How My Practice Of Using Manipulatives In Teaching Multiplying And Dividing Fractions Influences The Students' Conceptual Unders

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HOW MY PRACTICE OF USING MANIPULATIVES IN TEACHING MULTIPLYING AND DIVIDING FRACTIONS INFLUENCES THE STUDENTS’ CONCEPTUAL UNDERSTANDING OF THESE OPERATIONS

by

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B.S. University of Central Florida, 1999

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Education in the department of Teaching and Learning Principles in the College of Education at the University of Central Florida Orlando, Florida

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**ABSTRACT**

This qualitative study examined how my practice of using manipulatives to teach multiplying and dividing fractions to 8th grade students facilitated their conceptual understanding of those operations. The students who participated in the study were enrolled in one of my intensive mathematics classes.

Before the lessons began, I interviewed the students and gave them a pre-assessment to determine their content knowledge and comfort level with manipulatives. The students engaged in activities that included solving problems using various manipulatives. During the activities, I made observations of their problem solving techniques and how they used the manipulatives. At the conclusion of the unit I gave them a post assessment and conducted post interviews to determine any change in their content knowledge and comfort level with using manipulatives.

I concluded through my research that by giving the students a hands-on, minds-on approach to learning they were able to develop an understanding of the concepts and apply that knowledge to multiplying and dividing fractions.
This thesis is dedicated to my husband, Will, and my children for believing in and encouraging me. Your unconditional love and support has seen me through many difficult times and I thank you.
ACKNOWLEDGMENTS

I would like to thank Dr. Juli Dixon for her unending patience and guidance throughout this project. You have been an exemplary example for me as a professional and a person. I would also like to thank my husband, Will and my cohort for providing me with for all of the feedback and words of encouragement at all the right times.
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CHAPTER ONE INTRODUCTION

Research suggests that students who use manipulatives to solve real-life problems are able to construct their own knowledge about fraction concepts and operations thereby acquiring a higher level of understanding of fractions (Naiser, Wright, & Capraro, 2004). Students, who are taught procedurally, only know how to follow rules to solve multiplication and division of fractions and are not able to understand the meaning of the computation, and therefore, are unable to apply the knowledge to complex problems (Wu, 2001).

An example of students using manipulatives to solve multiplying and dividing fractions would be if they used pattern blocks to model two-thirds of two-thirds, which would be $\frac{2}{3} \times \frac{2}{3}$ (see Figure 1). They could model the whole by using three red trapezoids then they could place 2 green triangles each on two of the red trapezoids to model the two-thirds of two-thirds. This would show that there are four green triangles out of a possible total of nine green triangles on the three red trapezoids. The students could see that two-thirds multiplied by two-thirds is equivalent to four-ninths by counting the green triangles.
Research has shown that children who use manipulatives outperform those who do not and manipulative use also increases scores on retention and problem solving tests (Clements 1999). Clements also states that students’ attitudes toward mathematics are improved after instruction with manipulatives.

The theory of how and when to teach fractions has been a topic of debate for many years. According to National Council of Teachers of Mathematics, (NCTM) students in middle school should have an in-depth understanding of fractions, including the ability to add, subtract multiply and divide fractions and be able to use them in problem solving. However, some researchers such as Groff (1996) feel that the emphasis for middle school students should not include fractional operations. According to Groff, teaching fractional operations is a “dead-end activity” and “seemed like a complete waste of time” (p. 177). Groff stated that he could not find any empirical evidence that substantiates the claims that knowing how to perform operations of fractions is closely related to the successful conduct of their everyday lives, as students or adults.
Rationale

Every school year I find myself spending an inordinate amount of time reviewing fractions. After teaching eighth grade mathematics for several years, I have discovered that out of all of the topics I teach throughout the year, fractions causes the most anxiety to a good number of students. Some of the students know how to apply the procedures to perform basic operations but they do not understand fraction concepts enough to apply that knowledge to other problems. Within just a few years of teaching, I became interested in finding ways to relieve this anxiety for my students and discovering what exactly causes the students to become so confounded by fractions. I strongly feel that if I can find a way for the students to understand the simple concepts of fractions then they will be able to solve all of the various types of problems that involve fractions.

My research was completed with one of my intensive math classes this year but I have been attempting to develop a successful series of lesson to teach the various concepts of fractions. In past years, when I taught honors courses, I found that most of the students could solve all types of problems until they encountered a problem that contained fractions. Since I teach 8th graders and they are supposed to learn fractions by the fifth grade, I had assumed that my students would come to me with an understanding of fractions and that they would be ready to solve complex problems containing fractions, such as solving equations with fractions rather than whole numbers.

I have become more motivated with each passing year to find a means to facilitate the understanding of the concepts of multiplying and dividing fractions. Applying the use of manipulatives to teach multiplying and dividing fractions coincides with my belief that if
I make mathematics tangible, my students will be able to develop a deeper understanding of the concepts. Creating this deeper sense of understanding should also foster a deeper desire to learn more advanced mathematics as well as instill a sense of achievement in the students.
Purpose of the Study

The purpose of this study was to examine the effects of my practice of using manipulatives to teach multiplying and dividing fractions on 8th grade students’ conceptual understanding of these operations. I believe that through the use of manipulatives, students will be able to gain conceptual knowledge of fractions and make the connections needed to apply that knowledge to all types of problems while also honing their problem solving skills. As with any high-quality lesson using manipulatives, I plan to continue the lessons through the pictorial and abstract stages.

Research Question #1

In what ways does my use of manipulatives affect my 8th grade students understanding of the concept of fractions?

Research Question #2

In what ways does my use of manipulatives affect my 8th grade students understanding of multiplying and dividing fractions?
Definitions

- Conceptual understanding: For the purpose of this study, Conceptual understanding means having relational knowledge as opposed to procedural knowledge and the ability to apply an adapt the concept to new situations.

- Manipulatives: For the purpose of this study, manipulatives refer to any type of object that is used in teaching to help the student see and understand the concepts being taught (Rust, 1999).

- Growth – as it applies to this research, a student being able to solve problems at an abstract level and understand the process of multiplying and dividing fractions defines growth in student understanding.

Some examples of manipulatives are uncooked beans, Cuisenaire rods, two-color counters, pattern blocks, tangrams, base-ten blocks and algebra tiles. For this study, manipulatives refer to any object a student uses to aide in solving problems. The most common manipulatives my students chose to use were pattern blocks, Cuisenaire rods, unifix cubes and two-color counters (see Figure 2).
Figure 2: Examples of some of the math manipulatives used in this study.
Significance

According to the 2003 Trends in International Mathematics and Science study, (TIMSS) American 8th grade students scored significantly lower on the mathematics test than 12 of the 23 schools that participated in the study. TIMSS found that the countries that scored higher taught their students using conceptually-based lesson plans where teachers in the U.S. taught their students mostly procedurally-based lessons. Research shows that time on task is an important contributor to achievement but time alone is not enough, it has to be combined with time spent on high-quality instruction (Moyer, 2001).

Based on observations I have made over the past several years, I believe the nature of our students is changing and we need to move away from the traditional methods of teaching. Students of today have much different learning styles due to immersion in technology such as the Internet, computers and video games. According to the learning pyramid (National Training Laboratories [NTL], 2004) students who practice by doing retain 75% of the learned skills and those that have immediate use of newly learned skills have a retention rate of 90% (see figure 3). According to NTL students must use a hands-on, minds-on approach through the use of concrete manipulatives and problem-solving discovery lessons. Therefore, the direction of my study is to explore how my practice of using manipulatives in teaching multiplying and dividing fractions influences the students’ conceptual understanding of these operations.
Figure 3: Learning Pyramid for the National Training Laboratories.
In the next chapter I will describe the methods I used to teach multiplying and dividing fractions and collect my data. I will give a detailed explanation of the interview process, assessments, activities and portfolios.
CHAPTER TWO LITERATURE REVIEW

“Fraction concepts continue to be one of the most challenging mathematics topics for middle school students” (Kouba, Zawojewski & Strutchens, 1997, p.87). Introducing abstract symbols, terminology and forms of representation without first developing meaning on the basis of student’s experiences and readiness may result in some of the difficulties they exhibit with fraction concepts and operations (NCTM, 1994).

The purpose of this study was to examine the effects of students using manipulatives to learn the concepts of multiplying and dividing fractions. I also wanted to look at why students are not exposed to manipulatives more frequently. There has been an immense amount of research on using manipulatives to teach and learn the concepts of multiplying and dividing fractions. (Becker, 1998; Clements, 1999; Hartshorn& Boren, 1990; Rust, 1999)

I will report on students’ understanding of the concepts of fractions, such as, finding equivalent fractions, identifying fractions, and the importance of using equal parts. I will also take account of the research on students understanding of the operations on fractions, in particular multiplying and dividing and the use of manipulatives to teach and learn the before mentioned fraction concepts.

Students’ understanding of the concepts of fractions

Middle school students’ misconceptions with multiplying and dividing fractions are for the most part conceptual not procedural (Greer, 1992). For example, from their
experience with whole numbers, many students develop a belief that multiplication makes bigger and division makes smaller.

“By giving students concrete ways to multiply and divide fractions such manipulatives as pattern blocks, tiles, and cubes can contribute to the development of well grounded, interconnected understandings of mathematical ideas” (Stein & Bovalino, 2001, p.356). Stein and Bovalino believe that getting students to think about mathematics in ways that go beyond using procedures to solve routine problems is an important goal of the mathematics reform. Teachers must recognize the importance of conceptual knowledge in their efforts to help students build procedural knowledge (Carpenter, 1986).

