Mechanics of Low Dimensional Biomimetic Scale Metamaterials

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MECHANICS OF LOW DIMENSIONAL BIOMIMETIC SCALE METAMATERIALS

by

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Combination of topology and material could play an important role in giving rise to nontraditional behavior in mechanical structures and is a typical strategy in nature. Topology is concerned with the geometrical and spatial properties of the objects, which are preserved under continues mechanical deformation of the object such as stretching, bending, twisting, etc. In this work, we focus on structures based on fish scale inspired surface topology. The utilized idea for surface topology is a bioinspired design on the substrate of biological scale-covered systems. Scales are a path breaking evolutionary adaptation that accompanied vertebrate evolution for the past 500 million years. Fish scales are inherently lightweight with diverse shapes, sizes, materials, and distribution, and they provide remarkable architecture-material enhancement, typical of metamaterials. Here we provide a perspective on mechanical behavior of fish scale inspired structures and quantify the origins of some of their striking mechanical properties that include nonlinear and directional strain stiffening in both bending and twisting, dual nature of friction which combines both resistance as well as adding stiffness to motion. We will provide derivation of mathematical laws that govern structure-property relationships that can help guide design. The response of biomimetic scale under twisting, bending and combined load is tailorable through the geometric arrangement and orientation of the scales. Also, the analytical models have been validated by the finite element analysis. We outline and explain the progress in understanding the complexities of these structures in global and local deformation modes and conclude by offering future perspectives and challenges.
To my parents, my late uncle, and all who always believed in me.
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CHAPTER 1: INTRODUCTION

1.1 Motivation

Combination of topology and material could play an important role in exhibiting nonlinear and nontraditional behavior in mechanical structures exceptionally biological systems. Topology is concerned with the geometrical and spatial properties of the objects, which are preserved under continues mechanical deformation of the object such as stretching, bending, twisting, etc. Several area of studies such as theoretical physics, quantum mechanics, biology, computer science, robotics, and cosmology are dealing with the topological study and its effects on the systems. Topology could be applicable to the bulk materials, surface of the structure, or interfaces in multi-material systems at various length scales. From mechanical point of view, the tailorability of topological properties can increase the multifunctionality of the structure.

In biological systems, surface topology can engage in the stiffness, mobility, adhesion, protection, wave reflection, camouflaging, etc. in macro- and mesoscales. Also, from mechanical and material science point of view, surface and interface topology could affect permeability, diffusivity, adhesion, electrostaticity, anisotropy, fluid flow, fracture and degradation, etc. in meso-, micro- and nanoscale. Surface topology could affect the local properties of the material as well as the global response of the structure. These aforementioned aspects indicate the necessity of using multiscale approach to study the role of surface geometry on the various low dimensional multi-material systems. Low dimensional materials are the materials that have at least one dimension small enough for the physical properties of the material. This low dimensionality could lay somewhere between the scale of individual atoms to the scale of the bulk material. Materials like ceramic coatings, graphene, bioinspired coatings, carbon fibers, and nanotubes are including in this category, as shown in Figure 1.1 for spectrum of the length scale and disciplines.
Multiscale modeling is a style of modeling of multiple phenomena at different length scales simultaneously to describe a complex system. These models sometimes originate from physical laws of different nature, including continuum mechanics to the molecular dynamics and quantum mechanics. Multiscale modeling can be applied on modeling fluids, solids, polymers, composites, etc., as well as various physical and chemical phenomena (like chemical reactions, diffusion, thermophysics, etc.). The necessity of multiscale modeling is on the base of this fact that the available macroscale models are not accurate enough to consider the micro-phenomena. Also, the microscale models are not efficient enough with available current computational technology, and offer too much information that can deviate the focus of system description from its global behavior. The aim of multiscale modeling is delivering a balance between feasibility, accuracy and efficiency. In traditional approaches, we tend to focus on one particular scale. For example to focus on just macroscale phenomena, we just consider some constitutive relations for the effects of smaller scales, or to focus on microscale behavior, the effects of larger scales are neglected by assuming the system homogeneous at larger scales. Despite traditional modeling, multiscale
approach uses a variety of models at various resolution and complexity, linked together either analytically or numerically.

In this work, we focus on bioinspiration for surface topology in the low dimensional multi-materials. The idea for surface topology is a bioinspired design on the base of biological scale-covered systems. Scales have been a recurring dermal feature in the evolutionary history of complex vertebrates. One reason for their success is their tremendous multifunctional benefits, including hydrodynamic, mechanical stiffness, chemical, and optical advantages. This evolutionary requirement has made them hierarchical, structurally hybrid, and composite in nature capable of engaging multiple length scales. From a mechanical standpoint, scales traditionally provide protection against foreign objects and organism attacks. This feature is inspiration for using these principles to develop bioinspired coating conception as protective glove and armor design. However, another structural feature is of immense importance in addition to localized loads common in protective applications. It is the role of scale engagements as topological surface features in modifying the global deformation behavior of the underlying structure. This feature deepens the role of scales in aiding both locomotion and swimming. The representative volume element (RVE) is the smallest volume in a multi-material system, which if a measurement can be made over it that will yield a value representative of the whole system. In this work, an RVE-based model will be developed to investigate the local and global deformation behavior of a scale-covered bioinspired metamaterial to unveil the effect of the scales in natural systems at meso- and macroscale resolution.

1.2 State of the Art

Topological surfaces have been granted a great attention in the past decades due to the exceptional mechanical properties they provide. Their advantages extend well into a variety of applications. For example, topography on the surface of a soft grippers aid grasping objects more efficiently [7].
In addition, surface topography similar to the gecko feet can also help adhesion with various objects [8]. In biomedicine, it has been presented that adding topographical features to a smooth surface reduce the phenomena of bacterial fouling [9]. Interestingly, anisotropic topological features is known to significantly influence the surface wettability of aluminum, which is typically hydrophilic [10]. The benefit of surface topology also appears in aerospace applications. Dermal skin of sharks was found to enhance the aerodynamic performance of airfoils [11]. In polycrystalline materials, interface diffusion creep between two phase material is highly affected by the interface topology [12]. Even modifying mesoscale surface topology could lead to noise reduction in helical gears [13]. Thus, it concluded that endowing surface features promises a remarkable enhancement in functions in wide range of engineering applications.

Biomaterials have many interesting properties due to their unique combination of topology and material. These properties can be used directly or as inspiration to give benefits to the mechanical systems. In bio-systems, we can find lightweight structures, which have high mechanical resistance against mechanical or thermal loads that can be occurred in nature. In recent years, a lot of researchers got inspiration from biomaterials and this bioinspiration leads to develop biomimetics interdisciplinary field, which is the imitation of the structures, models, systems, and elements of nature for the purpose of solving complex problems [14]. Also by development of 3D printing technology during recent years, designing and fabrication of 3D printed metamaterials and structures increased and there are efforts to propose new designed metamaterials, which can show novel properties or applications that barely can be found in the regular materials like metals, minerals, plastics or even composites. This leads to develop architectured materials, which can be called bioinspired metamaterials [4, 15–18]. Many biological and biomimetic structures use geometrically pronounced features to produce highly nonlinear behavior [19–22]. These materials include seashells, hierarchical honeycombs, snail spiral, seahorse tail, fish scales, lobster exoskeleton, crab exoskeleton, butterfly wings, armadillo exoskeleton, sponge skeleton, etc. [23–27].
Among these structures, dermal scales have garnered special attention recently due to complex mechanical behavior in small and large deflections both at the material and at the scale level [28–34]. The materials making up scales are hybrid, hierarchical, composite and can engage at multiple length scales [35–45]. Scales are naturally multifunctional [46, 47] and protective for the underlying substrate, which has been an inspiration of armor designs, where overlapping scales can resist penetration and provide additional stiffness [32, 34, 42, 48, 49]. Fabrication methods such as synthetic mesh sewing and stretch-and-release have been recently developed to produce overlapping scale-covered structures in 2D and 1D configuration [4, 50]. These fabricated structures show almost ten times more puncture resistance than soft elastomers. In addition to protection against external objects, which is mechanically an indentation type local deformation problem, global deformation modes such as bending [51–58] and twisting [58–60] of a substrate reveal equally interesting properties. These include small strain reversible nonlinear stiffening and locking behavior due to the sliding kinematics of the scales in one-dimensional substrates [52, 54, 56, 57, 59]. Clearly, friction between sliding scales can significantly alter the nature of nonlinearity in both bending and twisting [54, 60, 61].

As mentioned earlier, surface topology could affect the behavior of a system at different length scales due to its multiscale nature. The associated length scale of the feature are inherent within the idea of multiscale modeling to gain the accurate physics of the problem [62]. Two different multiscale methodologies are originated from the paradigm of solid mechanics continuum theory: hierarchical method and concurrent method. Concurrent methods provide the missing information in the macroscale model from on-the-fly computations at the microscale level. In this method, the computational domain divided with various regions associated with different simulation methods [63]. In the hierarchical methods, a set of different numerical techniques are used sequentially at disparate length scales and connect through a bridging methodology, such as statistical analysis methods, homogenization techniques, or optimization methods [64]. Before the multiscale model-
ing is defined, bridging method were being addressed in the solid mechanics community to bring lower length information into a continuum model at a higher length scale [65]. Eshelby [66, 67] created the modern “micromechanics” analysis, modeling, and simulation by declaring a self-consistent theme by assigning microscale heterogeneity to an effect within the continuum level. The continuum model could be assigned as a “Representative Volume Element” or RVE. In the self-consistent method, the RVE should be determined and how the boundary conditions work down from higher scales to lower scales. In Mechanics, the restriction variable is strain or strain derivatives, which are transferred from the higher scale to the lower scale [68, 69]. Different types of displacement boundary conditions were examined, and concluded the periodic boundary conditions are the best in most cases, for the best representing of macroscopic behavior by the RVEs [70, 71]. The fundamental assumption in homogenization theories is that the ratio of the small-scale to the macro-scale representative lengths must be small enough [70, 72].

For linear problems, asymptotic homogenization theories with periodic boundary conditions have become into the most used and successful approach enabling obtaining both the local state in the RVE and the global solution in the two scales [71, 73, 74]. As a matter of fact, extending this theory to microstructure evolving nonlinear problems implies the need of modelling the whole domain and each RVE, for each load or time step. Therefore, it allows obtaining the homogenized (averaged) macroscopic material properties at each local point and time [75–77]. After solving the macroscopic problem, the values of the connecting variables are transferred to the micro-RVE, that is then solved and updated its microstructure and properties, if needed. This procedure, often denoted as computational homogenization or, when using finite element approximation called as multi-level finite element method (FE2). This method is usually formulated in an iterative way that makes the computational process expensive, although cost is strongly reduced when using parallel computation for which this approach is especially well suited [76, 77]. It worth to mention that FE2 multiscale approach is a novel type of model falls within the general category of multiscale models.
in which, the constitutive equations are written only on microscopic scale, and homogenization and localization equations are used to compute the macroscopic strains and stresses, knowing the mechanical state at microscopic level [72, 75, 78].

1.3 Intellectual Merit

There are some intellectual merits in this work that we discuss briefly in this section. The main intellectual merit is the tailorability of structure behavior using their surface topology. Stiff scale features on the soft skin in some animals and biological organism have topological effect on their locomotion, protection and camouflaging. Dermal scales have attracted tremendous scrutiny in last decades due to their unusual mechanical properties such as strain-stiffening and locking behavior due to sliding and friction between the scales. Bioinspired deigns on the base of dermal scales can endow a novel tailorable nonlinear elasticity in the structures, which can be useful in wide range of engineering application. We can obtain a lightweight stiffening mechanism by adding relatively rigid scale-like topological features on a surface of a flexible substrate. The contact between the scales can provide a geometrical nonlinear behavior for a linear elastic structure.

1D substrates with stiff scales revealed strain stiffening in the bending mode of deformation [51]. Later simplification revealed the distinct nonlinear regimes of elasticity with and without friction between scales contact [52, 54]. More studies revealed the limits of theoretical assumptions underpinning the models and their effect on predicted relationships [55, 57]. Extending the dimensionality of the problem, 2D substrates were also investigated, showing several similarities with their 1D counterparts in bending [4, 53, 58, 79]. However, the mechanics of twisting, a critical and fundamental mode of global deformation has not yet been discussed in detail. Twisting mode of deformation is perceptible for some soft robotic applications and can also arise due to boundary defects and bending-twisting coupling in structures. Furthermore, this mode is an important
first step towards more complex combined deformation modes and two-dimensional scale-covered metamaterials. We address these nonlinear problems for the first time by investigating the response of stiff scale-covered slender biomimetic substrates under torsional and thermal loads and outline the gamut of topological tailorability of the system.

This work is critical to understanding how to address the development of advanced modern materials including bioinspired metamaterials. This is important because the bioinspired metamaterials signify an altogether new paradigm of materials development. Unlike traditional materials, development of these modern materials rely extensively on a-priori computational methods, materials modeling and nonlinear optimization to reveal the accurate distribution of matter at multiple scales needed for high performance and tailor made applications. These models can then be fed directly to digital manufacturing platforms such as additive manufacturing and robotic assembly to yield materials which transcend the envelope of traditional materials. This will result in development of unique and novel materials for most industries across the United States from healthcare to military and space industries.

Furthermore, the analysis and testing of these materials will bring about a fundamental advancement in our understanding of these novel low dimensional multi-material systems. Without this work, we will not be able to progress in developing a new class of biologically inspired - topologically complex, multifunctional structures at all scales which have a broad spectrum of applications such as space structures as mesh reflector and booms, bioinspired anti-fouling surfaces, fuselage of aircraft, medical devices such as stent systems, and unmanned aerial vehicles (UAVs).

Development of new materials has been an ancient endeavor which retains its relevance in the modern world. Counterintuitively, development and adoption of novel materials do not necessarily arise from scarcity. On contrary, historically, most materials such as bronze, iron, composites, etc. signaled a paradigm shift in material processing and design once the initial serendipitous
discovery of the base material had been made. Appearance of novel materials technology had often signaled tremendous societal, economic and geopolitical upheavals and realignments. This general disruptive nature of materials discovery continues till today. Thus, the direct fruits of this research help maintain a scientific edge to deepen the success of this new frontier metamaterials through accelerated computing and additive manufacturing, which could lead to multifunctional, durable and robust material production at an industrial scale.

1.4 Dissertation Outline

The dissertation is consisted of seven chapters. Chapter 1 is the introduction, which includes motivation of this research, state of the art, and intellectual merit of this work. Chapter 2 obtains a literature review on the bioinspired metamaterials, biomimetic scale-covered systems, developed fabrication methods, and different studies which have been conducted on these low dimensional multi-materials. Multiple research papers including peer-reviewed journal papers and conference proceedings are already published or under publication process in accordance with this research, which four of them are consisting the chapters 3 to 6. Each chapter presents a single research paper, which is cited under the first page of the chapter. Finally, chapter 7 states the comprehensive conclusion of this research and provides the perspective of future works.

1.5 List of Publications

This research resulted in the following peer-reviewed journal publications:


Also, this work resulted in the following conference proceedings:


CHAPTER 2: BACKGROUND AND LITERATURE REVIEW

2.1 Abstract

Exoskeletons, such as scales on fishes and snakes were a critical evolutionary adaptation. Honed by millions of years of evolutionary pressures, they are inherently lightweight and yet multifunctional, aiding in protection, locomotion and optical camouflaging. This makes them an attractive candidate for biomimicry to produce high performance multifunctional materials with applications to soft robotics, wearables, energy efficient smart skins and on-demand tunable materials. Canonically speaking, biomimetic samples can be fabricating by partially embedding stiffer plate like segments on softer substrates to create a bi-material system, with overlapping scales. The origins of exoskeleton’s mechanical behaviors are not merely due to load distribution but because of an intricate interplay of deformation, sliding and interfacial behavior. Such interplay give rise to property combinations that are typically not visible in the parent material of either the scales or the substrates. Here we review and present the origins of some of their fascinating behavior which include nonlinear and directional strain stiffening in. We outline and explain the progress in understanding the complexities of these structures in global and local deformation modes.

Citation 1:


Citation 2:

2.2 An Ancient Inspiration

Fishes are primitive animals, appearing early during the Cambrian explosion of life forms about 500 million years ago [80–82]. Although the first fishes likely did not possess scales [82], they appeared soon [80] and eventually made their way forward to reptiles such as snakes and alligators, and mammals [29, 32]. Natural scales are lightweight, stiff, and multifunctional [83, 84]. Protection, locomotion, thermal regulation, and camouflaging have been attributed to scales [81, 85, 86]. From a physical standpoint, their lightweight structure, stiffness tunability, and damage tolerance are of great interest for designing bio-inspired advanced materials. Scales endow unusual properties to the underlying substrate by two pathways – via unusual parent (scale) material properties, and the complexity of organization and mutual engagement.

Properties of the parent scale material have been intensely scrutinized over the last decades. Scanning Electron Microscopy and X-Ray diffraction studies of naturally occurring scales show that they are essentially complex composites with intricate microstructures [34, 48, 87, 88], to provide high strength and toughness against indentation through enhanced energy dissipation before failure [89] and excellent balance between strength and flexibility [34]. These mechanical properties are strongly dependent on hygroscopic states and loading rates [90, 91]. In contrast to scale material behavior, scales’ organization and engagement can also produce emergent multifunctionality. Overlapping or discrete arrangements of scales guarantee flexible locomotion and protection, Figure 2.1 (a) [92, 93], by enhance drag reduction and regulate body undulations [81, 94]. On land, scales in snakes help in locomotion by manipulating friction [95]. Scales aid in protection against penetration by dispersing localized forces such as bites across greater areas via scale contact for damage mitigation [32, 40, 45, 79]. Different overlap patterns e.g. stacked, linear overlapped, and staggered overlap, influence these behaviors [42, 96].

Such enhanced survivability have made scales, and their geometric forms, universally prevalent in
the animal kingdom. Yet the scales themselves and their organization are diverse, responding to the predators, environment, and the biology of the organism itself. Even within the organisms, they can vary in material, shape, size, arrangement and color depending on the functionality [33,34,44,88]. Thus, interest in scales architecture arose early in history.

Among these applications was the direct mimicry of the scaled integuments for producing lamelllar and scaled armors in the mediaeval and ancient times [48, 49]. In spite of this early interest, little was known about the fundamental mechanics behind superior performance, which resulted in similar designs reappearing over years. Only recently has greater scrutiny ensued, buoyed by
rapid advances in 3D printing and the need for novel soft structural materials for tailored protection [34, 48, 88, 98, 99], and in wearables and robotics [100, 101]. Here, the focus has been on creating novel mechanical behavior from the sliding and interlocking of scales, which leads to unusual combinations of force–displacement nonlinearity, anisotropy, penetration and structural damping [51–61]. Such unusual behavior originate from scales geometrical arrangements, rather than directly from the parent materials. This distinctly geometric or topological characteristic puts these biomimetic structures in the class of a fast-developing frontier of synthetic materials called metamaterials.

2.3 Mechanical Metamaterials and Emergence

Mechanical metamaterials typically consist of periodic assemblies of multiple elements or composites, which exhibit incredible property combinations distinct from their parent materials [102, 103]. These include negative Poisson’s ratio [104–106], tailorable elasticity [52, 56, 59], shape morphing [103, 107, 108], unprecedented damage and fracture tolerances [98, 109], acoustic and photonic band gaps [110, 111], tailored anisotropy and abnormal penetration resistance [42, 45, 97, 112]. The origins of these phenomena are tied to the idea of emergence. Emergence is a phenomena where the overall properties of a system at the macroscopic scale are different from the microscopic scale due to the collective interactions of elements constituting the system [113]. Although synthetically conceived, some of the best examples of using geometry and topology to enhance properties are found in the natural world. Devoid of advanced parent materials that humans have mastered, other organisms are forced to improve mechanical properties using topological routes in response to evolutionary pressures over millions of years [88, 114]. Thus, it is common to see complex patterns in natural structures such as honeycombs [111, 115], fish scales [51–61], and nacre structure [116, 117].
This makes bioinspiration a natural path to develop metamaterials. For instance, bioinspired honeycombs can provide unusual hardening and auxetic behavior controlled by the structure’s geometry [111, 115]. Hierarchical structure and instability induced by compression can further tune their band gaps [111]. Similarly the structure of nacre has been utilized to provide strain hardening, high toughness and impact resistance [116, 117]. Biomimetic scales are in the same vein where emergence occurs due to the intricate mutual scales sliding kinematics. Such substrates exhibit complex strain stiffening, interlocking stages, anisotropy, and strange dissipation behavior, even in the deformation range where the scales and the substrates individually do not exhibit them, as characteristics of emergent behavior.

2.4 Biomimetic Scale Metamaterial Fabrication

Scales have been adopted to make primarily two types of biomimetic structures. The first type is an exoskeletal architecture, includes both the overlapping architecture, where a soft substrate is covered with partially-embedded overlapping scales Figure 2.1 (b) [52, 56, 57, 59], as well as the non-overlapping or segmented architecture, Figure 2.1 (c) [97]. One fabrication strategy is to glue 3D printed stiff plates [52, 56, 57, 59], or steel sheets [57], into prefabricated grooves of a Vinylpolysiloxane (VPS) elastomer [52, 56, 57, 59], or 3D printed flexible material [57]. Other strategies comprise multi-material 3D printing [118], laser engraving of alumina strip for scales and stretch-and-release fabrication method [50], and sewing cellulose acetate butyrate (CAB) scales on a polypropylene mesh using cotton thread [4].

Other strategies include sequential molding and casting [119–122]. Similar strategies and along with laser engraving can be used for fabricating segmented samples architecture [97]. Topologically interlocked materials (TIMs) [123] is also another approach which uses either building blocks of various shapes or craving the interfaces of interlocking elements from the bulk of glass panel
using focused laser beam [124–126]. In the second type of architecture, scales are fully embedded inside the matrix to form composite imbricated stiff inclusions into a thick soft substrate, Figure 2.1 (d) [42]. This architecture can be manufactured using a multi-material 3D printer [38], or embedding 3D printed Acrylonitrile butadiene styrene (ABS) plates into a rubber substrate [42]. In spite of these differences in architecture, these metamaterials share many typical behaviors. In this research, we fabricated the samples as shown in Figure 2.2.

![Fabricated 1D scale-covered prototypes](image1.png)  ![Fabricated 2D scale-covered prototypes](image2.png)

Figure 2.2: (a) Fabricated 1D scale-covered prototypes using glue the stiff scales into prefabricated grooves of deformable substrate. (b) Fabricated 2D scale-covered prototypes using female mold to arrange scales in a 2D overlapped arrangement.

2.5 Bending of Scale-covered 1D Beam

2.5.1 Mechanics of Global Bending Deformation

Bending is one of the most important deformation mode for slender structures. In a plain beam, the moment–curvature characteristics remain linear for small deflections, reflecting the linear elastic behavior of the parent materials. This behavior changes when scales are integrated into the structure. Here samples are partially-embedded with scales on one side of substrate to highlight the differential properties, with and without scales engagement, Figure 2.1 (b) and Figure 2.2. As the substrate deforms, scales begin engaging and their mutual sliding gives rise to geometrically
dictated nonlinearity. The nature and origin of the nonlinearity can be best introduced in bending [51, 52]. The substrate bent shape can be envisaged as circular arc of a beam, Figure 2.3 (a). Assuming periodic scale engagement, a representative volume element (RVE) can be isolated, Figure 2.3 (a). One can assume rigid scales if they are sufficiently stiff. The RVE geometry, Figure 2.3 (a), reveals two kinematic variables – the local angular deflection of the scales $\theta$, and the substrate angular change $\psi$ (related to curvature $\kappa$). These variables can be interrelated by imposing periodicity [52, 54]:

$$
\eta \psi \cos \frac{\psi}{2} - \sin (\theta + \frac{\psi}{2}) = 0.
$$

(2.1)

Here, $\psi = \kappa d$ and $\eta = l/d$, where $d$, $l$ are the scales’ spacing and exposed length respectively [52, 54]. This relationship can be plotted for varied $\eta$ generating a kinematic mechanisms map, Figure 2.3 (b). This map indicates that the system performance spans three kinematic regimes of operations including linear (before the scale engagement), nonlinear stiffening, and finally a kinematically locked configuration. Note that this locked configuration is independent of friction and of purely kinematic origin. Realistically speaking, near this locked state, the internal forces would rise substantially, leading to a transition from substrate deformation to scale deformation, which is significantly stiffer. The moment-curvature response for this system can be derived by a micro-macro energy balance (Hill-Mandel condition) [127].

