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Development of High-power Single-mode Yb-doped Fiber Amplifiers and Beam Analysis

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DEVELOPMENT OF HIGH-POWER SINGLE-MODE YB-DOPED FIBER AMPLIFIERS
AND BEAM ANALYSIS

by

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ABSTRACT

High-power fiber laser systems enjoy a widespread use in manufacturing, medical, and defense applications as well as scientific research, due to their remarkable power scalability, high electrical to optical efficiency, compactness and ruggedness. However, single-mode fiber power scaling has stagnated in the past years, primarily due to the onset of nonlinear effects such as stimulated Brillouin/Raman scattering and transverse modal instabilities.

This thesis addresses the analysis and mitigation of transverse modal instabilities in high-power fiber amplifiers. I describe the high-power fiber amplifier testbed that I set up to test fibers fabricated in house. I will show our results of a Yb-doped fiber amplifier with more than 2.2 kW signal power and beam quality of 1.1 M2. In consequence, I demonstrate mode-selective amplification in a large mode-area Yb-doped fiber using a 3-mode photonic lantern. All three modes were amplified to above 4 W with OSNRs higher than 16 dB. In addition, I show a novel high-speed beam analysis technique to study transverse modal instabilities. To guide fiber designs, I developed a GPU accelerated simulation suite to study the dynamics that occur in high-power fiber amplifiers. A $64 \times 64$ spatial grid, with 6000 time- and 20000 distance-steps can be solved at $12 \text{ min} \cdot \text{m}^{-1}$ on a GeForce GTX 1080 Ti. Based on these simulations, I will show dynamic transverse modal instability mitigation strategies that rely on mode modulation.
To those fighting against SARS-CoV-2.
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CHAPTER 1: INTRODUCTION

Undoubtedly, one of the greatest innovations in laser science has been the development of Yb-doped fiber lasers that in turn led to unprecedented advances in various defense, industrial and research settings. Throughout history, every mastery of energy source by mankind, tremendously impacted the quality of life. From mechanical energy, thermal, electric, and nuclear we finally reached optical energy in 1960, when Maiman demonstrated the first laser and its unique features such as: high intensity, directionality, monochromaticity and coherence. In the decades following the invention of the laser, the solution to a unknown problem has transformed itself into an enabling key technology in the sectors of bio-tech health-care, telecommunications and transportation. From the advent of the laser, the prospect of highly directed energy transfer immediately resulted in studies of lasers for material processing applications and was incorporated into industrial machines for cutting and drilling by the 70s [1]. What makes fiber laser systems so successful, is their remarkable power scalability, high electrical to optical efficiency, compactness and ruggedness, their simplified thermal management and their most striking characteristic: brightness enhancement. Dramatic brightness enhancement is made possible through a pump guided by a secondary cladding of lower refractive index, while the signal is guided in the core [2]. In such systems, the amplified signal may exhibit a three to four orders of magnitude higher brightness than the pump itself. Asymmetric fiber geometries further increase the overlap between pump and core and hence the pump absorption along the fiber. Today, single-moded diode pumped amplifiers have reached nearly diffraction-limited output powers of 5 kW [3, 4]. Further single-moded power scaling to 10 kW was possible with tandem-pumping, which reduces the heat load through the naturally smaller quantum defect. Incoherent and coherent beam combing have allowed to reach powers of up to 100 kW [5, 6] and the trend of coherent beam combing promises yet higher powers with applications in directed energy. However, single-mode single-fiber power scaling has
stagnated in the past years, primarily due to the onset of nonlinear effects such as SBS/SRS and transverse modal instabilities (TMI) [7]. TMI occurs as a sudden degradation of beam quality at high average powers. The effect itself has been observed experimentally for different fiber geometries, at different IR wavelengths, pumping and seeding conditions and fiber arrangements (see the review by Zervas [8] and Jauregui et al. [9]). At this point it is still unclear whether there are viable pathways to not only mitigate but completely stop the detrimental effects of TMI from occurring.

This thesis addresses the analysis and mitigation of transverse modal instabilities in high-power fiber amplifiers. In the first chapter I introduce the theoretical backbone such as fiber amplifiers, transverse modal instabilities and the solution of the heat equation. In sequence, in the second chapter, I lay out the GPU acceleration of a dynamic BPM model to simulate TMI effects. GPU acceleration enables fast TMI threshold estimation of fiber design and the study of dynamic effects to suppress TMI. The third chapter, lays out the GPU acceleration of a steady-state model that can be used to obtain TMI thresholds. This model is then used to study the impact of doping concentration and pump cladding diameters. In the fourth chapter, I discuss both passive and active TMI mitigation strategies such as confined doping, frequency detuning of a higher order mode, and spatial-temporal control. I present a novel mitigation technique that doesn’t rely on feedback and increases the TMI threshold by 1.4-times. In the last chapter I present a high-power fiber amplifier test-bed that I set up and the critical optical fiber components required for high-power fiber lasers. In consequence, I demonstrate a fiber amplifier reaching an output power of 2.3 kW and excellent beam quality. To study TMI I deliberately reduced the TMI threshold and propose a novel beam-analysis scheme operating at MHz-speeds. Finally, I show our work of selective modal amplification in a fiber amplifier [74]. In addition to the presented work, I worked on record breaking spatial-division-multiplexing schemes in telecommunications in collaboration with Bell Labs and in the field of topological photonics. All my co-authored contributions are stated in the list of contributions at the end of this thesis.
In the 1960’s raising bandwidth requirements for telecommunication systems lead to the development of hollow-waveguides for millimeter wave communication. Ultimately, optical fibers won the day supporting much higher bandwidths and, lower loss and ease of deployment. First demonstrations of guided light in optical fibers based on total-internal-reflection were done by Snitzer et al. [10, 11], followed by Kao’s landmark paper about the prospect of using optical fiber for communication in 1966 [12]. Kao’s measurement of 4 dB/km bulk silica attenuation of a Schott glass at around 850 nm [13], was quickly followed by a 20 dB/km optical fiber produced by Corning in 1970. In the past decades this loss has been constantly reduced and reached a ultra-low loss of 0.1419 dB/km at 1560 nm wavelength in 2018 [14]. With a total contribution of 80 %, Rayleigh scattering eclipses other losses such as infrared absorption, absorption due to OH ions and transition metals, and scattering due to waveguide imperfections. While all fibers discussed in this thesis are based on the guiding principle of total-internal-reflection other mechanisms exist such as photonic-bandgaps [15, 16]. Fibers based on this principle offer tremendous opportunities such as supercontinuum generation, guidance for light in spectral regions where SiO₂ is heavily absorbing, and the prospect of lower latency optical communication. In the following, I will discuss the underlying models that I will use throughout my thesis.

Maxwell’s equations

Maxwell’s equations are a set of differential equations that describe how electric \( (E) \) and magnetic \( (B) \) fields are generated and how they propagate. Variants of these equations enable us to propagate
beams through waveguides and obtain mode profiles. The macroscopic Maxwell equations read

\[ \nabla \cdot D = \rho \] (2.1)

\[ \nabla \cdot B = 0 \] (2.2)

\[ \nabla \times E = -\frac{\partial B}{\partial t} \] (2.3)

\[ \nabla \times H = J + \frac{\partial D}{\partial t} \] (2.4)

Here, \( \rho \) and \( J \) denote charge and current density. Additionally, two auxiliary fields, namely the displacement field \( D \) and magnetizing field \( H \) were introduced. They are related to the electric and magnetic fields through

\[ D = \varepsilon_0 \varepsilon E + P_{NL} = \varepsilon_0 (1 + \chi^1)E + \varepsilon_0 (\chi^2 E^2 + \chi^3 E^3 + \ldots) \] (2.5)

\[ H = \frac{1}{\mu_0} B - M \] (2.6)

\( \chi \) is the susceptibility, \( \varepsilon \) the permittivity, \( \mu \) the permeability and \( M \) the magnetic polarization. The nonlinear part \( P_{NL} \) of the displacement field accounts for the modification of the optical material by the light itself and the ensuing effects such as second harmonic generation [17]. In nonmagnetic media such as optical fibers \( M = 0 \). The combination of Eq. 2.1 through 2.4, and the assumption of no charges and currents, results in the wave equation which describes light propagation in optical fibers.

\[ \nabla \times \nabla \times E = -\frac{n_0^2}{c^2} \frac{\partial^2 E}{\partial t^2} - \frac{\partial^2 P_{NL}}{\partial t^2} \] (2.7)

The nonlinear part is neglected here. A Fourier transform to frequency-space results in the scalar Helmholtz equation, which will be used in Sec. 2 to obtain the modes in an optical fiber and in
Sec. 2 to propagate these modes through an optical fiber.

\[ \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} + \frac{\omega^2}{c^2 n_0^2} E = 0 \]  
(2.8)

**Fiber characteristics**

Fibers with low contrast index profiles represent waveguides with weak guiding and the corresponding modes are known as linearly polarized (LP) modes. If the assumption of weak guidance does not hold, one has to distinguish TE and TM modes. Here we concentrate on weakly-guiding fibers and obtain the modes from the Helmholtz equation. Assuming a z-invariant transverse mode-pattern we insert the Ansatz \( E = \mathcal{E}(x,y) \exp(-j\beta z) \), with \( \beta = n_{\text{eff}} k_0 \) into Eq. 2.8 and obtain

\[ \frac{\partial^2 \mathcal{E}}{\partial x^2} + \frac{\partial^2 \mathcal{E}}{\partial y^2} + k_0^2 (n_0^2 - n_{\text{eff}}^2) \mathcal{E} = 0 \]

(2.9)

Depending on whether \( n_{\text{eff}} \) is larger or smaller than \( n_0 \) the field is decaying or guided. The solution to this eigenvalue problem results in the eigenmodes (mode fields) and eigenvalues (\( n_{\text{eff}} \)) of the optical fiber given some index profile \( n_0 \). We can used second-order differences to discretize Eq. 2.9

\[ \frac{\mathcal{E}_{x-1,y} - 2\mathcal{E}_{x,y} + \mathcal{E}_{x+1,y}}{\Delta x} + \frac{\mathcal{E}_{x,y-1} - 2\mathcal{E}_{x,y} + \mathcal{E}_{x,y+1}}{\Delta y} + k_0^2 n_0^2 \mathcal{E}_{x,y} = k_0^2 n_{\text{eff}}^2 \mathcal{E}_{x,y} \]  
(2.10)

This equation can be efficiently solved using sparse eigenmode solvers and utilized for the analysis of fibers of varying geometries and index profiles. For demonstration, I determined the modes and their eigenvalues of a graded index fiber of 20 \( \mu \text{m} \) and 0.2 NA and compared them to the eigenvalues of a step index fiber (SIF) with the same NA and diameter. The index profile of a
Figure 2.1: Fiber mode properties in dependence on refractive index profiles, core size and mode number. a) Effective indices for step- and graded-index fiber of 20 µm core diameter and 0.2 NA. b) mode field area of fundamental mode of a step-index fiber vs. core size. (c) first three mode groups in a graded-index fiber. Note, the color change indicates a phase jump of $\pi$

graded-index fiber is given by

$$n^2 = \begin{cases} 
    n_{\text{core}}^2 \left( 1 - 2\Delta \left( \frac{r}{r_{\text{core}}} \right)^\alpha \right) & r \leq r_{\text{core}} \\
    n_{\text{clad}}^2 & r > r_{\text{core}} 
\end{cases} \quad (2.11)$$

$$\Delta = \frac{n_{\text{core}}^2 - n_{\text{clad}}^2}{2n_{\text{core}}^2} \quad (2.12)$$

