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Two-dimensional buoyancy driven thermal mixing in a horizontally partitioned adiabatic enclosure

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The dynamics of the transient, two-dimensional buoyancy driven thermal mixing of two fluid masses at different temperatures, initially at rest and confined to separate portions of a horizontally partitioned adiabatic enclosure, is investigated numerically within the framework of the Boussinesq approximation. The fluids are allowed to mix through a centrally located opening or vent in the partition. Apart from the geometric parameters, the dynamics is governed by the Rayleigh (Ra) and Prandtl (Pr) numbers. Spanning the range $500 \leq Ra \leq 10^4$ at $Pr=0.71$ and unity aspect ratios of the vent and the enclosures, the dominant spatial and temporal flow structures, in the asymptotic approach of the system towards a state of thermomechanical equilibrium, have been identified. These dominant modes have been utilized to classify the flow dynamics observed at different Ra into three distinct flow regimes. An approach utilizing new scalar norms to quantify the instantaneous state of mixing and to track the mixing process in time has been utilized to identify the flow modes favoring or opposing the mixing process. It is shown that the flow mode comprising of counterflowing streams in the vent yields the highest mixing rate. It is also shown that this flow mode results in a large build-up of enstrophy in the system. For $Ra \leq 5000$, an increase in Ra brings about an increase in the overall mixing rate. However, for $Ra > 5000$, there exists a vortex trapped in the vent for a significant length of time, preventing the two fluid masses to mix directly, thereby slowing down the overall mixing rate in comparison to the flows for $Ra \leq 5000$. © 2008 American Institute of Physics.

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I. INTRODUCTION

The problem of buoyancy driven mixing of initially thermally stratified fluids in adiabatic enclosures has received much less attention. Studies by Conover *et al.*,¹ Epstein,² Myrum,³ and Kumar *et al.*⁴ are some of the few efforts to understand buoyancy induced mixing in enclosures.

Two fluid masses, initially separated and in a state of rest at different temperatures with heavier fluid mass stacked above the lighter one, in an adiabatic enclosure, undergo a transient buoyancy induced motion during the mixing process eventually culminating into a state of thermal and mechanical equilibrium. The fluid motion during the transient phase, governs the rate of the mixing process. In an attempt to model such physical scenarios, most of the previous studies have considered horizontally partitioned enclosures with a vent (Fig. 1).

Kumar *et al.*⁴ carried out a computational study of the transient laminar flow of air in an enclosure, which is similar to the one depicted in Fig. 1. The critical control parameter was found to be the Rayleigh number. For a vent of aspect ratio of unity, three flow regimes, namely, conduction, countercurrent, and oscillatory flow regimes, were identified

as the Rayleigh number (Ra) was varied from 1×10^3 to 5×10^4 . These regimes were identified on the basis of the dominant flow structures observed through the vent. A numerical study of the unsteady buoyant flow through a horizontal vent placed slightly asymmetrically between the two enclosures was also performed by Singhal and Kumar.⁵

An interesting flow feature found to exist in these unsteady mixing problems is the formation of buoyant plumes and/or thermals. This is essentially the outcome of the instability of the horizontal fluid interface separating the two fluid masses which is a well studied problem.^{6,7} The problem is referred to as a Rayleigh–Taylor stability problem hereinafter referred to as the RT problem and the instability as RT instability. The turbulent mixing in channels and rectangular enclosures caused by the RT instability has been studied both experimentally and numerically.^{8–12} These studies have primarily focused on the initial development of the characteristic thermal plumes (bubbles and spikes) on either side of the density discontinuity and the entrainment and mixing caused by such flow structures.

The proper orthogonal decomposition (POD) is an established technique for extracting the most “energetic” or dominant spatial/temporal structures in the average sense from a spatiotemporal data ensemble.¹³ In open literature, there are a few studies involving the application of the POD technique

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FIG. 1. A schematic diagram of the generic geometrical and physical aspects of the class of problems under consideration.

to unsteady laminar flows with heat transfer.^{14–18} Recently, Hasan *et al.*¹⁹ applied POD to demonstrate that the oscillatory instability (supercritical Hopf bifurcation) for two-dimensional thermally driven convection in a horizontal heated circular cylinder at $Ra \approx 8.83 \times 10^3$ takes place in the form of standing waves. In another recent study, Hasan and Sanghi²⁰ demonstrated that the POD analysis effectively revealed the presence and structure of the traveling waves in thermally driven flow in a horizontal rotating cylinder.

The present work involves a computational investigation of two-dimensional buoyancy induced mixing of a $Pr=0.71$ fluid (air) in a vented geometry (Fig. 1) within the framework of the Boussinesq approximation. The individual aspect ratios of the enclosures and the vent are fixed at unity. The ratio of width of the vent to that of the enclosures is taken to be 0.2. The geometry has been selected to constitute a *model mixing problem* in large enclosures connected by a vent. Initially, an interface (thermal) lying midway across the height of the vent separates stationary fluid masses, at higher and lower temperatures, at the bottom and lower enclosures, respectively. All physical surfaces are adiabatic. The fluid motion is triggered via the RT instability and evolves under the action of buoyancy forces. Thus, the transient flow and mixing process is characterized by the ratio of destabilizing effects of buoyancy to the stabilizing effects of diffusion. For buoyancy driven flows, such a ratio is represented by the Rayleigh number defined as

$$Ra = \frac{g\beta\Delta TH^3}{\nu\kappa}, \quad (1)$$

where g is the acceleration due to gravity, $\Delta T=(T_H-T_C)$ is the initial temperature difference between the hot and the cold fluids, and H is the height of the vent (Fig. 1). The coefficient of volume expansion is denoted as β , while kinematic viscosity and thermal diffusivity are denoted as ν and κ , respectively.

