Mechanical Characterization of Anisotropic Fused Deposition Modeled Polylactic Acid Under Combined Monotonic Bending and Torsion Conditions

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Mechanical Characterization of Anisotropic Fused Deposition Modeled Polylactic Acid Under Combined Monotonic Bending and Torsion Conditions

by

Aaron Santomauro

A thesis submitted in partial fulfillment of the requirements for the Honors in the Major Program in Mechanical Engineering in the College of Engineering and Computer Science and in the Burnett Honors College at the University of Central Florida
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Thesis Chair: Ali P. Gordon, Ph.D.
Abstract

Mechanical strength of polylactic acid (PLA) is increasingly relevant with time because of its attractive mechanical properties and 3D printability. Additive manufacturing (AM) methods, such as fused deposition modeling (FDM), stereolithography (SLA), and selective laser sintering (SLS), serve a vital role in assisting designers with cheap and efficient generation of the desired components. This document presents research to investigate the anisotropic response of multi-oriented PLA subjected to multiple monotonic loading conditions. Although empirical data has previously been captured for multi-oriented PLA under tensile and compressive loading conditions, the data has yet to be applied with regard to a representative component geometry. The tensile and compressive empirical data were ultimately used to develop elastic and yield constitutive models which aided in the characterization of PLA under torsion and bending. This representative component geometry is expected to experience a combined torsion and bending load condition in an effort to address this integral gap in the mechanical properties of multi-oriented PLA. In addition to the acquired empirical data, finite element analysis (FEA) and analytical modeling are employed to supplement the accurate modeling of future component analysis. As a result of the proposed array of experiments, the torsional and bending capabilities of PLA are forecasted to vary based on the print orientation. Lastly, the broader impact of this work is dedicated to addressing the material’s capability to operate in environments which possess significant torsion and bending such as model aircraft wings and shafts for remote controlled cars.
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1.0 Introduction

Since the initial conception of additive manufacturing (AM), a multitude of private sector companies and government organizations specializing in regions of technical competency (including those of engineering and hard sciences) have made successful use of 3D printing technology for the purpose of fabricating functional prototypes. The primary objective for AM techniques focuses on providing cheaper methods of manufacturing, expeditious generation of components, and mechanically viable prototypes for immediate design integration. Despite other forms of manufacturing such as subtractive manufacturing (SM), AM allows for the relatively simple incorporation of various parameters including an ample selection of materials, patterns, orientations, layer resolutions, etc. to ensure desired print quality and efficiency. Additionally, SM is conducive to superfluous material waste while AM promotes a significant decrease in material waste by building in a layer-by-layer format rather than sculpting a pre-existing block of material. As shown in Fig. 1-1, the fused deposition modeling (FDM) process begins as a stereolithography (STL) file generated by a 3D modeling software, then the STL file is converted to a G-code file within a slicer software which contains the code used to autonomously operate the 3D printer, then the file is sent to the 3D printer where filament is extruded through a nozzle and the 3D printing process is initiated and reaches completion. This advantageous characteristic of

![Figure 1-1. FDM process.](image-url)
AM ultimately makes for an effective manufacturing process and allows users to accurately introduce parameters of salient measure.

The principal reason for conducting this research is to augment the information already shown for the mechanical performance of polylactic acid (PLA). Combined torsion and bend tests were conducted on PLA component specimens manufactured at an assortment of orientations. Acquired data during this study was utilized to examine the correlation between mechanical response and print orientation. Moreover, a failure analysis of the fracture surfaces associated with each orientation was completed to provide a more comprehensive understanding of the anatomical nature and consequences of layer-by-layer deposition. Also, the stiffnesses obtained from the experimental results were compared with computationally generated models such as finite element analysis (FEA) and analytical modeling.

Commencement of this thesis supplies a synopsis of AM from a historical perspective as well as previous research conducted on the topic of mechanical characterization of FDM materials. Furthermore, the theoretical mechanics section will serve to give information on the mathematical justification behind the proposed theorems and show the relationship between analytical methods and experimental methods. Next, the experimental approach section will reveal the specific parameters used during the manufacturing and testing phases of the project while the experimental results section will enumerate and compare the quantifiable values for the desired mechanical properties. Finally, the modeling section will be dedicated to implementing select theorems from the theoretical mechanics section as a direct application to the data procured in this thesis.
2.0 Background

2.1 Additive Manufacturing

The history of 3D printing began in the early 1980s with a pioneering idea to convert digital renderings to real-life physical components with the stereolithography (SLA) printing technique which soon sparked another type of 3D printing technique known as selective laser sintering (SLS) in 1988 [Bensoussan, 2016]. Following this discovery, SLS 3D printing proliferated for the purpose of extruding plastic and metallic material [Bensoussan, 2016]. In 1992, FDM, also known as fused filament fabrication (FFF), was born which prompted the accretion of dynamic models to be used in intricate systems [Bensoussan, 2016]. Among the previously discussed 3D printing methods, FDM is known to be the cheapest with applications ranging from prototype design to food and drug packaging for the medical industry. Furthermore, FDM is known for extensive use of support material such as polyvinyl alcohol (PVA) to ensure structural integrity and aesthetically pleasing features. On the other hand, SLA uses a slightly different approach for layer adhesion opposed to the typical FDM interface, including the use of a UV laser to bind each layer acquired from a resin-filled tank via a process known as photopolymerization. The printing process for SLA 3D printing incorporates relatively low forces on parts which translates to a lesser chance of aberration propagation and hence a lower chance of print failure, however even the smallest inconsistency in the print may cause catastrophic failure as a result of the re-coater mechanism. Moreover, SLS 3D printing integrates a high-power carbon dioxide (CO$_2$) laser which serves to bind layers of powdered material (e.g. plastics, metals, glass, ceramics, and various composites) into solid parts as in FDM and SLA 3D printing processes. The differentiating factor between SLS 3D printing and FDM and SLA 3D printing resides in the ability for the unfused powder to serve as the support for the entire printing process. As a result, this unfused material is able to be
recycled and thus SLS 3D printing is preferred in regard to reclaiming material. Fused deposition modeling is known to utilize a variety of materials such as acrylonitrile butadiene styrene (ABS), polyethylene terephthalate glycol (PETG), and polylactic acid (PLA). Among other thermoplastics, PLA is known to be a bioplastic which is indicated by the fact that it is derived from biomass. This characteristic of PLA allows for the material to be easily biodegradable after its end use as opposed to ABS which is known to be highly toxic to the environment. Aside from the environmentally friendly properties of PLA, the thermoplastic is currently used in a variety of engineering applications including prototypical models and biodegradable medical devices. Holistically speaking, 3D printed components are more frequently considered for load-bearing applications.

2.2 Mechanical Characterization

The dearth of knowledge in the ever-growing field of AM FDM materials may exist due to the excessive amount of ambient factors (environmental, electronic, and mechanical) exhibited by the AM device. These factors are prime candidates for influencing the mechanical performance of the proposed specimens. A wealth of literature has been dedicated to capturing the mechanical response of FDM PLA subjected to purely bending or purely torsion; however, very little research has embraced the influence of both conditions simultaneously. Hence, the need for complimentary research is fulfilled by this study.

Upon seeking to supplement the research already conducted in this field, the following sources of literature are presented to illustrate the effects of process variables which include both printer settings and sample orientation. The topic of printer settings is synonymous with user assigned characteristics such as print speed, layer resolution, extrusion temperature, etc. On the
other hand, the topic of sample orientation simply refers to the resulting mechanical properties associated with varying the direction by which the samples are printed relative to each other.

Based on a previous study executed by Torres et al. [2015], the torsional properties of heat-treated PLA were found and assessed based on American Society for Testing and Materials (ASTM) E143 [2013]. The standard ASTM E143 [2013] will be used to provide accurate calculations in regard to the shear modulus and polar moment of inertia of the respective specimen geometry at room temperature. Although the cross-section of the specimens in the study by Torres and coauthors are known to be circular in nature, similar concepts may be applied to this study as the segments to be subjected to torsion will possess a rectangular cross-section. The processing parameters utilized by Torres and coauthors were varied to satisfy the use of a Taguchi L9 orthogonal array which consisted of print layer thickness from 0.1 mm - 0.3 mm in increments of 0.1 mm; infill relative densities from 20% - 100% in increments of 40%; and heat treatments at 100°C for 0 minutes, 5 minutes, and 20 minutes. The specimens were manufactured using a Makerbot Replicator 2 (printing setup shown in Fig. 2-1) while the experiments were performed on an MTS Bionix Electromechanical Torsion tester where each specimen was fixed at one end and with the other end torqued at a constant rate of 0.1 RPM [Torres et. al, 2015]. Moreover, the shear stress and shear strain of the candidate specimens were defined based on Eq. (2.2.1) and Eq. (2.2.2):

\[ \tau = \frac{T_D}{2J} \]  

\( (2.2.1) \)
\[ \gamma = \frac{\theta D}{2L} \]  

(2.2.2)

Notable results from this particular study included the greatest shear modulus at a value of 1265 MPa and an ultimate shear stress at approximately 62 MPa which included the parameters of 0.1 mm layer thickness, 100% infill density, and 20-minute heat-treatment time.

Furthermore, a study by Perkowski [2017] addressed the mechanical response of anisotropic FDM PLA under monotonic tension and compression conditions. Within this study, it is said that FDM materials have been treated as homogeneous and linear elastic where the term linear elastic can be characterized as the proportional relationship between the applied stress and corresponding strain. Various failure theories and criteria were utilized to serve as a prognostication for the analytical analysis of the material such as Von Mises, Drucker-Prager, Hill, and Tsai-Wu [Perkowski, 2017]. The Tsai-Wu failure criterion was developed under the notion that it can predict the failure criterion for tensile-compressive asymmetric materials. This study was dedicated to filling the knowledge gap associated with generating stress-strain data for twelve (12) orientations in both tension and compression as defined by Fig. 2-2. Testing standards included the use of ASTM D638 [2014] for tensile testing and ASTM D695 [2015] for compression testing.

Notable results from this study include a maximum elastic modulus of 3146.136 MPa for the 90° XY orientation at 0° relative to the print bed (XY plane) and a minimum elastic modulus.
of 541.142 MPa for the 45° ZX+45° plane at 45° relative to the print bed (XY plane) for tensile testing. Additional notable results from this study include a maximum compressive strength of 63.638 MPa for the 45° XY orientation at 0° relative to the print bed (XY plane) and a minimum compressive strength of 21.035 MPa for the 45° ZX orientation at 45° relative to the print bed (XY plane) for compression testing. These results are indicative that anisotropic FDM PLA is relatively stronger in tension than in compression which may give researchers and industry personnel another reason to use FDM PLA in tension rather than compression. Finally, this study includes a future work section which alludes to the component level study to be tested in this study as shown in Fig. 2-3.

Moreover, a study by Domingo-Espin et al. [2015] introduced data for a polycarbonate (PC) material which focused on creating a constitutive orthotropic model with regard to the FEA analysis [2015]. Similar to the aforementioned study by Perkowski [2017], tensile tests were performed per ASTM D638 [2014] for the PC material which were manufactured with a Stratasys Fortus 400mc 3D printer. Printing parameters included a 0.254 mm diameter nozzle, 100% infill density, and a cross-hatch pattern layup of +45°/-45°. According to the results for the tensile testing, the greatest elastic modulus was experienced with the specimens printed within the XY plane.
plane at 90° relative to the x-axis in Fig. 2-2. As a continuation of the study with the same material (PC), a component level study was initiated to assess the torsional and bending strength which consisted of the setup as illustrated in Fig. 2-4. During this continuation, a total of six (6) print orientations were evaluated as shown in Fig. 2-5. Proceeding mechanical testing, results of this component level study yielded a maximum stiffness of 1.43 N/mm from orientation 1.02 and minimum stiffness of 1.33 N/mm from orientation 2.02 in Fig. 6. Complimentarily, upon assessment of the FEA model, the maximum stiffness of 1.39 N/mm occurred for orientation 5.02 while the minimum stiffness of 1.26 N/mm occurred for orientation 2.01. Comparatively, it appears that the FEA model proposed by Domingo-Espin et al. [2015] cannot accurately predict the stiffness relative to each orientation. Although this study is meant to address the mechanical properties of PC, it may be possible to

![Figure 2-4. Empirical study (left) and FEA analysis (right) conducted by Domingo-Espin et al. [2015].](image)

![Figure 2-5. Various print orientations for the component level study (adapted from Domingo-Espin et al. [2015]).](image)
obtain higher relative accuracy with regard to the component level study of PLA as mentioned by Fig. 2-3. It is salient to note that the work conducted in this thesis was partially derived from the study conducted by Domingo-Espin et al. [2015].

Another relevant study conducted by Zou et al. [2016] proceeded to evaluate the isotropic and anisotropic elasticity of 3D printed ABS tensile components manufactured at 0°, 30°, 45°, 60°, and 90° on the XY plane per ISO 527-2-2012 [2012]. A constitutive model was established for both isotropic and anisotropic elasticity where the axial and transverse stresses and strains were compared, and elastic constants were determined. Results for this study rendered a maximum elastic modulus of 2425.94 MPa for the 90° orientation and a minimum elastic modulus of 2339.42 MPa for the 30° orientation. Also, the Poisson’s ratio was recorded for each manufacture orientation to be 0.37 for 0°, 0.38 for 30°, 0.31 for 45°, 0.37 for 60°, and 0.42 for 90° which is indicative of the fact that Poisson’s ratio is dependent upon the manufacturing orientation. The primary difference between the study conducted by Perkowski [2017] and the study conducted by Zou et al. [2016] is that the 22.5° and 67.5° manufactured specimens were replaced by the 30° and 60° specimens, respectively. Additionally, the study conducted by Perkowski [2017] consisted of gauging the asymmetry associated with specimens manufactured in varying planes while the study conducted by Zou et al. [2016] only assessed the effect of manufacturing in one plane.

Conclusively, all studies reviewed in this section did not incorporate the idea of measuring and tabulating data pertaining to the torsion and bending capabilities of 3D printed PLA manufactured at varying orientations. Knowledge acquired from the study conducted by Torres et al. [2015] yielded information about the torsional performance of 3D printed PLA manufactured at only one orientation under a specified heat treatment temperature. Moreover, the study orchestrated by Perkowski [2017] employed the use of 3D printed PLA manufactured at varying
orientations subjected to tensile and compressive loading conditions, however it did not address the properties of the material under the influence of torsion or bending conditions. Furthermore, the experimentation executed by Domingo-Espin et. al [2015] supplied intelligence pertaining to the torsional and bending capabilities of 3D printed polycarbonate material manufactured at varying orientations. Although the latter study includes similar testing performed within the corresponding project detailed by this thesis, it does not delineate the associated properties with 3D printed PLA manufactured at multiple orientations. Finally, the study initiated by Zou et al. [2016] communicated the results attained by exposing 3D printed ABS manufactured at five (5) orientations to tensile loading conditions, but it does not investigate the torsional or bending capabilities of the material. All of the aforementioned studies are indicative of an array of knowledge affiliated with 3D printed PLA, varying print orientations, and torsional and bending loading, however none of them integrate all three aspects which is the reason for this project.

2.3 Theoretical Mechanics

Mechanical modeling of structures is a viable approach for assessing mechanical properties. The American Society for Testing and Materials (ASTM) standards ASTM D5023 [2015] and ASTM D6272 [2017] provides testing guidelines for assessing modeling and strength of a plastic under three-point and four-point bending conditions, respectively. The analytical formulations associated with energy methods are integral to fully capturing the mechanical response of the material. According to Budynas [1999], transverse and shear deflections can be calculated by Castigliano’s Theorem (also known as the complementary energy theorem). Firstly, Eq. (2.3.1) considers the existence of transverse deflections within the structure given by
\[ \delta_i = \frac{\partial \Phi}{\partial P_i} \]  

(2.3.1)

where \( \delta \) is the component of total deflection (transverse deflection) in the direction of \( P \), \( \Phi \) is the complementary energy of the system, \( P \) is a gradually applied force, and \( i \) represents the tabulated point/location along the structure where the loading conditions are valid. Next, Eq. (2.3.2) is responsible for defining the rotational deflection in a member under torsion conditions given by

\[ \theta_i = \frac{\partial \Phi}{\partial M_i} \]  

(2.3.2)

where \( \theta \) is the shear deflection about a specified axis and \( M \) is the concentrated moment at a particular point on the structure. It is important to note that Eq. (2.3.2) is only valid for angles supported by the small angle approximation \[ \sin(\theta) \cong \theta \text{ for } \theta \cong \pm 15^\circ \cong \pm 0.2618 \text{ rad} \] if this value is to contribute towards total transverse deflection. Consequently, Eq. (2.3.3) shows the resulting complementary work generated by \( P \) which is dependent upon the total deflection and the applied force,

\[ W_c = \int_0^{P_s} \delta \, dP \]  

(2.3.3)

Notably, Eqn. (3) and Eqn. (4) are initially derived from the complementary energy theorem as shown by Eqn. (5) where \( W_c \) is the complementary work. Alternatively, in the event that the load-displacement relation is linear, the strain energy is shown as a differential by Eq. (2.3.4) and Eq. (2.3.5),

\[ \delta = \frac{\partial U}{\partial P} \]  

(2.3.4)

\[ \theta = \frac{\partial U}{\partial M} \]  

(2.3.5)

which collectively represent Castigliano’s Second Theorem [Budynas, 1999; Castigliano, 1873; Lobontiu et. al, 2008].
Furthermore, application of these relationships toward this particular study include the use of geometrically dependent constants such as area moment of inertia and polar moment of inertia from Eq. (2.3.6) and Eq. (2.3.7),

\[ I = \frac{bh^3}{12} \quad (2.3.6) \]

\[ J = \frac{bh(b^2 + h^2)}{12} \quad (2.3.7) \]

where \( I \) is the area moment of inertia, \( J \) is the polar moment of inertia, \( b \) is the horizontal width of any given specimen segment, and \( h \) is the vertical height of any given specimen segment.

Additionally, application specific equations are needed to supplement the general framework for accurate and thorough analysis, hence introducing the bending, torsion, direct shear, and total deflection at yielding calculations in Eq. (2.3.8), Eq. (2.3.9), Eq. (2.3.10), and Eq. (2.3.11) respectively;

\[
\delta_{Bending} = \frac{\int_0^{L_B} P_y \cdot x^2 \, dx}{E \cdot I} \quad (2.3.8)
\]

\[
\delta_{Torsion} = \frac{2 \cdot (1 + \nu) \cdot P_y \cdot L_T^2 \cdot L_B}{E \cdot J} \quad (2.3.9)
\]

\[
\gamma = \frac{P_y \cdot \sum_{i=1}^{n} L_i}{G \cdot b \cdot h} \quad (2.3.10)
\]

\[
\delta_y = \delta_{Bending} + \delta_{Torsion} + \gamma \quad (2.3.11)
\]

where \( \delta_{Bending} \) is the deflection due to bending, \( \delta_{Torsion} \) is the deflection due to torsion, \( \gamma \) is the deflection due to direct shear, \( L_B \) is the length of the segment being bent, \( P_y \) is the applied load at the commencement of yielding, \( x \) is the variable of integration, \( E \) is the elastic modulus of the material, \( \nu \) is the Poisson’s ratio of the material, \( L_T \) is the length of the moment arm responsible for generating the torque, \( \sum_{i=1}^{n} L_i \) is the summation of segment lengths involved in the direct shear
deflection, \( G \) is the shear modulus of the material, and \( \delta_y \) is the total elastic deflection incurred by Eq. (2.3.8) through Eq. (2.3.11).

More in-depth analysis of Eq. (2.3.8) through Eq. (2.3.11) will be shown in section 5.2 which is provided to supplement the empirical data modeling and FEA modeling. Also, as a critical component to stiffness calculations, Eq. (2.3.12) will be used to calculate the stiffness in each specimen within the elastic region of deformation;

\[
k = \frac{P_y}{\delta_y}
\]

(2.3.12)

where \( k \) is the stiffness of the specimen. Notice that all three (3) components of the total deflection at yielding, \( \delta_y \), are equally dependent upon the applied load at yielding, \( P_y \), which supports the physical assumption that the specimen stiffness, \( k \), is actually independent of \( P_y \) in the elastic region, hence implying that all values of \( k \) within the elastic region must be equivalent. In other words, the slope of the load versus displacement curve must be constant until commencement of the plastic region.

2.4 Knowledge Gaps

As enumerated in section 2.2, prior work in this particular field has been completed, but is lacking an additional effort toward the evaluation of anisotropic FDM PLA under combined torsion and bending conditions. Factors such as layer resolution, layer thickness, heat treatment, testing standards (compression, tension, pure torsion, etc.), material (PLA, ABS, PC, etc.), and select orientations were modified for their respective studies.

For instance, the work conducted by Torres et al. [2015] discovered the torsional rigidity of FDM PLA only under pure torsion conditions and only manufactured at one orientation. Furthermore, the work performed by Perkowski [2017] successfully identified the compressive
and tensile behavior of anisotropic FDM PLA with failure surfaces for both uniaxial compression and tensile conditions, but did not address the behavior of the material with respect to combined torsion and bending conditions. Moreover, the study conducted by Domingo-Espin et al. [2015] explored the response of PC material manufactured at only six (6) different orientations under the influence of combined torsion and bending conditions and does not unveil characteristics of the entire spectrum of orientations.