The alpha-factor $\alpha$ can used to adjust the group delay, which is set to $\alpha = 2$ here for a parabolic index profile. The eigenvalues of both SI and GI multi-mode fibers are shown in Fig. 2.1 (a). In contrast to the modes in the SIF, the modes in the GIF appear in well defined groups where higher order groups have more members. In addition, the groups are nearly equally spaced, akin to the modes in a harmonic oscillator in quantum mechanics. The groups are classified by the principal mode number $g = 2m + l - 1$ [18]. Where $l$ and $m$ are the azimuthal and radial mode numbers of the LP$_{lm}$ modes. The few degenerate modes for the SIF fibers are due to modes which only differ
by a rotation. The mode field areas (MFAs), defined in Eq. 2.13, do not scale linearly, but exhibit a minimum which position is inversely related to the NA. This can be seen in Fig. 2.1 (b) where the effective areas \( A_{\text{eff}} \) of the fundamental mode of a step-index fiber for the NAs 0.06, 0.14 and 0.22 are plotted in dependence of core size, using the definition

\[
A_{\text{eff}} = \frac{\left( \int |\mathbf{E}|^2 dA \right)^2}{\int |\mathbf{E}|^4 dA} \tag{2.13}
\]

Figure 2.1 (c) displays the LP modes of the first four groups of a GI fiber (20 µm core diameter and 0.2 NA) without mode-profile degeneracies. Another important measure of an optical fiber is its numerical aperture \( \text{NA} = n \sin \theta_{\text{max}} = \sqrt{n_{\text{core}}^2 - n_{\text{cladding}}^2} \), that gives the angular acceptance of incoming light. NA requirements for Double-clad fibers often exceed those achievable with conventional all-glass designs. While low-refractive index coatings are one possible remedy, air-clad structures allow an even larger tuning range with the possibility of reaching NAs beyond 0.8. Such air-clad structure is shown in the inset of Fig. 2.2 (a). While the NA of the cladding can be measured, a simple slab-waveguide model allows the estimation of the effective indices in the web structure and the corresponding NAs can be found, as shown in Fig. 2.2 (a) [19]. A air-cladding drawn in house is shown in Fig. 2.2 (b). Measuring the bridge thickness, the model is in accordance with the measurement: at 1 µm a bridge thickness of 180 µm corresponds to a NA of 0.9.

**Beam propagation**

To propagate an electric field \( \mathbf{E} \) along a waveguide, we make the assumption that \( \mathbf{E} \) satisfies the scalar Helmholtz Eq. 2.8 [20]. Expressing the field as the product of a complex field amplitude and carrier wave \( \mathbf{E}(\omega, x, y, z) = \mathcal{E}(\omega, x, y, z) \exp(-ikz) \), we can rewrite Eq. 2.8 to

\[
-\frac{\partial^2 \mathcal{E}}{\partial z^2} + 2in_0k_0 \frac{\partial \mathcal{E}}{\partial z} = \frac{\partial^2 \mathcal{E}}{\partial x^2} + \frac{\partial^2 \mathcal{E}}{\partial y^2} + k_0^2(n(x,y)^2 - n_0^2)\mathcal{E} \tag{2.14}
\]
Neglecting the first term on the left-hand side leads to the Fresnel form of the wave equation. This equation describes the field subject to diffraction ($\partial_{xx} + \partial_{yy}$) and index-guiding in the waveguide $k_0^2(n(x,y)^2 - n_0^2)$. Using the Fresnel equation, the field can be propagated using the Fourier split-step method described by Agrawal et al. [21]. The method splits every propagation step $dz$ into two steps: a diffraction and guiding part. Due to its structure it can be accelerated on the GPU, which is especially beneficial for the simulation of mode-multiplexers where many fiber modes have to be propagated through the device to be designed. As an example the modes $LP_{01}$ and $LP_{11}$ are propagated with the split-step method through a stretched fiber, i.e. Fig. 2.3. While the fundamental mode adapts its field size to the taper, the higher-order mode couples to radiating modes and is completely lost. As the computational boundaries are not absorbing the light, they reflect the rejected light from the waveguide in the case of the $LP_{11}$ mode.
Figure 2.3: Beam propagation through a stretched fiber. The field intensity of the modes LP01 (a) and LP11 (b) along a 3 mm long section of fiber stretched as indicated by the grey lines. While the fundamental mode adapts, the higher-order mode is completely lost. Note the numerical artifact due to non-absorbing boundaries for (b).

Loss

Optical fibers endow systems with several benefits such as a small footprint, and a reduced sensitivity to perturbations. However, in addition to the loss introduced by the glass material itself, fiber splices and bends not only introduce additional loss but also couple modes. The ensuing section describes transmission loss introduced by mode size mismatch, beam offset and cleave angles for both single-mode and multi-mode fibers. Furthermore, confinement loss is discussed which is important in e.g. amplifiers where modes are displaced from the core and, hence, have their overlap with the doped region reduced.
Figure 2.4: Splice/coupling loss for fundamental and higher order mode. (a) Mode size mismatch. (b) Parallel beam offset. (c) Angular offset.

**Splice/coupling loss**

Theoretical expressions to determine splice/coupling loss for single-mode fibers were derived by Marcuse in 1977 [22], and are stated for mismatch and tilt in Eq. 2.15 and beam offset in Eq. 2.16.

\[
T(w_1, w_2, \theta) = \left( \frac{2w_1w_2}{w_1^2 + w_2^2} \right)^2 \exp \left( -2 \left( \frac{\pi nw_1 w_2 \theta}{w_1^2 + w_2^2} \right)^2 \right) \tag{2.15}
\]

\[
T(d) = \left( \frac{2w_1w_2}{w_1^2 + w_2^2} \right)^2 \exp \left( -\frac{2d^2}{w_1^2 + w_2^2} \right) \tag{2.16}
\]

\(w_i\) denotes 1/e\(^2\) beam waist, \(\theta\) the angular mismatch of the two fibers, \(n\) the refractive index of glass, and \(d\) the parallel mode offset. The combination of both equations at the point where the transmitted power decreases to 1/e from the point of no loss, we obtain the relation

\[
d_e \theta_e = \frac{\lambda}{n\pi} \tag{2.17}
\]

\[
d_e = \left( \frac{w_1^2 + w_2^2}{2} \right)^{1/2} \tag{2.18}
\]

\[
\theta_e = \left( \frac{w_1^2 + w_2^2}{2} \right)^{1/2} \frac{\lambda}{\pi nw_1 w_2} \tag{2.19}
\]
The equation Eq. 2.17 shows that the tolerance of mode loss to beam offset and tilt are inversely related to each other; a decreased beam offset, increases the tolerance to beam tilt and vice-versa. Figure 2.4 (a-c) shows the impact of mode mismatch, parallel offset and angle offset on the losses introduced at a fiber splice for a single-mode fiber. Analytically estimated losses are in good agreement with losses obtained from mode overlaps. The use of mode overlaps enables the estimation of these losses for higher-order modes and is shown for mode LP\textsubscript{11} also in Fig. 2.4 (a-c). As can be seen, offset, tilt and mismatch have a much greater negative impact on the mode LP\textsubscript{11} than on the fundamental mode LP\textsubscript{01}. This trend continues for even higher-order modes and accentuates the importance of good cleaves and splices for multi-moded high-power fiber amplifiers.

Bending loss

The environment can have a great impact on the physical properties of optical fibers, a desired commodity in the case of sensors and undesired in e.g transmission. Fibers can be intentionally bend to push higher-order modes into cutoff and enable single single-mode operation in amplifiers. To study the effect of bending on the fiber modes, I will employ a finite-element-method mode solver, which excels for fibers with complex geometries that require local mesh refinements. Once the modes are determined for a given bend diameter, the bending loss can either be calculated from the imaginary part of the effective refractive index or the outflowing power (Poynting vector $\vec{S} = \vec{E} \times \vec{H}$).

To terminate the commonly occurring open boundaries in these studies, the region of interest is surrounded with an artificial absorbing layer, a perfectly matched layer (PML). This layer introduces losses for modes that extend into the boundary as it strongly absorbs incoming radiation and reflects little over a large angular bandwidth. This feature is achieved through impedance matching at the interface of the computational region and the PML. The original concept was introduced
by Berenger in 1994 [23], and further extended by Sacks through endowing the boundary material with a complex and anisotropic permittivity and permeability [24]. The new permittivity and permeability tensors depend on the anisotropy tensor $\overline{\Lambda}$.

$$\overline{\varepsilon}_{\text{PML}} = \varepsilon * \overline{\Lambda} \quad (2.20)$$

$$\overline{\mu}_{\text{PML}} = \mu * \overline{\Lambda} \quad (2.21)$$

As the outgoing field mostly propagates in radial direction a cylindrical coordinate system guarantees a mostly perpendicular incidence and an optimum use of computational space. However, packages like COMSOL use the Cartesian coordinate system. Hence, the anisotropy tensor is defined in the cylindrical coordinate system and then transformed to the Cartesian coordinate system. For a derivation of the anisotropic cylindrical PML see Teixeira and Chew [25].

$$\overline{\Lambda}_{xyz} = \begin{pmatrix}
\frac{t_r}{r_S}\cos^2(\phi) + \frac{i}{r_S}\sin^2(\phi) & \sin(\phi)\cos(\phi)\left(\frac{t_r}{r_S} - \frac{r}{r}\right) & 0 \\
\sin(\phi)\cos(\phi)\left(\frac{t_r}{r_S} - \frac{r}{r}\right) & \frac{t_r}{r_S}\sin^2(\phi) + \frac{i}{r_S}\cos^2(\phi) & 0 \\
0 & 0 & \frac{t_r}{r_S}
\end{pmatrix} \quad (2.22)$$

$$S_r = 1 - i\zeta \left(\frac{r-r_i}{t}\right)^m \quad (2.23)$$

$$t = r_i + \left(1 - i\frac{\zeta}{m+1} \left(\frac{r-r_i}{t}\right)^m\right)(r-r_i) \quad (2.24)$$

$\zeta$ is the absorbing coefficient, $r_i$ the radius where the PML starts, $t$ the thickness of the PML at given azimuthal position, the order of propagation loss is typically $m = 2$ [26].

To verify the method I determined the bending loss of a fiber presented and characterized by Beier et al. [3], Figure 2.5 (a). In addition, the effective area of the fundamental mode shows the characteristic increase for small bend diameters when a large percentage of the mode extends into the cladding. With these results at hand, we can investigate the bending characteristics of fibers.
commonly used in double-clad fiber amplifiers such as a 20 µm/400 µm core/clad diameter 0.06 NA fiber. Figure 2.6 (a) and (b) show the dependence on bending diameter of the loss, effective area and core overlap for the modes LP$_{01}$, LP$_{11a}$, and LP$_{11b}$. The simulation clearly indicates, that at a bending diameter of less than 15 cm the HOM modes are cutoff and not supported anymore by the fiber - a good operation regime for single-mode amplifiers. However, during operation the incurred heat load might change the refractive index profile such that the mode loss decreases and more HOMs are supported.

