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Deviations from perfect integer quantum Hall effect

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Deviations from perfect quantization of the Hall resistivity ρ_{yx} at integer filling factors have recently been observed in Si metal-oxide-semiconductor field-effect transistors. We demonstrate how scattering between degenerate states at the edges in a quantum Hall system can lead to size-dependent deviations in ρ_{yx} from its quantized values. While the Hall resistivity fluctuates, the current flows without dissipation along the channel due to the absence of backscattering.

The integer quantum Hall effect (IQHE) is manifested by plateaus in the Hall resistivity ρ_{yx} centered at electron densities (in a fixed magnetic field) such that the filling factor ν is integer. On these plateaus, ρ_{yx} attains the value h/je^2 , with j an integer, to within 0.001 parts per million (ppm).¹ At the same time, the longitudinal resistivity ρ_{yy} vanishes, which means that the transport of current along the channel is dissipationless.² The exact quantization of ρ_{yx} and the concomitant vanishing of ρ_{yy} can be understood in terms of a single-particle picture and an argument due to Laughlin,³ based on gauge invariance. It has been argued^{3,4} that this holds in the presence of disorder or edges. Extended states near the edges play an important role, and more recent work⁵⁻⁷ using a Landauer formalism stresses that the IQHE at low currents is in fact controlled by edge currents. Recently, deviations from the perfect quantization of ρ_{yx} have been observed^{8,9} in the values of ρ_{yx} measured at the center of a plateau (i.e., integer ν) on Si metal-oxide-semiconductor field-effect transistor (MOSFET) samples, relative to values measured on GaAs heterojunctions, while the longitudinal resistivity remains at zero to better than a few parts per billion in ρ_{xx}/ρ_{yx} . Both metastable plateaus with values of ρ_{yx} higher than the quantized value, as well as undulations of ρ_{yx} about its quantized value vs filling factor have been observed.⁹ These deviations are at about 0.1 ppm, well above the background noise level.

It is an important fact that the deviations were observed in Si MOSFET's, and not in GaAs heterojunctions. In Si MOSFET's, the electrons may be scattered by ionized impurities in the inversion layer, by interface charges, and by interface roughness. Due to these scattering processes, the highest attainable mobilities in Si MOSFET's are about two orders of magnitude lower than those in GaAs heterojunctions. A simple calculation reveals that single impurities in Si MOSFET will give rise to potentials with length scales of the order of a magnetic length, substantially smaller than in GaAs, and strengths at least of the order of the magnetic energy $\hbar\omega_c$. The facts that the deviations observed by Yoshihiro *et al.*⁹ were made on Si MOSFET's and that they were sensitive to thermal cycling and thunderstorm-induced electrical disturbances suggest that impurity scattering is responsi-

ble for the deviations. Yoshihiro *et al.*⁹ also apply a calculation by Hirai and Komiyama¹⁰ based on the Landauer-Büttiker formalism to obtain a bound on the error in the Hall resistivity due to the voltage probes. The conclusion is that the relative error in the Hall resistivity due to the probes should be less than 0.001 ppm. In this Rapid Communication, we will present arguments and explicit calculations showing how single short-range impurity scattering between discrete degenerate edge states can give rise to deviations from the perfect quantization of ρ_{yx} .

Impurity scattering in the integer quantum Hall effect has been studied before (see Prange and Girvin in Ref. 2). For example, the problem of a smoothly varying impurity potential can be solved in the guiding-center approximation, and the perfect quantization of ρ_{yx} is not violated. However, in Si MOSFET's this approximation is a poor one, since impurities and interface roughness can give rise to scatterers with strengths of the order of the cyclotron energy and ranges of the order of the magnetic length $\ell_B = \sqrt{\hbar c/eB}$. The effect of short-range scatterer has been studied before in systems without edges, notably by Prange (see Prange in Ref. 2) and Ando.¹¹ These authors showed that impurity scattering should not violate the perfect quantization of the Hall conductivity at the center of a Landau level. Recently, Trugman and Nicopoulos^{12,13} have also studied scattering from short-range scatterers in an infinite system. They argue¹³ that the presence of scatterers may lead to a breakdown of the dissipationless transport. The reason for this is the mixing of degenerate states in different Landau levels by the scatterers. The degenerate states are, however, separated spatially, and any matrix element between two such states is essentially zero unless their spatial separation is of the order of a magnetic length. In order to achieve this, the applied electric field has to be rather large. In a finite system, however, the confining potential of the system will bend the Landau levels upward near the edges. As a consequence, there will always be degenerate states in different Landau levels within one magnetic length of one another at the edges (see Fig. 1). Hence, any perturbation will mix such states. This was noted by Viehweger *et al.*,¹⁴ who investigated a system of noninteracting electrons with impurities on the surface of a cylinder. These

