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Scattering by a perfectly conducting strip loaded with a dielectric cylinder (TM case)

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Abstract: A rigorous analysis of the problem of scattering from an incident E -polarised wave on a dielectric-loaded perfectly conducting strip is presented for a line source and plane wave excitations. The vector Green's theorem is used to derive an integral equation for the unknown electric and magnetic field components. The singular component of the magnetic field is then cancelled out by properly choosing the auxiliary fields and boundary conditions. The unknown expansion coefficients of the scattered electric field are determined by numerically solving an infinite set of simultaneous linear equations after truncation. The accuracy of the proposed solution is verified for the loaded and unloaded strip by comparing the resulting values of the scattered field with those based on the method of moments. For the loaded strip, the dependence of the scattered field as well as the scattering cross-sections on the strip width, dielectric constant and angle of incident wave is investigated and presented for plane wave incidence.

1 Introduction

The scattering by a thin perfectly conducting strip has been studied by many authors using different techniques. The exact solution of a plane wave scattered by a conducting strip was presented by Morse and Rubenstein in terms of Mathieu functions by solving the Helmholtz equation in elliptic cylindrical co-ordinates [1]. Approximate solutions for the low and high frequencies were also investigated [2-7]. Previous attempts to maximise or minimise the scattering cross-section of thin strips include the use of resistive rather than conducting materials [8, 9] and loading the edges by cylindrical dielectric shells of homogeneous or inhomogeneous permittivity profiles [10]. This paper examines the scattering properties of a thin perfectly conducting strip loaded by a dielectric cylinder as shown in Fig. 1. The dielectric medium is assumed to be linear, homogeneous, isotropic and free from losses and characterised by relative permittivity ϵ_r . The analysis is based upon integral equations derived from Green's second identity. The reciprocity theorem is used along with appropriate boundary condi-

tions to transform the resulting integral equations into an equivalent matrix equation of infinite order. The matrix equation is then solved numerically, after truncation to retrieve the unknown expansion coefficients of the scattered field.

Fig. 1 Perfectly conducting finite strip loaded with dielectric cylinder

2 Basic formulation

Consider the E -polarisation case (TM with respect to the z -axis) where the electric field has a z component only of the circular cylindrical co-ordinates (ρ, ϕ, z) . A time dependence $e^{j\omega t}$ is assumed and suppressed throughout. Furthermore, consider two Maxwellian fields $(\mathbf{E}, \mathbf{H})$ and $(\mathbf{E}_m, \mathbf{H}_m)$ of the same frequency, supported, respectively, by sources $(\mathbf{J}, \mathbf{M})$ and $(\mathbf{J}_m, \mathbf{M}_m)$ which might exist within a surface S , enclosing a volume V , of a linear, homogeneous and isotropic medium. An application of Green's second identity, or Green's vector theorem yields [11]

$$\int_S (\mathbf{E}_m \times \mathbf{H} - \mathbf{E} \times \mathbf{H}_m) \cdot \hat{n} dS = \int_V (\mathbf{E} \cdot \mathbf{J}_m - \mathbf{E}_m \cdot \mathbf{J} - \mathbf{H} \cdot \mathbf{M}_m + \mathbf{H}_m \cdot \mathbf{M}) dV \quad (1)$$

where $\hat{n}$ is a unit normal directed out of V . Let $\mathbf{E}$ and $\mathbf{H}$ represent the unknown fields due to the sources $\mathbf{J}$ and $\mathbf{M}$, while $\mathbf{E}_m$ and $\mathbf{H}_m$ are the chosen fields, the sources of which will always be situated outside the region of application. To obtain a solution for the real fields, we divide the space into three regions where the integer m denotes the region number. Regions I and II are inside the dielectric cylinder and are separated by the conducting strip,

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whereas region III is outside the cylinder. We also assume that the strip lies in the z - x plane, the axis of the cylinder coincides with the z -axis and the radius of the cylinder is equal to half the width of the strip (denoted by a) as shown in Fig. 1. It is convenient to make a different choice of E_m and H_m for the different regions, though in all regions they are source free, i.e. J_m and M_m are zero. If the real sources J and M are in region III and with $M = 0$, an application of eqn. 1 to region I yields

$$\int_0^\pi (E_z H_{1\phi} - E_{1z} H_\phi) d\phi = 0 \quad \rho = a \quad (2)$$

similarly for region II, we have

$$\int_\pi^{2\pi} (E_z H_{2\phi} - E_{2z} H_\phi) d\phi = 0 \quad \rho = a \quad (3)$$

while for region III, eqn. 1 reduces to

$$\int_0^{2\pi} (E_{3z} H_\phi - E_z H_{3\phi}) d\phi = \frac{-I}{a} E_{3z}(\rho_0, \phi_0) \quad \rho = a \quad (4)$$

