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A SYNCHRONOUS PHASE DETECTION SYSTEM
FOR AN OPTICAL INTERFEROMETRIC SENSOR

BY

IRA JEFFREY BUSH

THESIS

Submitted in partial fulfillment of the requirements
for the Master of Science degree in Engineering
in the Graduate Studies Program of the College of Electrical Engineering
University of Central Florida
Orlando, Florida

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1981

A SYNCHRONOUS PHASE DETECTION SYSTEM
FOR AN OPTICAL INTERFEROMETRIC SENSOR

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IRA JEFFERY BUSH

ABSTRACT

A system has been developed to accurately detect phase produced in optical interferometric sensors. The system employs optical heterodyning and synchronously detects optical phase by feeding back an error signal to a phase modulator in the reference leg of the interferometer. This system is seen to have properties similar to a phase-locked loop used for the demodulation of FM signals. The system is modeled and found to be of second order and nonlinear. A linear approximation to the original model serves to accurately describe the system in synchronous operation and is corroborated with well matched empirical data. The nonlinear model is simulated via computer techniques and is used to describe the system's parameters that lead to loss and reacquisition of synchronization.

A handwritten signature in dark ink, reading "Ronald L. Phillips", written over a horizontal line.

Dr. Ronald L. Phillips
Director of Thesis

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CHAPTER I

REVIEW OF INTERFEROMETRIC SENSORS

General Interferometric Phase Measurement

Real time measurement of optical phase fluctuations provides information in the areas of: 1) measuring refractive index variations in plasmas or gasses; 2) fiber optic sensors used for hydro-acoustic sensing; 3) fiber optic sensors used for determining rates of rotation.

All of these processes involve interferometric detection of optical phase to produce the measurement. A schematic representation of a Mach-Zehnder interferometer¹ is shown in Figure 1. Light from a single frequency laser is split up into two beams via a beam-splitter, BS 1. The upper beam is denoted as the signal leg while the lower beam inherits the name of reference leg. The sensor leg experiences optical phase modulation, while the reference leg maintains a constant optical phase. The two optical beams are recombined with a second beamsplitter, BS 2, and the interference pattern is detected with an optical "square-law" detector. If assumptions of collinear, plane wave propagation and parallel polarizations for the recombined beams are made, it is an established fact that the output

¹Eugene Hecht and Alfred Zajac, Optics (Reading, Massachusetts: Addison Wesley Publishing Company, 1979), p. 290.

Fig. 1. Mach-Zehnder interferometric system used to measure optical phase modulation.

of the detector (current or voltage response) produces a signal, $s(t)$, that is proportional to the relative optical phase difference between the signal and reference leg of the interferometer.

To quantitatively determine how the detector output signal is proportional to the relative phase difference of the two optical signals, consider the following simple mathematical model for the interferometer shown in Figure 1.

Assuming plane wave, single frequency operation of the laser, the electric fields for the signal and reference legs may be expressed:

$$\overline{E}_s = E_s \exp \left[i \left(\omega t - kz + \theta_{s1} + \theta_i(t) + \theta_{n1}(t) \right) \right] ; \text{ signal leg} \quad (1.1)$$

$$\overline{E}_r = E_r \exp \left[i \left(\omega t - kz + \theta_{s2} + \theta_{n2}(t) \right) \right] ; \text{ reference leg,}$$

where both beams are assumed to be propagating in the z direction with electric field amplitudes E_s and E_r where:

k = the wave number of the optical frequency;

θ_{s1} = the static phase shift produced by the optical path length of the signal leg;

θ_{s2} = the static phase shift produced by the optical path length of the reference leg;

$\theta_i(t)$ = the phase modulation produced by the signal source;

ω_o = the optical radian frequency;

$\theta_{n1}(t)$ = any noise phase produced by the signal leg;

$\theta_{n2}(t)$ = any noise phase produced by the reference leg.

The optical detector is modeled as a "square-law" device which produces a signal output, $s(t)$, proportional to the incident power of the electric field. In the case of the interferometer in Figure 1, the incident field, $\overline{E}_i$, is composed of the vector sum of the fields from the sensor and the reference leg or

$$\overline{E}_i = \overline{E}_s + \overline{E}_r . \quad (1.2)$$

If E_s and E_r are assumed to be linearly polarized in the same direction, the power of the incident field is linearly proportional to $|E_i|^2$ and the detected output, $s(t)$, is linearly proportional to the field power or

$$s(t) \propto |E_i|^2 = E_i E_i^* , \quad (1.3)$$

where $*$ represents the complex conjugate. After some algebraic manipulations and the use of Eulers formula, Equation (1.3) results in

$$s(t) = K \left[E_r^2 + E_s^2 + 2E_r E_s \cos (\theta_s + \theta_i(t) + \theta_n(t)) \right] , \quad (1.4)$$

where K is a constant of proportionality relating to the efficiency of the detector. The term θ_s represents the difference between the static phase terms ($\theta_s = \theta_{s1} - \theta_{s2}$) while the term $\theta_n(t)$ represents

the difference between the noise phase terms

$$(\theta_n(t) = \theta_{n1}(t) - \theta_{n2}(t)) .$$

Inspection of Equation (1.4) shows that the output of the simple passive interferometer in Figure 1 produces a signal output that is a nonlinear function of $\theta_i(t)$, $\theta_n(t)$, and θ_s . It is also a function of E_r and E_s . The idea behind the interferometer is to measure θ_i . The rest of the terms serve to obfuscate this measurement of θ_i ; these terms will be discussed individually.

The field amplitude terms, E_r and E_s are seen to vary as the laser intensity varies. They also may vary as a function of variable interferometer attenuation. The static phase θ_s is a constant and generally takes on a value of $0 - 2\pi$ with a uniform probability. The noise phase, $\theta_n(t)$, will be seen to vary from interferometer to interferometer. For Mach-Zehnder interferometers, the thermal expansion and contraction of materials is usually the term manifest in producing $\theta_n(t)$, which can generally be assumed to be a random variable. In summary, the simple passive interferometric phase measurement process is nonlinear, subject to two random variables in phase and unable to distinguish the difference between wavefront and field amplitude variations.

Fiber Optic Interferometric Sensors

The scope of interferometric systems discussed in this paper will be limited to that of fiber optic sensors. This choice stems primarily from the author's sponsor being quite active in this area; also, the technology is rather new and generates a lot of interest

in the scientific community. The two most common fiber optic sensors are the acoustic sensor^{2,3} and the rate gyroscope.^{4,5} The basic concepts of these two sensors are described in the following passages.

The Acoustic Sensor

The basic configuration of the fiber optic acoustic sensor is shown schematically in Figure 2. This diagram is seen to be very similar to the general Mach-Zehnder interferometer shown in Figure 1. Light from a laser is split up by a beamsplitter, BS 1, and is coupled into single-mode fibers via proper focusing optics (generally microscope objectives for a laboratory setup). The upper leg is the signal leg and is comprised of a coil of fiber generally 10 to 100 m in length rolled up into a "sensing coil" as to be compact enough to be much smaller in diameter than the acoustic wavelengths of interest (to be detected). The lower leg is the reference leg and is generally a piece of fiber as long as the sensor leg (unless the

²Joseph A. Bucaro, Henry D. Dardy, and Edward F. Carome, "Optical Fiber Acoustic Sensor," Applied Optics 16 (September 1977): 1761-65.

³J.H. Cole, R.L. Johnson, and P.G. Bhuta, "Fiber Optic Detection of Sound," Journal of the Acoustic Society of America 62 (April 1977): 1136-38.

⁴S. Ezekiel and S.R. Balsamo, "Passive Ring Resonator Laser Gyroscope," Applied Physics Letters 30 (March 1977): 478-82.

⁵W.C. Goss et al., "Fiber-Optic Rotation Sensor Technology," Applied Optics 19 (June 1980): 852-58.

laser has an unusually long coherence length) as to match its optical path length. The beams are coupled out of the fibers and recollimated via the proper optics and recombined with a beam-splitter, BS 2, to produce an interference pattern on the detector. If it is assumed that the light in the fiber is propagating single mode (HE_{11}) and the cladding modes are stripped off along with all the preliminary assumptions leading up to the development of Equation (1.4), the phase terms describing the input signals and the noise signals may be defined. The static phase term, θ_s , may be arbitrarily assumed to be 0. The input phase term $\theta_i(t)$, is described as

$$\theta_i(t) = \left[\left(\frac{dn}{dP} \right) + \frac{n}{l} \left(\frac{dl}{dP} \right) \right] k l P(t) ,^6 \quad (1.5)$$

where l is the length of the fiber exposed to the sound field, n is the effective refractive index of the single-mode fiber, k is the optical wave number, P represents static pressure and $P(t)$ represents the acoustic pressure that the fiber is subjected to. This relationship assumes that the sensing coil's largest dimension (generally the diameter) is much smaller than the acoustic wavelength. The bracketed term represents the effective index change for a one meter (or unity length, depending on the convention used)

⁶M.R. Layton and J.A. Bucaro, "Optical Fiber Acoustic Sensor Utilizing Mode-Mode Interference," Applied Optics 18 (March 1980): 666.

piece of fiber exposed to a static pressure. The first term in the brackets describes the true index change or dispersion resulting from the incident pressure while the second term relates the length change of the fiber (as a function of pressure) to the effective index change.

The system noise may be represented with both amplitude and phase terms. The amplitude noise is manifested in the E_r and E_s terms of Equation (1.4) and is either a function of the shot, intermodulation or spontaneous emission noise from the laser or the shot, thermal (Johnson) or dark current produced by the photo-detector. These "noise floor" terms are of little significance when considering the functional operation of the acoustic sensor (albeit it does represent the minimum detectable signal level). The phase noise of the system primarily consists of three components: 1) phase noise produced by acoustic background noise; 2) phase noise produced by a path length mismatch resulting in loss of coherence; and 3) phase noise resulting from temperature fluctuations. The first two terms are basically "noise floor" type phenomenon and if certain precautions are taken in the laboratory (or setup) these terms do not greatly inhibit system operation. The third term, the thermal-induced phase fluctuation is capable of producing very large phase shifts. In fact, it is not uncommon to see phase shifts of approximately 500 radians/meter $^{\circ}\text{C}$ in jacketed

fiber.^{7,8} This number is deduced by taking Hockers bare fiber number and multiplying by a scaler described by Tateda to generalize the jacketed fiber. This thermal-phase noise, because of its large magnitude poses a problem in the passive interferometric detection shown in Figure 2. As was discussed in an earlier section of this chapter, the detected acoustic phase (of a relatively small magnitude) will become a nonlinear function of thermal-induced noise phase (as seen in Equation (1.4)).

The thermal-induced noise phase, θ_{nT} , may be modeled as a function of index dispersion and length change such that

$$\theta_{nT}(t) = \left[\left(\frac{dn}{dT} \right) + \frac{n}{l} \left(\frac{dl}{dT} \right) \right] k l T(t) , \quad (1.6)$$

where T is static temperature and $T(t)$ is the dynamic temperature representing environmental fluctuations. If one has any understanding at all of the dynamics of heat flow in materials, he/she would scoff at Equation (1.6). It is obvious that provisions have not been made to describe the diffusion time for heat flow through the fiber. Equation (1.6) provides a mere superficial insight to the mechanisms of thermal-induced phase shifting. Due to the 2π

⁷G.B. Hocker, "Fiber-Optic Sensing of Pressure and Temperature," Applied Optics 18 (May 1979): 1445-48.

⁸M. Tateda, S. Tanaka, and Y. Sugawara, "Thermal Characteristics of Phase Shift in Jacketed Optical Fibers," Applied Optics (March 1980): 770-73.

degeneracy ($n = 0, \pm 1, 2, \dots$) of $\cos(\theta_{nT}(t))$ and the large temperature sensitivity of the fiber, θ_{nT} may be considered a random variable uniformly distributed between 0 and 2π .

The Rate Gyroscope

The basic configuration of the fiber optic rate gyroscope is shown schematically in Figure 3. This interferometer will produce a phase shift (via the sagnac effect) that is a function of rotation rate, Ω , of the sensor. Equal nonreciprocal phases (of equal magnitude and opposite sign) are induced in the two counter-propagating beams in the fiber and are seen to be proportional to the area, A , of the coil, the number of fiber loops, N , that comprise the coil and the angular rotation rate, Ω , of the coil expressed in radians/second. The phase shift, ϕ , measured by the interferometer may be expressed in equation form as

$$\phi = \mu NA\Omega . \quad (1.7)$$

The coefficient μ is a constant of proportionality and is equal to $4/\lambda c$, where λ is vacuum wavelength and c is the vacuum speed of light.⁹ Equation (1.7) is an ideal relationship and assumes a circular loop and collinear polarization between the two counter-propagating beams (so that the fiber birefringence does not alter

⁹ Grant R. Fowles, Introduction to Modern Optics (New York: Holt, Rinehart and Winston, 1975), p. 315.

Fig. 2. Basic configuration for fiber-optic acoustic sensor.

Fig. 3. Basic configuration for fiber-optic rotational rate sensor.

the expression).

