Testing of Active Feedback Applied to Piezoelectric Devices

Fall 1983

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TESTING OF ACTIVE FEEDBACK APPLIED TO PIEZOELECTRIC DEVICES

BY

WILLIAM H. BARROW
B. S. E., University of Central Florida, 1981

RESEARCH REPORT

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ABSTRACT

The purpose of this research is to test a unique implementation of active feedback applied to a low frequency piezoelectric transducer. Understanding that active feedback can be used to shape the frequency response of an electrical system a method for applying active electroacoustical feedback will be used to obtain a wider acoustical bandwidth in a piezoelectric transmitter. By applying active feedback to a resonant device its bandwidth was increased by a factor of 2.5. This is a significant improvement for applications where minimum electrical phase shift versus frequency is desired.
ACKNOWLEDGEMENTS

I would like to express my appreciation for the help and guidance given to me by Bob Martin, Dr. Don Malocha, Dr. Michael Harris, and Madjid Belkerdid in preparing this research report.
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1. INTRODUCTION

Piezoelectric devices have a long history of excellent performance as acoustical transducers. One well known characteristic of these devices is that they can convert mechanical energy into electrical energy and vice-versa. This dual affect is commonly called reciprocity. By applying this concept with that of active feedback we can adjust the system’s parameters.

The purpose of this research is to analyze a low frequency piezoelectric system to verify that active feedback can improve the system’s performance. The improvement of the system will be judged on the increase in bandwidth obtained through different types of compensation. Various closed-loop systems will be compared to the open-loop system to determine the relative bandwidth improvement.
2. THE EFFECTS OF FEEDBACK

There are several parameters of a system which are affected by the implementation of feedback. Some of the more noted aspects include the reduction of sensitivity to system parameter variations and output disturbances, control of the system's transient response and control of the system's bandwidth [1].

The major goal is to increase the bandwidth of the system's frequency response which can also be interpreted as a decrease in the phase change versus frequency of the response. An example of how this is achieved follows.

Consider the block diagram in Figure 1 where \( T(s) \) is the transfer function of the system and \( A \) is the gain of the system. Let \( T(s) \) be a single pole low-pass function with a cut-off frequency of one radian per second. The transfer function for the system \( T(s) \) is

\[
T(s) = \frac{V(s)_{\text{out}}}{V(s)_{\text{in}}} = \frac{A}{s+1}. \tag{1}
\]

Using this same transfer function in a closed loop system with a gain of \( B \) in the feedback loop as shown in Figure 2 the resulting transfer function is
Figure 1. Block diagram for a single-pole low-pass function

\[ \frac{A}{s+1} \]

\[ V_{in} \rightarrow A \frac{1}{s+1} \rightarrow V_{out} \]

Figure 2. Block diagram for a single-pole low-pass function with feedback.

\[ V_{in} \rightarrow + \rightarrow \frac{A}{s+1} \rightarrow V_{out} \rightarrow - \rightarrow B \rightarrow V_{in} \]

\[ V_{in} \rightarrow + \rightarrow \frac{A}{s+1} \rightarrow V_{out} \rightarrow - \rightarrow B \rightarrow V_{in} \]
\[ T(s) = \frac{A}{s + (1 + AB)}. \] \hspace{1cm} [2]

The cutoff frequency of the feedback system has increased to \((1 + AB)\) rad/sec whereas the system without feedback has a cutoff frequency of 1 rad/sec. As long as the product of \(A\) and \(B\) is positive an increase in bandwidth can be realized but there is a drawback to this. By comparing the dc gain of the two systems notice that the open-loop system has a gain of \(A\) and the closed-loop system has a gain of \(A / (1 + AB)\). Assuming \(AB\) is positive so as to increase the bandwidth, the gain has decreased by a corresponding amount. Figure 3 illustrates the effect of feedback on the gain and bandwidth of the system. Fortunately the system of interest has sufficient gain which will be sacrificed for extra bandwidth.

