shared repertoire of resources, experiences, and tools to address recurring challenges in teaching fractions.

Thus, the CoP comprises teachers with a shared domain of interest in mathematics education and a focus on teaching fractions. These teachers engage in collaborative practices, share experiences, and build a collective understanding of effective teaching strategies to enhance student learning outcomes in this domain. While CoP constituted an integral part of the conceptual framework aimed at fostering the collective sharing of information among teachers, lesson study served as the practical, operational tool that was employed to drive meaningful change at ABC Elementary.

Historicity of Lesson Study

The historicity of lesson study traces its origins back to Japan, where it emerged as a powerful method for improving teaching and learning. The term 'lesson study,' or 'jugyokenkyu' in Japanese, encompasses an iterative process of observing, reflecting, revising, and reteaching a lesson based on pre-determined objectives and outcomes (Wiburg & Brown, 2006). This educational practice gained momentum in Japan during the 1860s as the country transitioned from an agricultural society to an industrial one, necessitating advancements in its educational system. Given limited government funding for extensive reforms, a seemingly simple yet innovative approach emerged: experienced teachers modeling lessons for novice teachers (Wiburg & Brown, 2006).

Ashida, an influential Japanese teacher, played a pivotal role in expanding this idea by incorporating student discourse as a crucial element of lesson study. This introduced a framework for conducting experimental lessons, followed by intensive observations, deep reflections, and a commitment to revision with teachers of varying experience and expertise, all aimed at improving students' outcomes in mathematics (Wiburg & Brown, 2006).

In Japan, lesson study is deeply ingrained in the national mathematics curriculum, with implementation occurring at three levels: school, regional, and national. Specific dates are designated for lesson study, usually at the end of the academic year in February or March, and pre-determined objectives guide the process. At the study's conclusion, artifacts and documents are created to document the lessons learned and guide subsequent presentations.

The emergence of lesson study on the international stage can be traced back to 1999 when it garnered attention in the United States through a video study conducted as part of the Trends in International Mathematics and Science Study. The exceptional performance of Japanese students in mathematics during this international study drew scholars' attention, including Stigler and Hiebert (2009), Fernandez and Yoshida (2004), Olson and Wolner (2017), and Lewis (2005), who conducted their own research and disseminated their findings in English, thus bringing lesson study to a global audience.

Initially, in the United States, PLCs were utilized to facilitate teacher interactions and enhance teachers' understanding of instructional content. However, the effectiveness of these meetings was questioned when administrative tasks overshadowed the essential processes of knowledge dissemination and instructional reconstruction (Wiburg & Brown, 2006). In contrast, lesson study provided a unique approach involving teachers in collaborative action research (Wiburg & Brown, 2006). Consequently, lesson study held great potential as a valuable tool for empowering teachers and fostering meaningful collaboration within a community of practitioners.

Furthermore, traditional PLCs typically do not involve a group of teachers collaboratively collecting data during a live lesson, analyzing student responses, and examining the effectiveness of specific instructional approaches (Akiba & Howard, 2021). In contrast, lesson study provides a structure for teachers to gather real-time data on student learning, collectively analyze this information, and make connections between student outcomes and the instructional strategies employed during the lesson.

Additionally, teachers' participation in the lesson study empowered them to see themselves as lesson developers instead of lesson executors (Akiba & Howard, 2021). Lesson study engaged educators' knowledge and not just imposed 'proven' strategies on them (Lewis & Perry, 2017). Thus, lesson study served as a platform for collaborative lesson planning and professional development among teachers. Figure 3 shows a lesson study flowchart that summarizes the various stages.

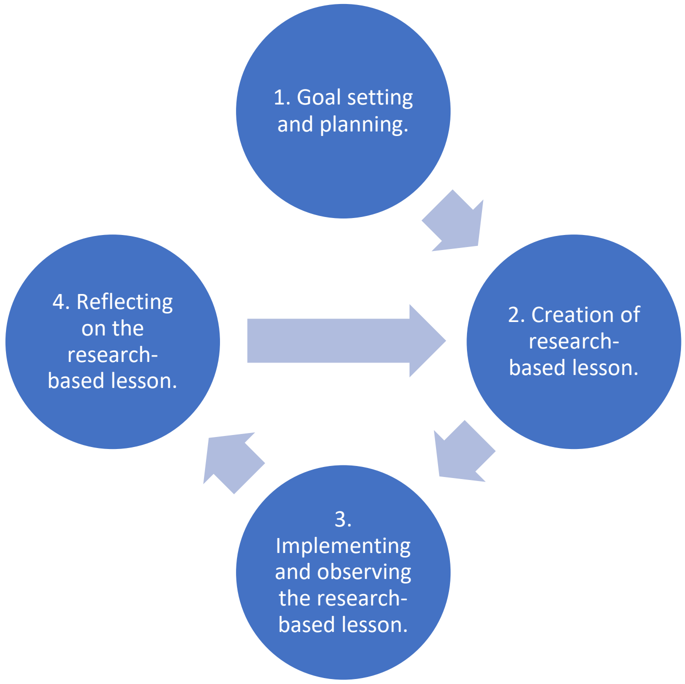

Figure 3 Summary of Lesson Study Process

Challenges in Practicing Lesson Study in the United States

Despite the success of lesson study in the Eastern world, there is limited empirical evidence on the effectiveness of lesson study in the United States. A large-scale empirical study conducted by Lewis and Perry (2014, 2017) found that the sustainability and impact of lesson study heavily depend on the level of support afforded to the lesson study teachers by the school and district authorities (Lewis & Perry, 2017).

The Florida Department of Education (FLDOE) won a competitive grant, securing $700 million through the Race to the Top (RTTT) federal program in 2010 to promote lesson study (Akiba & Wilkinson, 2015). Of 68 school districts, 58 adopted and implemented lesson study on their campuses (Akiba et al., 2019). During the funding period from 2010 to 2014, a total of 434 schools participated in the lesson study initiative (Akiba & Howard, 2021).

However, maintaining the same level of engagement in lesson study proved challenging, as only 250 schools continued with the practice after the funding period concluded in 2016 (Akiba & Howard, 2021). According to a survey conducted among teachers in Florida who participated in lesson study, the average time spent on one lesson study cycle was twelve hours over a period of approximately two months (Akiba & Wilkinson, 2015). Thus, the extensive time requirements of lesson study and the associated costs for the provision of substitutes were major deterrents in the sustainability of the lesson study. In contrast to Japan, where time is dedicated within the school schedule for lesson study, in the United States, lesson study is often included as an additional initiative for teachers to juggle. Within the current system, teachers' work schedules do not allow for the continuous learning process needed to sustain lesson study (Akiba & Wilkinson, 2015).

Another challenge with implementing lesson study in the United States is that many teachers are unfamiliar with the research process involved in studying the curriculum, collecting and interpreting data, and drawing conclusions and implications for teacher and student learning (Akiba & Howard, 2021; Akiba & Wilkinson, 2015). Transitioning teachers into this new role of generating professional knowledge to inform their pedagogical approaches requires the extensive support of leadership coupled with access to adequate resources.

One such resource needs to be the inclusion of knowledgeable experts. Including experts becomes necessary as many teachers may need more opportunities and resources to engage in professional development to deepen their content and pedagogical content knowledge (Akiba & Howard, 2021; Lewis & Perry, 2017). Japanese teachers engage in an average of two lesson study cycles annually and enjoy opportunities to observe various research lessons (Wiburg & Brown, 2006). However, in the United States, opportunities to improve professional development by observing pedagogical experts during a live lesson is the exception rather than the rule (Akiba & Wilkinson, 2015).

Understanding the purpose of lesson study and its' implementations and challenges is beneficial to ensuring its sustainability at ABC Elementary. Using lesson study can create a dynamic environment for collaborative learning, growth, and the exchange of ideas. This is especially important as both teachers and students grapple with fraction acquisition. Therefore, lesson study can potentially support teachers as they re-examine the curriculum and seek to develop their pedagogical strategies within the domain of fractions. This can create more impactful learning experiences for students, thereby catalyzing change.

Catalyzing Change

Nationally and internationally, numerous studies share the commonality of an overemphasis on procedural knowledge in mathematics lessons (Boaler, 1998; Chirove & Ogbonnaya, 2021; Özpınar & Arslan, 2021). This unbalanced approach impedes students' ability to develop a deep understanding of mathematical principles and apply their knowledge to real-world situations. Furthermore, the reliance on rote memorization and repetitive practice in mathematics instruction limits students' critical thinking skills and problem-solving abilities, which are vital in a rapidly evolving world (Boaler, 1998). *Catalyzing Change* (NCTM, 2020) acknowledges that educational reforms "are shifting priorities and moving toward more inquiry-based approaches that build on what is known about how children learn mathematics" (NCTM, 2020, p. 5). These shifts, though, need to be consistently implemented by districts, schools and classroom teachers.

Therefore, it is essential for teachers to align their pedagogical approaches with students' learning needs. This can be achieved by prioritizing students' conceptual understanding. The shift from traditional teacher-centered to student-centered approaches can help close the achievement gap. Thus, by embracing innovative pedagogical strategies such as those recommended by NCTM, students in the United States can be better prepared to compete in and contribute to the global arena.

Transformative change in mathematics instruction in the United States is of paramount importance if we want to maintain our dominance as a world superpower. American students' persistently low mathematics achievement levels necessitate transformative change if we hope to graduate students capable of thriving in this ever-changing world. *Catalyzing Change in Early Childhood and Elementary Mathematics* challenges educational stakeholders to critically examine "systemic approaches that ensure access, equity, and excellence for each and every child" (NCTM, 2020, p. 2). This study accepted this challenge by examining two prevalent issues: the need to ensure that mathematics lessons are robust by integrating conceptual and procedural knowledge and the importance of providing teachers opportunities to deepen their content knowledge and enhance their pedagogical skills through lesson study.

Conclusion

The literature review explored the importance of balancing procedural and conceptual knowledge in mathematics learning and instruction. It discussed their individual significance and then advocated for a purposeful interplay between these knowledge types. Within this literature review, the global prevalence of procedural instruction in mathematics and its subsequent limitations in fostering deep understanding and problem-solving skills among students was highlighted. Furthermore, the classifications of procedural and conceptual knowledge were examined, revealing the need for a more nuanced understanding of these concepts and their interrelationships.

Empirical studies support the iterative view, which emphasizes the simultaneous development of conceptual and procedural knowledge, where each informs the progress of the other. This foundational insight forms the bedrock of this research. This review also investigated the challenges students face when learning fractions. The importance of teachers' content knowledge and pedagogical strategies as a precursor to improving students' learning outcomes was also examined.

The concept of CoP as a framework for fostering collaboration and shared learning among educators, with lesson study serving as a tool for operationalizing meaningful change in mathematics instruction was also discussed. Finally, the review underscored the urgent need for transformative change in mathematics education as advocated by the *Catalyzing Change* framework.

CHAPTER 3: METHODOLOGY

The methodological approach of this research utilized Robert Yin's (2013) single-case study design. This allowed for an in-depth investigation of contemporary phenomena within a real-world context. Although multiple teachers were involved, the study was considered a single-case study because it occurred within a single location, ABC Elementary School. The case study was grounded in principles that allowed for the generation of rich, meaningful insights by employing rigorous analysis. It offered a holistic and in-depth approach to the complex, contemporary issues ABC Elementary teachers face when co-constructing mathematics lessons. In this case study, the case was defined as the fraction-based instructional mathematics lessons. The unit of analysis was the teachers who were engaged in reconstructing the mathematics lessons through lesson study. The term mathematics lessons refer to a curated collection of mathematical tasks and deliberate instructional strategies designed with a specific educational goal in mind.

The data collection techniques employed included:

1. Semi-structured interviews with participants;
2. Observations of the lesson study process, including the planning, implementation, and debriefing stages; and
3. Analysis of participants' written reflections on their experiences and insights gained through the lesson study.

Triangulation was used to increase the trustworthiness of the findings. The data collection methods are discussed in detail later in this chapter. The chapter concludes with an outline of the data analysis strategies to ensure the process was transparent and replicable.

Conceptual Framework Alignment with Case Study

The intersection of the conceptual frameworks underpinned the research and shaped the methodological choices. The *Catalyzing Change* (NCTM, 2020) framework emphasizes the necessity for transformative change in mathematics education by concentrating on equitable structures, expanding the purpose of mathematics, and cultivating deep mathematical thinking. Hiebert's (1984) Site Theory contributed to understanding the dynamic interplay between procedural and conceptual knowledge. His work offers strategies to foster deep mathematical understanding, including:

1. Assigning meaning to mathematical symbols,
2. Using visuals to explain rules,
3. Encouraging students to assess the reasonableness of their answers.

These strategies help bridge the gap between procedural and conceptual knowledge, enabling students to develop a more comprehensive understanding of mathematical concepts. Hiebert's (1984) research provided the content-specific framework for the study, while the *Catalyzing Change* (NCTM, 2020) framework addressed the broader context of mathematics education reform.

The CoP (Wenger, 1998) framework emphasizes the value of collaboration and shared learning among educators. The alignment further corresponded with Yin's (2013) case study design because it enabled the exploration of a phenomenon within a community of teachers—which is a real-life context. Lesson study's iterative cycles of planning, implementation, and reflection aligned well with the case study design. The recurring weekly meetings and teaching-feedback cycles provided multiple data collection points, which is consistent with Yin's (2013) methodology for single case study designs.

The alignment between Yin's case study approach and the conceptual frameworks of *Catalyzing Change* (NCTM,2020), Hiebert's (1984) Site Theory, and Wenger's (1998) CoP contributed to shaping a comprehensive, contextually grounded, and cyclic approach to the pedagogical approaches employed by ABC Elementary's educators when teaching fractions.

Restatement of Research Questions and Propositions

This research focused on exploring how educators use pedagogical strategies to combine procedural and conceptual knowledge in lessons about fractions within the context of lesson study. In line with Yin's (2013) case study methodology, propositions were derived from the conceptual framework to guide the research and ensure a thorough, detailed, and comprehensive investigation of the research questions. The following research questions and propositions guided this study:

Research Question 1:

- How does the engagement of teachers in the creation and analysis of fraction-based mathematics tasks in lesson study influence their instructional decision-making processes?

Proposition 1:

- The iterative nature of lesson study empowers teachers to enhance their instructional practices by fostering a deep understanding of how procedural and conceptual knowledge complement each other in mathematics instruction, thus enabling teachers to refine their pedagogical strategies.

Research Question 2:

- How do teachers construct pedagogical strategies that integrate procedural and conceptual knowledge through lesson study?

  Proposition 2:

- The active involvement of teachers in the lesson study would cultivate a reflective mindset enabling them to develop a repertoire of pedagogical strategies that effectively integrate procedural and conceptual knowledge while positively impacting their content knowledge.

These research questions and propositions highlighted the expected outcomes and provided a focused direction for the study, ensuring a thorough investigation of the role of lesson study in shaping educators' pedagogical strategies for integrating procedural and conceptual knowledge in fraction instruction.

Population

The population consisted of mathematics teachers at ABC Elementary. During the 2023 – 2024 school year, 43 teachers and instructional staff taught mathematics. The teaching positions included kindergarten through 5th grade, Exceptional Student Education (ESE), and interventionists. Approximately 70% of the staff identified as Black, 22% are Hispanic, and 8% are White.

Setting

The school adheres to the prescribed curriculum provided by Aspen County. The administration at ABC Elementary strongly recommends that teachers employ the backward design framework for lesson planning. With students' performance scores in mind, educators aim to leverage data-driven insights for optimized small-group sessions.

To support this data-driven approach, professional development opportunities at ABC Elementary are focused on data analysis trends. Teachers receive training on how to effectively interpret and utilize student performance data to inform their instructional decisions. Therefore, professional development opportunities are solely focused on data analysis trends.

Recruitment

Following approval from the Institutional Review Board (see Appendix A) at the university as well as the school district research office and principal approval, an emailed invitation to participate in this study was sent to the selected instructional staff (see Appendix B). The email outlined each participant's rights, requested confirmation of their interest in participating, and encouraged them to reach out with any questions to the principal investigator (myself). In this study, I had dual roles.

Although I was a third-grade teacher, I actively engaged in the lesson study process alongside my colleagues. Simultaneously, as an observer, I collected data and made observations about the lesson study process and the pedagogical strategies employed by the participants to combine procedural and conceptual knowledge in fraction lessons.

Participants

Within the target population, my sample that agreed to participate consisted of three third-grade teachers, one third-grade interventionist, one fourth-grade teacher, and two mathematics coaches who taught mathematics during the 2023 – 2024 school year. The three third-grade teachers and one interventionist were responsible for teaching mathematics to all third-grade students at ABC Elementary, while the fourth-grade teacher taught mathematics to one class of fourth-grade students. The two mathematics coaches provided support and guidance to all mathematics teachers at the school. I employed convenience sampling to select all the third-grade teachers, interventionist, fourth grade teacher and mathematics coaches (n = 7) because they were accessible, willing to participate in the study, and shared the same context, which was necessary for this study. Although they had varying titles during the lesson study period, all participants served as mathematics teachers in a classroom at ABC Elementary. The characteristics of the participants are shown in Table 2.

Table 2 Characteristics of Participants

Demographics Categories	Statistics *n* = 7
Age	
20 - 30	1
30 – 39	5
40 – 49	1
Gender	
Male	0
Female	7
Educational Attainment	
Bachelor’s degree	4
Master’s degree	3
Years of Teaching	
< 5	1
6 – 10	6
>10 years	0
Ethnicity	
Black	4
White	3
Grade Placement	
Third Grade	4
Fourth Grade	1
Administrative	2

In this study, each participant’s pseudonym and years of service as a classroom teacher are important pieces of information. Pseudonyms were used to protect the participants' identities and maintain confidentiality. The years of service as a classroom teacher provide insight into each participant's level of experience in teaching mathematics. Table 3 reveals this information:

Table 3 Participants' Pseudonyms and Years of Service as Classroom Teachers

Pseudonym of Teacher	Number of Years as a Classroom Teacher
Ms. Emily	1
Ms. Paul	9
Ms. Mark	7
Ms. Lane	7
Ms. Peters	6
Ms. Bob	10
Ms. Deron	8

When discussing the group as a whole, the seven members in the lesson study are collectively referred to as 'participants.' When referring to each individual participant, their respective pseudonyms are used to protect their anonymity.

The inclusion of knowledgeable others or experts is a requirement of the lesson study process (Wiburg & Brown, 2006). Within the group of participants in the lesson study, two teachers were identified as knowledgeable others based on their academic pedigree and professional mathematics education accomplishments: Ms. Bob and Ms. Deron. Both teachers were esteemed alumni of a local university with master's degrees in mathematics education. In addition to teaching students' mathematics at ABC Elementary, one served as a mathematics coach, while the other was a teacher leader. Their expertise and experience in the field of mathematics education made them valuable assets.

The first expert or knowledgeable other, Ms. Deron, was a veteran mathematics coach with three years of service at ABC Elementary. Ms. Deron was an invaluable source of hands-on experience and understood the school's pedagogical practices. The second knowledgeable other or expert – Ms. Bob, was a member of a prestigious doctoral program designed to enhance the expertise and foster leadership qualities for K-8 mathematics teachers.

Both experts taught mathematics for more than eight consecutive years in an elementary school setting, specifically covering third-grade fraction content. Together, the experts served as influential agents of positive change within the academic ecosystem of ABC Elementary as they brought unique insights and perspectives to the group.

Rationale for Adapting District Pacing Guide

The allotted research period for the lesson study at ABC Elementary spanned the first and second semesters of third grade. However, the district's pacing guide had fraction instruction scheduled in the third semester. I chose to adapt the district's pacing guide and introduce fraction instruction earlier than scheduled for several reasons.

First, empirical data from ABC Elementary revealed that third graders struggled with fractions more than any other mathematical concept. Thus, this domain warranted closer examination. I recognized that lesson study constituted a pivotal element of this research; therefore, I seized the opportunity offered by these sessions to facilitate teacher collaboration to address the specific challenges encountered in teaching fractions with the hope of improving students' outcomes.

Even though the unit on fractions fell outside the conventional timing, careful analysis of the prerequisites for third-grade fraction learning revealed that students had already amassed the necessary experiences and foundational knowledge essential for engaging with the benchmark. This meant that, despite the unconventional timing, students were well-prepared to dive into the unit on fractions. By making this adjustment, the research could more effectively investigate the impact of lesson study on fraction instruction and work toward deepening conceptual understanding in this challenging mathematical domain. A robust data collection process was utilized to ensure a rigorous investigation.

Data Collection Procedures

Yin's (2013) case study design includes the triangulation of data from multiple sources, thus allowing for a deep and nuanced understanding of teachers' pedagogical strategies for designing, implementing, analyzing, and reflecting on fraction-based mathematics lessons. In this study, the following data collection tools were used:

Interviews

In Yin's (2013) case study design, interviews serve as a foundational data collection tool as they allow for rich, nuanced information. I developed an interview protocol (see Appendix C) based on the research questions and propositions. These questions aimed to provide insights into the participants' individual experiences, perspectives, and feelings regarding the pedagogical strategies they employed in the lesson study to co-create the mathematics lessons.

Questions one to three aligned with the first research question and focused on how the participants construct pedagogical strategies that align procedural and conceptual knowledge through lesson study. These questions prompted the participants to provide specific examples of the alignment of conceptual and procedural knowledge within mathematics tasks and future approaches they would utilize to ensure continuity with this approach.

Questions four through seven aligned with the second research question and examined how the participants' engagement in analyzing mathematics tasks that incorporated elements of procedural and conceptual concepts through lesson study contributed to their instructional decision-making. These questions delved into the impact of lesson study on instructional decision making, professional development, and overall teaching enhancement in relation to fractions.

The individual interviews took place face-to-face, and the participants permitted it to be audio recorded. Otter.ai was used to transcribe digital files into textual transcripts. The transcripts were rechecked to ensure that the data was consistent with the interview and that nonverbal cues were included in my field notes. The interviews required approximately one hour of participants' time, and they received no compensation for their participation.

Artifacts

Yin (2013) recognized artifacts as a potential source of evidence in case studies and highlighted the importance of using them to corroborate findings and enhance the study's validity. Artifacts were tangible objects or documents that held educational significance to participants' instructional strategies. Artifacts included lesson plans; to aid in documenting the lesson, the participants were supplied with a lesson template (see Appendix D). In this template, they were expected to record their revised mathematics lesson. The template was provided to the participants with the understanding that they could modify it as needed during the process of creating the lesson. The co-created lesson plans allowed me to trace changes in instructional strategies over time and provided insights into how teachers' and participants' reported strategies evolved through the lesson study.

Another artifact I created was a 3-2-1 reflection template (see Appendix F). At the end of each lesson study session, the participants were asked to respond in writing, listing three things they learned during the lesson study, two ways it affected their teaching, and one new thing they would like to try. This open-ended written reflection elicited the participants' experiences with the lesson study and provided insights into how they reflected on their instruction and planned to improve.

Other examples of artifacts included classroom materials or manipulatives used to teach fractions. The district-issued curriculum provided detailed information about fraction-based mathematical lessons as well as expected instructional strategies. It illuminated the broader educational framework within which the participants operate. This background was essential to understanding why specific pedagogical approaches were adopted.

Observations

Observations are another component of Yin's (2013) case study design. Observations aided in documenting actual behavior rather than self-reported behavior, which may sometimes be subjected to bias. The richness and depth of data derived from observations of the participants interactions during the lesson study allowed me to explore and gather information in a natural setting. Based on the study's research questions, I created two observation protocols that served as data collection tools.

The Researcher's Observation Protocol (see Appendix F) I created allowed me opportunities to collect real-time data and understand the context in which the participants operated while they co-created mathematics lessons during the lesson study framework. This protocol consisted of several components:

1. Pre-Observation Details: This section captured the date, time, location, participants' names, and the lesson study stage being observed.
2. Use of Resources: This section focused on how the participants incorporated materials, manipulatives, and technology to support the teaching and learning of fractions.

3. Observation Focus Areas: This section included two main areas of focus - *Lesson Planning* (how the participants constructed pedagogical strategies aligning procedural and conceptual knowledge in lesson study) and *Lesson Reflection* (insights or realizations expressed by the participants about the fraction tasks).
4. Field Notes: This section allowed me to record any additional observations or notes during the lesson study.
5. Next Steps: This section provided space for me to note any follow-up actions or considerations for future lesson study sessions.

The Participant's Observation Protocol (see Appendix G), which I created, enabled the participants to record their observations about the lesson's implementation. It helped the participants concentrate on the execution and strategies within the lesson enactment. It ensured that the participants' attention was on the implementation of the lesson's content and progression rather than assessing the performance of the teacher delivering it. This protocol also served as a valuable tool during the subsequent debrief, assisting the participants in recalling and reflecting on their observations. The Participant's Observation Protocol consisted of several components:

1. Observation Details: This section captured the date, time, location, the participant's name, and the name of the lesson being observed.
2. Use of Resources: This section focused on the materials, manipulatives, and technology used to support the teaching and learning of fractions.
3. Observation Focus Areas: This section included four main areas of focus - *Lesson Implementation* (the goal of students assessing the reasonableness of their solutions), *Lesson Reflection* (insights about the fraction tasks), *Student Errors*

(any observed student errors), and *Questions to Elicit Connections* (questions that promoted a deeper understanding of fractions).

4. Noticing and Wonderings: This section allowed the participants to record any additional observations, thoughts, or questions that arose during the lesson implementation.

These observation protocols served as structured tools to guide data collection. They ensured that both the participants and I focused on key aspects of the lesson study process, particularly in relation to the teaching and learning of fractions. Furthermore, I utilized audio recording during the lesson study sessions to ensure that all dialogue was captured when the participants discussed their pedagogical and instructional decision-making processes with their peers.

