High Speed Turbo Tcm Ofdm For Uwb And Powerline System

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HIGH SPEED TURBO TCM OFDM

FOR UWB AND POWERLINE SYSTEM

by

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Major Professor: Lei Wei
ABSTRACT

Turbo Trellis-Coded Modulation (TTCM) is an attractive scheme for higher data rate transmission, since it combines the impressive near Shannon limit error correcting ability of turbo codes with the high spectral efficiency property of TCM codes. We build a punctured parity-concatenated trellis codes in which a TCM code is used as the inner code and a simple parity-check code is used as the outer code. It can be constructed by simple repetition, interleavers, and TCM and functions as standard TTCM but with much lower complexity regarding real world implementation. An iterative bit MAP decoding algorithm is associated with the coding scheme.

Orthogonal Frequency Division Multiplexing (OFDM) modulation has been a promising solution for efficiently capturing multipath energy in highly dispersive channels and delivering high data rate transmission. One of UWB proposals in IEEE P802.15 WPAN project is to use multi-band OFDM system and punctured convolutional codes for UWB channels supporting data rate up to 480Mb/s. The HomePlug Networking system using the medium of power line wiring also selects OFDM as the
modulation scheme due to its inherent adaptability in the presence of frequency selective channels, its resilience to jammer signals, and its robustness to impulsive noise in power line channel. The main idea behind OFDM is to split the transmitted data sequence into $N$ parallel sequences of symbols and transmit on different frequencies. This structure has the particularity to enable a simple equalization scheme and to resist to multipath propagation channel. However, some carriers can be strongly attenuated. It is then necessary to incorporate a powerful channel encoder, combined with frequency and time interleaving.

We examine the possibility of improving the proposed OFDM system over UWB channel and HomePlug powerline channel by using our Turbo TCM with QAM constellation for higher data rate transmission. The study shows that the system can offer much higher spectral efficiency, for example, 1.2 Gbps for OFDM/UWB which is 2.5 times higher than the current standard, and 39 Mbps for OFDM/HomePlug1.0 which is 3 times higher than current standard. We show several essential requirements to achieve high rate such as frequency and time diversifications, multi-level error protection. Results have been confirmed by density evolution. The effect of impulsive noise on TTCM coded OFDM system is also evaluated. A modified iterative bit MAP decoder is provided for channels with impulsive noise with different impulsivity.
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CHAPTER I

INTRODUCTION

1.1 Background and Motivation

Information transferred within an electronic communication channel is always liable to be corrupted by noise within the channel. The need therefore arises to be able to preserve information accurately as it journeys through a noisy channel. Addressing this problem, Shannon [1] in 1948, showed that arbitrarily reliable transmission is possible through the noisy channel if the information rate in bits per channel use is less than the channel capacity of the channel. Furthermore, Shannon et al. proved the existence of codes that enable information to be transmitted through a noisy channel such that the probability of errors is as small as required, providing that the transmission rate does not exceed the channel capacity. This is now known as the channel coding theorem. The codes referred to in the channel coding theorem do not
prevent the occurrence of errors, but rather allow their presence to be detected and corrected. These codes are known as *error-detecting* and *error-correcting codes*, or in short *error-control codes*.

In today’s telecommunications market there are dramatically increasing demands for capacity, high data rates, service diversity, and service quality, which have to be achieved with spectrum utilization efficiency and low complexity of technologies. The error control coding plays a key role in the design of such digital communications systems. The aim of the error control is to ensure that the received information is as close as possible to the transmitted information, with as low as possible complexity. A well known result from Information Theory is that a randomly chosen code of sufficiently large block length is capable of approaching channel capacity [1]. However, the optimal decoding complexity increases exponentially with block length up to a point where decoding becomes physically unrealizable. Much of communications and coding research has been driven by the problem of efficient data communications over transmission medium impaired by noise and interference over the past half century.

As the landmark developments in coding area, the invention of turbo error control codes [2] and the rediscovery of low-density parity-check (LDPC) codes [3] [4] have created tremendous excitement since the gap between the Shannon capacity limit and practically feasible channel utilization is essentially closed. Since then, much attention has been drawn to theoretically understand the essence of turbo codes and LDPC codes. Motivated by the principle of turbo codes, researchers have come
up with many compound codes, such as: serially concatenated codes [5], parallel concatenated codes [6] [7], various product code [8], Turbo Trellis-coded Modulation (TTCM) [9] [10] [11] [12] [13] [14], multilevel codes [15] [16] [17], and parity-concatenated codes [18] [19] [20]. Among aforementioned compound codes, TTCM is an attractive scheme for higher data rate transmission, since it combines the impressive near shannon limit error correcting ability of turbo codes with the high spectral efficiency property of TCM.

Motivated by [18] [19], which concatenate convolutional codes with low-density parity-check codes and obtain the performance within 0.45 dB of the Shannon limit, we explore the concatenation of trellis-coded modulation with low-density parity-check codes and build the corresponding decoding structure. The objective is to develop a novel coding/decoding scheme suitable for current or desired communication systems with superior bit error rate performance over existing systems at a high bandwidth efficiency with low complexity.