The phase, \(\phi_{ol}\), of the frequency response for the open-loop system is

\[ \phi_{ol}(w) = \arctan(w) \] \hspace{1cm} [3]

For the closed-loop system, we find that the phase is

\[ \phi_{cl}(w) = \arctan \left( \frac{w}{1 + AB} \right) \] \hspace{1cm} [4]

Assuming that the product \(AB\) is positive, the argument of the arctangent function in equation 3 increases slower than the one in equation 4. Therefore the closed-loop system does not reach the maximum phase shift until some frequency greater than that for the open-loop system.
Figure 3: The effect of feedback on gain and bandwidth.
3. UNIQUE IMPLEMENTATION OF FEEDBACK

Along with testing the concept that feedback can be used to adjust the bandwidth of the system, the feedback will be implemented in a unique manner [2].

Normally, to monitor the output of an acoustical transmitter, a separate receiver is required. The output voltage of the receiver may then be used to close the loop of the system. In most cases, the transmitter and receiver employ a piezoelectric device to send and to receive the acoustical signal. This method of monitoring the acoustical signal illustrates the theorem of reciprocity. If a piezoelectric device is deformed or in some way subjected to a strain, an electrical potential is developed across it. The reverse is also true, i.e. applying an electrical potential across the material will deform or change its shape in a particular manner. This opposite effect is called electrostriction but both phenomena are usually understood as the piezoelectric effect [3].

This feedback scheme takes advantage of this dual effect and monitors the output of the device by letting the transmitter and receiver be separate areas of the same physical device. By using a three terminal device as shown in Figure 4, one terminal can be used as the ground of the device and the other two will be designated as the forward and the feedback terminals. A voltage applied across the forward terminal is converted into mechanical energy; this energy is converted
Figure 4. Three terminal piezoelectric device for implementation of feedback.
into electricity across the feedback terminal. This is essentially the same
method discussed in the previous paragraph except that the transmitter
and receiver are now integral parts of the same device. The output of
each terminal is related to the surface area it is connected to. By
normalizing the total surface area of the device to unity, the area of the
feedback terminal can be designated as α and the area of the forward
terminal will be (1−α). In this way the electrical response at the
feedback terminal can be related to the response at the forward terminal
in terms of magnitude assuming that they both have the same frequency
response to within a scaling factor.

A significant advantage of this application is the consolidation of the
system. The transmitter and receiver would always be subjected to the
same environmental conditions so that the mechanical properties of each
section of the device would vary in accord with each other. Also, since
this feedback system only makes the transition from electrical to
mechanical to electrical energy. The variable losses due to acoustical
coupling would be overcome.
4. DEVELOPMENT OF MODEL FOR PIEZOELECTRIC DEVICE

To analyze the piezoelectric device, a suitable model is needed that describes the electrical, mechanical, and acoustical properties which the device exhibits.

There exist analogies to Ohm's law and Kirchoff's laws in mechanical systems which can be derived beginning with Newton's second law of motion [4]. These analogies are possible because of the similarity between the mathematics used in analyzing the two types of systems. Because of this similarity, the analysis can be facilitated by the use of an electrical equivalent circuit.

In this analogy, a force is modeled as a current and a velocity as a voltage. For the mechanical system we are concerned with elements which describe quantities such as mass, friction and compliance which are all forms of mechanical impedances. Their corresponding electrical impedance elements using the above analogy are inductance, resistance and capacitance, respectively.

A mechanical system will have a resonant motion when its mechanical impedance is at a minimum in much the same way that an electrical system resonates when its electrical impedance is at a minimum.
There are various modes of vibration which may be set up in a piezoelectric ceramic. Understanding that there are many modes which may be excited in the ceramic, we shall limit our model to account for only one of them [5]. In doing so the assumption is made that the frequencies of vibration of the other modes are outside our range of interest and will not affect the analysis.

The acoustical output of the device due to the mechanical motion is modeled by a current controlled current source which is properly scaled to show the efficiency of the system in converting mechanical motion into acoustical pressure. This controlled source is identical to an ideal transformer with a particular turns ratio.

The electroacoustic transducer is essentially a capacitor with the equivalent circuit describing the piezoelectric effect in shunt to it.

The electrical capacitor is made up of the ceramic piezoelectric material which is plated on opposite sides with a conductive material to form the electrodes. These two metallic surfaces being separated by a dielectric form the electrical capacitance which is called \( C_e \).