Optical amplifiers

Similar to laser oscillators, optical amplifiers require a gain medium and a pumping process to excite the constituting atoms into higher energy levels. However, optical feedback, a necessary commodity for a laser oscillator, is in general unwanted and can lead to catastrophic damage for high-gain amplifier systems. Optical amplification is either based on stimulated emission or optical nonlinearities and takes place in gain media such as doped insulators (crystals, glasses) or semi-
Figure 2.6: Bending characteristics of 20 µm/400 µm core/clad diameter 0.06 NA fiber. (a) Bending loss of the modes LP
\text{01}, LP_{11a}, and LP_{11b}. (b) Effective mode area and core power overlap of the modes LP_{01}, LP_{11a}, and LP_{11b}. Power overlaps are denoted through dashed lines. Note the cutoff of the HOM modes at a bending diameter of 15 cm.

conductors [27]. Here, we focus on amplification through stimulated emission in doped optical fibers - in particular the rare-earth metal Ytterbium (Yb).

Yb is one of the most prominent rare-earth metals used for amplifiers at 1 µm wavelength. When pumped at the point of highest absorption at 976 nm the amplifier behaves as a three-level system with a ground and excited manifold at $^2F_{7/2}$ and $^2F_{5/2}$. Lasing around 1030 nm exhibits a low quantum defect and hence enables high efficiencies with relatively low thermal impediments - unless high powers are achieved. The high upper-state lifetime of 1 ms results in a high small signal gain. Quenching and photo-darkening (PD) negatively affect the performance of the amplifier; quenching results from clustering of the dopants and can cause an extreme shortening of the upper-life time due to multi-phonon transitions and energy transfer processes. PD generally lowers the efficiency of the amplifier and can represent an additional heat source in addition to the quantum defect [28]. In order to design amplifiers, rate and propagation equations can be employed and numerically integrated. Giles and Desurvire discuss the derivation of a spectrally resolved model involving the upper state level and power propagation for a two-level amplifier [29]. In this model,
a number of optical channels of frequency bandwidth $\Delta \nu_k$ at wavelength $\lambda_k$ are propagated through a gain medium. Pump and signal are considered to have zero, and ASE a finite bandwidth and the doping ions are assumed to be uniformly distributed across the fiber core. The time-dependence of the upper state population $n_2$ is described by

$$\frac{dn_2}{dt} = \sum_k \frac{P_k(z) a_k}{A_k h \nu_k} n_1 - \sum_k \frac{P_k(z) g_k}{A_k h \nu_k} n_2 - \frac{n_2 N_{\text{Yb}}}{\tau} \quad (2.25)$$

$$n_1 + n_2 = 1$$

$$a_k = \sigma_a^k N_{\text{Yb}} \Gamma_k$$

$$g_k = \sigma_e^k N_{\text{Yb}} \Gamma_k$$

with modal area $A_k$, overlap factor $\Gamma_k$, upper-state lifetime $\tau$ and Yb-doping concentration $N_{\text{Yb}}$, absorption $a_k$ and gain $g_k$. The amplifier propagation equation is

$$\frac{dP_k}{dz} = u_k ([a_k + g_k] n_2 - [a_k + l_k]) P_k + u_k m g_k h \nu_k \Delta \nu_k n_2 \quad (2.26)$$

with extra loss $l_k$, propagation direction $u_k$ and number of polarization modes $m$. The second term on the right-hand-side accounts for amplified spontaneous emission. Under the steady-state approximation of the upper-state population $n_2$, Eq. 2.25 turns into

$$n_2 = \frac{\sum_k \frac{P_k(z) a_k}{h \nu_k A_k}}{\frac{N_{\text{Yb}}}{\tau} + \sum_k \frac{P_k(z) (a_k + g_k)}{h \nu_k A_k}} \quad (2.27)$$

Equation 2.26 and 2.27 can be numerically integrated once initial values are specified. For example a core-pumped 3 m long Yb-doped fiber (doping of $2.5 \times 10^{25}$ m$^{-3}$) of 3 µm core radius is seeded with a beam of 1 mW and pumped with 500 mW. The seed is amplified to approximately 200 mW (23 dB gain) while nearly 100 mW of ASE are generated, i.e. Fig. 2.7 (a). After 2 m the pump is completely absorbed and the signal is amplified through reabsorbed forward ASE. Note also
that the backward ASE sees a net-gain towards the end of the fiber and exists the amplifier with more than twice the power of the forward ASE. Increasing the input signal saturates the gain, allows higher power extraction from the amplifier and drastically reduces the ASE content. Gain saturation for three different pump powers: 100, 300 and 500 mW is shown in Fig. 2.7 (b). The power of the input signal which reduces the small signal gain to half its value is the saturation power $P_{\text{sat}}$ and for low-gain amplifiers can be determined according to

$$P_{\text{sat}} = \frac{A h v}{(\sigma_c + \sigma_a) \tau} \quad (2.28)$$

Note the maximum of 0.5 inversion in Fig. 2.7 (a), due to the equivalence of the emission and absorption cross-section at the pump wavelength, using the cross-sections reported by Paschotta et al. [30]. The maximum population possible is reached at infinite pump power and without spontaneous emission

$$n_u^{\text{max}} = N_{\text{Yb}} \frac{\sigma_p^2}{\sigma_p^2 + \sigma_c^2} \quad (2.29)$$
A full inversion is only possible for a three-level system where the excited upper-state quickly relaxes to the lasing level.

Figure 2.8: Amplification and heat load for three pump configurations: co-pumped, bi-directionally pumped, and counter-pumped. (a)-(c) Power evolution of pumps (dashed) and signal (solid). (d)-(e) heat load associated with amplifier depicted on top. The dashed lines in the lower plots indicate the average heat load.

Heat sources and effects

Optical amplification is inherently lossy, where the quantum defect, photo-darkening and background losses contribute to an overall heat load along the amplifier. The difference in energy of pump and signal is in general lost to heat (quantum defect). Photo-darkening is a highly-wavelength dependent absorption through color-centers in the doped core and can significantly increase the heat load. Concentrating on quantum defect heating alone, I studied the heat load in three pumping configurations (Fig. 2.8): co-pumped, bi-directionally pumped, and counter-pumped. The simulation used a 50/250 µm core/clad diameter fiber, 0.1 core NA, pump wave-
length of 976 nm and signal wavelength of 1064 nm and an Yb concentration of $3.5 \times 10^{25} \text{ m}^{-3}$. The heat load in the co-pumped amplifier peaks about mid-way, monotonously rises in the counter-pumped case, and exhibits a combination of both in the bi-directional case. While the heat-load profile is drastically different in all cases, they share a similar average heat-load. Heating a fiber modifies its cold refractive index profile through the thermo-optic effect. The heat loads shown in Fig. 2.8 ((d) through (f)) can be used in the time-independent heat-equation to solve for the temperature variation across the core, and use the changed refractive index profile, in turn, in a mode-solver. Figure 2.9 shows two effects associated with an increasing heat load. First, the mode field area of the modes $\text{LP}_{01}$ and $\text{LP}_{11}$ decreases (thermal lensing). Second, the effective index difference between the two modes increases, effectively decreasing their beat-length. Thermal lensing leads to a localization of higher-order modes and in strong cases may even lead to the guidance of more modes [31]. In addition, a varying heat-load can imprint a moving refractive index grating that can couple power between modes along the amplifier: transverse mode instability. This effect will be discussed in the next section. A simulation of the temperature imprint of a sinusoidally oscillating heat load (similar to interfering FM and HOM) is shown in Fig. 2.10. The temperature profile and in consequence the refractive index lags behind the interference pattern - supporting
the possibility of a coherent modal energy exchange.

![Figure 2.10: Phase lag between moving heat load and temperature profile. The vertical grey line indicates the centroid position of the heat load and the inset the overall location of the heat load in the simulation domain.](image)

Transverse mode instability

The scaling of average-power and peak-power in optical amplifiers went along with mitigation strategies to prevent intensity dependent nonlinear effects such as Raman, Brillouin scattering and SPM. In these strategies, the signal intensity is reduced through temporal stretching of laser pulses [32] and/or the increase of core size. Unlike the first approach, the second works for CW and pulsed lasers. While an increased core diameter size is effective in suppressing intensity-dependent nonlinearities, advanced designs are required to maintain high beam quality [33, 34, 35, 36].

The introduction of large-mode area fibers has lead to ever shorter fibers and in turn an explosion in peak-power and average power of amplifier systems. However, these increases came at the cost of higher thermal loads, which enabled thermal effects such as mode-field shrinking and transverse mode instability (TMI). TMI exhibits a threshold-like onset above which chaotic coupling between fundamental and higher order modes occurs. After the first reported observations of TMI around the 2010s [7], the field has witnessed extensive studies of the effect, the underlying physics and
possible mitigation strategies. For an overview of the historical developments the reader is referred to the review by Jauregui et al. [9].

Experimental observations suggest that a thermally induced long period grating couples different modes and as such causes TMI, also referred to as stimulated thermal Rayleigh scattering (STRS) [37, 38]. There are two main requirements to obtain mode coupling induced by TMI: a thermally induced refractive index grating (RIG) and a phase offset between the RIG and the modal interference pattern (MIP). Coupled mode theory can be used to study the effect of both RIG and MIP on modes described by their amplitudes/phase alone. The corresponding coupled mode equations can be stated as

$$\frac{\partial}{\partial z} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = i \begin{pmatrix} \beta_1 & \kappa_{12}(z) \\ \kappa_{21}(z) & \beta_2 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$

(2.30)

where $\beta_i$ denote the propagation constants and $\kappa_{ij}$ denote the coupling coefficients. In general, modes $E_1$ and $E_2$ are orthogonal and do not couple to each other unless a perturbation occurs. TMI facilitates this coupling through a refractive index of varying strength $\Delta n$ and phase offset $\Phi$ between RIG and MIP. Here, the coupling is assumed to share its period with the MIP, given by the beat $\Delta \beta = \beta_1 - \beta_2$.

$$\kappa_{ij} \propto \Delta n (1 + \cos (\Delta \beta z + \Phi))$$

(2.31)

Equation 2.30 can be integrated to study the influence of the phase offset between MIP and RIG. Figure 2.11 shows the coupling between the modes $E_1$ and $E_2$ for phase offsets between $\pi/2$ and $-\pi/2$. The maximum energy transfer occurs at a phase shift $\Phi = \pi/2$[39] and a sign change in the phase is interestingly accompanied by a reversal in power flow. For a zero phase-offset there is no power flow at all. At this point, it has to be emphasized that the TMI threshold is
Figure 2.11: Perturbation-induced coupling between modes in dependence of phase-offset between MIP and RIG. Phase-offset is indicated at top-left in each graph.

not determined by the active fiber alone, but is influenced by various amplifier parameters such as pump and seed wavelength [28, 40] pump direction, seed power [40], gain saturation [41], grating strength and amount of phase shift [42]. In any event, there is three regimes that can be clearly distinguished in systems that suffer from TMI: a stable, an intermediate and a chaotic regime as shown in Figure 2.12. All three traces exhibit a similar transient for the first milliseconds. Above a particular threshold, the output beam quality starts to degrade quickly. Currently, there is two widely employed numerical models to study TMI based on different assumptions with regards to the origin of the RIG. The first assumes different central frequencies of the transverse modes that lead to a traveling interference wave [43, 44, 45]. The second model, hinges on a RIG that increases in intensity at higher thermal-loads and hence only requires small phase perturbations for energy transfer between the modes [46, 42]. In addition, the analytical model by Zervas shows that at high average powers an amplifier might become unstable, such that perturbations might trigger TMI [31]. I will discuss and GPU accelerate both models, but naturally the dynamic mitigation techniques outlined in Chapter 5 rely on the second model.
Figure 2.12: Transient behavior of fiber amplifier below and above TMI threshold. (a) Stable regime at below threshold. (b) intermediate regime around threshold (c) Chaotic regime above TMI threshold. Note, all three graphs show a transient following the sudden pump onset at time 0.