The Third International Mathematics and Science Study, (TIMSS) has confirmed that 8th grade students in Japan and 17 other countries, had a higher achievement rate in mathematics than 8th grade students in the United States (Beaton et al., 1996; Mullis et al., 1997). The study also found there were differences in the way mathematics is taught in the United States versus the higher achieving countries. Teachers in the United States teach most of their lessons using a procedurally based method while teachers in the higher performing countries teach mostly conceptually based lessons (TIMSS, 2003).

“Students who only know procedures have a difficult time in generalizing the information in order to transfer their knowledge to new problems” (Wu, 2001, p. 174). They also have a difficult time recalling algorithms and explaining the concepts taught three weeks after a procedural lesson but the students’ were able to explain how to solve a problem after a conceptually-taught lesson (Philipp & Vincent, 2003).

NCTM states that students in middle school should acquire a deep understanding for fractions and be able to use them competently in problem solving (Naiser et. al, 2004). In
order for this deep understanding to occur and for the students’ learning to be lasting, multiplying and dividing fractions need to be taught at the concrete level (May, 1994).

Another study found that early elementary students solved multiplying and dividing fraction problems in a natural way that was very sensible but once they had been exposed to the traditional, procedural methods in the later grades they became confused and were unable to solve these same types of fractional problems (Riddle & Rodzwell, 2000).

“When teachers forgo teaching for understanding in order to expedite the process then students are subjected to years of memorization without meaning, which can have disastrous effects” (Bezuk & Armstrong 1992, p. 729).

**Manipulative Use**

Even as far back as the 19th century, Pestalozzi advocated the active involvement of children in the learning process (Hartshorn & Boren, 1990). He also is quoted as saying “Children should not be given ready-made answers but should arrive at answers themselves. To do this their own powers of seeing, judging and reasoning should be cultivated, their self-activity encouraged” (Silber 1965, p. 134). I believe that this implies that children learn through the use of concrete manipulatives with well thought out lessons.
Benefits of Manipulative Use

“It is found, for the most part, that when manipulatives are used properly to teach the concepts of multiplying and dividing fractions, students outperform those students who do not use manipulatives” (Raphael & Wahlstrom, 1989, p.173). Manipulative use also gets the students more engaged in the lesson and provides a way for students to represent their thinking (Naiser, Wright, & Capraro, 2004).

According to Naiser et al., (2004) teachers must find a variety of strategies, including the use of manipulatives, to use in the classroom for teaching multiplying and dividing fractions. “Manipulatives can be used to induce understanding of concepts in the middle school curriculum when they are used properly” (Moyer & Jones, 2004, p. 32). Glidden (2002) also believes when used properly, manipulatives are valuable tools for helping students understand mathematical concepts and operations, and their use should be part of every teacher’s practice. Manipulatives also provide a good method for teachers to understand what their students are thinking (Naiser, Wright & Capraro, 2004). Naiser et al. reported that by watching what the students are doing with the manipulatives and how they construct meaning, a teacher can tell if a student has conceptual understanding or not.

Kieren (1988) recommends that instruction on multiplying and dividing fractions build on children’s intuitive understanding of the concepts of fractions and on interaction with objects rather than merely following procedures. “Teachers should help students generalize symbolic algorithms from their experiences with real world problem contexts, manipulatives and pictures” (Bezuk & Armstrong, 1992, p. 729). This will allow students
to construct their own mathematical understanding, which is vital to a satisfactory foundation for future understanding (NCTM, 1989).

Similar to what Raphael and Wahlstrom found, Clements (1999) found that when teachers, who are knowledgeable about manipulative use, teach mathematics using manipulatives the students usually outperform students who do not use manipulatives. Clements reports that these benefits hold true no matter what the grade level, ability or topic. He also writes that students’ attitudes toward mathematics improved when knowledgeable teachers taught them with concrete materials.

Naiser et al (2004) found through their research that using manipulatives to teach multiplying and dividing fractions made the lessons more active and provided an effective way for the students to represent their thinking. They also found that teachers were able to understand student thinking by observing them use manipulatives where as with paper and pencil the students “genuine thinking” is not insured. Students who were exposed to a procedural lesson prior to a conceptual lesson scored significantly lower on an assessment than students who were only taught the conceptual lesson (Philipp & Vincent, 2003).

Krech (2000) found through his research that hands-on activities are the best way for students to learn about multiplying and dividing fractions. He also states that teaching with manipulatives permits students to experience fractions on a concrete level. May (1994) states that the best way for students to learn multiplying and dividing fractions is to have them create models that give meaning to the operation. She believes that the best way to do that is to provide them with good activities and the right kind of manipulative. When manipulatives are not used and students merely learn the symbolic patterns they do
not progress beyond simple computation and they do not learn to use the patterns to solve problems (Moyer & Jones, 2004).

CRA (Concrete, Representational & Abstract) is a “highly touted” instructional method for teaching mathematics to students with learning disabilities (Witzel, 2001). Witzel has developed a workshop to implement the CRA model in any mathematics classroom. This sequence of instruction, starting with concrete, then moving to pictorial representation and then on to abstract symbols, has many benefits for students (Witzel, 2006). Witzel states “Three of the benefits of the CRA model are students develop mathematical concepts, a greater procedural understanding and it makes mathematics fun.”

Another study by Hartshorn & Boren (1990), describes a sequence similar to the CRA model. Hartshorn explains a sequence by Heddens (1986) that has four stages instead of three but they are very comparable. The middle pictorial step is divided into two levels, the semi concrete and the semi abstract. In the semi concrete level, the students use pictures that represent the real items used and in the semi abstract level the students use pictures that do not look like the objects (Hartshorn & Boren, 1990).

Manipulative pitfalls

Why are more teachers not using manipulatives to teach fraction concepts? Some of the reasons I have found through my review of literature were:

- Lack of understanding of the concepts themselves (Moyer, 2001)
- Not wanting to give up control (Moyer, 2004)
- Teachers’ attitudes and beliefs towards teaching fractions (Groff, 1996)
• The lack of time required to use manipulatives properly (Gilbert & Bush, 1988)

One study I found states that the “effectiveness of using manipulatives is directly related to the teacher’s depth of understanding of fraction concepts and how to use manipulatives to show these concepts” (Sowell, 1989, p. 498). According to findings by Moyer (2001), using manipulatives was little more than a diversion when teachers who used them were not able to represent the concepts themselves. But she also found in a later study, that research in mathematics education and cognitive psychology suggests that teachers need to make a switch from rote memorization of facts and procedural teaching methods to methods that give students more control over their learning (Moyer, 2004). Leaving students on their own for too long is another downfall that can happen when manipulatives are used (Stein & Bovalino, 2001).

Setting up manipulatives and developing a good lesson to enhance the learning of any concept requires a substantial amount of time. Time also plays a part in the teacher learning how to use the manipulative. Some people just do not have any more time to go to another workshop (Moyer, 2001). This comes back to prioritizing and teacher attitudes towards fractions (Moyer, 2001). However, I feel that if more teachers see the importance for students to be exposed to manipulatives when they are learning fractions, more teachers are going to find the time to learn how to use them. In one study it was found that teachers who used manipulatives reported doing so primarily because they feel that manipulatives benefit the students’ learning and that students enjoy using them (Howard, Perry & Tracey, 1997). Howard et al. also stated that teachers feel confident in using the manipulative they know but would appreciate further training on manipulative use.
The amount of time teachers spend using manipulatives to teach multiplying and dividing fractions is relatively low compared to the availability and familiarity that has been reported to researchers about manipulatives (Hatfield, 1994). One would think that with the research showing that students learn how to multiply and divide fractions best through the use of manipulatives, teachers would learn to use them and pull them out of the closet. Moyer (2001) did a study on using manipulatives in the classroom and found that the teachers who used manipulatives less often were the ones who were slower to relinquish full control of their classroom. She found that teachers who were more control oriented tended to talk more, give more directives, criticize more frequently and ask more controlling questions. It was found through this research that to promote autonomous thinking in students, the teachers needed to relinquish some of the control over students’ learning and encourage them to think and make sense of the mathematics at hand.

There are many articles, books, videos and workshops that train educators how to use manipulatives to teach concepts. The National Council of Teachers of Mathematics (NCTM) organization is an exceptional resource for finding articles and workshops on using manipulatives as well as multiplying and dividing fractions. Even if the activity or lesson found is not focused on fractions, or even mathematics for that matter, most of the hands-on activities can be modified to use with whatever concept we are teaching. There is a plethora of resources out there in the form of articles written by teachers for teachers that give detailed descriptions of activities that can be used and adapted. For example, there is an article called “Putting the pieces together”, in which the author details five different activities from grades k-8 (Naylor, 2003). Another article gives hands-on ideas and discusses the importance of students verbally sharing what they have learned (Krech,
There is a website that has several virtual manipulatives the students can use online to learn the concepts of multiplying and dividing fractions (National Science Foundation, 2005).

According to the National Council of Teachers of Mathematics (NCTM, 2000), when it is time to teach fractions, teachers feel a level of frustration not unlike that of the students. Some teachers lack the understanding of the concepts of fractions, so it is difficult to show students how to use the manipulatives. Some teachers find it hard to admit they do not know something that relates to their field, so they are not even asking co-workers for help. NCTM also states “effective teaching requires understanding of what students know” (2000, p.11). Since some teachers do not feel competent using fractions in the classroom, students miss out on this exceptional teaching tool. Most teachers are willing to make the necessary changes to their teaching methods but they need the necessary professional development experiences. Through these experiences teacher are willing and able to make changes to their practices (Andreasen, Swan, & Dixon, in press).