The mechanical resonance, having a band-pass characteristic, is modeled as an RLC segment. The current \( I_m \) that flows through this mechanical segment is analogous to a force. The mechanical force that is transmitted into acoustical pressure is usually modeled as a transformer but for simplicity it is shown as a current controlled current source with a factor \( R \), which shows the coupling efficiency of the device. The resulting model is shown in Figure 5 where \( I_a = RI_m \).
Figure 5. Model of piezoelectric device near resonance.
5. ANALYSIS OF MODEL FOR PIEZOELECTRIC DEVICE

To obtain the numerical values of the elements in the electrical equivalent model for the device there are different types measurements that need to be made. First measure the input impedance of the device and correlate its behavior to the analytical derivation of the impedance. A review of A-matrix methods for this derivation are shown in appendix A.

Using this technique the input impedance can be determined from the product of transfer impedance and current gain.

\[
\text{Input impedance} = \frac{V_{\text{in}}}{I_{\text{in}}^2} = \frac{A_{12}}{A_{22}}
\]  

[5]

Taking the model for the device shown in Figure 5 we can derive the A-matrix and solve for the input impedance. Assuming that the responsitivity of the system is unity the matrix elements for the remaining system are shown in equation 6.

\[
\begin{bmatrix}
V_{\text{in}} \\
I_{\text{in}}
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
\frac{1}{sC_e} & 1
\end{bmatrix} \begin{bmatrix}
\frac{1}{s} \left[ s^2 L_m + sR_m + \frac{1}{C_m} \right] \\
\frac{1}{s} \left[ s^2 L_m + sR_m + \frac{1}{C_m} \right]
\end{bmatrix} \begin{bmatrix}
V_{\text{out}} \\
I_{\text{out}}
\end{bmatrix}
\]

[6]

If we simplify the equation so as to express the systems parameters directly we obtain.

\[
\begin{bmatrix}
V_{\text{in}} \\
I_{\text{in}}
\end{bmatrix} = \begin{bmatrix}
1 & \frac{1}{s} \left[ s^2 L_m + sR_m + \frac{1}{C_m} \right] \\
\frac{1}{sC_e} & 1 + \frac{1}{sC_e} \left[ s^2 L_m + sR_m + \frac{1}{C_m} \right]
\end{bmatrix} \begin{bmatrix}
V_{\text{out}} \\
I_{\text{out}}
\end{bmatrix}
\]

[7]
The input impedance $Z_{in}$ may be obtained directly and with some simplification is found to equal.

$$Z_{in} = \frac{s^2 L_m C_m + s R_m C_m + 1}{s [C_m + C_e] \left[ s^2 \left( \frac{L_m C_m}{C_m + C_e} \right) + s \left( \frac{R_m C_m}{C_m + C_e} \right) + 1 \right]} \quad [8]$$

By plotting the input impedance versus frequency, notice some significant points on the curve. There is a frequency for which the impedance of the device is a minimum and will be designated as $f_z$ because this is where the zeros are located. The second frequency of interest is at the relative maximum which will be called $f_p$ because it shows the location of the poles. Consider a general impedance function.

$$F(s) = \frac{\left[ \frac{s}{w_{z_1}} \right]^2 + \left[ \frac{s}{w_{Q_{z_1}} w_{z_1}} \right] + 1}{\left[ \frac{s}{w_{o_1}} \right]^2 + \left[ \frac{s}{w_{Q_{o_1}} w_{o_1}} \right] + 1} \quad [9]$$

Equating coefficients between equations 8 and 9 we can solve for $C_m$ and $L_m$. This is assuming that we already know what the value of $C_e$ is from a low frequency capacitance measurement.

$$f_z = \frac{1}{2\pi \sqrt{L_m C_m}} \quad [10]$$

$$f_p = \frac{1}{2\pi \sqrt{\left( \frac{L_m C_m}{C_m + C_e} \right)^{1/2}}} \quad [11]$$
If \( w_{p1} \) is sufficiently less than \( w_{p2} \) the plot of the input impedance is similar to the one in Figure 6.

