Annular Beam Shaping And Optical Trepanning

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ANNULAR BEAM SHAPING AND OPTICAL TREPANNING

by

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ABSTRACT

Percussion drilling and trepanning are two laser drilling methods. Percussion drilling is accomplished by focusing the laser beam to approximately the required diameter of the hole, exposing the material to one or a series of laser pulses at the same spot to melt and vaporize the material. Drilling by trepanning involves cutting a hole by rotating a laser beam with an optical element or an x–y galvo-scanner. Optical trepanning is a new laser drilling method using an annular beam. The annular beams allow numerous irradiance profiles to supply laser energy to the workpiece and thus provide more flexibility in affecting the hole quality than a traditional circular laser beam.

Heating depth is important for drilling application. Since there are no good ways to measure the temperature inside substrate during the drilling process, an analytical model for optical trepanning has been developed by considering an axisymmetric, transient heat conduction equation, and the evolutions of the melting temperature isotherm, which is referred to as the melt boundary in this study, are calculated to investigate the influences of the laser pulse shapes and intensity profiles on the hole geometry. This mathematical model provides a means of understanding the thermal effect of laser irradiation with different annular beam shapes.

To take account of conduction in the solid, vaporization and convection due to the melt flow caused by an assist gas, an analytical two-dimensional model is developed for optical trepanning. The influences of pulse duration, laser pulse length, pulse repetition
rate, intensity profiles and beam radius are investigated to examine their effects on the recast layer thickness, hole depth and taper.

The ray tracing technique of geometrical optics is employed to design the necessary optics to transform a Gaussian laser beam into an annular beam of different intensity profiles. Such profiles include half Gaussian with maximum intensities at the inner and outer surfaces of the annulus, respectively, and full Gaussian with maximum intensity within the annulus. Two refractive arrangements have been presented in this study.

Geometric optics, or ray optics, describes light propagation in terms of rays. However, it is a simplification of optics, and fails to account for many important optical effects such as diffraction and polarization. The diffractive behaviors of this optical trepanning system are stimulated and analyzed based on the Fresnel diffraction integral. Diffraction patterns of the resulting optical system are measured using a laser beam analyzer and compared with the theoretical results. Based on the theoretical and experimental results, the effects of experimental parameters are discussed.

We have designed the annular beam shaping optical elements and the gas delivery system to construct an optical trepanning system. Laser drilling experiments are performed on the Stainless Steel-316 (SS 316) plate and the Inconel 718 (IN 718) plate. The geometry of the trepanning holes with different sizes is presented in this study.
To my parents and friends
ACKNOWLEDGMENTS

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# TABLE OF CONTENTS

LIST OF FIGURES .................................................................................................................. xi

LIST OF TABLES .................................................................................................................... xx

LIST OF ACRONYMS/ABBREVIATIONS .............................................................................. xxi

CHAPTER 1: INTRODUCTION AND LITERATURE REVIEW ............................................ 1

1.1. Motivation .................................................................................................................. 1

1.2. Literature Reviews .................................................................................................. 7

  1.2.1. Experimental studies on laser drilling ........................................................... 7

  1.2.2. Theoretical studies on laser drilling ............................................................... 8

  1.2.3. Annular beam shaping .................................................................................. 10

1.3. Objectives ................................................................................................................. 14

CHAPTER 2: TEMPERATURE DISTRIBUTIONS DUE TO ANNULAR LASER BEAM HEATING ........................................................................................................ 16

2.1. Introduction .............................................................................................................. 16

2.2. Intensity Profiles for Annular Laser Beams .......................................................... 16

2.3. Thermal Modeling .................................................................................................. 19

2.4. Results and Discussion .......................................................................................... 23

2.5. Conclusions ............................................................................................................ 38

CHAPTER 3: TWO-DIMENSIONAL MODEL FOR MELTING AND VAPORIZATION DURING OPTICAL TREPANNING ............................................................ 39

3.1. Introduction ............................................................................................................ 39

3.2. Mathematical Model for Optical Trepanning ...................................................... 39
3.3. Mathematical Formulation ................................................................. 43
  3.3.1. Energy balance at the interfaces ..................................................... 43
  3.3.2. Heat conduction in the solid phase ............................................... 44
  3.3.3. Mass and energy balances in the liquid metal layer ....................... 45

3.4. Results and Discussions ................................................................. 48
  3.4.1. Recast layer and cavity depth ......................................................... 50
  3.4.2. Taper .............................................................................................. 55

3.5. Conclusions ....................................................................................... 61

CHAPTER 4: RAY TRACING FOR ANNULAR LASER BEAM SHAPING .......... 62
  4.1. Design Consideration ........................................................................ 62
  4.2. Two Steps Transformation of a Gaussian Beam into an Annular Beam ...... 63
    4.2.1. Transformation of a Gaussian Beam into a Uniform Circular Beam ...... 63
    4.2.2. Transformation of a uniform circular beam into an annular beam ........ 67
    4.2.3. Design analysis ............................................................................. 73
    4.2.4. Design results ............................................................................. 80
  4.3. Single Step Transformation of a Gaussian Beam into an Annular Beam ...... 85
    4.3.1 Conservation of energy ................................................................. 85
    4.3.2 Design analysis ............................................................................. 87
    4.3.3 Design results ............................................................................. 93
  4.4. Conclusions ....................................................................................... 96

CHAPTER 5: DIFFRACTION ANALYSIS FOR ANNULAR LASER BEAM
SHAPING ................................................................................................. 97
  5.1. Introduction ...................................................................................... 97
6.4.2. Variation of the size of the trepanning hole................................. 156
6.5. Conclusions...................................................................................... 160

CHAPTER 7: SUMMARY ......................................................................... 161
7.1. Conclusions..................................................................................... 161
7.2. Future Work.................................................................................... 164
REFERENCES .......................................................................................... 166
LIST OF FIGURES

Figure 1.1 Schematic diagrams of different laser drilling techniques. ......................... 3

Figure 1.2 Typical setup with laser, fast galvo shutter, telescope, quarter wave plate, scan head, and x-y-z-stage. The quarterwave plate ensures circular polarization of the beam incident on the sample [Herbst et al. (2003)]. .................................................... 4

Figure 1.3 Trepanning heat optics system. The focal spot on the sample draws a circular path [Herbst et al. (2003)]. ........................................................................................... 4

Figure 1.4 Photograph of a turbine blade with laser drilled holes for an aircraft engine. . 5

Figure 1.5 Generation of a Bessel beam and an annular beam with an axicon. ............... 12

Figure 2.1 Transient temperature distributions due to a full Gaussian annular laser beam with uniform pulse ($P_a = 150 \text{ W}, f = 2 \text{ kHz}, t_{on} = 100 \text{ µs}$). ........................................ 25

Figure 2.2 Transient temperature distributions due to a full Gaussian annular laser beam with triangular pulse ($P_a = 150 \text{ W}, f = 2 \text{ kHz}, t_{on} = 100 \text{ µs}$). ................................ 26

Figure 2.3 Radial temperature variations due to a full Gaussian annular laser beam with uniform pulse ($P_a = 150 \text{ W}, f = 2 \text{ kHz}, t_{on} = 100 \text{ µs}$). ........................................ 28

Figure 2.4 Radial temperature variations due to a full Gaussian annular laser beam with triangular pulse ($P_a = 150 \text{ W}, f = 2 \text{ kHz}, t_{on} = 100 \text{ µs}$). ........................................ 29

Figure 2.5 Axial temperature variations due to a full Gaussian annular laser beam with uniform pulse ($P_a = 150 \text{ W}, f = 2 \text{ kHz}, t_{on} = 100 \text{ µs}$). ........................................ 30

Figure 2.6 Axial temperature variations due to a full Gaussian annular laser beam with triangular pulse ($P_a = 150 \text{ W}, f = 2 \text{ kHz}, t_{on} = 100 \text{ µs}$). ........................................ 31
Figure 2.7 Evolution of the workpiece melt boundary for different laser intensity profiles and uniform laser pulse ($P_a = 50\,\text{W}, f = 1\,\text{kHz}, t_{on} = 100\,\mu\text{s}$, drilling time ($t$) = 21 $\mu\text{s}$). .......................................................... 33

Figure 2.8 Evolution of the workpiece melt boundary for different laser intensity profiles and uniform laser pulse ($P_a = 12\,\text{W}, f = 1\,\text{kHz}, t_{on} = 100\,\text{ns}$, drilling time ($t$) = 6 $\mu\text{s}$). .......................................................... 34

Figure 2.9 Evolution of the workpiece melt boundary for different laser intensity profiles and uniform laser pulse ($P_a = 12\,\text{W}, f = 1\,\text{kHz}, t_{on} = 10\,\text{ns}$, drilling time ($t$) = 2 $\mu\text{s}$). 36

Figure 2.10 Evolution of the workpiece melt boundary for different laser intensity profiles and triangular laser pulse ($P_a = 50\,\text{W}, f = 1\,\text{kHz}, t_{on} = 100\,\mu\text{s}$, drilling time ($t$) = 21 $\mu\text{s}$). .......................................................... 37

Figure 3.1 Schematic diagram of two–dimensional model for melting and vaporization during optical trepanning. .......................................................... 41

Figure 3.2 Control volume for mass and energy balances in the metal layer. ............. 42

Figure 3.3 Variation of recast layer thickness and cavity depth with laser intensity for full Gaussian annular beam ($f = 1\,\text{kHz}, t_{on} = 100\,\text{ns}$, drilling time ($t$) = 100 ms, $R_{oa} = 85\,\mu\text{m}$ and $R_{ia} = 65\,\mu\text{m}$). .......................................................... 51

Figure 3.4 Variation of recast layer thickness and cavity depth with laser intensity ($f = 1\,\text{kHz}, t_{on} = 100\,\text{ns}$, drilling time ($t$) = 100 ms, $R_{oa} = 85\,\mu\text{m}$ and $R_{ia} = 65\,\mu\text{m}$). ............... 53

Figure 3.5 Variation of recast layer thickness and cavity depth with inner radius of the annular laser beam for full Gaussian annular beam ($f = 1\,\text{kHz}, t_{on} = 100\,\text{ns}$, drilling time ($t$) = 100 ms, $R_{oa} = 85\,\mu\text{m}$). .......................................................... 54
Figure 3.6 Geometrical description of taper angle for a trepanning hole. ........................ 55

Figure 3.7 Variation of taper and laser drilling time with the laser intensity for full
Gaussian annular beam ($S_{\alpha} = 632 \, \mu m$, $f = 1$ kHz, $R_{oa} = 85 \, \mu m$ and $R_{ia} = 65 \, \mu m$). .. 57

Figure 3.8 Variation of taper and drilling time with laser intensity for different laser
intensity profiles ($S_{\alpha} = 632 \, \mu m$, $f = 1$ kHz, $R_{oa} = 85 \, \mu m$ and $R_{ia} = 65 \, \mu m$). ........ 58

Figure 3.9 Variation of taper and drilling time with the inner radius of annular beam for
full Gaussian annular beam ($S_{\alpha} = 632 \, \mu m$, $t_{on} = 100 \, ns$, $f = 1$ kHz and $R_{oa} = 85 \, \mu m$). ................................................................. 60

Figure 4.1 Geometrical configuration of laser beam shaping system for circular beam. . 64

Figure 4.2 Geometrical configuration of laser beam shaping system............................... 65

Figure 4.3 Geometrical configuration of axicon refractive system. ................................. 70

Figure 4.4 Geometrical configuration of vaxicon refractive system. ............................... 72

Figure 4.5 Distance ($D^*$) between the input and output lens versus central ray refraction
angle.......................................................................................................................... 76

Figure 4.6 Minimum slope of lens surface versus central ray refraction angle. ............ 77

Figure 4.7 Lens surface curvature parameter $\gamma$ versus central ray refraction angle. ...... 78

Figure 4.8. Maximum slope of lens surfaces versus central ray refraction angle.......... 79

Figure 4.9 Input and output surface of lens for converting a Gaussian beam into a uniform
beam. ......................................................................................................................... 81

Figure 4.10 Input and output surface of axicon lens for converting a uniform circular
beam into an annular beam. ...................................................................................... 82

Figure 4.11 Input and output surface of vaxicon lens for converting a uniform circular
beam into an annular beam. ...................................................................................... 83
Figure 4.12 Distance ($D^*$) between the input and output lens versus central ray refraction angle.......................................................................................................................... 89

Figure 4.13 Minimum slope of lens surface versus central ray refraction angle........... 90

Figure 4.14 Lens surface curvature parameter $\gamma$ versus central ray refraction angle........ 91

Figure 4.15 Maximum slope of lens surfaces versus central ray refraction angle.......... 92

Figure 5.1 Geometrical configuration of an axicon refractive system......................... 98

Figure 5.2 Irradiance profile of an input Gaussian laser beam before the first axicon lens. (a) Transverse cross section of the annular beam profile. (b) Laser irradiance profile in the X-direction on the transverse plane. (c) Laser irradiance profile in the Y-direction on the transverse plane. Radius of the input Gaussian beam $r_w = 0.327$ mm. .................................................................................................................................................. 105

Figure 5.3 A small part of the transverse cross section of a large annular beam showing radial variation of the irradiance profile after passing through the first axicon lens. $z' = 71$ mm, $r_w = 0.327$ mm.................................................................................................................. 107

Figure 5.4 Variations of the theoretical laser irradiance distributions $I(\theta, z)$ along the $z$-axis ($r = 0$) after the first axicon lens for different base angles, to check where the spot of Arago disappears (point $A^*$ in this Fig.) so that the second axicon lens can be placed there. $r_w = 0.327$ mm and $I_0 = 1$ W/mm$^2$. ......................................................... 108

Figure 5.5 A small section of a large annular beam showing radial variation of the irradiance profile after passing through the second axicon lens. $r_w = 0.327$ mm, $z' = 95$ mm and $Z' = 50$ mm. ........................................................................................................................................ 110
Figure 5.6 Propagation of an annular beam after passing through the second axicon lens, showing the variation of the irradiance in the radial direction. $r_w = 0.327 \text{ mm}$, $z' = 95 \text{ mm}$ and $Z' = 110 \text{ mm}$.

Figure 5.7 Development of diffraction patterns along axial direction in annular beam shaping with a refractive axicon system. (a) Experimental diffraction patterns produced by an imperfect refractive axicon system. An imperfect axicon has a blunt vertex. (b) Theoretical diffraction patterns produced by a perfect refractive axicon system. A perfect axicon has a pointed vertex.

Figure 5.8. Irradiance distributions of an annular beam before the focal plane at $L = 47$ mm for a convex lens of focal length $f_i = 50$ mm. (a) Transverse cross section of the annular beam profile. (b) Beam profile in the $X$–direction on the transverse plane. (c) Beam profile in the $Y$–direction on the transverse plane. $R_o = 0.665 \text{ mm}$, $r_w = 0.327 \text{ mm}$, $z' = 95 \text{ mm}$ and $Z' = 110 \text{ mm}$.

Figure 5.9 Irradiance distributions along the radial direction at the focal plane. $f_i = 50$ mm, $r_w = 0.327 \text{ mm}$, $z' = 95 \text{ mm}$ and $Z' = 110 \text{ mm}$.

Figure 5.10 Irradiance distributions of the annular beam at 2 mm after the focal plane (i.e., at $L = 52$ mm). (a) Transverse cross section of the annular beam profile. (b) Beam profile in the $X$–direction on the transverse plane. (c) Beam profile in the $Y$–direction on the transverse plane. $f_i = 50$ mm, $L = 52$mm, $r_w = 0.327$ mm, $z' = 95$ mm and $Z' = 110$ mm.

Figure 5.11. Outer diameter of an annular beam at different axial locations after passing through a convex lens. $r_w = 0.327 \text{ mm}$, $z' = 95 \text{ mm}$, $Z' = 110 \text{ mm}$ and $f_i = 50$ mm, 49.5 mm and 45.3 mm.
Figure 5.12 Outer diameter of an annular beam at different axial locations after passing through a convex lens (expanded scale). $r_w = 0.327$ mm, $z' = 95$ mm, $Z' = 110$ mm and $f_i = 50$ mm, 49.5 mm and 49.3 mm. ................................................................. 124

Figure 5.13 The normalized intensity against the axial distance L along the optical axis for a converging full Gaussian annular beam and a converging Gaussian circular beam. For the full Gaussian annular beam, $R_{ca} = 13.5$ mm, $R_w = 0.52$ mm, $\rho_1 = 50$ mm, $\lambda = 1064$ nm and $f_i = 50$ mm. For the Gaussian circular beam, $r_w = 0.52$ mm, $\rho_1 = 50$ mm, $\lambda = 1064$ nm and $f_i = 50$ mm. .................................................................................. 127

Figure 5.14 The normalized intensity against the axial distance L along the optical axis for a converging full Gaussian annular beam and a converging Gaussian circular beam. For the full Gaussian annular beam, $R_{ca} = 13.5$ mm, $R_w = 0.52$ mm, $\rho_1 = 50$ mm, $\lambda = 1064$ nm and $f_i = 50$ mm. For the Gaussian circular beam, $r_w = 0.52$ mm, $\rho_1 = 50$ mm, $\lambda = 1064$ nm and $f_i = 200$ mm................................. 128

Figure 5.15 Variations of the diffraction limits of the converging annular beam. Where $R_{ca} = 10$ mm and $\lambda = 1064$ nm. ................................................................. 130

Figure 5.16 Variations of the diffraction limits of the converging annular beam. Where $R_w = 1.5$ mm and $\lambda = 1064$ nm. ................................................................. 131

Figure 5.17 Variations of the outer radius of the annular beam along the optical axial distance. Where $f_i = 50$ mm and $\lambda = 1064$ nm................................. 133

Figure 5.18 Variations of the outer radius of the annular beam along the optical axial distance. Where $f_i = 150$ mm and $\lambda = 1064$ nm................................. 134

Figure 6.1 Schematic experimental setup for optical trepanning ................................. 137
Figure 6.2 Schematic representation of an optical trepanning system. $h_1$ is the distance between two axicon lenses, $h_2$ is the distance between the flat surface of the second axicon lens and the focusing lens, and $h_3$ is the distance between the focusing lens and the sample surface.

Figure 6.3 Photographs of the annular nozzle used for optical trepanning.

Figure 6.4 Micrograph of the etch pattern on the workpiece surface due to annular beam heating. $h_1 = 95$ mm, $h_2 = 110$ mm, $f_l = 50$ mm and $Z = 48.3$ mm.

Figure 6.5 Micrograph of the melt etch pattern on the workpiece surface due to annular beam heating. $h_1 = 95$ mm, $h_2 = 110$ mm, $f_l = 50$ mm and $Z = 48.6$ mm.

Figure 6.6 Micrograph of the etch pattern on the workpiece surface due to annular beam heating. $h_1 = 95$ mm, $h_2 = 110$ mm, $f_l = 50$ mm and $Z = 49.2$ mm.

Figure 6.7 Micrograph of the etch pattern of the annulus centre on the workpiece surface. $h_1 = 95$ mm, $h_2 = 110$ mm, $f_l = 50$ mm and $Z = 48.6$ mm.

Figure 6.8 Top view of irradiance profiles for a multimode laser beam, where $D_s = 3.0$ mm and $P_a = 35$ W.

Figure 6.9 Top view of irradiance profiles for a multimode laser beam. (a) $D_s = 2.0$ mm and $P_a = 10$ W. (b) $D_s = 2.0$ mm and $P_a = 25$ W.

Figure 6.10 (a) Top view of irradiance profiles for a Gaussian beam. (b) 3-D view of the irradiance profile for a Gaussian beam. Where $D_s = 1.8$ mm, $P_a = 3$ W and $r_0 = 0.806$ mm.

