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Increase Spatial Sampling For Wave Front Mid-Spatial Frequency Error Recovery

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A differential curvature sensing device for measuring a wavefront curvature by employing increased spatial sampling for wavefront testing with mid-frequency error recovery. The device includes a sampling sensor having an output beam, an optical element to split said output beam, a lenslet array in the path of each beam to generate corresponding sampling grids, a shearing element for shifting the grid points in horizontal and vertical directions to produce plural sampling grids having plural grid points for use generating a spatial sampling grid having a density for mid-spatial frequency recovery. The displacement of the shifting less than a pitch size of the lenslet array, and a measuring device measuring plural slopes of plural wavefronts at each grid point to obtain a wavefront normal curvature and corresponding twist curvature terms to determine a principal curvature and directions. The sensor is a Shack-Hartman sensor, shearing interferometer sensor and other discrete-point sampling sensors.
Fig. 1 Prior Art

Fig. 2 Prior Art
Fig. 3

Fig. 4a
FIG. 4b

FIG. 4c
**Fig. 5**

Lenslet A: \((x^{(0)}, y^{(0)})_x\) → Lenslet B: \((x^{(0)}, y^{(0)})_y\)

\[\Delta x \rightarrow C_{0,x}\]

Lenslet A conjugate: \((x^{(0)}, y^{(0)})\)

**Fig. 7a**

**Fig. 7b**
INCREASE SPATIAL SAMPLING FOR WAVE FRON T MID-SPATIAL FREQUENCY ERROR RECOVERY

This application is a continuation-in-part of U.S. patent Ser. No. 11/414,586 filed on Apr. 28, 2006 and claims the benefit of priority to U.S. Provisional Patent Application No. 60/692,884 filed on Jun. 22, 2005.

FIELD OF THE INVENTION

This invention relates to sensors and, in particular, to methods, systems, apparatus and devices for a differential Shack-Hartmann curvature sensor to measure the differentials of wave front slope, i.e., wave front curvatures, to determine the wave front shape and for wave front testing with mid-spatial frequency error using increased spatial sampling for wave front mid-spatial frequency error recovery.

BACKGROUND AND PRIOR ART

The Shack-Hartmann sensor is one of the most popular wavefront sensors presently available. The sensor measures the slope data of the wavefront by comparing the coordinates of Hartmann grid points from the measurement beam with coordinates from the reference beam. FIG. 1 is a schematic layout of a prior art Shack-Hartmann sensor. Hartmann invented the Hartmann test in 1900, and Roland Shack improved this technique by introducing a lenslet array in 1971. The reference beam was generated by a pin-hole light source and injected into this system as ideal wavefront to calibrate the systematic error. In FIG. 1, reference beam 105 is represented by a dashed line, and the continuous line represents the measurement beam 120. The wavefront is first collimated by the collimation lens then passes through a lenslet array 110. The Hartmann screen divides the wavefront into many sub-apertures, the micro-lens array 110 focuses the wavefront of each sub-aperture into a group of Hartmann grid points. Comparing the coordinate differences between the measured wavefront and the reference wavefront, the wavefront slopes can be calculated according to the following formula:

\[
\frac{\partial W}{\partial x} = \frac{x_{mea} - x_{ref}}{f} \\
\frac{\partial W}{\partial y} = \frac{y_{mea} - y_{ref}}{f}
\]

where \((x_{mea}, y_{mea})\) is the Hartmann grid coordinates of the reference beam, \((x_{mea}, y_{mea})\) is the Hartmann grid coordinates of the measurement beam, and \(f\) is the focal length of the lenslet array.

Compared to the Hartmann test, the Shack-Hartmann wavefront sensor provides improved photon efficiency because the position of the focal spot is proportional to the average wavefront slope over each sub-aperture and the position is independent of higher-order aberrations and intensity profile variations. The Shack-Hartmann sensor is a parallel wavefront sensor operating in real time. It has application in active/adaptive optics, optical testing, ophthalmology, telescope image analysis and atmosphere and random media characterizations.

However, the usage of the Shack-Hartmann sensor is restrictive because it is sensitive to vibration, tilt and whole body movement. Therefore, a need exists for a sensor that provides the benefits of the Shack-Hartmann sensor without the limitations.

