Rr Interval Estimation From An Ecg Using A Linear Discrete Kalman Filter

Arun Janapala
University of Central Florida

Part of the Electrical and Electronics Commons

Find similar works at: https://stars.library.ucf.edu/etd

University of Central Florida Libraries http://library.ucf.edu

This Masters Thesis (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Electronic Theses and Dissertations, 2004-2019 by an authorized administrator of STARS. For more information, please contact STARS@ucf.edu.

STARS Citation
https://stars.library.ucf.edu/etd/339
RR INTERVAL ESTIMATION FROM AN ECG USING A LINEAR DISCRETE KALMAN FILTER

by

ARUN N JANAPALA
B.Tech. Nagarjuna University, 2002

A thesis submitted in partial fulfillment of the requirements for the degree Master of Science in the Department of Electrical and Computer Engineering in the College of Engineering and Computer Science at the University Of Central Florida
Orlando, Florida

Spring Term
2005
ABSTRACT

An electrocardiogram (ECG) is used to monitor the activity of the heart. The human heart beats seventy times on an average per minute. The rate at which a human heart beats can exhibit a periodic variation. This is known as heart rate variability (HRV). Heart rate variability is an important measurement that can predict the survival after a heart attack. Studies have shown that reduced HRV predicts sudden death in patients with Myocardial Infarction (MI). The time interval between each beat is called an RR interval, where the heart rate is given by the reciprocal of the RR interval expressed in beats per minute. For a deeper insight into the dynamics underlying the beat to beat RR variations and for understanding the overall variance in HRV, an accurate method of estimating the RR interval must be obtained. Before an HRV computation can be obtained the quality of the RR interval data obtained must be good and reliable. Most QRS detection algorithms can easily miss a QRS pulse producing unreliable RR interval values. Therefore it is necessary to estimate the RR interval in the presence of missing QRS beats. The approach in this thesis is to apply KALMAN estimation algorithm to the RR interval data calculated from the ECG. The goal is to improve the RR interval values obtained from missed beats of ECG data.
ACKNOWLEDGEMENTS

I am honored to have had the opportunity to work with Dr. Weeks as my thesis advisor. I thank him for the best gift of all, that of learning.

I would like to thank Dr. Bauer and Dr. Klee for their time and support in serving as my thesis committee members.

I would like to thank my parents who have sacrificed so much to give me a good education. I thank them for all they have done for me.

I would also like to thank my friends Pavan, Akarsh, Rahul and Sash for their constant support and help throughout the period of my research.

Finally I would like to thank Deepthi. Without her constant love and support throughout my time here, this thesis would not have been possible.
# TABLE OF CONTENTS

LIST OF FIGURES ........................................................................................................... vi

LIST OF TABLES ............................................................................................................... viii

CHAPTER 1: INTRODUCTION .......................................................................................... 1

- Objective ..................................................................................................................... 1
- Functioning of heart ................................................................................................. 2
- ECG background ....................................................................................................... 5
- Heart Rate .................................................................................................................. 8
- Arrhythmia ................................................................................................................ 8
- Heart rate detection .................................................................................................. 9

CHAPTER 2: DISCUSSION OF THE KALMAN FILTER ...................................................... 12

- Introduction ................................................................................................................ 12
- Mean squared error .................................................................................................. 14
- MSE solution ............................................................................................................. 15
- Steepest Descent Algorithm ..................................................................................... 18
- Least mean square (LMS) algorithm ........................................................................ 21
- KALMAN Filter Derivation ....................................................................................... 22
- Filter Algorithm ....................................................................................................... 27

CHAPTER 3: KALMAN FILTER IMPLEMENTATION ...................................................... 30

- Implementation ........................................................................................................ 30
- Filter Parameters ....................................................................................................... 30
- Filter adaptation ....................................................................................................... 31
- Performance in noisy environment ......................................................................... 37
- Filter performance with different parameters ....................................................... 39
- Statistics .................................................................................................................... 46

CHAPTER 4: IMPLEMENTATION ON ECG RECORDINGS ............................................. 48

- Implementation setup ............................................................................................... 48
- A Real Time QRS Detection Algorithm ................................................................. 48
- Results for real ECG data ....................................................................................... 51

CONCLUSION AND FUTURE WORK .......................................................................... 73

- Conclusion ................................................................................................................ 73
- Future Work .............................................................................................................. 74
LIST OF FIGURES

Figure 1.1: HRV analysis ........................................................................................................................... 1
Figure 1.2: HRV analysis with estimated RR intervals ................................................................. 2
Figure 1.3: Anatomy of Human Heart ................................................................................................. 3
Figure 1.4: Cardiac conduction system ......................................................................................... 4
Figure 1.5: Pacemaker cell action potential .................................................................................... 6
Figure 1.6: Non Pacemaker cell action potential ........................................................................... 6
Figure 1.7: Action potentials at different parts of the heart ....................................................... 7
Figure 1.8: ECG wave ......................................................................................................................... 7
Figure 2.1: Mean Square Error ........................................................................................................ 15
Figure 2.2: Non Recursive Adaptive Filter .................................................................................... 17
Figure 2.3: KALMAN Estimator ..................................................................................................... 28
Figure 3.2: Test Signal....................................................................................................................... 33
Figure 3.3: Intervals of the test signal ............................................................................................. 34
Figure 3.4: Estimated Interval .......................................................................................................... 34
Figure 3.5: Comparison of the estimated interval with actual interval ......................................... 35
Figure 3.6: Estimation Error ............................................................................................................. 36
Figure 3.7: KALMAN Gain ............................................................................................................... 37
Figure 3.8: Performance of the filter with a noisy input ............................................................... 38
Figure 3.9: Effect of the filter estimation with variation of the Measurement noise ................. 40
Figure 3.10: Variation in estimation Error with variation in measurement Noise Covariance 41
Figure 3.11: Defining the filter delay ............................................................................................ 42
Figure 3.12: Variation of the filter delay with variation in Measurement Noise Covariance .. 42
Figure 3.13: Filter performance with the process noise covariance assumed to be 10 .......... 43
Figure 3.14: Variation in mean estimation error with variation in Process Noise Covariance .. 44
Figure 3.15: Variation of the filter delay with variation in the Process Noise Covariance ....... 45
Figure 4.1: Estimation of RR intervals from ECG recordings ............................................. 48
Figure 4.3: ECG and the QRS detector plot for file 100.dat ................................................ 53
Figure 4.4: Measured Intervals and Estimated intervals for file 100.dat .............................. 54
Figure 4.5: ECG and the QRS detector plot for the file 101.dat ........................................... 55
Figure 4.6: Measured Intervals and Estimated intervals for file 101.dat .............................. 56
Figure 4.7: ECG and QRS detector plots for file 102.dat .................................................... 57
Figure 4.8: Measured Intervals and Estimated intervals for file 102.dat .............................. 58
Figure 4.9: ECG and QRS detector plot for file 103.dat ..................................................... 59
Figure 4.10: Measured Intervals and Estimated intervals for file 103.dat ......................... 60
Figure 4.11: ECG and QRS detector plot for file 104.dat ................................................... 61
Figure 4.12: Measured Intervals and Estimated intervals for file 104.dat ......................... 62
Figure 4.13: ECG and QRS detector plot for file 105.dat ................................................... 63
Figure 4.14: Measured Intervals and Estimated intervals for file 105.dat ......................... 64
Figure 4.15: ECG and QRS detector for file 106.dat ......................................................... 65
Figure 4.16: Measured Intervals and Estimated intervals for file 106.dat ......................... 66
Figure 4.17: ECG and QRS detector for file 107.dat ......................................................... 67
Figure 4.18: Measured Intervals and Estimated intervals for file 107.dat ......................... 68
Figure 4.19: ECG and QRS detector plot for file 118.dat .................................................... 69
Figure 4.20: Measured Intervals and Estimated intervals for file 118.dat ......................... 70
Figure 4.21: ECG and QRS detector plot for file 119.dat .................................................... 71
Figure 4.22: Measured Intervals and Estimated intervals for file 119.dat ......................... 72
LIST OF TABLES

Table 3.1: Statistics of measured and estimated RR intervals .................................................. 46
Table 4.1: Filter parameters for the ECG recordings ..................................................................... 52
Table A1: Comparison of Estimated intervals with Measured Intervals for beat 1 to 36 .......... 76
Table A2: Comparison of Estimated intervals with Measured Intervals for beat 37 to 73 ....... 77
Table A3: Comparison of Estimated intervals with Measured Intervals for beat 74 to 107 ..... 78
Table A4: Comparison of Estimated intervals with Measured Intervals for beat 108 to 140 ... 79
Table A5: Comparison of Estimated intervals with Measured Intervals for beat 141 to 171 ... 80
Table A6: Comparison of Estimated intervals with Measured Intervals for beat 172 to 204 ... 81
Table A7: Comparison of Estimated intervals with Measured Intervals for beat 205 to 230 ... 82
CHAPTER 1: INTRODUCTION

Objective

The rate at which the human heart beats exhibits a periodic variation. This is known as heart rate variability (HRV). Heart rate variability is an important measurement that can predict the survival after a heart attack. Studies have shown that reduced HRV predicts sudden death in patients with Myocardial Infarction (MI).

The HRV is originally measured from the R-R interval data calculated from the ECG. Figure 1.1 gives the steps in calculating HRV. First an ECG recording of a patient is obtained. Next, the QRS part of the ECG is detected and the RR interval is calculated. From the RR interval the HRV is found.

![HRV analysis diagram](image)

Figure 1.1: HRV analysis

However before the R-R interval undergoes the HRV computation we have to make sure that the quality of the R-R interval data is good and reliable. There might be a possibility for a QRS
detection algorithm to miss a beat. This type of error usually results in unreliable HRV measurement. Therefore it is necessary to first filter the R-R interval sequence based on some algorithm and then submit the newly obtained data for HRV analysis.

The KALMAN estimation algorithm has been used on the R-R interval data, which estimates the R-R intervals with minimum prediction error based on the previous values of the R-R interval. This algorithm was tested on different ECG recordings and its performance has been studied in this thesis. Figure 1.2 shows the addition of the RR interval estimation in the HRV process hence improving the HRV analysis.

![Diagram of HRV analysis with estimated RR intervals](image)

**Functioning of heart**

The *Cardiovascular system* is responsible for the supply of blood which is full of oxygen and nutrients to each and every part of the human body. The most crucial organ in this system is the human *heart*. It is the heart that pumps blood through the entire body by contracting and relaxing its muscular walls continuously. The human heart is made up of four chambers: Right Atrium (RA), Left Atrium (LA), Right Ventricle (RV), Left Ventricle (LV). The impure blood
(deoxygenated blood) from the body first comes into the RA through the Superior and Inferior Vena Cavae (SVC & IVC).

This blood then flows into the RV through the Tricuspid valve when the RA contracts. When the RV contracts, blood is then pumped into the Pulmonary Artery (PA) which leads to the lungs where the deoxygenated blood picks up oxygen. The oxygenated blood comes into the LA through Pulmonary Veins (PV). The LA then contracts and pushes the blood into the RV through Mitral Valve. Blood is pumped into the Aorta through the Aortic Valve when the RV contracts. Aorta is the main artery of the body. Through the Aorta, all the blood that is pumped from the heart is distributed to the entire body. The RV has a thick wall which enables it to pump with enough force so that blood is carried to all the parts of the body. The flow of blood is thus controlled by organized contractions and relaxations of the four
chambers of the heart. The Electrical impulses generated by the heart tissue allow these heart chambers to contract and relax in a continuous fashion. The heart has its own electrical system called as cardiac conduction system.

The Sinoatrial Node (SA) which is also called as the natural pacemaker initiates the heart beat by sending out an electrical impulse. The movement of these electrical signals makes the muscles contract. Once the signal diminishes, the heart muscles relax. The signal generated by the SA Node first passes through the muscles of LA and RA, thus making it contract. The electrical signal then proceeds to the Atrioventricular Node (AV) and pauses there for while - giving the Ventricles sometime to fill with blood. The electrical signal is then relayed down to the Ventricles through a conducting tissue called Bundle of His, which branches into two pathways -
left and right bundle branches that supply to the Ventricles - making it to contract and pump the blood.

**ECG background**

As mentioned above it is the heart’s electrical activity that enables the heart muscles to contract and relax. The heart’s electrical activity starts at the SA node also known as the *natural pacemaker*. The electrical excitation at the SA node is due to *depolarization* and *repolarization* of its cells. The pacemaker cells have various ions within and outside the cell membrane (mainly $K^+$, $Na^+$ and $Ca^+$). But these ions exist in different concentrations across the cell membrane. This chemical gradient enables the ions to flow across the membrane. The $Ca^+$ ion channels in the membrane open up allowing an inward flow of $Ca^+$ ions thus making the inside of the cell more positive, and the outside of the cell more negative. The voltage difference within the cell across the membrane is now more positive. The voltage across the membrane is called as the *membrane potential*. The membrane potential reaches a particular point from when it starts opposing further inflow of $Ca^+$ ions. This inflow of $Ca^+$ into the cell making the membrane potential more positive is called as *depolarization*. Now the $K^+$ ion channels in the membrane open up allowing the outflow of the $K^+$ ions. Because of the outflow of positive charge from the cell, it becomes more negative within the cell membrane and more positive across the cell membrane. This phase is called as *repolarization*. When we plot the combined effect of both depolarization and repolarization of the pacemaker cell it looks like an impulse as shown in Figure 1.5. This impulse is called as an *action potential*. This action potential now triggers the adjacent heart cells causing it to depolarize its cells. The depolarization and repolarization mechanisms in the rest of the heart cells (*non pacemaker cells*) are quite similar to that of the pacemaker cells except that depolarizations in these cells occur due to the inflow of $Na^+$ ions.
Another notable difference is the duration of the repolarization phase. During the repolarization phase of non-pacemaker cells, there is a balanced inflow and outflow of Ca\(^+\) and K\(^+\) ions respectively, for some time, after which the Ca\(^+\) ion channels are closed.