I shared my post-observation field notes with the participants as a basis for subsequent reflection and dialogue. This process enabled the participants to further elaborate on their reported instructional strategies. This cycle of observation, documentation, and subsequent discussion created an iterative process of reflection and learning, aligning with Yin's (2013) methodology for case study design.

Data Analysis

Overall, data analysis in a case study involves detailed description, analysis, and interpretation (Creswell, 2017). Yin's (2013) case study methodology emphasized a rigorous approach to data analysis, focusing on patterns, connections, and theoretical insights. My analysis journey commenced by using deductive coding based on the existing literature and theories to identify key concepts, phrases, or categories relevant to both research questions. The a priori codes served as a structured starting point for data analysis by ensuring focused attention to the specific aspects of the data I wanted to explore. Within Microsoft Excel, I created two tables to organize the a priori codes data: the first column represented the priori codes, and the second column defined its meaning (see Appendix H). This tabular representation provided a systematic approach to understanding and analyzing the initial data.

I utilized my set of pre-determined codes (for example ‘pedagogical insights’ and ‘task-adaption’) and combed through the dataset with a focus on identifying segments that resonated with these pre-determined codes (Creswell, 2018). This approach enabled a progression from the a priori codes to the broader themes and patterns that emerged during the analysis process. I then started to discern patterns and recurring themes within the codes. This process facilitated the identification of similarities and connections across the different data segments, which bolstered the validation of my conceptual framework.

Subsequent steps involved cross-referencing my initial propositions with the identified findings to corroborate or challenge them. To ensure the robustness of my conclusions, I validated the outcomes through triangulation – a process involving the comparison of findings across various data sources (Yin, 2013). I did this by comparing my field notes to the participants’ written reflections and interview answers.

This meticulous approach further bolstered the credibility of my conclusions. Finally, I reflected on my assumptions and potential biases that might have influenced the analysis. I engaged in member checking by sharing my preliminary findings with the participants to ensure the accuracy and authenticity of the interpretations (Creswell, 2018; Yin, 2013). Figure 4 outlines the data analysis process.

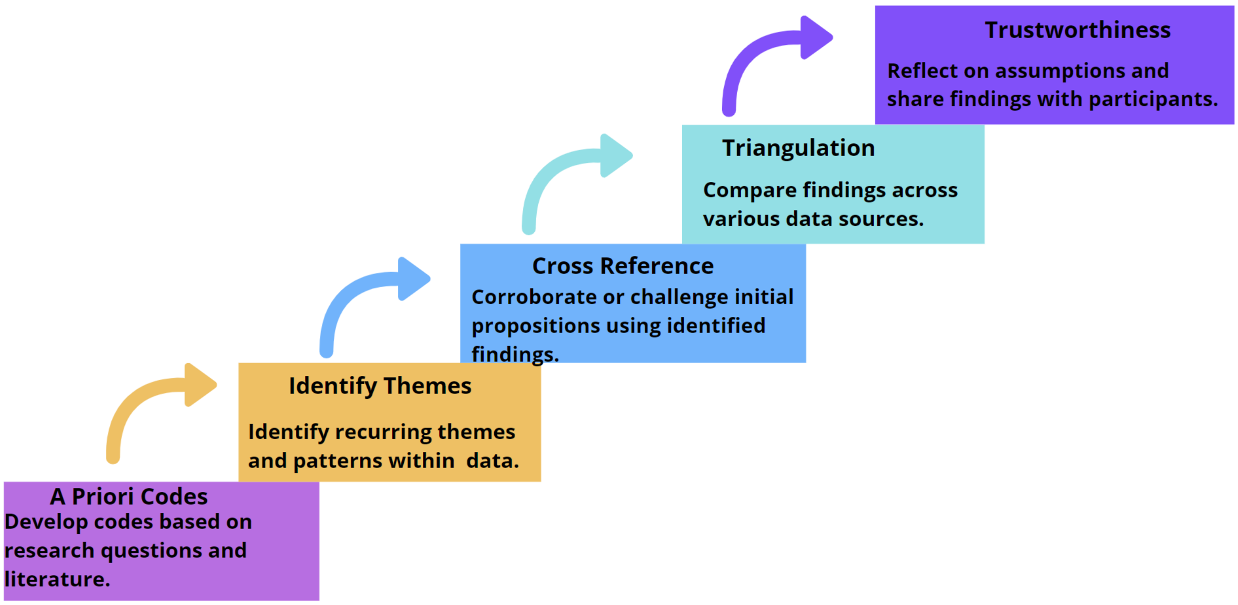

Figure 4 Data Analysis Process

Lesson Study Overview

To gain a better sense of the study, a more detailed explanation of the lesson study is warranted. The lesson study process involved several structured stages aimed at improving mathematics instruction and enhancing the participants' collaboration. The initial research plan served as a blueprint to guide their collaborative efforts. However, classroom interactions cannot be precisely pre-planned, and some degree of flexibility was warranted due to the unpredictability of live lessons.

Thus, within the lesson study, participants were afforded the autonomy to respond to real-time feedback and evolving scenarios. This adaptability empowered the participants to tailor the research-based lessons to best meet the learning objectives of the lesson study. Therefore, while the initial plan provided a structured approach, it was created with the understanding that adjustments and modifications would be necessary based on the evolving needs of the participants and students. The deviations from the initial plan, along with the justifications, are closely examined in Chapter 4.

Initial Research Plan for Each Stage of Lesson Study

Stage 1 – Developing students' learning goals and lesson study objectives.

The lesson study process began with the knowledgeable others establishing norms, expectations, and roles. I furnished the participants with pertinent literature from *Catalyzing Change* (NCTM, 2020), Hiebert's (1984) Site Theory, and Wenger's (1998) CoP models (see Figure 2). Specifically, the recommendations from *Catalyzing Change* (NCTM, 2020) outlined three research-based practices for effective mathematics instruction centered on conceptual understanding and equitable access. Hiebert's (1984) Site Theory detailed the process of linking procedural and conceptual knowledge by utilizing three distinct sites. The CoP models (Wenger, 1998) explained the collaborative nature of lesson study. These carefully curated reading materials underscored the role of academic discourse, justification, multiple representations, identification of early conceptions, and differentiating instruction to develop conceptual knowledge before engaging in procedural fluency.

In a traditional lesson study, the lesson study's goals and objectives are created jointly by the participants. However, due to time restraints and this research study's focus, I supplied the objective of the lesson study, which was to develop mathematics lessons to improve students' learning of fractions through the integration of conceptual and procedural components. I also supplied the overarching, broader goal as required by a lesson study, which was to encourage students to assess the reasonableness of their solutions through academic dialogue. The participants then delved into the district-issued curriculum fraction-based mathematics lessons to discuss modifications.

Stage 2 – Developing and engaging in the research-based lesson plan.

During this stage, the knowledgeable others collaboratively interacted with the participants to further dissect and examine the existing curriculum mathematics tasks on fractions. The participants focused on the hands-on activities, the anticipated students' responses and errors, instructor strategies, and evaluative questions. Through collective inquiry, the participants explored the nuanced interpretations of conceptual and procedural knowledge. These actions resulted in the development of two new lessons: *Lesson A* and *Lesson B* (see Appendixes I and J).

In this study, the term lesson refers to the curated mathematics tasks, deliberate instructional strategies, and tailored student experiences, which the participants designed for the specific educational benchmark, "*Represent and interpret unit fractions in the form 1/n as the quantity formed by one part when a whole is partitioned into n equal parts"* (Citation withheld to preserve confidentiality). These lessons were created with the understanding that further modifications based on students' interactions might be warranted. Central to the lesson study process was the integration of both conceptual and procedural elements within the lessons.

Transitioning from the collaborative phase, the participants then shifted their perspective to mirror that of their students. This technique is rooted in constructivist learning theories. The works of Lev Vygotsky and Jean Piaget underscore the idea that learning through social interactions and collaboration enables learners to actively construct and expand their knowledge (Lourenço, 2012). Thus, by experiencing the learning tasks as students, the participants reinforced their understanding of these principles and could better incorporate them into their teaching practices.

Furthermore, educational theorist John Dewey posited that when children's interests are piqued, they learn more effectively (Dewey, 1913). Hence, by immersing themselves in the student experience, the participants ensured that the lessons embedded concepts and activities that could captivate and engage the learners.

Employing this approach allowed the participants to critically evaluate *Lessons A* and *B* from the students' vantage points and helped them to notice elements of the lesson that they might have overlooked during the planning process. The participants focused on identifying effective strategies, crafting suitable scaffolding support, formulating higher-order thinking questions, and selecting relevant contexts that resonated with their students' real-life experiences. Additionally, they incorporated manipulatives to augment students' comprehension of fractions. Thus, assuming the students' perspectives was a way to identify any gaps in either the lesson's learning goals or activities.

My primary focus as the researcher was to observe how the participants interacted. I also participated in the co-creation of the mathematics lesson as needed. Participants ended each session by completing a written reflection on their experiences with the lesson study. My field notes, coupled with the participants' reflections, were later transcribed and analyzed to identify trends in the data.

Stage 3 – Implementing and observing the research-based lesson.

The third and fourth stages of this process were iterative in the sense that we cycled through the process for each of the expert teachers. Prior to the lesson's implementation, it was decided that Ms. Bob would teach two iterations of *Lesson A* and Ms. Deron would teach two iterations of *Lesson B*. The other participants would observe how the students interacted with the lessons regarding the study objectives and goals and document their feedback in the observation protocol.

Stage 4 – Reflecting and improving the research-based lesson.

After *Lesson A* implementation, a debrief session followed where the participants met to share their findings and observations. Based on the recommendations and insights, *Lesson A* (see Appendix I) was improved and renamed *Lesson A -Version 2* (see Appendix K). Participants ended the session by completing a written reflection on their experiences with the lesson study.

A few days later, Ms. Bob taught *Lesson A -Version 2* to a new group of students while the other participants observed and documented their thoughts on Participant's Observation Protocol. The participants then reconvened to debrief and discuss how the modifications had impacted the second iteration *of Lesson A Version 2.*

The lesson study process repeated Stages 3 and 4 with Ms. Bob teaching *Lesson B* (see Appendix J); this process was informed by the decisions and modifications that were associated with *Lesson A Version 2*. In essence, the lesson study process for *Lesson B* was different from *Lesson A* in that the first iteration was already a revised lesson. Participants ended the lesson study process by completing a written reflection on their experiences with the lesson study.

In summary, in Stage 1 of the initial plan, norms and roles were determined. I presented information about the *Catalyzing Change* (NCTM, 2020) framework and Hiebert's (1984) Site Theory. The objectives and goals for the lesson study were established. In Stage 2, the participants focused on examining existing mathematics tasks housed within the prescribed curriculum. They restructured those tasks so that they included the integration of procedural and conceptual components.

In Stage 3, the expert teachers implemented iterations of *Lessons A* and *Lesson B*, while the other participants observed the live lessons and recorded evidence of student learning on their observation protocol. In Stage 4, the participants reflected on the lesson and collaborated to make improvements in the subsequent iterations of the lessons. Figure 5 was informed by Wenger's (1998) work and provides an overview of the lesson study procedure.

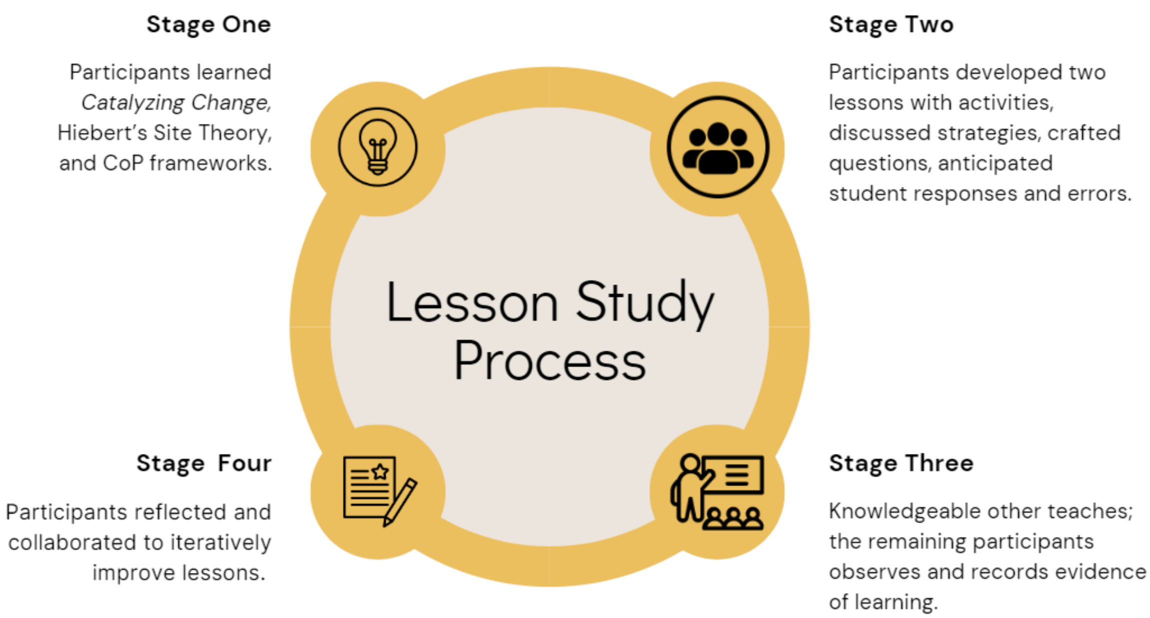

Figure 5 Overview of Lesson Study Procedure

<u>Initial Timeline for Data Collection</u>

Case study involves systematic collection of data, scheduling of dates and corresponding activities for the data collection and analysis processes to ensure a structured approach to gathering and examining the relevant information (Yin, 2013). The timeline for data collection and subsequent analysis of the data commenced during the fall of 2023 and concluded in the spring of 2024. Table 4 contains the initial planned dates and scheduled activities for data collection.

Table 4 Initial Data Collection Plan

Date Range (September to December)	Scheduled Activity
September	Institutional Review Board (IRB) and district research approval. Formal emails eliciting participants' engagement.
September Meeting 1	Stage 1: Discussion on *Catalyzing Change*, Hiebert and CoP conceptual frameworks; development of a student learning goal and lesson study objective; examination and discussion of curriculum. Stage 2: Develop two lesson plans, engage with tasks and lessons as students, and complete written reflection.
September Meeting 2	Stage 3: Ms. Deron teaches *Lesson A* to Class 1; observation by other team members.
October Meeting 3	Stage 4: Debriefing on *Lesson A*; modifications lead to the creation of *Lesson A Version 2*.
October Meeting 4	Stage 3: Ms. Deron teaches *Lesson A Version 2* to Class 2; observation by other participants.
October Meeting 5	Stage 4: Debriefing on *Lesson A Version 2*; enhancements to create *Lesson B* based on recommendations
November Meeting 6	Stage 3: Ms. Bob teaches *Lesson B* to Class 3; observations by other participants.
November Meeting 7	Stage 4: Debriefing on *Lesson B*; modifications lead to the creation of *Lesson B Version 2*.
November Meeting 8	Stage 3: Ms. Bob teaches *Lesson B Version 2* to Class 4; observation by other participants.
December Meeting 9	Stage 4: Debriefing and recommendations for *Lesson B Version 2*.
December Meeting 10	Individual, face-to-face interviews took place with each participant. The data collection concluded.

Role of Researcher/Positionality

Reflexivity refers to a researcher's conscious recognition and open dialogue regarding their position within the study in a manner that pays due regard to the research site and participants (Creswell, 2017). Primarily, I served as a researcher. My duties were anchored in the principles of objective inquiry, adhering to the methodologies ascribed by Yin's (2013) case study design. In this capacity, I focused on the careful collection and thoughtful data analysis.

Simultaneously, I held an insider role as a third-grade teacher at the same school where the study was conducted. This position imbued me with an intimate understanding of the school environment, the educational dynamics, and the unique challenges inherent in modifying curriculum to align with our students' contextual understanding. This proximity to the field of study offered a depth of contextual understanding, which enabled me to interpret findings with a nuanced familiarity that might otherwise be inaccessible to an external researcher.

Additionally, I graduated from the same master's program as the two expert teachers. In navigating these roles, I remained mindful of the potential overlaps and intersections, carefully balancing objectivity with insight and ensuring that each role enriched rather than compromised the other. The various capacities I served throughout this research - as a third-grade teacher, an expert mathematics teacher, and a researcher - all coalesced to provide a holistic, informed, and comprehensive approach to this study.

However, I recognized that my multiple roles could potentially influence the study's findings due to my personal and professional involvement in the setting. To ensure the integrity and validity of the study, I committed to implementing rigorous data collection and analysis methods, as Yin's (2013) case study design outlined. I continually reflected on my assumptions, actions, and impact on the research to mitigate any potential bias and enhance credibility.

I maintained transparency in my role and relationship with the participants, ensuring they were aware of my multiple roles as a fellow teacher, expert mathematics teacher, and researcher. The ethical considerations of this research were strictly adhered to, and all participant information was kept confidential. All participants were informed about the purpose of the study, the procedures involved, and their rights, including the right to withdraw at any time.

Reliability

Yin defines reliability as "the consistency and repeatability of the research procedures used in a case study" (Yin, 2008, p. 240). I strictly adhered to Yin's (2013) case study design framework, which emphasized a well-constructed and rigorously followed protocol. The data collection procedures and tools were maintained consistently, ensuring that each participant's interactions were handled in the same meticulous manner. This promoted the reliability of the research by minimizing the chances of variability in the data collection process.

Furthermore, Yin (2013) emphasized the importance of establishing a case study database to improve reliability. I created a systematic database to store and organize the research data. This included interview transcripts, observation notes, and documents, all meticulously cataloged and cross-referenced. Each step taken in the collection, storage, and analysis of data was carefully documented to ensure the transparency of the process, allowing other researchers to follow the same steps and arrive at similar results, thus enhancing the reliability of this study.

Trustworthiness and Validity

To ensure the trustworthiness and validity of this qualitative case study, I employed a combination of strategies. Triangulation of different data sources, such as interviews, observations, and document/artifact analysis, increased the robustness and validity of the findings (Yin, 2013). This involved comparing and contrasting data from multiple sources to cross-validate the findings.

I engaged in member-checking with the participants by presenting initial findings to them. This process validated their perspectives, minimizing the risk of misrepresentation and enhancing the alignment of their experiences with the conclusions drawn from the data. Member checks played a significant role in maintaining the study's authenticity.

Furthermore, I regularly consulted with peers and mentors, particularly my academic advisor, who added an external perspective to the research process (Creswell & Miller, 2000). These discussions enabled critical feedback, which validated my interpretations and refined the conclusions.

To ensure full transparency, I linked findings to specific theoretical and conceptual evidence throughout the data collection and analysis process (Yin, 2013). This approach strengthened the credibility and dependability of the study's outcomes.

Limitations

While this case study design provided valuable insights into the effects of lesson study on the participants' pedagogical strategies for integrating procedural and conceptual knowledge in fraction instruction, it is essential to acknowledge the study's limitations. One limitation is my positionality. As a third-grade teacher and a graduate of the same master's program as the two knowledgeable others, my multifaceted roles might have influenced the findings due to my personal and professional affiliations. It is important to acknowledge that through peer-checking and data triangulation, I actively strived to eliminate or reduce these limitations.

Another limitation stemmed from the specific timeframe for data collection (September to December). This short timeframe may not have captured the long-term changes that the lesson study evoked.

Furthermore, this study relied heavily on interviews and accounts from self-reflection. With self-reported data, participants can intentionally or unintentionally provide biased data.

Delimitations

To maintain a focused and feasible research design, this case study has several delimitations that should be considered when interpreting the findings. The study is limited to a single elementary school (ABC Elementary) in a specific school district (Aspen County).

The study focused on a specific group of participants: three third-grade teachers, one third-grade interventionist, one fourth-grade teacher, and two mathematics coaches. These participants were selected using convenience sampling, which may not be representative of the larger population of elementary school teachers. Furthermore, the study focused on a single case (the fraction-based instructional mathematics lessons within the district-issued curriculum) at ABC Elementary, which allowed for an in-depth investigation but may have limited the applicability of the findings to other cases or settings.

Conclusion

The purpose of this study was to investigate how the participants co-constructed and implemented mathematical lessons that align procedural and conceptual knowledge of fractions and to explore how their engagement in lesson study influenced their instructional decision-making. The study was grounded in the conceptual frameworks of *Catalyzing Change* (NCTM, 2020), Hiebert's (1984) Site Theory and CoP (Wenger, 1998), which collectively provided a structured, iterative approach to problem-solving and emphasized the value of collaboration and shared learning among educators.

The study employed Yin's (2013) single-case study design, focusing on a group of seven participants at ABC Elementary School. The lesson study process involved several structured stages aimed at improving mathematics instruction and enhancing participants' collaboration. Participants engaged in developing students' learning goals, creating and modifying lesson plans, implementing and observing research-based lessons, and reflecting on and improving the lessons based on their observations and insights.

Data collection methods included semi-structured interviews, observations of the lesson study process, and analysis of artifacts such as lesson plans and participants' reflections. The data analysis process involved deductive coding based on existing literature and theories, identifying patterns and themes, and cross-referencing initial propositions with the findings. The research findings are disseminated in Chapter 4.

CHAPTER 4: RESULTS

The challenges facing students' acquisition of fraction knowledge have been widely documented (Bruce et al., 2023; Charalambous & Pitta-Pantazi, 2006; Namkung & Fuchs, 2019). One possible cause that exacerbates students' difficulties within this domain is when mathematical instruction prioritizes procedural skills over conceptual understanding, which hinders students' grasp of fractions (Hiebert, 1984). Consequently, to investigate how the participants' engagement in lesson study influences their instructional decision-making and construction of pedagogical strategies that integrate procedural and conceptual knowledge in fraction-based mathematics lessons, this study employed an exploratory case study approach following Yin's (2013) methodology. The following research questions were investigated:

- How does the engagement of teachers in the creation and analysis of fraction-based mathematics lessons in lesson study influence their instructional decision-making processes?
- How do teachers construct pedagogical strategies that integrate procedural and conceptual knowledge through lesson study?

To answer these questions, I launched a lesson study initiative at ABC Elementary to explore its impact on the participants' pedagogy and capacity to cultivate a deeper understanding of fractions. Specifically, the lesson study investigated how these participants adapted mathematics lessons to incorporate conceptual components integrated with procedural elements related to fractions.

As an observer, I meticulously documented the pedagogical transformations and evolving strategies employed by the participants throughout the collaborative lesson study process. This enabled direct observation of the case in its natural context, as recommended for case study research by Yin (2013). After completing all observations from across all four stages of the lesson study, I engaged in one-on-one interviews with each participant.

Conducting these interviews was a crucial step in Yin's (2013) case study methodology and proved to be a valuable tool for getting insights and perspectives from the participating teachers on how the lesson study process influenced their teaching methods and practices. Upon completion of the interviews, my next step was to triangulate the data by analyzing the information collected from my own observations during the lesson study stages, the data gathered from the one-on-one interviews, and the transcripts of the lesson study sessions and discussions. The predetermined codes, which were derived from the research questions and the conceptual framework, served as a lens through which I analyzed the triangulated data.

This chapter begins by explaining the a priori codes I created. Later in the chapter, I share my observations and results of the participants' instructional actions across the four stages of the lesson study cycle. Stages 1 and 2 focus on the development of the research-based lessons, while Stages 3 and 4 revolve around the implementation and revision of the lessons. Due to the iterative nature of Stages 3 and 4, my observations and the results of the participants' actions are interwoven in this section, as these two stages deeply inform each other. The chapter concludes by identifying trends from the a priori codes. These trends led to the discovery of two critical overarching themes.

Utilization of A Priori Codes

Employing the case study methodology, these pre-determined codes (see Appendix H) served as an analytical lens through which the data collected during the lesson study could be scrutinized for patterns, themes, and insights related to the participants' practices and instructional strategies in teaching fractions. The pre-determined codes were:

- *Lesson Study*
- *Feedback Integration*
- *Reflective Practice*
- *Shift in Teachers' Thinking*
- *Pedagogical Adaptation*
- *Procedural-Conceptual Integration*
- *Student-Centered Considerations*
- *Real-World Connections*
- *Utilizing Visual Representations.*
- *Task Adaptation*
- *Promoting Deep Mathematical Understanding*

As I analyzed the data using the pre-determined codes, I noticed some codes were interrelated. Specifically, the *Lesson Study* caused the participants to be more attentive to the *Feedback Integration* generated from observing their students' interactions with the mathematics lessons. This insightful feedback was thoroughly discussed, fostering a culture of *Reflective Practice*. The reflection catalyzed a *Shift in Teachers' Thinking* about the design of mathematics lessons. This ideological shift consequently influenced the *Pedagogical Adaptations* they made. I grouped these four interconnecting codes under the umbrella category of *Teacher Development Codes.*

Furthermore, the participants' focus on *Procedural – Conceptual Integration* was deeply influenced by *Student-Centered Considerations*, which in turn were informed by *Real-World Connections*. This included *Utilizing Visual Representations*, which led to *Task Adaptations* that promoted *Deep Mathematical Thinking*. The interrelated codes illuminated how lesson study catalyzed shifts in the participants' beliefs and practices toward more student-centered, conceptually grounded instruction aimed at developing meaningful mathematical understanding. I referred to this group of codes as *Student-Focused Instructional Shift Codes.* Figure 6 summarizes the codes for *Teacher Development* and *Student-Focused Instructional Shift.*

Figure 6 Summary of Codes

The *Teacher Development Codes* encapsulated the transformation in participants' approaches, particularly the processes, outcomes, and impacts of their instructional decisions regarding introducing conceptual fraction-based mathematics lessons. The *Student-Focused Instructional Shift Codes* focused on the shift towards more student-centered pedagogical strategies that the teachers developed through the lesson study process. These codes allowed for a focused and purposeful analysis of the data, ensuring that the findings were relevant and pertinent to the research objectives. With this framework in place, I embarked on Stage 1 of the lesson study.

