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Adjustable Low Frequency Servo Compensation Using Operational Amplifiers

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ADJUSTABLE LOW FREQUENCY SERVO COMPENSATION USING OPERATIONAL AMPLIFIERS

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ABSTRACT

This paper defines a variable transfer function that is used to compensate for low frequency structural reasonances in a turret so that stabilization of a closed loop servo system can be achieved. Three circuits for implementing this compensation are presented. They are: the feedforward three amplifier biquad, the summing four amplifier biquad, and the single amplifier biquad with pole-zero cancellation. Design equations allowing the engineer to go directly from the given transfer function to the actual component values are developed for each circuit. A comparison of the final circuit designs is also presented.

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To my wife, whose encouragement, patience, and sacrifices made this possible and to Dr. Robert Walker for his expert advice.

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INTRODUCTION

The purpose of this paper is to examine circuits capable of realizing a specific servo compensation transfer function and to provide a bridge between that transfer function and the selection of component values for a circuit. This requires the development of equations that solve for the circuit component values in terms of the given servo parameters. Thus, the many and tedious steps to go from the transfer function in servo form to the final circuit are eliminated. By doing this, a test tool can be built which allows quick and easy compensation within the design range required.

The design requirements are for a transfer function:

$$T(S) = \left[\frac{\frac{S^2}{\omega_z^2} + \frac{2\zeta_z}{\omega_z} S + 1}{\frac{S^2}{\omega_p^2} + \frac{2\zeta_p}{\omega_p} S + 1}\right] K$$

where the variables take on the ranges

$0.1 \geq K \leq 10$

$188 \text{ r/s} \leq \omega_z \leq 1257 \text{ r/s}$

$160 \text{ r/s} \leq \omega_p \leq 1257 \text{ r/s}$

$0.045 \leq \zeta_z \leq 0.060$

$0.20 \leq \zeta_p \leq 0.35$

ζ_z (nominal) = 0.050

ζ_p (nominal) = 0.30

Further, either $\omega_p = \omega_z$

or $\omega_p = 0.85\ \omega_z$

Additional requirements are independent dc gain adjustment and a minimum number of adjustable components.

This transfer function implements a notch filter but not in the traditional band reject sense. This is because of the non-zero S^1 term in the numerator of the transfer function. Therefore, many of the widely used band reject circuits were not acceptable for this design. In addition, when $\omega_p = 0.85\ \omega_z$ the high frequency gain is three dB less than the low frequency gain.

Figures 1 and 2 show normalized magnitude and phase plots of T(S) with norminal values for ζ_p and ζ_z when $\omega_p = \omega_z$ and $\omega_p = 0.85\ \omega_z$ respectively.

The given transfer function is used to provide compensation for a closed loop servo system which controls the motion of a stabilized turret. In particular, this transfer function is used to cancel unwanted structural resonances caused by friction, wire torques, and inertial differences between systems. These variations are in part due to manufacturing tolerances on castings and machinings, different wire routing, and wire lacing. It was determined empirically that the required compensation would be of

