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Mathematical Knowledge for Teaching: Instructional Reasoning in High-Density Black Populations

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MATHEMATICAL KNOWLEDGE FOR TEACHING: INSTRUCTIONAL REASONING IN HIGH-DENSITY BLACK POPULATIONS

by

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ABSTRACT

To be economically competitive, U.S. citizens must be mathematically competent (Wang et al., 2010). However, students in the United States have consistently underperformed those in other industrialized nations in mathematics (Program for International Student Assessment [PISA], 2018), which threatens the economic health of the nation (Achieve, 2013; Auguste et al., 2009; Harbour et al., 2018; Mickelson et al., 2013). Federal education reform was implemented and failed to improve the mathematics achievement of U.S. pupils (Cheong Cheng, 2020). Researchers have found links between teacher knowledge and student achievement; however, factors mediate this relationship (Hatisauri & Erbas, 2017). As a result, non-significant and inconsistent research findings are common. The purpose of this phenomenological research study was to build an understanding of the meaning elementary mathematics educators with average mathematical knowledge for teaching in high-density Black schools (EMEs) ascribe to their instructional reasoning. The EMEs participated in an interview or focus group to explore their lived experiences and understand the essence of their instructional reasoning. The EME participating in this research accredited their instructional reasoning to their schemata for teaching and learning. The EMEs held schemata for how students learned mathematics, the availability or lack of resources available to teach mathematics, their knowledge of mathematics content progressions, and their understanding of students' knowledge. The EME schemata for teaching and learning must be understood to deepen the conceptualization of mathematical knowledge for teaching (MKT) and inform policymakers to enhance federal and state mandates and stakeholders interested in teacher development and training.

Keywords: mathematical knowledge for teaching, instructional reasoning, instructional decisions, high-density Black schools, schemata for teaching and learning

I dedicate this dissertation to my late father Robert Rumph, my mother Minnie Rumph, my sister Rita Rumph, and my children Brianna, Nazaria, and Ian Wallace. My family is my inspiration. My father, although he has transitioned from this world, guides me still today with his words of wisdom. My children motivate me to be my best. Thank you, Brianna, Nazaria, and Ian, for your patience and understanding. Rita, we began our dissertation journey together. Thank you for your encouragement and support. I would not have endured this journey without you walking by my side.

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that the knowledge and expertise you have imparted to this community of research encourages changes and practices that will support urban educators' growth and development in high-density Black school

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LIST OF MEDIA/ABBREVIATIONS/NOMENCLATURE/ACRONYM

AYP - annual yearly progress

BLS - behaviorism learning theories

CRM - curriculum resource materials

EME - mathematics educators in high-density Black schools, with average mathematical knowledge for teaching

ESEA - The Elementary and Secondary Education Act

ESSA - Every Student Succeeds Act

FLDOE - Florida Department of Education

IRB - institutional review board

IRT - item response theory

KCS - knowledge of content and students

LMT - learning Mathematics for Teaching Project

MKT - mathematical knowledge for teaching

MKTA - Mathematical Knowledge for Teaching Assessment

NCEE - National Commission on Excellence in Education

NCES - National Center for Education Statistics

NCLB - No Child Left Behind

NCTM - National Council of Teachers of Mathematics

NAEP - National Assessment of Educational Progress

PCK - pedagogical content knowledge

PISA - Program for International Student Assessment

SCK - specialized content knowledge

SMK - subject matter knowledge

STEM - science, technology, engineering, and mathematics

CHAPTER ONE: INTRODUCTION

Background

Over the last few decades, the United States federal government has enacted several educational reform initiatives to improve education equality and the mathematical performance of Black students (e.g., Goals 2000: Educate America Act, No Child Left Behind Act of 2001 [NCLB], American Recovery and Reinvestment Act of 2009). Yet, investigations of students' achievements have continually reflected a gap among the mathematics achievements of Black and White students (Bohrnstedt et al., 2015; Kuhfeld et al., 2018). Researchers have found that teachers' mathematics knowledge impacts student achievement; however, mediating factors limit the identification of direct relationships among the two (Hatisauri & Erbas, 2017). As a result, the research community would benefit from understanding the meaning that elementary mathematics educators (EMEs) in high-density Black schools ascribe to their instructional reasoning to identify factors mediating teachers' mathematical knowledge for teaching (MKT) and to inform policymakers to enhance federal and state mandates as well as stakeholders interested in teacher development and training.

Purpose of the Study

This phenomenological study aimed to describe the meaning EMEs accredited to their instructional reasoning. Earlier investigations of MKT explored teacher mathematical knowledge, student outcomes, or other related factors. However, little to no MKT research examined how EMEs attribute meaning to their instructional reasoning. Providing rich descriptions of the meaning educators ascribe to their instructional reasoning contributes to the body of MKT research by providing the insights needed by policymakers involved in education reform efforts, research practitioners investigating teacher mathematical knowledge, and stakeholders interested in teacher training and development in mathematics.

Significance of the Study

To be economically competitive, U.S. citizens must be mathematically competent (Wang et al., 2010). However, students in the United States have consistently underperformed in mathematics compared to students in other industrialized nations (Program for International Student Assessment [PISA], 2018), which threatens the economic health of the nation (Achieve, 2013; Auguste et al., 2009; Harbour et al., 2018; Mickelson et al., 2013). Federal education reform initiatives have been implemented in an attempt to improve the mathematics achievement of U.S. pupils. However, a Black–White achievement gap exists, and this persistent gap in achievement has a negative effect on the nation's economy (Auguste et al., 2009; Mickelson et al., 2013).

Teachers have a profound impact on the academic achievement of students, and their mathematical knowledge influences students' mathematics performance (Ball et al., 2008; Fennema & Franke, 1992; Turner & Rowland, 2011). Studies that address teacher knowledge have primarily focused on classifying teacher knowledge, the effects of teacher knowledge on student outcomes, and the relationship between teacher knowledge, instructional quality, and student achievement (Hoover et al., 2016). However, these studies yielded mixed results, and conceptualizing teachers' mathematical knowledge was challenging (Hoover et al., 2016). Given this continued gap in understanding, researchers would benefit from further examining the meaning EME's ascribe to their instructional reasoning. Building the research base around the experiences of mathematics teachers will help decision makers within the field understand the factors that mediate teachers' MKT, inform investigations of teachers' mathematical knowledge, and provide policymakers involved in the education equality reform effort the rationale to support mathematics training and development for teachers.

Research Question

The research question for this study was: What meaning do mathematics educators in high-density Black schools, with average mathematical knowledge for teaching, ascribe to their instructional reasoning?

Methodology

This study was conducted using a phenomenological research methodology. The researcher used this qualitative methodology to build an understanding of the essence of how EMEs attribute meaning to their instructional reasoning when teaching mathematics in high-density Black schools. The Mathematical Knowledge for Teaching framework, a practice-based theory of MKT, was conceptualized by Ball et al. (2008). Ball and colleagues (2008) sought to explain the knowledge educators need to teach mathematics and develop reliable and valid measures of teachers' mathematical knowledge. The mathematical knowledge for teaching framework is described in the conceptual framework section. The framework informed the concept of elementary mathematics teachers' mathematical knowledge, which explained the content and pedagogical knowledge used by educators to teach mathematics. In this study, the researcher used the mathematical knowledge for teaching framework to conceptualize teachers' mathematical knowledge.

Conceptual Framework

Teacher knowledge has been a subject of research for decades and has been found to have a profound impact on student performance (Ball & Bass, 2003; Ball et al., 2008; Fennema & Franke, 1992; Petrou & Goulding, 2011; Turner & Rowland, 2011). One of the most accepted frameworks for teacher content knowledge is the Shulman (1986) pedagogical content knowledge (PCK) (Ball et al., 2008; Hill et al., 2008; Gess-Newsome et al., 2019). Shulman (1986) described PCK as the knowledge that links content and pedagogy. Supporters of PCK favor the framework because it captures teachers' "knowledge and practice within the teaching of a specific discipline" (Gess-Newsome et al., 2019, p. 945). However, critics of PCK argue that the framework lacks clear operational definitions for research and measurement (Hill et al., 2008; Petrou & Goulding, 2011). Hill et al. (2008) explained that research yielded limited evidence to explain what PCK is and its relation to the mathematics achievement of students. Nevertheless, PCK is cited in over 1,200 journal articles across disciplines (Petrou & Goulding, 2011) and is commonly used to demonstrate the multidimensional relationships between content knowledge, teaching, and learning (Ball et al., 2008; Gess-Newsome et al., 2019; Petrou & Goulding, 2011).

Criticism of Shulman's work has influenced the emergence of additional teacher knowledge models (Petrou & Goulding, 2011), including the mathematical knowledge for teaching framework developed by Ball and colleagues (2008). Mathematical knowledge for teaching is a theory of practice aimed to conceptualize the content knowledge needed to teach mathematics (Ball et al., 2008; Hill et al., 2008; Petrou & Goulding, 2011). Ball and colleagues (2008) attempted to clarify Shulman's work by distinguishing between subject matter knowledge (SMK) and PCK (Ball et al., 2008; Petrou & Goulding, 2011). In Figure 1 the connections between the Ball et al. (2008) conceptualization of content knowledge for teaching and the Shulman (1986) classifications of SMK and PCK are illustrated.

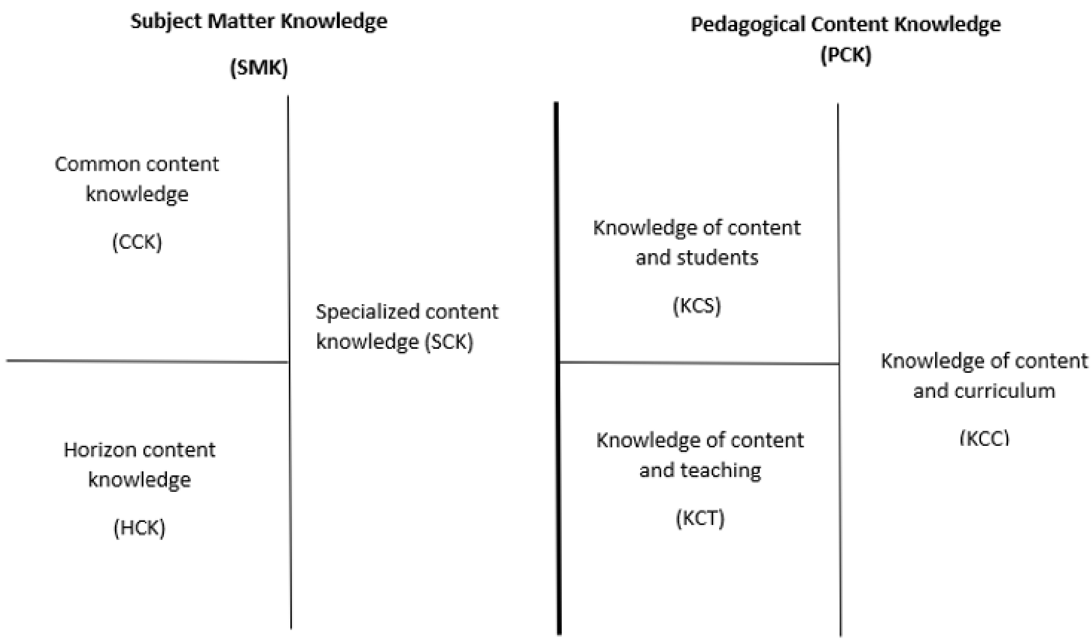


Figure 1: Ball, Thames, and Phelps (2008) Domains of Mathematical Knowledge for Teaching

Note. Shulman's category scheme compared to Ball, Thames, and Phelps. From "Content Knowledge for Teaching," by D. Ball, M. Thames, and G. Phelps, 2008, *Journal of Teacher Education, 59,* p. 403.

However, Ball and colleagues (2008) also faced critics who believed that, similar to Shulman (1986) work, the mathematical knowledge for the teaching framework lacked distinction between its knowledge classifications and who felt clarification was critical for gaining an adequate understanding of teachers' mathematical knowledge (Petrou & Goulding, 2011). Despite the criticism, the mathematical knowledge for teaching framework developed measures of teachers' MKT and identified relationships between teachers' mathematical knowledge and students' performance (Petrou & Goulding, 2011). As a result, the mathematical knowledge for teaching framework is recurrent in mathematics PCK research and has been used to establish measures of teachers' mathematical knowledge, identifiers of teachers' mathematics knowledge, and the connections to gains in students' mathematics achievement (Hill et al., 2011).

Research Design

In this study, the researcher used a qualitative design. Creswell (2013) defined qualitative research as beginning with" assumptions and the use of interpretive/theoretical frameworks that inform the study of research problems addressing the meaning individuals or groups ascribe to a social or human problem" (p.44).To study this problem, qualitative researchers use an emerging qualitative approach to investigate the collection of data in a natural setting sensitive to the people and places under study and the data analysis that is both inductive and deductive and establishes patterns or themes. The final written report or presentation includes the voices of participants, the researcher's reflexivity, a complex description and interpretation of the problem, and its contribution to the literature or a call for change.

Furthermore, Creswell and Poth (2018) classified qualitative approaches into five categories: (a) narrative research, (b) phenomenology, (c) grounded theory, (d) ethnography, and (e) case study. All five approaches follow the same procedure for conducting research; however, differences exist in the foci on the research questions addressed (Creswell & Poth, 2018). The onus is on the researcher to provide rich descriptions of the phenomenon for the people or culture being examined (Newman & Benz, 1998).

The researcher used a phenomenological research design to explore the research question. "A phenomenological study is designed to describe and interpret an experience by determining the meaning of the experience as perceived by the people who have participated in it" (Ary et al., 2009, p. 461). Phenomenological research yields rich narratives aimed at building an understanding of the concept being explored (Ary et al., 2009, 2019). A phenomenological design was used to describe and interpret the meaning EMEs ascribe to their instructional reasoning.

Procedures

The researcher defined the study population and outlined the characteristics that would be used to select the subjects for the study sample. Bracketing, a process of eliminating personal experiences to develop an unbiased perspective of the phenomena, was used to assure the researcher's personal biases did not affect the study (Ary et al., 2019; Creswell & Poth, 2018).

The target population consisted of teachers in a state in the Southeastern United States who taught mathematics in schools with high-density Black populations that held Title I classifications. Teachers were required to hold certification from the state's Department of Education with an active certificate to teach mathematics at their assigned grade level (i.e., elementary education, middle grade mathematics, or mathematics). Additional criteria for participant selection included having at least one year of teaching experience in a kindergarten through fifth-grade class within a general education setting.

Instrumentation and Data Collection

The researcher completed training for (see Appendix A) and used the Mathematical Knowledge for Teaching Assessment (MKTA; Learning Mathematics for Teaching Project [LMT], 2008) to screen study volunteers and identify participants with average MKT. The MKTA is designed to measure teachers' content knowledge and PCK in six mathematics content domains: (1) numbers and operations; (2) patterns, functions, and algebra; (3) geometry; (4) rational number; (5) proportional reasoning; and (6) data, probability, and statistics (LMT, 2019). In this study, the researcher used the MKTA to assess teachers' MKT in one specific domain: number and operations. Educators who scored within one standard deviation above or below the mean on the MKTA received a classification of having average MKT. Participants classified with average MKT were randomly interviewed (n = 15) or invited to participate in a focus group.

The researcher used semi-structured interviews for data collection. In semi-structured interviews, the researcher sets the area of focus and creates the questions before the interviews. However, modifications were allowed during the interview process (Ary et al., 2019). Interviews were conducted by telephone on a one-on-one basis, and the focus group was held virtually. The researcher analyzed the data, identifying significant statements, and combined the data into themes to characterize the phenomenon (Creswell & Poth, 2018). To illustrate the phenomenon's overall essence, the researcher used textural and structural descriptions (Creswell & Poth, 2018). Using significant statements, themes, and textural and structural descriptions, the researcher collectively explored the meaning EMEs attributed to their instructional reasoning. In Chapter 3, the researcher will describe the methodology and research design in detail.

Delimitations

This study was delimited to five EMEs. Participants in this study worked in schools with large populations of minorities and taught in a public school with more than 40% of the student population identified as low-income in Florida. Participants were certified by the Florida Department of Education (FLDOE) in elementary education and met the minimum requirements by the FLDOE to teach in a public-school setting in Florida.