![Input impedance graph](image)

**Figure 6.** Example of frequency response of input impedance for a piezoelectric device.
Solving for $C_m$ and $L_m$ from equation 10 and 11 we find that

\[
C_m = C_0 \left[ \frac{f_p}{f_z} \right]^2 \left( \frac{1}{\left( \frac{f_p}{f_z} \right)^2 - 1} \right) \tag{12}
\]

\[
L_m = \frac{1}{\left( \frac{2\pi f_z}{2\pi} \right)^2 C_m} \tag{13}
\]

In order to determine the mechanical resistance, $R_m$, it is necessary to measure the bandwidth of the acoustical output. Define the bandwidth as the -3dB point from the maximum output of the device. Drive the device with a voltage source and measure the output with a sound pressure level meter or microphone to obtain this bandwidth. This is shown in Figure 7 where the bandwidth is $f_h - f_l$. The mechanical resistance can be calculated from

\[
R_m = 2\pi L_m BW \tag{14}
\]

where BW is the bandwidth in hertz.
Figure 7. Example of frequency response of acoustical output of a piezoelectric device.
6. TEST EQUIPMENT

The sound pressure level (SPL) of the piezoelectric device must be accurately monitored in order to verify the bandwidth of the device. To measure the SPL, an audio microphone whose bandwidth is much greater than the bandwidth of the projector is used. The projector and the microphone are mounted on a piece of wood to keep their orientation precise. To assure that the sound is not coupled to the microphone through the wood, the microphone is isolated with a piece of rubber tubing. The spacing between the projector and the microphone is about two inches and they are directly facing each other.

To drive the projector network, the sweep function generator of a spectrum analyzer will be utilized. This sweep function generator also drives the horizontal channel of the spectrum analyzer. The acoustic signal measured by the microphone will be used to drive the vertical channel. The output level of the microphone is very small, therefore a pre-amplifier with a gain of 20dB is used before the connection to the vertical amplifier of the spectrum analyzer. The pre-amplifier is shown in Figure 8.

Since the spectrum analyzer provides external outputs for both horizontal and vertical channels, an analog XY plotter is used to obtain hard copies of the frequency response of the acoustic projector. A block diagram of the test configuration is shown in Figure 9.
Figure 8. Preamplifier used to amplify signal from microphone to spectrum analyzer.
Figure 9. Block diagram of test configuration for measuring the acoustical output of the piezoelectric transmitter.
The data measured by this arrangement will be used to characterize the performance of the acoustical transmitter. Though this measurement system is not calibrated, it can be used to obtain data for comparisons of the system's response.
7. MODEL IDENTIFICATION

Consider the piezoelectric device with the three terminals as shown in Figure 4. There are three capacitances which model the electrical characteristics of this device. One exists across each pair of terminals. \( C_{e1} \) is across the input terminal and ground, \( C_{e2} \) across the input and feedback terminals and \( C_{e3} \) across the feedback terminal and ground.

Using a capacitance meter and consecutively shorting two of the three terminals and measuring between the two remaining ones isolates two parallel capacitances at a time and the three resulting equations can be solved for each value.

Since parallel capacitances add, the resulting measurements give the sum of two capacitances at a time shown below.

\[
C_{e1} + C_{e2} = 15.42 \text{ nF} \\
C_{e1} + C_{e3} = 16.59 \text{ nF} \\
C_{e2} + C_{e3} = 1.17 \text{ nF}
\]

By solving these equations for each capacitance it is found that \( C_{e2} \) is
much less than \( c_{e1} \) or \( c_{e3} \) such that

\[
C_{e1} \approx 15.42 \text{ nF}
\]

\[
C_{e2} \approx 0 \text{ nF}
\]

\[
C_{e3} \approx 1.17 \text{ nF}
\]

From the capacitance measurement it was found that the total electrical capacitance \( C_e \) equals 16.59 nF.

The frequency response of the acoustical output was measured and a plot of the magnitude of the forward section is shown in Figure 10. This information is used to obtain the bandwidth of the acoustical output. In the open-loop system, the bandwidth \( BW \) of the device is 275 Hz with a center frequency \( f_0 \) of 5500 Hz. This defines the \( Q \) of the device where

\[
Q = \frac{f_0}{BW}
\]

Therefore

\[
Q = 20.0
\]

From the impedance measurement, \( f_z \) was equal to 5200 Hz and \( f_p \) was equal to 5570 Hz.