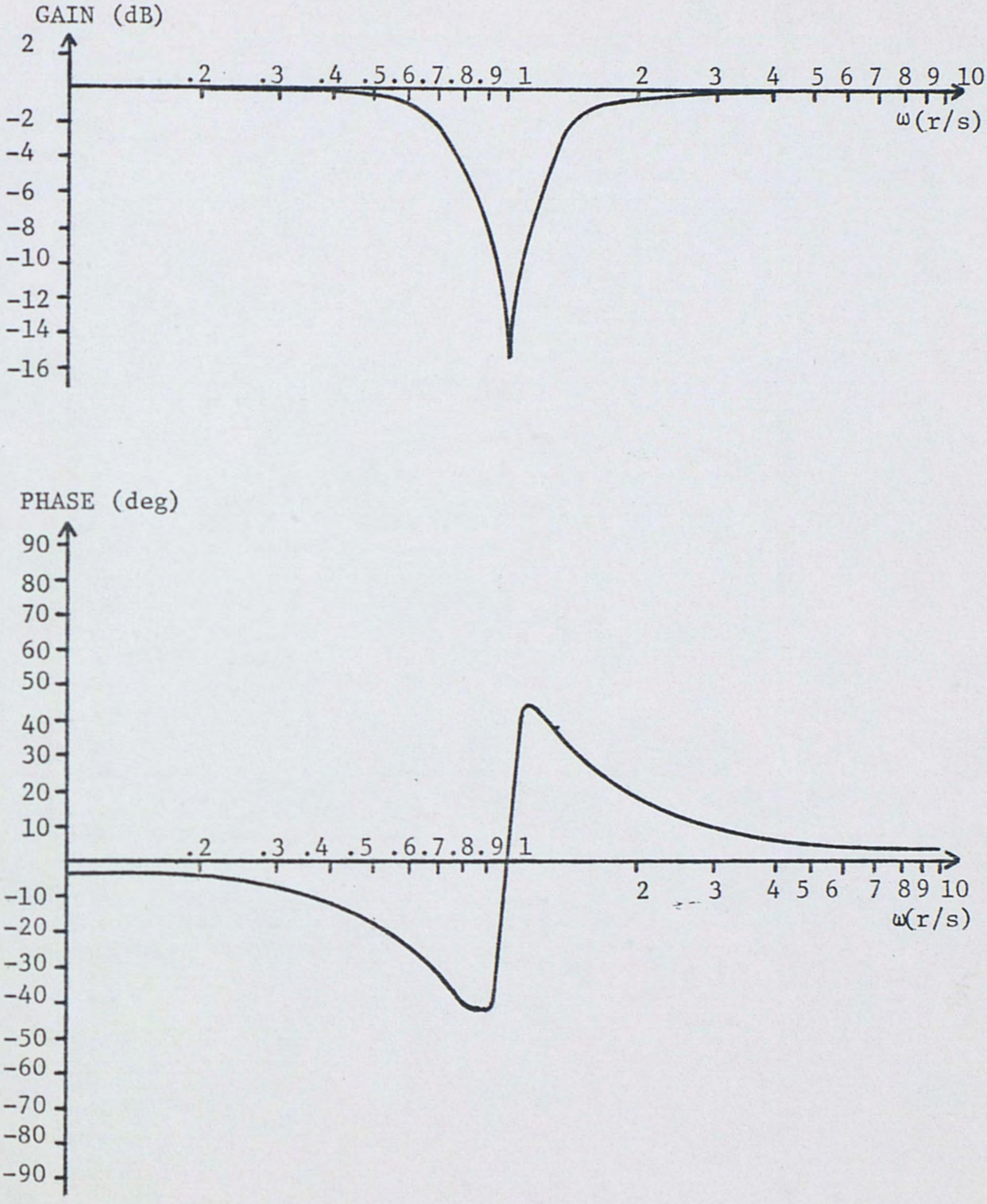

Figure 1. GAIN AND PHASE PLOTS FOR T(S); $\omega_z = \omega_p$

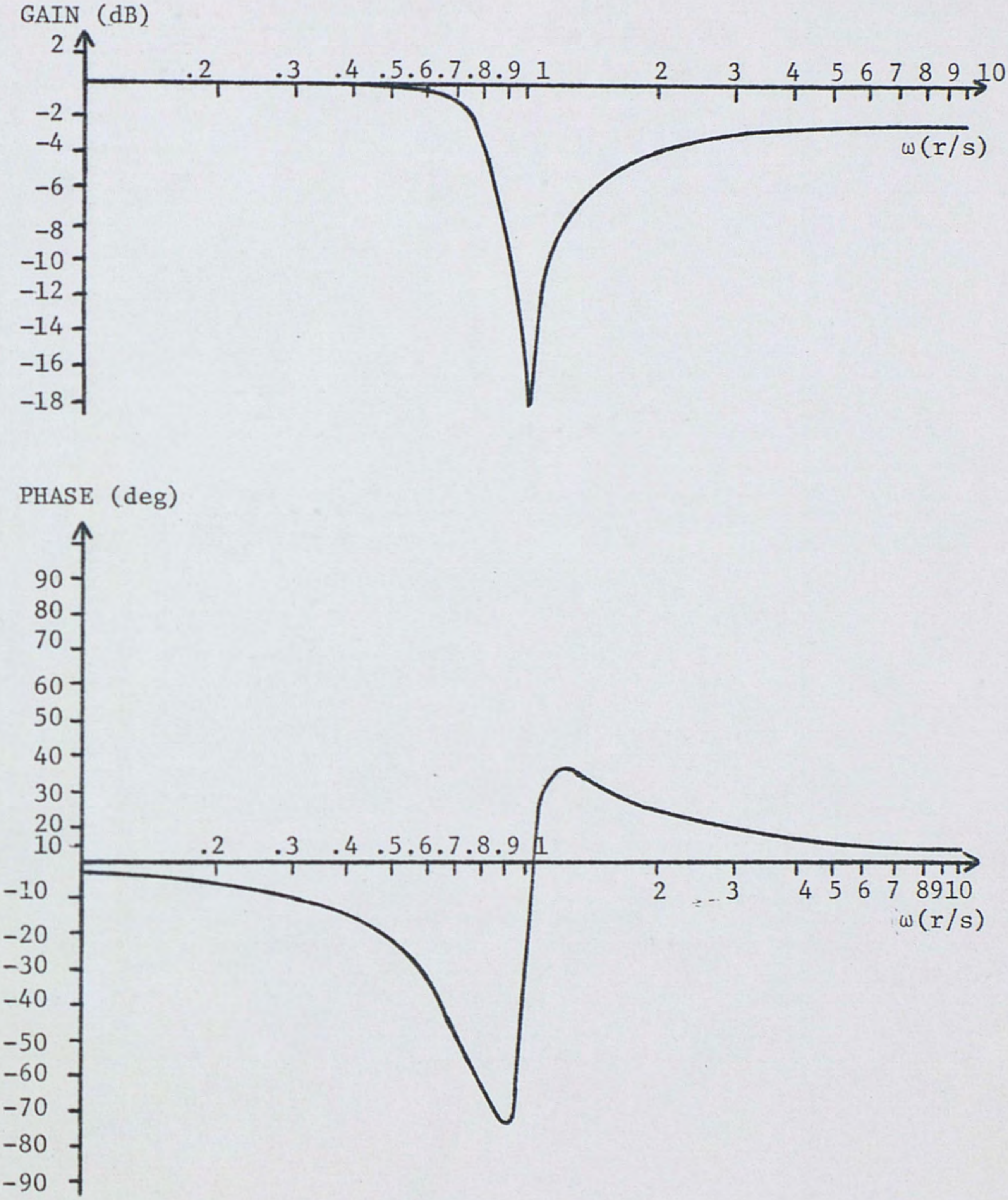

Figure 2. GAIN AND PHASE PLOTS FOR T(S); 0.85 $\omega_z = \omega_p$

the form given for T(S) and would be located between 188 r/s and 1257 r/s.

As mentioned previously, the resonance to be eliminated varies from unit to unit. For this reason an adjustable notch filter that is capable of operating over the entire range is desirable. This is especially true during the initial debug phases of system integration because it allows the engineer to set a compensation transfer function quickly , run a frequency response on the turret, and recompute a new compensation transfer function when required. This significantly reduces the testing time, while allowing the designer to fine-tune the compensation for each stabilized platform. After the required compensation is found, it is implemented into the servo compensation circuitry.

One of the major design problems is to find a circuit that uses a minimum number of adjustable components (one for each parameter) while still keeping the range of adjustment from being too large or obtaining negative component values. These problems can be reduced by not fixing any circuit components, but the adjustment time for each transfer function goes up as does the time required for computation of the new values. However, the equations for the circuits discussed herein have been solved such that no negative component value results over the design range and a minimum of adjustable components are required. The equations are simple to solve and are given in terms of the servo parameters in the transfer function to be implemented. The solution was

obtained in this manner so that the design engineer could go directly from the information given him by the system engineer to the final circuit values.

It should be noted that this paper considers only single component realizations over the required range of resistance adjustment. Obviously this minimizes the overall parts count, but in some cases the resistor values required for implementation are too large. This problem can be overcome, at the expense of increased component count, by utilizing the resistive tee (Moschytz 1975). The substitution of this network for a single resistor increases the effective resistance in the circuit, and therefore a large resistance can be simulated by three smaller ones.

It is obvious that a circuit with these qualities is a valuable test tool for finding and confirming the exact compensation required for turret stabilization. In addition, a circuit implementing these characteristics must be permanently installed in the turret compensation electronics. It is therefore desirable that the same circuit be utilized for that purpose. This means that the circuit must be capable of operation in the system environment, which includes humidity, altitude, and temperature extremes.

The three circuits examined in this paper are the feedforward three amplifier biquad (FTAB) design, the summing four amplifier biquad (SFAB) design, and the single amplifier biquad with pole-zero

cancellation (SABPZC). Each circuit is developed in a similar manner. The result in each case is a circuit with specified fixed components and a minimum number of adjustable components; equations are derived to solve for their values. The equations are such that they yield practical component values for realization of the specified transfer function anywhere within the design range. Each circuit is examined separately in the order listed above, and the results are compared in the summary that follows.

FEEDFORWARD THREE AMPLIFIER BIQUAD DESIGN

A general biquadratic function can be implemented using three operational amplifiers connected as shown in Figure 3. This approach, which utilizes a feedforward scheme to form the zeros of the transfer function, is described by Fleischer and Tow (1973). In order to form the zeros, the input signal is applied to the ground potential nodes of the circuit i.e., the negative inputs of the operational amplifiers.

Daryanani (1976) states that the transfer function of the feedforward three amplifier biquad is

$$T(S) = \frac{Vo(S)}{V_I(S)} = -\frac{R_8}{R_6}\left[\frac{S^2+S[\frac{1}{R_1C_1}-\frac{R_6}{R_4C_1R_7}]+\frac{R_6}{R_7R_3R_5C_1C_2}}{S^2+S[\frac{1}{R_1C_1}]+\frac{R_8}{R_7R_2R_3C_1C_2}}\right] \qquad (1)$$

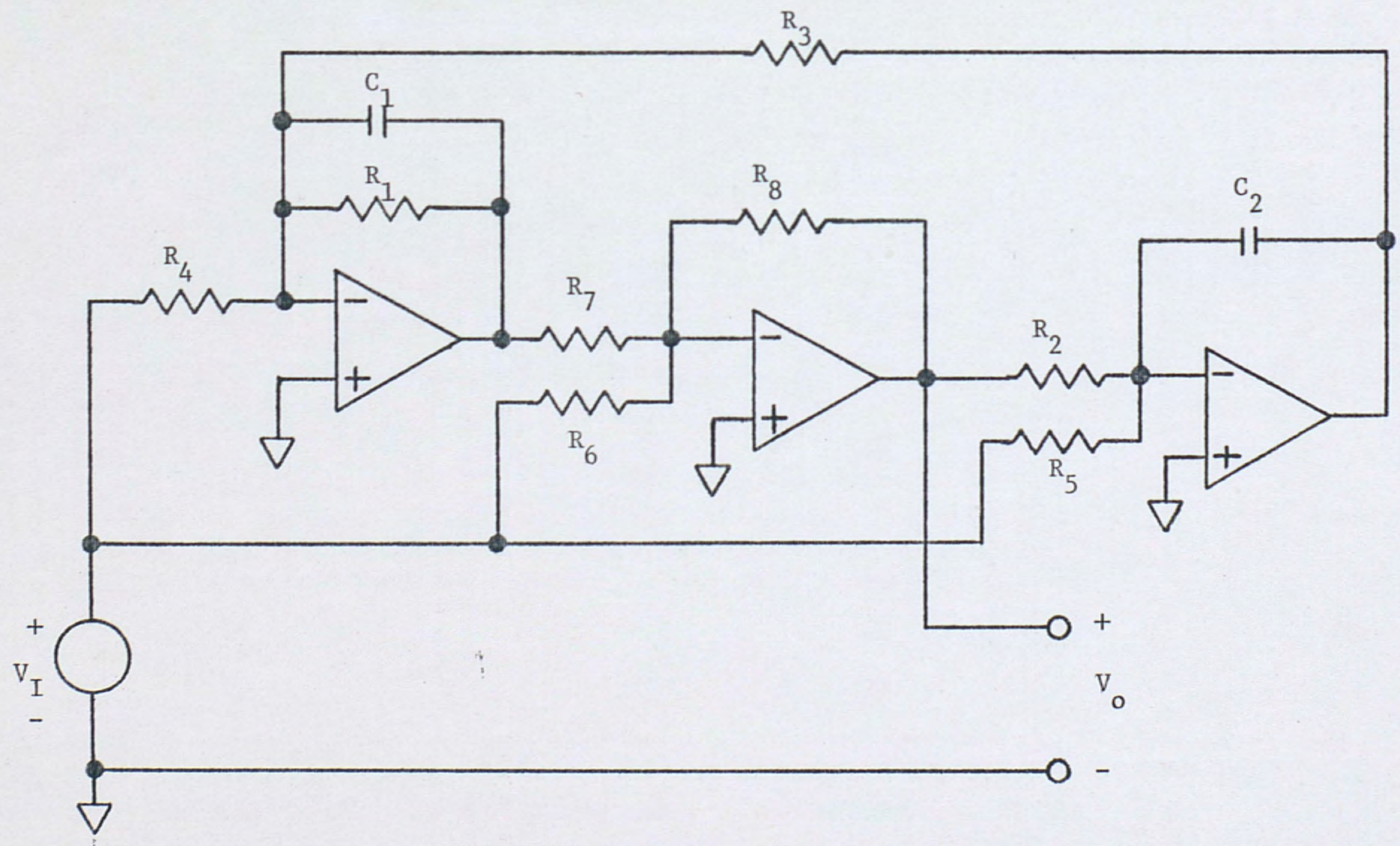

Figure 3. FEEDFORWARD THREE AMPLIFIER BIQUAD

After rearranging into the servo form, we get

$$T(S) = -\frac{R_2}{R_5}\left[\frac{S^2\left[\dfrac{1}{\dfrac{R_6}{R_3R_5R_7C_1C_2}}\right] + S\left[\dfrac{(R_4R_7-R_1R_6)R_3R_5C_2}{R_1R_4R_6}\right] + 1}{S^2\left[\dfrac{1}{\dfrac{R_8}{R_2R_3R_7C_1C_2}}\right] + S\left[\dfrac{R_2R_3R_7C_2}{R_1R_8}\right] + 1}\right] \quad (2)$$