Observations of Stage 1: Developing students' learning goals and lesson study objectives.

This stage of the lesson study took place with the help of administrative support and spanned approximately two hours after school. In Stage 1, I shared the roles and responsibilities needed for the productive functioning of the group. These roles included:

- A facilitator who leads the group through the creation of the lesson plan and actively monitors the group norms.
- A notetaker who records and uploads the lessons into the lesson template.
- A timekeeper who keeps the participants on track and transitions them from one activity to the next.

Ms. Deron volunteered to be the facilitator, Ms. Emily volunteered to be the notetaker, and Ms. Lane volunteered as the timekeeper. Ms. Bob, with the support of the other participants, established the norms and expectations. These norms included:

- Be present.
- Be open to new ideas.
- Be respectful of each other.

I then furnished the participants with pertinent literature from *Catalyzing Change* (NCTM, 2020) and outlined three research-based practices for effective mathematics instruction centered on conceptual understanding and equitable access. I also shared Hiebert's (1984) Site Theory, which details the process of linking procedural and conceptual knowledge by utilizing three distinct sites. The participants also learned about the fundamentals of the CoP models

(Wenger, 1998), given the collaborative nature of lesson study. These carefully curated reading materials underscored the role of academic discourse, justification, multiple representations, identification of early conceptions, and differentiating instruction to develop conceptual knowledge and procedural fluency.

Unlike a traditional lesson study where the participants collectively determined objectives, our process was adapted due to rigid scheduling constraints. With limited available times that worked for all participants, efficiency became paramount. Therefore, given my research focus, I supplied the learning objective, which was centered on developing fraction lessons by integrating conceptual and procedural elements. I also provided the participants with the broader goal of encouraging students to assess the reasonableness of their solutions through academic discourse.

The participants, with the exception of the knowledgeable others, shared that they initially thought the lesson study was a way to build interesting ‘cool activities’ into the mathematics lessons. I challenged them to “reconceptualize lessons in terms of what they wanted students to understand, how they would assess this understanding, and finally, which procedures or activities they might use to lead the students towards understanding” (Wiburg & Brown, 2006, p. 11). The participants, excluding the knowledgeable others, shared that they did not think of themselves as lesson designers, as they typically implemented curricula developed by an outside expert.

Ms. Emily, Ms. Peters, Ms. Lane, and Ms. Mark originally believed that they needed to follow the district-issued curriculum rigidly, believing that every slide in the PowerPoint lesson needed to be covered. They admitted that their focus was on completing the entirety of the lesson (quantity) and not necessarily on ensuring students developed a conceptual understanding of the content (quality). These participants seemed uncomfortable with deviating from scripted lesson plans and were hesitant and skeptical of modifying the standardized mathematics tasks.

In fact, at the start of the lesson study, Ms. Peters questioned, *"Why upset the cart? With everything teachers have to do, why change the lesson? Isn't that just adding more work for us?"* Ms. Lane expressed similar wariness about altering the established curriculum, stating, *"An expert made that curriculum, so why bother changing it?"* This question was immediately addressed by Ms. Deron, a knowledgeable other, who shared, *"By adapting lessons to meet the conceptual needs of our students, we can help close the achievement gaps at our school."* Her statement seemed to spur the participants to adopt a more open mindset as the conversation changed from questioning why the alterations to the mathematical content were needed to considering what content needed to be changed and the rationale for that change.

For this study, the participants focused on the following mathematics benchmark: "*Represent and interpret unit fractions in the form* $\frac{1}{n}$ *as the quantity formed by one part when a whole is partitioned into 'n' equal parts"* (Citation withheld to preserve confidentiality of source). Specifically, students needed to represent and interpret unit fractions in the form $\frac{1}{n}$ as the quantity formed by one part when a whole is partitioned into *'n'* equal parts.

Ms. Deron then shared with the other participants that this benchmark required students to conceptually understand that fractions are parts of a whole. She further explained that when the whole is divided into *'n'* equal pieces, each individual unit is part of the whole. Ms. Deron surmised that the conceptual components for students to understand were that unit fractions are the foundation for all fractions and that fractions are numbers. She added that within this benchmark, students' understanding must also encompass that the greater a unit fraction's denominator, the greater its number of parts.

The participants then unpacked the vertical and horizontal fraction progression from curriculum standards. Analyzing the progression of concepts from grade to grade and across mathematical domains helped identify essential fraction concepts that students should articulate as evidence of their mastery of the lesson's learning goals. These key principles from standards included:

- that fractions signify parts of a whole,
- that a unit fraction symbolizes one equal part of the whole,
- that the numerator represents the quantity of parts considered within a whole, and
- the denominator denotes the total number of equivalent parts into which the whole is divided.

The participants spent considerable time discussing the different components that would be included in the conceptual lessons. They collaboratively brainstormed and shared ideas about what elements should be present to foster conceptual understanding of fractions among students. Their focus was to develop conceptual components based on the procedural elements housed within the curriculum. I listened to their discussions and surmised their collective thinking by creating a conceptual evidence graphic (see Figure 7) that encapsulated the key components they identified as crucial for conceptual learning.

The conceptual evidence graphic aligned with *Catalyzing Change* (NCTM, 2020) because it broadened the scope of mathematical learning by ensuring students grasped the foundational significance of fractions as representations of quantities. Additionally, it merged seamlessly with Hiebert's (1984) Site Theory by incorporating 'Procedural Execution' (Site 2) through visual representations, reinforcing the notion of fractions as numbers.

Therefore, exploring the vertical and horizontal learning progressions guided the development of a conceptual evidence graphic rooted in research-based principles. Having this research-grounded understanding of essential fraction concepts, the participants then shifted focus to exploring various models that could further build students' conceptual understanding.

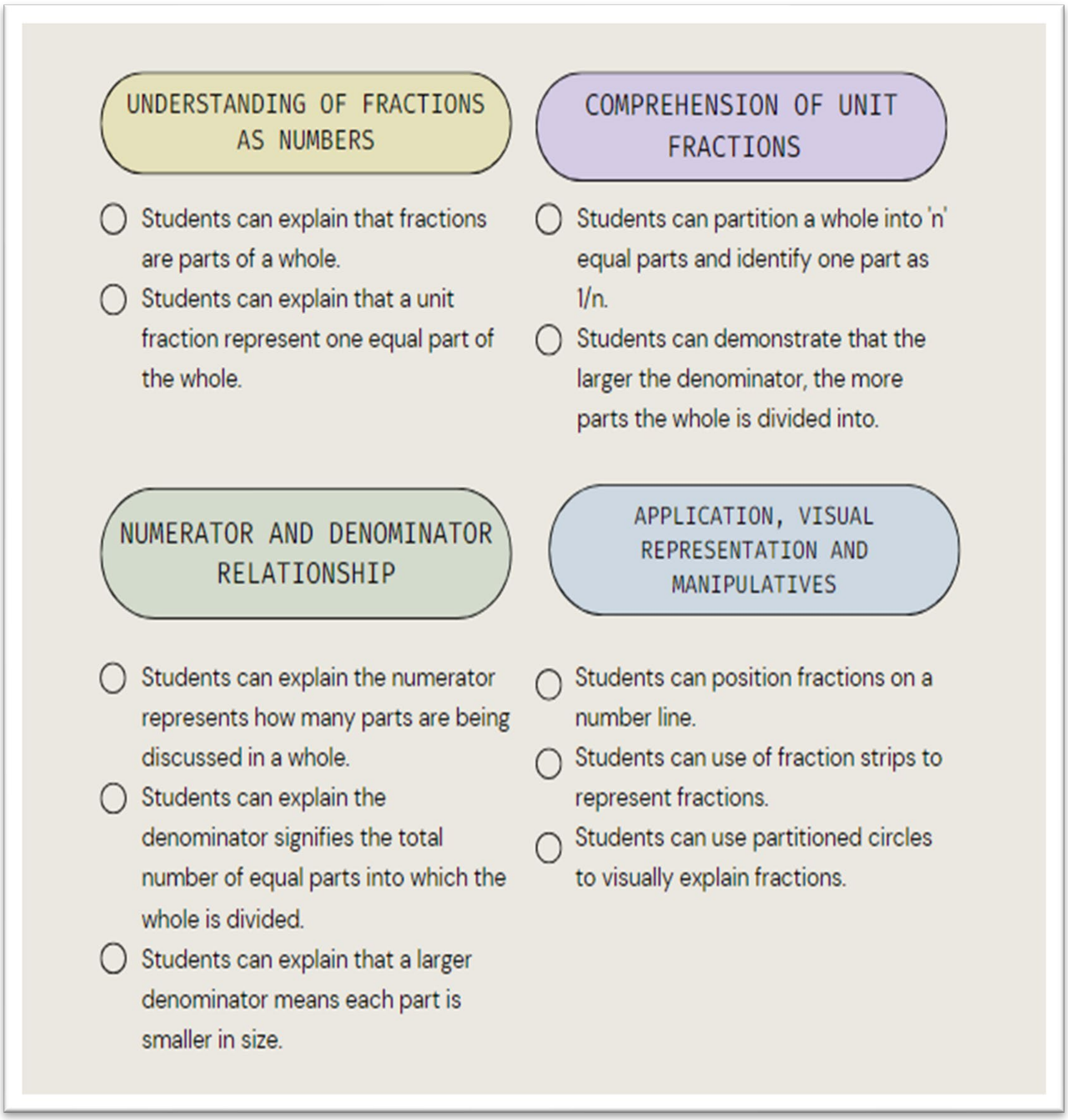

Figure 7 Evidence of Conceptual Understanding

<u>Observations of Stage 2: Creating the research-based lesson.</u>

Before developing the mathematics lessons, the participants engaged in a discussion about the context in which mathematics problems were presented to students. They raised concerns about the lack of connection between the mathematics lessons and the students' real-life experiences. Ms. Peters expressed, *"The examples in the mathematics textbook – using things like snow and skyscrapers – are not things our students intuitively understand."* Ms. John shared that her students "*were not motivated and interested in solving mathematics problems that seem so far removed from their everyday experiences."* In response to these concerns, the participants made a collective decision to make every aspect of the modified lessons relatable to their students' lived experiences.

At first, Ms. Emily questioned how relating the mathematics lessons to the students' lives could be accomplished. Ms. Lane responded to her inquiry by stating:

> Let's make the math tasks relate to things the students know. Florida Classic weekend is approaching....and our community is excited about that. We could use examples from that in our math tasks instead. We could discuss unit fractions by identifying one person who scores a goal out of the entire team. The point is.... let's connect to what the kids are familiar with when creating these tasks.

Ms. Lane's idea resonated with the other participants, who smiled in approval. Ms. Bob synthesized Ms. Lane's perspective by stating:

> Tapping into contexts and examples that students already understand would help make the mathematics more realistic and engaging for them. It can empower the students to see themselves as mathematicians because they may already be unconsciously thinking about math in their daily lives.

Rather than relying on rote memorization and procedures, the participants wanted activities fostering deep mathematical understanding. Ms. Deron stated, *"Fractions are so confusing partly because our students do not get enough time to actively explore fractions through concrete experiences."* Ms. Bob added, *"The way the student workbooks are written, kids are exposed to fraction symbols without fully understanding what it means."* To compensate for these deficiencies and cultivate deep mathematical understanding, Ms. Bob advocated for using visual representations. She bolstered her argument by explaining, *"Since visual representations can be tangible, partitioning tasks with concrete items encourages students to reason through the math, like partitioning equal-sized pieces and then counting how many of those pieces make up the whole."*

The participants conducted a thorough analysis of the prescribed curriculum, focusing on the hands-on activities and tasks that students would be expected to complete. Armed with the knowledge of what they wanted students to do, explain, or produce to show conceptual understanding of the benchmark (see Figure 7), the participants began to explore potential adaptations and modifications to the existing lesson plans. Their goal was to enhance the lessons in ways that would foster a deeper conceptual understanding among students.

During their initial exploration of the standards, the participants discovered that students in the second grade were exclusively exposed to the area model for fractions. Ms. Deron recognized the need to expand students' understanding of fractions beyond this limited exposure, so she challenged the other participants to incorporate various representations for unit fractions into the modified lessons, including set and linear models.

The lesson study fostered a culture of open inquiry as it promoted open dialogue and mutual support among the participants, enabling them to address their uncertainties and challenges in teaching fractions. Ms. Emily, a first-year teacher who admitted to being typically reserved during PLCs, ventured to ask how the set model could build a student's conceptual understanding of unit fractions.

Ms. Deron used the lesson study platform to provide the rationale for incorporating set models. She shared that using discrete, countable objects catered to students who naturally think in terms of whole numbers and individual units. Ms. Deron connected this approach with *Catalyzing Change's* mission of 'developing deep mathematical understanding' (NCTM, 2020) by explaining that set models allow students to physically manipulate and visualize fraction concepts, which promotes a deeper understanding of the underlying principles. She also linked this strategy to Hiebert's (1984) 'Procedural Execution' (Site 2), which emphasizes the importance of tying concrete examples to abstract mathematical theories. Ms. Deron explained to Ms. Emily and the other participants that by using set models, students can tangibly perceive what a unit fraction represents and how it relates to the whole, thereby bridging the gap between the concrete and the abstract. Ms. Deron further elaborated on this connection, stating:

> When students can physically divide a set of objects into equal parts and see how each part relates to the whole, they are better equipped to understand the procedural steps involved in working with fractions. This hands-on experience provides a solid foundation for understanding the abstract concepts later on.

Through these explanations, Ms. Deron demonstrated how incorporating set models supported students' development of deep mathematical understanding.

This explanation resonated with Ms. Emily as she shared that she could use a set model with the cookie jar in her classroom. By filling the jar with an assortment of ten cookies and asking students to identify what one cookie represents, students would engage with the concept of a unit fraction—as each cookie represents $\frac{1}{10}$ of the total set of cookies. According to Ms. Emily, *"The concrete representation would help students understand that one cookie is part of the whole."* This statement revealed how Ms. Emily hoped to turn an abstract concept into a tangible one. Therefore, Ms. Deron's explanations had profound implications for Ms. Emily as she began seeing the lesson study as a way to enhance her own pedagogy.

Building on this idea, the participants decided to employ everyday items such as pens, pencils, and cookies to explain unit fractions within a set model. They discussed how real-life examples of identifying one item out of a set would resonate with the students, thereby making the abstract concept of fractions more approachable and understandable. These scenarios subtly conveyed to students that they engage in mathematical thinking in their daily lives, often without realizing it.

The participants also decided to integrate another real-life analogy that students could easily relate to, such as the sharing of a chocolate bar. This decision was based on the understanding that every child had likely experienced sharing at some point in their school or home life. By incorporating these familiar contexts, the participants aimed to create an interactive lesson that intuitively illustrated the concept of fractions in a way that was both engaging and grounded in student-centered considerations. The participants opted for the linear model for *Lesson A* and the set model for *Lesson B*.

Deviating from the prescribed tasks embedded within the curriculum, the participants also changed the generic multiplication bell-ringer activity. They opted for a real-world sharing scenario that better aligned with the learning goals. Ms. Peters admitted in her reflections that *"Creating warm-up activities that were directly aligned with the learning objectives was meaningful and engaging. It made more efficient use of instructional minutes and effectively* *propelled the students towards achieving those objectives."* The participants' collaborative decision to refine the fraction lesson to emphasize conceptual understanding rather than generic computation evidenced the lesson study's capacity to cultivate intentional redesign centered on conceptual learning.

As the participants continued to develop their lesson plan, they discussed creating opportunities for students who experienced initial difficulties to get additional time and support for learning in a timely manner, which would not detract from them missing new instruction. They also planned activities that would enrich and extend the learning of students who already understood the learning targets. They also intentionally planned for students to engage in academic discourse.

Planning for Academic Discourse

For *Lesson A*, the participants purposefully selected a chocolate bar to represent the linear fraction model. They felt that the chocolate bar manipulative helped students tangibly enact splitting a whole into fractional parts. While their focus on using a tangible and relatable model was evident, the context did not align with a linear model. The implications of this misalignment between the chosen model and the mathematical concept will be further discussed later in this chapter.

At this point in the research-based lesson creation, the participants' focus was to build opportunities for academic discourse to facilitate conceptual connections. With the exception of the knowledgeable others, the other participants admitted that having students share their thinking aloud was a novel concept. Ms. Deron suggested that using sentence stems to encourage students' verbalization was warranted.

The participants agreed that many students struggled to coherently verbalize their ideas and problem-solving processes when working in groups. They acknowledged that the novelty of the mathematics lesson—which required students to give voice to their thinking using academic language—was not a skill previously practiced in the classroom.

Since the overarching goal of the lesson study was to seamlessly integrate conceptual and procedural knowledge within the lesson, the participants realized that students needed scaffolding to verbalize their conceptual thinking effectively. Ms. Deron proposed using structures like sentence stems to further guide students' understanding of key mathematical concepts like numerators and denominators. She suggested providing partially complete sentences like the one in Figure 8.

There is _____ whole chocolate.

____ friends share the chocolate.

Each friend will get _____ out of _____ pieces.

Figure 8 Sentence Stem

The following excerpt captured Ms. Deron's explanation to the lesson study participants on how the sentence stems could be effectively utilized to deepen conceptual understanding:

> The first sentence guides the students to identify that there is one whole unit we are working with. The second sentence then helps students partition the number of equal parts that the

> whole unit will be divided into. The third sentence promotes understanding that each person receives one share out of that total number of equal parts. Through hands-on exploration with manipulatives, our students realize that the numerator in a fraction represents the share they are specifically examining— in other words, their individual piece. The denominator represents the total number of equal pieces that make up the whole. When the students are filling in those blanks, they're doing two things; they're practicing how to talk about fractions and they're watching us connect those words to the actual fraction symbols - the numbers above and below the line. So as the kids are saying 'one out of five equal pieces' in the sentence frame, we're also writing $\frac{1}{5}$. Instead of just seeing numbers and lines, those fraction notations start to make sense to the students. This ties back to the conceptual list [see Figure 7] we developed as evidence of students' understanding.

At the end of Ms. Deron's detailed explanation, the participants agreed that the use of sentence stems would ensure students articulate their understanding of the fraction in context, thereby fostering a deeper conceptual grasp of the fraction parts as they relate to the whole. Ms. Bob stated:

> By having students fill in the blanks in this sentence stem, we're pushing them to think about the meaning of the fraction in the context of the problem. It's not just about following a procedure, but about understanding how the parts relate to the whole.

This statement demonstrated Ms. Bob's awareness of the distinction between conceptual and procedural knowledge. The participants then developed activities where students were challenged to craft word problems based on their partitioning of the chocolate bar. The participants felt that having students generate novel problems would further position them as active meaning-makers in applying fraction concepts rather than passively replicating what was already modeled. Ms. Deron added:

> When students generate their own problems, they have to think deeply about the concepts and how they apply them in different contexts. It's not just about solving problems we give them, but about being able to create and reason with fractions on their own.

The participants' decision to have students create their own word problems based on the partitioning of the chocolate bar was a conscious choice to position students as active meaning-makers.

Planning for Questions

Ms. Bob and Ms. Deron, the knowledgeable experts, then challenged the other participants to consider the type of questions they were posing to the students. They encouraged them to develop quality questions that would stimulate mathematical thinking and promote deeper conceptual understanding. In response to this challenge, the participants collaborated to create higher-order, open-ended questions that would guide students through the open exploration activity they were creating. These questions were designed to provoke critical thinking and elicit responses that would help students construct their own understanding of the terms 'numerator' and 'denominator.' For example, students would be prompted to engage in academic discussions guided by prompts such as, *"Can someone tell me what the 1 in the answer represents? Why is it there? What would be a suitable name for the top number, the 1, that represents the piece each friend gets?"* The participants wanted the students to generate a definition for the numerator, such as 'share' or 'one piece,' and the denominator, such as 'the whole' as having students ascribe meaning to the formal terms would serve to bolster their conceptual understanding.

Furthermore, using these types of questions would provide students with opportunities to justify their thinking, thus fostering a deeper understanding of the underlying mathematical concepts. Prior to the lesson study, the participants admitted to only using the questions posted on the prescribed PowerPoint slides during instruction. As Ms. Peters shared:

> I was trained to implement the 'I do, we do, you do' model of instruction. I explained how to solve the problem using a particular procedure and then waited for the student to replicate the steps on their own. This approach focused more on rote repetition rather than building conceptual understanding.

At this juncture, Ms. Deron stated, *"The use of higher order thinking questions, questions that push kids to provide a justification for their solutions*, *can build up the academic rigor of a math task."*

Ms. Deron then introduced the Instructional Quality Assessment (IQA) toolkit as a valuable tool for creating high-quality, rigorous questions in the classroom (Boston et al., 2019). She highlighted that one of the key areas measured by the IQA toolkit is the use of open-ended, higher-order questioning strategies, which prompted students to justify their answers. She explained that research has consistently shown that asking more open-ended questions improved critical thinking skills, created richer classroom discourse, and promoted deeper learning. Ms. Deron further elaborated:

> The use of why and how questions such as "Why did you use the strategy?" or "How can you solve using another strategy? This encourages students to verbalize their thinking aloud, which in turn deepens their comprehension and that of their peers.

Ms. Deron then led the participants in crafting sample open-ended questions that could be utilized during upcoming lessons. Shifting from closed-ended to open-ended questions allowed the participants to uncover errors and assess learning more effectively, as students were encouraged to justify their models and explain their reasoning. The carefully crafted higher-order, open-ended thinking questions were recorded in the lesson plan template (see Appendixes I and J). These questions included:

- *What does the denominator tell you about a fraction?*
- *What does the numerator tell you about a fraction?*
- *If we keep increasing the denominator in a unit fraction, what happens to the size of each part? Why?*
- *If you combine two-unit fractions from a whole divided into three parts, what fraction do you get?*

The experience of crafting these types of questions had a lasting impact that extended beyond the lesson study as the participants applied this questioning technique in other disciplines. Ms. Emily shared in her written reflection:

> Encouraging students to provide a justification for their answers was not a strategy I was familiar with. I have since used this strategy across all other subject areas. The quality of the students' responses is so much deeper than when asking closed-ended questions.

Ms. Mark, in her post-lesson interview, added:

> I know how to present content to the class, and I know how to teach in small groups, but engaging students in a whole-class conversation about mathematics was not something I did, until now. Asking the higher-order thinking questions and asking for the justification piece really elevated the students' responses.

Therefore, according to Ms. Emily and Ms. Mark, the experiences with justifying solutions seemed to be instrumental in deepening students' comprehension of foundational mathematical principles as they enabled the students to perform procedures, articulate their thinking, and defend their mathematical reasoning.

Ms. Peters then shared insights from her previous years of teaching fractions, noting that an anticipated error students might make would be incorrectly assuming all fractions must have a '1' in the numerator. To preemptively plan for this, the participants proactively included questions exploring variations in fractions. For instance, they asked, *"What if one friend got two pieces of the whole, how would you label that?"*

Planning for Procedural Fluency

As each participant was assigned various roles to facilitate the process, the timekeeper, who was responsible for keeping the group on track, alerted the others that in order to stay on schedule, they needed to move on to creating *Lesson B*. As the participants transitioned into crafting *Lesson B*, they agreed that revisions to *Lesson A* were likely to occur after its implementation.

For *Lesson* B, the participants decided that the concept of sharing a packet of cookies would be used as the hook to draw the students into the lesson on unit fractions for set models. In the exploratory activity, each cookie represented part of the total set. For example, given a package with six cookies (five chocolate and one golden vanilla), students would model each cookie with six counters, using five of one color and 1 of another to denote the vanilla. By connecting the set model to friends sharing actual cookies, the participants wanted students to make sense of fraction symbols in a realistic situation.

The participants also recognized the crucial role of procedural fluency in developing students' mathematical proficiency. They wanted to design an activity that not only reinforced the conceptual understanding of unit fractions but also provided students with opportunities to practice the procedure of combining unit fractions to form a whole. They planned the lesson to include the concept that unit fractions could be combined to form a whole, thereby introducing procedural fluency, for example $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6}$. By incorporating this procedural aspect, the participants aimed to help students develop the ability to accurately and efficiently perform fractional operations.

To further support the development of procedural fluency, the participants decided to include multiple examples of combining unit fractions to form a whole. They wanted students to practice this procedure with various denominators, such as $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3}$. By providing students with ample opportunities to practice this procedure in different contexts, the participants hoped to reinforce the conceptual understanding while simultaneously building students' procedural skills.

The participants connected this activity with *Catalyzing Change* (NCTM, 2020) as it broadened the scope of mathematical learning by emphasizing fractions as quantifiable representations. This approach also integrated with Hiebert's Site Theory (1984) by incorporating 'Procedural Execution' (Site 2) through visual representations. By using concrete models, such as counters, to represent fractions, the participants were providing students with a tangible way to understand and execute fractional procedures. This visual approach served as a bridge between conceptual understanding and procedural fluency, allowing students to see the connection between the two aspects of mathematical proficiency. To proactively identify potential student errors within the lesson, the teachers simulated completing the lesson activities from the learner's perspective.