Using this information, return to equations 12, 13 and 14 to calculate the equivalent electrical impedances of the device.
Figure 10. Open loop frequency response of acoustical output for piezoelectric device.
\[ C_m = 16.59 \times 10^{-9} \left[ \frac{5570}{5200} \right]^2 - 1 \]

\[ C_m = 2.445 \text{ nF} \]

\[ L_m = \frac{1}{(2\pi 5200)^2 \left[ 2.445 \times 10^{-9} \right]} \]

\[ L_m = 383.2 \text{ mH} \]

\[ R_m = 2\pi \left[ 383.2 \times 10^{-3} \right] 275 \]

\[ R_m = 662.1 \text{ \Omega} \]

By taking the ratio of the feedback capacitance to the forward capacitance we can estimate the relative surface areas of the forward and feedback portions of the device.

\[ \alpha = \frac{C_e3}{C_e1} = \frac{1.17 \text{ nF}}{16.59 \text{ nF}} \]

\[ \alpha = 0.07 \]

\[ (1 - \alpha) = 0.93 \]
8. OPTIMIZATION OF FEEDBACK DESIGN

Three types of compensation are analyzed to compare the bandwidth improvement. They are

1. varying the relative sizes of the forward and feedback sections (alpha compensation)
2. amplifier compensation
3. pole-zero cancellation (beta compensation)

The bandwidth improvement was compared for the maximally flat case [6] of each type of compensation. The criterion for a maximally flat magnitude response is discussed in appendix B. Because of the resulting order of the transfer functions obtained for the feedback systems we will use the band-pass to low-pass transformation and normalize the cutoff frequency to one radian. This is done by substituting

\[ P = Q \left( \frac{s}{w_o} + \frac{w_o}{s} \right) \]  \hspace{1cm} [16]

in equation 9. Therefore

\[ \frac{1}{(p+1)} = \frac{s}{w_o Q} \left( \frac{s}{w_o} \right)^2 + \frac{s}{w_o Q} + 1 \]  \hspace{1cm} [17]
This will reduce a second order function to a first order function and still retain all the information needed to analyze the frequency response while making the algebraic manipulations easier.
9. COMPENSATION BY ADJUSTING ALPHA

Consider varying $\alpha$ (alpha), the percent of feedback surface area relative to the overall device surface area. Although this is not a variable in our case, it would be interesting to see where the device could be segmented during the manufacturing process to obtain a wider bandwidth. Assuming no other compensation is to be used, a means for summing the feedback and the input signals would still be required to close the loop of the system. To see how this affects the system, look at the transfer function in equation 18 which was obtained from the block diagram in Figure 11. The system transfer function is found to be

$$T(p) = \frac{(1-\alpha)H(p)}{1+\alpha(1-\alpha)H^2(p)}$$

[18]

where $H(p) = \frac{H_0}{p+1}$ in low-pass form and $T(p) = \frac{SPL_{out}(p)}{V_{in}(p)}$

By substituting for $H(p)$ we obtain

$$T(p) = \left[\frac{H_0}{1+\alpha(1-\alpha)H_0^2}\right] \frac{1+p}{\left[1 + \frac{2p}{1+\alpha(1-\alpha)H_0^2} + \frac{p^2}{1+\alpha(1-\alpha)H_0^2}\right]}$$

[19]
Figure 11. Block diagram of alpha compensated system.
The magnitude of the transfer function is found by letting \( p = jw \) in equation 19, multiplying it by its complex conjugate and taking the square root of the result. Equation 20 shows the magnitude of the frequency response.

\[
|T(\Omega)| = \left[ H_0 - \frac{(1-\alpha)}{1+\alpha(1-\alpha)H^2_0} \right] \left[ \frac{1 + \Omega^2}{1 + \frac{2(1-\alpha+\alpha^2)}{2\Omega^2} + \frac{1}{[1+\alpha(1-\alpha)H^2_0]^2}} \right]^{1/2}
\]