**Heat equation**

To incorporate heat effects in the amplifier model the heat equation has to be solved (here in two spatial dimensions only). I neglected longitudinal heat flow as the temperature in the transverse plane changes varies faster than along a beat length along the fiber.

$$\rho c \frac{\partial T}{\partial t} = Q + k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$  \hspace{1cm} (2.32)

To date, numerous different algorithms have been developed to solve this equation due to its ubiquitous appearance in physics. In particular, algorithms to solve this equation in two spatial and one time dimension have been demonstrated by Douglas and Peaceman [47, 48]. In this thesis I
will employ an alternating direction (ADI) scheme to obtain tridiagonal matrices that can be effi-
ciently solved on both CPU and GPU platforms [49]. We can derive the ADI matrices from the
discretization proposed by Crank and Nicolson that gives second-order time accuracy [50]. While
the method itself is unconditionally stable, spurious solutions might cause oscillating solutions if
the following Neumann stability criterion is not met

\[
\frac{\Delta t}{\Delta x^2 \rho c} < 0.5
\]  

(2.33)

The Crank-Nicolson method averages the position derivative for the old and future temperature
and uses forward differences for the time derivative. This turns Eq. 2.32 into

\[
k \left[ \frac{(T_{-x} - 2T + T_{+x}) + (T'_{-x} - 2T' + T'_{+x})}{2\Delta x^2} + \frac{(T_{-y} - 2T + T_{+y}) + (T'_{-y} - 2T' + T'_{+y})}{2\Delta y^2} \right] = \rho c \frac{T' - T}{\Delta t} + Q
\]  

(2.34)

With ADI we apply the Crank-Nicolson method in one spatial dimension at a time and move all
other terms to the right-hand-side (rhs). To efficiently iterate between both spatial directions we
can keep the lhs-vectors for both spatial steps and flip between x- and y-steps by transposing the
temperature and heat load arrays and update only the rhs-vector. I obtained the temporal tem-
perature profiles with a central heating source and Dirichlet boundary conditions \( T = 0 \) for two
methods: FEM and ADI. As can be seen in Fig. 2.13 the two methods result in exactly the same
evolution of temperature at three equidistant points in time. The distribution of the heat source is
shown in the inset of the first subplot.
Figure 2.13: Comparison of ADI and FEM solutions to the 2D heat-equation at three time-steps. The constant heat source distribution is shown in the first subplot.
CHAPTER 3: TMI: GPU IMPLEMENTATION OF DYNAMIC MODEL

In order to numerically study the behavior of high-power amplifiers and the detrimental mode-coupling above the transverse mode instability (TMI) threshold, I implemented a model able to resolve transients and react to modulations in either seed or pump. This model is based on the work by Jauregui et al. [40, 51]. Including a BPM, this model naturally accounts for gain saturation, thermal lensing and mode coupling. For the present studies the only considered heat source depends on the quantum defect (QD) and absorption losses. Photodarkening (PD) can be included according to Jauregui et al. [40]. This chapter describes the TMI model and its GPU implementation and measures employed to decrease computation time.

Model

Jauregui et al. [51] proposed a semi-analytical solution for the heat equation (Eq. 3.15), which relies on the pre-computed eigenvalues and eigenvectors of the laplacian - bad for a GPU implementation as GPUs are often memory-bandwidth limited. Once the heat equation is discretized with finite-differences I chose the alternating direction implicit method (ADI) [49] for efficient solutions of the heat-equation across a total time of 80 ms in time-steps of 12.5 μs. The ADI method separates the steps in x and y and results in tridiagonal matrices that can be solved efficiently using tri-diagonal matrix algorithms (TDMA). The model itself interleaves rate equations, the heat equation and beam propagation. At every point along z, the spatially resolved upper state population is calculated to determine the heat deposition and its impact on temperature. The resulting temperature profile changes the refractive index, which in turn, impacts diffraction in the sequential beam propagation. All steps are repeated until the end of the fiber is reached.
\[ E_s(x,y,t) = \sqrt{P_{01}} E_{01}(x,y) + \sqrt{P_{11}} E_{11}(x,y) \exp(-i2\pi\Delta\omega t) \]  

(3.1)

At every spatial step the population inversion is obtained for all temporal steps. The corresponding rate equations describing the lasing action are

\[
\begin{align*}
\frac{\partial N_1}{\partial t} & = - (\sigma_s^a \phi_s + \sigma_p^a \phi_p) N_1 + \left( \sigma_s^c \phi_s + \sigma_p^c \phi_p + \frac{1}{\tau} \right) N_2 \\
\frac{\partial N_2}{\partial t} & = (\sigma_s^a \phi_s + \sigma_p^a \phi_p) N_1 - \left( \sigma_s^c \phi_s + \sigma_p^c \phi_p + \frac{1}{\tau} \right) N_2
\end{align*}
\]  

(3.2)

(3.3)

with \( \phi, \sigma \) and \( \tau \) denoting the fluences, the cross-sections and upper-state life-time of signal (s) and pump (p) respectively. In this dynamic model, the rate equations are time-dependent, basically introducing a hysteresis in the upper level population.

\[
\begin{align*}
n_{u}^f(x,y,z,t) & = n_{u}^i(x,y,z) \left[ 1 - \exp(-\gamma \Delta t) \right] + n_{u}^i(x,y,z) \exp(-\gamma \Delta t) \\
\gamma & = (\sigma_s^c + \sigma_s^a) \phi_s + (\sigma_p^c \phi_p + \frac{1}{\tau}) \]
\end{align*}
\]  

(3.4)

(3.5)

\[
\begin{align*}
n_{u}^f & = \frac{N_2}{N_1} = \frac{\sigma_s^a \phi_s + \sigma_p^a \phi_p}{\gamma} \\
N_{Yb} & = N_1 + N_2
\end{align*}
\]  

(3.6)

(3.7)

Where \( n_{u}^i \) is the initial state before the next pump/signal update, \( n_{u}^f \) represents the steady-state of the upper population ratio after the pump/signal update (Eq. 3.6), and \( \gamma \) is defined in Eq. 3.5.

Considering the loss mechanisms of photodarkening and background losses we can determine the
total gain and heat load. To determine photodarkening, we’ll use the heuristic formula

\[
\alpha_{PD} (\text{dB/m}) = \left( 175 \left( \frac{N_{\text{Yb}}}{8.74 \cdot 10^{25}} \right)^{2.09} \right) \frac{n_{\text{u}}}{0.46 \cdot \gamma} \quad (3.8)
\]

which is a fit to the data presented in [52, 53]. Note, the photodarkening loss has a nearly quadratic dependence on the Ytterbium concentration \(N_{\text{Yb}}\) and a linear dependence on the upper laser level population.

This allows to obtain the total gain \(g\) and the deposited heat load \(Q\)

\[
g^{\text{inv}} = 0.5 \left(-\sigma_s^a + (\sigma_s^e + \sigma_s^a) n_u\right) N_{\text{Yb}} \quad (3.9)
\]

\[
g(x, y, t) = g^{\text{inv}} - 0.5 \frac{\ln 10}{10} \alpha_{PD} - 0.5 \alpha_s \quad (3.10)
\]

\[
Q^{QD} = \left(1 - \frac{V_s}{V_p}\right) \left(-\sigma_p^a + (\sigma_p^e + \sigma_p^a) n_u\right) N_{\text{Yb}} \frac{P_p}{A} \quad (3.11)
\]

\[
Q^{PD}_{s} = \frac{\ln 10}{10} \alpha_{PD} I_s \quad (3.12)
\]

\[
Q^{PD}_{p} = \frac{\ln 10}{10} \alpha_{PD} P_p \quad (3.13)
\]

\[
Q(x, y, t) = Q^{QD} + Q^{PD}_{s} + Q^{PD}_{p} \quad (3.14)
\]

While the heat load is imparted instantaneously, the temperature response is delayed and, in turn, the effect on the refractive index. In order to determine the impact of the thermal load \(Q = Q(x, y, t)\) on the refractive index of the fiber we have to solve the heat equation to obtain the spatially and temporally resolved temperature across the core.

\[
\rho c \frac{\partial T}{\partial t} = Q + k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (3.15)
\]

Where \(\rho\) is the mass density, \(c\) specific heat capacity and \(k\) the scalar thermal conductivity. In
sequence, the refractive index is updated through the thermo-optic effect

\[ n_{xy}(x,y,t) = n_{xy}(x,y,0) + \frac{dn}{dT} T(x,y,t) \]  

(3.16)

\( \frac{dn}{dT} \) denotes the thermo-optic coefficient. For the small index contrasts considered, I use the Fresnel equation with an added gain term \( g \) to describe the propagation of the beam inside the fiber.

\[ 2i n_0 k_0 \frac{\partial \varepsilon}{\partial z} = \frac{\partial^2 \varepsilon}{\partial x^2} + \frac{\partial^2 \varepsilon}{\partial y^2} + k_0^2 (n(x,y)^2 - n_0^2) \varepsilon + g \varepsilon \]  

(3.17)

In consequence, we can integrate this equation using the FFT split-step method [21], which alternates the operation of two transfer matrices: diffraction \( H \) and guiding \( S \):

\[ H = \exp \left( -i \frac{\Delta z}{2 k_c} \left( k_x^2 + k_y^2 \right) \right) \]  

(3.18)

\[ S = \exp \left( i \frac{\Delta z}{2 k_c} (k^2(x,y,t) - k_c^2) + g(x,y,t) \right) \]  

(3.19)

Here, \( k = n(x,y,t) k_0 \) and \( g(x,y,t) \) are updated with Eq. 3.16 and 3.10.

The general outline of the algorithm is shown in Fig. 3.1 (a) and time required to complete one time-step in a 64 × 64 grid in Fig. 3.1 (b). The values of the constants and simulation parameters are stated in Tbl. 3.1.