Thus, the total applied work on the beam can be written as $W = L_B \int_0^\kappa M d\kappa$ where $L_B$ is the substrate length, $M$ is the applied moment, and $\kappa$ is the substrate curvature. This work is absorbed by the beam and scales leading to $W = \frac{1}{2} E_B I_B \kappa^2 + \epsilon_{scales}$, where $E_B$ and $I$ are the Young’s modulus and the second moment of area of the beam, respectively. Also, $\epsilon_{scales}$ is the energy from scales rotation on the substrate. Assuming a torsional spring to capture this rotational stiffness, $\epsilon_{scales} = \frac{1}{2} N K_B (\theta - \theta_0)^2$, where $N = L_B / d$, $\theta_0$, and $K_B = 1.75 E_B D^2 (L/D)^{0.66}$ are the number of scales, initial scale’s inclination, and scales torsional stiffness of the scale–substrate joint, respectively.
Here, $D$ and $L$ are the scale’s thickness and scale’s embedded length [52]. Plugging in $\theta = \theta(\psi)$ from Equation (2.1), we get the bending moment response from work–energy balance as follows:

$$M(\kappa) = E_B I \kappa + K_B (\theta - \theta_0) \frac{\partial \theta}{\partial \psi} H(\kappa - \kappa_e).$$  \hspace{1cm} (2.2)

Here $\kappa_e$ and $H(\kappa - \kappa_e)$ are the engagement curvature and Heaviside step function to track engagement, respectively. This relationship shows that after scales engagement, the moment response of the structure displays nonlinear stiffening, as seen in Figure 2.3 (c) for the frictionless case denoted by $\mu = 0$. In the above relationships, key simplifications arise in estimating the rotational stiffness of the scale–substrate joint, substrate material nonlinearity, scale rigidity assumption, scale distribution uniformity, and reference and post-engagement periodicity. This can raise concerns about the universality of results.

However, extensive follow-up studies relaxing these restrictive simplifications have confirmed the surprisingly robust nature of the overall nonlinear strain stiffening and locking behavior [55–57]. In addition to overlapping exoskeletal architecture, investigations on the fully-embedded designs also indicate distinct strain-stiffening behavior in bending [38, 42, 128]. However, material compression between fully-embedded scales is the governing mechanism. Inclusions provide nonlinear stiffening, dependent on the volume fraction of stiff scales right from the beginning of the loading [38, 42, 128].

### 2.5.2 Frictional Effects in Bending

For exoskeletal architecture, friction between the sliding scales is of critical significance. The locking curvature is a kinematic idealization. In reality, normal forces start rising sharply as this configuration is approached. For rough surfaces, if Coulomb friction force is assumed between
Figure 2.3: (a) The isolated RVE and free body diagram for bending load. (b) The kinematic mechanisms map. (c) The moment–curvature response. (d) nondimensionalized friction force vs curvature for various friction coefficients. (e) The kinematic mechanisms map with frictional effects. (f) The post-engagement moment–curvature for various friction coefficients.

For simplicity, bending is again used to exemplify the analysis. In this case, using the same RVE concept introduced earlier, a force balance can be carried out with friction force \( f_{fr} = \mu N \), where \( N \) is the normal force and \( \mu > 0 \) is the coefficient of friction, Figure 2.3 (d). This leads to frictional force dependence on curvature, shown in Figure 2.3 (d). Here friction force is normalized by \( K_B/l \), and subscript \( e \) indicates engagement values. Clearly, the frictional force is singular in nature,
indicating a point of “frictional lock”, beyond which the scales cannot move. This lock is found to occur earlier than the frictionless kinematic lock. Juxtaposing this on the kinematic mechanism maps, we get a progressively advancing locking envelope with increasing $\mu$, as shown in Figure 2.3 (e) with dashed lines. Although the architecture–dissipation relationships are considerably more complicated than frictionless case, friction can be included using extended energy balance for bending, [54,60]:

$$M(\kappa) = E_B I \kappa + (K_B (\theta - \theta_0) \frac{\partial \theta}{\partial \psi} + f_{fr} \frac{dr}{d\psi}) H(\kappa - \kappa_e).$$

(2.3)

Here, $\kappa_e$ is the engagement curvature value, and $dr$ is the relative differential displacement in the direction of sliding, shown in Figure 2.3 (a). Increasing the coefficient of friction leads to an increase in nonlinear strain-stiffening in a scaled system, Figure 2.3 (f). This increase in strain-stiffening is tailorable with respect to the geometric parameters of the system [54,60].

Thus, friction can enhance resistive forces due to additional internal forces but also restrict range of motion by advancing lock. Therefore, increasing $\mu$ may not necessarily increase total dissipative work in a deformation cycle. This is shown in Figure 2.4, as the relative energy dissipation (RED), which is the ratio of frictional or dissipative work calculated as $w_{fr} = \frac{1}{\kappa_e} \int K_{lock} f_{fr} dr$, to the total work calculated as $w_{sys} = w_{fr} + \frac{1}{2} EI \kappa_{lock}^2 + \frac{1}{2} \frac{K_B}{d} (\theta_{lock} - \theta_0)^2$ for different values of $\mu$, and $\eta$ [54].

Here, the substrate is considered linear elastic material and the scales are rigid. Also, additional dissipation could come from material sources such as plasticity, viscoelasticity, Mullins effects in rubber, etc. [129], which is not considered here. As shown in Figure 2.4, dissipation is maximized only at intermediate $\mu$ in bending.
In this section, we provide an emergent dynamical behavior that arises due to the free oscillation of scale-covered 1D beams. The equation of motion of such beams can be obtained using the variational energy equation that is derived from Hamilton principle of least action $\delta \int_{t_i}^{t_f} (\dot{T} - \dot{V}) dt = 0$. In this formula, $\ddot{T}$ refers to the kinetic energy per unit length and $\ddot{V}$ denotes the strain energy per unit length of both substrate and scales [61]. The resulting equation of motion can be written as $m \frac{\partial^2 \ddot{y}}{\partial x^2} + \frac{\partial^2 M(\kappa)}{\partial x^2} = 0$, where $m$ is the mass of scale-covered beam and $M(\kappa)$ was presented earlier in Equation (2.3). Without loss of generality, we assume a simply supported beam and an initial velocity conditions that ensures the activation of only the first mode of vibration [61]. We solve the equation of motion numerically using the direct numerical integration method called Newmark along with Newton-Raphson [130].

We plot the deflection of the middle point of the beam, $\ddot{y}$ normalized by the radius of gyration against the time which is also normalized by the undamped period of a plain beam, $\ddot{t}$ for the case of
$\eta = 10$ and various coefficients of frictions, $\mu$ as shown in Figure 2.5. The free oscillation response highlights that the most interesting feature of the simple structure is the exponential decay, Figure 2.5, that arises due to the interfacial friction between scales unlike the most common spring-mass system, which indicates a linear decay in the response due to dry friction [131].

Figure 2.5: Free vibration of the midpoint of a simply supported biomimetic scale-covered beam with 25 scales, $\theta_0 = 3^\circ$, $\eta = 10$, and various coefficients of frictions $\mu$. The deflection is normalized by vertical length scale (radius of gyration) and the time is normalized by the natural frequency and time period of the plain elastic substrate. The increase in the interfacial friction between neighboring scales leads an anisotropy in the exponential decay of both smooth and scaled side of the beam, illustrated by exponential decay $\delta_1$ and $\delta_2$. Inset illustrates the case study tested.

The difference in response implies a geometric regime transformation of damping from dry friction to viscous drag. The revealed dampening phenomena, also called “viscous emergence”, which resembles viscous dumping behavior of vibrating structures, was pronounced with postulation of only dry Coulomb friction. However, the similarities with viscously damped system is limited. For conventional damped oscillators, the decaying behavior continues forever until motion stops, but in this biomimetic system the decay will not lead to a complete stop of the system but rather until deflection is small enough that scales may not engage any more, returning the system to a
conservative system (i.e., the slightly artificially damped system). Note that the increase in interfacial friction provides different stiffening mechanisms on each side of the beam due to the steady loss energy in each cycle. Thus, the topology-induced nonlinearity brought about the overlapping of scales provides a significant anisotropy of frictional behavior, giving rise to emergent behavior tailorability through interfacial and geometric parameters. Here, the control parameters can be the overlap ratio of scales $\eta$ and the coefficient of friction between neighboring scales $\mu$.

2.7 Penetration and Contact Response

Geometrical effects made possible by segmentation, overlap, and scales interaction can enhance mechanical properties beyond what is possible for monolithic materials at small or large strains. Here, failure mechanisms and elastic response are of interest. The penetration resistance of natural scales systems confirm the contributions of both the microstructure of the scales themselves as well as their organization [45, 89]. Metamaterials inspired by these designs are investigated by applying indentation loading to scales attached to a soft elastic substrate [38, 40, 45, 50, 79, 118] or by performing ballistic impact simulations or experiments [132–134]. In general, the puncture resistance of segmented systems, Figure 2.1 (c), are significantly greater than a continuous glass plate attached to soft substrate [97].

Additionally, these segmented scaled systems are capable of sustaining damage in different sections and maintaining functionality, unlike a monolithic glass plate that could completely shatter [79]. Several failure modes have been identified, including substrate material shearing [42], scale bending [42], scale puncture [44, 45], and scale fracture [97]. However, in segmented systems, the most notable is the tilting failure mode [79, 96, 97], Figure 2.6 (a). This failure mode occurs when scales rotate excessively during contact, allowing the indenter to slip through the gaps and penetrate the underlying substrate material. This mode is consistent regardless of the
strength of the scale materials, and depends on the scale stability [79, 96, 97, 114] as the scales resist rotation. It has been found that the tilting mechanism is more prevalent with decreasing scale size [79, 97] and substrate stiffness [79], Figure 2.6 (a).

The engagement of scales dramatically increase the scale stability, with the contacting surfaces providing support [96, 114] as well as friction between scale and indenter [50]. However, the friction between scales doesn’t seem to provide additional benefits to the system [45]. This highlights the synergistic importance of material and geometry in determining the behavior of these systems including scale overlap, scale angle and volume fraction [4, 50, 112]. Some overlapping architectures can disperse the loads over much larger areas [4, 45], even after a partial puncture, reducing damage to the underlying substrate [45]. The anisotropy of contact area depends on indenter radius, relative to the scale length [112]. The energy dissipation of a partially-embedded system has been shown significant increase via the addition of surface grit and elastic cover, which delays the scales disengagement, maintains contact with the indenter tip and pulls on the surrounding scales through the indentation [121]. In a partially-embedded systems, the effective contact area increases beyond the radius of the indenter [112], and increasing the load continues to increase the redistribution of strain through a greater portion of the sample until scale failure [51, 112].

It has also been found that contact stiffness increases with scale density and decreasing interface rotational stiffness [51, 112]. For fully-embedded scales, Figure 2.1 (d), flexible scales deform around a blunt indenter, increasing the stiffness and acting locally as a composite material [42]. The scale overlap and initial scale angle of a fully-embedded system have been varied to determine the governing variables in the failure modes of the system [42]. The flexibility and protective properties of embedded systems have also been found to be tunable by altering the geometric properties and distribution of the scales [38, 42, 112]. For instance, low initial scale angle and high volume fraction display greatly reduced flexibility [38], Figure 2.6 (b).
As described earlier, some of the earliest uses of scaly substrates have been for armor protection. However, non-penetrative contact typically possible in the robotic gripping applications are equally fascinating. One way to highlight the interplay of scales and indenter, is by carrying out indentation experiments with different indenter nose radius, representing various types of contacts ranging from the very sharp to blunt. As seen in Figure 2.7, a sharp indenter can induce an anisotropic contact area, which is mitigated as the indenter radius increases. The origin of the anisotropy is due to the arrangement and orientation of the scales. A sharp indenter induces preferential engagement of scales on one side (right in the figure) compared to other. This breaks the symmetry of the contact area [112].

It is observed that as the indenter radius increases, for a given scale length, the anisotropy in the contact surface decreases. This is due to the ability of larger indenters to come into contact with additional scales to the left as the substrate deforms. From this, it is anticipated that larger indenters, capable of contacting multiple scales at the onset, could become significant in the force-displacement response. This requires further study.
Figure 2.7: (a) Decrease in loading surface anisotropy with increasing indenter radius, where $R = 2.5$, 25, 37.5, and 50 mm. (b) Strain concentration, and propagation from embedded scale tips, displayed in finite element and DIC analyses. (c) Plain, $\eta = 1.2$, $\eta = 1.6$, and $\eta = 2.4$ samples shown under equal loading of 80 N show reduced strain concentrations with increasing $\eta$.

We compare our FE simulations experimentally with full field 3D Digital Image Correlation (DIC) measurements. The 3D DIC analyses were run to confirm the finite element simulation results for the experimental configurations, where $R = 37.5$ mm and $\eta = 1.2$, 1.6 and 2.4. The strains in the samples were observed to follow the same distribution, with the strain propagating from the embedded scale ends, as shown in Figure 2.7 (b). Additionally, it is shown that the addition of scales, with increasing $\eta$, increases the distribution and reduces strain concentration, as shown in Figure 2.7 (c), where the four experimental samples are shown under a load of 80 newtons.

### 2.8 Mass Deposition on Topographic Surfaces

Surfaces of semi-aquatic mammals are highly vulnerable to biofilm growth, yet furry mammals that reside at the interface of water and sunlight practically escape this burden, despite the available provisions for micro-organism proliferation. The complex but conspicuous scaly structures on the surface of the furs, as evidenced by scanning electron microscope images have long been
hypothesized to be a very important factor mediating this behavior [135]. Hence, surfaces with topographic features, like biomimetic scales, provide a promising avenue for antifouling strategies. In light of this, a simple model at quiescent flow is described which can shed some light on the role of deformation and topology.

Here, a steady state model is constructed where the external fouling transport is assumed to be diffusive in nature. The steady-state diffusion is modeled by the Laplacian $\nabla^2 C = 0$ in an infinite domain surrounding the fur. The fouling is assumed to be a general first order rate equation. This means the flux of mass deposited on the surface $q$ is proportional to its concentration $C$, i.e., $q = kC$. Using these assumptions, the role of deformation and topology of a beam with scale-like topography on its surface can be seen in Figure 2.8. The plot illustrates the total mass aggregated per unit length along the surface against various configurations of deformation.

![Figure 2.8](image)

Figure 2.8: Mass deposition per unit length along the upper and lower surface of plain beam and scale-like topographic beam with various scales density surrounded by infinite media and the concentration at the far-filed is constant $C_\infty = 10$ while $k = 1$. The cases presented include 20 and 40 scales arranged on the surface with an initial angle of $5^\circ$. 
Here it is assumed that the beam deforms in a sinusoidal shape, where the amplitude of the shape function of the beam is normalized by the thickness $\tilde{A}$ and varied from convex to concave. The total mass is expressed as $M_t = \frac{1}{L_s} \int_{L_3} q ds$, where $ds$ is a line element length of either the topographic or smooth surface, $L_s$ refers to the total arc length of the surface, and $q$ represents the flux of deposited mass. The figure evidently ensures that mass deposition per unit length can be significantly reduced on the upper surface of convex configuration.

In particular, we notice that density of topography (i.e., more scales added to the surface) also leads to less deposition when presented along the upper surface of convex configuration and the opposite is true for concave configurations. Interestingly, the imposition of topography on only one surface provides anisotropy in the amount of deposited mass as compared to smooth surfaces on both sides of the beam. In summary, topography and deformation provide an imperative bearing on mass deposition or biofouling of surfaces, giving rise to antifouling surface design tailorability through topographic surfaces and deformation.

2.9 Applications

As described earlier, some of the earliest uses of scaly substrates have been for armor protection. However, non-penetrative contact typically possible in the robotic gripping applications are equally fascinating. However, another structural feature is of immense importance in addition to localized loads common in protective applications. It is the role of scale engagements as topological surface features in modifying the global deformation behavior of the underlying structure. Topological surfaces advantages extend well into a variety of applications. For example, topography on the surface of a soft grippers aid grasping objects more efficiently. In addition, surface topography similar to the gecko feet can also help adhesion with various objects. In biomedicine, it has been presented that adding topographical features to a smooth surface reduce the phenomena of bacterial
fouling. Interestingly, anisotropic topological features is known to significantly influence the surface wettability of aluminum, which is typically hydrophilic. The benefit of surface topology also appears in aerospace applications. Dermal skin of sharks was found to enhance the aerodynamic performance of airfoils. In polycrystalline materials, interface diffusion creep between two phase material is highly affected by the interface topology. Even modifying mesoscale surface topology could lead to noise reduction in helical gears. Thus, it concluded that endowing surface features promises a remarkable enhancement in functions in wide range of engineering applications.

Bioinspired deigns on the base of dermal scales can endow a novel tailorable nonlinear elasticity in the structures, which can be useful in wide range of engineering application. We can obtain a lightweight stiffening mechanism by adding relatively rigid scale-like topological features on a surface of a flexible substrate. The contact between the scales can provide a geometrical nonlinear behavior for a linear elastic structure. Also, twisting mode of deformation is perceptible for some soft robotic applications and can also arise due to boundary defects and bending-twisting coupling in structures. In this work, we develop a new class of biologically inspired - topologically complex, multifunctional structures at all scales which have a broad spectrum of applications such as space structures as mesh reflector and booms, bioinspired anti-fouling surfaces, fuselage of aircraft, medical devices such as stent systems, unmanned aerial vehicles (UAVs), CubeSats, Ornithopter or “flapping wing” (drone close to surface), and torque-twist controls in space structures like telescopes. All of these possible applications makes them an attractive candidate for biomimicry to produce high performance multifunctional materials with applications to soft robotics, wearables, energy efficient smart skins and on-demand tunable materials.
CHAPTER 3: TAILORABLE TWISTING OF BIOMIMETIC SCALE-COVERED SUBSTRATE

3.1 Abstract

In this letter, we investigate the geometrically tailorable elasticity in the twisting behavior of biomimetic scale-covered slender soft substrate. Motivated by qualitative experiments showing a significant torsional rigidity increase, we develop an analytical model and carry out extensive finite element simulations to validate our model. We discover a regime differentiated and reversible mechanical response straddling linear, nonlinear, and rigid behavior. The response is tailorable through the geometric arrangement and orientation of the scales. The work outlines analytical relationships between geometry, deformation and kinematics, which can be used for designing bioinspired scale-covered materials.

Citation:

Scales have been a recurring dermal feature in the evolutionary history of complex vertebrae [28–33]. One reason for their success is their tremendous multifunctional benefits, including hydrodynamic, chemical, and optical advantages [46, 47, 80, 136]. From a mechanical standpoint, scales traditionally provide protection against foreign objects and organism attacks [34, 49]. This evolutionary requirement has made them structurally hybrid [37, 38, 137], hierarchical [35, 138, 139], and composite in nature [36, 83, 140] capable of engaging multiple length scales [33, 39–45]. This feature was an inspiration for recent work on using these principles for armor design [32, 34, 141].

However, another structural feature is of immense importance in addition to localized loads common in protective applications. It is the role of scale engagements in modifying the global deformation behavior of the underlying structure. This feature deepens the role of scales in aiding both locomotion [142] and swimming [143, 144]. The mechanics underscoring this behavior have been an area of intense scrutiny since the last few years. 1D substrates with stiff scales revealed strain stiffening due to sliding, scale deformation as well as friction in the bending mode of deformation [51]. Later simplification revealed the distinct nonlinear regimes of elasticity even without scale deformation or friction [52]. Nonlinearity due to frictional effects were further isolated and their effect on locking and dissipation quantified [54]. More studies revealed the limits of theoretical assumptions underpinning the models and their effect on predicted relationships [55, 56].

Extending the dimensionality of the problem, two-dimensional substrates were also investigated [4, 53, 58, 96]. These showed several similarities with their one-dimensional counterparts in bending. However, the mechanics of twisting, a critical and fundamental mode of global deformation has not yet been discussed in detail. Twisting mode of deformation is perceptible for some robotic applications [145–151] and can also arise due to boundary defects and bending-twisting coupling in structures [152, 153]. Furthermore, this mode is an important first step towards more complex
combined deformation modes and two-dimensional metamaterials of this type. In this letter, we study the response of stiff scale-covered slender biomimetic substrates under torsional loads and outline the gamut of geometrical tailorable of twisting elasticity.

3.3 Prototype Fabrication and Qualitative Experiment

We carry on qualitative and motivational experiments and the twisting deformation of the uniform prismatic beam covered with scales and the plain sample are shown in Figure 3.1(a). For these samples, the scales were 3D printed out of Poly Lactic Acid (PLA) thermoplastic and the substrate was made from a silicone elastomer known as Vinylpolysiloxane (VPS) by casting into a 3D printed mold with prefabricated grooves for scale insertions. Then the scales were embedded and adhered to the grooves using a silicone glue (Permatex Corporate).

Young’s modulus of PLA and VPS tested under tensile test by MTS Insight®(Electromechanical – 50 kN Standard Length), were found as 2.86 GPa and 1.5 MPa, respectively. We subjected the samples to twist loading, contrasting their response using an MTS Bionix EM®(Electromechanical Torsion – 45 Nm) with similar boundary conditions. Note that the engagement happens only in the clockwise twist direction of the substrate. The applied twist load speed was 0.085 RPM and the experiments have been done up to 2.4 rad. The significant gains in torsional stiffness in the scale-covered samples were apparent when compared to a plain counterpart as shown in Figure 3.1(a) with dashed lines as uncertainties from different tests.

3.4 Kinematics of Twisting for 1D Scaled Beam

Using this motivating experiment, we investigate this twisting behavior by developing an analytical model. We consider a rectangular prismatic bar as a linear elastic substrate. Due to the high
contrast in the elastic modulus of substrate and scales, we use a rigid scale assumption. We first simplify the geometry of this system by considering each scale as a rectangular plate with thickness $t_s$, width $2b$ and total length $l_s = L + l$, where $L$ and $l$ are the length of the substrate embedded section and the exposed section, respectively. The patterned row of scales, spaced by $d$ and embedded on a rectangular prismatic substrate, is quantified in a general orientation defined with angles $\theta$, $\alpha$, and $\gamma$. $\theta$ is the dihedral angle between the top surfaces of substrate and scale, $\alpha$ is the angle between the substrate’s rectangular cross section and the edge of scale’s width, and $\gamma$ is the dihedral angle between the side surfaces of substrate and scale (angle between normal unit vectors $\hat{m}$ and $\hat{n}$ of the side surfaces), Figure 3.1(b).

![Figure 3.1](image_url)

Figure 3.1: (a) Twisting responses of the scale-covered and plain samples. The substrate’s and scale’s dimensions were $200 \times 25 \times 12.5 \text{ mm}$, and $36 \times 36 \times 1 \text{ mm}$. The scales spacing, inclusion length, initial scale inclination and alpha were $d = 14 \text{ mm}$, $L = 10 \text{ mm}$, $\theta_0 = 10^\circ$, and $\alpha = 45^\circ$, respectively. (b) Perspective top view and dimetric view of two consecutive scales geometrical configuration, showing the three orientational angles $\theta$, $\alpha$, and $\gamma$. $\hat{m}$ and $\hat{n}$ are the normal unit vectors of the scale’s and substrate’s side surface, respectively.
We assume that the length of the beam and number of scales are large enough to satisfy the periodicity in the scale engagement under pure torsion with negligible edge effects. This allows the isolation of a representative volume element (RVE) formed one scale and the underlying substrate (scale’s thickness is neglected) at distance \(d\) from adjacent scales embedded in the substrate, Figure 3.1(b). To develop the kinematics of the scale at the RVE level, consider the RVE scale (1st scale in Figure 3.1(b)) fixed with respect to the one immediately preceding it (2nd scale in Figure 3.1(b)).