**Educators Perspectives**

Not all educators agree on when we should introduce certain fractional concepts, and this causes some lack of continuity as a student progresses through the grades (Groff, 1996). Even though, by way of standardized testing, there is an enormous amount of pressure put on teachers to teach certain concepts in a given year, some teachers will not put the emphasis on those concepts. For example, Groff (1996) feels that the emphasis for
middle school students should not include fractional operations such as multiplying and dividing fractions. He states that the focus at this age should be on learning the concepts of fractions. Students should learn what 1/2 really is and what it means to eat 1/3 of a pizza or how to use a ruler and a measuring cup.

Professional mathematics teaching organizations, like NCTM, feel that middle school students should learn to add, subtract, multiply and divide fractions. The pressure to teach these operations is still very strong in the middle school. What usually ends up happening is middle school teachers teach both the concept of fractions and the operations, with such little time neither one gets developed full (Rust, 1999).

**Summary**

Overall, I found that the majority of the research supports the use of manipulatives to teach mathematics concepts and in particular to this study, multiplying and dividing fractions. The reasons I found during my research for the use of manipulatives not to work were due to teacher issues not students’ misunderstandings. In the next chapter, I will outline the methodology I used to interview the students, give assessments, teach lessons and gather necessary data.
CHAPTER THREE METHODOLOGY

The purpose of my action research was to investigate my students’ conceptual understanding of multiplying and dividing fractions. Through my research, I wanted to determine if the use of manipulatives to multiply and divide fractions impacted the students’ conceptual understanding of these operations. A good lesson should incorporate multiple instructional methods such as the use of manipulatives, technology and paper/pencil activities (Alejandre, 2000).

In this chapter I will describe the school and class settings, my students and the procedures I used to collect my data. I will also describe the lessons that were covered with the students as part of this study.

Design

All though the majority of my research was qualitative, I did collect quantitative data from a pre and post-assessment. As a participant-observer, I was able to interact with my students and document their reactions, findings and behaviors first hand. Action research in education is described as a study conducted by colleges in a school setting where the results of their activities improve instruction (Glickman, 1992). During this study I examined the use of manipulatives in my teaching of multiplying and dividing fractions to improve the students’ conceptual understanding of those concepts.
School Setting

I teach at a middle school in central Florida, this school is an established school and has been in the community, as a middle school, since 1979. The school site used to be the local high school prior to it becoming the middle school. The school is named for a teacher at the high school who was then a principal and finally superintendent of schools for the county.

Most of the teachers in this school have taught here for an extended period of time. Only 18% of the teachers are in their first year at this school and almost half, 43%, of the teachers have advanced degrees such as masters, specialist and doctorate.

In this school, 53.4% of the students are considered economically disadvantaged and receive free or reduced lunch. The ethnicity of the students at this school are as follows; 52% white, 26% Hispanic, 15% African American, 4% multiracial, 2% Asian and 1% listed as other. Eighteen percent of the students are English Speakers of other Languages (ESOL).

Because of the large special needs population at the school the teacher to student ratio is 1 to 15. The total student population is 1170, which includes 374 6th graders, 409 7th graders and 387 8th graders. These numbers are constantly changing because of the high mobility rate.
Class Setting

I teach intensive mathematics, this course is for students who are below grade level. Students in this class also attend a regular eighth grade mathematics class. I have learning stations set up in my classroom and the students are divided into groups of three to four students per group. I used one of the stations to complete the activities on fractions so that the students could rotate through the other stations to continue reinforcing what was being taught in their regular mathematics class. I usually have four stations set up and the students go to two stations a day, in the course of the week they participate in each of the stations twice. Some examples of the stations I use are, computer programs and Internet sites, a games stations where students play mathematics oriented games, a hands on station where they use manipulatives to solve various problems to support the learning in their regular mathematics class and a homework help station where they get help on their homework from their regular mathematics class. I change the activities in these stations each week so that I can keep the interest of the students.

There are 28 desks in the classroom set up in four groups of seven. I have four computer stations for student use and one for teacher use that are connected to the Internet. The two bookshelves are full of novels, resource books and manipulatives for students’ use. There is a large television connected to the teacher computer, which is a great teaching tool. I use it to show power point slideshows, Internet sites, demonstrate virtual manipulatives and to stream videos from discovery learning (see figure 4).
Figure 4: Picture of Classroom where study took place

**Students**

The students in this study were members of my third period classroom. I randomly chose this class at the beginning of the school year. There are 15 students that range from thirteen to fifteen years old, five girls and ten boys. Seven of the students are of Hispanic descent, three are African American and five are Caucasian. One of my students in this class is an ESOL student. Eleven OR 73% of these students receive free or reduced lunch and there is one student who is listed as homeless.
When it comes to Maslow’s hierarchy of needs (Huitt, 2004), most of my students are still trying to get their needs met from the bottom four areas (see Figure 5). According to Maslow (1967), a person has to have their needs met from the bottom of the pyramid, their physiological and safety needs, before they can function at the next level. The students in this class are faced with trying to get their basic needs met like where they are going to sleep and what they are going to eat. I face the challenge every day of trying to encourage my students to learn, one way I do this is by creating an environment where mathematics is fun and interesting.

Figure 5: Maslow’s Hierarchy of needs

The students were placed in my classes because they scored below grade level on the Florida Comprehensive Assessment Test (FCAT), the state standardized test, during the
previous school year. They also are in a regular 8\textsuperscript{th} grade mathematics class; a few of the students are in an advanced mathematics class but for some reason they did not score well on the FCAT test. I have found through discussions with my students that some of the reasons as to why they did not score well were: they were tired and did not feel like trying, they felt frustrated, and they did not care.

There are three reasons I believe these students are not performing well in mathematics. Using manipulatives will not help the first two reasons but I fully believe manipulative use will have an impact on students’ conceptual understanding of multiplying and dividing fractions. The first reason I believe the students are not performing well is that they have so many personal issues going on in their young lives that learning mathematics is just not a priority. Many of them have to baby-sit younger siblings, cook dinner, take care of parents, and worry about what they will eat or where they will sleep. Second, all of the students read below grade level, which makes it difficult to comprehend the higher level questions and read the instructions (see Table 1). The FCAT reading score is on a scale of 1 to 5, a level 3 is considered at grade level. The students who score below a 50\% are considered high risk on the MAZE comprehension test. Last, they did not grasp mathematical concepts taught to them at an earlier age and they are struggling with basic concepts that make it difficult to transfer any knowledge to new situations.
Table 1: Students Reading Levels

<table>
<thead>
<tr>
<th>Student</th>
<th>FCAT Level</th>
<th>Comprehension level (MAZE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shawn</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>Matt</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>Lacy</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
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<tr>
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<td>10</td>
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<tr>
<td>Andrew</td>
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<tr>
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</tr>
<tr>
<td>Jose</td>
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<td>Robert</td>
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<td>Sammy</td>
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<td>Darius</td>
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<tr>
<td>Jessica</td>
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</tr>
<tr>
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<td>Kevin</td>
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<tr>
<td>Mean</td>
<td>1.3</td>
<td>23.6</td>
</tr>
</tbody>
</table>

 Procedures

The students were given an informed consent form that was signed by their parent or guardian, which was returned to me before the study began. (Appendix C) I read the assent form to the class and received verbal assent from each of the students. (Appendix B) At this time I also explained to my students that all of their work and comments would be kept confidential and that I would be using pseudonyms instead of their real names.
Pre-interview

Directly following the collection of signed consent forms and reading the assent script I interviewed all of the students that were going to participate in this study. I asked them questions with four main objectives in mind which were to determine:

- their current levels of understanding of the concept of a fraction,
- their current levels of understanding of the concepts of multiplying and dividing fractions,
- if students had previously used manipulatives to learn multiplication and division of fractions, and
- students’ attitudes towards using manipulatives.

I developed five basic questions to function as a starting point for the interviews, and then I asked other questions based on their previous answers (see Appendix D). These interviews lasted about 10 – 15 minutes each. I planed to ask the same questions upon completion of the lessons on multiplying and dividing fraction.

During the interviews I asked the students to view two problems that a fictitious student solved and I asked them if they thought the student had solved the problems correctly. I then asked my students to explain the steps the fictitious student had taken to solve these problems. For the last three questions I ask them if they had ever used manipulatives, and if they had, then I ask them to explain how they felt about using manipulatives. Then I asked them how they would solve a particular problem using manipulatives (see figure 6).
4. How would you use manipulatives to solve the following problem \( \frac{1}{3} \times \frac{3}{5} \)?

Figure 6: Sample interview question.

In the future I would change this question to a contextually based question instead of a simple abstract multiplication problem. I believe I would obtain a greater understanding of what the students knew if I would have used a contextually based question.