It is possible to model the rotational rate sensor with the general interferometer Equation (1.4). The phase shift, ϕ , related to the rotation rate represents the input signal, $\theta_i(t)$; the static phase θ_s will be zero as there is no path length mismatch; the field amplitudes E_r and E_s will be a function of the laser output power and the input coupling efficiencies. The phase noise sources for the rate gyro will be much difference than those experienced in the acoustic sensor while the amplitude noise will be exactly the same. The difference in the phase noise stems from the interferometric paths (medium) for both beams being the same. For the system shown in Figure 3, thermal and acoustic fluctuations will not produce any phase noise as the phase shift they produce is reciprocal (that is, the phase produced by one propagation direction is canceled out by the same phase produced in the opposite propagation direction). The only appreciable phase noises in this type of system are 1) distributed Rayleigh backscattering over the entire fiber creating a superposition of signals out of phase with the transmitted beams,¹⁰ and 2) fiber birefringence which can cause the counter-propagating polarized beams to take different paths, therefore, altering the value of θ_s . If the fiber's birefringence is related to a randomly changing environment than the random path

¹⁰C.C. Cutler, A.S. Newton, and J.H. Shaw, "Limitation of Rotation Sensing by Scattering," Optics Letters 5 (November 1980): 488-90.

length difference will create a random drift phase.

Albeit the rotational rate sensor does not have the drift problem that the acoustic sensor has, it has problems that are just as complicated. They may be itemized as follows:

1. There is no way to tell the direction of rotation.
2. The system is nonlinear and demonstrates a poor dynamic range.
3. The measurement is a function of laser intensity.

These problems are easily identified when Equation (1.4) is used to model the fiber gyro in Figure 3. Some reasonable assumptions are taken allowing $\theta_s = 0$, $\theta_i = \phi$, $E_R = E_S = E$ and $\theta_n = 0$, to produce signal output,

$$s(t) = K \left[2E^2 (1 + \cos(\phi)) \right] . \quad (1.8)$$

It is indeed evident as the cosine operation produces an even function, that this signal is independent of the sign of ϕ . The sign of ϕ is most important in that the specifications for most rate gyroscopes mandate magnitude and direction. The system produces a nonlinear output as the cosine function is nonlinear and also suffers from a poor dynamic range in phase measurement resulting from the $2n$ ($n = 0, \pm 1, \pm 2, \dots$) degeneracy. It is also obvious that $s(t)$ varies as the optical intensity (which is proportional to E^2) varies.

CHAPTER II

SYSTEM DESCRIPTION AND OPERATION

System Introduction

It should now be evident, after a most laborious discussion, that passive fiber-optic interferometric sensors (or any interferometer) suffer from a wide variety of problems. In order to overcome these problems, it is obvious that the interferometer must be modified.

This paper introduces, describes and models a synchronous detection system used to detect phase in an optical interferometer. This system operates linearly, has the ability to function independently of large fluctuations in E_r and E_s , operates independent of θ_s and exhibits a large dynamic range. The system is called a heterodyne phase feedback interferometer (HPFI). The HPFI is a closed loop system and operates on the principal of feeding back an error signal to a phase shifter in the reference leg of the interferometer to maintain synchronization or phase lock to the signal leg.

The concept of the HPFI may be applied to any interferometric system. Each interferometric system has its own set of specifications such as dynamic range requirements, noise floor, bandwidth considerations, environmental conditions, packaging requirements, and

physical configuration. These specifications serve as an indicator on how the HPFI should be configured for the given system. For example, if one were to apply the technology of the HPFI to the case of the fiber-optic interferometric acoustic sensor, he/she would have to employ a phase shifter that can track out the large thermal induced phase shifts as it is seen that this phase may well be a few thousand radians. Suppose an interferometer is used to measure the refractive index of various samples. This system would be much different than the previously mentioned system in that this one would use an air path and would not need to track phase much more than a few radians in a very limited bandwidth. This system may employ a linear crystal as the phase shifter such as lithium niobate (which most certainly would not be suitable for the acoustic sensor).

It should indeed be obvious that the HPFI's configuration may vary from system to system. Enumeration on how the HPFI adapts itself to each different system is superfluous to the intent of this paper; the underlying motivations are more concerned with the fundamental behavior of the HPFI and the mathematical modeling used to describe its operating characteristics. The author wishes only to discuss (for reasons described earlier) fiber-optic interferometric systems used to detect acoustic signals and rates of rotation.

The Heterodyned Phase Feedback Interferometer (HPFI) is a system developed to accurately measure optical phase fluctuations in an active interferometric configuration. It overcomes many problems experienced by passive interferometers as discussed earlier in this paper. It acts as a linear detector, operates independently over a

wide range of optical intensity and polarization fluctuations, exhibits a very large dynamic range and is of a simple design configuration.

The HPFI is a second order synchronous detection system that employs a correction signal feedback. The term synchronous pertains to the system being locked in phase to the signal input. The HPFI's operation is similar to that of an electronic Phase Locked Loop (PLL) commonly used for detecting frequency-modulated signals. The PLL accomplishes demodulation and phase lock by feeding back an error signal (resulting from the difference of synchronization between the system and its input signal) to a voltage/current controlled frequency shifter. Since optical interferometric sensors produce phase-modulation, the HPFI accomplishes demodulation and phase lock by feeding an error signal back to a voltage/current controlled phase shifter.

System Operation

Figures 4, 5, 6, and 7 show the block diagrams that represent the HPFI in both acoustic or rotational rate detection configurations. The fundamental difference between Figures 4 and 5 and Figures 6 and 7 is the method utilized in producing the carrier signal. Figures 4 and 6 demonstrate creation of a carrier signal using phase-modulation techniques while Figures 5 and 7 demonstrate creation of a carrier frequency by using a Bragg diffractor (which is somewhat akin to a frequency modulation technique).

Figures 4 and 5 demonstrate the HPFI setup for acoustic detection. In Figure 4, light from a single frequency laser is split

Fig. 4. The HPFI configured for acoustic sensing employing a phase modulator to produce the carrier signal.

Fig. 5. The HPFI configured for acoustic sensing employing a Bragg diffractor to produce the carrier signal.

up to provide inputs to both legs of the interferometer. One leg, the signal leg, is optically phase modulated by the fiber optic coil exposed to the acoustic pressure field; while the reference leg is phase modulated by Mod 1 and Mod 2. (The methods of optical phase modulation will be discussed later in this text.) The beams are recombined, having collinear propagation, to produce an interference pattern on the detector. Mod 2 is sinusoidally driven at ω_m to produce a phase modulated signal, with a sufficient depth of modulation to produce phase excursions that exceed π radians to utilize the full contrast of the interference signal produced by the interferometer. Mod 2 essentially shifts the signal information up in frequency as sidebands to carrier frequencies of ω_m and its harmonics. The number and amplitude of the harmonics naturally depend on the phase excursion produced by the sinusoidal drive signal. The output signal from the optical square-law detector is cross correlated (via multiplication with an electronic mixer) to a reference signal at the carrier frequency ω_m . The low frequency portion of the cross correlated signal is fed back to Mod 1, the feedback phase modulator, and is denoted the error signal. When the loop gain of the feedback system is of a sufficient magnitude, it is seen that the phase produced by Mod 1, effectively duplicates the phase in the signal leg created by the acoustic perturbances. If the phase modulator in the reference leg is linear, then the output voltage, $f(t)$, is described as being a linear function of the phase produced in the signal leg (i.e., the demodulated output). It will be shown in subsequent sections that as the loop gain is increased,

the phase in the reference leg becomes a closer estimate of the phase in the signal leg. It is also seen that as the loop gain increases, the system damping (or stability) decreases. This indicates the need to modify the loop (low pass) filter of the HPFI for high gain operation.

It will be shown that this modification for a suitable system is simple to implement.

Inspection of Figure 5 reveals that the carrier frequency for signal information is produced in a different manner. In this design, an electro-optic modulator, a Bragg diffractor (which will be discussed in more detail later in this text), is used to produce two optical beams of different frequency spatially removed from each other. Despite the difference of the methods of producing the carrier signal, it is seen that the operation of both system (in Figures 4 and 5) in terms of system transfer function or output response are synonymous.

Figures 6 and 7 demonstrate the HPFI setup for rotational rate sensing utilizing the Sagnac effect. These systems are somewhat different than the acoustic sensors but the measurement system (HPFI) maintains the identical function as before. In the rotational rate sensor systems, a carrier signal is generated; the beams counter-propagate through the sensor coil to be recombined on the detector or detectors (the choice of one or two detectors is optional). The detected signal is cross-correlated with an unmodulated signal at the fundamental of the carrier frequency with the resultant low frequency component being processed and fed back to a phase modulator in the

Fig. 6. The HPFI configured for rotational rate sensing employing a phase modulator to produce the carrier signal

Fig. 7. The HPFI configured for rotational rate sensing employing a Bragg diffractor to produce the carrier signal.

interferometer. As in the case of the acoustic sensor, the feedback signal also constitutes the output signal of the measurement system.

The particular block diagrams shown in Figures 6 and 7 utilize evanescent field type coupling^{1,2} to constitute physical beam-splitting. This is a well known phenomenon in dielectric waveguides where the guided mode has wave energy beyond the propagation boundaries. This energy is directionally transferred to another waveguide by placing it close enough to the primary waveguide, thereby performing the task of a bulk beamsplitter. The evanescent field type coupling, depicted as dc (directional coupler) is schematically used in Figures 6 and 7 to give insight to an integrated optical device. These evanescent couplers may also be used in lieu of beamsplitters for the acoustic sensors represented in Figures 4 and 5.

All of the HPFI configurations discussed up to now employ processes that shift the information up in frequency. This feature is of tantamount importance when considering the effectiveness of an interferometric measurement system. This frequency shift of signal information enhances the interferometers ability in two ways:

1. It allows system operation in a region where detector

¹S.K. Sheem and T.G. Giallorenzi, "Single-Mode Fiber-Optical Power Divider: Encapsulated Etching Technique," Optics Letters 7 (January 1979): 29-31.

²R.A. Bergh, G. Kotler, and H.J. Shaw, "Single Mode Fiber Optic Directional Couplers," Electronic Letters 12 (February 1980): 18.

1/f noise and laser intermodulation noise is low.

2. It allows amplitude stabilization techniques (a clipper or AGC circuit) to be applied to prevent optical intensity fluctuations or acute polarization from altering the system dynamics.

For semiconductor optical detectors, the main contributions to 1/f noise relate to the properties of the surface of the material and how the generation and recombination of carriers occur.³ Laser intermodulation results from the multiline operation of most lasers. This multiline phenomenon is well known in lasers resulting from "inhomogeneously broadened" medias⁴ where the frequency spacing of the laser lines within the broadened gain curve is approximately equal to $c/2\ell$ where c is the speed of light in the laser medium and ℓ is the cavity length. This frequency spacing is constantly shifting as the spacing is a function of both random environmental conditions and the mismatch between the cavity resonance and the inverted mediums natural resonance. This constant shifting of the frequency spacing along with optical square-law detection techniques produces an intermodulation noise which dominates the lower frequencies.⁵ Both 1/f noise and intermodulation noise manifest

³C.C. Matchenbacher and F.C. Fitchen, Low-Noise Electronic Design (New York: John Wiley and Sons, 1973), p. 19.

⁴Amnon Yariv, Introduction to Optical Electronics, 2d ed, (New York: Holt Rinehart and Winston, 1976), p. 90.

⁵Spectra Physics, Model 124A Helium-Neon Gas Laser Instruction Manual (Mountain View California: Spectra Physics Inc., 1969), p. 8.

themselves as amplitude fluctuations. The HPFI can be easily configured to operate at frequencies well away from these noise sources.

Since the HPFI operates with a carrier frequency, it is a simple matter to stabilize the carrier level independent of optical intensity fluctuations or acute polarization rotations. The importance of the carrier amplitude stabilization will be elucidated in a subsequent chapter in this paper where it will be demonstrated that the loop gain, G , of the system is a function of the input carrier level. The most desirable HPFI configurations rely on G being a constant.

Carrier amplitude stabilization techniques employ either an automatic gain control (AGC) device or a hard limiter. Slow random phase fluctuations manifest themselves such that a hard limiter (which is much simpler than an AGC) cannot be used for HPFI's configured with the carrier signal being generated by a phase modulator. This stems chiefly from the insight that the phase modulated carrier generation technique generates harmonics (in fact, for maximum signal contrast, the second harmonic is almost the same size as the fundamental). These harmonics are also subject to the hard limiter. This type of process obfuscates the information to a point that proper operation of the system is impossible. Automatic gain control must be used for the case of phase modulated carrier generation. The only HPFI configuration where hard limiting is viable is where only one carrier exists, which obviously restricts operation to systems employing the Bragg diffractor.