In the context of the present problem, some remarks on the definition of Ra in Eq. (1) are worth making. The Ra number is usually defined by taking the product of *imposed finite thermal gradient* and the *fourth power of the vertical length scale* across which the finite gradient is imposed (Ref. 6). However, for the present problem, as the initial vertical temperature profile is discontinuous, the definition of Ra employs the initial temperature difference. Further, the chosen

length scale is H as the initial discontinuous vertical thermal profile would spread *first inside the vent*, which in turn influences the fluid penetration into the enclosures. As the initial localized infinite thermal gradient at the interface would spread to become finite with the passage of time, Ra can also be referred to as the *initial* Ra number in the context of the present study.

As revealed by the literature survey, not much is known regarding the nature of spatiotemporal dynamics exhibited during buoyancy driven mixing in adiabatic enclosures, as shown in Fig. 1. The generic problem has a fairly large parametric space (aspect ratios of vent and enclosures, relative sizes of vent and enclosures, and Rayleigh and Prandtl numbers). Therefore, the study of the effect of all of the parameters on the flow dynamics is a potentially large undertaking. Hence, for the present study, the effect of Ra on the flow dynamics is investigated. Specifically, the objectives of this work are

- identification of the modes (spatiotemporal patterns) that the system exhibits in the process of approaching equilibrium for different values of Ra in the range $[500, 10^4]$ at a fixed $Pr=0.71$,
- investigation of the influence of these modes on the mixing process, and
- application of POD analysis to detect possible wavelike flow modes.

The earlier works, in similar geometries, have not investigated the effects of the transitional flow patterns on the *rates of mixing*. The present study aims to address this very relevant and practical issue from an engineering point of view. For the model mixing problem, the values of Ra have been constrained to a few thousand as even for these values, there are no previous studies detailing the complex unsteady spatiotemporal dynamics.

The paper is organized into five sections. An introduction to the problem along with relevance to prior research is presented in Sec. I. The mathematical formulation, the numerical scheme employed, and the related validation studies are covered in Sec. II. The different flow patterns observed for $Ra \in [500, 10^4]$ are analyzed and categorized into different flow regimes in Sec. III. Section IV pertains to the application of POD to study the oscillatory behavior observed in the formation of the thermals. Finally, the salient conclusions drawn from the study are presented in Sec. V.

II. MATHEMATICAL FORMULATION AND NUMERICAL SCHEME

A. Governing equations

The governing equations of mass, momentum, and energy that are subject to the Boussinesq approximation for a Cartesian frame¹⁹ are as follows.

In continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (2)$$

In x momentum,

FIG. 2. Geometries of the flow problems employed for validation.

$$\frac{\partial u}{\partial \tau} = -\frac{\partial p}{\partial x} + \text{Pr} \nabla^2 u - u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y}. \quad (3)$$

In y momentum,

$$\frac{\partial v}{\partial \tau} = -\frac{\partial p}{\partial y} + \text{Ra} \text{Pr} \theta + \text{Pr} \nabla^2 v - u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y}. \quad (4)$$

In energy,

$$\frac{\partial \theta}{\partial \tau} = \nabla^2 \theta - u \frac{\partial \theta}{\partial x} - v \frac{\partial \theta}{\partial y}. \quad (5)$$

The equations have been made dimensionless by employing H , κ/H , and H^2/κ as the reference scales for length, velocity, and time, respectively. The dimensionless parameters in Eqs. (3) and (4) are Prandtl number ($\text{Pr} = \nu/\kappa$) and Rayleigh number (Ra), as defined in Eq. (1).

The dimensionless space and time coordinates are symbolically represented as x , y , and τ , respectively. The symbols u and v represent the dimensionless Cartesian velocity components in the x and y directions, respectively. The dimensionless pressure and temperature are defined as

$$p = \frac{(P - P_C)H^2}{\rho_C \kappa^2}, \quad (6)$$

$$\frac{T - T_C}{T_H - T_C}. \quad (7)$$

In Eqs. (6) and (7), T_C , ρ_C , and P_C are the temperature, density, and pressure, respectively, of the cold fluid mass. The temperature of the warmer fluid mass is denoted as T_H .

B. Initial and boundary conditions

The solid boundaries are taken to be adiabatic with no-slip conditions. Initially, the fluid is taken to be in a quiescent state ($u=v=0$), with θ taken to be uniformly 0 and unity in the upper ($y>0$) and lower half ($y\leq 0$) of the domain (Fig. 1), respectively.

C. Numerical scheme

Computations have been carried out utilizing a semi-implicit, pressure-correction scheme^{21,22} on a nonstaggered mesh. The concept of “momentum interpolation” has been employed in order to avoid grid-scale pressure oscillations

TABLE I. Comparison of the Nusselt number data for the differentially heated square enclosure with that in Refs. 26 and 27.

Ra	Nu_{avg}	Nu_{max}	Nu_{min}	Source
10^5	4.51	7.71	0.73	Present
	4.51	7.72	0.73	Reference 26
	4.52	7.73	0.72	Reference 27
10^6	8.84	17.89	0.97	Present
	8.82	17.92	0.99	Reference 26
	8.82	17.56	0.98	Reference 27
10^7	16.51	39.39	1.38	Present
	16.52	39.47	1.38	Reference 27

that can result on a nonstaggered mesh.^{23,24} The scheme is described in detail in the work of Hasan and Sanghi.²⁵

Equations (3)–(5) are discretized on a Cartesian collocated grid by using a finite difference methodology. The convective terms are discretized by using a hybrid scheme of a fourth-order central differencing, or a third-order Taylor series based upwinding scheme depending on the local cell Peclet number. If $|\text{Pe}| < 2$, then central differencing is preferred, otherwise upwinding is utilized. At near boundary nodes, the convective terms are *always* discretized by using a second-order central differencing scheme. The viscous terms are discretized by using a fourth-order central differencing in the interior while a second-order central differencing scheme is utilized for near boundary nodes.