This causes a flat region in the action potential of a non pacemaker cell called as the plateau, as shown in Figure 1.6.

The depolarization and repolarization of various elements of the heart’s electrical system generate a voltage pattern which can be measured by placing few electrodes on the skin over the chest. These voltage measurements are called *Electrocardiograms* or *ECGs*. 
The ECG wave has various segments as shown in Figure 1.8 each representing some particular action of the heart.

*P wave:* This part of the ECG wave is the result of the atrial contraction.

*QRS complex:* The QRS complex represents the ventricular activation.

*T wave:* This portion of the ECG wave represents the ventricular relaxation.
The popular ECG systems at present are the 12 lead, 5 lead and 3 lead ECGs [15]. The leads record the heart's electric field in different angles.

**Heart Rate**

It is the number of heart beats per minute. The time interval between each beat is the R wave to R wave interval, in short called as RR interval. Therefore heart rate is given by the reciprocal of the RR interval expressed in beats per minute. A normal heart beats 60 to 100 times a minute.

**Arrhythmia**

Any deviation from the normal heart rhythm or irregular heart beat due to a malfunction in the heart’s conduction system is called as arrhythmia. The two major types of arrhythmia are *Bradyarrhythmia* and *Tachyarrhythmia*.

*Bradyarrhythmia*: If the heart beat is below 60 beats per minute which is considered to be very slow then the heart is said to be suffering from Bradycardia.

*Tachyarrhythmia*: If the heart beat is above 100 beats per minute which is considered to be very high then the heart is said to be suffering from Tachycardia.

Both Bradycardia and Tachycardia are medically significant. There are other arrhythmias like the *sinus arrhythmia* and *sinus tachycardia* which are not medically significant.

*Sinus arrhythmia*: It is the variation in the heart rhythm during breathing.

*Sinus tachycardia*: It is the speeding up of the heart rate during any physical exercise.
Heart rate detection

The detection of QRS complexes (or the beat detection) precedes any type of ECG analysis. In order to measure the heart rate, the QRS complexes must first be detected from a noise corrupted ECG wave thus eliminating the false peaks and false negatives. Various algorithms have been brought forward till today but none of them are perfect [5]. This is because of the diverse nature of the ECG characteristics and its morphology.

Gary M Friesen et al [5] compared the noise sensitivity of nine QRS detection algorithms based on

- Amplitude and first derivative
- First derivative
- First and second derivative
- Digital filters

None of the algorithms evaluated in this study were perfect for various types of noise degraded ECG signals. A tuning procedure was employed on the algorithms to determine the preset constants used by the algorithm, for it to give the best results when applied on noise corrupted data. But this tuning procedure would not be practical in clinical setting. Thresholds that have an adaptive approach will be more appropriate.

Pan J and Tompkins W [6] proposed a real-time QRS detection algorithm which uses an adaptive threshold. This threshold adjusts its levels to adapt to the periodic changes in the ECG signal. It detects the QRS complexes based on the slope, amplitude and width information. This adaptive approach proves to be best to the ECG signals which vary a lot in its characteristics, morphologies and the heart rate.
Ivaylo I Christov [7] has developed a real-time QRS detection using combined adaptive threshold. The adaptive threshold used in this algorithm combines three parameters.

- An adaptive slew rate
- A parameter which rises when noise with very high frequency occurs
- A parameter to avoid missing heart beats with unusually low amplitudes

This algorithm compares the absolute value of this adaptive threshold and the sum of the differentiated ECG from different leads, for the QRS detection. Sensitivity (Se) of 99.69% and Specificity (Sp) of 99.66% has been achieved using this algorithm.

Se and Sp can be defined as

\[
Se = \frac{TP}{TP+FN} \quad (1.1)
\]

\[
Sp = \frac{TP}{TP+FP} \quad (1.2)
\]

TP is true positive, FN and FP are the false negative and false positive respectively.

Afonso et al [8] have developed a multi rate digital signal processing algorithm to detect heart beats in the ECG. This algorithm incorporates a filter bank which contains a set of filters. These filters decompose the incoming ECG signal into various sub bands. Each sub band will carry some particular information regarding the ECG. These sub bands are then processed individually based on the specific application. For example major proportion of the QRS complex energy extends to the 40 Hz frequency. If a QRS complex is originated at the SA node its energy will be appearing in the higher frequency sub bands. Therefore by analyzing different sub bands of the ECG, beat can be detected.
In this thesis, based on the pulses generated by the QRS detection algorithm by Pan J, Tompkins W the **R-R interval** has been estimated using a **KALMAN estimator**. KALMAN filter derivation and its applications are discussed in detail in the following chapter. The KALMAN filter presented in this thesis was implemented in MATLAB.

The filter presented here was first tested on a few test signals and its performance was studied. The results are included in Chapter 3. The filter was then used on the ECG recordings available in the MIT/BIH ECG data base [20]. Ten ECG recordings were selected from the database to test the KALMAN filter. The results are shown in chapter 4.
CHAPTER 2: DISCUSSION OF THE KALMAN FILTER

Introduction

The estimation theory started to develop when the need to estimate the motion of the heavenly bodies from a set of noisy telescopic measurements [9] arose. To solve this problem, the method of least squares was used which was invented by Karl Friedrich Gauss. One method is based on KALMAN filtering [10] and is considered a modern form of this least squares estimation technique.

The purpose of the KALMAN filter is to estimate the accurate information from inaccurate data. It essentially estimates the state of a system based on the system output measurements which contain random errors. It is extensively used in the areas of navigational science, tracking, signal processing and control theory. The major applications where the KALMAN filter plays a vital role are vehicle navigation, satellite orbit estimation, radar tracking, filtering and estimation of signals and real time control of machines. In the following few paragraphs interesting proposals of the application of the KALMAN filter are discussed.

M.P.Taravainen et al [11] in their paper "Estimation of Non Stationary EEG spectrum with KALMAN Filter" published in 2003, discuss an adaptive spectrum estimation method for non-stationary EEG. Since the EEG signals are non-stationary, a parametrical spectral analysis method was used which is based on a time varying auto regressive moving average (ARMA) model. The KALMAN filter algorithm was used in the estimation of the parameters for the above time varying model.
The KALMAN filter also has interesting applications in the field of Communications. It can be used in the estimation of the round trip time in communication networks. This application has been discussed in detail in the paper "Round Trip Time Estimation in Communication Networks using adaptive KALMAN filtering" by Krister Jacobsson et al [12]. The round trip time in the communication network plays an important role in the congestion control. Therefore the more accurate the round trip time estimates are the more efficient is the congestion control. The round trip time is usually measured from packet acknowledgements, which include delays caused by the network transient effects. This makes the measured round trip time less accurate. The KALMAN filter in conjunction with a change detection algorithm was used to get more accurate estimates of the round trip times and the results showed that the estimates are more accurate when compared to the round trip time estimator currently used in TCP [12].

The paper "Real time Estimation of Human Body Postures using KALMAN filter" by Kazuhiko Taka Hashi et al [13] proposes the usage of KALMAN filter while estimating the postures of human body, which is very important especially for machine-user interface. While estimating the human body postures using the silhouette based estimation method problems such as self occlusion occur. In silhouette based estimation method, the feature points of the human body (like the top of the head, tips of hands and feet, and elbow, knee joints) are obtained from the human silhouettes (outline images). The problem of self occlusion occurs in certain postures like when the hands overlap the body. It is difficult to obtain the feature points in such cases. An autoregressive model has been used to track and obtain these feature points. The parameters of the AR model are estimated using the KALMAN filter [13].
R.E. KALMAN published his famous paper [10] in 1960, “A new approach to linear filtering and prediction problems” in which he described a recursive algorithm that solves the discrete data linear filtering problem. It is an efficient computational solution of the method of least squares. The filter was constructed with the goal to minimize the mean squared error [10].

**Mean squared error**

Consider a signal $y_k$ and its estimate from the estimator is $\hat{y}_k$. The error in estimation is given by

$$e_k = y_k - \hat{y}_k \quad (2.1)$$

Therefore the error is amount of deviation of the estimated signal from the true signal.

The squared error function is defined as

$$f(e_k) = (y_k - \hat{y}_k)^2 \quad (2.2)$$

Since we have to consider the ability of the filter over a period of time, a more appropriate metric would be the expected value of the squared error function. This metric is named as the loss function.

$$loss\ function = E\{f(e_k)\} \quad (2.3)$$

This leads us to the mean square error (MSE)

$$\varepsilon_k = E\{e_k^2\} \quad (2.4)$$
The mean square error serves as an index to how best our estimator performs. Estimators with low MSE are considered to be good because low mean square error indicates a very low deviation of our estimated values from the actual values.

**MSE solution**

Mean squared error solution serves as one of the best performance criterion for most of the adaptation algorithms used on adaptive filters. The Mean squared error solution is also known as *Weiner* solution, and the class of filters that apply algorithms which lead to the weiner solution are called the weiner filters [14].

The simplest structure that can be used for adaptive filters is as shown in Figure 2.2. The T block represents a delay by one step. $y(n)$ represents the output and $x(n)$ the input. $d(n)$ is the desired response and $e(n)$ is the error. $a_0, a_1, \ldots, a_{N-1}$ represent the filter coefficients.

The output of the filter $y(n)$ can be mathematically represented as
\[ y(n) = x^T_n a_n \] \hspace{1cm} (2.5)

\( N \) is the length of the filter.

\[ a_n = [ a_0, a_1, a_2, \ldots, a_{N-1}]^T \] is the filter coefficient vector at any time instant \( n \).

\[ x_n = [ x_1, x_2, x_3, \ldots, x_{N}]^T \] is the input vector at any time instant \( n \).

The error function is given by (2.6) where

\[ e(n) = d(n) - y(n) \] \hspace{1cm} (2.6)

d(n) is the desired response.

Based on Equation 2.4 the mean square error (MSE) is given by

\[ \varepsilon(a_n) = \mathbb{E}\{ e(n)^2 \} \] \hspace{1cm} (2.7)

substituting Equation 2.6 in 2.7

\[ \varepsilon(a_n) = \mathbb{E}\{ d^2(n) - 2d(n)y(n) + y^2(n) \} \] \hspace{1cm} (2.8)

for a filter with fixed coefficients the MSE function is given by

\[ \varepsilon(a_n) = \mathbb{E}\{ d^2(n) \} - 2\mathbb{E}\{ d(n)\} a_n^T x_n + \mathbb{E}\{ a_n^T x_n x_n^T a_n \} \] \hspace{1cm} (2.9)

If \( p_n \) is the cross correlation of the desired response and the input then it is given by

\[ p_n = \mathbb{E}\{ d(n) x_n \} \] \hspace{1cm} (2.11)

If the correlation of the input \( x(n) \) is given by \( R_n \) then it is given by

\[ R_n = \mathbb{E}\{ x_n x_n^T \} \] \hspace{1cm} (2.12)

Substituting Equations 2.11 and 2.12 in 2.10 the MSE can be expressed as

\[ \varepsilon(a_n) = \mathbb{E}\{ d^2(n) \} - 2a_n^T p_n + a_n^T R_n a_n \] \hspace{1cm} (2.13)
Figure 2.2: Non Recursive Adaptive Filter

The filter coefficient vector should take values such that MSE is minimum.
The gradient of MSE is

\[ \text{Gradient of } \varepsilon(a_n) = g_n = -2p_n + 2R_n a_n \]  \hspace{1cm} (2.14)

Equating the elements of the gradient vector \( g_n \) to zero we get the coefficient vector that minimizes the MSE.

\[ a_{\text{opt}} = R_n^{-1} p_n \]  \hspace{1cm} (2.15)

This is the Weiner solution. The filter should adapt its coefficients based on the above equation such that the mean squared error reaches its minimum. Some of the available algorithms that lead to the Weiner solution are Least mean square algorithm and Steepest descent algorithm.

**Steepest Descent Algorithm**

The method of steepest descent uses the following updating formula to reach the MSE solution.

\[ a_{k+1} = a_k + \mu (g_k) \]  \hspace{1cm} (2.16)

\( a \) is the filter coefficient vector, \( k \) is the iteration number and \( g \) is the gradient vector of the mean squared error \( \varepsilon(a) \). \( \mu \) is the convergence factor that determines the convergence behavior of the algorithm.