Pre-assessment

After I had interviewed everyone, I gave the students a pre-assessment (appendix E). The first four problems asked the students to solve the problems using drawings and the last six asked the students to solve the problems using manipulatives and then draw a picture of how they used the manipulative to solve the problem. The students were given pattern blocks, Cuisenaire rods and two color counters for the test. I used a rubric to score this assessment. I used a 3-point rubric to score the students responses to the assessment so 30 points was the highest score possible. Students earned a score of 3 for responses that were correct and showed complete understanding of the concept, a 2 for responses that were partially correct, a 1 for correct answers with no explanation and a zero for incorrect answers.
Activities

Activity One: Exploring Manipulatives

The first time I gave students manipulatives, I allowed time for exploration. I allowed the students an entire 52-minute class period to explore the manipulatives. I set up four learning stations each with the following manipulatives, pattern blocks, Cuisenaire rods, two color counters, and tangrams. I placed one set of manipulatives at a station and the students spent 10 minutes at a station then rotated to the next until they had visited all four. The students built things, created patterns and compared sizes of the various manipulatives. I believe that this assisted the students in becoming more familiar with the manipulatives and allowed for them to become more comfortable using these learning aides to solve mathematical problems. My reasoning for allowing this time for exploration was two fold; the first was so that the students became familiar with the manipulatives and second to eliminate some of these behaviors during the lessons.

Activity Two: Identifying Fractions

The next day the lesson focused on identifying fractions. I wanted to ensure the students had a basic understanding of what a fraction is. In this lesson, the students used pattern blocks to explore fractions. The basic set of pattern blocks includes six types of shapes: yellow hexagons, blue rhombi, orange squares, green triangles, red trapezoids and the beige rhombi. Four of the shapes have certain relationships: green triangle are the smallest; a blue rhombus’s area is exactly twice that of a green triangle, a red trapezoid’s
area is exactly three times that of a green triangle, and a yellow hexagon has an area six times that of a green triangle.

I used a lesson from a summer workshop offered through my county for intensive mathematics teachers. The students were given pattern blocks and asked questions like “How many green triangles are in the blue rhombus?” and “If the yellow hexagon equals one then the green triangle equals what?” Then the questions started getting more difficult, for example, “If the yellow hexagon and the blue rhombus equal one, what is the green triangle?” At the end of the lesson students were shown various designs on the overhead and then asked them questions about what fraction of the design is a certain color (Lanius, 2000) (see Figure 7).

Figure 7: An example problem from a lesson focusing on the concepts of fraction.

To move from purely concrete activities towards abstract problem solving, I gave the students a paper with various shapes that they had to tell what fraction of the shape was shaded or not shaded.
Activity Three: Equivalent Fractions

The lesson before moving to multiplication and division of fractions was to find equivalent fractions. Again we started with activities using pattern blocks. Then they were given a choice of two-color counters and Cuisenaire rods to demonstrate equivalent fractions. To move them towards representing fractions pictorially they used manipulatives to find equivalent fractions then drew pictures to illustrate how they used the manipulatives. The students have moved from representing fractions concretely to representing fractions pictorially.

Activity Four: Multiplying and Dividing lessons

To begin the lessons on multiplying and dividing fractions, we went back to the pattern block activities. The students were asked to solve problems like what is 1/4 of 1/2 using the various pattern blocks. During subsequent lessons the students were given a choice of manipulatives to solve various contextually based problems that involved multiplying and dividing fractions. Some examples of the contextual problems are in figure 8.

Example A:

If Jack has 5/7 of a candy bar left over, how much of the original candy bar will Jack have left if he gives his friend Shawn 1/3 of the candy bar?

Example B:

Sarah has 5 yards of material and she needs to cover dining room chairs. If each chair requires ¾ of a yard, how many chairs can she cover?

Figure 8: Example of contextual problems.
While the students were solving problems, I wanted them to explore the use of manipulatives and discover how to model the problem. Once the students solved the problems they drew illustrations of how they used the manipulative. We spent about a week on activities involving multiplying and dividing fractions. We concluded the unit on multiplying and dividing fractions by solving problems on activity sheets that started out with pictures and ended with abstract number problems. I did this to take the students from concrete to representational and finally to the abstract level of problem solving with multiplying and dividing fractions.

Post Interview

After the activities, I conducted interviews with the students. In order to have one-on-one time to conduct the interviews, I had the students rotate through classroom stations and I made myself one of the stations to conduct the interviews. This way I could conduct the interviews without pulling the students from other classes and the rest of the students were actively involved in other learning activities. I used the same interview questions from the pre-interview (see appendix D). I interviewed the students before I gave them the pre-assessment so that I could have a better understanding for the way the students solved problems.

Post assessment

For the post-assessment, I gave the students the same test they took for the pre-assessment. I did this so I could compare how they solved the problems before and after
the lessons using manipulatives. The next time I do a pre and post-assessment, I will revise the questions on the pos-assessment to reduce some of the effect of taking the same assessment twice. I believe I can still look for growth with problems that are similar in nature but using different numbers and settings.

Portfolios

During this study, the students kept portfolios of their activities and writings. I asked the students to keep all of their work in the order that they completed the activities so I could observe any growth that occurred in their performance from the beginning to the end of the lessons. I believe this process also helped the students see that they were learning and making changes to the way they solved fraction problems.

Observations

I also kept a journal of observations of the students’ reactions and comments as they participated in the activities. Keeping a journal enabled me to go back and look at how the students were solving problems after class so that I could plan how I could help them come to a better understanding of the concepts being taught. In order to keep this journal I placed empty paper in a three ring binder and I kept the binder with me during the lessons so that when I heard or saw something of interest I could write it down immediately.
Summary

I used the information from the pre and posttest, pre and post interviews, my observations and the students’ portfolios to determine any changes in the students’ conceptual understanding of multiplying and dividing fractions. I also looked for attitude changes and comfort levels in regards to the students’ use of manipulatives.

The results from the various forms of data that I collected are explained in the next chapter. I looked for growth in the students’ understanding of the concepts taught and for any patterns that emerged from the data. I also looked for a change in their attitude about using manipulatives.
CHAPTER FOUR DATA ANALYSIS

In this chapter I will describe the information I collected and make a note of any themes and patterns that emerged. My research in studying the use of manipulatives to teach multiplying and dividing fractions required the collection of data from several different sources. Pre-assessment and post-assessment, pre-interviews and post interviews, student portfolios, and teacher anecdotal observations were all used to determine the students’ conceptual understanding of multiplying and dividing fractions and to determine their attitude towards using manipulatives before and after the study. I analyzed the data to find the answers to my guiding questions, which are:

1-In what ways does my use of manipulatives affect my 8th grade students understanding of the concept of fractions?

2- In what ways does my use of manipulatives affect my 8th grade students understanding of multiplying and dividing fractions?

Interviews

Pre-Interview

I came to the realization rather early in the interview process that most of the students did not know what the term “manipulative” meant. After I explained the definition of manipulative and provided the students with some examples, almost all of the students seem to recall using some form of manipulatives. I imagine that most were using manipulatives without realizing it. The most common seemed to be manipulatives used in elementary school, for example, counting bears, plastic money, beans, and a cardboard
clock for learning how to tell time. Through the interview process I found that the only manipulatives some of the students remember ever having used to learn mathematics in middle school are the *Hands on Equations* sets (Borenson, 1997).

I also wanted to ascertain how they felt about using manipulatives. Before the lessons began, most of the students felt that the manipulatives would not help them learn how to multiply and divide fractions. The students could not foresee how using a concrete item could help them understand a mathematics concept. Most of the students believed that using manipulatives was for younger students only. They wanted me to teach them the procedures again, and then be done with the whole issue of fractions. After we had a discussion about manipulatives and how they could help the students understand the concept of fractions they were open to the idea of using manipulatives. I reminded the students how they probably learned to add whole numbers in kindergarten. They were first given two sets of bears that they could count to get a total number and then pretty soon they were given a paper with boxes that contained pictures of objects then finally they were given actual addition problems. I then explained this is what we were going to do with multiplying and dividing fractions. We were going to start with manipulatives and work through problems until they were ready to move to pictures and then finally to abstract algorithms.

During the process of these interviews, I found that only two of the students, Kevin and Lacy, reported that they had used manipulatives to learn how to multiply and divide fractions. They both had an intensive mathematics teacher in the 7th grade that employed the use of manipulatives to teach certain topics. But when I asked them to describe how they would use manipulatives to solve “1/3 of 3/5” neither one could explain the method.
in a way that would solve the problem nor did they show a conceptual understanding of the multiplication problem. When I posed this question to the other students, ten of the students responded with an “I don’t know.” Lacy and Anyelo said they would use two-color counters and show 1/3 and 3/5 but they did not know what to do next. Robert, Darius and Kevin gave explanations of how they would use Cuisenaire rods but their explanations did not explain how to solve this multiplication problem. For example, Robert said he would get three rods and take 1/3 of them and then times by 5 blocks. I asked Robert how he would multiply by five blocks then he replied “I guess you can’t, what I mean is I would show 1/3 five times.” After some discussion with Robert he was able to see that his method could have been used to solve 5 times 1/3 by modeling repeated addition but he could not offer a further explanation of how to solve 1/3 times 3/5.

During the pre-interview the students were shown a problem that a fictitious student had worked and they were asked if they thought the students answer was correct (see Figure 9). Then they were asked to explain what the student had done in his first two steps. Six of the students were able to say that the student had the correct answer and of those six, four were able to explain what the student was doing, the other two said they just guessed. Even though four students were able to give an explanation it was procedural in nature and none of the four could tell me why the student performed each of the steps.
1. A student used the following procedures to solve $\frac{1}{5} \div 3$

   Stage 1    \[ \frac{1}{5} \div \frac{3}{1} \]

   Stage 2    \[ \frac{1}{5} \times \frac{1}{3} \]

   Stage 3    \[ \frac{\frac{1}{5}}{\frac{3}{3}} = \frac{1}{15} \]

Do you think the student’s answer is correct?