From equation (2) the servo parameters K, ω_z, ζ_z , ζ_p, and ω_p can be expressed in terms of the circuit components by using the coefficient matching technique. Solving for K gives

$$K = -\frac{R_2}{R_5} \quad (3)$$

which is the dc gain of the circuit. The zero frequency, ω_z, is

$$\omega_z = \left[\frac{R_6}{R_3R_5R_7C_1C_2}\right]^{1/2} \quad (4)$$

and the pole frequency, ω_p, is

$$\omega_p = \left[\frac{R_8}{R_2R_3R_7C_1C_2}\right]^{1/2} \quad (5)$$

Matching coefficients of the S term in the numerator yields

$$\frac{2\zeta_z}{\omega_z} = \frac{(R_4R_7 - R_1R_6)\ R_3R_5C_2}{R_1R_4R_6} \quad (6)$$

from which the numerator damping ratio, ζ_z, is found to be

$$\zeta_z = \frac{R_4R_7 - R_1R_6}{2\ R_1R_4}\left[\frac{R_3R_5C_2}{R_6R_7C_1}\right]^{1/2} \quad (7)$$

similarly,

$$\frac{2\zeta_p}{\omega_p} = \frac{R_2R_3R_7C_2}{R_1R_8} \quad (8)$$

which is solved for the denominator damping ratio ζ_p. Rearranging and substituting for ω_p gives

$$\zeta_p = \frac{1}{2R_1} \left[\frac{R_2 R_3 R_7 C_2}{R_8 C_1} \right]^{1/2} \tag{9}$$

Table 1 shows the dependences between the servo parameters and the circuit components. By using this table and practical constraints the components to be fixed and those to be adjustable are chosen. The variable components will be those which have a minimum interaction between the servo parameters, such as R_4, which results in changing only the numerator damping ratio when it is adjusted. The next step is to solve for the variable components in terms of the servo parameters. These are known quantities which come from the transfer function to be implemented.

The capacitors, C_1 and C_2, will be fixed and, at present, assume a normalized value of one farad. The actual value will be determined later when the impedance scale factor has been chosen. However, they will be in the range of 0.01 to 1 microfarads since these values are easy to obtain, inexpensive, and stable. Also, capacitance values in this range allow smaller resistance values to be used in the circuit. This reduces the effect of high impedance leakage in the circuit. Had either or both of the capacitors been chosen for parameter adjustment, a large variation in value would have been required to meet the design specification. This would have been prohibitive in both size and cost.

TABLE 1

FEEDFORWARD THREE AMPLIFIER BIQUAD; SERVO PARAMETERS VS. CIRCUIT COMPONENTS

Parameters	Components									
	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	C_1	C_2
K		X			X					
ω_z			X		X	X	X		X	X
ζ_z	X		X	X	X	X	X		X	X
ω_p		X	X				X	X	X	X
ζ_p	X	X	X				X	X	X	X

Independent adjustment of the dc gain, K, is a requirement. Since K can not be adjusted without disturbing the frequency response of the circuit in Figure 3 another operational amplifier is added as shown in Figure 4. Its connection at the output of the frequency dependent network gives independent dc gain adjustment along with increased flexibility. This is important because it allows the ac compensation to remain as is regardless of how the dc gain is changed. However, changing the ac gain can result in a new dc gain.

From circuit topology considerations and ease of debug R_6, R_7, and R_8 will be fixed resistors, and they shall be made identical. The debug of the stage is now trivial. This leaves R_1 through R_5 available for the adjustment of the four ac parameters. From table 1 it is found that R_3 has the most interaction with the servo parameters, so it is made a fixed resistor. This leaves R_4 for the adjustment of ζ_z, R_5 for the adjustment of ω_z, R_2 for the adjustment of ω_p, and R_1 for the adjustment of ζ_p. Note that the only independently adjustable parameter is ζ_z. When adjusting R_1 both ζ_z and ζ_p change. If R_2 is varied, then ω_p and ζ_p will change. Both ω_z and ζ_z change when R_5 is adjusted. However, the final design equations take into account the parameter interactions, thus eliminating this as a design problem.

Values for the fixed components must be obtained, and then equations for the variable components, expressed in terms of the fixed components and the servo parameters, are solved. As stated

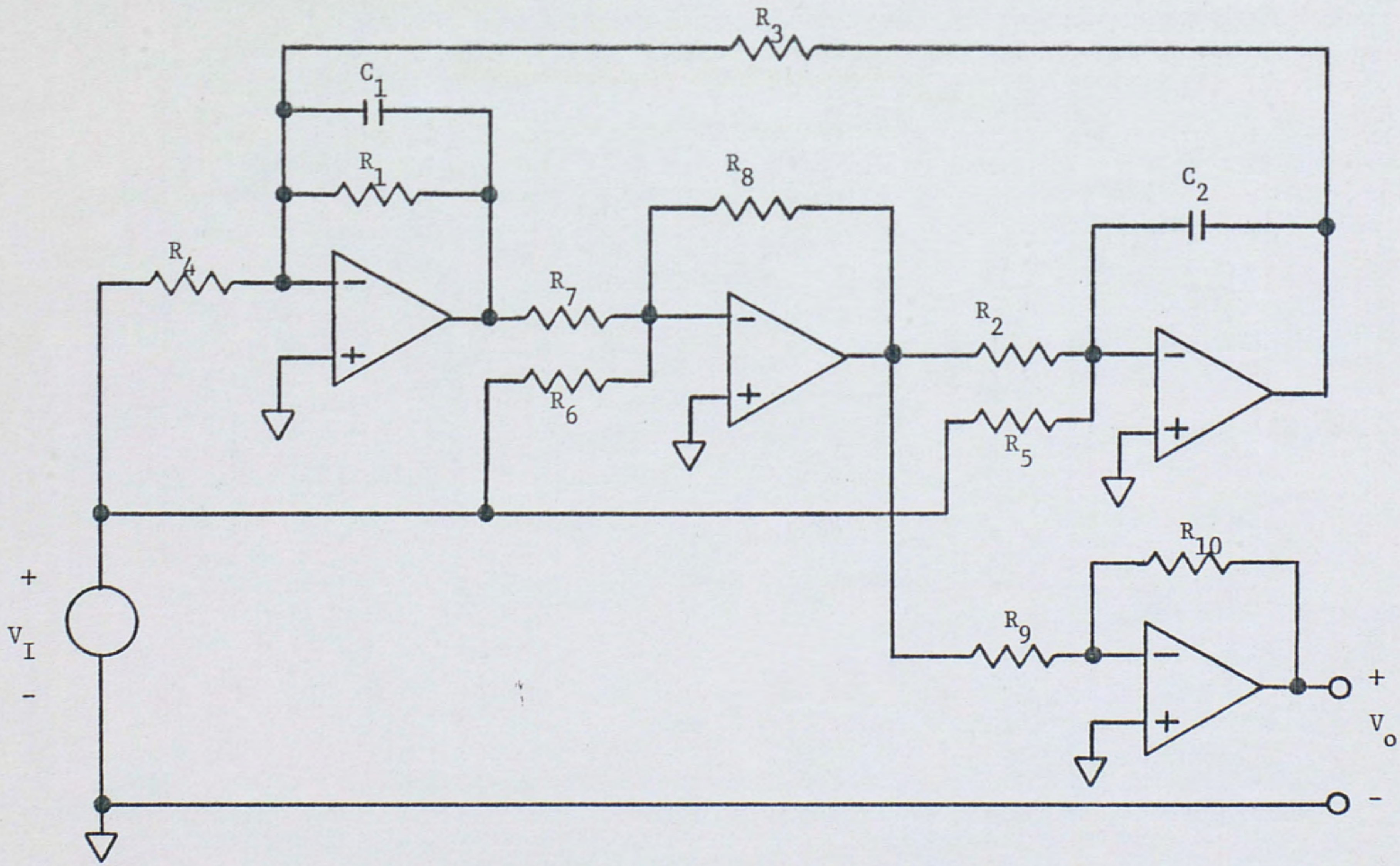

Figure 4. MODIFIED FEEDFORWARD THREE AMPLIFIER BIQUAD

above, $R_6 = R_7 = R_8 = R$, and R will be set equal to 10.5 KΩ. This is a standard value, large enough to have low power dissipation while still small enough to keep offsets and bias effects to a minimum. In addition, changes caused by temperature and moisture will be minimal.