Since there is no z variation in either region, the three regions are assumed to be of arbitrary finite length in the z direction. As a result, the integration with respect to z on both sides of eqn. 1 results in the assumed finite length which is then cancelled out from both sides. The surface integrals over the end caps of a cylindrical region also cancels each other since the outward normals are oppositely directed. Also, the fields E_z , E_{1z} and E_{2z} are assumed to be zero on the strip surface (to satisfy the Dirichlet boundary condition) while the far field radiation condition is applied. In eqn. 4 I is the intensity of the electric line source located at (ρ_0, ϕ_0) . Eqns. 2-4 represent three integral equations to be solved for the real electric and magnetic fields E_z and H_ϕ on the boundary $\rho = a$. However, due to the singularity of the magnetic field at the edge of the conducting strip, it is better to eliminate the unknown magnetic field from these equations, leaving the finite electric field to be determined. In order to extract H_ϕ one may choose for convenience the following boundary conditions:

$$E_{1z} = E_{3z} \quad \rho = a \quad 0 \leq \phi \leq \pi \quad (5)$$

$$E_{2z} = E_{3z} \quad \rho = a \quad \pi \leq \phi \leq 2\pi \quad (6)$$

Hence by subtracting eqns. 2 and 3 from eqn. 4 and using the relations given by eqns. 5 and 6, we obtain

$$\int_0^\pi E_z H_{1\phi} d\phi + \int_\pi^{2\pi} E_z H_{2\phi} d\phi - \int_0^{2\pi} E_z H_{3\phi} d\phi = \frac{-I}{a} E_{3z}(\rho_0, \phi_0) \quad \rho = a \quad (7)$$

where the integrals containing the unknown H_ϕ cancel out.

The auxiliary fields E_{1z} , E_{2z} may be expanded as Fourier series equated to E_{3z} over the relevant ranges. Hence, we assume that

$$E_{1z} = \sum_{n=1}^{\infty} A(n, p) J_n(k_1 \rho) \sin n\phi \quad (8)$$

$$E_{2z} = \sum_{m=1}^{\infty} B(m, p) J_m(k_1 \rho) \sin m(\phi - \pi) \quad (9)$$

where E_{3z} may take the following form

$$E_{3z} = H_p(k\rho) e^{jp\phi} \quad p = 0, \pm 1, \pm 2, \dots \quad (10)$$

In the above equations $J_n(x)$ is the Bessel function of the

first kind of order n and argument x , k is the wave number in free space, $k_1 = \sqrt{\epsilon_r} k$. $H_p(x)$ is the Hankel function of order p and argument x , while the superscript (2) is implied and suppressed throughout. It should be noted that the expansion coefficients A and B are determined by imposing the boundary conditions given by eqns. 5 and 6 which yield

$$A(n, p) = \frac{2H_p(ka)}{\pi J_n(k_1 a)} \int_0^\pi e^{jp\phi} \sin n\phi d\phi \quad (11)$$

$$B(m, p) = \frac{2H_p(ka)}{\pi J_m(k_1 a)} \int_\pi^{2\pi} e^{jp\phi} \sin m(\phi - \pi) d\phi \quad (12)$$