Assuming Students' Perspectives

Taking on students' roles helped the participants to better understand students' early conceptions and refine their questioning techniques. By immersing themselves in the students' perspectives, the participants could anticipate the thought processes and challenges that their students might face when engaging with the mathematical tasks. This role-playing exercise allowed the participants to step back from their roles as educators and experience the learning process from the students' point of view, particularly in relation to students' zone of proximal development (ZPD). By considering the students' ZPD, the participants could better understand the appropriate level of support and scaffolding needed to facilitate learning. As the participants attended to the tasks as students, they quickly recognized that a potential error could be that students would confuse 'one packet' with 'one unit fraction.' To preemptively address this confusion, they devised guiding questions, such as:

- *What fraction represents one whole packet?*
- *What would the numerator and denominator be?*
- *How many cookies are in one packet, and how would that relate to our unit fraction representation?*
- *When you see the fraction* $\frac{1}{6}$, *what part of the packet are you thinking about?*

These questions were formulated to help the students focus on the differences between the whole packet and individual items within it, ensuring clarity in understanding unit fractions in the task context. Ms. Emily shared that this was her first experience seeing a mathematics task through her students' eyes and expressed that she did not typically experience this learning in traditional PLC. Ms. Peters shared with the group:

> Working through these tasks as a student is powerful. It solidifies my own understanding of the content and reveals areas that commonly lead to misunderstandings. By identifying areas for misconceptions, I can proactively plan targeted scaffolding.

By experiencing the tasks as students, the participants could identify areas where additional guidance or clarification might be required. This insight allowed them to refine their questioning techniques and develop a more student-centered approach to teaching. Therefore, the participants admitted to finding value in this immersive experience as it provided them with a fresh perspective on how students might interpret and engage with mathematical tasks.

Documenting Lesson Plans

This lesson study session concluded after the participants documented the newly created research-based lesson questions in the lesson plan template (see Appendixes N and O). They also included the required materials for the lessons and the expected outcomes of students' actions. The participants departed with the mutual understanding that modifications to both lessons would be determined after implementation.

<u>Observations of Stages 3 and 4: Implementing and Revising the Research-Based Lesson.</u>

Prior to the implementation of *Lesson A*, the participants decided that only Ms. Deron would respond to students' questions during the lesson. The remaining participants who were in the room during the lesson's implementation would refrain from answering any students' questions, even if the students looked to them for assistance. This strategic decision to avoid the remaining participants intervening in the lesson was a conscious part of the lesson plan, designed to simulate a regular classroom environment and observe the lesson's natural flow.

To discourage students from seeking help from the other participants, they decided to avoid making eye contact with the students and to position themselves with their backs to the students. This served to minimize potential distractions for the students and allowed the participants to discreetly listen to student discussions, enabling them to better assess the effectiveness of the lesson. Since the lesson called for students to work in small groups, each participant aligned themselves with a student group to carefully observe what was transpiring and the academic discourse unfolding among the students.

The team used their Observation Protocol to carefully record objective notes about what they saw or heard and not their personal evaluations or interpretations of those observations. It is important to note that the participants' objective was to discern how the lesson's activities advanced the students toward the lesson goals. As a result, their focus was not on critiquing Ms. Deron's teaching but on the efficacy of the lesson. Participants keenly observed students looking for their 'aha' moments or early assumptions. This move allowed for diverse and exciting discussions during the debrief. In the following sections, I share my observations of the aftermath of *Lesson A's* implementation.

Critical Incidence

The reflective debrief of *Lesson A* took place several hours post-lesson to allow the participants time to process their thoughts. I asked each participant to document what went well with the lesson and things that should be improved. Each participant retreated into a quiet space for solitary reflection and utilized the Observation Protocol to document and synthesize their thoughts. The written reflection afforded the participants the time to consider the observed lesson and instruction in relation to their past experiences.

At the start of the debrief for *Lesson A*, all the participants, excluding Ms. Deron, shared that watching a lesson they co-created unfold in real-time, without the weight of attending to students' responses, freed their mental capacity to focus on how the lesson impacted students' understanding. Ms. Marks called this *"liberating,"* explaining that not having to manage the classroom herself allowed her to better analyze the effectiveness of the planned lesson. Similarly, Ms. Peters shared that observing the 'teacher moves' without intervening allowed her to *"soak in the lesson and critically assess what was working"* in a way she did not have the capacity for when teaching her full classroom.

As part of my reflective process, prior to the *Lesson A* debrief, I had previously shared with my advisor the participant's use of the linear model. Based on my advisor's feedback, I realized we were inaccurately using the linear model. We were focused on the benchmark standards of teaching students to identify a unit fraction and, therefore, did not consider that with the linear model, we needed to focus specifically on length. I first shared this feedback specifically with Ms. Deron.

Through reflective dialogue, Ms. Deron and I identified an area for improvement in our instruction and refined our understanding of the linear model's application in teaching fractions. When all the other participants reconvened to engage in collaborative reflection, Ms. Deron asked the participants to begin with a discussion of the concrete descriptions of what was seen and heard. Ms. Deron posed the question, *"Who noticed an error in the partitioning of the shape?"* Six of the teachers looked perplexed, as Ms. Emily stated, *"I thought the kids did a great job!"* This statement was met with murmurs of agreement from the remaining participants, which underscored the fact that differentiating between linear and area models was challenging.

At this juncture, Ms. Emily quietly asked how the shape was incorrectly partitioned. Since the goal of the lesson study was to build a deeper understanding of fraction concepts in the participants, Ms. Deron addressed the difference between the linear and area models. She illustrated that a vertical partition of the linear model, as depicted on the left of Figure 9, would look like taking a ruler and marking off sections—each section represents a unit of length (such as centimeters or inches). The total length is the sum of these units. It is a straightforward, linear representation of length, and it is one-dimensional since it does not utilize width or height.

Ms. Deron continued her explanation by stating that if the linear model was partitioned like the example on the right in Figure 8, where there was a grid or array of rectangles or squares, this would be an area model. Since this model contained height and width, it would be considered two-dimensional. Figure 9 reveals her illustration of an example and a non-example using the linear and area models.

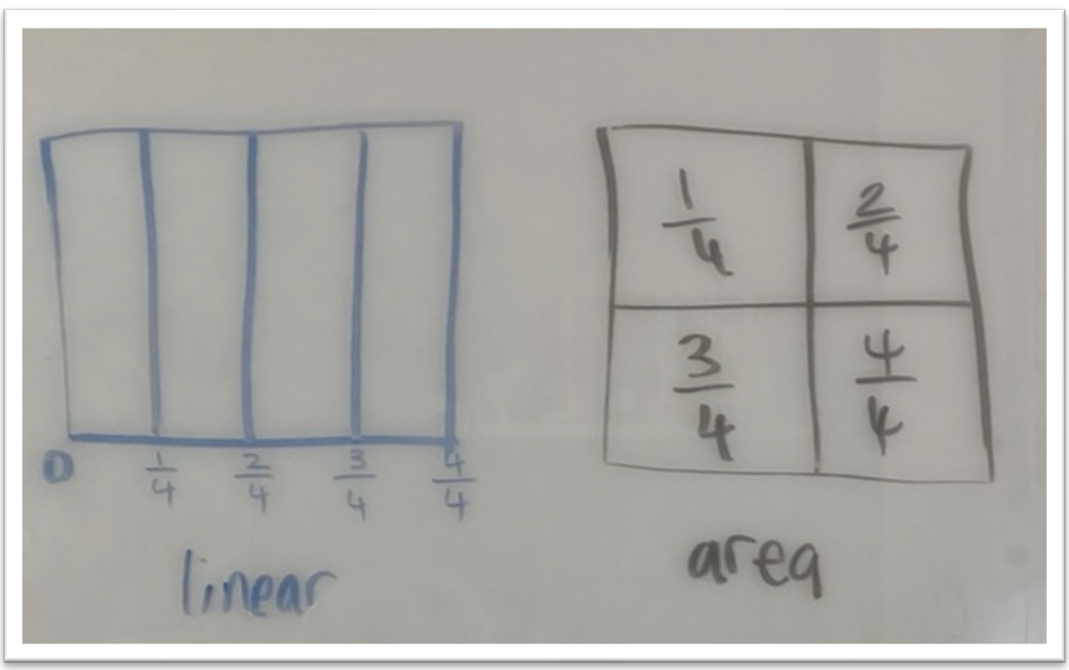

Figure 9 Linear and Area Models

The discussion that followed was deeply impactful, as Ms. Deron added:

> The linear model divided a whole into fractional parts along a single dimension - like the length of a strip, whereas in the area model, it divides a whole shape into fractional parts of its total area using a 2D shape. The linear model must start at zero since it measures length, unlike the area model, which represents space. So, when we are using the linear model, the representation should look like this.

Ms. Bob furthered the conversation by adding that in the one-dimensional, vertical partition of the linear model, the whole length of the model represents the whole unit, whereas in the two-dimensional, horizontal partition, such as the rectangle or array, the whole area of the model represents the whole unit. She summarized the discussion by stating that in terms of the lesson's objective, the key concept for unit fractions is that the whole, whether a line or an area, is divided into equal parts. Each equal part is a unit fraction of that whole. She reminded the participants that since the lesson called for the use of linear models, the students should be prompted to partition their whole lengthwise.

Ms. Deron then reiterated to the participants the importance of clear, precise explanations to help students grasp the concept of unit fractions and their representation of a linear model. She noted, *"While the purpose here is identifying unit fractions, improperly differentiating between models will undoubtedly cause confusion down the road. Students may not struggle with it in this lesson, but the conceptual gap will resurface in later grades."*

This revelation sparked a chorus of 'a-ha' and nods of understanding from the other participants. Ms. Mark confessed, *"I did not understand the differences. Since the objective of the lesson was to identify a unit fraction, I really did not pay attention to the specificity of the models. Now I know better."* Ms. Peters, a veteran teacher, remarked with surprise, *"I've been* *teaching math for so long, but I never fully grasped the key difference between the linear and area models until now."* While the definition of the linear model was explained and discussed, the lesson the teachers created (see Appendix I) did not reflect a linear model but rather an area model. The implications of this are discussed in Chapter 5.

Further Analysis of Lesson A

The participants then transitioned into discussing other aspects of *Lesson A*. Ms. Peters highlighted the instance when Ms. Deron asked the students to divide their chocolate bar (fraction strip) into equal parts. She shared that the students immediately started to cut the fraction paper strip into pieces without ensuring the pieces were the same size. She was intrigued when Ms. Deron made slight modifications and engaged in spontaneous interactions with students to inform them that folding the paper prior to cutting it would ensure that the pieces partitioned would be of equal size.

The participants also noted how Ms. Deron refrained from directly answering students' questions, choosing instead to pose questions that would guide the students toward solving the problems themselves. This approach resonated with Ms. Emily, who, during the debrief, remarked:

> I usually ask a question, and if no one answers promptly, I repeat the question but louder. Then, I give the students the answers instead of waiting for them to come up with them on their own. After observing Ms. Deron, I realized I needed to give more wait time.

Furthermore, in her written reflections, Ms. Emily wrote:

> I never saw wait time modeled in a classroom setting before. Usually, I call on a kid to provide an answer, and if he or she doesn't, I pick someone else or give them the answer. Wait time was not a strategy I had in my toolbox.

Thus, Ms. Emily realized she needed to give her students sufficient opportunity to engage in productive struggle, which was a crucial pedagogical adaptation spearheaded by Ms. Deron. Since Ms. Deron modeled this instructional move, the other participants admitted to adopting it in their classrooms, along with allowing students the opportunity to engage in productive struggle. As Ms. Mark stated:

> When I pose a question, I start a mental countdown from fifteen. This ensures that I give enough wait time. I also have started scaffolding the questions I ask the kids. This way I'm coaching them to the right answer and allowing them to productively struggle.

Thus, Ms. Marks's action revealed that due to her involvement in the collaborative learning process, the lessons she learned carried over into other aspects of her instructional repertoire.

While the participants highlighted evidence of what went well with the lesson they planned, they also suggested some areas that could be improved. For example, after *Lesson A* implementation, Ms. John observed that while the smaller group format was somewhat effective, it did not guarantee active engagement from all students. Ms. John, in her reflection, wrote that she noticed *"larger group sizes made it difficult for everyone to get an equal chance to participate."* She brought this to the attention of the other participants. This realization catalyzed a discussion on the adoption of more inclusive strategies to leverage all students' inputs. Building on Ms. John's observation, Ms. Lane then raised a concern about the amount of instructional time students spent creating word problems to match the partitioning scenario.

Creating Lesson A Version 2

To address the need to engage all learners, the participants discussed forming smaller groups (4 students as opposed to 6 students) in the subsequent iterations of the lesson so that all students could participate in the learning process. Furthermore, within these reduced-size groups, Ms. Bob suggested assigning specific roles for each student, such as *'Captain, Resource Manager, Speaker, and Recorder.'* Her idea was that assigning roles would help ensure active student engagement by providing each student with a sense of responsibility and ownership over the learning process. Since Ms. Emily had not previously employed this strategy, she was unsure how it needed to be implemented. Ms. Bob elaborated on the characteristics of each role, stating:

> The *Captain's* role empowers a student to oversee the group and keep the group on task; the *Resource Manager* allows a student to gather and distribute the materials needed to complete the activity; the *Speaker* vocalizes the group's ideas, and the *Recorder* captures the group's collective thinking in writing.

Ms. Deron added another layer of sophistication to this strategy by urging the teachers to strategically consider each student's strengths and areas for growth before assigning them to a specific role. They agreed on a general framework to assign the roles: The *'Captain'* role was assigned to the students who exhibited strong leadership potential; the *'Resource Manager'* role was assigned to students who the participants thought were detail-oriented and well-organized. The *'Speaker'* role went to the students who were comfortable with public speaking, as they would vocalize the group's ideas to the rest of the class during presentations. The *'Recorder'* role was assigned to students with well-developed writing skills. Each participant then spent a few minutes assigning specific roles to students from their class, as they understood their students' capabilities better than anyone else.

The pedagogical strategies gleaned from this lesson study session resonated deeply with Ms. Emily. In her reflections, she wrote:

> As a new teacher, I struggle with classroom management. Redirecting kids all the time used up a lot of my math block. However, when I gave each kid an assigned role, I noticed

their off-task behaviors were reduced, and they were excited about fulfilling their obligation to the team. Students who did not typically work together started to under this new grouping system.

Ms. Emily later shared with the other participants that the instructional change she made helped to curtail off-task behaviors and freed additional instructional time for more focused and effective mathematics teaching. Thus, the feedback generated from the collective debrief, coupled with the reflective practice, enhanced the classroom management in Ms. Emily's room. Her statement energized and encouraged the team to celebrate her efforts.

Ms. Lane, who had raised a concern about the amount of instructional time students spent creating word problems to match the partitioning scenario, suggested that, given the limited mathematics block, removing the activity would allow students to focus more on achieving the learning target. The other participants agreed with this suggestion. To reflect the revised changes, which included the introduction of student roles and the removal of the word problem activity, the team updated *Lesson A* and renamed it *Lesson A Version 2*. Consequently, the original *Lesson A* was renamed *Lesson A Version 1*. A few days after creating *Lesson A Version 2*, the participants implemented the second research-based lesson. The ensuing section details my observation of the implementation, revisions, and subsequent name change of the lesson.

Creating Lesson A Version 3

After implementing *Lesson A Version 2,* the participants saw value in including a warm-up activity titled 'Solve and Share' which would be placed at the beginning of the lesson. They felt that this activity would activate students' prior knowledge and engage them with the concept of fractions before delving into the primary lesson. For the 'Solve and Share' activity, the participants designed a real-world scenario in which students were asked to determine how to divide a pizza among friends equally. This problem was crafted to encourage students to think about fractions in a familiar context, setting the stage for a more detailed exploration of unit fractions in the main lesson. By adding the 'Solve and Share' component, the participants aimed to create a more cohesive and engaging lesson structure that would better support students' understanding of the key concepts.

Another key point the participants found impactful during the implementation of *Lesson A Version 2* was when the students divided the same-sized chocolate bar (paper fraction) based on their choice of denominator. In their reflections, Ms. Bob, Ms. Lane, and Ms. John had varying perspectives on the use of paper fractions. Ms. Bob found this activity particularly engaging, stating, *"The students were thoroughly engaged in the lessons, everyone was making sense of the unit fraction – the chocolate manipulatives made the lesson more interactive and engaging."*

However, Ms. Lane suggested that using paper strips of varying lengths could lead to more diverse student responses and richer discoveries. She specifically noted, "*Comparing unit fractions of different wholes helps illustrate that while unit fractions share the same name, their actual size depends on the whole."* I encouraged Ms. Lane to share this reflection with the other participants, who agreed with her thinking and decided to incorporate her suggestion of using unequal paper lengths in future iterations of the lesson.

Ms. John felt that partitioning wholes into equal pieces was consuming an excessive amount of instructional time. In her written reflection, she captured the nuances of the problem, stating, *"While the students understood the concept of sharing intuitively, the actual partitioning of the paper fractions detracted from the lesson's goal as kids had a difficult time using the scissors."* Ms. John shared her thinking with the other participants. The participants saw value in both Ms. John's and Ms. Lane's perspectives. They wanted to keep the hands-on activity embedded in the lesson, but they also wanted to explore how technology could be used to enhance the lesson. To effectively utilize all instructional minutes, Ms. Bob suggested the adoption of digital fraction bars, where students could select a whole and partition it into equal-sized pieces. The participants decided to incorporate digital tools into the instructional process. They discovered digital manipulatives on the Didax website (Didax, n.d.) that allowed students to visualize and interact with a whole before dividing it into various partitions.

Adapted Lesson Study Plan

Based on the established research schedule plan, the participants' next instructional move should have been to improve *Lesson B.* Given that the primary objective was to develop a lesson that seamlessly integrated conceptual and procedural elements, the participants agreed that these additional enhancements should take precedence over adhering to strict timelines. As a result, they adapted the research plan to include a third iteration of *Lesson A*, which they named *Lesson A Version 3*. These modifications made it impractical to implement the second version of *Lesson B*. Moreover, two of the participants held administrative positions and had prior commitments that restricted their ability to extend the lesson study beyond the scheduled time. Consequently, the schedule did not accommodate the second implementation of *Lesson B* in a classroom setting. Nevertheless, the participants fully developed, updated and documented *Lesson B* after its implementation and renamed it *Lesson B Version 2*.

Another adaptation involved the knowledgeable others responsible for modeling the lessons. Originally, the plan was for Ms. Deron to teach both versions of *Lesson A* and for Ms. Bob to teach two iterations of *Lesson B* (see Table 4). However, the team concluded that having a single teacher consistently teach the lessons would better inform the lesson study. Ms. Deron graciously volunteered to teach all the lessons, as she possessed the necessary experience, knowledge, and leadership skills for this role. All revisions to *Lesson A Version* 2 were documented and resulted in the creation of *Lesson A Version 3*. A few days after the creation of this lesson, it was implemented in the classroom. The following section details my observations of the implementation of *Lesson A Version 3.*

Lesson A Version 3

I noticed that during the implementation *of Lesson A Version 3,* particularly in the 'Solve and Share' segment, instances of misunderstanding emerged. The students needed to complete a task that read:

> Ms. Thomas has a pizza and wants to share it with her 5 friends. Everyone gets an equal slice. Create a model to show how many slices the pizza has and how much of the pizza each person gets.

The participants designed this lesson with the intention that students would divide the pizza into six equal shares. However, some students interpreted the lesson as sharing the pizza between five friends. This misinterpretation led to discussions among the students about how to partition the pizza correctly. While these student conversations were relevant to the topic, they detracted from the main learning goal of the lesson, which was to identify a unit fraction.

The participants noted how the students misconstrued the words in the text. This observation helped to ensure that further tasks were worded more carefully to avoid confusion. Thus, the students' affective responses provided opportunities for the participants to better understand the connections between the student's experiences and the lesson content.

The participants then shared their thoughts on the adoption of the digital manipulatives in the lesson. Ms. Mark shared in her interview that:

> Using digital manipulatives helped to make the learning deeper and more insightful. I heard kids having conversations such as the more the whole is divided, the smaller the pieces are. This type of conversation did not occur when they were partitioning using scissors.

Then Ms. Lane spoke about how the technological modification enabled insightful comparisons:

> The students from two different groups divided their whole into four pieces, but the lengths of each whole were significantly different. Both groups were discussing that a unit fraction meant one piece of the whole. However, they were able to make cross-group observations that the unit fraction differed in size. This led to a conversation between both groups that even though the name of the fraction was the same ($\frac{1}{4}$), the length of the piece was different. From this activity, students understood that the size of the whole affects the size of the unit fraction.

Ms. Lane and Ms. Mark's observations provided qualitative evidence that *Lesson A Version 3* adjustments promoted the deeper mathematical thinking the team sought to foster through visual representations. Ms. Mark referenced the 'Comprehension of Unit Fractions' section from the Evidence of Conceptual Understanding graphic. She remarked, *"Since students were able to demonstrate that the more the whole is partitioned, the smaller the pieces will become, this demonstrates that they were developing a conceptual understanding about the learning content."* The other participants smiled as they realized the efficacy of their work for *Lesson A Version 3*. The participants then transitioned into reviewing *Lesson B.* The following section describes my observation of the implementation of *Lesson B*, which took place several days after the debrief and modification of *Lesson A Version 3*.

Improvements to Lesson B

An example of the participants' renewed commitment to embedding conceptual components in the mathematics lesson was made evident during the implementation *Lesson B*, when Ms. Deron asked students to identify a unit fraction using a set model. She focused on helping students recognize the whole (a packet of four cookies), composed of individual units, and then identify a single cookie as representing one equal share of the whole set. During the implementation of *Lesson B*, Ms. Deron said:

> So what does this one vanilla cookie represent when you are thinking about all the cookies? [Wait for students' response] That's right, this 'special' cookie represents one equal part out of 4 parts. How do you think we can write that?

During the debrief, Ms. Emily reflected upon the conceptual components of the lesson. She shared:

> The way the task was implemented helped students understand that a single cookie was not a whole but a part of the whole. While each cookie seems whole on its own, it is actually a fractional piece of the entire pack. The use of cookies in a pack made the mathematics make sense to the kids and me!

Her remarks reinforced how leading with a conceptual understanding based on tangible items helped build the mental models needed for students to grasp more complex fractional relationships. Ms. Lane added that the responses the students were giving were tied back to the evidence for the conceptual learning document. She stated:

> The students could identify that the whole was divided into equal parts. They realized that the numerator referred to the parts being discussed [vanilla cookie] while the denominator was the sum of all the parts in the whole. This shows that the students had conceptual understanding about the learning content.

Her remarks spurred the group to acknowledge that their work positively impacted student learning and encouraged them to celebrate that victory.

Ms. Lane made an astute observation regarding the implementation of *Lesson B*. She noticed that students struggled transitioning from modeling set fractions with the two-colored counters and cookies to modeling set fractions with classroom items. To address this challenge, Ms. Lane suggested that instead of having students encase the cookies with yarn, they should simply create a visual representation of the cookies. This change would help students focus on the concept of set fractions without getting distracted by the physical manipulation of the yarn.

Secondly, Ms. John proposed that when students work with their chosen classroom items, they should place all the items from the package onto the table and then use the yarn to circle the items. This approach would create a clear visual representation of the 'whole' and help students better understand the relationship between the individual items (unit fractions) and the complete set.

The incorporation of Ms. Lane and Ms. John's recommendations led to the development of *Lesson B Version 2*. This revised version aimed to provide a more structured and targeted approach to teaching unit fractions using set models, as it included several improvements, such as clearer examples, more specific guiding questions, and additional instructional support. Due to scheduling constraints explained earlier in the chapter, *Lesson B Version 2* could not be implemented in the classroom.

The collaborative efforts of the participants in developing *Lesson B Version 2* demonstrated the effectiveness of the lesson study process in enhancing pedagogical content knowledge. To further explore the impact of lesson study, the following section analyzes the teacher development codes identified throughout the study.

<u>Analysis of Teacher Development Codes</u>

As ascribed by the case study methodology, I systematically captured evidence related to each a priori code. The individual codes *(Lesson Study, Feedback Integration, Reflective Practice, Shift in Teachers' Thinking, and Pedagogical Considerations)* helped me spotlight specific lesson improvements. However, by interconnecting these codes, I could better understand the various components that needed to work together to enhance the participants' pedagogical content knowledge. The codes were multilayered and complex; the isolated codes captured 'what' improved, while the connections explained 'how' it was improved. This was then analyzed more deeply to uncover trends in the data.

Lesson Study

One important impact I uncovered was a collective shift in the mindsets and collaborative practices of all seven participants, which created supportive conditions for pedagogical and instructional improvements. Evidence of this change occurred after a few lesson study sessions when I checked in with Ms. Lane about her thoughts on adjusting the curriculum. She reported feeling confident in making necessary changes to address her students' learning needs. When I asked how she would approach adapting future mathematics tasks, she outlined concrete steps:

> I would start by analyzing benchmarks to identify what students need to know, understand, and do. Next, I would preview materials and identify needed tools and manipulatives. Finally, I would help students connect the concrete learning to the abstract concepts by the lesson's end.

Similarly, when I posed this question to Ms. Emily, she responded, "*Hands-on activities must be a precursor to abstract content.*" She explained that using manipulatives or real-world objects to represent fractions can help students grasp the concept of parts and wholes before moving on to the abstract representation of fractions using symbols.

However, it's crucial to note that while hands-on activities serve as a valuable starting point, the goal is to integrate conceptual understanding with procedural fluency. Once students have developed a solid conceptual foundation through hands-on experiences, the participants gradually introduced more abstract representations and procedures. This integration helped students connect their concrete understanding to the symbolic representations and algorithms used in mathematics (Hiebert, 1984). During her interview, Ms. Lane admitted to feeling empowered to make research-informed instructional changes in the best interest of her students.

Each participant I interviewed reported similar takeaways - that the lesson study had helped them critically reflect on their teaching and empowered them with the autonomy to adjust their lessons. Ms. Emily described the experience as *"eye-opening and refreshing,"* while Ms. Peter agreed, *"Changing the curriculum just made sense - if we do things the way we always have, our kids will perform like they always do...then nothing will ever get fixed."* These statements demonstrated the impact that lesson study had on the participants' thinking. At the onset of the lesson study, they were initially hesitant and skeptical about altering the district-prescribed curriculum; however, through the collaborative inquiry process with colleagues, they shifted their mindset.