SUMMARY OF THE INVENTION

A primary objective of the invention is to provide a new method, system, apparatus and device for measuring differentials of the wavefront slopes to determine the wavefront curvature without the use of an external reference or reference light source after calibration.

A secondary objective of the invention is to provide a new method, system, apparatus and device measuring differentials of the wavefront slopes to determine the wavefront curvature independent of vibration, tilt or whole body movement so it is useful for measurements on a moving stage.

A third objective of the invention is to provide a new method, system, apparatus and device for a differential curvature sensor that is scale tunable by varying differential values.

A fourth objective of the invention is to provide a new method, system, apparatus and device for a differential curvature sensor that can be applied in optical testing with vibration.

A fifth objective of the invention is to provide a new method, system, apparatus and device for a differential curvature sensor without moving parts for increased reliability.

A sixth objective of the invention is to provide a new method, system, apparatus and device for a differential curvature sensor for use in optical testing, active and adaptive optics, shape-extraction in bio-optics and ophthalmology, such as corneal measurement.

A seventh objective of the invention is to provide a new method, system, apparatus and device for a differential curvature sensor for measuring the normal curvatures and the twist curvature terms.

An eighth objective of the invention is to provide a new method, system, apparatus and device for a differential curvature sensor for measuring the wavefront principal curvatures and directions.

A ninth objective of the invention is to provide a new method, system and apparatus for increased spatial sampling for mid-spatial frequency error recovery.

A first preferred embodiment of the invention provides a differential curvature sensing device for measuring a wavefront curvature using increased spatial sampling for wavefront testing with mid-frequency error recovery. The device includes a sampling sensor having an output beam, an optical element to split said output beam into a first, second and third beam, each of said first, second and third beams traveling in one of an x, y and z direction, respectively, and a first, second and third lenslet array in the path of the first, second and third beam, respectively, to generate a corresponding first, second and third sampling grid each having plural grid points. A shearing element for shifting said plural grid points a distance in a horizontal and vertical direction to produce plural sampling grids each having plural grid points for generating a spatial sampling grid having a density for mid-spatial frequency recovery. The displacement of the shifting is less than a pitch size of the lenslet array, and a measuring device for measuring plural slopes of plural wave fronts at each grid point of each spatial sampling grid to obtain a wavefront normal curvature and corresponding twist curvature terms for determining the principal curvatures and directions, wherein said plural slopes at each of the plural grid points are measured approximately simultaneously before and after differentially shifting the sampling grid. The sampling sensor is
selected from a group including a Shack-Hartman sensor, a curvature sensor, a shearing interferometer sensor and a 4-dimensional technique.

A second preferred embodiment provides a system for measuring differentials of wave front slopes to determine a wavefront curvature without the use of a reference. The system includes a light source for generating an input light beam, a beam splitter splitting the input light beam to produce at least three output beams in different directions, at least three lenslet arrays, one in a path of each of the at least three output beams for generating a corresponding at least three sampling grids, at least one optical element for shearing two of the at least three output beams in a horizontal direction and a vertical direction.

A third preferred embodiment provides a method for measuring a normal curvature of a wavefront with mid-spatial frequency error detection with a differential curvature sensor, including the steps of providing a sensing system having a first, second and third output beam in different directions for generating an original sampling grid having plural grid points, measuring differentials of the plural wave front slopes at plural grid points of the original sampling grid, moving the sampling grid in a horizontal direction with a displacement less than a pitch size of the lenslet compared to said original sampling grid, measuring differentials of the plural wavefront slopes at plural grid points of the horizontally displaced grid to produce a second sampling grid, moving the original grid in a vertical direction with a displacement less than a pitch size of the lenslet compared to said original sampling grid to produce a third sampling grid, measuring differentials of the plural wave front slopes at plural grid points of the third sampling grid, the wavefront local curvature is determined, from which the principal curvatures and directions of the wavefront are computed.

Further objects and advantages of this invention will be apparent from the following detailed description of preferred embodiments, which are illustrated schematically in the accompanying drawings.

BRIEF DESCRIPTION OF THE FIGURES

FIG. 1 shows a prior art Hartmann screen and lenslet array and their coordinate differences.

FIG. 2 shows an example of a prior art Hartmann grid.