D. Validation

The numerical scheme and the associated code has been validated through two well studied problems involving two-dimensional buoyancy driven flow in (i) a differentially heated square enclosure (problem 1) and (ii) a circular enclosure with a continuous periodic distribution of temperature (problem 2), as shown in Figs. 2(a) and 2(b). The computations in both geometries have been carried out for $\text{Pr}=0.71$ and for a range of Ra as depicted in Tables I and II, respectively.

In the steady regime, the values of the Nusselt number over the heated wall/surface are compared for problems 1 and 2 in Tables I and II with the numerical data of de Vahl Davis,²⁶ Nonino and Croce,²⁷ and Xin *et al.*,²⁸ respectively.

TABLE II. Comparison of the Nusselt number data over the heated portion of the wall of the differentially heated circular enclosure with that in Ref. 28.

Ra	Nu	Source
1×10^4	2.615	Present
	2.615	Reference 28
5×10^4	4.076	Present
	4.078	Reference 28
1×10^5	4.899	Present
	4.912	Reference 28
5×10^5	7.512	Present
	7.523	Reference 28

FIG. 3. Time histories of $v(-0.715, 0)$ (a) showing the oscillatory growth of the instability and (b) the long term stable oscillatory pattern at $Ra = 1.2 \times 10^6$ and $Pr = 0.71$.

From Tables I and II, it can be seen that the agreement with the computational data is excellent. It was shown by Xin *et al.*²⁸ that at $Ra \cong 1.06 \times 10^6$, the steady flow undergoes a supercritical Hopf bifurcation. Figure 3 shows the time history of $v(-0.715, 0)$ at $Ra = 1.2 \times 10^6$. An exponential oscillatory growth with a period of oscillation equal to 3.2×10^{-3} dimensionless units is observed. This value is within 1% of the value reported by Xin *et al.*²⁸

III. FLOW REGIMES AND DISCUSSION

The computations are carried out for $500 \leq Ra \leq 10^4$ and $Pr = 0.71$. The transient flow patterns at any Ra are studied by generating snapshots of streamlines and isotherms at different time instants. The time history of the y component of velocity has been recorded at a set of nine sample points whose coordinates are listed in Table III.

A quantitative measure of the level of thermal mixing E_T defined as

$$E_T = \langle (\theta - \theta_{eq})^2 \rangle^{1/2}, \quad (8)$$

where $\langle \cdot \rangle$ is the averaging operator defined as

$$\langle \cdot \rangle \equiv \frac{1}{A} \int \int_D (\cdot) dA, \quad (9)$$

is proposed to monitor the mixing process in time. In Eq. (8), θ_{eq} is the final uniform state (perfectly mixed) equilibrium temperature that would be attained by the system in the asymptotic limit of infinitely large times. The term $dA (=dx \cdot dy)$ refers to the differential area element and symbol D represents the entire flow domain. For the present choice of initial conditions, $\theta_{eq} = 0.5$. Thus, values of E_T quantitatively reflect different levels of mixing during the transient mixing process. Denoting the deviation $(\theta - \theta_{eq})$ as θ' , it can be readily observed that θ' satisfies Eqs. (5). Utilizing Eqs. (8) and (5), it can be shown that the mixing measure E_T satisfies the following equation:

$$\frac{d(E_T^2/2)}{d\tau} = \langle \theta' \nabla^2 \theta' \rangle - \langle \theta' (\vec{V} \cdot \nabla) \theta' \rangle. \quad (10)$$

From the above equations, the following observations can be made.

- (i) The temporal evolution of the mixing measure is directly controlled by the combined processes of diffusive (irreversible) and convective mixing (reversible).
- (ii) The second law of thermodynamics, as applied to the adiabatic system, essentially requires that $d(E_T^2/2)/d\tau \leq 0$, i.e., temperature differences in the fluid must reduce in time. Thus, it can be deduced that during mixing, the possible flow modes at any instant are constrained by the inequality

$$\langle \theta' \nabla^2 \theta' \rangle \leq \langle \theta' (\vec{V} \cdot \nabla) \theta' \rangle. \quad (11)$$

- (iii) Any changes in the flow modes or patterns during the mixing process may show up as discontinuous slope changes in the $E_T - \tau$ characteristic.

TABLE III. Coordinates of the various spatial points at which the time history of the flow variables is recorded.

Region	Sample point locations
Upper enclosure	(-1.0, 2.0)
	(0.0, 2.0)
	(+1.0, 2.0)
Vent	(-0.25, 0)
	(0, 0)
	(+0.25, 0)
Lower enclosure	(-1.0, -2.0)
	(0.0, -2.0)
	(+1.0, -2.0)

FIG. 4. (a) sketch showing the formation of a thermocline with line sources of buoyancy and (b) isotherm pattern for $Ra=1500$ at $\tau=4.0$ depicting the penetration of the thermal into the enclosures.

In the light of these observations, the scalar measure as defined via Eq. (8) appears to be a suitable quantitative indicator for the class of problem under study. However, it should be noted that the definition in Eq. (8) can only be applied to scenarios where (a) the mixing domain is finite and (b) the equilibrium state is known *a priori*.