The second scale rotates about the torsion axis by twist angle of \(\phi\) caused by the twisting of the underlying slender substrate. This leads to an RVE (local) twist rate \(\Phi = \frac{\phi}{d}\). From Figure 3.1(b), if the second scale twists around the torsion axis, then engagement would only happen when one edge of rotated scale, \(D'C'\), contacts at a point with the subsequent edge \(BC\) of the fixed scale. The continual twisting of the underlying beam progresses the contact between two consecutive scales, increasing the scale’s inclination angle from an initial angle \(\theta_0\) to the current angle \(\theta\). This contact imposes kinematic constraints on scale sliding, leading to the following nonlinear relationship between the substrate’s twist angle \(\phi\) and scale’s inclination angle \(\theta\) (see Supplementary Material of this chapter for derivation):

\[
\begin{align*}
&\left(\cos \phi - 1\right)\left(\beta \sin 2\alpha \sin \theta + \eta \cos^2 \alpha \sin 2\theta + 2\lambda \cos 2\alpha \cos \theta\right) - 2\cos \alpha \cos \phi \sin \theta \\
&2\sin \alpha \sin \phi (\eta + \lambda \sin \theta) + 2\cos \alpha \sin \phi \cos \theta (\beta - \sin \alpha) = 0. \quad (3.1)
\end{align*}
\]

Where \(\eta = l/d\), \(\beta = b/d\), and \(\lambda = t/d\) are the overlap ratio, dimensionless scale width, and dimensionless substrate thickness, respectively. In the small twist regime \((\theta \ll 1, \phi \ll 1)\), the implicit constraint equation simplifies to the explicit \(\theta = \phi (\eta \tan \alpha + \beta - \sin \alpha)\). Furthermore for \(\alpha, \beta \ll 1\) (thin substrate with grazing scales), we get \(\theta \sim \phi (\eta \alpha + \beta - \alpha)\). For higher \(\eta\) range \((\eta \gg 1)\), then for fixed \(\alpha\) and \(\beta\) the first term will dominate and we will get \(\theta \sim \eta \phi\). This is
similar to the scaling law obtained from bending deformation [52] and underscores the universal importance of scale overlap ratio. Unlike bending, we also find an additional amplification factor $\theta \sim \beta \phi$ underscoring the more general nature of this system.

The governing nonlinear Equation (3.1) establishes a phase map spanned by $(\theta - \theta_0)/\pi$ and $\phi/\pi$, which is shown in Figure 3.2(a) for different $\eta$ along with FE simulation results (see Supplementary Material of this chapter). Here, $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, $\beta = 1.25$, and $\lambda = 0.45$. This phase diagram maps out three kinematic regimes of operations for the structure under twisting, which includes linear, nonlinear and rigid regions of operation. The linear region is a direct result of the non-engagement of scales. However, as soon as the scales begin engaging, a distinctly nonlinear regime emerges, tuned by $\eta$.

The angle of engagement $\phi_e$ decreases with increasing the overlap ratio. For relatively smaller deformation, an explicit relationship emerges between $\phi_e$ and other kinematic parameters, $\phi_e = \frac{\theta_0}{\eta \tan \alpha + \beta - \sin \alpha}$. The stiffening increases with scale sliding, ultimately leading to a point where no more sliding is possible without significantly deforming the scales themselves. This is the third regime of deformation called “locking”, which the system begins to behave almost as a rigid body. We find this rigidity envelope mathematically by satisfying $\partial \phi / \partial \theta = 0$.

Locking signals a sharp rise in contact forces, which violates the scale rigidity condition near the envelope due to local scale deformation. Around that phase boundary, the stiffness of the whole system transitions towards the stiffness of the scales, which are significantly stiffer than the substrate. This is consistent with previously published work [52, 53]. Also, if $\eta$ is smaller than critical value of $\eta_c$, no engagement is possible due to the geometrical limit of engagement. This computes to $\eta_c = \frac{1 - 2 \beta \sin \alpha}{\cos \alpha \cos \theta_0}$ and is physically meaningful if $\eta_c > 0$ (see Supplementary Material of this chapter). The agreement of the analytical relationship with FE simulations in Figure 3.2(a) shows minimal effect of substrate warping on the kinematics.
We explore the geometric tailorability of elasticity using another phase map, parametrized by $\alpha$, Figure 3.2(b), with $\eta = 3$, $\theta_0 = 10^\circ$, $\beta = 1.25$, and $\lambda = 0.45$. This phase map shows that increasing $\alpha$ not only leads to a quicker engagement but also steeper nonlinearity. Also, there exists a critical $\alpha_c$, below which no locking would be possible for a given set of geometrical parameters. Although increasing $\eta$ always leads to decreasing locking twist angle, this trend does not hold for $\alpha$.

These effects are summarized using two other phase diagrams, both spanned by $\eta$ and $\beta$ in Figure 3.3. Figure 3.3(a) indicates that locking angles decrease for higher $\eta$ and it increases with $\beta$ for low enough $\eta$; however, higher $\eta$ depresses the sensitivity of locking angle to $\beta$. Interestingly, this phase plot shows that although higher $\eta$ always decreases the range of deformation, the influence of $\beta$ is strongly dependent on $\eta$. In Figure 3.3(b), which tracks the critical angle $\alpha_c$ below which locking would not take place, similar tuning behavior of $\beta$ is apparent.
Figure 3.3: (a) Phase plot of locking $\varphi$ ($\varphi_{\text{lock}}$), spanned by $\eta$ and $\beta$ with the given values of $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, and $\lambda = 0.45$. (b) Phase plot of critical $\alpha$ ($\alpha_c$), spanned by $\eta$ and $\beta$ with the given values of $\theta_0 = 10^\circ$ and $\lambda = 0.45$.

3.5 Mechanics of 1D Scaled Beam under Twisting

These kinematic nonlinearities ensure that mechanical response would also be nonlinear even when the materials themselves are in the linear elastic regime. We consider the twisting of the biomimetic scale-covered substrate as a combination of plain beam twisting and scales rotation. The scale rotation in 3D space can be defined by change in angles $\theta$, $\alpha$, and $\gamma$, Figure 3.1(b). As the scales engage and begin rotating, the elastic substrate resists scales rotation. The substrate resistance is modeled as linear torsional springs corresponding to the change in each of the angles $\theta$, $\alpha$, and $\gamma$. Thus the energy absorbed due to the 3D rotation of each scale can be described as $U_{\text{scale}} = \frac{1}{2}(K_\theta(\theta - \theta_0)^2 + K_\alpha(\alpha - \alpha_0)^2 + K_\gamma(\gamma - \gamma_0)^2)$, where $K_\theta$, $K_\alpha$, and $K_\gamma$ are the corresponding rotational spring constants.

Extensive FE simulations indicate that the contribution of both $K_\alpha$ and $K_\gamma$ terms are negligible (see Supplementary Material of this chapter). This leads to $U_{\text{scale}} \approx \frac{1}{2}K_\theta(\theta - \theta_0)^2$. The most
significant parameters of the scale-substrate joint stiffness $K_\theta$ is the Young’s Modulus of substrate $E_B$, the scale’s embedded length $L$, the scale’s width $2b$, the scale’s thickness $t_s$ as well as $\theta_0$ and $\alpha$. We further assume that $t_s \ll l_s$ and $0 \ll L \ll 2t$. Considering this system as a single scale embedded in a semi-infinite beam in length and thickness, we postulate the following scaling expression:

$$\frac{K_\theta}{E_B t_s^2} = C_B(\alpha) b \left( \frac{L}{t_s} \right)^n f(\theta_0),$$

where $n$ is a dimensionless constant, and $f(\theta_0)$ and $C_B(\alpha)$ are dimensionless angular functions.

We carried out FE simulations on a single scale embedded in a semi-infinite media and varied the relevant geometric variables of Equation (3.2) to ascertain the fit of this empirical relationship. We find an excellent fit in the region of $12 < L/t_s < 80$, yielding $n = 1.55$, $C_B(\alpha) = 3.62$, and $f(\theta_0) \approx 1$, indicating negligible angular dependence (see Supplementary Material of this chapter).

In non-circular cross sections, warping leads to an out-of-plane displacement even in small deformation [154]. Although warping’s effect in kinematics was negligible, its effect on mechanics must be accounted. Typically, the effect of warping is addressed using a non-dimensional pre-multiplier $C_w$ in the relationship between torque and twist rate leading to $T = C_w G_B I \Phi$, where $T$ is the RVE (local) torque, $G_B$ is the shear modulus of elasticity, and $I$ is the moment of inertia of the beam cross section. $C_w$ can be found from literature for standard cross sections [155]. The embedding of rigid inclusions leads to an increase in stiffness even before engagement.

This composite-like effect can be estimated through either computations or analytically using homogenization models. In this work, this is modeled numerically using an inclusion correction factor $C_f$, which would depend on the volume fraction, shape and size of the inclusion. This leads to a modified torque-twist relationship $T = C_f C_w G_B I \Phi$. Motivated by elasticity arguments, we postulate that $C_f = 1 + C_0(\alpha) \left( \frac{\xi}{\lambda} \right)^m h(\theta_0)$, where $\xi = L/d$, $h(\theta_0)$ and $C_0(\alpha)$ are dimensionless angular functions for dependency of $C_f$ to $\theta_0$ and $\alpha$. We ascertained the fit, using FE numerical simula-
tions (see Supplementary Material of this chapter), yielding \( m = 1, C_0(\alpha) = 1.33, \) and \( h(\theta_0) \approx 1, \) indicating negligible angular dependence. With these assumptions, work-energy balance for the unit length of the substrate can be described as:

\[
\Phi \int_0^\Phi T(\Phi')d\Phi' = \frac{1}{2}C_fC_wG_BI\Phi^2 + \frac{1}{2}K_\theta(\theta - \theta_0)^2H(\Phi - \Phi_e),
\]

(3.3)

where \( H(\Phi - \Phi_e) \) is the Heaviside step function to track scales engagement. The right-hand side of this equation is the summation of the energy absorbed by the substrate’s elastic torsion \( U_{\text{substrate}} = \frac{1}{2}C_fC_wG_BI\Phi^2, \) and the scales engagement \( U_{\text{scales}} = \frac{1}{2}K_\theta(\theta - \theta_0)^2H(\Phi - \Phi_e). \) The torque-twisting rate relationship for the system could be found by differentiating Equation (3.3) with respect to \( \Phi \) and is written as:

\[
T(\Phi) = C_fC_wG_BI\Phi + \frac{K_\theta}{d}(\theta - \theta_0)\frac{\partial \theta}{\partial \Phi}H(\Phi - \Phi_e). 
\]

(3.4)

This nonlinear expression is plotted for different \( \eta \) with \( \theta_0 = 10^\circ, \alpha = 45^\circ, \beta = 0.6, \) and \( \lambda = 0.32 \) in Figure 3.4. The substrate’s properties are considered as \( G_B = 10 \) GPa and \( v = 0.25 \) with the cross section’s dimension of \( 32 \times 16 \) mm. The scale spacing, the thickness and the embedded length of the scales are assumed as \( d = 25 \) mm, \( t_s = 0.1 \) mm, and \( L = 4.5 \) mm.

The results are compared to FE simulations and we find a remarkable fit with our model. The plot clearly demonstrates the sharp rise in nonlinear stiffening. The plot also highlights the inclusion effect in significantly increasing the torsional stiffness even before the engagement and underscores the accuracy of our model.

The linearized torque-twist rate for small \( \theta \) is:

\[
\frac{T(\Phi)}{G_BI} = C_fC_w\Phi + 2C_B\beta(1 + v)\frac{i^2d^2}{L}\left(\frac{L}{t_s}\right)^n(\eta \tan \alpha + \beta - \sin \alpha)^2(\Phi - \Phi_e)H(\Phi - \Phi_e), 
\]

(3.5)
eta = 5
eta = 8
eta = 15
eta = 20

Rectangular Beam with Cw
Beam with Cf

Figure 3.4: Phase map of dimensionless torque \( \frac{T(\Phi)}{G_B l} \) versus twist rate (\( \Phi \)) for different \( \eta \) with the given values of \( \theta_0 = 10^\circ \), \( \alpha = 45^\circ \), \( \beta = 0.6 \), and \( \lambda = 0.32 \). Black dotted lines represent FE results.

where we recall that \( G_B = \frac{E_B}{2(1+v)} \). This linearized analytical expression sheds important light on the role of geometric parameters in enhancing the torsional stiffness of the substrate in small deformation. Particularly apparent is the stiffening effect of the lateral \( \beta \) parameter, which has a nearly cubic relationship to torque. The effect of overlap ratio \( \eta \) is quadratic, similar to bending behavior. This highlights the distinctness of the twisting response of the biomimetic scale-covered substrate. This plot also conforms with the more gentle slope of the experimental samples which corresponds to \( b = 18 \text{ mm}, l = 26 \text{ mm}, t = 6.25 \text{ mm}, d = 14 \text{ mm}, \alpha = 45^\circ \), and \( \theta_0 = 10^\circ \), leading to \( \eta = 1.86, \beta = 1.29, \) and \( \lambda = 0.45 \).

Interestingly, although under large deformation, some amount of material nonlinearity can be expected to arise, the substrate’s material was found to behave at least nominally linear up till the twisting limits considered in this paper. This is evident in the experimental results, Figure 3.1(a). Further FE simulations with nonlinear material models (Neo-Hookean and Mooney-Rivlin) also showed little effects on our results (see Supplementary Material of this chapter).
3.6 Conclusion

In conclusion, our work shows the geometrical tailorability of elastic response under twisting loads including stiffness, envelopes of operations and the overall energy landscape. We find that stiffness increase brought about by scales is highly nonlinear, reversible, and tailorable, distinct from simply coating or embedding with a stiffer material or a composite. This system exhibits a very specific nonlinear behavior including a seamless straddling between linear elastic, nonlinear elastic and a quasi-rigid behavior which exhibited by neither the PLA nor the silicone elastomer material. Each of these regimes can be tailored using a different geometrical arrangement. This study strengthens the arguments of using biomimetic scales for designing structural metamaterials in a wide range of applications beyond simple bending. The current analytical model is aimed primarily to obviate the need for detailed fully resolved FE simulations for some aspects of design and analysis. These FE simulations become prohibitive for larger number of scale contacts, larger twists or for future work on dynamics, which would require repeated FE simulations on the structure.

3.7 Supplementary Material

3.7.1 Derivation of Kinematic Formula

To find a contact criterion between the two consecutive scales shown in Figure 3.5, the 3D-equations of line $BC$ from the fixed scale and line $D'C'$ from the rotated scale, are established with respect to the coordinates $xyz$ placed on the torsion axis. The coordinates $xyz$ is the transformation of coordinates $XYZ$ in the $Y$–axis direction up to half the thickness of the substrate, $t$. Therefore, the $y$–axis still passes through the midpoint of the fixed scale’s width, $O$. Using this coordinate system, the parametric equation of the line $BC$ (red line shown in Figure 3.5) can be written as:
where \( p \) is the line parameter which can vary from \(-b\) to \(b\), and \( t \) is the half thickness of the beam.

To write equation of the line \( D'C' \) (blue line in Figure 3.5), which is on the rotated scale, first the point \( D' \) is located after rotation, using rotation matrix \( R(\varphi) \) by angle \( \varphi \) about torsion axis shown in Figure 3.5 (here is \(-z \) axis). This rotation matrix can be written as:

\[
R(\varphi) = \begin{bmatrix}
\cos \varphi & \sin \varphi & 0 \\
-\sin \varphi & \cos \varphi & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]  

(3.7)

Then, the unit vector of line \( D'C' \) is rotated by angle \( \varphi \) about torsion axis using Rodrigues’ rotation formula. This formula states that if a vector \( \vec{V} \) rotates about a unit vector \( \hat{s} \) by angle \( \varphi \), according to the right hand rule the vector after rotation \( \vec{V}_{rotated} \) should be:

\[
\vec{V}_{rotated} = \vec{V} \cos \varphi + (\hat{s} \times \vec{V}) \sin \varphi + (\hat{s}) (\hat{s} \cdot \vec{V}) (1 - \cos \varphi).
\]  

(3.8)

By this method, the parametric equation of line \( D'C' \) from the second scale after rotation was derived as:

\[
x = (\tan \theta \tan \varphi - \sin \alpha)q + (t \sin \varphi - b \cos \alpha \cos \varphi),
\]

\[
y = (\tan \theta + \sin \alpha \tan \varphi)q + (t \cos \varphi + b \cos \alpha \sin \varphi),
\]

\[
z = (\frac{\cos \alpha}{\cos \varphi})q + (d - b \sin \alpha),
\]

(3.9)
where \( q \) is the line equation parameter and can vary from 0 to \( l \). To find a contact point between these two lines, Equation (3.6) and Equation (3.9) must coincide for \( x \), \( y \) and \( z \) respectively. This will lead to \( p \) and \( q \) from the first two equations of derived system of equations. These parameters can then put into the third equation, leading to an analytic relationship between \( \theta \) and \( \phi \). The distance between scales denoted by \( d \) determines the line density of scales, which can be used to nondimensionalize the geometrical parameters. The nonlinear geometric relation between the substrate local twist angle \( \phi \) and scale inclination angle \( \theta \) can be written as:

\[
(\cos \phi - 1) \left( \beta \sin 2\alpha \sin \theta + \eta \cos^2 \alpha \sin 2\theta + 2\lambda \cos 2\alpha \cos \theta \right) - 2\cos \alpha \cos \phi \sin \theta \\
2\sin \alpha \sin \phi(\eta + \lambda \sin \theta) + 2\cos \alpha \sin \phi \cos \theta(\beta - \sin \alpha) = 0,
\]

where \( \eta = l/d \), \( \beta = b/d \), and \( \lambda = t/d \).

Figure 3.5: Perspective top view of two consecutive scales.

### 3.7.2 Finite Element (FE) Simulation Details

The finite element (FE) simulations of twisting was carried out using commercially available software ABAQUS/CAE 2017 (Dassault Systèmes). For FE models, an assembly of two different parts including rectangular prismatic substrate and the scales was made. Both were 3D deformable
solids. Thereafter rigid body constraint was imposed on the scales obviating any need for material properties for scales. In this model, a sufficient long rectangular linear elastic beam considered to satisfy periodicity. This resulted in 25 scales embedded into the substrate. A static step with active nonlinear geometry option was used to model the twisting of the structure. The twisting load was applied on the end cross section of the beam and the other end was fixed. The contact mechanics was modeled using the self-contact option for the entire structure with frictionless sliding between every two neighboring scales. To obtain mesh convergence and get accurate numerical results, sufficient mesh density was considered for different regions of the model. Due to the inclusion of scales and complexity of the beam’s top surface, the beam’s upper layer was meshed utilizing tetrahedral quadratic element C3D10 and the regions far from the embedded part and the scales was meshed using the quadratic hexahedral element C3D20. These elements were standard quadratic 3D stress elements. About 110,000 elements were employed for the simulations.

3.7.3 Derivation of $\eta_c$

By looking to Figure 3.5, it’s obvious that if the $\eta$ be a small value as much as only the last possible point of line $BC$ (which is $B$) can coincide with the last possible point of rotated line $D'C'$ (which is $D'$), this is the geometrical limit of engagement. This configuration leads to the critical value of $\eta_c = \frac{1 - 2\beta \sin \alpha}{\cos \alpha \cos \theta_0}$ for the overlap ratio, and the derived $\eta_c$ for given $\theta_0$, $\alpha$ and $\beta$ should be physically meaningful ($\eta_c > 0$). If $\eta < \eta_c$, scale engagement never happen and the structure behaves always as a linear elastic beam under twisting.

3.7.4 $U_{scale}$ Scaling Law

As the scales engage and begin rotating, the elastic material into which the beam is embedded, resists against the scales rotation. Considering small scale rotation, the substrate resistance is
modeled as linear torsional springs corresponding to the change in each of the angles $\theta$, $\alpha$ and $\gamma$. Thus the energy absorbed due to 3D rotation of each scale per unit depth of the beam can be described as $U_{scale} = \frac{1}{2}K_\theta(\theta - \theta_0)^2 + \frac{1}{2}K_\alpha(\alpha - \alpha_0)^2 + \frac{1}{2}K_\gamma(\gamma - \gamma_0)^2$, where $K_\theta$, $K_\alpha$ and $K_\gamma$ are the rotational spring constant for change corresponding to each angle. Using FE simulations on several test cases, we found that the variation of $\alpha$ and $\gamma$ is about 1% and 2% relative to the variation of $\theta$ as shown in Figure 3.6 for a certain case, thus they were neglected in the calculations. This leads to $U_{scale} \approx \frac{1}{2}K_\theta(\theta - \theta_0)^2$.

![Figure 3.6: Comparison between the change of three orientational angles (a) $\theta$, (b) $\alpha$ and (c) $\gamma$.](image)
3.7.5 $K_\theta$ Scaling Law

To find the dimensionless constants and functions in the $K_\theta$ Formula, we carried out FE simulations on a single scale embedded on a semi-infinite beam in length and thickness and varied the relevant geometric variables to ascertain the fit of this empirical relationship. As shown in Figure 3.7, we find an excellent fit in the region of $12 < L/t_s < 80$, which yields to $n = 1.55$, $C_b(\alpha) = 3.62$, and $f(\theta_0) \approx 1$, which means there is no angular dependency for rotational stiffness due to symmetry.

![Diagram](image)

Figure 3.7: FE results to find a dimensionless equation for rotational stiffness of a single scale placed on a semi-infinite beam in length and thickness.

3.7.6 $C_f$ Scaling Law

Another effect of rigid scales on the elastic substrate can be defined as “Inclusion Effect”. Consider a row of rigid scales embedded into a long enough elastic beam without any exposed length for scales ($l = 0$, therefore $\eta = 0$). This system can be considered as a particulate composite and its mechanical properties such as torsional rigidity is affected by the volume fraction, shape and size of inclusions. In this situation, there is no scale engagement and the rigid scales will not act as torsional springs, but due to inclusion of rigid planes into the elastic substrate, torsional
rigidity will increase and this effect can be described by dimensionless coefficient $C_f$ as “Inclusion Correction Factor” in the formula of $T = C_f C_w G_B I \Phi$. When there is no rigid inclusion into elastic beam, inclusion correction factor is $C_f = 1$ and then by adding rigid inclusions to the system $C_f$ will increase, therefore $C_f \geq 1$. This coefficient should be dependent on the volume fraction of the rigid inclusions with respect to the elastic substrate’s volume. Thus, it will be dependent of number of inclusions ($\sim 1/d$), inclusion length ($\sim L$), inclusion width ($\sim b$) and their size ratio with respect to the substrate thickness ($\sim 1/t$). These dependencies can be described with dimensionless coefficients, Equation (3.11):

$$C_f = 1 + C_0(\alpha)(\frac{\zeta \beta}{\lambda})^m h(\theta_0),$$  \hspace{1cm} (3.11)

where $\zeta$ is dimensionless inclusion length as $\zeta = L/d$, $h(\theta_0)$ and $C_0(\alpha)$ are dimensionless angular functions to describe the dependence of $C_f$ to $\theta_0$ and $\alpha$. To find these dimensionless constants and functions, many FE simulations were carried out on a row of inclusions on a long rectangular beam with varying parameters of $\zeta$, $\beta$ and $\lambda$ and then tried to fit the best equations on the derived results. The numerical results are shown in Figure 3.8.

![Figure 3.8](image-url)

**Figure 3.8:** Plot of dimensionless inclusion correction factor versus geometrical dimensionless variable group dependent on the volume fraction of the rigid inclusions.
According to the results of numerical simulations, these values were found: \( m = 1, C_0(\alpha) = 1.33, \) and \( h(\theta_0) \approx 1. \) It means correction factor is not dependent on the orientation of inclusion.

### 3.7.7 Effects of Material Nonlinearity

We considered two FE models with Neo-Hookean and Mooney-Rivlin material, which are commonly available hyperelastic material models in ABAQUS/CAE 2017. To obtain material parameters \( C_{10}, D_1 \) for the Neo-Hookean material, we fit the two material constants with linear elastic moduli in small deformation. This would lead to \( C_{10} = G_B/2 \) and \( D_1 = 6(1 - 2\nu)/E_B, \) where \( G_B \) is the substrate shear modulus and \( E_B \) is the substrate Young’s modulus and \( \nu \) is the substrate Poisson’s ratio. Similar approach for Mooney-Rivlin model with material constants \( C_{10}, C_{01} \) and \( D_1, \) leads to \( C_{10} + C_{01} = G_B/2 \) and \( D_1 = 6(1 - 2\nu)/E_B. \) The torque-twist responses of these two nonlinear material were match with the torque-twist response of linear substrate, Figure 3.9. Therefore the material nonlinearity is negligible confirming the trends of the experimental work.