FIG. 3 shows a Shack-Hartmann grid with displacements in the x and the y directions.

FIG. 4a shows a direct implementation layout of the differential Shack-Hartmann curvature sensor according to an embodiment of the present invention.

FIG. 4b shows another direct implementation layout of the differential Shack-Hartmann curvature sensor according to another embodiment of the present invention.

FIG. 4c shows yet another direct implementation of the differential Shack-Hartmann curvature sensor according to another embodiment of the present invention.

FIG. 5 shows a calibration block diagram according to the present invention.

FIG. 6 shows an experimental apparatus for the differential Shack-Hartmann curvature sensor according to the first embodiment of the present invention.

FIG. 7a shows the image space of Roddier’s curvature sensing technique of the prior art.

FIG. 7b shows the object space of Roddier’s curvature sensing corresponding to the image space of FIG. 7a.

FIG. 8 shows a sampling grid from a Differential Shack-Hartmann sensor at (0, 0).

FIG. 9 shows the sampling grid of FIG. 8 with the grid shifted to by 0.6 along the x-axis to (0, 0.6).

FIG. 10 shows the sampling grid of FIG. 9 with the grid shifted to by 1.2 along the x-axis to (1.2, 0).

FIG. 11 shows the sampling grid of FIG. 10 with the grid shifted by -0.6 along the y-axis to (0, -0.6).

FIG. 12 shows the sampling grid of FIG. 11 with the grid shifted by 0.6 along the x-axis to (0.6, -0.6).

FIG. 13 shows the sampling grid of FIG. 12 with the grid shifted by 1.2 along the x-axis to (1.2, -0.6).

FIG. 14 shows the sampling grid of FIG. 13 with the grid shifted by -0.6 along the y-axis to (0, -1.2).

FIG. 15 shows the sampling grid of FIG. 14 with the grid shifted by 0.6 along the x-axis to (0.6, -1.2).

FIG. 16 shows the sampling grid of FIG. 15 with the grid shifted by 1.2 along the x-axis to (1.2, -1.2).

DESCRIPTION OF THE PREFERRED EMBODIMENTS

Before explaining the disclosed embodiments of the present invention in details it is to be understood that the invention is not limited in its application to the details of the particular arrangements shown since the invention is capable of other embodiments. Also, the terminology used herein is for the purpose of description and not of limitation.

The following is a list of the reference numbers used in the drawings and the detailed specification to identify components:

<table>
<thead>
<tr>
<th>Reference Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>Shack-Hartmann sensor</td>
</tr>
<tr>
<td>105</td>
<td>reference beam</td>
</tr>
<tr>
<td>110</td>
<td>Hartmann screen and lenslet array</td>
</tr>
<tr>
<td>120</td>
<td>measurement beam</td>
</tr>
<tr>
<td>200</td>
<td>Hartmann grid</td>
</tr>
<tr>
<td>310</td>
<td>Hartmann grid</td>
</tr>
<tr>
<td>320</td>
<td>horizontal displacement (S_x)</td>
</tr>
<tr>
<td>330</td>
<td>vertical displacement (S_y)</td>
</tr>
<tr>
<td>400</td>
<td>differential Shack-Hartmann</td>
</tr>
<tr>
<td>402</td>
<td>curvature sensor</td>
</tr>
<tr>
<td>404</td>
<td>first lenslet array</td>
</tr>
<tr>
<td>406</td>
<td>second lenslet array</td>
</tr>
<tr>
<td>408</td>
<td>third lenslet array</td>
</tr>
<tr>
<td>410</td>
<td>beam splitters</td>
</tr>
<tr>
<td>412</td>
<td>first camera</td>
</tr>
<tr>
<td>414</td>
<td>second camera</td>
</tr>
<tr>
<td>416</td>
<td>third camera</td>
</tr>
<tr>
<td>417</td>
<td>lenslet shearing device (x)</td>
</tr>
<tr>
<td>418</td>
<td>lenslet shearing device (y)</td>
</tr>
<tr>
<td>610</td>
<td>first light source</td>
</tr>
<tr>
<td>620</td>
<td>second light source</td>
</tr>
</tbody>
</table>