Figure 6.11 Micrograph of a through-hole by optical trepanning on the substrate SS 316. (a) Micrograph of the entrance (top) surface of a through-hole. (b) Micrograph of the exit (bottom) surface of a through-hole. (c) Micrograph of the top surface of the...
central drop-out disk. (d) Micrograph of the bottom surface of the central drop-out disk. Where $h_1 = 120$ mm, $h_2 = 70$ mm, $h_3 = 47$ mm, $f_i = 50$ mm, $f = 2$ kHz and $P_a = 5$ W .......................................................... 152

Figure 6.12 Micrograph of an optical trepanning hole on the entrance surface of the substrate SS 316. Where $h_1 = 120$ mm, $h_2 = 70$ mm, $h_3 = 47$ mm, $f_i = 50$ mm, $f = 2$ kHz and $P_a = 5$ W. .......................................................... 154

Figure 6.13 Micrograph of an optical trepanning hole on the entrance surface of the substrate SS 316 for the case of misalignment of the optical elements. Where $h_1 = 120$ mm, $h_2 = 70$ mm, $h_3 = 48$ mm, $f_i = 50$ mm, $f = 2$ kHz and $P_a = 5$ W. ............... 154

Figure 6.14 Micrograph of the cross section of drilling holes by optical trepanning on the substrate IN 718. (a) Longer focal length for the focusing lens. Where, $h_1 = 110$ mm, $h_2 = 75$ mm, $h_3 = 103.5$ mm, $f_i = 120$ mm, $f = 2$ kHz and $P_a = 8$ W. (b) Shorter focal length for the focusing lens. Where, $h_1 = 120$ mm, $h_2 = 70$ mm, $h_3 = 48$ mm, $f_i = 50$ mm, $f = 2$ kHz and $P_a = 8$ W .......................................................................................... 155

Figure 6.15 Micrograph of a through-hole by optical trepanning on the substrate SS 316. (a) Entrance (top) surface of a through-hole. (b) Exit (bottom) surface of a through-hole. Where $h_1 = 120$ mm, $h_2 = 70$ mm, $h_3 = 46.5$ mm, $f_i = 50$ mm, $f = 2$ kHz and $P_a = 5$ W. .......................................................................................... 157

Figure 6.16 Micrograph of a through-hole by optical trepanning on the substrate SS 316 (a) Entrance (top) surface of a through-hole. (b) Exit (bottom) surface of a through-hole. Where $h_1 = 120$ mm, $h_2 = 70$ mm, $h_3 = 48$ mm, $f_i = 50$ mm, $f = 2$ kHz and $P_a = 5$ W .......................................................................................... 158
Figure 6.17 Micrograph of a through-hole by optical trepanning on the substrate SS 316
(a) Entrance (top) surface of a through-hole. (b) Exit (bottom) surface of a through-hole. Where $h_1 = 120 \text{ mm}$, $h_2 = 70 \text{ mm}$, $h_3 = 49 \text{ mm}$, $f_i = 50 \text{ mm}$, $f = 2 \text{ kHz}$ and $P_a = 3 \text{ W}$.
LIST OF TABLES

Table 2.1 Material properties of Inconel 718 and laser parameters used for this study. .. 24

Table 3.1 Material properties of Inconel 718, laser parameters and O₂ assist gas used for this study .......................................................... 49
LIST OF ACRONYMS/ABBREVIATIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Thermal diffusivity ($m^2.s^{-1}$)</td>
</tr>
<tr>
<td>$a_s$</td>
<td>Thermal diffusivity of solid ($m^2.s^{-1}$)</td>
</tr>
<tr>
<td>$A$</td>
<td>Absorptivity</td>
</tr>
<tr>
<td>$C$</td>
<td>Uniform intensity profile of the laser beam ($J.kg^{-1}.K^{-1}$)</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat capacity ($J.kg^{-1}.K^{-1}$)</td>
</tr>
<tr>
<td>$C_{pe}$</td>
<td>Effective specific heat capacity ($J.kg^{-1}.K^{-1}$)</td>
</tr>
<tr>
<td>$C_{ps}$</td>
<td>Specific heat capacity of solid ($J.kg^{-1}.K^{-1}$)</td>
</tr>
<tr>
<td>$C_{pl}$</td>
<td>Specific heat capacity of liquid ($J.kg^{-1}.K^{-1}$)</td>
</tr>
<tr>
<td>$C_{eff}$</td>
<td>Effective specific heat capacity of solid ($J.kg^{-1}.K^{-1}$)</td>
</tr>
<tr>
<td>$D$</td>
<td>Distance between the input and output lenses along their optical axis (m)</td>
</tr>
<tr>
<td>$D^*$</td>
<td>Dimensionless distance between the input and output lenses along their optical axis, $D / R_{ia}$</td>
</tr>
<tr>
<td>$D_G$</td>
<td>Distance between the input and output lenses along their optical axis (m)</td>
</tr>
<tr>
<td>$D_{hi}$</td>
<td>Hole diameter at the top surface of the workpiece on which the laser beam is incident (m)</td>
</tr>
<tr>
<td>$D_{ho}$</td>
<td>Hole diameter at the bottom of the workpiece (m)</td>
</tr>
<tr>
<td>$D_{ia}$</td>
<td>Inner diameter of an annular beam (m)</td>
</tr>
<tr>
<td>$D_{oa}$</td>
<td>Outer diameter of an annular beam (m)</td>
</tr>
<tr>
<td>$D_s$</td>
<td>Diameter of a spatial filter (m)</td>
</tr>
<tr>
<td>$E$</td>
<td>Activation energy for viscous flow ($J.mol^{-1}$)</td>
</tr>
<tr>
<td>$f$</td>
<td>Laser pulse repetition rate</td>
</tr>
</tbody>
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\( f_g \) \hspace{1cm} \text{Friction factor}

\( f_l \) \hspace{1cm} \text{Focal length of the focusing lens (m)}

\( F' \) \hspace{1cm} \text{Optical path length (m)}

\( I \) \hspace{1cm} \text{Irradiance of the laser beam (W.m}^{-2}\text{)}

\( I_0 \) \hspace{1cm} \text{Maximum intensity of the laser beam (W.m}^{-2}\text{)}

\( I_{0, \text{bessel}} \) \hspace{1cm} \text{Zero order modified Bessel function of the first kind}

\( \overline{I} \) \hspace{1cm} \text{Hankel transform of } I

\( k \) \hspace{1cm} \text{Thermal conductivity (W.m}^{-1}.\text{K}^{-1}\text{)}

\( k_l \) \hspace{1cm} \text{Thermal conductivity of the liquid phase (W.m}^{-1}.\text{K}^{-1}\text{)}

\( k_s \) \hspace{1cm} \text{Thermal conductivity of solid (W.m}^{-1}.\text{K}^{-1}\text{)}

\( k' \) \hspace{1cm} \text{Wave number}

\( L_m \) \hspace{1cm} \text{Latent heat of melting (J.kg}^{-1}\text{)}

\( L_v \) \hspace{1cm} \text{Latent heat of vaporization (J.kg}^{-1}\text{)}

\( n \) \hspace{1cm} \text{Refractive index of the lens}

\( n_0 \) \hspace{1cm} \text{Refractive index of the surrounding medium}

\( P \) \hspace{1cm} \text{Total of laser power (W)}

\( P_a \) \hspace{1cm} \text{Laser average power (W)}

\( q_l \) \hspace{1cm} \text{Average heat flux along the melting front (W.m}^{-2}\text{)}

\( q_s \) \hspace{1cm} \text{Average heat flux along the vapor front (W.m}^{-2}\text{)}

\( r \) \hspace{1cm} \text{Radial distance (m)}

\( r_h \) \hspace{1cm} \text{Hole diameter ratio}

\( r_0 \) \hspace{1cm} \text{Radial distance of the } 1/e^2 \text{- point (m)}

\( R \) \hspace{1cm} \text{Radial distance (m)}
$R_{ca}$  
Radial distance of the point of maximum intensity $I_0$ from the center of the annulus (m)

$R_g$  
Gas constant (J.mol$^{-1}$.K$^{-1}$)

$R_{ia}$  
Inner radius of the annular laser beam (m)

$R_{oa}$  
Outer radius of the annular laser beam (m)

$R_{0G}$  
Radial distance of the $1/e^2$- point for an input Gaussian laser beam (m)

$R_w$  
Characteristic width of the laser beam (m)

$R'$  
Dimensionless radial distance, $R / R_{ia}$

$s$  
Shape of lens surface (m)

$S_m$  
Depth of the liquid front (m)

$\dot{S}_m$  
Velocity of the melting front along z direction (m.s$^{-1}$)

$S_{vt}$  
Depth of the vapor front (m)

$S_{vv}$  
Depth of the vapor front due to vaporization (m)

$t$  
Time variable representing the drilling time (s)

$t_{on}$  
Laser pulse duration (s)

$t_m$  
Time at which the substrate begins to melt after the beginning of laser irradiation (s)

$t_p$  
Laser pulse period (pulse-on plus pulse-off time) (s)

$t_{pk}$  
Time at which the pulse attains its peak intensity (s)

$t_1$  
Thickness of the input lens (m)

$t_2$  
Thickness of the output lens (m)

$t_3$  
Thickness of the focusing lens (m)

$T$  
Temperature (K)
\( T_b \) Boiling temperature (K)

\( T_l \) Temperature of the liquid phase (K)

\( T_m \) Melting temperature (K)

\( T_s \) Temperature of the solid phase (K)

\( T_{st} \) Temperature at the vaporization front (K)

\( T_0 \) Workpiece temperature before it is irradiated with the laser beam (K)

\( \overline{T} \) Laplace transform of T

\( \overline{U}_r \) Average velocity of the liquid flow along r direction (m.s\(^{-1}\))

\( \overline{U}_z \) Average velocity of the liquid flow along z direction (m.s\(^{-1}\))

\( v_g \) Gas flow velocity (m.s\(^{-1}\))

\( z \) Axial distance variable along axial (z) axis (m)

\( Z \) Axial distance along optical axis (m)

**Greek Symbols**

\( \alpha \) half-apex angle of the axicon lenses (degree)

\( \alpha_c \) Central ray refraction angle (degree)

\( \alpha_1 \) half-apex angle of the first axicon lenses (degree)

\( \alpha_2 \) half-apex angle of the second axicon lenses(degree)

\( \beta \) Base angle of the first axicon (degree)

\( \beta_1 \) Base angle of the second axicon (degree)

\( \beta_2 \) Base angle of the axicon(degree)
\( \phi \) Laser pulse shape function describing the temporal structure of the laser pulse energy with \( t \) as the time variable

\( \rho \) Density (kg.m\(^{-3}\))

\( \rho_g \) Density of gas (kg.m\(^{-3}\))

\( \rho_l \) Density of liquid (kg.m\(^{-3}\))

\( \rho_s \) Density of solid (kg.m\(^{-3}\))

\( \theta_t \) Taper angle (degree)

\( \lambda \) Wavelength of the laser beam (m)

\( \chi \) Lens surface curvature parameter

\( \gamma \) \( n_0 / n \)

\( \tau_i \) Shear stress at the interface of the liquid metal and outgoing assist gas (kg.m\(^{-1}\).s\(^{-2}\))

\( \mu \) Viscosity of the liquid metal. (kg.m\(^{-1}\).s\(^{-1}\))

\( \mu_0 \) Pre-exponential viscosity of the liquid metal (kg.m\(^{-1}\).s\(^{-1}\))

\( \delta \) Recast layer thickness (m)
CHAPTER 1: INTRODUCTION AND LITERATURE REVIEW

1.1. Motivation

Lasers have been used for melting and vaporizing materials for a variety of applications including microhole drilling. The aerospace industry, in particular, has been employing this technique for drilling large number of closely spaced cooling holes in turbine engine components, such as, airfoils, nozzle guide vanes and combustion chambers [Ready et al. (2001); Bech et al. (1997)].

Percussion drilling and trepanning are two laser drilling methods (Fig. 1.1). Percussion drilling is accomplished by focusing the laser beam to approximately the required diameter of the hole, exposing the material to one or a series of laser pulses at the same spot to melt and vaporize the material. Drilling by trepanning involves cutting a hole by rotating a laser beam with an optical element or an x–y galvo-scanner. Optical elements are used to scan the beam in either spiral or circular orbits while maintaining the focused laser spot on the workpiece. The laser beam is much smaller than the desired hole. Complex patterns can be produced in the workpiece using multi-axis machining systems including galvoscanner to effectively mill away the material. Trepanning also can be achieved by rotating the workpiece. There are several types of trepanning. Simple trepanning involves piercing the material with the laser beam and then cutting out the
hole using a single circular cut. In helical drilling the material is not pierced immediately and many circular cuts are used, each one deeper than the previous until the material is fully penetrated. Fig. 1.2 shows the typical trepanning setup by using a scanner system [Herbst et al. (2003)]. The laser beam is switched with a fast Lambda Physik galvoshutter, with an open / close response time of $< 10^{-3}$ s. A beam expansion telescope is used to increase the diameter of the beam, which is then guided through a quarterwave plate to ensure circular polarization of the beam on the target. For drilling precise circular holes the scanner head is replaced with a trepanning system as shown in Figure 1.3 [Herbst et al. (2003)]. The main parts of a trepanning system are rotating wedges to shift the laser beam. By rotating the wedges the beam draws a circular path on the work piece. More advanced systems consist of up to three wedges to control the laser beam to achieve varying tape in holes.

These conventional trepanning processes can be referred to as mechanical trepanning. Often an assist gas is used to expel the molten material and induce metal burning at the laser–matter interaction zone in order to increase the drilling speed. Circular laser spots are generally used in conventional laser drilling.
Figure 1.1 Schematic diagrams of different laser drilling techniques.

(a) Percussion drilling (one pulse)  (b) Percussion drilling (more pulses)

(c) Mechanical trepanning  (d) Optical trepanning
Figure 1.2 Typical setup with laser, fast galvo shutter, telescope, quarter wave plate, scan head, and x-y-z-stage. The quarterwave plate ensures circular polarization of the beam incident on the sample [Herbst et al. (2003)].

Figure 1.3 Trepansing heat optics system. The focal spot on the sample draws a circular path [Herbst et al. (2003)].
Modern aerospace gas turbines require large numbers of small diameter holes to provide cooling in the turbine blades (Fig. 1.4), nozzle guide vanes, combustion chambers and afterburner. A typical modern engine will have more than 100,000 holes. Such holes can be successfully produced by laser trepanning, but this is a relatively slow process compared with laser percussion drilling [French et al. (1998)]. With percussion drilling however the control of hole parameters such as taper, entrance hole variation and
roundness is much more difficult than trepanning, and these parameters usually are the utmost important for such applications.

An annular laser beam can provide a new drilling mechanism. When an annular beam is focused on the workpiece surface, the material around the periphery of the annulus is heated, melted, vaporized and removed, leading to the formation of a hole. This process is very similar to the conventional trepanning, which we refer to as optical trepanning, does not involve any rotating optics or rotating workpiece. Optical trepanning is expected to have the following advantages over the conventional laser drilling techniques.

- Optical trepanning can produce high quality holes since its mechanism is very similar to the conventional trepanning method. Compared to the whole circular spot focused on the workpiece for the percussion drilling, most of laser energy is focused on the periphery of the annulus for the optical trepanning. By adjusting the optical components, the thickness of the annular beam, i.e., (the difference between the inner radius and the outer radius of the annular beam) can be varied. These thin annular beams can achieve thinner recast layer and smaller taper than percussion drilling.

- Optical trepanning can improve trepanning speed since it does not involve any rotating optics or rotating workpiece.

- Optical trepanning can provide more flexibility in affecting the hole quality than percussion drilling since an annular beam allows numerous irradiance profiles to supply laser energy to the workpiece. For percussion drilling, the laser beams are usually either Gaussian or uniform. However, for optical trepanning, the nature of the
hole’s taper can be modified by supplying annular laser beams with different irradiance profiles.

1.2. Literature Reviews

1.2.1. Experimental studies on laser drilling

Major concerns in laser drilling are geometrical and metallurgical characteristics [Bandyopadhyay et al. (2002); Yilbas et al. (1997)]. Geometrical characteristics include hole size, taper, circularity and repeatability, and metallurgical characteristics refer to heat-affected zone, recast layer, spatter formation and micro-cracking. These characteristics are influenced by several factors including average laser power, pulse energy, pulse duration, pulse repetition rate, pulse temporal and spatial profiles, intensity profiles, focus settings and optical as well as thermal properties of the workpiece. Effective utilization of lasers depends very much upon proper understanding of the effects of these factors.

Chen et al. (1996) examined the effects of laser peak power, pulse format and wavelength for drilling three advanced materials: NiAl, N5 and SiC CMC, and observed that cracking in NiAl was greatly reduced when high peak power with short laser format was used. Recast layers in all three materials were generally thinner when high peak powers, short pulse formats or long pulse bursts were employed. Yilbas (1997) attempted to identify a few dominant process variables that affect the hole quality and reported that the effect of
the location of the laser focal plane was very significant in most case, and that the taper formation in percussion laser drilling can be significantly reduced by suitable control of laser variables. Low et al. (2001) investigated the effects of assist gas on the physical characteristics of spatter formation during percussion laser drilling on NIMONIC 263 alloy by using a fiber optics-delivered Nd:YAG Laser. Ng and Li (2001) studied the effects of laser peak power and pulse width on the repeatability of hole geometry and found that melt ejection and spatter formation contributed to the poor repeatability of the drilling process. Ghoreishi et al. (2002) investigated the effects of six controllable laser variables on the hole taper and circularity in percussion laser drilling of stainless steel workpiece and showed that the pulse width and peak power affected the hole diameter, taper and circularity significantly, whereas, the pulse repetition rate had no effect on these three hole characteristics. Voisey et al. (2003) analyzed the melt ejection, angle of the eject trajectory and molten layer thickness during laser drilling of metals. Kamlage et al. (2003) showed that at high laser intensity, well above the ablation threshold, femtosecond lasers can drill deep, high-quality holes in metals without any post-processing or special gas environment.

1.2.2. Theoretical studies on laser drilling

Percussion drilling is very popular for producing high quality holes in aero-engine applications and, consequently, this method has been investigated by many researchers. Chan and Mazumder (1987) discussed the physics of vaporization and liquid metal expulsion during laser-materials interaction and presented a one-dimensional steady-state
mathematical model for material damage with a single pulse. Armon et al. (1989) formulated a two-dimensional metal drilling problem based on enthalpy balance. Radley et al. (1992) presented a computational model for drilling holes with focused Gaussian laser beams and compared with experimental results. Kar and Mazumder et al. (1990 and 1992) studied the effects of pulse duration on cavity formation during laser drilling. They developed a theoretical model for gas-assisted low-power laser drilling. Yilbas et al. (1996) investigated the evaporation effects induced in metals by a laser beam with intensity relevant to the drilling process. Semark and Matsunawa (1997) showed the role of recoil pressure during laser materials processing by carrying out the theoretical analysis of the energy balance in the laser-metal interaction zone. Shannon (1998) obtained reasonable analytical estimates of laser energy coupling due to the formation of a cavity using observables that can be measured easily. Zhang et al. (1999) investigated the melting and vaporization phenomena during the laser drilling and obtained the locations of the solid-liquid and liquid-vapor interfaces by solving energy conservation equations at the interfaces.

Sankaranarayanan et al. (1998) presented a pin-hole experiment to measure the laser energy absorbed by the plasma during laser drilling. Sankaranarayanan and Kar (1999) carried out a quasi-steady-state analysis for the plasma formed during laser processing to determine its height, diameter and temperature. Solana et al. (1999) presented an analytical model for the laser drilling of metals with absorption within the vapor. Cheng et al. (2000) investigated the effects of intrapulse structure on hole geometry in laser drilling. Ruf et al. (2001) proposed an analytical model for laser drilling which included
three-dimensional heat conduction in a simplified manner. Using this simplified analytical ablation model, several geometrical influences on laser drilling were investigated. A mathematical model describing the drilling process was presented for different temporal profiles of pulsed laser beams. Low et al. (2002) developed a one-dimensional steady-state hydrodynamic physical model based on the realistic material removal mechanisms associated with the laser drilling of metals using medium laser intensities. Ghoreishi et al. (2002) employed statistical modeling to investigate the relationships and interactions among six controllable variables on the hole taper and circularity in percussion laser drilling. Setia and May (2003) trained neural networks using the error back-propagation algorithm to model the average values of the responses in the percussion drilling. The prediction error for all the neural network models was less than 5.5%. Ho and Wen (2004) derived an analytic model by considering the effects of the Inverse bremsstrahlung absorption within the plasma and Fresnel absorption at the cavity wall.

1.2.3. Annular beam shaping

Laser beams with annular transverse cross-sections have been investigated for different types of applications such as atom trapping and guiding [Manek et al. (1998); Molloy et al. (2002)], optical confinement of cold atoms [Kulin et al. (2001); Metcalf et al. (2003)], laser machining [Rioux et al. (1978); Belanger et al. (1978)] and optical data storage [Descour et al. (1999)]. Such an annular beam can be generated by a variety of methods. A laser resonator operating in a higher-order circularly symmetric mode intrinsically has
an output annular beam shape. It is, however, difficult to change the radius of the ring for this type of mode. Wang et al. (1993) designed a special unstable resonator to generate a ring output whose radius could be changed by tuning the laser frequency. Optical elements can also be used for such beam shaping applications. Computer-generated holograms can generate the same phase values as those produced by axicon lenses, to reconstruct both zeroth-order and higher-order Bessel beams [Davis et al. (1993); Tao et al. (2004)]. A ring-toric lens can image a point to a ring instead of to another point [Descour et al. (1999)]. Micro-collimation of the output beam from a small hollow optical fiber can also produce an annular laser spot [Yin et al. (1997)]. The resulting annular spot can then be focused to micrometer size in the near-field using the diffracted field of the linear polarization LP_{11} mode of a hollow-core optical fiber [Shin et al. (2001)]. Nematic liquid crystal generates annular beams using self-phase modulation [Shevchenko et al. (2004)]. This method enables the creation of beams with sub-millimeter diameter and annular “radial width”, or $\Delta R$ of a few tens of microns for propagation distances longer than 10 mm. Conic lenses such as axicon and waxicon lenses are, however, used most frequently to generate annular beams. Such optical elements provide flexibility in tailoring the size of the focused annular laser spot. Annular laser beams with variable inner and outer radii can be generated using an optical system consisting of axicon lenses and a convex lens [Mcleod (1954); Song et al. (1999); Lloyd et al. (2003); Angelis et al. (2003); Jaroszewicz et al. (2005)].
Figure 1.5 Generation of a Bessel beam and an annular beam with an axicon.
Axicon lenses are usually defined as optical elements that image a point into a line segment along the optical axis [Mcleod (1954); Jaroszewicz et al. (2005)]. As shown in Fig. 1.3, the axicon lens is a conical surface of revolution capable of blending light from a point source, which is located on the axis of revolution, by reflection or refraction or both [Flores (2001); Thaning et al. (2002)]. Refractive axicon lens was described by Mcleod in 1954. A glass cone refracts all rays at the same angle relative to the optical axis. Similar effect can be obtained using a reflecting cone. Flores (2001) presented a method for designing spherically symmetric gradient-index (index of refraction) axicon lenses, which produce a variety of different irradiance patterns along the optical axis, and with a boundary refraction index larger than or equal to the refraction index of the surrounding medium. The diffractive version of an axicon lens is rather common today [Sochacki et al. (1992); Popov et al. (1998); Thanining et al. (2002 and 2003)]. Sochacki et al. (1992) employed a ray tracing technique together with the conservation of energy in ray bundles to design diffractive axicon lenses having the optimal phase retardation function that produces the desired on-axis irradiance. Lens axicons [Jaroszewich et al (1999)] have been proposed as an easier, cheaper, and more efficient alternative to other types of axicons. The simplest forward-type lens axicon is composed of a diverging third-order spherical aberrated lens and a perfect converging lens.

The axicons and their combinations have been used for many applications. Rioux et al. (1978) combined an axicon lens and a convex lens to form an optical system producing an annular beam for drilling good quality large diameter holes using a high power laser beam. The ring beams have also attracted increasing interests in the field of laser cooling
and trapping of neutral atoms [Manek et al. (1998)]. Axicons have been used to generate intense non-diffracting beams [Durnin (1987); Herman et al. (1991); Garces-Chavez et al. (2002)]. Studies on non-diffracting beams by Durine et al. (1987) drew interest in axicon optics. It has been shown that an axicon can generate a Bessel beam, a so-called non-diffracting beam, which means the irradiance pattern of a beam propagating in free space remains unchanged in the transverse plane. The central irradiance profile of such a beam can be extremely narrow with effective diameter as small as several wavelengths and yet possess an infinite depth of field. They can be applied to imaging, metrological applications, dispersionless optical system design and the production of plasma waveguides [and Song et al. (1999); Garces-Chavez et al. (2002)].

### 1.3. Objectives

The purpose of this project is to design the optical elements for annular beam shaping and develop an optical trepanning system for laser drilling. The research includes the following aspects:

- Design and construction of an annular laser beam shaping system to transform an input circular Gaussian beam into an annular beam with different intensity profiles.
- Diffraction analysis for the beam propagation along the optical trepanning system.
- Measurements of annular beam irradiance profiles along the optical trepanning system and comparison with theoretical results.
- Temperature distributions due to annular beam heating.
• Two–dimensional model for melting and vaporization during optical trepanning.

• Design and construction of an optical trepanning system for laser drilling.
CHAPTER 2: TEMPERATURE DISTRIBUTIONS DUE TO ANNULAR LASER BEAM HEATING

2.1. Introduction

An analytical thermal model for optical trepanning is discussed in this section by considering an axisymmetric, transient heat conduction equation, and the evolutions of the melting temperature isotherm, which is referred to as the melt boundary in this study, are calculated to investigate the influences of the laser pulse shapes and intensity profiles on the hole geometry. Heating depth is important for drilling application. Since there are no good ways to measure the temperature inside substrate during the drilling process, this mathematical model provides a means of understanding the thermal effect of laser irradiation with different annular beam shapes.