Substituting R and $C_1 = C_2 = 1$ into equations (4), (5), (7), and (9) normalizes them and gives

$$\omega_p = [R_{2N}R_{3N}]^{-1/2} \tag{10}$$

$$\omega_z = [R_{3N}R_{5N}]^{-1/2} \tag{11}$$

$$\zeta_p = \frac{1}{2\ R_{1N}} [\ R_{2N}R_{3N}]^{\ 1/2} \tag{12}$$

$$\zeta_z = \frac{R_{4N}-R_{1N}}{2R_{1N}R_{4N}} [\ R_{3N}R_{5N}]^{1/2} \tag{13}$$

where the subscript N denotes a normalized value. Solving equations (10) through (13) for R_{2N}, R_{5N}, R_{1N}, and R_{4N} respectively gives

$$R_{2N} = \frac{1}{R_{3N}\omega_p^2} \tag{14}$$

$$R_{5N} = \frac{1}{R_{3N}\omega_z^2} \tag{15}$$

$$R_{1N} = \frac{1}{2\omega_p\zeta_p} \tag{16}$$

$$R_{4N} = \frac{1}{2[\omega_p\zeta_p - \omega_z\zeta_z]} \tag{17}$$

By setting $R_6 = R_7 = R_8 = R$ the parameter equations have been simplified greatly. It is seen that these equations do not depend on the actual values of these resistors but only that they be equal. The normalized solutions for R_1 and R_4 are complete, but R_2 and R_5 are still dependent on the normalized value of R_3. It is also seen that if $\omega_p = \omega_z$ then $R_2 = R_5$. A normalized value of R_3 will be chosen such that R_2 and R_5 are neither excessively large nor very small over the design range. This eliminates problems caused by high resistance leakage paths and circuit loading. However, a value of R_{3N} that will allow this may not exist.

Since R_2 and R_5 have similar ranges only the range of one is found and will be used for computing the normalized value of R_3. But first the denormalizing factor, α, must be determined because the actual circuit values are obtained using it. This factor can be found by choosing the value of the capacitors to be used in the circuit and working backwards. Equations (18) and (19) show the relationships between the actual and normalized values of capacitance and resistance.

$$C_{act} = \frac{C_N}{\alpha} \tag{18}$$

$$R_{act} = \alpha R_N \tag{19}$$

The selection of C_1 and C_2 is based on a stable, common value that is large enough to allow reasonable resistor values. Several

values fit this catagory and a capacitor of 0.47 microfarads will be used in the circuit. Since the values of C_1 and C_2 were normalized to one farad when solving the equations for R_{1N}, R_{2N}, R_{4N}, and R_{5N} the denormalizing factor, α, becomes $2.1276X10^6$.

Now the range of in-circuit resistance values for R_5 can be shown to be

$$R_{5MIN} = \frac{2.1276X10^6}{R_{3N}[1257]^2} = \frac{1.3474}{R_{3N}} \tag{20}$$

and

$$R_{5MAX} = \frac{2.1276X10^6}{R_{3N}[188]^2} = \frac{59.878}{R_{3N}} \tag{21}$$

Substitution of R_{3N} into equations (20) and (21) yields

$$R_{5MIN} = \frac{1.3474\alpha}{R_{3\ act}} = \frac{2.8667X10^6}{R_{3\ act}} \tag{22}$$

and

$$R_{5MAX} = \frac{59.878\alpha}{R_{3\ act}} = \frac{1.2739X10^8}{R_{3\ act}} \tag{23}$$

By substituting in suitable values for R_3 in equations (22) and (23) the corresponding ranges of R_5 can be determined. Any value of R_3 can be used that meets the criteria previously stated; in this design R_3 is set equal to 2150 ohms. The range of R_5 then becomes 1330 Ω to 59.3KΩ and the normalized value of R_3 is $1.0105 \text{ X } 10^{-3}$. Therefore, equations (14) and (15) become

$$R_{2N} = \frac{989.61}{\omega_p^{\ 2}} \tag{24}$$

and

$$R_{5N} = \frac{989.61}{\omega_z^2} \tag{25}$$

By denormalizing equations (16), (17), (24), and (25) the equations for the values of R_1, R_2, R_4, and R_5 to be used in the circuit are obtained and are

$$R_1 = \frac{1.0638X10^6}{\omega_p \zeta_p} \tag{26}$$

$$R_2 = \frac{2.10549X10^9}{\omega_p^2} \tag{27}$$

$$R_4 = \frac{1.0638X10^6}{\omega_p \zeta_p - \omega_z \zeta_z} \tag{28}$$

$$R_5 = \frac{2.10549X10^9}{\omega_z^2} \tag{29}$$

Now all the resistors used to adjust the frequency dependent portion of the transfer function have been expressed in terms of the given parameters. The equations are simple and easily solved for the required circuit values.

The dc gain adjustment has yet to be derived. Note that the dc gain through the frequency dependent network is equal to $-R_2/R_5$. This was shown in equation (3) and is now denoted as K_1. The dc gain for the circuit in Figure 4 is

$$K = K_1 K_2 \tag{30}$$

where K_2 is the dc gain of the inverting buffer amplifier. Substituting K_1 into equation (30) and solving for K_2 yields

$$K_2 = -\frac{KR_5}{R_2} \tag{31}$$

Further substitution for R_2 and R_5 yields

$$K_2 = -\frac{K\omega_p^2}{\omega_z^2} \tag{32}$$

From the circuit topology the dc gain of the inverting buffer stage is

$$K_2 = -\frac{R_{10}}{R_9} \tag{33}$$

Substituting equation (33) into (31) and solving for R_{10} gives

$$R_{10} = \frac{K R_9 \omega_p^2}{\omega_z^2} \tag{34}$$

Using good design practice R_9 is chosen as 4700Ω, and R_{10} becomes a 50KΩ potentiometer in series with a small resistor (330Ω). This allows the dc gain of the circuit to be set between 0.1 and 10, which meets the design requirements. Therefore the final equation for R_{10} is

$$R_{10} = \frac{4700K\omega_p^2}{\omega_z^2} - 330\ \Omega \tag{35}$$

Similarly the other adjustable components have small resistors in series with them. They must be small enough so that the circuit is still capable of operating over its design range.

The derivation of the design equations in terms of the given servo parameters for the feedforward three amplifier biquad circuit is complete. These equations, including the adjustment for the

small series resistor, are summarized in Table 2. Table 3 gives the individual component value or range where applicable. This will be the only design to include the small series resistor since good design procedure was demonstrated. The resistor is not required for comparison of the final circuit designs.

SUMMING FOUR AMPLIFIER BIQUAD DESIGN

The circuit shown in Figure 5 is the summing four amplifier biquad circuit (Daryanani 1976). Comparison of this circuit topology to the modified feedforward three amplifier biquad of Figure 4 shows many similarities. This is because both circuits were developed based on analog computer implementations for realizing linear transfer functions. However, a major difference exists. The summing four amplifier biquad uses a summing technique to implement the zeros of the transfer function. Tow (1969) gives complete details of the synthesis equations for this circuit.

The transfer function of the summing four amplifier biquad as given by Daryanani (1976) is

$$V_o = \frac{R_{10}}{R_8} V_{LP} - \frac{R_{10}}{R_7} V_{BP} - \frac{R_{10}}{R_9} V_{IN} \tag{36}$$

where

V_{LP} = standard low-pass transfer function

V_{BP} = standard band-pass transfer function

V_{IN} = input signal voltage

V_o = output signal voltage

TABLE 2

FEEDFORWARD THREE AMPLIFIER BIQUAD; DESIGN EQUATIONS

$$R_1 = \frac{1.0638\text{X}10^6}{\omega_p \zeta_p} - 750$$

$$R_2 = \frac{2.10549\text{X}10^9}{\omega_p^2} - 1100$$

$$R_4 = \frac{1.0638\text{X}10^6}{\omega_p \zeta_p - \omega_z \zeta_z} - 750$$

$$R_5 = \frac{2.10549\text{X}10^9}{\omega_z^2} - 1100$$

$$R_{10} = \frac{4700\ K\ \omega_p^2}{\omega_z^2} - 330$$

TABLE 3

FEEDFORWARD THREE AMPLIFIER BIQUAD; COMPONENT VALUES/RANGES

Component	Value/Range
C_1	0.47μf
C_2	0.47μf
R_1	$2000\Omega \leq R_1 \leq 22K\Omega$
R_2	$200\Omega \leq R_2 \leq 82K\Omega$
R_3	2150Ω
R_4	$2600\Omega \leq R_4 \leq 27K\Omega$
R_5	$200\Omega \leq R_5 \leq 59K\Omega$
R_6	10.5KΩ
R_7	10.5KΩ
R_8	10.5KΩ
R_9	4700Ω
R_{10}	$950\Omega \leq R_{10} \leq 47K\Omega$