Substituting the gradient vector (Equation 2.14) in Equation 2.16

\[ a_{k+1} = a_k + \mu (-2p + 2R_a) \] \hspace{1cm} (2.17)

From the Weiner solution Equation we have

\[ a_{\text{opt}} = R^{-1} p \] \hspace{1cm} (2.18)

That implies

\[ p = R^{-1} a_{\text{opt}} \] \hspace{1cm} (2.19)

Substituting Equation 2.19 in 2.17 will give us
The error in the filter coefficients when compared to the optimal filter coefficients can be represented as

\[ v_k = a_k - a_{\text{opt}} \]  \hspace{1cm} (2.21)

Equation 2.20 can now be represented as

\[ v_{k+1} = (I - 2\mu R) v_k \]  \hspace{1cm} (2.22)

The above Equation is pre-multiplied by \( Q^T \) where \( Q \) is a unitary matrix that diagonalizes \( R \).

\[ Q^T v_{k+1} = (I - 2\mu Q^T R Q) Q^T v_k \]  \hspace{1cm} (2.23)

The coefficient error vector \( \mathbf{v} \) is rotated to the principal axis by the following transformation

\[ \mathbf{v}_{k+1}' = Q^T \mathbf{v}_{k+1} \]  \hspace{1cm} (2.24)

Equation 2.23 now can be rewritten as

\[ \mathbf{v}_{k+1}' = (I - 2\mu \Lambda) \mathbf{v}_k' \]  \hspace{1cm} (2.25)

\( \Lambda \) is the eigenvalue matrix of the input correlation matrix \( R \). [15] By mathematical induction the solution to Equation 2.25 must be

\[ \mathbf{v}_k' = (I - 2\mu \Lambda)^k \mathbf{v}_0' \]  \hspace{1cm} (2.26)

From the above result we can see that the steepest descent algorithm is stable and convergent when

\[ \lim_{k \to \infty} (I - 2\mu \Lambda)^k = 0 \]  \hspace{1cm} (2.27)

\[ \begin{bmatrix}
\lim_{k \to \infty} (I - 2\mu \lambda_0)^k & 0 & \cdots & 0 \\
0 & \lim_{k \to \infty} (I - 2\mu \lambda_1)^k & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \lim_{k \to \infty} (I - 2\mu \lambda_N)^k 
\end{bmatrix} = 0 \]  \hspace{1cm} (2.28)
where $\lambda_0, \lambda_1, \ldots, \lambda_N$ are the eigen values of $R$.

From the above form we see that the convergence condition is satisfied by choosing $\mu$ so that

$$0 < \mu < 1/\lambda_{\text{max}}$$

(2.29)

where $\lambda_{\text{max}}$ is the largest eigen value of $R$.

Consider Equation 2.26 and 2.27

$$\lim_{k \to \infty} v'_k = 0$$

(2.30)

that implies

$$\lim_{k \to \infty} a_k = a_{\text{opt}}$$

(2.31)

Therefore by appropriate selection of $\mu$ and a finite number of iterations the coefficient vector converges to the optimal value. So basically the algorithm is designed in such a way that the optimal solution is achieved by descending toward the minimum on the error performance surface $\epsilon(a_n)$. The optimal values for the filter coefficients reside at the minimum of the error surface. In the above algorithm, at each iteration the measurement of the gradient vector was required. In most of the applications it is difficult to get an exact measurement, so the gradient is estimated. Hence the steepest descent algorithm is implemented using the gradient estimate instead of the actual gradient values. The gradient is estimated by taking differences between short term averages of $e_k^2$ where $e_k$ is the error vector of the filter output at $k^{th}$ iteration as defined in Equation 2.6. The usage of the gradient estimate instead of actual gradient will have some effect on the filter performance.
Least mean square (LMS) algorithm

This is another popular adaptive algorithm for descending on the error performance surface [15]. The LMS algorithm uses a more effective and simple method to estimate the gradient vector.

From Equation 2.5 and 2.6 we can define error at $k^{th}$ iteration as

$$e_k = d_k - x^T_k a_k$$  \hspace{1cm} (2.32)

Instead of taking differences between short term averages of $e_k^2$ to estimate the gradient of MSE $\varepsilon$, $e_k^2$ itself is taken as the estimate of $\varepsilon$ which will give us the following expression for the gradient of $\varepsilon$.

$$\hat{g}_k = \begin{bmatrix}
\frac{\partial e_k^2}{\partial a_0} \\
\frac{\partial e_k^2}{\partial a_1} \\
\frac{\partial e_k^2}{\partial a_2} \\
\vdots \\
\frac{\partial e_k^2}{\partial a_N}
\end{bmatrix} = \begin{bmatrix}
\hat{e}_k^2 \\
\hat{e}_k^2 \\
\hat{e}_k^2 \\
\vdots \\
\hat{e}_k^2
\end{bmatrix} = -2e_k x_k$$ \hspace{1cm} (2.33)

$\hat{g}_k$ is the estimate of the gradient vector. The derivatives in the above Equation can deduced from Equation 2.32. Therefore the gradient estimate from the above Equation can be deduced by multiplying the input vector with the error. Substituting the above result in Equation 2.16 we get updating formula for this algorithm which is

$$a_{k+1} = a_k + 2\mu e_k x_k$$ \hspace{1cm} (2.34)

Using the above updating formula the algorithm leads to the MSE solution.

The KALMAN filter was constructed based on the principal to reduce the mean square error.
**KALMAN Filter Derivation**

In order to use the KALMAN filter on any signal, the process measured must be able to be described by a linear dynamic system. The KALMAN filter has been developed using state space techniques.

Consider a discrete time controlled process of a linear dynamic system that is governed by the linear stochastic difference Equation.

\[ g_{k+1} = Ag_k + Bu_k + w_k \]  

(2.35)

A linear difference Equation relates a variable to its previous values linearly as shown above. Using this Equation a solution to \( g_{k+1} \) is found. \( g \) is the state variable of the system that describes its dynamics. \( u \) is a known input that controls the system and \( B \) a constant matrix. \( A \) is the state transition matrix that describes the changes in the state due to system dynamics. \( w_k \) is a random variable that represents the process noise.

The output of the system can be modeled as below [3]

\[ y_k = Hg_k + v_k \]  

(2.36)

\( H \) is the measurement matrix that relates the output of the system to the system state. \( y_k \) is the measured value or the output of the system. \( v_k \) is the random variable that represent the measurement noise.

The state variable \( g \) describes the system but this system state cannot be measured directly. So we use the measured value \( y \) which is a function of \( g \) to obtain the best possible estimate of the system state. A criterion [17] has to be employed while estimating the state \( g \) because the measured value \( y \) cannot be trusted completely as it is corrupted by the measurement noise \( v \).
The criteria for the estimator to determine the best estimate of the state, is to satisfy the following two requirements. First the expected value of our state estimate should be equal to the expected value of the true state. Second, the state estimate should vary as little as possible from the true state which means that the estimator should have the error variance as low as possible.

The KALMAN filter solution satisfies the above requirements when we make certain assumptions about the noise that affects our system. The measured noise $v_k$ and the process noise $w_k$ are assumed to be white noise, independent and have a normal probability distribution. The covariance matrices of the process and measurement noise are denoted as $Q$ and $R$ respectively and the error covariance matrix is denoted by $P$.\[3\]

\[
Q = E\{w_kw_k^T\} \quad (2.37)
\]

\[
R = E\{v_kv_k^T\} \quad (2.38)
\]

\[
P = E\{e_ke_k^T\} \quad (2.39)
\]

$e_k$ is the estimation error (Equation 2.1). The above expressions can be deduced as follows.

Let us denote $x_i, x_j$ to be two random variables. The covariance of the two random variables is defined as

\[
cov (x_i, x_j) = E\{[x_i - E(x_i)] [x_j - E(x_j)]\} \quad (2.40)
\]

if we consider the random variables to be white, zero mean, normal and independent then

\[
E(x_i) = 0 \quad (2.41)
\]

\[
E(x_j) = 0 \quad (2.42)
\]

$E(x_i), E(x_j)$ represent the mean values of $x_i$ and $x_j$ which are equal to zero because the mean of normal white random variables is zero.

Therefore,
\[ \text{cov} (x_i, x_j) = E(x_ix_j) \quad (2.43) \]

Consider the case where \( i = j \)

\[ \text{cov} (x_i, x_j) = E(x_i^2) \quad (2.44) \]

If \( x \) is vector with random variables \( x_1, \ldots, x_n \)

\[ \text{cov} (x, x) = E(x^2) \quad (2.45) \]
\[ x^2 = xx^T \quad (2.46) \]

Therefore,

\[ \text{cov} (x, x) = E(xx^T) \quad (2.47) \]

From the above deduction we can conclude that the covariance matrices \( Q, R \) and \( P \) can be expressed as shown in Equation 2.37, 2.38 and 2.39.

Based on the above assumptions a set of Equations are defined for the estimator. These Equations are divided into two groups.

- **Time Update Equations**

  \[ \dot{\hat{g}}_k = A \dot{\hat{g}}_{k-1} + Bu_{k-1} \quad (2.48) \]
  \[ P_k = AP_k A^T + Q \quad (2.49) \]

- **Measurement Update Equations**

  \[ \dot{\hat{g}}_k = \dot{g}_k + K_k (y_k - H \hat{g}_k) \quad (2.50) \]
  \[ P_k = (1 - K_k H)P_k \quad (2.51) \]
  \[ K_k = P_k H^T (HP_k H^T + R)^{-1} \quad (2.52) \]

\( \dot{\hat{g}}_k \) is the a priori state estimate, \( \hat{g}_k \) is the a posteriori state estimate, \( P_k \) is the a priori estimate of the error covariance, \( P_k \) is the a posteriori estimate of the error covariance.
\[ P_k = \mathbb{E}\{e_k e_k^T\} \quad (2.54) \]

\( K_k \) is the KALMAN gain. \( y_k \) is the measurement and \( H\hat{g}_k \) is the measurement prediction.

The origin of the filter is based on the following. We make a measurement \( y_k \). We also make a prediction of this measurement using the a priori estimate \( \hat{g}_k \) which will be \( H\hat{g}_k \) (\( H \) is the matrix that relates the output of the system to the system state). \( (y_k - H\hat{g}_k) \) is called the residual or innovation [3]. It is the deviation of the predicted measurement from actual measurement. Equation 2.48 from the set of Equations defined for the estimator predicts a better estimate for the current state based on a linear combination of the a priori estimate \( \hat{g}_k \) and a weighted version of the residual.

\[ \hat{g}_k = \hat{g}_k + K_k (y_k - H\hat{g}_k) \]

The weight \( K_k \) (KALMAN gain) is chosen in such a way that it minimizes the a posteriori error covariance \( P_k \). From Equation 2.54 we have

\[ P_k = \mathbb{E}\{e_k e_k^T\} \]

From Equation 2.1 we can rewrite \( P_k \) as

\[ P_k = \mathbb{E}\{(g_k - \hat{g}_k)(g_k - \hat{g}_k)^T\} \quad (2.55) \]

Substituting Equation 2.50 in 2.55

\[ P_k = \mathbb{E}\{[(1-K_k H)(g_k - \hat{g}_k) - K_k v_k][((1-K_k H)(g_k - \hat{g}_k) - K_k v_k)^T]\} \quad (2.56) \]

Taking the derivative of the trace from the above Equation with respect to \( K \) and equating it to zero will give us value for \( K \) which will minimize the error covariance.

\[ K_k = P_k H^T (HP_k H^T + R)^{-1} \]

This is one of the measurement update Equations defined in the set of Equations for the estimator (Equation 2.52).

Equation 2.56 can be expressed as [3]
\[
P_k = (1-K_kH)P_k(1-K_kH)^T + K_kR_k^T \tag{2.57}
\]

Substituting Equation 2.56 in 2.57 we get the measurement update Equation for the error covariance (Equation 2.53).

\[
P_k = (1 - K_kH)P_k^* \tag{2.58}
\]

The time update Equation for the error covariance can be deduced as following.

\[
e_{k+1}^* = g_{k+1} - \hat{g}_{k+1} \tag{2.59}
\]

Equation 2.58 represents the error in our a priori estimate.

State projection is done as

\[
\hat{\mathbf{g}}_{k+1} = A\hat{\mathbf{g}}_{k-1} \tag{2.60}
\]

where \(\hat{\mathbf{g}}_{k} \) is the a priori state estimate and \(\hat{\mathbf{g}}_{k-1} \) is the previous a posteriori state estimate.

Based on Equation 2.59 and 2.35

\[
e_{k+1}^* = (Ag_k + w_k) - (A\hat{g}_k) \tag{2.61}
\]

Rearranging Equation 2.60

\[
e_{k+1}^* = A(g_k - \hat{g}_k) + w_k \tag{2.62}
\]

Based on Equation 2.1 we can express Equation 2.61 as

\[
e_{k+1}^* = (Ae_k + w_k) \tag{2.63}
\]

From Equation 2.53 we can express the a priori error covariance as

\[
P_{k+1}^* = E\{(e_{k+1})^T(e_{k+1})\} \tag{2.64}
\]

Substituting Equation 2.62 in 2.63 we get

\[
P_{k+1}^* = E\{(Ae_k + w_k)(Ae_k + w_k)^T\} \tag{2.65}
\]

The cross correlation between \(w_k\) and \(e_k\) will be zero [3]. Therefore

\[
P_{k+1}^* = E\{(Ae_k)(Ae_k)^T\} + E\{w_kw_k^T\} \tag{2.66}
\]

Equation 2.65 can be rearranged as

\[
P_{k+1}^* = AE\{e_ke_k^T\}A^T + E\{w_kw_k^T\} \tag{2.67}
\]
Based on Equations 2.37 and 2.39

\[ P_{k+1} = AP_kA^T + Q \]  \hspace{1cm} (2.67)

From the above Equation we can express the time update Equation for error covariance as shown in Equation 2.51

\[ P_k^* = AP_kA^T + Q \]

**Filter Algorithm**

Figure 2.3 shows the block diagram of the steps required in implementing the KALMAN filter. Initially a posteriori state \( \hat{g}_k \) and a posteriori error covariance \( P_k \) are assigned a guess value. These values are used to predict forward in time the a priori state \( \hat{g}_{k+1} \) and the a priori error covariance \( P_{k+1}^* \) based on Equations 2.45 and 2.46. The estimation of the measurement is given by the product of a priori state \( \hat{g}_{k+1} \) and measurement matrix \( H \) as shown in the Figure 2.3. The error in our estimation can be obtained. This error which is also called the residual is substituted in Equation 2.47 to obtain updated a posteriori state \( \hat{g}_{k+1} \) which will be again used in the next cycle of prediction. KALMAN gain \( K_k \) which was a part of Equation 2.47 is calculated based on the a priori error covariance \( P_{k+1}^* \) predicted in the first step. It is here that the actual error minimization takes place because the KALMAN gain is calculated based on the principal to reduce the error covariance. The expression for KALMAN gain is given in Equation 2.49. An updated a posteriori error covariance \( P_{k+1} \) is calculated based on Equation 2.48 and this value will be used in the next iteration for predicting the a priori error covariance. KALMAN gain adjusts its value for every iteration and slowly converges to an optimal value such that the error covariance reaches its minimum.
Therefore based on the KALMAN equations the KALMAN filter recursively calculates the current estimate based on the past measurement and slowly converge its parameters such that the estimated value deviates very little from the actual value.
Chapter 3 discusses one implementation of the KALMAN filter used to estimate the ECG RR interval.
CHAPTER 3: KALMAN FILTER IMPLEMENTATION

Implementation
The KALMAN filter was implemented in Matlab based on the KALMAN Equations discussed in the previous chapter. The filter was first tested using test signals whose characteristics are known. Various aspects regarding the filter performance were studied with these test signals as our inputs to the filter. Later the filter was tested on ECG recordings from the MIT/BIH database.