2.2. Intensity Profiles for Annular Laser Beams

An annular laser beam of different intensity profiles is investigated in this section. Such profiles include half Gaussian with maximum intensities at the inner and outer surfaces of the annulus, respectively, and full Gaussian with maximum intensity within the annulus. The intensities \( I(r,t) \), for these three cases are given by

\[
I(r,t) = I_0 \exp \left( -2 \frac{(r - R_{ca})^2}{R_w^2} \right) \phi(t) \quad [2.1]
\]

where \( \phi(t) \) is the laser pulse shape function describing the temporal structure of the laser pulse energy with \( t \) as the time variable. For a uniform pulse shape, \( \phi(t) \) is given by
\[ \phi(t) = \begin{cases} 
1 & \text{for } n t_p \leq t \leq n t_p + t_{on}, \quad n = 0, 1, 2, 3, \cdots, \infty \\
0 & \text{for } n t_p + t_{on} < t < (n+1) t_p 
\end{cases} \] \quad \text{[2.2]}

For a triangular shape, \( \phi(t) \) is given by

\[ \phi(t) = \begin{cases} 
\frac{(t - n t_p)}{t_{pk}} & \text{for } n t_p \leq t \leq n t_p + t_{pk}, \quad n = 0, 1, 2, 3, \cdots, \infty \\
\frac{(t - t_{on} - n t_p)}{t_{on} - t_{pk}} & \text{for } n t_p + t_{pk} < t \leq n t_p + t_{on} \\
0 & \text{for } n t_p + t_{on} \leq t \leq (n+1) t_p 
\end{cases} \] \quad \text{[2.3]}

Here \( t_{on} \) and \( t_p \) are laser pulse-on time and period (pulse-on plus pulse-off time) respectively. \( t_{pk} \) is the time at which the pulse attains its peak intensity. \( r \) is the radial distance from the center of the annulus, \( r_0 \) is the characteristic width of the annular laser beam, i.e., the distance between the point of maximum intensity \( I_0 \) to the point where the intensity \( I(r) \) is \( I_0 / e^2 \). \( R_{ca} \) is the radius of the point of maximum intensity \( I_0 \). \( R_{ca}, R_w \) and \( I_0 \) are given by the following expressions for different intensity profiles.

Case 1 – Full Gaussian beam with maximum intensity located at the central radius of the annulus (full Gaussian beam)

\[ R_w = \frac{R_{ia} + R_{oa}}{2}, \] \quad \text{[2.4]}

\[ R_{ca} = \frac{R_{oa} - R_{ia}}{2}. \]
Substituting Eqs. 2.2, 2.3 and 2.4 into Eq. 2.1 to determine the pulse energy of the annular beam, \( I_0 \) can be obtained based on the energy balance. For a uniform pulse shape,

\[
I_0 = \frac{2P_a \phi(t)}{\pi \alpha^2 \left[ \exp(-2R_w^2 / R_{ca}^2) + \sqrt{2\pi} \left( R_w / R_{ca} \right) \text{erfc} \left( -\sqrt{2}R_w / R_{ca} \right) \right] t_{on} f}.
\]  

[2.5]

For a triangular pulse shape,

\[
I_0 = \frac{4P_a \phi(t)}{\pi \alpha^2 \left[ \exp(-2R_w^2 / R_{ca}^2) + \sqrt{2\pi} \left( R_w / R_{ca} \right) \text{erfc} \left( -\sqrt{2}R_w / R_{ca} \right) \right] t_{on} f},
\]  

[2.6]

where \( P_a \) is the total laser power, \( f \) is the laser pulse repetition frequency, and \( R_{ia} \) and \( R_{oa} \) are the inner and outer radii of the annular beam respectively.

Case 2 - Half Gaussian beam with maximum intensity located at the outer radius of the annulus (outer half Gaussian beam)

\[
R_w = R_{oa},
\]  

[2.7]

\[
R_{ca} = R_{oa} - R_{ia}.
\]

For a uniform pulse shape,

\[
I_0 = \frac{2P_a \phi(t)}{\pi \alpha^2 \left[ \exp(-2R_w^2 / R_{ca}^2) -1 + \sqrt{2\pi} \left( R_w / R_{ca} \right) \text{erf} \left( \sqrt{2}R_w / R_{ca} \right) \right] t_{on} f}.
\]  

[2.8]

For a triangular pulse shape,

\[
I_0 = \frac{4P_a \phi(t)}{\pi \alpha^2 \left[ \exp(-2R_w^2 / R_{ca}^2) -1 + \sqrt{2\pi} \left( R_w / R_{ca} \right) \text{erf} \left( \sqrt{2}R_w / R_{ca} \right) \right] t_{on} f}.
\]  

[2.9]
Case 3 - Half Gaussian beam with maximum intensity located at the inner radius of the annulus (inner half Gaussian beam)

\[ R_w = R_{in}, \]  \[ R_{ca} = R_{oa} - R_{ia}. \]

For a uniform pulse shape,

\[ I_0 = \frac{2P_a \Phi(t)}{\pi r_0^2 [1 + \sqrt{2\pi (R_w / R_{ca})}] t_{on} f}. \]  \[ [2.11] \]

For a triangular pulse shape,

\[ I_0 = \frac{4P_a \Phi(t)}{\pi r_0^2 [1 + \sqrt{2\pi (R_w / R_{ca})}] t_{on} f}. \]  \[ [2.12] \]

2.3. Thermal Modeling

The mathematical model for the temperature distribution in the substrate during optical trepanning is based on the following assumptions:

(1) Melting and vaporization are neglected; only a simple heat conduction model is developed for the temperature distribution in the substrate to present an analytical tool to understand the effects of various laser characteristics in annular laser beam heating.

(2) All the thermophysical properties are taken to be independent of temperature.

(3) Heat loss to the environment due to convection and surface radiation is not taken into account.
During laser irradiation of the workpiece, a significant fraction of the laser energy is reflected and the rest of the energy is absorbed by the workpiece. The absorbed energy heats up the material and the thermal energy propagates in the workpiece by the heat conduction mechanism until melting occurs. For optical trepanning, the transient heat conduction equation is solved in cylindrical coordinates for a semi-infinite medium which is heated with a stationary, pulsed annular laser beam. The origin of the chosen coordinate system lies on the surface of the semi-infinite medium and coincides with the laser beam center. The radial (r) axis lies on the substrate surface and the axial (z) axis points into the workpiece from the workpiece surface. Both r and z extend to infinity. The governing heat conduction equation is given by [Ozisik (1993)]

$$
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T(r,z,t)}{\partial r} \right) + \frac{\partial^2 T(r,z,t)}{\partial z^2} = \frac{1}{a} \frac{\partial T(r,z,t)}{\partial t},
$$

[2.13]

for \( 0 \leq r \leq \infty \), \( 0 \leq z \leq \infty \) and \( t > 0 \), and the boundary and initial conditions can be written as

1. \( T(0,z,t) \) is finite \hspace{1cm} [2.14]
2. \( T(\infty,z,t) = T_0 \), \hspace{1cm} [2.15]
3. \( T(r,\infty,t) = T_0 \), \hspace{1cm} [2.16]
4. \( -k \frac{\partial T}{\partial z} \bigg|_{z=0} = AI(r)\phi(t) \), \hspace{1cm} [2.17]
5. \( T(r,z,0) = T_0 \). \hspace{1cm} [2.18]

Where \( k \), \( a \) and \( A \) are the thermal conductivity, thermal diffusivity and absorptivity of the workpiece, respectively. \( T_0 \) is the workpiece temperature before it is irradiated with the laser beam.
To solve this problem, the temperature $T(r, z, t)$ is scaled in the following way:

$$T_1(r, z, t) = T(r, z, t) - T_0,$$  \[2.19\]

and the integral transform technique is applied for the time and radial variables.

Application of the Laplace transform with respect to $t$ to Eq. 2.13 with the initial condition gives [Ozisik (1993)]

$$a \left( \frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial z^2} \right) = sT_1,$$  \[2.20\]

where $s$ is the Laplace transform variable.

The boundary and initial conditions can be written as

$$\bar{T}_1(0, z, s) \text{ is finite},$$  \[2.21\]

$$\bar{T}_1(\infty, z, s) = 0,$$  \[2.22\]

$$\bar{T}_1(r, \infty, s) = 0,$$  \[2.23\]

$$-k \frac{\partial \bar{T}}{\partial z} \bigg|_{z=0} = AI(r)g(s).$$  \[2.24\]

Application of the zero-order Hankel transform with respect to $r$

$$\Psi(K, z, s) = \int_{0}^{\infty} rJ_0(Kr)\bar{T}_1(r, z, s)dr,$$  \[2.25\]

to Eq. [2.20] gives

$$\frac{\partial^2 \Psi}{\partial z^2} - K^2 \Psi = \frac{s}{a} \Psi,$$  \[2.26\]

which must satisfy the following boundary conditions:

$$\Psi(K, \infty, s) = 0,$$  \[2.27\]
\[-k \frac{\partial \Psi(K,z,s)}{\partial z} |_{z=0} = A \tilde{I}(K)g(s). \] 

\[\tilde{I}(K)\] is the Hankel transform of \( I(r) \), that is

\[\tilde{I}(K) = \int_0^\infty r J_0(Kr)I(r)dr.\] 

where \( K \) is the Hankel transform variable. \( J_0(Kr) \) is the Bessel function of the first kind of order zero.

The solution of the Eq. 2.26 can be written as

\[\Psi(K,z) = \frac{\tilde{I}(K)g(s)\exp(-z\sqrt{K^2 + s/a})}{k\sqrt{K^2 + s/a}}.\] 

The inverse Hankel transform give the following solution

\[\bar{T}_1 = \int_0^\infty KJ_0(Kr)\tilde{I}(K)g(s)\exp(-\sqrt{K^2 + s/a})dK.\] 

Applying the inverse Laplace transform, the temperature distribution, \( T_1(r,z,t) \) is found to be

\[T_1(r,z,t) = \int_0^\infty \int_0^\infty \frac{K\sqrt{\alpha}J_0(Kr)\tilde{I}(K)\exp(-K^2a(t-\tau))\exp\left(-\frac{z^2}{4a(t-\tau)}\right)g(\tau)}{k\sqrt{\pi(t-\tau)}}d\tau dK,\] 

where \( \tilde{I}(K) \) is the Hankel transform of \( I(r) \), that is

\[\tilde{I}(K) = \int_0^\infty r J_0(Kr)I(r)dr.\] 

Substituting the expression of \( \tilde{I}(K) \) from Eq. 2.33 into Eq. 2.32, the temperature distribution, \( T(r,z,t) \) is found to be
$T(r,z,t) = \int_{0}^{\infty} \frac{AI_{0}}{2k\sqrt{a\pi(t-\tau)^3}} \exp\left(-\frac{2(l-r_{c})^2}{r_{0}^2}\right) \exp\left(-\frac{(l^2+r^2+z^2)}{4a(t-\tau)}\right) I_{0,bessel}\left(\frac{rl}{2a(t-\tau)}\right) \phi(\tau) d\tau + T_{0}$,

[2.34]

where $I_{0,bessel}\left(\frac{rl}{2a(t-\tau)}\right)$ is the zeroth order modified Bessel function of the first kind.

By neglecting the laser beam energy outside the annular region, the integral of $T(r,z,t)$ can be approximately from $R_{ia}$ to $R_{oa}$ for the parameter $l$.

2.4. Results and Discussion

Eq. 2.34 is used to calculate the temperature distribution in an Inconel 718 substrate. The values of the material properties [Sylvan (1972); Spittle (1989)] and laser parameters used for this study are listed in Table 2.1.
Table 2.1 Material properties of Inconel 718 and laser parameters used for this study.

<table>
<thead>
<tr>
<th>Substrate material</th>
<th>Inconel 718</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho$ (kgm$^{-3}$)</td>
<td>7840</td>
</tr>
<tr>
<td>Thermal conductivity, $k$ (Wcm$^{-1}$K$^{-1}$)</td>
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</tr>
<tr>
<td>Thermal diffusivity, $a$ ($10^{-6}$m$^2$s$^{-1}$)</td>
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<tr>
<td>Melting temperature, $T_m$ (K)</td>
<td>1623</td>
</tr>
<tr>
<td>Specific heat capacity, $C_p$ (Jkg$^{-1}$K$^{-1}$)</td>
<td>569</td>
</tr>
<tr>
<td>Latent heat of melting, $L_m$ (kJkg$^{-1}$)</td>
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<tr>
<td>Effective specific heat capacity, $C_{pe}$ (Jkg$^{-1}$K$^{-1}$)</td>
<td>658</td>
</tr>
<tr>
<td>Absorptivity, $A$</td>
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<tr>
<td>Nd:YAG laser wavelength, $\lambda$ ($\mu$m)</td>
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<td>Inner radius of the laser beam, $R_{ia}$ ($\mu$m)</td>
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<tr>
<td>Outer radius of the laser beam, $R_{oa}$ ($\mu$m)</td>
<td>75</td>
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Figure 2.1 Transient temperature distributions due to a full Gaussian annular laser beam with uniform pulse ($P_a = 150$ W, $f = 2$ kHz, $t_{on} = 100$ µs).
Figure 2.2 Transient temperature distributions due to a full Gaussian annular laser beam with triangular pulse ($P_a = 150$ W, $f = 2$ kHz, $t_{on} = 100$ µs).
Figs. 2.1 and 2.2 show the temperature rise at various points in the substrate during laser irradiation with the uniform pulse shape and triangular pulse shape, respectively. The transient temperature profiles are plotted at four points on the substrate surface: (i) laser beam center \((r = 0 \text{ and } z = 0)\), (ii) inner radius of the annular laser spot on the substrate surface \((r = 60 \text{ µm and } z = 0)\), (iii) mid-point of the annular spot \((r = 67.5 \text{ µm and } z = 0)\), and (iv) outer radius of the annular spot \((r = 75 \text{ µm and } z = 0)\). For Case 1 (full Gaussian beam with maximum intensity located at the mid-point of the annular spot), it can be seen from Fig. 2.1 that the temperature at the beam center is much lower than at other points on the substrate surface. The temperature at the mid-point of the annulus is higher than at other regions of the workpiece. Therefore, the material around the laser beam center will not be melted if it receives insufficient amount of energy from the annular region by conduction. The rapid temperature rise and fall in Figs. 2.1 and 2.2 are due to rapid heating by the laser pulse during the pulse-on time and rapid cooling of the hot region during the pulse-off time by rapid conduction of heat to other parts of the substrate respectively. The substrate surface begins to melt at \(t_m = 62 \text{ µs}\) and \(t_m = 55 \text{ µs}\) during the first pulse for the uniform and triangular laser pulse shapes respectively.

Figs. 2.3 and 2.4 represent the radial variation of the temperature at the substrate surface \((z = 0)\) for two different times \((t = 100 \text{ and } 200 \text{ µs})\) and inside the substrate \((z = 10 \text{ µm})\) for two different times \((t = 100 \text{ and } 200 \text{ µs})\). The temperature is much lower outside the annular region than inside the annular region. This suggests that melting and vaporization occur mainly within the annulus in optical trepanning. The material around the center of
the annulus will generally not be melted in optical trepanning. For the full Gaussian beam, the maximum temperature occurs at \( r = 65 \, \mu\text{m} \) as shown in Fig. 2.4. The radial distance of the melting region on the substrate surface is 79 \( \mu\text{m} \) and 81 \( \mu\text{m} \) at time \( t = 100 \, \mu\text{s} \) for the uniform and triangular pulse shapes respectively.

![Graph showing radial temperature variations due to a full Gaussian annular laser beam with uniform pulse.](image)

**Figure 2.3** Radial temperature variations due to a full Gaussian annular laser beam with uniform pulse \( (P_a = 150 \, \text{W}, f = 2 \, \text{kHz}, t_{on} = 100 \, \mu\text{s}) \).
Figure 2.4 Radial temperature variations due to a full Gaussian annular laser beam with triangular pulse ($P_a = 150$ W, $f = 2$ kHz, $t_{on} = 100 \mu s$).
Figs. 2.5 and 2.6 show the temperature distributions along the depth of the substrate at the radial locations $r = 0$ and 67.5 $\mu$m (mid-point of the annular laser spot) for two different times $t = 0.4$ and 2.4 ms. It can be seen from these figures that the temperatures are higher at $r = 67.5$ $\mu$m than at $r = 0$. For the radial location $r = 67.5$ $\mu$m, the influences of the irradiation extend axially up to about 90 $\mu$m and 180 $\mu$m at $t = 400$ $\mu$s and $t = 2400$ $\mu$s respectively. The melting temperature isotherm extends axially up to 20 $\mu$m and 24 $\mu$m at the radial location $r = 67.5$ $\mu$m and time $t = 2400$ $\mu$s for the uniform and triangular pulse shapes respectively.

Figure 2.5 Axial temperature variations due to a full Gaussian annular laser beam with uniform pulse ($P_a = 150$ W, $f = 2$ kHz, $t_{on} = 100$ $\mu$s).
Figure 2.6 Axial temperature variations due to a full Gaussian annular laser beam with triangular pulse ($P_a = 150$ W, $f = 2$ kHz, $t_{on} = 100$ µs).
Fig. 2.7 shows that the melt boundary extends up to 85 µm in the radial direction for the inner half Gaussian laser beam of uniform pulse shape with 100 µs pulse duration at the drilling time $t = 21\mu s$. Due to heat conduction, the melt boundary will increase in the radial direction and thus the hole diameter will increase if the melt is expelled with an assist gas or removed by vaporization. To obtain a fixed diameter hole, such as 170 µm diameter in this study, we choose different drilling times ($t$) for spatially three different intensity profiles and temporally uniform pulse shape. The maximum melt depths are 12.5 µm, 16.5 µm and 17 µm for the full Gaussian, inner half Gaussian and outer half Gaussian laser beams respectively, and the full Gaussian annular beam produced the smallest melt volume. Fig. 2.8 shows that the melt boundary extends up to 85 µm in the radial direction for the inner half Gaussian laser beam of uniform pulse shape with 100 ns pulse duration at the drilling time $t = 6 \mu s$. The maximum melt depths are 13 µm, 15 µm and 15.5 µm for the full Gaussian, inner half Gaussian and outer half Gaussian laser beams respectively.
Figure 2.7 Evolution of the workpiece melt boundary for different laser intensity profiles and uniform laser pulse ($P_a = 50 \text{ W}, f = 1 \text{ kHz}, t_{on} = 100 \mu\text{s}$, drilling time ($t$) = 21 $\mu\text{s}$).
Figure 2.8 Evolution of the workpiece melt boundary for different laser intensity profiles and uniform laser pulse ($P_a = 12$ W, $f = 1$ kHz, $t_{on} = 100$ ns, drilling time ($t$) = 6 $\mu$s).

Fig. 2.9 shows that the melt boundary extends up to 85 $\mu$m in the radial direction for the inner half Gaussian laser beam of uniform pulse shape with 10 ns pulse duration at the drilling time $t = 2$ $\mu$s. The maximum melt depths are 9.5 $\mu$m, 10 $\mu$m and 10.2 $\mu$m for the full Gaussian, inner half Gaussian and outer half Gaussian laser beams respectively. Outer half Gaussian annular laser beams produce deeper melt depth than the inner half Gaussian and full Gaussian beams. This is because the maximum temperature occurs at the outer circumference of the annulus and heat is conducted from there towards the
center of the annulus, producing deeper melt depth. While for the inner half Gaussian beam, the maximum temperature occurs at the inner circumference of the annulus and some of the thermal energy is transferred towards the outer circumference away from the central region. The full Gaussian annular laser beams produce smaller melt depth than the other two types of beam. This is because the maximum intensity of full Gaussian annular laser beam is located inside the annular region where the temperature is maximum, and the thermal energy is transferred towards both sides (inner and outer circumferences) of the annular region. This causes more heat conduction in the radial direction than along the thickness of the workpiece.

Fig. 2.10 shows the evolution of the melt boundary in the axial direction for different laser intensity profiles and triangular pulse of average laser power 50 W. The melt boundary extends up to 78.5 µm, 80 µm and 84.5 µm in the radial direction with 100 µs pulse duration at the drilling time \( t = 21 \) µs for the full Gaussian, outer half Gaussian and inner half Gaussian laser beams respectively. The maximum melt depths are 7 µm, 10 µm and 11 µm for the full Gaussian, inner half Gaussian and outer half Gaussian beams respectively. The triangular pulse produces small melt volume and large tapered melt boundary compared to the case of uniform pulse.
Figure 2.9 Evolution of the workpiece melt boundary for different laser intensity profiles and uniform laser pulse ($P_a = 12$ W, $f = 1$ kHz, $t_{on} = 10$ ns, drilling time ($t) = 2$ $\mu$s).
Figure 2.10 Evolution of the workpiece melt boundary for different laser intensity profiles and triangular laser pulse ($P_a = 50$ W, $f = 1$ kHz, $t_{on} = 100$ µs, drilling time ($t$) = 21 µs).
2.5. Conclusions

An analytic expression for the temperature distribution is presented to examine the effects of annular laser beam heating for three types of intensity distribution in optical trepanning. On the basis of the numerical results of this study dealing with pulsed laser heating of Inconel 718 material, the following conclusions can be drawn:

- The heating of the material around the center \((r = 0)\) of the annular laser spot on the surface of the substrate is minimal. The melting and vaporization of the material are expected to occur within the annulus leaving an unmelted piece at the center similar to what is observed in mechanical trepanning.

- Annular laser beams with outer half Gaussian intensity profile produce larger melt volume and deeper holes than the inner half Gaussian and outer half Gaussian beams for uniform pulse shapes, whereas annular laser beams with full Gaussian intensity profile produce smaller melt volume than the other two intensity profiles of uniform pulse shape.

- Uniform pulse shapes are found to produce less tapered and deeper melt boundary than triangular pulses.
CHAPTER 3: TWO-DIMENSTIONAL MODEL FOR MELTING AND VAPORIZATION DURING OPTICAL TREPANNING

3.1. Introduction

In this chapter, an analytical two-dimensional model is developed for optical trepanning. The analysis accounts for conduction in the solid, vaporization and convection due to the melt flow caused by an assist gas. Based on the model, the influences of pulse duration, laser pulse length, pulse repetition rate, intensity profiles and beam radius are investigated to examine their effects on the recast layer thickness, hole depth and taper.