Explain stages 1 and 2

Figure 9: Explaining how to solve a division problem

The four students who described what the student was doing were Kevin, Matt, Robert, and Anyelo. The students all told me the “keep it, change it, flip it” method that they had memorized. The “keep it, change it, flip it” method refers to the following procedures: you keep the first fraction the way it is, change the division to multiplication, find the multiplicative inverse of the second fraction then multiply numerator by numerator and denominator by denominator. I asked all four students why they would solve the problem this way and they all said, “Just because that’s the way you do it”.

Matt’s response to this question is shown in Figure 10.
Matt’s response

Stage 1  \textit{Turn 3 into a fraction}

Stage 2  \textit{Do the keep it, change it, flip it thing.}

Do you think the student’s answer is correct? Yes

Figure 10: One student’s response to an interview question.

Post interview

I conducted post interviews with the students after spending two weeks using manipulatives to teach lessons on the concepts of fractions and then on multiplying and dividing fractions. After working with these students for two weeks, I have to say that I am not surprised by the outcomes of the post interviews. Through anecdotal observation of the students progressing through each lesson I could see that they were embracing and even enjoying using manipulatives to learn.

Obviously, since we just finished the lesson, they all answered yes to the question “Have you ever used manipulatives to solve multiplying and dividing fractions?” For the question “How do you feel about using manipulatives?” all but one student, Shawn, reported positive feelings towards the use of manipulatives. They no longer felt that it is childish or pointless to use manipulatives in mathematics class. Shawn still reported feeling like the manipulatives were unnecessary and that he could have learned how to multiply and divide fractions without them. I asked him if he had been taught how to
multiply and divide fractions before and he said, “Yes” then I asked him how he was
taught previously. Shawn said that when he was taught about fractions in the past the
teacher showed him the steps to solve the problem worked “a bunch of problems on the
chalkboard” and then gave him several problems to complete on his own for homework.
My next question to Shawn was if he knew how to solve fraction problems before we
started our lessons and he said “not really”. I then asked if he thought that he really had
learned the concepts in the past without the manipulatives and if he now feels that he
understands the concepts since we have completed the lessons using manipulatives.
Shawn responded to me by saying “You’ve got a point.”

Most of the students were now able to explain why the fictitious student took those
steps to solve the division problem. They were able to explain that you had to start with
something that represented 1/5 and then divide that into three equal parts. Then the
students could figure out that one of those three parts was equal to 1/15. This is the same
as saying 1/3 of 1/5, which is a multiplication problem and since multiplication is the
inverse of division that is why the keep it, change it, flip it method works.

One other interview question I asked was if the students thought you could compare the
two fractions 11/13 and 14/17 by cross multiplying and comparing the answers. During
the pre interview only five students answered correctly but during the post interview
twelve students answered correctly. Through the use of two color counters, several of the
students were able to determine that by cross-multiplying they were finding a common
denominator then comparing the numerators to find the larger fraction. They knew that
they could multiply the denominators to get a multiple of each other, this would not
always be the least common denominator but it would be a common one. So from there
they found equivalent fractions using the two color counters for the fractions they were comparing. (see figure 11). Once they did this a few times, some of the students were able to see why they were taught to cross multiply to find the greater fraction.

Which fraction is greater $1/2$ or $3/4$?

*With manipulatives*

![Fraction Manipulatives](image)

*By cross multiplying*

\[
\begin{align*}
\frac{1}{2} \times \frac{3}{4} &= 4 \\
2 \times 3 &= 6
\end{align*}
\]

6 is bigger than 4, so $3/4$ is bigger than $1/2$.

Figure 11: Finding equivalent fractions.
Pre/Post Assessment

After I had interviewed all of my students, I gave them a pre-assessment in which they had to explain how to solve the problems using manipulatives or pictures. There were ten questions on the assessment, on the first four problems the students had to make a drawing of how they solved each problem. For problems five through ten, the students had to use pattern blocks, two-color counters or Cuisenaire rods to solve the problems then draw a picture of how they used the manipulatives.

I used a 3-point rubric to score the students responses to the assessment so 30 points was the highest score possible. Students earned a score of 3 for responses that were correct and showed complete understanding of the concept, a 2 for responses that were partially correct, a 1 for correct answers with no explanation and a zero for incorrect answers. As you can see in Table 2, all of the students made a gain in their score from the pre-assessment to the post-assessment. I had expected their pre-assessments to be low but I was surprised at how low the scores really were. Three of the students were unable to answer any of the questions and three more were only able to answer one question but were unable to offer any explanation.
Table 2: Pre and Post-assessment comparisons

<table>
<thead>
<tr>
<th></th>
<th>Pre-assessment</th>
<th>Post-assessment</th>
<th>Change</th>
</tr>
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<tbody>
<tr>
<td>Shawn</td>
<td>1</td>
<td>24</td>
<td>+23</td>
</tr>
<tr>
<td>Matt</td>
<td>6</td>
<td>23</td>
<td>+17</td>
</tr>
<tr>
<td>Lacy</td>
<td>3</td>
<td>23</td>
<td>+20</td>
</tr>
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<td>21</td>
<td>+20</td>
</tr>
<tr>
<td>Britney</td>
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<td>21</td>
<td>+21</td>
</tr>
<tr>
<td>Andrew</td>
<td>0</td>
<td>12</td>
<td>+12</td>
</tr>
<tr>
<td>Anyelo</td>
<td>3</td>
<td>23</td>
<td>+20</td>
</tr>
<tr>
<td>Jose</td>
<td>3</td>
<td>24</td>
<td>+21</td>
</tr>
<tr>
<td>Robert</td>
<td>0</td>
<td>20</td>
<td>+20</td>
</tr>
<tr>
<td>Sammy</td>
<td>8</td>
<td>29</td>
<td>+21</td>
</tr>
<tr>
<td>Darius</td>
<td>4</td>
<td>23</td>
<td>+19</td>
</tr>
<tr>
<td>Jessica</td>
<td>7</td>
<td>15</td>
<td>+8</td>
</tr>
<tr>
<td>Chealsy</td>
<td>3</td>
<td>25</td>
<td>+22</td>
</tr>
<tr>
<td>Ruth</td>
<td>1</td>
<td>27</td>
<td>+26</td>
</tr>
<tr>
<td>Kevin</td>
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</tr>
<tr>
<td>Mean</td>
<td>3.5</td>
<td>22.5</td>
<td>+19</td>
</tr>
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</table>

As you can see on table 2, the students mean score on the pre-assessment was 3.5. When the students were able to solve a problem correctly on the pre assessment, most the work the students had shown was procedural with no explanations. When they did use pictures, the drawings were of the fractions listed in the problem separately and not a representation of the problem itself (see Figure 12). Only one student attempted to use manipulatives and sketch how he used them on the pre assessment. The remaining students worked most of problems procedurally with a few circles used to represent the given fractions. In figure four, Ruth attempted problem 1 on the pre-assessment by drawing 2 pizzas. On the first pizza she shaded 1/2 of the pizza and she shaded 3/8 of the second pizza. Then she just wrote the answer 1/8, when I ask her how she arrived at that answer she admitted that she just guessed.
Emily ate \( \frac{1}{2} \text{ of } \frac{3}{8} \text{ of a pizza that was left over. How much of the whole pizza did Emily eat?} \)

\[
\begin{array}{c}
\text{Figure 12: Ruth’s solution to problem 1 on the pre assessment.}
\end{array}
\]

After the pre assessment, I asked the students to tell me why they solved the problems the way they did. The students could not give me a specific reason the only response they had was that is the way they were shown.

The students were given the same assessment again about two weeks later after we finished the lessons on multiplying and dividing fractions with the use of manipulatives. This time the students were able to answer most of the questions correctly, and more importantly, the students were able to explain their solutions. In figure 13, you can see how one student answered question number 1 before and after the lessons.
Pre-assessment:

For problems 5 – 10, use pattern blocks, two-color counter or Cuisenaire rods to model each situation. Draw a picture of how you used the manipulative.

Four workers each painted $\frac{3}{5}$ of a wall. How many walls did they paint in all?

Post-assessment:

5. Four workers each painted $\frac{3}{5}$ of a wall. How many walls did they paint in all? $2\frac{2}{5}$ walls

Figure 13: Darius’s solution to problem 5 on the pre assessment and post assessment.

On the pre-assessment Darius did not display an understanding of the concepts presented in the question. He drew 5 hexagons and shaded three so that he could represent $\frac{3}{5}$, Darius did not show the four workers each painting $\frac{3}{5}$ of a wall. He also did not identify that this was a multiplication problem. On the post assessment, you can
see that Darius has represented each of the four workers and shown the four 3/5 of a wall that each worker painted by using Cuisenaire rods. He then started making wholes and found there were 2 whole and 2 parts left. From this he was able to determine that the four workers painted 2 2/5 walls. After the assessment I asked him to explain his illustration and how he used Cuisenaire rods because at first it looked like he drew his own pictures and did not use the manipulatives.

Mrs. Bale: “Darius, can you explain how you used the Cuisenaire rods to solve problem number 5?”

Darius: “Yea. I used 20 of these blue rods because there were 4 workers and the walls were divided into fifths and 4 times 5 is 20.”

Mrs. Bale: “OK, then what did you do?”

Darius: “I separated the 20 blue rods into four piles with five rods to show the four walls, then I separated out 3 from the 5 to show 3/5 out of each wall. Then I just started putting them together until I got 2 whole walls and 2 rods left over.”

Mrs. Bale: “How did you know that the 2 blue rods left over were 2/5 of a wall?”