Limitations

Studies that target specific populations for investigation are not generalizable to other populations (Ary et al., 2019; Bordens & Abbott, 2011). Therefore, transferability to populations outside this context may be limited. Interviews were conducted virtually. Interviews were conducted through technology because of COVID-19—face-to-face opportunities were not available.

Assumptions

Two assumptions existed in this study. The researcher proceeded following the assumptions that (1) all participants answered all questions truthfully and (2) participant responses to interview and focus group questions were reflective of their instructional reasoning.

Definitions of Terms

- High-density Black schools are schools with a Black student population between 60 and 100% (Bohrnstedt et al., 2015).
- Horizon content knowledge is the ability to make connections with how topics interrelate with other topics taught later in the curriculum (Ball et al., 2008; Hill et al., 2008).
- Instructional reasoning is "the activity through which teachers attached their actions to the purpose that undergirds them" (Kavanagh et al., 2020, p. 3). The term instructional reasoning is used interchangeably with the term *pedological reasoning* in the literature (Kavanagh et al., 2020).
- Knowledge of content and students (KCS) is the knowledge that combines teachers' knowledge of mathematics and students (e.g., misconceptions students have about a

specific mathematics concept). It intertwines what teachers know about math content with the teachers' knowledge of common misconceptions and mistakes that arise when students interact with mathematics topics (Ball et al., 2008; Hill et al., 2008).

- Knowledge of content and teaching is the knowledge of mathematics and the knowledge of teaching (e.g., the teacher's ability to determine which instructional options would enhance comprehension of the content; Ball et al., 2008).
- Mathematical knowledge for teaching (MKT) is the mathematical knowledge and skill needed to teach mathematics (Ball et al., 2008).
- Pedagogical content knowledge (PCK) is the "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (Shulman, 1987, p. 8). Furthermore, it is the knowledge that aids teachers in understanding what makes specific content difficult or easy (Shulman, 1986).
- Specialized content knowledge (SCK) is the unique knowledge required to teach mathematics (Ball et al., 2008).
- Subject matter knowledge (SMK) is "the amount of and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p. 9).

Summary

This phenomenological study sought to describe the universal essence teachers assigned to their instructional reasoning through structured interviews. The participants were elementary mathematics teachers who taught mathematics in high-density Black schools. Previous investigations of mathematical knowledge in teaching explored teacher mathematical knowledge, student outcomes, or other related factors (Hoover et al., 2016). To date, little to no mathematical knowledge for teaching research focused on EMEs or sought to understand the meaning EMEs attribute to their instructional reasoning.

Organization of the Manuscript

This study is organized into five chapters, references, and appendices. Chapter One includes a description of the study background, a statement of the problem, the study's purpose, the significance of the study, definition of terms, the conceptual framework, research questions, the methodology, delimitations, and study assumptions. In Chapter Two, the researcher reviews the literature on the conceptualization of teachers' mathematics knowledge, the emergence of mathematical content knowledge modifications, and mathematical knowledge contributions for teaching. In Chapter Three, the researcher explains the research design, instruments, the methodology process, the sample and sampling method, the data collection process and procedure, and the data analysis process. Chapter Four includes an analysis of the study data. In Chapter Five, the researcher discusses the findings of the study, implications of the findings, and recommendations for future research. The study concludes with references and appendices.

CHAPTER TWO: REVIEW OF THE LITERATURE

For over three decades, the U.S. federal government has enacted multiple education reform efforts to safeguard its global position in the international marketplace, encompassing goals to address issues of education equality. However, many education reform initiatives fail (Cheong Cheng, 2020). Literature shows Black students in high-density Black schools are at a greater risk of underachievement in mathematics than their White counterparts (Bohrnstedt et al., 2015; Mickelson et al., 2013), and their underperformance negatively impacts the U.S. gross domestic product (Auguste et al., 2009). Additionally, parallel to the mutual goal of education equality, education reform initiatives sought to increase the overall academic achievement of all students in the United States (Every Student Succeeds Act [ESSA], 2015; Goals 2000: Educate America Act; NCLB, 2002). Little to no research has focused on the lived experiences of EMEs in high-density Black schools with average mathematical knowledge for teaching. In this literature review, the researcher describes the importance of mathematics achievement, education reform efforts to improve equality and academic achievement, the history of the Black–White achievement gap, the research around the emergence and conceptualization of MKT, and the relationships among teacher mathematical knowledge, student achievement, and instructional reasoning.

Importance of Mathematics and Education Reform History

For over 30 years, policymakers have enacted federal education reform initiatives to elevate the academic achievement of U.S. pupils in mathematics and to gain a competitive advantage in the global marketplace (Wang et al., 2010). However, students in the United States have consistently underperformed other industrialized nations in mathematics (PISA, 2018), which threatens the economic health of the United States (Achieve, 2013; Harbour et al., 2018; Auguste et al., 2009; Mickelson et al., 2013). The Organization for Economic Co-operation and Development (OECD) is an internationally recognized organization that provides a range of social, economic, and environmental evidence-based solutions to improve policies (OECD, n.d.). The OECD conducts a worldwide study every three years, the Program for International Student Assessment (PISA), which measures and ranks 15-year-old students' average performance across OECD-participating countries in mathematics, reading, and science. In 2018, PISA ranked the United States 39th out of 70 OECD countries in mathematics, which was similar to findings from 2012 and 2003. The United States was ranked poorly by PISA, which identified the United States as below average in mathematics performance. In 2012 and 2003, over one-fourth of U.S. students scored below baseline proficiency in mathematics, and an insignificant number of students ranked as high-performing (PISA, 2018).

Similar to PISA, the National Assessment of Educational Progress (NAEP) measures, analyzes, compares, and ranks students' performance in the United States in reading and mathematics (National Center for Education Statistics [NCES], n.d.). However, the NAEP is "congressionally mandated" and administered by the U.S. Department of Education's NCES within the Institute of Education Sciences (NCES, n.d., para 2.). It is the only assessment that measures U.S. students' performance by state and provides performance outcomes disaggregated by geographic location and racial, ethnic, and socioeconomic status. The NCES administers the NAEP in reading and mathematics every two years. A relative sample of the U.S. population representing the fourth, eighth, and 12th graders are assessed (NCES, n.d.). In 2019, the NAEP found no significant change since the administration of the 2017 NAEP in the percentages of fourth and eighth graders performing at or above NAEP proficiency in mathematics (NCES, n.d.).

To be economically competitive, U.S. citizens must be mathematically competent at a global level (Mickelson et al., 2013; Wang et al., 2010). Current mathematics findings suggest that U.S. citizens are not mathematically literate, which places the U.S. workforce and economy at risk (Wang et al., 2010). Occupational demands for personnel in the science, technology, engineering, and mathematics (STEM) fields are proliferating at ratios approximately twice the rate of non-STEM related professions (Achieve, 2013). Equally important, over 50% of U.S. occupations require practical knowledge and application of skills in arithmetic, algebra, geometry, calculus, and statistics, including 45% of low-skilled jobs, approximately 50% of middle-skilled jobs, and over 80% of high-skilled jobs (Achieve, 2013). With such a demand for mathematics proficiency, the United States cannot afford its students to graduate from high school with inadequate mathematics skills. According to Achieve (2013), a bipartisan group of governors and business leaders leading the effort to support states in making college and career readiness a priority for all students, improving the performance of U.S. students has the potential to increase the country's gross domestic product by 36%. Nevertheless, despite the overwhelming demand and the benefits to the economy, the United States has been unable to create a workforce with the necessary skills (Hanushek et al., 2010; Wang et al., 2010).

Unfortunately, these paradigms are not novel. The United States has known about its pupils' inadequate mathematics performance and the potential impact this has on its economy for decades. In 1981, the Secretary of Education, Terrell Howard Bell, created the U.S. National Commission on Excellence in Education (NCEE; 1983). The NCEE was responsible for the examination of the quality of education in the United States. The NCEE investigation results yielded a report titled *A Nation at Risk: The Imperative for Educational Reform* (Gardner et al., 1983). In the report, NCEE highlighted the following national risk indicators:

- A 72% increase in remedial mathematics coursework in public 4-year colleges.
- One-quarter of all mathematics courses taught in public 4-year colleges were remedial courses.
- The average performance of high school students was lower than students' performance when Sputnik launched over two decades earlier in 1957.
- There were complaints about millions of dollars in costly remediation of basic skills in reading, writing, spelling, and computation from military and corporate leadership.

The outcome of the investigation indicated that the decline in the U.S. standing was associated with four inadequacies in education processes: (a) content, (b) expectations, (c) time, and (d) teaching (NCEE, 1983). The report fueled a need for education reform and encouraged federal involvement in supporting students at risk for low academic achievement, higher education and research, and civil rights safeguards (Strauss, 2018). As a result, the findings became the foundation for many education and legislative mandates following publication (e.g., Goals 2000: Educate America Act, NCLB, American Recovery and Reinvestment Act of 2009).

Goals 2000: Educate America Act (1994) was passed by Congress in 1994. The purpose of the Goals 2000: Educate America Act was to improve teaching and learning quality. It was enacted to provide a national framework for education reform that would encourage research consensus, build the development of national standards and teaching certifications, and promote systematic change for equitable educational opportunities and high achievement for all students (Goals 2000: Educate America Act of 1994, HR.1804, Cong. § 103). The Goals 2000: Educate America Act (1994) had eight goals to be met by the year 2000:

- All students would enter school ready to learn.
- A national high school graduation rate equal to or greater than 90%.
- "All students will leave grades four, eight, and 12 having demonstrated competency over challenging subject matter, including English, mathematics, science, foreign languages, civics and government, economics, arts, history, and geography, and every school in America will ensure that all students learn to use their minds well, so they may be prepared for responsible citizenship, further learning, and productive employment in our Nation's modern economy" (Goals 2000: Educate America Act of 1994, p. 10).
- Continuous learning opportunities for teachers to develop the skills and knowledge needed to educate and prepare the nation's students for the next century.
- The nation's students would rank first in mathematics and science achievement.
- One hundred percent of the nation's adults would be literate and possess the knowledge and skills needed to compete in the global economy and exercise the rights and responsibilities of citizenship.
- The nation's schools would be free of drugs, violence, unlawful presence of weapons, and alcohol and provide an environment conducive to learning.
- Schools would collaborate with stakeholders to increase parent participation and involvement to foster students' social, emotional, and academic growth (Goals 2000: Educate America Act of 1994).

Earley (1994) explained that one of the foundational goals of the Goals 2000: Educate America Act was to identify universal guidelines for student achievement and public-school systems, which caused controversy in regard to federal entities creating policies around "performance" and "opportunity-to-learn" standards (p. 4). Earley (1994) further explained that the ratification of the Goals 2000: Educate America Act of 1994 affirmed the National Education Goals as federal policy, setting precedence for subsequent education legislation associated with one or more of the eight National Education Goals.

The Elementary and Secondary Education Act (ESEA; 1965), reauthorized and retitled in 2002 as NCLB 2002 and reauthorized again in 2015 retitled the Every Student Succeeds Act (ESSA), was seen as a dramatic federal improvement in K–12 education (Heise, 2017). The purpose of NCLB was to ensure that the nation's students had an equitable and impartial opportunity to meet rigorous academic standards. The law supported standard-based education reform and established high accountability standards for states, schools, and teachers on the belief that measurable goals would improve education (Education Commission of the States, 2002). Implemented in 2002, NCLB was written to accomplish its purpose through systems of accountability (e.g., teacher assessment, student assessment); teacher preparation and development; curriculum and instruction resource alignment to academic standards; monitoring of students' progress with common benchmarks for student achievement; meeting the needs of low-performance students in our highest-poverty schools (e.g., second language learners, students with disabilities); closing achievement gaps among minority and nonminority students, disadvantaged and advantaged students, and high- and low-performing students; increased autonomy and flexibility for decision makers, schools, and teachers for greater responsibility; advanced and accelerated education programs; improving instructional quality through staff professional development; coordinating services; and providing parents with valuable opportunities to engage in their students' education (NCLB, 2002). These provisions expanded the federal role in public education by executing a system of accountability for student achievement (Heise, 2017; Klein, 2015).

According to Jacob (2017), the central focus of the NCLB was to resolve racial, ethnic, and socioeconomic inequities among subgroups of students. Nevertheless, it is important to note that NCLB was not solely focused on racial and ethnic differences but also emphasized socioeconomic disparities among different ethnic and racial groups. One of the legislation's goals was to improve the performance of disadvantaged populations, thereby closing achievement gaps by 2014 that have long persisted for students according to their race, ethnicity, and socioeconomic group (Jacob, 2017). To monitor school progress against these goals, schools were required to achieve adequate yearly progress (AYP) in all subgroups (e.g., race, students with disabilities, lowest-achieving students within the school). Schools unable to attain AYP were sanctioned (Heise, 2017; Klein, 2015).

Additionally, beginning in the 2002–2003 school year, schools were mandated to hire *highly qualified* teachers. Individuals received the highly qualified classification after passing teacher certification examinations and earning a bachelor's degree (Klein, 2015). Despite the consequences to schools and the highly qualified teacher requirements, districts struggled to meet AYP. By 2010, it was evident that many schools would not meet the NCLB mandates; therefore, the Obama administration offered waivers for many of the prescriptive mandates of NCLB (Jacob, 2017; Klein, 2015).

In 2015, the legislation originally coined ESEA (1965) and reauthorized as NCLB (2002) was reauthorized as ESSA. With the passage of ESSA, the commitment to equal access to quality education for all students was renewed by the United States (U.S. Department of Education, n.d.). Many of the prescriptive mandates of the NCLB were abandoned (Heise, 2017). The two federal legislations have distinct differences. In ESSA, more local control was given to the states, allowing states to create, assess, and measure academic standards, and it eliminated federal consequences to schools unable to meet AYP. Nevertheless, the enactment of ESSA did not abolish all federal oversight. The act provided provisions for federal supervision of high schools with low graduation rates and the lowest five percent of each states' schools (Heise, 2017).

The Goals 2000: Educate America Act of 1994, ESSA, and NCLB represent three federal education reform efforts to improve the academic achievement of the nation's students over the last 25 years. However, students in the United States continue to underperform compared to their international counterparts, especially students navigating poverty and those from historically underserved racial, ethnic, and linguistic groups.

Over three decades have passed since the NCEE made a plea for a public commitment to education reform that provides high-quality educational opportunities that yield the possibility of lucrative employment options, resulting in productive participating members of society serving interests of self and society, regardless of race, class, or economic status (NCEE, 1983). The NCEE stated:

> We do not believe that a public commitment to excellence and educational reform must be made at the expense of a strong public commitment to the equitable treatment of our diverse population. The twin goals of equity and high-quality schooling have profound and practical meaning for our economy and society, and we cannot permit one to yield to the other either in principle or in practice. To do so would deny young people their chance to learn and live according to their aspirations and abilities. It also would lead to a generalized accommodation to mediocrity in our society on the one hand or the creation of an undemocratic elitism on the other. (NCEE, 1983, p. 13)

Since the publication of *Nation at Risk*, multiple federal education reform mandates were enacted that pledged to improve the achievement of the nation's pupils through quality and equitable education reform efforts.

Despite the educational reform efforts of the United States, its pupils have made minimal progress in the last 20 years (Hanushek et al., 2010; Mickelson et al., 2013). Present-day students in the United States still underperform pupils in other industrial nations (PISA, 2018). With students' performance in mathematics predictive of the nation's economic competitiveness, education reform efforts must produce students with strong mathematics abilities to improve the economic trajectory of the United States (Hanushek et al., 2010; Mickelson et al., 2013).

History of Black Education and the Black–White Achievement Gap

After the Civil War, the United States Congress passed the 13th, 14th, and 15th Amendments, also known as the reconstruction amendments. The reconstruction amendments abolished slavery and supplied former slaves the rights of citizenship, equal protection of the law, and right to vote. Nevertheless, Whites resented Black equality and used brutal and oppressive tactics to undercut Blacks' provisions under the reconstruction amendments. During the next 30 years, following the enactment of the reconstruction amendments, rulings of the Supreme Court annulled many federal law protections (e.g., *Berea College v. Kentucky* [1908]). *Plessy v. Ferguson* (1896) set a judicial precedent for the *separate but equal* doctrine endured by Blacks during the Jim Crow era. *Plessy v. Ferguson* (1896) was the first test of the 14th Amendment equal protection clause and upheld the constitutionality of separate but equal facilities based on race. The ruling prohibited racial integration within public transportation, schools, and facilities. Therefore, by the 1900s, schools were segregated entirely by race.