3.2. Mathematical Model for Optical Trepanning

The optical trepanning process considered in this model is schematically illustrated in Figs. 3.1 and 3.2. An annular laser beam of intensity $I(r, t)$ is illuminated on the workpiece surface. Due to intense heat flux, the material around the annulus laser spot is heated, melted and vaporized and the melt is removed by an assist gas. Melting and vaporization occur mainly within the annulus. Melting generally does not occur around the center of the annulus in optical trepanning. The melt expulsion and laser heating also affect the shape of the solid–liquid interface. The geometrical shapes of the liquid-vapor and solid-liquid interfaces around the periphery of the annulus are shown in Fig. 3.1. The assist gas exerts a force on the melt and expels it upwards. This melt expulsion and laser vaporization creates a cavity, i.e., a liquid-vapor interface.
The mathematical model utilizes the conservation of energy, i.e., the Stefan condition at the solid-liquid and liquid-vapor interfaces, taking the effects of the liquid metal flow and the assist gas flow into account. The model is based on the following assumptions:

(1) No plasma is generated in the cavity. Only liquid metal and metal vapor are considered to form during the optical trepanning process.

(2) The temperature distribution in the liquid metal is assumed to be linear.

(3) The thermophysical properties of the liquid and solid phase are taken to be constant.

(4) Heat loss to the environment due to convection and surface radiation is neglected. This assumption leads to the determination of maximum material damage, i.e., maximum thermal effect of laser irradiation.

(5) The material removal from the melt layer is considered to be mainly due to the melt expulsion by the assist gas. The vaporization rate is considered negligible compared to the melt expulsion rate in the model.

(6) The temperature distribution in the solid phase is assumed to be locally one-dimensional and it is obtained under the quasi-steady state approximation.

(7) The shape of the cavity does not change during the laser pulse-off time.
Figure 3.1 Schematic diagram of two-dimensional model for melting and vaporization during optical trepanning.
Figure 3.2 Control volume for mass and energy balances in the metal layer.
3.3. Mathematical Formulation

3.3.1. Energy balance at the interfaces

Assuming that the solid-liquid (melting front) and liquid-vapor (vaporization front) interfaces have been formed, their geometrical shapes are expressed respectively as

\[ z = S_{vt} (r, t), \quad [3.1] \]

\[ z = S_m (r, t), \quad [3.2] \]

where \( S_{vt} \) and \( S_m \) are the depths of vaporization and melting fronts respectively. \( S_{vt} \) denotes the total depth of the cavity which is formed due to vaporization and liquid metal expulsion. At the solid-liquid interface \( z = S_m \), the energy balance and boundary conditions can be expressed as [Kar and Mazumder (1990)]

\[
-k_i \frac{\partial T_i}{\partial z} \left[ 1 + \left( \frac{\partial S_m}{\partial r} \right)^2 \right] = -k_s \frac{\partial T_s}{\partial z} \left[ 1 + \left( \frac{\partial S_m}{\partial r} \right)^2 \right] + \rho_s L_m \frac{\partial S_m}{\partial t}, \quad [3.3]
\]

\[ T_i = T_s = T_m \quad \text{at} \ z = S_m (r, t), \quad [3.4] \]

where \( \rho_s \) is the density of the solid phase, \( L_m \) is the latent heat of melting, \( k_i \) and \( k_s \) are the thermal conductivities of liquid and solid phases respectively, \( T_i \) and \( T_s \) are the temperatures of liquid and solid phases respectively.

At the liquid-vapor interface \( z = S_{vt} \), the energy balance and boundary conditions can be expressed as
\[ AI(r, t) + k_i \frac{\partial T_i}{\partial z} \left[ 1 + \left( \frac{\partial S_{av}}{\partial r} \right)^2 \right] = \rho_i L_v \frac{\partial S_{av}}{\partial t}, \]  
\[ [3.5] \]

\[ T_i = T_{st} \text{ at } z = S_{av}(r, t), \]  
\[ [3.6] \]

where \( \rho_i \) is the density of the liquid phase, \( A \) is the absorptivity of the liquid metal for the incident laser beam, \( I(r, t) \) is the laser intensity, \( L_v \) is the latent heat of vaporization, \( S_{av} \) is the cavity depth due to vaporization only.

\( T_{st} \) can be estimated by the following expression [Sankaranarayanan et al. (1999)]

\[ T_{st} = AI \sqrt{\frac{4t}{\pi \rho_i C_{eff} k_i}}, \]  
\[ [3.7] \]

where \( C_{eff} \) is the effective heat capacity that accounts for the latent heat of melting and the specific heat capacity of the substrate \( C_{ps} \), which is taken as

\[ C_{eff} = C_{ps} + \frac{L_m}{T_m}. \]  
\[ [3.8] \]

It should be noted that this model is based on an assumption that the melt layer is continuously expelled, but Eq. 3.7 assumes heat conduction in a stationary liquid layer.

### 3.3.2. Heat conduction in the solid phase

To simplify the heat conduction analysis in the solid phase, a one-dimensional heat diffusion model is assumed,

\[ \frac{\partial T_s}{\partial t} = a_s \frac{\partial^2 T_s}{\partial z^2}, \]  
\[ [3.9] \]

\[ T_s = T_m \text{ at } z = S_m(r, t), \]  
\[ [3.10] \]
where $a_s$ is the thermal diffusivity of the solid phase.

Letting $z' = z - \dot{S}_m(r, t)$,

where $\dot{S}_m(t)$ is the velocity of the melting front along z direction. Eq. 3.9 can be written in a moving coordinate system with the origin being fixed at the solid-liquid interface, and then, under quasi-steady state condition [Ozisik (1993)], the temperature distribution in the solid phase can be obtained as follows

$$T_x = T_0 + (T_m - T_0) \exp\left(-\frac{\dot{S}_m(r, t)}{a_s}(z - S_m(r, t))\right). \quad [3.11]$$

### 3.3.3. Mass and energy balances in the liquid metal layer

Following the conservation of mass and assumption (5), the amount of material melted at the solid-liquid interface is equated to the sum of the amount of melt expelled by the assist gas through the inner and outer surfaces of an annular control volume of inner and outer radii $R_{io}$ and $R_{oa}$ respectively and thickness $\Delta z$ as shown in Fig. 3.2. This control volume is obtained by projecting the melt layer onto a plane parallel to the radial direction.

$$\pi \rho_l \bar{U}_z (R_{oa}^2 - R_i^2) = 2 \pi \rho_l \bar{U}_r R_{io} (S_m - S_{vi}) + 2 \pi \rho_l \bar{U}_r R_{oa} (S_m - S_{vi}), \quad [3.12]$$

where $\bar{U}_z$ and $\bar{U}_r$ are the average velocities of the liquid metal along the $z$ and $r$ directions respectively. Eq. 3.12 can be simplified as

$$\bar{U}_z (R_{oa} - R_{ia}) = 2 \bar{U}_r (S_m - S_{vi}). \quad [3.13]$$

Similarly, the energy balance for the melt layer can be written as
where \( \bar{q}_v \) and \( \bar{q}_m \) are the average heat fluxes at the vaporization and melting fronts respectively, \( C_{pl} \) is the specific heat capacity of the liquid phase and \( \bar{T}_i \) is the average temperature of the melt layer. \( \bar{T}_i \) and \( \bar{q}_m \) can be approximated as

\[
\bar{T}_i = \frac{T_{st} + T_m}{2},
\]

and

\[
\bar{q}_m = \frac{(\bar{T}_i - T_m)k_i}{(S_m - S_{st})}.
\]

By substituting Eq. 3.16 into Eq. 3.14, the following expression is obtained,

\[
\bar{q}_v = k_i \frac{T_{st} - T_m}{S_m - S_{st}} \frac{(S_m - S_{st})(T_{st} + T_m)}{R_o - R_i} \rho_l C_{pl} \bar{U}_r.
\]

The average velocity profile of the liquid metal in the radial direction can be estimated as

\[
\bar{U}_r = \frac{\tau_i}{2\mu} (S_m - S_{st}),
\]

where \( \tau_i \) is the shear stress at the interface of the liquid metal and outgoing assist gas, \( \mu \) is the viscosity of the liquid metal. \( \tau_i \) can be estimated from the drag force [Kar et al. (1992); Bird et al. (1960)], i.e.,

\[
\tau_i = \frac{1}{2} \rho_g v_g^2 f_g,
\]
where \( \rho_g \) and \( v_g \) are the density and the average velocity of the outgoing assist gas respectively and \( f_g \) is the friction factor. The viscosity of the liquid metal \( \mu \) can be estimated by the following expression [Sylvan (1972); Spittle et al. (1989)]

\[
\mu(T) = \mu_0 \exp \left( \frac{E}{R_g T} \right),
\]

where \( \mu_0 \) is the pre-exponential viscosity and \( E \) is the activation energy for viscous flow, which are both constants. \( T \) is the absolute temperature of the liquid metal and \( R_g \) is the universal gas constant.

When the assist gas jet enters into the cavity, it creates a stagnation point at the bottom of the cavity, and then the jet reverses its flow direction outward along the side wall of the cavity. In other words, the assist gas jet is envisioned as flowing into the cavity along the central core of the cavity and then the jet flows out of the cavity through the annular region between the central incident assist gas jet and the side wall of the cavity.

During the cavity formation, the liquid metal is removed by a combination of evaporation and melt expulsion. However, the mass fraction removed by evaporation is typically less than a tenth of the total mass removed [Smith (2002)]. Also the cavity grows mainly in the \( z \) direction compared to the \( r \) direction as shown in Fig. 3.1 (i.e., along the direction of laser beam propagation) during laser drilling. Therefore, neglecting the vaporization rate and considering the drilling speed in the \( z \)-direction, the following expression for the recast layer thickness \( (\delta) \) can be obtained by combining Eqs. 3.5, 3.16, 3.17 and 3.18.
\[
\delta = \frac{1}{6B_4} \left\{ -108B_2 + 12\sqrt{3} \left( \frac{4B_3^3 + 27B_2^2B_3}{B_3} \right)^{\frac{1}{2}} \right\}^{\frac{1}{3}} \frac{2B_1}{\left\{ -108B_2 + 12\sqrt{3} \left( \frac{4B_3^3 + 27B_2^2B_3}{B_3} \right)^{\frac{1}{2}} \right\}^{\frac{1}{3}}}
\]

where

\[
B_1 = AI, \quad B_2 = k_i(T_{\text{st}} - T_m), \quad \text{and} \quad B_3 = \frac{\rho_i C_{\text{pl}} \tau_i (T_{\text{st}} + T_m)}{2\mu(R_o - R_i)}.
\]

Similarly, by considering the motion of the melting front to be mainly in the z-direction compared to the r direction, Eqs. 3.3, 3.11 and 3.16 can be combined to obtain the following expression for the depth of the melting front

\[
S_m(r, t) = \int_0^t \frac{k_i(T_{\text{st}} - T_m)}{\delta \left( \frac{k}{a_s}(T_{\text{st}} - T_0) + \rho_s L_m \right)} dt.
\]

### 3.4. Results and Discussions

The above mathematical model was used to investigate the effects of various process parameters such as laser intensity, pulse-on time, intensity profile and annular beam radius. Inconel 718 was used as the substrate. The values of the material properties [Pottlacher et al. (2002); Basak et.al (2003)], laser parameters and O\textsubscript{2} assist gas properties [Bird et al. (1960); Low et al. (2002)] used for this study are listed in Table 3.1. Three types of annular intensity profiles have been used in this study.
Table 3.1 Material properties of Inconel 718, laser parameters and O₂ assist gas used for this study

<table>
<thead>
<tr>
<th>Substrate material</th>
<th>Inconel 718</th>
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<tr>
<td>Density of solid, $\rho_s$ (kg/m$^3$)</td>
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<td>Nozzle exit diameter, $d_n$ (mm)</td>
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3.4.1. Recast layer and cavity depth

Fig. 3.3 shows the variation of the maximum recast layer thickness and cavity depth at the bottom of the cavity with the laser intensity at the drilling time $t = 100$ ms for various laser pulse-on times. The results exhibit a common trend, i.e., the recast layer thickness and the cavity depth increase with the increase in the laser intensity. This is because the material melting rate is higher with the increase in the laser intensity causing more melting of materials. Much of the melt is expelled from the cavity by the assist gas, which leads to a steady state condition when the melting and melt expulsion rates are equal. The recast layer thickness increases when the melting rate is higher than the expulsion rate. The effects of laser pulse-on time on the recast layer thickness are significant as shown in Fig. 3.3. Generally, the recast layer thickness decreases rapidly with the increase in the laser pulse-on time, because more material is melted during each laser pulse and the melt is expelled by the assist gas efficiently. This produces thinner recast layer and deeper cavity depth.
Figure 3.3 Variation of recast layer thickness and cavity depth with laser intensity for full Gaussian annular beam ($f = 1 \text{ kHz}$, $t_{on} = 100 \text{ ns}$, drilling time ($t$) = 100 ms, $R_{oa} = 85 \mu \text{m}$ and $R_{ia} = 65 \mu \text{m}$).
Fig. 3.4 shows the variation of the maximum recast layer thickness and cavity depth at the bottom of the cavity with the laser intensity at the drilling time $t = 100$ ms for various laser intensity profiles. The recast layer thickness and the cavity depth increase with the increase in the laser intensity for three types of intensity profiles. These three intensity profiles generate the same maximum cavity depth. For high laser intensities, the inner half Gaussian beam produces the thickest recast layer and the outer half Gaussian beam produces the thinnest recast layer. This is because more melting occurs at the inner circumference of the annulus for inner half Gaussian beam compared to the amount of melting in the other two types of intensity profiles, and the assist gas is less effective in removing the melt in the former case.

Fig. 3.5 shows the variation of the maximum recast layer thickness and the cavity depth at the bottom of the cavity with the inner radius of the annular beam at the drilling time $t = 100$ ms for various laser intensities. The recast layer thickness decreases linearly and the cavity depth increases as the inner radius of the annular beam increases. This is because the melt front propagates more along the depth direction than along the radial direction as the inner radius of the annular beam increases. The melt is expelled by the assist gas efficiently, causing thinner recast layer and deeper cavity depth. Compared to the effects of laser intensity in Fig. 3.4, the effects of the inner radius of the annular beam are more significant, because the width of the annulus decreases for higher inner radius of the annular beam, resulting in lower melt volume to produce holes of a given depth than in the case of lower inner radius.
Figure 3.4 Variation of recast layer thickness and cavity depth with laser intensity ($f = 1$ kHz, $t_{on} = 100$ ns, drilling time ($t$) = 100 ms, $R_{oa} = 85$ µm and $R_{ia} = 65$ µm.)
Figure 3.5 Variation of recast layer thickness and cavity depth with inner radius of the annular laser beam for full Gaussian annular beam ($f = 1 \text{ kHz}$, $t_{on} = 100 \text{ ns}$, drilling time ($t = 100 \text{ ms}$, $R_{oa} = 85 \mu\text{m}$).
3.4.2. Taper

As shown in Fig. 3.6, a measure of the taper of laser-drilled holes is the hole diameter ratio \( D_{hi}/D_{ho} \), where \( D_{hi} \) is the hole diameter at the top surface of the workpiece on which the laser beam is incident and \( D_{ho} \) is the hole diameter at the bottom of the workpiece. This diameter ratio, which is referred to as taper in this study, is plotted in Figs. 3.7-3.9. Perfectly cylindrical holes are drilled when \( D_{hi}/D_{ho} = 1 \). \( D_{hi}/D_{ho} > 1 \) indicates convergent, i.e., convergent nozzle-shaped, holes and \( D_{hi}/D_{ho} < 1 \) implies divergent, i.e., divergent nozzle-shaped, holes.

Figure 3.6 Geometrical description of taper angle for a trepanning hole.
Fig. 3.7 shows the variation of taper and laser drilling time with the laser intensity for various laser pulse-on times for laser-drilled holes of depth 632 µm. The increase in the laser pulse-on time or the laser intensity reduces the drilling time, but increases the hole taper. This is because more melt is produced as the laser intensity increases, resulting in thicker melt layer in the radial direction. Generally the assist gas is ineffective in removing the melt from the bottom portion of blind holes. This decreases the hole diameter at the bottom of the workpiece, forming more tapered holes. Similarly, thicker melt layer in the radial direction with the increase in the laser pulse-on time results in more tapered holes.

Fig. 3.8 shows the variation of taper and laser drilling time with the laser intensity for different intensity profiles for laser-drilled holes of depth 632 µm. Full Gaussian annular beams and inner half Gaussian annular beams generate convergent holes \((D_{hi}/D_{ho} > 1)\), i.e., the hole diameter decreases along the depth direction. Outer half Gaussian annular beams generate divergent holes \((D_{hi}/D_{ho} < 1)\), i.e., the hole diameter increases along the hole depth. This is because the maximum laser intensity occurs inside the annular region for the full Gaussian annular beam and at the inner circumference of the annulus for the inner half Gaussian annular beam, which causes higher melting rate along the thickness of the workpiece in these two regions of the annulus. Thus a convergent hole is formed when the workpiece is melted preferentially in these two regions over its entire thickness. On the other hand, the maximum laser intensity occurs at the outer circumference of the annulus for the outer half Gaussian annular laser beam, leading to the divergent holes.
Figure 3.7 Variation of taper and laser drilling time with the laser intensity for full Gaussian annular beam ($S_v = 632 \, \mu m, f = 1 \, kHz, R_{oa} = 85 \, \mu m$ and $R_{ia} = 65 \, \mu m$).
Figure 3.8 Variation of taper and drilling time with laser intensity for different laser intensity profiles ($S_{vt} = 632 \, \mu m$, $f = 1 \, kHz$, $R_{oa} = 85 \, \mu m$ and $R_{ia} = 65 \, \mu m$).
Fig. 3.9 shows the variation of taper and drilling time with the inner radius of annular beam for different laser intensities for laser-drilled holes of depth 632 μm. The taper formation in optical trepanning is reduced significantly by increasing the inner radius of an annular beam. This is because smaller melt volume is produced as the inner radius of an annular beam increases, forming thinner recast layer leading to less tapered holes. Compared to the effects of laser intensity and laser pulse-on time, the effects of the inner radius of the annular beam are more significant because the melt front propagates in the radial direction more than in the former two cases. An increase in the laser intensity or the laser pulse-on time widens the melt front in the radial direction due to more energy input to the workpiece. For very short laser pulse-on time, the heat conduction along the radial direction is limited and the melt front propagates a small distance in the radial direction.

The tapers shown in Figs. 3.7, 3.8 and 3.9 are large (e.g., $\tau > 1.1$ for a full Gaussian annular beam) because our model is based on the blind hole geometry and the tapers have been calculated by considering the recast layer thickness at the bottom of the cavity. Optical trepanning is expected to produce holes with less tapers for through holes, because in such cases, the melt can be removed efficiently by the assist gas through the bottom of the workpiece. This will reduce the differences in the hole diameters at the top and bottom surfaces of the workpiece and produce less tapered holes.
Figure 3.9 Variation of taper and drilling time with the inner radius of annular beam for full Gaussian annular beam ($S_{\text{rt}} = 632 \, \mu\text{m}$, $t_{\text{on}} = 100 \, \text{ns}$, $f = 1 \, \text{kHz}$ and $R_{oa} = 85 \, \mu\text{m}$).
3.5. Conclusions

An analytic two-dimensional model is developed for optical trepanning. The analysis accounts for conduction in the solid, vaporization, convection due to the melt flow and the effects of an assist gas. On the basis of the results ensuing from the present study dealing with pulsed Nd:YAG laser optical trepanning of IN 718 material, the following conclusions can be drawn:

1. The effects of annular beam radius are significant in most cases. It significantly influences the drilling hole qualities due to the widening of the melt layer in the radial direction. For a fixed outer radius of an annular beam, thinner recast layer, smaller taper and higher drilling speed are obtained with the increase in the inner radius of the annular beam.

2. By using different types of intensity profiles, the nature of the hole taper can be modified, i.e., convergent or divergent holes can be produced. Full Gaussian beam and inner half Gaussian beam generate convergent holes and outer half Gaussian beam produces divergent holes. This shows that annular beams can provide more flexibility in affecting the hole quality than tradition circular beams since an annular beam allows numerous irradiance profiles to supply laser energy to the workpiece.

3. An increase in the laser intensity generates thicker recast layer, deeper cavity depth and larger taper.

4. An increase in the laser pulse-on time generates thinner recast layer, deeper cavity depth and larger taper.
4.1. **Design Consideration**

Laser beam shaping is the process of redistributing the irradiance and phase of a beam of optical radiation. The irradiance distribution defines the beam profile, such as Gaussian, multimode, annular, rectangular, elliptical or circular. We present a laser beam shaping analysis to transform a Gaussian laser beam into an annular beam of different intensity profiles with two different approaches. Such profiles include half Gaussian with maximum intensities at the inner and outer radii of the annulus, respectively, and full Gaussian with maximum intensity within the annulus. In the first approach, refractive optical elements are designed to convert a circular Gaussian beam to a uniform circular beam first and then transformed into an annular beam with desired intensity profiles. In the second approach, refractive optical elements are designed to convert a circular Gaussian beam to an annular beam with desired intensity profiles only in one step. The ray tracing technique of geometrical optics is used to design the required lens system. The incident laser beam is considered as a Gaussian TEM\(_{00}\) beam with a known intensity profile.
4.2. Two Steps Transformation of a Gaussian Beam into an Annular Beam

4.2.1. Transformation of a Gaussian Beam into a Uniform Circular Beam

4.2.1.1 Conservation of energy

Noting that the powers of the input and output laser beams must be equal, the energy balance can be written as follows in polar coordinates for rotationally symmetric systems shown in Fig. 4.1 [Dickey and Holswade (2000)],

\[ \iint_{0}^{2\pi} \int_{0}^{r_{0G}} I_{in,G}(r) rdrd\theta = \iint_{0}^{2\pi} \int_{0}^{r_{0u}} I_{out,u}(r) rdrd\theta \]  \[4.1\]

where \( I_{in,G} \) and \( I_{out,u} \) are the irradiances of the input Gaussian and output uniform laser beams respectively, \( r_{0G} \) is the distance between the point of maximum intensity \( I_0 \) to the point where the intensity \( I_{in,G}(r) \) of the input Gaussian laser beam is \( I_0/e^2 \) and \( r_{0u} \) is the radius of the output uniform laser beam. \( I_{in,G}(r) \) and \( I_{out,u}(r) \) are given by the following expressions:

\[ I_{in,G}(r) = I_0 e^{-\frac{r^2}{\sigma_{0G}^2}} \]  \[4.2\]

\[ I_{out,u}(r) = C \]  \[4.3\]

where \( I_0 = 2P/\pi\sigma_{0G}^2 \), \( P \) is the total laser power, \( C \) is the uniform (i.e., constant) intensity profile of the output laser beam. Eq. 4.1 is an energy balance over the entire laser beam cross-section (i.e., over the radii \( r_{0G} \) and \( r_{0u} \)). Similarly an energy balance equation can be written over two arbitrary radii \( r_G \) and \( r_u \) (Fig. 4.1). Substituting Eqs. 4.2 and 4.3 into
these two energy balance equations, we obtain the following expression for the radial distance of the output laser beam\cite{Dickey and Holswade (2000)}

\begin{equation}
    r_u = \frac{r_{0u}[1 - e^{(-2r^2 / r_{0u}^2)}]^{1/2}}{[1 - e^{(-2r^2 / r_{0u}^2)}]^{1/2}}
\end{equation}

\[4.4\]
4.2.1.2 The constancy of optical path length

Based on Fig. 4.1, the optical path length of the central ray can be written as

\[ nt_1 + n_0 D_G + nt_2 = F \]  \[ 4.5 \]

where \( F \) is a positive constant, \( t_1 \) and \( t_2 \) are thicknesses of the input and output lenses at the optical axis respectively, \( D_G \) is the distance between the input and output lenses along their optical axis, and \( n \) and \( n_0 \) are the refractive indices of the lens and the surrounding medium respectively.
For an arbitrary ray, the optical path length is

\[ nz + n_0 \sqrt{(r_u \pm r_G)^2 + (z_u - z_G)^2} + n(t_1 + t_2 + D_G - z_u) \]  \[ 4.6 \]

which can be written as follows since the optical path length must be the same for all the rays,

\[ n(z_G - z_u) + n_0 \sqrt{(r_u \pm r_G)^2 + (z_u - z_G)^2} = F' \]  \[ 4.7 \]

where \( F' = (n_0 - n)D_G \).