In addition, the transformations the participants experienced through the lesson study process sparked a wave of broader impacts across the school. Prior to the initiation of the lesson study, the weekly PLC meetings at ABC Elementary focused primarily on sorting through data to determine the lowest-performing 25% of students and then determining what resources would be utilized to provide additional instructional assistance to this group. According to Ms. Peters' written reflections, these meetings unwittingly created isolation. She stated that *"even though we met as a team to discuss this data, based on the information I received, I prioritized topics my kids had difficulty with."*

However, after participating in the first lesson study session, the mathematics coach, who was one of the knowledgeable others, immediately restructured the PLC meetings to be more collaborative, mirroring the lesson study approach. I observed that during the PLC meetings that followed the lesson study, ABC Elementary teachers were empowered to review the prescribed tasks embedded within the curriculum and make accommodations or adjustments based on their classroom needs—just as the participants had done in the lesson study cycle. The participants were instructed by the mathematics coach to look at the curriculum's activities as if they were students. This way, they could spot any issues, assumptions, or confusing parts that students might face.

Ms. Emily immediately recognized the palpable difference in the PLC that followed the first lesson study session. In her subsequent written reflection, she noted that the positive transformation of the PLCs could have been influenced by lesson study. She wrote, *"This is the first time I left the PLC meeting knowing how to teach the lesson."* Evidence curated from observations, interviews, and the participants' reflections all point to the lesson study's impact on enhancing the participant's teaching practices.

Feedback Integration and Reflective Practice

The lesson study allowed the participants to observe and adapt their collaboratively designed lessons, utilizing real-time student feedback. The synthesizing of feedback occurred during the debriefs following each lesson's implementation. Through this iterative process, each version of the lesson was continuously improved. During the reflective discussions, the participants analyzed areas for improvement. Engaging in reflective practices was a vital step in the participants' professional growth, as it enabled them to think critically about their own teaching methods and their impact on student learning. *Reflective Practice* enabled participants to share a platform where they could learn from their and their colleagues' experiences. Thus, *Reflective Practice* emerged as a complementary component to *Feedback Integration,* enabling the participants to attend to and integrate feedback. While feedback offered crucial insights, reflective practice created the necessary collaborative space for the participants to analyze and apply those insights to transform their instruction.

Ms. Peters, in her interview, shared, *"The process of observing students' interactions with the content and each other provided new insights that I never considered during the normal course of teaching."* She credited the lesson study with providing opportunities for her *"To deeply explore how the mathematical content impacts students in a way typical instructional did not."*

Through in-depth conversations like these, the participants deepened their comprehension of content in ways that traditional PLCs did not facilitate. Propelled by insights uncovered through reflective examination of *Lesson A* implementation, the team unanimously decided to pivot from the initial research plan. While this action step (discussed earlier in the chapter) adjusted the lesson plan schedule, it showcased the participants' fervent commitment to the lesson study as the participants were more interested in incorporating revisions into the lesson rather than adhering to a rigid scheduling guideline. As a result, the participants had the space to discuss improvements to the lessons and opportunities to improve their own understanding of the mathematical content.

During the exit interviews, all participants pointed to Ms. Deron's poignant explanation of the models as a pivotal turning point in furthering their comprehension of the linear model. Thus, reflecting on their practice unearthed the participants' unfinished learning and offered them opportunities to actively further their conceptual understanding. Ms. Lane shared in her interview, *"I definitely feel the lesson study enhanced my teaching…it provided me with a new lens to look through. I did not understand the differences or purpose of the models, and now I have a better understanding of them."*

Shift in Teachers' Thinking

The combination of feedback integration and reflective practice catalyzed shifts in the participants' thinking. Analysis of their reflections and interviews showed that they developed a deeper conceptual understanding of fractions instruction. At the inception of the lesson study, Ms. Peters and Ms. Lane expressed overt hesitation about modifying the standardized curriculum, with Ms. Peters questioning if it would *"upset the cart"* and Ms. Lane doubting the necessity by asking, *"Why bother changing it?"* Their stance reflected fixed mindsets rooted in curricular compliance and fidelity.

However, during their post-lesson study interviews, Ms. Lane openly admitted feeling empowered to adapt tasks intentionally, "*changing things to better align with* [her] *students' needs."* Ms. Peters also conveyed that she selectively *"deletes extraneous slides and hones in on critical information"* during lessons. Similarly, Ms. John stated that she adopted an *"improvement over compliance mindset."* She explained that she previously focused exclusively on coverage out of fidelity to daily lesson plans, but now she transitioned to modifying activities with intentionality to actively *"accelerate learning."* Ms. Mark stated that with lesson study, *"we can go from remediation to intervention."* This instructional move was birthed because the participants understood the lesson conceptually. Thus, they were able to modify the lesson to focus on the conceptual elements that could help their students better achieve the learning goal.

Another notable shift in participants' mindsets stemmed from the collaboration that the lesson study necessitated. In every post-lesson study interview, each participant spotlighted the immense value of collective planning, implementation, feedback, and reflection to enhance their instructional practices. Ms. Mark credited professional collaborations for expanding her pedagogical repertoire, admitting, *"Through collaboration, I learned how powerful it was to utilize technology in a lesson."* Ms. Lane also noted a dramatic difference in the quality of lessons constructed, finding that *"Usually I plan my own tasks and homework for mathematics, but through this process* [lesson study], *I saw the value in collective planning."* Based on their exit interviews, the participants shared the collective notion that the lesson study was not another 'add-on' they felt they had to engage in. This indicated that all participants saw value in the lesson study experience as it was an effective way to "support increased achievement, district and school curriculum alignment, data-based decision making, and quality teaching" (Wiburg & Brown, 2006, p. 126).

This shift in the participants' thinking revealed how they moved from a culture of isolation to a culture of collaboration. Data gathered from the interviews revealed that these changes helped the participants feel connected to their colleagues and the work at hand.

Novice teacher Ms. Emily offered an insightful comparison of the lesson study to the school's traditional PLC structure, noting, *"In PLC, we usually don't get to go through the mathematics tasks and make changes as a group. I don't always know what strategy to use, so this* [lesson study] *was beyond helpful."* Ms. Peters and Ms. John, who were accustomed to independent planning in isolation, acknowledged how the collaborative nature of lesson study yielded more robust lessons compared to their previous *'private practice.'* Thus, the growth in the participants' conceptual understandings of fractions acted as a stimulus for pedagogical improvements.

Pedagogical Adaptations

The participants' shifting mindsets catalyzed their pedagogical adaptations, which extended beyond the unit on fractions. During one session, Ms. John shared that she now "*Thoroughly reviews each task in the curriculum to ensure she includes conceptual components."* She further expressed feeling more confident adjusting or discarding tasks that did not meet the learning objective. Thus, her pedagogical adaptations stemmed from a shift in how she thought about instruction. Ms. Lane also experienced this shift in thinking, as evidenced by this excerpt from her post-lesson study interview:

> Before [the lesson study], I would just teach what was on the slides. I never really bothered with digital manipulatives. However, after I observed the students in the digital group engaging in discussions we did not anticipate, I knew I had to get on board with digital manipulatives.

This candid reflection revealed a compelling narrative of professional growth as Ms. Lane purposefully included a technological component to enhance engagement and understanding, something she did not do prior to the lesson study.

Another instrumental pedagogical adaptation was the participants' utilization of the districts' standards as a planning tool to identify the vertical and horizontal benchmark alignments of a mathematical domain. Before the lesson study initiative, most participants conceded that they did not actively engage with or consult the standards for mathematics when designing instruction. By the end of the lesson study, utilizing the standards was a more familiar activity. Ms. Deron shared the importance of vertical and horizontal planning by stating:

> Using the standards can enable teachers to design tasks that are accessible yet challenging with a low floor and high ceiling. This can ensure that all students, regardless of their skill level, find success and be challenged in the tasks.

At the conclusion of the lesson study, I checked in with all participants about their use of the standards. They all shared that they had begun utilizing the district issued standards document to plan further iterations of their mathematics lessons.

Summary of Teacher Development Codes

From my observations of the lesson study process, the participants' reflections, and interviews, the data revealed that the *Lesson Study* approach caused the participants to pay closer attention to the *Feedback Integration* produced by the students after interacting with the lesson. This feedback, when analyzed by the participants through *Reflective Practice,* ultimately led them to have *A Shift in Teachers' Thinking* about how content should be disseminated in the classroom. Over time, this reflective analysis and evolution in perspective led to observable changes in the participants' *Pedagogical Adaptations.* The data suggested that the lesson study process, alongside thoughtful reflection, supported meaningful professional growth for the participants. The following concept map summarizes my analysis of the *Teacher Development Codes.*

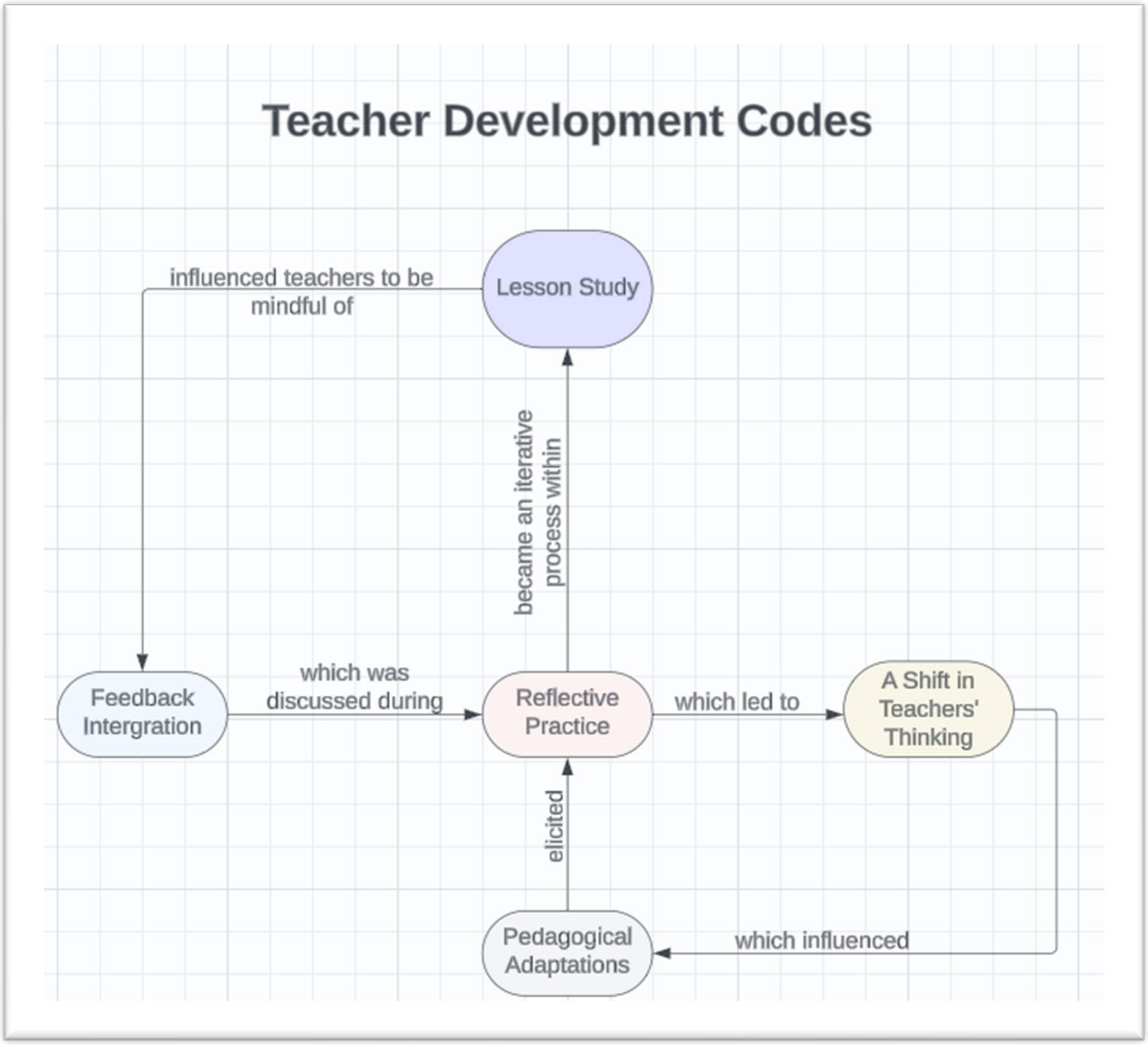

Figure 10 Teacher Development Codes

<u>Analysis of Student-Focused Instructional Shift Codes</u>

The lesson study process influenced the participants' pedagogical practices, specifically, their focus on integrating conceptual and procedural understanding in fraction-based lessons. The interconnected codes (*Procedural-Conceptual Integration, Student-Centered Considerations, Real-World Connections, Utilizing Visual Representations, Task Adaptations, and Promoting Deep Mathematical Thinking*) that surfaced during data analysis were deeply interconnected, with each one informing and influencing the others. For instance, the participants' focus on

Procedural-Conceptual Integration was heavily shaped by their *Student-Centered Considerations*, which in turn were informed by their efforts to make *Real-World Connections* for their students. Similarly, *Utilizing Visual Representations* and *Task Adaptations* was driven by the desire to promote deeper thinking and conceptual understanding among students.

Through the lens of these *Student-Focused Instructional Shift Codes*, I explored how the lesson study process fostered changes in the participants' pedagogical strategies and content knowledge.

Conceptual – Procedural Integration Through Task Adaptations

According to the mathematics coach at ABC Elementary, many teachers seemed to prioritize procedural mathematics instruction. During the post-lesson study interview, she shared:

> Teachers, especially in grades 3–5, tend to emphasize the procedural fluency of mathematics tasks as they are in testing grades. Students are primarily taught to memorize procedures without grasping the conceptual 'why' behind these procedures. Since third grade is a retention grade, the teachers tend to feel pressured to ensure that students are passing both their formative and summative assessments. Everyone is looking at their data; instructional decisions are made because of their data, and the pressure to perform is real.

The mathematics coach was not alone in her thinking; during the third debrief session, Ms. John brought up the contentious issue of high-stakes testing to the group, admitting that she was *"feeling pressured to teach to the test, as administration watches mathematics scores with hawk eyes."* She further went on to add, "*I do prioritize procedure over conceptual understanding. However, I know this is not the best practice."* Ms. Lane echoed a similar sentiment by stating:

> I know it is meant to be a form of accountability when the proficiency level of each class is shared after we take a test. However, it is demoralizing when my class is constantly in

> last place. That does something to my ego, and it is hard not to take it personally. So yes, I teach to the test, because of the hidden expectations that exist.

This statement was met with nods and murmurs of agreement from the participants, with Ms. Peters adding:

> There is so much data we are expected to be on top of….. end of unit tests, reassessments test, practice tests…. We are constantly testing the students. From January until the FAST testing, I push test-taking strategies because, at the end of the day, what people care about is how many kids passed.

These candid revelations highlighted the unrelenting tensions between best teaching practices and standardized testing pressures that the participants face.

Nevertheless, through the collaborative power of lesson study, all the participants were willing to engage in a transformative practice that prioritized building conceptual understanding as a precursor to procedural knowledge. Ms. Bob stated during the second debrief:

> Despite most teachers in testing graders prioritizing procedural knowledge to improve students' mathematical scores, the sad reality is that every year, there are huge gaps in students' mathematical understanding. To break these patterns, we need to revamp how math is taught.

This statement further spurred the participants to invest in building students' conceptual knowledge since it laid the essential groundwork for students to deeply understand procedural skills in mathematics (Hiebert, 1984).

Ms. Peters shared that, prior to the lesson study, she pre-taught mathematical vocabulary by posting and defining mathematical terms on the classroom wall before students actively used or understood those terms. However, after watching Ms. Deron's strategy of having students ascribe meaning to terms such as the ‘numerator’ and ‘denominator,’ she adjusted her instructional repertoire. During the debrief, she rationalized that:

> Having students explain what the terms mean, instead of directly telling them, is much more powerful. The goal of the lesson is not just that they learn the words 'numerator' or 'denominator.' The goal is for them to identify that one piece out of a whole is a unit fraction. Pushing vocabulary without sense-making is a waste of time.

The other members of the group echoed her sentiment, with Ms. Mark adding, *“Memorizing vocabulary when the concept isn’t there is not productive.”* Ms. Lane summarized that *“showing by using manipulatives and telling through student discourse provided a valuable framework for teaching conceptually rich mathematics.”*

Another adaptation to the mathematics lesson that aided in developing deeper conceptual learning was utilizing sentence stems. Ms. Deron, ABC Elementary's mathematics coach, had noticed during her classroom walkthroughs that many students struggled to articulate their ideas and problem-solving processes coherently when working in groups. The participants recognized that the mathematics lesson, which required students to express their thinking using academic language, introduced a skill that was not commonly practiced in the classroom.

Realizing that students needed support to effectively verbalize their conceptual thinking, the participants decided to provide scaffolding. Ms. Deron suggested using structures like sentence stems (see Figure 11) to guide students’ understanding of key mathematical concepts, such as numerators and denominators.

> There is _____ whole chocolate.
>
> ____ friends share the chocolate.
>
> Each friend will get _____ out of _____ pieces.

Figure 11 Sentence Stem

The following excerpt captured Ms. Deron’s explanation to the lesson study participants on how the sentence stems could be effectively utilized to deepen conceptual understanding:

> The first sentence guides the students to identify that there is one whole unit we are working with. The second sentence then helps students partition the number of equal parts that the whole unit will be divided into. The third sentence promotes understanding that each person receives one share out of that total number of equal parts.
>
> Through hands-on exploration with manipulatives, the students realize that the numerator in a fraction represents the share they are specifically examining— in other words, their individual piece. The denominator represents the total number of equal pieces that make up the whole. When the students are filling in those blanks, they're doing two things; they're practicing how to talk about fractions and they're watching us connect those words to the actual fraction symbols - the numbers above and below the line. So as the kids are saying 'one out of five equal pieces' in the sentence frame, we’re also writing $\frac{1}{5}$. Instead of just seeing numbers and lines, those fraction notations start to make sense to the students. This ties back to the conceptual list we developed as evidence of students’ understanding.

This detailed explanation about the use of sentence stems encouraged the participants to implement this adaptation as a way to help students communicate their ideas more clearly. To further develop the conceptual aspect of the lesson on unit fractions, Ms. Deron reminded the group of *Catalyzing Change's* (NCTM, 2020) emphasis on student-centered instruction and real-world connections.

Student-Centered Considerations and Real–World Connections

The a priori codes *Student-Centered Considerations* and *Real-World Connections* were found to be interconnected and complementary in the lesson study. The participants recognized that the lack of connection between the mathematics lessons, and the students' lived experiences was a barrier to engagement and understanding. To address this, they made a concerted effort to relate every aspect of the modified lessons to the students' everyday experiences, such as using examples from the upcoming Florida Classic weekend and everyday items like pens, pencils, and cookies to explain unit fractions.

Another student-centered consideration was the need for smaller group sizes to ensure equal participation. Ms. John suggested assigning specific roles (*Captain, Resource Manager, Speaker, and Recorder*) to each student within the reduced-size groups to promote active engagement and a sense of ownership over the learning process. Ms. Deron added that the participants should strategically assign these roles based on each students' strengths and areas for growth.

Thus, the interplay between the codes *Student-Centered Considerations* and *Real-World Connections* demonstrated that grounding mathematics lessons in contexts familiar to students' lived experiences facilitated greater engagement and attention, aligning with the goals of student-centered instruction. A similar interplay was found between the codes *Promoting Deep Mathematical Thinking* and *Utilizing Visual Representation*.

Promoting Deep Mathematical Thinking and Utilizing Visual Representations

The participants in the lesson study aimed to foster deep mathematical understanding by using visual representations and open-ended questioning. They recognized that relying on procedures was insufficient, and students needed more time to actively explore fractions through concrete experiences. Ms. Bob advocated for using visual representations, such as partitioning tasks with concrete items, to encourage students to reason through the mathematics lesson.

After implementing an activity where students divided same-sized paper fractions, Mrs. Lane suggested using unequal paper lengths to elicit more diverse responses and richer discoveries. This modification led to insightful comparisons and a deeper understanding of how the size of the whole affects the size of the unit fraction.

The participants also shifted from using closed-ended questions to open-ended, higher-order questioning strategies. Ms. Deron introduced the Instructional Quality Assessment (IQA) toolkit, which emphasized the importance of asking open-ended questions that prompt students to justify their answers. The participants crafted sample open-ended questions and applied this technique across various disciplines. Ms. Emily and Ms. Mark observed that asking students to justify their solutions deepened their comprehension of foundational mathematical principles, as they were able to perform procedures, articulate their thinking, and defend their mathematical reasoning. After leading the team through crafting these new question types, the participants applied this questioning technique in other disciplines.

Summary of Student-Focused Instructional Code

The *Student-Focused Instructional Shift Codes* that emerged from the data analysis were deeply interconnected and influenced the participants' pedagogical practices in the lesson study. The participants recognized the need to prioritize building conceptual understanding as a precursor to procedural knowledge despite the pressures of standardized testing. They adapted their instructional strategies, such as using sentence stems and manipulatives, to support students in verbalizing their conceptual thinking and developing a stronger grasp of mathematical concepts. The *Student-Focused Instructional Shift Codes* highlighted the transformative impact of the lesson study process on the participants' pedagogical strategies and content knowledge as they collaborated to create a more student-centered and conceptually rich mathematics learning experience. Figure 12 summarizes the interplay of the *Student-Focused Instructional Shift Codes*.

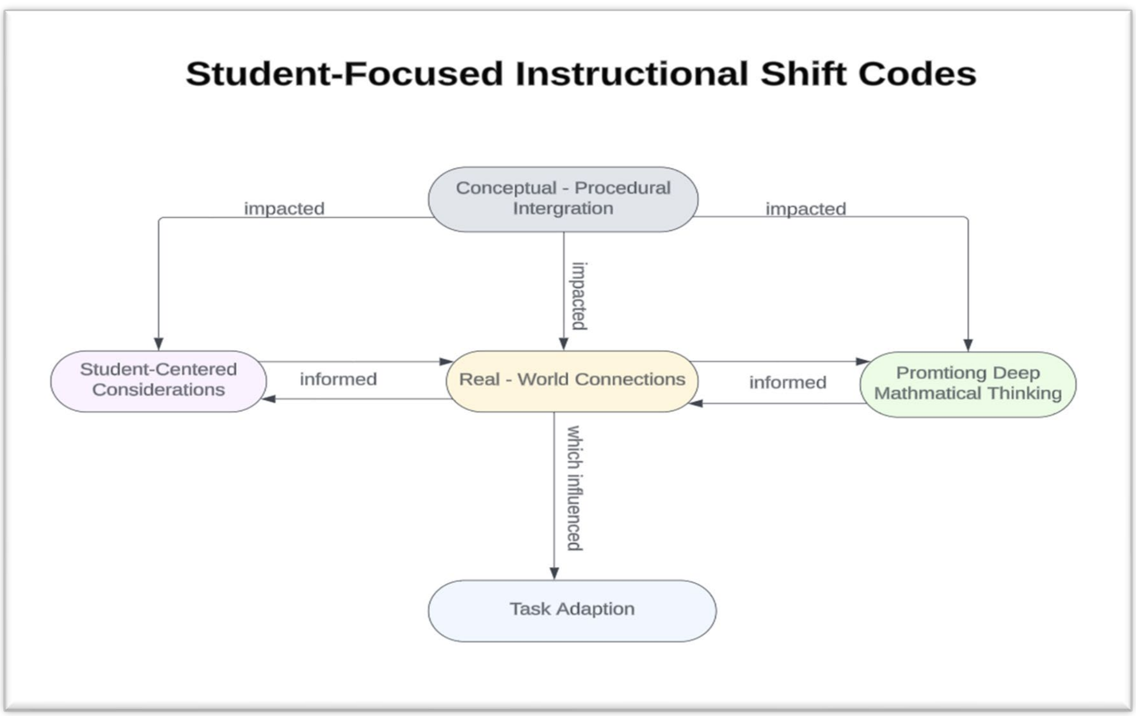

Figure 12 Student-Focused Instructional Shift Codes

Trends Showcasing Participants' Development

The data collected and analyzed throughout this study provided valuable insights into how the participants' engagement in the lesson study influenced their instructional decision-making processes and the construction of pedagogical strategies that integrate procedural and conceptual knowledge. By employing Yin's (2013) case study methodology and utilizing a priori codes, I could identify specific trends that emerged from the data that aligned with my initial propositions.

The first research question in this study was: *"How does the engagement of teachers in the creation and analysis of fraction-based mathematics tasks in lesson study influence their instructional decision-making processes?"* The primary focus of this study was to investigate whether and to what extent the participants' engagement in a lesson study centered on fractions influenced their instructional decision-making processes when teaching fractions in their classrooms. My proposition posited that the iterative lesson study approach would empower the participants to blend procedural and conceptual knowledge more effectively by facilitating reflective pedagogical refinements. Using that position as my lens, I analyzed the data that resulted from the a priori *Teacher Development Codes*.

The data resonated with my proposition and substantiated it. The participants' engagement in creating and analyzing fraction-based mathematics lessons influenced their instructional decision-making processes, such as what examples to use, how to sequence ideas, what questions to ask students, and how to check for understanding. In addition, the participants reflected on their teaching practices through the lesson study, observed the outcomes, and adjusted their strategies accordingly.