Optical Modulation

Bragg Diffraction

Figure 8 schematically represents the operation of a Bragg diffractor. A single frequency ultrasonic wave is produced by a piezoelectric transducer and coupled into an isotropic homogeneous medium. The effect that this traveling wave has on the interaction medium is that of a sinusoidally varying index distribution and may be conceptualized with the analogy of the device being a dynamic diffraction grating which doppler shifts the diffracted light. A commonly used transducer to produce the ultrasonic signal is lithium niobate (LiNbO_3): a common diffraction interaction medium is lead molybdate (PbMoO_4).

When input light to the Bragg diffractor is at the Bragg angle, ϕ_β , where

$$\phi_\beta = \frac{1}{2} \lambda / \Lambda, \quad (2.1)$$

where λ is the free-space wavelength of the light and Λ is the acoustic wavelength in the interaction medium, for certain given conditions such as sufficient ultrasonic power and long enough interaction length, all of the diffracted light will constructively interfere in one direction giving rise to a single diffracted beam at a shifted optical frequency.⁶

⁶Robert Adler, "Interaction Between Light and Sound," IEEE Spectrum 42 (May 1967): 42-54.

If this diffraction phenomenon is modeled on the particle level, the theory of conservation of momentum predicts that the acoustically produced phonon and the input photon are annihilated with the simultaneous creation of a new photon at a new propagation direction. Figure 9 shows the momentum diagram for propagation. Conservation of energy describes the upshift of frequency for the scattered light which is simply equal to the acoustic frequency. The angle, ϕ , the scattered light is shifted from the unscattered light is described by Yariv⁷ as

$$\phi = \lambda / \Lambda = 2 \phi_{\beta} \quad . \quad (2.2)$$

Optical Phase Modulation

The optical phase modulators shown in Figures 4, 5, 6, and 7 will be defined as any transducer that produces a change in optical phase that varies linearly with its input signal. From previous discussion, it was elucidated that different interferometric measurement systems see a large range of phase magnitudes to synchronize to. The acoustic sensor may have to track thousands of radians while the rotation rate sensor might only require a trackability of around ten radians.

For the fiber optic acoustic sensor, a very effective modulation technique is accomplished by wrapping the fiber tightly around a thin

⁷Amnon Yariv, Quantum Electronics, 2d ed. (New York: John Wiley and Sons, 1975), p. 359.

Fig. 8. Bragg diffractor.

Fig. 9. Momentum diagram for Bragg diffractor.

walled piezoelectric cylinder. This type of modulator induces an optical phase shift by stretching the fiber resulting from an electro-mechanical radial displacement of the cylinder.⁸ If these piezoelectric devices are operating at frequencies well below their resonant frequency, they may be modeled as linear devices exhibiting a linear relationship between the applied voltage and the induced optical phase shift.⁹ In the optics laboratory at the Naval Research Laboratory's Underwater Sound Reference Detachment in Orlando, FL, a 2-in. diameter PZT-4 (lead zirconate titanate) cylinder with a one-eighth of an inch thickness wall 2-in. long with approximately 10 m of fiber wrapped around it is capable of producing a phase shift (using the 6328Å line from a helium-neon laser) of approximately 10 radians/volt. This cylinder is capable of sustaining voltages of up to ± 500 V and operates linearly up to approximately 2000 Hz. This predicts a trackability of this phase modulator of ± 5000 radians.

The rotational rate sensor does not require such a large tracking range and a suitable phase modulator may be realized by utilizing the Pockels effect¹⁰ seen in certain crystals. The Pockels

⁸Warren P. Mason, ed., Physical Acoustics (New York: Academic Press, 1964), p. 220-22.

⁹D.A. Jackson, A. Dandridge, and S.K. Sheem, "Measurements of Small Phase Shifts Using a Single-Mode Optical Fiber Interferometer," Optics Letters 14 (April 1980): 139-41.

¹⁰T. Tamir, ed., Topics in Applied Physics, Integrated Optics (New York: Springer-Verlag, 1975), p.148.

effect is a linear electro-optical effect observed in certain crystals that produces a birefringence linearly proportional to the applied transverse electric field. To produce just phase-modulation and not polarization rotation the input beam (which is assumed to be linearly polarized) is lined up to have its polarization parallel to one of the crystals birefringent axes. The crystal most commonly used for this type of optical phase-modulation is lithium niobate (LiNbO_3). This is due in part to its inherent large electrooptic coefficient and also to the ability to use the crystal in an integrated optic configuration. These lithium niobate modulators have the ability to operate linearly up to tens of megohertz with a range up to about ± 5 radians using conventional components that generate voltage swings of ± 20 V. These numbers are based on general handbook values and currently achievable crystal geometries.^{11,12}

¹¹Walter G. Driscoll, ed., Handbook of Optics (New York: McGraw-Hill Book Company, 1978), p. 17.7-13.

¹²James H. Cole, interview held during office hours at Naval Research Laboratory, Washington, DC, March 1981.

CHAPTER III

LINEAR ANALYSIS

Earlier in this paper, the HPFI was described as a second-order nonlinear system. The nonlinearity results from the correlation technique (the mixer) used in comparing the interferometer's phase to the reference phase (from the oscillator) which produces the sine of the difference of both of the phases. As can be expected, this nonlinear term manifests itself as a malignant menace in the system modeling such that simple solutions are completely out of the question. Careful inspection of the system equation shows that when one is interested in the steady-state response, the nonlinearity may be replaced with a first-order, linear approximation providing certain provisions on system parameters are met. This linearization creates an amenable environment for the linear-minded engineering community and standard control theory techniques may be utilized in analyzing the system. This chapter develops the model for the HPFI and uses the linear approximation to analyze and discuss the system performance and operation.

The Model

General Development

A statement was made in the last chapter to the effect that the

modeling for the HPFI is consistent regardless of the interferometer or method of producing the carrier signal (this will become evident as the discussion continues). With this in mind, the system in Figure 4 (the acoustic sensor with a phase-modulated carrier signal) will arbitrarily be chosen to be modeled.

The interference pattern produced by the interferometer in Figure 4 may be generalized by Equation (1.4) with the introduction of two more phase terms, ϕ_m and θ_m , produced by the respective phase modulators Mod 1 and Mod 2. The output of the optical square-law detector will yield

$$s(t) = \left[K E_r^2 + E_s^2 + 2E_r E_s \left(\cos(\theta_s + \theta_i(t) + \theta_n(t) - \phi_m(t) - \theta_m(t)) \right) \right]. \quad (3.1)$$

From a previous discussion, ϕ_m , is the feedback phase to keep the interferometer synchronized to the input while θ_m is phase produced to generate the carrier and may be represented by the expression

$$\theta_m = \beta \sin \omega_m t, \quad (3.2)$$

where ω_m is the radian frequency of the oscillator. For the case of the acoustic sensor, θ_n will take on the value of θ_{nT} (the temperature-induced phase shift). If the phase terms are lumped together

such that

$$\theta_i + \theta_{nT} + \theta_s - \phi_m = \phi_i - \phi_m = \phi_e \quad (3.3)$$

and only being interested in the ac component of the interference pattern,

$$s(t)_{ac} = 2E_r E_s K \left[\cos(\beta \sin \omega_m t - \phi_e) \right] . \quad (3.4)$$

This expression may be rewritten with the aid of some trigonometric identities to reveal

$$\begin{aligned} s(t)_{ac} = 4KE_r E_s \left[\sum_{n=1}^{\infty} J_{2n}(\beta) \cos(2n\omega_m t) \cos\phi_e \right. \\ \left. + \sum_{n=0}^{\infty} J_{2n+1}(\beta) \sin \left\{ (2n+1)\omega_m t \right\} \sin\phi_e \right] ; \end{aligned} \quad (3.5)$$

or by rearranging the terms

$$\begin{aligned} s(t)_{ac} = 4KE_r E_s \left\{ (J_1(\beta) \sin\phi_e) \sin\omega_m t + (J_2(\beta) \cos\phi_e) \cos 2\omega_m t \right. \\ \left. + (J_3(\beta) \sin\phi_e) \sin 3\omega_m t + (J_4(\beta) \cos\phi_e) \cos 4\omega_m t + \dots \right\} . \end{aligned} \quad (3.6)$$

Where $J_K(X)$ represents a Bessel function of the first kind of order K and argument X .

If it is known that $\omega_m \gg \omega_c$, where ω_c is the cutoff frequency (-3 dB point) for the low-pass filter, it is most obvious that the only significant term in $s(t)_{ac}$ (Equation (3.6)) is the first one ($J_1(\beta) \sin \phi_e \sin \omega_m t$). The rest of the terms will be greatly attenuated. It can therefore be stated that for the case of $\omega_m \gg \omega_c$,

$$s(t)_{eff} = 4KE_r E_s J_1(\beta) \sin(\phi_e) \sin(\omega_m t) . \quad (3.7)$$

The mixer is modeled as a multiplier whose inputs are $s(t)$ and $\sin \omega_m t$. It is a well known fact that multiplication of sinusoids produces sinusoids at the sum and difference frequencies of the input frequencies (or product terms). If the same logic as before ($\omega_m \gg \omega_c$) is employed it is seen that the only term that is passed by the low-pass filter is the difference frequency. The effective input signal to the low-pass filter is

$$f_o(t)_{eff} = 2E_r E_s K J_1(\beta) K_1 \sin(\phi_e) = G_1 \sin(\phi_e) , \quad (3.8)$$

where K_1 is the gain of the AGC and amplifier and $G_1 = 2E_r E_s K J_1(\beta) K_1$. This term can also be expressed in the frequency domain via the Laplace transform

$$F_o(s)_{eff} = G_1 \mathcal{L}\{\sin \phi_e(t)\} ; \quad (3.9)$$

which makes it rather easy to obtain the expression for the system output

$$F(s) = L(s)G_1 \mathcal{L}\{\sin\phi_e(t)\} , \quad (3.10)$$

where $L(s)$ is the transfer function that models the low-pass filter. The expression for the feedback phase, $\phi_m(t)$, may be simply modeled by the equations

$$\phi_m(t) = Cf(t)$$

or

$$\phi_m(s) = CF(s), \quad (3.11)$$

where C may be described as the modulator constant expressed in radians/volt. Note that the modulator is assumed to be constant over all frequencies. This is generally not true, but will be assumed to hold for the frequencies of interest. Combining Equations (3.10) and (3.11) complete the feedback loop and produce the system equation

$$\phi_m(s) = CG_1L(s) \mathcal{L}\{\sin\phi_e(t)\} . \quad (3.12)$$

The low-pass filter used in this analysis is shown in Figure 10, where its transfer function is described as

Fig. 10. Second order low-pass filter.

$$\frac{Y(s)}{X(s)} = L(s) = \frac{\lambda^2}{s^2 + 2s\lambda + \lambda^2}, \quad (3.13)$$

where λ is defined to be $1/RC$ or the inverse time constant of one of the filters. It is assumed that the second filter poses no loading problems on the first. This is guaranteed by placing a unity gain buffer amplifier between the two filter sections which operates with a large input impedance and a low output impedance. The combination of Equations (3.3), (3.12), and (3.13) result in the expression

$$(s^2 + 2s\lambda + \lambda^2) \Phi_m(s) = G\lambda^2 \mathcal{L}\{\sin(\phi_i(t) - \phi_m(t))\}, \quad (3.14)$$

where $G = CG_1$. If both sides of this equation are inverse Laplace transformed, one obtains

$$\begin{aligned} \mathcal{L}^{-1}\{s^2 \Phi_m(s)\} + 2\lambda \mathcal{L}^{-1}\{s \Phi_m(s)\} + \lambda^2 \mathcal{L}^{-1}\{\Phi_m(s)\} \\ = G\lambda^2 \sin(\phi_i(t) - \phi_m(t)); \end{aligned}$$

or

$$\begin{aligned} \ddot{\phi}_m(t) + \mathcal{L}^{-1}\{s\phi_m(0)\} + \mathcal{L}^{-1}\{\dot{\phi}_m(0)\} + 2\lambda[\dot{\phi}_m(t) \\ + \mathcal{L}^{-1}\{\phi_m(0)\}] + \lambda^2\phi_m(t) = G\lambda^2 \sin(\phi_i(t) - \phi_m(t)). \end{aligned} \quad (3.15)$$

If it is assumed that the feedback phase, $\phi_m(t)$, and its time derivative are zero, the resulting terms of Equation (3.15) are

$$\ddot{\phi}_m + 2\lambda\dot{\phi}_m + \lambda^2\phi_m = G\lambda^2 \sin(\phi_i - \phi_m) , \quad (3.16)$$

where the time variation for both variables, ϕ_i and ϕ_m , is assumed.

Equation (3.16) will also result when modeling the system in Figure 5 (which uses a Bragg diffractor to produce the carrier signal). To demonstrate how the Bragg diffractor works, the interferometer equations will be rederived.