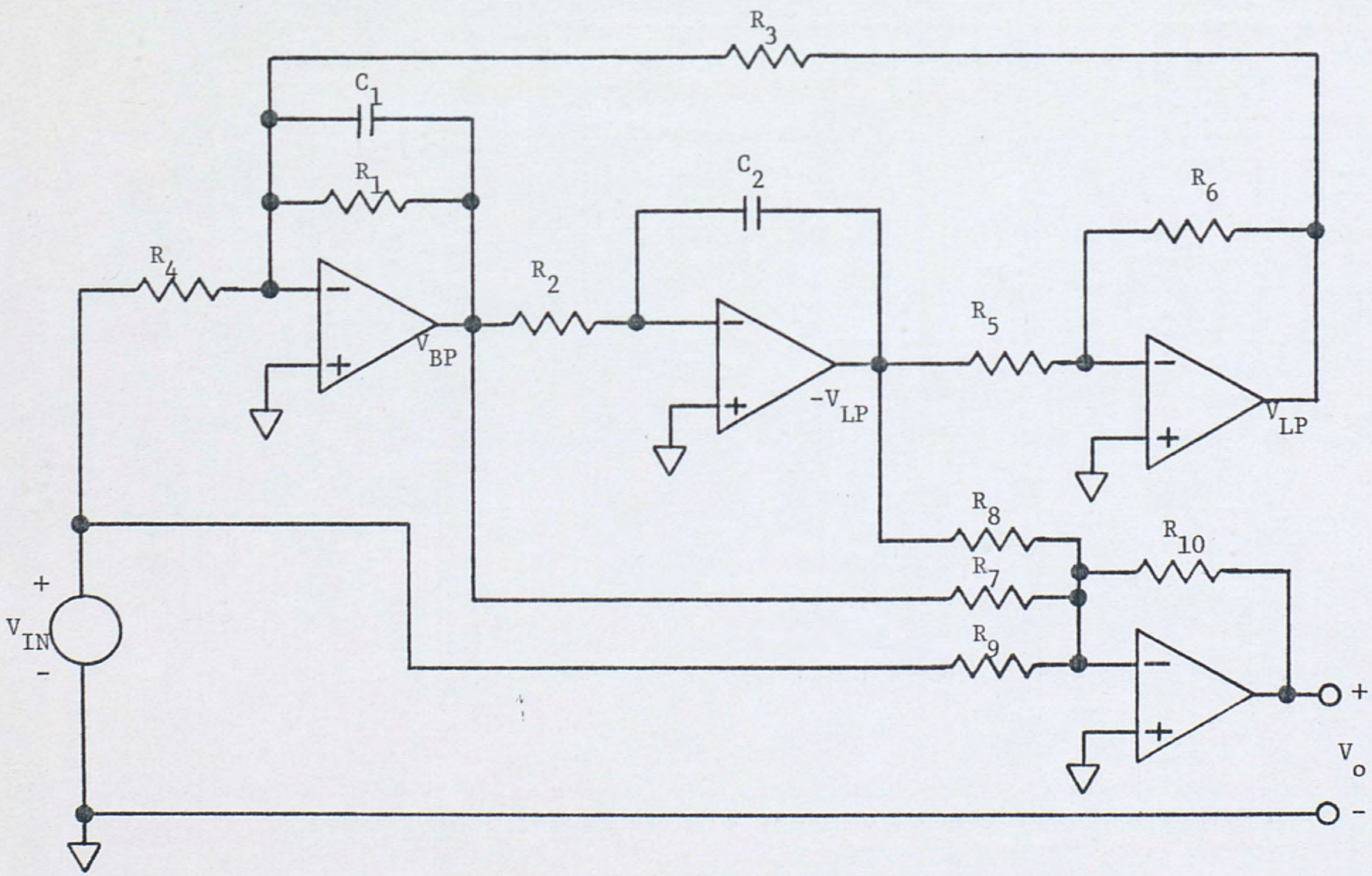

Figure 5. SUMMING FOUR AMPLIFIER BIQUAD

After substituting for V_{LP} and V_{BP}, we obtain after much manipulation

$$\frac{V_o}{V_{IN}} = -\frac{R_{10}}{R_9}\left[\frac{S^2+S[\frac{1}{R_1C_1}-\frac{R_9}{R_4R_7C_1}]+\frac{1}{R_2R_3C_1C_2}+\frac{1}{R_2R_4R_8C_1C_2}}{S^2+S\,[\frac{1}{R_1C_1}]+\frac{1}{R_2R_3C_1C_2}}\right] \tag{37}$$

Rearranging (37) into the servo form and solving for the servo parameters in terms of the components, we get

$$\omega_p^2 = \frac{1}{R_2R_3C_1C_2} \tag{38}$$

$$\zeta_p = \frac{1}{2R_1}[\frac{R_2R_3C_2}{C_1}]^{1/2} \tag{39}$$

$$\omega_z^2 = \omega_p^2 + \frac{R_9}{R_2R_4R_8C_1C_2} \tag{40}$$

$$\zeta_z = \frac{\frac{1}{R_1C_1}-\frac{R_9}{R_4R_7C_1}}{2[\omega_p^2+\frac{R_9}{R_2R_4R_8C_1C_2}]^{1/2}} \tag{41}$$

$$K = \frac{R_{10}}{R_9}[1+\frac{R_3R_9}{R_4R_8}] \tag{42}$$

The relationships between the servo parameters and the circuit components are shown in Table 4. It is seen that R_5 and R_6 are independent from the transfer function. However, it is appearent from the circuit that R_5 must equal R_6 to provide a gain of negative one. Any reasonable values can be used for R_5 and R_6; in this design they are set equal to 14KΩ. As in the feedforward three amplifier biquad, the capacitors are a fixed value. This leaves either R_2 or

TABLE 4

SUMMING FOUR AMPLIFIER BIQUAD; SERVO PARAMETERS VS. CIRCUIT COMPONENTS

Parameters	Components											
	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	C_1	C_2
K			X	X				X	X	X		
ω_z		X	X	X				X	X		X	X
ζ_z	X	X	X	X			X	X	X		X	X
ω_p		X	X								X	X
ζ_p	X	X	X								X	X

R_3 for adjusting ω_p. However, it was shown by Muir and Robinson (1968) that the sensitivity of the pole Q due to variations in finite amplifier gain can be minimized if $R_2C_2 = R_3C_1$. Therefore, both R_2 and R_3 will be used to adjust ω_p, and they will be kept equal in value. The adjustments of ω_z, ζ_p, and ζ_z are accomplished by R_4, R_1, and R_7 respectively, while R_{10} is used to independently adjust K.

Let

$$C_1 = C_2 = 1$$

$$R_2 = R_3 = R$$

$$R_8 = R_9 = R_x$$

Then equations (38) through (42) become

$$\omega_p = \frac{1}{R} \tag{43}$$

$$\zeta_p = \frac{R}{2R_1} \tag{44}$$

$$\omega_z = \frac{1}{R}\left[\frac{R_4+R}{R_4}\right]^{1/2} \tag{45}$$

$$\zeta_z = \frac{\frac{1}{R_1} - \frac{R_x}{R_4R_7}}{2\left[\frac{1}{R^2} + \frac{1}{RR_4}\right]^{1/2}} \tag{46}$$

$$K = \frac{R_{10}}{R_x}\left[1 + \frac{R}{R_4}\right] \tag{47}$$

Solving equations (43) through (47) for the resistors in terms of the servo parameters, we obtain

$$R = \frac{1}{\omega_p} \tag{48}$$

$$R_1 = \frac{1}{2\omega_p \zeta_p} \tag{49}$$

$$R_4 = \frac{\omega_p}{\omega_z^2 - \omega_p^2} \tag{50}$$

$$R_7 = \frac{R_x[\omega_z^2 - \omega_p^2]}{2\omega_p[\omega_p \zeta_p - \omega_z \zeta_z]} \tag{51}$$

$$R_{10} = \frac{\omega_p^2 R_x K}{\omega_z^2} \tag{52}$$

Examining equation (51) for the case $\omega_z = \omega_p$ gives $R_7 = 0$. This condition is not allowed since improper circuit operation would occur: a non-zero summing resistor is required at the negative input of an operational amplifier. Therefore, another set of resistors for parameter adjustment must be chosen.