This reflective mindset was crucial for developing and refining a repertoire of pedagogical strategies that blend procedural and conceptual knowledge. As the participants became more reflective, they could better assess the effectiveness of their instructional tasks and make informed decisions that enhanced their knowledge and the learning experience for their students. The impact of the lesson study was not limited to fractions but also extended to other topics in mathematics and as well as cross-curricular activities. This analysis of the coded data revealed the following trends:

- *Authentic Collaborative Inquiry and Professional Growth:* True collaboration, coupled with reflective practice, reduced the participants' isolated instructional planning and promoted pedagogical and content knowledge growth.
- *Shift towards Growth Mindset and Adaptability:* The lesson study empowered the participants to break away from rigidly adhering to a prescribed curriculum, fostering instead a growth mindset that enabled them to experience more autonomy to make informed instructional changes aligned with their students' diverse needs.

The second research question that guided this study was: *How do teachers construct pedagogical strategies that integrate procedural and conceptual knowledge through lesson study?* My proposition was that the active involvement of the participants in the lesson study would cultivate a reflective mindset. It would enable them to develop a repertoire of pedagogical strategies that would effectively blend procedural and conceptual knowledge.

Data collected revealed that through engagement in the lesson study, the participants were more attuned to the specific mathematical content that needed improvement or reinforcement to support student comprehension. The participants realized that by integrating real-world connections and examples reflective of their students' lives into the mathematics lessons, they could make the content more accessible, relatable, and overall engaging. This led to adapting mathematics lessons and instructional approaches to incorporate increased student-centered considerations and concrete links to existing experiences outside of the classroom. Analyzing the data for the *Student-Focused Instructional Shift Codes* revealed the following trends:

- *Shift Towards Conceptual Grounding:* The participants prioritized aligning conceptual knowledge in the fraction-based lessons.
- *Task Adaptation for Deeper Understanding:* The participants' task adaptations involved intentional redesigns aligned with learning objectives while embedding real-world connections and student-centered considerations.

Thus, the data analysis corresponding to these codes not only reinforced but also lent credence to my propositions that the reflective mindset fostered by lesson study encouraged the participants to develop a repertoire of pedagogical strategies that made the mathematics lessons more engaging and meaningful.

Overarching Themes

Identifying the trends from the *Teacher Development Codes* and *Student-Focused Instructional Shift* Codes led to the discovery of two critical overarching themes. The first theme I identified was the *transformative power of lesson study.* The lesson study approach facilitated meaningful changes in the participants' mindsets, pedagogical skills, and instructional practices.

Participating in collaborative inquiry during the lesson study encouraged the participants to move away from rigidly following the prescribed curriculum and instead take ownership of adapting mathematics tasks to meet students' learning needs. By working jointly to re-conceptualize procedural tasks, the participants aligned conceptual foundations with procedural fluency when adapting curriculum materials. This professional growth then rippled outwards as the mathematics coach adopted lesson study elements into school-wide PLCs.

The second theme identified was *a shift toward more conceptual, student-centered mathematics instruction*. The participants realized the need to align students' conceptual understanding before emphasizing procedural fluency. Their task adaptations focused on connecting mathematical ideas to students' real lives, using multiple representations, improved questioning, and technology integration to stimulate deeper engagement. They compared their co-created activities with the Evidence for Conceptual Understanding graphic (see Figure 7) to ensure that students were afforded the opportunity to build their conceptual understanding. The lesson planning became centered around student considerations - their lived experiences, needs, skills, and early conceptions. This conceptual, student-centered approach facilitated richer learning.

Both themes centered on growth in instructional mindsets, catalyzed by repeated peer collaboration, observation, and knowledge exchange. Evidence from my observations, the participants' reflections, and interviews revealed that the lesson study strengthened the interplay between conceptual and procedural understanding. Thus, the case study offers evidence supporting my initial propositions. Figure 13 provides a visual of how the trends were condensed into themes.

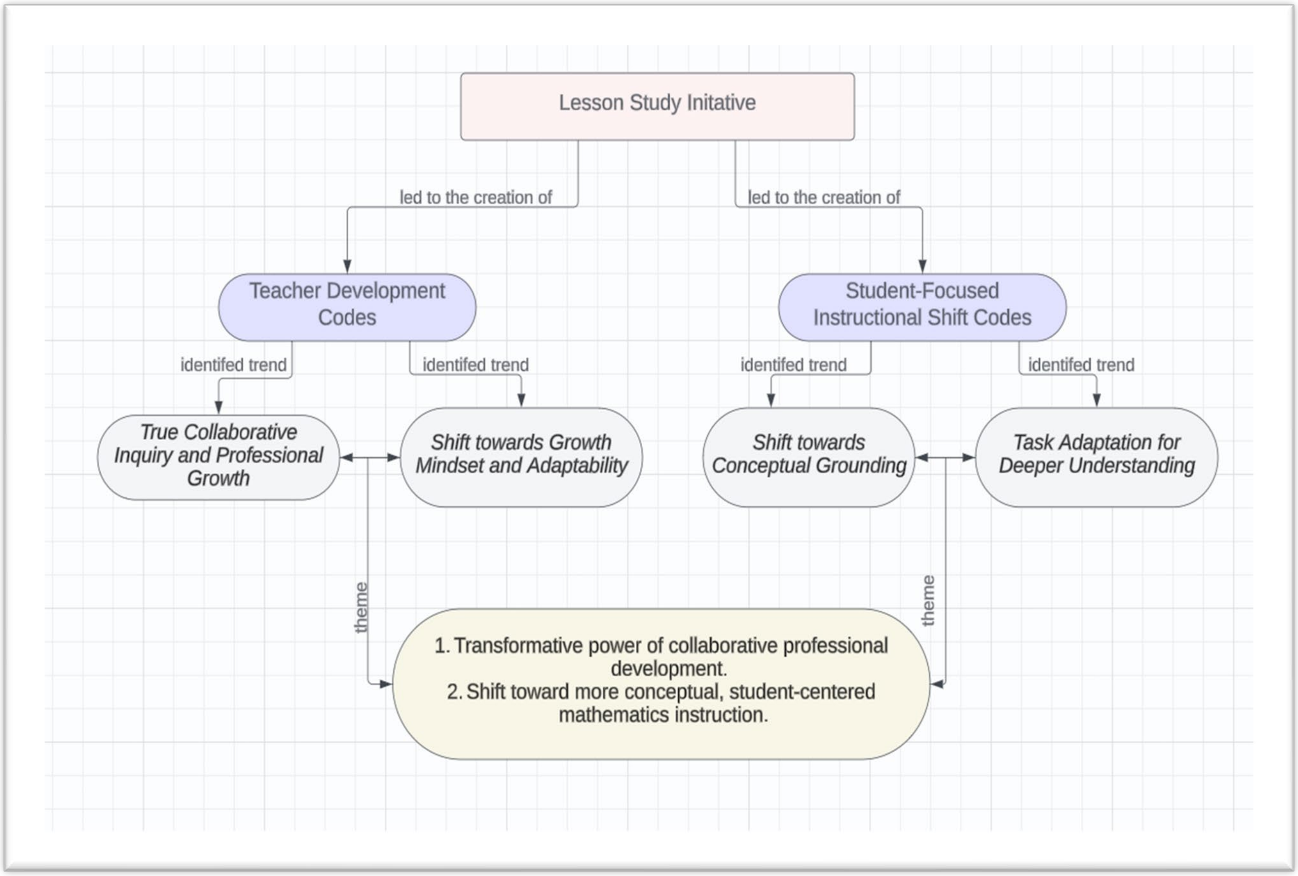

Figure 13 Process of Trends to Themes

<u>Conclusion</u>

This study explored two interconnected questions related to the impact of lesson study on the participants' development, explicitly situated in the domain of fraction concepts. The first research question was:

- How does the engagement of teachers in the creation and analysis of fraction-based mathematics tasks in lesson study influence their instructional decision-making processes?

This question required taking a broad view to examine how immersive engagement in collaborative planning, teaching, observing, and debriefing of fraction-focused lessons could influence overall instructional decision-making processes. The findings suggest that repetitive collaboration cultivated a growth mindset that empowered the participants to make adaptive instructional decisions that aligned with students' needs. Additionally, by working jointly to analyze learning and refine teaching, the participants benefited from professional growth in both pedagogical and content knowledge realms.

The second research question that guided this study was:

- How do teachers construct pedagogical strategies that integrate procedural and conceptual knowledge through lesson study?

This question examined how the participants used the lesson study process to collaboratively create pedagogical strategies that blend conceptual understanding with procedural fluency. The results pointed to prioritizing conceptual grounding before building procedural knowledge as a key pedagogical approach nurtured through lesson study.

This chapter demonstrated how iterative and reflective participation in lesson study impacted instructional approaches. Specifically, the participants' growth in adaptability, conceptual integration, and student-centered task design were highlighted. Thus, the overarching themes identified through this case study were the transformative power of collaborative lesson study and a shift toward more conceptual, student-centered mathematics instruction.

In Chapter 5, I discuss these themes' implications and significance for educational practice and future research.

CHAPTER 5: CONCLUSION

The United States, as a global leader in education, recognizes the significance of mathematics education in preparing students for the challenges and opportunities of the 21st century. The nation's commitment to continuous improvement and innovation in mathematics education is evident through its investments in reform policies and modernizing instructional practices (U.S. Department of Education, 2019). According to the Goals 2000: Educate America Act (P.L. 103-227), which was signed into law on March 31, 1994, by President Bill Clinton, "by the year 2000, United States students will be first in the world in mathematics and science achievement" (US Metric Association, 2015, p. 1). To achieve this ambitious goal Education Secretary Miguel Cardona asserted that, "We need a math revolution" given that "math is critical to our global competitiveness and leadership" (Saric, 2023). Thus, transformative change is required in both the content that is taught and how it is executed. To tackle these concerns, my qualitative case study conducted at ABC Elementary School with seven participants was guided by these two research questions:

- How does the engagement of teachers in the creation and analysis of fraction-based mathematics tasks in lesson study influence their instructional decision-making processes?
- How do teachers construct pedagogical strategies that integrate procedural and conceptual knowledge through lesson study?

Through my first research question, I explored if and how participating in a collaborative lesson study that focused explicitly on fractions impacted how the participants made instructional decisions. These decisions impacted the types of fraction tasks they chose to use, the sequence and structure of fraction lessons, the questions they asked students about fractions, the learning materials used in fraction instruction, and how student learning was assessed.

The second research question probed how lesson study participation enabled the participants to build pedagogical strategies that aligned conceptual understanding with procedural fluency. Specifically, I looked at the strategies the participants employed with students to build their understanding of fractions, their identification of unit fractions, the relationship between the numerator and denominator, and the various representations and manipulatives utilized for this process.

To fully capture the lesson study process and its impacts, I employed a multifaceted qualitative data collection strategy. This included conducting in-depth interviews with the participants to understand their perspectives and experiences in detail. I also directly observed their process during the lesson study and the subsequent implementation of the lesson as it unfolded in order to document real-time interactions and engagement. Furthermore, I observed the reflective debriefing sessions where the participants analyzed and discussed key aspects of the lesson and their students' learning with peers. The findings from this study were nuanced and comprehensive, with substantial intersection across the two key themes that emerged.

The first finding showed that repetitive collaboration among educators cultivated a growth mindset that empowered them to make adaptive decisions aligned with students' needs while experiencing professional pedagogical and content knowledge growth. The second finding revealed that the lesson study approach helped teachers prioritize conceptual grounding before building procedural knowledge by incorporating student-centered considerations cultivated through iterative collaborative planning.

In this final chapter, I expound on these findings as they relate to the literature on utilizing a balanced approach to instruction to enrich students' mathematical comprehension and learning outcomes. I also share the implications related to the use of lesson study, as well as its strengths and limitations. The chapter concludes with suggestions for future research.

Discussion of Major Findings

The findings of this study underscore the potential of lesson study to catalyze transformations and cultivate robust mathematical agencies in the participants. These findings align with the conceptual framework, which incorporates *Catalyzing Change* (NCTM, 2020), Hiebert's (1984) Site Theory research on aligning procedural and conceptual knowledge, and Wenger's (1998) CoP theory. The findings from this study were analyzed through the lens of how lesson study can serve as a catalyst for transformative change in mathematics education.

Collaborative Vulnerability

Reflection is a powerful tool that orchestrates change. Collet (2019) described reflection as "a deliberate and thoughtful act where one revisits and critically assesses their experiences to extract learning for professional practice" (p. 75). Through lesson study, all participants had opportunities to engage in this type of purposeful reflection, which catalyzed a shift in their mindsets and subsequently influenced their approach to teaching. It also helped them become more attuned to their own learning needs in mathematics (Wenger et al., 2002).

A critical incident highlighted the importance of vulnerability and empathy within the lesson study process. When I shared the feedback about the misalignment of the linear model with Ms. Deron and the other participants, it required me to be vulnerable and acknowledge my own oversight. This vulnerability opened the door for empathy and mutual learning, as it created a safe space for all participants to admit their own unfinished learning without fear of judgment.

While it was personally uncomfortable to acknowledge this oversight, it ultimately provided an opportunity for my own professional growth. This experience underscores the fact that even those deemed as experts can have areas of unfinished learning. This suggests that a more nuanced definition of the knowledgeable other is warranted. An updated definition should include that the knowledgeable other is not necessarily the sole source of expertise but is someone who is willing and capable of resourcing information, as well as learning, unlearning, and relearning as necessary. This perspective allows for professional growth and acknowledges that expertise is not static but develops through continuous reflection and collaboration.

My experience should serve as a cautionary tale to any teachers engaging in lesson study. The role of an expert does not necessitate having all the answers. Rather it requires adaptability, the ability to resource new information, and openness to learning from others. I encourage teachers not to be intimidated by the prospect of being an expert, as it is an opportunity for continuous learning and growth.

Moreover, errors can serve as springboards to new understanding. When the discrepancy between the intended use of a linear model and the actual use of an area model persisted, it provided an opportunity for deeper exploration and learning. By openly discussing and reflecting on these errors, the participants gained a more comprehensive understanding of the models and their applications. Thus, embracing vulnerability and empathy within the lesson study allows for a more authentic and transformative learning experience for all involved.

Even when the correct use of the linear model was explained, the discrepancy between the intended use of a linear model and the actual use of an area model persisted. This suggests that while the participants were exposed to and acknowledged their new learning about the linear model, this limited exposure may not have been sufficient to fully transform their content knowledge.

The participants recognized that their superficial understanding of the content encouraged their adherence to the prescribed curriculum because “when teachers do not feel confident and fluent with the content, they are teaching, they tend to just follow the textbook and engage in a teacher-directed model only” (Wiburg & Brown, 2006, p. 111). By fostering a space where teachers could openly discuss their unfinished learning, the lesson study embodied a key aspect of CoP: developing a shared repertoire of resources, experiences, and ways of addressing recurring problems (Wenger, 1998).

Professionalization of Teaching

Furthermore, this honest self-reflection, which served as a powerful stimulus for growth, motivated the participants to learn by doing instead of being told what to do. Thus, the expectation that professional learning of mathematics was limited only to the scheduled days for PLC was eliminated. This shift towards professionalizing teaching meant that the participants actively sought opportunities for more relevant learning opportunities embedded into their daily work lives. This aligns with the wealth of research evidence that points to the effectiveness of Japanese lesson study approaches for transforming instructional practices in the United States when sufficient support systems are in place (Akiba & Wilkinson, 2015; Lewis & Perry, 2017).

Furthermore, the lesson study afforded the teachers opportunities to observe live interactions between the students and the learning content. According to Collet, “Those who observe the research lesson are freed from the ongoing, intensive brainwork of on-the-spot decision-making" (2019, p. 63). This freedom allowed the participants to focus on students' learning needs and adapt their instructional approach accordingly.

The decision to implement *Lesson A*, a third time, rather than rigidly moving to *Lesson B*, highlighted the participants' newfound commitment to responsive, meaningful instruction tailored to student needs. The participants' improvement ideas embodied the iterative ethos of lesson study, where continuous refinement is valued over a fixed number of revisions (Fernandez & Yoshida, 2004). This sent a powerful message about prioritizing mathematics education as they extended the research timeline to optimize *Lesson A* despite scheduling constraints. This move aligned with the real intentionality behind lesson study, which is "not the lessons themselves, but rather how the lessons relate to student learning" (Wiburg & Brown, 2006, p. 11).

Communities of Practice

The initial structure of PLCs reinforced the 'privatization of teaching' as it focused on desegregating data to identify clusters of students for remedial work. DuFour et al. (2009) referred to this practice as DRIP – data rich, but information poor as the "data alone will neither inform nor improve a teacher's practice" (DuFour et al., 2009, p. 26). As a result, teachers concentrated on aspects of their own teaching that contributed to poor scores, leading each teacher to prioritize the learning needs of their respective classes in isolation.

Furthermore, some participants acknowledged that conceptual knowledge was more effective than procedural teaching. However, they admitted to teaching procedurally due to perceived constraints within their educational settings. Pfeffer and Sutton (2000) described this phenomenon as the 'knowing-doing gap,' where individuals are aware of best practices but fail to implement them. This discrepancy often stems from an unspoken expectation to conform to established norms within the institution.

However, the lesson study facilitated a transformation to this mindset by encouraging the participants to change their thinking of ‘my kids, your kids’ when discussing students to an assumption that these were ‘our kids.’ Perhaps “the single most important factor for a successful school restructuring and the first order of business for those interested in increasing the capacity of their schools is building a collaborative internal environment that fosters cooperative problem-solving and conflict resolution” (Eastwood & Louis, 1992, p. 215).

The insights gained through the collaborative lesson planning and analysis fostered a shift in the dynamics of the PLC meetings at ABC Elementary. The mathematics coach reshaped the structure of the PLC meetings to align with the principles and practices of lesson study. For example, the participants were tasked with examining the activities housed within the curriculum through the perspectives of a student so they could pinpoint potential obstacles, early conceptions, or areas of confusion that students might encounter. These insights created opportunities for the participants to share strategies and interventions that could address and preemptively counteract the anticipated challenges, thus increasing the potential for a smoother learning trajectory for students. This transformative change to PLC was not limited to third grade as it resonated across all ABC Elementary’s campus.

This signaled the participants’ alignment with *Catalyzing Change's* (NCTM, 2020) vision of promoting responsive, equitable instruction grounded in building students’ conceptual mathematical understanding. This finding also demonstrated the power of CoP in sharing insights and promoting the spread of effective practices.

Newfound Pedagogical Strategies

Furthermore, the participants learned how to adopt new pedagogical strategies through the collaborative power of the lesson study. These strategies aligned with Level 4 of the Instructional Quality Assessment (IQA) (Boston et al., 2019) and required students to make conjectures, provide reasoning to support their ideas, and justify their solutions. Rather than just providing answers, the students needed to demonstrate the reasoning verbally or physically behind their responses. This required them to explain their thought processes and engage intellectually with the materials.

Thus, analysis of students' responses within lesson study cycles cultivated the participants' agency to create opportunities to offer rich, mathematical tasks that foster meaning-making and prompt students to explore their curiosity (Akiba & Howard, 2021; Hiebert, 1986; Stein et al., 1996). As they made adjustments to specific lessons and witnessed the positive impact on student learning, they celebrated the small victories they achieved.

DuFour et al. (2009) suggest that maintaining the momentum for change involves more than simply achieving minor successes. It also requires recognizing and celebrating these achievements while acknowledging the contributions of the people who made them possible. By engaging in reflective practice and collaboration, the participants' recognition of the small victories or positive changes in their classrooms helped reinforce the efficacy of the lesson study and sustained momentum for change, aligning with the principles of CoP theory (Wenger, 1998). This celebration of their collaborative efforts was something new to them, and it fostered a sense of camaraderie that had previously been lacking in their PLCs.

Recognition of the Diverse Needs of Students

The lesson study increased the participants' attention to their students' needs. They better understood where their students were in the learning process and the gaps between the state standards, benchmarks, and classroom instruction (Wiburg & Brown, 2006). Their understanding of their students' unfinished learning empowered them to develop instructional content that effectively addressed the discrepancies between the students' current knowledge and the expected learning outcomes.

Instead of solely relying on data from standardized tests, they started to place greater emphasis on evaluating students' thought processes via academic discourse. This approach provided a more comprehensive picture of the students' learning progress and enabled the participants to make informed decisions about their instructional strategies.

Increased Autonomy

By better understanding their students' needs, the participants became empowered to exercise their professional judgment in prioritizing meaningful mathematical content over strict adherence to the prescribed curriculum (Lewis & Perry, 2017; NCTM, 2020). In their post-lesson study interviews, all participants shared that having the confidence to adjust instruction—whether by incorporating supplemental representations or examples, providing individual remediation, or re-teaching essential concepts—was more impactful for enriching their students' conceptual understanding than rigidly adhering to scripted curricular lessons. Thus, they felt that quality superseded quantity, as they measured success not by the number of tasks or slides presented but by the depth of students' conceptual understanding.

They further added that they shifted their focus to providing opportunities where students could see themselves as 'the doers of mathematics' (NCTM, 2020). This intentional effort to position students at the center of the learning process directly corresponds to one of the key principles outlined in *Catalyzing Change* (NCTM, 2020), which calls for developing deep mathematical understanding by engaging students in meaningful, student-centered learning experiences.

The participants added that they now restructure their mathematics block to create opportunities for students to delve deeper into the underlying principles and ideas behind mathematical concepts instead of solely focusing on procedural tasks. They continue to draw from Hiebert's (1984) Site Theory to ensure students can connect the visual aspects of unit fractions to their symbolic, procedural representations (Site 1).

Instead of emphasizing memorizing rules, they now encourage students to develop a deep understanding of unit fractions by relating them to familiar objects and experiences, thereby facilitating meaning-making. They then strengthen students' conceptual grasp of the material by reinforcing the reasoning behind mathematical rules through these models (Site 2). Finally, they encourage students to provide justification for their solutions or assess the reasonableness of their answers (Site 3). According to the participants, this balance between concrete, representational, and abstract thinking within a mathematical task is paramount for fostering holistic student comprehension (Hiebert, 1986; Osana & Pitsolantis, 2011; Rittle-Johnson, 2019; Star, 2005).

Creation of Equitable Structures

The participants worked towards transforming their classrooms into engaging and exploratory spaces where students could experience the inherent beauty, wonder, and joy of mathematics (NTCM, 2020). By incorporating manipulatives, models, and technology into their lessons, the participants provided students with tangible ways to represent abstract mathematical concepts and forge connections between ideas.

This approach not only made mathematics more accessible for students but also fostered a sense of equity by ensuring that all learners, regardless of their background or knowledge, had the opportunity to engage with the learning content in a meaningful and relevant way. By creating these inclusive and stimulating learning environments, the participants actively positioned students as active meaning-makers in the learning process rather than passive recipients of mathematical knowledge (NCTM, 2020), thus contributing to the *Catalyzing Change* vision of equitable and high-quality mathematics education for all students.

The major findings of this study underscore the transformative potential of collaborative professional engagement through lesson study and the shift toward conceptual, student-centered mathematics instruction. By integrating the conceptual framework components – *Catalyzing Change*, Hiebert's (1984) Site Theory, and CoP (Wenger, 1998) theory – the findings contribute to the existing literature and theories in the field of mathematics education, highlighting the importance of collaborative, reflective, and responsive teaching practices in fostering equitable and meaningful learning experiences for students.

Implications of Findings

Based on the findings from this study, several key implications emerged. The first implication highlights how genuine collaboration among the participants can positively transform mathematics education. Collaboration inspired changes in the participants' mindsets and encouraged them to prioritize more student-centered and conceptually focused instruction, aligning with the recommendations outlined in the *Catalyzing Change* framework (NCTM, 2020). Thus, investing in lesson study cycles can create and sustain a culture of continuous learning among educators (Wiburg & Brown, 2006). Therefore, the administration should create environments where lesson study can thrive. They can do so by placing value on the participants' learning, making resources and time available for their work, encouraging participation, and removing barriers to learning (Akiba & Howard, 2021; Lewis & Perry, 2017; Wenger et al., 2002).

Another implication of this study revealed that the participants needed opportunities to enhance their pedagogical content knowledge about mathematical topics (Huinker & Bill, 2017). Having a non-evaluative environment allowed the participants to openly acknowledge their unfinished learning, such as a lack of awareness regarding the distinctions between the linear and area models for fractions. It also highlighted the fact that the participants needed opportunities to explore linear models, experiment with different representations, and discuss their understanding with colleagues to build a more robust conceptual foundation.

To change ingrained pedagogical content knowledge takes time and effort (Akiba & Howard, 2021). Therefore, building and maintaining a safe space where participants are vulnerable enough to acknowledge their own limitations and receive sufficient time to internalize new learning with support from knowledgeable others can open avenues where they feel comfortable discussing their challenges and seeking support (Akiba et al., 2019; Wiburg & Brown, 2006).

In addition, as the participants learned how to become more autonomous within the safe space cultivated through lesson study, the wealth of collective content knowledge amongst the participants empowered them to become content creators and not just content enforcers (Akiba & Howard, 2021). This shift instilled a deeper sense of ownership over their lessons, fostering an environment where they felt empowered and invested in crafting tailored instructional experiences to meet the needs of their students (Lewis & Perry, 2017).

One of the most notable implications was the shift from individual teaching practices to a collective mindset of shared responsibility for student learning. They referred to the students as 'our kids' and worked collaboratively to ensure that they were delivering high-quality mathematics instruction for all students. Their combined efforts provided tangible results that each participant later witnessed within their classroom (Fernandez & Yoshida, 2004). This encouraged them to celebrate their small victories, reinforced their purpose, and fostered a sense of camaraderie among the participants. This action helped maintain the participants' motivation and commitment to the lesson study because when the participants feel valued and supported in their efforts to transform their teaching practices, they are more likely to persist in the face of challenges and strive for excellence (NCTM, 2020).