**GPU acceleration**

To efficiently execute the algorithm all steps outlined in Fig. 3.1 (a) are executed on a GPU to benefit from massively parallel calculations. In general, any computation-offloading to the GPU should minimize data transfer between host and the device due to the severe speed penalties introduced.
<table>
<thead>
<tr>
<th>variable</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yb$^{+3}$ doping concentration</td>
<td>$N_{\text{Yb}}$ = $3.5 \times 10^{-25}$ m$^{-3}$</td>
</tr>
<tr>
<td>Signal absorption crosssection</td>
<td>$\sigma_s^a$ = $3.58 \times 10^{-25}$ m</td>
</tr>
<tr>
<td>Signal emission crosssection</td>
<td>$\sigma_s^c$ = $6.00 \times 10^{-27}$ m</td>
</tr>
<tr>
<td>Pump absorption crosssection</td>
<td>$\sigma_p^a$ = $1.87 \times 10^{-24}$ m</td>
</tr>
<tr>
<td>Pump emission crosssection</td>
<td>$\sigma_p^c$ = $1.53 \times 10^{-24}$ m</td>
</tr>
<tr>
<td>Upper state lifetime</td>
<td>$\tau$ = 850 $\mu$s</td>
</tr>
<tr>
<td>Thermo-optic coefficient</td>
<td>$\frac{dn}{dT}$ = $1.29 \times 10^{-5}$</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$k$ = 1.38 W$\cdot$m$^{-1}$$\cdot$K$^{-1}$</td>
</tr>
<tr>
<td>Heat capacity</td>
<td>$c$ = 703 J$\cdot$kg$^{-1}$$\cdot$K$^{-1}$</td>
</tr>
<tr>
<td>glass density</td>
<td>$\rho$ = $2.2 \times 10^3$ kg$\cdot$m$^{-3}$</td>
</tr>
<tr>
<td>time-step</td>
<td>$\Delta z$ = 15 $\mu$s</td>
</tr>
<tr>
<td>x-y grid</td>
<td>64</td>
</tr>
<tr>
<td>Pump cladding diameter</td>
<td>400 $\mu$m</td>
</tr>
</tbody>
</table>

Table 3.1: Constants and parameters used in TMI simulation.

by the PCIexpress bus. Hence, the presented algorithm copies all required data to the GPU in the beginning of the simulation, executes the algorithm solely on the GPU and only transfers data at selected steps back to the host. The heart of the algorithm uses pycuda to interface python with the C++ code of the GPU kernels [54]. To reach the timings, shown in Fig. 3.1 (b), I implemented several optimizations of a naive implementation. For the heat-equation solver I chose cyclic reduction (CR) and extended an algorithm presented by Zhang et al. to support $64 \times 64$ matrices [55]. Every CR step is followed by a transpose of the solution which is then used to update the rhs-vector. To be able to store all data-vectors in shared memory I use half-precision for the upper and lower diagonal and single-precision for the main diagonal and the rhs-vector. The rhs-vector is progressively overwritten by the solution vector to save cache memory. In the future, implementations as proposed by Wei et al. can be implemented to support larger matrices and make use of hybrid schemes [55, 56, 57]. Generally, this parallelism cannot be exploited as successive fields (temperature, upper state population, etc.) depend on previous solutions. Luckily, we can remove these dependencies by operating on the fields at different position and time as indicated in Fig. 3.1.
Furthermore, I increased integration speeds of spatial field pixels (as needed for the pump update) through atomic reductions across warps. In such way, the GPU-solver is able to process hundreds of systems in parallel. For a $64 \times 64$ grid, 6000 time-steps and 20000 spatial steps the algorithm runs at $12 \text{ min} \cdot \text{m}^{-1}$ on a GeForce GTX 1080 Ti. Despite 100-times more time-steps, the running time only increases 6-fold, compared to the steady-state model described in Chapter 4. The method described therein relies on pre-calculated Green’s functions that have to be accessed from global memory at each step of the heat-solver, severely slowing down the solver.

![Figure 3.1: Computational scheme and timings for dynamic algorithm.](image)

I used the algorithm to determine the TMI threshold for a Nufern 20/400 fiber in co-pumped configuration. The results and parameters can be seen in Tbl. 3.2.

**Summary**

In this chapter I implemented a GPU accelerated model to study transverse modal instabilities. Run on a GeForce GTX 1080 Ti, the dynamic model runs at $12 \text{ min} \cdot \text{m}^{-1}$ on a $64 \times 64$ grid, 6000
Table 3.2: Computed thresholds for various fibers. In all cases the pump and seed wavelength were 977 nm and 1064 nm.

time-steps and 20000 spatial steps. I used the code to determine the TMI threshold of a 13 m 20/400 µm core/clad diameter fiber to be 780 W.
CHAPTER 4: TMI: GPU IMPLEMENTATION OF STEADY-STATE MODEL

The main change from the dynamic to the steady-state model is the physical origin of the refractive index grating. Smith and Smith introduce a frequency offset between propagating modes to generate a traveling wave [58]. The frequency difference itself might be due to e.g. amplitude noise, quantum noise, or spontaneous Rayleigh scattering [44]. Despite its plausibility, as of today, there has been no experimental confirmation that this effect is indeed the main cause to trigger transverse mode instabilities. Nonetheless, due to its steady-state nature, the model allows a fast trend analysis of TMI thresholds. In this approach, the threshold-estimation is a two-step process. First, find the offset frequency that gives maximum gain for the higher order mode. Second, increase the pump power until the HOM reaches a set percentage of the FM power, here 5%. In their paper, Smith and Smith stated an algorithm speed of 20 min·m$^{-1}$ for a non-parallelized version of their code. The here presented implementation reduces the computational cost to 2 min·m$^{-1}$ for a 64×64 grid. Finally, I used the model study doping geometries and the impact of doping concentrations (similar to the work by Naderi et al. [59] and Xia et al. [60].

Model

The dynamic and steady-state algorithms differ in the heat-equation solver and rate equation solver. While the heat load is imparted instantaneously, the temperature response is delayed and, in turn, the effect on the refractive index. In order to determine the impact of the thermal load $Q = Q(x, yt)$ on the refractive index of the fiber we have to solve the heat equation to obtain the spatially and
temporally resolved temperature across the core.

\[ \rho c \frac{\partial T}{\partial t} = Q + k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]  

(4.1)

Where \( \rho \) is density, \( c \) specific heat capacity and \( k \) scalar thermal conductivity.

As steady-periodic heat conduction is considered heat flow along the z-axis is not accounted for. This assumption is valid for periodic forcing and when the transients are not of interest. Under this steady-periodic assumption, Eq. 4.1 can be efficiently solved using Green’s functions as explained in Ref. [58, 61].

Figure 4.1: Outline of steady-state algorithm and timing overview. (a) Algorithm sections that are executed for all time-steps and repeated until the end of the fiber is reached. (b) Distribution of time needed to calculate sections indicated in (a) for one spatial-step along the fiber (64 \times 64 spatial grid and 64 time-steps, run on a GeForce GTX 1080 Ti). (c) Performance optimization through limiting source terms for temperature calculation to region around doped core.

GPU acceleration

Major speedups were gained by (not in order): half2-datatypes to store pre-calculated Green-functions, planning FFT’s, fast float32 cuda-math, block size and count dependent on available SMs, atomic reductions across warps for e.g. updating the pump and restricting source terms in
the convolution of heat equation solver to doped regions (Fig. 4.1 (c)). For a $64 \times 64$ grid, 64 time-steps and 20000 spatial steps, and two frequencies, the algorithm runs at 2 min · m$^{-1}$ on a GeForce GTX 1080 Ti.

Studies

Figure 4.2: Influence of Yb-doping geometry on TMI-threshold for confined, triangle, tri-star, and conventional doping. The individual doping-profile shape is indicated inside each bar. All doping profiles were adjusted in size to reach a pump-absorption of 95% with the same fiber length.

In the following I use the steady-periodic method to test conventional approaches to mitigate TMI in fiber amplifiers. Along with the conventional approach where the Yb-doped section coincides with the core itself, other doping shapes were studied in the past. LMA fibers where the doping is confined to a fraction of the core (confined-doping) hold great promise for further single-mode power-scaling below TMI-threshold [62, 63, 64, 65]. Furthermore, asymmetric dopings where the doping doesn’t exhibit full rotational symmetry has been explored by Naderi et al. [59]. Figure 4.2 contrasts the TMI thresholds of these different approaches, where the individual doping profiles were adjusted in size to reach a pump-absorption of 95% at the end of a fiber of same length.

As indicated in Fig. 4.2, confined doping shows the greatest promise for increased TMI thresholds through the engineering of the doping profile. The more complex to fabricate doping patterns such
as a triangular or tri-star profiles do not surpass the TMI-threshold. However, these designs might show benefits in SBS suppression as their acoustic mode profiles will have a lower overlap with the optical modes. To quantify the threshold-scaling of the confined-doping approach, I studied two fibers of 50/200 µm and 30/200 µm core/clad diameter with $r_{\text{doping}}/r_{\text{core}}$ ratios of 0.5, 0.75 and 1. The length of the amplifier was adjusted for 95% pump absorption, the Yb-concentration was $3.5 \times 10^{25}$ m$^{-3}$, and the amplifier was co-pumped.

Figure 4.3: Influence of doping confinement on TMI-threshold. (a)-(b) TMI threshold and fiber length for 15/100 µm and 25/200 µm core/clad radius fiber in dependence of doping ratio. (c)-(d) Power, and temperature for 15/100 µm core/clad fiber at 0.75 $r_{\text{doping}}/r_{\text{core}}$ ratio. The grey lines in (d) indicate the core boundary.

Figure 4.3 (a) clearly demonstrates that confining the doped region of the core can tremendously increase the TMI-threshold. For both core diameters (30 µm and 50 µm), halving the $r_{\text{doping}}/r_{\text{core}}$ ratio doubles the TMI-threshold. However, this threshold-doubling is accompanied by an up to 3 times longer fiber (Fig. 4.3 (b)). Longer fiber negatively effect SBS thresholds. At this point, the question arises whether a reduced Yb-doping concentration or smaller, fully Yb-doped core will have the same effect: longer fibers with lower heat loads and in turn, a higher TMI threshold.
Figure 4.4 (a) shows the impact of different doping concentrations \((1 \times 10^{25} \text{ m}^{-3} \text{ to } 5 \times 10^{25} \text{ m}^{-3})\) on the TMI threshold for a 50/200 µm core/clad diameter fiber. The individual fiber lengths were again adjusted for 95% pump-absorption. Doping concentration alone does not appear to have a strong effect on the TMI threshold itself when photo-darkening is not considered. The heat load distribution over longer fiber lengths does not increase the threshold as in the confined doping case. If photo-darkening were considered, the TMI threshold would drop for higher concentrations due to the square-dependence of photo-darkening loss on doping-concentration [40, 60].

Figure 4.4: Influence of doping concentration and core radius on TMI-threshold. (a) TMI-threshold and fiber length for 95% pump absorption in dependence of doping concentration \(N_{\text{Yb}}\) (b) \(r_{\text{core}}\).

Summary

In this chapter I describe the steady-state model proposed by Smith and Smith, which I in consequence implement on the GPU. The heat solver relies on pre-computed Green’s functions and as such is memory bound. In general, the algorithm runs 6-times faster than the dynamic model. The model was used to compare the effect of doping profiles in fibers on the TMI threshold and show a doubling in TMI threshold for confined doping fibers.
CHAPTER 5: TMI MITIGATION TECHNIQUES

TMI mitigation techniques can be broadly classified into passive and active. Passive techniques solely rely on system design such as fiber geometry, doping profiles, and pumping direction. In contrast, active techniques employ dynamic control mechanisms such as AOMs and phase-shifters that are part of feedback loops. This chapter employs the GPU accelerated dynamic model to study both passive and active TMI mitigation techniques, such as confined doping in fibers, the unidirectional energy flow imposed by a traveling wave, and amplitude and phase control of the spatial modes supported by the fiber. Interestingly, for straight fibers the $L_{02}$ mode is the strongest competitor for the fundamental mode around TMI threshold for confined doping fibers.