Darius: “Because the wall was broken down into five pieces and I had 2 of them left over which is 2/5.”
The mean score on the post assessment was 22.5, which was a 19-point increase from the pre assessment. I feel that I can attribute most of this increase to the lessons the students were taught with the use of manipulatives. The students began at a concrete level of understanding and solving problems then moved to the representational and abstract levels of understanding and problem solving.

During the first multiplication lesson the students were using pattern blocks to solve problems and it did not take some of them very long to figure out how to solve the problems. The students were able to see that they were finding groups of fractions and therefore made sense of \( \frac{3}{4} \) of \( \frac{1}{2} \) being the multiplication problem \( \frac{3}{4} \times \frac{1}{2} \). The students were able to apply the concept that they had “x groups of y” so they could figure out future problems rather quickly (see figure 14). This helped them with the contextual problems on the post-assessment because they were able to read the problem and determine what operation they needed to perform from the way the problem was worded.

Some of the increase might be explained by the fact that the students took the same test twice, and the students were refreshed on the concepts of fractions.

\[
3 \times 4 (\text{3 groups of 4}) = 12
\]

![Figure 14: Multiplying as “x groups of y”](image-url)
Students’ understanding of fraction concepts

When the students began working on the assignments they were confused about how to use the manipulatives. Starting with the concepts of a fraction instead of jumping right into multiplying and dividing turned out to be a good idea. I thought they might become bored or disinterested in this type of a lesson since they should have mastered those skills by now. It turned out that the students needed to be re-exposed to identifying fractions, finding fractional parts of various wholes and finding equivalent fractions.

The following findings were based on data collected from students’ portfolios and anecdotal observations of students solving problems using manipulatives.

Identifying Fractions

At the beginning of the day when I asked students to model 2/3 with the manipulatives, several of them had a difficult time identifying 2/3 (see figure 15). We spent time discussing what 2/3 meant such as 2/3 of a whole or 2 out of 3 parts then I had the students explore the manipulatives to find pieces that fit together to make thirds. I also had the students demonstrate various ways to model 2 out of 3. After the students had explored with the manipulatives for several minutes I had everyone chose one way to model 2/3, this time (see figure 16) they were all able to accurately model the fraction.
Students Concept of the Whole

I believe solving and discussing problems involving equivalent fractions and the concept of one whole facilitated their learning how to multiply and divide fraction and understanding the concepts of those operations. Evidence to support this theme can be found from comparing pre-assessment and pre-interview responses to post-assessment and post-interview responses. On the pre assessment and pre interview the students were unable to choose a whole that would support their arriving at a solution. For example, on the pre-assessment the students were drawing 3 circles and shading 2 of them to illustrate 2/3 of a pizza, then when they needed to find 1/2 of 2/3 they were giving an answer of 1 instead of 1/3 (see figure 17)
2/3 of a pizza

When the students have this set up then they take 1/2 of what is shaded they see that they half is 1 full circle so they answer 1/2 x 2/3 = 1 instead of seeing that the 1 circle is 1 of 3 or 1/3 of the pizza.

Figure 17: Example of the importance of using correct whole.

Students’ Working Knowledge of Equivalence

They also were unable to determine an equivalent fraction that would aide in their finding a solution. In the following pre-assessment problem: 2/3 of a pizza is left over. Jim ate 3/4 of the left over pizza. How much of the pizza did Jim eat? The most common answer to this was 3/5 when I ask the students to explain how they came to that answer, Jessica replied “I split the pizza into three pieces so I could have get 2/3, then I cut the 2 pieces in half so now I have four pieces. Now I can take three of those that Jim ate and since I had a total of 5 pieces that means he ate 3/5.” Most of the students agreed with her explanation. It helps to solve this problem if the students understand that the whole pizza is their whole and that 2/3 is equivalent to 4/6 so that they can arrive at the answer of 3/6 or 1/2 instead of 3/5. On the post-assessment, most of the students used the hexagon to
represent the whole pizza then they used the blue rhombus to represent \( \frac{2}{3} \) of the pizza. From there they used the green triangles to represent individual pieces because 6 of them fit on a hexagon. Now they were able to show that he ate 3 out of 6 or \( \frac{1}{2} \) of the original pizza (see figure 18).

![Figure 18: Using manipulatives to solve the pizza problem.](image)

**Importance of Having Equal Parts**

When the lesson on the concepts of fractions began the questions that I asked them were fairly simple and the students were able to easily demonstrate their answers. One of the questions was: “How many green triangles are in the blue rhombus?” The students continued to do well when the questions were switched to represent parts of a whole such as: “If the yellow hexagon is equal to 1, the green triangle equals what?” The students were able to correctly tell me the answer was \( \frac{1}{6} \) and show this with their pattern blocks.

More discussion and time was needed by the students to solve the next set of questions. An example of one of the harder questions is “If the blue rhombus plus the green triangle is equal to \( \frac{3}{8} \), what is \( \frac{3}{4} \)?” The students had to manipulate the pattern blocks for several minutes in order to determine the answer. The students would put the
blue rhombus and the green triangle together but then they wanted to use other pattern blocks to create $\frac{3}{4}$. Ruth said the answer was 2 rhombuses because she put 2 green triangles with the rhombus and triangle to represent fourths then removed three pieces, the blue rhombus and two green triangles, to represent $\frac{3}{4}$. We had a discussion about the importance of equal parts.

Mrs. Bale: “If we were to share halves of a piece of cake

Would it be fair if my half was bigger than yours?”

Ruth: “No, that wouldn’t be fair.”

Mrs. Bale: “What do you think we need for it to be fair?”

Ruth: “I guess we would need both halves to be the same.”

Mrs. Bale: “Think about the problem again. If you have three green triangles and one red rhombus, do you have equal parts that you can name fourths?”

Ruth: “No.”

Mrs. Bale: “What do you think you could use to model fourths?”

Ruth: “I get it. I can use equivalent fractions and use 8 green triangles so that I can show $\frac{3}{8}$ with 3 triangles and $\frac{3}{4}$ with 6 triangles because $\frac{3}{4}$ is equal to $\frac{6}{8}$. “
By allowing Ruth the time to explore with the pattern blocks she was able to discover that she needed to change all of the pieces to green triangles. She was able to see that she needed 8 green triangles and then she could separate them into fourths and finally see that 3/4 is 6 green triangles.

Several other students had this difficulty with using equal parts. They seemed to divide their parts into the number they needed instead of making sure they had equal parts. Several more conversations such as the one I had with Ruth occurred during this class period. The students spent the entire class period solving these types of questions and by the end of the period most of the students were able to represent the problems rather quickly and arrive at the correct answers.

The next day I concentrated the lesson on finding equivalent fractions. I gave the students pattern blocks, Cuisenaire rods and two color counters to represent equivalent fractions. I believe the work they did during the previous two days enabled them to understand how to find equivalent fractions using these manipulatives. Toward the end of the period I gave them a sheet with three problems on it where they had to find equivalent fractions. I asked them to use pattern blocks to solve the first problem 2/3 =? /6, Cuisenaire rods to solve the second 3/5 =? /15 and two-color counters to solve the third 3/4 =? /16. Jose used the yellow hexagon as the whole in the first problem then he used the rhombi to split it into thirds, on two of the rhombi he placed four green triangles and from there he was able to see that 2/3 is equal to 4/6. For the second problem he played with the rods until he found that 15 light green rods were the same length as 5 blue ones. From there he could see that he needed to count how many light green rods was the same length as 3 blue rods. He was able to determine that 3/5 is equal to 9/15. On the third
problem Jose was easily able to use the two color counters to show that $\frac{3}{4}$ is equivalent to $\frac{12}{16}$ (see figure 19).

Figure 19: Jose illustrations of equivalent fractions.
Like Jose, most of the students in the class were able to find an equivalent fraction by using pattern blocks and they were all able to solve the third problem using the two color counters but most of the students had difficulties with the second problem. It took several minutes for anyone to discover how to use the Cuisenaire rods to find an equivalent fraction for the second problem. Jose was the first so I ask him to explain why he chose to use the green and the blue rods. Jose said “I had to find rods that when I lined 15 of one color up they would equal 5 of another color because I am finding equivalent fractions.” Once he said this you could see the light come on for several other students and they started experimenting with the other rods to see if they could find another combination of 5 and 15 that were the same length. Britney was able to find that three dark green rods was the same length as 1 red rod. I believe most of the difficulties using Cuisenaire rods came from being unfamiliar with them because we spent more time using pattern blocks than any other manipulatives during the previous lessons.

Students’ understanding of fraction operations

Once I felt the students had an understanding of the concepts of a fraction, which is to say they could represent 1/3 and 3/5 using manipulatives and justify their representations and find equivalent fractions, then I moved the class on to multiplying and dividing fractions.

We used pattern blocks with the first lesson on multiplying and dividing fractions. Students were asked to solve problems such as: “Pat needs to cut boards measuring 3/4-foot for some picture frames. How many lengths can she cut from and 8-foot board?”
What part of the picture frame would she have left?” and “Four friends buy a total of 3/4 pound of chocolate. How much will each person get if they share the chocolate equally?”

Kevin chose to use Cuisenaire rods on the first problem with the picture frame. He found 4 red rods were equal in length to one brown rod. Kevin lined up 8 brown rods to represent the 8-foot board then he lined up the red ones next to it to represent the fourths. He then put the red rods into groups of three to represent the 3/4 and counted 10 groups. Kevin was then able to see that Pat had 2 pieces left over and at first he wanted to call this 2/4 or 1/2. Then I ask him what how many fourths he needed for a frame, when he said “3” you could see he understood then he said “Oh, you have 2 out 3 pieces left so that would be 2/3 of a picture frame” (see figure 20).