Consequently, the candidate FDM material can be characterized in terms of its acquired mechanical properties (stiffness, yield force, peak force, resilience, toughness, and ductility) and resulting failure modes. The assessment of anisotropic FDM PLA subjected to combined torsion and bending conditions in a component specimen has not been completed thus far. It is critical to note that high quality accuracy and precision of data has a direct correlation with the ability to maintain the exact printer and loading device settings for all specimens used in this study.
3.0 Experimental Approach

Among the various approaches utilized to evaluate the performance of polylactic acid (PLA) exposed to pure bending or pure torsion, the following tactic proved advantageous to assessing the behavior of PLA exposed to combined bending and torsion conditions. The subsequent experiments were derived from the parameters and primary setup stated in ASTM D638 [2014] with modification to be examined in a proceeding section.

3.1 Specimen Fabrication

Amid the myriad of additive manufacturing methods, FDM was chosen for this study in order to maintain consistency with the disquisition composed by Perkowski [2017] as this thesis is meant to serve as a continuation of that effort. Consequently, an Ultimaker 2+ FDM printer (Fig. 3-1) was employed to meet the demand for relatively precise layer resolution and consistent generation of necessary geometries per manufacturing orientation. Specifications associated with the Ultimaker 2+ FDM printer include a 0.4 mm nozzle diameter which is suitable for a maximum temperature of 260 °C, heated glass build plate acceptable for a maximum temperature of 100 °C, and maximum XYZ build volume of 223 x 223 x 205 mm.
For the purposes of this particular study, the parameters defined in Table 3-1 were chosen to complete the specimen fabrication and to maintain consistency from the study conducted by Perkowski [2017]. In order to maintain strict uniformity among all specimens, the same filament (Ultimaker NFC PLA with blue color) was used throughout all print sessions. Parameters that are salient to note for this study include layer height, infill density, and filament diameter. It is integral that the parameters enumerated in Table 3-1 should be maintained as listed for every print while varying the filament manufacturer or brand could potentially result in discernable contrasts in structural performance. Even a relatively miniscule discrepancy in filament properties may cause significant deviation from the most accurate results.
In regard to the FDM process, there are multiple steps and precautions that need to be taken for the purpose of ensuring a desirable output. Firstly, a computer aided design (CAD) model is developed within a 3D modeling software where all measurements and geometric constrains must be finalized prior to importation to a print parameter modification software. Here, a stereolithography (STL) file is generated which will be used in the following step. Next, the STL file will be imported to the print parameter modification software and printing parameters will be applied per the requirements in Table 3-1. In this software, the STL file will be converted to a G-code file in which a numerical control programming language will be implemented for this conversion process. Within this G-code file contains the instructions for the printer to follow every millisecond of operation. Following file conversion, the file is then transferred to the 3D printer via memory storage device where the registered instructions are read and executed by the printer.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer Height</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>Extrusion Width</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>Perimeters</td>
<td>0</td>
</tr>
<tr>
<td>Infill Density</td>
<td>100%</td>
</tr>
<tr>
<td>Infill Pattern</td>
<td>Rectilinear</td>
</tr>
<tr>
<td>Infill Angle (Within XY Plane)</td>
<td>0°</td>
</tr>
<tr>
<td>Infill Print Speed</td>
<td>30 mm/s</td>
</tr>
<tr>
<td>Support Print Speed</td>
<td>60 mm/s</td>
</tr>
<tr>
<td>Travel Print Speed</td>
<td>120 mm/s</td>
</tr>
<tr>
<td>Filament Brand</td>
<td>Ultimaker NFC PLA - Blue</td>
</tr>
<tr>
<td>Filament Diameter</td>
<td>2.85 ± 0.10 mm</td>
</tr>
<tr>
<td>Filament Density @ 25°C</td>
<td>1.236 g/cm³</td>
</tr>
</tbody>
</table>

Table 3-1. Combined bending and torsion specimen fabrication specifications.
until the printer reaches termination of the code. The object should then be ready to remove from
the bed of the printer and be used for its primary intent.

Before the FDM process could take place, it was necessary to generate a model of the
specimen which served to digitally represent the realistic geometry to be implemented during the
chosen test. Resultantly, the chosen geometry represented in Fig. 3-2 was the proposed component
specimen. This particular component specimen was chosen to typify a common element exposed
to combined bending and torsion conditions as illustrated by Domingo-Espin et. al. [2015]. In the
study conducted by Domingo-Espin et. al. [2015], a polycarbonate (PC) material was evaluated
under torsion and bending conditions in six (6) orientations with an “L” shaped geometry. The
factors of differentiation between the specimens used in the previously stated study and this

![Figure 3-2. Dimensions of component specimen. The component is symmetric about the line AA.](image)

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Honors in the Major (HIM) study are simply the amount of print orientations, material selection, and geometry. The specifications of the study conducted by Domingo-Espin et. al [2015] contained six (6) print orientations, PC material, and a geometry similar to the continuous “L” shaped segment from point a to point c in Fig. 3-3. Analogously, the specifications of this HIM study corresponded to twelve (12) print orientations, PLA material, and the precise geometry displayed in Fig. 3-2.

Based on the consideration of geometry in Fig. 3-2 and imparted loading in Fig. 3-4, the determination of loading modes in each section of the geometry can be complete. This relationship between segment and loading condition is shown in Fig. 3-3 where the loading conditions are symmetric about its central axis along the line AA as conveyed in Fig. 3-2. Notice that no torsion exists in segment de of the geometry because there is a distance of 0 mm between the neutral axis of segment de and the applied load. Point a is meant to be located along the line AA halfway between points b and b’ which follows the symmetric relation of one side to the other. In summary, bending and torsion is expected to occur in segments ab, bc, and cd while pure bending is expected to occur in segment de.

![Figure 3-3](image)

**Figure 3-3.** Sketch of loading in each section of the selected geometry. The letter B represents the bending loading condition while the letter T represents the torsion loading condition. The conditions are symmetric about the centerline through point a.
3.2 Experimental Setup

Previous studies integrated the use of testing standards such as ASTM D638 [2014] and ASTM D695 [2015] which have strict guidelines on procedural and preparational aspects of experimentation, however this HIM study does not strictly follow a set of ASTM guidelines due to the idiosyncratic nature of the specimen geometry and manufacturing process. Following the 3D printing process, the specimens were prepared by removing support material as needed for each orientation transcending the XY plane. Four (4) specimens were manufactured per all twelve (12) orientations for the purpose of instituting a reasonable sample size which optimized the amount of filament and, therefore, print time. The orientations considered in this study include 0° XY, 22.5° XY, 45° XY, 67.5° XY, 90° XY, 22.5° ZX, 45° ZX, 67.5° ZX, 90° ZX, 22.5° ZX+45, 45° ZX+45, and 67.5° ZX+45. These 48 specimens were experimentally tested under the same parameters as defined in ASTM D638 [2014] by which a displacement rate of 5 mm/min was imposed to ensure that all specimens would rupture within 30 seconds to 5 minutes. Also, a data acquisition rate of 200 Hz was used to record the vertical load as well as the displacement of the crosshead.

![Component specimen ANSYS FEA (left) and experimental setup (right) [Perkowski, 2017]. Imparted vertical loading occurs as shown by the red arrows.](image)

**Figure 3-4.** Component specimen ANSYS FEA (left) and experimental setup (right) [Perkowski, 2017]. Imparted vertical loading occurs as shown by the red arrows.
The experiments were performed on an MTS Criterion Model 42 equipped with a Model LSB.203 2 kN load cell and sensitivity of 2.208 mv/V which is shown in Fig. 3-5(a). Supplementary G240G Mechanical Vice Action Grips rated to 5 kN were used to secure the augmented c-channel bracket (Appendix E, Fig. E-1) for affixation of the component specimen. The terminal ends of the specimen as shown in Fig. 3-3 (points e and e’) were inserted into the c-channel bracket extending from both the top and bottom fixtures. The 3.3 mm diameter holes near the terminal ends of the specimen comply with an M3x0.5 threaded hole in the c-channel bracket for intercalation of a 25 mm long 18-8 Stainless Steel Socket Head Screw to disable slippage from the fixtures.

Figure 3-5. (a) MTS Criterion Model 42 test frame. (b) Isometric view of basic setup. (c) Front view of basic setup.

Moreover, a sample sketch of the output data and extractable mechanical properties were predetermined based on the extent of previous relevant literature. A total of six (6) mechanical properties were of paramount interest to this study per each specimen including the achieved peak force, $F_{Peak}$, yield force at which the slope of the curve deviates by 2%, $F_y$, resilience, $U_R$,
toughness, $U_T$, stiffness, $k$, and ductility, $\delta$ as shown in Fig. 3-6. Note that the forces of interest ($F_{\text{Peak}}$ and $F_y$) are extrapolated from a specific point along the curve to the y-axis of the plot area. Also notice that the energy associated with resilience, $U_R$, and toughness, $U_T$, are based upon the area underneath the curve taken from a deflection of 0 mm until the material begins to yield and when the material reaches failure, respectively. Additionally, the stiffness, $k$, of the material was determined by assessing the slope of the curve in the elastic region of the plot while the ductility, $\delta$, was determined by simply measuring the amount of vertical deformation from a deflection of 0 mm until failure.

**Figure 3-6.** Sample representation of output data and mechanical properties to be extracted from said data.
4.0 Experimental Results

Following the successful completion of specimen manufacturing and experimental endeavors, the corresponding data must be explored for purposes of comparing print orientations and determining the optimal orientation for applicatory knowledge. Loading conditions at the grip terminals were similar to that of a tensile test, however, the location of the load relative to the geometry of the specimen produced combined torsion and bending conditions throughout most members of the model shown in Fig. 3-3. From these experiments, the stiffness was calculated based upon the load reading from the universal testing machine (UTM) and the corresponding grip distance gradient. Results were analyzed in a systematic fashion using Microsoft Excel and MATLAB (code located in Appendix A). Four (4) specimens were tested per orientation to provide more precise results when reporting the data. A brief synopsis of the general relationship between print orientation and desired mechanical properties will serve to satisfy the first part of the discussion. Next, the sample data for the respective plane will be discussed along with calculated mechanical properties enumerated within section 3.2 in Fig. 3-6. Moreover, a comparison between the desired mechanical properties per each orientation of the plane in question and those of the other planes will serve to fulfill the final part of the discussion. Lastly, a plot comparing each mechanical property per each orientation will be reported to complete the discussion. The preceding aspects will be reported in the same manner for all three (3) planes. All ancillary data is captured within Appendix B.

4.1 XY Plane Results

With regard to the build orientation relative to the loading direction, all XY plane specimens consisted of horizontal consecutive layers which were orthogonal to the vertical loading
direction. Although all XY plane specimens possessed similar build orientation relative to the loading direction, it is important to note that these orientations do not necessarily lead to similar stiffnesses when considering the reactionary loads associated with torsion and bending. Due to these components of torsion and bending in addition to the varying cross-sections of the specimens, it is expected that each sample along the XY plane will react differently under the specified conditions. It is, however, paramount to note that stiffness symmetry will be prevalent for XY plane specimens as the orientations diverge from the 45° XY orientation. For instance, the stiffness associated with the 0° XY orientation will be expected to approximately match the stiffness in the 90° XY orientation and the stiffness in the 22.5° XY orientation will be expected to approximately match the stiffness in the 67.5° XY orientation.

From the experimentation phase of the XY plane samples, the data reveals that the 0° XY plane specimens performed at the lowest average stiffness of 0.653 N/mm while the 45° XY plane specimens performed at the greatest average stiffness of 1.013 N/mm. Unexpectedly, the data symmetry was compromised by the performance of the 90° XY plane specimens with an average stiffness of 0.952 N/mm. This unanticipated spike in stiffness may have been prompted by aberrations in the 3D printing process such as slightly more material extruded per the cross-section of the specimen.
4.2 ZX Plane Results

Similar to the XY plane samples, the ZX plane sample orientations were swept within a given plane. Printing along the ZX plane prompted the need for copious support material to provide sufficient fortification underneath the specimen. This support material was crucial to maintaining geometric consistency of the specimens to avoid potential deformation from a combination of gravity and relatively high temperature (tending to yield comparatively viscous material which will not solidify immediately after extrusion). These specimens corresponding to the ZX plane possess infill layers which were deposited at orientations approaching (i.e. as the component orientation approaches 90° ZX from 0° ZX) the direction parallel to the loading direction. For instance, the 22.5° ZX sample layer orientation is situated 67.5° away from the loading direction when inserted into the grips while the 67.5° ZX sample layer orientation is
situated 22.5° away from the loading direction when inserted into the grips. Additionally, the 90° ZX sample layer orientation is situated 0° away from the loading direction which implies that the imparted torsion will result in rotational shearing of the adjacent layers along the outer segments (100 mm segments perpendicular to the 47.5 mm segments shown in Fig. 3-2) and bending within the same segments will result in peeling of layers.

From the experimentation phase of the ZX plane samples, the data in Fig. 4-2 reveals that the 22.5° ZX plane sample performed with the greatest average stiffness at 1.015 N/mm, while the 0° ZX plane sample performed with the least average stiffness at 0.653 N/mm. Notice that the average stiffness of 0.653 N/mm is shared between both Fig. 4-1 and Fig. 4-2. This is because the 0° XY samples and 0° ZX samples are precisely synonymous with each other due to the print orientation. The reason for the 22.5° ZX orientation generating the greatest stiffness may be attributed to the optimal amount of interfacial material among adjacent layers relative to the loading direction.

<table>
<thead>
<tr>
<th>Values (Units Dependent Upon Mechanical Property)</th>
<th>0° Zx</th>
<th>22.5° Zx</th>
<th>45° Zx</th>
<th>67.5° Zx</th>
<th>90° Zx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness (N/mm)</td>
<td>63.12</td>
<td>43.394</td>
<td>37.642</td>
<td>37.679</td>
<td>39.017</td>
</tr>
<tr>
<td>Yield Force (N)</td>
<td>86.025</td>
<td>68.174</td>
<td>70.422</td>
<td>78.224</td>
<td>88.168</td>
</tr>
<tr>
<td>Resilience (J)</td>
<td>3.622</td>
<td>2.598</td>
<td>2.814</td>
<td>2.887</td>
<td>2.814</td>
</tr>
<tr>
<td>Peak Force (N)</td>
<td>6.871</td>
<td>6.168</td>
<td>6.004</td>
<td>6.024</td>
<td>6.310</td>
</tr>
<tr>
<td>Ductility (mm)</td>
<td>114.670</td>
<td>114.670</td>
<td>114.670</td>
<td>114.670</td>
<td>114.670</td>
</tr>
<tr>
<td>Toughness (J)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Figure 4-2.** Average results for all five (5) ZX in-plane orientations.
4.3 ZX+45° Plane Results

This ZX+45° plane is unique in that it bisects the XY plane while intersecting the Z axis. The ZX+45° plane was chosen to follow the procedure per the disquisition written by Perkowski (2017) which proved to supplement the results gathered from the XY and ZX planes. Similarities between the ZX plane samples and ZX+45° plane samples include necessary use of support material as well as the gradual approach toward layers being deposited parallel to the print orientation (i.e. as the component orientation approaches 90° ZX+45° from 0° ZX+45°). Furthermore, the ZX+45° plane sample orientations were expected to react mechanically analogous to their ZX plane sample counterparts, experiencing rotational shear between layer interfaces and peeling of layers. The latter statement was predicated upon the fact that, such as the ZX plane orientations, the ZX+45° plane samples would possess a Z support component whereas the XY plane samples possessed no Z support component.

From the experimentation phase of the ZX+45° plane samples, the data in Fig. 4-3 reveals that the 0° ZX+45° plane sample performed with the greatest average stiffness at 1.013 N/mm, while the 45° ZX+45° plane sample performed with the least average stiffness at 0.825 N/mm. Notice that the average stiffness of 1.013 N/mm is shared between both Fig. 4-1 and Fig. 4-3. This is because the 45° XY samples and 0° ZX+45° samples are precisely synonymous with each other due to the print orientation. Also notice that the average stiffness of 0.856 N/mm is shared between both Fig. 4-2 and Fig. 4-3. This is because the 90° ZX samples and 90° ZX+45° samples are, again, synonymous with each other due to the print orientation.
Among all twelve (12) print orientations, it is obvious that the 22.5° ZX orientation performed with the greatest average stiffness at 1.015 N/mm while the 0° XY (also known as 0° ZX) orientation performed with the least average stiffness at 0.653 N/mm. Among all three (3) print planes, it is obvious that the ZX+45° plane collectively performed the greatest with an average stiffness of 0.903 N/mm while the ZX plane collectively performed the least with an average stiffness of 0.842 N/mm as delineated in Fig. 4-4.
4.4 Failure Analysis

Following empirical data acquisition, the component specimen failure surfaces were visually inspected in an effort to further characterize the material under the chosen loading conditions. With regard to combined bending and torsion, the failure modes tended to vary based on the print orientation. Moreover, the location of failure for the candidate specimens were shown in Appendix B (Fig. B-1 through Fig. B-12) to exist relatively close to the corners of the specimens where comparatively high stress concentration factors dominated because of the sharp geometry. The primary failure modes present in this project were noticed to resemble those of layer-to-layer interface debonding, adjacent in-plane trace-to-trace interface debonding, direct shear, relatively miniscule tensile and compressive interactions surrounding the neutral axis of the rectangular cross-section as a result of bending, and subsequent combinations of all four (4). It is clear that all four (4) failure modes exist in both a ductile and brittle format based upon the corresponding evidence via deflection-force plots and visual inspection of fracture surfaces. The holistic

![Figure 4-4. Average stiffness values per each planar orientation. Error bars represent the minimum and maximum stiffness values attained per each planar orientation.](image-url)
consideration of the aforementioned numerical and physical evidence will serve to fulfill comprehension of material performance.

When considering the XY plane results, the specimens exhibited mostly layer-to-layer interface debonding. A consequential byproduct of the bending endured by all specimens promoted failure by tensile and compressive forces as stated in the previous paragraph. It can be seen in Fig. 4-5 through Fig. 4-9 that the angle of the fracture surface was either parallel or orthogonal to the corresponding print angle. For instance, in Fig. 4-5(a) (0° XY specimen), the fracture surface was both parallel and orthogonal to the print orientation. Also considering the fact that the print orientation alternated per each sequential layer (0° for first layer, 90° for second layer, 0° for third layer, etc.), it makes intuitive sense that the fracture surface would reveal propagation in both directions. It is also important to note that the fracture pattern in Fig. 4-5(b) does not resemble the fracture pattern shown in Fig. 4-5(a), hence giving rise to the loss in directional linear contour as the fractures occur further away from the corner.

![Fracture Surface Images](image)

**Figure 4-5.** (a) 0° XY specimen fracture surface following failure near specimen corner. (b) 0° XY specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-1.

Successive orientations in Fig. 4-6 (22.5° XY specimen), Fig. 4-7 (45° XY specimen), Fig. 4-8 (67.5° XY specimen), and Fig. 4-9 (90° XY specimen) tend to follow suit in a similar manner.
The fracture pattern in Fig. 4-6(a) shows an increase in serrated edges from Fig. 4-5 which primarily stems from induction of layer-to-layer interface debonding, adjacent in-plane trace-to-trace interface debonding, and tensile stresses from bending which tend to pull apart the individual traces. Furthermore, the white region indicated by the red contour in Fig. 4-6(b) displays an area of relatively strong adhesion, which ultimately affects the ability of the material orientation to plastically deform in lieu of brittle deformation. In comparison with Fig. 4-6(a), it is evident that most plastic deformation occurs toward the center of any given segment because it is free to rotate, bend, and translate further with the same amount of the applied load than that of regions closer to the corners. This distance-plasticity dependency is subject to alternate between corner and center locations based on the print orientation, however. For example, the red contour in Fig. 4-8(a) shows the opportunity for plastic deformation to also occur nearest the corner location, this time for the 67.5° XY specimen. A similar phenomena occurred for the 90° XY specimen in Fig. 4-9.

Figure 4-6. (a) 22.5° XY specimen fracture surface following failure near specimen corner. (b) 22.5° XY specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-2.
Figure 4-7. (a) 45° XY specimen fracture surface following failure near specimen corner. (b) 45° XY specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-5.

Figure 4-8. (a) 67.5° XY specimen fracture surface following failure near specimen corner. (b) 67.5° XY specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-8.

Figure 4-9. (a) 90° XY specimen fracture surface following failure near specimen corner. (b) 90° XY specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-7.
As the orientations transcend from the XY plane into the ZX plane, the complexity of the fracture surfaces tended to increase. This increase in complexity is defined by the amount of failure modes observed to be present at that particular surface. For instance, Fig. 4-10(a) shows signs of a combination of three (3) out of four (4) failure modes stated above. The layer-to-layer interface debonding can be observed to have occurred within the region denoted by the green contour where small squares show the underlying layer. Direct shear can be observed to have occurred just to the right of the green highlighted region (within the orange highlighted region) where the surface is rougher. Additionally, since the surface is viewed to change elevation from left to right, this is an indication that tensile stresses were greatest toward the planar region (within the green highlighted region) and least toward the top (within the orange highlighted region), therefore leaving some residual traces above that plane.