In 1954, *Brown v. Board of Education of Topeka* (1954) sparked new legislation and became the basis for educational inequality case law. *Brown v. the Board of Education of Topeka* is credited as the "impetus" in the movement for equality in education (Osborne & Russo, 2007, p. 7). The ruling in *Brown v. Board of Education of Topeka* deemed segregation within public schools unlawful; therefore, it legally ended segregation based on the race-setting precedent that the clause—separate but equal—used in the *Plessy v. Ferguson* case violated the equal protection clause of the 14th Amendment (Forte, 2017).

Following the decision of *Brown v. Board of Education of Topeka*, the ESEA was enacted to support the needs of students who had been educationally underserved (Forte, 2017). The enactment of ESEA is a seminal event in education legislation history and addressed the educational inequalities for economically disadvantaged children in the United States. It is an extensive statute that sanctioned funds for primary and secondary education (Paul, 2016). Title I, a provision of the ESEA, allocated 83% of its budget to schools and school districts with high percentages of students from low-income households. The purpose of Title I was to improve students' achievement in the core subject areas of reading, mathematics, and writing for students in urban and rural school systems (Paul, 2016).

The following year, (1955), the Office of Education of the U.S. Department of Health, Education, and Welfare appointed James Coleman and colleagues to complete an investigation of educational opportunity in the United States (Coleman et al., 1966; U.S. Department of Education, 2016). The outcome of his inquiry, *The Coleman Report: Equality of Education Opportunity*, set precedence for racial and social gaps in achievement, now known as *the achievement gap* (U.S. Department of Education, 2016).

The academic achievement gap between different races has been an area of concern in the United States for decades. Much of the achievement gap research has focused on one of three constructs: (a) poverty gap, (b) income gap, or (c) race and ethnicity gap (Kuhfeld et al., 2018). Depending on the theory, descriptions of student achievement progress vary from narrowing to widening. Kuhfeld et al. (2018) investigated the relationship between race and ethnicity and poverty gaps. Kuhfeld et al. (2018) found no evidence existed that the Black–White achievement gap has narrowed. Additionally, they explained that the interaction between race and income revealed alternative explanations for the perceived Black–White achievement gap decreases, and past investigations of the student achievement gap used one-dimensional examinations of race, ethnicity, or poverty; therefore, research findings led to the conclusion that the race and ethnicity gap, or poverty gap, has narrowed over time. When researchers considered all three factors, the achievement gap had not narrowed over time (Kuhfeld et al., 2018). As a result, the single dimension analyses of race and ethnicity or poverty may draw pseudo claims to the progress of equity in education (Kuhfeld et al., 2018). Furthermore, Kuhfeld et al. (2018) explained additional findings that revealed performance gaps between poor White students and poor Black students, which refuted the notion that income is a proxy for resources, and suggested that resources were not evenly distributed by race and ethnicity.

Mickelson et al. (2013) conducted a study to investigate the relationship between the racial composition of schools and students’ mathematics achievement. Mickelson and colleagues (2013) performed a meta-regression analysis of 25 studies and controlled for the students’ socioeconomic status. Their findings revealed a statistically significant and negative relationship between densely populated minority schools and students' mathematics achievement. However, the negative relationship was not examined because it was outside of their investigation (Mickelson et al., 2013).

Bohrnstedt et al. (2015) investigated the relationships among Black student density, the percentage of Black students within a school population, the Black–White achievement gap, and student achievement for NCES. Bohrnstedt and colleagues (2015) used the 2015 Mathematics Grade 8 Assessment NAEP data and the Common Core of Data to analyze the effects of Black student density on the Black–White achievement gap (Bohrnstedt et al., 2015). An analysis of the data revealed that nationally, on average, Black students scored 30–37 percentage points below their White peers, and the achievement of Black and White students was lower in high Black density schools while the Black–White achievement gap was not.

When accounting for socioeconomic status, educator, pupil, and school characteristics, Bohrnstedt et al. found the following:

- The higher the Black student density was, the more significant the Black–White achievement gap was.
- The lower the Black student density was, the smaller the achievement gap was.

Furthermore, numerous investigations of the Black–White achievement gap have found discrepancies in the performance of Black and White students (e.g., Braun et al., 2010; McDonough, 2015).

After the publication of the *Coleman Report: Equality of Educational Opportunity* (1966), various federal education reform laws (e.g., NCLB, ESSA) were enacted to close the achievement gap and improve the academic achievement of all students within the core content areas of reading, mathematics, science, and social studies. However, it is important to note, the global standing of the United States is dependent on the mathematics performance of its pupils

(Achieve, 2013; Hanushek et al., 2010; Wang et al., 2010), and federal education reform efforts have been unsuccessful in improving the global competitiveness of its pupils in mathematics and in closing the achievement gap that persists among Black and White students in the United States.

The Emergence of Knowledge Domains

Educational literature supports the claim that strong content knowledge is a foundational component of teacher knowledge (e.g., Ball et al., 2008; Baumert & Kunter, 2013; Charalambous & Hill, 2012; Copur-Gencturk, 2018; Goodwin & Kosnik, 2013; Hoover et al., 2016). As such, investigations of teachers' knowledge have increased, and teacher knowledge analysis has led to teachers' knowledge distinctions. Thus, content knowledge, PCK, and generic pedagogical knowledge exist in educational literature as distinctive dimensions of teacher knowledge (Baumert & Kunter, 2013).

Pedagogical Content Knowledge

Shulman (1986) published *Those Who Understand: Knowledge Growth in Teaching*, in which he discussed the need for a theoretical framework for understanding the complexities of teacher knowledge. He argued that there are distinct categories of knowledge for teaching. In this publication, Shulman (1986) explained the importance of distinguishing between content knowledge categories, establishing PCK as the knowledge that builds on SMK and general knowledge. Shulman (1987) continued this line of research, further discussing teacher knowledge classifications, and he identified eight categories:

- content knowledge

- general pedagogical knowledge
- curriculum knowledge
- PCK
- knowledge of learners and their characteristics
- knowledge of educational contexts
- knowledge of educational ends, purposes
- values, and their philosophical and historical grounds

Shulman (1987) concluded PCK is "a special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 8), which is likely to provide insights into the differences between content specialists and educators. Shulman's work, focused on PCK, continued (e.g., Shulman, 1998; Shulman, 2000; Shulman, 2013) and endured as the most well-known classification of teacher knowledge (Hurrell, 2013).

Although the educational research community generally accepts the concept of PCK, several researchers have challenged, modified, and elaborated on the concept (e.g., Hurrell, 2013; Copur-Gencturk et al., 2018; Hodgen, 2011). In the Shulman (1987) article, *Knowledge of Teaching: Foundations of the New Reform*, he encouraged further investigation of what he referred to as the "*wisdom of practice,"* that is, the real-world pedagogical knowledge of teachers (p.8). Shulman (1987) concluded that teachers, from novice to master, provide an extensive opportunity to understand pedagogical knowledge. He argued that future research was needed to collect, assemble, and interpret teachers' pedagogical knowledge to create case literature and codify the principles of teachers' knowledge to use as exemplars (Shulman, 1986, 1987, 1998). In his view, knowledge for teaching was not rigid or absolute and, therefore, required continuous exploration, discovery, and refinement (Shulman, 1987).

Mathematics Pedagogical Content Knowledge

In response to Shulman's call for action, researchers have attempted to conceptualize teachers' mathematical knowledge. Fennema and Franke (1992), *The Knowledge Quartet*, and Ball et al. (2008), *mathematical knowledge for teaching framework*, are examples of mathematics knowledge frameworks. The same theoretical foundation, Shulman's PCK, inspired each of these models.

In 2005, Rowland et al. published *Elementary Teachers' Mathematics Subject Knowledge: The Knowledge Quartet and the Case of Naomi*, which reported findings from the Subject Matter Knowledge in Mathematics project (Rowland et al., 2005). The knowledge quartet is context-oriented and emerged to support teachers' mathematical knowledge development. The framework's goal was to reflect on mathematical teaching and knowledge for enhancement (Rowland, 2013; Turner & Rowland, 2011). Turner and Rowland (2011) stated that the knowledge quartet could "be an effective tool in developing teachers' mathematical content knowledge through focused reflections on their mathematics teaching" (p. 211). Unlike other models, the focus of the knowledge quartet is mathematical knowledge development. While not the primary focus of this research study, it offers one example of the variation of mathematics knowledge-based conceptualizations.

Two other models, the Fennema and Franke (1992) model and the Ball et al. (2008) mathematical knowledge for teaching model have a similar purpose—these models attempt to clarify and expound upon Shulman's knowledge classifications for teaching in mathematics. Fennema and Franke (1992) developed a mathematical knowledge model that expanded and modified Shulman's original framework. Their model identified educators' knowledge needs for teaching mathematics and addressed context within the classroom. The Fennema and Franke

(1992) model included four categories: (a) knowledge of content, (b) knowledge of pedagogy, (c) knowledge of student cognition, and (d) teacher beliefs, and all of the components were described as equally important for effective mathematics teaching. Furthermore, in the Fennema and Franke (1992) model, teacher practices were contingent upon teacher beliefs about student thinking. Teacher knowledge was developed through interactions with students and subject matter content (Fennema & Franke, 1992). Later researchers interpreted this to indicate teacher knowledge is both “interactive and dynamic” (Petrou & Goulding, 2011, p. 14). Finding the measures of this model is challenging because researchers have yet to discover a methodological process to measure this model (Petrou & Goulding, 2011).

A team of researchers from the University of Michigan developed the mathematical knowledge for teaching framework under the direction of the Mathematical Teaching and Learning Project and the Learning Mathematics for Teaching project (LMT; 2019; Petrou & Goulding, 2011). The framework elaborated on two of Shulman’s knowledge-based classifications: (a) content knowledge and (b) PCK (Petrou & Goulding, 2011). In the mathematical knowledge for teaching model, Shulman’s content knowledge classification was segmented into two parts: (1) common content knowledge, the mathematical knowledge used in teaching and other professions, and (2) specialized content knowledge (SCK), the unique knowledge used to teach mathematics. PCK was partitioned into two different parts: (1) knowledge of content and students (KCS) and (2) knowledge of content and teaching (Ball et al., 2008; Petrou & Goulding, 2011). The mathematical knowledge for teaching framework was a practice-based theory of mathematics knowledge for teaching and was designed to validate Shulman’s work by developing “reliable and valid measures of mathematical knowledge for teaching” (Petrou & Goulding, 2011, p. 15). The mathematical knowledge for teaching framework advanced the agenda for mathematics teacher knowledge (Copur-Gencturk et al., 2018; Petrou & Goulding, 2011). Therefore, the mathematical knowledge for teaching framework is one of the most widely used frameworks.

The mathematical knowledge for teaching framework arose as an evolutionary response to the emergence of PCK to better measure the capability of teachers in effectively delivering mathematics instruction (Ball et al., 2008). The instructional decisions of teachers are complex processes tied, in part, to their levels of mathematical knowledge. Teacher-level factors, such as teacher knowledge, affect their instructional decisions and practices. As described in this chapter, there has been very little research that seeks to explore the intersection of those factors as it relates to teachers' mathematical knowledge.

Hoover et al. (2016) conducted a review of the literature for "distinctive mathematical knowledge requirements for teaching" (p. 4). The authors searched six databases for empirical peer-reviewed journals, using broad search terms for mathematics, content knowledge, and teaching. Of the 3,000 articles found, 190 addressed the specific mathematics knowledge needs for teaching. Two researchers coded and evaluated five elements of each article: (a) study genre, (b) research problem, (c) study variables, (d) causality process and procedure, and (e) findings. As a result, three domains emerged: (a) the structure of mathematical knowledge for teaching, (b) improving teachers' mathematical knowledge for teaching, and (c) contributions of mathematical knowledge for teaching (Hoover et al., 2016). The mathematical knowledge structure for teaching includes the various definitions and conceptions of mathematical knowledge for teaching and the relationships among these conceptualizations and other variables. The domain for teacher improvement included teacher professional development, education programs, curriculum, practices, knowledge for teaching development, and how to improve teaching and learning. The last domain, contributions of mathematical knowledge, encompassed questions about if and what contributes to student learning or teacher practice (Hoover et al., 2016). Hoover and colleagues argued that the field continued to struggle with developing meaningful measures of mathematical knowledge for teaching because of challenges with competing views on (1) conceptualizations of knowledge; (2) knowledge relationship, use, and achievement; (3) ways to measure claims about professional knowledge; and (4) shared definitions as well as an inadequate body of knowledge (Hoover et al., 2016). As a result, Hoover et al. (2016) suggested three research priorities enhance the MKT body of knowledge: (1) focused studies on developing an understanding of what MKT is, how it is acquired, and what it does; (2) investigation methods; and (3) studies of "mathematical knowledge for fluent and equitable teaching" (Hoover et al., 2016, p. 25). Considering the findings of Hoover et al. (2016), additional MKT investigations will be important to clarify, deepen, and advance the understanding of MKT and its impact on mathematics teaching and learning.

Mathematical Knowledge for Teaching

Within the MKT research base, the most frequently addressed question is the investigation of how teachers' MKT impacts student achievement (Hoover et al., 2016). Ball et al. (2005) conducted an evaluation of 700 first- and third-grade teachers and approximately 3,000 students to determine the effect of teachers' specialized knowledge for teaching and content knowledge for teaching on gains in student performance in mathematics. The investigation revealed that teachers' specialized knowledge for teaching and content knowledge for teaching significantly predicted student academic growth in mathematics (Ball et al., 2005). Additional researchers have investigated the impact of differing levels of MKT on student achievement and found that students of teachers with high levels of MKT demonstrated higher gains in achievement outcomes (e.g., Ball et al., 2008; Ball et al., 2005; Ottmar et al., 2015).

Nevertheless, investigations of teachers’ MKT and students’ achievement are not definitive. Hatisauri and Erbas (2017) conducted a study on teachers’ mathematical knowledge related to teaching functions and student learning outcomes. Similar to the Ball et al. (2005) findings, connections were found between teachers’ MKT and student achievement. However, Hatisauri and Erbas (2017) concluded that the relationship between student achievement and teachers’ MKT lacked direct associations because mediating factors may have contributed to the relationship. Teachers’ SCK and KCS appeared to influence the teachers’ instructional practices and the learning experiences provided to students, which, in turn, may have affected student achievement (Hatisauri & Erbas, 2017).

Investigations of the relationship between MKT, teachers’ instructional practice, and student achievement are the next most prominent MKT studies (Hoover et al., 2016). Examinations of MKT, instructional practice and student achievement relationships have yielded mixed results. Although some studies have revealed connections among these variables, others have not (Hoover et al., 2016).

Shechtman et al. (2010) investigated teachers’ mathematical knowledge, instructional practices, and student achievement. They conducted two separate experimental design studies. The first study focused on seventh-grade mathematics teachers, and the second focused on eighth-grade mathematics teachers (Shechtman et al., 2010). Similar to previously mentioned studies, Shechtman and colleagues (2010) used the mathematical knowledge for teaching framework to conceptualize teachers’ mathematical knowledge but measured only teachers’ content and SCK (Shechtman et al., 2010). Shechtman and colleagues (2010) measured teachers’

MKT over a two-year timeframe. Their investigation yielded mixed results—the authors found a relationship between teachers' MKT and student achievement in the first year of the study when the students were in seventh grade; however, no relationship existed between teachers' MKT and student outcomes at any point of measurement for eighth-grade mathematics teachers or in the second year of study for seventh-grade mathematics teachers (Shechtman et al., 2010).

Complementary to those findings, Ottmar et al. (2015) identified a statistically significant relationship between teacher instructional practices and student achievement but not between teachers' MKT and student outcomes. Based on these findings, Ottmar and colleagues (2015) argued that a possible distinction exists between teachers' MKT and their ability to use their mathematical knowledge to implement effective mathematics instruction (Ottmar et al., 2015).