Filter Parameters
The main filter parameters are Q and R, where Q and R represent the assumed process and measurement noise covariance. The filter can be properly tuned by adjusting the associated parameters. The KALMAN filter discussed in chapter 2 is used to measure the states of a system based on noisy output measurements. The filter presented here is implemented to estimate the RR intervals from measured values as shown in Figure 3.1. As discussed earlier the rate at which the heart beats is not constant. Various factors like respiration, stress, age etc will influence the manner in which the heart rate changes. The current RR interval value will be almost equal to the past value except for some deviation because of various medical factors. Hence the heart rate can be expressed as

\[
RR_{interval_{k+1}} = RR_{interval_k} + \text{some deviation}
\]

Equation 3.1 can now be treated as a linear difference Equation (Equation 2.35). The deviation at each beat can be considered as process noise. Based on Equation 3.1 we can assume the state transition matrix \( A = 1 \). The state transition matrix relates the previous state of the system to the current state. Our noisy measurement is of the state directly so \( H(\text{measurement} \)
matrix) = 1. Some guess values are taken for the apriori state and apriori error covariance. The filter will eventually converge irrespective of the guess values.

**Filter adaptation**

Figure 3.1 briefly outlines the procedure to test the filter adaptation. A test signal is generated which will serve as our filter input. As the QRS detector will produce an impulse train by detecting and placing an impulse wherever a QRS complex occurs in the ECG and the intervals between them will be the RR interval, it would be appropriate for us to select an impulse train as our test signal. The intervals between the impulses in our test signal were varied according to a known pattern as shown in Figure 3.2. The RR intervals are measured. These varying RR intervals are shown in Figure 3.3. The measured intervals were then passed through our filter for it to estimate the time interval. The estimated intervals were then compared to the measured intervals. Because of our prior knowledge of the nature of the interval variation we can best describe the filter performance from its estimated values. From the output of the filter as shown in Figure 3.4 we can observe the rate at which the filter adapts itself to best estimate from the measured data. The comparison is better seen in Figure 3.5. This shows us that the filter implemented here can adapt to any time interval and it is a desired feature because the heart rates vary from one person to other and the filter should adapt to those changes.
Generate the test signal

Intervals of the test signal are measured

Based on the parameters and the input intervals the KALMAN filter Equations are used to compute the estimated intervals

Estimated intervals

Measured intervals

The error in the estimation is computed and sent as a feedback to the filter for optimization

Figure 3.1: KALMAN Filter Implementation on the test signal
Figure 3.2: Test Signal
Figure 3.3: Intervals of the test signal

Figure 3.4: Estimated Interval
The estimation error is plotted and shown in Figure 3.6. We can see that the error shoots up when there is a drastic change in the input interval but it quickly drops back as the filter learns the change. It can be shown that the KALMAN gain converges to an optimum value from the plot shown in Figure 3.7.
Figure 3.6: Estimation Error
Figure 3.7: KALMAN Gain

**Performance in noisy environment**

The input in the previous section was noise free. There is always a possibility to make errors while measuring the intervals. This noise is called as the measurement noise. Instead of a clean
input, we feed the filter with a noise corrupted input. Usually the measurement noises tend to be Gaussian by nature, so we corrupt the input intervals with Gaussian noise. Now we have noisy interval measurements as our filter input. The filter performance is studied by observing the estimated values from the filter output. The filter output is compared to the noisy input and the actual input as shown in Figure 3.8.

Figure 3.8: Performance of the filter with a noisy input

From the comparison in Figure 3.8 we can observe that the estimated values are closer to the actual values than the measured values which are corrupted with noise. Therefore the KALMAN filter will reduce the noise that will cause our measurements to deviate from the actual values.
Filter performance with different parameters

The filter parameters were varied and its performance on various aspects has been observed. First the filter was tested on our noise corrupted signal for different assumptions of the measurement noise. The plots are shown in Figure 3.9. It is shown from the estimated values that the filter estimates follow closely the noisy measurements when the assumed measurement covariance is a low value (here $V = 0.5$). But if the measurement noise covariance is considered to be high the filter estimates were slower in following the measurements and that’s the reason why the output deviates a lot from the actual interval.

The effect on the performance of the filter by varying the filter parameters was also studied on our noise free test signal (Figure 3.2). The measurement noise covariance is varied through a range of values and the absolute mean estimation error for each value was observed and plotted as shown in Figure 3.10 and as expected we see that the mean estimation error increases with the increase in the measurement noise covariance.
Figure 3.9: Effect of the filter estimation with variation of the Measurement noise ($V=40, 10, \text{ and } 0.5$)
It was discussed that the assumption of a high measurement noise covariance will slow down the filter estimates in following the measurements. This will result in large estimation errors at high measurement noise covariance. It was shown from the output of the filter (Figure 3.5) when a noise free test signal was taken as the input, the filter took some time to adapt to sudden changes in the input. We define the filter delay as the time taken for the filter to reach a value that is 90% of our actual input as shown in Figure 3.11. The behavior of the filter delay was studied by varying the measurement noise covariance as shown by the plots in Figure 3.12. The delay keeps increasing with the increase in the measurement noise covariance because of the degradation in the filter performance with the increase in the measurement noise covariance. After a certain value of the measurement noise covariance the estimated values from the filter output never reaches the 90% value of the input and the delay becomes infinity.
Figure 3.11: Defining the filter delay

This can be justified from our previous discussion that a high measurement noise covariance increases filter estimates. This behavior can be shown from the plot shown in Figure 3.12.

Figure 3.12: Variation of the filter delay with variation in Measurement Noise Covariance
All the previous results were also obtained by varying the process noise covariance. The results of the filter performance in the presence of noise, absolute mean estimation error and the time delay is shown in figures 3.13 – 3.15.

By assuming a high process noise covariance \( (Q) \) it can be shown in Figure 3.13 that the filter estimates follow the measurements very closely.

![Figure 3.13: Filter performance with the process noise covariance assumed to be 10](image)

Figure 3.13: Filter performance with the process noise covariance assumed to be 10

While assuming high uncertainty (high \( Q \)) in the process it is desirable to have process measurements that are reliable because the filter will assume a high process noise covariance
and be able to follow the actual measurements very closely. This behavior was also reflected in our study of the effect on absolute mean estimation error with the variation of process noise covariance through a range of values. It can be shown in Figure 3.14 that the estimation error decreases with the increase in the process noise covariance.

![Figure 3.14: Variation of the absolute mean estimation error with variation in Process Noise Covariance](image)

Figure 3.14: Variation of the absolute mean estimation error with variation in Process Noise Covariance
This can be justified from our discussion that when the filter assumes a high process noise covariance it will be able to follow the actual measurements very closely and hence the estimation error would obviously be less. It is exactly the opposite when we vary the measurement noise covariance (Figure 3.12) where the estimation error increases with an increase in the measurement noise covariance because when the filter assumes high measurement noise covariance it will not be able to follow the actual measurements closely.

Figure 3.15 shows the behavior of the filter delay with the variation of the process noise covariance.

Figure 3.15: Variation of the filter delay with variation in the Process Noise Covariance
The filter delay decreases with the increase in the process noise covariance because the filter estimates follow the measurements very closely when the process noise covariance was assumed to be high and any change in the filter input, the filter will quickly learn them.

When it comes to the practical implementation of the filter, the measurement noise covariance (R) can be known beforehand. It can be determined by taking few offline sample measurements. It is difficult to have an accurate value of the process noise covariance (Q) as we cannot directly observe the process we are estimating. However superior filter performance can be obtained by properly tuning the filter parameters Q and R though we do not have a rational basis in choosing them.

Statistics

The values for the measured intervals, estimated intervals from the filter output and the actual intervals for the plot shown in Figure 3.8 is listed through tables A1 to A7 in appendix. The average and standard deviation of the measured intervals and estimated intervals shown in Table 3.1 are computed from this simulated data.

Table 3.1: Statistics of measured and estimated RR intervals

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of actual intervals</td>
<td>12.95652</td>
</tr>
<tr>
<td>Average of measured intervals</td>
<td>13.11652</td>
</tr>
<tr>
<td>Average of estimated intervals</td>
<td>13.00651</td>
</tr>
<tr>
<td>Standard Deviation of measured intervals</td>
<td>7.953382</td>
</tr>
<tr>
<td>Standard Deviation of estimated intervals</td>
<td>6.855498</td>
</tr>
<tr>
<td>Standard Deviation of actual intervals</td>
<td>7.114032</td>
</tr>
</tbody>
</table>
From the statistics in Table 3.1 it is shown that filter was able to estimate the intervals close to the actual intervals, from the interval measurements which deviate from the actual intervals due to the addition of measurement noise.

As shown in Table A1 the filter started out with an initial guess value of 2 for the interval. The actual interval was 10. The filter took just seven iterations to reach an estimated value of 10.05 which is indeed very close to the actual interval. As shown in Table A3 at the 100th beat the actual interval changes from 10 to 30. It is shown that the filter took nine iterations to reach an estimated value of 30.579. And at 134th beat (Table A4) when the actual interval dropped back to 10 it took eleven iterations for the filter to estimate a value of 10.127 (Table A5). The filter was quick in learning in spite of a large variance in the input.
CHAPTER 4: IMPLEMENTATION ON ECG RECORDINGS

Implementation setup

The filter presented above is now tested on ECG recordings available in the MIT/BIH database [20]. First an ECG recording from the MIT/BIH database is obtained. Next the QRS part of the ECG is detected by a QRS detector which was implemented by using the algorithm developed by Pan and Tompkins [6]. The RR intervals are then calculated. Next, the RR interval is passed through the KALMAN filter for estimation. The block diagram of the steps required is shown in Figure 4.1.

![Figure 4.1: Estimation of RR intervals from ECG recordings](image)

A Real Time QRS Detection Algorithm

A real time QRS detection algorithm was developed by Tompkins and Pan in 1985 [6]. A summary of the paper is shown in Figure 4.2.

In the early days, the QRS complexes were detected by setting a threshold level and comparing the R wave slope with the threshold. If the slope crosses the threshold value then it is classified as a QRS complex. But this method was not very reliable because the chances of classifying
noise peaks as QRS complexes were very high. The method developed by Dr. Tompkins takes into consideration other parameters of the signal such as the signal amplitude and the width of the QRS to determine if a signal peak is QRS complex or not.

First the ECG signal is passed through a band pass filter to remove noises like the muscle noise, 60HZ interference, baseline wander and T-wave interference. An adaptive threshold is used on this filter output to determine the QRS complexes based on the signal amplitude. This filtered output is then passed through a differentiator to obtain the QRS complex slope information. The output of the differentiator is then squared to make all the data points positive and also emphasize the high frequencies which are usually the QRS components. Then the squared signal is passed through a moving window integrator to extract out the width of the QRS complex. The duration of the rising edge of the integration waveform gives the width of the QRS complex. Therefore the rising edge of the integrator output waveform can be used to find a QRS complex. An adaptive threshold is applied on this output to classify the QRS complex.

Both the classifications made by using the band pass filter and integrator outputs are AND ed together to give out the final output.

The QRS detector output will be an impulse train with the impulses located wherever the detector locates a QRS complex. The RR intervals can now be computed from this impulse train and passed through the KALMAN filter for estimation.
The ECG recording from the MIT/BIH database is read

The read signal is passed through a band pass filter

The filter output is differentiated

The differentiated signal is squared making all the data points positive

A moving integration filter is applied on the differentiated output

Threshold

Processing delay

AND

Output

Figure 4.2: Summary of Pan and Tompkins QRS detection algorithm [6]
Results for real ECG data

The KALMAN filter was tested on 10 ECG recordings and the results are shown through Figures 4.3 to 4.24. Based on the quality of the QRS detection signal the filter was tuned by varying its parameters to best estimate the RR interval and as well filter out any measurement noise. The criteria for the parameters was to select high R (measurement noise covariance) value if the detector output is erratic and low R value if the detector accurately classifies the QRS complexes.