Moreover, failure modes experienced by the surface generated in Fig. 4-11(b) contains a slightly different combination of three (3) failure modes including layer-to-layer interface debonding, adjacent in-plane trace-to-trace interface debonding, and visible tensile failure of
individual traces. Once again, the layer-to-layer interface debonding is indicated by the green highlighted boxes. The yellow boxes indicate the tensile separation of individual traces as seen to protrude out of the serrated surface below them. Also, visual inspection from inside the yellow boxes reveals that the individual traces are not packed at the same linear density as the layer which precedes them, hence showing evidence of trace-to-trace interface debonding.

![Figure 4-11](image)

**Figure 4-11.** (a) 45° ZX specimen fracture surface following failure near specimen corner. (b) 45° ZX specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-6.

Surprisingly unusual fracture surfaces exist in Fig. 4-12 (67.5° ZX specimen) as they deviate from the rising trend of increased complexity in the preceding specimens (22.5° ZX and 45° ZX specimens). The fracture surfaces have relatively low roughness likely induced by layer-to-layer interface debonding which is rare considering the predominant nature of high complexity in surfaces among the other specimen orientations. Comparatively, Fig. 4-13(b) (90° ZX specimen) possesses a similar characteristic with an added few layers.

One more plane to consider with regard to the XY and ZX planes includes the ZX+45° plane. This plane introduces interesting fracture surface contours where select few of the micrographs contain all four (4) of the proposed failure modes, hence suggesting greater complexity. For instance, Fig. 4-14(b) (22.5° ZX+45° specimen) depicts the debonding of layers
(green box), adjacent individual traces along layer plane (black box), tensile failure of individual traces (yellow boxes), and direct shear effects (orange box) to create the denticulate surface. The debonding of layers exists because of the step-like surface contour which shows a lack of plane beyond that point. The adjacent individual trace separation exists because the region just to the left and above the black box is missing its adjacent traces in the print direction which means that those traces were torn away from the residual traces. The tensile failure of individual traces exists because of the out-of-page discontinuation of traces facing the viewing direction. Finally, direct shear effects exist due to the out-of-plane propagation of layer tearing relative to the layer print orientation. Although Fig. 4-15 (45° ZX+45°) and Fig. 4-16 (67.5° ZX+45°) do not follow the exact same pattern and contour, another surface such as the one depicted in Fig. 4-16(b) may possess all four (4) failure modes due to its complexity.

![Image](image.png)

**Figure 4-12.** (a) 67.5° ZX specimen fracture surface following failure near specimen corner. (b) 67.5° ZX specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-9.
Figure 4-13. (a) 90° ZX specimen fracture surface following failure near specimen corner. (b) 90° ZX specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-12.

Figure 4-14. (a) 22.5° ZX+45° specimen fracture surface following failure near specimen corner. (b) 22.5° ZX+45° specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-10.

Figure 4-15. (a) 45° ZX+45° specimen fracture surface following failure near specimen corner. (b) 45° ZX+45° specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-7.
With regard to the relationship between fracture surface area and required energy needed to propagate the fracture, the toughness measurement is indicative of the how well the material is able to absorb energy during both elastic and plastic deformation until failure. Take, for instance, the white regions in Fig. 4-8(a) [67.5° XY specimen (Specimen ID: 67.5_XY_1)] and Fig. 4-9(a) [90° XY specimen (Specimen ID: 90_XY_4)] where the fracture propagated from those regions to the regions more blue in color. As the length of the fracture increases by some factor, the energy required to maintain consistent plane stress also must increase by that same factor. By the latter statement, the toughness values for those two specific specimens [7.789 J (Chart B-8) and 5.138 J (Chart B-8), respectively] must be greater than the toughness values exhibited by a specimen such as that of Fig. 4-12 [67.5° ZX specimen (Specimen ID: 67.5_ZX_2)] by which the toughness was measured at 2.249 J (Chart B-9). This comparison can be completed between any given specimens as long as the fracture surface is clearly comprehensible (able to see difference between white and blue filament color) and the appropriate data is available.

Figure 4-16. (a) 67.5° ZX+45° specimen fracture surface following failure near specimen corner. (b) 67.5° ZX+45° specimen fracture surface following failure further away from corner. Corresponding failure diagram located in Appendix B, Fig. B-10.
5.0 Modeling

5.1 Finite Element Analysis

The finite element analysis (FEA) modeling portion of this study was included to provide an attempt at utilizing digital computational energy for the sole purpose of predicting material behavior with a partial collection of known values such as elastic moduli extracted from the work by Perkowski [2017]. The computer application of choice for this study was Analysis Systems (ANSYS). An FEA simulation model coupled with an analytical model (section 5.2) and empirical model (chapter 4.0) represents an important cornerstone of verifying and capturing accurate results for future reference by field engineers and scholars.

First, the mathematical justification for the FEA simulation model must be shown in order to fully understand the results. The simplified stiffness tensor (originally a fourth-order tensor composed of 81 constants), $C$, represented by Eq. (5.1.1), is shown as reference for implementation into Eq. (5.1.5) [Perkowski, 2017; Kelly, 2013; Bos et al., 2004]. In the isotropic case, the units of the $C_{ij}$ ($i = 1,2,3,4,5,6$ and $j = 1,2,3,4,5,6$ independently of each other) components will be in terms of stress (Pa, psi or some variation preceded by a Greek prefix) which will ultimately relate to the elastic moduli in the specified directions.

\[
C = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\
C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\
C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}
\end{bmatrix} \quad (5.1.1)
\]

The simplified compliance tensor, $S$, represented by Eq. (5.1.2), is shown to supplement $C$ by the relationship shown in Eq. (5.1.3) and Eq. (5.1.4). By this relationship, $S$ is simply the inverse of $C$ and vice versa. Conversely, in the isotropic case, the units of the $S_{ij}$ components will be in terms
of inverse stress (Pa\(^{-1}\) or psi\(^{-1}\)) which will ultimately relate to the inverse of the elastic moduli in the specified directions.

\[
\mathbf{S} = \begin{bmatrix}
S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\
S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\
S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\
S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\
S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\
S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66}
\end{bmatrix}
\]  

(5.1.2)

Furthermore, the stress vector, \(\mathbf{\sigma}\), and strain vector, \(\mathbf{\varepsilon}\), are expanded from Eq. (5.1.3) to show their overall relationship with each other via \(\mathbf{C}\) and \(\mathbf{S}\) in Eq. (5.1.5) and Eq. (5.1.6), respectively.

\[
\mathbf{\sigma} = \mathbf{C} \mathbf{\varepsilon} = \mathbf{S}^{-1} \mathbf{\varepsilon}
\]  

(5.1.3)

\[
\therefore \quad \mathbf{C} = \mathbf{S}^{-1}
\]  

(5.1.4)

\[
\begin{bmatrix}
\sigma_1 = \sigma_{xx} \\
\sigma_2 = \sigma_{yy} \\
\sigma_3 = \sigma_{zz} \\
\sigma_4 = \sigma_{yz} \\
\sigma_5 = \sigma_{xz} \\
\sigma_6 = \sigma_{xy}
\end{bmatrix} = \mathbf{C} \begin{bmatrix}
\varepsilon_1 = \varepsilon_{xx} \\
\varepsilon_2 = \varepsilon_{yy} \\
\varepsilon_3 = \varepsilon_{zz} \\
\varepsilon_4 = \varepsilon_{yz} \\
\varepsilon_5 = \varepsilon_{xz} \\
\varepsilon_6 = \varepsilon_{xy}
\end{bmatrix}
\]  

(5.1.5)

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6
\end{bmatrix} = \mathbf{S} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix}
\]  

(5.1.6)

Using Eq. (5.1.6) to devise Eq. (5.1.7), the setup for \(\mathbf{S}\) is made up of 9 independent constants by which the matrix components are symmetric about the diagonal [Bertoldi et al., 1998].
Moreover, the compliance terms within Eq. (5.1.7) may be represented by the corresponding terms in Eq. (5.1.8). The values of these terms are determined based on experimentally obtained mechanical properties \( (E, v, \text{ and } G) \) with numbered subscripts which give rise to the anisotropic nature of the material. For instance, \( E_1 \) represents the elastic modulus along any given direction, \( E_2 \) represents the elastic modulus along any given direction whose axis is orthogonal to \( E_1 \), and \( E_3 \) represents the elastic modulus in the direction orthogonal to the plane created by \( E_1 \) and \( E_2 \).

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\
S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix}
\] (5.1.7)

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_1} & -\frac{v_{12}}{E_2} & -\frac{v_{13}}{E_3} & 0 & 0 & 0 \\
-\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{23}}{E_3} & 0 & 0 & 0 \\
-\frac{v_{13}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2G_{23}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2G_{13}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{12}}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix}
\] (5.1.8)

Since it is critical to identify the stiffness matrix in lieu of the compliance matrix, the compliance matrix must undergo an inverse operation as shown in Eq. (5.1.9).
Now that the stiffness matrix is defined, a program capable of solving for the inverse of a six by six matrix must be used to find the values associated with these terms. Note that the factors of 2 in the terms associated with the shear modulus, $G$, were omitted in Eq. (5.1.9) to account for symmetry in the deflection analysis. The mechanical properties needed for accurate stiffness matrix calculations ($E$, $v$, and $G$) were systematically entered into a program developed in MATLAB (Appendix A) to determine the inverse of the compliance matrix. The resulting stiffness matrices were critical to functionality of the FEA model.

The optimal value for stiffness per each orientation were determined by a method of trial and error. By similar standards to Eq. (5.1.8), Eq. (5.1.10) represents the relationship between stress and strain via another compliance matrix which is now defined by theoretical formulations.

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_1} & -\frac{v_{21}}{E_2} & -\frac{v_{31}}{E_3} & 0 & 0 & 0 \\
-\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{32}}{E_3} & 0 & 0 & 0 \\
-\frac{v_{13}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
\] (5.1.9)

Where Eq. (5.1.11), Eq. (5.1.12), Eq. (5.1.13), Eq. (5.1.14), and Eq. (5.1.15) represent the components for the compliance matrix in Eq. (5.1.10) along arbitrary axis measured at an angle, $\theta$, relative to the datum associated with the original set of axes [Gordon, 2019].
\[
\bar{S}_{11} = S_{11} \cos^4 \theta + S_{22} \sin^4 \theta + (2S_{12} + S_{66}) \sin^2 \theta \cos^2 \theta \tag{5.1.11}
\]
\[
\bar{S}_{22} = S_{11} \sin^4 \theta + S_{22} \cos^4 \theta + (2S_{12} + S_{66}) \sin^2 \theta \cos^2 \theta \tag{5.1.12}
\]
\[
\bar{S}_{12} = (S_{11} + S_{22} - S_{66}) \cos^2 \theta \sin^2 \theta + S_{12} (\cos^4 \theta + \sin^4 \theta) \tag{5.1.13}
\]
\[
\bar{S}_{66} = 2 (2S_{11} + 2S_{22} - 4S_{12} - S_{66}) \cos^2 \theta \sin^2 \theta + S_{66} (\cos^4 \theta + \sin^4 \theta) \tag{5.1.14}
\]
\[
\bar{S}_{16} = \bar{S}_{26} = 0 \tag{5.1.15}
\]

And Eq. (5.1.16), Eq. (5.1.17), Eq. (5.1.18), and Eq. (5.1.19) represent the constants within the above equations which are dependent upon \(E\), \(v\), and \(G\) [Gordon, 2019].

\[
S_{11} = \frac{1}{E_L} \tag{5.1.16}
\]
\[
S_{22} = \frac{1}{E_T} \tag{5.1.17}
\]
\[
S_{12} = -\frac{v_{LT}}{E_L} = -\frac{v_{TL}}{E_T} \tag{5.1.18}
\]
\[
S_{66} = \frac{1}{G_{LT}} \tag{5.1.19}
\]

Where \(E_L\) is the elastic modulus in the longitudinal direction relative to the datum axis, \(E_T\) is the elastic modulus in the transverse direction relative to the datum axis, \(v_{LT}\) and \(v_{TL}\) are the Poisson’s ratios measured within the plane generated by the longitudinal and transverse axes, and \(G_{LT}\) is the shear modulus within that same plane. From this, Eq. (5.1.11) through Eq. (5.1.19) are the framework for determining theoretical values of shear moduli, \(G\), and Poisson’s ratio, \(v\), which have not been reported in literature for all of the twelve (12) orientations for FDM PLA referenced in this thesis. By this logic, \(\theta\) will simply be replaced by the Euler angle associated with each orientation measured from the longitudinal axis. The elastic moduli reported by Perkowski [2017] are shown in Table 5-1 which were critical to the calculation of the stiffness matrices described by the preceding equations.
<table>
<thead>
<tr>
<th>Orientation</th>
<th>Elastic Modulus (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0° XY</td>
<td>3055.527</td>
</tr>
<tr>
<td>22.5° XY</td>
<td>2585.438</td>
</tr>
<tr>
<td>22.5° ZX</td>
<td>2354.121</td>
</tr>
<tr>
<td>22.5° ZX+45°</td>
<td>1063.47</td>
</tr>
<tr>
<td>45° XY</td>
<td>2446.893</td>
</tr>
<tr>
<td>45° ZX</td>
<td>1332.84</td>
</tr>
<tr>
<td>45° ZX+45°</td>
<td>541.142</td>
</tr>
<tr>
<td>67.5° XY</td>
<td>2758.835</td>
</tr>
<tr>
<td>67.5° ZX</td>
<td>796.965</td>
</tr>
<tr>
<td>67.5° ZX+45°</td>
<td>739.247</td>
</tr>
<tr>
<td>90° XY</td>
<td>3146.136</td>
</tr>
<tr>
<td>90° ZX</td>
<td>636.127</td>
</tr>
</tbody>
</table>

Table 5-1. Tabulated elastic modulus (Young’s modulus) per orientation [Perkowski, 2017].

In addition to Table 5-1, it is appropriate to include a 3D surface plot which measures the elastic modulus as a function of the orientation angle, E(Θ). Hence, the plot shown in Fig. 5-1 is indicative of this relationship for the 0° XY orientation. The MATLAB code used to generate the plot is located in Appendix A while surface plots for the other eleven (11) orientations can be seen in Appendix C.

![Surface plot of E(Θ) for the 0° XY orientation. Red regions indicate higher strength and blue regions indicate lower strength.](image-url)
The FEA results for deformation and stress of the 0° XY specimen orientation are shown in Fig. 5-2 and Fig. 5-3, respectively. These figures are meant to display the general mechanical response of the material to deformation for all twelve (12) orientations. As shown in Fig. 5-2, the deformation increases as the model is pulled in the positive Z direction (downward direction in the context of the image). Here, the blue color indicates regions of negligible (minimum) deflection whereas the red color indicates regions of extreme (maximum) deflection. It is important to note that the maximum deflection of 78.562 mm is measured from the datum (commencement of deflection at 0 mm). The multi-colored scale bar on the left hand side of the figure shows the continuous progression of deformation over the resulting deformation.

Furthermore, the stress map in Fig. 5-3 reveals the stress imparted on the specimen in the XY plane based on the specified loading conditions. The stress incurred in Fig. 5-3 is contributed by a combination of bending and torsional stresses where the blue color indicates regions of compression and the red color indicates regions of tension, hence the equal and opposite values indicated in the minimum and maximum areas of the scale bar (-15.189 MPa and 15.189 MPa, respectively).

Figure 5-2. Front view of FEA simulation with directional deformation along the Z-axis (vertical direction).
Moreover, the stiffness matrix associated with the resulting FEA simulation in Fig. 5-2 and Fig. 5-3 is shown in Table 5-2. The inputs required to calculate this stiffness matrix are shown in Table 5-3 including the values for mechanical property constants such as $E$, $v$, and $G$. Note that Table 5-2 is required to emulate $C$ from Eq. (5.1.1) in order to provide proper values for a valid FEA model. Also, Table 5-3 reveals the mechanical property constants which were utilized in calculating the six by six matrix in Eq. (5.1.9). The stiffness matrix calculation MATLAB code in Appendix A was used to take the inverse of the compliance matrix and, hence, acquire the stiffness matrix.

**Figure 5-3.** Top view of FEA simulation with stress map shown in the XY plane.
Supplementary data such as force and displacement from the 0° XY specimen tested by means of FEA is essential to the comparison of experimental stiffness and FEA stiffness, $k$. According to Fig. 5-4, the recorded FEA stiffness for the 0° XY orientation (measured from the slope of the curve) was tabulated at 0.636 N/mm which exhibited an experimental error of 0.590% when compared to the 0.640 N/mm average stiffness acquired during experimental observations. This relatively small experimental error merits the FEA model accurate for this particular orientation. Further comparison of stiffness results for additional XY plane, ZX plane, and ZX+45° plane orientations are enumerated in Fig. 5-5, Fig. 5-6, and Fig. 5-7, respectively.

Table 5-2. Tabulated stiffness matrix values for the 0° XY orientation.

Table 5-3. Tabulated stiffness matrix input values for the 0° XY orientation.
Commencing the analysis with the XY plane, the FEA stiffness results are in relatively close proximity to the experimental stiffness results which deems the FEA model acceptable for this application. According to Fig. 5-5, minimum experimental error exists in the 22.5° XY orientation at 0.136% error and maximum experimental error exists in the 45° XY orientation at 3.425% error. This minimal error is proper justification for the use of FEA for this particular plane. The average experimental error for the XY plane was reported to be 1.164%.

Figure 5-4. Resulting load versus displacement curve of the 0° XY orientation for the FEA model.
To continue the analysis, the stiffness results for the FEA simulation in the ZX plane will be presented along with their experimental counterparts. According to Fig. 5-6, minimum experimental error exists in the 0° ZX (0° XY) orientation at 0.590% error and maximum experimental error exists in the 22.5° ZX orientation at 89.045% error. The average experimental error for the ZX plane was reported to be 24.512%. Among further investigation, the 22.5° ZX orientation may have experienced a deviation in input values ($E$, $v$, and $G$) for calculation of the stiffness matrix elements which may have been responsible for the relatively extreme increase in experimental error. This discrepancy yielded the least FEA reported stiffness at 0.537 N/mm while the experimental stiffness existed at the greatest value at 1.015 N/mm among all the ZX plane orientations.

Figure 5-5. XY plane experimental versus FEA results comparison.
Termination of this analysis is indicated by the FEA versus experimental results in the ZX+45° plane. According to Fig. 5-7, minimum experimental error exists in the 0° ZX+45° (45° XY) orientation at 3.425% error and maximum experimental error exists in the 45° ZX+45° orientation at 50.318% error. The average experimental error for the ZX plane was reported to be 24.701%. In comparison with the ZX plane average experimental error, the ZX+45° plane average experimental error is relatively close which shows that the FEA results are at approximately the same level of reliability for both planes. This perturbation in experimental error from the XY plane may have been impacted by the use of arbitrarily selected values for ν and G outlined in Appendix C which is a direct result of the dearth in experimental knowledge of ν and G at the twelve (12) orientations.

Figure 5-6. ZX plane experimental versus FEA results comparison.
Figure 5-7. ZX+45° plane experimental versus FEA results comparison.
5.2 Castigliano’s Theorem

The analytical model of this study was conducted in MathCAD and meant to predict the deflection and stiffness based on previously acquired empirical data such as elastic moduli, Poisson’s ratio, shear moduli, and geometric properties as well as theoretically proven equations. The appropriate equations will be referenced from section 2.3 which contains important information about the deflection and stiffness. This section will remain in constant reference to Appendix D.

As implied by section 2.3, Castigliano’s Theorem is meant to analytically predict the deflection for relatively simple or complex geometries. In order to do so, the deflection must be dependent upon select parameters which ultimately contribute to the overall response of the geometry in question. For this particular application, the deflection is dependent upon parameters such as the applied force, material constants, and geometric constraints. It is important that either Fig. 3-2 or Fig. 3-3 is used for the purpose of reference to provide a visual aid when selecting specimen segments.

First and foremost, the geometric and material constants must be properly defined for accurate calculations. This means that the segment width, $b$, segment height, $h$, length of segment $\overline{ab}$, $L_{ab}$, length of segment $\overline{bc}$, $L_{bc}$, length of segment $\overline{cd}$, $L_{cd}$, length of segment $\overline{de}$, $L_{de}$, area moment of inertia, $I$, polar area moment of inertia, $J$, elastic moduli, $E$, Poisson’s ration, $\nu$, shear moduli, $G$, and applied load, $P$, must all be given a value.