Impact on Teacher Instructional Practices

Studies of MKT that have contributed to teacher instructional practice literature are the least prevalent in MKT research (Hoover et al., 2016). As previously noted, regardless of the investigative focus, researchers continuously grapple over how to conceptualize MKT, how to improve it, and how it influences student achievement and teacher practice. Similar to the aforementioned investigations, research into the relationship between teachers' instructional practice and MKT has also yielded mixed results (Charalambous & Hill, 2012). MKT and instructional practice studies (e.g., Ball et al., 2008; Charalambous & Hill, 2012; Copur-Gencturk et al., 2015; Hill et al., 2012; Ottmar et al., 2015) often use instructional quality measures to assess the relationship among these factors. These studies found teachers with higher levels of MKT produced lessons of higher quality, and teachers with lower MKT implemented lessons of poorer quality. However, the researcher only found this consistency when comparing extreme cases, such as when comparing teachers with low MKT to teachers with high MKT scores. When comparing teachers with average and low or average and high levels of mathematical knowledge, researchers were unable to draw the same conclusions (e.g., Ball et al., 2008; Charalambous & Hill, 2012; Copur-Gencturk et al., 2015).

Copur-Gencturk et al. (2015) conducted a longitudinal study to investigate the relationship between the change in MKT and instruction over a 3-year time span. She classified mathematics instruction into five categories: (1) inquiry-based lesson, (2) student engagement, (3) worthwhile mathematics tasks, (4) mathematics sense-making agenda, and (5) classroom climate (Copur-Gencturk et al., 2015). The study results revealed that, except for the engagement component of mathematical instruction, when changes in MKT were statistically significant, changes in instructional practices were statistically significant (Copur-Gencturk et al., 2015).

Summary

Current investigations of MKT suggest more investigations of teacher mathematical knowledge are needed to create a practice-based theory of teacher mathematical knowledge. Frameworks of mathematical knowledge lack uncontested definitions. Therefore, a common language for practice and investigation is unfounded, and findings are inconsistent. This makes codifying principles for teaching and learning unlikely. Furthermore, the present bodies of knowledge lack success in improving teachers' knowledge for teaching mathematics. To understand the influence that teachers' MKT has on their day-to-day practices, the research community needs additional explorations of teachers' MKT.

From this literature review, many gaps exist in the body of research on MKT. Despite the continually growing field of mathematical knowledge for teaching research, many factors are unknown. The studies described in this review focused on MKT improvement. Therefore, an exploration of MKT emphasized monitoring changes in the educators' knowledge, instructional skills, and student performance. As a result, nonsignificant and inconsistent research findings were common in these studies. These inconclusive findings may have been because of a variety of extraneous factors. While it may appear obvious that the improvement of teachers' MKT is vital to positively affect student learning outcomes, how teacher MKT interacts with the common experiences of teaching may be an interesting link. Furthermore, examining these interactions may support the codification of teacher practices and development of empirically supported teacher development programs and establish a consensus in mathematical knowledge classifications, definitions, and MKT models.

CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY

In this chapter, the researcher will present the methodology for this qualitative study. The study was conducted using a phenomenological design to explore the lived experiences of EMEs in high-density Black schools with average mathematical knowledge for teaching and to describe the essence of their instructional reasoning decisions. All participants in the study taught in high-density Black schools. The research design, data collection procedure, and data analysis process will be explained in this chapter.

Statement of the Problem

Over the last four decades, the United States federal government enacted several education reform initiatives to improve the education equality and the mathematical performance of Black students (e.g., Goals 2000: Educate America Act, NCLB, American Recovery and Reinvestment Act of 2009). Nevertheless, investigations of student achievement have continually reflected a gap between Black and White students' mathematics achievement (Bohrnstedt et al., 2015; Kuhfeld et al., 2018). Researchers have found that teachers' mathematics knowledge impacts student achievement; however, mediating factors limit direct relationships among the two (Hatisauri & Erbas, 2017). As a result, the research community would benefit from understanding the meaning EMEs ascribe to their instructional reasoning to identify factors mediating teachers' mathematical knowledge for teaching and to inform federal and state officials of factors that might drive policy and stakeholders interested in teacher development and training in high-density Black populations.

Research Design

For this study, the researcher used a phenomenological research design. Phenomenology is a qualitative research methodology. Qualitative research is carried out in natural settings (Creswell & Poth, 2018; McMillian, 2008). The researcher participates in data collection, which includes the participants' perspectives, and the researcher creates in-depth narratives through data analysis (Creswell & Poth, 2018; McMillian, 2008). Specific to phenomenological studies, the investigator emphasizes understanding the participants' personal perspectives through their conscious experiences. In order words, phenomenological researchers seek to build an understanding of the way participants "experience and give meaning to an event, concept, or phenomenon" (Ary et al., 2019, p. 410; Creswell, 2003).

A phenomenological design is a suitable qualitative research methodology when shared experiences are required to understand elements of a problem or to enhance policies or practices (Ary et al., 2019). In this study, the researcher used phenomenology to understand the instructional reasoning of EMEs. This addition to the mathematics education research base will aid in building an understanding of the factors that mediate teachers' MKT and inform the investigations of teacher's mathematical knowledge and provide the policymakers involved in education equality reform efforts the rationale to support mathematics training and development for teachers.

Population and Sample

The target population of this phenomenological study included all EMEs teaching at high-density Black Title I schools. Part A (Title I) of the Elementary and Secondary Education Act, as amended by the ESSA, classified schools with more than 40% of the enrolled student population as low-income under Title I. Ary et al. (2019) explained that the target population consists of the group to which the researcher will generalize research outcomes. The study participants were recruited from the accessible population (Ary et al., 2009; Ary et al., 2019). Within the framework of the target population, an accessible population was drawn, meeting the same definition upon which the target population was drawn. The accessible population included 422 elementary mathematics educators teaching in one of 12 high-density Black schools within an urban school district in Florida.

Sample

After determining the accessible population, the researcher used criterion sampling to identify participants. In this study, the researcher used criterion sampling to identify participants. The initial criteria were (1) having at least one year of teaching experience and (2) holding a state teaching certification in mathematics at the level they taught. To further delineate, a standardized questionnaire was used to identify potential participants for the study. The researcher completed training (see Appendix A) to administer the Mathematical Knowledge for Teaching Assessment (MKTA; Learning Mathematics for Teaching Project [LMT], 2008) to determine potential participants' MKT. The MKTA is a survey designed to measure teachers' MKT (LMT, 2019). Survey prompts represent the common problems associated with teaching mathematics. Survey questions included explanations to rule in mathematics, untraditional methods for solving mathematics, and exploring teachers' understanding of appropriate grade-level mathematics vocabulary (LMT, 2019). The MKTA results are presented as an item response theory (IRT) score. The range for IRT scores was three standard deviations above and below the mean. The researcher used IRT scores as the final criterion to classify potential participants into one of three MKT classifications: low (less than or equal to one standard deviation below the mean), average (greater than one standard deviation below the mean, and less than or equal to one standard deviation above the mean), and high (greater than one standard deviation above the mean). The educators selected for study participation scored in the average MKT range.

The MKTA was available in two implementation formats: (a) paper–pencil and (b) web-based. According to Bordens and Abbott (2011), web-based questionnaires increase accessibility and data collection speed, expanding the size of the data set. The researcher elected to administer the MKTA using the Teacher Knowledge Assessment System, the web-based method for all participants.

Potential participants (*n* = 114) received a notification by email one week before the survey was released to overcome a nonresponsive bias. A second notice was released on the first scheduled day of the study and included the link to the Teacher Knowledge Assessment System, directions for completing the MKTA online, the rationale for participant selection, confidentiality disclosure, point of contact to address questions, and the official signature of the researcher. On the first day of the third week of data collection, potential participants were administered the MKTA. The researcher sent a third announcement to reduce the number of nonresponsive study participants (Bordens & Abbott, 2011). At the end of the screening of possible participants, the researcher sent a thank you notice to all respondents (Bordens & Abbott, 2011).

Implementation of the criterion sampling procedure led to a group of qualified possible participants (n=15). All 15 potential participates received an invitation to participate in the second phase of the study, interviews, or focus groups; five agreed to participate. The final sample included five EMEs in a large urban school district in Florida with a Title 1 classification. Teachers were certified by the FLDOE for an approved certificate in mathematics (e.g., elementary education, middle grade mathematics, mathematics) with one or more years of teaching experience. Teachers who were not selected to participate in a one-on-one interview ($n = 15$) were selected to participate in a focus group.

Research Question

The following research question was developed to drive this phenomenological study: What meaning do mathematics educators in high-density Black schools, with average mathematical knowledge for teaching, ascribe to their instructional reasoning?

Data Collection and Procedures

Having selected the study sample, the researcher requested and obtained approvals from the University of Central Florida's Institutional Review Board (IRB; see Appendix B) and the school district in which the research was to be conducted before beginning data collection. The National Research Act of 1974 mandated a sanctioned board review all human research and safeguard subjects (Ary et al., 2009; Ary et al., 2019). To fulfill this directive, institutions of higher education have an IRB. The IRB is appointed to monitor research involving human subjects. Three fundamental principles guide their work: (a) to protect participants from mental or physical harm, (b) to protect the subjects' right to know the context of the study and consent or to withhold participation, and (c) to ensure respect to participants' privacy (Ary et al., 2019; Ary et al., 2009). Once approved, the researcher submitted a request to the school district IRB for inclusion in the study. Upon receiving approval, the study began.

Informed Consent

The researcher provided and reviewed the informed consent with participants before the interviews began (see Appendix C). The researcher provided participants with the option to withdraw from the study before, during, or after the interview process, and all signed consent forms were obtained from subjects before proceeding with interviews. Upon receiving signed informed consent forms and validating the interview questions, the researcher interviewed the selected participants.

Participant Classification

The researcher used the results from the MKTA to classify teachers by one of three levels of MKT: (1) low, (2) average, or (3) high. After classification, teachers who demonstrated average MKT were placed on a list and randomly sorted using Random.org, a randomizing list website. The researcher used list positions for participant selection. As an example, the first educator on the list, identified as participant one, was interviewed first, and the second teacher, identified as participant two, was interviewed second. Teachers who were not selected to participate in a one-on-one interview (n = 15) were selected to participate in a focus group. Due to the limitations imposed by restrictions related to COVID-19, a non-probability sampling method was used. The participate selection was based on potential participates availability. Ary et al. (2019) stated non-probability might be the only option available to researchers, and in such a situation, convenience sampling provides a viable option to obtain the data.

Data Analysis

The researcher followed a data analysis procedure as outlined by Ary et al. (2019) to analyze the qualitative data set. The qualitative data analysis consisted of three steps: (a) comprehension and organization, (b) codification and categorization, and (c) interpretation and representation (Ary et al., 2019).

During the first step of the analysis, the researcher organized and read participant transcripts to improve understanding. First, the researcher organized transcripts by participant and type (i.e., interviews vs. focus group). Next, the researcher read each participant's transcripts multiple times, in order of the interviews, for comprehension. Then, the researcher read the focus group transcription multiple times for comprehension.

During the second step of the analysis, the researcher coded and categorized the transcripts. Coding is the staple of qualitative analysis. In phenomenological research, coding is essential for identifying the essence of the phenomenon. During coding, categories and themes develop within the data set and are consistently refined. In this study, the researcher used open coding. Open coding simplifies and categorizes qualitative data into manageable chunks (Ary et al., 2019). To begin, the researcher read each line of transcription and coded important or repeated phrases, words, sentences, thoughts, activities, and behaviors. Then, the researcher merged units with the same codes into clusters. The researcher coded, categorized, and reviewed clusters to ensure relatedness. Then, the researcher combined the clusters into significant categories to determine the emergent themes for the data set. The researcher repeated the same process for every interview and focus group transcription. Next, the researcher compared and selected common emergent themes from each data set (i.e., interviews and focus group) to interrupt and represent the data set.

In the final step, the researcher interpreted and represented the data set. According to Ary et al. (2019), qualitative data interpretation is not confined to a specific procedure. However, data interpretation is supported by what is known through data and addresses misconceptions, questions unconfirmed knowledge, and highlights discoveries (Ary et al., 2019; Creswell & Guetterman, 2019; Creswell & Poth, 2018).

Finally, the researcher represented the phenomenon through textual and structure descriptions. Ary et al. (2019) and Creswell and Poth (2018) explained that, when reporting on themes, topics, or cases, descriptive details are frequently used to represent the data sets in qualitative studies. In this study, the researcher used textual and structural descriptions to illustrate the meaning and essence that EMEs ascribe to their instructional reasoning.

Reliability and Validity

In qualitative studies, validity and reliability are conveyed through four principles of trustworthiness: (a) creditability, (b) transferability, (c) dependability, and (d) confirmability. In this study, the researcher used all four principles to improve the trustworthiness of the research findings.

The integrity of a study depends on the researcher's ability to ensure that the data collected are credible. Credible data are truthful and accurate. In this study, the researcher used member checking to establish study credibility. Member checking is a process used by researchers to share developing conclusions with participants to prompt participant feedback (Lochmiller & Lester, 2017). Then, participants reviewed the transcripts and the researcher's data interpretations for accuracy. The researcher revised inaccuracies and interpretations to ensure the findings accurately conveyed the participants' viewpoints.

In qualitative research the goal is transferability, not external validity. Transferability is the extent to which the study findings are generalizable to other groups or contexts. In this study, the researcher developed rich descriptions of participants' settings, backgrounds, and self-reflections to increase the study transferability. According to Ary et al. (2019), transferability is increased when sufficient details of the study context and cases are available and reactivity is limited. Additionally, the researcher used a cross-case comparison and a detailed methodology. Detailed methodologies increase the transferability of qualitative studies because a rich description provides potential investigators with data to analyze whether the study context is transferable to their setting and population. When findings are comparable, cross-case comparisons increase transferability (Ary et al., 2019).

Dependability is defined as the constancy or stability of research findings (Ary et al., 2019). Dependability is similar to reliability in quantitative research. However, in qualitative studies, dependability refers to monitoring or explaining the difference between data findings. In this study, the researcher used data triangulation and the coding agreement strategy, code-recode, to ensure dependability. The researcher coded every transcription twice and compared each transcript for similar interpretations.

Additionally, the researcher triangulated data by collecting the data from focus groups and interviews for multiple participants to improve the study's dependability. The researcher compared and selected common emergent themes from interviews and the focus group to interrupt and represent the data set. The same strategies were used for confirmability. Confirmability in qualitative research ensures that data interpretations are free from bias (Ary et al., 2019). The researcher used the code-recode method to confirm the procedures and interpretations were free from bias. Additionally, bracketing, a process commonly used in phenomenological studies, was used to ensure the researcher's personal biases did not affect the study.

Interviews and Focus Groups

The researcher used semi-structured interview questions for data collection. Interview questions were reviewed by a group of experts using the Delphi technique and are included in Appendix D. The researcher recorded interviews using a digital recorder, an Olympus VN-S41PC, and used Rev.com, a transcription service to transcribe the audio files. Rev.com completed a nondisclosure agreement before the interview data transmission to their platform. The uploads were username and password protected. Transport Layer Security 1.2., a feature of Rev.com, was used to encrypt the files and communication between the researcher and Rev.com (e.g., emails and audio files). Additionally, a two-factor authentication was an added security measure.

The interview and focus group settings and times were arranged at the convenience of the participants. Interviews and focus groups were conducted virtually. The participants confirmed the room conditions were conducive for an interview with minimal background noise and free from distractions. The researcher used member checking to ensure that the data collected was reflective of the participants' perspectives. Based on the results of member checking, participants' responses were amended to accurately convey their viewpoints.

The researcher categorized data by collection method (e.g., interview, focus group) and participant number, if applicable). Then, interview and focus group data sets were compared. The researcher used themes in common among interviews and the focus group to represent and interrupt the data set. The researcher used data encryption and a secure password-protected workspace to protect the privacy of the participants. The researcher will store all data securely for five years and destroy all data after five years.

Limitations

Studies that target special populations for investigation are not generalizable to other populations (Ary et al., 2019; Bordens & Abbott, 2011). The participants in this study teach mathematics in schools with large populations of minority students and teach in public schools with more than 40% of the student population identified as low-income. Therefore, transferability to populations outside of this context may be limited.