Transverse mode instability mitigation through confined doping fibers

In this section I study fibers where doping is confined to a fraction of the core (confined-doping). These fibers hold great promise for further TMI-threshold scaling [62, 63, 64, 65]. To quantify the threshold-scaling of the confined-doping approach, I studied a fiber of 50/200 $\mu$m core/clad diameter with $r_{\text{doping}}/r_{\text{core}}$ ratios of 0.5, 0.75 and 1. The length of the amplifier was adjusted for 95% pump absorption, the Yb-concentration was $3.5 \times 10^{25}$ m$^{-3}$, and the amplifier was co-pumped. As indicated in Fig. 5.1 (a), at the expense of a 2-times longer amplifier, the TMI-threshold is increased two-fold for a $r_{\text{doping}}/r_{\text{core}}$ ratio of 0.5. To obtain the TMI threshold from the time-traces I follow the definition by Otto et al. [66], which tracks the normalized RMS-deviation of the time-traces with respect to output power. The point where the derivative of a exponential fit to the data reaches $0.1 \% \cdot W^{-1}$ corresponds to the TMI threshold. To disregard the initial transient, I only consider the last third of each time-trace. Figure 5.1 (b) illustrates the method. The other commonly used threshold definition introduced by Johansen et al. [67] leads to a similar
threshold. Interestingly, the mode-resolved time-traces (Fig. 5.1 (c)) around threshold for the cases $r_{doping}/r_{core}$ ratios of 0.75 and 0.5 indicate the change from coupling between LP$_{01}$ and LP$_{11}$ in the first case to coupling between LP$_{01}$ and LP$_{02}$ in the second. While the LP$_{11}$ modes are next-in-line to see gain in the homogeneously doped case, the confined doping naturally benefits the LP$_{0m}$ modes. Depending on the fiber arrangement, the higher-order modes belonging to this symmetry class are also more susceptible to bending than the LP$_{11}$ modes and as such can be suppressed more easily. The interference with the fundamental mode leads to an asymmetric-grating (LP$_{11}$) and symmetric grating (LP$_{02}$) respectively.

Figure 5.1: TMI thresholds for confined doping fiber obtained with dynamic model. (a) Thresholds for different doping ratios (b) The TMI-threshold determination for the case of 0.75 $r_{doping}/r_{core}$. (c) time-traces for cases $r_{doping}/r_{core}$ of 0.75 and 0.5 in the intermediate regime. The doping profiles are shown for each time-trace in the insets (the outer contour indicates the core boundary).
Transverse mode instability mitigation through a traveling wave

Smith and Smith hypothesized that a traveling wave is causing TMI and leads to uni-directional energy flow between the FM and HOMs. While Stihler et al. do not agree with this hypothesis, they pointed out that such a traveling wave can be used to stabilize a beam above TMI threshold and force energy flow towards the FM with a negative detuning of the HOM with respect to the FM [68]. A sign reversal of the detuning causes flow from the FM to the HOM instead. A rough estimate of the expected frequency of maximum coupling is given by thermal diffusion time across the fiber core [69].

\[ f = \frac{1}{\tau} = \frac{r^2 C \rho_{\text{Silica}}}{k} \approx 1.12r^2 \]  

(5.1)

For a 50 µm diameter fiber, the expected frequency is around 1400 Hz. Photonic lanterns or MPLC’s can be used for the mode-multiplexing and a AOM to introduce the frequency shift. To simulate the effect, I used a fiber 50/200 µm core/clad diameter of 1.8 m length. To only permit the first three modes to propagate in a cold fiber, I set the NA to 0.2. A higher NA (more guided modes), and negative detuning leads to energy flow to modes other than to the fundamental mode. Bending can be used to increase the losses of the HOMs to force the energy flow towards the fundamental. Figure 5.2 (a) shows the free running system at 1.2-times the TMI threshold 380 W for co-pumping conditions. A detuning of \(-1400\) Hz and 1400 Hz causes energy flow into the FM or HOM respectively (Fig. 5.2 (b) and (c)). A stabilization of FM power at two-times the TMI threshold has been reported by Stihler for a bi-directional pumping scheme. A bi-directional pumping scheme more equally distributes the heat load and results in a similar optimum-coupling frequency along the fiber amplifier. However, in all cases, the required frequency shifts are orders of magnitudes smaller than the linewidths typically encountered in beam-combinable amplifier systems (>25 GHz), unlike in the simulations where both pump and seed have no spectral extent. If this
the effect can be exploited in experiments has to be shown in the future.

Figure 5.2: Traveling wave stabilization technique at 1.2-times above TMI threshold. (a) Free running system (b) Negative frequency detuning, energy flow to FM. (c) Positive frequency detuning, energy flow to HOMs.

Transverse mode instability mitigation through modal control of the seed signal

In contrast to the previous method, the method presented here relies on the modulation of the spatial content of the seed laser. The TMI mitigation technique of amplitude and phase control was first implemented by Otto et al. where they used a feedback-controlled acoustic-optic modulator to sweep a beam across a fiber core to stabilize a output beam above TMI threshold [70]. Here, the output of the amplifier was monitored for beam fluctuations to control the input to the amplifier in such a way as to stabilize the output beam using a standard control loop. In another demonstration, Montoya et al. used a photonic lantern to selectively excite modes at the amplifier input to retain
single-mode operation above TMI threshold - again this demonstration relied on a feedback loop to stabilize the output beam [71]. Naturally, a feedback loop increases the complexity of the amplifier design, and as such it is desirable to develop a new mitigation technique that does not require active control loops. Here, we investigate the time-dependent excitation of selected modes to suppress TMI without relying on any feedback. In other words, I propose the suppression of TMI by modulating the modal distribution of the seed laser at the input of the amplifier. In order to demonstrate this approach, I analyzed the performance of a high-power fiber amplifier. To this end, I employed the GPU accelerated amplifier simulations discussed in Chapter 3. These simulations capture the dynamic behavior of the output signal as the modal content of the seed is temporally modulated. The output of the amplifier was decomposed and analyzed for its model content for different seed modulation conditions. It is important to note that fiber considered in my investigation is similar to fibers investigated in previous TMI studies. However, this method presented here is not limited to this particular fiber and amplifier parameters and can be implemented in other fiber types and amplifier architectures. As an example, the amplifier consists of a 50/500 µm core/cladding diameter Yb-doped fiber of core NA 0.02 and pump cladding NA of 0.44. The fiber length was 1.8 m. The amplifier was co-pumped at 976 nm considering a uniform distribution across the pump cladding and core. First, we obtained a nominal TMI threshold of 330 W, without any modulations of the seed signal. I then investigated the effects that the modulation of the signal has on the beam quality of the amplified seed at the output. In particular, the phase and amplitudes of the LP_{01} and LP_{11a} mode of the seed were modulated at a frequency 2f and f as shown in Fig. 5.3 (a). This modulation represents a beam oscillating uni-axially across the core. However, it is important to note, that the method itself is not limited to the frequencies and modulation chosen here. The total seed power was 10 W at 1061 nm wavelength, and the pump was propagated in the same direction as the seed with an input power of 460 W. This pump power, corresponds to an operation of the amplifier at 1.4-times above the TMI threshold. At modulation frequencies of 400 Hz and 800 Hz we see an unstable output beam (Fig. 5.4 (a) and (b)). If we increase the modulation frequency to around
Figure 5.3: Modulation scheme and possible system settings. (a) Modulation employed for presented system. Experimental implementation for the system presented with two modes (b), for a system with a multitude of controlled modes (c) and a few-mode oscillator where the modal content is controlled inside the cavity (d).

2000 Hz the output beam stabilizes (Fig. 5.4 (c) and (d)). For even higher modulation frequencies the output beam returns to an unstable regime (Fig. 5.4 (e)). This result clearly indicates, that there is a range of frequencies at which TMI can be successfully suppressed. For this amplifier configuration, we found that the optimum frequency range was between 1500 Hz to 3500 Hz. Here we only analyzed a co-propagating pump, but the method is also applicable to other pumping configurations such as bi-directional and counter-propagating pumps. When the modulation frequency is too slow, the refractive index grating formed by the interference between modes has enough time to develop and efficient coupling of energy between the FM and HOM occurs (Fig. 5.4 (a) and (b)) [9]. On the other hand, for too large frequency of the modulation, the refractive index grating
Figure 5.4: Spatial-temporal control to stabilize output above TMI threshold. Here shown at five different modulation frequencies 400, 800, 2000, 3000 and 4000 Hz. Only around the modulation frequency of 2000 Hz is the refractive index grating washed out and the output beam stable.

assumes the average of the interference pattern which is likely to be inhomogeneous and again decreases the output beam quality (Fig. 5.4 (e)). In an experimental realization of the proposed TMI mitigation scheme (Fig. 5.3 (b)), the modal control of the seed can be achieved using mode multiplexers such as photonic lanterns or multi-planar light conversion devices [72, 73, 74]. This approach can be extended to a multitude of modes of the input seed where TMI can be effectively mitigated for a wide range of fibers and amplifier settings (Fig. 5.3 (c)). Moreover, a few-mode
seed oscillator could be developed using a photonic lantern within the cavity that naturally exhibits the modulation required to suppress TMI in an amplifier down-stream (Fig. 5.3 (d)). Here, we presented a proof-of-concept where the output beam was stabilized above TMI threshold by modulating the phase and amplitude of at least two modes.

Summary

In this chapter, I studied the TMI mitigation technique of confined doping and found a two-fold TMI threshold increase for a confined doping fiber when compared to a conventional homogeneously doped fiber. If the confinement level is reduced enough, the fundamental mode starts to couple to the LP\(_{02}\) mode instead of the LP\(_{11a,b}\) modes. In sequence, I studied two dynamic modulation techniques. Firstly, a traveling wave to encourage energy flow towards the fundamental mode which lead to a 1.2-fold increase in TMI-threshold for a co-propagating pump configuration. Secondly, I propose a novel method where the phase and amplitude of the LP\(_{01}\) and LP\(_{11a}\) are modulated and TMI is effectively suppressed. Although I present results considering only two spatial modes, the proposed spatial modulation of the seed signal can be extended to a larger number of modes in order to further increase the TMI threshold in various amplifier configurations, including pumping schemes such as co-, counter- and bi-directional pumping and various fiber parameters. In summary, the combination of utilizing novel fibers and dynamic control of the modal content of the seed signal promises an enormous potential for power scaling and TMI mitigation.
CHAPTER 6: HIGH-POWER FIBER AMPLIFIERS

The impressive growth experienced by fiber lasers and amplifiers has been made possible due to their remarkable power scalability, excellent thermal management, and ability to be integrated into modular systems [75, 8, 76]. As such, systems with kilowatt average power or megawatt peak power are now readily available. However, due to the high optical intensity in the fiber core, both pulsed and continuous wave (CW) laser systems are limited by nonlinear effects such as four-wave mixing, self-phase modulation, stimulated Brillouin scattering, and stimulated Raman scattering [77]. In order to mitigate these undesirable nonlinear effects, extensive investigations have been concentrated on the development of large mode area (LMA) fibers. The use of LMA fibers is an effective strategy to mitigate optical nonlinearities (such as SBS and SBS) by reducing the optical field intensity within the Yb-doped core, along with shortening the required fiber length. However, as the mode field diameter of LMA fibers increases, it becomes more difficult to maintain single-mode operation and to inhibit excitation of higher-order modes (HOMs). The propagation of HOMs produces thermally induced refractive index changes, which recently have been identified to underpin the onset of thermal mode instability (TMI) at high thermal loads. This nonlinear mechanism ultimately limits average power scaling in fiber amplifier systems [7, 78]. TMI manifests itself as a sudden transition from stable single-mode operation to a regime in which the output spatial mode profile fluctuates rapidly due to power coupling between the fundamental and higher order modes [7, 78, 45, 79].