Assuming a loss-less system would allow \( H_0 \) to equal unity. To obtain maximally flat magnitude the coefficients of \( \omega^2 \) must be equal. By equating coefficients it is found that \( \alpha = 0.618 \) or about 62\% of the total device must be segmented for feedback. If this is compared to the normalized open-loop cut-off frequency of one radian/second the alpha compensated system has almost fifty percent more bandwidth at 1.49 radians/second. Plots of the response versus alpha are shown in Figure 12.
Figure 12. Theoretical frequency response of acoustical output for alpha compensation.
10. AMPLIFIER COMPENSATION

Consider the transmitter in the feedback configuration with a simple amplifier for compensation in the feedback loop. Let the amplifier gain be equal to \( A \). The block diagram for this system is shown in Figure 13. The loop transfer function is

\[
\frac{\text{SPL}(p)_{\text{out}}}{V(p)_{\text{in}}} = \frac{H(1-\alpha)}{1+\frac{2p}{1+\alpha(1-\alpha)A^2} + \frac{p^2}{1+\alpha(1-\alpha)A^2_H}}
\]

To simplify analysis, let \( A_0 = 1 + \alpha(1-\alpha)H^2_0 \). The magnitude is found to be

\[
|T(\Omega)| = \frac{(1-\alpha)}{A_0} \left[ \frac{1+\Omega^2}{1 + \frac{4-2A_0}{A_0^2} \Omega^2 + \frac{1}{A_0^2} \Omega^4} \right]^{1/2}
\]

For maximally flat magnitude it is required that

\[
1 = \frac{4-2A_0}{A_0^2}
\]

It is found that for \( H_0 = 1 \) and alpha equalling 0.07, which is what the capacitance measurements determined, \( A \) must equal 3.625 and that the cut-off frequency has been increased to 1.49 radians. Notice that if the device is sectioned properly, the same bandwidth improvement would be obtained but the gain of this configuration is greater. Plots of the
Figure 13. Block diagram of amplifier compensated system.
frequency response with different loop gains are shown in Figure 14.

The increase in bandwidth can also be seen by looking at the time response of the system. From Figure 15 we can derive the transfer function to be

\[ H(p) = \frac{2}{(p+1)} \]  \hspace{1cm} [24]

where the time response is found to be

\[ h(t) = 2e^{-t} \]  \hspace{1cm} [25]

The closed loop transfer function from Figure 16 is

\[ H(p) = \frac{2(p+1)}{2+2p+p^2} \]  \hspace{1cm} [26]

and its time response is

\[ h(t) = 2e^{-t} \cos(t) \]  \hspace{1cm} [27]

Plots of these responses are shown in Figure 17 and the closed loop system is seen to have a more narrow time response which can be translated as a wider bandwidth in the frequency domain.
Figure 14. Theoretical frequency response of acoustical output for amplifier compensation.
Figure 15. Block diagram of open-loop system.

Figure 16. Block diagram of feedback system.
Figure 17. Theoretical time response for open loop and Amplifier compensated system.
11. COMPENSATION BY ADJUSTING BETA

What is termed β (beta) compensation is actually a pole-zero cancellation method for compensation. By implementing a compensation network in the feedback loop of the system, we can further extend the bandwidth of the system.

The compensation network will be of the form \((p+1)/(p+β)\) where the denominator, when denormalized is

\[
\frac{1}{(p+β)} = \frac{sQ}{W} = \frac{1}{1+\left(\frac{sQ}{W}\right)^2}
\]

where \(Q_d\) is the Q of the device and we shall define the Q of the compensation as \(Q_c\). By comparing equation 17 with equation 28 we can write that

\[
Q_c = \frac{Q_d}{β}
\]

From the block diagram in Figure 18 the transfer function of the beta compensated circuit is found to be

\[
\text{\textit{T}(Ω)} = \left[\frac{\beta(1-α)H_0}{\text{\textit{βAa}(1-α)H}_0^2}\right] \left[\frac{1-α^2}{α^2}\right]^{1/2}
\]

\[
\frac{1}{1 + \frac{(β+1)^2-2[β+Αα(1-α)H_0^2]}{[β+Αα(1-α)H_0^2]}} \left[\frac{1-α^2}{α^2}\right]^{1/2}
\]

\[
\left[\frac{1}{1 + \frac{[β+Αα(1-α)H_0^2]}{[β+Αα(1-α)H_0^2]}}\right]^{1/2}
\]
Figure 18. Block diagram of Beta compensated system.
By selecting values of $\beta$ greater than 1 the gain $A$ of the system can be increased to obtain a maximally flat response. As $\beta$ is increased, the bandwidth is increased which results in more gain needed in the feedback loop to attain maximally flat magnitude. This feedback method should yield the most significant results provided the gain in the feedback loop can be increased enough and not cause instability problems. Figure 19 shows the affect of increasing $\beta$ on bandwidth.
Figure 19. Theoretical frequency response of acoustical output for beta compensation.
12. RESULTS