The second set of resistors is the same as the first except that ω_z is adjusted by R_8 instead of R_4 and $R_4 = R_9 = R_x$. This means equations (48), (49), and (52) are the same as before but equation (50) becomes

$$R_8 = \frac{\omega_p}{\omega_z^2 - \omega_p^2} \tag{53}$$

and equation (51) becomes

$$R_7 = \frac{1}{2\omega_p \zeta_p - 2\omega_z \zeta_z} \tag{54}$$

This gives a complete set of normalized equations in terms of the servo parameters. They are listed here for convenience; the subscript N has been added to denote normalized values.

$$R_N = \frac{1}{\omega_p} \tag{50}$$

$$R_{1N} = \frac{1}{2\omega_p \zeta_p} \tag{56}$$

$$R_{8N} = \frac{\omega_p}{\omega_z^2 - \omega_p^2} \tag{57}$$

$$R_{7N} = \frac{1}{2\omega_p \zeta_p - 2\omega_z \zeta_z} \tag{58}$$

$$R_{10N} = \frac{\omega_p^2 K \quad R_{XN}}{\omega_z^2} \tag{59}$$

Equation (59) also depends on R_x. So R_x is selected such that a reasonable value of R_{10} is obtained when the dc gain is varied between 0.1 and 10. Also recall that the relationship between ω_z and ω_p is restricted to be either $\omega_z = \omega_p$ or $0.85\ \omega_z = \omega_p$. This gives

$$R_{10N\ MIN} = 0.07225\ R_{XN} \tag{60}$$

and

$$R_{10N\ MAX} = 10\ R_{XN} \tag{61}$$

Denormalizing yields

$$R_{10}\ \text{MIN} = 0.07225\ R_x \tag{62}$$

$$R_{10}\ \text{MAX} = 10\ R_X \tag{63}$$

Defining the minimum allowable value of R_{10} to be 1000Ω, and solving equation (62) for R_x gives R_x = 13.84KΩ which is not a standard value. The next higher standard value, 14KΩ, is selected for R_x since R_{10} must be at least 1000 Ω. This yields $R_x = R_4 = R_9$ = 14K Ω, and a good range of values for R_{10} (1KΩ to 140KΩ) results.

Now the remaining equations can be denormalized and put into their final forms. The denormalizing factor, α, will be the same as the one used for the feedforward three amplifier biquad and therefore $C_1 = C_2$ = 0.47 microfarads. Denormalizing equation (59) and substituting in for R_x gives

$$R_{10} = \frac{1.4 \times 10^4\ \omega_p^2 K}{\omega_z^2} \tag{64}$$

and equations (55) through (58) become

$$R = R_2 = R_3 = \frac{2.1276\text{X}10^6}{\omega_p} \tag{65}$$

$$R_1 = \frac{1.0638\text{X}10^6}{\omega_p \zeta_p} \tag{66}$$

$$R_8 = \frac{2.1276\text{X}10^6}{\omega_z^2 - \omega_p^2} \tag{67}$$

$$R_7 = \frac{1.0638X10^6}{\omega_p\zeta_p - \omega_z\zeta_z} \tag{68}$$

By utilizing the design requirements, the range of values that these variable components must take on can be computed. The value or range of each component is given in Table 5.

From Table 5 it is seen that the maximum value of R_8 is infinity. This occurs when $\omega_z = \omega_p$. Obtaining this value poses no problem for a test tool since a switch can be put in series with the potentiometer. The infinity setting is then obtained by opening the switch. When $0.85\omega_z = \omega_p$ a 50KΩ potentiometer is sufficient.

SINGLE AMPLIFIER BIQUAD WITH POLE-ZERO CANCELLATION

Another method of implementing a biquadratic equation is with a single operational amplifier and utilizing a pole-zero cancellation technique. This circuit is shown in Figure 6. It was obtained by using an operational amplifier in the inverting mode and choosing the appropriate input and feedback networks from a transfer impedance table (Truxal 1958, p. 6-4.) Therefore, the transfer function is

$$T(S) = -\frac{Z_F(S)}{Z_I(S)} \tag{69}$$

TABLE 5

SUMMING FOUR AMPLIFIER BIQUAD; COMPONENT VALUES/RANGES

Component	Value/Range
C_1	$0.47\mu f$
C_2	$0.47\mu f$
R_1	$2500\Omega \leq R_1 \leq 22.5K\Omega$
R_2	$1600\Omega \leq R_2 \leq 11.5K\Omega$
R_3	$1600\Omega \leq R_3 \leq 11.5K\Omega$
R_4	$14K\Omega$
R_5	$14K\Omega$
R_6	$14K\Omega$
R_7	$3100\Omega \leq R_7 \leq 28K\Omega$
R_8	$5000\Omega \leq R_8 \leq$ 50KΩ *
R_9	$14K\Omega$
R_{10}	$1000\Omega \leq R_{10} \leq 140K\Omega$

* Largest finite value. $R_8 = \infty$ when $\omega_p = \omega_z$

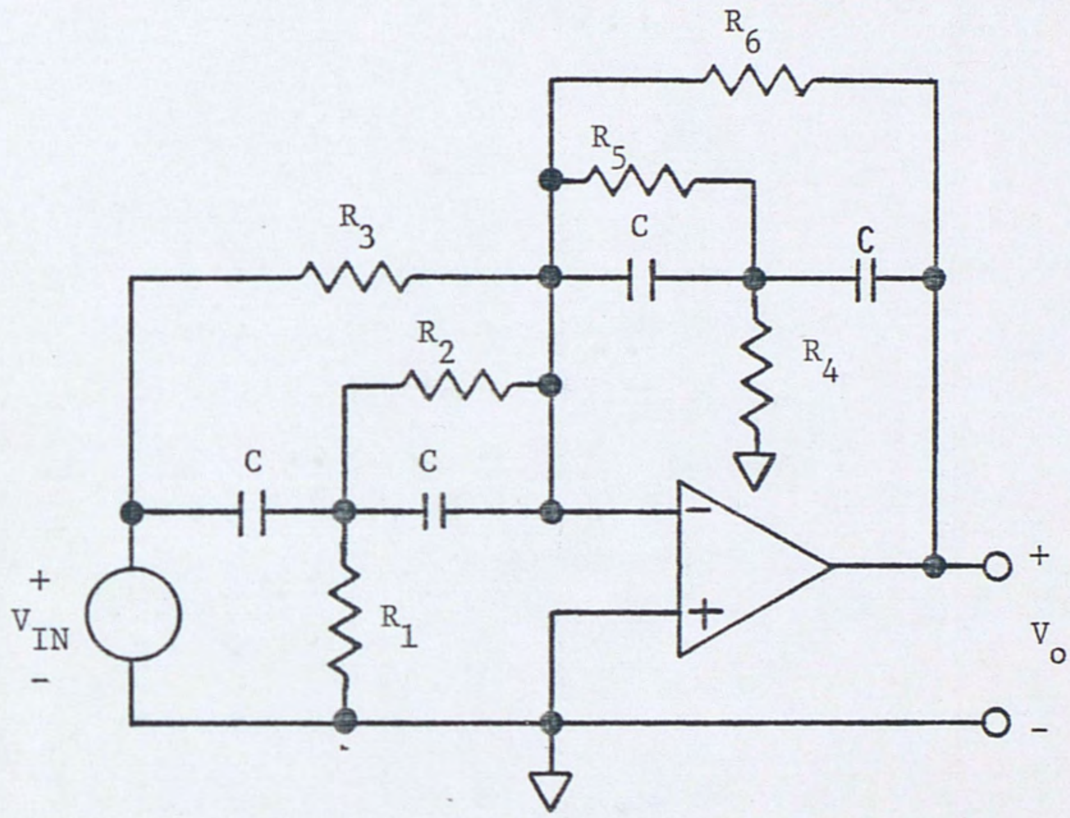

Figure 6. SINGLE AMPLIFIER BIQUAD WITH POLE-ZERO CANCELLATION

where Z_I (S) and Z_F (S) are obtained directly from the transfer impedance table. They are

$$Z_I(S) = R_3\left[\frac{S[\frac{2R_1R_2C}{R_1+R_2}] + 1}{S^2[\frac{R_1R_2R_3C^2}{R_1+R_2}] + S\,[\frac{R_1C[2R_2+R_3]}{R_1+R_2}] + 1}\right] \tag{70}$$

and

$$Z_F(S) = R_6\left[\frac{S[\frac{2R_4R_5C}{R_4+R_5}] + 1}{S^2[\frac{R_4R_5R_6C^2}{R_4+R_5}] + S[\frac{R_4C[2R_5+R_6]}{R_4+R_5}] + 1}\right] \tag{71}$$

which gives

$$T(S) = -\frac{R_6}{R_3}\left[\frac{\left[S[\frac{2R_4R_5C}{R_4+R_5}] + 1\right]\left[S^2[\frac{R_1R_2R_3C^2}{R_1+R_2}] + S\,[\frac{R_1C[2R_2+R_3]}{R_1+R_2}]+1\right]}{\left[S[\frac{2R_1R_2C}{R_1+R_2}] + 1\right]\left[S^2[\frac{R_4R_5R_6C^2}{R_4+R_5}] + S\,[\frac{R_4C[2R_5+R_6]}{R_4+R_5}]+1\right]}\right] \tag{72}$$

In order to make equation (72) into a biquadratic, a pole-zero cancellation must occur. This requires that

$$\frac{1}{\omega} = \frac{2R_1R_2C}{R_1+R_2} = \frac{2R_4R_5C}{R_4+R_5} \tag{73}$$

Additionally, the pole-zero cancellation must take place at a frequency, ω, given by

$$\omega \geq 10\ \omega_z \tag{74}$$

This minimizes the effects of a less than complete pole-zero cancellation.