The minus sign in Eqs. 4.6 and 4.7 is for the Galilean case where no rays cross the optical axis, and the positive sign is for the Keplerian case where all rays cross the optical axis [Hoffnagle and Jefferson (2003)]. The minus sign is used in the present study.

Eq. 4.7 yields the following relationship between \( z_u \) and \( z_G \)

\[ z_u - z_G = \frac{-nF' + \sqrt{n^2F'^2 - (n^2 - n_0^2)[F'^2 - n_0^2(r_u - r_G)^2]}}{n^2 - n_0^2} \]  \[ 4.8 \]

### 4.2.1.3 Snell’s law

Considering the input and output rays as parallel to the optical axis, as shown in Fig. 4.2, we can write

\[ \frac{dz_G}{dr_G} = \frac{dz_u}{dr_u} = \tan \theta_{11} = \tan \theta_{22} \]  \[ 4.9 \]

\[ \tan(\theta_{11} - \theta_{12}) = \tan(\theta_{21} - \theta_{22}) = (r_u - r_G)/(z_u - z_G) \]  \[ 4.10 \]

Combining Eqs. 4.8, 4.9 and 4.10 with Snell’s law, the slopes of the input and output lens surfaces can be determined by using the following expression [Karim and Cherri (1987)]
\[
\frac{dz_G}{dr_G} = \frac{dz_u}{dr_u} = \frac{\gamma}{\left( \frac{F_{n^2}}{(r_u \pm r_d)^2} - \gamma^2 + 1 \right)}
\]  

where \( \gamma = n_0 / n \).

### 4.2.2. Transformation of a uniform circular beam into an annular beam

#### 4.2.2.1. Conservation of energy

The uniform laser beam obtained in the last section will be transformed into an annular beam with different intensity profiles in this section. Noting that the powers of the input and output laser beams must be equal, the energy balance can be written as follows in polar coordinates for rotationally symmetric systems (Fig. 4.3) [Dickey and Holswade, (2000)]

\[
\int_0^{2\pi} \int_0^{\pi} I_{in,u}(R)Rd\theta dR = \int_0^{2\pi} \int_{R_u}^{\pi} I_{out,a}(R)Rd\theta dR
\]

where \( I_{in,u}(R) \) and \( I_{out,a}(R) \) are laser irradiances of the input uniform circular beam and the output annular beam, which are given by

\[
I_{in,u}(R) = C
\]  

and

\[
I_{out,a}(R) = I_0 e^{-\frac{2(R-R_u)^2}{R_w^2}}
\]

where \( C \) is a constant for uniform intensity profile, \( R \) is the radial distance from the center of the annulus, \( R_w \) is the characteristic width of the laser beam, i.e., the distance between the point of maximum intensity \( I_0 \) to the point where the intensity \( I_{out,a}(R) \) is \( I_0/e^2 \). \( R_w \) is
the radial distance of the point of maximum intensity \( I_0 \) from the center of the annulus. 

\( R_{ca}, R_w \) and \( I_0 \) are given by the following expressions for different intensity profiles.

Case 1 – Full Gaussian beam with maximum intensity located at the center of the annulus

\[
R_{ca} = \frac{R_{ia} + R_{oa}}{2} \quad [4.15]
\]

\[
R_w = \frac{R_{oa} - R_{ia}}{2}
\]

\[
I_0 = \frac{2P}{\pi R_w^2 \exp(-2R_{ca}^2 / R_w^2) + \sqrt{2\pi} (R_{ca} / R_w) \text{erfc}(\sqrt{2R_{ca} / R_w})}
\]

where \( P \) is the total laser power, \( R_{ia} \) and \( R_{oa} \) are the inner and outer radii of the annular laser beam respectively.

Case 2 – Outer half Gaussian beam with maximum intensity located at the outer radius of the annulus

\[
R_{ca} = R_{oa} \quad [4.16]
\]

\[
R_w = R_{oa} - R_{ia}
\]

\[
I_0 = \frac{2P}{\pi R_w^2 \exp(-2R_{ca}^2 / R_w^2) - 1 + \sqrt{2\pi} (R_{ca} / R_w) \text{erf} (\sqrt{2R_{ca} / R_w})}
\]

Case 3 – Inner half Gaussian beam with maximum intensity located at the inner radius of the annulus,

\[
R_{ca} = R_{ia} \quad [4.17]
\]

\[
R_w = R_{oa} - R_{ia}
\]

\[
I_0 = \frac{2P}{\pi R_w^2 [1 + \sqrt{2\pi} (R_{ca} / R_w)]}
\]
So Eq. (12) can be rewritten as

\[
C \int_0^{2\pi R_{uw}} \int_0^{R_{uw}} RdRd\theta = \int_0^{2\pi R_{uw}} \int_0^{R_{uw}} I_0 e^{-\frac{(R-R_{uw})^2}{R_u^2}} RdRd\theta \quad [4.18]
\]

which yields

\[
C \int_0^{2\pi R_{uw}} RdRd\theta = -\frac{R_w^2}{4} \left\{ \exp\left[ -\frac{2(R_{ua} - R_{ca})^2}{R_w^2} \right] - \exp\left[ -\frac{2(R_{ia} - R_{ca})^2}{R_w^2} \right] \right\} + \\
\frac{\sqrt{\pi} R_w R_{ca}}{2\sqrt{2}} \left\{ \text{erf}\left[ \frac{\sqrt{2}(R_{ua} - R_{ca})}{R_w} \right] - \text{erf}\left[ \frac{\sqrt{2}(R_{ia} - R_{ca})}{R_w} \right] \right\} \quad [4.19]
\]

where \( R_{uw} \) is the maximum radius of the input uniform laser beam.

Eq. 4.12 represents an energy balance over the entire laser beam. Considering energy balance over an arbitrary radius \( R_u \) (Fig. 4.3) for the uniform beam and \( R_{ia} \) to \( R_{ia} \) for the annular beam, we can write

\[
C \int_0^{2\pi R_u} RdRd\theta = -\frac{R_w^2}{4} \left\{ \exp\left[ -\frac{2(R_{ua} - R_{ca})^2}{R_w^2} \right] - \exp\left[ -\frac{2(R_{ia} - R_{ca})^2}{R_w^2} \right] \right\} + \\
\frac{\sqrt{\pi} R_u R_{ca}}{2\sqrt{2}} \left\{ \text{erf}\left[ \frac{\sqrt{2}(R_{ua} - R_{ca})}{R_w} \right] - \text{erf}\left[ \frac{\sqrt{2}(R_{ia} - R_{ca})}{R_w} \right] \right\} \quad [4.20]
\]

Combining Eqs. 4.19 and 4.20, the radial distance of the input beam can be expressed in terms of the corresponding radial distance of the output beam, i.e.,

\[
R_u = R_{uw} \left[ \frac{R_w^2}{4} \left\{ \exp\left[ -\frac{2(R_{ua} - R_{ca})^2}{R_w^2} \right] - \exp\left[ -\frac{2(R_{ia} - R_{ca})^2}{R_w^2} \right] \right\} + \frac{\sqrt{\pi} R_w R_{ca}}{2\sqrt{2}} \left\{ \text{erf}\left[ \frac{\sqrt{2}(R_{ua} - R_{ca})}{R_w} \right] - \text{erf}\left[ \frac{\sqrt{2}(R_{ia} - R_{ca})}{R_w} \right] \right\} \right]^{1/2}
\]

[4.21]
Figure 4.3 Geometrical configuration of axicon refractive system.
4.2.2.2. Constancy of optical path length conservation of energy

As shown in Fig. 4.3, the optical path length of the central ray can be written as

\[ nt_1 + n_0 \left( \frac{R_u}{\sin \alpha} \right) + n(D - R_u \cot \alpha) + nt_2 = F \]  \[ \text{[4.22]} \]

where \( F \) is a positive constant, \( D \) is the distance between the input and output lenses along their optical axis, \( \alpha \) is the central ray refractive angle, \( t_1 \) and \( t_2 \) are thicknesses of the input and output lenses along their optical axis respectively, and \( n \) and \( n_0 \) are the refractive indices of the lens and the surrounding medium respectively.

Similarly, the optical path length for an arbitrary ray can be written as

\[ nt_1 + nZ_u + n_0 \sqrt{(R_a \pm R_u)^2 + (Z_a - Z_u)^2} + n(D + t_2 - Z_a) \]  \[ \text{[4.23]} \]

The positive sign in Eq. 4.23 is for the axicon lens [Arnold et al. (1977)] where all rays cross the optical axis as shown in Fig. 4.3 and the minus sign is for the “vaxicon” lens where no rays cross the optical axis as shown in Fig. 4.4. It should be noted that a vaxicon lens is a compound axicon lens with a w-shaped cross section [Arnold et al., 1977], and thus vaxicon is actually a w-shaped (i.e., w-axicon) lens. The term “v-axicon” is referred to as vaxicon in this study to designate a v-shaped conical lens.

The optical path length for all the rays must be the same, i.e.,

\[ n(Z_u - Z_u) + n_0 \sqrt{(R_u \pm R_a)^2 + (Z_u - Z_a)^2} = F' \]  \[ \text{[4.24]} \]

where \( F' = n_0 \left( \frac{R_u}{\sin \alpha} \right) - n(R_u \cot \alpha) \), which yields the following relation for the axial distance of the two lens surfaces \( Z_a(R_a) \) and \( Z_u(R_u) \):

\[ Z_a - Z_u = \frac{-nF' + \sqrt{n^2F'^2 - (n^2 - n_0^2)(F'^2 - n_0^2(R_u \pm R_a)^2)}}{n^2 - n_0^2}. \]  \[ \text{[4.25]} \]
Figure 4.4 Geometrical configuration of vaxicon refractive system.
4.2.2.3. Snell’s law

Considering the input and output rays as parallel to the optical axis as shown in Fig. 4.2, we obtain

\[ \frac{dZ_u}{dR_u} = \frac{dZ_a}{dR_a} = \tan \theta_{11} = \tan \theta_{22} \]  \hspace{1cm} [4.26]

\[ \tan(\theta_{11} - \theta_{12}) = \tan(\theta_{21} - \theta_{22}) = (R_u - R_a)/(Z_a - Z_u) \]  \hspace{1cm} [4.27]

Combining Eqs. 4.25, 4.26 and 4.27 with Snell’s law, the slope of the input and output lens surfaces can be determined by using the following expression [Karim and Cherri, (1987); Jahan and Karim (1989)]

\[ \frac{dZ_a}{dR_a} = \frac{\gamma}{\left( \frac{F'^2}{(R_u \pm R_a)^2} - \gamma^2 + 1 \right)} \]  \hspace{1cm} [4.28]

where \( \gamma = n_0 / n \). Substituting the expression of \( R_a \) from Eq. 4.21 into Eq. 4.28 and numerically integrating Eq. 4.28, the profile of the output lens surface \( Z_a(R_a) \) can be determined and then the profile of the input lens surface \( Z_u(R_u) \) can be obtained from Eq. 4.25.

4.2.3. Design analysis

The variable \( F' \) in Eqs. 4.7 and 4.24 is known as the importance parameter for the refractive systems, which can be determined by the distance \( (D) \) between the input and output lenses, the central ray refraction angle \( \alpha_c \), and the refractive indices of the lens material \( (n) \) and surrounding medium \( (n_0) \). The refractive indices are fixed for a given lens material and surrounding material. Fused Silica is chosen as the lens material in this
study. The refraction index of the fused silica is 1.46 for 532 nm wavelength light [Ready and Farson (2001)]. The surrounding medium is considered air for which $n_0$ is taken as 1 in the present study. For optimizing and simplifying the beam shaping lens design procedure, the following constraints may be followed:

1) The distance between the input and output lens surfaces should be as small as possible.

2) The lens surface curvature parameter $\chi$, where $\chi = \left( \frac{d^2 z}{dr^2} \right)_{\text{max}}$, must be as small as possible.

3) The tangent of the central ray refraction angle ($\tan \alpha_c$) also should be less than the minimum slope of the lens surface.

The first constraint determines a physically small refracting system. The second constraint yields a lens design that can be fabricated fairly easily. The third constraint helps in achieving beam shaping through refractive optics. If the tangent of the central ray refraction angle ($\tan \alpha_c$) is larger than the minimum slope of the lens surface for the vaxicon lens design as in Fig. 4.3, the input light ray will experience total internal reflection. For the axicon lens design as in Fig. 4.4, the input light ray also will experience total internal reflection if the tangent of the central ray refraction angle ($\tan \alpha_c$) is larger than the minimum slope of the lens surface. These two situations need to be avoided to maximize the laser power in the output beam.

We can get suitable parameters for the lens design by varying the central ray refraction angle $\alpha_c$. Defining a dimensionless variable $x^* = x/R_0$, we can write $R^*_0 = R_0 / R_0$, $R^*_0 = 1$, 

74
\[ R'_{oa} = R_{oa}/R_{oa} \text{ and } D' = D/R_{oa}. \] The beam shaping analysis is carried out by taking \( R'_{oa} = 0.5 \) and \( R'_{oa} = 1.5. \) From Fig. 4.5, the distance \( D' \) between the input and output lenses decreases with the increase in the central ray refraction angle \( \alpha_c \) and this effect becomes insignificant when the central ray refraction angle exceeds 20°. According to the third constraint, \( \tan \alpha_c \) should be smaller than the minimum slope of the input lens surface for both axicon and vaxicon lenses. Therefore, from Fig. 4.6, the angle \( \alpha_c \) should be less than 42° for both axicon and vaxicon lens designs.

In Fig. 4.7, the lens surface curvature parameter \( \chi \) reaches a maximum value when the central ray refraction angle is 25°, and then \( \chi \) decreases as the central ray refraction angle increases. Based on the second constraint, the optimal angle should be 0 - 10° and 30 - 40° for the vaxicon lens, while it should be 30 - 40° for the axicon lens since \( \chi \) is smaller in this range as shown in Fig. 4.7. In Fig. 4.8, the maximum slope of the lens surface is a monotonically increasing function of the central ray refraction angle \( \alpha_c \). However, the maximum slope of the lens surface does not change significantly after the central ray refraction angle reaches 30°.

Based on all the three constraints, the optimal central ray refraction angle \( \alpha_c \) can be in the range of 30 - 40°. We select the central ray refraction angle as 40° to obtain a small lens surface curvature parameter \( \chi \) in order to simplify the lens fabrication process. We also select the central ray refraction angle as 40° for converting the Gaussian beam into a half Gaussian beam with maximum intensity at the inner or outer radius of the annulus.
Figure 4.5 Distance ($D^*$) between the input and output lens versus central ray refraction angle.
Figure 4.6 Minimum slope of lens surface versus central ray refraction angle.
Figure 4.7 Lens surface curvature parameter $\gamma$ versus central ray refraction angle.
Figure 4.8. Maximum slope of lens surfaces versus central ray refraction angle.
4.2.4. Design results

The input and output lens surface profiles can be calculated for converting a Gaussian circular beam into a uniform circular beam as shown in Fig. 4.9. We choose the following dimensionless parameters: $r_{0G}^* = \frac{r_{0G}}{R_{ia}} = 0.5$, $r_{0u}^* = \frac{r_{0u}}{R_{ia}} = 1$ and $D_G^* = D_G/R_{ia} = 1$.

By means of polynomial curve fitting, the input and output lens surface profiles are found to be

Input lens surface: $z(r_G) = -1.30r_G^5 + 4.04r_G^4 - 4.74r_G^3 + 2.67r_G^2 - 0.02r_G$, \[4.29\]

Output lens surface: $z(r_u) - D_G = -4.71r_u^5 + 19.86r_u^4 - 22.75r_u^3 + 9.18r_u^2 - 0.10r_u$. \[4.30\]

For converting the uniform circular beam into a full Gaussian annular beam, we choose the central ray refraction angle $\alpha_c = 40^\circ$ and the dimensionless maximum radius of the input lens surface as $r_{0u}^* = \frac{r_{0u}}{R_{ia}} = 1$, $R_{ia}^* = \frac{R_{ia}}{R_{ia}} = 1$ and $R_{oa}^* = \frac{R_{oa}}{R_{ia}} = 1.5$. The dimensionless distance between the input and output lenses is $D^* = D/R_{ua} = 1.92$.

The profiles of the input and output lens surfaces can be calculated for converting the uniform circular beam into a full Gaussian annular beam as shown in Fig. 4.10. For axicon lens, polynomial curve fitting yields the following expressions for the input and output lens surfaces:

Input lens surface: $Z_u(R_u) = 10^{-2} \times (0.24R_u^5 - 0.97R_u^4 + 1.68R_u^3 - 1.75R_u^2 - 0.93R_u)$. \[4.31\]
Output lens surface:

\[ Z_a(R_a) - D = -0.04R_a^5 + 0.26R_a^4 - 0.74R_a^3 + 1.06R_a^2 + 0.17R_a - 0.71 \quad \text{(for } R_{ia} \leq R_a \leq R_{oa}) , \]

\[ Z_a(R_a) - D = 0 \quad \text{(for } 0 \leq R_a \leq R_{ia}) . \]

[4.32]

Figure 4.9 Input and output surface of lens for converting a Gaussian beam into a uniform beam.
Figure 4.10 Input and output surface of axicon lens for converting a uniform circular beam into an annular beam.
Figure 4.11 Input and output surface of vaxicon lens for converting a uniform circular beam into an annular beam.
For vaxicon lens, the profiles of the input and output lens surfaces are shown in Fig. 4.11, and the corresponding polynomial expressions are obtained as follows:

Input lens surface: 
\[ Z_u(R_u) = 0.05R_u^5 - 0.13R_u^4 + 0.10R_u^3 - 0.04R_u^2 + 0.93R_u. \]  [4.33]

Output lens surface:
\[ Z_a(R_a) - D = 0.76R_a^5 - 4.22R_a^4 + 9.26R_a^3 - 10.02R_a^2 + 6.28R_a - 2.06 \quad \text{(for } R_{ia} \leq R_a \leq R_{oa}), \]
\[ Z_a(R_a) - D = 0 \quad \text{(for } 0 \leq R_a \leq R_{oa}). \]  [4.34]

For converting a uniform beam into an outer half Gaussian beam, i.e., the maximum laser intensity is located at the outer radius of the annular beam, the input and output axicon lens surface profiles are found to be

Input lens surface: 
\[ Z_u(R_u) = 10^{-2} \times (0.27R_u^5 - 1.09R_u^4 + 1.87R_u^3 - 1.91R_u^2) - 0.92R_u. \]  [4.35]

Output lens surface: 
\[ Z_a(R_a) - D = -0.05R_a^5 + 0.32R_a^4 - 0.87R_a^3 + 1.19R_a^2 + 0.12R_a + 0.71 \quad \text{(for } R_{ia} \leq R_a \leq R_{oa}), \]
\[ Z_a(R_a) - D = 0 \quad \text{(for } 0 \leq R_a \leq R_{oa}). \]  [4.36]

Vaxicon lens profiles to convert a uniform beam into an outer half Gaussian beam are given by

Input lens surface: 
\[ Z_u(R_u) = 10^{-3} \times (0.05R_u^5 - 8.05R_u^4 + 4.04R_u^3 - 8.74R_u^2) + 0.92R_u. \]  [4.37]

Output lens surface: 
\[ Z_a(R_a) - D = 0.03R_a^5 - 0.27R_a^4 + 0.79R_a^3 - 1.09R_a^2 + 1.65R_a - 1.12 \quad \text{(for } R_{ia} \leq R_a \leq R_{oa}), \]
\[ Z_a(R_a) - D = 0 \quad \text{(for } 0 \leq R_a \leq R_{ia}). \]  [4.38]
For converting a uniform beam into an inner half Gaussian beam, i.e., the maximum laser intensity is located at the inner radius of the annular beam, the input and output axicon lens surface profiles are found to be:

Input lens surface: \( Z_\alpha (R_\alpha) = -0.19R_\alpha^2 + 0.41R_\alpha^4 - 0.35R_\alpha^3 + 0.11R_\alpha^2 - 0.94R_\alpha \). \[4.39\]

Output lens surface: \( Z_\alpha (R_\alpha) - D = -0.11R_\alpha^5 + 0.71R_\alpha^4 - 1.19R_\alpha^3 + 2.44R_\alpha^2 - 0.67R_\alpha - 0.51 \) (for \( R_{ia} \leq R_\alpha \leq R_{oa} \)),

\[ Z_\alpha (R_\alpha) - D = 0 \] (for \( 0 \leq R_\alpha \leq R_{ia} \)). \[4.40\]

Vaxicon lens profiles to convert a uniform beam into an inner half Gaussian beam are given by:

Input lens surface: \( Z_\alpha (R_\alpha) = 0.12R_\alpha^5 - 0.28R_\alpha^4 + 0.21R_\alpha^3 - 0.09R_\alpha^2 + 0.94R_\alpha \). \[4.41\]

Output lens surface:

\( Z_\alpha (R_\alpha) - D = -0.75R_\alpha^5 + 4.77R_\alpha^4 - 11.87R_\alpha^3 + 14.40R_\alpha^2 - 7.61R_\alpha + 1.05 \) (for \( R_{ia} \leq R_\alpha \leq R_{oa} \)),

\[ Z_\alpha (R_\alpha) - D = 0 \] (for \( 0 \leq R_\alpha \leq R_{ia} \)). \[4.42\]

4.3. Single Step Transformation of a Gaussian Beam into an Annular Beam

4.3.1 Conservation of energy

Noting that the powers of the input and output laser beams must be equal, the energy balance can be written as follows in polar coordinates for rotationally symmetric systems (Fig. 1) [Dickey and Holswade (2000)]
where \(I_{in,G}\) and \(I_{out,a}\) are the irradiances of the input Gaussian circular and output annular laser beams respectively, \(R_{0G}\) is the distance between the point of maximum intensity \(I_0\) to the point where the intensity \((I_{in,G}(R))\) of the input Gaussian laser beam is \(I_0/e^2\) and \(R_{ia}\) and \(R_{oa}\) are the inner and outer radius of the output annular laser beams respectively. \(I_{in,G}(R)\) and \(I_{out,a}(R)\) are given by the following expressions:

\[
I_{in,G}(R) = I_0 e^{-\frac{R^2}{R_{0G}^2}},
\]

for uniform annular beams, 

\[
I_{out,a}(R) = C
\]

for Gaussian annular beams,

where \(I_0 = 2P/\pi R_{0G}^2\), \(P\) is the total laser power, \(R\) is the radial distance from the center of the annulus. For uniform annular beams, \(C\) is the uniform (i.e., constant) intensity profile of the output laser beam.