Making the same assumptions as before as to the monochromatic planar nature of both optical fields, $\overline{E}_{r1}$ and $\overline{E}_{s1}$, in Figure 5 may be expressed

$$\overline{E}_{s1}(t) = E_{s1} \exp[i((\omega_o + \omega_m)t + \theta_{s11} + \theta_{i1}(t) + \theta_{n1}(t))]$$

and

(3.17)

$$\overline{E}_{r1}(t) = E_{r1} \exp[i(\omega_o t + \theta_{s12} + \theta_{n2})] .$$

If the same optical detector is used and is ac coupled, the output voltage, $s(t)$, of the square-law detector yields

$$s_1(t) = 4KE_{r1}E_{s1} \cos(\omega_m t + \phi_{e1}) , \quad (3.18)$$

where $\phi_{e1} = \phi_{i1} - \phi_{m1}$.

If the gain of the amplifier is negative and the input to the mixer (multiplier) is $\sin \omega_m t$,

$$\begin{aligned} f_{ol}(t) &= -4KE_{r1}E_{s1}K_1 \cos(\omega_m t + \phi_e) \sin \omega_m t \\ &= -2KE_{r1}E_{s1}K_1 [\sin(2\omega_m t + \phi_e) - \sin \phi_e] . \end{aligned} \quad (3.19)$$

For $\omega_m \gg \omega_c$, the equation reduces to

$$f_{ol}(t) = 2K_1E_{r1}E_{s1} \sin(\phi_e) = G_{11} \sin \phi_e , \quad (3.20)$$

where $G_{11} = 2K_1E_{r1}E_{s1}$.

It is clearly obvious that Equation (3.8) is very much like Equation (3.20). The only difference between the two is how the individual amplitude terms make up the respective gain terms, G or G_{11} . This is irrelevant to the model in that the terms are treated as a constant and may be set by changing the gain of an amplifier in either of the feedback loops. The feedback modulators to both systems are functionally identical, hence the models representing both systems are functionally identical. If one were to model the rate sensors in Figures 6 and 7, the end result (as should be expected) will be an expression just like the one of Equation (3.16).

Linearization of the Model

Inspection of the system Equation (3.16) shows a second-order

nonlinear system. The nonlinearity arises from the dependent variable being a part of the argument of the sine function (a nonlinear operator).

If one, for the time being, assumes a slowly varying input phase (ϕ_i), such as a thermal drift phase, the only term on the left-hand side of Equation (3.16) (assuming steady state conditions) of any significant value will be the $\lambda^2 \phi_m$ term; remember, the main objective for this system is for ϕ_m to duplicate ϕ_i so ϕ_m will be slowly varying also. An approximation to this occurrence may be written as

$$\phi_m \approx G \sin(\phi_i - \phi_m) \quad \left| \begin{array}{l} 1. \quad \lambda \phi_m \gg 2\dot{\phi}_m \\ 2. \quad \lambda^2 \phi_m \gg \ddot{\phi}_m \end{array} \right. \quad (3.21)$$

where the λ^2 term has been cancelled out of both sides. It is fairly obvious to see that the maximum value of the right side of this equation is limited to G , the loop gain (expressed in radians). This allows a maximum value for ϕ_m to be established,

$$|\phi_m|_{(\max)} = G. \quad (3.22)$$

This is a most interesting consequence (although not so surprising) in that this equation is very similar to the lock range

equation¹ for a PLL which is

$$\Delta\omega_L = K_v, \quad (3.23)$$

where $\Delta\omega_L$ represents the frequency tracking ability of the PLL and K_v represents its loop gain expressed in radians/second.

Equation (3.21) provides some very interesting insight to the nonlinearity of the HPFI. It provides especially interesting results for sufficiently large G . For example, if G is arbitrarily chosen to be 100, Equation (3.22) produces $|\phi_m|_{(\max)} = 100$. From Equation (3.21), for $\phi_m = 100$, it isn't difficult to determine that $\phi_i - \phi_m = \pi/2$; hence $\phi_i = 101.57$. This suggests a fairly linear relationship between ϕ_i and ϕ_m (recall that the sine function is linear for small angles and does not bring in the higher ordered terms of its power series until the angle gets larger). To test this idea, the nonlinear part of Equation (3.21) is replaced with a linear substitute, or specifically

$$G(\phi_i - \phi_m) \leftarrow G \sin (\phi_i - \phi_m) . \quad (3.24)$$

For the case of $G = 100$, this substitution, when placed into Equation (3.21) yields $\phi_i = 101.0$. The worst case error (as the nonlinearity of the sine function becomes worst case) between the

¹Floyd M. Garner, Phaselock Techniques (New York: John Wiley and Sons, 1966) p. 36.

linear approximation and the nonlinear equation is 0.57%. It can be easily demonstrated that as G increases this worst case error will decrease.

This past example makes a pretty powerful statement about the steady state operation of the HPFI operating with a G of about 100 or more and adhering to the conditionalities of Equation (3.21). It says that the nonlinearity (for large enough G) may be for the most part ignored when studying the steady-state behavior of the HPFI when it experiences a slowly varying input phase.

Up to this point, the linearization of the system equation for the HPFI has only been considered for the case of slowly varying input phase (ϕ_i). This analysis obviously does not cover the general case for all frequencies as it would violate the constraints of Equation (3.21). It would be most desirable to generate the criteria for linearization of the system equations for all frequencies. This turns out to be possible and will be seen to work quite well. Before plunging into this analysis it would be beneficial to describe some parameters esoteric to the fiber optic sensors being discussed.

1. Analysis shows that the input radian frequency never needs to be more than 4λ .
2. These higher frequency signals will be of a much smaller magnitude (in phase excursion) than the earlier described low-frequency phase. These signals for both the acoustic

and rotational rate sensors rarely need to exceed levels larger than a radian.

A worst case analysis will be performed for a system operating at a frequency of 4λ . Suppose an input signal is generated to produce some output signal

$$\phi_m(t) = A \sin \omega t ,$$

then

$$\dot{\phi}_m(t) = A\omega \cos \omega t , \quad (3.25)$$

and

$$\ddot{\phi}_m(t) = -A\omega^2 \sin \omega t .$$

If these values are plugged into Equation (3.16) and, allowing $\omega = 4\lambda$, one sees

$$-A\omega^2 \sin \omega t + A/2 \omega^2 \cos \omega t + \frac{A\omega^2}{16} \sin \omega t = G \frac{\omega^2}{16} \sin(\phi_i - A \sin \omega t)$$

or

$$-15 A \sin \omega t + 8 A \cos \omega t = G[\sin(\phi_i - A \sin \omega t)]$$

simplying to get

$$17 A \left(\frac{-15}{17} \sin \omega t + \frac{8}{17} \cos \omega t \right) = G \sin (\phi_i - A \sin \omega t); \quad (3.26)$$

letting

$$\cos \alpha = \frac{15}{17} ; \quad \sin \alpha = \frac{8}{17} . \quad (3.27)$$

Equation (3.26) is reduced to

$$17A (\cos \omega t \sin \alpha - \sin \omega t \cos \alpha) = G \sin (\phi_i - A \sin \omega t)$$

or

$$-17A \sin(\omega t - \alpha) = G \sin (\phi_i - A \sin \omega t); \quad (3.28)$$

solving for ϕ_i , one finds

$$\phi_i = \sin^{-1} \left[\frac{-17A}{G} \sin(\omega t - \alpha) \right] + A \sin \omega t. \quad (3.29)$$

Notice here that if the largest value of A is assumed (1 radian) and also the largest value of $\sin (\omega t - \alpha)$ is assumed (unity), the worst case condition for describing the nonlinearity of the system is met, leaving

$$\phi_{i(wc)} = \sin^{-1} \left[\frac{17}{G} \right] + \sin(\alpha - \pi/2) \quad (3.30)$$

from the CRC math tables,² the power series expansion for the inverse sine function is described for an argument x as

$$\sin^{-1}(x) = x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} x^5 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7} x^7 + \dots \quad (3.31)$$

$$(x^2 < 1; -\pi/2 < \sin^{-1}(x) < \pi/2)$$

²Samuel L. Selby and Brian Girling, eds., CRC Standard Mathematical Tables, 14th ed., (Cleveland, Ohio: The Chemical Rubber Company, 1964), p. 409.

If G is assumed to be 100 or greater, as in the low-frequency analysis, one finds that all of the higher order terms of the power series may be neglected as they are less than one percent of the magnitude of the first order term. This indeed implies an accurate approximation for a linear function, which implies a linear system. It is therefore permissible to rewrite the linear version of Equation (3.16) as an accurate approximation of the HPFI

$$\ddot{\phi}_m + 2\lambda \dot{\phi}_m + \lambda^2 \phi_m = G\lambda^2(\phi_i - \phi_m) . \quad (3.32)$$

Applications and Verification of the Linear Model

System Analysis

Frequency domain techniques utilized to analyze linear closed-loop systems seem to require less energy (calculations) than time domain techniques. It is a well known fact that the author most dutifully follows the lowest energy approach and is self justified by performing the system analysis in the frequency domain.

If "0" initial conditions are assumed for ϕ_m and $\dot{\phi}_m$, Equation (3.32) may be Laplace transformed to reveal

$$(s^2 + 2s\lambda + \lambda^2)\phi_m(s) = G\lambda^2(\phi_i(s) - \phi_m(s)) , \quad (3.33)$$

which may be arranged to produce the transfer function, $H(s)$, describing the ratio of the system output, $\phi_m(s)$, to the system input, $\phi_i(s)$, obtaining

$$\overline{H(s)} = \frac{\phi_m(s)}{\phi_i(s)} = \frac{G\lambda^2}{s^2 + 2s\lambda + \lambda^2(1 + G)} \quad (3.34)$$

From the theory of Laplace transforms, one may recall that the system transfer function represents the Laplace transform of the impulse response of the system. Since the impulse has a flat spectrum, $\overline{H(s)}$ represents the normalized steady state frequency response of the system. The amplitude response is determined by solving for the magnitude of $\overline{H(s)}$; the phase is determined by standard techniques from complex variable algebra. For $s = i\omega$,

$$\text{Mag } \overline{H(s)} = H(s) = \frac{G\lambda^2}{\left\{4\omega^2\lambda^2 + [\lambda^2(1 + G) - \omega^2]^2\right\}^{1/2}} \quad (3.35)$$

and

$$\text{Phase } \overline{H(s)} = H(s)\angle\theta = \tan^{-1} \left[\frac{2\omega\lambda}{\lambda^2(1 + G) - \omega^2} \right] \quad (3.36)$$

It is quite obvious that the system's magnitude and phase vary with the loop gain, G , the inverse time constant, λ , and the radian frequency, ω .

The natural frequency, f_n , for the HPFI will be defined as the frequency at which $H(s)$ is a maximum. To determine the natural frequency, one simply differentiates $H(s)$ with respect to ω and sets it equal to zero. The resulting equation is

$$\omega_n = \lambda \sqrt{G - 1} \quad \text{or} \quad f_n = \frac{\lambda \sqrt{G - 1}}{2\pi} . \quad (3.37)$$

The natural frequency will be an aid in determining the system bandwidth and estimating the system performance.

The system bandwidth may be described by the cutoff frequency, f_c , which is defined to be the frequency where the output signal is attenuated by 3 dB from the input signal. To determine f_c , $H(s)$ is equated to 0.707 (-3 dB) to explicitly solve for ω_c ($=2\pi f_c$) which reveals

$$f_c = \lambda/2\pi \left\{ G - 1 + (2G^2 - 4G)^{1/2} \right\}^{1/2} \quad (3.38)$$

A close inspection shows that for $G \geq 10$, the cutoff frequency may be accurately approximated as

$$f_c = 1.55 f_n \quad (G \geq 10) . \quad (3.39)$$

It turns out (and is lucidly demonstrated) that the HPFI requires G to be much larger than 10 to produce accurate detection. A useful parameter to describe the stability of a system is the damping ratio, δ . It is determined by the placement of the complex poles of the system transfer function. It is defined as the sine of the angle made from the imaginary axis to the pole in the second quadrant with

the origin at the vertex.³ For the transfer function in Equation (3.34), δ may be expressed as

$$\delta = \frac{1}{\sqrt{1 + G}} . \quad (3.40)$$

The important feature to note here is that the damping is only a function of G .

A typical system in the laboratory operates with a λ of any value and G between 50 and 2000. Figures 11, 12, and 13 show plots of $H(s)$ and $H(s)\angle\theta$ for $\lambda = 2100$, $G = 1000, 500, 200$, and 50. Figure 11 demonstrates increased bandwidth with increased G ; Figure 12 is the same as Figure 11 but is normalized to f_n . One may take interest in noting that when G is increased, the system amplitude at f_n is increased. This is inversely related to the damping of the system. Inspection of Equation (3.35) produces

$$H(f_n) = \sqrt{G/2} . \quad (3.41)$$

The plot in Figure 13 is an expanded version of Figure 11 which exemplifies the system's output for a flat input spectrum in amplitude and phase. The dashed line for the plot of $H(s)$ represents

³James L. Melsa and Donald G. Schultz, Linear Control System (New York: McGraw-Hill Book Company, 1969), p. 154.

Fig. 11. Theoretical response; $\lambda=2100$, G is variable.

Fig. 12. Theoretical response; $\lambda=2100$, G is variable.