The following set of equations is specifically dedicated to the bending and direct shear components, respectively, of deflection for segment $\overline{ab}$ of the $0^\circ$ XY orientation, but can be replicated accordingly for the additional eleven (11) orientations. Observe that Eq. (5.2.1) and Eq. (5.2.2) are analogous to Eq. (2.3.8) and Eq. (2.3.10), respectively.
\[
\delta_{ab}^{\text{Bending}_{e_{XY}}} = \frac{\int_0^L P \cdot x^2 \, dx}{E_{0_{XY}} \cdot I} \quad (5.2.1)
\]
\[
\gamma_{0_{XY}} = \frac{P \cdot \sum_{i=1}^n L_i}{G_{0_{XY}} \cdot b \cdot h} = \frac{P \cdot (L_{ab} + L_{bc} + L_{cd} + L_{de})}{G_{0_{XY}} \cdot b \cdot h} \quad (5.2.2)
\]

Note that the only values which need to be modified in Eq. (2.3.8) to account for the other orientations and segments include the upper limit of integration and the elastic modulus. Also notice that Eq. (5.2.2) can accommodate for any given number of segments which implies that relatively complex geometries can be assessed using this equation and the only values which need to be modified include the shear modulus per each orientation.

Moreover, the torsion induced in each segment is different due to the varying length of the moment arm discussed in section 2.3. Recall from section 2.3 that the torsional deflection equation can be used only if the small angle approximation is applied to the structure. The torsion for segments \( \overrightarrow{ab}, \overrightarrow{bc}, \) and \( \overrightarrow{cd} \) are delineated in Eq. (5.2.3), Eq. (5.2.4), and Eq. (5.2.5), respectively, which, again, can be applied to the additional eleven (11) orientations. Notice that segment \( \overrightarrow{de} \) is omitted from the torsion calculations. This is due to the fact that segment \( \overrightarrow{de} \) is predicted to experience pure bending as a result of the applied force existing directly on that segment.

\[
\delta_{ab}^{\text{Torsion}_{e_{XY}}} = \frac{2 \cdot (1 + \nu) \cdot P \cdot L_{bc}^2 \cdot L_{ab}}{E_{0_{XY}} \cdot J} \quad (5.2.3)
\]
\[
\delta_{bc}^{\text{Torsion}_{e_{XY}}} = \frac{2 \cdot (1 + \nu) \cdot P \cdot L_{cd}^2 \cdot L_{bc}}{E_{0_{XY}} \cdot J} \quad (5.2.4)
\]
\[
\delta_{cd}^{\text{Torsion}_{e_{XY}}} = \frac{2 \cdot (1 + \nu) \cdot P \cdot L_{de}^2 \cdot L_{cd}}{E_{0_{XY}} \cdot J} \quad (5.2.5)
\]
Therefore, the total deflection in the system for the 0° XY specimen must be

$$\delta_{Total_{0XY}} = \delta_{ab\text{Bending}_{0XY}} + \gamma_{0XY} + \delta_{ab\text{Torsion}_{0XY}} + \delta_{bc\text{Torsion}_{0XY}}$$

$$+ \delta_{cd\text{Torsion}_{0XY}}$$

(5.2.6)

This calculated deflection may be incorporated into Eq. (2.3.12) to achieve the stiffness where

$$k_{0XY} = \frac{P}{\delta_{Total_{0XY}}}$$

(5.2.7)

Subsequent results of the preceding analytical analysis may be compared to the experimentally obtained values to view the validity of the analytical model. For instance, according to Fig. 5-8, the trend for the experimental stiffness from 0° XY to 90° XY reaches a peak at 45° XY while the analytically determined stiffness yields a noticeable trough. This discrepancy in comparison is evident that some external factors are at play which could include sample size per orientation (currently at n = 4 per orientation) and a multitude of printing factors to be discussed in section 6.1. The smallest experimental error in the XY plane (Fig. 5-8) results from the comparison of stiffnesses in the 0° XY orientation at 13.664% error while the largest experimental error results from the comparison of stiffnesses in the 45° XY orientation at 82.863% error. The average experimental error for the XY plane was reported to be 37.329%.
Surprisingly, however, the ZX plane orientations prove to act much differently than the XY plane orientations. Based on the results displayed in Fig. 5-9, the analytically acquired stiffness tend to descend when transitioning from 0° ZX to 90° ZX which is certainly an unintended consequence of the analytical model. With regard to experimental error, the smallest originated from the 0° ZX orientation at 13.664% error while the largest was yielded from the 90° ZX orientation at 452.356% which was an exceedingly unacceptable response. The average experimental error for the ZX plane was reported to be 206.027%.

Figure 5-8. XY plane experimental versus numerical results comparison.
With regard to the ZX+45° plane, the stiffnesses behaved in a similar manner along with the ZX plane when transcending away from the XY plane as revealed in Fig. 5-10. That is, the analytically obtained values tend to decrease as the orientation begins to increase. Respective experimental error is the least at the 0° ZX+45° (45 XY°) orientation at an error of 82.863% while the greatest experimental error occurs at the 45° ZX+45° orientation at an error of 525.187%, even greater than the error experienced by the 90° ZX orientation. The average experimental error for the ZX plane was reported to be 342.708%.

Figure 5-9. ZX plane experimental versus numerical results comparison.
Figure 5-10. ZX+45° plane experimental versus numerical results comparison.
6.0 Conclusions

6.1 Closing Statements

Similar to nearly all scientific experiments, this project required human-machine interface which is likely to include relatively minimal inconsistencies regarding the validity and reproducibility of the project as a whole. Per the latter statement, the author would like to share a brief inventory of recommendations which may improve said validity and reproducibility in analogous projects.

Firstly, the AM process requires the use of a slicer software which allows the user to modify various parameters including print speed, layer resolution, and infill pattern. These parameters must be chosen properly to yield the ideal print and subsequently produce accurate experimental data. For instance, if the infill print speed is chosen to be relatively high, then the linear density of extruded traces, and hence the entire layer, may be compromised due to the lack of material present within the component. Secondly, the aforementioned slicer software includes the ability to modify the G-code for purposes of including additional movements and/or temporal variations in the extrusion nozzle. For instance, if the G-code is unintentionally modified to include a decrease in the extrusion nozzle temperature throughout the printing process, the nozzle may fail to extrude material due to potentially severe material coagulation. Not only can this affect the amount of extruded material per unit time, but it can also lead to localized filament degradation immediately adjacent to the knurled feeder wheel which can paralyze the remainder of the printing process. Thirdly, patience after a completed print is key to successful retrieval of the desired object. For instance, a user’s first instinct promptly following completion of the print may be to remove the component from the print bed. While it will depend upon the make and model of the employed printer, the print bed may or may not have the ability to heat. For the printers that have
heated beds in use during prints, it is crucial to note that the print bed will be relatively close to the operating temperature immediately following termination of the print which means that the material-bed interface will still be warm. It is, therefore, not in the user’s best interest to remove the part immediately after the print is complete due to the material’s propensity of deforming at high temperatures. Failure to wait a sufficient time after print completion may result in undesired changes in geometry and consequently unacceptable data acquisition. Lastly, prudent removal of support material can also cause undesired changes in geometry. Careful utilization of subtractive manufacturing tools such as blades, drills, and rotary equipment may be used to take away support material. However, a relatively simple and innocuous solution of removing material may be to implement a soluble support material (different from the infill material) which dissolves in water.

In summary, this compendium of recommendations serves the purpose of assisting future researchers interested in the propagating scope of mechanics of additively manufactured components.

6.2 Future Work

In an effort to supplement the content provided in this thesis, there are numerous facets which can benefit from future ancillary studies.

For instance, this work can benefit from the generation of 3D failure and yield surfaces similar to those produced by Perkowski [2017] for tensile and compression failure data. When applied to the content in this thesis, the 3D failure surfaces would capture results for numerous properties at each of the twelve (12) orientations including stiffness, yield force, resilience, peak force, ductility, and toughness. Once these surfaces are manufactured, the performance of the material per each orientation will be visually comprehensible. Candidate failure theories include
the Von Mises-Hencky’s failure criterion as well as the Tsai-Hill failure criterion which are able to incorporate the characteristics of bending and torsion to achieve a valid failure model [Avalos and Sánchez, 2014; Jonas et al., 2012].

Furthermore, additional experimental data for another plane can be acquired to augment the three (3) existing planes. One candidate plane would be the ZX+22.5° plane which would bisect the existing ZX plane and ZX+45° plane. Acquiring empirical data for this particular plane would serve fruitful due to the fact that a similar plane (ZX+67.5° plane) would also be satisfied by this experimental endeavor. Since mechanical properties are expected to be symmetric about the ZX+45° plane (i.e. the ZX plane properties should be equivalent to the ZX+90° plane), the ZX+22.5° plane should be equivalent to the ZX+67.5° plane. The same six (6) mechanical properties mentioned earlier would be measured and applied accordingly which would also mean testing FDM PLA in tension (using ASTM D638) for that particular orientation to acquire the elastic moduli and ultimate tensile strength (UTS).

Moreover, experimental acquisition of shear moduli and Poisson’s ratio are yet to be discovered for FDM PLA at the varying orientations. One method to consider would be coincident with that carried out by Torres et al. [2015] without the heat treatment dependency, variation in layer thickness, and variation in infill densities. Replication of this study with the exception of the aforementioned parameters at the desired orientations will be critical to achieving the shear moduli at the varying orientations which are necessary to calculate the stiffness matrix described in the modeling chapter of this thesis (chapter 5.0, section 5.1). The Poisson’s ratio may be experimentally procured by, once again, using ASTM D638 where the strains in axial and transverse directions must be measured since it depends upon both.
Finally, two more supplementary experiments associated with this thesis include exposure of FDM PLA to flexural fatigue and torsion fatigue (individual and combined) conditions. These experiments are meant to simulate environments which involve remote-controlled (RC) machines such as RC aircraft and RC ground vehicles. Since RC aircraft wings experience flutter in turbulent conditions, flexural fatigue testing would be ideal for this application via ASTM D7774 [ASTM, 2017; Peck, 1959; Hodges et al., 2002]. Since RC ground vehicles experience torsion in the drive shafts and axles for all driving conditions, either monotonic torsion or torsion fatigue would be ideal for this application via a similar standard to ASTM E2207 [2015] and ASTM D5279 [2013]. These experimental observations and data analyses can yield important information which may contribute to possible machine operation applications such as military reconnaissance aids, geological surveying, and leisure/hobby activities.
Appendix A: Code
Mechanical Property Calculation:

% Developed by: Samuel Kleespies
% Modified by: Aaron Santomauro

% Code justification: this code is responsible for generating
% various data plots and mechanical properties which serve
% relevance to the data analysis and results

% Language: MATLAB

% read in data for each curve in from excel sheet
data1 = xlsread("67.5_ZX+45_Data.xlsx","67.5_ZX+45_1","A9:B16837");
stiffdata1 = xlsread("67.5_ZX+45_Data.xlsx","67.5_ZX+45_1","A9:B15000");
data2 = xlsread("67.5_ZX+45_Data.xlsx","67.5_ZX+45_2","A9:B14281");
stiffdata2 = xlsread("67.5_ZX+45_Data.xlsx","67.5_ZX+45_2","A9:B15000");
data3 = xlsread("67.5_ZX+45_Data.xlsx","67.5_ZX+45_3","A9:B15577");
stiffdata3 = xlsread("67.5_ZX+45_Data.xlsx","67.5_ZX+45_3","A9:B15000");
data4 = xlsread("67.5_ZX+45_Data.xlsx","67.5_ZX+45_4","A9:B15374");
stiffdata4 = xlsread("67.5_ZX+45_Data.xlsx","67.5_ZX+45_4","A9:B15000");

% calculations for first curve
% ----------------------------
% x and y values for entire curve
x = data1(:,1);
y = data1(:,2);
% x and y values for first linear part of curve
% to find stiffness
xs = stiffdata1(:,1);
ys = stiffdata1(:,2);

% find equation of line for stiffness
stiffeq = polyfit(xs,ys,1);
yf = polyval(stiffeq,x);
dif = y - yf;
% xn is the x value for which the yield force is located
xn = find(dif > 0, 1, 'last');

figure(1)
plot(x,y)
hold on
plot(x,yf)
plot(x(xn),y(xn),'+r')
hold off
grid
xlabel("Extension(mm)")
ylabel("Load(N)")

% save all the mechanical properties for the first curve
peakforce1 = max(y);
ductility1 = x(end);
yieldforce1 = data1(xn,2);
stiffness1 = stiffeq(1,1);
toughness1 = trapz(x,y);
resilience1 = trapz(data1(1:xn,1),data1(1:xn,2));
% calculations for second curve
% ----------------------------
x = data2(:,1);
y = data2(:,2);
xs = stiffdata2(:,1);
ys = stiffdata2(:,2);

stiffeq = polyfit(xs,ys,1);
yf = polyval(stiffeq,x);
dif = y - yf;
xn = find(dif > 0, 1, 'last');

figure(2)
plot(x,y)
hold on
plot(x,yf)
plot(x(xn),y(xn),'+r')
hold off
grid
xlabel("Extension(mm)")
ylabel("Load(N)")

peakforce2 = max(y);
ductility2 = x(end);
yieldforce2 = data2(xn,2);
stiffness2 = stiffeq(1,1);
toughness2 = trapz(x,y);
resilience2 = trapz(data2(1:xn,1),data2(1:xn,2));

% calculations for third curve
% ----------------------------
x = data3(:,1);
y = data3(:,2);
xs = stiffdata3(:,1);
ys = stiffdata3(:,2);

stiffeq = polyfit(xs,ys,1);
yf = polyval(stiffeq,x);
dif = y - yf;
xn = find(dif > 0, 1, 'last');

figure(3)
plot(x,y)
hold on
plot(x,yf)
plot(x(xn),y(xn),'+r')
hold off
grid
xlabel("Extension(mm)")
ylabel("Load(N)")

peakforce3 = max(y);
ductility3 = x(end);
yieldforce3 = data3(xn,2);
stiffness3 = stiffeq(1,1);
toughness3 = trapz(x,y);
resilience3 = trapz(data3(1:xn,1),data3(1:xn,2));
% calculations for fourth curve
% ----------------------------
x = data4(:,1);
y = data4(:,2);
xs = stiffdata4(:,1);
ys = stiffdata4(:,2);
stiffeq = polyfit(stiffdata4(:,1),stiffdata4(:,2),1);
yf = polyval(stiffeq,x);
dif = y - yf;
xn = find(dif > 0, 1, 'last');

figure(4)
plot(x,y)
hold on
plot(x,yf)
plot(x(xn),y(xn),'+r')
hold off
grid
xlabel("Extension (mm)")
ylabel("Load (N)")

peakforce4 = max(y);
ductility4 = x(end);
yieldforce4 = data4(xn,2);
stiffness4 = stiffeq(1,1);
toughness4 = trapz(x,y);
resilience4 = trapz(data4(1:1:xn,1),data4(1:1:xn,2));

% create arrays with properties using all 4 curves
peakarray = [peakforce1, peakforce2, peakforce3, peakforce4];
ductarray = [ductility1, ductility2, ductility3, ductility4];
yieldarray = [yieldforce1, yieldforce2, yieldforce3, yieldforce4];
stiffarray = [stiffness1, stiffness2, stiffness3, stiffness4];
tougharray = [toughness1, toughness2, toughness3, toughness4];
resilarray = [resilience1, resilience2, resilience3, resilience4];

% average the values of the properties of the 4 curves
PEAKFORCE = mean(peak);  
DUCTILITY = mean(duct);  
YIELDFORCE = mean(yield);  
STIFFNESS = mean(stiff);  
TOUGHNESS = mean(tough);  
RESILIENCE = mean(resil);

Stiffness Matrix Calculation:

% Developed by: Navindra Wijeyeratne
% Modified by: Aaron Santomauro

% Code justification: this code is responsible for generating a
% stiffness matrix which is a necessary input for successful and
% accurate rendering of a finite element analysis (FEA) model.

% Language: MATLAB
% Units for E's and G's are in MPa.
% Units for v's are unitless.

% Due to symmetry of the compliance matrix, elastic modulus of direction
% 2 is equivalent to that of direction 1 for the XY plane only.
% The ZX and ZX+45° planes will require different elastic moduli.

% Insert elastic modulus of direction 1 here:
E1 = 000.000;

% Insert elastic modulus of direction 2 here:
E2 = 000.000;

% Insert elastic modulus of direction 3 here:
E3 = 000.000;

% Insert Poisson's ratios here (subsequent equations were devised
% to preserve symmetry among the off-diagonal elements in the compliance
% matrix):

v12 = 000.000;

v21 = (E2*v12)/E1;  % = (E1*v12)/E2 for the XY plane orientations.

v13 = 000.000;

v31 = (E3*v13)/E1;  % = v13 for the XY plane orientations.

v23 = 000.000;

v32 = (E3*v23)/E2;  % = (E3*v23)/E2 for the XY plane orientations.

% Insert shear modulus of plane 12 here:
G12 = 000.000;

% Insert shear modulus of plane 23 here:
G23 = 000.000;

% Insert shear modulus of plane 13 here:
G13 = G12;  % = G23 for the XY plane orientations.

% S is Compliance.
S = [1/E1 -v21/E2 -v31/E3 0 0 0;
     -v12/E1 1/E2 -v32/E3 0 0 0;
     -v13/E1 -v23/E2 1/E3 0 0 0;
     0 0 0 1/G23 0 0;
     0 0 0 0 1/G13 0;
     0 0 0 0 0 1/G12];

% C is stiffness.
C=inv(S)
E(Θ) Surface Plotting:

function Anisotropic_Elastic_Constants
clc; clear; clear all; close all;

% This is the only place in the program where the user makes the changes
% based on users wishes

function Anisotropic_Elastic_Constants
clc; clear; clear all; close all;

% Displays title information
disp(sprintf('
Analysis About Anisotropic Elastic Constants Based On MATLAB'))

disp(sprintf('University of Central Florida'))
disp(sprintf('pei@knights.ucf.edu'))
disp(sprintf('**********************************************************************'))
disp(sprintf('***** Introduction *****'))
disp(sprintf('**********************************************************************'))

disp(sprintf('Users can choose the type of anisotropy from metal alloy in 3D: '))
disp(sprintf('orthotropic, tetragonal, transversely isotropic, cubic, isotropic. '))
disp(sprintf('The users should be able to supply the elastic constants, as well. '))
disp(sprintf('In addition to generating the plot, the code should export the data to a text file of some sort. '))

disp(sprintf('This displays title information'))
disp(sprintf('This is the only place in the program where the user makes the changes'))

disp(sprintf('This displays title information'))

disp(sprintf('Please input an integer: [1-Orthotropic, 2-Tetragonal, 3-Transversely isotropic, 4-Cubic, 5-Isotropic]')

Input = input('Please input an integer: [1-Orthotropic, 2-Tetragonal, 3-Transversely isotropic, 4-Cubic, 5-Isotropic]');
if Input>5 || Input<0
warning('Please input an integer among 1-5.')
end
if Input<6 && Input > 0
disp(sprintf('Your input number is: %g 
', Input));
end