Because of COVID-19, interviews were conducted through technology as face-to-face opportunities were not available. The possible impact of this change was approved before data collection commenced.

The purpose of this chapter was to outline the research methodology. The research design, data collection method, and data analysis process were explained. The purpose of Chapter 4 is to report the study results and confirm the compliance of methodology procedures.

CHAPTER FOUR: FINDINGS

Introduction

The purpose of this phenomenological research study was to build an understanding of the meaning that EMEs in high-density Black schools with average mathematical knowledge for teaching, ascribe to their instructional reasoning. Specifically, this study was conducted to investigate the reasoning for instructional decisions made by teachers with average MKT. In Chapter Four, the researcher presents the interview and focus group findings related to the research question: What meaning do mathematics educators in high-density Black schools, with average mathematical knowledge for teaching, ascribe to their instructional reasoning?

Interviews and a focus group were used to understand the meaning EMEs ascribed to their instructional reasoning. Five educators participated in this research (n= 5). Four themes emerged:

- descriptions about how students learn
- resources
- understanding of content progression
- understanding of students' knowledge

The findings are organized by theme with the related subthemes where applicable.

Participant Summary

The sample included five EMEs in a large urban school district in the Southeast United States. All teachers, with one or more years of teaching experience, were certified by the FLDOE in an approved certificate inclusive of mathematics (i.e., elementary education, middle grade mathematics, or mathematics) who were actively teaching kindergarten through fifth grade.

Participants varied in years of teaching experience. Teaching experience ranged from 3 to 19 years. There was limited variation in racial demographics, and all of the participants were female. A full description of participant demographics is included in Table 1. For this manuscript, pseudonyms were used to protect the identities of the participants.

Description of Participants

Joanna. Joanna was a first-grade teacher with 4 years of experience as an elementary educator. She has a background in political science and a master's degree in applied social sciences. First grade is the only grade in which she has teaching experience.

Jennifer. Jennifer was a second-grade teacher with 19 years of experience as an elementary educator. She has experience teaching fourth, third, and second grades. She began her teaching career as a fourth-grade teacher and has been teaching second grade for the last 8 years.

Michelle. Michelle was a fourth-grade teacher with 7 years of experience as an elementary educator. She was an education major with a bachelor's degree in elementary education with advanced coursework in middle school mathematics. Six of her 7 years of experience were teaching fourth grade.

Susan. Susan was a second-grade teacher with 11 years of experience as an elementary educator. She has a bachelor's degree in exercise science and a master's degree in educational leadership. Her teaching experiences included second grade, third grade, and K–8 physical education. Of her 11 years of teaching experience, she spent 6.5 years teaching physical education, less than 1 year teaching third grade, and 3 years teaching second grade.

Faith. Faith was a second-grade teacher with 4 years of experience as an elementary educator. She has a bachelor's degree in interdisciplinary studies with a minor in social science. She was experienced in teaching second and first grade. She has 3 years of experience teaching first grade, and this was her first-year teaching second grade.

Table 1:Participant Demographics

Participant	Teaching Experience	Race	Gender	Grade Taught
Joanne	4 years	Black	female	1
Jennifer	19 years	White	female	2
Michelle	6 years	Black	female	4
Susan	11 years	Black	female	2
Faith	4 years	Black	female	2

Data Analysis Findings

Research approval was received by four schools in a large urban district in the Southeastern United States. Invitations to participate in the study were sent to 114 teachers employed in the approved schools. There were 24 responses from educators who agreed to participate in the study, and 19 completed screener assessment, the MKTA. Of those who completed the MKTA, 79% (*n* = 15) scored average. Participants with average MKT were randomly selected to participate in an interview (*n* = 6) or a focus group (*n* = 9). Five participants consented to participation.

Participant interviews (*n* = 2) ranged from 23 to 46 minutes in length, and the focus group (*n* = 3) lasted 64 minutes. Audio from each interaction was recorded, transcribed, and analyzed. The data were organized and read one interaction at a time to improve understanding of the data collected. Each line of transcription was read, and important or repeated phrases, sentences, thoughts, activities, and behaviors were coded within the data. Units with the same codes were merged into clusters. Codes and clusters were reviewed to ensure relatedness. The same process was repeated for every interaction. The clusters were combined into significant categories to determine the emergent themes (see Appendix E).

Research Question

The research question for this study was: What meaning do mathematics educators in high-density Black schools, with average mathematical knowledge for teaching, ascribe to their instructional reasoning?

Mathematics educators are tasked daily with making instructional decisions and find themselves grappling with their descriptions and knowledge of their students, content, and resources. As a result, the following four themes emerged as influences on the instructional reasoning of EMEs.

Theme One: Descriptions About How Students Learn

Educators have descriptions about how students learn. These descriptions influenced the instructional decisions educators made in all stages of lesson implementation. The first theme, descriptions about how students learn, included two subthemes: (a) models and (b) connections to prior knowledge.

Theme One Subtheme: The Use of Models

Throughout the interviews and focus group, models repeatedly emerged as a subtheme to the theme of descriptions about how students learn. Educators perceived models as an important element in their lesson. Teachers proactively prepared and implemented models into their lessons because of their descriptions about how students learn. Joanne explained her students' need for visual models and teacher modeling:

> I know they needed visuals and lessons that would kind of break it down. I needed to present the lesson in different ways. So, I used videos. I used T-charts. I used, Kami, the online extension so that they can see my work as I do it. Because I think they're, they are visual, in a way that they have been exposed so much to video games and things like that. So, that kind of leads to that.

Michelle described her students' need for content to be modeled through demonstration:

> Kids learn in different ways, so some will learn better by me just modeling it, some will be able to just watch someone else do it, like watch a video, if they're more visual learners. Modeling how to do it and actually have them also watch a video on how to do it as well. The extra modeling, the videos, and all of that helps them to like gain some kind of confidence in the knowledge that they already have.

Faith described her students need for concepts to be demonstrated to them through modeling:

> I found a video on YouTube, and it was more like a kindergarten, first-grade video and . . . what it was they had an example, I guess. And the guy would teach—if . . . you have a ball, or you have stairs, or things like that, you would use a measuring tape to measure the curves. As you can see, if I put a ruler here, it's not going to work. If I put the yardstick here, it's not going to work. You can't tell if you go through the grooves or if

> you're going around the curve, you won't be able to tell the exact distance. So, they took the measuring tape and actually measured the ball. They put it on the ball for them to see it. And that was in the small groups, and then they kind of got the concept better. Them seeing it, visualizing it, was much better in the small group and watching that kindergarten first-grade video . . . I didn't let them know it wasn't second grade, but that helped them a whole lot more.

Jennifer described her students' success when using visual and student modeling:

> I kind of grouped them up with ones that I knew what we were doing well and ones that needed help. And that if they could see the other person on the page using their strategy that kind of gave them a strategy to use if they didn't already have one. As they collaborated . . . they can use the same page online, and I think that helped a lot of them be able to see . . . and I find that other kids will jump on the microphone and say do it the way… I did it was this ….and instantly, that student, they'll go, "Oh. Great. Thank you. That's what I needed." And they're able to . . . many times, they said, "Oh, I get it now." And I'm like, okay.

Theme One Subtheme: Connections to Prior Knowledge

Educators consider students' previous content exposure and their personal experiences as factors that influence how students learn. Michelle explained the importance of bringing in students' personal experiences to improve students' understanding of concepts:

> I try to use their logical reasoning to explain, like bringing their personal experience, things that they would do that has to do with time. So, putting it in a way that brings in their experience to kind of make it easier to understand.

Joanne explained, on multiple accounts, her need to aid students in making connections to concepts and skills previously taught:

> I think I needed to present the lesson in a way that ties basics. I mean what they have already been taught. You know, sometimes I think they think it's something new. Something that they haven't seen before, so I have to think about the things that they have been exposed to already. And tie that in and use the same wording sometimes. Because if I use some other wording, it might . . . I think it may bring them to a state of confusion, maybe. Because I might be bringing in something that they haven't heard before. So, I try to use the same vocabulary, the same little words that we've used in the past to introduce or even to continue on with the lesson we are focusing on at the time. So, I think that helps them to hear the same things over and over, as opposed to bringing in or introducing some new language without going over it first or without them hearing it numerous times. So, that same language, using that same language, I think, is beneficial for them.

Another description from Joanne about helping her students make connections,

> I think they have to have that feedback going and kind of get them to think about things that they already learned, you know, and bring that to the forefront. Because if not, you know, I don't want them to think this is something brand new. This is something that we're just adding on, I want to make sure I use those things that we've learned before to bring it to this lesson so that they can tie it all in. The anchor charts, I use because those are like a routine that we go through. So, they see these anchor charts often. So that is

like a reminder for them to look back into their minds about things that we already learned.

Faith explained adjusting vocabulary and the context of word problems to support students in making connections to content, “So I had to change the wording of the word problems to get them to relate”

Theme Two: Resources

Participants interacted with many curricula, assessments, and instructional resources. These interactions, or lack thereof, influenced their instructional reasoning. Three resource types emerged as factors influencing participants’ instructional reasoning: (a) assessment, (b) curriculum support, and (c) accessible digital teaching tools.

Theme Two Subtheme: Assessment

The participants’ instructional reasoning was influenced by assessment. Faith explained the impact the criterion assessments had on her instructional decisions as “looking at the assessment for the standard.” Michelle also shared the significance that the criterion-based assessment had on her content focus, “Being that it’s 35% of the Florida Standard Assessment math questions. So that was one of the things that influenced my decision.

Theme Two Subtheme: Curriculum Support

The participants’ instructional reasoning was influenced by curriculum resources and support. Susan explained that she uses the district curriculum resources to determine her lesson sequence, “I pretty much use the curriculum resources material (CRM) guideline for sequencing and also the lessons that have already been created to kind of go off of to teach that standard." Jennifer shared that her lesson planning was also guided by district curriculum resources; however, she further explained that planning is also impacted by collaboration with others:

> I know that there are changes that I make sometimes because of suggestions I've gotten or maybe things I didn't think of when I had planned it that . . . or adding to it, or maybe taking things away, so I know that also impacts how I plan what I'm going to teach. I guess the other one is maybe collaboration with your team or . . . we do a lot of collaboration with our APs [assistant principals]. And I know that sometimes, that will influence my lesson or how I . . . we go over lessons that we've already planned to look at them before we teach them.

Faith explained her reasoning for modifying the CRM used at her school:

> When I looked at the CRM and I saw some of the questions, well… I saw some of the wording, I was like, okay so yeah, no. I knew that I had to change the terminology and put them where they needed to be in order to grasp the concept of measurement a little bit more.

Joanne explained her rationale for using a curriculum resource:

> I think Go Math is less complex. And it's focused on the lesson that they started with focused on the 120 chart as a tool to add tens and then, we did add ones in the second lesson. We looked at visual examples of tens and one's digits. We went over examples of adding, how to add one-digit numbers, because they were about to get into two digits adding 10s to two-digit numbers.

Theme Two Subtheme: Accessible Digital Teaching Tools

The participants' instructional reasoning was influenced by accessible technology-based resources, or a lack thereof. Joanne explained her rationale for using technology-based learning tools:

> I think I'm always trying to keep that in consideration that I don't have all my students face-to-face, so I have to do things in a different way. They don't have the manipulatives that . . . all the manipulatives that they would have if they were in the classroom, like the ones and tens, the little blocks that we usually have for them in previous years. So, there's a lot of things that we have to do differently in that way, so that we can maybe teach them better, without the use of manipulatives so you have to find different ways. A lot of times I try to bring in visuals or any type of manipulatives and not manipulatives that we're really used to, like the hands on, because it's hard to do that now but, things that I guess, make it plain for them. So that's why I brought in Kami, I brought in anchor charts. Different things that, I mean, I would use anchor charts and have them displayed. But you know, we have to do it in a different way, because we don't have a wall that we can just have up and then you know, they can refer to, throughout the day or throughout the lesson. If there's a PDF of the page that I'm working in Go Math, I select my Kami and I'm able to write on Kami, on the page with them watching me from their Google Classroom, from the Google Meet I've tried to find different games, like Kahoot or Nearpod, things like that, that have more hands activities on because they don't have that advantage right now being online.

Jennifer explained how she used digital teaching tools to teach students online:

> The way it's set up is because everybody was online before, they got used to the system . . . we're online the entire day. And we are in an online lesson either in Nearpod or in SMART Learning the entire day. So, it's basically the same thing that I might have done before in a SMART Learning or in a power point, but it is loaded into the program. So, they're able to see—I use all day are Nearpod and SMART Learning. And in Nearpod, I can see everybody's page all at once. And in SMART, I can kind of careen through and go through each child like, right there at the board. I can just sift through every student's page wherever they are, even my students in the room now.

Susan explained how she contemplated the teaching tools available for online teaching:

> I felt that the measuring, when we were doing measuring before we went into the word problems, it was kind of difficult to get them to kind of grasp it because I feel like measurement is one area where you have to kind of use the tools and use the objects. A lot of the kids being online, it's difficult to get them to grasp the concept of what actual . . . you know, how long is an actual yardstick or meter stick and things like that because they didn't have the stuff at home to . . . you need the tools in order to actual . . . actually do the measuring part of it. And I knew that most of my kids wouldn't have anything besides a small ruler, and I knew that they had a small ruler because I sent it home with them, you know, over Christmas break. But as far as having the measuring tape, the yardstick, meter stick, those kinds of things they didn't have. So, I did have to throw in into the planning that, okay, for this section, you're going to have to actually measure this with them using this tool so they can see what it is, what it looks like, and you know, about how long whatever you're measuring is using that specific tool.

Theme Three: Knowledge of Content Progression

Joanne explained the selection of a curriculum resource because of the review of prerequisite skills:

> We reviewed the 120 chart . . . looking at numbers and patterns because they have to understand the concept of when they're adding numbers, they have to know that they're adding, not just the number itself, but they're adding tens and where they're moving the 120 chart.

Michelle explained that she scaffolded a time conversation lesson to include foundational knowledge before presenting grade-level content, "I knew that we had to start with the basics, talking about what those hands mean and how to count the time of the minutes and the hours." Jennifer explained a lesson she taught on line plots and the need to add prerequisite skills to aid students in their understanding:

> I kind of also noticed that the way that the lessons were written, it kind of already assumed that they knew what line plots were, they knew everything about number lines, and that's not something that they knew. So, I kind of had to work backwards a little bit and go into some I-Ready materials to see where I could add in some prerequisite ideas that weren't in the CRMs when I was planning.

Susan explained the progression of measurement concepts:

> I could see that they were not getting the concept. They were not getting it at all, and if they couldn't get past that, then how were we going to add and subtract measurements. Then we went through and talked about there's about this number of inches in one foot. And then, we compared the paper clip to an actual ruler and then just kind of built onto it like that.

Theme Four: Understanding Students' Knowledge

Participants analyzed their lessons and performed checks for comprehension to verify the students' understanding of the content. The theme, understanding students' knowledge, yields two subthemes: (a) check for understanding and (b) lesson analysis.

Theme Four Subtheme: Checking for Understanding

The participants' instructional reasoning was influenced by the knowledge gained from checking students' understanding of the content. Jennifer described making instructional decisions by monitoring students' responses to tasks and questioning students to determine their understanding of the content:

> I can see as they're writing on the page and exactly what they're doing at that exact moment. So, if I want to stop and say, "Hey, you're writing this number. How did you come up with that number? How did you make that decision?" And then they can . . .. They'll go through the steps on how they got . . .. And I can see, and then I can write in real time right on their paper or I can help them, say maybe, "Did you look at this piece here?" I like to be able to see as they're writing it out, what their thinking is, and then have them literally explain it, right there.

Susan explained her use of repetition and questioning to check for student understanding:

> I took the computer and walked around with my . . . you know, with it being projected on myself, using the yardstick, trying to show them, you know, as much as I could in the camera view. And we measured the door, we measured the window, and then just, you know . . . questioning . . . questioning them through the process. I think they kind of grasped it a lot because I use repetition. It's something that I would always constantly,

> you know, talk about when we were in that area of the standard. And having them call it back to me. So, I would . . . you know, I would say it and then . . . I would, you know, at random times, I'd say, "Okay, and . . . how many inches are in a foot?" Or, you know, things like that. Just repetition. Me saying it and having them also repeat it back to me.