Furthermore, the corresponding ϕ components of the magnetic fields are

$$H_{1\phi} = \frac{1}{j\eta_1} \sum_{n=1}^{\infty} A(n, p) J'_n(k_1 \rho) \sin n\phi \quad (13)$$

$$H_{2\phi} = \frac{1}{j\eta_1} \sum_{m=1}^{\infty} B(m, p) J'_m(k_1 \rho) \sin m(\phi - \pi) \quad (14)$$

$$H_{3\phi} = \frac{1}{j\eta} H'_p(k\rho) e^{jp\phi} \quad (15)$$

where the primes denote differentiation with respect to the full argument, η is the intrinsic impedance of free space and $\eta_1 = \eta/\sqrt{\epsilon_r}$. Now, substitution of the auxiliary field expression into eqn. 7 yields

$$\begin{aligned} & \frac{1}{j\eta_1} \sum_{n=1}^{\infty} A(n, p) J'_n(k_1 a) \int_0^\pi E_z \sin n\phi d\phi \\ & + \frac{1}{j\eta_1} \sum_{m=1}^{\infty} B(m, p) J'_m(k_1 a) \int_\pi^{2\pi} E_z \sin m(\phi - \pi) d\phi \\ & - \frac{1}{j\eta} H'_p(ka) \int_0^{2\pi} E_z e^{jp\phi} d\phi = \frac{-I}{a} H_p(k\rho_0) e^{jp\phi_0} \quad (16) \end{aligned}$$

The solution of the above equation gives the values of the field E_z at the boundary $\rho = a$. Assuming the following series for E_z in region III

$$E_z = \sum_{q=-\infty}^{\infty} C_q H_q(k\rho) e^{-jq\phi} \quad (17)$$

and substituting eqn. 17 into eqn. 16, we obtain

$$\begin{aligned} & \sum_{q=-\infty}^{\infty} C_q H_q(ka) \left[\sum_{n=1}^{\infty} A(n, p) J'_n(k_1 a) f_a(n, q) \right. \\ & \quad \left. + \sum_{m=1}^{\infty} B(m, p) J'_m(k_1 a) f_b(m, q) \right] - \frac{2\pi}{\sqrt{\epsilon_r}} \\ & \quad \times C_p H'_p(ka) H_p(ka) = \frac{I\eta_1}{ja} H_p(k\rho_0) e^{jp\phi_0} \quad (18) \end{aligned}$$

where C_q and C_p are the unknown coefficients and f_a and f_b are given by

$$f_a(n, q) = \begin{cases} \frac{n[1 - e^{jp\pi}(-1)^n]}{(n^2 - p^2)} & q \neq \pm n \\ \pm \frac{\pi}{2j} & q = \pm n \end{cases} \quad (19)$$

$$f_b(m, q) = \begin{cases} \frac{m[e^{-jp\pi} - (-1)^m]}{(m^2 - p^2)} & q \neq \pm m \\ \pm \frac{\pi}{2j} e^{-jq\pi} & q = \pm m \end{cases} \quad (20)$$

The two summations inside the square brackets in eqn. 18 are functions of p and q and can therefore be represented by a function $f_c(p, q)$. In the above two summations, the number of terms are set to be at least equal to $3k_1a + 5$ to insure the convergence. Thus eqn. 18 can be written as

$$\sum_{q=-\infty}^{\infty} C_q H_q(ka) f_c(p, q) - \frac{2\pi}{\sqrt{\epsilon_r}} \times C_p H'_p(ka) H_p(ka) = \frac{1}{k_1 a} H_p(k\rho_0) e^{jp\phi_0} \quad (21)$$

where I is given the value $j/(\eta k)$. This infinite set of equations can be approximated by truncating the variation in p and q from $-N$ to $+N$, where N is chosen large enough to give the accuracy required.

3 Scattered field

The field of an electric current filament of intensity I located at (ρ_0, ϕ_0) and parallel to the z -axis is given by

$$E_l^i = \begin{cases} -\frac{\eta k I}{4} \sum_{n=-\infty}^{\infty} H_n(k\rho_0) J_n(k\rho) e^{-jn(\phi-\phi_0)} & \rho < \rho_0 \\ -\frac{\eta k I}{4} \sum_{n=-\infty}^{\infty} J_n(k\rho_0) H_n(k\rho) e^{-jn(\phi-\phi_0)} & \rho > \rho_0 \end{cases} \quad (22)$$

where the subscript l refers to a line source. The scattered field is then written as

$$E_l^s = -\frac{\eta k}{4} \sum_{n=-\infty}^{\infty} d_n H_n(k\rho_0) H_n(k\rho) e^{-jn(\phi-\phi_0)} \quad \rho_0 \geq a \quad (23)$$