The remaining servo parameters expressed in terms of the circuit components are obtained from equation (72) by coefficient matching. They are

$$\omega_z^2 = \frac{R_1+R_2}{R_1R_2R_3C^2} \tag{75}$$

$$\zeta_z = \frac{R_1[2R_2+R_3]}{2[R_1R_2R_3[R_1+R_2]]^{1/2}} \tag{76}$$

$$\omega_p^2 = \frac{R_4+R_5}{R_4R_5R_6C^2} \tag{77}$$

$$\zeta_p = \frac{R_4[2R_5+R_6]}{2[R_4R_5R_6[R_4+R_5]]^{1/2}} \tag{78}$$

$$K = \frac{R_6}{R_3} \tag{79}$$

Table 6 shows the interaction between the components and the servo parameters. From Table 6 it is seen that components in both the input and feedback network are dependent on ω. This is because a pole-zero cancellation in T(S) is required. Therefore, both R_1 and R_4 must be used to adjust ω so that a cancellation does occur. The adjustment of ω_z, ζ_z, ω_p, and ζ_p will be by R_3, R_2, R_6, and R_5 respectively, and C is set equal to one farad, thereby normalizing the equations. However, this does not leave any component by which to vary K. To control K, another operational

TABLE 6

SINGLE AMPLIFIER BIQUAD WITH POLE-ZERO CANCELLATION;
SERVO PARAMETERS VS. CIRCUIT COMPONENTS

Parameters	Components						
	R_1	R_2	R_3	R_4	R_5	R_6	C
K			X			X	
ω_z	X	X	X				X
ζ_z	X	X	X				
ω_p				X	X	X	X
ζ_p				X	X	X	
ω	X	X		X	X		X

amplifier must be added in a manner similar to the one added to the feedforward three amplifier biquad circuit discussed previously. This allows independent adjustment of the dc gain without changing the ac response of the circuit. In addition, the components are solved in terms of ω so that optimum component values can be obtained.

Using equation (75) and solving for R_3 yields

$$R_3 = \frac{R_1+R_2}{R_1 R_2 \omega_z^2} \tag{80}$$

and from equation (73)

$$\frac{R_1+R_2}{R_1 R_2} = \frac{R_4+R_5}{R_4 R_5} = 2\omega \tag{81}$$

By substitution R_3 becomes

$$R_3 = \frac{2\omega}{\omega_z^2} \tag{82}$$

which is in terms of servo parameters only. Also, from equation (80)

$$R_1+R_2 = R_1 R_2 R_3 \omega_z^2 \tag{83}$$

which will be used in a later equation.

To solve for R_2, equation (76) is rearranged to read

$$4\zeta_z^2 \; R_1 R_2 R_3 [R_1+R_2] = R_1^2 [2R_2+R_3]^2 \tag{84}$$

Substituting in equation (83) yields

$$2\zeta_p R_2 R_3 \omega_z = 2R_2 + R_3 \tag{85}$$

and substitution for R_3 gives

$$\frac{4\zeta_z R_2 \ \omega}{\omega_z} = 2R_2 + \frac{2\omega}{\omega_z^2} \tag{86}$$

After further manipulation of equation (86), the value of R_2 in terms of servo parameters is given as

$$R_2 = \frac{\omega}{\omega_z[2\zeta_z \omega - \omega_z]} \tag{87}$$

Equation (81) will be used to solve for R_1. Rewriting gives

$$R_1 + R_2 = 2\omega \ R_1 R_2 \tag{88}$$

therefore,

$$R_1 = \frac{R_2}{2\omega R_2 - 1} \tag{89}$$

However, R_1 is not yet in terms of only the servo parameters. The substitution of equation (87) into equation (89) yields

$$R_1 = \frac{\omega}{2\omega^2 - \omega_z[2\zeta_z \omega - \omega_z]} \tag{90}$$

The solutions for R_4, R_5, and R_6 are obtained in a similar fashion by starting with equations (77) and (78). The results are

listed below:

$$R_4 = \frac{\omega}{2\omega^2 - \omega_p[2\zeta_p \omega - \omega_p]} \tag{91}$$

$$R_5 = \frac{\omega}{\omega_p[2\zeta_p \omega - \omega_p]} \tag{92}$$

$$R_6 = \frac{2\omega}{\omega_p^2} \tag{93}$$

Note the similarities that exist between equations (91), (92), and (93) and equations (90), (87), and (82) respectively.

Now the dc gain adjustment is considered. Figure 7 shows the additional circuitry required to obtain this flexibility. The overall dc gain K that is required is equal to K_1 times K_2, where K_1 is equal to equation (79), the dc gain of the frequency dependent network, and K_2 is the dc gain of the added buffer amplifier. Substituting equations (82) and (93) for R_3 and R_6 yields

$$K = \frac{\omega_z^2 K_2}{\omega_p^2} \tag{94}$$

and from Figure 7

$$K_2 = - \frac{R_8}{R_7} \tag{95}$$

Combining equations (94) and (95) and solving for R_8, the final

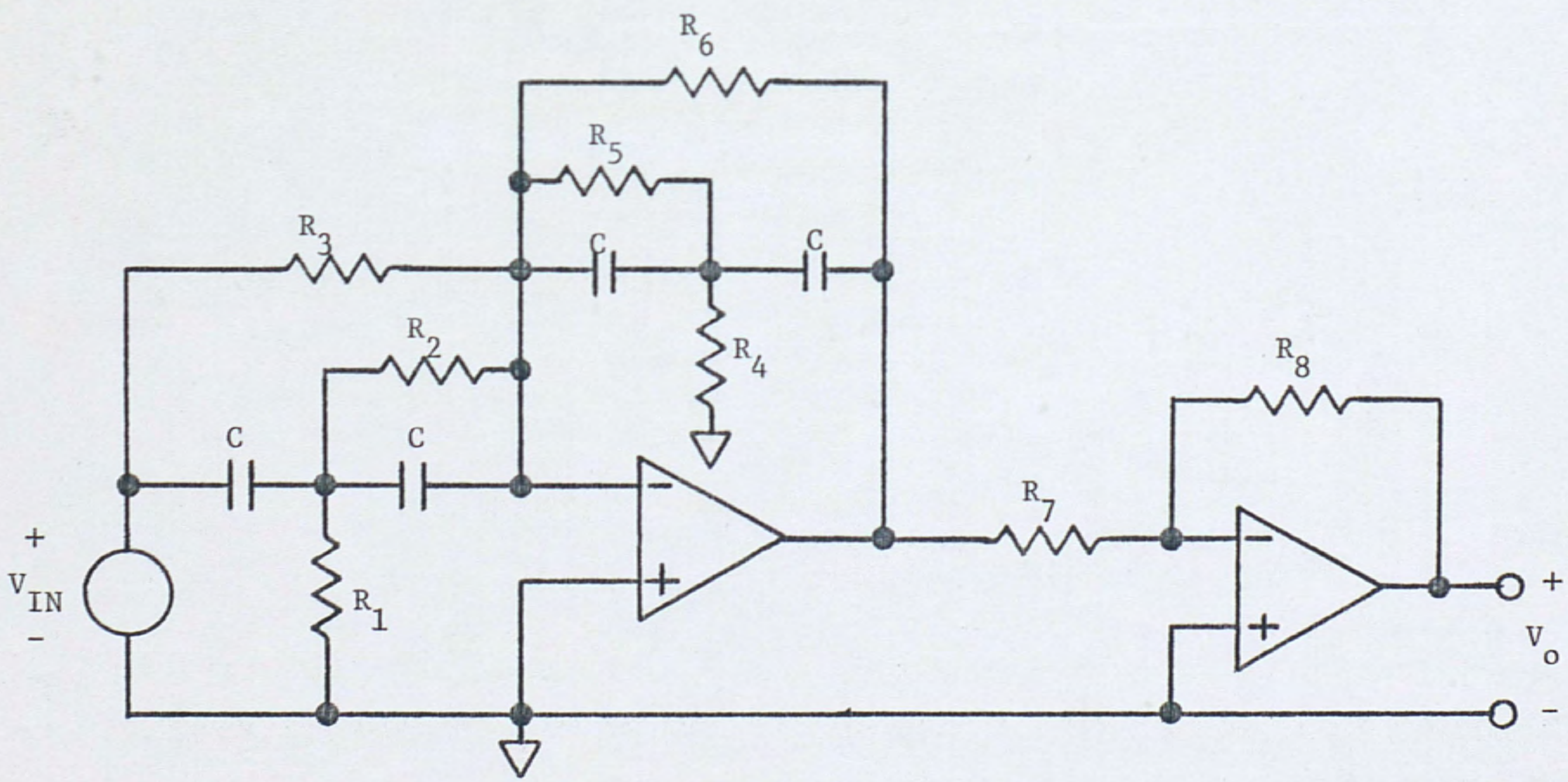

Figure 7. MODIFIED SINGLE AMPLIFIER BIQUAD WITH POLE-ZERO CANCELLATION

form for the dc gain adjustment is obtained:

$$R_8 = \frac{R_7 K \omega_p^2}{\omega_z^2} \tag{96}$$

where R_7 is a fixed resistor whose value is chosen to yield reasonable values of R_8 over the entire range of dc gain adjustment. Therefore, R_7 can be set equal to 4700Ω, and R_8 becomes a 50 KΩ potentiometer.