Michelle explained her rationale for using questioning to check for student understanding:

> I first gave them the question, so kind of using the discovery method, as seeing like, having them kind of do it first before I tell them how to do it. I started out by just giving them a question with the clock and just trying to figure out if they knew how to read it and what each one meant. What operations are required, how many steps. So, I knew that they had issues with multistep problems, so that was something that was influenced by decision to start at the basics.

Joanne described that her instructional reasoning was influenced by checking for student understanding:

> We went through the problems that I added in Kami. And we went through it together, and I walked them through it. We did okay, asked some questions, "Okay, what's our next step? Somebody tell me." And I usually use the chat box. And everybody has a chance to answer. So, I'm looking for that. I'm looking for, who has a good understanding? Who is still not answering the question? And then I know how to proceed—if I need to do another example or if they need to have another practice problem. I saw that there were still a few that were confused as how to go up or down the 120 chart. So, I went back to the 120 chart, and I called on persons who were not answering.

Theme Four Subtheme: Lesson Analysis

The participants' instructional reasoning was influenced by their lesson analysis. Jennifer explained examining students' abilities during a lesson to make immediate instructional decisions:

> And more lately, I've been letting where my kids are, kind of guiding them past that because there are things that I find that they're missing now that we're getting into the second half of the year that I didn't have a problem with at the beginning of the year. I find that what I planned for isn't the best because I've been planning with other people and maybe what they do isn't always what I do. I find honestly, most of my days kind of, that's not working. Change it now. I don't wait till tomorrow to fix it, I fix it now so that we can learn it today . . . . I find that my kids online kind of guide that because I have several that love to come on and help other people if they're struggling. And they love to explain it to them so that they can understand because they want them to be able to get it. Number one because they want us to move on and turn the page. And two, they like to be able to discuss that with someone. They like that someone's listening to them, like, "Yes, please tell me. I want to know how to get this." So I find that I let them kind of guide where we need to go, and then I'm willing to change whatever's not working right then and there and then I fix it. So, it's right then and there. It's not always something I planned for. It's not even generally something I plan for.

Susan explained while reflecting on a lesson she taught to make instructional decisions for upcoming lessons:

> I think it . . . it just kind of goes with the flow of teaching. Like if you're . . . you're teaching something and . . . you know, this math concept and you noticed that they're not

> getting . . . because I've had times when I'm thinking, oh, this is a great lesson, you know, they're going to get this. And then we get to get though the lesson and it's crickets. You know, nobody understands it, so then you kind have to, as a teacher, you kind of got to reflect on that and say, "Okay, so what did I . . . what did I do that didn't work? What worked? You know, and how can I change it for the next time?" You know, do I need to, like she said, do I need to throw in more of that vocabulary that they might be used to that they could understand it better? You know, do I need to show a video where they can see it, or is it something that I can show myself doing? And then try to apply it to . . . I think it's kind of reflection, I guess, is what I'm trying to say.

Faith explained making an instructional decision to modify her teaching structure after analyzing the success of the lesson taught:

> With teaching it in whole groups. And then, I found out that some got the concept faster than the others, and then I just started having a number talk, as whole group, and then teaching it in my small groups. I needed them to be able to grasp that . . . that concept. But I . . . I don't know, I guess I did think about it when I was preplanning for it, but it didn't really hit until it was . . . it was a lot harder to teach. And a lot harder to get them to answer my questions in . . . in the math block. For the next day or . . . I had to plan a different route in order to have a successful math block the next day.

Summary

The purpose of this chapter was to report the meaning that mathematics teachers with average mathematical knowledge for teaching, in high-density Black schools, ascribed to their instructional reasoning. A focus group and interviews were analyzed to provide insight into the meaning participants ascribed to their instructional reasoning. Four themes emerged: (a) descriptions about how students learn, (b) resources, (c) knowledge of content progression, and (d) understanding of students' knowledge. Subthemes emerged within three of the four themes. The theme descriptions about how students learn yielded two subthemes: (1) models and (2) connections to prior knowledge. The second theme, resources, produced three subthemes: (1) assessment, (2) curriculum support, and (3) accessible digital learning tools. Within the theme, understanding of students' knowledge, two subthemes were identified: (1) checks for understanding and (2) lesson analysis. Themes and subthemes will be discussed in Chapter Five.

CHAPTER FIVE: CONCLUSION

Conclusion

The purpose of this phenomenological research study was to understand the meaning that EMEs, with average mathematical knowledge for teaching in high-density Black schools, ascribed to their instructional reasoning. In this chapter, the researcher will review the problem statement, the purpose of the study, the methodology, and the findings of the study.

Statement of the Problem

Over the last four decades, the government of the United States enacted several education reform initiatives to improve the education equality and mathematical performance of Black students (e.g., Goals 2000: Educate America Act, NCLB, American Recovery and Reinvestment Act of 2009). Nevertheless, Black students continue to perform poorly in mathematics in comparison to their White and international peers (Auguste et al., 2009; Mickelson et al., 2013). Researchers who investigated student performance have found links between teacher knowledge and student achievement (Hatisauri & Erbas, 2017). Little to no research has focused on understanding teacher knowledge in high-density Black populated schools. Studies that address teacher knowledge targeted general populations and primarily focused on classifying teacher knowledge, the effects of teacher knowledge on student outcomes, and the relationship between teacher knowledge, instructional quality, and student achievement (Hoover et al., 2016). However, these studies yielded mixed results, and conceptualizing teachers' mathematical knowledge was challenging (Hoover et al., 2016). Therefore, uncontested conceptualizations of teachers' mathematical knowledge do not exist, and researchers have yet to discover how teachers apply their mathematical knowledge and pedagogical reasoning, which is essential to developing an uncontested theory of teachers' knowledge and understanding the application of teacher knowledge in high-density populated schools (Ball et al., 2008) Given this continued gap in understanding, researchers would benefit from further examining the meaning EMEs ascribe to their instructional reasoning.

Purpose of the Study

This phenomenological study was conducted to describe the meaning EMEs ascribed to their instructional reasoning. Previous investigations of educators' MKT investigated teacher's mathematical knowledge, student outcomes, or other related factors. However, little to no MKT research sought to understand the meaning EMEs ascribed to their instructional reasoning. Rich descriptions of the meaning EMEs assign to their instructional reasoning support the body of MKT research by providing the insights needed to inform the policymakers involved in education reform efforts, research practitioners investigating teacher mathematical knowledge, and stakeholders interested in teacher training and development in mathematics.

Methodology

This study was conducted using a phenomenological research methodology. This qualitative methodology was used to understand the meaning EMEs ascribed to their instructional reasoning. The mathematical knowledge for the teaching framework was used to conceptualize teachers' MKT and identify teachers with average MKT.

Discussion of Findings

Four themes emerged from an analysis of interview and focus group data: (1) descriptions about How Students Learn, (2) resources, (3) understanding of content progression, and (4) understanding of students' knowledge. Subthemes emerged within three of the four themes: (1) descriptions about how students learn yielded two subthemes: (a) models and (b) connections to prior knowledge; (2) resources yielded three subthemes: (a) assessment, (b) curriculum support, and (c) accessible distance learning tools; and (4) understanding of students' knowledge yielded two subthemes: (a) checks for understanding and (b) lesson analysis.

Summary of Themes

A primary responsibility of an educator is to support students in knowledge acquisition. For this reason, decades of research focused on understanding teachers' knowledge, instructional practice, and its impact on student achievement. Many of these investigations found that teachers' knowledge and practices affected student achievement; however, these relationships were mediated by factors—known and unknown.

When considering the education community, educational initiatives are influenced by educational reform mandates, learning theories, learning principles, and standards, some of which are rooted in empirical evidence and others formed from theories of practitioner practice. As educators are confronted with these influences, schemata are developed that are the foundation of instructional reasoning for EMEs. The EMEs participating in this research accredited their instructional reasoning to their schemata for teaching and learning. The EMEs had schemata for How Students Learn, resources, content progression, and determining students' knowledge.

Theme One: Descriptions about How Students Learn

The EME accredited their instructional reasoning to their descriptions of How Students Learn mathematics. The EME descriptions about How Students Learn mathematics aligned to the Mathematical Knowledge for Teaching (MKT) framework, National Council of Teachers of Mathematics (NCTM) principles and standards for mathematics and behaviorism, cognitivism, and constructivism learning theory.

EMEs use their schemata to describe their knowledge and pedagogical perspective for how students learn mathematics. The EMEs believed students grasped mathematics when concepts were taught with models and when content was connected to students' prior knowledge. Instructional reasoning of EMEs was supported by Ball and colleagues (2008) MKT framework. The MKT framework has six domains, two of which align to EMEs instructional reasoning: (a) knowledge of content and students, and (b) knowledge of content and teaching.

The EMEs used their knowledge of content and students and knowledge of content and teaching to make instructional decisions before lesson implementation. They anticipated student challenges before lesson were taught and made adjustments to lesson content and their instructional approaches to accommodate their beliefs about Black student content knowledge, prior experiences, and learning styles. As an example, EMEs believed students understood mathematics concepts when the concepts were taught with models; however, they described the models broadly. The EMEs defined models visually, concretely, and cognitively. Visual models included graphic organizers, pictures, and videos. Concrete models were described as tangible objects, such as mathematics manipulatives, and cognitive models were explained as modeling the thinking process, known as a *think aloud* (Powerup What Works, n.d). The EMEs used visual models in their lessons because they believe that their students were visual learners and visual models were needed to made lesson content easier for students to comprehend. As an additionally example, EMEs also believed that students learn mathematics when content is connected to their prior knowledge. Prior knowledge was defined as making connections to familiar language, real-world experiences, and previously taught content. The EMEs selected curriculum resources and lessons tasks and modified questions and mathematical story problems to support Black student understanding of content. Therefore, EMEs instructional reasoning aligned to MKT domains (1) knowledge of content and students and (2) knowledge of content and teaching. However, EMEs descriptions of how students learn could not be isolated within one MKT domain. The EMEs instructional reasoning aligned to their knowledge of their students, content and teaching. Descriptions from the EMEs always included their perspective of their students. The EMEs considered how to represent the content, modify the content, and present content within the context of how their students learned.

When evaluating EMEs knowledge and perceptive about how students learn through the lens of the MKT domain, knowledge of content and teaching, EMEs knowledge of content and teaching was also supported by the NCTM Principles and Standards for School Mathematics (2000) and learning theory. The EMEs described students learning mathematics with models and when connected to their prior knowledge. The EMEs descriptions revealed instructional models used when EMEs taught elementary mathematics.

The NCTM principles and standards are recognized as a guide for quality school mathematics programs. The NCTM guidance contains a list of principles, content, and process standards for school mathematics. The NCTM principles and process standards support EMEs descriptions about how students learn. The EMEs descriptions for how students learn included the belief that students learned mathematics with visual models. The NCTM process standards stated that mathematical content could be represented in various ways, including pictures, concrete objects, and numbers. Additionally, standards included the description of when "students gain access to mathematical representations and the ideas they express and when they can create representations to capture mathematical concepts or relationships, they acquire a set of tools that significantly expands their capacity to model and interpret physical, social, and mathematical phenomena" (NCTM, n.d., p. 4). The EMEs also believed that students learned mathematics when content was connected to prior knowledge which is also support by NCTM principles for school mathematics. The NCTM principles for school mathematics state that new knowledge is built from experience and prior knowledge to develop student mathematical understanding. Therefore, EMEs descriptions of how students learn revealed their use of instructional practices in alignment with NCTM principals and standards of mathematics.

The EMEs descriptions about how students learn mathematics were also supported by behaviorism, cognitivism, and constructivism learning theories. When evaluating EMEs knowledge and perceptive about how students learn through the lens of the MKT domain, knowledge of content and teaching, EMEs explanations for making connections to students' prior knowledge, use of visual and models (i.e., demonstration, visual illustrations, manipulatives) revealed their use of learning theory; however, there are different purposes for using these representations (Ertmer & Newby, 2013).

Behaviorism learning theories focus on observable behaviors and seek ways to reinforce desired behaviors and decrease undesirable behaviors (Kennedy et al., 2008; Reys et al., 1998). Skinner's (1948) theory of child development and Pavlov's Classical Conditioning Theory of Learning are examples of behaviorism learning theories. Learning objectives are met when desired responses are met (Ertmer & Newby, 2013). EMEs used of models aligned to behaviorism learning theory. EMEs used anchor charts, visual examples and videos to review and reinforce mathematical procedures and skills. EMEs used these models to support students in mastery of mathematics skills and processes (e.g., measuring objects, replacing ten and ones on a place value chart). Strategies for behaviorism learning theories emphasize mastery learning; producing observable, measurable outcomes; sequenced practice; and prompts to support a strong stimulus-response association (i.e., repetitive practice to strengthen associations, illustrative examples.) EMEs schemata for using models in mathematics lessons were revealed through their descriptions of How Students Learned mathematics. Therefore, schemata for using models aligned to behaviorism teaching practices.

Cognitivism learning theories focus on knowledge acquisition and mental structures. Similar to BLS, cognitivism learning theories are learner and environment-centered. However, the learner is an active participant, and cognition is emphasized. Learning is defined by what students know and how the knowledge was developed (Ertmer & Newby, 2013). EMEs modeled content, used graphic organizers, and mnemonics, and made connections to students prior learning experiences. Cognitivism learning theories emphasize metacognition, advanced organizers, demonstrations, and connections to prior learning (Ertmer & Newby, 2013). EMEs used modeling and processes to make their and their student thinking visible. Therefore, EMEs descriptions for How Students Learn revealed EME schemata for modeling their thinking and using graphic organizers, which aligned to cognitivism teaching practices.

Constructivism learning theories are classified as a subsection of cognitivism. Constructivists believe that learning is created from meaningful experiences. Constructivism learning theories consider both the learner and the environment; however, the learning context is realistic and relevant to the students' experiences (Ertmer & Newby, 2013). Bruner's (1966) learning theory and Piaget's (1976) cognitive stages are examples of constructivism theories for learning. EMEs used cooperative learning structures and reciprocal teaching strategies to elicit student prior knowledge and apply student knowledge. Kennedy et al. (2008) explained that constructivists create opportunities for students to activate their prior knowledge to support their understanding of application and reflection. Therefore, EMEs descriptions for how Black student learn revealed EME schemata for using cooperative structure and reciprocal teaching strategies, which aligned to constructivist teaching practices.

EMEs accredited their instructional reasoning to their descriptions of How Students Learn mathematics. EMEs used their knowledge of content, students, and teaching to make instructional decisions. Their descriptions of How Students Learn also revealed instructional practice usage. As EMEs described their knowledge and perceptive of How Students Learned mathematics, their schemata for instructional practices were also revealed. EMEs schemata was diverse and aligned to various learning theories. However, it is also important to note, although all three learning theories were found among EMEs, behaviorism and cognitivism descriptions were more prevalent than constructivism.

Theme Two: Resources

The EMEs participating in this research accredited their instructional reasoning to available resources. Three subthemes emerged as the rationale for EMEs' instructional decisions: (a) assessment, (b) curriculum support, and (c) accessible digital teaching tools. The EMEs' instructional decisions were influenced by local and state testing, curriculum support provided (i.e., coaching, curriculum resources, professional learning communities), and accessible digital teaching tools.

Instructional decisions made by EMEs were influenced by unit- and state-based assessments. The EMEs emphasized content that they knew would be assessed on unit and state assessments. Hatfield et al. (2005) explained, with a demand for measuring students' progress, more instructional time is used to teach content related to state and district testing.

The EME instructional decisions were influenced by peer collaboration and the available curriculum support. Peer collaboration influenced the instructional decisions EMEs made during planning (i.e., grade-level team, instructional coaches, principals, and assistant principals). The EMEs explained the feedback from the influenced decisions peers made during planning. The EMEs also acknowledged using district- and school-based resources as their primary instructional resource. Choppin (2011) conducted a study on teacher adaptation to curriculum resources, and he found that teachers use resources according to their understanding, attention to student thinking, and ability to make connections between their knowledge of their students' thinking and the curriculum.