![Figure 3.9: Dimensionless torque \( \frac{T(\Phi)}{G_B l} \) versus twist rate (\( \Phi \)) for different substrate’s material including Linear Elastic, Neo-Hookean and Mooney-Rivlin with the given values of \( \theta_0 = 10^\circ, \alpha = 45^\circ, \beta = 0.6, \eta = 8, \) and \( \lambda = 0.32. \)](image-url)

Figure 3.9: Dimensionless torque \( \frac{T(\Phi)}{G_B l} \) versus twist rate (\( \Phi \)) for different substrate’s material including Linear Elastic, Neo-Hookean and Mooney-Rivlin with the given values of \( \theta_0 = 10^\circ, \alpha = 45^\circ, \beta = 0.6, \eta = 8, \) and \( \lambda = 0.32. \)
CHAPTER 4: COULOMB FRICTION IN TWISTING OF BIOMIMETIC SCALE-COVERED SUBSTRATE

4.1 Abstract

Biomimetic scale-covered substrates provide geometric tailorability via scale orientation, spacing and also interfacial properties of contact in various deformation modes. No work has investigated the effect of friction in twisting deformation of biomimetic scale-covered beams. In this work, we investigate the frictional effects in the biomimetic scale-covered structure by developing an analytical model verified by finite element simulations. In this model, we consider dry (Coulomb) friction between rigid scales surfaces, and the substrate as the linear elastic rectangular beam. The obtained results show that the friction has a dual contribution on the system by advancing the locking mechanism due to change of mechanism from purely kinematic to interfacial behavior, and stiffening the twist response due to sharp increase in the engagement forces. We also discovered, by increasing the coefficient of friction potentially using engineering scale surfaces to a critical coefficient, the system could reach to instantaneous post-engagement locking. The developed model outlines analytical relationships between geometry, deformation, frictional force and strain energy, to design biomimetic scale-covered metamaterials for a wide range of applications.

Citation:
4.2 Introduction

Many biological and biomimetic structures possess geometrically pronounced features. Such geometric features include for instance scales and intricate topological arrangement in their interior. This leads to nonlinear behavior such as nonlinear strain-stiffening in bending [51, 52, 56–58], nonlinear strain-stiffening in twisting [59], nonlinear stress-strain behavior in nature inspired cellular architecture [27, 103, 156–159], and nonlinear dispersion relationships in honeycomb structures leading to acoustic band gaps [111]. These structures include seashells, hierarchical honeycombs, snail spiral, seahorse tail, fish scales, lobster exoskeleton, crab exoskeleton, butterfly wings, armadillo exoskeleton, sponge skeleton, etc. [23–27]. Among these structures, dermal scales have garnered special attention recently due to complex mechanical behavior in bending and twisting [28–32, 34, 160]. Scales in nature are naturally multifunctional, lightweight [33, 35–39, 46, 47, 83, 138], and protective of the underlying substrate, which has been an inspiration of armor designs [32, 34, 49, 141], where overlapping scales can resist penetration and provide additional stiffness [32, 34, 42, 48]. Fabrication methods such as synthetic mesh sewing and stretch-and-release have been recently developed to produce overlapping scale-covered structures in 2D and 1D configurations [4, 50]. These fabricated structures show almost ten times more puncture resistance than soft elastomers.

In addition to these localized loads, global deformation modes – such as bending and twisting – can be important for a host of applications that require structural modes of deformation, namely soft robotics, prosthetics, and morphing structures. In this context, characterizing bending and twisting plays an important role in ascertaining the benefits of these structures. Prior research has shown that bending and twisting of a scale-covered substrate show small-strain reversible nonlinear stiffening and locking behavior, due to the sliding kinematics of the scales embedded in the substrates [51–59, 61, 101]. Such sliding interlocking structures possess certain unique character-
istics, which give biological structures advantages without sophisticated parent materials. These include sharp and rapid increase in stiffness, leading to an almost rigid final shape (i.e. locking [52, 56, 57, 59]). This type of behavior is known to assist the entire body of the fish as an external tendon [161, 162]. A good biological example here is the Arapaima fish, which lives in the Amazon river, shown in Figure 4.1(a) [163]. Their body’s inner layer can twist and compress under stress, while their scales reorient themselves to help resist against external force and increase their strength [141, 164]. These dermal scales are also known to affect snake motion on surfaces [142,165,166]. Sliding behavior is also exploited in the tail of seahorses, which helps in its prehensile functionalities [26, 167, 168]. From an engineering perspective, such preferential locking behavior is critical for a range of applications. For example, in soft and collaborative robotics, a robotic appendage must balance flexibility and range of motion with stiffness to preserve an arm shape [169–171]. Thus, manipulating stiffening behavior is among the most important goals of such advanced applications.

Locked states guarantee the intermediate nonlinear behavior. Thus, the universality of such behaviors across deformation regimes needs to be ascertained. Several recent publications have probed this phenomena in depth for bending modes in both uniform [52] and non-uniform scales distributions [56, 57], and for both frictionless [52] and frictional cases [54]. However, the literature for the torsional deformation is somewhat less developed. Here, only the frictionless case has been probed into, which showed that locking is possible, but only for certain oblique angles of scales [59]. Therefore, the role of friction and its possible universal role has not been established in literature. In other words, questions remain about the parallels of properties modification brought about by friction in bending with twisting. For instance, Coulomb friction in bending regime advances the locking envelopes but at the same time, limits the range of operation [54]. In the dynamic regime, Coulomb friction can lead to damping behavior, which mimics viscous damping [61]. Clearly, friction between sliding scales can significantly alter the nature of nonlinearity. However, in spite
of these studies, the role of friction in influencing the twisting behavior of a scale-covered structure has never been investigated before.

In this paper we investigate the role of friction in affecting the twisting behavior of biomimetic scale-covered systems under pure torsion for the first time. We establish an analytical model aided by finite element (FE) computational investigations. We assume rigid scales, linear elastic behavior of the substrate, and Coulomb model of friction between scales’ surfaces. We compare our results with FE model to verify the proposed analytical model.

4.3 Materials and Methods

4.3.1 Materials and Geometry

We consider a rectangular deformable prismatic bar with a row of rigid rectangular plates embedded on substrate’s top surface. For the sake of illustration, we fabricate prototypes of 3D-printed scales made from the polymer Polylactic acid (PLA) \(E_{PLA} \sim 3 \text{ GPa}\), partially-embedded into the top surface of a silicone substrate and adhered with silicone glue (Permatex Corporate) to prefabricated grooves on the molded slender Vinylpolysiloxane (VPS) (Zhermack SpA) substrate \(E_{VPS} \sim 1.5 \text{ MPa}\), as shown in Figure 4.1(b). The Young’s modulus of these materials were obtained by tensile tests and the substrate’s material was found to behave linearly for moderate torsional deformation [59]. In our lab scale testing, the silicone based polymeric substrate material did not exhibit appreciable anisotropy. This is consistent with previous reports in literature [172, 173]. However, for the biomimetic scale-covered substrate, anisotropy between directions of twisting (engaged vs non-engaged) would obviously arise. This is an example of emergent behavior, which is typical in many topologically and geometrically complex structures including the current biomimetic fish scale system [103, 159]. The prototype is illustrated under twisted
configuration in Figure 4.1(b). The rigidity assumption for the scales is valid in the limit of much higher stiffness of the scales, away from the locking state [34, 161]. Note that we did not perform physical torsion experiments in this paper, but used real prototypes only for aiding visualization.

Figure 4.1: (a) Arapaima fish which can twist their body’s inner layer and their scales reorient themselves to help resisting against external force and increase their strength. The image is adapted under CC BY 2.0 license [163]. (b) The fabricated prototype were made from 3D-printed PLA scales and molded slender Vinylpolysiloxane (VPS) substrate in initial and twisted configuration.

The pure twisting behavior allows us to assume periodicity, letting us isolate a fundamental representative volume element (RVE) for modeling the system, Figure 4.2(a). The scales are considered to be rectangular rigid plates with thickness $t_s$, width $2b$, and length $l_s$, and oriented at angles $\theta$ and $\alpha$ as shown in Figure 4.2(b) with respect to the rectangular prismatic substrate. $\theta$ is the scale inclination angle defined as the dihedral angle between the substrate’s top surface and the scale’s bottom surface, and $\alpha$ is the angle between the substrate’s cross section and the scale’s width. The length of exposed section of scales is denoted as $l$, and the length of embedded section of the scales is $L$. Therefore, the total length of the scale is $l_s = L + l$. The spacing between the scales is constant and denoted by $d$, which is a geometrical parameter reciprocal to the density of scales. We assume that the scale’s thickness $t_s$ is negligible with respect to the length of the scales, $l_s$ ($t_s \ll l_s$), and the scale’s embedded length is also negligible with respect to the substrate’s thickness ($0 \ll L \ll 2t$). This thin-plate idealization for the biomimetic scales is appropriate for this case and typically used in literature for analogous systems [52, 54–57, 59, 61].
Figure 4.2: The schematic of three consecutive scales geometrical configuration: (a) Top view of scales configuration. (b) Dimetric view showing scales orientational angles of $\theta$ and $\alpha$, and the embedded part of each scale. Note that angle $\theta$ is exaggerated and the scale’s thickness $t_s$ is not illustrated in this figure.

4.3.2 Kinematics

For global deformation modes such as pure bending and twisting, the scale periodicity is a good approximation [52, 59]. Periodicity assumption allows us to consider just three consecutive scales configuration at the RVE level, we call these scales as “zeroth scale”, “1st scale”, and “2nd scale” respectively from left to right. The corners of these scales are marked likewise in Figure 4.1(b) and Figure 4.2. Without loss of generality, we consider 1st scale is fixed locally with respect to other scales. A twisting deformation with twist rate $\Phi$, is applied to the rectangular prismatic substrate about torsion axis, which passes through the beam cross section center. Due to this underlying deformation, the 2nd scale rotates by twist angle of $\varphi = \Phi d$, and the zeroth scale rotates in reverse direction about the torsion axis with $-\varphi = -\Phi d$, because 1st scale assumed locally fixed. The continual twisting of the substrate progresses the contact between each two consecutive scales simultaneously due to periodicity, by coincidence between lines $C_1B_1$ and $D_2C_2$, as well as lines $D_1C_1$ and $C_0B_0$. 

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To find a contact criterion between 1st scale and 2nd scale, the 3D-equations of lines $C_1B_1$ and $D_2C_2$ would be established. We place the coordinates $XYZ$ on the midpoint of 1st scale’s width as shown in Figure 4.2. Then we place coordinates $xyz$ on the torsion axis at point $O = (0, -t, 0)$ measured from the coordinates $XYZ$. Hereafter, coordinates $xyz$ is our reference frame. Note that, we do not show coordinates $xyz$ and scale’s thickness $t_s$ in Figure 4.2(b) to avoid visual complexity. We establish local coordinates on each scale, denoted as “local coordinates of $i$th scale”, and coordinates origin is located on the corner of the scale at point $D_i$.

In these local coordinates, the unit vector of $x$-axis ($n_{Xi}$) is on the edge $D_iC_i$, the unit vector of $y$-axis ($n_{Yi}$) is on the edge $D_iA_i$, and the unit vector of $z$-axis ($n_{Yi}$) is out of plane and perpendicular to $n_{Xi}$ and $n_{Yi}$. Figure 4.2(b). On each scale, edges $D_iC_i$ and $A_iB_i$ are parallel and in direction of $n_{Xi}$, and edges $C_iB_i$ and $D_iA_i$ are parallel and in direction of $n_{Yi}$. Point $M_i$ is located in the middle of edge $C_iB_i$. Using these established coordinates, equations of line $C_1B_1$ of 1st scale can be obtained on the base of the unit vector $n_{Y1}$ and the location of point $M_1$, which is located in the middle of the edge $C_1B_1$. Line $D_2C_2$ is located on the 2nd scale, which is rotating with angle $\phi$ about torsion axis. To find the equation of this line, first we locate the corners of 2nd scale as shown in Figure 4.2(a), before and after rotation using rotation matrix. Thus, we find the location of point $D_2$ and $C_2$ after rotation, which are located at ends of the line $D_2C_2$. Then, the rotating local coordinates on this scale and the unit vector in direction $D_2C_2$ ($n_{X2}$) can be established. Finally, the equations of line $D_2C_2$ is delivered by using the unit vector $n_{X2}$ and point $D_2$.

To find the contact point of these two lines, we solve their equations together as a system of equations, which yields a nonlinear relationship between $\phi$ and $\theta$. To represent a general form for this relationship, we define dimensionless geometric parameters $\eta = l/d$, $\beta = b/d$, and $\lambda = t/d$ as the overlap ratio, dimensionless scale width, and dimensionless substrate thickness, respectively. The governing relationship between the substrate local twist angle $\phi$ and the scale inclination angle $\theta$ can be written as:
\[(\cos \varphi - 1) \left( \beta \sin 2\alpha \sin \theta + \eta \cos^2 \alpha \sin 2\theta + 2\lambda \cos 2\alpha \cos \theta \right) - 2 \cos \alpha \cos \varphi \sin \theta
\]
\[2 \sin \alpha \sin \varphi (\eta + \lambda \sin \theta) + 2 \cos \alpha \sin \varphi \cos \theta (\beta - \sin \alpha) = 0. \quad (4.1)\]

The details of derivation of this relationship can be found in Supplementary Material of Chapter 3. We can use the same procedure to establish the locations of zeroth scale’s corners and its local coordinates after rotating with angle \(-\varphi\) about torsion axis. We find the same nonlinear relationship between \(\varphi\) and \(\theta\) as shown in Equation (4.1) due to the periodicity of the system. Also, the location of point \(P_{12}\) as the intersection between lines \(D_2C_2\) and \(C_1B_1\), and the location of point \(P_{10}\) as the intersection between lines \(D_1C_1\) and \(C_0B_0\), can be calculated as described in Supplementary Material of Chapter 3. These points are illustrated in Figure 4.3, which shows the twisted state of the RVE.

To express the dimensionless geometric parameters qualitatively in a biological scale-covered system, it can be mentioned that the overlap ratio \(\eta = l/d\) determines how much fish scales are long, and \(\beta = b/d\) is representing fish scales width. Also, \(\lambda = t/d\) determines the thickness of the effective fish skin layer.

From the beginning of contact between the scales (scales engagement), the relationship (4.1) is established between the substrate local twist angle \(\varphi\) and the scales inclination angle \(\theta\). After starting the scales engagement, scales slide over each other and \(\theta\) starts to increase from its initial value \(\theta_0\) according to the nonlinear relationship (4.1). Note that scales engagement starts at a point with relatively small twist angle which we call it engagement point \((E_p)\). Therefore, to find an explicit relationship for the twist angle \(\varphi\) at this point known as the engagement twist angle \(\varphi_e\), we linearize (4.1) by considering small twist regime \((\varphi \ll 1, \theta \ll 1)\) which leads to \(\varphi_e = \theta_0 / (\eta \tan \alpha + \beta - \sin \alpha)\).
Using the kinematic relationship (4.1), we probe the existence of a singular point where locking can take place. This would be the envelope defined by \( \frac{\partial \phi}{\partial \theta} = 0 \), and beyond which no more sliding is possible without significant deformation of the scales. This point is called the “kinematic locking” of the system [59]. The locking point \((L_p)\) happens when the scale-covered structure, even though it has a deformable substrate, can not be twisted anymore due to the kinematic contact between relative-rigid scales (scales engagement) and the established geometrical arrangement (singular point). Note that the kinematic locking state can occur even without friction since it is a result of singularity in the governing kinematic relationship of the system. Beyond this point, stiffness increases sharply as it is determined by the stiffness of the scales.

4.3.3 Mechanics

To investigate the role of friction in twisting behavior of biomimetic scale-covered substrate, we investigate the free body diagram of the RVE (here 1\(^{\text{st}}\) scale) during engagement as shown in Figure 4.3. The forces on the 1\(^{\text{st}}\) scale are as follows. At contact point between zeroth scale and 1\(^{\text{st}}\) scale \(P_{10}\), there are two reaction forces including friction force \(f_{10}\) acting in the plane of 1\(^{\text{st}}\) scale by angle \(\chi_{10}\) with respect to the unit vector \(nx_{1}\), and normal force \(N_{10}\) acting perpendicular to this plane in direction \(-nz_{1}\) as shown in Figure 4.3. Also, at contact point between 1\(^{\text{st}}\) scale and 2\(^{\text{nd}}\) scale \(P_{12}\), two reaction forces are acting including friction force \(f_{12}\) in the plane of 2\(^{\text{nd}}\) scale by angle \(\chi_{12}\) with respect to the unit vector \(nx_{2}\), and normal force \(N_{12}\) perpendicular to the plane of 2\(^{\text{nd}}\) scale in direction \(nz_{2}\) as shown in Figure 4.3.

Note that the direction of friction forces are dependent on the direction of relative motion between each scale pairs. Due to the periodicity, the value of friction forces are equal \(f_{fr} = f_{10} = f_{12}\), and also the value of normal forces are equal \(N = N_{10} = N_{12}\). According to the described free body diagram, the balance of moments at the base of 1\(^{\text{st}}\) scale can be described in the vectorial format
as follows:

\[
K_\theta(\theta - \theta_0) = \left( O_1P_{10} \times \left( - (f_{fr} \cos \chi_{10})n_{X1} - (f_{fr} \sin \chi_{10})n_{Y1} - (N)n_{Z1} \right) + O_1P_{12} \times \left( (f_{fr} \cos \chi_{12})n_{X2} + (f_{fr} \sin \chi_{12})n_{Y2} + (N)n_{Z2} \right) \right) \cdot n_{Y1}. 
\] (4.2)

Where \( O_1P_{10} \) and \( O_1P_{12} \) are the position vector of contact points \( P_{10} \) and \( P_{12} \) with respect to the base of the 1\textsuperscript{st} scale, respectively, as shown in Figure 4.3. \( K_\theta \) is the “rotational spring constant” or the “rigid scale–elastic substrate joint stiffness”. As the scales engage, they tend to push each other and increase their inclination angle \( \theta \), but the elastic substrate resists against scales rotation. This resistance is modeled as linear torsional spring \([51,52,59]\), and the absorbed energy due to the rotation of each scale is \( U_{\text{scale}} = \frac{1}{2}K_\theta(\theta - \theta_0)^2 \), thus the local reaction moment would be \( M_{\text{scale}} = K_\theta(\theta - \theta_0) \). According to developed scaling expression in \([59]\), \( K_\theta = 3.62E_Bt_3^2b(L/t_s)^{1.55} \), where \( E_B \) is the elastic modulus of substrate.

![Figure 4.3: Twisted state of the RVE and free body diagram of each pair of scales representing their contact points \( P_{10} \) and \( P_{12} \), normal force \( N \), and friction force \( f_{fr} \) at the contact points.](image)

According to the Coulomb’s Law of Friction, scales do not slide while \( f_{fr} \leq \mu N \), where \( \mu \) and \( N \) are coefficient of friction and normal force, respectively, while sliding regime is marked by the equality. Note that we use the same value for static coefficient of friction as well as the kinetic
coefficient of friction in this study, although typically static coefficient of friction is slightly higher [174, 175]. On the basis of scales relative motion expressed in Supplementary Material of this chapter, the angle between the friction force $f_{fr}$ in the 1\textsuperscript{st} scale plane and the unit vector $n_{x1}$ is equal to the angle between friction force $f_{fr}$ in the 2\textsuperscript{nd} scale plane and the unit vector $n_{x2}$. This means $\chi_{10} = \chi_{12}$, and can be shown as $\chi$. This finding also conforms the periodicity in the system. Using these considerations, we can derive the following expression as the non-dimensionalized friction force $f_0$, with respect to the free body diagram shown in Figure 4.3:

$$f_0 = \frac{f_{fr} l}{K_{\theta}} \leq \frac{(\theta - \theta_0) l}{\left( O_1 P_{12} \times (\cos \chi_{n_{x2}} + \sin \chi_{n_{y2}} + \frac{1}{\mu} n_{z2}) - O_1 P_{10} \times (\cos \chi_{n_{x1}} + \sin \chi_{n_{y1}} + \frac{1}{\mu} n_{z1}) \right) \cdot n_{y1}}.$$  \hspace{1cm} (4.3)

Due to the nature and the geometrical configuration of the system, the magnitude of the friction force derived in (4.3), may exhibit singularity at a certain twist rate. This rise in friction force may lead to a “frictional locking” mechanism, observed in the bending case too [54]. If predicted, the frictional locking should happen at the lower twist rate compared to kinematic locking, because of the limiting nature of friction force. We call the twist rate in which locking happens as $\Phi_{\text{lock}}$, and the twist angle and the scale inclination angle would be as $\varphi_{\text{lock}} = \Phi_{\text{lock}} d$ and $\theta_{\text{lock}}$, respectively.

The friction force computed above will lead to dissipative work in the system during sliding. The non-dissipative component of the deformation is absorbed as the elastic energy of the biomimetic beam. This elastic energy is composed of elastic energy of the beam and the scales rotation. To calculate this elastic energy of the beam, we consider a linear elastic behavior for the beam with a warping coefficient $C_w$ for a non-circular beam [59,155]. Furthermore, due to the finite embedding of the scales, there will be an intrinsic stiffening of the structure even before scales engagement. This stiffening can be accurately captured by using an inclusion correction factor $C_f$ [59]. $C_f$ is a function of the volume fraction of the rigid inclusion into the elastic substrate, and postulated as $C_f = 1 + 1.33(\zeta \beta / \lambda)$, where $\zeta = L/d$ for an analogous system [59]. With these considerations,
modified torque–twist relationship of the beam is \( T = C_f C_w G_B I \Phi \), and the elastic energy of the beam can be considered as \( U_B = \frac{1}{2} C_f C_w G_B I \Phi^2 \). As mentioned earlier, the energy absorbed by the scales can be obtained by assuming the scale’s resistance as linear torsional spring and the absorbed energy due to the rotation of each scale will be \( U_{scale} = \frac{1}{2} K_\theta (\theta - \theta_0)^2 \). Similarly the dissipation can be given as the product of the sliding friction and distance travelled by the point of application per scale. Then we use the work–energy balance to arrive at:

\[
\int_\Phi^0 T'(\Phi')d\Phi' = \frac{1}{2} C_f C_w G_B I \Phi^2 + \left( \frac{1}{2} K_\theta (\theta - \theta_0)^2 + \frac{1}{d} \int_{\Phi_e}^{\Phi} f_{fr} dr \right) H(\Phi - \Phi_e),
\]

(4.4)

where \( \Phi, \Phi_e = \varphi_e/d \), \( G_B \), and \( I \) are the current twist rate, the engagement twist rate, the shear modulus of elasticity, and the beam cross section’s moment of inertia. \( H(\Phi - \Phi_e) \) is the Heaviside step function to track scales engagement. Also, \( C_f, C_w, \) and \( K_\theta \) are inclusion correction factor, warping coefficient, and rotational spring constant of scale–substrate joint stiffness, respectively. In (4.4), \( f_{fr} \) represents the friction force between scales, and \( dr \) is the relative differential displacement traveled by the point of friction application. Derivation of \( dr \) has been described in Supplementary Material of this chapter.

The torque–twist rate relationship per substrate’s unit length could be obtained by taking the derivative of (4.4) with respect to the twist rate \( \Phi \), while considering \( \varphi = \Phi d \), as follows:

\[
T(\Phi) = C_f C_w G_B I \Phi + \left( K_\theta (\theta - \theta_0) \frac{\partial \theta}{\partial \varphi} + f_{fr} \frac{dr}{d\varphi} \right) H(\Phi - \Phi_e).
\]

(4.5)

We also compute the maximum possible dissipation of the system by computing the frictional work done till locking \( (W_{fr}) \) and compare it with the total work done \( (W_{lock} = U_{el} + W_{fr}) \), where \( U_{el} \) is the elastic energy of the system). These energies can be computed per substrate’s unit length as:
\[
U_{el} = \frac{1}{2} \left( C_f C_w G_B I(\Phi_{lock})^2 + \frac{1}{d} K_\theta (\theta_{lock} - \theta_0)^2 \right),
\]

\[
W_{fr} = \frac{1}{d} \int_{\Phi_e}^{\Phi_{lock}} f_{fr} dr.
\]

We define the relative energy dissipation (RED) factor as the ratio of the frictional work per unit length \(W_{fr}\), to the total work done on the system per unit length \(W_{lock}\):

\[
RED = \frac{W_{fr}}{W_{lock}}.
\]

Generally, RED is dependant on the coefficient of friction \(\mu\), dimensionless geometric parameters of the system \(\eta, \beta, \text{and} \lambda\), scale spacing \(d\), scales initial orientation angles \(\alpha\) and \(\theta_0\), substrate elastic properties \(G_B, I, \text{and} C_w\), and scale–substrate joint parameters \(K_\theta \text{and} C_f\), but the most important parameters are \(\mu, \eta, \text{and} \alpha\).