The method, system, apparatus and device of the present invention provides a Shack-Hartmann curvature sensor that shears the Shack-Hartmann grid, horizontally and vertically, as shown in FIG. 3 to measure the wavefront curvatures at each Hartmann grid point. The curvature is the derivative of slopes. The differential Shack-Hartmann curvature sensor of the present invention is based on the prior art Shack-Hartmann sensor and the Hartmann grid shown in FIGS. 1 and 2, respectively. By measuring the differentials of the wavefront slopes at each Hartmann grid point, the wavefront local curvatures (normal curvatures and the twist term curvature) are determined, from which the principal curvatures and directions of the wavefront are computed.
A. Normal Curvature Measurements

Differential displacements are made in the x and y directions to obtain slope differentials of the wavefront, the wavefront curvatures. The curvature sensor of the present invention is implemented as a Shack-Hartmann sensor with three output channels for achieving the Hartmann grid shearing in two perpendicular directions. The slope is measured in the three channels as well as the curvature of the wavefront at each Hartmann grid point.

The Hartmann grid is shifted a lateral differential distance in the x and in the y direction, which are suggested in this invention to adopt 1/10 to 1/2 of the pitch size of the Hartmann grid according to each specific application. As shown in FIG. 3, for example, the slopes at each point are measured before and after the differential shift to obtain the slope differentials, the curvature in the x direction and in the y direction can be obtained by applying

\[ c_{0x}(i) = \frac{\partial^2 W}{\partial x^2} \bigg|_{x_i} = \frac{1}{f} \left( \frac{\partial^2 W}{\partial x \partial y} \right) \bigg|_{x_i} = \frac{1}{f} \left( \frac{W_{i+1} - W_{i-1}}{2} \right) - c_{0y}(i) \]  

\[ c_{0y}(i) = \frac{\partial^2 W}{\partial y^2} \bigg|_{y_i} = \frac{1}{f} \left( \frac{\partial^2 W}{\partial x \partial y} \right) \bigg|_{y_i} = \frac{1}{f} \left( \frac{W_{i+1} - W_{i-1}}{2} \right) - c_{0x}(i) \]

where \( c_{0x}(i) \) and \( c_{0y}(i) \) are obtained by

\[ c_{0x}(i) = \frac{1}{f} \left( \frac{\partial^2 W_{i+1} - \partial^2 W_{i-1}}{2} \right) \]

\[ c_{0y}(i) = \frac{1}{f} \left( \frac{\partial^2 W_{i+1} - \partial^2 W_{i-1}}{2} \right) \]

and in this paper \( S_x \) and \( S_y \) are the differential shifts in the x- and y-directions, respectively. Theoretically, \( c_{0x}(i) \) and \( c_{0y}(i) \) are \( "1/f" \), because the lenslet array is moved a lateral distance \( S_x \) in x-direction, for example, the reference Shack-Hartmann grid moves exactly the same distance \( s_y \) accordingly, therefore, \( \chi^{ref}=\chi^{ref}=s_x \) and \( c_{0x}(i)-1/f \); similarly, \( c_{0y}(i)-1/f \). However, in practice \( c_{0x}(i) \) and \( c_{0y}(i) \) are not \( "1/f" \), because the coordinates are measured by different CCD cameras, so they belong to different coordinate system and therefore \( c_{0x}(i) \) and \( c_{0y}(i) \) are obtained by calibration.

The slope differentials in the cross directions, referred to as the corresponding twist curvatures, are given by

\[ c_{0xy}(i) = \frac{\partial^2 W}{\partial x \partial y} \bigg|_{x_i,y_i} = \frac{1}{f} \left( \frac{\partial^2 W}{\partial x \partial y} \right) \bigg|_{x_i,y_i} = \frac{1}{f} \left( \frac{W_{i+1} - W_{i-1}}{2} \right) - c_{0yx}(i) \]

\[ c_{0yx}(i) = \frac{\partial^2 W}{\partial y \partial x} \bigg|_{x_i,y_i} = \frac{1}{f} \left( \frac{\partial^2 W}{\partial x \partial y} \right) \bigg|_{x_i,y_i} = \frac{1}{f} \left( \frac{W_{i+1} - W_{i-1}}{2} \right) - c_{0xy}(i) \]

where \( c_{0xy}(i) \) and \( c_{0yx}(i) \) are constants given by

\[ c_{0xy}(i) = \frac{1}{f} \left( \frac{\partial^2 W_{i+1} - \partial^2 W_{i-1}}{2} \right) \]

\[ c_{0yx}(i) = \frac{1}{f} \left( \frac{\partial^2 W_{i+1} - \partial^2 W_{i-1}}{2} \right) \]