Another implication of this study was the combined use of the conceptual frameworks—*Catalyzing Change* (NCTM, 2020), Hiebert's (1984) Site Theory, and CoP (Wenger,1998)—to advocate for sustainable change in mathematics education. The intersection of these frameworks provided valuable insights and strategies for promoting conceptual understanding and equitable learning in the classroom. Finally, sharing the successful practices and insights gained through collaborative professional development with the wider school community at ABC Elementary contributed to the system-wide improvement, as evidenced by the restructuring of the PLC meetings. When cohorts of participants with varying experience and content knowledge work together and share their experiences and knowledge with others, they can help to create a culture of continuous learning and improvement that benefits all students (Wenger et al., 2002; Wiburg & Brown, 2006).

Implications for the Field and Profession

At the local level, the insights gained can potentially shape instructional practices at ABC Elementary. Because teachers are arguably one of the most important factors impacting student outcomes, the role of administrators and mathematics coaches can be modified to actively support them by providing resources, time, and space to engage in discussions about their work (Wiburg & Brown, 2006). For example, the purpose and function of PLC meetings can be revitalized to align with the lesson study principles of collaborative planning of activities and tasks, which foster conceptual understanding.

Furthermore, teachers may need additional support to teach mathematics "in a manner that supports student-directed, problem-based learning" (Wiburg & Brown, 2006, p. 12). This can occur by providing opportunities where teachers can observe and learn from knowledgeable others from their campus or other educational institutions (Akiba & Howard, 2021; Wiburg & Brown, 2006). These partnerships could involve university faculty providing professional learning workshops, consulting on curriculum design, or even co-teaching lessons with classroom teachers. By working closely with university experts, teachers can gain valuable insights into best practices and stay up-to-date with the latest research findings. Learning from observing and reflecting on the practices of knowledgeable others, can provide fluid and adaptive best practices that may differ from the traditional procedural approaches the teachers experienced themselves (Akiba & Howard, 2021; Akiba et al., 2019; Wiburg & Brown, 2006). By working with knowledgeable others, a community of practice can develop where teachers are actively supported in creating conceptual mathematics.

Additionally, sharing the findings regarding the efficacy of lesson study for facilitating conceptual teaching could reshape teacher training programs and professional learning opportunities. For example, teacher preparation programs could consider incorporating lesson study as a core component of their curriculum, if they have not already done so, giving pre-service teachers hands-on experience with collaborative planning and reflective practice. Similarly, if not already implemented, PLCs could be redesigned to focus on conceptual understanding and student-centered learning. This would develop adaptive skillsets in educators and empower them to become critical consumers of curriculum. Therefore, instead of presenting the prescribed curriculum, they would have the knowledge and skills necessary to adapt instructional materials to meet the learning needs of their students. By fostering this autonomy, teachers would be encouraged to be more vested in executing instructional strategies focusing on conceptual growth (Akiba & Howard, 2021).

Another implication from this study reflects the importance of incorporating conceptual elements alongside procedural practice in mathematics curricula, as highlighted in the *Catalyzing Change* (NCTM, 2020) framework. Even with a well-designed curriculum that balances conceptual and procedural knowledge, teachers can still benefit from collaboratively identifying missed opportunities to develop student understanding and revising lessons to better address those needs (Stein & Smith, 1998). This underscores the critical role of providing teachers with the necessary support, resources, and professional training necessary to enhance their pedagogical content knowledge, regardless of the curriculum being used (Akiba & Howard, 2021; Huinker & Bill, 2017).

While there are curriculum standards that already prioritize conceptual understanding, it is crucial for State Departments of Education officials, school administrators, and instructional coaches to provide teachers with the necessary resources, professional development opportunities, and ongoing support to effectively implement these standards in their classrooms. This includes offering targeted training sessions, facilitating collaborative learning communities, and ensuring access to high-quality instructional materials that promote conceptual understanding. They should also allocate adequate funding and resources to support these initiatives at the district, state, and national levels.

If these recommendations are implemented more broadly, these findings could spark gradual transformation across the educational landscape - from how teachers instruct to the activities prioritized within mathematics curricula. These implications can potentially address persistent student challenges through meaningful, equitable instruction and possibly impact instruction, and assessment. This could better prepare future student populations to attain higher mathematics proficiency.

President Clinton had ambitious goals for American students, as outlined in the Goals 2000: Educate America Act (P.L. 103-227), which aimed for United States students to be the world leaders in mathematics and science achievement by the year 2000. By leveraging the insights discovered in this study, educators, policymakers, and other stakeholders can implement instructional changes that prioritize conceptual understanding, teacher agency, and equitable access. This can put our mathematics education on a trajectory to finally make President Clinton’s vision a reality, while fostering the beauty, wonder, and joy of mathematics.

Strengths of Study

This study demonstrated several strengths. First, the utilization of the conceptual frameworks, including *Catalyzing Change* (NCTM, 2020), Hiebert’s (1984) Site Theory and CoP (Wenger, 1998) provided a strong, theoretical foundation for this study. The framework ensured the findings were grounded in established literature and aligned with current trends in the field.

Second, by utilizing the case study methodology, I ensured that triangulation through diverse data sources, such as interviews, observations, and document analysis, bolstered the study's validity as this technique established consistent and well-supported conclusions (Yin, 2013).

Another notable strength involved member-checking, where participating teachers validated my findings (Yin, 2013). This process diminished the potential for misinterpretation and elevated my credibility by accurately representing their voices. Regular consultations with peers and mentors, including my academic advisor, also offered an external viewpoint, enhancing the study's objectivity.

Finally, the research unfolded in a real-world third-grade classroom environment at ABC Elementary School. This practical setting allowed for an authentic investigation into how lesson study could address the challenges teachers face regarding fraction instruction. The real-world context enhanced the ecological validity and direct applicability of the findings (Yin, 2013).

Limitations of Study

While the study makes valuable contributions, it is essential to acknowledge its limitations. There was a small sample size of seven educators at one elementary school. Because the research focused solely on third-grade fraction acquisition, the findings may not directly apply to other grade levels, and the specificity could limit relevance to other subjects. In general, a case study is not generalizable to the overall population.

Lastly, while positive instructional changes occurred, sustaining such transformations over the years would require ongoing structured collaboration time and the repeated implementation of the lesson study cycle. However, dedicating time to these activities may disrupt the established pacing guide, potentially leading to conflicts with curriculum coverage expectations and time constraints.

Recommendations for Future Research

While this study makes essential contributions to enrich fraction instruction, ample opportunities exist to substantiate and build upon these initial findings through further research. Future lesson study research could implement comparative analyses across grade levels, investigating whether transformations in pedagogical approaches occur when lesson study targets fields other than mathematics, such as science. Another potential study could examine whether video recordings of the lessons would produce similar positive outcomes as real-time observations.

Another avenue for future research could involve longitudinal studies that follow teachers over an extended period to examine the long-term effects of lesson study on their instructional practices and student learning outcomes. These studies could also investigate the factors that support or hinder the sustainability of the changes in teaching practices over time.

Based on the findings and implications within this study, further research questions can be posed. For example, how can the use of resources and personnel affect the sustainability of lesson study? How can lesson study be effectively implemented and sustained in different school contexts? How can technology be leveraged to support collaborative planning and reflective practice among teachers? Exploring these questions can add to the findings of this study and advance our understanding of effective mathematics teaching and learning.

Conclusion

Data derived from the case study revealed two key findings. The first finding was that repetitive collaboration within lesson study cycles had the power to transform the participants' mindsets- from being focused on procedural compliance- towards adopting more adaptive, student-centered approaches. The participants enhanced their professional growth in content knowledge and pedagogy through the lesson study (Wiburg & Brown, 2006), and this empowered them with the autonomy to make informed curricular changes.

The second finding revealed that collaborative planning, lesson implementation, and honest reflection enabled the participants to prioritize conceptual understanding by utilizing student-centered considerations through real-world connections when adapting fraction tasks. This change occurred because they went from working independently to working interdependently; they changed from creating individual goals, to having mutual accountability (DuFour et al., 2009).

These results have implications for enhancing mathematics instruction at ABC Elementary and the broader school district. Findings can lead to improved professional learning opportunities, where teachers work collaboratively in iterative cycles to improve lessons based on live observations from the classroom. Lesson study can be an invaluable tool for advocating for change by gradually shifting instructional pedagogical approaches from rote procedural methods to more meaningful, equitable learning and establishing teacher agency and autonomy in the classroom.

This research makes important strides in showcasing lesson study's efficacy as a transformative vehicle for driving collaborative teacher development and responsiveness. The qualitative insights contribute to a valuable understanding of how repeated engagement in lesson study cycles can catalyze positive changes in educators' mindsets, practices, and sense of empowerment regarding mathematics instruction.

Lesson study positions teachers at the forefront of educational reform. Reform that empowers educators as collaborative decision-makers and focuses on student-centered instruction can establish a foundation for transforming mathematics instruction into a more adaptive, equitable approach, which is necessary for promoting success in an increasingly complex and technological world.

APPENDIX A
IRB APPROVAL

UNIVERSITY OF CENTRAL FLORIDA

Institutional Review Board
FWA00000351
IRB00001138, IRB00012110
Office of Research
12201 Research Parkway
Orlando, FL 32826-3246

MODIFICATIONS REQUIRED TO SECURE APPROVAL

August 25, 2023

Dear Nisha Phillip-Malahoo:

On 8/25/2023, the IRB reviewed the following protocol:

Type of Review:	Initial Study, Exempt Category 1
Title:	LESSON STUDY AS A CATALYST FOR INTEGRATING CONCEPTUAL AND PROCEDURAL COMPONENTS IN FRACTION-BASED MATHEMATICS TASKS: AN ELEMENTARY SCHOOL CASE STUDY
Investigator:	Nisha Phillip-Malahoo
IRB ID:	STUDY00005772
Funding:	None
Documents Reviewed:	• 6. Observation Protocol for Use by Researcher.docx, Category: Other; • EXPLANATION OF RESEARCH - parents 8-23-23.pdf, Category: Consent Form; • Form 254, Category: Consent Form; • Form 255, Category: IRB Protocol; • Interview Protocol, Category: Interview / Focus Questions; • Observation Form, Category: Other; • Order & Compare Fractions Curriuclm.pdf, Category: Other; • Recruitment Email, Category: Recruitment Materials;

The IRB determined that modifications are required to approve the protocol. The modifications required and their reasons are listed here:

1. Upload external study team personnel CITI training documentation and completed Individual Investigator Agreements
2. Attach the OCPS IRB approval/determination letter when it is available.

Should you disagree with these requested changes, your response will be reviewed by the convened IRB. At your request, you can respond in person to the IRB.

UCF

UNIVERSITY OF CENTRAL FLORIDA

Institutional Review Board
FWA00000351
IRB00001138, IRB00012110
Office of Research
12201 Research Parkway
Orlando, FL 32826-3246

Sincerely,

Renea Carver
IRB Manager

APPENDIX B
RECRUITMENT EMAIL

Recruitment Email

Dear Mrs. _____________:

I am writing to see if you would like to participate in a research study being conducted by myself as part of my milestone requirement from [redacted] Research plays an important role in advancing our understanding of students' mathematical identities and helps lead to improvements in mathematical outcomes. This research project has been approved by [redacted] and follows IRB guidelines.

The following information summarizes the study and what it involves:

Research Title: LESSON STUDY AS A CATALYZING FORCE FOR INTEGRATING CONCEPTUAL AND PROCEDURAL COMPONENTS IN FRACTION-BASED MATHEMATICS TASKS: AN ELEMENTARY SCHOOL CASE STUDY

Study Purpose: This study examines the effectiveness of lesson study as an approach to integrating conceptual and procedural knowledge in fraction-based mathematics tasks at Pinewood Elementary. The research investigates how lesson study empowers teachers to enhance their instructional practices through reflection and active engagement and promotes a comprehensive understanding of fractions by enhancing mathematics tasks.

Participation Requirements: The research study asks you to participate in a lesson study which meets eleven times for 45 minutes per session for six weeks. At the completion of the lesson study, you will be asked to participate in an interview. The interview should last approximately 30 – 45 minutes. It will be conducted face-to face in the faculty conference room.

Contact Information: If you are interested in participating in this study, please click on the link provided in this email. If you would like to learn more about the study, you can contact me at:

[redacted]

Participating in research is voluntary. There are no known risks involved in this research.

If you have any questions, please let me know.

Principal Investigator: [redacted] Contact Phone: [redacted]

APPENDIX C
INTERVIEW PROTOCOL

Introduction

"Thank you for taking the time to speak with me today. This interview is part of a study aiming to understand how third-grade teachers intertwine procedural and conceptual knowledge in lesson studies, specifically focusing on fraction-based tasks. Your insights and experiences are valuable and will greatly contribute to this research."

Research Question 1: How do educators construct pedagogical strategies that intertwine procedural and conceptual knowledge through lesson study?

1. Can you walk me through your process of modifying a mathematics task so that it incorporates both procedural and conceptual knowledge of fractions?
2. Could you provide an example of a mathematics task where you intertwined procedural and conceptual knowledge of fractions?
3. Looking at future topics in the curriculm, how might you approach modifying the lesson.

Research Question 2: How do teachers' engagement in the analysis of the mathematics tasks that incorporate elements of procedural and conceptual concepts through lesson study contribute to their instructional decision making?

4. How, if at all, has your engagement in analyzing mathematics tasks during lesson study influenced your instructional decision making in regard to creating lessons on fractions?
5. Can you share an instance where a lesson study involving fractions had an impact on your instructional decision making and how you teach fractions?
6. How has your professional development been influenced by the analysis of mathematics tasks within the framework of a lesson study, particularly in relation to fractions?
7. Did you feel the lesson study process enhanced your teaching and if so, how. If it didn't, why do you suppose that is?

Conclusion

"Thank you for sharing your experiences and insights. Your responses will greatly contribute to the understanding of how lesson studies may enhance the quality of teaching fractions in third grade. If there is anything else you would like to add, please feel free to do so."

APPENDIX D
LESSON PLAN TEMPLATE

<table>
<tr><td colspan="2">Mathematics Learning Goal:</td></tr>
<tr><td colspan="2">•</td></tr>
<tr><td colspan="2">Evidence of Student’s Conceptual Thinking</td></tr>
<tr><td colspan="2">•</td></tr>
<tr><td colspan="2">Students Prior Knowledge</td></tr>
<tr><td colspan="2">Students have prior knowledge about:
•</td></tr>
<tr><td colspan="2">Instructional Support- Tools, Resources, Materials</td></tr>
<tr><td colspan="2">•</td></tr>
<tr><td colspan="2">Task</td></tr>
<tr><td colspan="2">Objective:
Task Launch:</td></tr>
<tr><td>Open Exploration:
•</td><td>Notes:</td></tr>
</table>

<table>
<tr><td colspan="2">Class Discussion:</td><td>Notes:

Extension Activity:</td></tr>
<tr><th colspan="3">Assessment</th></tr>
<tr><td colspan="3"></td></tr>
<tr><th>Solutions Paths with Errors and Anticipated Student Errors</th><th>Questions to Address Errors</th><th>Instructional Support – Teacher Questions</th></tr>
<tr><td></td><td></td><td></td></tr>
</table>

APPENDIX E
3-2-1 REFLECTION

Lesson Study Reflection

3 Things I Learned During Lesson Study

1. ______________________________
2. ______________________________
3. ______________________________

2 Ways This can Affect My Teaching

1. ______________________________
2. ______________________________

1 thing I'd like to try

1. ______________________________

APPENDIX F
RESEARCHER'S OBSERVATION PROTOCOL

<table>
<tr><td>Pre-Observation Details:

Date of lesson study observation:
Time of observation:
Location:
Teachers' names:
Lesson Study Stage:</td><td>Use of Resources:

How do the teachers include materials, manipulatives, and technology to support the teaching and learning of fractions?</td></tr>
<tr><td colspan="2">Observation Focus Areas</td></tr>
<tr><td>Lesson Planning:

How do third-grade educators construct pedagogical strategies intertwining procedural and conceptual knowledge in lesson study?</td><td>Lesson Reflection:

Insights or realizations expressed by the teachers about the fraction tasks</td></tr>
<tr><td colspan="2">Field Notes:</td></tr>
<tr><td colspan="2">Next Steps:</td></tr>
</table>

APPENDIX G
PARTICIPANTS' OBSERVATION PROTOCOL

<table>
<tr><td>Observation Details:

Date of lesson observation:
Time of observation:
Location:
Teachers’ name:
Lesson’s name:</td><td>Use of Resources:

What materials, manipulatives, and technology support the teaching and learning of fractions?</td></tr>
<tr><td colspan="2">Observation Focus Areas</td></tr>
<tr><td>Lesson Implementation:

Goal: The students will assess the reasonableness of their solutions.</td><td>Lesson Reflection:

Insights about the fraction tasks</td></tr>
<tr><td>Student Errors:</td><td>Questions to Elicit Connections:</td></tr>
<tr><td colspan="2">Noticing and Wonderings:</td></tr>
</table>

APPENDIX H
A PRIORI CODES

Initial Codes for Research Question 1

Research Question: **How does the engagement of teachers in the creation and analysis of fraction-based mathematics tasks in lesson study influence their instructional decision-making processes?**	
Deductive Codes	Meaning
1. Task Adaptation	how teachers adapt existing tasks or create new ones during lesson study.
2. Procedural-Conceptual Integration:	how lesson study participation influences teachers' strategies to effectively integrate both procedural and conceptual knowledge when teaching fractions.
3. Student-Centered Considerations	how teachers tailor their tasks to the needs, abilities, and learning styles of their students.
4. Conceptual Shift in Teacher's Thinking	changes in teachers' conceptual understanding of fractions and how these shifts influence their instructional decision-making.
5.Lesson Study	how engagement in lesson study influences teachers' instructional decisions regarding fraction-based mathematics tasks.

Initial Codes for Research Question 2

Research Question: **How do teachers construct pedagogical strategies that integrate procedural and conceptual knowledge through lesson study?**	
Deductive Codes	Meaning
1. Pedagogical Adaptation	how teachers modify their teaching methods and approaches based on lesson study insights to incorporate both procedural and conceptual aspects into mathematics lessons.
2. Real-World Connections	teachers' efforts to relate mathematical concepts to real-life scenarios.
3. Promoting Deep Mathematical Understanding:	how teachers design pedagogical strategies to foster deep mathematical understanding, emphasizing the development of conceptual knowledge among students.
4. Feedback Integration	incorporation of feedback from peers or students into teaching practices.
5. Reflective Practice	teachers' self-reflection on their strategies and continuous improvement through lesson study.
6.Utilizing Visual Representations:	use of visual aids, such as symbols and models, to bridge the gap between procedural and conceptual knowledge

APPENDIX I
EXPLORING UNIT FRACTIONS LESSON A VERSION 1

Mathematics Learning Goal:

MA.3.FR.1.1

Represent and interpret unit fractions in the form as the quantity formed by n one part when a whole is partitioned into n equal parts.

- Students will understand that a unit fraction represents one part of a whole that has been divided into equal parts.
- Using **fraction strips**, area models, set models, or other manipulatives, students should be able to show unit fractions visually.

Evidence of Student's Conceptual Thinking

- Students can articulate a unit fraction, represents one part of a divided whole.
- When using fraction strips, students can show a unit fraction and name that fraction as "1 out of n parts."

Students Prior Knowledge

Students have prior knowledge about:

- Partitioning area models such as circles and rectangles into two, three, and four equal-sized parts. Naming the parts using appropriate language, including halves and fourths (1^{st} grade).
- Partitioning area models such as circles and rectangles into two, three, or four equal-sized parts. Naming the parts using appropriate language and describing the whole as two halves, three-thirds, or four-fourths (2^{nd} grade).
- Partitioning area models such as rectangles into two, three, or four equal-sized parts in two different ways, showing the equal-sized parts of the whole may have different shapes (2^{nd} grade).

Instructional Support- Tools, Resources, Materials

- Fraction Strips, Index Cards, Scissors

Task

<table>
<tr><td colspan="2">

Objective: Using linear models, students will learn that a unit fraction is one part of an evenly divided whole.

Task Launch: (3 minutes)

1. Hook: begin with a discussion about chocolates. Ask, "Who likes chocolates? I love chocolates like the Hersey bar (show a Hersey bar). Have you ever had to share a chocolate bar?

</td></tr>
<tr><td>

Open Exploration: (10 minutes)

1. Interactive Group Activity: Organize students into small groups and provide each group with a paper fraction strip and a model representing a chocolate bar. Instruct each group to decide how many friends they want to share the chocolate with. Encourage them to discuss and decide collaboratively.
2. Hands-on Activity: Have students use their fraction strip to physically divide the chocolate bar into equal parts, demonstrating how they would share it among their chosen number of friends.
3. After sharing the chocolate, ask each group to explain how they divided it and how much each friend received. Encourage them to create a word problem that aligns with their sharing model. This should be written on an index card.

Expected Outcomes:

- **Hands-on Fraction Understanding:** Students should demonstrate the ability to physically partition the fraction strip into equal parts to represent their sharing scenario.
- **Conceptual Understanding:** They should be able to articulate that each friend receives one piece, emphasizing the concept of unit fractions.
- **Word Problem Creation:** Students will successfully generate word problems corresponding to their sharing models, reinforcing their understanding of unit fractions.

</td><td>

Notes:

*If a group has difficulty creating a word problem, provide the following sentence prompt:

______ friends share a chocolate bar. How much of the bar would one friend receive?

Possible Answer:

A friend would receive one out of ________ pieces.

</td></tr>
<tr><td>

Class Discussion: (10 minutes)

1. After asking each group to explain how many pieces they divided the chocolate into and why they made that choice, write both the number of pieces and the reasoning on the

</td><td>

Notes:

Extension Activity:

</td></tr>
</table>

board. For example, "*We divided it into 5 pieces because we had 5 friends. Each friend would get 1 out of 5 pieces*." 2. Ask: "What do you notice about all these answers? Can you find a pattern?" Possible answers: - There is a one in the front of every answer. - The whole chocolate was divided into different pieces. **Introducing Numerator and Denominator:** 3. **Interactive Discussion:** Instead of directly explaining the terms numerator and denominator, engage students in a guided conversation. Ask, "Can someone tell me what the 1 in the answer represents? Why is it there?" Encourage students to articulate their understanding. 4. **Definition from Students:** Ask, "What would be a suitable name for the top number, the 1, that represents the piece each friend gets?" Allow students to generate their own terms for the numerator, such as "share" or "portion," and write these terms on the board. 5. **Discussion on Denominator:** Similarly, ask about the second number and its significance. Instead of providing the term "denominator" right away, ask students to brainstorm what this number represents. Let them suggest terms like "total pieces" or "whole." 6. Refer back to the initial answer you wrote on the board that the students provided. Say: "What do you notice about all these answers?" Say, "How can we write these ideas using fractions? Let's work together to create a fraction format." Encourage students to suggest formats like "1/total share" or "1/total pieces." **7.** Say: "This is a **unit fraction**. Can someone explain what a unit fraction is?" Possible answer: **one** equal piece out of the **whole**. **Expected Outcomes:** • Students will learn how to write a unit fraction. • Students will also develop a more conceptual grasp of the numerator and denominator and their role in representing parts of a whole.	Students can rotate around the room to solve the different groups' word problem.
Assessment	

Independent:

Four friends, Ms. Malahoo, Ms. Braham, Ms. Rosenburg and Ms. Thrift shared this bar equally.

Can you complete the sentences using words from the word bank?

Denominator, one, numerator, four

Ms. Braham will receive ___________ piece out of _______ pieces.

The top number is the ___________, and this refers to one piece of out the equal-sized whole.

The bottom number is the _____________ and this refers to how many pieces the whole is equally divided into.

As a fraction, it can be written as: $\frac{\quad}{\quad}$

Solutions Paths with Errors and Anticipated Student Error	Questions to Address Errors	Instructional Support – Teacher Questions
# 1: The student believes any fraction with the same numerator and denominator is a unit fraction. For example, when asked what a unit fraction is, a student responds with 3/3. **Correction:** Reinforce the concept that unit fractions have a numerator of 1. Guide the student to see that $\frac{2}{3}$ means two parts out of three, whereas a	• What does the denominator tell you about a fraction? • What does the numerator tell you about a fraction? • I noticed you wrote "$\frac{2}{3}$ " when referring to one part of a whole split into three. Can you show me how "$\frac{2}{3}$ " looks and compare it to one part? • What does it mean for the parts to be "equal"?	• What is a whole? • What does equal pieces mean? • Why do you think the number '1' is always in the numerator in unit fractions? • If we keep increasing the denominator in a unit fraction, what happens to the size of each part? Why?

unit fraction like $\frac{1}{3}$ represents just one of those parts. **# 2:** When asked to represent $\frac{1}{3}$, a student divides a shape into three parts but makes one part significantly larger than the others. **Correction:** Discuss the concept of "equal" and how it applies to fractions. Use tools like rulers or grid paper to assist the student in making more accurate partitions.		• If you combine two unit fractions from a whole divided into 3 parts, what fraction do you get? • When might you encounter unit fractions in everyday life?