To test the predicted performance of the amplifier compensation, a circuit was used that would duplicate the characteristics of the block diagram in Figure 13. The actual circuit used for the test is shown in Figure 20.

U1, U2, and U3 are LF351 operational amplifiers. The input and feedback signals are summed with the appropriate phases at the inverting input of U2. By varying the resistor \( R_{FB} \), the gain in the feedback loop can be adjusted according to \( A = \frac{R_2}{R_{FB}} \).

When the open-loop frequency response was measured, higher order modes were found to exist of significant amplitude to limit the maximum gain allowable in the feedback loop before oscillations occurred. In an attempt to minimize the amplitude of these higher order modes, the transmitter was placed across U2 which causes the higher frequencies to be rolled off due to the electrical capacitance of the device. The electrical capacitance causes U2 to behave as an integrator.

Several values of \( R_{FB} \) were tried and the results of the frequency response are shown in Figure 21. Oscillations occurred when the feedback gain A was increased to approximately 20 volts/volt. The maximum bandwidth achieved was 2.55 times the open-loop bandwidth.
Figure 20. Electrical network used to measure bandwidth increase.
Figure 21. Actual frequency response of acoustical output for amplifier compensated system. (Magnitude levels are relative)
A comparison of the theoretical and test results are shown in Table 1. As the feedback gain $A$ is increased, the bandwidth of the acoustical output increased. The rises at the edge of the passband predicted by the calculations do not appear. This could be the result of the assumption that $H_0 = 1$ and would mean that the calculated gain in the feedback loop could be in error. According to the calculations, to obtain a 2.55 improvement in bandwidth would result in a 1.5 dB peak at the edge of the passband. The gain of the test circuit would be on the conservative side meaning to match the calculated response requires a higher than calculated gain in the actual circuit.
## Table 1
### AMPLIFIER COMPENSATION

THEORETICAL AND ACTUAL RESULTS

<table>
<thead>
<tr>
<th>Feedback Gain (A V/V)</th>
<th>Bandwidth Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
</tr>
<tr>
<td>3.625</td>
<td>N/A</td>
</tr>
<tr>
<td>6.25</td>
<td>2.09</td>
</tr>
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<td>2.48</td>
</tr>
<tr>
<td>16.67</td>
<td>2.55</td>
</tr>
</tbody>
</table>
CONCLUSION

It has been demonstrated that active electroacoustic feedback can be implemented to adjust the performance of the system in terms of expanding the bandwidth.

An instability problem in the amplifier compensated system arose which was unexpected. By looking at the modes of vibration for a ceramic disk it is found that the solution to the set of boundary conditions that exist are in the form of Bessel functions. A few of these modes of vibration are shown in Figure 22. The axes that are shown are on the plane \( z=0 \). Taking a pie slice of the circle and looking at it on edge will reveal that part of the slice is positive and part is negative with respect to \( z \). The negative part can be related to negative voltage and vise-versa such that the total area under the curve, being positive or negative, determines the sign of the feedback signal. This transition from negative to positive feedback is believed to be the cause of the oscillation problems. With a properly sectioned device the feedback gain could be increased to well beyond what was attained in this testing.
Figure 22. Shapes of vibrational modes for a circular plate clamped at edges.
APPENDIX A

REVIEW OF A-MATRIX ANALYSIS

Considering the two port network shown in Figure 23 and writing two equations which characterize its behavior, one can derive all of the parameters needed to describe the system. These equations are well known in two port network theory and will be included here for completeness.

\[ V_{in} = A_{11} V_{out} + A_{12} I_{out} \]  \[ I_{in} = A_{21} V_{out} + A_{22} I_{out} \]