Several measures such as instantaneous available potential energy and the minimum potential energy attainable in a closed system through adiabatic processes have been proposed in earlier studies.^{29,30} These measures are primarily designed to separate the effects of adiabatic large scale mixing or “stirring” (reversible) from the diabatic irreversible mixing processes. These methods are quite general and can be applied to any general scenario where the domain is unbounded and/or the final equilibrium state is not known *a priori*. However, the measures are somewhat awkward to compute, particularly the background or minimum potential energy. Recently, Coffey and Hunt^{31,32} proposed measures of ventilation effectiveness, which represent the efficiency, of removal/flushing of buoyancy from a given space. The main limitation of these measures is that they are not direct indicators of instantaneous level of mixing in relation to the asymptotic equilibrium state of homogeneous mixing which is an important statistic from the application point of view.

At this stage, some general remarks regarding the development of the flow in the initial stages are worth making. During the initial phases, the perturbations generated by the RT instability *quickly* pass through the vent into the enclosures leaving essentially *conductive processes in the vent*. This leads to the formation of a typical thermocline³³ or a three-layer temperature profile in the y direction inside the vent, as shown in Fig. 4(a). With the passage of time, depending on the magnitude of Ra , the conductive processes in the vent may prevail or lead to bulk motion or convection inside the vent.

As the thermal and the flow perturbations enter the enclosures centrally on either side of the vent, some transient vertical velocities develop in the enclosures whose action is similar to that of an emerging jet of fluid into a large quiescent environment, causing entrainment of the surrounding

FIG. 5. Streamline pattern for $Ra=500$ at $\tau=2.0$.

fluid in the enclosures. This results in a cap-/bulblike isotherm pattern (hereinafter referred to as starting cap) in the enclosures. This phenomenon is observed at all of the Rayleigh numbers considered and is a characteristic of the initial conditions.

It is also worth mentioning that the present study utilizes initial conditions that are quite difficult to replicate in laboratory. Thus, no exact experimental analogues exist for the present numerical study.

A. Grid independence studies

The grid independence is carried out at a value of $Ra=10^4$ on three Cartesian structured uniform grids having 51×111 , 101×221 , and 201×441 mesh points. It is observed that the difference in the time histories of the flow velocities, recorded at locations given in Table III, in going from the 101×221 grid onto the 201×441 grid is within 1%. To check the time independence, dimensionless time steps of 1×10^{-6} and 0.5×10^{-6} were employed. No significant deviations were recorded for the two steps in the time histories. In order to monitor the level of accuracy of the divergence free constraint, the net volume flow rate across the planes $y=0$, $+3.5$, and -3.5 was monitored in time for the grid independence studies. It was found that the value of the volume flow rate did not exceed 1×10^{-3} dimensionless units.

B. The conductive regime

For low Rayleigh numbers, $500 \leq Ra \leq 1500$, a transient flow is observed in the two enclosures as a result of RT instability induced perturbations, as discussed earlier. Apart from this initial flow phenomenon that lasts for a very short

FIG. 6. The isotherms for $Ra=500$ at (a) $\tau=2$ and (b) $\tau=20$.

duration ($\tau \sim 2$), there is *no flow* through the vent and the mode of heat transfer across it is conductive.

The initial flow into the enclosures leads to the creation of a horizontal temperature gradient in the enclosures. The horizontal thermal gradients act as a source of generation of vorticity in the enclosures, as depicted in Fig. 5. An analogous flow pattern is observed in the lower enclosure. These cells or convective loops are transient after which the mode of heat transfer in the entire system is conductive. The isotherms in the system at the initial state and after a long interval are shown in Fig. 6.

The flow patterns evolve in a similar manner at $Ra=1000$ and $Ra=1500$, with the flow dying out quickly after the initial flow phenomenon. Figure 7 compares the time history of v at the position $(0,2)$ at $Ra=500$, 1000 , and 1500 , respectively. As this point lies in the midregion of the upper enclosure, it is directly influenced by the initial pen-

FIG. 7. Comparison of time histories of $v(0,2)$ in the conduction regime.

FIG. 8. Mixing characteristics in the conduction regime.

etration of the fluid in the enclosures. This is evident from the sudden increment of the velocity in the first few time steps. An increase in the Rayleigh number increases the magnitude of this initial peak in velocity, as would be expected since the driving force is increased.

Figure 8 shows the time histories of the parameter E_T at $Ra=500$, 1000 , and 1500 , respectively. During the early stages ($\tau \leq 2$), the three curves are coincident and start to deviate for later times with mixing getting faster as Ra is increased. This is so as the process in the initial phase ($\tau \leq 2$) is essentially governed by the vent geometry and the initial conditions, and hence, largely independent of Ra . Later, as the flow disturbance penetrates and generates thermal gradients in the enclosures, the buoyancy starts to affect the subsequent mixing process in the enclosures.

C. The counterflow convective regime

For higher values of the Rayleigh number, between 2000 and 5000 , after the initial evolution of the unstable perturbations, a flow pattern emerges in the system comprising of counterflowing streams along the sides of the vent. Figures 9(a) and 9(b) show the streamline patterns at $\tau=5$ and $\tau=10$, respectively, for $Ra=2000$. The flow patterns indicate two distinct modes of exchange of fluid between the two enclosures. In both modes, the convective loops drive the fluid between the two enclosures in some sort of a diagonal exchange. The flow pattern switches between the two modes in time resulting in direction switching of the counterflowing streams in the vent [Fig. 9(b)]. Further, while two saddle points are observed at $\tau=5$ in the vent, only one saddle point is observed at $\tau=10$. The flow pattern, during the mode-switching phase, exhibits space-time symmetry in the sense that one mode along with its mirror image (reflectionally symmetric pattern), are observed at two different time instants. In this diagonal exchange, two-mode switching behavior has not been reported in the earlier studies on similar configurations and is a new finding. The time histories of the vertical velocities at two points in the vent exposed to the

FIG. 9. Instantaneous streamline patterns for $Ra=2000$ at (a) $\tau=5.0$ and (b) $\tau=10.0$.

counterflowing streams are shown in Fig. 10. The flow reversal in the counterflowing streams is very much evident from the time histories.