Substituting Eqs. 4.44, 4.45 and 4.46 into the energy balance equation, we obtain the following expression for the radial distance of the output laser beam [Dickey and Holswade (2000)]

\[
R_{oa}^2 = -2R_{0G}^2 \ln \left\{ 1 - \left[ 1 - \exp(-2) \right] \left( \frac{R_{oa}^2 - R_{ia}^2}{R_{ia}^2} \right) \right\}
\]

for uniform annular beams, 

\[\text{(4.47)}\]
Similarly, substituting the expression of $R_G$ from Eqs. 4.47 and 4.48 into Eq. 4.17 and numerically integrating Eq. 4.17, the profile of the output lens surface $Z_a(R_a)$ can be determined and then the profile of the input lens surface $Z_G(R_G)$ can be obtained from Eq. 4.14.

4.3.2 Design analysis

We can get suitable parameters for the lens design by varying the central ray refraction angle $\alpha_c$. Defining a dimensionless variable $x^* = x/R_{ia}$, we can write $R^*_{ia} = 1$, $R^*_{oa} = R_{oa}/R_{ia}$ and $D^* = D/R_{ia}$. The beam shaping analysis is carried out by taking $R^*_{oa} = 1.5$. From Fig. 4.12, the distance $D^*$ between the input and output lenses decreases with the increase in the central ray refraction angle $\alpha_c$ and this effect becomes insignificant when the central ray refraction angle exceeds 20°. According to the third constraint, $\tan \alpha_c$ should be smaller than the minimum slope of the input lens surface for both axicon and vaxicon lenses. Therefore, from Fig. 4.13, the angle $\alpha_c$ should be less than 43° for both axicon and vaxicon lens designs.
In Fig. 4.14, the lens surface curvature parameter $\chi$ reaches a maximum value when the central ray refraction angles are $25^\circ$ and $15^\circ$ for vaxicon and axicon lenses respectively, and then $\chi$ decreases as the central ray refraction angle increases. Based on the second constraint, the optimal angle should be $0 - 10^\circ$ and $35 - 43^\circ$ for the vaxicon lens, while it should be $35 - 43^\circ$ for the axicon lens since $\chi$ is smaller in this range as shown in Fig. 4.14. In Fig. 4.15, the maximum slope of the lens surface is a monotonically increasing function of the central ray refraction angle $\alpha_c$. However, the change of the maximum slope of the lens surface is less than $10\%$ between $35^\circ$ and $43^\circ$ central ray refraction angle $\alpha_c$.

Based on all the three constraints, the optimal central ray refraction angle $\alpha_c$ can be in the range of $35 - 43^\circ$. We select the central ray refraction angle as $40^\circ$ to obtain a small lens surface curvature parameter $\chi$ in order to simplify the lens fabrication process. We also select the central ray refraction angle as $40^\circ$ for converting the Gaussian beam into a half Gaussian beam with maximum intensity at the inner or outer radius of the annulus.
Figure 4.12 Distance ($D^*$) between the input and output lens versus central ray refraction angle.
Figure 4.13 Minimum slope of lens surface versus central ray refraction angle.

Figure 4.13 Minimum slope of lens surface versus central ray refraction angle.
Figure 4.14 Lens surface curvature parameter $\gamma$ versus central ray refraction angle.
Figure 4.15 Maximum slope of lens surfaces versus central ray refraction angle.
4.3.3 Design results

We choose the central ray refraction angle $\alpha_c = 40^\circ$ and the dimensionless maximum radius of the input lens surface as $R_{0G}^* = R_{0G} / R_{ia} = 1$, $R_{ia}^* = R_{ia} / R_{ia} = 1$ and $R_{oa}^* = R_{oa} / R_{ia} = 1.5$. The dimensionless distance between the input and output lenses is $D^* = D / R_{ia} = 1.19$.

The profiles of the input and output lens surfaces can be calculated for converting the Gaussian beam into a uniform annular beam as shown in Fig. 4.10. For axicon lens, polynomial curve fitting yields the following expressions for the input and output lens surfaces:

Input lens surface:

\[
Z_G(R_G) = 10^{-2} \times (0.26R_G^4 + 1.03R_G^3 - 1.58R_G^2) - 0.93R_G. \quad [4.49]
\]

Output lens surface: $Z_a(R_a) - D = -0.08R_a^5 + 0.54R_a^4 - 1.42R_a^3 + 1.88R_a^2 - 0.32R_a + 0.59$

(for $R_{ia} \leq R_a \leq R_{oa}$),

\[
Z_a(R_a) - D = 0 \quad (for \ 0 \leq R_a \leq R_{ia}). \quad [4.50]
\]

For axicon lens, the profiles of the input and output lens surfaces are shown in Fig. 4.11, and the corresponding polynomial expressions are obtained as follows:

Input lens surface: $Z_G(R_G) = -0.01R_G^5 + 0.01R_G^4 - 0.01R_G^3 + 0.93R_G$. \quad [4.51]

Output lens surface:

\[
Z_a(R_a) - D = -0.31R_a^5 + 1.75R_a^4 - 3.99R_a^3 + 4.49R_a^2 - 1.58R_a + 0.82 \quad (for \ R_{ia} \leq R_a \leq R_{oa}),
\]

\[
Z_a(R_a) - D = 0 \quad (for \ 0 \leq R_a \leq R_{ia}). \quad [4.52]
\]
For converting a Gaussian beam into a full Gaussian annular beam, i.e., the maximum laser intensity is located at the outer radius of the annular beam, the input and output axicon lens surface profiles are found to be

**Input lens surface:**

\[
Z_G(R_G) = 10^{-2} \times (0.42R_G^3 - 1.49R_G^5 + 2.23R_G^3 - 2.00R_G^2) - 0.93R_G. \tag{4.53}
\]

**Output lens surface:**

\[
Z_a(R_a) - D = -0.02R_a^5 + 0.13R_a^4 - 0.39R_a^3 + 0.58R_a^2 + 0.49R_a + 0.39 \quad (\text{for } R_a \leq R_a \leq R_{oa}), \tag{4.54}
\]

\[
Z_a(R_a) - D = 0 \quad (\text{for } 0 \leq R_a \leq R_a). \tag{4.54}
\]

**Vaxicon lens profiles to convert a Gaussian beam into a full Gaussian beam are given by**

**Input lens surface:**

\[
Z_G(R_G) = 10^{-2} \times (0.99R_G^3 - 2.87R_G^4 + 1.53R_G^3 - 0.01R_G^2) + 0.93R_G. \tag{4.55}
\]

**Output lens surface:**

\[
Z_a(R_a) - D = 0.60R_a^5 - 3.73R_a^4 + 9.09R_a^3 - 10.98R_a^2 + 7.52R_a - 1.31 \quad (\text{for } R_{ia} \leq R_a \leq R_{oa})
\]

\[
Z_a(R_a) - D = 0 \quad (\text{for } 0 \leq R_a \leq R_{ia}). \tag{4.56}
\]

For converting a Gaussian beam into an outer half Gaussian beam, i.e., the maximum laser intensity is located at the inner radius of the annular beam, the input and output axicon lens surface profiles are found to be

**Input lens surface:**

\[
Z_G(R_G) = 10^{-2} \times (0.55R_G^3 - 1.90R_G^4 + 2.75R_G^3 - 2.30R_G^2) + 0.93R_G. \tag{4.57}
\]

**Output lens surface:**

\[
Z_a(R_a) - D = -0.03R_a^5 + 0.19R_a^4 - 0.52R_a^3 + 0.73R_a^2 + 0.41R_a + 0.41 \quad (\text{for } R_{ia} \leq R_a \leq R_{oa})
\]
\[ Z_a(R_a) - D = 0 \quad (\text{for } 0 \leq R_a \leq R_{ia}). \]  [4.58]

Vaxicon lens profiles to convert a Gaussian beam into an outer half Gaussian beam are given by

Input lens surface:
\[ Z_G(R_G) = 10^{-2} \times (1.22R_G^5 + 1.64R_G^4 - 1.53R_G^3 - 0.29R_G^2) + 0.92R_G. \]  [4.59]

Output lens surface:
\[ Z_a(R_a) - D = -0.51R_a^5 + 3.02R_a^4 - 7.18R_a^3 + 8.51R_a^2 - 4.11R_a + 1.45 \quad (\text{for } R_{ia} \leq R_a \leq R_{oa}), \]
\[ Z_a(R_a) - D = 0 \quad (\text{for } 0 \leq R_a \leq R_{ia}). \]  [4.60]

For converting a Gaussian beam into an inner half Gaussian beam, i.e., the maximum laser intensity is located at the inner radius of the annular beam, the input and output axicon lens surface profiles are found to be

Input lens surface:
\[ Z_G(R_G) = 10^{-2} \times (0.05R_G^5 - 0.04R_G^4 + 0.97R_G^3 - 1.45R_G^2) + 0.93R_G. \]  [4.61]

Output lens surface:
\[ Z_a(R_a) - D = -0.09R_a^5 + 0.57R_a^4 - 1.51R_a^3 + 2.00R_a^2 + 0.40R_a + 0.61 \quad (\text{for } R_{ia} \leq R_a \leq R_{oa}), \]
\[ Z_a(R_a) - D = 0 \quad (\text{for } 0 \leq R_a \leq R_{ia}). \]  [4.62]

Vaxicon lens profiles to convert a Gaussian beam into an inner half Gaussian beam are given by

Input lens surface:
\[ Z_G(R_G) = 10^{-2} \times (1.60R_G^5 - 3.77R_G^4 + 2.61R_G^3 - 2.42R_G^2) + 0.93R_G. \]  [4.63]

Output lens surface:
Using geometric optics, two refractive arrangements have been developed that transform a Gaussian circular laser beam into an annular laser beam with required intensity distribution. Two lens systems using axicon and vaxicon lenses, respectively, are presented in this study. The lens surface profiles can be calculated by considering the conservation of energy, constancy of optical path length and Snell’s law. An optical path length constant $F'$ is found to be an important parameter to determine the lens profile. For a specific lens system, however, the suitable lens profiles can be found by varying the central ray refraction angle. To simplify the lens fabrication procedure and achieve beam shaping through refractive optics, three constraints are listed and the influences of different central ray refraction angles on the lens profiles are presented in this paper. The optimal central ray refraction angles are in the range of $35 - 43^\circ$ for our cases. We have chosen the central ray refraction angle as $40^\circ$ to determine the input and output lens surfaces in this study.
5.1. Introduction

In the previous chapter, based on geometrical optics, two refractive arrangements have been developed that transform a Gaussian circular laser beam into an annular laser beam with required irradiance profiles. Geometric optics, or ray optics, describes light propagation in terms of rays. Rays are bent at the interface between two dissimilar media, and may be curved in a medium in which the refractive index is a function of position. The ray in geometric optics is an abstract object which is perpendicular to the wavefronts of the actual optical waves. Geometric optics provides rules for propagating these rays through an optical system, which indicates how the actual wavefront will propagate. Note that this is a significant simplification of optics, and fails to account for many important optical effects such as diffraction and polarization. In this chapter, the diffractive effects of the axicon system and a convex lens focusing the collimated annular beam have been studied using the Fresnel diffraction integral. The theoretical diffraction patterns are compared to the patterns measured with a laser beam analyzer.

Compared with an axicon lens, a vaxicon lens is much more difficult to machine due to its concave conical shape. A common way to machine this vaxicon lens is to drill a small hole on the central axis of the axicon lens. To limit the effects of the central hole, the laser beam can be expanded and a mask used to cover the hole of the vaxicon lens
thereby blocking any portion of the beam from passing through the hole. An annular beam can thus be generated without the loss of laser energy. Considering the manufacturing feasibility, the refractive axicon lenses are chosen in our study to construct a beam shaping system to transform an input Gaussian beam into an annular beam.

Figure 5.1 Geometrical configuration of an axicon refractive system.
5.2. Diffraction Analysis of an Axicon Refractive System

5.2.1. Fresnel approximation of the diffraction field after the first axicon lens

Fig. 5.1 shows the propagation of an arbitrary ray through a refractive axicon system. The incident ray $A_1B_1$ intersects the optical axis at point D, and then it is collimated to the ray $B_2A_2$ by the second axicon lens. $\beta_1$ and $\beta_2$ are the base angles of the first and the second axicon lenses respectively. The surface constraint for the axicon refractive system is $\beta_1 = \beta_2$ to achieve collimation. For diffractive analysis of the beam propagation, we consider an input Gaussian electric field illuminating the flat surface of the first axicon. The amplitude of the Gaussian electric field, $U_0(r)$, can be written as follows in polar coordinates for rotationally symmetric systems:

$$U_0(r) = \sqrt{I_0} e^{-\frac{r^2}{r_w^2}},$$

[5.1]

where $r_w$ is the radius between the point of maximum irradiance $I_0$ to the point where the irradiance of the input Gaussian laser beam is $I_0/e^2$ and $r$ is the radius of any point on a transverse plane.

To simplify the calculation of the optical phase function, the Gaussian beam is assumed to enter the axicon lens as a plane wave. This assumption is strictly true only if the input plane of the axicon lens coincides with the beam waist within the Rayleigh zone. As a
ray travels from left \((A_1B_1)\) to right \((B_1C_1)\) in Fig. 5.1, the optical phase delay \(\phi_1(r)\) introduced by the first axicon lens and the air is[Goodman (1996)]

\[
\phi_1(r) = k^* n(t_1 - r_1 \tan \beta_1) + k^* n(r_1 - r) \tan \beta_1 + n_\perp k^* r \tan \beta_1 / \cos \theta_1 ,
\]

where

\[
\theta_1 = \arcsin\left(\frac{n \sin \beta_1}{n_\perp}\right) - \beta_1 ,
\]

\(t_1\) is the thickness of the first axicon lens along its optical axis, \(r_1\) is the radius of the first axicon lens, \(k^*\) is the wave number of the incident laser beam, and \(n\) and \(n_\perp\) are the refractive indices of the axicon lens and the surrounding medium respectively. So the transmittance function \(T_1(r)\) of the first axicon lens is

\[
T_1(r) = \begin{cases} 
\exp(i\phi_1(r)) & \text{for } r < r_1 \\
0 & \text{for } r \geq r_1 
\end{cases}.
\]

Neglecting the constant phase factor \(\exp(ikn\delta_1)\) [Perez et al. (1986)], the diffraction field \(U_1(r,z)\) at a distance \(z'\) from the axicon tip \(O_1\) along the optical axis can be written as follows in polar coordinates for rotationally symmetric systems by the Fresnel diffraction integral[Perez et al. (1986)],

\[
U_1(R,z') = \frac{k^* \exp(k^* z')}{iz'} \exp\left(-\frac{r^2}{2z'}\right) \exp\left(-\frac{-r^2}{2z'}\right) \exp\left(-\frac{jk^* r^2}{2z'}\right) \exp\left[i k^* r \tan \beta_1 \left(\frac{1}{\cos \theta_1} - n\right)\right] J_0\left(\frac{k^* R}{z'}\right) rdr .
\]
5.2.2. Fresnel approximation of the diffraction field after the second axicon lens

As a ray travels from left (C₂B₂) to right (B₂A₂) in Fig. 5.1, the optical phase delay \( \phi_2(R) \) introduced by the second axicon lens and the air is [Goodman (1996)]

\[
\phi_2(R) = k^* n(t_2 - R \tan \beta_2) + k^* n(R - R) \tan \beta_2 + k^* n_0 R \tan \beta_2 / \cos \theta_2 , \tag{5.6}
\]

where

\[
\theta_2 = \arcsin\left( \frac{n \sin \beta_2}{n_0} \right) - \beta_2 , \tag{5.7}
\]

\( \delta_2 \) is the thickness of the second axicon lens along its optical axis and \( R_1 \) is the radius of the second axicon lens. So the transmittance function \( t_2(R) \) of the second axicon lens is

\[
T_2(R) = \begin{cases} 
\exp(i\phi_2(R)) & \text{for } R < R_1 \\
0 & \text{for } R \geq R_1 .
\end{cases} \tag{5.8}
\]

Neglecting the constant phase factor \( \exp(ik'nt_2) \) [Perez et al. (1986)], the diffraction field, \( U_2(\rho, Z') \), at a distance \( Z' \) from the second axicon tip O₂ along the optical axis can be written as follows in polar coordinates for rotationally symmetric systems by the Fresnel diffraction integral [Perez et al. (1986)]

\[
U_2(\rho, Z') = \frac{k^* \exp(ik'Z')}{iZ'} \exp\left( \frac{ik^* \rho^2}{2Z'} \right) \int_0^R U_1(R, z') \exp\left( \frac{ik^* R^2}{2Z'} \right) \exp\left[ ik^* R \tan \beta_2 \left( \frac{1}{\cos \theta_2} - n \right) \right] J_0\left( \frac{k^* R \rho}{Z'} \right) RdR . \tag{5.9}
\]

5.2.3. Fresnel approximation of the diffraction field after the focusing lens

As a ray travels from left (B₂A₂) to right (A₃B₃) in Fig. 1, the optical phase delay \( \phi_3(\rho) \) introduced by the focusing lens and the air is [Goodman (1996)]
\[
\phi_3(\rho) = k^* n_2 \delta_3 - k^* n_0 \frac{\rho^2}{2f_i}.
\]  
[5.10]

where \( t_3 \) is the thickness of the focusing lens along its optical axis and \( f_i \) is the focal length of the focusing lens.

So the transmittance function \( T_3(\rho) \) of the focusing lens is

\[
T_3(\rho) = \begin{cases} 
\exp(i\phi_3(\rho)) & \text{for } \rho < \rho_1, \\
0 & \text{for } \rho \geq \rho_1 
\end{cases}.
\]  
[5.11]

Neglecting the constant phase factor \( \exp(ikn_3t_3) \) [Goodman (1996)], the diffraction field \( U_3(\eta, L) \) at a distance \( L \) from the focusing lens tip \( O_3 \) along the optical axis can be written as follows in polar coordinates for rotationally symmetric systems by the Fresnel diffraction integral [Goodman(1996)]

\[
U_3(\eta, L) = \frac{k^*}{iL} \exp(ik^*L) \exp(\frac{ik^*\eta^2}{2L}) \int_0^{\rho_1} U_2(\rho, Z) \exp(\frac{ik^*\rho^2}{2L}) \exp(-\frac{ik^*\rho^2}{2f_i}) J_0(\frac{k^*\rho \eta}{L}) \rho d\rho.
\]  
[5.12]

### 5.3. Laser Irradiance Distributions at Different Locations of the Axicon Refractive System

#### 5.3.1. Laser irradiance distributions after the first axicon lens

The laser irradiance profiles are calculated using the above-mentioned diffraction patterns. As shown in Fig. 5.1, an input Gaussian (TEM\(_{00}\)) Nd:YAG laser beam of wavelength \( \lambda = 1.064 \text{ \mu m} \) is incident on the flat surface of the first axicon lens with the beam axis lying on the principal axis of the axicon lens. The waist of the Gaussian beam
was located inside the Nd:YAG laser cavity. A collimated horizontal beam from the exit of the laser system was turned to a vertical beam with a 45° mirror to direct it to the first axicon lens. Both axicon lenses and the focusing lens are made of fused silica of refractive index $n = 1.46$. The half-apex angle of the axicon lenses are $\alpha_1 = \alpha_2 = 72^\circ$ and their base angles are $\beta_1 = \beta_2 = 18^\circ$ as shown in Fig. 5.1. The focal length of the convex lens is $f_i = 50$ mm. The incident beam profiles are shown in Fig. 5.2. The annular beam irradiance profiles were measured with a CCD camera (Laser Cam II, $4.7 \times 5.5 \, \mu\text{m}$ pixel size). To obtain the diffraction patterns of the annular beam at different axial locations of the axicon lens system, the CCD camera was placed on a translation stage to adjust its position in the optical system.

Fig. 5.3 shows the measured irradiance profile after the beam passes through the first axicon lens. Several rings are generated around the annular region, i.e., several low irradiance inner diffraction rings are formed near the main high irradiance thin outer ring. Most of the laser energy is focused onto the outer main ring. The performance of this “narrower annulus” is actually beneficial for optical trepanning, since the width of the annular beam (i.e. the difference between the inner and outer annular beam radii) significantly influences the drilled hole quality. This effect has been found to be very important, because a wider annular laser spot increases the growth of melt layer in the radial direction. Thinner recast layer, smaller taper and higher drilling speed are obtained for a fixed outer radius of a given annular beam with smaller annular width, i.e., with larger inner radius of the annular beam.
Figure 5.2 Irradiance profile of an input Gaussian laser beam before the first axicon lens. (a) Transverse cross section of the annular beam profile. (b) Laser irradiance profile in the X-direction on the transverse plane. (c) Laser irradiance profile in the Y-direction on the transverse plane. Radius of the input Gaussian beam $r_w = 0.327$ mm.
The measured width of the main annular ring is smaller than that predicted by the Fresnel diffraction integral. The discrepancy between the experimental and theoretical results may be due to several reasons: the blunt tip of the axicon, the imperfect incoming Gaussian beam, aberrations and imperfections of the optical elements. The phase shift due to optical elements is calculated in this study by considering the incident beam as a plane wave at the input plane of each optical element. This approximation is strictly applicable for the beam waist plane within the Rayleigh zone. Outside of this plane, the beam involves a quadratic phase function in the paraxial approach. Therefore, the existence of a quadratic phase could also be responsible for the discrepancy between the experimental and theoretical results.