Fig. 13. Theoretical response; $\lambda=2100$, G is variable.

the system input. One may take note that the system with a G of 1000 produces the most desirable output (in that ϕ_m more accurately duplicates ϕ_i) response for a 2 kHz input signal bandwidth. The system with $G = 1000$ also has the highest bandwidth and the smallest damping ratio. From Equation (3.40) the damping ratio for $G = 1000$ is calculated to be 0.0316. This is a rather small δ and may create some system disadvantages such as overshoot causing system instability or loss of lock. One might be inclined to use the results in Figure 13 as a design aid (as the author did some time ago). This practice will not yield much information as each curve on the plot contains a different natural frequency. If one looks at Figure 12, where the curves are normalized to their respective natural frequencies, it would become lucidly evident that the curves for $H(s)$ are almost identical when the frequency is not near the natural frequency. This would tend to indicate that the natural frequency governs the system response (at lower frequencies than f_n) while the loop gain governs the damping. With all this information at hand, the design engineer can meet various specifications by choosing the proper f_n for system response and G for dynamic range and damping ratio, while λ will "pop out" of the design equations. Figures 14 and 15 demonstrate how f_n can be used to pick the system response. In Figure 14, f_n is seen to be the same for all curves. This is accomplished by picking various pairs of G and λ that satisfy Equation (3.27) for a constant f_n . In this figure, f_n is approximately 10 kHz ($f_c \approx 15.5$ kHz). Note that $H(f_n)$ varies

Fig. 14. Amplitude response for $f_n = 10$ kHz.

Fig. 15. Expanded view of Figure 14.

for various G but that $H(f \ll f_n)$ is relatively independent of G . The design engineer is interested in the system response where the output duplicates the input. This is seen to occur for $f \leq .1f_c$. Figure 15 demonstrates this phenomenon; it shows the first 2 kHz of the curves in Figure 14. In this expanded view, it is very obvious that the curves are almost identical.

Verification of the Linear Model

Up to this point, the development of this paper has been completely theoretical. The author now wishes to present some laboratory results to provide a little corroboration to this most enlightening theory.

A system was configured to demonstrate the HPFI with a Bragg diffractor (Figure 5) at the Naval Research Laboratory's Underwater Sound Reference Detachment (Orlando, FL). The laser used was a Tropel-100 single frequency (which provides a coherence length well over 100 m) helium neon 1-mw source of wavelength 0.6328 microns. The Bragg diffractor used was manufactured by Isomet and consists of a series 230 driver operating at 80 MHz and a model #232-1 lead molybdate modulator. The fiber used was ITT-T16-01 step index fiber which has the property of propagating single mode (HE_{11}) optical radiation of wavelengths greater than 0.6 microns. A RCA 4840 photomultiplier tube with a pinhole input was used to detect the recombined beams. The output current from the photomultiplier is ac coupled, converted to a voltage, multiplied by the carrier signal, filtered and fed back to Mod 1. Mod 1 consists of a length

of fiber wrapped around a piezoelectric cylinder. Its modulator constant was measured to be 4.08 radians/volt.

To test the theory generated for the HPFI, one must have a controlled (or known) input signal. This is most easily implemented by replacing the acoustic environment with another phase modulator. This modulator was measured to produce a phase of 7.87 radians per applied volt. This acoustic simulator modulator was driven with a known voltage producing a known phase shift and swept from 100-2000 Hz in 100 Hz increments. This was done for two different system configurations where in the first configuration f_n is set to 6000 Hz and the second configuration is operated at $f_n = 4000$ Hz. The natural frequencies of the individual system configurations were determined by observing the system output, $f(t)$, on a spectrum analyzer. The hump produced in the noise floor at the natural frequency was easily discernable in both cases and probably accurate to $\pm 3\%$. Once f_n is known, G may be determined from Equation (3.37) providing λ is known (λ was measured and is 2200 in both cases). Once G is determined, then Equation (3.35) may be utilized to theoretically predict the system's amplitude response.

Figures 16 and 17 compare the theoretical and actual responses of both systems. The theoretical response is represented by the dashed line while the laboratory data is shown with the solid line. It is no surprise that the theoretical and the empirical data are highly correlated. This evidence should provide corroboration to the carefully drawn assumptions implemented to generate the linear

Fig. 16. Amplitude response for $G = 295$, $\lambda = 2200$.

Fig. 17. Amplitude response for $G = 131$, $\lambda = 2200$.

estimation of the nonlinear model.

Still, the most cynical of researchers may raise their eyebrows at the fact that the entire transfer function was not tested. This was done mainly in part relating to the lower (frequency) end of the transfer function being the important (or of most interest) area for it contains the useful demodulation band (where $\phi_i = \phi_m$). The rest of the transfer function isn't that particularly important, except for locating the natural frequency. To be completely thorough, a test of the entire transfer function was performed (at a much later date). In this experiment, $\lambda = 3225$, $f_n = 4800$ Hz, and $G = 38.4$. This experimental setup was identical to the other two, except that the feedback modulator was changed to one with a constant of 2.4 radians/volt. Figure 18 compares the theoretical and actual response of the system; again both are highly correlated.

The Solution to the Step Response and Its Effect on System Damping

It is a well known fact that any linear second order feedback system will "ring" at its fundamental frequency upon the receipt of a discontinuous input signal. The two most common input signals are the step and impulse. Since the step function is more physically realizable, it is chosen to be used to investigate the HPFI's behavior when responding to a discontinuity.

The step response to the HPFI may be easily obtained by starting with Equation (3.34). Dividing Equation (3.34) by s and taking the inverse Laplace transform of $H(s)/s$ with the assumption of "zero" initial conditions yields the step response, $r(t)$, to a unit step of

Fig. 18. Theoretical and actual response of HPFI; $\lambda = 3230$, $G = 38.4$.

phase starting at $t = 0$. Described in equation form

$$r(t) = -1 \left\{ \frac{H(s)}{s} \right\} = -1 \left\{ \frac{G\lambda^2}{s(s^2 + 2s\lambda + \lambda^2(1 + G))} \right\} \quad (3.42)$$

simplifying the expression

$$r(t) = -1 \left\{ \frac{G^2}{s[s + \lambda(1 + i\sqrt{G})][s + \lambda(1 - i\sqrt{G})]} \right\} \quad (3.43)$$

The technique of partial fraction expansion will be used to recognize known transform pairs.

$$\frac{G^2}{s[s + \lambda(1 + i\sqrt{G})][s + \lambda(1 - i\sqrt{G})]} = \quad (3.44)$$

$$\frac{A}{s} + \frac{B}{s + \lambda(1 + i\sqrt{G})} + \frac{C}{s + \lambda(1 - i\sqrt{G})}$$

where

$$A = \frac{G}{1 + G} ;$$

$$B = \frac{-(G + i\sqrt{G})}{2(1 + G)}$$

$$C = \frac{-(G - i\sqrt{G})}{2(1 + G)}$$

The three inverse transforms are taken and are combined via laws of superposition for linear systems to obtain

$$r(t) = \frac{1}{1+G} \left[G - \left(\frac{G + i\sqrt{G}}{2} \right) e^{-\lambda(1 + i\sqrt{G})t} - \left(\frac{G - i\sqrt{G}}{2} \right) e^{-\lambda(1 - i\sqrt{G})t} \right]; \quad (3.45)$$

which may be simplified using some trigonometric relations to produce

$$r(t) = \frac{1}{1+G} \left[G - \sqrt{G^2 + G} e^{-\lambda t} \cos(\lambda\sqrt{G}t - \theta) \right]; \quad (3.46)$$

where $\theta = \tan^{-1}(G^{-1/2})$.

This equation represents the step response of the linearized model of the HPFI. Inspection of the equation shows no great surprises. At $t = 0$, the system output $r(t)$ is zero (as is expected relating to the boundary conditions), and as time increases to infinity, the resulting steady state solution reduces to

$$r(t)_{ss} = \frac{G}{1+G}. \quad (3.47)$$

If one goes back to Equation (3.46) and defines the fundamental radian frequency of the step response to be $\omega_f (= \lambda\sqrt{G})$; λ may be expressed in terms of the damping ratio, δ , as

$$\lambda = \frac{\omega_f \delta \sqrt{G+1}}{\sqrt{G}}. \quad (3.48)$$

If $G \geq 50$, an accurate approximation to this equation is seen as

$$\lambda = \omega_f \delta \quad ; \quad (3.49)$$

$$G > 50$$

rewriting Equation (3.46) one obtains

$$r(t) = \frac{1}{1+G} \left[G - \sqrt{G^2 + G} e^{-\omega_f \delta t} \cos(\omega_f t - \theta) \right] . \quad (3.50)$$

It is quite obvious that the decay time of the oscillation at the system's fundamental frequency is a function of ω_f and δ (one might also note that for large G ($G > 50$) $\omega_f = \omega_n$).

Figures 19 and 20 show the theoretical step response for various combinations of λ and G ($f_n = 10,000$) chosen for Figure 14. It simply demonstrates a quicker decay time for a larger δ . This indicates that if one desires a system that stabilizes quickly, δ should be large (or G is small) which sacrifices system dynamic range (recall Equation (3.22)). This obviously indicates that a compromise be made between δ and the dynamic range when designing for the optimum system. Figure 21 shows some photographs of the HPFI setup in the laboratory under various conditions of f_n with λ constant. Note that the decay time for all three configurations is the same (as theory predicts).

Fig. 19. Theoretical step response for $f_n = 10$ kHz and various δ .

Fig. 20. Theoretical step response for $f_n = 10$ khz and various δ .

HORIZ: 500 μ SEC/DIV

VERT: 100 mV/DIV

$f_n = 8$ kHz

$= .044$

HORIZ: 500 μ SEC/DIV

VERT: 200 mV/DIV

$f_n = 6.5$ kHz

$= .053$

HORIZ: 500 μ SEC/DIV

VERT: 200 mV/DIV

$f_n = 5$ kHz

$= .070$

HORIZ: 500 μ SEC/DIV

VERT: 200 mV/DIV

$f_n = 4$ kHz

$= .086$

Fig. 21. System step response for $\lambda = 2,200$ and various f_n .

A Compensation Technique for Increasing System

Damping When Large Loop Gains Are Involved

A most obvious feature of the HPFI as modeled up to now is that when the system requires any substantial loop gain the damping becomes quite low. This response is primarily due to the choice of low-pass filter in the feedback loop which is the simplest 2-pole low-pass combination possible. This simple filter is used as a vehicle to analyze the operation and performance characteristics of the HPFI by using relatively simple and uncluttered mathematics. A more sophisticated low-pass filter design in the feedback loop of the HPFI will allow much more flexibility in controlling the loop gain and damping ratio. One such filter commonly utilized by designers of phase-locked loops is the lag-lead filter.⁴ This filter when implemented in the HPFI provides some shunt resistance in one of the single pole filters and is shown in Figure 22.

The transfer function for this filter is

$$\overline{L(s)}_c = \frac{K\lambda_1(s + \lambda_3)}{(s + \lambda_1)(s + \lambda_2)} \quad , \quad (3.51)$$

where

$$\lambda_1 = 1/R_1C_1; \quad \lambda_2 = 1/(R_2 + R_3)C_2; \quad \lambda_3 = 1/R_3C_3; \quad K = \lambda_2/\lambda_3 \quad .$$

⁴Signetics Corporation, Applications (Sunnyvale, California: Signetics Corporation, 1974), p. 6.

Fig. 22. Lag-lead filter.

If Equation (3.13) is replaced with Equation (3.51) the closed loop transfer function for the HPFI may be solved (assuming all of the linearizing assumptions hold true) to be

$$H(s)_c = \frac{G\lambda_1 K(s + \lambda_3)}{s^2 + s(\lambda_1 + \lambda_2 + \lambda_1 GK) + \lambda_1[\lambda_2 + GK\lambda_3]} . \quad (3.52)$$

This equation is seen to duplicate Equation (3.34) (as it should) in the limit as R_3 is taken to zero.

As before, it is most desirable to break up the transfer function into its magnitude and phase terms. With a substantial amount of algebra, one will find

$$\text{Mag } \overline{H(s)}_c = H(s)_c = \frac{G\lambda_1 K \sqrt{\ell^2 + m^2}}{c^2 + d^2} , \quad (3.53)$$

where

$$\begin{aligned} c &= 2\omega^2 \lambda_1 (2 + GK) , \\ d &= 2\omega(\omega^2 - \lambda_1^2 - GK\lambda_3) , \\ \ell &= 2c\lambda_3 - 2d\omega , \\ m &= 2d\lambda_3 - 2c\omega , \\ \lambda_1 &= \lambda_2 , \end{aligned}$$

and

$$H(s)_c \angle \theta = \tan^{-1}(\ell/m) . \quad (3.54)$$

It is most difficult to look at Equation (3.53) and interpret the HPFI's behavior for various values of compensation as this expression is somewhat complicated. Figures 23 and 24 show computer-generated plots for the theoretical amplitude response Equation (3.53) of the HPFI for various compensations at two respective loop gains of 500 and 1500. One may take note that when K approaches 0 (or is very small), Equation (3.53) will predict responses very similar to that of Equation (3.35) (if K takes on the value of 0 the two equations are equal). It should become clearly obvious in Figures 23 and 24 that when K is increased the magnitude of the natural frequency becomes lower. This indicates that there is an increase in system damping (indicating better stability). Just how much the damping is increased by adding the compensation may be realized by observing the closed loop poles of the transfer function in Equation (3.52). Some algebraic manipulations reveal that the damping ratio for the lag lead system is

$$\delta_c = \frac{\lambda_1(1 + GK) + \lambda_2}{2\sqrt{\lambda_1(\lambda_2 + GK\lambda_3)}} \quad (3.55)$$

or when $\lambda_1 = \lambda_2$, a simpler equation results

$$\delta_c = \frac{2 + GK}{2\sqrt{1 + G}} \quad \left| \quad \lambda_1 \approx \lambda_2 \right. \quad (3.56)$$

Fig. 23. Amplitude response for various values of K ; $G = 500$, $\lambda = 200$.