FIG. 1. A system in a confining potential $V_c(x)$ is translationally invariant in the y direction, so the eigenstates of the Hamiltonian can be taken to be eigenstates of $\hat{p}_y$. Here the energies are plotted vs $\ell_B^2 k$. The confining potential forces the Landau levels to bend upward at the edges. As a consequence, states such as the ones indicated at momenta $\hbar k$ and $\hbar(k - \delta k)$ which belong to different Landau levels become degenerate with a lateral separation of the order of ℓ_B .

authors did not examine the possibility that mixing of the edge states may lead to deviations in the Hall resistivity from its quantized values. We will here show that such mixing may lead to deviations from the perfect quantization of ρ_{yx} while the transport remains dissipationless. To demonstrate how this could happen, we first review Laughlin's argument³ in a language suitable for our presentation. We take the system to be periodic in the y direction with an applied magnetic field $B\hat{z}$, and write the Hamiltonian in the Landau gauge as

$$H = \frac{1}{2m} \left[\mathbf{p} - \frac{eBx\hat{y}}{c} - \frac{e}{c} \mathbf{A}_{\text{gauge}} \right]^2 + V_c(x) + V_i(x, y). \quad (1)$$

Here, $V_c(x)$ is the confining potential, $V_i(x, y)$ is the scattering potential, and $\mathbf{A}_{\text{gauge}}$ is a vector potential which we take to be $\mathbf{A}_{\text{gauge}} = \frac{\hbar c}{e} q \hat{y}$, with q of the dimension of a wave number. This vector potential does not give rise to a magnetic field, but is a convenient device¹⁵ to generate a current in the sample by adiabatically increasing q . Without the potential $V_i(x, y)$, the eigenstates are $\phi_{nk}(x)e^{iky}$, where the index n denotes the Landau level that the state belongs to. These states are eigenstates of $\hat{p}_y$, and so are extended in the y direction, but are well localized in the transverse (x) direction. In the presence of the scattering potential, the extended states are also well localized in the transverse direction. We label these states $\psi_{n,\alpha}$. The index n is obtained from the index of the states $\phi_{n,k}$ (fixed n) with which $\psi_{n,\alpha}$ has the biggest overlap, and the spatial ordering of the states $\psi_{n,\alpha}$ for fixed n in the transverse direction determines the index α . We write the energy eigenvalues of the electron states

$\psi_{n,\alpha}$ of H as $E_{n,\alpha}$. From the Feynman-Hellman theorem we find that