B. Principal Curvature Computations

The normal curvature is the change of the surface normal in an osculating plane, and the principal curvatures of a non-umbilical point are the maximum and minimum values of normal curvatures, say \( k_1 \) and \( k_2 \), in two perpendicular directions. Regarding a local patch of surface, the principal curvatures are invariants, which are insensitive to the surface orientation. In order to evaluate the principal curvatures, it is assumed that the neighborhood of a Hartmann grid point is represented by a “Monge patch” of the form:

\[ X=x_0 + x_1 y_1 + x_2 y_2 + x_3 y_1 y_2, \]
where \((\vec{e}_1, \vec{e}_2, \vec{e}_3)\) is an orthogonal frame in Euclidean 3-space. Then the second fundamental form has a matrix to describe the local surface shape as

\[
[[w]]=\begin{pmatrix}
\frac{\partial^2 w}{\partial x^2} & \frac{\partial^2 w}{\partial x \partial y} \\
\frac{\partial^2 w}{\partial x \partial y} & \frac{\partial^2 w}{\partial y^2}
\end{pmatrix}
\]

and

\[
[[c]]=\begin{pmatrix}
c_{xx}(i) & c_{xy}(i) \\
c_{xy}(i) & c_{yy}(i)
\end{pmatrix},
\]

where \(c_{xx}(i)\) and \(c_{yy}(i)\) are the wave front normal curvatures in the \(x\)-direction and in the \(y\)-direction, i.e.

\[
c_{xx}(i)=\frac{\partial^2 w}{\partial x^2}(i)
\]

and

\[
c_{yy}(i)=\frac{\partial^2 w}{\partial y^2}(i);
\]

off-diagonal terms \(c_{xy}(i)\) and \(c_{yx}(i)\) are the corresponding twist curvature terms, i.e.

\[
c_{xy}(i)=\frac{\partial^2 w}{\partial x \partial y}(i)
\]

and

\[
c_{yx}(i)=\frac{\partial^2 w}{\partial y \partial x}(i).
\]

It is assumed that \(c_{xy}(i)=-c_{yx}(i)\).

The determinant of matrix \([[w]]\), denoted as \(K\), is known as the Gaussian curvature. The trace of the matrix \([[w]]\), denoted as \(2H\), is known as the mean curvature. Both Gaussian curvature and mean curvature are algebraic invariants, which do not change with rotation of the orthogonal frame \((\vec{e}_1, \vec{e}_2, \vec{e}_3)\) about the normal.

By diagonalizing the matrix \([[w]]\) to rotate the orthogonal frame about \(\vec{e}_3\), the off-diagonal terms disappear, and a new matrix \([[II']]\) is obtained by

\[
[[II']]=[[P]][[w]][[P]^T],
\]

where \(P\) is an orthogonal matrix defined by

\[
P=\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\]

where angle \(\theta\) is defined as the frame rotation angle. The new matrix \([[II']]\) is a diagonal matrix, which is

\[
[[II']]=\begin{pmatrix}
k_1(i) & 0 \\
0 & k_2(i)
\end{pmatrix},
\]

where \(k_1(i)\) and \(k_2(i)\) \((k_1(i)>k_2(i))\) are the eigenvalues of the matrix \([[w]]\), also known as the first and second principal curvatures that define the maximum and minimum normal curvatures at a given point patch, and \(i=1, 2, \ldots, m\), where \(m\) is the total number of Shack-Hartmann grid points. Plug the Equations (8), (10) and (11) into Equation (9), to obtain the principal curvatures \(k_1\) and \(k_2\) at each grid point as

\[
k_1(i)=\frac{c_{xx}(i)+c_{yy}(i)-\sqrt{(c_{xx}(i)-c_{yy}(i))^2+4c_{xy}(i)^2}}{2},
\]

and the rotation angle \(\theta\) is the angle between the first principal curvature and the \(x\)-direction, which can be obtained by

\[
\theta(i)=\frac{1}{2}\tan^{-1}\left(\frac{-c_{xy}(i)}{c_{xx}(i)-c_{yy}(i)}\right),
\]