4.4 Finite Element Simulations

We have developed an FE model for verification of the developed analytical model for the biomimetic scale-covered system under twisting deformation. The FE simulations are carried out using commercially available software ABAQUS/CAE 2017 (Dassault Systèmes). We considered 3D deformable solids for scale and substrate. However, for the scales, rigid body constraint was imposed. A sufficient substrate length is considered for rectangular prismatic substrate to satisfy the periodicity. Then an assembly of substrate with a row of 25 scales embedded on its top surface is created. The scales are oriented at angles of \(\theta_0\) and \(\alpha\) as defined in the analytical model. Linear elastic material properties including \(E_B\) and \(\nu\) are applied to the substrate part which leads to the shear modulus of \(G_B = \frac{E_B}{2(1+\nu)}\).
The simulation was considered as a static step with nonlinear geometry option. The left side of the beam is fixed and the twisting load was applied on the other side of the beam. A frictional contact criteria is applied to the scales surfaces with coefficient of friction $\mu$ for a twisting simulation. The top layer of substrate is meshed with tetrahedral quadratic elements C3D10 due to the geometrical complexity around scales inclusion. Quadratic hexahedral elements C3D20 are used for other regions of the model. A mesh convergence study is carried out to find sufficient mesh density for different regions of the model. A total of almost 70,000 elements are employed in the FE model.

4.5 Results and Discussion

To study the frictional force behavior in this system, we use (4.3) to plot non-dimensionalized friction force $f_0$ for different $\mu$ values at various non-dimensionalized twist rate $\Phi/\Phi_e$. In a real biological scale-covered system like fish scales, the geometry of fish scales, scales’ surface roughness, and the epidermal mucus, which typically covers the fish scales, significantly affect the values of coefficient of friction $\mu$ [176]. The non-dimensionalized friction force is shown in Figure 4.4 for scale-covered system with $\eta = 3$, $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, $\beta = 1.25$, and $\lambda = 0.45$. From this figure, it is clear that increasing twist rate leads to a rapid increase in the friction force for any coefficient of friction. There is a singular characteristic for this load as shown with dashed lines for each $\mu$ in Figure 4.4, which indicates a friction based locking mechanism. This is in addition to the purely kinematic locking mechanism reported earlier in literature for frictionless counterparts [59]. We call the twist rate at the locking point ($L_p$), as the locking twist rate $\Phi_{lock}$.

Next, we investigate the scale rotation in response to applied twist. This is achieved by plotting the scale inclination angle $\theta$ versus substrate local twist angle $\varphi$. Using the nonlinear relationship (4.1), two plots are established spanned by $(\theta - \theta_0)/\pi$ and $\varphi/\pi$ as shown in Figure 4.5 for different $\eta$ and $\alpha$, respectively. Note that in a biological fish scales system, $\eta$ roughly translates to the extent
of scales overlap. Whereas, $\alpha$ describes the angle between the direction of scales arrangement and the twist axis.

Figure 4.4: Non-dimensionalized friction force vs Non-dimensionalized twist rate ($\Phi_e$ is the engagement twist rate) for various coefficients of friction with the given values of $\eta = 3$, $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, $\beta = 1.25$, and $\lambda = 0.45$. This figure shows that the friction forces approach singularity near a certain twist rate as the frictional locking configuration for each $\mu$.

In Figure 4.5(a), the given geometrical parameters are as follows $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, $\beta = 1.25$, and $\lambda = 0.45$. For $\mu = 0$, which indicates frictionless case, we obtain purely kinematic locking points for each $\eta$ by using $\partial \phi / \partial \theta = 0$ to obtain rigidity envelope [59]. We juxtapose this with plots of the rough interfaces ($\mu > 0$), where the locking limits are found via the singularity point of friction force described in (4.3). Clearly, friction advances the locking configuration. However, the locking line does not merely translate downwards as observed in the bending case [54]. This is an important distinction from the pure bending of rough biomimetic scale-covered beams reported earlier [54]. As coefficient of friction increases, the frictional locking envelope can intersect the horizontal axis. This is the instantaneous locking or the “static friction locking” case. Interestingly, this is a geometrically-dictated static friction locking in contrast to the actual static friction coefficient.
mediated locking. This once again highlights the contrast and interplay of material and geometry in this class of structures.

In Figure 4.5(b), the effect of scales orientation with angle $\alpha$ is investigated. This angle serves as an important geometric tailorability parameter of the system [59]. In this plot, $\eta = 3$, $\theta_0 = 10^\circ$, $\beta = 1.25$, and $\lambda = 0.45$. For higher angles $\alpha$, a quicker engagement occurs with steeper nonlinear gains and earlier locking. As shown in Figure 4.5(b), the curves for lower $\alpha$ ($\alpha = 15^\circ$ and $\alpha = 0^\circ$) fail to reach the rigidity envelope for $\mu = 0$, because mathematically there is no singularity point for Equation (4.1) for these cases. This means, by decreasing $\alpha$ sufficiently, the system would not reach to the kinematic locking. However, frictional locking is universal and will determine the locking behavior. In this aspect, this system again differs from bending case, since in twisting, friction can cause locking even when kinematic locking is not possible. This figure also shows the possibility of static friction locking by increasing $\mu$. However, note that as $\alpha$ increases, such static friction locking becomes more difficult to achieve, because it requires much higher frictional coefficients. Overall, the frictional locking envelope is a highly nonlinear function admitting no closed form solution unlike the pure bending case [54].

In order to understand the effect of friction force on the mechanics of the system, we use (4.5) to plot the non-dimensionalized post-engagement torque-twist rate plot for various coefficients of friction, Figure 4.6(a). Dimensionless geometrical parameters for this case are $\eta = 3$, $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, $\beta = 1.25$, $\lambda = 0.45$, $\zeta = 0.35$, and $L/t_s = 35$. As shown in Figure 4.6(a), higher coefficient of friction significantly increases the torsional stiffness of the structure. Therefore, the friction force has a dual contribution to the mechanical response of the biomimetic scale-covered system – while advancing locking state, thereby limiting range of motion, but also increasing the torsional stiffness of the system.

To verify the analytical model, we have developed an FE model as described in section 4.4. Then
Figure 4.5: The plot representation of the biomimetic scale-covered beam under twisting differentiated to three distinct regimes of performance including: linear region (before scales engagement) which is from 0 to engagement point ($E_p$) shown on e.g. curves $\eta = 3$ and $\alpha = 45^\circ$; nonlinear region (during scales engagement) which is from engagement point ($E_p$) to locking point for $\mu = k$ ($L_{pk}$) shown on e.g. curves $\eta = 3$ and $\alpha = 45^\circ$; and rigid region which is after locking point for $\mu = k$ ($L_{pk}$) shown on e.g. curves $\eta = 3$ and $\alpha = 45^\circ$. (a) Plot of the system for different $\eta$ with the given values of $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, $\beta = 1.25$, and $\lambda = 0.45$. (b) plot of the system for different $\alpha$ with the given values of $\eta = 3$, $\theta_0 = 10^\circ$, $\beta = 1.25$, and $\lambda = 0.45$.

we have performed FE simulations for different $\eta$ and $\mu$ values and extracted torsional response of the structure $T(\Phi)/G_{BI}$, versus twist rate from the beginning of the simulation as shown in Figure 4.6(b). The following dimensionless parameters are used for this model: $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, $\beta = 0.6$, $\lambda = 0.32$, $\zeta = 0.18$, and $L/t_s = 45$. Also the following elastic properties are considered for substrate: $E_B = 25 \text{ GPa}$, $\nu = 0.25$, with a cross section dimension of $32 \times 16 \text{ mm}$. In this figure, the dotted lines represent FE results. The plot highlights remarkable agreement between analytical and FE results for two different overlap ratios along with different coefficients of friction. The small deviation between results could be caused by edge effects and numerical issues. As it is shown in Figure 4.6(b), we have performed multiple FE simulations for different cases. We have
presented a contour plot of von Mises stress with real-scale deformation for one of these cases in Supplementary Material of this chapter, to give a better perspective about the FE investigations which have been performed in this work.

Note that there are differences in visual appearance between these two torque–twist curves (Figure 4.6(a) and 4.6(b)). This is not mutually contradictory. The Figure 4.6(a) captures the twisting response only after the scale engagement, which visually amplifies the nonlinearity, whereas 4.6(b) plots from the reference configuration. Since Figure 4.6(b) captures only a small portion of the nonlinearity, it appears visually linear after engagement. Due to extreme convergence issue with FE software used for model verification, we are limited to relatively small twisting angles. The excellent match between FE and model is due in part to model accuracy, and also the relatively small geometric nonlinearity affecting the FE simulations. We expect significantly more deviations from the theory if the twisting were to proceed to relatively large values or near locking where scale deformations would be significant.

As we have mentioned earlier in section 4.3.3, we have considered the same value for static and kinetic coefficient of friction in this study, but in general the coefficients of static friction $\mu_s$ and kinetic friction $\mu_k$ are always slightly different, with $\mu_s > \mu_k$ [174, 175]. From the mechanics point of view, vastly different coefficients of friction can lead to jumps in Torque–twist behavior after initial contact is made. Thus, the twisting motion will momentarily stop till the applied torque leads the internal forces to a sufficient value to overcome static friction. At this point, the motion will start again and resume on the path calculated from kinetic friction assumption. This can lead to a momentary stick-slip motion. Once sliding begins again, the rest of the plot would be similar.

Only a fraction of applied work goes into elastic storage, whereas the rest is dissipated or lost. The proportion of lost energy is a critical quantity of interest in typical inelastic materials such as polymers and metals [177, 178]. Such losses are typically material properties, which are dependent
Figure 4.6: Torque–twist rate curve derived from Equation (4.5) for different cases: (a) Non-dimensionalized post-engagement torque–twist rate curves for various coefficients of friction with the given values of $\eta = 3$, $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, $\beta = 1.25$, $\lambda = 0.45$, $\zeta = 0.35$, and $L/t_s = 35$, showing the perceptible effect of friction in the effective torsional stiffness of the biomimetic scale-covered structure. Here, $h$ is the substrate’s thickness ($h = 2t$). (b) Verification of analytical model using numerical results through the plot of $T(\Phi)/G_BI$ versus twist rate ($\Phi$) for various coefficients of friction and two different $\eta$ with the given values of $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, $\beta = 0.6$, $\lambda = 0.32$, $\zeta = 0.18$, and $L/t_s = 45$. Black dotted lines represent FE results.

on molecular or crystal structure. Thus, they tend to vary drastically across material classes. This lost proportion is useful in designing damping structures or determine possible temperature rise during loading. For biomimetic fish scale structures currently under study, the fundamental nature of this loss is geometric in origin. Therefore, we can tailor this loss from geometric arrangement of scales. Such tailorable can be of great interest in designing structures that can lie within acceptable range of energy dissipation.

Interestingly, in our structure, the frictional forces may not always lead to increase lost work, since friction can also decrease range of motion. In order to quantify the dual contribution of friction,
we investigate the frictional work during twisting by using the relative energy dissipation (RED), described in (4.8). According to the geometric origins of friction, the scale overlap ratio $\eta$ and the oblique angle $\alpha$ come into the play. We set our analysis by fixing all parameters involved in RED, except $\mu$, $\eta$, and $\alpha$. This leads to contour plots shown in Figure 4.7. In these contour plots, we have considered $\theta_0 = 10^\circ$, $\beta = 1.25$, $\lambda = 0.45$, $\zeta = 0.35$, $L/t_s = 35$, and the substrate’s properties as follows $E_B = 25 \, GPa$, $\nu = 0.25$, and the cross section dimension of $32 \times 16 \, mm$.

In Figure 4.7(a), we consider $\alpha = 45^\circ$ to obtain an energy dissipation contour plot spanned by $\eta$ and $\mu$. This plot indicates that RED increases for higher $\mu$, and also increases very slightly with $\eta$. This contour plot shows that $\eta$ does not have as strong effect as coefficient of friction, on frictional energy dissipation of the system. Thus, for this oblique configuration, interscale sliding friction dominates overall dissipation. In fact, beyond a certain coefficient of friction, the increment in lost work is minimal. Thus, while designing the system, it would serve little to aim for very high frictional coefficients. However, this plot alone is an incomplete description of the problem since it may be an artifact of particular oblique configuration. Therefore, in the next figure, Figure 4.7(b), we fix $\eta = 3$ varying $\alpha$ and $\mu$. Here, we find that until some value of $\alpha$, the effect of increase in friction coefficient leads to higher proportion of energy loss at intermediate $\mu$. This finding indicates that for very rough surfaces, locking begins to severely limit the range of motion and thus overall frictional work in a cycle. This is consistent with bending analogs [54]. However, as $\alpha$ increases to beyond $40^\circ$, this intermediate maxima effect begins to disappear, appearing again at higher ($> 60^\circ$) angles. This is a surprising result and shows how the obliqueness can be tuned to get frictional behavior as desired. This plot also shows that very low frictional coefficients (quasi-smooth regime) leads to low energy loss no matter what the oblique angle is. However, the geometrical effects from the angle dramatically amplify frictional effects even when frictional coefficients increase moderately. The white region in this contour plot is related to the instantaneous post-engagement frictional locking, which happens at lower $\alpha$ and
higher $\mu$. At this condition, the system locks statically at the engagement point and the friction force does not work on the system.

Figure 4.7: Non-dimensional relative energy dissipation (RED) factor contour plot with given values of $\theta_0 = 10^\circ$, $\beta = 1.25$, $\lambda = 0.45$, $\zeta = 0.35$, $L/t_s = 35$, $E_B = 25$ GPa, $\nu = 0.25$, and the substrate’s cross section of $32 \times 16$ mm for two different cases: (a) spanned by $\mu$ and $\eta$ by fixing $\alpha = 45^\circ$. (b) spanned by $\mu$ and $\alpha$ by fixing $\eta = 3$.

4.6 Conclusion

We investigate for the first time, the effect of Coulomb friction on the twisting response of a biomimetic beam using a combination of analytical model and FE simulations. We established the extent and limits of universality of frictional behavior across bending and twisting regimes. The analytical model would help in obviating the need for full-scale FE simulations, which are complicated for large number of scales and for large deflection. We find that several aspects of the mechanical behavior show similarity to rough bending case investigated earlier [52]. At the same time, critical differences in response were observed, most notably the effect of the additional oblique angle. This work shows the dual contribution of frictional forces on the biomimetic scale-
covered system, which includes advancing the locking envelope and at the same time increasing the torsional stiffness. Interestingly, if the coefficient of friction is large enough for a given configuration, it can lead to the instantaneous post-engagement frictional locking known as the static friction locking. This investigation demonstrates that engineering of the scale’s surfaces, which produce wide range of coefficients of friction, can play an important role in tailoring the deformation response of biomimetic scale-covered systems under a variety of applications.

Our investigation shows the possibility of using surface roughness to tailor stiffness and dissipation behavior during twisting of biomimetic scale-covered substrates. Thus, a wide range and characteristic of friction behavior can arise by specially engineering surfaces of the scales. Combined with scale geometry, scale orientation, substrate combinations and distribution can potentially provide highly tailorable behavior, unprecedented for conventional substrates.

4.7 Supplementary Material

4.7.1 Derivation of Parameters of Scales Relative Motion

To describe the relative motion between zeroth scale and 1st scale, we would need the relative motion of contact point $P_{10}$ on the edge $D_1C_1$ and edge $C_0B_0$. Motion of point $P_{10}$ on the edge $D_1C_1$ can be described as the change in the length of vector $P_{10}C_1$, which is always in direction of $n_x$, and the change in the length of vector $P_{10}C_0$, which is always in direction of $n_y$. By using the superposition principle, the total differential displacement of point $P_{10}$ can be described in vectorial format as $dR_{10} = (d|P_{10}C_1|)n_x + (d|P_{10}C_0|)n_y$. Figure 4.3. The unit vector $n_y$ can be described in the local coordinate established on 1st scale as follows:

$$n_y = (n_y.n_x)n_x + (n_y.n_y)n_y + (n_y.n_z)n_z.$$  (4.9)
By projecting \( n_{y0} \) on the 1\(^{st} \) scale plane, we can describe relative motion of zeroth scale with respect to 1\(^{st} \) scale as the planar relative displacement, as follows:

\[
dr = \left( d|P_{10}C_1| + d|P_{10}C_0|(n_{y0}.n_{x1}) \right)n_{x1} + \left( d|P_{10}C_0|(n_{y0}.n_{y1}) \right)n_{y1}. \tag{4.10}
\]

The length of (4.10) can be described as the relative differential displacement value:

\[
dr = |\dr| = \sqrt{\left( d|P_{10}C_1| + d|P_{10}C_0|(n_{y0}.n_{x1}) \right)^2 + \left( d|P_{10}C_0|(n_{y0}.n_{y1}) \right)^2}. \tag{4.11}
\]

To find the angle between the friction force \( f_{fr} \) acting in the plane of 1\(^{st} \) scale and the unit vector \( n_{x1} \), we can use (4.10) and (4.11) as the relative displacement vector and its value, then angle \( \chi_{10} \) is derived as:

\[
\chi_{10} = \arccos \left( \frac{1}{\dr} \left( d|P_{10}C_1| + d|P_{10}C_0|(n_{y0}.n_{x1}) \right) \right). \tag{4.12}
\]

If we repeat similar steps for the relative motion between 1\(^{st} \) scale and 2\(^{nd} \) scale, it will lead to the similar relationship for the angle between the friction force \( f_{fr} \) acting in the plane of 2\(^{nd} \) scale and the unit vector \( n_{x2} \). Finally by computing the values of these relationships, we find that \( \chi_{10} = \chi_{12} \), and can be shown as \( \chi \). This finding also conforms the periodicity in the system.

4.7.2 Stress and Deformation Obtained by FE Simulation

The finite element (FE) simulations of twisting of a scale-covered beam was carried out using commercially available software ABAQUS/CAE 2017 (Dassault Systèmes). An assembly was made including two different 3D deformable solids including rectangular prismatic substrate and the scales. Then we applied rigid body constraint on the scales obviating any need for material
properties for scales. In this model, a sufficient long rectangular linear elastic beam considered to satisfy periodicity. The twisting load was applied on the right end of beam cross section and the other end was fixed as shown in the Figure 4.8. The contact mechanics was modeled using the self-contact option for the entire structure, with a defined coefficient of friction between every two neighboring scales for each simulation. The following figure shows a contour plot of von Mises stress in a twisted scale-covered beam with deformation scale factor 1. The geometrical parameters of the structure are as follows $\eta = 3$, $\theta_0 = 10^\circ$, $\alpha = 45^\circ$, $\beta = 1.25$, $\lambda = 0.45$, $\zeta = 0.35$, in the following Figure.

Figure 4.8: Contour plot of von Mises stress in a twisted scale-covered beam with deformation scale factor 1. The unit for stress is MPa in this contour plot.
5.1 Abstract

We develop the mechanics of one dimensional filamentous structure with protruding fish scale like features embedded on to the surface under combined bending and twisting load using Cosserat kinematic formulation. This model allows us to bypass the limitations of typical finite element computations inherent in these systems when deflections are large. The model reveals for the first time the combined effect of bending and twisting on fish scale elastica. The model subsumes previous models on pure bending and twisting but also shows previously unobserved phenomena that arises due to the coupled effects of these loads. This includes modulation of locking and ‘kinked’ nonlinear behavior in one deformation direction by the other and lockless behavior of certain geometries. This cross coupling enhances stiffnsses and reveals new regimes of nonlinear stiffening. The model is general and highly useful for future design and optimization.

Citation:
5.2 Introduction

Fishes are now almost synonymous with scales, although scales are far more versatile in nature. They also cover numerous reptiles and can also be intermittently found in mammals such as in pangolins and armadillos [31, 32]. More interestingly, there are scale-like features in the wings of butterflies, human hair and papillae on feline tongues [179,180] indicating the singular importance of the scale morphology in enhancing functions. Scales are generally a lightweight addition to a substrate due to low volume fraction and yet enhance stiffness, and multifunctionality [83, 84]. Several critical properties including protection, locomotion, camouflaging, and thermal regulation have been attributed to scales [81, 86, 181]. Thus they are now being intensely studied as material templates for make armors, smart skins, soft robotics and multifunctional surfaces. However, the understanding the mechanics of such systems are critical towards the ultimate goal of development and design. Mechanically, scales give rise to fascinating emergent behavior such as strain stiffening, directionality and anomalous frictional behavior [52,54–57,59–61,182]. These behaviors aid the organism in balancing multiple complex functions such as locomotion with protection.

Such possibilities have lead to numerous studies in the past to understand both the material and the structural response of these systems. Several studies highlighted and confirmed the essential behaviors of 1-dimensional beam like substrates covered uniformly with scales, Figure 5.1 [52, 59, 93]. These included establishing precise structure-property relationships in bending for both smooth and rough sliding between scales [52,54]. The essential characteristics of bending behavior such as strain stiffening and locked states were found to be universally valid even when scales were not uniform (e.g. functionally graded [57]) or loaded under non-uniform bending [56]. Recent studies in twisting have also indicated that locking and strain stiffening is possible under uniform torsion [59,60]. However, torsion calculations while underlining the universality also highlighted the striking differences from bending. For instance, the kinematic locking envelopes turned out
to be a complex nonlinear functions of geometry, unlike the bending case. In addition, the tilt angle of the scales, Figure 5.1 had a significant impact on the nature of locking, with some angles precluding any locking behavior. When friction was included in the work, more differences with the bending case were evident [54, 60]. For instance, frictional locking is universal, but in twisting the frictional locking envelope is highly nonlinear without closed form solution unlike the bending case. Also, the relative energy dissipation is monotonically increase with $\mu$ for twisting case, even though for bending case increasing $\mu$ may not necessarily increase total dissipative work in a cycle [54, 60]. Such differences are expected as the mechanical behavior are structure, load and geometry driven with no intrinsic guarantees of universality. Thus it is important to investigate individual canonical load cases carefully and rigorously.

Figure 5.1: (a) Natural fish scales under deformation mode, adapted under CC BY 2.0 [93]. (b) Fabricated biomimetic scale metamaterials under bending deformation. (c) Fabricated biomimetic scale metamaterials under twisting deformation. (d) Fabricated biomimetic scale metamaterials under combined bending and twisting deformation.
In this context, the mechanics of combined loading, specifically combined bending and twisting has not been studied. There is virtually no knowledge of the behavior of fish scale substrates in 3-D or spatial deflections, which is a significantly more complex but of great practical utility due to nature of practical loads and possibility of defects on the overall filament leading to cross curvatures. Prior experience shows that existing models are not universally valid or even extrapolate well across geometry-loading combinations and rigorous calculations need to be done.

We address this lacuna by investigating the mechanics of a frictionless tilted elastica, Figure 5.1. We address the presence of spatial curvature within the framework of Cosserat kinematics. We develop a rigorous Cosserat rod model for the biomimetic scale filament and use it to reveal the nature and regimes of nonlinearity, the interplay of bending and twisting and locking behavior. In the process we obtain structure-property relationships for this type of loading. We use FE simulations and existing results in literature to validate our model. This paper is organized as follows, we first develop the kinematics of filament in bending and twisting loads. Assuming small strains and additive energies, next we derive the moment-curvature relationships. In the next section, we discuss the results and their significance, and finally we conclude the paper summarizing the distilling the insights.

5.3 Contact Mechanics of Scales

5.3.1 Global Kinematics of Elastica

We develop the global kinematics in terms of Cosserat description of the scale covered slender substrate. A schematic of this biomimetic system is shown in Figure 5.2 (a). We model the substrate as a Cosserat rod [183] whose centroidal curve in the undeformed reference configurations is defined by \( R(s) \), where \( s \) represents the arc-length along the undeformed configuration. We as-
sume a flat reference configuration of the rod pointing along the z-direction as shown in the Figure 5.2 (a), thus \(\mathbf{R}(s) = s\mathbf{e}_3\), where \(\mathbf{e}_1\), \(\mathbf{e}_2\), and \(\mathbf{e}_3\) are the unit cartesian basis vectors along the \(x\), \(y\), and \(z\) directions, respectively. We identify the directors of the undeformed rod with these vectors.

Let \(\mathbf{r}(s)\) denote the deformed position of the centroidal curve, and \(\mathbf{d}_1(s)\), \(\mathbf{d}_2(s)\), and \(\mathbf{d}_3\) are the orthonormal directors moving along the deformed rod.