Fig. 5.4 shows the theoretical longitudinal irradiance distributions along the z-axis for different base angles of the axicon lenses. A spot with high irradiance is formed when the distance $z'$ less than 20 mm for the axicon lens with base angle $\beta_1 = 5^\circ$. This is because most of the incident laser energy is focused into the optical axis of the axicon lens over a certain distance of the optical axis. Axicon lenses with small base angles have longer focused line over which high irradiance spots are generated. To maximize the laser energy in the annular region, the second axicon lens should be placed after this region of focused line during optical trepanning. Axicon lenses with relatively large base angles should be chosen for compact optical trepanning system. It should be noted that the high irradiance spots on the optical axis may be the spots of Arago. The apparition of the bright line along the axis could also be due to the spherical aberration of the system. This bright spot is referred to as Arago’s spot in this study although, historically, Arago’s spot
appears at the region of the geometrical shadow of an obstacle when light is diffracted by the obstacle [Harvey and Forgham (1984)].

Figure 5.3 A small part of the transverse cross section of a large annular beam showing radial variation of the irradiance profile after passing through the first axicon lens. $z' = 71$ mm, $r_w = 0.327$ mm.
Figure 5.4 Variations of the theoretical laser irradiance distributions $I(0, z)$ along the $z$-axis ($r = 0$) after the first axicon lens for different base angles, to check where the spot of Arago disappears (point $A^*$ in this Fig.) so that the second axicon lens can be placed there. $r_w = 0.327$ mm and $I_0 = 1$ W/mm$^2$. 

A'----Spot of Arago disappear at this point
5.3.2. Laser irradiance distributions after the second axicon lens

Fig. 5.5 shows the annular profile after the beam passes through the second axicon lens. As in the case of the first axicon lens, several inner diffraction rings and a main outer ring with maximum irradiance are found by the second axicon lens as well. The thickness of the main outer ring is much larger than that of the ring obtained after the first axicon lens. On the annular cross-sectional plane of the laser beam, both the experimental and theoretical irradiance profiles of the main ring exhibit a Gaussian-like profile (i.e., a Gaussian-like beam with maximum irradiance located at the center of the annulus).

Fig. 5.6 shows the theoretical development of the annular beam after the second axicon lens. The width of the annulus increases and the maximum value of the irradiance decreases with the increase of distance $Z'$ between the first and the second axicon lenses. However, the radius of the center of the annulus remains almost the same as the distance $Z'$ increases from 5 mm to 155 mm based on the collimation of the laser beam after the second axicon.
Figure 5.5 A small section of a large annular beam showing radial variation of the irradiance profile after passing through the second axicon lens. $r_w = 0.327$ mm, $z' = 95$ mm and $Z' = 50$ mm.
Figure 5.6 Propagation of an annular beam after passing through the second axicon lens, showing the variation of the irradiance in the radial direction. $r_w = 0.327$ mm, $z' = 95$ mm and $Z' = 110$ mm.
5.3.3. Development of imperfect annular beams due to imperfect axicons

and the effect of focusing lens on the irradiance profiles

The laser irradiance distributions after the focusing lens are investigated in this section. The blunt tip of an axicon lens produces an imperfect annular beam, i.e., an annulus with multiple diffraction rings as illustrated in Fig. 5.7. Such rings are absent in the theoretical result because the model is based on a perfect axicon lens, i.e., an axicon lens with pointed vertices, resulting in a perfect annular beam. The imperfection of the annular beam, i.e., the presence of multiple rings in the annulus, generates Arago’s spot over a certain axial length around the focal point although the theory predicts Arago’s spot only on the focal plane as sketched in Fig. 5.7. The imperfect annular beam, i.e., an annulus with multiple diffraction rings, reappears after the focal plane in the experimental results; whereas a perfect annular beam appears in the theoretical result because the incident beam on the focusing lens is a perfect annular beam. These three observations and the laser irradiance profiles before the focal plane, on the focal plane and after the focal plane are analyzed in Figs. 5.8, 5.9 and 5.10 respectively.

Fig. 5.8a shows the annular profile after the beam passes through the focusing lens. Here again, Arago’s spot appears after the focusing lens due to the diffraction effect, even when the incident collimated annular beam does not contain any energy on the axis of the beam. Several diffraction rings disappear downstream after being converged by the focusing lens, leaving only one main outer ring with maximum irradiance. Figs. 5.8b and 5.8c show the laser irradiance profiles along both the longitudinal direction and
transverse plane of the annular beam. On the transverse plane, the irradiance profile is a Gaussian-like distribution. The outer radius $R_o$ of the annular beam is defined as the distance between the center of the annular beam to the point where the irradiance of the laser beam is $1/e^2$ of the maximum irradiance $I_0$. A laser beam analyzer was used to plot the irradiance profile of the annular beam on a computer screen where the irradiance point corresponding to $1/e^2$ of the maximum irradiance $I_0$ can be selected on the radial axis and the radius of this point can be determined using data acquisition software. The value of this radius is taken as the outer radius $R_o$.

Optical trepanning is expected to provide more flexibility in affecting the hole quality than traditional circular beam laser drilling, since an annular beam enables shaping of the laser irradiance profile to supply laser energy to the workpiece in a variety of way. For percussion drilling, the irradiance profiles are usually either Gaussian or uniform. In optical trepanning, the geometry of the hole taper can be modified, i.e., convergent or divergent holes can be produced using different types of irradiance profiles. Different profiles also can have important effects on the recast layer thickness, heat-affected zone, (HAZ) and drilling speed. Fig. 5.8 shows a Gaussian-like annular beam generated from an input Gaussian beam. Annular beams with various irradiance profiles can be obtained using axicon lenses with different curvatures of the conic surface.

It should be noted that the alignment of the optical elements is critical to obtain a good quality annular beam. The elements must be illuminated at exactly normal incidence with coincident optical and laser beam axes. Even a slightly oblique incidence will change the
quality of the annular beam and affect the presence or disappearance of Arago’s spot [Thaning et al. (2003); Arimoto et al. (1992)]. The measured non-uniformly wide annular beam shown in Fig. 5.8 is mainly due to the imperfect alignment of the axicon lens system. Also the incident laser beam must be perfectly circular to obtain a good quality annular beam.

Experimental results show higher irradiance in Arago’s spot and smaller beam radius than the corresponding theoretical results. This may be because the larger rings, which were considered in the theoretical calculation as the beam passes through the second axicon lens, do not affect the performance of the laser beam analyzer significantly owing to the insensitivity of the instrument to the low energy content of the larger rings. The experimental annular ring was found to contain several narrow rings in the annulus before the focusing lens. When such multiple rings of the annulus are focused, some of the rings may be diffracted to the central spot. This creates Arago’s spot with high irradiance in a region around the focal plane. According to the theoretical model, the annular beam is made of a single wide ring before the focusing lens. The focusing lens causes diffraction of this ring and produces Arago’s spot only on the focal plane.

Fig. 5.9 shows the theoretical diffraction pattern at the focal plane for an annular beam. The annular ring disappears and a bright circular spot with very high irradiance appears at the laser beam center. In experiments, however, multiple diffraction rings are produced in the annulus after the second axicon lens due to the several reasons as mentioned earlier. These rings disappear after being converged by the focusing lens. Several
diffraction rings, however, reappear after the focal plane due to the divergence of the beam as shown in Fig. 5.10.

Figure 5.7 Development of diffraction patterns along axial direction in annular beam shaping with a refractive axicon system. (a) Experimental diffraction patterns produced by an imperfect refractive axicon system. An imperfect axicon has a blunt vertex. (b) Theoretical diffraction patterns produced by a perfect refractive axicon system. A perfect axicon has a pointed vertex.
Theoretical results
Experimental results

Laser Irradiance, $I/I_{\text{max}}$

X (mm)
Figure 5.8. Irradiance distributions of an annular beam before the focal plane at $L = 47$ mm for a convex lens of focal length $f_l = 50$ mm. (a) Transverse cross section of the annular beam profile. (b) Beam profile in the $X$–direction on the transverse plane. (c) Beam profile in the $Y$–direction on the transverse plane. $R_o = 0.665$ mm, $r_w = 0.327$ mm, $z' = 95$ mm and $Z' = 110$ mm.
Figure 5.9 Irradiance distributions along the radial direction at the focal plane. $f_i = 50$ mm, $r_w = 0.327$ mm, $z' = 95$ mm and $Z' = 110$ mm.
Experimental results

Theoretical results

(a)

(b)
Figure 5.10 Irradiance distributions of the annular beam at 2 mm after the focal plane (i.e., at $L = 52$ mm). (a) Transverse cross section of the annular beam profile. (b) Beam profile in the $X$–direction on the transverse plane. (c) Beam profile in the $Y$–direction on the transverse plane. $f_i = 50$ mm, $L = 52$mm, $r_w = 0.327$ mm, $z' = 95$ mm and $Z' = 110$ mm.
5.3.4. Effect of focal length on the beam radius

Figs. 5.11 show the variation of the outer radius of the annular beam with distance $L$ between the focusing lens and the observation plane. Annular beams of different diameters are obtained by adjusting $L$. For example, the outer radius of the annular beam is $R_o = 1.02 \text{ mm}$ for $L = 46 \text{ mm}$, and $R_o = 85 \mu\text{m}$ for $L = 49 \text{ mm}$. The importance of this capability is that it provides tremendous flexibility to trepan holes having outer radius variations over an order of magnitude using a single optical system. Since the outer radius is a very sensitive function of $L$, this capability also enables drilling tapered holes by systematically varying $L$ using a servo-system during the drilling process.

Fig. 5.11a shows that the theoretical and experimental values of $R_{oa}$ do not match for the nominal focal length $f_i = 50 \text{ mm}$. The discrepancy could be due to the manufacturing tolerance in the focal length of the convex lens. This type of focal shift is also observed in diffracted converging spherical waves [Li and Wolf (1981)], where the focal shift due to this effect is given by $\Delta f_i = -f_i/(1 + \pi^2 N^2)$ and $N = a^2 / \lambda f_i$ for a Gaussian beam. $\Delta f_i = 1.337 \mu\text{m}$ for a typical case with $a = 14.2 \text{ mm}$, $\lambda = 1.064 \mu\text{m}$ and $f_i = 50 \text{ mm}$ in this study.

On the other hand, the specification on the focal length of the focusing lens was $50 \text{ mm} \pm 2\%$ due to the manufacturing tolerance. So theoretical calculations were carried out to determine the annular beam radius $R_{oa}$ for different values of the focal length $f_i$. The values of $R_{oa}$ were found to be closer to the experimental results for $f_i = 49.3$ than for $f_i = 49.5$ and $50 \text{ mm}$. The discrepancy between the experimental and theoretical results may be due to spherical aberration, which is the most important of all primary aberrations,
caused by different focal positions for marginal meridional and paraxial rays [Malacara et al. (1994)]. For lenses with spherical surfaces, the rays that are parallel to the optical axis but at different distances from the axis fail to converge to the same point after passing through the lens. This effect changes the annular profiles and the corresponding diffraction patterns. According to the theoretical model, the minimum diameter of the annular beam is $R_{oa} = 90 \, \mu m$ for an axicon system with $r_w = 0.327 \, \text{mm}$, $z' = 95 \, \text{mm}$, $Z' = 110 \, \text{mm}$ and $f_i = 49.3 \, \text{mm}$. This value of $R_{oa}$ is very close to the experimental radius of 85 $\mu m$ (Fig. 5.11b).

Smaller radius annular beams can be achieved with lasers having shorter wavelengths. Increasing the diameter of the input laser beam or decreasing the focal length of the focusing lens can also generate smaller radius annular beams. The annular beam radius decreases as the plane of observation approaches the focal plane of the focusing lens. However, more and more laser energy accumulates on the optical axis near the focal plane, which limits the minimum annular beam size.
Figure 5.11. Outer diameter of an annular beam at different axial locations after passing through a convex lens. \( r_w = 0.327 \text{ mm}, z' = 95 \text{ mm}, Z' = 110 \text{ mm} \) and \( f_i = 50 \text{ mm}, 49.5 \text{ mm} \) and 45.3 mm.
Figure 5.12 Outer diameter of an annular beam at different axial locations after passing through a convex lens (expanded scale). $r_w = 0.327$ mm, $z' = 95$ mm, $Z' = 110$ mm and $f_i = 50$ mm, 49.5 mm and 49.3 mm.
5.4. **Focal Shifts for a Converging Annular Beam**

When a monochromatic, uniform, converging spherical wave is diffracted at a circular aperture in an opaque screen, the point of maximum intensity of the diffractive wave is usually considered to be located at the geometrical focus. However in fact, the waist of the focusing beam is in the plane of the lens and is much smaller than the linear size of the lens when a monochromatic Gaussian beam is focused by a thin lens [Li, Y. and Wolf, E. (1981)]. The point of maximum intensity is not at the geometrical focus but is somewhat closer to the lens. We refer to this effect as the focal shift. In this section, we consider a case when an annular beam is focused by a thin lens. To simplify the calculation, the waist of the annular beam is assumed to be the plane of the lens, and the input annular beam is considered as a collimated full Gaussian annular beam. Neglecting the constant phase factor \( \exp(i k n \delta_3) \) [Goodman (1996)], the diffraction field \( U_3(\eta, L) \) at a distance \( L \) from the focusing lens tip \( O_3 \) along the optical axis can be written as follows in polar coordinates for rotationally symmetric systems by the Fresnel diffraction integral [Goodman(1996)]

\[
U_3(\eta, L) = \frac{k^* \exp(i k^* L)}{i L} \int_0^{R_w} \exp \left( -\frac{(\rho - R_{ca})^2}{2 R^2} \right) \exp \left( -\frac{i k^* \rho^2}{2 f_i} \right) J_1 \left( -\frac{k^* \rho \eta}{L} \right) \rho d\rho.
\]

\[\text{[5.13]}\]

Where, \( R_w \) is the characteristic width of the full Gaussian annular beam, \( R_{ca} \) is the radial distance of the point of maximum intensity \( I_0 \) from the center of the annulus. The diffraction field \( U_3 \) at the optics axis can be written as follows

\[
U_3(0, L) = \frac{k^* \exp(i k^* L)}{i L} \int_0^{R_w} \exp \left( -\frac{(\rho - R_{ca})^2}{2 R^2} \right) \exp \left( -\frac{i k^* \rho^2}{2 L} \left( \frac{1}{L} - \frac{1}{f_i} \right) \right) \rho d\rho.
\]

\[\text{[5.14]}\]
Its solution is given by

\[
U_3(0, L) = \frac{2\sqrt{-A} \left[ \exp(\rho_1^2 A + \rho_1 B) - 1 \right] + B\sqrt{\pi} \exp\left( \frac{-B^2}{4A} \right) \left\{ \text{erf}\left( \frac{2\rho_1 A + B}{2\sqrt{-A}} \right) - \text{erf}\left( \frac{B}{2\sqrt{-A}} \right) \right\}}{4A\sqrt{-A}},
\]

\[
\times \exp \left[ -\frac{R_w^2}{R_w^2} \right] \frac{k^* \exp(ik^* L)}{iL},
\]

[5.15]

where \( A = \left[ \frac{1}{R_w^2} - \frac{i k^*}{2} \left( \frac{1}{L} - \frac{1}{f_i} \right) \right], \quad B = \frac{2R_w^2}{R_w^2}\).

The irradiance distribution of the diffracted wave is defined by

\[
I(0, L) = U_3(0, L) \cdot \overline{U_3(0, L)}.
\]

[5.16]

Figs. 5.13 and 5.14 show the normalized intensity against the axial distance \(L\) along the optical axis for a converging full Gaussian annular beam and a converging Gaussian circular beam. When the wavelength of the input laser beam \(\lambda = 1064\) nm and the focal length of the focusing lens \(f_i = 50\) mm, there are no obvious effects for both circular and annular beams as shown in Fig. 5.13. As the increase of the focal length, the focal shift becomes obvious as shown in Fig. 5.14 for \(f_i = 200\) mm. However, in our study, compared with a converging Gaussian beam, focal shifts of a converging annular beam can be ignored.
Figure 5.13 The normalized intensity against the axial distance $L$ along the optical axis for a converging full Gaussian annular beam and a converging Gaussian circular beam. For the full Gaussian annular beam, $R_{ca} = 13.5$ mm, $R_w = 0.52$ mm, $\rho_1 = 50$ mm, $\lambda = 1064$ nm and $f_l = 50$ mm. For the Gaussian circular beam, $r_w = 0.52$ mm, $\rho_1 = 50$ mm, $\lambda = 1064$ nm and $f_l = 50$ mm.
Figure 5.14 The normalized intensity against the axial distance $L$ along the optical axis for a converging full Gaussian annular beam and a converging Gaussian circular beam. For the full Gaussian annular beam, $R_{ca} = 13.5$ mm, $R_w = 0.52$ mm, $\rho_1 = 50$ mm, $\lambda = 1064$ nm and $f_l = 200$ mm. For the Gaussian circular beam, $r_w = 0.52$ mm, $\rho_1 = 50$ mm, $\lambda = 1064$ nm and $f_l = 200$ mm.
5.5. **Diffraction Limits for a Converging Annular Beam**

Optical trepanning will be applied to drill the hole for different materials. It is very important to know the minimum achieved size of the annular beam for an optical trepanning system. In this section, the diffraction limits of a converging annular beam are calculated by using the Fresnel approximation. To simplify the calculation, we assume an ideal full Gaussian annular beam converged by a focusing lens and the waist of the annular beam is the plane of the focusing lens. Therefore, the laser irradiance profiles after the focusing lens are calculated by Eq. 5.13.

Figs. 5.15 and 5.16 show the variations of the achieved minimum annular beam with different parameters. Those parameters include the focal length of the focusing lens $f$, the size parameters of the full Gaussian annular beam, $R_w$ and $R_{ca}$. Smaller annular beams can be obtained by increasing the width of the input annular beam $R_w$ as shown in Fig. 5.15. For example, the minimum outer radius of the annular beam $R_{oa}$ is 190 $\mu$m if $R_w = 0.5$ mm and $f_i = 100$ mm, it can be reduced to 46 $\mu$m if $R_w = 2.5$ mm and $f_i = 50$ mm. Smaller annular beam can also be achieved by using the focusing lens with the shorter focal length. For example, the minimum outer radius of the annular beam $R_{oa}$ can be reduced to 20 $\mu$m if $R_w = 2.5$ mm and $f_i = 50$ mm. The diffraction limits of the converging annular beam are almost same with variation of $R_{ca}$ as shown in Fig. 5.16. However, the converging rate of an annular beam after the focusing lens increase as the increase of $R_{ca}$, which will be discussed in the next section.
Figure 5.15 Variations of the diffraction limits of the converging annular beam. Where \( R_{ca} = 10 \text{ mm} \) and \( \lambda = 1064 \text{ nm} \).
Figure 5.16 Variations of the diffraction limits of the converging annular beam. Where $R_w = 1.5 \text{ mm}$ and $\lambda = 1064 \text{ nm}$.
5.6. **Converging Profiles of an Annular Beam**

The refractive axicon system provides flexible way for adjustments of the size of annular beams. The size of the annular beam is varied with the distance $h_3$ between the focusing lens and the substrate surface. The converging profiles of annular beams along optical axis near the focal plane are calculated using Eq. 5.13.

The converging rates of annular beams along the optical axial direction are varied with the beam parameters $R_w$ and $R_{ca}$. The converging rate of annular beam will increase as the increase of the annular beam parameter $R_w$ shown in Fig. 5.17. Large converging rate of annular beams along the optical axial direction can also be achieved as the increase of the annular beam parameter $R_{ca}$. However, the converging rate of annular beams can be modified by using the focusing lens with the different focal length. Compared with the Fig. 5.17 and 5.18, the optical system with the longer focal length ($f_i = 150$ mm) have lower converging rate than the optical system with the shorter focal length ($f_i = 50$ mm).
Figure 5.17 Variations of the outer radius of the annular beam along the optical axial distance. Where $f_t = 50$ mm and $\lambda = 1064$ nm.
Figure 5.18 Variations of the outer radius of the annular beam along the optical axial distance. Where $f_i = 150$ mm and $\lambda = 1064$ nm.
5.7. **Conclusions**

An axicon refractive system has been designed to convert an input Gaussian beam into an annular beam. The diffraction patterns generated by different optical elements of the system are investigated by using the Fresnel diffraction integral. The laser irradiance profiles produced by the system were measured using a laser beam analyzer. The following conclusions can be drawn based on the numerical solutions and measured annular profiles:

- A Gaussian-like annular beam can be produced with an axicon refractive system. The annular beam diameter decreases as the plane of observation approaches the focal plane of the focusing lens. However, the minimum value of the annular beam diameter is limited by the diffraction effect.

- A thin outer annulus with maximum irradiance and several inner diffraction rings with lesser irradiance were found to form due to diffraction.

- More and more laser energy accumulates on the optical axis of the axicon lens system over a small region around the focal plane. The annular ring disappears at the focal plane and reappears after the focal plane.

- The optical elements need to be aligned to generate radically symmetric irradiance profiles, which is important for drilling circular holes. Optical trepanning can be applied to drill small holes with low power lasers since relatively high irradiance can be obtained with small area of the annular beam. Large area of the annular beam reduces the irradiance for a given laser power. Therefore, high power lasers would be necessary for optical trepanning of large holes in thick samples.
CHAPTER 6: EXPERIMENTAL STUDIES OF OPTICAL TREPANNING

6.1. Experimental Setup

Figure 6.1 displays the experimental setup used in optical trepanning experiments. Lee Series 800 Nd:YAG laser (1064 nm) was the input laser source. The Lee Laser Q-switching system offers considerable flexibility for external (remote) control of Q-switch operation from a customer-supplied computer. With a pulse generator and a function generator, the laser pulse shape, the pulse repetition rate and the drilling time can be dictated. The laser pulse shape and the pulse repetition rate were captured by a Silicon photodetector. Experimental parameters also include the laser average power and the laser beam profiles. Average power was measured using a high power detector and a power meter, with a manufacturer quoted accuracy of ±2.5%. The minimum measured average power for the power detector is 60 mw. The laser beam profiles along this optical trepanning system were characterized by a LaserCam-II 1/4" CCD camera (4.7 × 5.5 µm pixel size) and an analog beamview analyzer system. The instruments used in these investigations were:

- Lee Laser Series 800 Nd:YAG (λ = 1064 nm) Q-Switched laser system
- Electro-Optic Technologies Silicon Photodetector (Model ET-2020)
- SRS 4-Channel Digital Delay/Pulse Generator (Model DG535)
- BK Precision Function Generator (Model 4040)
- High power detector (Model 818P-250-25)
- Hand-Held Optical Power and Energy Meter (Model 841-PE)
- Tektronics Two Channel Digital Oscilloscope
- LaserCam-II 1/4” CCD Camera and Analog Beamview Analyzer System
- Optical Trepanssion System

Figure 6.1 Schematic experimental setup for optical trepanning
The Nd:YAG Lee Laser System is designed for use in industrial contexts. To avoid Q-switch triggering by electrical interference from other systems, a robust input signal greater than +5 Volts is required to trigger the Q-switch power supply. Laser drilling time was dictated by the SRS Pulse Generator. A TTL pulse of the required drilling time was sent to the BK Precision Function generator. The TTL pulse controls the burst input of the function generator, specifying the time that the function generator will output a signal. The frequency of the function generator output signal sets the Q-switch modulation frequency. Function generator output is split, with one output to the Ext. Var. BNC connector of the Q-switch power supply, and a second output to the Tektronics oscilloscope. Function generator output signal can also be fed to the Analog BNC connector of the Q-switch power supply to control Q-switch frequency. However, the millisecond response time of the Analog BNC compared to the nanosecond response time of the Ext. Var. BNC makes such a connection impractical for both pulse shaping and the current experiments with laser drilling.