The principal curvatures can also be computed by evaluating the eigenvalues of matrix \([[w]]\) with its characteristic equation as

\[
det(\lambda I-[[w]])=0,
\]

the result is the same as Equation (12). The rotation angle \(\theta\) can also be computed with Euler’s formula (1760) by

\[
\cos 2[\theta(i)]=\frac{2c_{xy}(i)-2H(i)}{k_1(i)-k_2(i)},
\]

where \(H\) is the mean curvature given by

\[
H(i)=(c_{xx}(i)+c_{yy}(i))/2,
\]

Then the angle \(\theta\) is given by

\[
\theta(i)=\frac{1}{2}\cos^{-1}\left(\frac{c_{xx}(i)-c_{yy}(i)}{k_1(i)-k_2(i)}\right),
\]

Apply Equation (12) into Equation (17), to obtain

\[
\theta(i)=\frac{1}{2}\cos^{-1}\left(\frac{|c_{xy}(i)-c_{yx}(i)|}{\sqrt{(c_{xx}(i)-c_{yy}(i))^2+4c_{xy}(i)^2}}\right),
\]

which is equivalent to Equation (13).
C. Comparisons with the Previous Arts

With the system calibrated, the discrepancies between the image de-magnifying systems are cancelled, the discrepancies between the two arms in the cube beam splitter are cancelled, and the aberrations in the collimator and the cube prisms are also cancelled. Besides the error from the ideal flat mirror used for introducing the reference beam, which can be very small, and the nominal errors in applying the shearing the focus of a lens. The idea of the Roddier’s curvature sensor makes it more reliable.

As shown in the configuration in FIG. 6, a point light source 610 is used to generate the beam for measurement and the point light source 620 is used to generate the beam for calibration. After calibration, no reference light beam is necessary.

The following description compares the differential Shack-Hartmann curvature sensor with a prior art curvature sensor. In 1988, Francois Roddier proposed a method to measure the local curvature of the wavefront surface by measuring the difference in illumination of the two planes before and after the focal point as shown in FIG. 7. The Roddier’s curvature sensor is based on the irradiance Poisson equation, which is derived from Teague’s irradiance transport equation. Roddier obtained

\[
\frac{l_1 - l_2}{l_1 + l_2} = \left( \frac{\partial W}{\partial z} \right) \Delta z
\]

(5)

where \(\Delta z\) is the distance from the pupil plane of the defocused plane \(P_1\) or \(P_2\) viewed from the object space. A plane at a distance from the focus is conjugated to a plane at a distance \(\Delta z\) from the pupil. Roddier proved that

\[
\Delta z = \frac{fs}{l + f}
\]

(6)

So Roddier obtained the well-known equation

\[
\frac{l_1 - l_2}{l_1 + l_2} = \left( \frac{\partial W}{\partial z} \right) \Delta z
\]

Curvature sensing is a technique used typically in adaptive optics to measure the Laplacian of the wavefront by subtracting intensity profiles from an exact distance before and after the focus of a lens. The idea of the Roddier’s curvature sensor is that the normalized differential intensity change along the optical axis provides the information of the local Laplacian curvature of the wavefront.

For adaptive optics systems, the image before and after the focus is usually switched mechanically, making the systems quite noisy during operation. In contrast, the differential Shack-Hartmann curvature sensor contains no moving parts making it more reliable.

Foucault knife-edge testing involves moving a knife-edge through the focus of a beam and observing the intensity pattern on a screen. Like interferometry, knife-edge testing allows high-spatial frequency aberrations to be observed. But it requires very accurate alignment of the knife-edge to the beam focus, and it is qualitative.

In summary, the differential Shack-Hartmann curvature sensor shares the important features with the Shack-Hartmann sensor, such as it is a real-time wavefront measurement, measurements are inherently two-dimensional and parallel, it is independent of higher-order aberrations and intensity profile variations, has good photon efficiency and is good for the all wavelength bands.

The differential Shack-Hartmann curvature sensor also provides some unique features such as eliminating the need for external references after calibration, the sensor is independent of vibrations, tilt and whole body movements, which makes it a good choice for measurements with moving objects, and is scale tunable by changing differential values.