The directors of the deformed configuration are related to those of the undeformed configuration through a rotation matrix \(\mathbf{Q}(s)\):

\[
\mathbf{d}_\alpha(s) = \mathbf{Q}(s)\mathbf{e}_\alpha, \text{ for } \alpha = 1, 2, 3. \tag{5.1}
\]

Differentiating (5.1) and substituting for \(\mathbf{e}_\alpha\) using (5.1), we obtain:

\[
\mathbf{d}'_\alpha(s) = \mathbf{Q}'(s)\mathbf{Q}^T(s)\mathbf{d}_\alpha. \tag{5.2}
\]

Let us define \(\kappa\) to be the axial vector associated with the skew-symmetric matrix \(\mathbf{K} := \mathbf{Q}'\mathbf{Q}^T\). We can rewrite (5.2) using the cross-product, as follows:

\[
\mathbf{d}'_\alpha = \kappa \times \mathbf{d}_\alpha. \tag{5.3}
\]

The axial vector \(\kappa := \kappa_\alpha\mathbf{d}_\alpha\) contains the bending and twisting “strain” measures of the rod and the associated skew-symmetric matrix \(\mathbf{K}\), in terms of the bending strain variables, is given by:

\[
\mathbf{K} = \begin{pmatrix}
0 & -\kappa_3 & \kappa_2 \\
\kappa_3 & 0 & -\kappa_1 \\
-\kappa_2 & \kappa_1 & 0
\end{pmatrix}. \tag{5.4}
\]
Since $Q' = KQ$ for rods with uniform curvatures, $K$ is independent of $s$, and $Q'$ can be explicitly integrated to obtain:

$$Q(s) = e^{sK},$$  \hspace{1cm} (5.5)

where we have assumed that the scale at $s = 0$ is fixed and does not undergo any change in orientation as the scales deform. Thus, $Q(0) = I$.

![Figure 5.2: (a) Schematic of flat reference configuration of the scale-covered rod. (b) Schematic of deformed configuration of the scale-covered rod under coupled bend-twist load.](image)

Note that the definition of strain in the Cosserat sense is distinct from the actual 3 dimensional strain in the substrate, which is assumed to be fairly small. For pure twisting or bending the strain in the Cosserat sense is identical to respectively twist angles and curvatures. In more general case the matrix description is appropriate. For the rest of the paper, we discuss “strains” strictly in the Cosserat sense. The overall information of the deformed configuration of the center line of the elastica is fully contained within the entries of matrix described in Equation (5.4). In general, the rod undergoes stretching and shear. The strain variables associated with these modes of deformation...
are given by \( \nu_1, \nu_2, \) and \( \nu_3, \) respectively and are related to the deformation through:

\[
\mathbf{r}' = \sum_{\alpha=1}^{3} \nu_\alpha \mathbf{d}_\alpha. \tag{5.6}
\]

Using (5.1) to express \( \mathbf{d}_\alpha \) in terms of \( \mathbf{e}_\alpha \) and integrating the resulting equation, we obtain

\[
\mathbf{r}(s) = \sum_{\alpha=1}^{3} \int_0^s \nu_\alpha \mathbf{Q}(\tau) \mathbf{e}_\alpha \, d\tau, \tag{5.7}
\]

where we used the boundary condition \( \mathbf{r}(0) = \mathbf{0}. \) In what follows, we assume inextensibility and unshearability of the rod and set \( \nu_1 = \nu_2 = 0 \) and \( \nu_3 = 1. \) These assumptions have been validated based on the results of extensive numerical solutions, which have been conducted according to Sec. 5.4. According to the numerical results, the length of rod’s center line remains constant under the combined loading, which means the rod is inextensible. No shear in implied in the Cosserat sense was observed.

### 5.3.2 Local Kinematics and Contact Conditions

We assume that the scales are much stiffer than the underlying substrates which allows us to postulate rigidity of the scales. This assumption is common in literature and mimics the relatively large stiffness difference between the substrate and the scales. In addition, it is common in literature to impose periodicity of contact for both bending and twisting [52, 59]. Non periodic or functionally graded systems have been investigated earlier which indicated the salient features of these systems are still preserved [56, 57]. We model the scales as rigid two-dimensional rectangles of length \( l \) and width \( 2b. \) In their reference configuration, they are equally separated and oriented parallel to each other with orientation described by dihedral angles \( \alpha_0 \) and \( \theta_0 \) as shown in Figure 5.2a. The separation of two adjacent scales measured along the centroidal curve of the undeformed configu-
ration is \( d \). As the rod deforms, we assume that the scales rigidly rotate with the rod until contact between scales occurs. After this point the scales continue to remain rigid, but with dihedral angles \( \theta \) and \( \alpha_0 \). The assumption that the angle \( \alpha_0 \) remains unchanged due to contact is based on FEM simulations that we present in section 5.4. Rigidity of the scales implies that the engaged configurations of the scales can be mapped to a fictitious reference configuration where the normal to the plane(s) of the scales (pointing in the direction as shown in the figure) is therefore given by

\[
N = \sin \theta \sin \alpha_0 \mathbf{e}_1 + \cos \theta \mathbf{e}_2 - \sin \theta \cos \alpha_0 \mathbf{e}_3.
\]

We now focus on the scales which we assume to be located at predetermined values of \( s_i \) \((i = 1, 2, \ldots)\) as shown in Figure 5.2. Let \( \Omega_i \subset \mathbb{R}^3 \) be the set of all points lying on the rectangle constituting the scale located at \( s_i \) in the undeformed configuration and \( \omega_i \subset \mathbb{R}^3 \) be the set of all points on the same, but in the deformed configuration as shown in Figures 5.2 (a) and (b), respectively. The midpoint of the base of the scales are located at \( R_i := R(s_i) \) in the reference configuration and \( r_i := r(s_i) \) in the deformed configuration. Note that for prescribed bending strains \( \kappa_i \) \((i = 1, 2, 3)\), \( r(s_i) \) is given by (5.7). It is clear from Figure 5.2 that for any \( X_i \in \Omega_i \) and \( x_i \in \omega_i \), the vectors \( X_i - R_i \) and \( x_i - r_i \) lie on the scales in the undeformed and deformed configurations, respectively. It then follows that, for any \( X_i \in \Omega_i \),

\[
N \cdot [X_i - R_i] = 0. \tag{5.8}
\]

Under deformation, the scales rotate rigidly such that:

\[
x_i - r_i = Q_i [X_i - R_i], \tag{5.9}
\]

where \( Q_i := Q(s_i) \) is the rotation matrix given by (5.5) and relates the orientation of the directors of the rod at \( s_i \) to their reference state (5.1). The condition for intersection of the two adjacent
scales (say, \( i = 0 \) and \( i = 1 \)) is given by:

\[ x_1 - x_0 = 0, \]  
\[ (5.10) \]

which using (5.9) can be equivalently written as:

\[ \mathbf{r}_1 + Q_1 [\mathbf{X}_1 - \mathbf{R}_1] - \mathbf{X}_0 = 0, \]
\[ (5.11) \]

where we have used \( Q_0 = I \) and the boundary condition \( \mathbf{R}_0 = \mathbf{r}_0 = 0 \). Thus the conditions for finite planes \( \omega_0 \) and \( \omega_1 \) to intersect is summarized by the following conditions:

\[ \mathbf{X}_0 - Q_1 [\mathbf{X}_1 - \mathbf{R}_1] = \mathbf{r}_1, \]  
\[ (5.12a) \]

\[ \mathbf{N} \cdot \mathbf{X}_0 = 0, \text{ for } \mathbf{X}_0 \in \Omega_0, \] 
\[ (5.12b) \]

\[ \mathbf{N} \cdot [\mathbf{X}_1 - \mathbf{R}_1] = 0, \text{ for } \mathbf{X}_1 \in \Omega_1. \]  
\[ (5.12c) \]

Solution(s) \( \mathbf{X}_0 \) and \( \mathbf{X}_1 \) of (5.12) give the coordinates of the points of intersection of the scales located at \( s_0 \) and \( s_1 \) pulled back to the reference configuration of the scales. For a computational viewpoint, while it is easy to solve this linear system of equations, verifying that these solutions lie on the planes \( \Omega_0 \) and \( \Omega_1 \), respectively is not as straightforward. This is due to the complicated three-dimensional orientation of the scales. To make this verification process computationally easy, we introduce new variables \( \mathbf{\hat{X}}_0 \) and \( \mathbf{\hat{X}}_1 \) defined by:

\[ \mathbf{\hat{X}}_i := T [\mathbf{X}_i - \mathbf{R}_i], \quad i = 0, 1, \]
\[ (5.13) \]
where $T$ is the rotation matrix:

$$
T = \begin{pmatrix}
\cos \alpha & 0 & \sin \alpha \\
0 & 1 & 0 \\
-\sin \alpha & 0 & \cos \alpha 
\end{pmatrix}.
$$

(5.14)

The transformation (5.13) transforms the coordinates of points in $\Omega_i$ to coordinates with respect to the new axes $\hat{X}$ and $\hat{Z}$, as shown in Figure 5.3. The transformation $T$ rotates the projection of the scales, shown in gray in the figure, such that the bounds of this projection can be written as the cartesian product $[-b, b] \times [0, l \cos \theta]$.

![Figure 5.3: Schematic showing the coordinate system for $\hat{X}$.

Thus the condition $\mathbf{X}_i \in \Omega_i$ can be equivalently expressed as $\hat{\mathbf{X}}_i \in \mathcal{B}$, where,

$$
\mathcal{B} := [-b, b] \times [0, \infty] \times [0, l \cos \theta].
$$

(5.15)

Rewriting (5.12) in terms of the new variables, (5.13), results in the following equivalent conditions
for the intersection of two adjacent scales:

\[
T^T \hat{X}_0 - Q_1 T^T \hat{X}_1 = r_1, \quad (5.16a)
\]

\[
N^T T^T \hat{X}_0 = 0, \quad (5.16b)
\]

\[
N^T T^T \hat{X}_1 = 0, \quad (5.16c)
\]

where \( \hat{X}_0, \hat{X}_1 \in \mathcal{B} \).

These equations can be equivalently written as:

\[
\begin{bmatrix}
T^T & -Q_1 T^T \\
N^T T^T & O \\
O & N^T T^T 
\end{bmatrix}
\begin{bmatrix}
\hat{X}_0 \\
\hat{X}_1
\end{bmatrix}
= \begin{bmatrix}
r_1 \\
0 \\
0
\end{bmatrix}, \quad (5.17)
\]

where \( O = [0 \ 0 \ 0] \).

Note that this is a system of five equations in six unknowns. If \( \hat{X}_0 \) and \( \hat{X}_1 \) remain unrestricted, then we have two possibilities—no solutions, corresponding to the two planes being parallel or infinitely many solutions with a line of intersection. However, the scales are finite planes, and \( \hat{X}_0 \) and \( \hat{X}_1 \) must lie in \( \mathcal{B} \). When finite planes intersect, we have three possibilities: 1) no solution, when the planes are either parallel or do not intersect within the domain, 2) infinitely many solutions when the two planes interpenetrate each other with a line-segment intersection or the edge of one plane intersects either the face or edge of the other plane, or 3) a unique solution, otherwise.

The third case of a unique solution corresponds to the physically relevant configurations of the
scales when they are engaged. Any such configuration falls in the following classes – It is either a corner of one of the scales or is a point of intersection of two edges of different scales, Figure 5.2.

To find these solutions, we can fix two of the coordinates—\( \hat{X}_i \) and \( \hat{Z}_i \) for the case of corner contact or \( \hat{X}_i \) and \( \hat{Z}_j \) (\( i \neq j \)), for example, for edge contacts between different scales. We now list the various possibilities of contact.

1. Corner of a one scale makes contact with the face of the second. There are various possibilities here. If the right corner of the base edge of the first scale makes contact with the face of the second, we have \( (\hat{X}_0, \hat{Z}_0) = (-b, 0) \), if the left corner of the base edge of the first scale makes contact with the face of the second, we have \( (\hat{X}_0, \hat{Z}_0) = (b, 0) \), if the right corner of the top edge of the first scale makes contact with the face of the second, \( (\hat{X}_0, \hat{Z}_0) = (-b, l \cos \theta) \), if the left corner of the top edge of the first scale makes contact with the second, \( (\hat{X}_0, \hat{Z}_0) = (b, l \cos \theta) \). Of these four possibilities we only consider the last two as they lead to configurations that don’t cause interpenetration of the scales. Similar set exits of the corner of the second scale makes contact with the face of the first.

2. The edge of the first scale makes contact with the edge of the second scale. There are again several possibilities here. If the right edge of the first scale makes contact with the right edge of the second we have \( (\hat{X}_0, \hat{X}_1) = (-b, -b) \). If the right edge of the first makes contact with the top edge of the second we have \( (\hat{X}_0, \hat{Z}_1) = (-b, l \cos \theta) \). Similarly if the left edge of the first scale makes contact with left and top edges of the second scale, we have the conditions \( (\hat{X}_0, \hat{X}_1) = (b, b) \) and \( (\hat{X}_0, \hat{Z}_1) = (b, l \cos \theta) \). Other possibilities are also possible.

All the above cases can be captured by the conditions:

\[
\begin{align*}
\mathbf{a}_0^T \hat{X}_0 + \mathbf{a}_1^T \hat{X}_1 &= \hat{\alpha}, \\
\mathbf{b}_0^T \hat{X}_1 + \mathbf{b}_1^T \hat{X}_1 &= \hat{\beta}.
\end{align*}
\]
Where \( a_0, a_1, b_0, b_1 \) are each three dimensional vectors that select the appropriate coordinates of \( \hat{X}_0 \) and \( \hat{X}_1 \) to impose conditions on and \( \hat{\alpha} \) and \( \hat{\beta} \) are the conditions on the coordinates. For example, for the case when the left corner of the top edge of the first scale makes contact with the face of the second we saw in (1) above that \((\hat{X}_0, \hat{Z}_0) = (b, l \cos \theta)\). These conditions can be imposed by choosing \( a_0 = (1, 0, 0) \), \( a_1 = (0, 0, 0)^T \) and \( b_0 = (0, 0, 1) \), \( b_1 = (0, 0, 0)^T \) which pick coordinates \( \hat{X}_0 \) and \( \hat{Z}_0 \), respectively, and \( \hat{\alpha} = b, \hat{\beta} = l \cos \theta \). For the cases discussed in (2), for instance when \((\hat{X}_0, \hat{X}_1) = (b, b)\), we choose \( a_0 = (1, 0, 0) \), \( a_1 = (0, 0, 0)^T \) and \( b_0 = (0, 0, 0) \), \( b_1 = (1, 0, 0)^T \), and \( \hat{\alpha} = b, \hat{\beta} = b \).

To obtain an implicit expression for the dependence of \( K \) on \( \theta \), we now solve (5.16) simultaneously with (5.18). This is done by the following steps. First, we solve (5.16a) for \( \hat{X}_1 \), i.e.,

\[
\hat{X}_1 = TQ_1^T T \hat{X}_1 - TQ_1^T r_1.
\]

Plug this into (5.18), together with (5.16b), we obtain the linear 3x3 system of equations \( A\hat{X}_0 = c \), where:

\[
A = \begin{pmatrix}
(TN)^T \\
(a_0 + TQ_1^T a_1)^T \\
(b_0 + TQ_1^T b_1)^T
\end{pmatrix},
\quad
\begin{pmatrix}
0 \\
\hat{\alpha} + a_1^T TQ_1^T r_1 \\
\hat{\beta} + b_1^T TQ_1^T r_1
\end{pmatrix}.
\]

Plugging the solution \( \hat{X}_0 \) obtained from the previous system with \( \hat{X}_1 = TQ_1^T T \hat{X}_1 - TQ_1^T r_1 \) in (5.16a), we obtain the following implicit dependence of \( \theta \) in terms of \( K \):

\[
f(K, \theta) = N^T Q_1^T T \hat{X}_0 - N^T Q_1^T r_1 = 0,
\]

where \( \hat{X}_0 \) solves the equation.
These equations finally link the local to the global kinematics completing the multiscale kinematics description of the system. These highly nonlinear “bridging” laws can only be solved numerically.

5.3.3 Mechanics of Biomimetic Scale Elastica

In order to understand the mechanics of this structure, we take recourse to energy balance between the global loads and local deformation. We model the beam to be made up of a linearly elastic material whose strain energy is given by:

$$\varepsilon_{\text{beam}} = \int_0^L \left[ \frac{1}{2} EI_1 \kappa_1^2 + \frac{1}{2} EI_2 \kappa_2^2 + \frac{1}{2} GJ \kappa_3^2 \right] ds. \quad (5.21)$$

To include the contribution from the scales, we note that a scale’s rotations is captured by the angles $\theta$, $\alpha$ and $\gamma$. As the substrate deforms under applied strain, the scales rotate freely until a critical threshold of curvatures is reached when the scales engage. Let $\Omega_e \subset \mathbb{R}^3$ denote the bending strains $(\kappa_1, \kappa_2, \kappa_3)$ when the scales are engaged which can be obtained by solving the equation (5.20) for $\theta = \theta_0$. Beyond this point the scales are engaged and their rotations are resisted by substrate into which the former are built in. We model the substrate resistance using linear torsional springs with the elastic energy stored in the springs for each scale given by:

$$\varepsilon_{\text{scale}}(\theta, \alpha, \gamma) = \frac{1}{2d} \left[ K_{\theta}(\theta - \theta_0)^2 + K_{\alpha}(\alpha - \alpha_0)^2 + K_{\gamma}(\gamma - \gamma_0)^2 \right] H_{\Omega_e}(\mathbf{k}), \quad (5.22)$$

where $H_{\Omega_e}$ is the indicator function on $\Omega_e$ (i.e., $H_{\Omega_e}(\mathbf{k}) = 1$, if $\mathbf{k} \in \Omega_e$, and zero, otherwise). The total energy per RVE can be additively written as:

$$\varepsilon(\kappa_1, \kappa_2, \kappa_3) = \varepsilon_{\text{beam}}(\kappa_1, \kappa_2, \kappa_3) + \varepsilon_{\text{scale}}(\theta, \alpha, \gamma). \quad (5.23)$$
Finite element simulations presented in Sec. 5.4 show that the change in $\alpha$ and $\gamma$ from $\alpha_0$ and $\gamma_0$ is minimal even when the scales are engaged. This observation is in agreement with our earlier findings for pure bending and twisting [52, 59]. We drop their dependence from energy calculations. Note that $\theta$ is itself a function of $(\kappa_1, \kappa_2, \kappa_3)$, hence the dependence of $\mathcal{E}$ on the same.

The spring constant of the torsional spring, $K_{\theta}$ is related to the Young’s modulus of the substrate $(E)$, scale thickness $(t_s)$, inclusion length $(L)$ and $\theta_0$. As we have shown in [52, 59] the following non-dimensional scaling exists:

$$\frac{K_{\theta}}{Et_s^2} = C(\alpha) \left( \frac{L}{t_s} \right)^n f(\theta_0),$$  

(5.24)

where $n$ is non-dimensionless constant that we estimate using FE simulations, and $C$ and $f$ are functions of angular parameters. For the results presented below, these were $C_B = n = f(\theta_0) =$. Our FE simulations indicate the effect of $K_{\alpha}$ and $K_{\gamma}$ are negligible (see Supplementary Material of Chapter 3) and can therefore be neglected. The moments in the three directions are computed by differentiating (5.23) with respect to $\kappa_1$, and $\kappa_3$:

$$M_1 = C_f EI \kappa_1 + \frac{K_{\theta}}{d}(\theta - \theta_0) \frac{\partial \theta}{\partial \kappa_1} [H(\kappa_1 - \kappa_e^1)\left| H(\kappa_3 - \kappa_e^3) \right|],$$  

(5.25a)

$$M_3 = C_f C_w GI \kappa_3 + \frac{K_{\theta}}{d}(\theta - \theta_0) \frac{\partial \theta}{\partial \kappa_3} [H(\kappa_1 - \kappa_e^1)\left| H(\kappa_3 - \kappa_e^3) \right|].$$  

(5.25b)

These relations can be computed numerically to obtain the moment-curvature relationships.

### 5.4 Finite Element Analysis

We develop a Finite Element (FE) model to compare the numerical results with the results derived from the developed analytical model for the biomimetic scale-covered system. For this purpose, the
FE simulations are carried out using ABAQUS/CAE 2017 (Dassault Systèmes), the commercially available software for Finite Element Analysis (FEA). In these FE models, the 3D deformable solids have been considered for the scales and the substrate. To avoid the edge effects and to satisfy the periodicity, a sufficient length is considered for the rectangular prismatic substrate. With this substrate’s length, an assembly of a row of 19 scales and the substrate has been considered, which the scales are embedded on the substrate’s top surface. In this assembly, the scales are oriented with angle of $\theta_0$ with respect to the substrate’s top surface, and angle of $\alpha$ with respect to the substrate’s rectangular cross section. The substrate’s material model has considered as linear elastic properties with the elastic modulus $E_B$ and the Poisson’s ratio $\nu$, which leads to the shear modulus of $G_B = \frac{E_B}{2(1+\nu)}$ for the substrate. Because the scales are considered rigid with respect to the substrate, rigid body constraints are imposed to the scales.

The mechanical loads of the bending and twisting are applied quasi-statically to the system. For this purpose, the simulation is considered as two static steps with active nonlinear geometry option. In the first step, the bending rotations has been applied to the both end cross-sections with reverse directions, linearly increasing from 0 to $\Psi$ during the step time. Here, $\Psi \approx \kappa_1 L_B / 2$, where $L_B$ is the substrate’s length. This approximation ($\approx$) is due to the edge effect, which violate the uniformity of curvature along the whole beam. Thus in the FE results, We will find the average curvature of the beam far from the beam ends to remove edge effect, as it is explained in next paragraph. In the second step, the bending rotations is fixed at the final value of $\Psi$, and the twisting rotations has been applied to the both end cross-sections with reverse directions, linearly increasing from 0 to $\Phi \approx \kappa_3 L_B / 2$ during the step time. A frictionless surface-to-surface contact is applied to the scales surfaces. To have a reliable numerical results, a mesh convergence study is carried out to discover a sufficient mesh density for different regions of the model. This mesh convergence leads to a total number of almost 230,000 elements. In this mesh, because the geometry is complex due to the scales inclusions, the top layer of substrate is meshed with the tetrahedral quadratic elements.
C3D10. Other regions of the model are meshed with the quadratic hexahedral elements C3D20.

To compare the FE simulations with theory presented in Section 5.3, we extract the bending curvatures $\kappa_1$, $\kappa_2$, and $\kappa_3$ in the following manner. First, we extract the position vector for the mid-point of the top face of the beam from the numerical simulations. This vector is an estimate for $\mathbf{r}(s)$ as given in (5.7). The information of the directors $\mathbf{d}_1(s)$ and $\mathbf{d}_2(s)$ is extracted by subtracting the position vectors of center-line and the right-edge of the beam, respectively, from the estimate for $\mathbf{r}(s)$ and normalizing the resulting quantities to produce unit vectors. Since $\mathbf{d}_3 = \mathbf{r}'(s)$, this director is estimated by computing $\mathbf{r}'(s)$ using finite differences along the beam. It follows from (5.1) that rotation matrix mapping the cross-sections of the beam is given by $\mathbf{Q}(s) = [\mathbf{d}_1(s), \mathbf{d}_2(s), \mathbf{d}_3(s)]$ where the directors are taken to be the column vectors of the matrix. The skew-symmetric matrix $\mathbf{K}$ containing the bending strains $\kappa_1(s)$, $\kappa_2(s)$, $\kappa_3(s)$ (5.4) along the length of the beam is computed using $\mathbf{Q}' = \mathbf{KQ}$ as $\mathbf{Q}'(s)\mathbf{Q}^{-1}(s)$, where $\mathbf{Q}'(s)$ is estimated using finite differences. To avoid boundary effects, we average the bending strains over the middle half of the beam to obtain estimates for average bending strains in the beam.

5.5 Results and Discussion

In Figure 5.4, we plot of cross strain effects, with various types of scales contact color coded differently in the plots. Without loss of generality, we take the initial scale angle $\theta_0 = 0$ (grazing scales). A $\theta_0 > 0$ would only shift the plots forward till engagement occurs, with the curves thereafter following the same trajectories [52,59]. In Figure 5.4, we plot $\theta$ at contact as a function of $\kappa_3$ (for $\kappa_1 = 0.2$, $\kappa_2 = 0$, $\eta = 3$, $\beta = 1.25$, and $\alpha = 30^\circ$). We observe four distinct modes of contact which are color coded in the figure as follows:

- **Black:** Left edge of the first scale makes contact with top edge of the second. Configuration is shown as scheme A in Figure 5.4.
• Red: Corner of the first scale makes contact with face/edge of the second scale. Configurations are shown as the scheme B, and C in Figure 5.4.