If the RR intervals are measured from an erroneous QRS detector output, it is obvious that there will be errors in the interval measurements. Because of these errors the interval measurements deviate widely from the actual intervals. From our discussion in chapter 3 it has been shown that the assumption of high R (measurement noise covariance) will slow down the filter estimates in following the measurements. Therefore the interval measurements are less trusted and the filter will tend to estimate values that do not vary as widely as the measurements. Thus the filter estimates would be more close to the actual values. This leads us to tune the parameter R to high values if the QRS detector output is erroneous.

If the QRS detection was accurate a very low value for R and a high value for Q were assumed. If the QRS detection was accurate then there would be a very less chance for the RR interval measurements to be erroneous. From our discussion in chapter 3 we determined how the filter behaved for low values of R and for high values of Q. By assuming high Q, the filter estimates follow the measurements very closely. Since our measurements here have fewer errors, this is a desired feature. Additionally a very low value for R will also make the filter follow the measurements more quickly.
Therefore based on the performance of the QRS detector on the ECG signals, appropriate values for Q and R have been assumed. Table 4.2 lists the parameters chosen for different ECG signals from the MIT/BIH database [20].

Table 4.1: Filter parameters for the ECG recordings

<table>
<thead>
<tr>
<th>MIT/BIH (database)</th>
<th>Process Noise Covariance</th>
<th>Measurement Noise Covariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>file name</td>
<td>(Q)</td>
<td>(R)</td>
</tr>
<tr>
<td>100.dat</td>
<td>3</td>
<td>20.0</td>
</tr>
<tr>
<td>101.dat</td>
<td>4</td>
<td>15.0</td>
</tr>
<tr>
<td>102.dat</td>
<td>5</td>
<td>0.1</td>
</tr>
<tr>
<td>103.dat</td>
<td>4</td>
<td>10.0</td>
</tr>
<tr>
<td>104.dat</td>
<td>4</td>
<td>1.0</td>
</tr>
<tr>
<td>105.dat</td>
<td>4</td>
<td>12.0</td>
</tr>
<tr>
<td>106.dat</td>
<td>4</td>
<td>12.0</td>
</tr>
<tr>
<td>107.dat</td>
<td>4</td>
<td>0.1</td>
</tr>
<tr>
<td>118.dat</td>
<td>4</td>
<td>15.0</td>
</tr>
<tr>
<td>119.dat</td>
<td>5</td>
<td>20.0</td>
</tr>
</tbody>
</table>
Figure 4.3: ECG and the QRS detector plot for file 100.dat
The RR interval should be fluctuating somewhere around 0.8 seconds. But because of bad performance of the QRS detection (Figure 4.3) it is shown in Figure 4.4 there are a few noise pulses in the RR interval measurements. This is because of the failure of the QRS detector to detect all the QRS complexes in the ECG signal. Missed beats caused the noise pulses in our RR interval measurement. So a selection of high R would cause less deviation in our estimated values. Here R was assumed a value of 20 and Q a value of 3. It can be shown that the estimated values are still closer to our heart rate (0.8 seconds) than RR interval measurements.
Figure 4.5: ECG and the QRS detector plot for the file 101.dat
Q was assumed to be 4 and R was assumed a high value 15 because of the erroneous QRS detector output shown in Figure 4.5. The KALMAN filter provided the estimated intervals which are less erroneous than our RR interval measurements (Figure 4.6).
Figure 4.7: ECG and QRS detector plots for file 102.dat
Figure 4.8: Measured Intervals and Estimated intervals for file 102.dat

It is shown in Figure 4.7 that the QRS detector output was very accurate which leads us to assume a very low value for R (0.1) and relatively higher value for Q (5). Figure 4.8 shows the estimated values of the filter which follow our RR interval measurements very closely. Since the measurements have no errors they can be considered as the actual intervals and the KALMAN filter was able to estimate them accurately.
Figure 4.9: ECG and QRS detector plot for file 103.dat
Figure 4.10: Measured Intervals and Estimated intervals for file 103.dat

Figure 4.9 show that the QRS detector misses many beats in the ECG. When the RR interval is computed, it is shown in Figure 4.10 that the extended interval resulted in noise pulses at instances where the QRS detector missed a beat. The filter estimation shows that the estimated intervals are kept low at the occurrence of the noise pulses thus decreasing the error.
Figure 4.11: ECG and QRS detector plot for file 104.dat
Here the parameter R was chosen a low value of 1 because the QRS detection was accurate thus giving us less erroneous RR intervals. Q was assigned a value of 4. As shown in Figure 4.12 the filter was able to estimate the intervals from the given RR interval measurements. Since the measurements didn't have any errors, our filter estimation was close to the measurements as shown in Figure 4.12.
Figure 4.13: ECG and QRS detector plot for file 105.dat
Q was assumed to be 4 and R was assumed a high value of 12 because of the errors in the QRS detector output shown in Figure 4.13. Figure 4.14 show that the KALMAN filter provided the estimated intervals which seem to track the measured RR intervals.

Figure 4.14: Measured Intervals and Estimated intervals for file 105.dat
Figure 4.15: ECG and QRS detector for file 106.dat
Q was assumed to be 4 and R was assumed a high value 12 because of the errors in the QRS
detector output shown in Figure 4.15. Figure 4.16 show that the KALMAN filter provided the
estimated intervals which tracked the RR interval measurements.
Figure 4.17: ECG and QRS detector for file 107.dat
As shown in Figure 4.17, the QRS detector output was very accurate which leads us to assume a very low value for \( R \) (0.1) and relatively higher value for \( Q \) (4). Figure 4.18 shows the estimated values of the filter which follow our measured RR intervals very closely. Since the QRS detection has no errors, the measured RR interval and the KALMAN filter estimation had good agreement.
Figure 4.19: ECG and QRS detector plot for file 118.dat
Because of the errors in QRS detection we choose a higher value for R (15) and a value of 4 for Q. The filter estimation is shown in Figure 4.20.
Figure 4.21: ECG and QRS detector plot for file 119.dat
Q was assumed to be 5 and R was assumed a high value of 20 because of the errors in the QRS detector output as shown in Figure 4.21. Figure 4.22 shows that the KALMAN filter provided the estimated intervals that is in close agreement with the actual RR interval values.

Therefore from the results it was shown that KALMAN filter was able to estimate the RR intervals that are in close agreement to the actual intervals, by appropriate selection of the filter parameters.
CONCLUSION AND FUTURE WORK

Conclusion

Development of a KALMAN filter based estimation algorithm to estimate the RR intervals was the main contribution of this thesis. We considered an impulse train as the input to the KALMAN filter. The intervals between the impulses were estimated using the filter presented here. We were able to study the performance of the filter on this impulse train because of our prior knowledge of the intervals between the impulses. The filter was able to estimate the intervals with good accuracy. The filter was then tested on the same impulse train but the intervals between the impulses were now corrupted with Gaussian random noise. The filter was able estimate the RR intervals whose values were closer to the actual intervals than the noise corrupted intervals.

The KALMAN filter was then implemented on ECG recordings available in the MIT/BIH database. A QRS detection algorithm was applied on the ECG recordings to detect the QRS complexes. The RR intervals were computed from this detector output. Since the performance of the detector varied from one ECG recording to another, the RR intervals computed had noise spikes due to missed beats. The filter was able to reduce these errors and estimate the RR intervals close to the actual values.
Future Work

- A linear KALMAN filter was used in this thesis for estimation because the process here was taken to be a linear model. We can assume a non linear model for our process and use an Extended KALMAN filter for the estimation.

- Since the KALMAN filter implemented here is based on a discrete algorithm, it is well suited to be implemented on a DSP board. The actual performance and the theoretical performance can then be compared.
APPENDIX A

SIMULATED DATA
Table A1: Comparison of Estimated intervals with Measured Intervals for beat 1 to 36

<table>
<thead>
<tr>
<th>Beat number</th>
<th>Measured Interval</th>
<th>Estimated Intervals</th>
<th>Actual Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.546</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>14.184</td>
<td>6.4619</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>7.7225</td>
<td>9.0006</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>7.2187</td>
<td>8.6384</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>12.416</td>
<td>8.2703</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>12.484</td>
<td>9.2894</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>16.767</td>
<td>10.05</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>10.114</td>
<td>11.619</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>11.018</td>
<td>11.271</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>10.704</td>
<td>11.213</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>6.0853</td>
<td>11.097</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>10.009</td>
<td>9.9552</td>
<td>10</td>
</tr>
<tr>
<td>13</td>
<td>10.035</td>
<td>9.9675</td>
<td>10</td>
</tr>
<tr>
<td>14</td>
<td>12.252</td>
<td>9.9827</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>9.3714</td>
<td>10.498</td>
<td>10</td>
</tr>
<tr>
<td>16</td>
<td>13.744</td>
<td>10.242</td>
<td>10</td>
</tr>
<tr>
<td>17</td>
<td>8.4687</td>
<td>11.037</td>
<td>10</td>
</tr>
<tr>
<td>18</td>
<td>15.85</td>
<td>10.454</td>
<td>10</td>
</tr>
<tr>
<td>19</td>
<td>7.2201</td>
<td>11.679</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>9.6553</td>
<td>10.667</td>
<td>10</td>
</tr>
<tr>
<td>21</td>
<td>8.7482</td>
<td>10.437</td>
<td>10</td>
</tr>
<tr>
<td>22</td>
<td>10.839</td>
<td>10.054</td>
<td>10</td>
</tr>
<tr>
<td>23</td>
<td>10.815</td>
<td>10.232</td>
<td>10</td>
</tr>
<tr>
<td>24</td>
<td>9.7307</td>
<td>10.365</td>
<td>10</td>
</tr>
<tr>
<td>25</td>
<td>7.9574</td>
<td>10.221</td>
<td>10</td>
</tr>
<tr>
<td>26</td>
<td>5.9255</td>
<td>9.7069</td>
<td>10</td>
</tr>
<tr>
<td>27</td>
<td>12.888</td>
<td>8.8485</td>
<td>10</td>
</tr>
<tr>
<td>28</td>
<td>11.293</td>
<td>9.7655</td>
<td>10</td>
</tr>
<tr>
<td>29</td>
<td>9.3452</td>
<td>10.112</td>
<td>10</td>
</tr>
<tr>
<td>30</td>
<td>8.4886</td>
<td>9.9381</td>
<td>10</td>
</tr>
<tr>
<td>31</td>
<td>12.964</td>
<td>9.609</td>
<td>10</td>
</tr>
<tr>
<td>32</td>
<td>13.578</td>
<td>10.371</td>
<td>10</td>
</tr>
<tr>
<td>33</td>
<td>11.771</td>
<td>11.099</td>
<td>10</td>
</tr>
<tr>
<td>34</td>
<td>5.8119</td>
<td>11.251</td>
<td>10</td>
</tr>
<tr>
<td>35</td>
<td>12.244</td>
<td>10.017</td>
<td>10</td>
</tr>
<tr>
<td>36</td>
<td>8.7949</td>
<td>10.522</td>
<td>10</td>
</tr>
<tr>
<td>Beat number</td>
<td>Measured Interval</td>
<td>Estimated Intervals</td>
<td>Actual Interval</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------------</td>
<td>---------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>37</td>
<td>7.4105</td>
<td>10.13</td>
<td>10</td>
</tr>
<tr>
<td>38</td>
<td>8.6734</td>
<td>9.5128</td>
<td>10</td>
</tr>
<tr>
<td>39</td>
<td>13.432</td>
<td>9.3222</td>
<td>10</td>
</tr>
<tr>
<td>40</td>
<td>5.9669</td>
<td>10.255</td>
<td>10</td>
</tr>
<tr>
<td>41</td>
<td>10.627</td>
<td>9.2817</td>
<td>10</td>
</tr>
<tr>
<td>42</td>
<td>7.6393</td>
<td>9.587</td>
<td>10</td>
</tr>
<tr>
<td>43</td>
<td>9.5829</td>
<td>9.1448</td>
<td>10</td>
</tr>
<tr>
<td>44</td>
<td>8.2199</td>
<td>9.2443</td>
<td>10</td>
</tr>
<tr>
<td>45</td>
<td>9.1976</td>
<td>9.0117</td>
<td>10</td>
</tr>
<tr>
<td>46</td>
<td>13.204</td>
<td>9.0539</td>
<td>10</td>
</tr>
<tr>
<td>47</td>
<td>8.9804</td>
<td>9.9961</td>
<td>10</td>
</tr>
<tr>
<td>48</td>
<td>10.553</td>
<td>9.7655</td>
<td>10</td>
</tr>
<tr>
<td>49</td>
<td>8.7004</td>
<td>9.9442</td>
<td>10</td>
</tr>
<tr>
<td>50</td>
<td>13.708</td>
<td>9.6618</td>
<td>10</td>
</tr>
<tr>
<td>51</td>
<td>13.547</td>
<td>10.58</td>
<td>10</td>
</tr>
<tr>
<td>52</td>
<td>9.5315</td>
<td>11.254</td>
<td>10</td>
</tr>
<tr>
<td>53</td>
<td>8.4687</td>
<td>10.863</td>
<td>10</td>
</tr>
<tr>
<td>54</td>
<td>10.535</td>
<td>10.319</td>
<td>10</td>
</tr>
<tr>
<td>55</td>
<td>10.964</td>
<td>10.368</td>
<td>10</td>
</tr>
<tr>
<td>56</td>
<td>11.059</td>
<td>10.503</td>
<td>10</td>
</tr>
<tr>
<td>57</td>
<td>8.3054</td>
<td>10.63</td>
<td>10</td>
</tr>
<tr>
<td>58</td>
<td>13.867</td>
<td>10.102</td>
<td>10</td>
</tr>
<tr>
<td>59</td>
<td>9.343</td>
<td>10.957</td>
<td>10</td>
</tr>
<tr>
<td>60</td>
<td>14.976</td>
<td>10.59</td>
<td>10</td>
</tr>
<tr>
<td>61</td>
<td>9.7124</td>
<td>11.586</td>
<td>10</td>
</tr>
<tr>
<td>62</td>
<td>8.39</td>
<td>11.161</td>
<td>10</td>
</tr>
<tr>
<td>63</td>
<td>10.683</td>
<td>10.532</td>
<td>10</td>
</tr>
<tr>
<td>64</td>
<td>9.2329</td>
<td>10.566</td>
<td>10</td>
</tr>
<tr>
<td>65</td>
<td>10.767</td>
<td>10.263</td>
<td>10</td>
</tr>
<tr>
<td>66</td>
<td>11.411</td>
<td>10.378</td>
<td>10</td>
</tr>
<tr>
<td>67</td>
<td>6.4893</td>
<td>10.612</td>
<td>10</td>
</tr>
<tr>
<td>68</td>
<td>6.1929</td>
<td>9.6764</td>
<td>10</td>
</tr>
<tr>
<td>69</td>
<td>9.4481</td>
<td>8.8856</td>
<td>10</td>
</tr>
<tr>
<td>70</td>
<td>7.7626</td>
<td>9.0133</td>
<td>10</td>
</tr>
<tr>
<td>71</td>
<td>9.0425</td>
<td>8.7294</td>
<td>10</td>
</tr>
<tr>
<td>72</td>
<td>12.473</td>
<td>8.8005</td>
<td>10</td>
</tr>
<tr>
<td>73</td>
<td>10.974</td>
<td>9.6342</td>
<td>10</td>
</tr>
</tbody>
</table>
Table A3: Comparison of Estimated intervals with Measured Intervals for beat 74 to 107