Figure 20: Kevin’s solution to the picture frame problem.

Another student chose to solve the same problem using two color counters (see Figure 21)
Both of these students were able to show why $4 \times \frac{3}{5}$ is equal to $2 \frac{2}{5}$ through their modeling with the manipulatives. Most of the students in this class were able to solve this problem by using Cuisenaire rods, pattern blocks or two color counters. At first, Andrew was unable to solve this problem, he used 8 rods to show the 8 feet and he found 4 smaller rods to use to represent the fourths but he treated the 8 rods separately so his answer was that she could only make 8 picture frames and she had 8 pieces left over (see figure 22). One of the other students was able to help him understand the problem completely. The other student explained to Andrew that the girl in the problem would have 1 piece of wood that is 8-feet long and she would just keep cutting $\frac{3}{4}$-foot pieces so the girl would have more than 8 picture frames.
Through discussion and observation I was able to determine that most of the students were constructing their knowledge about multiplying and dividing fractions and they were able to invent algorithms that worked for them individually or they were able to make the connection with traditional algorithms. Most of the students chose to work with the pattern blocks for most of the problems. I believe the reason for this was because we used pattern blocks on the first lesson and they felt more comfortable solving problems with them. The students did use two color counters most frequently for finding equivalent fractions. The other manipulatives that were out for students’ use such as tangrams and fraction circles were used infrequently.
Summary

In conclusion, most of the data suggest that the students developed a greater understanding of multiplying and dividing fractions. This can be seen in the 19-point increase in the mean between the pre and post-assessment and the way the students answered the post interview questions. The students were taught operations on fractions using traditional, procedural methods prior to the lessons in my class but it was evident in the pre interviews and pre assessment that they did not have a sound knowledge of these concepts. The use of manipulatives helped the students construct their knowledge about fractions and operations on fractions.

The students developed an understanding of the importance of using equal parts, finding equivalent fractions and using the proper whole when multiplying and dividing fractions.

In the next chapter, I will review my findings and make recommendations for future research in using manipulatives in the middle grades mathematics classroom.
According to McCutcheon and Jung (1990), “action research is defined as a systematic form of inquiry that is collective, collaborative, self-reflective, critical, and undertaken by the participants of the inquiry.” (page 145) Action research in an education setting is a reflective process that allows for inquiry into finding solutions to everyday, real problems experienced in schools. It is also useful for researching ways to improve instruction and increase student achievement (Ferrance, 2000). I believe my research falls within the parameters of these definitions as I collected data from my students and reflected upon that data in order to improve instruction and increase student achievement on the concepts of multiplying and dividing fractions.

The purpose of this study was to investigate how my practice of using manipulatives in an eighth grade mathematics classroom influenced the students understanding of multiplying and dividing fractions. The study lasted for three weeks total from pre-interview to the post-assessment. There were 15 students who participated in the study who were in one of my intensive mathematics classes. Data were collected from pre and post-interviews, pre and post-assessments, students’ work collected in portfolios and anecdotal observations of the students solving problems.

Conclusion

The results support the contention of various articles I have read indicating that when manipulatives are used appropriately they help the students grasp the concepts of multiplying and dividing fractions (Moyer, 2001; NCTM, 2000; Stupiansky &
Stupiansky, 1990). Manipulatives help students grasp these concepts by developing a meaning for the operations rather than the application of rote procedures (Bezuk & Armstrong, 1992).

As I had mentioned in chapter four, when we first began the study, most of the students were not familiar with manipulatives and they were unable to demonstrate the meaning of various concepts of fractions. Thus choosing to start with a lesson on fraction concepts and equivalent fractions was a wise decision. The National Assessment of Educational Progress reports show that fractions are “exceedingly difficult for children to master” (NAEP, 2001, p.5). This was evident in my observations of students solving problems at the beginning of this study. The students in this study clearly did not have a solid concept of fractions before the study began.

I believe that teachers who continue to teach fractions using a traditional method need to adapt their methods to reflect current research findings and meets the needs of today’s students. This is in accord with Groff (1996) who states, “The teaching of fractions must be based on experimental research findings instead of on tradition and arbitrary opinions” (p. 177).

During my research I found explanations for why students are not exposed to manipulatives more frequently. I can understand how these issues would be a problem for teachers who are not using the manipulatives properly. While I agree it does take a little more time to plan a well-developed lesson than it does to read procedures and instructions from a textbook, I feel the benefits the children gain far out weigh the little extra time. Planning time can be greatly reduced if a resource book is developed and the manipulatives organized. I believe the noise level can be controlled to an extent to each
teacher’s preference. When the students are engaged in mathematical content and on task
the noise level is not an issue for me. I believe, through good staff development activities,
the issues of the teachers’ lack of understanding and some teachers believing that
manipulatives are as waste of time can be effectively addressed.

**Research Questions**

Research Question #1: In what ways does my use of manipulatives affect my students
understanding of the concept of fractions?

The results of this study are in accord with previous research that has found that
through the use of manipulatives, students are able to develop an understanding of the
concepts of fractions. I was able to determine this through the 19 point gain in their post
assessment, their answers to post interview questions and through the anecdotal
observations I made as they were solving problems during the lessons.

Research Question #2: In what ways does my use of manipulatives affect my students
understanding of multiplying and dividing fractions?

My response to this is similar to question number 1, most of the students developed an
understanding of multiplying, and dividing fractions by using manipulatives to solve
contextually-based problems.
Limitations

There were several limiting factors that occurred during this study that may have affected the results. These factors include; student attendance, time constraints, limited number of students (15), and county mathematics frameworks.

Several students have an ongoing attendance issue, which means they missed several lessons during this study. I attempted to work with them individually as much as possible but absences could have had an effect on their individual growth. I have compared student absences to their pre and post –assessment scores in Table 3. You can see that the students who had the lowest point increase from the pre-assessment to the post assessment were also absent the most days during the three weeks of this study. The two students continued to be absent more days than the other students in this class throughout the school year and they have also continued to have some of the lowest grades in the class.
Table 3: A comparison of student absences to their pre and post-assessment scores.

<table>
<thead>
<tr>
<th>Student</th>
<th>Pre-assessment</th>
<th>Post-assessment</th>
<th>Days Absent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shawn</td>
<td>1</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>Matt</td>
<td>6</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>Lacy</td>
<td>3</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>Chris</td>
<td>1</td>
<td>21</td>
<td>0</td>
</tr>
<tr>
<td>Britney</td>
<td>0</td>
<td>21</td>
<td>2</td>
</tr>
<tr>
<td>Andrew</td>
<td>0</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>Anyelo</td>
<td>3</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>Jose</td>
<td>3</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>Robert</td>
<td>0</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>Sammy</td>
<td>8</td>
<td>29</td>
<td>0</td>
</tr>
<tr>
<td>Darius</td>
<td>4</td>
<td>23</td>
<td>1</td>
</tr>
<tr>
<td>Jessica</td>
<td>7</td>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>Chealsy</td>
<td>3</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>Ruth</td>
<td>1</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td>Kevin</td>
<td>12</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td>Mean</td>
<td>3.5</td>
<td>22.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Due to the time constraint of a 54-minute class period, there were instances when we ran out of time before we made it completely through our day’s lesson. In most cases, I finished the lesson on the next day but due to the county mathematics framework we had a limited number of days to spend on fractions.

I believe that if the study involved more students it would add greater strength to the study. With more students I may have been able to find more trends and patterns in their problem solving.
Recommendations

Through the improved understanding and research, I will continue to use manipulatives to teach multiplying and dividing fractions. One change I have incorporated is to spend a little more time on the concepts of fractions before I begin the lesson on multiplying and dividing fractions. I believe this will have two benefits, one, the students will become more familiar with the different types of manipulatives and secondly, they will develop a greater understanding of fraction concepts.

Another recommendation deals with teacher knowledge of how to use manipulatives to teach mathematics. Teachers need to possess an understanding of the manipulatives as they pertain to the concept being taught or the manipulatives just become a diversion to the lesson (Moyer, 2001). Next year, before I use manipulatives to teach multiplying and dividing fractions, I will gain a better understanding of the different ways students can use these valuable learning aides to solve various problems. I have familiarized myself with all of the manipulatives that I use in my classroom and I will spend more time using them to solve problems so that I can be more prepared for some of the unusual methods the students find to solve their problems (see figure 23). In figure 23 the students were asked the following problem: There is 3/8 of Grant’s birthday cake left over. If he eats ½ of the left over cake for breakfast, how much of the original cake did he eat?” The student in this picture used two-color counters where most of the other students chose to use Cuisenaire rods. He first demonstrated 3/8 with the counters then spent some time trying to figure out how to find 1/2 of 3. Once he had spent some time thinking about this I asked him “When you find 1/2 of something what do you divide by?” He answered “2”
then he was able to “split” each of the counters, once he saw that he now had 16 counters
he made the connection to equivalent fractions. Now he was able to set up 6/16 and he
knew he could find half of 6. This is when he was able to discover that his answer to the
question was 3/16.

Figure 23: A student’s unusual response to a division question.

One thing I would change about the data collection would be to extend the study and
give post-assessments at various time intervals to determine if the level of understanding
is long lasting. I would give a post-assessment after the lessons then again around 6
weeks after the lessons ended.