D. Mid-Spatial Frequency Error Recovery

In another embodiment, for wave front testing with mid-spatial frequency error recovery increased spatial sampling is used. When spatial sampling with a given lenslet array is not adequate for mid-spatial frequency recovery, the sampling grid is moved with a displacement less than the lenslet pitch size to increase the sampling density. FIG. 8 shows a sampling grid from a Differential Shack Hartmann sensor at \((0, 0)\). For error recovery, a denser sampling is accomplished by first shifting the sampling grid of FIG. 8 by 0.6 along the x-axis to a position of approximately \((0.6, 0)\) to double the number sampling points. By shifting the sampling grid of FIG. 9 by 1.2 along the x-axis to approximately \((1.2, 0)\) triples the number of sampling points as shown in FIG. 10. At this point, the sampling is 3 times denser in the x-direction than the sampling shown in FIG. 8. To further increase the density, the sampling grid of FIG. 10 is shifted by -0.6 along the y-axis to approximately \((0, -0.6)\) as shown in FIG. 11, then is shifted by -1.2 along the y-axis to approximately \((0, -1.2)\) shown in FIG. 12. The sampling grid shown in FIG. 12 is three times denser than the original sampling grid in y-direction.

In a preferred embodiment, to achieve a sampling grid that is nine times denser than the original grid. First the grid shown in FIG. 12 is again shifted by 1.2 along the x-axis to approximately 1.2 -0.6 as shown in FIG. 13. Then the grid is shifted by -0.6 along the y-axis to approximately 0 -1.2 as shown in FIG. 14. Next, the grid shown in FIG. 14 is shifted by 0.6 along the x-axis to approximately 0.6, -1.2 as shown in FIG. 15. The final shift, shown in FIG. 16, shows the grid shifted right by 0.6 along the x-axis to approximately 1.2, -1.2, which increases the density of the sampling grid by nine times.

For wave front testing with mid-spatial frequency error, increasing the sampling points to nine times denser than the original sampling grid, provides wave front mid-spatial frequency error recovery. The prior art slope- or curvature-based wave front sensors that provide measurements are insensitive to vibration can benefit from the present invention using the mid-spatial frequency error recovery. The systems and methods of the present invention can be applied for different wave front sensing using a variety of sampling-grid-sensors including but not limited to Shack-Hartmann sensors, curvature sen-
sors, and shearing interferometers. Alternative sensors and techniques, such as 4-D technique will be obvious to those skilled in the art.

While the invention has been described, disclosed, illustrated and shown in various terms of certain embodiments or modifications which it has presumed in practice, the scope of the invention is not intended to be, nor should it be deemed to be, limited thereby and such other modifications or embodiments as may be suggested by the teachings herein are particularly reserved especially as they fall within the breadth and scope of the claims here appended.

We claim:

1. A differential sensing device for measuring a wavefront curvature using increased spatial sampling for wavefront testing comprising:
a sampling sensor having an output beam;
an optical element to split said output beam into a first, second and third beam, each of said first, second and third beams traveling in at least one of an x, y and z direction, respectively;
a first, second and third lenslet array in the path of the first, second and third beam, respectively, to generate a corresponding first, second and third sampling grid each having plural grid points;
a shearing element for shifting said plural grid points a distance in a horizontal and a vertical direction to produce plural sampling grids each having plural grid points the displacement of the shifting being less than a pitch size of the lenslet array; and
a measuring device for measuring plural slopes of plural wave fronts at each grid point on the spatial sampling grid to obtain wave front normal curvatures and corresponding twist curvature terms for determining a principal curvature and direction, wherein said plural slopes at each of the plural grid points is measured approximately simultaneously before and after differentially shifting the sampling grid.

2. The device of claim 1, wherein each of said first and second shearing elements comprise:
an opto-mechanical device.

3. The device of claim 1, wherein each of said first and second shearing elements comprise:
an electro-optics device.

4. The device of claim 1, wherein each of said first and second shearing elements comprise:
an optical parallel plate to make the differential displacements between the first and the second lenslet array.