<table>
<thead>
<tr>
<th>Beat number</th>
<th>Measured Interval</th>
<th>Estimated Intervals</th>
<th>Actual Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>74</td>
<td>4.1483</td>
<td>9.9385</td>
<td>10</td>
</tr>
<tr>
<td>75</td>
<td>15.246</td>
<td>8.6241</td>
<td>10</td>
</tr>
<tr>
<td>76</td>
<td>12.224</td>
<td>10.127</td>
<td>10</td>
</tr>
<tr>
<td>77</td>
<td>7.1703</td>
<td>10.603</td>
<td>10</td>
</tr>
<tr>
<td>78</td>
<td>13.079</td>
<td>9.8239</td>
<td>10</td>
</tr>
<tr>
<td>79</td>
<td>10.749</td>
<td>10.563</td>
<td>10</td>
</tr>
<tr>
<td>80</td>
<td>6.09</td>
<td>10.605</td>
<td>10</td>
</tr>
<tr>
<td>81</td>
<td>15.58</td>
<td>9.58</td>
<td>10</td>
</tr>
<tr>
<td>82</td>
<td>6.9246</td>
<td>10.942</td>
<td>10</td>
</tr>
<tr>
<td>83</td>
<td>7.4136</td>
<td>10.03</td>
<td>10</td>
</tr>
<tr>
<td>84</td>
<td>7.5324</td>
<td>9.4361</td>
<td>10</td>
</tr>
<tr>
<td>85</td>
<td>14.758</td>
<td>9.0039</td>
<td>10</td>
</tr>
<tr>
<td>86</td>
<td>14.68</td>
<td>10.31</td>
<td>10</td>
</tr>
<tr>
<td>87</td>
<td>7.8626</td>
<td>11.302</td>
<td>10</td>
</tr>
<tr>
<td>88</td>
<td>7.7654</td>
<td>10.521</td>
<td>10</td>
</tr>
<tr>
<td>89</td>
<td>9.2529</td>
<td>9.8957</td>
<td>10</td>
</tr>
<tr>
<td>90</td>
<td>12.566</td>
<td>9.7498</td>
<td>10</td>
</tr>
<tr>
<td>91</td>
<td>8.6881</td>
<td>10.389</td>
<td>10</td>
</tr>
<tr>
<td>92</td>
<td>14.078</td>
<td>10.003</td>
<td>10</td>
</tr>
<tr>
<td>93</td>
<td>8.9854</td>
<td>10.928</td>
<td>10</td>
</tr>
<tr>
<td>94</td>
<td>6.6298</td>
<td>10.487</td>
<td>10</td>
</tr>
<tr>
<td>95</td>
<td>9.1084</td>
<td>9.6113</td>
<td>10</td>
</tr>
<tr>
<td>96</td>
<td>11.602</td>
<td>9.4971</td>
<td>10</td>
</tr>
<tr>
<td>97</td>
<td>3.7696</td>
<td>9.9748</td>
<td>10</td>
</tr>
<tr>
<td>98</td>
<td>10.625</td>
<td>8.5662</td>
<td>10</td>
</tr>
<tr>
<td>99</td>
<td>5.135</td>
<td>9.0335</td>
<td>10</td>
</tr>
<tr>
<td>100</td>
<td>34.155</td>
<td>8.1485</td>
<td>30</td>
</tr>
<tr>
<td>101</td>
<td>31.578</td>
<td>14.052</td>
<td>30</td>
</tr>
<tr>
<td>102</td>
<td>29.811</td>
<td>18.031</td>
<td>30</td>
</tr>
<tr>
<td>103</td>
<td>35.783</td>
<td>20.705</td>
<td>30</td>
</tr>
<tr>
<td>104</td>
<td>26.086</td>
<td>24.128</td>
<td>30</td>
</tr>
<tr>
<td>105</td>
<td>31.231</td>
<td>24.572</td>
<td>30</td>
</tr>
<tr>
<td>106</td>
<td>34.593</td>
<td>26.084</td>
<td>30</td>
</tr>
<tr>
<td>107</td>
<td>32.451</td>
<td>28.016</td>
<td>30</td>
</tr>
</tbody>
</table>
Table A4: Comparison of Estimated intervals with Measured Intervals for beat 108 to 140

<table>
<thead>
<tr>
<th>Beat number</th>
<th>Measured Interval</th>
<th>Estimated Intervals</th>
<th>Actual Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>108</td>
<td>30.532</td>
<td>29.022</td>
<td>30</td>
</tr>
<tr>
<td>109</td>
<td>34.714</td>
<td>29.365</td>
<td>30</td>
</tr>
<tr>
<td>110</td>
<td>33.518</td>
<td>30.579</td>
<td>30</td>
</tr>
<tr>
<td>111</td>
<td>32.173</td>
<td>31.246</td>
<td>30</td>
</tr>
<tr>
<td>112</td>
<td>28.524</td>
<td>31.457</td>
<td>30</td>
</tr>
<tr>
<td>113</td>
<td>32.131</td>
<td>30.791</td>
<td>30</td>
</tr>
<tr>
<td>114</td>
<td>31.875</td>
<td>31.095</td>
<td>30</td>
</tr>
<tr>
<td>115</td>
<td>27.604</td>
<td>31.272</td>
<td>30</td>
</tr>
<tr>
<td>116</td>
<td>33.616</td>
<td>30.439</td>
<td>30</td>
</tr>
<tr>
<td>117</td>
<td>30.558</td>
<td>31.16</td>
<td>30</td>
</tr>
<tr>
<td>118</td>
<td>27.104</td>
<td>31.024</td>
<td>30</td>
</tr>
<tr>
<td>119</td>
<td>36.253</td>
<td>30.134</td>
<td>30</td>
</tr>
<tr>
<td>120</td>
<td>30.308</td>
<td>31.523</td>
<td>30</td>
</tr>
<tr>
<td>121</td>
<td>25.371</td>
<td>31.247</td>
<td>30</td>
</tr>
<tr>
<td>122</td>
<td>28.455</td>
<td>29.913</td>
<td>30</td>
</tr>
<tr>
<td>123</td>
<td>26.33</td>
<td>29.582</td>
<td>30</td>
</tr>
<tr>
<td>124</td>
<td>26.178</td>
<td>28.844</td>
<td>30</td>
</tr>
<tr>
<td>125</td>
<td>30.185</td>
<td>28.239</td>
<td>30</td>
</tr>
<tr>
<td>126</td>
<td>25.981</td>
<td>28.68</td>
<td>30</td>
</tr>
<tr>
<td>127</td>
<td>31.597</td>
<td>28.068</td>
<td>30</td>
</tr>
<tr>
<td>128</td>
<td>30.973</td>
<td>28.869</td>
<td>30</td>
</tr>
<tr>
<td>129</td>
<td>29.89</td>
<td>29.347</td>
<td>30</td>
</tr>
<tr>
<td>130</td>
<td>29.246</td>
<td>29.47</td>
<td>30</td>
</tr>
<tr>
<td>131</td>
<td>27.767</td>
<td>29.419</td>
<td>30</td>
</tr>
<tr>
<td>132</td>
<td>31.819</td>
<td>29.044</td>
<td>30</td>
</tr>
<tr>
<td>133</td>
<td>37.25</td>
<td>29.674</td>
<td>30</td>
</tr>
<tr>
<td>134</td>
<td>5.594</td>
<td>31.394</td>
<td>10</td>
</tr>
<tr>
<td>135</td>
<td>7.6004</td>
<td>25.537</td>
<td>10</td>
</tr>
<tr>
<td>136</td>
<td>5.4982</td>
<td>21.483</td>
<td>10</td>
</tr>
<tr>
<td>137</td>
<td>12.819</td>
<td>17.855</td>
<td>10</td>
</tr>
<tr>
<td>138</td>
<td>12.612</td>
<td>16.712</td>
<td>10</td>
</tr>
<tr>
<td>139</td>
<td>9.777</td>
<td>15.781</td>
<td>10</td>
</tr>
<tr>
<td>140</td>
<td>11.695</td>
<td>14.418</td>
<td>10</td>
</tr>
</tbody>
</table>
Table A5: Comparison of Estimated intervals with Measured Intervals for beat 141 to 171

<table>
<thead>
<tr>
<th>Beat number</th>
<th>Measured Interval</th>
<th>Estimated Intervals</th>
<th>Actual Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>141</td>
<td>8.5963</td>
<td>13.8</td>
<td>10</td>
</tr>
<tr>
<td>142</td>
<td>9.7002</td>
<td>12.619</td>
<td>10</td>
</tr>
<tr>
<td>143</td>
<td>7.9933</td>
<td>11.956</td>
<td>10</td>
</tr>
<tr>
<td>144</td>
<td>7.7162</td>
<td>11.056</td>
<td>10</td>
</tr>
<tr>
<td>145</td>
<td>13.469</td>
<td>10.298</td>
<td>10</td>
</tr>
<tr>
<td>146</td>
<td>6.9273</td>
<td>11.018</td>
<td>10</td>
</tr>
<tr>
<td>147</td>
<td>10.69</td>
<td>10.089</td>
<td>10</td>
</tr>
<tr>
<td>148</td>
<td>9.1015</td>
<td>10.226</td>
<td>10</td>
</tr>
<tr>
<td>149</td>
<td>6.163</td>
<td>9.9706</td>
<td>10</td>
</tr>
<tr>
<td>150</td>
<td>10.349</td>
<td>9.1062</td>
<td>10</td>
</tr>
<tr>
<td>151</td>
<td>11.703</td>
<td>9.3884</td>
<td>10</td>
</tr>
<tr>
<td>152</td>
<td>9.9774</td>
<td>9.9139</td>
<td>10</td>
</tr>
<tr>
<td>153</td>
<td>16.471</td>
<td>9.9283</td>
<td>10</td>
</tr>
<tr>
<td>154</td>
<td>11.625</td>
<td>11.414</td>
<td>10</td>
</tr>
<tr>
<td>155</td>
<td>12.491</td>
<td>11.462</td>
<td>10</td>
</tr>
<tr>
<td>156</td>
<td>9.067</td>
<td>11.695</td>
<td>10</td>
</tr>
<tr>
<td>157</td>
<td>7.5236</td>
<td>11.099</td>
<td>10</td>
</tr>
<tr>
<td>158</td>
<td>11.014</td>
<td>10.287</td>
<td>10</td>
</tr>
<tr>
<td>159</td>
<td>6.5627</td>
<td>10.452</td>
<td>10</td>
</tr>
<tr>
<td>160</td>
<td>14.165</td>
<td>9.5692</td>
<td>10</td>
</tr>
<tr>
<td>161</td>
<td>9.1841</td>
<td>10.613</td>
<td>10</td>
</tr>
<tr>
<td>162</td>
<td>15.945</td>
<td>10.288</td>
<td>10</td>
</tr>
<tr>
<td>163</td>
<td>7.6881</td>
<td>11.572</td>
<td>10</td>
</tr>
<tr>
<td>164</td>
<td>10.073</td>
<td>10.691</td>
<td>10</td>
</tr>
<tr>
<td>165</td>
<td>16.835</td>
<td>10.551</td>
<td>10</td>
</tr>
<tr>
<td>166</td>
<td>4.2327</td>
<td>11.977</td>
<td>10</td>
</tr>
<tr>
<td>167</td>
<td>5.2648</td>
<td>10.219</td>
<td>10</td>
</tr>
<tr>
<td>168</td>
<td>12.547</td>
<td>9.0945</td>
<td>10</td>
</tr>
<tr>
<td>169</td>
<td>8.2877</td>
<td>9.8782</td>
<td>10</td>
</tr>
<tr>
<td>170</td>
<td>13.615</td>
<td>9.5171</td>
<td>10</td>
</tr>
<tr>
<td>171</td>
<td>13.401</td>
<td>10.447</td>
<td>10</td>
</tr>
</tbody>
</table>
Table A6: Comparison of Estimated intervals with Measured Intervals for beat 172 to 204