Fig. 24. Amplitude response for various values of K ; $G = 1,500$, $\lambda = 200$.

These values are shown in Figures 23 and 24 with their respective amplitude responses.

The two plots in Figures 23 and 24 serve just to exemplify the improvements incurred by adding compensation, but do not serve to present total insight to the situation. The root-locus⁵ technique depicts the effect of the loop filter compensation more clearly (not to mention that it serves as an excellent design tool). The root-locus is simply a plot of the migration of the closed loop poles in the system as the loop gain is varied. This graphical technique can predict the damping and estimate the natural frequency for any system. Figure 25 shows the migration of the closed loop poles in the second and third quadrants of the complex plane for $K = 0.01$, $\lambda = 2$, and variable G . Since the system is second order, there are two branches (one in the second quadrant and one in the third quadrant); they start (for $G = 0$) at the open loop poles (for this case, both are at $s = -\lambda_1$) and form semicircles in the second and third quadrants with increasing G . As the gain is increased even further, the branches meet at the negative real axis at approximately $-2\lambda_3$ and split off in opposite directions staying on the real axis. Each point on the plot represents a gain spacing of 790; no values for gain are shown on the real axis.

This type of plot allows one to look at the pole migrations

⁵Melsa and Schultz, Linear Control Systems, p. 301.

and have a good feel for what system damping is. Figure 26 shows the damping ratio plotted as a function of gain for the system ($\lambda = 2$, $K = 0.01$, G variable). It does not require much effort to infer that any root-locus plot for the particular lag-lead configuration chosen ($\lambda_1 = \lambda_2$) leads to a circle of center $-(\lambda_1 + \lambda_3)$ with radius $\lambda_3 - \lambda_1$ that starts at $-\lambda_1$ with branches forming mirror image semi-circles about the negative real axis to meet at $-(2\lambda_3 + \lambda_1)$ and branch off in opposite directions along the real axis, one to terminate at $-\infty$ while the other terminates at $-\lambda_1$ (for infinite G). The tremendous advantage of the lag-lead filter is seen when it is compared to a system without compensation. The root-locus and resulting damping ratio for a system with no compensation (over the same range of gain depicted by Figure 25) is shown in Figures 27 and 28, respectively. Notice the extremely low damping implying poor stability. One might also take interest in noting that this situation (Figure 27) may also be created with a lag-lead configuration where λ_3 is placed at $-\infty$.

An interferometric system was set up to verify this theory. For this particular experiment, a setup similar to that shown in Figure 5 was used. The acoustic input was replaced by a linear phase modulator (fiber wrapped around a PZT-4 2-in. diameter thin-walled cylinder) driven with band limited (to 5 kHz) white (zero meaned), gaussian noise. The system gain, G , was determined by setting R_3 to 0, observing f_n and using Equation (3.37). For this

Fig. 25. Root locus plot for $K = 0.01$ and $\lambda = 2$.

Fig. 26. Damping ratio plotted versus loop gain; $K = 0.01$, $\lambda = 2$.

Fig. 27. Root locus for uncompensated HPFI with $\lambda = 2$.

Fig. 28. Damping ratio plotted versus loop gain for uncompensated HPFI with $\lambda = 2$.

case, G was found to be approximately 1200. The inverse time constant, λ_1 , was also set to be a constant at 500. Figure 29 illustrates the experimental results when K takes on values of 0, 0.0031, 0.0077, and 0.015 which predicts corresponding damping factors of 0.029, 0.082, 0.162, and 0.295 (from Equation (3.56)). Since white noise produces a flat spectrum, these plots represent the system transfer function $H(s)_c$.

It would not be out of reason for one to question the need for spending so much time (in analysis) on the noncompensated HPFI. The author's response to this question is that it is much easier to thoroughly analyze the noncompensated system and draw parallels to the compensated system. A detailed logical insight to this response follows.

If the compensated HPFI is modeled (assuming "0" initial conditions), the resulting nonlinear equation is

$$\ddot{\phi}_m + 2\lambda_1 \dot{\phi}_m + \lambda_1^2 \phi_m = G\lambda_1^2 \sin(\phi_i - \phi_m) \quad (3.57)$$

$$+ G\lambda_1 K \frac{d}{dk} \left[\sin(\phi_i - \phi_m) \right] .$$

Notice that this equation is exactly the same as Equation (3.16) except for the additive term on the right-hand side. It is relatively easy to observe (as before) that for slowly varying ϕ_m

$K = 0$
 $\delta = .029$

$K = .0031$
 $\delta = .082$

$K = .0077$
 $\delta = .162$

$K = .015$
 $\delta = .295$

Fig. 29. System response for various compensations; $\lambda_1 \approx \lambda_2 = 500$, $G = 1,200$; vertical axis at 10 dB per division, horizontal axis at 500 hertz per division.

$$\phi_m = G \sin(\phi_i - \phi_m) \quad (3.58)$$

leading to

$$\phi_{m \max} = G$$

which is exactly the same as the noncompensated system, which implies that for a large enough G , the system behaves linearly.

If one were to try to demonstrate linearity for the higher frequencies of ϕ_m , they would be presented with great difficulties when trying to use Equation (3.57) to explicitly solve for $\phi_{i(wc)}$ (as in Equation (3.30)). It is much easier to assume values for K to make the right-hand term of Equation (3.57) negligible. This leaves one with Equation (3.16) which has been shown to behave linearly for $\phi_i \leq 4\lambda_1$ and $\lambda \leq 1$.

The natural frequency for the compensated HPFI can also be estimated with solutions from the uncompensated HPFI. The author has yet to differentiate Equation (3.53) with respect to ω and set the result to zero to obtain an expression for ω_n . If he did, the expression promises to be lengthy. If one were to observe Figures 23 and 24, they would be able to see that when K is very small, the system becomes uncompensated. This is a reasonable assumption for $K = 0.0002$. Notice that for all values of K in Figures 23 and 24 that f_n remains relatively unchanged. Close inspection reveals that

$$f_n \approx \frac{\lambda_1 \sqrt{G-1}}{2\pi} \left| \begin{array}{l} G > 100 \\ \delta < 0.3 \end{array} \right. , \quad (3.59)$$

which is seen to be the same as Equation (3.57).

CHAPTER IV

NONLINEAR ANALYSIS

The theoretical development in the previous chapter was developed from the point of view that the HPFI was never operated in a region of instability or loss of synchronization. The most obvious example of the HPFI losing synchronization occurs when the input phase, ϕ_i , exceeds G , the loop gain. This is most easily observed in the low-frequency case where Equation (3.21) demonstrates that ϕ_m cannot possibly take on a value greater than G and hence, the system loses synchronization. The linear model of the HPFI does not address this loss of synchronization as it is a nonlinear process. This example suggests the HPFI's complete response to any input can only be observed via the nonlinear model. If the nonlinear model for the HPFI could be solved, only may be able to produce answers to the following questions:

1. When is the linear model a good approximation to the nonlinear system?
2. What conditions cause the system to lose synchronization?
3. How does the system respond to a loss of synchronization?

As there is no closed form technique to solve for the nonlinear expression in Equations (3.16) or (3.57), it is nearly impossible to

to provide generalized solutions. It is, however, possible to simulate this nonlinear system to provide specialized solutions for given inputs. This is accomplished by constructing a difference equation from the nonlinear differential equation and iteratively implementing it in a computer. This chapter develops the difference equation and utilizes it to demonstrate the system response for various inputs.

The Nonlinear Model Approximated With Discrete Modeling

Development of the Nonlinear Discrete Model

The most generalized expression that models the HPFI (with or without compensation) is described by Equation (3.57). This equation is nonlinear as the independent variable, ϕ_m , is contained within the argument of the sine function (which creates orders of ϕ_m higher than 1). The equation can be approximated in the discretized sense by constructing a difference equation.

If $\phi_m(t + k\Delta t)$ represented by a new variable, ϕ_{n+k} ($k = 0, \pm 1, 2, 3, \dots$), one sees that if Δt is kept constant that the subscript $(n + k)$ for the new variable denotes the value of ϕ_m at the discrete time $t + k\Delta t$. It is possible to define the discrete derivative to simulate a continuous derivative evaluated at a point in the following fashion:

$$\dot{\phi}_m(t) \approx \frac{\phi_{n+1} - \phi_{n-1}}{2\Delta t} \quad (4.1)$$

which is of course exactly true in the limit as Δt approaches "0".

In a similar manner, $\ddot{\phi}_m(t)$ may be approximated as follows

$$\begin{aligned}\ddot{\phi}_m(t) &\approx \frac{\frac{\phi_{n+1} - \phi_n}{\Delta t} - \frac{\phi_n - \phi_{n-1}}{\Delta t}}{\Delta t} \\ &= \frac{\phi_{n+1} - 2\phi_n + \phi_{n-1}}{(\Delta t)^2} .\end{aligned}\tag{4.2}$$

Equations (4.1) and (4.2) may be substituted in Equation (3.57) to construct a difference equation. Starting with Equation (3.57)

$$\ddot{\phi}_m + 2\lambda_1 \dot{\phi}_m + \lambda_1^2 \phi_m = G\lambda_1^2 \sin(\phi_i - \phi_m) + G\lambda_1 K \frac{d}{dt}[\sin(\phi_i - \phi_m)] ,$$

substituting the continuous variables with the discrete variables, one sees

$$\begin{aligned}&\frac{\phi_{n+1} - 2\phi_n + \phi_{n-1}}{(\Delta t)^2} + \frac{\lambda(\phi_{n+1} - \phi_{n-1})}{\Delta t} + G\lambda^2 \sin(\phi_i - \phi_n) \\ &+ G\lambda K \left[\frac{d}{dt}(\phi_i(t)) - \frac{\phi_{n+1} - \phi_{n-1}}{2\Delta t} \right] \cos(\phi_i - \phi_n)\end{aligned}\tag{4.3}$$

This equation may be arranged to solve explicitly for ϕ_{n+1} to yield

$$\frac{\Delta t^2 \lambda^2 G \sin(\phi_i - \phi_n) + GK/\lambda \frac{d}{dt} \phi_i(t) + \frac{\phi_{n-1}}{2\Delta t} \cos(\phi_i - \phi_n) - \phi_n}{1 + \lambda \Delta t + G\lambda \Delta t K \cos(\phi_i - \phi_n)/2} \quad (4.4)$$

$$+ \frac{\phi_{n-1} [\Delta t \lambda - 1] + 2\phi_n}{1 + \lambda \Delta t + G\lambda \Delta t K \cos(\phi_i - \phi_n)/2} = \phi_{n+1}$$

In this expression $\lambda_1 = \lambda_2 = \lambda$ and $K = \lambda_2/\lambda_3$. This relationship describes the "next state" of the system providing the two previous states (ϕ_n and ϕ_{n-1}) and the input value, ϕ_i , are given. It should be somewhat obvious that the equation be used in an iterative method to solve (or approximate) successive next states by simply letting the subscript n take on the value $n + 1$ or $n + 2$ or $n + 3$, and so on. A digital computer is a most useful tool for this type of implementation. One must be cautious, however, when selecting values for Δt as the computer may introduce error as it rounds off numbers. This occurs for values of Δt that are too small. Errors may also be introduced when the choice of Δt is too large. These occur primarily relating to the lack of resolution the system model provides when simulating some continuous operation. Before the discrete model is to be used to approximate system outputs relating to given inputs, one must insure that the parameters chosen for the model do not introduce extraneous effects.

Verification of the Nonlinear Discrete Model

To insure that the difference equation in Equation (4.1) is accurately predicting the system response, one must devise a method

to test it. This test should address such questions as:

1. Does the model work?
2. Are the chosen parameters for the model adequate?

One method called the phase-plane plot¹ used for demonstrating system stability or instability proves to be a good test for the difference equation. The phase-plane technique plots the trajectory of the HPFI's output (phase) versus its time derivative. For $\phi_1(t)$ being a step perturbation at $t = 0$, the phase-plane represents a plot of the systems step response versus its time derivative. The trajectory for a stable system would result in $\dot{\phi}_m = 0$ while t approaches infinity. If the linear model and the approximation to the nonlinear model are to show likenesses, then as t approaches infinity, the value for ϕ_m will approach $S \frac{G}{1 + G}$ (as predicted by Equation (3.47) as $\dot{\phi}_m$ approaches zero. For this case S represents the amplitude of the input step.