This chapter starts with the presentation of a high-power fiber amplifier test bed and critical fiber components for high-power setups. In consequence, I show our results for a single-mode amplifier reaching over 2 kW in amplified signal continuous-wave power. To study TMI we increased the bending diameter, and in consequence propose a novel method that allows high-speed beam-analysis. This method enables modal quantification in real time and furthermore indicates TMI
thresholds. Lastly, I present a scheme for selective spatial mode amplification in a few mode, double-clad Yb-doped large mode area (LMA) fiber which could enable dynamic spatial mode control in high power fiber lasers.

Fiber amplifier test bed

The test bed allows rapid testing of the performance of fibers drawn and fiber devices manufactured in house. To benefit from a flexible setup we developed several optical fiber technologies such as end-cap splices, passive-active splices, pump-combiners, and cooling mounts (Fig. 6.1). While losses at low power are usually not desired, they do not lead to catastrophic failure. This is completely different in high-power applications and in the following, I will discuss two critical technologies: end-caps and pump-combiners.

![Figure 6.1: Important optical fiber components of the fiber amplifier testbed.](image)

**End-caps**

In the advent of high-power fiber lasers, the energy density at the air-glass interface of the fibers drastically increased and as such end-caps were designed. End-caps are spliced to the end of fibers
and as such allow the beam to expand before experiencing the glass-air interface, greatly reducing the energy density. Furthermore, end-caps can be coated and hence further reduce undesirable back-reflections into the amplifier, naturally increasing free-lasing thresholds. Figure 6.2 (a) and (b) show such an end-cap after a splice and readily mounted in our test-bed.

![Figure 6.2: End-cap splicing and mounting. (a) End-cap after splicing in our vertical splicer. (b) Placement of the end-cap on a cooled mount.](image)

**Pump-combiners**

In our front-end we have three optical pumps, roughly 900 W each (delivery fiber: 220/240 µm core/cladding diameter, 0.22 NA). To combine these three beams we in-house fabricated efficient pump-combiners. As for any passive optical device, brightness has to be conserved in these devices to minimize coupling losses and minimize loss. For n input fibers brightness conservation is governed by

\[
d_{in}NA_{in} \leq d_{out}NA_{out} \quad (6.1)
\]

\[
B = \frac{d_{out}^2NA_{out}^2}{nd_{in}^2NA_{in}^2} \quad (6.2)
\]
In accordance to brightness conservation, we hosted the bundle of 3 lead fibers (220/240 µm core/cladding diameter, 0.22 NA) in a fluorine capillary, and tapered then down to a diameter of 400 µm including the fluorine capillary. At this stage two sets of images were taken: a micrograph image and the excitation of one pump lead, i.e. Fig. 6.3 (a) and (b). Even an over-saturated image does not show substantial light leakage out of the fiber. As visible in the facet view, the individual fibers are not greatly deformed. The taper was then spliced to a delivery fiber of 400 µm pump cladding diameter with a 0.27 NA. Using this pump combiner we reached a 98 % combination efficiency for our three pumps. With the delivery fiber used, the output beam can be readily free-space coupled into standard fibers such as the 20/400 0.44 NA Nufern fiber.

Figure 6.3: Facet view of the pump-combiner manufactured in house. (a) Micrograph image. (b) Non-saturated and (c) over-saturated image of one excited pump fiber revealing little light leakage into surrounding structure.

Characterization tools

For the characterization of the amplifier, we set up the following devices: a water cooled 4 kW power meter, forward and backward spectral analysis, M2 setup for beam characterization and modal decomposition at MHz-speeds. Our seed has a maximum power of 18 W at 1061 nm wavelength with a 0.2 nm bandwidth and our pump has a maximum output power of 2.5 kW delivered in a 0.27 NA 400 µm diameter fiber. To reach this pump power we combined three commercial
976 nm pumps in a pump combiner constructed in house. As both combined pump and signal had to be coupled free-space into the amplifier dichroic mirrors were used for spectral separation of seed and pump. A schematic of the setup is shown in Fig. 6.4. In addition, all devices are controlled through one graphical interface written in python. The same application can be used to stream/save recorded data to a file.

Figure 6.4: Schematic of high-power fiber amplifier test-bed. Abbreviations denote dichroic mirror (DM), power meter (PM), optical spectrum analyzer (OSA), and pump combiner (PC).

Single-mode operation at >2 kW

In this test-bed, we amplified the 18 W seed to 2.3 kW in a commercial Yb-doped fiber with a core/cladding diameter of 20/400 µm and 0.06 NA (Nufern LMA-YDF-20-400-Gen.8). We spliced a matched passive fiber to the active to move the hottest part of the counter-pumped amplifier away from the point of pump-coupling. We wrapped the active fiber on a water-cooled aluminum mandrel with a 10 cm bending diameter in order to increase the TMI threshold well above 2 kW [80]. A bending diameter of 10 cm pushes the higher-order modes into cutoff and only guides them at high heat loads as indicated in the simulation results in Fig. 6.5 (a). In addition to highly saturating the amplifier, ASE was suppressed by splicing a coated endcap to the passive lead of the amplifier. The amplifier characteristics such as signal power vs. absorbed pump power and M2 values are shown in Fig. 6.5 (b). Due to very low background losses in the fiber we reached a 90.9 % slope efficiency, only 1.1 % short of the 92 % limit imposed by the quantum defect. This is
also evidence of a low susceptibility to photo-darkening of this fiber. M2 was recorded for the output along the x- and y-axis at different signal power levels and averaged to 1.13, which corresponds to excellent mode-quality.

Figure 6.5: Fiber bend losses and Amplification performance of 14 m Yb-doped fiber amplifier. (a) Simulated bend losses for the first three modes for different bend radii. LP11a,b are cutoff at 10 cm bend diameter. (b) Signal power (left axis) and M2 values (right axis) in dependence of absorbed pump power.

Fast modal decomposition of optical beams

Optical beam analysis methods can be split into two groups: the first results in an overall metric not revealing the weights of the individual modal constituents while the other is able to retrieve these weights. A conventional method that results in a beam quality metric is the M2 analysis [81]. The decomposition of optical beams into their modal constituents is conventionally performed using various methods such as holography [82], offline retrieval from camera frames [83] and spatial filters [67]. Conventional spatial filter setups increase in complexity with the modal decomposition count. Digital holography is a powerful technique, but again requires a fast camera and furthermore requires a single-mode beam reference from the device - a requirement that cannot always be satisfied, especially if only the output beam of an optical system can be accessed.
Furthermore, certain modes may lose coherence reducing the information that can be obtained from the hologram. If one only relies on intensity information valuable information is lost and despite obtaining coincidences between reconstruction and original beam the modal weights of both can differ substantially, especially for a decomposition into many modes. Here, we propose to use a mode-demultiplexer to split the beam under test into its modal constituents and measure their individual weights with off-the-shelf photo-diodes whose signals, are in turn, sampled with analog-digital-converters. Photo-diodes with >GHz speeds are readily available. In contrast to previous methods we propose a method that retrieves the modal weights of an optical beam with high fidelity, high speeds and low spatial footprint. The method does not restrict the de-multiplexer used, and as such a photonic lantern or a device based on multi-plane light conversion (MPLCs) can be used [84, 73]. Both devices have been shown to support many modes, 15 modes for a mode-selective photonic lantern [85] and 210 modes in the case of the MPLC [86]. Both devices can be used as part of either a free-space (Fig. 6.6 (a)) or all-fiber setup (Fig. 6.6 (b)). The inset highlights the components of a potential implementation of the beam analysis. The system from which the beam originates can be treated as a black box and no access to obtain optical references is required. Using a low-percentage optical power pickup the analysis can be run while the main system is operational to provide real-time feedback. Our proposed method allows us to study modal dynamics ranging from Hz to GHz. In particular, for the analysis of the dynamics of the beam from a high-power fiber amplifier, this tool can give new insights into the dynamics around the TMI threshold at up to GHz-speeds and amends conventional methods such as spatial filters [67] and off-line algorithmic mode-retrieval from frames captured with a fast camera [83]. If desired, this method can be extended to retrieve the individual phases of each mode by using coherent-detection schemes such as e.g. heterodyning [87] to construct a camera operating at GHz speeds, greatly surpassing current high-speed cameras. In order to demonstrate the use of the method, we analyzed the modal-dynamics of the output of a high-power fiber laser. Single-mode high-power amplifiers often suffer from parasitic higher order modes that deteriorate the output beam. The
Figure 6.6: Placement of spatio-temporal beam analysis device in setup. Either in (a) free-space or (b) monolithic-fiber arrangement. The inset shows the components inside the beam analysis device used in (a) and (b).

exact and real-time temporal quantification of these modes has remained elusive however at high powers. To such end, we set up a Yb-doped fiber amplifier with a 20/400 µm diameter fiber from Nufern. Beier et al. found the TMI threshold of a similar fiber to be around 1 kW at a 50 cm bend-radius [80]. I spliced a 13 m long active fiber (Nufern LMA-YDF-20/400-M) to a 2 m long passive fiber (Nufern LMA-GDF-20/400-M) and equipped both ends with coated end-caps. I coupled a 8 W seed at 1061 nm wavelength into the active fiber and the pump into the passive section for a counter-pumping configuration. Water-cooling dissipated the accumulated heat of the active fiber. The majority of the amplified seed beam hit the powermeter, and I only directed a small portion to the optical spectrum analyzer, camera and mode de-multiplexer. The coupling into the beam analysis device was optimized by maximizing the power in the LP$_{01}$ mode. The amplifier performance and spectrum at 1.2 kW output power are shown in Fig. 6.7 (a) and (b). Due to a lack of oscilloscope channels, I only measured the first three modes (LP$_{01}$, LP$_{11a}$, and LP$_{11b}$) of the de-multiplexed output of a six-mode selective photonic lantern with 11-MHz photo-detectors in this proof-of-concept experiment. The oscilloscope was set to AC-coupling, a 20 MHz bandwidth-limit and a sampling-rate of 250 kSa/s. With another copy of the beam, I conducted a photo-diode measurement apertured to 3-times the MFD diameter, to determine the canonical TMI threshold [66].
I employed several fused silica wedges of the beam before hitting any detector to avoid saturation and the use of OD filters. Figure 6.8 shows time-traces at three different output powers of the amplifier: at low power (300 W), intermediate power (1200 W) and at the highest power (1500 W). From the intermediate regime onwards the method displays small coupling between the modes. Furthermore, an investigation of the frequency content reveals that the frequency content increases for an increasing pump. However, as the pin-hole method also corroborates, even at the highest power we did not reach the TMI threshold reported by Beier et al. for counter-pumping and around a 50 cm bending diameter [80]. Here, the large signal bandwidth of 300 GHz might have increased the TMI threshold above our pump limit as discussed by Tao et al. and Smith and Smith [88, 89].

Mode-selective amplification in a large mode-area Yb-doped fiber using a photonic lantern

One possible TMI mitigation technique is dynamic spatial mode control to steer the modal content during amplification. Photonic lanterns allow monolithic integration into amplifiers and were studied to control the three lowest order modes (LP_{01}, LP_{11a,b}) in a Yb-doped double-clad LMA fiber.