6.1.1. Geometrical configuration of an optical trepanning system

A refractive axicon lens system was designed to construct an optical trepanning system as shown in Fig. 6.2. The input Gaussian beam (TEM\(_{00}\), Nd:YAG laser, \(\lambda = 1.064 \, \mu\text{m}\)) was incident on the flat surface of the first axicon lens with the beam axis coincident with the principal axis of the axicon lens. Both input and output axicon lenses are fused silica. The half-apex angle of the axicon lens \(\alpha = 72^\circ\), refractive index \(n = 1.45\), and base angle \(\beta = 18^\circ\), (formed by the conical surface and the flat surface of the axicon lens), and the
focal length of the focusing lens \( f_l = 50 \) mm. Where, \( h_1 \) was defined as the separation between the vertices of the two axicon lenses, \( h_2 \) was defined as the distance between the flat surface of the second axicon lens and the focusing lens of focal length \( f_l \) and \( h_3 \) was defined as the distance between the focusing lens and the substrate surface. An annular nozzle [Fieret, et al. (1987)] was designed to deliver the assist gas as shown in Fig. 6.3. Assist gas helps eject molten material from the cavity during the laser pulse and also prevent it from spattering on the optical components. The oxygen gas with high purity (minimum purity 99.994%) was used in this study.
Figure 6.2 Schematic representation of an optical trepanning system. $h_1$ is the distance between two axicon lenses, $h_2$ is the distance between the flat surface of the second axicon lens and the focusing lens, and $h_3$ is the distance between the focusing lens and the sample surface.
Figure 6.3 Photographs of the annular nozzle used for optical trepanning.
6.2. Etch Test for Optical Trepanning

The optical trepanning system will ultimately be used to trepan precision holes through a variety of materials. However, for diagnostic purposes, it is useful to perform a “surface etch”, involving the removal of only about 10 µm of material. In this way both the outer and inner diameters remain clearly evident, in contrast to trepanning a hole, where information regarding the inner diameter is lost. Since determining the inner diameter is necessary to assess the “width” of the annular irradiance distribution, it was decided to perform some experimental “etching” tests.

Fig. 6.4 shows a diagnostic etch pattern in Inconel 718 resulting from the prototype optical trepanning system measured using an OMIS II optical measurement and inspection system at Laser Fare. For this case the distance from the center of the focusing lens to the surface of the Inconel 718 plate was $Z = 48.3$ mm, the outer diameter was measured at $D_{oa} = 585$ µm, the inner diameter at $D_{ia} = 540$ µm, corresponding to an annular radial “width” $R_w = (D_{oa} - D_{ia}) / 2 = 22.5$ µm.

Fig. 6.5 shows a second, smaller diagnostic etch pattern. Here $Z = 48.6$ mm, the average outer diameter was $D_{oa} = 360$ µm, and the average inner diameter was $D_{ia} = 300$ µm, corresponding to $R_w = (D_{oa} - D_{ia}) / 2 = 30$ µm. Note that in this case as the outer diameter was reduced, the annular radial width actually increased. Also, the annulus is not perfectly circular. This is likely due to tiny misalignment errors.
Figure 6.4 Micrograph of the etch pattern on the workpiece surface due to annular beam heating. $h_1 = 95$ mm, $h_2 = 110$ mm, $f_i = 50$ mm and $Z = 48.3$ mm.

Figure 6.5 Micrograph of the melt etch pattern on the workpiece surface due to annular beam heating. $h_1 = 95$ mm, $h_2 = 110$ mm, $f_i = 50$ mm and $Z = 48.6$ mm.
Finally, Fig. 6.6 shows a third diagnostic etch pattern. For this case $Z = 49.2$ mm, the average outer diameter was $D_{oa} = 172$ µm, the average inner diameter was $D_{ia} = 120$ µm, corresponding to $R_w = (D_{oa} - D_{ia}) / 2 = 26$ µm. Again, the annulus is not perfectly circular. This is most likely due to small optical misalignment errors.

The demands on alignment for the refractive axicon lens system are very high. To obtain an annular beam with good quality, the axicon must be illuminated strictly perpendicularly and the optical axes of all these lenses should be overlapped. Even a light inclination of the incident will change the quality of the annular beam and the spot of the Argo. The effects become more obvious when focusing to the small annular beam as shown in Figs. 6.5 and 6.6.

Furthermore, as shown in Figs. 6.4 - 6.6, the outer diameter of the melt spot increases as the distance, $Z$, between the axicon lens and the target plane, decreases. Figs. 6.4 - 6.6 show that the outer annular diameter is a very sensitive function of $Z$, with a change in $Z$ from 49.2 mm to 48.3 mm, resulting in a change of the outer annular diameter from 172 µm to 585 µm. This is because that the annular beams have been expanded to the relatively large size ring beam before the focusing lens. These results indicate that both the distance $h_1$ between two axicon lenses, and especially the distance $Z$, between the focusing lens and the target plane are critical parameters for the annular beam size.
Figure 6.6 Micrograph of the etch pattern on the workpiece surface due to annular beam heating. \( h_1 = 95 \text{ mm}, \ h_2 = 110 \text{ mm}, \ f_i = 50 \text{ mm} \) and \( Z = 49.2 \text{ mm} \).

Figure 6.7 Micrograph of the etch pattern of the annulus centre on the workpiece surface. \( h_1 = 95 \text{ mm}, \ h_2 = 110 \text{ mm}, \ f_i = 50 \text{ mm} \) and \( Z = 48.6 \text{ mm} \).
Fig. 6.7 shows the expand scale of the micrograph of the etch pattern due to heating of the spot of Argo on the workpiece surface. The spot of Argo is found to be an asteroidal shape as the result of the oblique illumination on the axicon lenses [Thaning et al. (2003); Arimoto et al. (1992)]. It should be noted that the central melt marker is due to heating of the spot of Argo. The spot of Argo with the circular shape should be generated if the axicon lenses are illuminated without any inclination of the incident light, i.e., the whole system of the refractive axicon lens is in perfect alignment.

6.3. Experimental Procedure

Optical trepanning experiments are performed in Stainless Steel-316 (SS 316) and Inconel 718 (IN 718). Metal samples are prepared by polishing the material surface with 320 grit sand paper and cleaning metal dust by light swabbing with an alcohol solution. The thickness of SS 316 and IN 718 sample are 104 µm and 632 µm respectively.

The pulse repetition rates of the laser beam were varied from 1.5 kHz ~ 10 kHz, measured by the silicon photodetector. Laser output average powers were varied from 2 W ~ 10 W. Oxygen was used as an assist gas, with a flow rate of 127 m/s. Separation between the annular nozzle and the substrate surface was 2 mm.

Beam profile measurements were measured with a CCD camera (Laser Cam II, 4.7 × 5.5 µm pixel size). Difference laser beam profiles are obtained by using different spatial filters inside the laser cavity. Spatial filters are usually designed for use either inside the
laser cavity as mode selectors, or outside the laser cavity to clean up the outer fringes of a propagating laser beam. In this study, the spatial filters are put inside the laser cavity as mode selectors.

Three different laser irradiance profiles are obtained by using 1.8 mm, 2.0 mm and 3.0 mm spatial filters inside the laser cavity respectively. Figs. 6.8 and 6.9 display the top view of the laser beam irradiances when $D_s = 3.0$ mm and $D_s = 2.0$ mm spatial filters are used. These multimode lasers are not strictly symmetry and can cause complex diffraction patterns along the refractive axicon system. Only some part of laser energy can be effectively focused on the periphery of the annulus. For obtaining a nice and clean output annular beam, the $D_s = 1.8$ mm spatial filter is used during optical trepanning to obtain a TEM$_{00}$ Gaussian laser beam shown in Fig. 6.10.
Figure 6.8 Top view of irradiance profiles for a multimode laser beam, where $D_s = 3.0$ mm and $P_a = 35$ W.
Figure 6.9  Top view of irradiance profiles for a multimode laser beam. (a) $D_s = 2.0$ mm and $P_a = 10$ W. (b) $D_s = 2.0$ mm and $P_a = 25$ W.
Figure 6.10 (a) Top view of irradiance profiles for a Gaussian beam. (b) 3-D view of the irradiance profile for a Gaussian beam. Where $D_s = 1.8$ mm, $P_a = 3$ W and $r_0 = 0.806$ mm.
6.4. Optical Trepanning

6.4.1. Geometry of drilling holes by optical trepanning

Fig. 6.11 shows the typical micrographs of a through-hole by optical trepanning in Stainless Steel-316. When an annular laser beam is focused on the substrate surface, the material around the periphery of the annulus is heated, melted, vaporized and removed, which trepans a through-hole and forms a central drop-out disk as shown in Figs. 6.11(c) and 6.11(d). The heat affect zone is found around the periphery of the annulus as shown in Figs. 6.11(a) and 6.11(b). This is because that most of laser energy is focused on the periphery of the annulus during the optical trepanning and some laser energy is transferred outwards the outer circumference from the annulus due to the heat conduction. The recast layer is formed around the periphery of the annulus. Melting and vaporization are also found on the centre of the central drop-out disk shown in Fig. 6.11. This is because the laser intensity of the centre spot is very high. The centre spot, which we referred as the spot of Arago in this study, although, historically, Arago’s spot appears at the region of the geometrical shadow of an obstacle when light is diffracted by the obstacle.
Figure 6.11 Micrograph of a through-hole by optical trepanning on the substrate SS 316. 
(a) Micrograph of the entrance (top) surface of a through-hole. (b) Micrograph of the exit (bottom) surface of a through-hole. (c) Micrograph of the top surface of the central drop-out disk. (d) Micrograph of the bottom surface of the central drop-out disk. Where $h_1 = 120$ mm, $h_2 = 70$ mm, $h_3 = 47$ mm, $f_i = 50$ mm, $f = 2$ kHz and $P_a = 5$ W.
Fig 6.12 shows the micrograph on the entrance surface of a drilling hole by optical trepanning in Stainless Steel-316. Melting and vaporization are found around the periphery of the annulus. However, those molten materials can not be effectively dragged out of the cavity by the assist gas with the low flow rate (35 m/s). Molten materials will be cooled down during laser pulse-off time and form the recast layer around the periphery of the annulus. The recast layer could weld the central drop-out disk as shown in Fig. 6.12. At the same time, more laser energy will be transferred from the circumference of the annulus to the centre disk due to the heat transfer, leading to more melting and vaporization on the centre.

The alignment is very important for a beam shaping system. The misalignment of optical elements could change the laser irradiance patterns on the target plane. For a refractive axicon lens system, low uniform annular beams or ellipse annular beams could be obtained as a result of the misalignment of the optical elements. Fig. 6.13 shows the micrograph on the entrance surface of a drilling hole by optical trepanning in stainless steel-316. Due to the misalignment of the optical elements, the annular beam is defocused on the substrate surface. It could generate the annular beam with very high intensity on one edge as shown in Fig. 6.13, which causes deeper drilling depth in one edge than others on the periphery of the annulus. The taper angle of drilling holes can be modified by changing the focusing length of focused lenses as shown in Fig. 6.14.
Figure 6.12  Micrograph of an optical trepanning hole on the entrance surface of the substrate SS 316. Where $h_1 = 120$ mm, $h_2 = 70$ mm, $h_3 = 47$ mm, $f_i = 50$ mm, $f = 2$ kHz and $P_a = 5$ W.

Figure 6.13  Micrograph of an optical trepanning hole on the entrance surface of the substrate SS 316 for the case of misalignment of the optical elements. Where $h_1 = 120$ mm, $h_2 = 70$ mm, $h_3 = 48$ mm, $f_i = 50$ mm, $f = 2$ kHz and $P_a = 5$ W.
Figure 6.14 Micrograph of the cross section of drilling holes by optical trepanning on the substrate IN 718. (a) Longer focal length for the focusing lens. Where, $h_l = 110 \text{ mm}$, $h_2 = 75 \text{ mm}$, $h_3 = 103.5 \text{ mm}$, $f_l = 120 \text{ mm}$, $f = 2 \text{ kHz}$ and $P_a = 8 \text{ W}$. (b) Shorter focal length for the focusing lens. Where, $h_l = 120 \text{ mm}$, $h_2 = 70 \text{ mm}$, $h_3 = 48 \text{ mm}$, $f_l = 50 \text{ mm}$, $f = 2 \text{ kHz}$ and $P_a = 8 \text{ W}$.
6.4.2. Variation of the size of the trepanning hole

The drilling hole’s diameter mainly depends on the size of the annular beam on the substrate surface. For the refractive axicon system, the ratio of the inner radius to the outer radius of the annular beam can be varied by adjusting the distance $h_2$ between two axicon lenses. And the annular beam diameter on the substrate surface can be varied by adjusting the distance $h_3$ between the focusing lens and the substrate surface. For example, as shown in Figs. 6.15, 6.16 and 6.17, through holes with 700 µm, 450 µm and 220 µm diameters are obtained for $h_3 = 46.5$ mm, 48 mm and 49 mm respectively.

The refractive axicon system provides a flexible way for adjustment of the size of annular beams. However, since the size of the annular beam is very sensitive on the distance $h_3$ between the focusing lens and the substrate surface, it will actually trepan a large taper hole rather than an expected zero taper hole by optical trepanning. To adjusting the taper angle of drilling holes, the size parameters $R_w$ and $R_{ca}$ of annular beams and the focal length $f_l$ of focusing lens should be optimized based on the manufacturing requirements.
Figure 6.15  Micrograph of a through-hole by optical trepanning on the substrate SS 316.

(a) Entrance (top) surface of a through-hole. (b) Exit (bottom) surface of a through-hole.

Where $h_1 = 120 \text{ mm}$, $h_2 = 70 \text{ mm}$, $h_3 = 46.5 \text{ mm}$, $f_i = 50 \text{ mm}$, $f = 2 \text{ kHz}$ and $P_a = 5 \text{ W}$. 
Figure 6.16  Micrograph of a through-hole by optical trepanning on the substrate SS 316
(a) Entrance (top) surface of a through-hole. (b) Exit (bottom) surface of a through-hole.

Where $h_1 = 120$ mm, $h_2 = 70$ mm, $h_3 = 48$ mm, $f_i = 50$ mm, $f = 2$ kHz and $P_a = 5$ W.
Figure 6.17  Micrograph of a through-hole by optical trepanning on the substrate SS 316
(a) Entrance (top) surface of a through-hole. (b) Exit (bottom) surface of a through-hole.

Where $h_1 = 120 \text{ mm}$, $h_2 = 70 \text{ mm}$, $h_3 = 49 \text{ mm}$, $f_i = 50 \text{ mm}$, $f = 2 \text{ kHz}$ and $P_a = 3 \text{ W}$. 
6.5. **Conclusions**

In this chapter, we constructed an optical trepanning system and performed the optical trepanning on the two different materials, IN 718 and SS 316. A through hole was trepanned by using the refractive axicon system and a central drop-out disk was found. This whole process is very similar to the mechanical trepanning. The refractive axicon system provides flexible way for adjustment of the size of annular beams. It can be used to drill the hole with different diameters. However, a tapering cavity was found due to the convergence of the annular beam after the focusing lens. The taper angle of drilling holes can be modified by variations of the focal length for the focusing lens. Imperfection of the input Gaussian laser beam, manufacturing tolerance of the optical elements or misalignment of optical system will cause the low uniformity of the irradiance of annular beams. It generates the annular beam with very high intensity on one edge, which causes deeper drilling depth in one edge than others on the periphery of the annulus.
CHAPTER 7: SUMMARY

7.1. Conclusions

Optical trepanning is very similar to the conventional trepanning, which we refer to as optical trepanning, does not involve any rotating optics or rotating workpiece. This study presents and constructs an optical trepanning system, and theoretically investigates the optical trepanning process, leading to the following conclusions:

(1) An axicon refractive system and a vaxicon refractive system can be designed to transform a Gaussian laser beam into a collimated annular beam of different intensity profiles. However, compared with an axicon lens, a vaxicon lens is much more difficult to machine due to its concave conical shape. The lens surface profiles can be calculated by considering the conservation of energy, constancy of optical path length and Snell’s law. An optical path length constant $F'$ is found to be an important parameter to determine the lens profile. For a specific lens system, however, the suitable lens profiles can be found by varying the central ray refraction angle. To simplify the lens fabrication procedure and achieve beam shaping through refractive optics, three constraints are listed and the influences of different central ray refraction angles on the lens profiles are presented.

(2) The diffraction patterns generated by different optical elements of the system are investigated by using the Fresnel diffraction integral. The laser irradiance profiles produced by the system were measured using a laser beam analyzer. A Gaussian-like annular beam can be produced with an axicon refractive system. The annular beam
diameter decreases as the plane of observation approaches the focal plane of the focusing lens. However, the minimum value of the annular beam diameter is limited by the diffraction effect.

(3) For a refractive axicon system, annulus with maximum irradiance and several inner diffraction rings with lesser irradiance were found to form due to diffraction. More and more laser energy accumulates on the optical axis of the axicon lens system over a small region around the focal plane. The annular ring disappears at the focal plane and reappears after the focal plane. The optical elements need to be aligned to generate radically symmetric irradiance profiles, which is important for drilling circular holes. Optical trepanning can be applied to drill small holes with low power lasers since relatively high irradiance can be obtained with small area of the annular beam. Large area of the annular beam reduces the irradiance for a given laser power. Therefore, high power lasers would be necessary for optical trepanning of large holes in thick samples.

(4) An analytic expression for the temperature distribution has been presented to examine the effects of annular laser beam heating for three types of intensity distribution in optical trepanning. It is found that the heating of the material around the center \((r = 0)\) of the annular laser spot on the surface of the substrate is minimal. The melting and vaporization of the material are expected to occur within the annulus leaving an unmelted piece at the center similar to what is observed in mechanical trepanning. Annular laser beams with outer half Gaussian intensity profile produce larger melt volume and deeper holes than the inner half Gaussian and outer half Gaussian beams for uniform pulse shapes, whereas annular laser beams with full Gaussian intensity profile produce smaller
melt volume than the other two intensity profiles of uniform pulse shape. Uniform pulse shapes are found to produce less tapered and deeper melt boundary than triangular pulses.

(5) An analytical two-dimensional model is developed for optical trepanning by taking accounts of conduction in the solid, vaporization and convection due to the melt flow caused by an assist gas. The effects of annular beam radius are significant in most cases. It significantly influences the drilling hole qualities due to the widening of the melt layer in the radial direction. For a fixed outer radius of an annular beam, thinner recast layer, smaller taper and higher drilling speed are obtained with the increase in the inner radius of the annular beam. By using different types of intensity profiles, the nature of the hole taper can be modified, i.e., convergent or divergent holes can be produced. Full Gaussian beam and inner half Gaussian beam generate convergent holes and outer half Gaussian beam produces divergent holes. This shows that annular beams can provide more flexibility in affecting the hole quality than traditional circular beams since an annular beam allows numerous irradiance profiles to supply laser energy to the workpiece. An increase in the laser intensity generates thicker recast layer, deeper cavity depth and larger taper. An increase in the laser pulse-on time generates thinner recast layer, deeper cavity depth and larger taper.

(6) We have constructed an optical trepanning system and performed the optical trepanning on the two different materials, IN 718 and SS 316. A through hole was trepanned by using the refractive axicon system and a central drop-out disk was found. This whole process is very similar to the mechanical trepanning. The refractive axicon system provides flexible way for adjustment of the size of annular beams. It can be used to drill the hole with different diameters.
7.2. **Future Work**

We have designed and constructed an optical trepanning system by using refractive axicon lenses. The refractive axicon system provides flexible way for adjustment of the size of annular beams. However, since the size of the annular beam is very sensitive on the distance between the focusing lens and the substrate surface, it will actually trepan a large taper hole rather than an expected zero taper hole by optical trepanning. Further investigation should involve how to optimize experimental parameters and design the optical arrangements to reduce the converging rate of the annular beam near the focal plane.

The uniformity of the irradiance of annular beam for different orientations is not very high. This may produce, in a real case environment, some irregularities in the quality of the edge of the trepanned hole. The low uniformity of the irradiance of annular beam is probably due to the impurity of the input Gaussian circular laser beam and the manufacturing tolerance of optical elements. Future work should investigate the effects of the manufacturing tolerance of the optical elements and evaluate the allowed manufacturing tolerance for optical trepanning.

We have conducted diffraction analysis for the optical trepanning system. However, we didn’t take the spherical aberration effects into account for this analysis. The spherical aberration effects could change the diffraction patterns of the laser beam near the focal plane and may defocus the annular beam. Aspherical lenses can be used to reduce the
aberration effects of converging annular laser beams on the near focal plane. The imperfection of the axicon tip is part of the possible reasons for the deviations between the theoretical and experimental diffraction patterns of the annular beam. The tip of axicon lens could be modeled as having a rounded tip. Then, the phase function would have two regions: one of them already modeled in the study and a second one modeled as a spherical surface.

The axicon and vaxicon refractive systems have been designed to transform a Gaussian laser beam into a collimated annular beam of different intensity profiles. Since the commercial axicon lens is available and much easy to be manufactured, the optical arrangements can be designed by using the commercial axicon lens to obtain the annular beam. However, there are some differences between the lens profiles of the commercial axicon and the theoretical axicon. In order to obtain the annular beam with required intensity profiles, shaping optical elements can be designed either before or after the commercial refractive axicon system.

To improve laser drilling speed and efficiency, it is very necessary to design an optical trepanning system with the output of multiply annular beams rather than single annular beam. To obtain the multiple annular beams, one way is the use of the beam splitters to split one laser beam into multiple beams. The other way is the use of diffraction optical elements (DOE) in the path of the laser beam to produce an array of micro-hole.
REFERENCES