The EMEs accessed district instructional materials (i.e., district assessments, adopted curriculum, and district-created lesson plans) to teach mathematics. Similarly, to Choppin (2011), EMEs used resources according to their understanding and their connections between their knowledge of their students and the resources. The EMEs reviewed accessible instructional materials and evaluated the relevance and potential effectiveness of the instructional material against their schema. The EMEs made decisions to implement lessons as written or to adapt the lesson to increase lesson effectiveness. When adaptions to lessons were made, the EMEs' lesson adaptations aligned with their schema for how students learn, knowledge of content progressions, and students' knowledge.

Accessible digital teaching tools influenced EMEs' instructional decisions. The EMEs described using digital teaching tools (i.e., Nearpod, SMART Learning, Kahoot, Google Classroom) to support their daily learning structure. The EMEs used digital teaching tools to monitor students' knowledge through games, monitor students' responses to questions and tasks, and provide visual lesson content. Additionally, the EMEs expressed the challenges of teaching with technology when the scheme had not been developed. This study was conducted during the coronavirus pandemic (i.e., COVID-19).

All EMEs taught classes online. Due to the COVID 19 pandemic, on average, 50% of their students attended class from home, and the remainder of their students attended school in person. All students had access to school-issued devices and were taught via technology regardless of the students' location for learning.

The NCTM (2015) states that technology can deepen students' understanding and proficiency in mathematics when technology is used effectively. The EMEs used I-Ready, Nearpod, videos, and games to support students' understanding in mathematics. One EME used virtual manipulatives. Kami and SMART Learning were used to supplement the traditional textbook.

Kennedy et al. (2008) stated that students benefit from the use of technology in the classroom. Virtual manipulatives and computer-based technology provided students with opportunities to construct meaning and explore foundational mathematics concepts (Kennedy et al., 2008). Hatfield et al. (2005) explained that technology could motivate students and teachers.

The EMEs instructional decisions were influences by their accessibility to resources. EMEs describe assessments, peer collaboration, curriculum resources guides, and tools for teaching as curricular resources that influenced their instructional decisions. The MKT framework domain is not board enough to support EMEs instructional reasoning for attribution for resources. How the knowledge and skills of teachers aligned to the MKT domain, Knowledge of content and curriculum is unclear (Koponen et al., 2016). Koponen and colleagues stated conceptualization of knowledge of content and curriculum should include teaching material, teacher instruments, technology. Therefore, these findings, align did not align with current conceptualization of MKT domain for knowledge of content and curriculum. However, these findings may provide additional insights into future conceptualization of EMEs knowledge of content and curriculum.

Theme Three: Knowledge of Content Progression

The EMEs who participated in this research accredited their instructional reasoning to their knowledge of content progression. Their knowledge of content progression influenced EMEs instructional decisions about the instruction of their lessons. Lessons were modified to access the prerequisite skills needed to present the content. Hull et al. (2011) stated that mathematical content must be taught sequentially and effectively to help students make connections between the sequential parts. They made decisions to present the lesson as written or to make adaptations.

Their knowledge of the prerequisite skills influenced the EMEs decisions to adapt lesson content or present lessons with adaptions. Learning progressions (i.e., learning trajectories) are developmental sequences to support students in learning mathematics. “Well-documented learning progression for all K–12 mathematics do not exist” (Common Core Standards,2013 p. 7); however, progressions are available from several mathematics topics (i.e., number and operations, counting and cardinality, geometry) in the primary sequence mathematics standards

(Common Core Standards, 2013). Wickstrom et al. (2012) conducted a case study to determine how teachers make sense of lesson projections and their teaching impact. The researchers found that, after being exposed to learning trajectories, the teacher developed a language for students' thinking, used lesson progressions to focus on students' strategies, shared knowledge with peers, and reflected on students' strategies and responses (Wickstrom et al., 2012). Sztajn et al. (2012) argued that learning trajectories are the foundation of instructional decisions. Emerging research on lesson progressions shows that when teachers use lesson progression, educators grow in their mathematical knowledge and ability to select instructional tasks and use student responses (Sztajn et al., 2012).

EMEs instructional reasoning was ascribed to their knowledge of mathematics content progressions. EMEs used their knowledge of previously taught content, whether the content was taught in previous years or during the same year, to make instructional decisions about the order and progression of content they taught. The MKT domain horizon content knowledge addresses educator awareness of content connections over time. The definition of Horizon content knowledge focused on mathematics projections, understanding mathematics content taught in the future, whether the same year or subsequent years. However, EMEs in this study primary used their horizon content knowledge to reflect on mathematics progression taught before lesson implementation to make conjectures about students' needs to scaffold lessons. Mosvold and Fauskaner (2014) investigated teachers' beliefs about mathematical horizon content knowledge and found that teachers "seemed to be more concerned about the mathematical content at the level they were teaching than the broader (more advanced) mathematical context" (p.12). Similar to the findings of Mosvold and Fauskaner (2014), EMEs focused on current content and were not focused on future connect connections.

Theme Four: Understanding Students' Knowledge

The EMEs accredited their instructional reasoning to their understanding of students' knowledge. Instructional decisions made by the EMEs were influenced by their understanding of their students' knowledge. The EMEs described completing an analysis of student understanding at two levels: (a) individual and (b) class.

The EMEs completed an individual analysis of student knowledge and comprehension by asking students questions about content and evaluating the tasks assigned. According to Kennedy et al. (2008), teachers gain knowledge of student understanding through formative assessment. Kennedy and colleagues (2008) further explained that, during lessons, teachers monitor students' thinking by questioning and reviewing tasks. Additionally, teachers have students use visual clues (i.e., thumb up, thumb down) to reflect their understanding (Kennedy et al., 2008).

Class level analysis takes place through EMEs reflecting on the general effectiveness of their lesson. Did most students struggle to understand the content? What were students able to do, what were they unable to do? Were all elements of learning present? If not, what could be added. However, EMEs varied on when modifications were made. Some EMEs adapted their lessons the following day, while others preferred to modify their lesson on the spot.

EME scheme for understanding students' knowledge aligns to MKT domain, knowledge of content and students. Ball et al. (2008) explained teachers' need to anticipate students' challenges, thinking, conceptions and misconceptions. The knowledge that combines teachers' knowledge of mathematics and students is known as the knowledge of content and students (Ball et al. 2008). EMEs used their knowledge of content and students to make instructional decisions. EME described the influence of students' conceptions, misconceptions, and questioning on their instructional decisions. EMEs used their understanding of their students' misconceptions to scaffold lesson content. EME s described previous learning experiences when students had trouble with lesson content and identified those challenges as potential barrios to student understanding of new content. EMEs also used questioning and problem-solving task to understand students' knowledge. EMEs measure students' knowledge based on their ability to solve and answer mathematical questions correctly. EMEs instructional decisions were influenced by their student's ability or inability to answer and solve problems posed during lesson. EMEs modified lessons real-time or the next day to increase the number of students who answered the mathematical questions correctly

Limitations

Studies that target special populations for investigation are not generalizable to other populations (Ary et al., 2019; Bordens & Abbott, 2011). The primary limitation to the generalization of these results is the scope of this study. This study was limited to participants with average MKT who taught in high-density Black schools. Therefore, transferability to populations outside this context is limited.

The second limitation concerns the interpretation of the study results. This study focused on understanding the meaning EMEs ascribe to their instructional reasoning. The purpose of the study was not to determine why EMEs ascribed meaning to this instructional reasoning. Therefore, it is important to note that EMEs did not explicitly attribute their instructional reasoning to NCTM principles or any learning theories. The EMEs used their schemata to justify their instructional decisions. Therefore, EMEs' instructional reasoning cannot be accredited to any principle or theory of learning to explain their schemata.

The third limitation concerns the timing of this study. This study was conducted during the coronavirus pandemic (COVID-19). During the pandemic, traditional schools were reimagined, which reshaped teaching and learning. Traditional teaching methods were transferred to the computer screen, where teachers recreated the classroom learning experience on virtual platforms. Families in the school district where this research was conducted could choose to have their children learn from home at their zoned school or attend onsite classes in their zones. Teachers were required to shift quickly to virtual platforms and convey their knowledge and pedagogy using the computer as a foundational curriculum resource. The uniqueness of teaching during the COVID-19 pandemic may have influenced EMEs instructional decisions.

Recommendations for Future Research

There are two limitations in this study that could be addressed in future research. The first limitation is the generalizability exclusive to EMEs. Because of the limited existing research around teacher's instructional reasoning, it is recommended that additional studies are conducted to confirm and expand these findings. Additionally, more investigations are needed to understand the instructional reasoning of EMEs at varying mathematical knowledge levels for teaching and to determine the commonalities and differences among varying levels of MKT. Little research exists on elementary educators' instructional reasoning in high-density populations. Therefore, additional studies to investigate the commonalities and differences among educators' instructional reasoning in high-density Black schools with those who teach high-density populations from other demographics are recommended to determine if unique experiences exist among the varying populations. Armed with this knowledge, the education community will be one step closer to codifying educational practices and informing policymakers involved in education reform efforts, research practitioners investigating teacher mathematical knowledge, and stakeholders interested in training and mathematics development for teachers.

The second limitation of this study is focused on understanding the meaning mathematics educators in high-density Black schools ascribed to their instructional reasoning. An analysis of the data yielded the meaning EME's attributed to their instructional reasoning; however, a deeper analysis is needed to understand what drives educators' accreditations. This additional perspective would provide the research community with an understanding of the theories and practices that are the most prevalent to the instructional reasoning of mathematics educators.

Educators are decision makers. They make decisions about student knowledge, the curriculum resources they use, the instructional strategies, and the content they teach. The EMEs' understanding of the what and the why of mathematics educators' instructional reasoning would advance the body of knowledge on teacher knowledge and practice.

Conclusions

Teachers make decisions on a daily basis. They decide how to assess students' knowledge, the appropriate instructional strategies to aid students in acquiring knowledge, the most effective resources to support students in knowledge building, and the proper sequence for appropriate knowledge acquisition. Each of these decisions is influenced by teacher schemata.

The EMEs who participated in this research ascribed their instructional reasoning to their schemata, which explained their underlying assumptions about teaching and learning. The EMEs developed schemata for how students learn mathematics, mathematics resources, mathematics content progression, and checking for understanding. However, their schemata could be altered through reflection after interactions or experiences with peers or students.

The EMEs' instructional reasoning can be ascribed to their schemata. Their schemata evolved through interactions and experiences from teaching and learning. There is limited evidence that EMEs' schemata for teaching and learning are specific to mathematics. It is possible that the EMEs used cross-curricular schemata to make instructional decisions when teaching mathematics.

Throughout the interviews, EMEs rarely used lexicon associated with mathematics theories for learning or mathematics principles and explained their instructional reasoning from a practical perspective. Therefore, additional investigations are needed to understand what drives the instructional reasoning of mathematics educators. Many factors, including teacher reflection, professional development, educational mandates, curricular resources and supports, success and failure of lesson implementation, and student assessment, can alter EMEs' schemata. The EMEs' schemata for teaching and learning must be understood and used to support teacher development and growth in mathematics teaching and learning.

APPENDIX A: CONFIRMATION OF TRAINING TO ADMINISTER MATHEMATICAL KNOWEDGE FOR TEACHING ASSESSMENT

From: noreply@qualtrics.com <noreply@qualtrics.com>
Sent: Sunday, October 27, 2019 1:54 PM
To: DeSheila Rumph
Subject: Teacher Knowledge Assessment System (TKAS) - Trained User Information

Dear DeSheila Rumph,

You are receiving this email as confirmation of your completion of the mandatory modules for the LMT/TKAS online training.

To access the measures, please go to the TKAS website *(web link)*

From TKAS, you can access the measures to download and use as paper and pencil forms, or you may choose instead to create an assessment plan for administration through TKAS.

Your access code is *(code)*

Your email address is *(address)*

Your password is the one you were given when you were approved for training or whatever you reset it to, if you reset it.

Please save this information in a safe place for future reference.

APPENDIX B: INSTITUTIONAL REVIEW BOARD APPROVAL

UNIVERSITY OF CENTRAL FLORIDA

Institutional Review Board

FWA00000351
IRB00001138, IRB00012110
Office of Research

12201 Research Parkway

Orlando, FL 32826-3246

EXEMPTION DETERMINATION

October 26, 2020

Dear DeSheila Rumph:

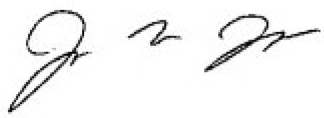
On 10/26/2020, the IRB determined the following submission to be human subjects research that is exempt from regulation:

Type of Review:	Initial Study, Category 3(i)(B)
Title:	Mathematical Knowledge for Teaching: Instructional Reasoning in High-Density Black Populations
Investigator:	Desheila Rumph
IRB ID:	STUDY00002237
Funding:	None
Grant ID:	None
Documents Reviewed:	• FSA page 2, Category: Faculty Research Approval; • FAS Page 1, Category: Faculty Research Approval; • DeSheila Rumph semi-structured interview guide 3.doc, Category: Interview / Focus Questions; • Email Scripts DeSheila Rumph 10222020.docx, Category: Recruitment Materials; • HRP-254-FORM Explanation of Research DeSHeila Rumph 10262020.pdf, Category: Consent Form; • HRP-255-FORM - Request for Exemption DeSheila Rumph 10222020.docx, Category: IRB Protocol; • MKTA Assessment , Category: Test Instruments; • MKTA Assessment , Category: Test Instruments;

This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made, and there are questions about whether these changes affect the exempt status of the human research, please submit a modification request to the IRB. Guidance on submitting Modifications and Administrative Check-in are detailed in the Investigator Manual (HRP-103), which can be found by navigating to the IRB Library within the IRB system. When you have completed your research, please submit a Study Closure request so that IRB records will be accurate.

If you have any questions, please contact the UCF IRB at 407-823-2901 or irb@ucf.edu. Please include your project title and IRB number in all correspondence with this office.

Sincerely,



Racine Jacques, Ph.D.
Designated Reviewer

APPENDIX C: INFORMED CONSENT

Informed Consent

You are being invited to take part in a research study. Whether you take part is up to you. This research aims to understand the instructional reasoning of elementary teachers who teach in densely populated Black schools.

The study has two phases. However, many participants will only participate in the first phase. All participants in this study will be asked to take a mathematics teaching assessment online. A limited number of participants who complete the assessment and receive an average score will also be invited to participate in a virtual interview or a focus group.

This study involves minimal risks. Participation in this study places participants at risk for the loss of data privacy or confidentiality. The following procedures will be executed to mediate these risks and ensure data security: The researcher will use data encryption and a secure password-protected workspace to protect participants' privacy. The researcher and participants will confirm room conditions are conducive for an interview with minimal background noise and free from distraction and ensure subject privacy. De-identifiable data will be used in the research finding.
If you decide to participate, the mathematics teaching assessment is predicted to take **approximately 20 minutes**. Those selected to participate in a focus group or interview will invest an additional 30 -60 minutes.

If you participate in the interview or focus group, you will be audio recorded. If you do not want to be recorded, you will not be able to participate in the study. Discuss this with the researcher. The recording will be kept in a locked, safe place and will be erased or destroyed following the University of Central Florida guidelines, which is after five years.

Your email address will be collected for a potential interview or focus group follow-up. However, the principal investigator will be the only individual with access to this information. Email addresses will also be destroyed following the University of Central Florida guidelines for five years.

You must be 18 years of age or older to take part in this research study. The target population for this study consist of teachers in Florida who teach mathematics in elementary schools with high-density Black populations that held Title I classifications. Teachers must hold a certification from the Florida Department of Education (FDOE) with an active certificate to teach mathematics at their assigned grade level (i.e., Elementary Education, Middle Grades Mathematics, or Mathematics). Additional criteria for participant selection included having at least one year of teaching experience in a kindergarten through fifth-grade class within a general education setting. Teachers who did not meet the criteria will be excluded from study participation.

For questions about the study or to report a problem:

If you have questions, concerns, or complaints, please see my information below: DeSheila Rumph, Graduate Student, Curriculum and Instruction Program, College of Innovation and Education, by email desheila@knights.ucf.edu or Dr. Suzanne Martin, Faculty Supervisor, Department of Learning Sciences and Educational Research at (407) 823-3859 or by email at Suzanne.martin@ucf.edu.

IRB contact about your rights in this study or to report a complaint: If you have questions about your rights as a research participant, or have concerns about the conduct of this study, please contact Institutional Review Board (IRB), University of Central Florida, Office of Research, 12201 Research Parkway, Suite 501, Orlando, FL 32826-3246 or by telephone at (407) 823-2901, or email irb@ucf.edu.