% Initial Matrix C, i.e. Stiffness Tensor C
% Hooke's law: Sigma = C Epsilon,
% where Sigma: stress, Epsilon: strain.
C=zeros(6);
if Input == 1
% Orthotropic Class

disp(sprintf('--------------------------------------------------'))
disp(sprintf('You choose Orthotropic.'))
disp(sprintf('9 constants in total for Orthotropic:'))
disp(sprintf('Ex, Ey, Ez, '))
disp(sprintf('nu_xy, nu_yz, nu_zx, '))
disp(sprintf('Gxy, Gyz, Gzx. '))
disp(sprintf('-----------Elastic Stiffnesses--------------------'))
disp('C(1,1)=115.9;C(1,2)=35.3; C(1,3)=46.8; C(1,4)=0; C(1,5)=0; C(1,6)=0;')
disp(' C(2,2)=174.1;C(2,3)=38.7; C(2,4)=0; C(2,5)=0; C(2,6)=0;')
disp(' C(3,3)=153.1;C(3,4)=0; C(3,5)=0; C(3,6)=0;')
disp(' C(4,4)=50.9;C(4,5)=0; C(4,6)=0;')
disp(' C(5,5)=70.2;C(5,6)=0;')
disp(' C(6,6)=26.6;')
disp(sprintf('--------------------------------------------------'))
disp(sprintf('You can update the parameters above based on the case.'))
disp(sprintf('Refer to the elastic constants surface plot in 3D, please.'))
Name='Orthotropic';
C(1,1)=115.9;C(1,2)=35.3; C(1,3)=46.8; C(1,4)=0; C(1,5)=0; C(1,6)=0;
C(2,2)=174.1;C(2,3)=38.7; C(2,4)=0; C(2,5)=0; C(2,6)=0;
C(3,3)=153.1;C(3,4)=0; C(3,5)=0; C(3,6)=0;
C(4,4)=50.9;C(4,5)=0; C(4,6)=0;
C(5,5)=70.2;C(5,6)=0;
C(6,6)=26.6;
end
if Input == 2
% Tetragonal Class

disp(sprintf('--------------------------------------------------'))
disp(sprintf('You choose Tetragonal.'))
disp(sprintf('6 constants in total for Tetragonal:'))
disp(sprintf('Ep = Ex = Ey ~= Ez, '))
disp(sprintf('nu_pz = nu_xz = nu_yz ~= nu_xy, '))
disp(sprintf('Gpz = Gxz = Gyz ~= Gxy. '))
disp(sprintf('-----------Elastic Stiffnesses--------------------'))
disp('C(1,1)=125;C(1,2)=87; C(1,3)=90; C(1,4)=0; C(1,5)=0; C(1,6)=0;')
disp(' C(2,2)=125;C(2,3)=90; C(2,4)=0; C(2,5)=0; C(2,6)=0;')
disp(' C(3,3)=138;C(3,4)=0; C(3,5)=0; C(3,6)=0;')
disp(' C(4,4)=36;C(4,5)=0; C(4,6)=0;')
disp(' C(5,5)=36;C(5,6)=0;')
disp(' C(6,6)=48;')
disp(sprintf('--------------------------------------------------'))
disp(sprintf('You can update the parameters above based on the case.'))
disp(sprintf('Refer to the elastic constants surface plot in 3D, please.'))
Name='Tetragonal';
\[
\begin{align*}
C(1,1) &= 1714.411; C(1,2) = 2904.125; C(1,3) = 2348.350; C(1,4) = 0; C(1,5) = 0; \\
C(1,6) &= 0; \\
C(2,2) &= 8445.497; C(2,3) = 5285.008; C(2,4) = 0; C(2,5) = 0; C(2,6) = 0; \\
C(3,3) &= 6847.317; C(3,4) = 0; C(3,5) = 0; C(3,6) = 0; \\
C(4,4) &= 14000.000; C(4,5) = 0; C(4,6) = 0; \\
C(5,5) &= 725.000; C(5,6) = 0; \\
C(6,6) &= 725.000.
\end{align*}
\]

if Input == 3
% Transversely Isotropic Class

disp(sprintf('--------------------------------------------------'))
disp(sprintf('You choose Transversely Isotropic.'))
disp(sprintf('5 constants in total for Transversely Isotropic:'))
disp(sprintf('Ep = Ex = Ey ~= Ez, '))
disp(sprintf('nu_p = nu_x = nu_y, '))
disp(sprintf('nu_pz = nu_xz = nu_yz, '))
disp(sprintf('Gpz = Gxz = Gyz.'))
disp(sprintf('-----------Elastic Stiffnesses--------------------'))
disp(sprintf('C(1,1)=581;C(1,2)=55; C(1,3)=121;C(1,4)=0; C(1,5)=0; C(1,6)=0;'))
disp(sprintf(' C(2,2)=581;C(2,3)=121;C(2,4)=0; C(2,5)=0; C(2,6)=0;'))
disp(sprintf(' C(3,3)=445;C(3,4)=0; C(3,5)=0; C(3,6)=0;'))
disp(sprintf(' C(4,4)=240;C(4,5)=0; C(4,6)=0;'))
disp(sprintf(' C(5,5)=240;C(5,6)=0;'))
disp(sprintf(' C(6,6)=48;'))
disp(sprintf('--------------------------------------------------'))
disp(sprintf('You can update the parameters above based on the case.'))
disp(sprintf('Refer to the elastic constants surface plot in 3D, please.'))
disp(sprintf('--------------------------------------------------'))
Name='TransIso';
C(1,1)=376;C(1,2)=260; C(1,3)=274;C(1,4)=0; C(1,5)=0; C(1,6)=0;
C(2,2)=376;C(2,3)=262;C(2,4)=0; C(2,5)=0; C(2,6)=0;
C(3,3)=395;C(3,4)=0; C(3,5)=0; C(3,6)=0;
C(4,4)=63;C(4,5)=0; C(4,6)=0;
C(5,5)=63;C(5,6)=0;
C(6,6)=58;
end

if Input == 4
% Cubic Class

disp(sprintf('--------------------------------------------------'))
disp(sprintf('You choose Cubic.'))
disp(sprintf('3 constants in total for Cubic:'))
disp(sprintf('E, nu, G.'))
disp(sprintf('-----------Elastic Stiffnesses--------------------'))
disp(sprintf('C(1,1)=231.40;C(1,2)=134.70;C(1,3)=134.70; C(1,4)=0; C(1,5)=0; C(1,6)=0;'))
disp(sprintf(' C(2,2)=231.40;C(2,3)=134.70; C(2,4)=0; C(2,5)=0; C(2,6)=0;'))
disp(sprintf(' C(3,3)=231.40; C(3,4)=0; C(3,5)=0; C(3,6)=0;'))
disp(sprintf(' C(4,4)=116.40;C(4,5)=0; C(4,6)=0;'))
disp(sprintf(' C(5,5)=116.40;C(5,6)=0;'))
disp(sprintf(' C(6,6)=116.40;'))
disp(sprintf('--------------------------------------------------'))
disp(sprintf('You can update the parameters above based on the case.'))
disp(sprintf('Refer to the elastic constants surface plot in 3D, please.'))
disp(sprintf('--------------------------------------------------'))
Name='Cubic';
% http://solidmechanics.org/text/Chapter3_2/Chapter3_2.htm
% C11=240.20; C12= 125.60; C44= 28.20; Name='Nb';
% C11=522.40; C12= 160.80; C44=204.40; Name='W';
% C11=107.30; C12= 60.90; C44= 28.30; Name='Al';
% C11=346.70; C12= 250.70; C44= 76.50; Name='Pt';
% C11=237; C12= 149; C44=100; Name='IN792-SX';
% C11=124.00; C12= 93.40; C44= 46.10; Name='Ag';
% C11= 49.50; C12= 42.30; C44= 14.90; Name='Pb';
% C11= 13.50; C12= 11.44; C44= 8.78; Name='Li';

C(1:3,1:3)=C12;
for i=1:3; C(i,i)=C11; end
for i=4:6; C(i,i)=C44; end
end
if Input == 5
% Isotropic Class
disp(sprintf('You choose Isotropic.'))

disp(sprintf('2 constants in total for Isotropic:'))
disp(sprintf('E and nu.'))
disp(sprintf('-----------Elastic Stiffnesses------------------------'))
disp('C(1,1)=107.30;C(1,2)=90.90; C(1,3)=90.90; C(1,4)=0; C(1,5)=0;
C(1,6)=0;')
disp('C(2,2)=107.30;C(2,3)=90.90; C(2,4)=0; C(2,5)=0; C(2,6)=0;')
disp('C(3,3)=107.30; C(3,4)=0; C(3,5)=0; C(3,6)=0;')
disp('C(4,4)=28.30; C(4,5)=0; C(4,6)=0;')
disp('C(5,5)=28.30; C(5,6)=0;')
disp('C(6,6)=28.30;')
disp(sprintf('--------------------------------------------------'))
disp(sprintf('You can update the parameters above based on the case.'))
disp(sprintf('Refer to the elastic constants surface plot in 3D, please.'))
disp(sprintf('--------------------------------------------------'))
disp(sprintf('Choose Fe mechanical properties to compute stiffness tensor.'))
Name='Isotropic';
C11=233.96; C12= 137.4; C44= 48.277;
C(1:3,1:3)=C12;
for i=1:3; C(i,i)=C11; end
for i=4:6; C(i,i)=C44; end
end

% Assign Matrix C symmetry constants
for i=2:6; for j=1:i-1; C(i,j)=C(j,i); end; end
% Apply surface plot in spherical coordinate (3D)
[theta, phi]=meshgrid( linspace(0,pi), linspace(0,2*pi) );
L1=sin(theta).*cos(phi);
L2=sin(theta).*sin(phi);
L3=cos(theta);
[V, A, Vmin, Vmax] = getE(C, L1, L2, L3); % FIND Elastic Modulus
%exp Nov29 move [V, A, Vmin, Vmax] = getG(C, theta, phi); % FIND Shear Modulus only
%for cubic
x=V.*L1; y=V.*L2; z=V.*L3;
% or apply function which transfer Spherical coordinate to Cartesian coordinate
% [x,y,z] = sph2cart(phi, pi/2-theta,V);
disp(sprintf('%s: Anisotropy =%6.2f, Min =%6.2f, Max =%6.2f\n', Name, A,
Vmin,Vmax));
disp(sprintf('--------------------------------------------------'))
SphericalPlot(x, y, z, V);
if Input == 4

[V, A, Vmin, Vmax] = getG(C, theta, phi); % FIND Shear Modulus only for cubic
end

%% Cartesian coordinate (3D), plot isosurface, run slower,
% [x,y,z]=meshgrid(linspace(-Vmax,Vmax));
% r=sqrt(x.^2+y.^2+z.^2);
% L1=x./r; L2=y./r; L3=z./r;
% theta=acos(L3); phi=atan2(L2,L1);
% [V, A, Vmin, Vmax] = getE(C, L1, L2, L3);
% [V, A, Vmin, Vmax] = getG(C, theta, phi);
% fprintf('%s: Anisotropy=%9.4f Emin=%9.4f Emax=%9.4f\n', Name, A, Vmin, Vmax);
% CartesianPlot(x, y, z, V-r, V);
%% Set output plot method or configuration
axis tight; title(sprintf('%s: Anisotropy=%6.2f',Name,A));
daspect([1 1 1]);
view(45,30); % View angle. update (30,30) if needed.
colormap jet; % default value for color in Matlab old version
cbar=colorbar; title(cbar, 'MPa');
camlight; lighting phong;
%% Set output figure format
set(gca, 'Position', [0.12, 0.05, 0.6, 0.85]);
set(gcf, 'Position', [500, 500, 380, 350]); % update size, for example 20,20,1000,900
set(gcf, 'PaperPositionMode', 'auto');

% Apply matrix C (Stiffness Tensor) and direction vector L1, L2, L3 to calculate elastic %constant E
function [E, A, Emin, Emax] = getE(C, L1, L2, L3)
S=inv(C); % Calculate Matrix S, i.e. Compliance Tensor
S11=S(1,1); S12=S(1,2); S13=S(1,3); S14=S(1,4); S15=S(1,5); S16=S(1,6);
S22=S(2,2); S23=S(2,3); S24=S(2,4); S25=S(2,5); S26=S(2,6);
S33=S(3,3); S34=S(3,4); S35=S(3,5); S36=S(3,6);
S44=S(4,4); S45=S(4,5); S46=S(4,6);
S55=S(5,5); S56=S(5,6);
S66=S(6,6);
% Triclinic system, degree of symmetry is the lowest in 7 systems, so % it has the most number of independent. 21. i.e. it is general system.

%ezpNov29
A=inv(C); %ezpNov29
S1111=S11 * L1.^4 + S22 * L2.^4 + S33 * L3.^4 ...
+ (S44+2*S23) * (L2.*L3).^2 + (S55+2*S13) * (L1.*L3).^2 + (S66+2*S12) * (L1.*L2).^2 ...
+ 2*(S14+S56) * L1.^2 + S24 * L2.^2 + S34 * L3.^2) .* L2.*L3...
+ 2*( S15 * L1.^2 + (S25+S46) * L2.^2 + S35 * L3.^2) .* L1.*L3 ...
+ 2*( S16 * L1.^2 + S26 * L2.^2 + (S36+S45) * L3.^2) .* L1.*L2;
% <On Anisotropic Elastic Materials for which Young's Modulus E(n) is
% Independent of
% n or the Shear Modulus G(n,m) is Independent of n and m>
% doi: 10.1007/s10659-005-9016-2
% ---------------------------------------------------------------
% Cubic Class, Elastic Constants, verify the triclinic system(general system)
% A=2*(S11-S12)/S44;
% E=S11+(1-A)*S44*( (L1.*L2).^2+(L2.*L3).^2+(L3.*L1).^2 );
E=1./E;
% FIND extreme value, (max and min value) for any class.
Emin=min(E(:)); Emax=max(E(:));
fprintf('A=%8.4f Emin=%8.4f Emax=%8.4f 
', A, Emin, Emax);
% Cubic system, compare the value above to verify it.
% Emin=1/S11; Emax=1/(S11+(1-A)*S44/3);
% if(A>1); Emin=1/S11; Emax=1/(S11+(1-A)*S44/3); end
end
% FIND Shear modulus G extreme value
% Based on matrix C and spherical coordinate theta, phi,
% ONLY available for Cubic System
function [G, A, Gmin, Gmax] = getG(C, theta, phi)
S=inv(C); % compute matrix S, i.e. Compliance Tensor
A=2*(S(1,1)-S(1,2))/S(4,4);
L1=sin(theta).*cos(phi);
L2=sin(theta).*sin(phi);
L3=cos(theta);
theta = theta(:); phi = phi(:);
G = zeros(length(phi),1);
for i=1:length(G)
[x, Gmin] = fminbnd(@(x) Gcubic(S, theta(i), phi(i), x), 0,pi); G(i) = Gmin;
%ezp [x, Gmax] = fminbnd(@(x) -Gcubic(theta(i), phi(i), x), 0,pi); G(i) = -
Gmax;
end
G = reshape(G, size(L1));
Gmin=min(G(:)); Gmax=max(G(:));
fprintf('A=%8.4f Gmin=%8.4f Gmax=%8.4f 
', A, Gmin, Gmax);
end
% Cubic Class
% Compute Shear Modulus G
function G = Gcubic(S, theta, phi, eta)
S11=S(1,1); S12=S(1,2); S44=S(4,4);
L1=sin(theta).*cos(phi);
L2=sin(theta).*sin(phi);
L3=cos(theta);
M1=cos(theta).*cos(phi).*cos(eta)-sin(phi).*sin(eta);
M2=cos(theta).*sin(phi).*cos(eta)+cos(phi).*sin(eta);
M3=-sin(theta).*cos(eta);
G = 4*S11* ( (L1.*M1).^2+(L2.*M2).^2+(L3.*M3).^2 ) ...
+ 8*S12* ( L1.*L2.*M1.*M2+L1.*L3.*M1.*M3+L2.*L3.*M2.*M3) ...
+ S44* ( (L2.*M3+M2.*L3).^2+(L1.*M3+M1.*L3).^2+(L1.*M2+M1.*L2).^2 );
G = 1./G;
end
% Shear Modulus theta, phi direction plot, extreme value
function [Gmin, Gmax] = plotGcubic(S, theta, phi)
%ezpNov29 function [Gmin, Gmax] =
plotGcubic(theta, phi)
eta=meshgrid(linspace(0, 2*pi)); %ezpNov29 eta=linspace(0, 2*pi, 200);
G = Gcubic(S, theta, phi, eta); %ezpNov29 G = Gcubic(theta, phi, eta);
Gmin=min(G(:));
Gmax=max(G(:));
figure;
plot(eta,G);
xlabel('Direction');
ylabel('Shear Modulus, MPa');
end
%% Plot 2D surface figure
function SphericalPlot(x, y, z, v)
% color mapping curved surface
surf(x,y,z,v, 'FaceColor','interp', 'EdgeColor','none'); %ezp
% curved surface project on coordinat system.
hold on
f=1.2; % set project position
xr=xlim; yr=ylim; zr=zlim;
mesh(zeros(size(x))+f*xr(1), y, z, v)
mesh(x, zeros(size(y)) - f*yr(1), z, v)
mesh(x, y, zeros(size(z)) + f*zr(1), v)
xlabel('X (MPa)');
ylabel('Y (MPa)');
zlabel('Z (MPa)');
hold off
% modulus cross sectional drawing, it is better to comment lines surf and
mesh above.
% Refer to https://www.zhihu.com/question/48734216
vmax=max(v(:)); %ezp
[X,Y,Z]=meshgrid(linspace(-vmax,vmax)); %ezp
contourslice(X,Y,Z, X, x,y,z,[0 0]); %ezp
contourslice(X,Y,Z, -Y, x,y,z,[0 0]); %ezp
contourslice(X,Y,Z, Z, x,y,z,[0 0]); %ezp
end
%% Plot 3D isosurface figure
function CartesianPlot(x, y, z, v, c)
p=patch(isosurface(x,y,z,v,c));
isocolors(x,y,z,v,p);
isonormals(x,y,z,v,p);
set(p,'FaceColor','interp', 'EdgeColor','none');
end
Appendix B: Data
### 0° XY Sample Results

![Sample Results Graph](image)

#### Raw Data

<table>
<thead>
<tr>
<th>Property</th>
<th>Specimen</th>
<th>Average</th>
<th>Std Dev.</th>
</tr>
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<tbody>
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<td>3</td>
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<td><strong>0° XY</strong></td>
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<td></td>
<td></td>
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<tr>
<td>Stiffness (N/mm)</td>
<td>0.708</td>
<td>0.650</td>
<td>0.612</td>
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<tr>
<td>Yield Force (N)</td>
<td>80.850</td>
<td>41.299</td>
<td>50.426</td>
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<tr>
<td>Resilience (J)</td>
<td>4.101</td>
<td>1.310</td>
<td>2.022</td>
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<tr>
<td>Peak Force (N)</td>
<td>80.850</td>
<td>75.566</td>
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<td>Ductility (mm)</td>
<td>110.203</td>
<td>116.742</td>
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<tr>
<td>Toughness (J)</td>
<td>4.100</td>
<td>4.196</td>
<td>4.830</td>
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</table>

#### Print Orientation

![Print Orientation Image](image)

#### Microstructure

![Microstructure Image](image)

Table B-1. 0° XY sample results, orientation, and cross-sectioned microstructure.
Figure B-1. 0° XY representative specimen failure diagram (Specimen ID: 0°_XY_1). Green X’s indicate location of failure while green measurements delineate the distance from a reference edge to the approximate centroid of each pair of fracture surfaces (all values measured with a pair of Vernier calipers in units of mm). Microscopy images are shown either above and below and/or left and right of each other for the purpose of displaying the direction from which the normal component of the fracture surface exists when looking directly at it. All microscopy images were taken at 30X magnification.
**22.5° XY Sample Results**

![Graph showing load vs. displacement for 22.5° XY samples.]

**Raw Data**

<table>
<thead>
<tr>
<th>Property</th>
<th>Specimen</th>
<th>Average</th>
<th>Std Dev.</th>
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<tbody>
<tr>
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<td>3</td>
</tr>
<tr>
<td><strong>Stiffness (N/mm)</strong></td>
<td>0.949</td>
<td>0.930</td>
<td>0.910</td>
</tr>
<tr>
<td><strong>Yield Force (N)</strong></td>
<td>65.010</td>
<td>62.040</td>
<td>58.332</td>
</tr>
<tr>
<td><strong>Resilience (J)</strong></td>
<td>2.225</td>
<td>2.060</td>
<td>1.869</td>
</tr>
<tr>
<td><strong>Peak Force (N)</strong></td>
<td>91.730</td>
<td>93.106</td>
<td>91.974</td>
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<tr>
<td><strong>Ductility (mm)</strong></td>
<td>109.882</td>
<td>115.393</td>
<td>124.061</td>
</tr>
<tr>
<td><strong>Toughness (J)</strong></td>
<td>5.514</td>
<td>6.324</td>
<td>6.534</td>
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</tbody>
</table>

Table B-2. 22.5° XY sample results, orientation, and cross-sectioned microstructure.
Figure B-2. 22.5° XY representative specimen failure diagram (Specimen ID: 22.5°_XY_1). Same description as Fig. B-1.
### 22.5° ZX Sample Results

![Graph showing 22.5° ZX Sample Results]

#### Raw Data

<table>
<thead>
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<th>Std Dev.</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
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<td>3</td>
</tr>
<tr>
<td><strong>Stiffness (N/mm)</strong></td>
<td>1.030</td>
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<tr>
<td><strong>Yield Force (N)</strong></td>
<td>76.865</td>
<td>70.250</td>
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<tr>
<td><strong>Resilience (J)</strong></td>
<td>2.868</td>
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<tr>
<td><strong>Peak Force (N)</strong></td>
<td>91.978</td>
<td>85.204</td>
<td>73.130</td>
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<tr>
<td><strong>Ductility (mm)</strong></td>
<td>97.935</td>
<td>87.882</td>
<td>71.067</td>
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<tr>
<td><strong>Toughness (J)</strong></td>
<td>4.860</td>
<td>3.875</td>
<td>2.691</td>
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</tbody>
</table>

Table B-3. 22.5° ZX sample results, orientation, and cross-sectioned microstructure.
Figure B-3. 22.5° ZX representative specimen failure diagram (Specimen ID: 22.5°_ZX_1). Same description as Fig. B-1.
### 22.5° ZX+45° Sample Results

![Graph showing load vs. displacement](image)

#### Raw Data

<table>
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<th>Property</th>
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<td>3</td>
</tr>
<tr>
<td>Stiffness (N/mm)</td>
<td>0.972</td>
<td>0.969</td>
<td>0.982</td>
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<td>Yield Force (N)</td>
<td>68.232</td>
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<td>Resilience (J)</td>
<td>2.384</td>
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<td>Peak Force (N)</td>
<td>101.113</td>
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<td>Ductility (mm)</td>
<td>124.813</td>
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<td>Toughness (J)</td>
<td>7.182</td>
<td>6.955</td>
<td>8.221</td>
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#### Print Orientation

![Print Orientation Diagram](image)

#### Microstructure

![Microstructure Image](image)