With the passage of time, as the thermal gradients weaken, this space-time symmetry is broken with the two rolls in the upper enclosure merging into a single roll while the counterflow streaming pattern still maintained through the vent, as shown in Fig. 11(a). At this stage, it is difficult to

FIG. 10. The time history of $v(+0.25, 0)$ and $v(-0.25, 0)$ in the vent at $Ra=2000$.

FIG. 11. (a) Streamline pattern at $\tau=14.0$ and (b) mixing characteristic at $Ra=2000$.

comment on the manner in which the symmetry breaking is observed to take place, with the other possible symmetry-breaking pattern of rolls merging in the bottom enclosure not being observed in the numerical simulations. The manner in which the space-time symmetry is destroyed is a phenomenon dictated by complex nonlinear interactions of perturbations with themselves and with the base flow (symmetric pattern).

The changes in the flow structure with time lead to the mixing characteristic, as shown in Fig. 11(b). Correlating the characteristic with the flow pattern/structure, it can be said that the mixing associated with the initial penetration of unstable disturbances and the subsequent generation of rolls has a linear character. As the diagonal exchange flow pattern is established at $\tau \sim 5$ [marked as A in Fig. 11(b)], there is a sudden increase in the rate of mixing. Further, another sudden increase in the rate of mixing is observed at $\tau \sim 10$ [marked as B in Fig. 11(b)]. This corresponds to the mode-switching event involving the diagonal exchange counterflow modes. As the mixing characteristic approaches the equilibrium ($E_T=0$) asymptotically, from a practical point of view, a value of 0.1 representing 80% mixing on the scale of E_T is utilized to estimate and compare the mixing times for different Ra .

Increasing the Rayleigh number from 2000 to 5000 essentially yields qualitatively similar flow patterns with the passage of time. However, as expected, some new transient flow structures are also observed. Figure 12 depicts the changes in the spatial flow structure with the passage of time at $Ra=3000$. A diagonal exchange counterflow mode is observed at $\tau=2.0$. However, the other reflectionally symmetric counterpart is not observed at this Ra . A new flow structure is observed at $\tau=3.5$, where the fluid in the vent interacts with the convective roll in the lower enclosure through a vortex formed in the vent. The vortex formed in the vent is observed for a short duration and a relatively stable flow structure with a single large roll in the upper enclosure, engaged in an exchange with the left roll in the lower enclosure via a counterflow pattern in the vent is observed for $6.0 \leq \tau \leq 10.0$.

Figure 13 compares the time histories at $(-0.25, 0)$ in the

FIG. 12. Snapshots of the evolving streamline patterns at different instants of time at $Ra=3000$.

vent at $Ra=3000$, 4000, and 5000 in order to highlight the quantitative differences with an increase in Ra . It is observed that with the increase in the Rayleigh number, higher velocities are achieved in the vent.

Figure 14 compares the mixing characteristics for the different Ra in the range $[2000, 5000]$. The characteristic for $Ra=2000$ has been included for comparison purposes. An increase in Ra essentially leads to higher mixing rates with the characteristic at $Ra=2000$ being different from the remaining ones owing to the phenomenon of *diagonal mode switching* observed for $Ra=2000$ and not at other Ra numbers. The mixing times corresponding to $E_T=0.1$ are reduced by a factor of 3 (approximately) as Ra is increased from 2000 to 5000. As shown earlier in Fig. 12, at $Ra=3000$, the flow pattern undergoes a change from vortex in the vent to a

FIG. 13. Comparison of time histories of $v(-0.25, 0)$ inside the vent in the counterflow convective regime.

FIG. 14. Mixing characteristics in the counterflow convective regime.

counterflow exchange mode at $\tau \sim 5.0$. This is also observed at $Ra=4000$ and $Ra=5000$. Figure 15 depicts these changes in the flow structure at $Ra=5000$. Therefore, it is this mode that is responsible for the *largest mixing rates*. To clearly differentiate between this counterflow exchange mode and the *diagonal* exchange counterflow modes encountered during the early phase of the mixing, the former is referred to as counterflow mode (type 2) while the latter is referred to as counterflow mode (type 1). It is further emphasized that in the Ra range of 2000–5000, only these two counterflow

FIG. 15. (a) Instantaneous streamline pattern at $\tau=4.0$ showing a vortex residing inside the vent and (b) the instantaneous streamline pattern at $\tau=6.0$ showing the counterflow mode (type 2). Both the patterns are taken at $Ra=5000$.

FIG. 16. Mixing characteristics as represented by thermal and normalized enstrophy norms at $Ra=5000$.

modes are observed that are responsible for convective mixing through the vent.

The counterflow exchange mode (type 2) through the vent yields highest mixing rates. On physical grounds, it can be argued that this is due to

- (i) direct physical interaction of fluid elements in the two enclosures, and
- (ii) larger convective loop spanning both the enclosures, which allows more time for the buoyancy effects to generate vorticity in the buoyant fluid elements.