<table>
<thead>
<tr>
<th>Beat number</th>
<th>Measured Interval</th>
<th>Estimated Intervals</th>
<th>Actual Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>172</td>
<td>10.399</td>
<td>11.118</td>
<td>10</td>
</tr>
<tr>
<td>173</td>
<td>8.4269</td>
<td>10.955</td>
<td>10</td>
</tr>
<tr>
<td>174</td>
<td>10.251</td>
<td>10.381</td>
<td>10</td>
</tr>
<tr>
<td>175</td>
<td>8.8628</td>
<td>10.352</td>
<td>10</td>
</tr>
<tr>
<td>176</td>
<td>7.1772</td>
<td>10.014</td>
<td>10</td>
</tr>
<tr>
<td>177</td>
<td>7.4068</td>
<td>9.3697</td>
<td>10</td>
</tr>
<tr>
<td>178</td>
<td>14.627</td>
<td>8.9241</td>
<td>10</td>
</tr>
<tr>
<td>179</td>
<td>10.644</td>
<td>10.219</td>
<td>10</td>
</tr>
<tr>
<td>180</td>
<td>14.013</td>
<td>10.315</td>
<td>10</td>
</tr>
<tr>
<td>181</td>
<td>14.797</td>
<td>11.155</td>
<td>10</td>
</tr>
<tr>
<td>182</td>
<td>9.2207</td>
<td>11.982</td>
<td>10</td>
</tr>
<tr>
<td>183</td>
<td>11.629</td>
<td>11.355</td>
<td>10</td>
</tr>
<tr>
<td>184</td>
<td>7.8006</td>
<td>11.417</td>
<td>10</td>
</tr>
<tr>
<td>185</td>
<td>6.8926</td>
<td>10.596</td>
<td>10</td>
</tr>
<tr>
<td>186</td>
<td>9.1369</td>
<td>9.7554</td>
<td>10</td>
</tr>
<tr>
<td>187</td>
<td>10.414</td>
<td>9.615</td>
<td>10</td>
</tr>
<tr>
<td>188</td>
<td>5.0793</td>
<td>9.7965</td>
<td>10</td>
</tr>
<tr>
<td>189</td>
<td>9.2386</td>
<td>8.7257</td>
<td>10</td>
</tr>
<tr>
<td>190</td>
<td>7.9213</td>
<td>8.8421</td>
<td>10</td>
</tr>
<tr>
<td>191</td>
<td>7.3737</td>
<td>8.6331</td>
<td>10</td>
</tr>
<tr>
<td>192</td>
<td>8.8217</td>
<td>8.3472</td>
<td>10</td>
</tr>
<tr>
<td>193</td>
<td>12.784</td>
<td>8.4549</td>
<td>10</td>
</tr>
<tr>
<td>194</td>
<td>2.5555</td>
<td>9.4376</td>
<td>10</td>
</tr>
<tr>
<td>195</td>
<td>11.532</td>
<td>7.8753</td>
<td>10</td>
</tr>
<tr>
<td>196</td>
<td>9.3205</td>
<td>8.7055</td>
<td>10</td>
</tr>
<tr>
<td>197</td>
<td>10.193</td>
<td>8.8451</td>
<td>10</td>
</tr>
<tr>
<td>198</td>
<td>7.0283</td>
<td>9.1511</td>
<td>10</td>
</tr>
<tr>
<td>199</td>
<td>7.3618</td>
<td>8.6692</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>5.903</td>
<td>8.3724</td>
<td>10</td>
</tr>
<tr>
<td>201</td>
<td>8.4746</td>
<td>7.8118</td>
<td>10</td>
</tr>
<tr>
<td>202</td>
<td>14.823</td>
<td>7.9623</td>
<td>10</td>
</tr>
<tr>
<td>203</td>
<td>10.781</td>
<td>9.5198</td>
<td>10</td>
</tr>
<tr>
<td>204</td>
<td>7.4183</td>
<td>9.806</td>
<td>10</td>
</tr>
</tbody>
</table>
Table A7: Comparison of Estimated intervals with Measured Intervals for beat 205 to 230

<table>
<thead>
<tr>
<th>Beat number</th>
<th>Measured Interval</th>
<th>Estimated Intervals</th>
<th>Actual Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>205</td>
<td>17.877</td>
<td>9.264</td>
<td>10</td>
</tr>
<tr>
<td>206</td>
<td>7.0688</td>
<td>11.219</td>
<td>10</td>
</tr>
<tr>
<td>207</td>
<td>11.893</td>
<td>10.277</td>
<td>10</td>
</tr>
<tr>
<td>208</td>
<td>9.471</td>
<td>10.644</td>
<td>10</td>
</tr>
<tr>
<td>209</td>
<td>13.056</td>
<td>10.378</td>
<td>10</td>
</tr>
<tr>
<td>210</td>
<td>8.5538</td>
<td>10.986</td>
<td>10</td>
</tr>
<tr>
<td>211</td>
<td>6.696</td>
<td>10.434</td>
<td>10</td>
</tr>
<tr>
<td>212</td>
<td>13.873</td>
<td>9.5851</td>
<td>10</td>
</tr>
<tr>
<td>213</td>
<td>7.8787</td>
<td>10.558</td>
<td>10</td>
</tr>
<tr>
<td>214</td>
<td>9.4321</td>
<td>9.9501</td>
<td>10</td>
</tr>
<tr>
<td>215</td>
<td>11.403</td>
<td>9.8325</td>
<td>10</td>
</tr>
<tr>
<td>216</td>
<td>11.112</td>
<td>10.189</td>
<td>10</td>
</tr>
<tr>
<td>217</td>
<td>4.6715</td>
<td>10.399</td>
<td>10</td>
</tr>
<tr>
<td>218</td>
<td>12.475</td>
<td>9.0985</td>
<td>10</td>
</tr>
<tr>
<td>219</td>
<td>11.741</td>
<td>9.8651</td>
<td>10</td>
</tr>
<tr>
<td>220</td>
<td>13.401</td>
<td>10.291</td>
<td>10</td>
</tr>
<tr>
<td>221</td>
<td>13.784</td>
<td>10.997</td>
<td>10</td>
</tr>
<tr>
<td>222</td>
<td>11.434</td>
<td>11.63</td>
<td>10</td>
</tr>
<tr>
<td>223</td>
<td>10.439</td>
<td>11.585</td>
<td>10</td>
</tr>
<tr>
<td>224</td>
<td>15.882</td>
<td>11.325</td>
<td>10</td>
</tr>
<tr>
<td>225</td>
<td>4.8837</td>
<td>12.359</td>
<td>10</td>
</tr>
<tr>
<td>226</td>
<td>3.179</td>
<td>10.662</td>
<td>10</td>
</tr>
<tr>
<td>227</td>
<td>14.773</td>
<td>8.9636</td>
<td>10</td>
</tr>
<tr>
<td>228</td>
<td>11.989</td>
<td>10.282</td>
<td>10</td>
</tr>
<tr>
<td>229</td>
<td>14.58</td>
<td>10.67</td>
<td>10</td>
</tr>
<tr>
<td>230</td>
<td>7.4579</td>
<td>11.558</td>
<td>10</td>
</tr>
</tbody>
</table>
A train of impulses were generated. The first 1000 impulses have a time interval of 10. The time interval for
the next 1000 impulses was set to 30. The final 1000 impulses have a time interval of 10 again. The test
signal was represented by the variable raw in the following code.

```matlab
% defining the length of the data
nlen=3000;

% Loading the array initially with zeros
raw=zeros(1,nlen);

% generating the impulses with varying intervals

pulloc=1; % pulse location
i=1;
compre=10; % interval
numpul=0; % number of pulses

while(pulloc <= 1000)
    pulloc = pulloc + compre;
    width=0;
    while(width <= 1)
        if((width+pulloc)>nlen)
            break
        end
        raw(width+pulloc)=5;
        width=width+1;
    end
    if(pulloc <= nlen)
        pulnum(i)=pulloc;
        numpul=numpul+1;
    end
    i=i+1;
end

compre=30;
while(pulloc <= 2000)
    pulloc = pulloc + compre;
    width=0;
    while(width <= 1)
        if((width+pulloc)>nlen)
            break
        end
```
raw(width+pulloc)=5;
width=width+1;
end
if(pulloc <= nlen)
pulnum(i)=pulloc;
numpul=numpul+1;
end
i=i+1;
end

compre=10;
while(pulloc <= nlen)
pulloc = pulloc + compre;
width=0;
while(width <= 1)
if((width+pulloc)>nlen)
    break
end
raw(width+pulloc)=5;
width=width+1;
end
if(pulloc <= nlen)
pulnum(i)=pulloc;
numpul=numpul+1;
end
i=i+1;
end

Calculation of the RR interval and implementation of the KALMAN filter on the test signal

The time interval between the impulses in the test signal are computed and stored in the variable orginterval. A copy of the interval data is taken into the variable interval so that the interval can be corrupted with the measurement noise for the noise performance study of the filter. The variable intervalesti gives the estimated values from the filter. Q and V represent the process and measurement noise covariance.

% calculating the intervals
for index=1:numpul-1
    orginterval(index)=pulnum(index+1)-pulnum(index);
end
plen=length(orginterval); % length of the interval array
error=randn(1,plen);
% process and measurement noise covariance
Q=0.5;
V=0.1;
error=error*sqrt(V); % measurement noise that corrupts the RR interval

for index=1:numpul-1
    % interval(index)=orginterval(index)+error(index);
    interval(index)=orginterval(index);
end

kallen=index; % length of the KALMAN filter

% estimating the measured intervals using KALMAN filter

% preallocate memory for all arrays
intervalesti=zeros(1,kallen);
xaposteriori=zeros(1,kallen);
residual=zeros(1,kallen);
papriori=ones(1,kallen);
paposteriori=ones(1,kallen);
k=zeros(1,kallen);

% define the system
a=1; % State transition matrix
h=1; % Measurement matrix

% initial guesses for state and aposteriori covariance
xaposteriori_0=2;
paposteriori_0=1;

% predictor equations
xapriori(1)=a*xaposteriori_0;
intervalesti(1)=h*xapriori(1);
residual(1)=interval(1)-intervalesti(1);
papriori(1)=a*a*paposteriori_0+Q;

% corrector equations
k(1)=h*papriori(1)/(h*h*papriori(1)+V);
paposteriori(1)=papriori(1)*(1-h*k(1));
xaposteriori(1)=xapriori(1)+k(1)*residual(1);

% calculating the rest of the values
for j=2:kallen

% predictor equations
xapriori(j)=a*xaposteriori(j-1);
intervalesti(j)=h*xapriori(j);
residual(j)=interval(j)-intervalesti(j);
papriori(j)=a*a*paposteriori(j-1)+Q;

% corrector equations
k(j)=h*papriori(j)/(h*h*papriori(j)+V);
paposteriori(j)=papriori(j)*(1-h*k(j));
xaposteriori(j)=xapriori(j)+k(j)*residual(j);
end
end

Implementation of the KALMAN filter on ECG recordings from MIT/BIH database.

The first section of the code reads the ECG recordings from the MIT/BIH database which are stored in a special file format (212 format). The second section of the code processes the ECG data obtained from the database and performs the QRS detection. The output of the QRS detection is a train of impulses, with impulses located at the occurrence of a QRS complex in the ECG. Finally, the RR intervals are computed from this data and passed on to the KALMAN filter algorithm for estimation.

% Reading the ECG files from the database

% Specify Data
PATH='C:\thesis\ecg\MIT-BIHArrhythmia Database'; % path, where data are saved
HEADERFILE='102.hea'; % header-file in text format
ATRFILE='102.atr'; % attributes-file in binary format
DATAFILE='102.dat'; % data-file
SAMPLES2READ=15000; % number of samples to be read

% load header data
fprintf(1,'\n$> WORKING ON %s ...
', HEADERFILE);
signalh= fullfile(PATH, HEADERFILE);
 fid1=fopen(signalh,'r');
z=fgetl(fid1);
A= sscanf(z, '%*s %d %d %d', [1,3]);
nosig= A(1); % number of signals
sfreq=A(2); % sample rate of data
clear A;
for k=1:nosig
    z= fgetl(fid1);
    A= sscanf(z, '%*s %d %d %d %d %d', [1,5]);
    dformat(k)= A(1); % format; here only 212 is allowed
    gain(k)= A(2); % number of integers per mV
    bitres(k)= A(3); % bitresolution
    zerovalue(k)= A(4); % integer value of ECG zero point
    firstvalue(k)= A(5); % first integer value of signal (to test for errors)
end;
fclose(fid1);
%
load binary data
if dformat~= [212,212], error('this script does not apply binary formats different to 212.'); end;

signald= fullfile(PATH, DATAFILE); % data in format 212
fid2=fopen(signald,'r');
A= fread(fid2, [3, SAMPLES2READ], 'uint8'); % matrix with 3 rows, each 8 bits long,
M2H= bitshift(A(:,2), -4);
M1H= bitand(A(:,2), 15);
PRL=bitshift(bitand(A(:,2),8),9); % sign-bit
PRR=bitshift(bitand(A(:,2),128),5); % sign-bit
M(:, 1)= bitshift(M1H,8)+ A(:,1)-PRL;
M(:, 2)= bitshift(M2H,8)+ A(:,3)-PRR;
if M(1,:) ~= firstvalue, error('inconsistency in the first bit values'); end;
switch nosig
    case 2
        M(:, 1)= (M(:, 1)- zerovalue(1))/gain(1);
        M(:, 2)= (M(:, 2)- zerovalue(2))/gain(2);
        TIME=(0:(SAMPLES2READ-1))/sfreq;
    case 1
        M(:, 1)= (M(:, 1)- zerovalue(1));
        M(:, 2)= (M(:, 2)- zerovalue(1));
        M=M';
        M(1)=[];
        sM=size(M);
        sM=sM(2)+1;
        M(sM)=0;
        M=M';
        M=M/gain(1);
        TIME=(0:2*(SAMPLES2READ-1))/sfreq;
        clear A M1H M2H PRR PRL;
        fprintf(1, '\\n\n$> LOADING DATA FINISHED$
');
        fprintf(1, '\\n$> ALL FINISHED$
');
% QRS DETECTION

x1=M(:,1);
time=TIME;