*Table B-4. 22.5° ZX+45° sample results, orientation, and cross-sectioned microstructure.*
Figure B-4. 22.5° ZX+45° representative specimen failure diagram (Specimen ID: 22.5°_ZX+45°_4). Same description as Fig. B-1.
### Raw Data

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<tr>
<td>Stiffness (N/mm)</td>
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**Table B-5.** 45° XY sample results, orientation, and cross-sectioned microstructure.
Figure B-5. 45° XY representative specimen failure diagram (Specimen ID: 45°_XY_4). Same description as Fig. B-1.
### Raw Data

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Table B-6. 45° ZX sample results, orientation, and cross-sectioned microstructure.
Figure B-6. 45° ZX representative specimen failure diagram (Specimen ID: 45°_ZX_1). Same description as Fig. B-1.
45° ZX+45° Sample Results

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</tr>
<tr>
<td>Toughness (J)</td>
<td>1</td>
<td>2.169</td>
<td>2.970</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.341</td>
<td>0.470</td>
</tr>
<tr>
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<td>3</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.109</td>
<td></td>
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</tbody>
</table>

Table B-7. 45° ZX+45° sample results, orientation, and cross-sectioned microstructure.
Figure B-7. 45° ZX+45° representative specimen failure diagram (Specimen ID: 45°_ZX+45°_2). Same description as Fig. B-1.
### Raw Data

#### 67.5° XY Sample Results

<table>
<thead>
<tr>
<th>Property</th>
<th>Specimen</th>
<th>Average</th>
<th>Std Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Stiffness (N/mm)</td>
<td>0.859</td>
<td>0.757</td>
<td>0.771</td>
</tr>
<tr>
<td>Yield Force (N)</td>
<td>56.977</td>
<td>48.407</td>
<td>56.727</td>
</tr>
<tr>
<td>Resilience (J)</td>
<td>1.882</td>
<td>1.541</td>
<td>2.040</td>
</tr>
<tr>
<td>Peak Force (N)</td>
<td>95.365</td>
<td>85.093</td>
<td>90.461</td>
</tr>
<tr>
<td>Ductility (mm)</td>
<td>137.202</td>
<td>140.063</td>
<td>137.012</td>
</tr>
<tr>
<td>Toughness (J)</td>
<td>7.789</td>
<td>6.876</td>
<td>7.701</td>
</tr>
</tbody>
</table>

#### Print Orientation

Table B-8. 67.5° XY sample results, orientation, and cross-sectioned microstructure.
Figure B-8. 67.5° XY representative specimen failure diagram (Specimen ID: 67.5°_XY_1). Same description as Fig. B-1.
### 67.5° ZX Sample Results

![Graph showing load-displacement relationship for 67.5° ZX samples](image)

**Raw Data**

<table>
<thead>
<tr>
<th>Property</th>
<th>Specimen</th>
<th>Average</th>
<th>Std Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td><strong>67.5° ZX</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiffness (N/mm)</td>
<td>0.807</td>
<td>0.819</td>
<td>0.732</td>
</tr>
<tr>
<td>Yield Force (N)</td>
<td>50.569</td>
<td>47.072</td>
<td>43.419</td>
</tr>
<tr>
<td>Resilience (J)</td>
<td>1.564</td>
<td>1.352</td>
<td>1.288</td>
</tr>
<tr>
<td>Peak Force (N)</td>
<td>51.651</td>
<td>57.568</td>
<td>44.412</td>
</tr>
<tr>
<td>Ductility (mm)</td>
<td>57.383</td>
<td>74.136</td>
<td>60.836</td>
</tr>
<tr>
<td>Toughness (J)</td>
<td>1.658</td>
<td>2.249</td>
<td>1.372</td>
</tr>
</tbody>
</table>

**Print Orientation**

![Print Orientation diagram](image)

![Microstructure images](image)

Table B-9. 67.5° ZX sample results, orientation, and cross-sectioned microstructure.
Figure B-9. 67.5° ZX representative specimen failure diagram (Specimen ID: 67.5°_ZX_4). Same description as Fig. B-1.
## 67.5° ZX+45° Sample Results

![Graph showing load vs. displacement](image)

### Raw Data

<table>
<thead>
<tr>
<th>Property</th>
<th>Specimen 1</th>
<th>Specimen 2</th>
<th>Specimen 3</th>
<th>Specimen 4</th>
<th>Average</th>
<th>Std Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness (N/mm)</td>
<td>0.957</td>
<td>0.839</td>
<td>0.826</td>
<td>0.865</td>
<td>0.872</td>
<td>0.051</td>
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<tr>
<td>Yield Force (N)</td>
<td>60.006</td>
<td>45.031</td>
<td>61.431</td>
<td>49.277</td>
<td>53.936</td>
<td>6.965</td>
</tr>
<tr>
<td>Resilience (J)</td>
<td>1.880</td>
<td>1.181</td>
<td>2.172</td>
<td>1.427</td>
<td>1.665</td>
<td>0.385</td>
</tr>
<tr>
<td>Peak Force (N)</td>
<td>60.296</td>
<td>45.031</td>
<td>61.755</td>
<td>49.277</td>
<td>54.089</td>
<td>7.115</td>
</tr>
<tr>
<td>Ductility (mm)</td>
<td>63.103</td>
<td>53.537</td>
<td>58.381</td>
<td>57.643</td>
<td>58.166</td>
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<tr>
<td>Toughness (J)</td>
<td>1.880</td>
<td>1.181</td>
<td>2.172</td>
<td>1.427</td>
<td>1.665</td>
<td>0.385</td>
</tr>
</tbody>
</table>

### Print Orientation

![Print orientation diagram](image)

### Microstructure

![Microstructure images](image)

Table B-10. 67.5° ZX+45° sample results, orientation, and cross-sectioned microstructure.
Figure B-10. 67.5° ZX+45° representative specimen failure diagram (Specimen ID: 67.5°_ZX+45°_2). Same description as Fig. B-1.
### Raw Data

<table>
<thead>
<tr>
<th>Property</th>
<th>Specimen</th>
<th>Average</th>
<th>Std Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Stiffness (N/mm)</td>
<td>0.933</td>
<td>0.976</td>
<td>0.970</td>
</tr>
<tr>
<td>Yield Force (N)</td>
<td>65.711</td>
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<td>69.863</td>
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<td>Resilience (J)</td>
<td>2.314</td>
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<td>2.507</td>
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<tr>
<td>Peak Force (N)</td>
<td>86.727</td>
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<tr>
<td>Ductility (mm)</td>
<td>105.459</td>
<td>104.022</td>
<td>101.934</td>
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<tr>
<td>Toughness (J)</td>
<td>5.008</td>
<td>5.247</td>
<td>4.936</td>
</tr>
</tbody>
</table>

### 90° XY Sample Results

![Graph showing load versus displacement for 90° XY sample results](image)

Table B-11. 90° XY sample results, orientation, and cross-sectioned microstructure.
Figure B-11. 90° XY representative specimen failure diagram (Specimen ID: 90°_XY_4). Same description as Fig. B-1.
90° ZX Sample Results

![Graph showing Load vs. Displacement for 90° ZX samples](image)

**Raw Data**

<table>
<thead>
<tr>
<th>Property</th>
<th>Specimen 1 (N/mm)</th>
<th>Specimen 2 (N/mm)</th>
<th>Specimen 3 (N/mm)</th>
<th>Specimen 4 (N/mm)</th>
<th>Average (N/mm)</th>
<th>Std Dev. (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness</td>
<td>0.919</td>
<td>0.798</td>
<td>0.930</td>
<td>0.777</td>
<td>0.856</td>
<td>0.069</td>
</tr>
<tr>
<td>Yield Force</td>
<td>58.342</td>
<td>32.565</td>
<td>38.972</td>
<td>20.689</td>
<td>37.642</td>
<td>13.633</td>
</tr>
<tr>
<td>Resilience</td>
<td>1.724</td>
<td>0.656</td>
<td>0.829</td>
<td>0.274</td>
<td>0.871</td>
<td>0.532</td>
</tr>
<tr>
<td>Peak Force</td>
<td>58.342</td>
<td>32.565</td>
<td>38.972</td>
<td>20.839</td>
<td>37.679</td>
<td>13.586</td>
</tr>
<tr>
<td>Ductility</td>
<td>50.517</td>
<td>36.977</td>
<td>42.126</td>
<td>26.446</td>
<td>39.017</td>
<td>8.720</td>
</tr>
<tr>
<td>Toughness</td>
<td>1.838</td>
<td>0.656</td>
<td>0.829</td>
<td>0.274</td>
<td>0.899</td>
<td>0.578</td>
</tr>
</tbody>
</table>

**Print Orientation**

**Microstructure**

Table B-12 90° ZX sample results, orientation, and cross-sectioned microstructure.
Figure B-12. 90° ZX representative specimen failure diagram (Specimen ID: 90°_ZX_1). Same description as Fig. B-1.
**XY Plane Average Results Comparison**

**XY Plane Average Stiffness**

![XY Plane Average Stiffness Chart]

Figure B-13. XY plane average stiffness data comparison. Error bars indicate minimum and maximum empirical values.

**XY Plane Average Yield Force**

![XY Plane Average Yield Force Chart]

Figure B-14. XY plane average yield force comparison. Same description as Fig. B-13.

**XY Plane Average Resilience**

![XY Plane Average Resilience Chart]

Figure B-15. XY plane average resilience comparison. Same description as Fig. B-13.
XY Plane Average Results Comparison (Continued)

**XY Plane Average Peak Force**

![Graph showing peak force comparison for different XY plane angles.]

Figure B-16. XY plane average peak force comparison. Same description as Fig. B-13.

**XY Plane Average Ductility**

![Graph showing ductility comparison for different XY plane angles.]

Figure B-17. XY plane average ductility comparison. Same description as Fig. B-13.

**XY Plane Average Toughness**

![Graph showing toughness comparison for different XY plane angles.]

Figure B-18. XY plane average toughness comparison. Same description as Fig. B-13.
**ZX Plane Average Results Comparison**

### ZX Plane Average Stiffness

![ZX Plane Average Stiffness Graph](image)

**Figure B-19.** ZX plane average stiffness comparison. Same description as Fig. B-13.

### ZX Plane Average Yield Force

![ZX Plane Average Yield Force Graph](image)

**Figure B-20.** ZX plane average yield force comparison. Same description as Fig. B-13.

### ZX Plane Average Resilience

![ZX Plane Average Resilience Graph](image)

**Figure B-21.** ZX plane average resilience comparison. Same description as Fig. B-13.
ZX Plane Average Results Comparison (Continued)

**ZX Plane Average Peak Force**

![ZX Plane Average Peak Force](image)

**Figure B-22.** ZX plane average peak force comparison. Same description as Fig. B-13.

**ZX Plane Average Ductility**

![ZX Plane Average Ductility](image)

**Figure B-23.** ZX plane average ductility comparison. Same description as Fig. B-13.

**ZX Plane Average Toughness**

![ZX Plane Average Toughness](image)

**Figure B-24.** ZX plane average toughness comparison. Same description as Fig. B-13.
ZX+45° Plane Average Results Comparison

ZX+45° Plane Average Stiffness

Figure B-25. ZX+45° plane average stiffness comparison. Same description as Fig. B-13.

ZX+45° Plane Average Yield Force

Figure B-26. ZX+45° plane average yield force comparison. Same description as Fig. B-13.

ZX+45° Plane Average Resilience

Figure B-27. ZX+45° plane average resilience comparison. Same description as Fig. B-13.
ZX+45° Plane Average Results Comparison (Continued)

**ZX+45° Plane Average Peak Force**

![Graph showing peak force comparison](image)

**Figure B-28.** ZX+45° plane average peak force comparison. Same description as Fig. B-13.

**ZX+45° Plane Average Ductility**

![Graph showing ductility comparison](image)

**Figure B-29.** ZX+45° plane average ductility comparison. Same description as Fig. B-13.

**ZX+45° Plane Average Toughness**

![Graph showing toughness comparison](image)

**Figure B-30.** ZX+45° plane average toughness comparison. Same description as Fig. B-13.
Appendix C: Surface Plots & FEA Results
Table C-1. 0° XY $E(\Theta)$ surface plot, stiffness matrix, relevant mechanical properties ($E$ in MPa, $v$ is unitless, and $G$ in MPa), and FEA simulation results.
22.5° XY [E(Θ)] Surface Plot

<table>
<thead>
<tr>
<th>Stiffness Matrix (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2980.391 1079.333 79.909 0.000 0.000 0.000</td>
</tr>
<tr>
<td>1079.333 2980.391 79.909 0.000 0.000 0.000</td>
</tr>
<tr>
<td>79.909 79.909 639.273 0.000 0.000 0.000</td>
</tr>
<tr>
<td>0.000 0.000 0.000 950.000 0.000 0.000</td>
</tr>
<tr>
<td>0.000 0.000 0.000 0.000 950.000 0.000</td>
</tr>
<tr>
<td>0.000 0.000 0.000 0.000 0.000 881.868</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>E₁        2585.438</td>
</tr>
<tr>
<td>E₂        2585.438</td>
</tr>
<tr>
<td>E₃        636.127</td>
</tr>
<tr>
<td>ν₁₂       0.360</td>
</tr>
<tr>
<td>ν₂₁       0.360</td>
</tr>
<tr>
<td>ν₁₃       0.080</td>
</tr>
<tr>
<td>ν₃₁       0.020</td>
</tr>
<tr>
<td>ν₂₃       0.080</td>
</tr>
<tr>
<td>ν₃₂       0.020</td>
</tr>
<tr>
<td>G₁₂       881.868</td>
</tr>
<tr>
<td>G₂₃       950.000</td>
</tr>
<tr>
<td>G₁₃       950.000</td>
</tr>
</tbody>
</table>

$k = 0.909 \text{ N/mm}$

Table C-2. 22.5° XY E(Θ) surface plot, stiffness matrix, relevant mechanical properties (E in MPa, ν is unitless, and G in MPa), and FEA simulation results.
### 22.5° ZX [E(Θ)] Surface Plot

![22.5° ZX [E(Θ)] Surface Plot](image)

### Stiffness Matrix (MPa)

<table>
<thead>
<tr>
<th></th>
<th>3936.439</th>
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<th>4569.936</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>3936.439</td>
<td>2227.768</td>
<td>4569.936</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2227.768</td>
<td>3424.223</td>
<td>6002.503</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>4569.936</td>
<td>6002.503</td>
<td>13845.927</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
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<tr>
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<td>0.000</td>
<td>0.000</td>
<td>453.469</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>453.469</td>
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</tr>
</tbody>
</table>

### Mechanical Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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</thead>
<tbody>
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<td>2354.121</td>
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<td>$E_2$</td>
<td>796.965</td>
</tr>
<tr>
<td>$E_3$</td>
<td>3146.136</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
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</tr>
<tr>
<td>$\nu_{21}$</td>
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</tr>
<tr>
<td>$\nu_{13}$</td>
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</tr>
<tr>
<td>$\nu_{31}$</td>
<td>0.790</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>0.400</td>
</tr>
<tr>
<td>$\nu_{32}$</td>
<td>0.400</td>
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<td>$G_{12}$</td>
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<td>$G_{23}$</td>
<td>2000.000</td>
</tr>
<tr>
<td>$G_{13}$</td>
<td>453.469</td>
</tr>
</tbody>
</table>

**Table C-3.** 22.5° ZX E(Θ) surface plot, stiffness matrix, relevant mechanical properties (E in MPa, $v$ is unitless, and $G$ in MPa), and FEA simulation results.
Table C-4. 22.5° ZX+45° E(Θ) surface plot, stiffness matrix, relevant mechanical properties (E in MPa, v is unitless, and G in MPa), and FEA simulation results.
Table C-5. 45° XY E(Θ) surface plot, stiffness matrix, relevant mechanical properties (E in MPa, v is unitless, and G in MPa), and FEA simulation results.

\[ k = 1.052 \text{ N/mm} \]
**45° ZX \([E(\Theta)]\) Surface Plot**

<table>
<thead>
<tr>
<th>Stiffness Matrix (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1797.223 977.391 1466.904 0.000 0.000 0.000</td>
</tr>
<tr>
<td>977.391 2451.830 2429.560 0.000 0.000 0.000</td>
</tr>
<tr>
<td>1466.904 2429.560 5730.095 0.000 0.000 0.000</td>
</tr>
<tr>
<td>0.000 0.000 0.000 14000.000 0.000 0.000</td>
</tr>
<tr>
<td>0.000 0.000 0.000 0.000 329.909 0.000</td>
</tr>
<tr>
<td>0.000 0.000 0.000 0.000 0.000 329.909</td>
</tr>
</tbody>
</table>

**45° ZX FEA Results**

![Graph showing load vs. displacement](image)

\[ k = 0.931 \text{ N/mm} \]

**Mechanical Properties**

<table>
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<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>(E_2)</td>
<td>1332.840</td>
</tr>
<tr>
<td>(E_3)</td>
<td>3146.136</td>
</tr>
<tr>
<td>(\nu_{12})</td>
<td>0.250</td>
</tr>
<tr>
<td>(\nu_{23})</td>
<td>0.250</td>
</tr>
<tr>
<td>(\nu_{13})</td>
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</tr>
<tr>
<td>(\nu_{31})</td>
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</tr>
<tr>
<td>(\nu_{23})</td>
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</tr>
<tr>
<td>(\nu_{32})</td>
<td>0.360</td>
</tr>
<tr>
<td>(G_{12})</td>
<td>329.909</td>
</tr>
<tr>
<td>(G_{23})</td>
<td>14000.000</td>
</tr>
<tr>
<td>(G_{13})</td>
<td>329.909</td>
</tr>
</tbody>
</table>

Table C-6. 45° ZX \(E(\Theta)\) surface plot, stiffness matrix, relevant mechanical properties (\(E\) in MPa, \(\nu\) is unitless, and \(G\) in MPa), and FEA simulation results.
45° ZX+45° [E(Θ)] Surface Plot

### Stiffness Matrix (MPa)

<table>
<thead>
<tr>
<th></th>
<th>913.334</th>
<th>480.420</th>
<th>1260.433</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>913.334</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>480.420</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1260.433</td>
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<td></td>
<td>2000.000</td>
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<td>0.000</td>
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</tbody>
</table>

### Mechanical Properties

<table>
<thead>
<tr>
<th></th>
<th>E₁</th>
<th>E₂</th>
<th>E₃</th>
<th>v₁₂</th>
<th>v₂₁</th>
<th>v₁₃</th>
<th>v₂₃</th>
<th>v₃₁</th>
<th>v₃₂</th>
<th>G₁₂</th>
<th>G₂₃</th>
<th>G₁₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>E₁</td>
<td>541.142</td>
<td>541.142</td>
<td>2446.893</td>
<td>0.250</td>
<td>0.250</td>
<td>0.200</td>
<td>0.200</td>
<td>0.904</td>
<td>0.904</td>
<td>321.281</td>
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</tr>
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</table>

Table C-7. 45° ZX+45° E(Θ) surface plot, stiffness matrix, relevant mechanical properties (E in MPa, v is unitless, and G in MPa), and FEA simulation results.
Table C-8. 67.5° XY E(Θ) surface plot, stiffness matrix, relevant mechanical properties (E in MPa, v is unitless, and G in MPa), and FEA simulation results.
67.5° ZX [E(Θ)] Surface Plot

Stiffness Matrix (MPa)

<table>
<thead>
<tr>
<th></th>
<th>1874.288</th>
<th>2274.373</th>
<th>2543.651</th>
<th>0.000</th>
<th>0.000</th>
<th>0.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1874.288</td>
<td>2274.373</td>
<td>5574.829</td>
<td>4403.328</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2274.373</td>
<td>5574.829</td>
<td>4403.328</td>
<td>7214.093</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2543.651</td>
<td>4403.328</td>
<td>7214.093</td>
<td>4000.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>0.000</td>
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<td>4000.000</td>
<td>453.469</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>453.469</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
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<td>0.000</td>
<td>0.000</td>
<td>453.469</td>
<td></td>
</tr>
</tbody>
</table>

67.5° ZX FEA Results

\[ k = 0.641 \text{ N/mm} \]

<table>
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<tr>
<th>Mechanical Properties</th>
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<tr>
<td>E_1</td>
</tr>
<tr>
<td>E_2</td>
</tr>
<tr>
<td>E_3</td>
</tr>
<tr>
<td>v_{12}</td>
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<tr>
<td>v_{21}</td>
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<tr>
<td>v_{13}</td>
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<tr>
<td>v_{31}</td>
</tr>
<tr>
<td>v_{23}</td>
</tr>
<tr>
<td>v_{32}</td>
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<tr>
<td>G_{12}</td>
</tr>
<tr>
<td>G_{23}</td>
</tr>
<tr>
<td>G_{13}</td>
</tr>
</tbody>
</table>

Table C-9. 67.5° ZX E(Θ) surface plot, stiffness matrix, relevant mechanical properties (E in MPa, v is unitless, and G in MPa), and FEA simulation results.
$67.5^\circ \text{ ZX}+45^\circ \text{ [E(}\Theta\text{)] Surface Plot}$

### Stiffness Matrix (MPa)

<table>
<thead>
<tr>
<th></th>
<th>1083.550</th>
<th>587.199</th>
<th>987.518</th>
<th>0.000</th>
<th>0.000</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1083.550</td>
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<tr>
<td>587.199</td>
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<td>987.518</td>
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### Mechanical Properties

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<th>$E_3$</th>
<th>$v_{12}$</th>
<th>$v_{21}$</th>
<th>$v_{13}$</th>
<th>$v_{31}$</th>
<th>$v_{23}$</th>
<th>$G_{12}$</th>
<th>$G_{23}$</th>
<th>$G_{13}$</th>
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</thead>
<tbody>
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<td>$E_2$</td>
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<td>$E_3$</td>
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</tbody>
</table>

$k = 0.683 \text{ N/mm}$

Table C-10. $67.5^\circ \text{ ZX}+45^\circ \text{ E(}\Theta\text{)}$ surface plot, stiffness matrix, relevant mechanical properties (E in MPa, v is unitless, and G in MPa), and FEA simulation results.
**90° XY [E(Θ)] Surface Plot**

### Stiffness Matrix (MPa)

<table>
<thead>
<tr>
<th></th>
<th>3624.566</th>
<th>1311.231</th>
<th>79.839</th>
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<th>0.000</th>
</tr>
</thead>
<tbody>
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<td>79.839</td>
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<tr>
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</tr>
</tbody>
</table>

### Mechanical Properties

- $E_1 = 3146.136$ MPa
- $E_2 = 3146.136$ MPa
- $E_3 = 636.127$ MPa
- $v_{12} = 0.360$
- $v_{21} = 0.360$
- $v_{13} = 0.080$
- $v_{31} = 0.016$
- $v_{23} = 0.080$
- $v_{32} = 0.016$
- $G_{12} = 800.000$ MPa
- $G_{23} = 1000.000$ MPa
- $G_{13} = 1000.000$ MPa

**90° XY FEA Results**

![Graph showing load vs. displacement]

$k = 0.933$ N/mm

### Table C-11

90° XY $E(\Theta)$ surface plot, stiffness matrix, relevant mechanical properties ($E$ in MPa, $v$ is unitless, and $G$ in MPa), and FEA simulation results.