In order to verify the second argument, a measure of system enstrophy, defined as,

$$E_W = \left\{ \frac{1.0}{A} \iint_D \omega^2 dA \right\}^{1/2}, \quad (12)$$

is devised, where ω is the instantaneous vorticity at any point in the enclosure. Figure 16 compares the time evolution of the mixing and the system enstrophy measures at $Ra=5000$. The enstrophy measure E_W has been normalized by the factor $(2 E_{Wmax})$. E_{Wmax} is the maximum value of E_W recorded during the mixing process. The largest peak in the enstrophy characteristic is remarkably coincident with the time instant $\tau \sim 5.0$ (approximately), which is the time at which the counterflow exchange mode (type 2) through the vent is established. Thus, this mode is accompanied by a large build-up of enstrophy in the system. This is a very interesting finding that has escaped the attention of the earlier workers. This also shows that quantitative measures such as those in Eqs. (8) and (12) can reveal interesting aspects of the mixing process.

D. The unsteady thermals and trapped-vortex regime

As Ra is increased further, the system starts to exhibit sustained formation and penetration of thermals or buoyant parcels of fluid one after the other from the vent into the enclosures at the start of the mixing process. A thermal refers to a sudden release of a finite volume/parcel of buoyant

FIG. 17. Instantaneous isotherm patterns showing the different stages of the development of a thermal at $Ra=6500$.

fluid from an instantaneous source of buoyancy. In the problem under consideration, both positively (upper enclosure) and negatively (lower enclosure) buoyant thermals were observed.

The different stages in the development of the thermal at $Ra=6500$ are shown in Fig. 17. As the starting cap-/bulblike structure is convected out of the vent, the entrainment with the enclosure fluid results in the formation of the mushroom-shaped structure for the head of the rising structure. Beyond this point, the process of pinch-off starts when the velocity of fluid in the stem becomes less than the velocity of the accelerating head due to buoyancy.³⁴ The pinch-off leads to the formation of the thermal from the head of the starting cap-/bulblike structure. As the thermal moves in the enclosure, entrainment causes an increase in its size. Finally, due to the limited extent of the enclosure, the thermal impinges on the horizontal walls. The formation of mushroom-shaped structures has been observed experimentally in the initial phases of the growth of the RT instability.^{9,12} This provides credibility to the numerical simulations presented in this study.

The successive formation of thermals is accompanied by an undulating wavelike motion in the vent, which is analogous to waves. Figure 18 shows the time history of $v(-0.25, 0)$ at $Ra=6500$. The undulating wavelike motions are clearly seen in the form of rapid oscillations of the

FIG. 18. Time history of v velocity in the vent during the thermal generation phase at $Ra=6500$.

FIG. 19. Instantaneous isotherms depicting the distortions caused by the oscillations of the flowfield inside the vent.

v -velocity component. In a time interval of $[1.5, 2.0]$ spanning these strong oscillations, spatial structure of the flow in the vent and its neighborhood is shown in Figs. 19(a)–19(f) and Figs. 20(a)–20(f), respectively. The snapshots have been recorded at a uniform interval of 0.1 dimensionless time units. From these snapshots, it can be concluded that the spatial patterns repeat in a periodic manner (approximately). Interestingly, in this phase, a single counterclockwise vortex is observed in the vent that appears to distort the thermal interface, on either side of the vent, in a manner so as to generate the disturbance necessary for the formation of the thermal [Fig. 19(a)]. The strong entrainment caused by the penetration of the thermals into the enclosures [Fig. 19(b)] appears to destroy the vortex in the vent as the entrained fluid is seen to enter partially into the vent [Fig. 20(b)]. The process of vortex creation and its subsequent destruction continues in a cyclic manner resulting in an oscillatory flow

FIG. 20. Snapshots of the streamline patterns during the oscillation phase inside the vent spanning the time interval $[1.5, 2.0]$ (top left to bottom right) at a uniform time increment of 0.1.

FIG. 21. Snapshots of the streamline patterns at different time instants for $Ra=6500$.

until the thermal gradients become less than a threshold value so that the process cannot be sustained. The process is quite complex involving growth of local instabilities and their interaction with the existing flow (mean flow) on the global scale. In the context of the geometry being considered, this phenomenon also has not been reported in the earlier works.

The wave motion in the vent lasts for a short time interval and gives way to a new evolving flow structure comprising of a single vortex *trapped* in the vent while the convective rolls in each of the enclosures gradually merge into a single primary roll, as shown in Fig. 21. This flow pattern persists for a significant length of time as the vortex is seen even for $\tau=10.0$. Finally, this trapped-vortex pattern transforms into the counterflow pattern (type 2). No further changes in the spatial flow structure are observed as the system approaches the equilibrium.

The mixing characteristic at $Ra=6500$ is shown in Fig. 22. In the initial phase of mixing, as expected, there is a large build-up of vorticity caused by the successive formation and penetration of the thermals. This is followed by a rather *prolonged* trapped-vortex phase. This is a phase where the mixing proceeds at a comparatively slow rate. It is also observed that the vorticity decays during this phase. Finally, as the thermal gradients weaken in the vent, the trapped vortex is not able to sustain itself in the vent and the counterflow exchange mode (type 2) is established. As observed earlier, this flow mode is accompanied by a sudden build-up of vorticity and subsequent increase in the mixing rate.

The prolonged phase of trapped vortex hinders the mixing process and, in fact, at $\tau \sim 11.5$, the value of E_T has dropped only to 0.23 whereas at a lower $Ra=5000$ at the same time instant, the corresponding value of E_T had

FIG. 22. Mixing characteristics as represented by thermal and normalized enstrophy norms at $Ra=6500$.

dropped to 0.1 (Fig. 14). Thus, contrary to intuitive thinking, mixing not only takes place at a slower rate at low values of Ra but also at sufficiently high values that are large enough for a vortex to form and reside in the vent.