% initialization area
ylp=zeros(1,length(x1));
yhp=zeros(1,length(x1));
yavg=zeros(1,length(x1));
ymov=zeros(1,length(x1));
yder=zeros(1,length(x1));
yout=zeros(1,length(x1));
thresholdf1=zeros(1,length(x1));
thresholdf2=zeros(1,length(x1));
thresholdi1=zeros(1,length(x1));
thresholdi2=zeros(1,length(x1));
spkf=zeros(1,500);
npkf=zeros(1,500);
spki=zeros(1,500);
npki=zeros(1,500);
period=max(time)/length(time);
t=0;
N=37; % this is the width of the integration window
latency=30; % this is the delay for which a second signal can't be used as a QRS
times=0;
qmax=zeros(2,length(x1));
wmax=zeros(2,length(x1));
out=zeros(1,length(x1));
fout=zeros(1,length(x1)+50);
iout=zeros(1,length(x1));
a=0;
b=0;
delay=0;
k=2; %filter peak variable
d=2; %integrator peak variable
set=1; %these next 2 flags allow for finding the peaks
let=1;
spkf(1)=0;
npkf(1)=0;
spki(1)=0;
npki(1)=0;
RRsum=0;
n=0;
count=0;
t=0;
e=0;
RRavg1=0;
RRavg2=0;
wcount=0;
countp=0;
intmax=0;
filtmax=0;

% this is the main loop of the program. the input is incrementally stepped
% through the filter, derivative, integration, etc. the delays in
% processing are taken into account using the if statements to make sure the
% data is used at the correct time.

for i=13:length(x1),
    ylp(i)=(x1(i)-2*x1(i-6)+x1(i-12)+2*ylp(i-1)-ylp(i-2));
    if i>32
        yhp(i)=(yhp(i-1)-ylp(i)/32+ylp(i-16)-ylp(i-17)+ylp(i-32)/32);
    end
    if i>35
        filtmax=filtmax+1;
        if filtmax==50
            qmax(1,k)=yhp(i);
            qmax(2,k)=i;
            for p=0:50
                if yhp(i-p)>qmax(1,k)
                    qmax(1,k)=yhp(i-p);
                    qmax(2,k)=i-p;
                end
            end
            filtmax=0;
            peakf=qmax;
            spkf(1)=qmax(1,2);
            %set=0;
            if n<3
                if n is small the peaks are based on the previous peaks
                if qmax(1,k)>qmax(1,k-1)
                    spkf(k)=.125*peakf(1,k)+.875*spkf(k-1);
                    npkf(k)=npkf(k-1);
                else
                    npkf(k)=.125*peakf(1,k)+.875*npkf(k-1);
                    spkf(k)=spkf(k-1);
                end
            else
                % once several peaks are determined, the signal peak is
%determined if it is within a certain range of a known peak
if (.85*qpk(n-1)<qmax(1,k))&(qmax(1,k)<1.15*qpk(n-1))
    spkf(k)=.125*peakf(1,k)+.875*spkf(k-1);
    npkf(k)=npkf(k-1);
else
    npkf(k)=.125*peakf(1,k)+.875*npkf(k-1);
    spkf(k)=spkf(k-1);
end
end

%determines the threshold value of this range
thresholdf1(k)=npkf(k)+.25*(spkf(k)-npkf(k));
thresholdf2(k)=.5*thresholdf1(k);
%determines if the signal of the specified range is greater or
%less than the threshold of that range and rewrites the output
%relative to the main i variable
for p=0:49
    if yhp(i-p)>thresholdf1(k)%&(wmax(2,d-1)+latency)<i
        fout(i-p)=1;
    else
        fout(i-p)=0;
    end
end
%increment the peak variable
k=k+1;
end
if i==500+35        %this corresponds to 2seconds(the largest time needed for 1 beat
[a t]=max(qmax(1,:));       %finds maximum point after several max locations
n=1;
for m=1:length(qmax)
    %qpk are the peaks that are within an allowable maximum,
%whereas qmax is all peaks which will include noise peaks
if (.85*a<qmax(1,m))&(qmax(1,m)<1.15*a)
    qpk(1,n)=qmax(1,m);
    qpk(2,n)=qmax(2,m);
    %once the first RR is found, the other variable based
%on this can be started
if n>1
    RR(n)=qpk(2,n)-qpk(2,n-1);
    RRsum=RRsum+RR(n);
    RRavg2=RRavg1;
    RRlow=.92*RRavg2;
    RRhigh=1.16*RRavg2;
    RRmiss=1.67*RRavg2;
end
    n=n+1;
end
end
if \( i \geq 500 + 35 \)
\[ \text{determines a range of acceptable peaks of qrs} \]
if \( .85 \times a < qmax(1,k) \) \& \( qmax(1,k) < 1.15 \times a \)
\[ qpk(1,n) = qmax(1,k); \]
\[ qpk(2,n) = qmax(2,k); \]
\[ RR(n) = qpk(2,n) - qpk(2,n-1); \]
\[ \text{this determines the averages if there are 8 or fewer RR} \]
\[ \text{intervals} \]
if \( n < 9 \)
\[ RRsum = RRsum + RR(n); \]
\[ RRavg1 = RRsum / (n-1); \]
\[ RRavg2 = RRavg1; \]
end
\[ RRavg1 = (RRavg1 \times 0.875) + 0.125 \times RR(n); \]
\[ \text{this if statement determines RRbar and keeps track of how} \]
\[ \text{often RRbar is equal to RR.} \]
if \( (RR(n) < RRhigh) \) \& \( (RR(n) > RRlow) \)
\[ RRbar(n) = RR(n); \]
\[ RRavg2 = 0.875 \times RRavg2 + 0.125 \times RRbar(n); \]
\[ count = count + 1; \]
if \( count == 8 \)
\[ RRavg2 = RRavg1; \]
end
else
\[ count = 0; \]
end
\[ RRlow = 0.92 \times RRavg2; \]
\[ RRhigh = 1.16 \times RRavg2; \]
\[ RRmiss = 1.67 \times RRavg2; \]
\[ n = n + 1; \]
end
\[ \text{once several RR's are established, we can start looking for} \]
\[ \text{missed peaks} \]
if \( n > 3 \)
if \( i - qpk(2,n-1) > RRmiss \)
\[ \text{thresholdf1(k-1) = thresholdf2(k-1);} \]
\[ \text{[peak t2] = max(peakf(1, qpk(2,n-2) + latency:i));} \]
\[ qpk(1,n-1) = \text{peak}; \]
\[ qpk(2,n-1) = t2 + qpk(2,n-2); \]
\[ spkf(k-1) = 0.25 \times \text{peakf}(1,k-1) + 0.75 \times \text{spkf}(k-2); \]
\[ RR(n-1) = qpk(2,n-1) - qpk(2,n-2); \]
if \( (RR(n-1) < RRhigh) \) \& \( (RR(n-1) > RRlow) \)
\[ RRbar(n-1) = RR(n-1); \]
\[ RRavg2 = 0.875 \times RRavg2 + 0.125 \times RRbar(n-1); \]
\[ count = count + 1; \]
if count==8
    RRavg2=RRavg1;
else
    count=0;
end
end
end
end

dis this is the start of the integration process with the derivative and
% the square outputs
if i>36
    yder(i)=0.125*(2*yhp(i)+yhp(i-1)-yhp(i-3)-2*yhp(i-4));
    ysq(i)=yder(i)*yder(i);
end
% this is the start of the moving average integrator where is the size
% of the window and yavg is the output that will be sent to the
% threshold detector
if i>72
    for j=0:N-1
        ymov(i)=ymov(i)+ysq(i-j);
    end
    yavg(i)=ymov(i)/(N);
% this section determines a maximum for a certain window(50samples)
intmax=intmax+1;
if intmax==50
    wmax(1,d)=yavg(i);
    wmax(2,d)=i;
    for p=0:50
        if yavg(i-p)>wmax(1,d)
            wmax(1,d)=yavg(i-p);
            wmax(2,d)=i-p;
        end
    end
    intmax=0;
end
% this initializer the first peak as a signal peak
spki(1)=wmax(1,2);
% let=0;
peaki=wmax;
thresholdi1(d)=npki(d-1)+.25*(spki(d-1)-npki(d-1));
thresholdi2(d)=.5*thresholdi1(d);
% average width of the integration window out from the threshold
% filter is equal to this delay
delay=40;
\%
\% runs the average signal through the threshold at the same
\% window interval that the threshold was determined
for p=0:49
  if yavg(i-p)>thresholdi1(d)
    iout(i-p)=1;
  else
    iout(i-p)=0;
  end
end
\%
\% this for loop determines the output based on the delay. it is
\% done here because we just calculated the iout (the most delayed
\% signal).
for p=0:49
  \%
  \% we want to delay the fout signal by half the delay of the
  \% integration output
  fdout(i-p)=fout(i-p-ceil(delay/2));
  out(i-p)=fdout(i-p)*iout(i-p);
end
\%
\% this determines whether the peak is a signal or noise peak
if wmax(1,d)>wmax(1,d-1)
  spki(d)=.125*peaki(1,d)+.875*spki(d-1);
  npki(d)=npki(d-1);
else
  npki(d)=.125*peaki(1,d)+.875*npki(d-1);
  spki(d)=spki(d-1);
end
\%
\% the d variable is only incremented when there is a new peak
d=d+1; \%
\% peak incrementer
end
end
\%
\% once d and n are large enough, a missed peak in the integration area
\% can be determined
if d>3 & n>3
  if wmax(2,d-2)+RRmiss<i
    thresholdi1(d-1)=thresholdi2(d-1);
    \[peak t3]=max(peaki(1,wmax(2,d-2)+latency:i));
    wmax(1,d-1)=peak;
    wmax(2,d-1)=t3+wmax(2,d-2);
    spki(d-1)=0.25*peaki(1,d-1)+0.75*spki(d-2);
  end
end
end
% KALMAN algorithm implementation

% calculating the RR Intervals from the QRS detector Output

nlen=length(x1);
numpul=0;
index=4;

while(index <= nlen)
    if (out(index-2)==0 && out(index)==1 && out(index-1)==0)
        numpul=numpul+1;
Pullocation(numpul)=index;
        index=index+160;
    else
        index=index+1;
    end
end

plen=length(pullocation);

for index=1:plen-1;
    orginterval(index)=time(pullocation(index+1))-time(pullocation(index));
    interval(index)=time(pullocation(index+1))-time(pullocation(index));
end

kallen=plen-1;

% estimating the measured intervals using kalman filter

% preallocate memory for all arrays

intervalest= zeros(1,kallen);
xaposteriori=zeros(1,kallen);
residual=zeros(1,kallen);
papriori=ones(1,kallen);
paposteriori=ones(1,kallen);
k=zeros(1,kallen);

% define the system
a=1; % State transition matrix
h=1; % Measurement matrix
Q=1; % Process noise covariance
V=0.02; % Measurement noise covariance

% initial guesses for state and aposteriri covariance
xaposteriori_0=0.6;
paposteriori_0=1;
% predictor equations
xapriori(1)=a*xaposteriori_0;
intervalesti(1)=h*xapriori(1);
residual(1)=interval(1)-intervalesti(1);
papriori(1)=a*a*paposteriori_0+Q;

% corrector equations
k(1)=h*papriori(1)/(h*h*papriori(1)+V);
paposteriori(1)=papriori(1)*(1-h*k(1));
xaposteriori(1)=xapriori(1)+k(1)*residual(1);

% calculating the rest of the values
for j=2:kallen
  % predictor equations
  xapriori(j)=a*xaposteriori(j-1);
  intervalesti(j)=h*xapriori(j);
  residual(j)=interval(j)-intervalesti(j);
  papriori(j)=a*a*paposteriori(j-1)+Q;

  % corrector equations
  k(j)=h*papriori(j)/(h*h*papriori(j)+V);
  paposteriori(j)=papriori(j)*(1-h*k(j));
  xaposteriori(j)=xapriori(j)+k(j)*residual(j);
end
LIST OF REFERENCES


2. Texas Heart Institute, http://www.texasheartinstitute.org/conduct.html


6. A real-time QRS detection algorithm. Pan J, Tompkins W.

7. Real time electrocardiogram QRS detection using combined adaptive threshold Ivaylo I Christov.

8. ECG beat detection using filter banks. Afonso VX, Tompkins WJ, Nguyen TQ, Luo S.


14. DIGITAL FILTERS analysis, design and applications. Andreas Antonio

15. ADAPTIVE SIGNAL PROCESSING. Bernard Widrow, Samuel D.Stearns


18. HEART RATE VARIABILITY, Marek Malik, A. John Camm


20. [http://ecg.mit.edu](http://ecg.mit.edu)

21. [http://physionet.org](http://physionet.org)