APPENDIX J
EXPLORING UNIT FRACTIONS LESSON B VERSION 1

Mathematics Learning Goal: ***MA.3.FR.1.1*** *Represent and interpret unit fractions in the form as the quantity formed by n one part when a whole is partitioned into n equal parts.*
• Students will understand that a unit fraction represents one part of a whole that has been divided into equal parts. • Using fraction strips, area models, **set models**, or other manipulatives, students should be able to show unit fractions visually.
Evidence of Student's Conceptual Thinking
• Students can articulate a unit fraction represents one part of a divided whole. • When using set models, students can show a unit fraction and name that fraction as "1 out of n parts."
Students Prior Knowledge
Students have prior knowledge about: • Partitioning area models such as circles and rectangles into two, three, and four equal-sized parts. Naming the parts using appropriate language, including halves and fourths (1st grade). • Partitioning area models such as circles and rectangles into two, three, or four equal-sized parts. Naming the parts using appropriate language and describing the whole as two halves, three-thirds, or four-fourths (2nd grade). • Partitioning area models such as rectangles into two, three, or four equal-sized parts in two different ways, showing the equal-sized parts of the whole may have different shapes (2nd grade).
Instructional Support- Tools, Resources, Materials
• Two color counters, yarn loop, white boards, and index cards with role icons. • Prior to the lesson, identify groups of four and assign roles to the group: a group leader, a speaker, a resource manager, and a recorder.
Task
Objective: Using set models, students will learn that a unit fraction is one part of an evenly divided whole. **Task Launch: (3 minutes)**

Introduction to Set Models: "Let's learn about Set Models! Imagine these 3 Oreo cookies are all in one pack. I use this yarn loop to show they are all together. Now, guess what? In this pack, there is one special Golden Oreo. How can we show that one special cookie in our pack? If we think about all the cookies, what part does the Golden Oreo make up?"

Solve and Share:

1. Draw the pack of Oreos, including the special Golden Oreo.
2. Write down the fraction that represents the Golden Oreo in relation to the whole pack.

Open Exploration: (15 minutes)

1. Crafting Personal Set Models:

- Divide students into collaborative small groups and provide each with a Ziplock bag filled with two-colored counters and a piece of yarn.
- **Creating their Own Set Models**:
    1. Allow students to gather items from around the classroom or use provided materials (a set of markers, a box of crayons, a package of pencils, a pack of Starburst) to create their own sets.
    2. Students should be able to describe the unit fraction represented by a single item in their collection.

2. Diving into Unit Fractions:

- Once they've created their 'whole' with the counters, challenge them: "Now, can you show just one item from your collection?

  Expected Outcome: They should turn over one of the two-color counters.

- Ask them to represent this as a fraction. For example, $\frac{1}{4}$ (if given 4 pencils in a pack).

3. Building up to the Whole:

- Next, guide them with: "How many of these individual items (or unit fractions) will you need to make up your entire collection?

Notes:

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- Provide a sentence prompt:

My collection has ______ pieces. Each piece is known as a unit fraction. It is written like _____________.

I will need _______ unit fractions to have a whole.

- Hand out whiteboards to each group and let them write the corresponding unit fractions to identify all the pieces in their whole.
- As they progress, encourage students to articulate their process with their peers. For example, if given a package with 10 markers: ($\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{10}{10}$ = 1 whole).

4. Reflect, Relate:

- After they've combined all the unit fractions, facilitate a group reflection: "What did you observe when you added all the unit fractions?

  Expected Outcome: Students should be able to articulate that when all unit fractions are combined, they make up the whole. For instance, if they have six pieces of $\frac{1}{6}$ each, combining them brings back the whole set of cookies.

  Expected Outcomes:

- **Conceptual Understanding:** Students will develop a clear understanding of what unit fractions represent and how multiple unit fractions can be combined to form a whole.
- **Recognition of Equal Parts:** They should recognize that each unit fraction is an equal part of the whole.
- **Hands-on Application:** Given the tactile nature of the lesson with counters and yarn, students should be able to physically represent a whole and its unit fractions, strengthening their conceptual understanding.

Extra Practice

- Answer the following question:

  You have a pack of ____ colored pencils (you choose a number). If you use one pencil to draw, what fraction of the pencil pack have you used? How many unit fractions will be in the whole pack?

  Sample response:

  If I use one pencil from my pack of 12 pencils, I have used 1 out of 12 pencils. That's the fraction $\frac{1}{12}$.

  There are 12 pencils, so there are 12 of these $\frac{1}{12}$ *pieces in the whole pack.*

Solutions Paths with Errors and Anticipated Student Error	**Questions to Address Errors**	**Instructional Support – Teacher Questions**
# 1: Whole Set Variation: Some students may struggle with the idea that the whole set can vary in number. For example, in one scenario, 8 counters could be the whole set, while in another scenario, 10 counters could be the whole. This can be confusing if students believe that the "whole" must always be the same number. **Correction:** Display multiple set models side by side, with differing quantities. Label each as a "whole." For instance, show a set of 8 apples and label it "whole collection of apples" and then show a set of 10 bananas and label it "whole collection of bananas." This visually reinforces the idea that the number constituting the "whole" can vary depending on the context.	• How does changing the denominator (but keeping the numerator as 1) affect the number of items you have from a set? • In a set of 6 oranges, what does $\frac{1}{6}$ mean? • Can you draw a set of 12 stars and show what $\frac{1}{12}$ of that set looks like?	• What does the word "unit" mean in "unit fraction"? • Can a "unit" be more than one item in a set? Why or why not? • If a set has 10 items and you take $\frac{1}{10}$ of the set, how many items do you have? • What fraction of the set would represent the entire set?

# 2: All unit fractions are equivalent because they all have a numerator of 1. **Correction:** Explain that while the numerator tells us how many parts we're looking at, the denominator tells us how many total parts the whole set is divided. Provide opportunities for students to reinforce this thinking.		

APPENDIX K
EXPLORING UNIT FRACTIONS LESSON A VERSION 2

Mathematics Learning Goal: ***MA.3.FR.1.1*** *Represent and interpret unit fractions in the form as the quantity formed by n one part when a whole is partitioned into n equal parts.*
• Students will understand that a unit fraction represents one part of a whole that has been divided into equal parts. • Using **fraction strips**, area models, set models, or other manipulatives, students should be able to show unit fractions visually.
Evidence of Student's Conceptual Thinking
• Students can articulate a unit fraction represents one part of a divided whole. • When using fraction strips, students can show a unit fraction and name that fraction as "1 out of n parts."
Students Prior Knowledge
Students have prior knowledge about: • Partitioning area models such as circles and rectangles into two, three, and four equal-sized parts. Naming the parts using appropriate language, including halves and fourths (1st grade). • Partitioning area models such as circles and rectangles into two, three, or four equal-sized parts. Naming the parts using appropriate language and describing the whole as two halves, three-thirds, or four-fourths (2nd grade). • Partitioning area models such as rectangles into two, three, or four equal-sized parts in two different ways, showing the equal-sized parts of the whole may have different shapes (2nd grade).
Instructional Support- Tools, Resources, Materials
• Fraction Strips, scissors, and index cards with role icons. • Prior to the lesson, identify groups of six and assign roles to the group: 2 partitioners, a group leader, 2 speakers, and a recorder.
Task

Objective: Using linear models, students will learn that a unit fraction is one part of an evenly divided whole.

Task Launch: (3 minutes)

2. Hook: begin with a discussion about chocolates. Ask, "Who likes chocolates? I love chocolates like the Hersey bar (show a Hersey bar). Have you ever had to share a chocolate bar?

Open Exploration: (10 minutes)	Notes:
4. Interactive Group Activity: Organize students into small groups and give each group a paper fraction strip. Instruct each group to decide how many friends they want to share the chocolate with. Encourage them to discuss and decide collaboratively. **5.** Hands-on Activity: Have students use their fraction strip to physically divide the chocolate bar into equal parts, demonstrating how they would share it among their chosen number of friends. **6.** After sharing the chocolate, ask each group to explain how they divided it and how much each friend received. Encourage them to create a word problem that aligns with their sharing model. Use the sentence stem to build the word problem. **Expected Outcomes:** • **Hands-on Fraction Understanding:** Students should demonstrate the ability to physically partition the fraction strip into equal parts to represent their sharing scenario. • **Conceptual Understanding:** They should be able to articulate that each friend receives one piece, emphasizing the concept of unit fractions. • **Word Problem Creation:** Students will successfully generate word problems corresponding to their sharing models, reinforcing their understanding of unit fractions.	*If a group has difficulty creating a word problem, provide the following sentence stem: There is _____ whole chocolate. ____ friends share the chocolate. Each friend will get _____ out of _____ pieces. **Possible Answer:** Each friend would receive ______ out of ________ pieces.

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Class Discussion: (10 minutes)

8. After asking each group to explain how many pieces they divided the chocolate into and why they made that choice, write both the number of pieces and the reasoning on the board. For example, "*We divided it into 5 pieces because we had 5 friends. Each friend would get 1 out of 5 pieces*."
9. Ask: "What do you notice about all these answers? Can you find a pattern?" Possible answers:
    - There is a one in the front of every answer.
    - The whole chocolate was divided into different pieces.

Introducing Numerator and Denominator:

10. **Interactive Discussion:** Instead of directly explaining the terms numerator and denominator, engage students in a guided conversation. Ask, "Can someone tell me what the 1 in the answer represents? Why is it there?" Encourage students to articulate their understanding.
11. **Definition from Students:** Ask, "What would be a suitable name for the top number, the 1, that represents the piece each friend gets?" Allow students to generate their own terms for the numerator, such as "share" or "one piece," and write these terms on the board.

    ***Explicitly teach that the numerator refers to the number of pieces from the whole that are taken or being discussed.** Ask students, "what would the numerator be if a friend had 2 pieces of the whole?"

    Discussion on Denominator: Similarly, ask about the second number and its significance. Instead of providing the term "denominator" right away, ask students to brainstorm what this number represents. Let them suggest terms like "total pieces" or "whole."

12. Refer back to the initial answer you wrote on the board that the students provided. Say: "What do you notice about all these answers?" Say, "How can we write these ideas using fractions? Let's work together to create a fraction format." Encourage students to suggest formats like "1/total share" or "1/total pieces."

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Notes:

Extension Activity:

Students can rotate around the room to solve the different groups' word problems.

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13. Explicitly teach: Say: “This is a **unit fraction**. Can someone explain what a unit fraction is?” Possible answer: **one** equal piece out of the **whole**.

Expected Outcomes:

- Students will learn how to write a unit fraction.
- Students will also develop a more conceptual grasp of the numerator and denominator and their role in representing parts of a whole.

Assessment

Independent: Pass out the exit slips. Allow students to read it in its entirety before completing the exit slip.

Four friends, Ms. Malahoo, Ms. Braham, Ms. Rosenburg, and Ms. Sebastien shared this bar equally.

Can you complete the sentences using words from the word bank?

Denominator, one, numerator, four

Ms. Sebastien will receive ___________ piece out of _______ pieces.

The top number is the ___________, and this refers to one piece out of the equal-sized whole.

The bottom number is the _____________, and this refers to how many pieces the whole is equally divided into.

As a fraction, it can be written as: [______]

Solutions Paths with Errors and Anticipated Student Error	Questions to Address Errors	Instructional Support – Teacher Questions
# 1: The student believes any fraction with the same numerator and denominator is a unit fraction. For example, when asked what a unit fraction is, a student responds with 3/3. **Correction:** Reinforce the concept that unit fractions have a numerator of 1. Guide the student to see that $\frac{2}{3}$ means two parts out of three, whereas a unit fraction like $\frac{1}{3}$ represents just one of those parts. **# 2:** When asked to represent $\frac{1}{3}$, a student divides a shape into three parts but makes one part significantly larger than the others. **Correction:** Discuss the concept of "equal" and how it applies to fractions. Use tools like rulers or grid paper to assist the student in making more accurate partitions.	• What does the denominator tell you about a fraction? • What does the numerator tell you about a fraction? • I noticed you wrote "$\frac{2}{3}$ " when referring to one part of a whole split into three. Can you show me how "$\frac{2}{3}$ " looks and compare it to one part? • What does it mean for the parts to be "equal"?	• What is a whole? • What does equal pieces mean? • Why do you think the number '1' is always in the numerator in unit fractions? • If we keep increasing the denominator in a unit fraction, what happens to the size of each part? Why? • If you combine two unit fractions from a whole divided into 3 parts, what fraction do you get? • When might you encounter unit fractions in everyday life?

APPENDIX L
EXPLORING UNIT FRACTIONS LESSON A VERSION 3

Mathematics Learning Goal: ***MA.3.FR.1.1*** *Represent and interpret unit fractions in the form as the quantity formed by n one part when a whole is partitioned into n equal parts.*
• Students will understand that a unit fraction represents one part of a whole that has been divided into equal parts. • Using **fraction strips**, area models, set models, or other manipulatives, students should be able to show unit fractions visually.
Evidence of Student's Conceptual Thinking
• Students can articulate a unit fraction represents one part of a divided whole. • When using fraction strips, students can show a unit fraction and name that fraction as "1 out of n parts."
Students Prior Knowledge
Students have prior knowledge about: • Partitioning area models such as circles and rectangles into two, three, and four equal-sized parts. Naming the parts using appropriate language, including halves and fourths (1st grade). • Partitioning area models such as circles and rectangles into two, three, or four equal-sized parts. Naming the parts using appropriate language and describing the whole as two halves, three-thirds, or four-fourths (2nd grade). • Partitioning area models such as rectangles into two, three, or four equal-sized parts in two different ways, showing the equal-sized parts of the whole may have different shapes (2nd grade).
Instructional Support- Tools, Resources, Materials
• Fraction Strips of differing lengths, scissors, and index cards with role icons. • Computer access for digital manipulatives (https://www.didax.com/apps/fraction-number-line/) • Prior to the lesson, identify groups of four and assign roles to the group: 1 partitioner, a group leader, 1 speaker, and a recorder.
Task

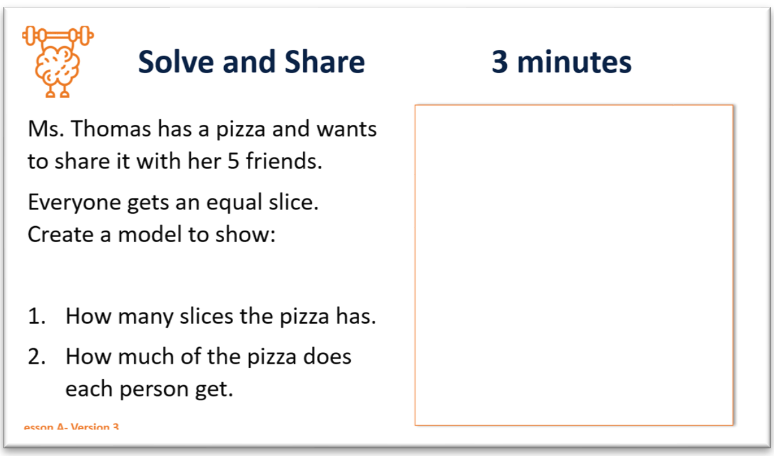
Objective: Using linear models (real and digital), students will learn that a unit fraction is one part of an evenly divided whole.

Lesson Opener: (1 minute)

Teacher: “Have you guys ever had to share food? In today’s lesson, we will investigate how to share food using both hands-on and digital resources.”

Solve and Share: (3 minutes)

**Allow students to struggle with this task productively. This is to elicit their thinking, not to teach explicitly.*



Expected Answer:

6 slices; one slice out of 6 slices.

Task Launch: (3 minutes)

3. Hook: begin with a discussion about chocolates. Ask, "Who likes chocolates? I love chocolates like the Hersey bar (show a Hersey bar). Have you ever had to share a chocolate bar? Today, you will be given either a piece of paper to represent Hershey’s chocolate or an online tool that models the Hershey's chocolate bar. Your job is to divide the chocolate among a group of friends. Your group needs to decide how many friends to share the chocolate with.

Open Exploration: (10 minutes)

Notes:

Sentence Stem:

There is _____ whole chocolate.

____ friends share the chocolate.

Each friend will get _____ out of _____ pieces.

7. Interactive Group Activity: Organize students into small groups and give each group a paper fraction strip. Instruct each group to decide how many friends they want to share the chocolate with. Encourage them to discuss and decide collaboratively. **8.** Hands-on Activity: Have students use their fraction strip to physically divide the chocolate bar into equal parts, demonstrating how they would share it among their chosen number of friends. The group on the computer will use the digital manipulatives to partition the model of the whole chocolate. **9.** After sharing the chocolate, ask each group to explain how they divided it and how much each friend received. Encourage them to use the sentence stems to capture their thinking. **Expected Outcomes:** • **Hands-on Fraction Understanding:** Students should demonstrate the ability to physically partition the fraction strip into equal parts to represent their sharing scenario. • **Conceptual Understanding:** They should be able to articulate that each friend receives one piece, emphasizing the concept of unit fractions. Students will successfully complete the sentence stems corresponding to their sharing models, reinforcing their understanding of unit fractions.	
Class Discussion: (10 minutes) 14. After asking each group to explain how many pieces they divided the chocolate into and why they made that choice, write both the number of pieces and the reasoning on the board. For example, "*We divided it into 5 pieces because we had 5 friends. Each friend would get 1 out of 5 pieces*." 15. Ask: "What do you notice about all these answers? Can you find a pattern?" Possible answers: - There is a one in the front of every answer. - The whole chocolate was divided into different pieces. **Introducing Numerator and Denominator:** 16. **Interactive Discussion:** Instead of directly explaining the terms numerator and denominator, engage students in a guided conversation. Ask, "Can someone tell me what the 1	**Notes:** **Extension Activity:** Students can rotate around the room to look at other students' models and sentence stems.

in the answer represents? Why is it there?" Encourage students to articulate their understanding. 17. **Definition from Students:** Ask, "What would be a suitable name for the top number, the 1, that represents the piece each friend gets?" Allow students to generate their own terms for the numerator, such as "share" or "one piece," and write these terms on the board. ***Explicitly teach that the numerator refers to the number of pieces from the whole that are taken or being discussed.** Ask students, "what would the numerator be if a friend had 2 pieces of the whole?" **Discussion on Denominator:** Similarly, ask about the second number and its significance. Instead of providing the term "denominator" right away, ask students to brainstorm what this number represents. Let them suggest terms like "total pieces" or "whole." 18. Refer back to the initial answer you wrote on the board that the students provided. Say: "What do you notice about all these answers?" Say, "How can we write these ideas using fractions? Let's work together to create a fraction format." Encourage students to suggest formats like "1/total share" or "1/total pieces." **19. Explicitly teach**: Say: "This is a **unit fraction**. Can someone explain what a unit fraction is?" Possible answer: **one** equal piece out of the **whole**. **Expected Outcomes:** • Students will learn how to write a unit fraction. • Students will also develop a more conceptual grasp of the numerator and denominator and their role in representing parts of a whole.	
Assessment – Solve and Share	

Independent: Pass out the exit slips.

Solve and Share: (3 minutes)

**Allow students to struggle with this task productively. This is to elicit their thinking, not to teach explicitly.*

Say: Let's revisit this task:

Ms. Beckett has a pizza and wants to share it with her 5 friends. Everyone gets an equal slice. How many slices will the pizza have? How much pizza does each person get?

Draw a model to show your thinking and write your answers as a fraction.

Expected Answer:

$\frac{6}{6}$, $\frac{1}{6}$

Solutions Paths with Errors and Anticipated Student Error	Questions to Address Errors	Instructional Support – Teacher Questions
# 1: The student believes any fraction with the same numerator and denominator is a unit fraction. For example, when asked what a unit fraction is, a student responds with $\frac{3}{3}$. **Correction:** Reinforce the concept that unit fractions have a numerator of 1. Guide the student to see that $\frac{2}{3}$ means two parts out of three, whereas a unit fraction like $\frac{1}{3}$ represents just one of those parts. **# 2:** When asked to represent $\frac{1}{3}$, a student divides a shape into three parts but makes one part significantly larger than the others.	• What does the denominator tell you about a fraction? • What does the numerator tell you about a fraction? • I noticed you wrote "$\frac{2}{3}$ " when referring to one part of a whole split into three. Can you show me how "$\frac{2}{3}$ " looks and compare it to one part? • What does it mean for the parts to be "equal"?	• What is a whole? • What does equal pieces mean? • Why do you think the number '1' is always in the numerator in unit fractions? • If we keep increasing the denominator in a unit fraction, what happens to the size of each part? Why? • If you combine two unit fractions from a whole divided into 3 parts, what

Correction: Discuss the concept of "equal" and how it applies to fractions. Use tools like rulers or grid paper to assist the student in making more accurate partitions.		fraction do you get? • When might you encounter unit fractions in everyday life?

APPENDIX M
EXPLORING UNIT FRACTIONS LESSON B VERSION 2

Mathematics Learning Goal: ***MA.3.FR.1.1*** *Represent and interpret unit fractions in the form as the quantity formed by n one part when a whole is partitioned into n equal parts.*
• Students will understand that a unit fraction represents one part of a whole that has been divided into equal parts. • Using fraction strips, area models, **set models**, or other manipulatives, students should be able to show unit fractions visually.
Evidence of Student's Conceptual Thinking
• Students can articulate a unit fraction represents one part of a divided whole. • When using set models, students can show a unit fraction and name that fraction as "1 out of n parts."
Students Prior Knowledge
Students have prior knowledge about: • Partitioning area models such as circles and rectangles into two, three, and four equal-sized parts. Naming the parts using appropriate language, including halves and fourths (1st grade). • Partitioning area models such as circles and rectangles into two, three, or four equal-sized parts. Naming the parts using appropriate language and describing the whole as two halves, three-thirds, or four-fourths (2nd grade). • Partitioning area models such as rectangles into two, three, or four equal-sized parts in two different ways, showing the equal-sized parts of the whole may have different shapes (2nd grade).
Instructional Support- Tools, Resources, Materials
• Two color counters, yarn loop, white boards, and index cards with role icons. • Prior to the lesson, identify groups of four and assign roles to the group: a group leader, a speaker, a resource manager, and a recorder.
Task
Objective: Using set models, students will learn that a unit fraction is one part of an evenly divided whole. **Task Launch: (3 minutes)**

Introduction to Set Models: "Let's learn about Set Models! I have a packet of Oreo cookies. There are 4 cookies in the pack. If one cookie is vanilla and the rest are chocolates, how can you describe the vanilla Oreo as a fraction? **Hint:** I ate ______ Oreo out of 4 Oreos. **Solve and Share:** 1. Draw the pack of Oreos. 2. Write down the fraction that represents the vanilla Oreo in relation to the whole pack.

Open Exploration: (15 minutes)	**Notes:**
1. Crafting Personal Set Models: • Divide students into collaborative small groups and provide each with a piece of yarn. • **Creating their Own Set Models**: 1. Allow students to gather items from around the classroom or use provided materials (a set of markers, a box of crayons, a package of pencils, a pack of Starburst) to create their own sets. 2. Students should place all the items from the package onto their table. They should then use the yarn to circle the items, thus making a 'whole.' **2. Diving into Unit Fractions:** • Once they've created their 'whole', challenge them: "Now, can you show just one item from your collection? Expected Outcome: They should hold or move one item from inside the yarn. • Ask them to represent this item as a fraction. For example, $\frac{1}{4}$ (if given 4 pencils in a pack). **3. Building up to the Whole:** • Next, guide them with: "How many of these individual items (or unit fractions) will you need to make up your entire collection?	.

• Provide a sentence prompt: My collection has ______ pieces. Each piece is known as a unit fraction. It is written like _____________. I will need _______ unit fractions to have a whole. • Hand out whiteboards to each group and let them write the corresponding unit fractions to identify all the pieces in their whole. • As they progress, encourage students to articulate their process with their peers. For example, if given a package with 10 markers: ($\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{10}{10}$ = 1 whole)	
4. Reflect, Relate: • After they've combined all the unit fractions, facilitate a group reflection: "What did you observe when you added all the unit fractions? **Expected Outcome:** Students should be able to articulate that when all unit fractions are combined, they make up the whole. For instance, if they have six pieces of $\frac{1}{6}$ each, combining them brings back the whole set of cookies. **Expected Outcomes:** • **Conceptual Understanding:** Students will develop a clear understanding of what unit fractions represent and how multiple unit fractions can be combined to form a whole. • **Recognition of Equal Parts:** They should recognize that each unit fraction is an equal part of the whole. • **Hands-on Application:** Given the tactile nature of the lesson with counters and yarn, students should be able to physically represent a whole and its unit fractions, strengthening their conceptual understanding.	
Extra Practice	

- Answer the following question:

  You have a pack of ____ colored pencils (you choose a number). If you use one pencil to draw, what fraction of the pencil pack have you used? How many unit fractions will be in the whole pack?

  Sample response:

  If I use one pencil from my pack of 12 pencils, I have used 1 out of 12 pencils. That's the fraction $\frac{1}{12}$.

  There are 12 pencils, so there are 12 of these $\frac{1}{12}$ *pieces in the whole pack.*

Solutions Paths with Errors and Anticipated Student Error	Questions to Address Errors	Instructional Support – Teacher Questions
# 1: Whole Set Variation: Some students may struggle with the idea that the whole set can vary in number. For example, in one scenario, 8 counters could be the whole set, while in another scenario, 10 counters could be the whole. This can be confusing if students believe that the "whole" must always be the same number. **Correction:** Display multiple set models side by side, with differing quantities. Label each as a "whole." For instance, show a set of 8 apples and label it "whole collection of apples" and then show a set of 10 bananas and label it "whole collection of bananas." This visually reinforces the idea that the number constituting the "whole" can vary depending on the context.	• How does changing the denominator (but keeping the numerator as 1) affect the number of items you have from a set? • In a set of 6 oranges, what does $\frac{1}{6}$ mean? • Can you draw a set of 12 stars and show what $\frac{1}{12}$ of that set looks like?	• What does the word "unit" mean in "unit fraction"? • Can a "unit" be more than one item in a set? Why or why not? • If a set has 10 items and you take $\frac{1}{12}$ of the set, how many items do you have? • What fraction of the set would represent the entire set?

# 2: All unit fractions are equivalent because they all have a numerator of 1. **Correction:** Explain that while the numerator tells us how many parts we're looking at, the denominator tells us how many total parts the whole set is divided. Provide opportunities for students to reinforce this thinking.		

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