In the future, I plan to study the affects of using manipulatives on other mathematics
concepts taught at the middle school level. Principles and Standards (NCTM, 2000) states
that “In the middle-grades mathematics classroom, young adolescents should regularly
engage in thoughtful activity tied to their emerging capabilities of finding and imposing
structure, conjecturing and verifying, thinking hypothetically, comprehending cause and effect, and abstracting and generalizing” (p.211). I believe that this can be accomplished through the appropriate use of manipulatives.

The students have a greater understanding of the importance of using the appropriate whole and of using equal parts when they are solving problems. The students are also able to determine the appropriate operation needed in a contextually based problem. After analyzing all of the data I have collected, I believe that the manipulatives used in this study made it possible for my students to learn the concepts of multiplying and dividing fractions.
July 27, 2005

Vickie Balc
528 Grandview Way East
Casselberry, FL 32707

Dear Mrs. Balc:

With reference to your protocol #05-2744 entitled, “The Effects of Using Manipulatives on Students’ Conceptual Understanding of Multiplying and Dividing Fractions” I am enclosing for your records the approved, expedited document of the UCFIRB Form you had submitted to our office. This study was approved by the Chairman on 7/23/05. The expiration date for this study will be 7/22/06. Should there be a need to extend this study, a Continuing Review form must be submitted to the IRB Office for review by the Chairman or full IRB at least one month prior to the expiration date. This is the responsibility of the investigator. Please notify the IRB when you have completed this study.

Please be advised that this approval is given for one year. Should there be any addendums or administrative changes to the already approved protocol, they must also be submitted to the Board through use of the Addendum/Modification Request form. Changes should not be initiated until written IRB approval is received. Adverse events should be reported to the IRB as they occur.

Should you have any questions, please do not hesitate to call me at 407-823-2901.

Please accept our best wishes for the success of your endeavors.

Cordially,

Barbara Ward, CIM
IRB Coordinator

Copy: IRB file

BW: cc
THE UNIVERSITY OF CENTRAL FLORIDA
INSTITUTIONAL REVIEW BOARD (IRB)

IRB Committee Approval Form

PRINCIPAL INVESTIGATOR(S): Vickie Bala
IRB #: 05-2744

PROJECT TITLE: The Effects of Using Manipulatives on Students' Conceptual Understanding of Multiplying and Dividing Fractions

[X] New project submission
[ ] Resubmission of lapsed project #
[ ] Continuing review of lapsed project #
[ ] Continuing review of #
[ ] Study expires
[ ] Initial submission was approved by expedited review
[ ] Initial submission was approved by full board review but continuing review can be expedited
[ ] Suspension of enrollment email sent to PI, entered on spreadsheet, administration notified

Chair

[X] Expedited Approval
Dated: 07/22/05

Cite how qualifies for expedited review:
minimal risk and

[ ] Exempt
Dated:

Cite how qualifies for exempt status:
minimal risk and

[X] Expiration
Date: 04/30/06

Complete reverse side of expedited or exempt form

[ ] Waiver of documentation of consent approved
[ ] Waiver of consent approved
[ ] Waiver of HIPAA Authorization approved

NOTES FROM IRB CHAIR (IF APPLICABLE): Approved with notes

[ ] Waiver of documentation of consent approved
[ ] Waiver of consent approved
[ ] Waiver of HIPAA Authorization approved

[ ] Waiver of documentation of consent approved
[ ] Waiver of consent approved
[ ] Waiver of HIPAA Authorization approved

Chair:

Signed:

Dr. Sophia Dziak

Co-Chairs:

Signed:

Dr. Jacqueline Byers

Complete reverse side of expedited or exempt form

[ ] Waiver of documentation of consent approved
[ ] Waiver of consent approved
[ ] Waiver of HIPAA Authorization approved

NOTES FROM IRB CHAIR (IF APPLICABLE): Approved with notes

[ ] Waiver of documentation of consent approved
[ ] Waiver of consent approved
[ ] Waiver of HIPAA Authorization approved

Chair:

Signed:

Dr. Sophia Dziak

Co-Chairs:

Signed:

Dr. Jacqueline Byers

Complete reverse side of expedited or exempt form

[ ] Waiver of documentation of consent approved
[ ] Waiver of consent approved
[ ] Waiver of HIPAA Authorization approved

NOTES FROM IRB CHAIR (IF APPLICABLE): Approved with notes

[ ] Waiver of documentation of consent approved
[ ] Waiver of consent approved
[ ] Waiver of HIPAA Authorization approved
Dear Students,

My name is Vickie Bale and I am a student at the University of Central Florida. I would like to ask you to participate in activities where you can learn about multiplying and dividing fractions. Before the activities/lessons I will give you a pretest and pre-survey and then after the lessons/activities I will give a posttest and post survey. If you choose to take part, you may stop at any time and you will not have to answer any questions you do not want to answer. Only group information will be shared and taking part in this exercise will not affect your grades. Would you like to participate?
August 2, 2005
Dear Parent/Guardian:

I am a graduate student at the University of Central Florida under the supervision of faculty member, Dr. Juli K. Dixon, conducting research on the effects of using manipulatives to teach students the concept of multiplying and dividing fractions. The purpose of this study is to help students gain conceptual knowledge of those skills. The results of the study may help teachers better understand the amount of knowledge gained and allow them to design instructional practices accordingly.

Your child, along with the others that participate, will be given a pretest and a presurvey to measure their knowledge and attitudes about fractions and their self-awareness of the use of manipulatives to solve mathematical problems. The students will then participate in a series of lessons about multiplying and dividing fractions that allow them to use manipulatives to solve problems. They will keep a portfolio of their work and be observed by me while they solve problems. After activities/lessons, they will be given a posttest and post survey to measure academic growth in the above mentioned topics.

Although the children will be asked to write their names on the tests, surveys and assignments for matching purposes, their identity will be kept confidential to the extent provided by law. We will replace their names with code numbers. Results will only be reported in the form of group data. Participation or nonparticipation in this study will not affect the children's grades or placement in any programs.

You and your child have the right to withdraw consent for your child's participation at any time without consequence. There are no known risks or immediate benefits to the participants. No compensation is offered for participation. Group results of this study will be available in April upon request. If you have any questions about this research project, please contact me at (407) 320-3850 or my faculty supervisor, Dr. Juli K. Dixon, at (407) 823-4140. Questions or concerns about research participants' rights may be directed to the UCFIRB office, University of Central Florida Office of Research, Orlando Tech Center, 12443 Research Parkway, Suite 207, Orlando, FL 32826. The hours of operation are 8:00 am until 5:00 pm, Monday through Friday except on University of Central Florida official holidays. The phone number is (407) 823-2901.

Sincerely,

Vickie l. Bale

I have read the procedure described above.

I voluntarily give my consent for my child, ____________________________ to participate in Vickie Bale’s study of the use of manipulatives on the conceptual understanding of multiplying and dividing fractions.

_________________________ /

Parent/Guardian Date

I would like to receive a copy of the procedure description.

I would not like to receive a copy of the procedure description.
Sample Interview Questions

1. A student used the following procedure to solve \( \frac{1}{5} \div 3 \).

   Stage 1
   \[
   \begin{array}{c}
   \frac{1}{5} \div \frac{3}{1} \\
   \end{array}
   \]

   Stage 2
   \[
   \begin{array}{c}
   \frac{1}{5} \times \frac{1}{3} \\
   \end{array}
   \]

   Stage 3
   \[
   \begin{array}{c}
   \frac{1}{15} \\
   \end{array}
   \]

   Stage 4
   \[
   \begin{array}{c}
   \frac{1}{15} \\
   \end{array}
   \]

   Do you think that the student's answer is correct? Yes/No

   Explain Stages 1 and 2.

2. A student used the following procedure to compare the fractions \( \frac{11}{18} \) and \( \frac{14}{17} \).

   He wrote: \( 14 \times 13 < 11 \times 17 \)

   Hence
   \[
   \begin{array}{c}
   \frac{14}{13} < \frac{11}{17} \\
   \end{array}
   \]

   Do you think that the student's answer is correct? Yes/No

   Do you think that the method he uses for comparing fractions is adequate? Yes/No

   Explain your answer.

3. Have you ever used manipulatives to solve multiplying and dividing fraction problems before?

4. How would you use manipulatives to solve the following problem

   \( \frac{1}{3} \times \frac{3}{5} \)?

5. How do you feel about using manipulatives?
APPENDIX E: PRE AND POST-ASSESSMENT
PRE AND POST ASSESSMENT
FRACTIONS

Make a drawing to show how you found each answer.

1. Emily ate 1/2 of 3/8 of a pizza that was left over. How much of the whole pizza did Emily eat?

2. Mrs. Nelson’s class had a pizza party. They had 1 2/3 pizza left over. Five teachers shared the left over pizza equally. How much of a pizza did each teacher get?

3. \( 2 \div \frac{3}{4} \)

4. \( \frac{1}{3} \times \frac{3}{4} \)

For problems 5 – 10, use pattern blocks, two-color counter or Cuisenaire rods to model each situation. Draw a picture of how you used the manipulative.

5. Four workers each painted 3/5 of a wall. How many walls did they paint in all?
6. Four friends but a total of \( \frac{3}{4} \) pound of chocolate. How much will each person get if they share the chocolate equally?

7. \( \frac{2}{3} \) of a pizza is left over. Jim ate \( \frac{3}{4} \) of the left over pizza. How much of the pizza did Jim eat?

8. \( \frac{3}{5} \times \frac{5}{6} \)

9. \( 1 \frac{3}{4} \times \frac{2}{3} \)

10. \( \frac{3}{4} \div \frac{1}{8} \)
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