• Green: Right edge of the first scale makes contact with top edge of the second. Configurations are shown as the scheme D and E in Figure 5.4.

• Blue: Top edges of the first and second scales make contact. Configurations are shown as the scheme F and G in Figure 5.4.

Figure 5.4: $\kappa_3$ vs $\theta$ for the assigned value of $\kappa_1 = 0.2$ showing various types of scales contact color coded. Here $\kappa_2 = 0$, $\eta = 3$, $\beta = 1.25$, and $\alpha = 30^\circ$.

In Figure 5.5 (a) we plot $\kappa_3$ (twisting) versus $\theta$ (scale angle rotation) for $\eta = 3$, $\beta = 1.25$, and $\alpha = 30^\circ$. The different curves correspond to different values of bending strains $\kappa_1$ as labeled in the figure. The red curves represent configurations where the corner of the first scale makes contact
with the face of the second scale, green curves are configurations where the top edge of the first scale makes contact with the right edge of the second scale, blue curves are configurations where the top edges of the two scales make contact, and black curves are configurations where the top edge of the first scale makes contact with the left edge of the second. A picture showing these configurations for the case of \( \kappa_1 = 0.2 \) is shown in Figure 5.4. This figure is fundamentally different from individual pure bending [52] or twisting [59] kinematics. Here, there are distinct regions of kinematics and they are dependent on the existing bending strain. Thus, bending can result in entirely new type of kinematic behaviors in twisting. The color coded plots are an indication that there would be kinks in twisting rigidity as contact regimes between scales undergo change as depicted in Figure 5.4. This plot can also be used to note the sensitivity of twisting to existing bending strains. We find that the sensitivity is quite high and thus any unintended bending either due to processing or loading asymmetries can drastically change the overall kinematics. Another interesting feature is that, locking angle for the scale seems to be insensitive to bending. Thus, the locking envelopes computed from pure twisting would still hold in this case. Thus, bending strains have a differential effect on the overall twisting kinematics, affecting the overall kinematics trajectory substantially while leaving the locked state unchanged.

In the same spirit, in Figure 5.5 (b), we highlight the effect of twisting strain on the bending behavior of the biomimetic beam. Here, we find that a twist in the system has a significant impact on the bending behavior. With increasing positive twist, the bending engagement occurs earlier, whereas the opposite occurs with negative twist. This overall trend reflects the fact that twisting on one side has an “opening” effect on the scale where they part from each other compared to the other. The slopes also change significantly indicating a potential effect on the overall stiffness of the system. The abrupt changes reflect sudden change of contact regimes and shown with different colors. Here again, interestingly the presence of twist does not affect the locking angle, analogous to the twisting case.
We now use our model to explore the twisting kinematics at a given bending strain, with changing geometric parameters, $\alpha$ and $\eta$. These parameters have shown to be critical in dictating the overall kinematics of pure twisting [59]. For this, we fix the bending curvature at $\kappa_1 = 0.1$. In Figure 5.6 (a), we plot $\theta$ vs $\kappa_3$ for different values of $\alpha$ (for $\beta = 1.25$, $\eta = 3$ and $\kappa_1 = 0.1$). The effect of the tile angle $\alpha$ is dramatic. For relatively small angles, i.e. $\alpha < 20^\circ$, there is a relatively “stiff” response on either direction of twisting. These reflect scales sliding on either direction of twisting. However, there is a marked anisotropy between the directions in both magnitude and contact regime. More interestingly, at higher tilt angles such bi-directional stiffness disappears altogether indicating the scales do not or only weakly engage in other other direction. The lack of engagement in the other direction can be visualized as “opening” of the scales in one direction vs closing. The effect of $\alpha$ on locking is also pronounced. At lower tilt angles, the stiffening effect disappears into a lockless behavior. This is due to scales sliding past each other without ever satisfying the contact constraints. Locking behavior emerges again for higher tilt angles.

Next, we probe the significance of overlap ratio $\eta$ on the overall kinematics of the system. In Figure
5.6 (b), we plot $\theta$ vs $\kappa_3$ for different values of $\eta = l/d$ (for $\beta = 1.25$, $\alpha = 30$ and $\kappa_1 = 0.15$). We see here again that $\eta$ influences the angle of locking. We find that $\text{eta}$ has a significant role to play in determining the overall kinematics of the system. Increasing $\text{eta}$ leads to an overall stiffer nonlinearity but at the same time can potentially change the overall contact regime sequence up until locking.

The last geometric parameter of significance is $\beta = b/d$ which measures the effect of substrate width on the kinematics. In Figure 5.7 we explore the role of $\beta = b/d$ in dictating the overall kinematics. Here we observe that locking occurs at the same value of $\beta$ and $\kappa_3$ (for $\kappa_1 = 0.1$, $\eta = 3$ and $\alpha = 30$) as long as $\beta$ is beyond a critical threshold. This observation can be interpreted as follows: below the a critical value of $\beta$, the scales are no wide enough that there is no locking and the scales simply lose contact as $\kappa_3$ is increased. But above this threshold, the scales are able to lock and the width of the scale has no influence on the angle of locking.

These analytical results are validated using finite element (FE) simulations shown in Figure 5.8.

Figure 5.6: (a) $\theta$ vs $\kappa_3$ for different values of $\alpha$ ($\eta = 3$, $\beta = 1.25$, $\kappa_1 = 0.1$). (b) $\theta$ vs $\kappa_3$ for different values of $\eta$ (for $\beta = 1.25$, $\alpha = 30$, $\kappa_1 = 0.1$).
Figure 5.7: $\kappa_3$ vs $\theta$ for different values of $\beta$. Here $\kappa_1 = 0.15$, $\eta = 3$ and $\alpha = 30^\circ$.

![Graph showing $\kappa_3$ vs $\theta$ for different $\beta$.]

Figure 5.8: (a) $\kappa_3$ vs $\theta$ different values of bending strains $\kappa_1$. Here $\eta = 3$, $\beta = 1.25$, and $\alpha = 30^\circ$. The Red dotted lines are the related FEM results. (b) $\kappa_3$ vs $\theta$ different values of overlap ratios $\eta$. Here $\kappa_1 \approx 0.075$, $\beta = 1.25$, and $\alpha = 30^\circ$. The Red dotted lines are the related FEM results.

We now look at the sensitivity of the bending kinematics in the presence of twist. Here again, we see that a twist significantly changes the kinematics when compared with pure bending [52]. Here, we see that kinks begin to appear indicative of different types of contacts as explained earlier. Interestingly, the engagement occurs at a negative bending value, which can again be attributed to the “opening” effect on scales due to twisting where scales are pushed apart in twist. Next, we see the effect of parameter $\alpha$. 
Figure 5.9: (a) $\theta$ versus $\kappa_1$ for $\kappa_3 = 0.1$, $\beta = 1.25$, $\eta = 3$ for different values of $\alpha$ (b) $\theta$ vs $\kappa_1$ for different values of $\eta$ (for $\beta = 1.25$, $\alpha = 30$, $\kappa_1 = 0.15$).

Finally, we focus our attention to the mechanical behavior of these systems. In Figure 5.10 we plot bending moment vs bending strains and discover the role played by the presence of twist for various values of twisting, when $\eta$ is fixed. Notice the critical role played by the twist in altering the bending response even when $\eta$ is fixed. Higher values of positive twist shift the engagement to earlier parts, whereas higher values of negative twist shift it in the opposite direction. Interestingly, unlike smooth plots seen for pure bending earlier [52], twist effectively changes the contact regime in terms of discontinuities and jumps in the plot indicating sudden changes in bending rigidity. When a twist is added in the positive direction, the neutral position (no bending strain) is fully engaged (due to the engagement brought about by the twist).

We see similar and even more dramatic effect in the torque-twist diagram in Figure 5.11. Here again, we see the disruptive role of the bending strains as the torque-twist response which are smooth nonlinear plots in pure twisting case [59] now turn into highly discontinuous plots with disparate twist modulus. Yet again, a non-zero bending strain can cause the neutral position (zero twist strain) to be pre-engaged due to bending. These highlight the tremendous differences that can be brought about from cross-coupling effects.
5.6 Conclusions

In this work, we addressed the cross coupling effects of bending and twisting in a biomimetic scale elastic beam for the first time. Here, the scales were plate like rectangular inclusions protruding at an angle from the surface of the elastic substrate. We find highly intricate and often surprising effect of one over the other across the kinematics and mechanics. We quantified these effects by
developing analytical relationships within the framework of Cosserat kinematics and global-local energy balance. This model reduced to the earlier developed model for pure bending and twisting in literature and was also validated with finite element simulations using a commercially available software. This study completes a significant missing piece in the mechanics of biomimetic scale elastica, which is of great practical importance in applications ranging from high performance structures to soft robotics.
CHAPTER 6: MECHANICS OF SCALE-COVERED PLATE UNDER BENDING DEFORMATION

6.1 Abstract

Biomimetic scale-covered substrates provide nontradiotional behavior in mechanical structure in various deformation modes. In this work, we investigate the effects of stiff scale plates on the behavior a 2D plate structure by developing an analytical model verified by finite element simulations. In this model, we consider frictionless contact between rigid scales surfaces, and the substrate is modeled as a deformable linear elastic rectangular plate. The results show that stiffening in the bending response of the structure after scales engagement in two in-plane directions. We considered a load case of both curvatures are upward and another load case of one of the curvature is upward and the other one is downward. The developed model outlines analytical relationships between geometry, deformation, and bending response of the system. The tailorability of the mechanical response with respect to the geometrical parameters provides valuable information on designing biomimetic scale-covered metamaterials for different applications.

Citation 1:

Citation 2:
6.2 Introduction

Exoskeleton elements such as fish scales were an early evolutionary innovation [29–31, 160]. Appearing initially in fishes as scales for protection against predators and rivals [34, 49], their functions grew rapidly to aid locomotion, swimming, camouflage and thermal regulation [24, 80, 88, 184, 185]. It is therefore not surprising that the exoskeleton form not only survived evolutionary honing but thrived in the form of variegated scales [32, 184, 186], furs [135, 187], papillae on feline tongues [188], and scales on hairs [189, 190]. Their mechanical advantages rest on not only their intricate material properties [40, 43, 44], but also their orientation, overlap and distribution. This interplay between material and geometry lies at the heart of high performance of this structural system. Prior research investigating the mechanics of simple one-dimensional beams with embedded scales have shown the emergence of remarkable nonlinearity.

These nonlinearities are often emergent, i.e. not found in the individual parts but arise at the system level form mutual interactions of discrete but regularly arranged exoskeleton elements. Such periodic mutual reinforcement are also the hallmark of metamaterials, and like them the exoskeletal systems also exhibit unique property combinations. By harnessing such emergence, we can obtain hitherto unprecedented property ranges, combinations, tailorability and tunability in performance, not possible by mere mixing or traditional composite systems. These properties although not properly understood, were recognized by early humans. For instance, the features of scales were an inspiration for principles of armor design [32, 34, 141], because overlapping scales can resist penetration and provide additional stiffness [32, 34, 48]. This inspiration is among the earliest interest in this area, which is based on the direct mimicry of the scaled integument for making scaled armors in the ancient times across the world [31, 48, 49, 191, 192]. Lamellar armors, fabricated out of lacing hard plates together, have been found in the ancient Egypt [31, 191], in the Scythian civilizations [191], and in the Persian empire [48, 191]. The scale armor also have been
used by the Assyrian and Mongolian armed forces \([48, 191]\), Roman troops \([31, 49]\), and Japanese Samurai \([49, 192]\). In spite of this early interest, little was known about the fundamental mechanics behind superior performance, which resulted in similar designs reappearing over years.

In addition to protection against external objects, which is mechanically an indentation type local deformation problem, global deformation modes such as bending and twisting of a substrate reveal equally interesting properties, for example a host of applications that require a structural mode of deformation such as soft robotics, prosthetics or morphing structures \([101, 147, 170, 171]\). In a scale-covered structure, it is the role of scale engagements in modifying the global deformation behavior of the underlying structure. These include reversible nonlinear stiffening and locking behavior due to the sliding kinematics of the scales in one-dimensional substrates \([51–54, 58–60]\).

1D substrates with stiff scales revealed strain stiffening due to sliding, scale deformation as well as friction in the bending mode of deformation \([51]\). Later simplification revealed the distinct nonlinear regimes of elasticity even without scale deformation or friction \([52, 59]\). Nonlinearity due to frictional effects were further isolated and their effect on locking and dissipation quantified \([54, 60]\). More studies revealed the limits of theoretical assumptions underpinning the models and their effect on predicted relationships \([55–57]\). Prior research has shown that bending and twisting of a substrate show small strain reversible nonlinear stiffening and locking behavior due to the sliding kinematics of the scales in one-dimensional substrates \([51–60]\). The universality of these behavior across bending of uniformly distributed scales, functionally graded scales and uniformly distributed twisting is an important discovery for these structures.

Fabrication methods for these scale-covered structures have been recently developed in 2D and 1D configuration. These fabricated structures show almost ten times more puncture resistance than soft elastomers. Generally, these scale-covered substrates can be fabricated in a number of ways. One strategy is using glue to attach stiff scales to deformable substrates. Here, the scales
could be 3D printed stiff plates [52, 56, 59, 61], or steel sheets [57], and the substrates could be made of Vinylpolysiloxane (VPS) elastomer [52, 56, 59, 61], or 3D printed flexible material [57], with prefabricated grooves for scales embedding. For emulating intricate and naturally inspired structures, additive manufacturing can be regarded [120], or a combination method of using female mold to arrange scales in a 2D overlapped arrangement, and casting silicone and then demolding can be utilized [121]. In addition, 3D printed mold to cast silicone [122], and multi-material 3D printing [119] have been used to mimic the scaled-like shark skin.

Extending the dimensionality of the problem, two-dimensional substrates were also investigated [4, 53, 58, 96]. In these published works, the focus is on the numerical simulations and experimental analysis and a detailed kinematic model has not established for scaled plate system. These researches showed several similarities with their one-dimensional counterparts in bending. In this paper, we represent the emergent behavior in scale-covered 2D structures under bending loads in two directions of the plate system. Then, we investigate the mechanics of these fish scale exoskeletal system under combined bending loads in two-dimensional plate with protruding scale-like features embedded on to the surface. Finally, we conclude the emergent behaviors in the biomimetic exoskeletal metamaterials and discuss the future development and challenges in this area.

6.3 Plate Deformation under 2D Bending

In this section we consider a deformable plate under two directional bending. $\kappa$ is the curvatures in $x$ direction, which is the change rate of the slope angle $\psi$ with respect to the arc length in longitudinal direction. Also, the curvatures in $z$ direction is $\tau$, which defines the rate of change of the slope angle $\omega$ with respect to the arc lengths in transverse direction. We consider two load cases as shown in Figure 6.1 (a) and (b). In load case (a), both curvatures are upward ($\kappa > 0$ and $\tau > 0$), and in the load case (b), the longitudinal curvature is upward ($\kappa > 0$) and the transverse
curvature is downward ($\tau < 0$). The curvatures are as follows:

$$\kappa = \frac{\partial^2 y/\partial x^2}{[1 + (\partial y/\partial x)^2]^{3/2}} \approx \frac{\partial^2 y}{\partial x^2},$$  \hspace{0.5cm} (6.1a)

$$\tau = \frac{\partial^2 y/\partial z^2}{[1 + (\partial y/\partial z)^2]^{3/2}} \approx \frac{\partial^2 y}{\partial z^2}. \hspace{0.5cm} (6.1b)$$

According to classical plate theory or the Kirchhoff plate theory [193], the moment–curvature response of two in-plane directions are as follows:

$$M_x = D\left(\frac{\partial^2 y}{\partial x^2} + \nu \frac{\partial^2 y}{\partial z^2}\right),$$  \hspace{0.5cm} (6.2a)

$$M_z = D\left(\frac{\partial^2 y}{\partial z^2} + \nu \frac{\partial^2 y}{\partial x^2}\right).$$  \hspace{0.5cm} (6.2b)

where $D = \frac{Eh^3}{12(1-\nu^2)}$, and $E$, $\nu$, and $h$ are the modulus, Poisson’s ratio, and thickness of plate.

Figure 6.1: (a) Deformation of plate when both curvatures are upward ($\kappa > 0$ and $\tau > 0$). (b) Deformation of plate when the longitudinal curvature is upward ($\kappa > 0$) and the transverse curvature is downward ($\tau < 0$).
From Equation (6.2, the curvatures can be obtained as follows:

\[
\kappa = \frac{\partial^2 y}{\partial x^2} = \frac{M_x - \nu M_z}{D(1 - \nu^2)},
\]

(6.3a)

\[
\tau = \frac{\partial^2 y}{\partial z^2} = \frac{M_z - \nu M_x}{D(1 - \nu^2)}.
\]

(6.3b)

According to equilibrium, the moments \(M_x\) and \(M_z\) are applied to all points constantly throughout the plate. Therefore, based on the moment–curvature equations (6.2), the deformed plate equation under bending loads \(M_x\) and \(M_z\) is as follows:

\[
y = \frac{1}{2} \frac{M_x - \nu M_z}{D(1 - \nu^2)} x^2 + \frac{1}{2} \frac{M_z - \nu M_x}{D(1 - \nu^2)} z^2.
\]

(6.4)

By substituting Equations (6.3a) and (6.3b) into Equation (6.4), the plate equation is as follows:

\[
y = \frac{1}{2} (\kappa x^2 + \tau z^2).
\]

(6.5)

### 6.4 Kinematics of Scale-covered Plate under Bending

In this section we consider a deformable plate with scales covering its surface. The scales are partially embedded on the top surface of the plate with staggered arrangement as shown in Figure 6.2. Without loss of generality, we consider the deformable plate as a square shape \((W_{sub} = L_{sub})\) with thickness \(2t\). The exposed length and the width of scales are \(l\) and \(2b\), respectively. The longitudinal and transverse distance between the scales are \(d\) and \(2a\). Therefore, in addition to previous dimensionless parameters \(\eta = l/d\), \(\beta = b/d\), and \(\lambda = t/d\), we also define \(\delta = a/d\), as the dimensionless clearance between the scales.
Figure 6.2: Scale-covered plate with the defined geometrical parameters and isolation of central scale and neighboring scales to demonstrate representative volume element (RVE) under two loading cases: (a) Both curvatures are upward ($\kappa > 0$ and $\tau > 0$). (b) Longitudinal curvature is upward ($\kappa > 0$), and transverse curvature is downward ($\tau < 0$).

According to the two directional pure bending, the periodicity assumption is a valid approximation for the scales engagement [52, 59, 60]. Periodicity assumption lets us to isolate the central scale of the structure and its neighboring scales to model the behavior of the whole structure. The central scale engages with 4 neighboring scales, and due to symmetry, the representative volume element (RVE) can be considered as the central scale and just one of the neighboring scale, as shown in Figure 6.2. The schematic of RVE is illustrated in Figure 6.3 (a) and (b) for two load cases.
Figure 6.3: The schematic of the RVE geometrical configuration: (a) Both curvatures are upward ($\kappa > 0$ and $\tau > 0$), which means both local bending angles are positive ($\psi > 0$ and $\omega > 0$). (b) Longitudinal curvature is upward ($\kappa > 0$) and the transverse curvature is downward ($\tau < 0$), which means the local bending angles in $x$ direction is positive ($\psi > 0$) and the local bending angles in $z$ direction is positive ($\omega < 0$).

In Figure 6.3, the central scales is denoted as “1st scale” and shown as a rectangular plate $A_1B_1C_1D_1$. The neighboring scale is denoted as “2nd scale” and shown as a rectangular plate $A_2B_2C_2D_2$. Due to the underlying deformation of substrate with longitudinal curvature $\kappa$ and transverse curvature $\tau$, the 2nd scale has been displaced and rotated as shown in Figure 6.3 for load cases (a) and (b).

In the load case (a), the corner $B_2$ of 2nd scale contacts with the top surface of 1st scale. We place the coordinates $xyz$ on the midpoint of 1st scale’s width, $M_1$, as shown in Figure 6.3 (a). To find a contact criterion between 1st scale and 2nd scale, the 3D-equations of plane $A_1B_1C_1D_1$ and coordinate of point $B_2$ need to be established. To establish 3D-equations of plane $A_1B_1C_1D_1$, the
coordinate of points \( N_1, D_1 \) and \( C_1 \) can be found, then the vectors \( M_1D_1 \) and \( M_1N_1 \) is obtained to calculate the normal vector of plane \( A_1B_1C_1D_1 \). Finally, the plate equation can be obtained by using the normal vector and coordinate of one of the points located in the plate. To find the coordinate of point \( B_2 \), located in the 2\(^{nd} \) scale, we start form the base point of 2\(^{nd} \) scale’s plate, \( M_2 \). The \( y \) coordinate of point \( M_2 \) is obtained from the deformed substrate’s equation (6.5). The \( x \) and \( z \) coordinates of point \( M_2 \) are \( d \) and \( -(b+a) \), respectively. This means in the deformed state of substrate, the local longitudinal and transverse arc lengths are equal to \( d \) and \( b+a \), respectively, which leads to \( \psi = \kappa d \) and \( \omega = \tau(b+a) \) for the local bending angles. This leads to the local curvature radius in longitudinal direction as \( R_x = \frac{1}{\kappa} = \frac{d}{\psi} \), and the curvature radius in transverse direction as \( R_z = \frac{1}{\tau} = \frac{b+a}{\omega} \), as shown in Figure 6.3.

Because the edge \( A_2D_2 \) is tangent to the substrate surface at the middle point \( M_2 \), the angle of this edge with the direction of \( z \) axis is equal to \( \omega \). According to this, the coordinate of points \( A_2 \) and \( D_2 \) can be found with respect to the point \( M_2 \). Also, the longitudinal symmetry line of 2\(^{nd} \) scale’s plate, \( M_2N_2 \), has an angle equal to \( \theta \) with respect to tangent line of the substrate at point \( M_2 \) in direction of \(-x\) axis. This mean the line \( M_2N_2 \) has an angle equal to \( \theta - \psi \) with respect to the direction of \(-x\) axis. From this, the coordinate of point \( N_2 \) can be found with respect to the point \( M_2 \). Because the line \( A_2D_2 \) is parallel to line \( B_2C_2 \), the locations of point \( B_2 \) and \( C_2 \), can be found with respect to the point \( N_2 \). After finding the location of point \( B_2 \), to satisfy the contact between this corner of 2\(^{nd} \) scale and surface of 1\(^{st} \) scale, the location of point \( B_2 \) must satisfy the \( A_1B_1C_1D_1 \) plate equation. This leads to the following nonlinear kinematic relationship between scale inclination angle \( \theta \), and the local bending angles \( \psi \) and \( \omega \):

\[
\eta \sin \psi - \sin \theta - \cos \theta \left( \frac{\psi}{2} + \left( \frac{\beta + \delta}{2} \right) \omega - \beta \sin \omega \right) = 0.
\]  

(6.6)

In the load case (b), instead of contacting the corner of 2\(^{nd} \) scale with the top surface of 1\(^{st} \) scale, the side edge of 1\(^{st} \) scale \( D_1C_1 \) contacts with the top edge of 2\(^{nd} \) scale \( C_2B_2 \). In this case, as mentioned
earlier the curvature in $z$ direction $\tau$, and the local bending angle $\omega$ are negative ($\tau < 0$ and $\omega < 0$). The procedure for finding coordinate of points $D_1$, $C_1$, $C_2$, and $B_2$ are same as load case (1). After finding coordinate of these point the directional vector of lines $D_1C_1$ and $C_2B_2$ can be calculated, then the 3D-equations of these lines are obtained by using the directional vectors and one points on the each lines. To find the contact between these two lines, we solve their equations together as a system of equations, which yields the following nonlinear kinematic relationship between scale inclination angle $\theta$, and the local bending angles $\psi$ and $\omega$, which has a slight difference with respect to Equation (6.6):

$$\eta \sin \psi - \sin \theta - \cos \theta \left( \frac{\psi}{2} + \frac{\beta + \delta}{2} \omega - \delta \tan \omega \right) = 0. \quad (6.7)$$

The details of mathematical derivation of Equation (6.6) and (6.7) can be found in Supplementary Material of this chapter, sections 6.9.1 and 6.9.2, respectively.