Figure 30 shows a phase-plane plot for a compensated system via iterative use of Equation (4.4). For this case, $\lambda = 100.0$, $K = 0.0001$, $G = 200$, $S = 1.000$, and $\Delta t = 0.0001$. The result here depicts what the linear model would predict. It shows a damped oscillation of $\phi_m(t)$ responding to a unity step input which decays to a steady-state value of $\dot{\phi}_m(t) = 0.000$ and $\phi_m(t) = 0.995$ which coincides exactly with what Equation (3.47) would predict.

¹Floyd M. Gardner, Phaselock Techniques (New York: John Wiley and Sons, 1967), p. 42.

Figures 31, 32, and 33 demonstrate a system simulation with parameters exactly the same as in Figure 30, but taken with larger time increments ($\Delta t = 0.004$, 0.0012 , and 0.002 seconds, respectively). The discrete nature of this iterative approximation is much more evident in these plots. One would think that such a large time increment would produce disastrous results, but from the point of view of the step response, Figures 31 and 32 do show stable configurations that attain the same steady-state value that the linearized model predicts. In Figure 33, the time increment finally becomes too large and the model generates an unstable system.

With the previous results at hand, a question is raised concerning a proper choice of Δt . It has been observed that three different choices of Δt generate stable predictable steady-state responses (albeit they appear to be very different). The most reasonable choice would be to go with the smallest time increment as it better approximates a continuous system (as the system actually operates).

The next question for one to address is how small an increment should one take for an adequate representation. It is most obvious that one would not like to take an infinitesimally small increment as computer time is expensive and it is also possible to generate computer round-off errors. The computer used to iteratively implement Equation (4.4) is a PDP 11-45 and it was found that computer time was much more a problem than round-off errors.

Fig 30. Phase-plane plot for a unity step input; $G = 200$, $\lambda = 100$ and $\Delta t = 0.0001$.

Fig. 31. Phase-plane plot for a unity step input; $G = 200$, $\lambda = 100$ and $\Delta t = 0.0004$.

Fig. 32. Phase-plane plot for a unity step input; $G = 200$, $\lambda = 100$ and $\Delta t = .0012$.

Fig. 33. Phase-plane plot for a unity step input; $G = 200$, $\lambda = 100$ and $\Delta t = 0.002$

The optimized time increment to choose would be one that is as large as possible without distorting the system (as the time increment chosen in Figures 31 and 32 does). The natural frequency f_n , as described by Equations (3.37) and (3.59), has been seen (in previous analysis) to be the highest unattenuated frequency found in the HPFI. It is also a good approximation to the resonant frequency of the HPFI. Since the HPFI responds to all discontinuities by exciting its resonant frequency, one need only pick a time increment for the discrete model to accurately represent this resonant frequency which is approximated by the natural frequency (Equation (3.59)). A reasonable time increment (deduced by inspection of the phase-plane plots) would be one that produces 50 data points per one cycle at the system's natural frequency (for $G > 50$). Choosing fewer data points would produce results shown in Figures 31, 32, and 33. Although the systems in Figures 31 and 32 are stable, the paucity of points per cycle at the higher frequencies will generate a signal distortion which results in harmonic distortion. From this point on, any implementation of the discrete model will use a time increment less than $(50 f_n)^{-1}$ or

$$\Delta t \leq \frac{1}{50 f_n} = \frac{2\pi}{50\lambda\sqrt{G-1}}, \quad (4.5)$$

which insures a stable system devoid of harmonic distortion.

Before leaving the subject of the phase-plane, it is interesting to observe the effects of system damping. Figure 34

demonstrates the HPFI setup with $G = 500$, $\lambda = 200$, $\Delta t = 2.5 \times 10^{-5}$, $S = 1.0$, and $K = 0.002$, 0.007 , and 0.02 , respectively, producing damping ratios of 0.047 , 0.12 , and 0.27 . Each system was simulated for a 5-msec duration starting at the time of the step input.

Notice how the systems with higher damping ratios stabilize faster.

Now that the nonlinear model for the HPFI has been constructed, one may judge its effectiveness by comparing it to the linearized model. Proper modeling will be evident if there is a high degree of similarity between the results of both models. To compare these models, their step and sinusoidal responses were investigated.

To obtain the step response of the linearized model, one starts with Equation (3.57), substitutes $\phi_i - \phi_m$ for $\sin(\phi_i - \phi_m)$ and solves the second order equation for ϕ_m assuming $\phi_i = 1$ starting at $t = 0$. If λ_1 is assumed to be equal to λ_2 , the resulting solution is

$$r(t)_c = \frac{KG\lambda_1\lambda_3}{B} + e^{-At} \left[\frac{z}{g} \cos(mt) - \frac{f}{g} \sin(mt) \right] ; \quad (4.6)$$

where

$$A = \lambda_1(2 + GK)/2,$$

$$B = \lambda_1^2(1 + G),$$

$$m = \sqrt{B - A^2},$$

$$a = KG\lambda_1(\lambda_3 - A),$$

$$b = KG\lambda_1 m,$$

Fig. 34. Phase-plane plot for unity step input to systems with varying damping factors; $G = 500$, $\lambda = 200$, $\Delta t = 2.5 \times 10^{-5}$.

$$c = -m^2 ,$$

$$d = -Am ,$$

$$e = ac - bd ,$$

$$f = bc - ad , \text{ and}$$

$$g = c^2 + d^2 .$$

This solution is compared to the prediction of the nonlinear model for various damping ratios and loop parameters. Figure 35 displays the comparison for $G = 1500$, $\lambda = 1000$ with $\delta = 0.05$, 0.10 and 0.32 ; Figure 36 displays the comparison for $G = 2500$, $\lambda = 1000$ with $\delta = 0.025$, 0.07 and 0.27 . Both figures show a very close comparison for the amplitude response but are off somewhat for the phase. This ambiguity in phase is understandable in that the HPFI is operating under non-steady-state conditions. When steady-state conditions prevail, both models predict the same value. To further corroborate this statement, the linearized sinusoidal response (Equation (3.54)) will be compared to the nonlinear model's sinusoidal response.

Figure 37 shows the responses of both models to a HPFI configured with $G = 5000$, $\lambda = 1000$ and $\delta = 0.05$ for a 50 Hz input signal. In this case, the results from both models are indistinguishable. Figure 38 shows the sinusoidal response at 1000 Hz for $G = 1000$, $\lambda = 1000$ and $\delta = 0.11$. Here one can see the transient behavior manifest in the nonlinear model. This behavior should similarly be evident in the linear model but it has been omitted as Equation (3.54) only predicts the steady-state

Fig. 35. Comparison of the step response of the linear and nonlinear models for the compensated HPFI; $G = 500$ and $\lambda = 1,000$; δ is varied.

Fig. 36. Comparison of the step response of the linear and nonlinear models for the compensated HPFI; $G = 2,500$ and $\lambda = 1,000$; δ is varied.

Fig. 37. Comparison of the sinusoidal response of the linear and nonlinear models for the compensated HPFI; $G = 5,000$, $\lambda = 1,000$, $\delta = 0.05$ and $f = 50$ hertz.

Fig. 38. Comparison of the sinusoidal response of the linear and nonlinear models for the compensated HPFI; $G = 1,000$, $\lambda = 1,000$, $\delta = 0.11$ and $f = 1,000$ hertz.

approximation of the linearized system. Notice as the nonlinear model approaches steady-state, the two plots converge to almost identical values.

This simple comparative analysis should indeed provide evidence that the linear and nonlinear model represent the same system.

Application of the Nonlinear Discrete Model

The Step Response

The step response of the HPFI was completely covered in the last section. One aspect that might not have been covered is the time domain response for the step function. Figure 39 demonstrates the time domain response of the HPFI configurations seen in Figure 34. The time for this duration is also 5 msec.

The Ramp Response

The ramp response of the HPFI provides some very interesting information particular to the system's loss of synchronization and its reacquisition. A straightforward method to induce the system to lose synchronization would be to allow $\phi_i(t)$ to be a ramp of a moderately slow slope (with respect to system parameters) and observe the model's behavior as ϕ_i exceeds $G + \pi/2$. An example of this is shown in Figure 40. Here $G = 200$, $\lambda = 400$, $K = 0.0001$ ($S = 0.071$), $\Delta t = 2 \times 10^{-5}$ and $r(t) = 250$ rad/sec. In this figure, ϕ_m is plotted versus ϕ_i . When ϕ_m attains its maximum value (G), the HPFI loses synchronization, ϕ_m responds by plummeting to a lower value where the system parameters allow reacquisition to continue tracking the ramp.

Fig. 39. Step response for $G = 500$ and $\lambda = 200$ for various damping ratios.

This loss of lock and reacquisition is a very complicated process (being nonlinear in nature). The curious researcher will naturally be interested in the mechanisms of this process. The results in Figure 40 are not very amenable in providing much information in describing these mechanisms; perhaps the phase-plane plot is more revealing. Figure 41 displays the phase-plane plot of a HPFI having the same parameters of the system modeled in Figure 40. This plot starts at the onset of the loss of synchronization ($\phi_m = 200$) and follows the system through to reacquisition and then steady-state tracking of the ramp input signal.

In this case, it is clearly easy to see that as soon as the system loses synchronization, ϕ_m becomes accelerated with a negative magnitude. This initial acceleration is so great that even when ϕ_m has regressed π radians, into a region where the system's acceleration ($\ddot{\phi}_m$) turns positive (in an attempt to stabilize) the system's negative momentum (described by $\dot{\phi}_m$) is too great to be reversed. It is clearly demonstrated that the acceleration which resists the declining motion of ϕ_m does not overcome it until ϕ_m drops to a value of approximately 20 radians. This phenomenon can readily be verified by observing the explicit relation for $\ddot{\phi}_m$ using the continuous uncompensated model (a simplification of the compensated HPFI) in Equation (3.16).

$$\ddot{\phi}_m = G\lambda^2 \sin(\phi_i - \phi_m) - 2\lambda\dot{\phi}_m - \lambda^2\phi_m . \quad (4.7)$$

Fig. 40. Ramp response for $g = 200$, $\lambda = 400$ and $K = 0.0001$.

Fig. 41. Phase-plane plot showing the loss and reacquisition of synchronization; $g(t) = 250$ radians/second, $G = 200$, $\lambda = 400$ and $K = 0.0001$.

One will notice that when ϕ_m becomes smaller, the acceleration term ($\ddot{\phi}_m$) becomes more and more positive, which opposes the negative motion of ϕ_m , in turn slowing down $\dot{\phi}_m$ to the point where the system regains synchronization. Figure 42 demonstrates a "close-up" view of the phase-plane plot shown in Figure 41. It shows the system regain synchronization, then exhibit a response that looks like the step response demonstrated in the previous section, but differs somewhat as the system is tracking the input signal which is a ramp. The system finally assumes steady-state at $\dot{\phi}_m = 250$ radians per second. Figures 43 and 44 show the phase-plane plot for two other configurations of the HPFI with the same f_n as in Figure 41 (900 Hz). In Figure 43, $G = 500$, $\lambda = 250$, $K = 0.0001$, and $r(t) = 250$ radians per second. In Figure 44, $G = 100$, $\lambda = 565$, $K = 0.0001$, and $g(t) = 250$ radians per second. Notice that all three systems regain lock at different values of ϕ_m but in each case, the range of $\dot{\phi}_m$ is the same being approximately $\pm 10,000$ radians per second. This type of phenomenon is very similar to what is described as the "capture range" of a phase-locked loop. This analogy may be corroborated when choosing a different f_n and observing the range of $\dot{\phi}_m$ required for reacquisition. Figure 45 shows the results of performing just such an experiment. In this case, f_n was chosen to be four times that seen in Figures 41, 43, and 44. The results show that the capture range for $\dot{\phi}_m$ is now approximately $\pm 40,000$ radians/second (four times the previous example) which agrees with the prediction as the bandwidth of the loop filter incurred a fourfold increase.

Fig. 42. Phase-plane plot, $g(t)=250$ radians/second $G=200$ $\lambda=400$ $K=.0001$

Fig. 43. Phase-plane plot for $g(t) = 250$ radians/second, $G = 100$, $\lambda = 565$ and $K = 0.0001$.

Fig. 44. Phase-plane plot for $g(t) = 250$ radians/second, $G = 100$, $\lambda = 565$ and $K = 0.0001$.

Fig. 45. Phase-plane plot for $g(t) = 1,000$ radians/second.