Figures 23(a) and 23(b) show the effect of increasing Ra on the time histories of $v(-0.25, 0)$ during the initial phase of mixing (thermals phase). From Fig. 23(a), it is observed that at $Ra=8000$, the oscillations have a higher amplitude as well as higher frequency. The effect of increasing Ra to much higher values is depicted in Fig. 23(b). Figure 23(b) depicts the time histories at $Ra=9000$ and $Ra=10\,000$. Again, the same trend of an increase in oscillation amplitudes as well as frequencies in the time interval^{2.5} is observed. However, an interesting difference is observed in the time histories at $Ra=9000$ and $Ra=10\,000$. The mean fluid v velocity at $(-0.25, 0)$ appears to have switched direction for the two cases. It has been shown that at $Ra=6500$ [Figs. 20(a)–20(f)], a negative mean v velocity in the left half of the vent is associated with a counterclockwise vortex created

FIG. 24. Streamline pattern during (a) the thermal phase ($\tau=4.4$), (b) the trapped-vortex phase, and (c) the counterflow phase in the vent at $Ra=10\,000$.

and destroyed cyclically in the initial phases and persisting at later times (trapped-vortex phase). A counterclockwise trapped vortex is also observed at $Ra=8000$ and $Ra=9000$ (figures not shown). This can be inferred from the time histories of $v(-0.25, 0)$ at $Ra=6500$, 8000 , and 9000 , respectively. However, Figs. 24(a) and 24(b) show that at $Ra=10\,000$, a *clockwise* vortex formation/destruction cycle as well as a clockwise trapped vortex is observed in the vent. Finally, as shown in Fig. 24(c), the trapped-vortex phase gives way to the counterflow mode (type 2) that is a mirror image of the counterflow mode (type 2) observed for $Ra=6500$, 8000 , and 9000 . The flow patterns observed at $Ra=10\,000$ (flow mode A) are mirror image counterparts of

FIG. 23. Effect of increasing Ra on the time history of v velocity in the vent at $(-0.25, 0)$ in the thermal regime.

FIG. 25. Comparison of mixing characteristics at different Ra in the unsteady thermals and trapped-vortex regime.

the flow patterns observed at $Ra=6500$, 8000 , and 9000 (flow mode B).

The two sets of flow patterns or flow modes observed in the numerical experiments in the range of $6500 \leq Ra \leq 10\,000$ represent two possible flow modes possessing reflectional symmetries about the y axis with the given boundary conditions (Fig. 1). The dynamic evolution of the system into one of these modes at a given Ra is a complex issue involving the growth of perturbations and is outside the scope of the present study.

Finally, to complete the picture, Fig. 25 compares the mixing characteristics for the three values of Ra in this regime. It is clearly seen that during the initial thermal phase followed by the trapped-vortex phase, the mixing rate increases with an increase in Ra . However, as Ra is increased, the trapped vortex is able to sustain itself in the vent for longer durations, thereby actually *slowing* down the mixing as the vortex does not allow the two fluid masses in the enclosures to mix directly. In fact, for target values of E_T in the range $(0.1-14)$, it is observed that the total mixing time (based on the target value of E_T) is slightly lower for lower values of Ra owing to the prolongation of the trapped-vortex phase. The trapped-vortex phase is succeeded by the counterflow mode (type 2) phase, which increases the mixing rate. As a remark, it is worth mentioning that three-dimensional flow perturbations may develop owing to instabilities of the vortex trapped in the vent. Capturing such a three-dimensional bifurcation is outside the scope of the present two-dimensional model.

As a final point of interest, the nature of the oscillations inside the vent is further investigated by using POD.

IV. POD ANALYSIS

A. Mathematical background

POD or Karhunen–Loève (KL) expansion is a statistical technique for obtaining an optimal spatial basis for decomposing the unsteady flowfield as

TABLE IV. Convergence of the leading five eigenvalues with an increase in number of flow field snaps at $Ra=9000$ in the time interval $[2, 6]$.

Eigenvalues	Snap=20	Snap=40	Snap=50
λ_1	550.0927	546.8760	546.1666
λ_2	44.1529	43.7216	43.6967
λ_3	34.1242	34.0862	34.1339
λ_4	18.4067	18.9314	19.0517
λ_5	14.0788	13.5210	13.3914

$$\mathbf{U}(\mathbf{x}, \tau) = \sum_{p=1}^{\infty} a^{(p)}(\tau) \boldsymbol{\phi}^{(p)}(\mathbf{x}), \quad (13)$$

where the superscripts in parentheses represent the POD mode quantum numbers. In the present case, $\mathbf{U}$ comprises of both the fluctuating velocity components and the fluctuating temperature field.

The KL basis function $\boldsymbol{\phi}$ can be obtained by maximizing the mean square projection of the data ensemble $\{\mathbf{U}^i, i=1, \dots, n\}$ onto the spatial basis function $\boldsymbol{\phi}$ where $\mathbf{U}^i$ are the realizations of the flowfield in n different experiments at the same time instant or realizations of an unsteady flowfield at n time instants in a single experiment. This leads to an integral eigenvalue problem possessing countably infinite, non-negative eigenvalues $\{\lambda\}_1^{\infty}$, and corresponding orthogonal eigenfunctions $\{\boldsymbol{\phi}\}_1^{\infty}$. If the eigenvalues $\{\lambda\}_1^{\infty}$ are ordered such that $\lambda^{(1)} > \lambda^{(2)} > \lambda^{(3)} \dots$, then the spatial modes $\boldsymbol{\phi}^{(1)}, \boldsymbol{\phi}^{(2)}, \dots$ represent the most dominant structures in the average sense.¹³ The vector space in which the decomposition is sought has an inner product or projection operation defined as

$$(\mathbf{f}, \mathbf{g}) = \int_D (f_1 g_1 + f_2 g_2 + \gamma f_3 g_3) dA, \quad (14)$$

where γ is the scaling factor whose value is based on the criteria proposed by Lumley and Poje.^{35,20}

FIG. 26. The eigenspectrum obtained from 50 snaps in the time interval $[2, 6]$.