6.5 Mechanics of Scale-covered Plate under Bending

Similar to the pure bending and pure twisting case, adding the rigid scales to the surface of the plate with provide additional stiffness response after scales engagement which can be modeled as rotational springs, here for two different in-plane direction as $K_{\theta,z}$ and $K_{\theta,x}$. Also, here we have the inclusion effect as the additional appreciable stiffness gain due to the scales inclusion into the substrate similar to the twisting case [59, 60]. This effect will model as the inclusion correction factors for two in-plane directions $C_{f,z}$ and $C_{f,x}$. According to these considerations, the moment–curvature response of scale-covered 2D plate for two in-plane directions will model as follows:

$$M_x(\kappa, \tau) = C_{f,x}D_B(\kappa + \nu \tau) + \frac{K_{\theta,x}}{d}(\theta - \theta_0) \frac{\partial \theta}{\partial \kappa} H(\kappa - \kappa_e), \quad (6.8a)$$
\[
M_z(\kappa, \tau) = C_{f,z}D_B(\tau + \nu \kappa) + \frac{K_{\theta,z}}{d}(\theta - \theta_0) \frac{\partial \theta}{\partial \tau} H(\kappa - \kappa_e),
\]
(6.8b)

where \(D_B = \frac{E_B L_B (2t)^3}{12(1-\nu^2)}\), and \(L_B\) is size of substrates width or length, \(L_B = W_{\text{sub}} = L_{\text{sub}}\). Also, \(\kappa_e\) and \(\tau_e\) are the engagement curvatures in \(x\) and \(z\) directions, respectively.

To obtain relationships for rotational springs \(K_{\theta,x}\) and \(K_{\theta,z}\), and inclusion correction factors \(C_{f,x}\) and \(C_{f,z}\), for two in-plane directions, extensive numerical simulations are required or previously developed scaling laws [52, 59] can be utilized, if these models provide sufficient accuracy. The details of developing these scaling laws can be found in the Supplementary Material of chapter 3, sections 3.7.4, 3.7.5, and 3.7.6.

6.6 Finite Element Modeling

We develop a Finite Element (FE) model to compare the numerical results with the results derived from the developed analytical model for the biomimetic scale-covered plate. The FE simulations are carried out using ABAQUS/CAE 2017 (Dassault Systèmes). The 3D deformable solids have been considered for the scales and the substrate. A sufficient length and width is considered for the square plate substrate to satisfy the periodicity and reducing the edge effect. An assembly of a staggered arrangement of 59 scales and the plate substrate has been considered, where the scales are partially embedded on the substrate’s top surface. The scales are oriented with inclination angle of \(\theta_0\) with respect to the substrate’s top surface in the longitudinal direction. The substrate’s is considered as linear elastic material with the elastic modulus \(E_B\) and the Poisson’s ratio \(\nu\).

To model the scales as rigid with respect to the deformable substrate, rigid body constraints are imposed to the scales geometry.

The bending mechanical loads are applied quasi-statically to the system through two static steps. In the results, we need to demonstrate the scales inclination angle \(\theta\) versus the longitudinal bending
angle $\psi$, while the transverse bending angle $\omega$ is fixed at different values, or vice versa. For this purpose, in the first step, the transverse rotational boundary conditions has been applied to the both lateral sides of substrate with reverse directions linearly increasing from zero to a desired value during the step time. Then in the second step, the transverse rotational boundary conditions are kept fixed at the assigned value from the first step, and the longitudinal rotational boundary conditions has been applied to the both front and back sides of substrate with reverse directions linearly increasing from zero. To model contact between scales, a frictionless surface-to-surface contact interaction has been applied to the scales surfaces.

The mesh convergence study is carried out to discover a sufficient mesh size and density for different regions of the model, to obtain a reliable numerical results. This leads to a total number of almost 354,000 elements in the mesh, including linear tetrahedral elements of type C3D4 and linear hexahedral elements of type C3D8. The top layer of substrate is meshed with the tetrahedral elements, because the geometry is complex due to the scales inclusions into the substrate, but the other regions of the model is meshed with the hexahedral elements.

6.7 Results and Discussion

In Figure 6.4, the change in the scales inclination angle $\theta$ from its initial value $\theta_0$ has been shown versus the local longitudinal bending angle $\psi$, for various local transverse bending angles $\omega$. Figure 6.4 (a) is related to the first load case ($\kappa > 0$ and $\tau > 0$), and Figure 6.4 (b) is related to the second load case ($\kappa > 0$ and $\tau < 0$). As it mentioned in section 6.4 and Equations (6.6) and (6.7), there is a slight difference in the kinematic relationships for these two load case, which does not have significant effect on the $\theta$ vs $\psi$ plot. According to this figure by increasing the transverse bending load, the scale engagement ($\theta = \theta_0$) happens earlier, but it does not have significant effect of change rate (slope) of the $\theta$ vs $\psi$. The black dotted lines represent FE results for each $\omega$ curve,
Figure 6.4: (a) Plot of $\theta - \theta_0$ vs $\psi$ of the scale-covered plate for different positive local transverse bending angle $\omega$ (load case 1) with the given values of $\theta_0 = 5^\circ$, $\eta = 3$, $\beta = 1.4$, and $\delta = 0.1$. Black dotted lines represent FE results. (b) Plot of $\theta - \theta_0$ vs $\psi$ of the scale-covered plate for different negative local transverse bending angle $\omega$ (load case 2) with the given values of $\theta_0 = 5^\circ$, $\eta = 3$, $\beta = 1.4$, and $\delta = 0.1$. Black dotted lines represent FE results.

which shows remarkable fit with our model.

In Figure 6.5, the change in the scales inclination angle $\theta$ from its initial value $\theta_0$ has been shown versus the local transverse bending angle $\omega$, for various local longitudinal bending angles $\psi$. Figure 6.5 (a) is related to the first load case ($\kappa > 0$ and $\tau > 0$), and Figure 6.5 (b) is related to the second load case ($\kappa > 0$ and $\tau < 0$). As it mentioned in section 6.4 and Equations (6.6) and (6.7), there is a slight difference in the kinematic relationships for these two load case, which does not have significant effect on the $\theta$ vs $\omega$ plot. According to this figure by increasing the longitudinal bending load, the scale engagement ($\theta = \theta_0$) happens earlier, but it does not have significant effect of change rate (slope) of the $\theta$ vs $\omega$. The black dotted lines represent FE results for each $\psi$ curve, which shows remarkable fit with our model.
Figure 6.5: (a) Plot of $\theta - \theta_0$ vs $\omega$ of the scale-covered plate for different positive local longitudinal bending angle $\psi$ (load case 1) with the given values of $\theta_0 = 5^\circ$, $\eta = 3$, $\beta = 1.4$, and $\delta = 0.1$. Black dotted lines represent FE results. (b) Plot of $\theta - \theta_0$ vs $-\omega$ of the scale-covered plate for different positive local longitudinal bending angle $\psi$ (load case 2) with the given values of $\theta_0 = 5^\circ$, $\eta = 3$, $\beta = 1.4$, and $\delta = 0.1$. Black dotted lines represent FE results.

Figure 6.6 (a) shows the moment–curvature response of the first load case ($\kappa > 0$ and $\tau < 0$) in comparison with the response of counterpart plane plate without scales. Clearly, the stiffness gains due to the inclusions (dashed curves) and emergent stiffness due to scales engagement can be observed. Figure 6.6 (b) displays the moment–curvature response of the second load case ($\kappa > 0$ and $\tau < 0$). In this plot similar to the first load case, inclusion effect and scales engagement have been observed. The results shown in Figure 6.6 are obtained from finite element simulations, and only the results of plain plate have been compared with theoretical results from Equation (6.2), which shows a great agreements.

In Figure 6.7 (a) and (b), the moment–curvature responses are displayed for a particular case with various $\eta$ in $x$ and $z$ directions, which is obtained from finite element analyses. The emergent stiffness gain after scales engagement is clearly observed which is dependent on $\eta$ value, and this
stiffness gain has higher effect in $x$ direction moment. These results are for the first load case the applied moments in both in-plane directions are positive ($\kappa > 0$ and $\tau > 0$).

In Figure 6.8 (a) and (b), the moment–curvature responses are displayed for a particular case with various $\delta$ in $x$ and $z$ directions. Figure 6.8 (a) shows that the emergent stiffness gain after scales engagement in $x$ direction moment is not dependence on $\delta$ value, but Figure 6.8 (b) shows slight dependency on the $\delta$ for the stiffness gain in $z$ direction moment. These results are obtained from finite element simulations for the first load case the applied moments in both in-plane directions are positive ($\kappa > 0$ and $\tau > 0$).

6.8 Conclusion

In this work, we addressed the coupling effects of two different bending in direction of in-plane axes, in a biomimetic scaled elastic plate. Here, the scales are considered rigid rectangular plates, partially embedded with an inclination angle from the surface of the elastic substrate. The scales
Figure 6.7: (a) The moment–curvature response for various \( \eta \) in X direction for first load case, obtained from FE results. (b) The moment–curvature response for various \( \eta \) in Z direction for first load case, obtained from FE results.

are arranged with staggered arrangement in two in-plane directions. We discovered highly intricate effects of scales engagement to the kinematics and mechanics of the system including nonlinear stiffening effect on the bending–curvature response of the system. We quantified these effects by developing analytical relationships within the framework of classical plate theory or the Kirchhoff plate theory. This model is validated with finite element simulations using a commercially available software, also the model results in absence of transverse bending loads shows compatibility with the earlier developed model for pure beam bending.

This study completes the pathway to extend the mechanics of biomimetic scale-covered systems from 1D beams to 2D plates, which is of great importance in real-world practical applications including soft robotics, architectured structural metamaterials, protective armors and wearable, and aerospace structures such as mesh reflector and booms.
Figure 6.8: (a) The moment–curvature response for various $\delta$ in X direction for first load case, obtained from FE results. (b) The moment–curvature response for various $\delta$ in Z direction for first load case, obtained from FE results.

6.9 Supplementary Material

6.9.1 Kinematic Derivation: Both Curvatures Upward

As shown in Figure 6.3 (a), the coordinate of points on the 1st scale can be obtained according to coordinates $xyz$, which is located on the midpoint of 1st scale’s width $M_1$. The coordinate of these points are as follows:

\[
M_1 = (0,0,0) \quad , \quad N_1 = (-l \cos \theta, l \sin \theta, 0), \nonumber \\
A_1 = (0,0,+b) \quad , \quad B_1 = (-l \cos \theta, l \sin \theta, +b), \quad (6.9) \\
D_1 = (0,0,-b) \quad , \quad C_1 = (-l \cos \theta, l \sin \theta, -b). 
\]
Similar to the 1st scale, the locations of points of the 2nd scale before the deformation of substrate can be obtained as follows:

\[ M_{2,0} = (d, 0, -(b+a)) \, , \, N_{2,0} = (d - l \cos \theta, l \sin \theta, -(b+a)), \]
\[ A_{2,0} = (d, 0, -a) \, , \, B_{2,0} = (d - l \cos \theta, l \sin \theta, -a), \]
\[ D_{2,0} = (d, 0, -(2b+a)) \, , \, C_{2,0} = (d - l \cos \theta, l \sin \theta, -(2b+a)). \]  

(6.10)

According to the section 6.3, and Equation (6.5), the equation of deformed substrate under longitudinal bending curvature \( \kappa = \frac{\psi}{d} \) and transverse bending curvature \( \tau = \frac{\omega}{b+a} \) is as follows:

\[ y = \frac{1}{2} \left( \frac{\psi}{d} x^2 + \left( \frac{\omega}{b+a} \right) z^2 \right). \]  

(6.11)

Based on the Equation (6.3), the location of the midpoint of 2nd scale’s width \( M_2 \) after the deformation of substrate is as follows:

\[ M_2 = \left( d, \frac{\psi d + \omega (b+a)}{2}, -(b+a) \right). \]  

(6.12)

The longitudinal symmetry line of 2nd scale’s plate, \( M_2N_2 \), has an angle equal to \( \theta \) with respect to tangent line of the substrate at point \( M_2 \) in direction of \(-x\) axis. This mean the vector \( M_2N_2 \) with the length \( l \) has an angle equal to \( \theta - \psi \) with respect to the direction of \(-x\) axis, which leads to:

\[ M_2N_2 = \left( -l \cos(\theta - \psi), l \sin(\theta - \psi), 0 \right). \]  

(6.13)

From Equation (6.12) and (6.13), the coordinate of point \( N_2 \) can be found as:

\[ N_2 = \left( d - l \cos(\theta - \psi), \frac{\psi d + \omega (b+a)}{2} + l \sin(\theta - \psi), -(b+a) \right). \]  

(6.14)
Because the edge $A_2D_2$ is tangent to the substrate surface at the middle point $M_2$, the angle of this edge with the direction of $z$ axis is equal to $\omega$. Also, the line $A_2D_2$ is parallel to line $B_2C_2$. Based on this, the following vectors can be found as follows:

$$\vec{A_2M_2} = \vec{M_2D_2} = \vec{B_2N_2} = \vec{N_2C_2} = (0, b \sin \omega, -b \cos \omega).$$ (6.15)

Based on the Equation (6.12) and (6.15), the coordinate of points $A_2$ and $D_2$ are as follows:

$$A_2 = \left( d, \frac{\psi d + \omega(b+b)}{2} - b \sin \omega, -(b+a) + b \cos \omega \right),$$
$$D_2 = \left( d, \frac{\psi d + \omega(b+b)}{2} + b \sin \omega, -(b+a) - b \cos \omega \right).$$ (6.16)

Also, according to the Equation (6.14) and (6.15), the locations of point $B_2$ and $C_2$ are as follows:

$$B_2 = \left( d - l \cos(\theta - \psi), \frac{\psi d + \omega(b+b)}{2} - b \sin \omega + l \sin(\theta - \psi), -(b+a) + b \cos \omega \right),$$
$$C_2 = \left( d - l \cos(\theta - \psi), \frac{\psi d + \omega(b+b)}{2} + b \sin \omega + l \sin(\theta - \psi), -(b+a) - b \cos \omega \right).$$ (6.17)

To find the equation of plate $A_1B_1C_1D_1$, the plate normal vector $\vec{n}_1 = (x_{n_1}, y_{n_1}, z_{n_1})$ is equal to:

$$\vec{n}_1 = \frac{\vec{M_1D_1} \times \vec{M_1N_1}}{|\vec{M_1D_1}||\vec{M_1N_1}|} = (\sin \theta, \cos \theta, 0)$$ (6.18)

The general form of plate equation with normal vector $\vec{n}_1$ and point $M_1$ located in the plate is described as $x_{n_1}(x - x_{M_1}) + y_{n_1}(y - y_{M_1}) + z_{n_1}(z - z_{M_1}) = 0$. By using the plate normal vector from Equation (6.18) and coordinates of point $M_1$, equation of plate $A_1B_1C_1D_1$ is as follows:

$$x \sin \theta + y \cos \theta = 0$$ (6.19)
To find the contact between the corner of 2nd scale \( B_2 \) and surface of 1st scale, the location of point \( B_2 \) shown in Equation (6.17), must satisfy the \( A_1B_1C_1D_1 \) plate equation, which yields:

\[
(d - l \cos(\theta - \psi)) \sin \theta + \left( \frac{\psi d + \omega(b + a)}{2} - b \sin \omega + l \sin(\theta - \psi) \right) \cos \theta = 0 \quad (6.20)
\]

By nondimensionalization the geometrical parameters with respect to the longitudinal spacing between the scale \( d \), we define dimensionless geometric parameters including \( \eta = l/d, \beta = b/d, \) and \( \delta = a/d. \) Using these dimensionless parameters, the Equation (6.20) is rewritten as:

\[
\eta \sin \psi - \sin \theta - \cos \theta \left( \frac{\psi}{2} + \left( \frac{\beta + \delta}{2} \right) \omega - \beta \sin \omega \right) = 0. \quad (6.21)
\]

6.9.2 Kinematic Derivation: Longitudinal Curvature Upward and Transverse Curvature Downward

In this section we the locations of points \( C_1, D_1, B_2, \) and \( C_2, \) which is provided in Equation (6.9) and Equation (6.17) to find the equation of lines \( D_1C_1 \) and \( B_2C_2. \) For this purpose, the directional vector of line \( D_1C_1 \) is called \( m_1 = (x_{m_1}, y_{m_1}, z_{m_1}), \) and the directional vector of line \( B_2C_2 \) is called \( m_2 = (x_{m_2}, y_{m_2}, z_{m_2}). \) These vectors are as follows:

\[
m_1 = \frac{D_1C_1}{|D_1C_1|} = (-\cos \theta, \sin \theta, 0), \quad (6.22)
\]

\[
m_2 = \frac{B_2C_2}{|B_2C_2|} = (0, \sin \omega, -\cos \omega). \quad (6.23)
\]

By using the directional vectors and one point located on each line, the general 3D-equation of the lines \( D_1C_1 \) and \( B_2C_2 \) are as follows:
The parametric form of the equation of line $D_1C_1$ as follows, where $p$ can vary from 0 to $l$:

\begin{align*}
  x(p) &= -p \cos \theta, \\
  y(p) &= p \sin \theta, \\
  z(p) &= -b. 
\end{align*}

The parametric form of the equation of line $B_2C_2$ as follows, where $q$ can vary from $-2b$ to 0:

\begin{align*}
  x(q) &= d - l \cos(\theta - \psi), \\
  y(q) &= q \sin \omega + \frac{\psi d + \omega(b + a)}{2} + b \sin \omega + l \sin(\theta - \psi), \\
  z(q) &= -q \cos(\omega - (b + a)) - b \cos \omega. 
\end{align*}

To find a contact point between these two lines, Equations (6.26) and (6.27) must be identical at $x$, $y$ and $z$ coordinate simultaneously, which means $x(p) = x(q)$, $y(p) = y(q)$, and $z(p) = z(q)$. From $x(p) = x(q)$, we find a relationship for $p$, and from $z(p) = z(q)$, we obtain a relationship for $q$. By substituting the obtained relationships for $p$ and $q$ into $y(p) = y(q)$, we find the following equation:

\begin{equation}
  \left(\frac{l \cos(\theta - \psi) - d}{\cos \theta}\right) \sin \theta = \left(\frac{a + b \cos \omega}{-\cos \omega}\right) \sin \omega + \frac{\psi d + \omega(b + a)}{2} + b \sin \omega + l \sin(\theta - \psi).
\end{equation}
By nondimensionalization the geometrical parameters with respect to \( d \), the Equation (6.28) can be rewritten as:

\[
\eta \sin \psi - \sin \theta - \cos \theta \left( \frac{\psi}{2} + \left( \frac{\beta + \delta}{2} \right) \omega - \delta \tan \omega \right) = 0.
\]  (6.29)

This relationship is slightly different than Equation (6.21).
CHAPTER 7: CONCLUSION

7.1 Summary of Conclusions

The main objective of this work was to investigate the mechanics of biomimetic scale-covered substrates. The work focused on discovering kinematic and mechanic relationships between geometry, architecture and materials. These biomimetic metamaterials which are inspired from fish scale-like structures, have potential applications in designing lightweight structures to tailor the mechanical response for various fields including protective applications like armors and wearables, soft robotics arms, high performance structures, and aerospace structures such as mesh reflector and booms. A summary with important findings and conclusions are highlighted below.

- In Chapter 3, our work showed the geometrical tailorability of elastic response under twisting loads including stiffness, envelopes of operations and the overall energy landscape. We find that stiffness increase brought about by scales is highly nonlinear, reversible, and tailorable, distinct from simply coating or embedding with a stiffer material or a composite.

- This system exhibits a very specific nonlinear behavior including a seamless straddling between linear elastic, nonlinear elastic and a quasi-rigid behavior which exhibited by neither the parent materials. Each of these regimes can be tailored using different geometrical arrangements of the structure.

- The developed analytical models are aimed primarily to obviate the need for detailed fully resolved FE simulations for design and analysis. These FE simulations are prohibitive for larger number of scale contacts, larger bending and twisting, or for future work on dynamics, which would require repeated FE simulations on the structure.
• In Chapter 4, we investigate for the first time, the effect of Coulomb friction on the twisting response of a biomimetic beam using a combination of analytical model and FE simulations. We established the extent and limits of universality of frictional behavior across bending and twisting regimes, to provide tailorable behavior, unprecedented for conventional substrates.

• We find that several aspects of the mechanical behavior of frictional effects under twisting show similarity to the bending case investigated earlier. At the same time, critical differences in response were observed, most notably the effect of the additional oblique angle $\alpha$.

• This work shows the dual contribution of frictional forces on the biomimetic scale-covered system, which includes advancing the locking envelope and at the same time increasing the torsional stiffness. Also, if the coefficient of friction is large enough, it can lead to the instantaneous post-engagement frictional locking known as the static friction locking.

• This investigation demonstrates that engineering of the scale’s surfaces, which can produce wide range of coefficients of friction, plays an important role in tailoring the deformation response of biomimetic scale-covered systems. This shows the possibility of using surface roughness to tailor stiffness and dissipation behavior under twisting deformation of biomimetic scale-covered substrates.

• In Chapter 5, we addressed the cross coupling effects of bending and twisting in a biomimetic scale elastic beam for the first time. We find highly intricate and often surprising effect of one over the other across the kinematics and mechanics.

• We quantified the bend-twist coupling effects by developing analytical relationships within the framework of Cosserat kinematics and global-local energy balance. This model reduced to the earlier developed model for pure bending and pure twisting. This study completes a significant missing piece in the mechanics of biomimetic scale elastica,
• In Chapter 6, we addressed the coupling effects of two different bending in direction of in-plane axes, in a biomimetic scaled elastic plate. Here, the scales are considered rigid rectangular plates, partially embedded with an inclination angle from the surface of the elastic substrate and the scales are arranged with staggered arrangement in two in-plane directions.

• We discovered highly intricate effects of scales engagement to the kinematics and mechanics of the system including nonlinear stiffening effect on the bending–curvature response of the system. We quantified these effects by developing analytical relationships within the framework of classical plate theory or the Kirchhoff plate theory, which is validated with finite element simulations. Also the model results in absence of transverse bending loads shows compatibility with the earlier developed model for pure beam bending.

• This study completes the pathway to extend the mechanics of biomimetic scale-covered systems from 1D beams to 2D plates, which is of great importance in real-world practical applications including soft robotics, architectured structural metamaterials, protective armors and wearables, and aerospace structures.

7.2 Perspective and Future Work

In spite of intense scrutiny for scale-covered system’s mechanical behavior, even now relatively little is known about the mechanics and dynamics of more general 2D systems such as plate and shell type metamaterials, heterogeneous scales distribution or the synergetic behavior under combined loads. Such complexities can give rise to new and potentially unanticipated emergent behaviors. For these developments, several existing challenges need addressing. These metamaterials often test the limits of commercial FE software due to the large number of contact pairs and significant contact nonlinearity. Thus, major advances in computer models and multiscale modeling are necessary. The scale separation in this case is between scales length and structure length scale (e.g.
m to cm). The time scales of structure and scales although assumed similar so far can also differ in case of high frequency or sharp local transients. Thus, the models can be applied in mesoscale length scale for few neighboring RVEs and in macroscale for the whole structure.

In addition to current paradigms, potentially novel behaviors can be accessed by using phase change materials in biomimetic scales, which can provide unprecedented on-demand functional programmability. Currently, very little is known of the time dependent behavior of such dynamically changing scales systems. For these structures, fabrication techniques need to advance considerably. Lastly, natural scales are inherently multifunctional. Multiphysics interactions such as coupling between fluids, heat and the nonlinear structural response still unexplored. Preliminary work on exploiting optical properties [108] show dramatic color changes through angular manipulations of scales. Studies on thermal or electromagnetic behavior have not been undertaken yet, representing a key future frontier.
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[79] R. Martini, F. Barthelat, Stability of hard plates on soft substrates and application to the
design of bioinspired segmented armor, Journal of the Mechanics and Physics of Solids 92

[80] E. H. Colbert, et al., Evolution of the vertebrates. a history of the backboned animals through
time., Evolution of the vertebrates. A history of the backboned animals through time.

steady swimming of longnose gar, lepisosteus osseus, Journal of Experimental Biology


[85] Z. Sun, T. Liao, W. Li, Y. Dou, K. Liu, L. Jiang, S.-W. Kim, J. H. Kim, S. X. Dou, Fish-
scale bio-inspired multifunctional zno nanostructures, NPG Asia Materials 7 (12) (2015)
e232–e232.

microstructural studies of the armor of the marine threespine stickleback (gasterosteus ac-

[87] T. Ikoma, H. Kobayashi, J. Tanaka, D. Walsh, S. Mann, Microstructure, mechanical, and
biomimetic properties of fish scales from pagrus major, Journal of structural biology 142 (3)


[160] H. Onozato, N. Watabe, Studies on fish scale formation and resorption, Cell and tissue research 201 (3) (1979) 409–422.