Therefore the total field is given by

$$E_l^i = \begin{cases} -\frac{\eta k}{4} \sum_{n=-\infty}^{\infty} H_n(k\rho_0) e^{-jn(\phi-\phi_0)} \\ \quad \times [I J_n(k\rho) + d_n H_n(k\rho)] & a \leq \rho < \rho_0 \\ -\frac{\eta k}{4} \sum_{n=-\infty}^{\infty} H_n(k\rho) e^{-jn(\phi-\phi_0)} \\ \quad \times [I J_n(k\rho_0) + d_n H_n(k\rho_0)] & \rho > \rho_0 \end{cases} \quad (24)$$

In comparing eqns. 17 and 24 at $\rho = a$ the unknown coefficients d_n can be determined, i.e.

$$d_n = -\frac{4C_n e^{-jn\phi_0}}{\eta k H_n(k\rho_0)} - \frac{I J_n(ka)}{H_n(ka)} \quad (25)$$

If the line source recedes to infinity, the corresponding incident field is recognised as a plane wave field E_p^i of constant amplitude E_0 , i.e.

$$E_p^i = E_0 e^{jk\rho \cos(\phi-\phi_0)} \quad (26)$$

and the subscript p refers to a plane wave. In such a case the right-hand side of eqn. 18 reduces to $4e^{jI p(\phi_0 + \pi/2) + \pi/2} / k_1 a$ with E_0 being equal to unity. Furthermore, E_p^i and the corresponding scattered field E_p^s can be expressed in terms of the cylindrical co-ordinate system as

$$E_p^i = E_0 \sum_{n=-\infty}^{\infty} j^n J_n(k\rho) e^{-jn(\phi-\phi_0)} \quad (27)$$

$$E_p^s = \sum_{n=-\infty}^{\infty} d_n H_n(k\rho) e^{-jn(\phi-\phi_0)} \quad (28)$$

Then the total field is given as

$$E_p^t = \sum_{n=-\infty}^{\infty} e^{-jn(\phi-\phi_0)} [E_0 j^n J_n(k\rho) + d_n H_n(k\rho)] \quad (29)$$

In comparing eqns. 17 and 29 at $\rho = a$, the coefficient d_n reduces to

$$d_n = C_n e^{-jn\phi_0} - \frac{E_0 j^n J_n(ka)}{H_n(ka)} \quad (30)$$

The far scattered field pattern $F(\phi)$ can be determined after using the large argument approximation of the Hankel function and normalising the resulting expression by the factor $\sqrt{2j} e^{-jk\rho} / \sqrt{\pi k\rho}$. Thus, the scattered field pattern due to a line source excitation $F_l(\phi)$, and due to an incident plane wave $F_p(\phi)$ are given, respectively, by

$$F_l(\phi) = \frac{-nk}{4} \sum_{n=-\infty}^{\infty} d_n H_n(k\rho_0) j^n e^{-jn(\phi-\phi_0)} \quad (31)$$

$$F_p(\phi) = \sum_{n=-\infty}^{\infty} d_n j^n e^{-jn(\phi-\phi_0)} \quad (32)$$

The scattering cross-section of the loaded strip is then given by Reference 12 as

$$\sigma(\phi) = \frac{2}{\pi} |F_p(\phi)|^2 \quad (33)$$

4 Numerical results and discussion

Examination of our analytical solution indicates that as ϵ_r approaches infinity the right-hand side and the second term of the left-hand side of eqn. 21 reduce to zero, which results in a zero value for the coefficients C_q . Thus, the coefficients d_n of the scattered field due to a line source excitation (eqn. 25) or an incident plane wave (eqn. 30) reduce to the well-known expressions for the scattering by a perfectly conducting cylinder. To further check the accuracy of the proposed solution, the scattered field patterns of loaded and unloaded conducting strip are calculated by using the well-known method of moments. As shown in Figs. 2 and 3 for $\phi_0 = 0^\circ$ and 90° , respectively, good agreement between the results based on the proposed technique and the method of moments is obtained for both $ka = 1$ and $ka = 5$. Fig. 4 shows the forward and back scattering cross-sections (σ_f and σ_b , respectively) versus N to check the convergence of the proposed solution for loaded strips. In this Figure $\epsilon_r = 9$ and $\phi_0 = 90^\circ$ while the parameter ka is set equal to 1, 2 and 5. It can be seen that the computed values of σ_f and σ_b converge rapidly with the first few expansion coefficients which is attributed to the efficiency of the proposed solution. Fig. 5 shows the scattered field pattern of a loaded strip excited by a normally incident plane wave where $ka = 1$ and 5 in Figs. 5a and 5b, respectively, and ϵ_r is a parameter. In Fig. 5a, the calculated values of the scattered fields, based on the method of moments are also shown for $\epsilon_r = 5$ and 9. Good agreement is observed between the numerical results based on the proposed method and the moment method, however one should point out that in the method of moment solution the orders of the impedance matrix was 100 and 130 for $\epsilon_r = 5$ and 9, respectively, while in the proposed method N was set to 11 for both cases. Although it is clear from Fig. 5b that the forward scattered field for $ka = 5$ increases with ϵ_r and the back scattered field decreases with ϵ_r , it is not possible to have this behaviour in general, as shown in Fig. 5a for $ka = 1$.