5. The device of claim 1, wherein the first and second shearing elements comprise:
a first and a second micro-screw.

6. The device of claim 1 said measuring device comprises:
a first, second and third camera in a corresponding path of each of said first, second and third beams, respectively, for measuring said wave front slopes at each one of the plural sampling grid point.

7. The device of claim 1, wherein the sampling grid sensor comprises:
a Shack-Hartman slope sensor.

8. The device of claim 1, wherein the sampling grid sensor comprises:
a shearing interferometer sensor.

9. A system for measuring differentials of wave front slopes to determine a wavefront curvature without the use of a reference, the system comprising:
a light source for generating an input light beam;
a beam splitter splitting the input light beam to produce at least three output beams in different directions;
determining said normal curvature and twist curvature terms of said wavefront from said measured differentials of said plural wavefront slopes for wavefront testing with mid-frequency error recovery.

15. The method of claim 14 wherein the normal curvature determination step comprises the step of:

\[ c_n(i) = \frac{1}{f} \left( \frac{x''_m(i) - y''_m(i)}{s_x} \right) - c_{0,n}(i) \]

and

\[ c_n(i) = \frac{1}{f} \left( \frac{y''_m(i) - x''_m(i)}{s_y} \right) - c_{0,n}(i), \]

where \( c_{0,n}(i) \) and \( c_{0,n}(i) \) are obtained by

\[ c_{0,n}(i) = \frac{1}{f} \left( \frac{x''_{ref} - y''_{ref}}{s_x} \right) \]

and

\[ c_{0,n}(i) = \frac{1}{f} \left( \frac{y''_{ref} - x''_{ref}}{s_y} \right), \]

to the measured coordinate differentials to obtain said normal curvature of said wavefront.

16. The method of claim 14, wherein the twist curvatures determination step comprises the step of:

\[ c_t(i) = \frac{1}{f} \left( \frac{x''_m(i) - y''_m(i)}{s_x} \right) - c_{0,t}(i) \]

and

\[ c_t(i) = \frac{1}{f} \left( \frac{y''_m(i) - x''_m(i)}{s_y} \right) = c_{0,t}(i), \]

where \( c_{0,t}(i) \) and \( c_{0,t}(i) \) are constants given by

\[ c_{0,t}(i) = \frac{1}{f} \left( \frac{y''_{ref} - y''_{ref}}{s_x} \right) \]

and

\[ c_{0,t}(i) = \frac{1}{f} \left( \frac{y''_{ref} - x''_{ref}}{s_y} \right), \]

to the measured coordinate differentials to obtain said twist curvatures of said wavefront.

17. The method of claim 14 further comprising the step of: calibrating said sensing system to reduce systematic errors.

18. The method of claim 14 wherein said differential measurement step comprises the steps of:

measuring coordinates of plural grid points of the plural wavefront without said differential displacement; and

measuring coordinates of plural grid points of the plural wavefronts with said differential displacement.

19. The method of claim 14, wherein the repeating the horizontal and vertical moving steps comprises the steps of:

measuring differentials of the original grid points of the original sampling grid;

moving the original sampling grid horizontally a first positive horizontal distance to increase a density of the sampling grid points in a horizontal direction;

measuring differentials of the first horizontally shifted grid points;

moving the first horizontally shifted sampling grid horizontally a second positive horizontal distance to further increase the density of sampling grid points in the horizontal direction;

measuring differentials of the second horizontally shifted grid points;

moving the original sampling grid vertically a first negative vertical distance to increase the density of the sampling grid points in vertical direction;

measuring differentials of the first vertically shifted grid points;

moving the first vertically shifted sampling grid vertically a second negative vertical distance to further increase the density of sampling grid points in vertical direction; and

measuring differentials of the second vertically shifted grid points.

20. The method of claim 19, further comprising the steps of:

moving the first horizontally shifted sampling grid vertically a first negative vertical distance to increase the density of the sampling grid points;

measuring differentials of the first horizontally and vertically shifted grid points;

moving the first horizontally and vertically shifted sampling grid a second horizontal positive direction to increase the density of the sampling grid points;

measuring differentials of the second horizontally and vertically shifted grid points;

moving the first horizontally and vertically shifted sampling grid a second negative vertical direction to further increase the density of sampling grid points; and

measuring differentials of the second horizontally and second vertically shifted grid points.

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