$$\partial E_{n,\alpha} / \partial q = (\hbar/e) I_{n,\alpha} L_y, \quad (2)$$

where $I_{n,\alpha}$ is the total current in the y direction carried by the electron state $\psi_{n,\alpha}$. Extended electron states have a value of $\partial E_{n,\alpha} / \partial q$ different from zero by the virtue of their single-valuedness. Usually, a vector potential such as $\mathbf{A}_{\text{gauge}}$ can be removed by a unitary transformation given by the operator $\hat{S} = \exp[-iq \sum_{n,\alpha} y_{n,\alpha}]$. This changes the wave function $\psi_{n,\alpha}$ to $\psi'_{n,\alpha} = \psi_{n,\alpha} \exp[-iq y_{n,\alpha}]$, but if $\psi_{n,\alpha}$ is extended in the y direction, then $\psi'_{n,\alpha}$ does not satisfy the periodic boundary condition if $\psi_{n,\alpha}$ does. This gives rise to the dependence of $E_{n,\alpha}$ on q . However, if $q = jq_0 = j2\pi/L_y$ with j an integer, the unitary transformation will map the energy spectrum back onto itself. It follows that if we increase q adiabatically from 0 to q_0 , which can be done by piercing the system by one flux quantum,³ each eigenstate $\psi_{n,\alpha}$ is mapped onto the eigenstate $\psi_{n,\alpha+1}$. The net effect will be to transfer one electron from each Landau level from one edge of the system to the other (see Fig. 2). This leads to a net change ΔE in total electronic energy and total current ΔI carried by the system, from Eq. (2). Let $f_{n,\alpha}$ be the occupancy (at 0 K) of the single-particle state $\psi_{n,\alpha}$. As q changes from 0 to q_0 , $f_{n,\alpha}$ is unchanged, but the eigenstate at q_0 is identical to the eigenstate $\psi_{n,\alpha+1}$ at $q = 0$, with energy $E_{n,\alpha+1}(q = 0)$. The relation between the change in total current and the change in electronic energy is, from Eq. (2),

$$\begin{aligned} I(q = q_0) - I(q = 0) &= \frac{e}{\hbar L_y} \left[\sum_{n,\alpha} f_{n,\alpha} \frac{\partial E_{n,\alpha}(q = q_0)}{\partial q} - \sum_{n,\alpha} f_{n,\alpha} \frac{\partial E_{n,\alpha}(q = 0)}{\partial q} \right] \\ &= \frac{e}{\hbar L_y} \sum_{n,\alpha} f_{n,\alpha} \left[\frac{\partial E_{n,\alpha+1}(q = 0)}{\partial q} - \frac{\partial E_{n,\alpha}(q = 0)}{\partial q} \right]. \end{aligned} \quad (3)$$

Only if we are allowed to linearize, i.e., if we may approximate

$$\frac{\partial E_{n,\alpha}(q = q_0)}{\partial q} \approx \frac{E_{n,\alpha}(q = q_0) - E_{n,\alpha}(q = 0)}{q_0}, \quad (4)$$

do we recover the IQHE,

$$|\sigma_{xy}| = e\Delta I / \Delta E = je^2/h, \quad (5)$$

with j an integer. If the energies of the extended states form a continuum, such linearization is permissible for an infinitesimal difference of the electrochemical potentials between the two edges, $\mu_R - \mu_L \Delta\mu$. Here, μ_R (μ_L) is the electrochemical potential on the right (left) edge. However, if these energies form a quasicontinuous set, or a discrete set, or if the spectrum of the edge states gives rise to a strong energy dependence of the density of states in the region of energy between μ_R and μ_L , such

linearization may not be permissible. The reason for this is fairly easy to understand from the point of first-order perturbation theory. Two unperturbed degenerate levels at $\phi_{n,k}$ and $\phi_{n,k-\delta k}$ will together carry the same amount of current with or without the scatterer because of current conservation. However, both will be shifted up in energy (for a repulsive scatterer), but the amount they each are shifted depends in detail on the diagonal and

FIG. 2. Total current (in units of $e\omega_c$) in the y direction per electron state in the Landau levels $n = 0$ and 1 is plotted vs single-particle energy in units of $\hbar\omega_c$ for (a) a system without impurity and (b) a system with impurity. In this figure, $\diamond$ corresponds to $q = 0$, $\square$ corresponds to $q = 0.25q_0$, $\times$ to $q = 0.5q_0$, and $+$ to $q = 0.75q_0$. Note that according to Eq. (3), the current is proportional to $\partial E_{n,\alpha}/\partial q$. In (a), $\partial E_{n,\alpha}/\partial q$ is approximately constant, but in (b), the energy and current carried by states degenerate in the absence of the impurity have been shifted, while nondegenerate states carry essentially the same current. Hence, $\partial E_{n,\alpha}/\partial q$ changes rapidly. With the Fermi level as indicated, adiabatically increasing q from 0 to q_0 will result in transferring electrons from the eigenstates at $q = 0$ to those at $q = q_0$. The net effect will be to empty the highest occupied states on the left edge, marked by filled diamonds, and to occupy the lowest available states on the right edge, as indicated by the arrows.