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1This was work was published in Optics Letters [74]
A mode-selective photonic lantern (MSPL) consists of several fibers of dissimilar core diameters and/or cladding diameters stacked in a lower refractive index capillary and adiabatically tapered to form a multimode waveguide. In this case, the modes from any excited input fiber are mapped to the corresponding modes of the multimode waveguide at the lantern output. This work extends previous work, that has been limited to <1 W of output power [90, 71].

The experimental setup is illustrated in Fig. 6.9. In order to analyze the amplifier performance, power meters, an optical spectrum analyzer (OSA), and a CCD camera were employed. A narrow-linewidth laser diode at 1064 nm was used as the seed. The 1064 nm seed wavelength was chosen, as it is spectrally distinct from amplified spontaneous emission (ASE) and self-lasing. The seed was first amplified in a 1 m long Yb-doped fiber (Nufern PM-YSF-HI), core pumped at 976 nm. Pre-amplification increased the seed power to 148 mW while maintaining an optical signal to noise.
Figure 6.9: Schematic of the selective mode amplifier. A 1064 nm narrow-linewidth laser was pre-amplified and then coupled to a three mode selective photonic lantern (MSPL), which is then free-space coupled to a cladding pumped LMA Yb-doped fiber.

ratio (OSNR) of 50 dB. The pre-amplifier output was then coupled to one of the input channels of the three mode selective lantern, which are mapped onto the LP$_{01}$, LP$_{11a}$, and LP$_{11b}$ modes at the lantern output. Figure 6.10 (a) depicts the photonic lantern employed in the experiment. The near field mode profiles corresponding to the different input ports, recorded at the photonic lantern end-facet, are depicted in Fig. 6.10 (b). Note that the slight asymmetry seen in the LP$_{11b}$ mode can be reduced by optimizing the photonic lantern fabrication process. In order to improve the coupling between the photonic lantern and the active LMA fiber, an intermediate few mode fiber (FMF) with a 15/125 µm core/cladding diameter was spliced to the photonic lantern output. The near field mode profiles after propagation through 1 m of the FMF are presented in Fig. 6.10 (c). Figures 6.10 (b) and 6.10 (c) clearly demonstrate that the LP$_{01}$, LP$_{11a}$, and LP$_{11b}$ modes can be excited by the photonic lantern with low modal cross-talk. Subsequently, the output from the FMF was free-space coupled into the 25/250 µm core/cladding diameter Yb-doped LMA fiber (Nufern PLMA-YDF-25/250-VIII) using two lenses, L1 and L2, with focal lengths of 2.86 and 4.6mm, respectively. A dichroic mirror coupled the signal into the amplifier and directed the unabsorbed pump to a power meter. The amplifier was cladding-pumped with counter-propagating light from a fiber coupled 976 nm diode with a 105/125 µm core/cladding diameter and 0.22 NA delivery fiber (nLight element E6). A dichroic mirror identical to that used at the amplifier input separated the output signal from the pump. The multimode pump was coupled to the active fiber using a pair of lenses, L3 and L4, with focal lengths of 6.24 and 8mm, respectively. A 10 nm bandpass
filter, centered at 1064 nm, was used to remove residual pump and ASE for the signal analysis as shown in Fig. 6.9. In order to investigate the amplification of the LP$_{01}$, LP$_{11a}$, and LP$_{11b}$ modes,

Figure 6.10: Photonic lantern with mode profiles before and after the device. (a) Illustration of the photonic lantern. (b) LP$_{01}$, LP$_{11a}$, and LP$_{11b}$ near field mode profiles at the photonic lantern end-facet, (c) at the FMF output, and (d) amplified mode profiles after 3 m Yb-doped LMA fiber at 5 W absorbed pump power.

each mode was individually excited with 77, 100 and 77mW, respectively. The fiber amplifier was pumped with a maximum power of 25 W. Up to an absorbed pump power of 5 W, all three modes retained good mode fidelity, as can be seen from the near field profiles presented in Fig. 6.10 (d). The output power versus the absorbed power for each mode is shown in Fig. 6.11 (a). All three modes exhibited similar amplification efficiency and output signal powers. Specifically at an absorbed pump power of 4.9 W the respective modal gains were 14 dB for LP$_{01}$, 12 dB for LP$_{11a}$, and 13 dB for LP$_{11b}$. For an absorbed pump power of 15 W, an output signal power of 4, 3.8 and 4.2 W was obtained for the three modes. In addition, the spectral dependence of the output signal on pump power for the LP$_{11a}$ mode is presented in Fig. 6.11 (b). The LP$_{01}$ and LP$_{11b}$ modes show a similar evolution of the spectrum with increased pump power and are not shown. At an absorbed power of 4.9 W, the OSNR was 16 dB. At this absorbed pump power, the LP$_{01}$ and LP$_{11b}$ modes exhibited higher OSNRs of 24 dB. The OSNR decreased with higher pump powers as self-lasing
Figure 6.11: Amplification performance of selective mode amplifier. (a) Measured signal output power as a function of the absorbed pump power for all three spatial modes LP\textsubscript{01}, LP\textsubscript{11a}, and LP\textsubscript{11b} amplified individually. (b) Output spectra for the LP\textsubscript{11a} mode amplification for absorbed pump power levels of 0, 4.9 and 8.2W. The OSNRs correspond to 29, 16 and 5dB.

occurred at 1030 nm and competed with the signal amplification at 1064 nm. Characterization of OSNR between the signal and ASE/self-lasing is critical in evaluating the amplifier performance. At around 9 W of absorbed pump power, the OSNRs were 12.4, 5 and 13 dB for the LP\textsubscript{01}, LP\textsubscript{11a}, and LP\textsubscript{11b} modes, respectively. The drastic increase of self-lasing at the absorbed pump powers above 9 W is the result of gain saturation at the seed (both spectrally and spatially). While the amplifier performance could be improved to suppress self-lasing, this is not critical for this proof of concept demonstration. As part of additional experiments, the output of the FMF was spliced

Figure 6.12: Simultaneous amplification of the mode groups: (case 1) LP\textsubscript{01} with LP\textsubscript{11a}, and (case 2) LP\textsubscript{11a} with LP\textsubscript{11b}.
to a 50:50 fiber splitter in which the two outputs were used to excite two arms of the photonic lantern simultaneously. This enabled two cases of bi-modal amplification to be investigated: (1) amplification of LP$_{01}$ together with LP$_{11a}$, and (2) amplification of LP$_{11a}$ together with LP$_{11b}$. The total power of both modes launched into the Yb-doped fiber was similar for both cases (70 mW for case 1 and 98 mW for case 2). The output power measurements as a function of the absorbed pump power are shown in Fig. 6.12 (a) for both launching conditions. The overall output efficiency is similar to the individual mode amplification, shown in Fig. 6.11 (a). At 5 W absorbed pump power, the total output power was 1.1 W for case 1 and 1.5 W for case 2. At this pump level, the observed OSNRs were 15.4 and 11.3 dB, respectively. For higher absorbed pump powers, the OSNRs further decreased in association with self-lasing near 1030 nm. For an absorbed pump power of 8.3 W, the OSNRs were 6 dB for case 1 and 4.1 dB for case 2. In both cases of simultaneous amplification, mode competition was observed. At an absorbed pump power of 100 mW the output mode profiles oscillate from the LP$_{01}$ to the LP$_{11a}$ mode at low Hertz-frequencies. We have separately confirmed the oscillation frequency by measuring the temporal output. In summary, we demonstrated mode selective amplification to multi-watt level average power using a three mode selective lantern to selectively excite the LP$_{01}$, LP$_{11a}$, and LP$_{11b}$ spatial modes in a LMA Yb-doped fiber. High spatial mode fidelity and high OSNRs were maintained for output powers of up to 2 W. With increased pump power, we observe ASE and self-lasing. Simultaneous amplification of two modes resulted in competition between the two excited modes. Research is ongoing to investigate mode competition in this amplifier configuration and to modulate the amplitude and phase of the spatial mode channels to control mode competition. Scaling to higher average power and improved modal purity can be achieved by directly splicing the photonic lantern to the active fiber. Likewise for power scaling, we are working to increase the number of spatial modes by using a photonic lantern with more input fibers.
In this chapter, I presented a test bed that I lead designed and set up. At the current stage, it features a 2.5 kW pump at 976 nm wavelength and a 18 W seed. To characterize the fibers drawn in house the following analysis tools are available: spatio-temporal mode characterization, M2 setup, spectral and temporal analysis of the backward traveling signal. This setup was used to demonstrate a 2.3 kW amplified seed with M2 value of 1.13 below TMI-threshold. I demonstrated a method of real-time modal decomposition complements the existing methods to characterize the modal content of fiber amplifiers in general, and the study of TMI in particular. Finally, I introduced the concept of modal control in fiber amplifiers to suppress TMI in a proof-of-concept experiment.
CHAPTER 7: FUTURE

Having only recently reached the capabilities to routinely our fibers at high-power and validate new TMI mitigation strategies, there are still many topics we have to investigate. The combination of confined-doping fibers and active mitigation technique seems like a good candidate to reach beyond the current single-mode amplifier records of 5 kW. To reach these power levels we are currently developing 7-1 pump-signal combiners that will not only allow us to free-space pump our amplifiers at 5 kW, but also enable monolithic all-fiber arrangements. In addition to the upgrade of pump capabilities, we are designing a bandwidth-tunable seed source that will allow us to explore the desirable region for coherent beam-combination of around 30 GHz. Such source will also allow us to experimentally probe the impact of signal bandwidth on TMI thresholds. Modifications at the seed-side will also allow us to experimentally verify the threshold scaling that I have obtained in my simulations and reach into the cw-power regimes of >5 kW.
APPENDIX A: SOLVING THE HEAT-EQUATION WITH FENICS
To solve the time-dependent heat equation using the open-source finite-element-solver FENICS we have to state the heat equation in variational form \cite{91,92,93}. While the interface can be easily scripted through python, the solvers themselves are compiled low-level routines. Our initial problem reads

\[
\frac{\partial T}{\partial t} = \nabla^2 T + f \quad \Omega \times 0, L \tag{A.1}
\]

\[
T = T_D \quad \text{on } \partial \Omega \times 0, L \tag{A.2}
\]

\[
T = T_0 \quad \text{at } t = 0 \tag{A.3}
\]

Discretizing the time-derivative with a backward difference

\[
\left(\frac{\partial T}{\partial t}\right)^{n+1} \approx \frac{T^{n+1} - T^n}{\Delta t} \tag{A.4}
\]

where \( n \) denotes the current time. Inserting Eq. A.4 in Eq. A.1 yields the time-discrete version of the heat equation

\[
\frac{T^{n+1} - T^n}{\Delta t} = \nabla^2 T^{n+1} + f^{n+1} \tag{A.5}
\]

Assuming \( T^n \) is known from the previous step and \( f \) at the next step we re-arrange the equation into known (right) and unknown (left)

\[
T^{n+1} - \Delta t \nabla^2 T^{n+1} = T^n + \Delta t f^{n+1} \tag{A.6}
\]
before turning it into the weak form $a(u, v) = L(v)$.

\[
a(u, v) = \int_{\Omega} (Tv + \Delta t \nabla T \cdot v) \, dx
\]

\[
L(v) = \int_{\Omega} (T^n + \Delta t f^{n+1}) \, v dx
\]

(A.7)
LIST OF PUBLICATIONS


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