The first step towards the solution has been completed; and the results are repeated for convenience.

$$R_1 = \frac{\omega}{2_{\omega}^{2} - \omega_z[2\zeta_z \omega - \omega_z]} \tag{97}$$

$$R_2 = \frac{\omega}{\omega_z[2\zeta_z \omega - \omega_z]} \tag{98}$$

$$R_3 = \frac{2\omega}{\omega_z^2} \tag{99}$$

$$R_4 = \frac{\omega}{2_{\omega}^{2} - \omega_p[2\zeta_p \omega - \omega_p]} \tag{100}$$

$$R_5 = \frac{\omega}{\omega_p[2\zeta_p \omega - \omega_p]} \tag{101}$$

$$R_6 = \frac{2\omega}{\omega_p^2} \tag{102}$$

$$R_8 = \frac{4700K\omega_p^2}{\omega_z^2} \tag{103}$$

Now the actual range of values over which the compoments must vary will be computed. The denormalizing factor used is the same one used in the previous analyses, and the component values are obtained by using the high and low design limits while varying ω. The component values obtained are then examined to determine the acceptability of their range of variation. By solving equations (97) through (103) in the above manner it can be shown that $\omega=12\omega_z$ provides a good compromise. Therefore the final design equations are given in Table 7, and Table 8 gives the component value or range where applicable. This completes the design of the single amplifier biquad with pole-zero cancellation.

TABLE 7

SINGLE AMPLIFIER BIQUAD; DESIGN EQUATIONS

$$R_1 = \frac{12}{\omega_z[289-24\zeta_z]}$$

$$R_2 = \frac{12}{\omega_z[24\zeta_z-1]}$$

$$R_3 = \frac{24}{\omega_z}$$

$$R_4 = \frac{12\omega_z}{288\omega_z^2 - \omega_p[24\omega_z\zeta_p-\omega_p]}$$

$$R_5 = \frac{12\omega_z}{\omega_p[24\omega_z\zeta_p-\omega_p]}$$

$$R_6 = \frac{24\omega_z}{\omega_p^2}$$

$$R_8 = \frac{4700K\omega_p^2}{\omega_z^2}$$

TABLE 8

SINGLE AMPLIFIER BIQUAD; COMPONENT VALUES/RANGES

Component	Value/Range
C	0.47μf
R_1	$70\Omega \leq R_1 \leq 475\Omega$
R_2	$100K\Omega \leq R_2 \leq 680K\Omega$
R_3	$40K\Omega \leq R_3 \leq 272K\Omega$
R_4	$99\Omega \leq R_4 \leq 482\Omega$
R_5	$3200\Omega \leq R_5 \leq 25.1K\Omega$
R_6	$56K\Omega \leq R_6 \leq 376K\Omega$
R_7	4700Ω
R_8	$100\Omega \leq R_8 \leq 50K\Omega$

SUMMARY

Three active networks have been examined and design equations derived that meet the design requirements stated previously. Now the three circuits are compared. From Table 9, a comparison of physical features, the SABPZC uses the smallest number of passive components and amplifiers, but the FTAB and SFAB are more easily tuned and require fewer adjustable components. A comparison of the range of component values is given in Table 10. Table 10 shows that the spread of component values for the FTAB and the SFAB are all reasonable except for R_8 of the SFAB. However, even R_8 is acceptable since the infinite value is required when $\omega_p=\omega_z$, and a 50KΩ potentiometer will satisfy the range of adjustment when $\omega_p=0.85\omega_z$. But, for the SABPZC, most of the ranges are undesirable. Resistors are too large or too small. This leads to granularity problems with adjustments and environmental problems due to the large resistance values.

By examining Table 11, a comparison of the final design equations, it is seen that the FTAB has the simplest equations. The SFAB equations are only slightly more complex than the FTAB but the SABPZC equations are considerably more complex than the SFAB equations. In addition, the parameter interaction on the component values is less for the FTAB and the SFAB.

TABLE 9

COMPARISON OF PHYSICAL FEATURES

Features	Circuit		
	FTAB	SFAB	SABPZC
Op Amps	4	4	2
Resistors	10	10	8
Capacitors	2	2	4
Adjustable Components	5	6	7
Tuning	Easy	Easy	Hard
Number of Components	16	16	14

TABLE 10

COMPARISON OF COMPONENT VALUES/RANGES

Component	Circuit		
	FTAB	SFAB	SABPZC
C	- - -	- - -	0.47μf
C_1	0.47μf	0.47μf	- - -
C_2	0.47μf	0.47μf	- - -
R_1	$2000\Omega \leq R_1 \leq 22K\Omega$	$2500\Omega \leq R_1 \leq 22.5K\Omega$	$70\Omega \leq R_1 \leq 475\Omega$
R_2	$200\Omega \leq R_2 \leq 82K\Omega$	$1600\Omega \leq R_2 \leq 11.5K\Omega$	$100K\Omega \leq R_2 \leq 680K\Omega$
R_3	2150Ω	$1600\Omega \leq R_3 \leq 11.5K\Omega$	$40K\Omega \leq R_3 \leq 272K\Omega$
R_4	$2600\Omega \leq R_4 \leq 27K\Omega$	14KΩ	$99\Omega \leq R_4 \leq 482\Omega$
R_5	$200\Omega \leq R_5 \leq 59K\Omega$	14KΩ	$3200\Omega \leq R_5 \leq 25.1K\Omega$
R_6	10.5KΩ	14KΩ	$56K\Omega \leq R_6 \leq 376K\Omega$
R_7	10.5KΩ	$3100\Omega \leq R_7 \leq 28K\Omega$	4700Ω
R_8	10.5KΩ	$5000\Omega \leq R_8 \leq \infty$	$100\Omega \leq R_8 \leq 50\Omega$
R_9	4700Ω	14KΩ	- - -
R_{10}	$950\Omega \leq R_{10} \leq 47K\Omega$	$1000\Omega \leq R_{10} \leq 140K\Omega$	- - -

TABLE 11

COMPARISON OF DESIGN EQUATIONS

Adjustable Component	Circuit		
	FTAB	SFAB	SABPEC
R_1	$R_1 = \frac{1.0638X10^6}{\omega_p \zeta_p}$	$R_1 = \frac{1.0638X10^6}{\omega_p \zeta_p}$	$R_1 = \frac{12}{\omega_z [289-24\zeta_z]}$
R_2	$R_2 = \frac{2.10549X10^9}{\omega_p^2}$	$R_2 = \frac{2.1276X10^6}{\omega_p}$	$R_2 = \frac{12}{\omega_z [24\zeta_z - 1]}$
R_3	- - - - - -	$R_3 = \frac{2.1276X10^6}{\omega_p}$	$R_3 = \frac{24}{\omega_z}$
R_4	$R_4 = \frac{1.0638X10^6}{\omega_p \zeta_p - \omega_z \zeta_z}$	- - - - - -	$R_4 = \frac{12\omega_z}{288\omega_z^2 - \omega_p [24\omega_z \zeta_p - \omega_p]}$
R_5	$R_5 = \frac{2.10549X10^9}{\omega_z^2}$	- - - - - -	$R_5 = \frac{12\omega_z}{\omega_p [24\omega_z \zeta_p - \omega_p]}$

TABLE 11
COMPARISON OF DESIGN EQUATIONS (CONTINUED)

Adjustable Component	Circuit		
	FTAB	SFAB	SABPEC
R_6	- - -	- - -	$R_6 = \frac{24\omega_z}{\omega_p^2}$
R_7	- - -	$R_7 = \frac{1.0638X10^6}{\omega_p\zeta_p - \omega_z\zeta_z}$	- - -
R_8	- - -	$R_8 = \frac{2.1276X10^6}{\omega_z^2 - \omega_p^2}$	$R_8 = \frac{4700K\ \omega_p^2}{\omega_z^2}$
R_9	- - -	- - -	- - -
R_{10}	$R_{10} = \frac{4700K\ \omega_p^2}{\omega_z^2}$	$R_{10} = \frac{1.4X10^4K\ \omega_p^2}{\omega_z^2}$	- - -

For the design requirements given, the SABPZC can be utilized but only in a 25^{o}C, low humidity environment. This is due to the large resistance values required for implementation. Also, its tuning procedure is more difficult, and since its operation is based on a pole-zero cancellation, which can not be perfect, it will contribute some error to the output. Both the FTAB and the SFAB are very similar in final results. They realize a general biquadratic function, easy to tune, and require less high frequency compensation (due to leading phase feedback) than single amplifier realizations (Thomas 1971). However, the FTAB is the most flexible circuit, and it has the simplest design equations. Therefore, the feedforward three amplifier biquad is the circuit to use.

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