FIG. 27. Time histories of temporal coefficients of the leading four POD modes.

The eigenfunctions or modes are obtained via a method of snapshots.³⁶ Once the eigenfunctions are known, they are further normalized so that $(\phi^{(p)}, \phi^{(p)}) = 1.0$. The total energy captured by the modal expansion in Eq. (13), in the average sense, is given as

$$E = \langle (\mathbf{U}, \mathbf{U}) \rangle = \sum_p \sum_r \langle a^{(p)} a^{(r)} \rangle (\phi^{(p)}, \phi^{(r)}) = \sum \lambda^{(p)}, \quad (15)$$

where the p^{th} mode contributes $\lambda^{(p)}$ to the mean energy of the fluctuating field. The temporal coefficients in Eq. (13) can be readily obtained by projection as

$$a^{(p)}(\tau) = (\mathbf{U}, \phi^{(p)}). \quad (16)$$

B. Snapshot independence

To investigate the structure of the waves observed in the unsteady thermal regime, snapshots of the flow field for the case of $Ra=9000$ over the time interval spanning, 2–6 dimensionless time units were taken. Table IV summarizes the variation of first five eigenvalues with increase in the number

of snapshots. These eigenvalues capture more than 90% of the total energy. The eigenvalues are nearly constant indicating the independence with respect to the number of snapshots.

C. Results

Figure 26 shows the eigenspectrum obtained with a set of 50 snaps. The eigenvalues do not occur in *pairs* indicating that the unsteady wavy motion observed in the vent does not represent a traveling wave. Instead, the flow pattern observed is the outcome of formation of stationary waves.^{19,37} The temporal structure of the leading four modes, as represented by $a^{(1)}(\tau), \dots, a^{(4)}(\tau)$, is shown in Fig. 27. The temporal coefficient $a^{(1)}$ captures the slowly varying, large temporal scale structure while the other three temporal coefficients capture the faster, relatively smaller temporal scale oscillations.

A POD analysis similar to the one described above was also carried out at $Ra=6500$, 8000, and 10 000, respectively. As for the case of $Ra=9000$, the POD eigenspectra again reveal the presence of standing waves.

V. OVERVIEW AND CONCLUSIONS

The observed phenomena and their analysis have been presented in the preceding sections. In the following paragraph(s), an overview of the work with an emphasis on the noteworthy results is presented.

In the approach from the initial state to the final equilibrium state, the system exhibits, in the range of $500 \leq Ra \leq 10^4$, the following three distinct dynamical regimes:

- (1) the conductive regime, $500 \leq Ra \leq 1500$,
- (2) the counterflow convective regime, $2000 \leq Ra \leq 5000$, and
- (3) the unsteady thermals and trapped-vortex regime, $6500 \leq Ra \leq 10\,000$.

The flow regimes have been distinguished on the basis of the observed dynamics and dominant flow patterns observed across the vent for a range of Ra . The initial penetration of disturbance into the enclosures triggered by RT instability and the subsequent smearing of the initial thermal discontinuity into a thermocline is observed at all the Rayleigh numbers and, hence, is common to all the regimes. Four dominant flow modes were observed, namely,

- (i) diagonal exchange counterflow modes (type 1),
- (ii) counterflow exchange mode (type 2),
- (iii) thermals and standing wave mode, and
- (iv) trapped-vortex mode.

With respect to geometry and boundary conditions, the earlier works have not identified all of the above modes. The solutions are shown to possess space-time symmetries with all the four modes exhibiting reflectional symmetries about the vertical symmetry axis either at different time instants at a given Ra or across different Ra (thermal regime).

A new scalar measure to monitor the mixing process quantitatively in time has been proposed. The governing equation for the measure has also been derived that leads to a thermodynamic constraint on the possible flow modes in the mixing process. In the different flow regimes, the mixing characteristics, while maintaining an overall decaying character, exhibit sharp or sudden changes in the mixing rates signaling the onset of a change from one flow mode to the other at a given Ra . On examining the mixing characteristics, it is concluded that the counterflow mode (type 2) yields the *highest mixing rate*. This is one of the major findings of the study. Further, it is shown that this flow mode leads to the largest build-up of enstrophy in the system which is believed to cause the most intense mixing.

The dynamically rich behavior observed offers enough motivation for carrying out a detailed investigation involving the effects of the size and geometry of the vents and the enclosures. For a wider thermal interface, i.e., at larger widths of the vent, initially, there is a distinct possibility of simultaneous generation of multiple starting caps/bulblike structures. On the other extreme, narrow and tall vents may lead to the formation of multiple trapped vortices, stacked vertically one on top of another.

The novelty of the study lies in identifying the dominant

flow modes in the mixing process and their impact on the mixing rates. In this regard, the effectiveness of the scalar norms defined in the study has been demonstrated. This approach has been largely ignored by the earlier efforts. The main limitation of the study lies in the fact that no scaling laws have been developed so that the results obtained cannot be directly applied to other similar configurations. It is worth pointing out that the measure proposed in the study can be utilized to define time-averaged local and overall mixing effectiveness measures, as proposed by Coffey and Hunt.^{31,32} Such measures would permit quantification and comparison of mixing efficiency during different events observed in different spatial regions of a system and across different Ra . Studies taking into account the above considerations would form a part of our future endeavors.

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