APPENDIX D: SEMI-STRUCTURED INTERVIEW PROTOCOL

Data	**Question**	**Prompts & elicitations**
To break the ice and provide some background.	Please tell me about your academic and professional experiences.	What experience did you have as an educator? Teaching experience What roles have you served
To understand influences on teachers' instructional decisions during planning	Tell me about a recent math concept you planned? Please included details about what influenced the planning decisions you made.	What influence the decisions you made during planning?
To understand influences on teachers' instructional decisions when teaching	Tell me about a recent math concept you taught? Please included details about influence how you taught the concept?	What impact the decisions you made while teaching?
Understanding the reason for selecting the strategies used to promote and monitor students understanding of the lesson	Tell me about the teaching procedures you use, and what influenced you to choose these procedures to teach this lesson?	Which strategy or strategies were used to teach this concept? Why did you select this strategy or strategies to teach this concept?
How does teacher knowledge of student thinking influence their instructional reasoning	What specific knowledge of your student thinking, if any, influenced your teaching of this concept?	How does teacher knowledge of student thinking influence their instructional reasoning
Contextual knowledge about students and general pedagogical knowledge that influences the teaching approach.	Where there any other factors that influence your teaching of this idea?	Is there anything else you considered that impact how you taught this concept?
Is the participant interested in providing any additional information	Is there anything else you would like me to know?	
Member checking	Paraphrase what was heard about the main data points 1. Influencing on planning 2. Influencing on concepts taught 3. Influences on procedures 4. Student thinking influences 5. Other factors that influence the concept taught	

APPENDIX E: SIGNIFICANT CATEGORIES

Significant Categories

	Descriptions about How Students to Learn
Visuals	
Jennifer	As they collaborated... They can use the same page online and I think that helped a lot of them be able to see...
Joanne	I know they needed visuals and lessons that would kind of break it down.
Joanne	I needed to present the lesson in different ways. So I used video's. I used T charts. I used, Kami, the online extension so that they can see my work as I do it
Joanne	Because I think they're, they are visual, in a way that they have, been exposed so much to, video games and things like that. So, that kind of leads to that
Faith	I found a video on YouTube and it was more like a Kindergarten, first grade video and... What it was they had an example, I guess. And the guy would teach, If... You have a ball, or you have stairs, or things like that, you would use a measuring tape to measure the... The curves. As you can see, if I put a ruler here, it's not going to work. If I put the yardstick here, it's not going to work. You can't tell if you go through the grooves or you're going around the curve, you won't be able to tell the exact distance. So, they took the measuring tape and actually measured the ball. They put it on the ball for them to see it. And that was in the small groups and then they kind of gotten the concept better Them seeing it, visualizing it was much better in the small group and watching that kindergarten first grade video... I didn't let them know it wasn't second grade, but that helped them a whole lot more.
Joanne	So, I'm shown' them, I'm projecting to them, the page and I'm writing on it and I'm filling' it out, just as they would on their, in their books So they're able to see-
Connecting Prior Knowledge	
Faith	So I had to change the wording of the word problems to get them to relate
Joanne	Because if I use some other wording, it might... I think it may bring them to a state of confusion, maybe. Because, I might be bringing in something that they haven't heard before. So I try to use the same vocabulary, the same, little words that we've used in the past to introduce or even to continue on with the lesson we are focusing on

	at the time. So I think that helps them to hear the same things over and over, as opposed to bringing in or introducing some new language without going over it first or without them hearing it numerous times. So that same language, using that same language, I think, is beneficial for them.
Joanne	I think I needed to present the lesson in a way that is tying into what they already know. I mean what they have already been taught. You know, sometimes I think they think it's something new. Something that they haven't seen before, so I have to think about the things that they have been exposed to already. And tie that in and use those same wording sometimes.
Joanne	. I think they have to have that feedback going and kind of get them to thinking' about things that they already learned, you know, and bring that to forefront. Because if not, you know, it... I don't want them to think this is something brand new. This is something that we're just adding on I want to make sure I use those things that we've learning before to, bring it to this lesson so that they can tie it all in.
Joanne	the anchor charts, I use because those are like a routine that we go through. So they see these anchor charts, often. So that is like a reminder for them to look back into their minds of things that we already learned
Michelle	I try to use their logical reasoning to explain, like bringing their personal experience, things that they would do that has to do with time
Michelle	So putting it in a way that, brings in their experience to kind of make it easier to understand.
Models	
Jennifer	And I find that other kids will jump on the microphone and say do it the way I did it was this and instantly, that student, they'll go, "Oh. Great. Thank you. That's what I needed." And they're able to... Many times, they said, "Ooh, I get it now." And I'm like, okay.
Jennifer	I kind of grouped them up with ones that I knew what we were doing and ones that needed help. And that if they could see the other person on the page using their strategy that kind of gave them a strategy to use if they didn't already have one.
Michelle	The extra modeling, the videos, and all of that helps them to like gain some kind of confidence in the knowledge that they already have.
Joanne	I go through the lesson and I tell them, why I'm doing certain things. I use the R-I-D-D. I read it twice. Identify the important information, determine the operation and then I determine the

	solution. So I would go through that and I just voice that out. And it's something that they've heard before, but I'm constantly doing it in that way so that they can have a good example of what they're doing, every time they do a problem. You know, it's something that is or should be routine to them at this point
Joanne	They're watching you complete the assignment that you are shown' during the, I do, the teacher does, the part the teacher does. So they can see, as you're talking, they can see you doing your 10s, like I was.
Joanne	they can see me work out the problem, going through all the steps from their devices because I have presented my screen to them.
Michelle	Modeling how to do it and actually have them watch, also watch a video on how to do it as well.
	Resources
Assessment	
Faith	Looking at the, assessment for the standard.
Michelle	being that it's 35% of the FSA math questions. So that was one of the things that influenced my decision
Curriculum Supports	
Sarah	I pretty much use the CPS guideline sequencing and also the lessons that have already been created to kind of go off of to teach that standard.
Jennifer	I know that there are changes that I make sometimes because of suggestions I've gotten or maybe things I didn't think of when I had planned it that... Or adding to it, or maybe taking things away so I know that also impacts how I plan what I'm going to teach.
Jennifer	I guess the other one is maybe collaboration with your team or... We do a lot of collaboration with our APs. And I know that sometimes, that will influence my lesson or how I... We go over lessons that we've already planned to look at them before we teach them.
Faith	that was just one of the factors that maybe contributing to changing my lesson for something.
Online Learning Tools	
Sarah	I felt that the measuring, when we were doing measuring before we go into the word problems, it was kind of difficult to get them to

	kind of grasp it because I feel like measurement is one area where you have to kind of use the tools and use the objects
Jennifer	I use all day are Nearpod and SMART Learning. And in Nearpod, I can see everybody's page all at once. And in SMART, I can kind of careen through and go through each child like, right there at the board. I can just sift through every student's page wherever they are, even the my ones in the room now-
Jennifer	The way it's set up is because everybody was online before, they got used to the system... We're online the entire day. And we are in an online lesson either in Nearpod or in SMART the entire day. So it's basically the same thing that I might have done before in a SMART learning or in a power point, but it is loaded into the program. So they're able to see-
Joanne	I've tried to find different games, like Kahoot or Nearpod, things like that, that have, more hands on, because they don't have that advantage right now, being on line
Joanne	If there's a PDF of the page, that I'm working in Go Math, I select my Kami and I'm able to write on Kami, on the page with them watching me from their Google Classroom, from the Google Meets.
Lack of Resources	
Sarah	a lot of the kids being online, it's difficult to get them to grasp the concept of what actual-... You know, how long is an actual yardstick or meter stick. And things like that, because they didn't have the stuff at home to...
Sarah	I knew. You... You kind of got... You have to... You need the tools in order to actual... Actually do the measuring part of it. And I knew that most of my kids wouldn't have anything besides a small ruler and I knew that they had a small ruler because I sent it home with them, you know, over Christmas break. But as far as having the measuring tape, the yardstick, meter stick. Those kinds of things, they didn't have. So I did have to throw in into the planning that, okay, for this section, you're going to have to actually measure this with them using this tool so they can see what it is, what it looks like, and you know, about how long whatever you're measuring is using that specific tool.
Joanne	I think I'm always try to keep that in consideration that I don't have all my students face-to-face, so I have to do things in a much different way. They don't the manipulatives that... all the manipulatives that they would have if they were in the classroom,

	like, the, ones and 10s, the little blocks that we usually, have for them in previous years. So, there's a lot of things that we have to do differently in that way, so that we can maybe teach them better, without the use of manipulatives so you have to find different ways
Joanne	A lot of times I try to bring in visuals or any type of manipulatives and not manipulatives that we're really used to, like the hands on, because it's hard to do that now. But, things that I guess, make it plain for them. So that's why I brought in Kami, I brought in anchor charts. Different things that ,I mean, I would use anchor charts and have them displayed. But you know, we have to do it in a different way, because we don't have a wall that we can just have up and them, you know, they can refer to, throughout the day or throughout the lesson
	Content progression
Sarah	I could see that they were not getting the concept. They were not getting it at all and if they couldn't get past that, then how were we going to add and subtract measurements
Sarah	." Then we went through and talked about there's about, this amount of inches in one foot. And then, we compared the paper clip to an actual ruler and then just kind of built onto it like that.
Faith	The attendance. that would have changed some of my lessons... Well, I mean, not changed it, but I would've had to get those students to where everyone else was when... When they were out. decided how deep we needed to go, how far in the basics we needed to go back.
Jennifer	They still need it. So I've added more measurement in as we've went and then I kind of also noticed that the way that the lessons were written, it kind of already assumed that they knew what line slots were, they knew everything about number lines, and that's not something that they knew. So I kind of had to work backwards a little bit and go into some I-Ready materials to see where I could add in some prerequisite ideas that weren't in the CRMs when I was planning.
Joanne	I think Go Math is less complex. And it's focused on the lesson that they started with focused on the 120 chart as a tool to add 10s. And then, we did, add ones in the second lesson
Joanne	We looked at visual and examples of 10s and ones digits. We went over examples of adding, how, how to add one digit numbers, because they were about to get into two, two digit adding 10s to two-digit numbers
Joanne	We reviewed the 120 chart. Looking at numbers and patterns, because they have to understand the concept of, when

	they're adding numbers they have to know that they're adding, not just the number itself, but they're adding 10s and where they're moving the 120 chart
Michelle	I knew that we had to start with the basic, talking about what those hands mean and how to count the time of the minutes and the hours. I have to like, break it down more To start out with the basics
Faith	I started teaching the whole lesson with making sure they knew the math vocabulary. They knew what a ruler was. They know what it's for. They knew... Know what a yardstick is, what is it for. You know what a measuring tape is, what is it for. So the math... That vocabulary... Oh, and the inches, the centimeters, Things like that, That were in the CRMs, I made sure that we did that first. Introducing... When I was introducing the... The standard.
Faith	I feel that like, you miss one step and you're completely lost For the next one. Those kids that were absent, I had to go back, re-teach that to get them moving with the rest of the group. Understanding Student Skills and Knowledge
Questioning	
Sarah	through videos and then also through you know, I took the computer and walked around with my... You know, with... With it being projected on myself, Using the yardstick, trying to show them, you know, as much as I should in the camera view. And we measured the door, we measured the window, and then just, you know... Questioning... Questioning them through the process
Faith	And... Were able to be more vocal about it and they were able to ask ,I think more in-depth questions. And I could also show them... The... The measurements and how it, you know... How it works that way much better in small groups than whole group.
Sarah	I think they kind of grasped it a lot because, I use repetition. It's something that I would always constantly, you know, talk about when we were in that area of the standard. And having them call it back to me. So, I would... You know, I would say it and then... I would, you know, at random times, I'd say, "Okay, and... How many inches are in a foot?" Or you know, things like that. Just repetition, me saying it and having them also repeat it back to me
Jennifer	would discuss what they did and how they came to that conclusion and it seemed to help them to understand a little bit better than just me telling them in my way. It gave them alternate strategies that maybe even I don't use, but still they were able to reach the correct

	answer from using someone else's strategy instead. Worked better for them
Jennifer	... And then if I need to help them more, I'll go to them. But it's a quicker way for me to go and immediately check and I can see as they're writing on the page exactly what they're doing at that exact moment. So if I want to stop and say, "Hey, you're writing this number. How did you come up with that number? How did you make that decision?" And then they can... They'll go through the steps on how they got... And I can see and then I can write in real time right on their paper or I can help them, say maybe, "Did you look at this piece here?" And it just... It seems to work better than taking that, you know, the extra walk over, now we have to explain. I like to be able to see as they're writing it out, what their thinking is and then have them literally to explain it to them , right there.
Michelle	What operations are required, how many steps. So I knew that they had issues with multi-step problems, so that was something that influenced by decision to start at the basics
Joanne	So that's a, we do. We went through the problems that I added in Kami. And we went through it together, and I walked them through it. We, did, okay, asked some questions. "Okay, what's our next step? Somebody tell me." And I usually use the chat box. And everybody has a chance to answer
Joanne	So I'm looking for that. I'm looking for, who is has a good understanding? Who is still not answering the question? And then I know how to proceed, if I need to do another example or if they need to have another practice problem
Joanne	I saw that there were still a few that were confused as how to go up or down the 120 chart. So I went back to the 120 chart and I called on persons that were not answering'
Lesson Analysis	
Jennifer	guided by the CRMs. And more lately, I've been letting where my kids are, kind of guiding them past that because there are things that I find that they're missing now that we're getting into the second half of the year that I didn't have a problem with at the beginning of the year.
Jennifer	I find that what I planned for isn't the best because I've been planning with other people and maybe what they do isn't always what I do. I find honestly, most of my days kind of, that's not working. Change it now. I don't wait till tomorrow to fix it, I'm fix it now so that we can learn it today.

Jennifer	I find that my kids online kind of guide that because I have several that love to come on and help other people if they're struggling. And they love to explain it to them so that they can understand because they want them to be able to get it. Number one because they want us to move on and turn the page. And two, they like to be able to discuss that with someone. They like that someone's listening to them, like, "Yes, please tell me. I want to know how to get this." So I find that I let them kind of guide where we need to go and then I'm willing to change whatever's not working right then and there and then I fix it. So [inaudible 00:33:02] right then and there. It's not always something I planned for. It's not even generally something I plan for.
Faith	With teaching it in whole groups. And then, I found out that some got the concept faster than the others and then I just started having a number talk, As whole group, and then teaching it in my small groups.
Sarah	I think it... It just kind of goes with the flow of teaching. Like if you're... You're teaching something and... You know, this math concept and you noticed that they're not getting... Because I've had times when I'm thinking, oh, this is a great lesson, you know, they're going to get this. And then we get to get in though the lesson and it's crickets. You know, nobody understands it, so then you kind have to, as a teacher, you kind of got to reflect on that and say, "Okay, so what did I... What did I do that didn't work? What worked? You know, and how can I change it for the next time?" You know, do I need to, um... Like see said, do I need to throw in more of that vocabulary that they might be used to that they could understand it better, You know, do I need to show a video where they can see it, or is it something that I can show myself doing And then try to apply it to... I think it's kind of reflection, I guess, is what I'm trying to say. I needed them to be able to grasp that... That concept. But I... I don't know, I guess I did think about it when I was preplanning for it, but it didn't really hit until it was... It was a lot harder to teach. And a lot harder to get them to answer my questions in... In the math block. For the next day, or... I had to plan a different route in order to have a successful math block the next day
Jennifer	I find being online actually has helped me to see because having the ability to see almost everyone's paper like, right then and there, I can see quickly if they're not getting it
Jennifer	With this group I have, they participate pretty much 100%, so if I see someone not participating, I'm like, okay, something's going

	wrong. They're not understanding, and then I ask if you need help, and they're like, "Yeah, I need help." I'm like okay, that's not working, let's see what we need to do to correct that or fix that and then start breaking it down.
Michelle	I started out by just give them a question with the clock and just trying to figure out if they knew how to read it and what each one meant.
Michelle	I first gave them the question, so kind of using the discovery method, as seeing like having them kind of do it first, before I tell them how to do it

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