off-diagonal matrix elements. Nearby states which are not degenerate, or at least not as near degeneracy, are not at all affected to the same degree by the scatterer. Hence, the derivative $\partial E_{n,\alpha}/\partial q \propto I_\alpha$ will not be constant near these degenerate levels. Note that there is no net transport of charge laterally, and there is no back-scattering. Hence, the diagonal components ρ_{xx} and ρ_{yy} still vanish, which implies that $|\sigma_{xy}| = \rho_{yx}^{-1}$ and that the diagonal components σ_{xx} and σ_{yy} vanish.

To explicitly demonstrate the validity of the adiabatic transfer of electrons from one edge to the other, while showing that the approximation Eq. (4) can fail, we have performed calculations on a system of length $L_y = 40\pi\ell_B$. The electrons were confined in the x direction by a potential which was flat in the interior and rose to several times $\hbar\omega_c$ within a fraction of a magnetic length at approximately $x = \pm 6.0\ell_B$ (at unit filling factor, this system contains 241 electrons, without spin degeneracy). The electron states were expanded in the Landau basis functions,

$$\phi_{kn} = (1/\sqrt{L_y})\phi(x - x_k)e^{iky}, \quad (6)$$

where $\phi_n(\xi)$ is a harmonic-oscillator function, and $x_k = \ell_B^2 k$, with $k = 2\pi n/L_y$. Each state was expanded in the four lowest Landau levels, $n = 0, \dots, 3$. The electron energies of the two lowest Landau levels for $q = 0$ in the absence of impurities are shown in Fig. 1. Note that at the edges, there are degenerate or near-degenerate states in the two levels, within approximately one magnetic length of one another. A repulsive Gaussian scatterer of strength $\hbar\omega_c$ and range $1.0\ell_B$ located at $(5.5, 60)\ell_B$ was then added to the system. The near-degenerate states at the right edge were then mixed as a result, affecting the current-voltage relation. We performed the calculation for values of q in Eq. (1) of $0, 0.25q_0, 0.5q_0$, and $0.75q_0$. The result is shown in Fig. 2, where we have plotted current vs energy for (a) the systems without and (b) with scatterer. This figure shows that the effect of adiabatically increasing q from 0 to q_0 moves electrons from the highest occupied states on the left edge in each level to the lowest unoccupied states in each level on the right edge. In both systems, the net effect will be an increase in current, since states on the right side carry positive current, and states on the left carry negative current. Also, there will be a net increase in electronic energy. For the system without scatterer, it is clear that the approximation in Eq. (4) is valid, and direct calculation verifies that if the Fermi energy initially is at $1.9\hbar\omega_c$, $\sigma_{xy} = 2e^2/h$ within numerical accuracy and finite-size corrections. However, it is also clear that for the system with the scatterer, the approximation Eq. (4) is not valid for states which were nearly degenerate in the absence of the scatterer. In the case that the Fermi level is at $1.9\hbar\omega_c$, for example, we obtain $\sigma_{yx} = (1.75 \pm 0.15)e^2/h$ (the errors are from numerical accuracy and finite-size corrections).

In conclusion, we have showed that the presence of strong short-range scatterers may lead to a deviation of ρ_{yx} from its ideal quantized value. The reason for this deviation is that the single-particle energies will not change linearly as the system is pierced adiabatically with