115
$90^\circ$ ZX [E(Θ)] Surface Plot

### Stiffness Matrix (MPa)

<table>
<thead>
<tr>
<th></th>
<th>1714.411</th>
<th>2904.125</th>
<th>2348.350</th>
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### Mechanical Properties

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<thead>
<tr>
<th></th>
<th>E1</th>
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<th>E3</th>
<th>ν12</th>
<th>ν23</th>
<th>ν31</th>
<th>ν13</th>
<th>ν21</th>
<th>ν32</th>
<th>G12</th>
<th>G23</th>
<th>G13</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>636.127</td>
<td>3055.527</td>
<td>3146.136</td>
<td>0.250</td>
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<td>0.742</td>
<td>0.150</td>
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<td>725.000</td>
<td>14000.000</td>
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</tbody>
</table>

$k = 0.905$ N/mm

Table C-12. $90^\circ$ ZX E(Θ) surface plot, stiffness matrix, relevant mechanical properties (E in MPa, $\nu$ is unitless, and $G$ in MPa), and FEA simulation results.
Appendix D: Analytical Model
**Analysis completed with PTC MathCAD 15.0***

Component Study Analysis (Deflection and Stiffness Evaluation) using Castigliano's Theorem

Goal: Find deflection of point e for all orientations. Find stiffness for all orientations.

**Geometric Properties**
- Base (m): \( b = 0.008 \)
- Height (m): \( h = 0.01 \)

Lengths of corresponding segments (m):
- \( l_{ab} = 0.046 \)
- \( l_{bc} = 0.092 \)
- \( l_{cd} = 0.0395 \)
- \( l_{de} = 0.046 \)

Constants used throughout entire problem (these are stated early to avoid repetitive iteration):

Area moment of inertia (m\(^4\)):
\[
I = \frac{b h^3}{12} = 6.667 \times 10^{-10}
\]

Polar Area Moment of Inertia (m\(^4\)):
\[
I_p = \frac{b h (0.25 h^2 + 0.5 h^2)}{12} = 1.093 \times 10^{-9}
\]

**Material Properties**
Elastic Moduli (Pa) [Perkowski, 2017]:

**XY Plane:**
- \( E_{xy} = 3055.52 \times 10^6 \)
- \( E_{xx} = 2585.43 \times 10^6 \)
- \( E_{yy} = 2446.89 \times 10^6 \)
- \( G_{xy} = 2758.83 \times 10^6 \)
- \( G_{xx} = 3146.35 \times 10^6 \)

**ZX Plane:**
- \( E_{zz} = 2354.12 \times 10^6 \)
- \( E_{xx} = 1332.84 \times 10^6 \)
- \( E_{yy} = 796.965 \times 10^6 \)
- \( G_{xz} = 636.127 \times 10^6 \)

**ZX+45° Plane:**
- \( E_{zz} = 1663.47 \times 10^6 \)
- \( E_{xx} = 541.14 \times 10^6 \)
- \( E_{yy} = 739.247 \times 10^6 \)

Poisson's Ratio (unitless):
\( v = 0.36 \)
Shear Moduli (Pa) [Theoretically acquired values]:

**XY Plane:**
- \( \sigma_{0,XY} = 800.000 \times 10^6 \)
- \( \sigma_{22.5,XY} = 881.868 \times 10^6 \)
- \( \sigma_{45,XY} = 1140.012 \times 10^6 \)
- \( \sigma_{67.5,XY} = 940.210 \times 10^6 \)
- \( \sigma_{90,XY} = 800.000 \times 10^6 \)

**ZX Plane:**
- \( \sigma_{22.5,ZX} = 453.469 \times 10^6 \)
- \( \sigma_{45,ZX} = 329.909 \times 10^6 \)
- \( \sigma_{67.5,ZX} = 453.469 \times 10^6 \)
- \( \sigma_{90,ZX} = 725.000 \times 10^6 \)

**ZX+45° Plane:**
- \( \sigma_{22.5,ZX+45} = 287.410 \times 10^6 \)
- \( \sigma_{45,ZX+45} = 321.281 \times 10^6 \)
- \( \sigma_{67.5,ZX+45} = 287.410 \times 10^6 \)

**Load:**
Test Load (N): \( P_x = 50.0 \)

***Note: All deflection calculations result in values with units of meters***
Deflection of Segment $ab$ (Bending + Torsion)

\[ \delta_{ab, \text{Bending, 0 XY}} = \frac{\int_{0}^{L} \frac{P_e x^2}{E_{0 \text{ XY}} I} \, dx}{E_{0 \text{ XY} I}} = 7.964 \times 10^{-4} \]

\[ \delta_{ab, \text{Bending, 22.5 XY}} = \frac{\int_{0}^{L} \frac{P_e x^2}{E_{22.5 \text{ XY} I}} \, dx}{E_{22.5 \text{ XY} I}} = 9.412 \times 10^{-4} \]

\[ \delta_{ab, \text{Bending, 45 XY}} = \frac{\int_{0}^{L} \frac{P_e x^2}{E_{45 \text{ XY} I}} \, dx}{E_{45 \text{ XY} I}} = 9.945 \times 10^{-4} \]

\[ \delta_{ab, \text{Bending, 67.5 XY}} = \frac{\int_{0}^{L} \frac{P_e x^2}{E_{67.5 \text{ XY} I}} \, dx}{E_{67.5 \text{ XY} I}} = 8.820 \times 10^{-4} \]

\[ \delta_{ab, \text{Bending, 90 XY}} = \frac{\int_{0}^{L} \frac{P_e x^2}{E_{90 \text{ XY} I}} \, dx}{E_{90 \text{ XY} I}} = 7.735 \times 10^{-4} \]

\[ \delta_{ab, \text{Bending, 22.5 ZY}} = \frac{\int_{0}^{L} \frac{P_e x^2}{E_{22.5 \text{ ZY} I}} \, dx}{E_{22.5 \text{ ZY} I}} = 1.034 \times 10^{-3} \]
Deflection of Segment ab (Bending + Torsion) [Continued]

\[ \delta_{ab,\text{Bending, 45\,ZX}} = \frac{\int_0^{l_{ab}} p(x) \phi^2(x) \, dx}{E_{45,\,ZX} I_{45,\,ZX}} = \frac{1.826 \times 10^{-3}}{} \]

\[ \delta_{ab,\text{Bending, 67.5\,ZX}} = \frac{\int_0^{l_{ab}} p(x) \phi^2(x) \, dx}{E_{67.5,\,ZX} I_{67.5,\,ZX}} = \frac{3.053 \times 10^{-3}}{} \]

\[ \delta_{ab,\text{Bending, 90\,ZX}} = \frac{\int_0^{l_{ab}} p(x) \phi^2(x) \, dx}{E_{90,\,ZX} I_{90,\,ZX}} = \frac{3.825 \times 10^{-3}}{} \]

\[ \delta_{ab,\text{Bending, 22.5\,ZX, 45\,ZX}} = \frac{\int_0^{l_{ab}} p(x) \phi^2(x) \, dx}{E_{22.5,\,ZX, 45,\,ZX} I_{22.5,\,ZX, 45,\,ZX}} = \frac{2.288 \times 10^{-3}}{} \]

\[ \delta_{ab,\text{Bending, 45\,ZX, 45\,ZX}} = \frac{\int_0^{l_{ab}} p(x) \phi^2(x) \, dx}{E_{45,\,ZX, 45,\,ZX} I_{45,\,ZX, 45,\,ZX}} = \frac{4.497 \times 10^{-3}}{} \]

\[ \delta_{ab,\text{Bending, 67.5\,ZX, 45\,ZX}} = \frac{\int_0^{l_{ab}} p(x) \phi^2(x) \, dx}{E_{67.5,\,ZX, 45,\,ZX} I_{67.5,\,ZX, 45,\,ZX}} = \frac{3.292 \times 10^{-3}}{} \]
Deflection of Segment $ab$ (Bending + Torsion) [Continued]

$$
\delta_{ab\text{, Torsion}_0, XY} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{XY} I_Y} = 1.885 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_22.5, XY} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{22.5, XY} I_Y} = 1.979 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_45, XY} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{45, XY} I_Y} = 1.755 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_67.5, XY} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{67.5, XY} I_Y} = 1.539 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_90, XY} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{90, XY} I_Y} = 2.053 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_22.5, ZX} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{22.5, ZX} I_Y} = 3.634 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_45, ZX} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{45, ZX} I_Y} = 6.077 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_67.5, ZX} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{67.5, ZX} I_Y} = 7.613 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_90, ZX} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{90, ZX} I_Y} = 4.554 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_22.5, ZX45} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{22.5, ZX45} I_Y} = 8.950 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_45, ZX45} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{45, ZX45} I_Y} = 6.551 \times 10^{-2}
$$

$$
\delta_{ab\text{, Torsion}_67.5, ZX45} = \frac{2(1 + \nu) \pi e (L_{bc})^2 (L_{ab})}{E_{67.5, ZX45} I_Y} = 6.551 \times 10^{-2}
$$
Total Deflection of Segment ab (Bending + Torsion)

\[ \delta_{ab\_Total\_0\_XY} = \delta_{ab\_Bending\_0\_XY} + \delta_{ab\_Torsion\_0\_XY} = 1.665 \times 10^{-2} \]

\[ \delta_{ab\_Total\_22.5\_XY} = \delta_{ab\_Bending\_22.5\_XY} + \delta_{ab\_Torsion\_22.5\_XY} = 1.907 \times 10^{-2} \]

\[ \delta_{ab\_Total\_45\_XY} = \delta_{ab\_Bending\_45\_XY} + \delta_{ab\_Torsion\_45\_XY} = 2.079 \times 10^{-2} \]

\[ \delta_{ab\_Total\_67.5\_XY} = \delta_{ab\_Bending\_67.5\_XY} + \delta_{ab\_Torsion\_67.5\_XY} = 1.844 \times 10^{-2} \]

\[ \delta_{ab\_Total\_90\_XY} = \delta_{ab\_Bending\_90\_XY} + \delta_{ab\_Torsion\_90\_XY} = 1.617 \times 10^{-2} \]

\[ \delta_{ab\_Total\_22.5\_ZX} = \delta_{ab\_Bending\_22.5\_ZX} + \delta_{ab\_Torsion\_22.5\_ZX} = 2.161 \times 10^{-2} \]

\[ \delta_{ab\_Total\_45\_ZX} = \delta_{ab\_Bending\_45\_ZX} + \delta_{ab\_Torsion\_45\_ZX} = 3.816 \times 10^{-2} \]

\[ \delta_{ab\_Total\_67.5\_ZX} = \delta_{ab\_Bending\_67.5\_ZX} + \delta_{ab\_Torsion\_67.5\_ZX} = 6.382 \times 10^{-2} \]

\[ \delta_{ab\_Total\_90\_ZX} = \delta_{ab\_Bending\_90\_ZX} + \delta_{ab\_Torsion\_90\_ZX} = 7.996 \times 10^{-2} \]

\[ \delta_{ab\_Total\_22.5\_ZX45} = \delta_{ab\_Bending\_22.5\_ZX45} + \delta_{ab\_Torsion\_22.5\_ZX45} = 4.783 \times 10^{-2} \]

\[ \delta_{ab\_Total\_45\_ZX45} = \delta_{ab\_Bending\_45\_ZX45} + \delta_{ab\_Torsion\_45\_ZX45} = 9.399 \times 10^{-2} \]

\[ \delta_{ab\_Total\_67.5\_ZX45} = \delta_{ab\_Bending\_67.5\_ZX45} + \delta_{ab\_Torsion\_67.5\_ZX45} = 6.881 \times 10^{-2} \]
Deflection of Segment bc (Bending + Torsion)

\[ \delta_{bc, \text{Bending}, 0, XY} = \frac{1}{E_{0, XY}} \int_{0}^{bc} P_{xy} \, dy = 6.371 \times 10^{-3} \]

\[ \delta_{bc, \text{Bending}, 22.5, XY} = \frac{1}{E_{22.5, XY}} \int_{0}^{bc} P_{xy} \, dy = 7.530 \times 10^{-3} \]

\[ \delta_{bc, \text{Bending}, 45, XY} = \frac{1}{E_{45, XY}} \int_{0}^{bc} P_{xy} \, dy = 7.956 \times 10^{-3} \]

\[ \delta_{bc, \text{Bending}, 67.5, XY} = \frac{1}{E_{67.5, XY}} \int_{0}^{bc} P_{xy} \, dy = 7.056 \times 10^{-3} \]

\[ \delta_{bc, \text{Bending}, 90, XY} = \frac{1}{E_{90, XY}} \int_{0}^{bc} P_{xy} \, dy = 6.188 \times 10^{-3} \]

\[ \delta_{bc, \text{Bending}, 22.5, ZY} = \frac{1}{E_{22.5, ZY}} \int_{0}^{bc} P_{xy} \, dy = 8.269 \times 10^{-3} \]
Deflection of Segment bc (Bending + Torsion) [Continued]

\[
\begin{align*}
\delta_{bc, \text{Bending}_{45, ZX}} &= \frac{\int_{0}^{\pi} P_{0} y^2 \, dy}{E_{45, ZX} I_{45}} = 1.461 \times 10^{-2} \\
\delta_{bc, \text{Bending}_{67.5, ZX}} &= \frac{\int_{0}^{\pi} P_{0} y^2 \, dy}{E_{67.5, ZX} I_{45}} = 2.443 \times 10^{-2} \\
\delta_{bc, \text{Bending}_{90, ZX}} &= \frac{\int_{0}^{\pi} P_{0} y^2 \, dy}{E_{90, ZX} I_{45}} = 3.090 \times 10^{-2} \\
\delta_{bc, \text{Bending}_{22.5, ZX45}} &= \frac{\int_{0}^{\pi} P_{0} y^2 \, dy}{E_{22.5, ZX45} I_{45}} = 1.831 \times 10^{-2} \\
\delta_{bc, \text{Bending}_{45, ZX45}} &= \frac{\int_{0}^{\pi} P_{0} y^2 \, dy}{E_{45, ZX45} I_{45}} = 3.597 \times 10^{-2} \\
\delta_{bc, \text{Bending}_{67.5, ZX45}} &= \frac{\int_{0}^{\pi} P_{0} y^2 \, dy}{E_{67.5, ZX45} I_{45}} = 2.633 \times 10^{-2}
\end{align*}
\]
Deflection of Segment bc (Bending + Torsion) [Continued]

\[ \text{def}_{\text{bc, X}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{XY} J_{0}} = 5.844 \times 10^{-3} \]

\[ \text{def}_{\text{bc, 22.5, X}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{22.5, XY} J_{0}} = 6.906 \times 10^{-3} \]

\[ \text{def}_{\text{bc, 45, X}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{45, XY} J_{0}} = 7.297 \times 10^{-3} \]

\[ \text{def}_{\text{bc, 67.5, X}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{67.5, XY} J_{0}} = 6.472 \times 10^{-3} \]

\[ \text{def}_{\text{bc, 90, X}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{90, XY} J_{0}} = 5.675 \times 10^{-3} \]

\[ \text{def}_{\text{bc, 22.5, Z}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{22.5, ZY} J_{0}} = 7.585 \times 10^{-3} \]

\[ \text{def}_{\text{bc, 45, Z}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{45, ZY} J_{0}} = 1.340 \times 10^{-2} \]

\[ \text{def}_{\text{bc, 67.5, Z}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{67.5, ZY} J_{0}} = 2.240 \times 10^{-2} \]

\[ \text{def}_{\text{bc, 90, Z}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{90, ZY} J_{0}} = 2.807 \times 10^{-2} \]

\[ \text{def}_{\text{bc, 22.5, ZX45}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{22.5, ZX45} J_{0}} = 1.679 \times 10^{-2} \]

\[ \text{def}_{\text{bc, 45, ZX45}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{45, ZX45} J_{0}} = 3.300 \times 10^{-2} \]

\[ \text{def}_{\text{bc, 67.5, ZX45}} = \frac{2(1 + \nu) P_e l_{cd}^2 (l_{bc})}{E_{67.5, ZX45} J_{0}} = 2.415 \times 10^{-2} \]
Total Deflection of Segment bc (Bending + Torsion)

\[ \delta_{bc\ Total\ 0\ XY} = \delta_{bc\ Bending\ 0\ XY} + \delta_{bc\ Torsion\ 0\ XY} = 1.221 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 22.5\ XY} = \delta_{bc\ Bending\ 22.5\ XY} + \delta_{bc\ Torsion\ 22.5\ XY} = 1.444 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 45\ XY} = \delta_{bc\ Bending\ 45\ XY} + \delta_{bc\ Torsion\ 45\ XY} = 1.525 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 67.5\ XY} = \delta_{bc\ Bending\ 67.5\ XY} + \delta_{bc\ Torsion\ 67.5\ XY} = 1.353 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 90\ XY} = \delta_{bc\ Bending\ 90\ XY} + \delta_{bc\ Torsion\ 90\ XY} = 1.186 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 22.5\ ZX} = \delta_{bc\ Bending\ 22.5\ ZX} + \delta_{bc\ Torsion\ 22.5\ ZX} = 1.585 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 45\ ZX} = \delta_{bc\ Bending\ 45\ ZX} + \delta_{bc\ Torsion\ 45\ ZX} = 2.800 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 67.5\ ZX} = \delta_{bc\ Bending\ 67.5\ ZX} + \delta_{bc\ Torsion\ 67.5\ ZX} = 4.683 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 90\ ZX} = \delta_{bc\ Bending\ 90\ ZX} + \delta_{bc\ Torsion\ 90\ ZX} = 5.867 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 22.5\ ZX15} = \delta_{bc\ Bending\ 22.5\ ZX15} + \delta_{bc\ Torsion\ 22.5\ ZX15} = 3.510 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 45\ ZX15} = \delta_{bc\ Bending\ 45\ ZX15} + \delta_{bc\ Torsion\ 45\ ZX15} = 6.897 \times 10^{-2} \]

\[ \delta_{bc\ Total\ 67.5\ ZX15} = \delta_{bc\ Bending\ 67.5\ ZX15} + \delta_{bc\ Torsion\ 67.5\ ZX15} = 5.049 \times 10^{-2} \]
Deflection of Segment cd (Bending + Torsion)

\[ \delta_{cd, \text{Bending}_0\ XY} = \int_0^{l_{cd}} \frac{P_e x^2}{E_0 \ XY^3} \, dx = 5.042 \times 10^{-4} \]

\[ \delta_{cd, \text{Bending}_{22.5\ XY}} = \int_0^{l_{cd}} \frac{P_e x^2}{E_{22.5\ XY^3}} \, dx = 5.959 \times 10^{-4} \]

\[ \delta_{cd, \text{Bending}_{45\ XY}} = \int_0^{l_{cd}} \frac{P_e x^2}{E_{45\ XY^3}} \, dx = 6.297 \times 10^{-4} \]

\[ \delta_{cd, \text{Bending}_{67.5\ XY}} = \int_0^{l_{cd}} \frac{P_e x^2}{E_{67.5\ XY^3}} \, dx = 5.585 \times 10^{-4} \]