To further investigate the behaviour of the forward and back scattered fields with respect to ϵ_r , we present Fig. 6 which shows the changes of σ_f and σ_b versus ϵ_r for

$\phi_0 = 90^\circ$, while the parameter ka is equal to 1, 2 and 5. Fig. 7 also shows the variations of σ_f and σ_b versus ϵ_r for different values of ϕ_0 , namely 0° , 30° and 60° while

Fig. 2 Normalised far scattered field pattern due to plane wave incident on conducting strip with $\phi_0 = 0^\circ$

a $ka = 1$
b $ka = 5$
— proposed solution
◇◇◇ method of moments

Fig. 3 Normalised far scattered field pattern due to plane wave incident on conducting strip with $\phi_0 = 90^\circ$

a $ka = 1$
b $ka = 5$
— proposed solution
◇◇◇ method of moments

Fig. 4 Scattering cross-section of loaded strip due to plane wave incident with $\phi_0 = 90^\circ$ and $\epsilon_r = 9$ against N

a Forward scattering cross-section at $\phi = 270^\circ$
b Back scattering cross-section at $\phi = 90^\circ$
○ ○ ○ $ka = 1$
△ △ △ $ka = 2$
× × × $ka = 5$

$ka = 2$. From Figs. 6 and 7 one can easily observe that the loaded strip resonates at different values of ϵ_r , ϕ_0 and ka while the number of resonances increases with k_1a for

the normally incident plane wave case. Finally, Fig. 8 shows the plots of σ_f and σ_b against ka for $\phi_0 = 90^\circ$ and ϵ_r as a parameter which is equal to 1, 5 and 9. From the

Fig. 5 Far scattered field pattern due to plane wave incident on loaded strip with $\phi_0 = 90^\circ$

a $ka = 1$
b $ka = 5$
— $\epsilon_r = 1$
--- $\epsilon_r = 5$
- - - $\epsilon_r = 9$
• • • • • method of moments

Fig. 6 Scattering cross-section of loaded strip due to plane wave incident with $\phi_0 = 90^\circ$ against relative permittivity ϵ_r

a Forward scattering cross-section at $\phi = 270^\circ$
b Back scattering cross-section at $\phi = 90^\circ$
— $ka = 1$
--- $ka = 2$
- - - $ka = 5$

Fig. 7 Scattering cross-section of loaded strip due to plane wave incident with $ka = 2$ against relative permittivity ϵ_r

a Forward scattering cross-section
b Back scattering cross-section
— $\phi_0 = 0^\circ$
--- $\phi_0 = 30^\circ$
- - - $\phi_0 = 60^\circ$

Fig. 8 Scattering cross-section of loaded strip due to plane wave incident with $\phi_0 = 90^\circ$ against ka

a Forward scattering cross-section at $\phi = 270^\circ$

b Back scattering cross-section at $\phi = 90^\circ$

— $\epsilon_r = 1$
 - - $\epsilon_r = 5$
 - · - $\epsilon_r = 9$

Figure one concludes that it is possible to increase σ_f and at the same time decrease σ_b for all values of ka , which are generally greater than 2 except at resonance, by increasing the value of k_1a . From the above results it seems that the scattering cross-section of a conducting strip can be greatly minimised or maximised by loading it with a dielectric cylinder. However, there may not be a simple relation between the optimal value of the scattering cross-section and the parameters ka , ϵ_r and ϕ_0 .

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