It can be stated that the capture range, $\dot{\phi}_c$ for the HPFI is approximately 1.75 times that of the natural radian frequency, or

$$\dot{\phi}_c \approx 1.75\omega_n \quad \left| \delta < 0.1 \right. \quad (4.8)$$

This relation has only been observed for systems with low damping (relatively uncompensated). When the systems are compensated, producing damping factors greater than 0.1, one would expect some change in Equation (4.8). A heuristic analysis of this situation will ensue. Figure 46 demonstrates a HPFI with $G = 200$, $\lambda = 400$, $g(t) = 250$ radians per second and $\Delta t = 20$ micro-seconds. In this case, ϕ_m is plotted versus ϕ_i for the ramp input. The solid line represents a system for which $\delta = 0.071$, the long dashed line represents $\delta = 0.105$ and the short dashed line represents $\delta = 0.371$. It is plainly evident that the larger the system damping the faster the HPFI regains synchronization. This phenomenon can only be accomplished by the HPFI having a larger capture range. This insight is proven true by observing the transfer functions seen in Figures 23 and 24. It is demonstrated that when the damping ratio is increased, the Q of the system is reduced or the bandwidth is broadened. It is this broadening of the bandwidth that may be attributed to the increase in capture range for the compensated HPFI.

It is very possible to construct a HPFI in the laboratory where the amplifier providing the feedback has a maximum output level that

Fig. 46. Ramp response of the HPFI for damping ratios of 0.071, 0.105 and 0.371; $G = 200$ $\lambda = 400$.

is less than what the system parameters allow. For example, suppose the system is constructed so that $G = 200$ but the feedback amplifier's maximum output only allows the HPFI an excursion of ± 60 radians. The discrete model incorporating the ramp response provides some very useful information concerning just such a system. If the discrete model is modified to accommodate the restricting limits of ϕ_m , the feedback amplifier limited system may be simulated. Figure 47 depicts just such a process. Here $G = 200$, $\lambda = 400$, $g(t) = 250$ radians/second, $|\phi_m| \leq 60$ (due to amplifier restrictions) and $\delta = 0.071$. It is surprising to note that the system does not lose lock immediately when ϕ_m becomes 60 radians. This result is quite obvious however when one observes Equation (3.16) assuming $\ddot{\phi}_m = \dot{\phi}_m = 0$ and setting the limit of ϕ_m to 60. It is seen that any value of ϕ_i between 60.3 and 62.4 will satisfy the relation.

Before leaving the ramp response, one more interesting point concerning the HPFI's performance may be investigated. When the system loses synchronization and resets itself, $\phi_m(t)$ may be approximated as a "step" change in value of some very large magnitude. It is interesting to note that the system does not recover by responding to such a large step, but responds to a step no greater than $\pi/2$. This is realized as the HPFI operates over a $\pm\pi/2$ degeneracy. Any signal it synchronizes on is only discernable by the value of the phase error ϕ_e . Since the phase error can only take on values of $-\pi/2 \leq \phi_e \leq \pi/2$, the HPFI can only respond to steps having a magnitude less than $\pi/2$. Figure 48 demonstrates

Fig. 47. Ramp response of the HPFI limited by the range of the feedback amplifier; $G = 200$, $\lambda = 400$ and $g(t) = 250$ radians/second.

Fig. 48. Ramp response of the HPFI as it regains synchronization; $G = 500$, $\lambda = 250$, $g(t) = 250$ radians/second and $\delta = 0.047$.

a system regaining synchronization with parameter values of $G = 500$, $\lambda = 250$, $g(t) = 250$ radians/second and $\delta = 0.047$.

The Sinusoidal Response

The sinusoidal response of the HPFI modeled discretely will serve to shed some light on determining a value for the maximum allowable sinusoidal signal that the HPFI is capable of tracking. It was brought out in the last section that the HPFI was slew rate limited as a result of the loop filter. This phenomenon is also evident as the HPFI operates in tracking a sinusoidal input phase of a certain amplitude and frequency. A straightforward way of finding the HPFI's maximum trackable sinusoidal amplitude at any given frequency is to input that frequency to the HPFI while slowly increasing the amplitude while watching for the system to lose synchronization. Figure 49 demonstrates this procedure implemented theoretically using the discrete model. For this simulation, $f = 400$ Hz, $\lambda = 2100$, $G = 900$, and $\delta = 0.033$. It is a simple matter of pinpointing the region where the system loses synchronization. If this process is repeated for different frequencies, one will be able to come up with a plot of maximum allowable amplitude as a function of frequency. This theoretical information is presented in Figure 50 along with experimental results for a HPFI configured for $G = 435$, $\lambda = 1800$, and $\delta = 0.048$. The experimental results follow the theory for the lower and higher frequencies but are a little off from about 100 Hz to 500 Hz. This is probably attributed to the fact that it was impossible to center the sinusoid about 0 radians

Fig. 49. Sinusoidal response of the HPFI as the input amplitude is linearly increased; $f = 400$ hertz, $\lambda \approx 2,100$, $G = 900$ and $\delta = 0.033$.

Fig. 50. Maximum sinusoidal amplitude response for $G = 435$ and $\lambda = 1,800$.

in the experiment as very large random thermal-induced phases (up to 50 radians/second) controlled the operating point. If one inspects Figure 50, one finds that $\dot{\phi}_m$ at the point where the system is losing synchronization (at any frequency) is about ten times larger than ω_n . For a different system configuration, this number may be smaller or larger. This type of sinusoidal analysis leads to the theoretical prediction of the dynamic lock range (author defined) of the system.

The computer program written to simulate the nonlinear differential equation will be included as an appendix.

CONCLUDING REMARKS

This text has been primarily devoted to the theoretical investigation of a unique interferometric stabilization technique. This investigation is by no means complete; its purpose is merely to lay the groundwork and theoretically generalize the noiseless operation of the HPFI. It was found that the analysis of the HPFI drew many parallels to the analysis of the phase-locked loop. This was no surprise as both systems use nonlinear phase comparison and feedback to synchronize to the input phase.

All of the analysis in this paper concerns second-order systems. This choice was intentional as HPFI's with first- or third-order loops place some severe limitations on system performance. The first-order system has practical value only at very low frequencies while the third-order system has operating regions where the system can become unstable. The detailed theoretical analysis to support these statements is omitted as the issue of choice of order of the HPFI is somewhat peripheral to the scope of this paper.

The noise performance of the HPFI is not included in this paper. This is an important topic not to be overlooked and will be investigated in the very near future.

APPENDIX

```
C*****C
C*****C
C.....THIS PROGRAM DISCRETELY MODELS A SECOND ORDER.....C
C.....NON-LINEAR DIFFERENTIAL EQUATION THAT REPRESENTS.....C
C.....A COMPENSATED HPFI SUBJECT TO STEP,RAMP,OR.....C
C.....SINUSOIDAL INPUTS.....C
C*****C
C*****C
C*****C
C*****C
C
C
C
C
C
C
C
REAL G,L,M,X,Y,I,N,LIMIT,X1(2050),Y1(2050),INC,K
REAL C1(2050),Z1(2050)
INTEGER U,P
C*****C
C.....INPUT SYSTEM PARAMETERS.....C
C*****C
      WRITE(S,10)
10     FORMAT(' ENTER LOOP GAIN,TIME CONST, PHI(I),D TIME,#POINTS,LIMIT:')
      READ(S,20)G,L,M,T,N,LIMIT
20     FORMAT(E20.0)
      WRITE(S,21)
C*****C
C.....ENTER THE OPEN LOOP ZERO TO POLE RATIO.....C
C*****C
21     FORMAT(' ENTER THE K VALUE ')
      READ(S,22)K
22     FORMAT(E20.0)
C*****C
C.....ENTER THE TWO PREVIOUS STATES OF THE SYSTEM.....C
C*****C
      WRITE(S,30)
30     FORMAT(' ENTER PHI(N),PHI(N-1): ')
      READ(S,40) PHI,PHI1
40     FORMAT(E20.0)
C*****C
C.....CHOOSE THE INPUT SIGNAL.....C
C.....STEP, RAMP, OR SINUSOIDAL INPUT?.....C
C*****C
      WRITE(S,50)
50     FORMAT(' $TYPE (1)STEP,(2)RAMP,(3)SINASQID: ')
      READ(S,60)TYPE
40     FORMAT(F10.0)
      IF(TYPE .EQ. 2)GOTO 150
      IF(TYPE .EQ.3)GOTO 200
C*****C
C....DOES THE OPERATOR WANT THE TIME DOMAIN RESPONSE.....C
C....OR THE PHASE PLANE PLOT FOR THE STEP INPUT.....C
C*****C
      WRITE(S,11)
11     FORMAT('$PHASE PLANE? Y/N ')
      READ(S,12)A
12     FORMAT(A2)
61     Z=1
        U=1
```



```

C*****C
C.....DOES THE OPERATOR WANT THE POINTS LISTED?.....C
C*****C
      WRITE(5,66)
46      FORMAT('DO YOU WANT POINTS LISTED: Y/N  ')
      READ(5,67)P
47      FORMAT(A2)
45      DO 100 I=1,N
          IF(TYPE .GE. 1 .AND. TYPE .LE. 4)GOTO(70,80,90),TYPE
C*****C
C.....SETS UP PROGRAM FOR A STEP INPUT.....C
C*****C
70      Q=M
          DQ=0
          GO TO 99
C*****C
C.....SETS UP PROGRAM FOR A RAMP INPUT.....C
C*****C
80      Q=S*I*T+M
          DQ=S
          GO TO 99
90      Q=(AMP+T*I*INC)*SIN(I*2*3.14159*FREQ*T+M)
C*****C
C.....SETS UP PROGRAM FOR A SINUSOIDAL INPUT.....C
C*****C
          DQ=(AMP+T*I*INC)*2.*3.14159*FREQ*COS(I*2.*3.14159*FREQ*T+M)
C*****C
C.....THE PROGRAM IMPLEMENTS THE DIFFERENCE EQUATION.....C
C*****C
99      FIR=G*SIN(Q-PHI)+(G*K/L)*(DQ+PHI1/2./T)*COS(Q-PHI)-PHI
          SEC=PHI1*(T*L-1.)/2.*PHI
          TOP=T*L*T*L*FIR+SEC
          DEN=1.+L*T+G*L*T*K*COS(Q-PHI)/2.
          X=TOP/DEN
          IF(ABS(X) .GT. LIMIT)X=SIGN(LIMIT,X)
          Y=X/T-PHI/T
          C=(X-2*PHI+PHI1)/(T*T)
          IF(A .EQ. 'Y')GO TO 112
          IF(B .EQ. 'Y')GO TO 2000
C*****C
C.....IF THE PROGRAM IS NOT SOLVING FOR THE PHASE-.....C
C.....PLANE PLOT, IT SETS UP AN ARRAY THAT REPRESENTS.....C
C.....EVERY HUNDREDTH VALUE CALCULATED BY ITERITIVELY.....C
C.....IMPLEMENTATION OF THE DIFFERENCE EQUATION.....C
C*****C
          IF(Z .LT. 100)Z=Z+1
          IF(Z .LT. 100)GO TO 115
          IF(TYPE .GE. 1 .AND. TYPE .LE. 4)GO TO(71,81,91),TYPE
71      Y1(U)=X
          X1(U)=I*T
          GO TO 92
81      Y1(U)=X
          X1(U)=Q
          GO TO 92
91      Y1(U)=X
          X1(U)=I*T
92      IF(P .EQ. 'Y')WRITE(5,120)X,Y,Q,C
120      FORMAT(' X= ',E20.10,' Y= ',E20.10,' Q= ',F10.5,' ACC= ',E20.10)
          U=U+1
          Z=1
          GO TO 115
2000      Y1(U)=Y
          X1(U)=X

```

```

2001      IF(P .EQ. 'Y')WRITE(5,2001)X,Y,Q,C
          FORMAT(' Y= ',F10.5,' Y= ',E20.10,' Q= ',F10.5,' ACC= ',E20.10)
          Z1(U)=C/(G*L*L)+Y*2./(G*L)+X/G
          C1(U)=I*T
          U=U+1
          Z=1
          GO TO 115
112      Y1(U)=Y
          X1(U)=X
          U=U+1
          Z=1
C*****C
C.....START THE NEXT ITERATION .....C
C*****C
115      PHI1=PHI
          PHI=X
100      CONTINUE
          U=U-1
          GO TO 1000
150      WRITE(5,151)
151      FORMAT('$INCREMENT BY 1? Y/N')
          READ(5,152)B
152      FORMAT(A2)
C*****C
C.....SET UP PARAMETERS FOR RAMP RESPONSE.....C
C*****C
          WRITE(5,155)
155      FORMAT('$ENTER SCALER: ')
          READ(5,160)S
160      FORMAT(E10.0)
          GO TO 61
C*****C
C.....SET UP PARAMETERS FOR SINUSOIDAL INPUT.....C
C*****C
200      WRITE(5,201)
201      FORMAT('$ENTER FREQUENCY,AMPLITUDE,INCREMENT: ')
          READ(5,220)FREQ,AMP,INC
220      FORMAT(3E10.0)
          GO TO 61
1000     CALL FILE(U,X1,Y1)
C*****C
C.....ADAPT ARRAY TO CONFORM TO PLOTTING SOFTWARE.....C
C*****C
          CALL FILE(U,C1,Z1)
          END

```

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