The matrix expression for these two equations now follows and is shown in equation 33.

\[
\begin{bmatrix}
V_{in} \\
I_{in}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
V_{out} \\
I_{out}
\end{bmatrix}
\]

To derive the variables \( A_{11}, A_{12}, A_{21}, \) and \( A_{22} \), we go back to the network in Figure 20 and open circuit the output terminals. This forces the output current to equal zero so that equations 31 and 32 can be used to derive the voltage gain and the transfer admittance of the network.

\[ \text{Voltage gain} = \frac{V_{out}}{V_{in}} = \frac{1}{A_{11}} \]
Transfer admittance \[ \frac{I_{\text{in}}}{V_{\text{out}}} = A_{12} \] \[35\]

Next we short circuit the output terminals which forces the output voltage to be equal to zero. We can now solve for the transfer impedance and the current gain.

Transfer impedance \[ \frac{V_{\text{in}}}{I_{\text{out}}} = A_{12} \] \[36\]

Current gain \[ \frac{I_{\text{out}}}{I_{\text{in}}} = \frac{1}{A_{22}} \] \[37\]

Figure 23. Two port network.
APPENDIX B
MAXIMALLY FLAT MAGNITUDE

One if the characteristics which we would prefer to have is a maximally flat magnitude response [6] in the frequency domain. An analytical method is discussed here.

By using the band-pass to low-pass transformation the center frequency is effectively relocated at dc or zero frequency. If we intend for our response to be maximally flat, we would like for the slope of the frequency response to be zero until we reach the cutoff frequency. This can be equivalently stated as requiring all of the derivatives of the response to be equal to zero at dc. Define the frequency response in general terms as

\[ H(\Omega) = \frac{H_H}{H_0} \left[ 1 + a_1 \Omega + a_2 \Omega^2 + a_3 \Omega^3 + a_4 \Omega^4 \ldots \right] \left[ 1 + b_1 \Omega + b_2 \Omega^2 + b_3 \Omega^3 + b_4 \Omega^4 \ldots \right] \]  

[38]

where \( H_0 \) is \( H(0) \). Now take the magnitude of the frequency response to obtain

\[ |H(\Omega)| = \left[ \frac{1 + a_1 \Omega^2 + a_2 \Omega^4 \ldots}{1 + b_1 \Omega^2 + b_2 \Omega^4 \ldots} \right]^1 \]

[39]
By dividing the denominator into the numerator yields

\[ H(\Omega) = H_0 \left[ 1 + \left( a_1 - b_1 \right) \Omega^2 + \left[ a_2 - b_2 - b_1 \left( a_1 - b_1 \right) \right] \Omega^4 + \cdots \right]^{1/2} \tag{40} \]

From complex variable analysis [7], a function can be uniquely defined by an infinite series of derivatives of the function called a Taylor series, or for our case a McLaurin series because we are going to expand the function about the origin, i.e. zero frequency. The series is of the form

\[
F(\Omega) = \sum_{n=0}^{\infty} \frac{F^{(n)}(0)}{n!} \Omega^n \tag{41}
\]

where \( F^{(n)} \) is the nth derivative of \( F(\Omega) \) with respect to \( \Omega \). If we evaluate the series at \( \Omega = 0 \) and write out the first few terms of the series we can equate coefficients between \( F(\Omega) \) and \( H(\Omega) \).

\[
F(\Omega) = F(0) + \frac{F^{(1)}}{1!} \Omega + \frac{F^{(2)}}{2!} \Omega^2 + \frac{F^{(3)}}{3!} \Omega^3 + \frac{F^{(4)}}{4!} \Omega^4 + \cdots \tag{42}
\]

If we require that all of the derivatives of \( H(\Omega) \) are equal to zero, then all of the coefficients of the powers of \( \Omega \) should be equal to zero. This will be true if \( a_1 \) equals \( b_1 \) for the first power of \( \Omega \), if this is true then for \( \Omega^2 \), \( a_2 \) must equal \( b_2 \). Continuing further in the same manner we find that

\[
a_k = b_k \tag{43}
\]

will meet the requirements for a maximally flat function.
REFERENCES