\[ \delta_{cd, \text{Bending}_{90\ XY}} = \int_0^{l_{cd}} \frac{P_e x^2}{E_{90\ XY^3}} \, dx = 4.897 \times 10^{-4} \]

\[ \delta_{cd, \text{Bending}_{22.5\ ZY}} = \int_0^{l_{cd}} \frac{P_e x^2}{E_{22.5\ ZY^3}} \, dx = 6.545 \times 10^{-4} \]
Deflection of Segment cd (Bending + Torsion) [Continued]

\[ \delta_{cd,Bending, 45\text{-}ZX} = \int_{0}^{\delta_{cd}} \frac{p_e^2 \, dx}{E_{45\text{-}ZX}I_{45\text{-}ZX}} = 1.156 \times 10^{-3} \]

\[ \delta_{cd,Bending, 67.5\text{-}ZX} = \int_{0}^{\delta_{cd}} \frac{p_e^2 \, dx}{E_{67.5\text{-}ZX}I_{67.5\text{-}ZX}} = 1.933 \times 10^{-3} \]

\[ \delta_{cd,Bending, 90\text{-}ZX} = \int_{0}^{\delta_{cd}} \frac{p_e^2 \, dx}{E_{90\text{-}ZX}I_{90\text{-}ZX}} = 2.422 \times 10^{-3} \]

\[ \delta_{cd,Bending, 22.5\text{-}ZX/45} = \int_{0}^{\delta_{cd}} \frac{p_e^2 \, dx}{E_{22.5\text{-}ZX/45}I_{22.5\text{-}ZX/45}} = 1.449 \times 10^{-3} \]

\[ \delta_{cd,Bending, 45\text{-}ZX/45} = \int_{0}^{\delta_{cd}} \frac{p_e^2 \, dx}{E_{45\text{-}ZX/45}I_{45\text{-}ZX/45}} = 2.847 \times 10^{-3} \]

\[ \delta_{cd,Bending, 67.5\text{-}ZX/45} = \int_{0}^{\delta_{cd}} \frac{p_e^2 \, dx}{E_{67.5\text{-}ZX/45}I_{67.5\text{-}ZX/45}} = 2.064 \times 10^{-3} \]
Deflection of Segment cd (Bending + Torsion) [Continued]

\[ k_{cd_{,\text{Torsion},0,\text{XY}}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{0_{,XY}}} = 3.403 \times 10^{-3} \]

\[ k_{cd_{,\text{Torsion},22.5,\text{XY}}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{22.5_{,XY}}} = 4.021 \times 10^{-3} \]

\[ k_{cd_{,\text{Torsion},45,\text{XY}}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{45_{,XY}}} = 4.249 \times 10^{-3} \]

\[ k_{cd_{,\text{Torsion},67.5,\text{XY}}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{67.5_{,XY}}} = 3.769 \times 10^{-3} \]

\[ k_{cd_{,\text{Torsion},90,\text{XY}}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{90_{,XY}}} = 3.305 \times 10^{-3} \]

\[ k_{cd_{,\text{Torsion},22.5,\text{ZX}}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{22.5_{,ZX}}} = 4.416 \times 10^{-3} \]

\[ k_{cd_{,\text{Torsion},45,\text{ZX}}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{45_{,ZX}}} = 7.800 \times 10^{-3} \]

\[ k_{cd_{,\text{Torsion},67.5,\text{ZX}}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{67.5_{,ZX}}} = 1.305 \times 10^{-2} \]

\[ k_{cd_{,\text{Torsion},90,\text{ZX}}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{90_{,ZX}}} = 1.634 \times 10^{-2} \]

\[ k_{cd_{,\text{Torsion},22.5,\text{ZX}45}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{22.5_{,ZX}45}} = 9.776 \times 10^{-3} \]

\[ k_{cd_{,\text{Torsion},45,\text{ZX}45}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{45_{,ZX}45}} = 1.921 \times 10^{-2} \]

\[ k_{cd_{,\text{Torsion},67.5,\text{ZX}45}} = \frac{2(1 + v)P_{cd}(h_{cd})^2}{E_{67.5_{,ZX}45}} = 1.406 \times 10^{-2} \]
Total Deflection of Segment cd (Bending + Torsion)

\[ \delta_{cd_{\text{Total}}_{0,XY}} = \delta_{cd_{\text{Bending}}_{0,XY}} + \delta_{cd_{\text{Torsion}}_{0,XY}} = 3.907 \times 10^{-3} \]

\[ \delta_{cd_{\text{Total}}_{22.5,XY}} = \delta_{cd_{\text{Bending}}_{22.5,XY}} + \delta_{cd_{\text{Torsion}}_{22.5,XY}} = 4.617 \times 10^{-3} \]

\[ \delta_{cd_{\text{Total}}_{45,XY}} = \delta_{cd_{\text{Bending}}_{45,XY}} + \delta_{cd_{\text{Torsion}}_{45,XY}} = 4.879 \times 10^{-3} \]

\[ \delta_{cd_{\text{Total}}_{67.5,XY}} = \delta_{cd_{\text{Bending}}_{67.5,XY}} + \delta_{cd_{\text{Torsion}}_{67.5,XY}} = 4.327 \times 10^{-3} \]

\[ \delta_{cd_{\text{Total}}_{90,XY}} = \delta_{cd_{\text{Bending}}_{90,XY}} + \delta_{cd_{\text{Torsion}}_{90,XY}} = 3.794 \times 10^{-3} \]

\[ \delta_{cd_{\text{Total}}_{22.5,ZX}} = \delta_{cd_{\text{Bending}}_{22.5,ZX}} + \delta_{cd_{\text{Torsion}}_{22.5,ZX}} = 5.071 \times 10^{-3} \]

\[ \delta_{cd_{\text{Total}}_{45,ZX}} = \delta_{cd_{\text{Bending}}_{45,ZX}} + \delta_{cd_{\text{Torsion}}_{45,ZX}} = 8.956 \times 10^{-3} \]

\[ \delta_{cd_{\text{Total}}_{67.5,ZX}} = \delta_{cd_{\text{Bending}}_{67.5,ZX}} + \delta_{cd_{\text{Torsion}}_{67.5,ZX}} = 1.498 \times 10^{-2} \]

\[ \delta_{cd_{\text{Total}}_{90,ZX}} = \delta_{cd_{\text{Bending}}_{90,ZX}} + \delta_{cd_{\text{Torsion}}_{90,ZX}} = 1.877 \times 10^{-2} \]

\[ \delta_{cd_{\text{Total}}_{22.5,ZX45}} = \delta_{cd_{\text{Bending}}_{22.5,ZX45}} + \delta_{cd_{\text{Torsion}}_{22.5,ZX45}} = 1.123 \times 10^{-2} \]

\[ \delta_{cd_{\text{Total}}_{45,ZX45}} = \delta_{cd_{\text{Bending}}_{45,ZX45}} + \delta_{cd_{\text{Torsion}}_{45,ZX45}} = 2.205 \times 10^{-2} \]

\[ \delta_{cd_{\text{Total}}_{67.5,ZX45}} = \delta_{cd_{\text{Bending}}_{67.5,ZX45}} + \delta_{cd_{\text{Torsion}}_{67.5,ZX45}} = 1.615 \times 10^{-2} \]
Deflection of Segment de (Pure Bending)

\[ \delta_{\text{de, Bending, 0, XY}} = \frac{\int_0^{de} Py^2 dy}{E_{0, XY} I} = 7.964 \times 10^{-4} \]

\[ \delta_{\text{de, Bending, 22.5, XY}} = \frac{\int_0^{de} Py^2 dy}{E_{22.5, XY} I} = 9.412 \times 10^{-4} \]

\[ \delta_{\text{de, Bending, 45, XY}} = \frac{\int_0^{de} Py^2 dy}{E_{45, XY} I} = 9.945 \times 10^{-4} \]

\[ \delta_{\text{de, Bending, 67.5, XY}} = \frac{\int_0^{de} Py^2 dy}{E_{67.5, XY} I} = 8.820 \times 10^{-4} \]

\[ \delta_{\text{de, Bending, 90, XY}} = \frac{\int_0^{de} Py^2 dy}{E_{90, XY} I} = 7.735 \times 10^{-4} \]

\[ \delta_{\text{de, Bending, 22.5, ZX}} = \frac{\int_0^{de} Py^2 dy}{E_{22.5, ZX} I} = -1.034 \times 10^{-3} \]
Deflection of Segment de (Pure Bending) [Continued]

\[ \delta_{de \text{ Bending } 45^\circ ZX} = \frac{L \int_0^L p e^2 dy}{I_{45^\circ ZX}} = 1.826 \times 10^{-3} \]

\[ \delta_{de \text{ Bending } 67.5^\circ ZX} = \frac{L \int_0^L p e^2 dy}{I_{67.5^\circ ZX}} = 3.053 \times 10^{-3} \]

\[ \delta_{de \text{ Bending } 90^\circ ZX} = \frac{L \int_0^L p e^2 dy}{I_{90^\circ ZX}} = 3.825 \times 10^{-3} \]

\[ \delta_{de \text{ Bending } 22.5^\circ ZX 45^\circ} = \frac{L \int_0^L p e^2 dy}{I_{22.5^\circ ZX \times 45^\circ}} = 2.288 \times 10^{-3} \]

\[ \delta_{de \text{ Bending } 45^\circ ZX 45^\circ} = \frac{L \int_0^L p e^2 dy}{I_{45^\circ ZX \times 45^\circ}} = 4.497 \times 10^{-3} \]

\[ \delta_{de \text{ Bending } 67.5^\circ ZX 45^\circ} = \frac{L \int_0^L p e^2 dy}{I_{67.5^\circ ZX \times 45^\circ}} = 3.292 \times 10^{-3} \]
Total Deflection of Segment de (Pure Bending)

\[ \delta_{\text{de Total,0,XY}} = \delta_{\text{de Bending,0,XY}} = 7.964 \times 10^{-4} \]
\[ \delta_{\text{de Total,22.5,XY}} = \delta_{\text{de Bending,22.5,XY}} = 9.412 \times 10^{-4} \]
\[ \delta_{\text{de Total,45,XY}} = \delta_{\text{de Bending,45,XY}} = 9.945 \times 10^{-4} \]
\[ \delta_{\text{de Total,67.5,XY}} = \delta_{\text{de Bending,67.5,XY}} = 8.820 \times 10^{-4} \]
\[ \delta_{\text{de Total,90,XY}} = \delta_{\text{de Bending,90,XY}} = 7.735 \times 10^{-4} \]
\[ \delta_{\text{de Total,22.5,ZX}} = \delta_{\text{de Bending,22.5,ZX}} = 1.034 \times 10^{-3} \]
\[ \delta_{\text{de Total,45,ZX}} = \delta_{\text{de Bending,45,ZX}} = 1.826 \times 10^{-3} \]
\[ \delta_{\text{de Total,67.5,ZX}} = \delta_{\text{de Bending,67.5,ZX}} = 3.053 \times 10^{-3} \]
\[ \delta_{\text{de Total,90,ZX}} = \delta_{\text{de Bending,90,ZX}} = 3.825 \times 10^{-3} \]
\[ \delta_{\text{de Total,22.5,ZX45}} = \delta_{\text{de Bending,22.5,ZX45}} = 2.288 \times 10^{-3} \]
\[ \delta_{\text{de Total,45,ZX45}} = \delta_{\text{de Bending,45,ZX45}} = 4.407 \times 10^{-3} \]
\[ \delta_{\text{de Total,67.5,ZX45}} = \delta_{\text{de Bending,67.5,ZX45}} = 3.292 \times 10^{-3} \]
Shear Deflection:

\[ \gamma_{0,XY} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{0,XY} b) b} = 1.746 \times 10^{-4} \]

\[ \gamma_{22.5,XY} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{22.5,XY} b) b} = 1.584 \times 10^{-4} \]

\[ \gamma_{45,XY} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{45,XY} b) b} = 1.225 \times 10^{-4} \]

\[ \gamma_{67.5,XY} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{67.5,XY} b) b} = 1.486 \times 10^{-4} \]

\[ \gamma_{90,XY} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{90,XY} b) b} = 1.746 \times 10^{-4} \]

\[ \gamma_{22.5,ZX} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{22.5,ZX} b) b} = 3.080 \times 10^{-4} \]

\[ \gamma_{45,ZX} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{45,ZX} b) b} = 4.234 \times 10^{-4} \]

\[ \gamma_{67.5,ZX} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{67.5,ZX} b) b} = 3.080 \times 10^{-4} \]

\[ \gamma_{90,ZX} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{90,ZX} b) b} = 1.927 \times 10^{-4} \]

\[ \gamma_{22.5,ZX45} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{22.5,ZX45} b) b} = 4.860 \times 10^{-4} \]

\[ \gamma_{45,ZX45} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{45,ZX45} b) b} = 4.860 \times 10^{-4} \]

\[ \gamma_{67.5,ZX45} = \frac{P_e (I_{ab} + I_{bc} + I_{cd} + I_{db})}{(G_{67.5,ZX45} b) b} = 4.860 \times 10^{-4} \]
Deflection at a From Neutral Position (m):

\[\delta_{e,0,XY} = \delta_{e, Total,0,XY} + \delta_{ed, Total,0,XY} + \delta_{be, Total,0,XY} + \delta_{ab, Total,0,XY} + \delta_{0,XY}\]

\[\delta_{e,0,XY} = 3.374 \times 10^{-2}\]

\[\delta_{e,22.5,XY} = \delta_{e, Total,22.5,XY} + \delta_{ed, Total,22.5,XY} + \delta_{be, Total,22.5,XY} + \delta_{ab, Total,22.5,XY} + \delta_{22.5,XY}\]

\[\delta_{e,22.5,XY} = 3.983 \times 10^{-2}\]

\[\delta_{e,45,XY} = \delta_{e, Total,45,XY} + \delta_{ed, Total,45,XY} + \delta_{be, Total,45,XY} + \delta_{ab, Total,45,XY} + \delta_{45,XY}\]

\[\delta_{e,45,XY} = 4.204 \times 10^{-2}\]

\[\delta_{e,67.5,XY} = \delta_{e, Total,67.5,XY} + \delta_{ed, Total,67.5,XY} + \delta_{be, Total,67.5,XY} + \delta_{ab, Total,67.5,XY} + \delta_{67.5,XY}\]

\[\delta_{e,67.5,XY} = 3.732 \times 10^{-2}\]

\[\delta_{e,90,XY} = \delta_{e, Total,90,XY} + \delta_{ed, Total,90,XY} + \delta_{be, Total,90,XY} + \delta_{ab, Total,90,XY} + \delta_{90,XY}\]

\[\delta_{e,90,XY} = 3.277 \times 10^{-2}\]

\[\delta_{e,22.5,ZX} = \delta_{e, Total,22.5,ZX} + \delta_{ed, Total,22.5,ZX} + \delta_{be, Total,22.5,ZX} + \delta_{ab, Total,22.5,ZX} + \delta_{22.5,ZX}\]

\[\delta_{e,22.5,ZX} = 4.387 \times 10^{-2}\]

\[\delta_{e,45,ZX} = \delta_{e, Total,45,ZX} + \delta_{ed, Total,45,ZX} + \delta_{be, Total,45,ZX} + \delta_{ab, Total,45,ZX} + \delta_{45,ZX}\]

\[\delta_{e,45,ZX} = 7.737 \times 10^{-2}\]

\[\delta_{e,67.5,ZX} = \delta_{e, Total,67.5,ZX} + \delta_{ed, Total,67.5,ZX} + \delta_{be, Total,67.5,ZX} + \delta_{ab, Total,67.5,ZX} + \delta_{67.5,ZX}\]

\[\delta_{e,67.5,ZX} = 1.290 \times 10^{-1}\]

\[\delta_{e,90,ZX} = \delta_{e, Total,90,ZX} + \delta_{ed, Total,90,ZX} + \delta_{be, Total,90,ZX} + \delta_{ab, Total,90,ZX} + \delta_{90,ZX}\]

\[\delta_{e,90,ZX} = 1.614 \times 10^{-1}\]

\[\delta_{e,22.5,ZX45} = \delta_{e, Total,22.5,ZX45} + \delta_{ed, Total,22.5,ZX45} + \delta_{be, Total,22.5,ZX45} + \delta_{ab, Total,22.5,ZX45} + \delta_{22.5,ZX45}\]

\[\delta_{e,22.5,ZX45} = 9.692 \times 10^{-2}\]

\[\delta_{e,45,ZX45} = \delta_{e, Total,45,ZX45} + \delta_{ed, Total,45,ZX45} + \delta_{be, Total,45,ZX45} + \delta_{ab, Total,45,ZX45} + \delta_{45,ZX45}\]

\[\delta_{e,45,ZX45} = 1.090 \times 10^{-1}\]

\[\delta_{e,67.5,ZX45} = \delta_{e, Total,67.5,ZX45} + \delta_{ed, Total,67.5,ZX45} + \delta_{be, Total,67.5,ZX45} + \delta_{ab, Total,67.5,ZX45} + \delta_{67.5,ZX45}\]

\[\delta_{e,67.5,ZX45} = 1.592 \times 10^{-1}\]
**Total Deflection at e (m):**

Here, we need to multiply the deflection at e by a factor of 2 in order to account for the symmetry.

\[
\begin{align*}
\delta_{e, \text{Final}, 0, XY} &= 2 \delta_{e, 0, XY} = 6.748 \times 10^{-2} \\
\delta_{e, \text{Final}, 22.5, XY} &= 2 \delta_{e, 22.5, XY} = 7.965 \times 10^{-2} \\
\delta_{e, \text{Final}, 45, XY} &= 2 \delta_{e, 45, XY} = 8.407 \times 10^{-2} \\
\delta_{e, \text{Final}, 67.5, XY} &= 2 \delta_{e, 67.5, XY} = 7.465 \times 10^{-2} \\
\delta_{e, \text{Final}, 90, XY} &= 2 \delta_{e, 90, XY} = 6.555 \times 10^{-2} \\
\delta_{e, \text{Final}, 22.5, ZX} &= 2 \delta_{e, 22.5, ZX} = 8.775 \times 10^{-2} \\
\delta_{e, \text{Final}, 45, ZX} &= 2 \delta_{e, 45, ZX} = 1.547 \times 10^{-1} \\
\delta_{e, \text{Final}, 67.5, ZX} &= 2 \delta_{e, 67.5, ZX} = 2.580 \times 10^{-1} \\
\delta_{e, \text{Final}, 90, ZX} &= 2 \delta_{e, 90, ZX} = 3.228 \times 10^{-1} \\
\delta_{e, \text{Final}, 22.5, ZX45} &= 2 \delta_{e, 22.5, ZX45} = 1.938 \times 10^{-1} \\
\delta_{e, \text{Final}, 45, ZX45} &= 2 \delta_{e, 45, ZX45} = 3.799 \times 10^{-1} \\
\delta_{e, \text{Final}, 67.5, ZX45} &= 2 \delta_{e, 67.5, ZX45} = 2.784 \times 10^{-1}
\end{align*}
\]
Stiffness (N/mm):

\[ k_{0\_XY} = \frac{\sigma_e}{\varepsilon_e\_Final\_0\_XY} \times 10^{-3} = 0.741 \]
\[ k_{22.5\_XY} = \frac{\sigma_e}{\varepsilon_e\_Final\_22.5\_XY} \times 10^{-3} = 0.628 \]
\[ k_{45\_XY} = \frac{\sigma_e}{\varepsilon_e\_Final\_45\_XY} \times 10^{-3} = 0.505 \]
\[ k_{67.5\_XY} = \frac{\sigma_e}{\varepsilon_e\_Final\_67.5\_XY} \times 10^{-3} = 0.670 \]
\[ k_{90\_XY} = \frac{\sigma_e}{\varepsilon_e\_Final\_90\_XY} \times 10^{-3} = 0.763 \]
\[ k_{22.5\_ZK} = \frac{\sigma_e}{\varepsilon_e\_Final\_22.5\_ZK} \times 10^{-3} = 0.570 \]
\[ k_{45\_ZK} = \frac{\sigma_e}{\varepsilon_e\_Final\_45\_ZK} \times 10^{-3} = 0.323 \]
\[ k_{67.5\_ZK} = \frac{\sigma_e}{\varepsilon_e\_Final\_67.5\_ZK} \times 10^{-3} = 0.194 \]
\[ k_{90\_ZK} = \frac{\sigma_e}{\varepsilon_e\_Final\_90\_ZK} \times 10^{-3} = 0.155 \]
\[ k_{22.5\_ZK45} = \frac{\sigma_e}{\varepsilon_e\_Final\_22.5\_ZK45} \times 10^{-3} = 0.258 \]
\[ k_{45\_ZK45} = \frac{\sigma_e}{\varepsilon_e\_Final\_45\_ZK45} \times 10^{-3} = 0.132 \]
\[ k_{67.5\_ZK45} = \frac{\sigma_e}{\varepsilon_e\_Final\_67.5\_ZK45} \times 10^{-3} = 0.180 \]
Appendix E: Parts
Figure E-1. Component test bracket. C-channel was generated for the purpose of wrapping around edges of the specimen to grip and pull.
References