a flux quantum, due to mixing between degenerate levels. At the same time, the diagonal components ρ_{xx} and ρ_{yy} remain zero, since there is no back-scattering and no charge is transferred laterally. In order to demonstrate that these deviations may be observable in macroscopic systems, and to compare with the results by Yoshihiro *et al.*, we estimate the magnitude of the deviation using first-order perturbation theory. Near the edges, the electron energies vary on the order of $\hbar\omega_c$ within a distance of the order of ℓ_B . Hence, we approximate the edge by a region with a linear potential with a slope $\hbar\omega_c/\ell_B$. Two degenerate states in two adjacent Landau levels are separated by a distance of the order of ℓ_B . We then add a delta-function impurity $V_0 r_0^2 \delta(x-x_0) \delta(y-y_0)$ located between these two states and calculate $\partial\epsilon/\partial q$, where ϵ is the shift in energy of the degenerate states due to the perturbation. The relative shift in the Hall resistivity $\Delta\rho_{yx}/\rho_{yx}$ can then be estimated as $q_0 \frac{\partial\epsilon}{\partial q} / \Delta E$, where ΔE is the energy separation between two adjacent states within the same Landau level in the absence of the scatterer, $\Delta E \sim \hbar\omega_c \ell_B / L_y$. The result is that $\Delta\rho_{yx}/\rho_{yx} \sim a V_0 r_0^2 / (\hbar\omega_c \ell_B L_y)$, where a is a numerical factor of order unity that depends only weakly on the magnetic field. To compare this estimate with the results by Yoshihiro *et al.*,⁹ we take $L_y = 1$ mm, $r_0 = 0.3\ell_B$, corresponding to a range of 20 Å at a magnetic field of 14 T, a separation of $2\ell_B$, and $V_0 = \hbar\omega_c$. The result we obtain is $\Delta\rho_{yx}/\rho_{yx} \sim 10^{-7}$. This estimate at least gives the order of magnitude of the deviations observed by Yoshihiro *et al.*⁹ We should, however, recognize that our estimate most likely overestimates the effect, since our simple calculation presumed the most favorable conditions possible for the effect to occur. On the other hand, the mechanism proposed here is consistent with the fact that the deviations are seen only in some Si MOSFET samples, but not in GaAs heterojunctions, since the mechanism discussed here can only be important if there is a strong short-range scatterer near degenerate states at one edge. This is extremely unlikely in GaAs heterojunctions, since the dopants are separated from the two-dimensional electron gas, which smooths out the impurity potentials. It may occur in Si MOSFET's, where the dopants reside in the inversion layer, and where the rough Si-insulator interface gives rise to scattering centers. One way to test the mechanism described here would be to measure the magnetic-field dependence of the deviation in a particular sample at fixed filling factor and compare it with the estimate above, in which the deviation scales with $B^{1/2}$

at fixed filling factor.

The calculations presented here have been performed for a closed system. Fluctuations in the Hall conductivity of closed systems have been noted previously by Aoki,¹⁶ who studied a disordered system on a torus. He further showed that averaging over all possible gauge fluxes is equivalent to ensemble averaging over impurities, and that the Hall conductivity averaged over gauge fluxes is exactly quantized. However, the sample dependence and the dependence on thermal history observed in the experiments by Yoshihiro *et al.*⁹ suggest that the observed deviations are a mesoscopic effect in the sense that they are not an ensemble average. Therefore, it is not correct in this case to ensemble-average the Hall conductivity. It would also be desirable to understand the deviations from the point of the Landauer formalism applied to an open system.⁵ The key point here may be that the scattering between degenerate states causes the density of states to depend strongly on energy in the range between the electrochemical potentials at the two edges. This energy dependence must be included explicitly in the calculation of the total current. Note, however, that the sum of transmission probabilities for electrons injected in edge channels on one edge is conserved in the presence of the scatterer. This is just a manifestation of the absence of backscattering ($\rho_{yy} = 0$) while the relation between the total current carried and the difference in electrochemical potential between the edges has been altered by the scatterer. We may also make a partial connection to scattering theory in the following way. Prange (see Prange in Ref. 2) have shown that in the case of a delta-function scatterer, the Hall conductivity is not altered. The reason for this is that the loss of current due to a bound state is precisely compensated for by states within one magnetic length of the scatterer, which speed up and increase the current. However, this compensation is exact only when all states are summed over. If the scatterer is located near one edge of the sample, the Fermi level may be located such that all these states are not occupied. This suggests that the Hall conductivity is not exactly quantized for a short-range scatterer near one edge. This situation will be the subject of a forthcoming publication.

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