Digital multimedia applications as they are getting common lately create an ever increasing demand for broad band communication systems. Ultra-wideband (UWB) communications has received great interest from the research community and industry due to its potential strength of leveraging extremely wide transmission bandwidths, to produce such desirable capabilities as extremely high data rate at short ranges, accurate position location and ranging, immunity to significant fading, high multiple access capability and potentially easier material penetrations [21] [22] [23] [24] [25].
It is essential for a wireless system to deal with the existence of multiple propagation paths (multipath) exhibiting different delays, resulting from objects in the environment causing multiple reflections on the way to the receiver. The large bandwidth of UWB waveforms significantly increases the ability of the receiver to resolve the different reflections in the channel [26] [27] [28] [29] [30]. Two basic solutions for inter-symbol interference (ISI) caused by multi-path channels are equalization and orthogonal frequency-division multiplexing (OFDM) [31] [32] [33].

In February 2002, the Federal Communications Commission allocated 7400 /MHz of spectrum for unlicensed use of commercial ultra-wideband (UWB) communication applications in the 3.1-10.6GHz frequency band. This move has initiated an extreme productive activity for industry and academic. Because of the restrictions on the transmit power, UWB communications are best suited for short-range communications: sensor networks and personal area networks (PANs). For highly dispersive channels like UWB, an orthogonal frequency-division multiplexing (OFDM) scheme is more efficient at capturing multipath energy than an equivalent single-carrier system using the same total bandwidth [34] [35] [36] [37] [38]. OFDM systems possess additional desirable properties, such as high spectral efficiency, inherent resilience to narrow-band RF interface and spectrum flexibility. IEEE P802.15 WPAN project [39] proposed a multiband OFDM system for UWB channel with data rate up to 480Mb/s by using punctured convolutional codes. We try to improve the system by using our TTCM functional parity-concatenated TCM code for offering much higher spectral
Increasing interest in smart home automation or home networks has driven the use of the low voltage power line as a high speed data channel. Powerline communications stands for the use of power supply grid for communication purpose. Power line network has very extensive infrastructure in nearly each building. Because of that fact the use of this network for transmission of data in addition to power supply has gained a lot of attention. Since power line was devised for transmission of power at 50-60 Hz and at most 400 Hz, the use this medium for data transmission, at high frequencies, presents some technically challenging problems. Besides large attenuation, power line is one of the most electrically contaminated environments, which makes communication extremely difficult. Further more the restrictions imposed on the use of various frequency bands in the power line spectrum limit the achievable data rates.

OFDM has been chosen as the modulation technique in Home Plug systems for high speed networking using the medium of power line wiring because of its inherent adaptability in the presence of frequency selective channels, its resilience to jammer signals, and its robustness to impulsive noise in power line channel. Again, we are trying to implement our parity-concatenated TCM coding/decoding scheme onto the current Home Plug system for offering higher data rate over power line channel.
1.2 Thesis Outline

The thesis is organized as follows. Chapter II first gives a technical review of previous work on TCM coding schemes, such as Ungerboeck’s TCM [40] [41] [42], multidimensional TCM [16] [43] [44] [45], and Forney’s concatenated TCM [46], followed by the description of the existed parity-concatenated TCM codes by [18] [19] [20]. As a natural extension of binary turbo codes, several turbo trellis coded modulation (TTCM) schemes have been developed for bandwidth-limited communications systems, and the remarkable error performance close to the Shannon capacity limit has been achieved. The corresponding decoding algorithms for coding schemes mentioned above will be explored thereafter, which will help build the iterative decoding scheme for our TTCM-functional parity-concatenated TCM codes in chapter III. Then, the principle of multicarrier modulation (MCM) will be highlighted and some notation specifically defined for MCM system will be introduced in this chapter for easy description in later chapters.

In chapter III, the architecture for UWB system based on multiband OFDM in IEEE P802.15 WPAN project proposal will be introduced first. Since our concern is the performance of the coded OFDM system, we follow the OFDM architecture in the standards and replace the punctured convolutional coding in standard by our parity-concatenated TCM codes. Then we will illustrate our proposed parity-concatenated TCM encoding structure, which is constructed by a punctured parity-concatenated
trellis codes in which a TCM code is used as the inner code and a simple parity-check code is used as the outer code. It functions as turbo TCM and has potential for offering much higher spectral efficiency when used in OFDM systems. The detailed advantage of our encoding scheme over Benedetto’s TTCM structure, such as how it functions as Turbo TCM, how it saves constituent codes and interleavers of conventional TTCM and how to extend the simple encoder structure to more complicated parity-concatenated TCM for coding rate diversity will be given sequentially. The corresponding iterative decoding algorithm extended from the standard binary turbo codes for our parity-concatenated TCM codes will be illustrated thereafter.

Then we will focus on the application performance of this Turbo TCM codes in OFDM system over UWB channel. We show several essential requirements to achieve high rate such as frequency and time diversifications, multi-level error protection and etc. OFDM modulation, UWB channel model, OFDM symbols passing through UWB channel, equalization at the receiver, information recovery and system performance evaluation through density evolution will all be elaborated in this chapter.

Chapter IV presents performance of our proposed Turbo TCM codes when applied to the HomePlug System. The HomePlug Power Line Networking system is specified only for operation on residential AC power lines carrying nominal AV voltages from 120 V to 240 V. The powerline channel characteristics, OFDM modulation scheme, interleaver design will be covered in this chapter. We replace the convolutional codes in forward error correction (FEC) part specified in the standard by our turbo TCM
encoder and evaluate the system’s performance through simulation over measured 60 feet powerline channel.

Chapter V illustrates the effect of impulsive noise on the TTCM coded OFDM system. The impulsive noise is an additive disturbance that arises primarily from the switching electric equipment. Therefore, bursty or isolated errors are usually generated by an impulsive noise affecting consecutive symbols in the trellis-based decoding algorithms since such decoders heavily rely on the history of the symbol sequence. We evaluate the system performance suffering impulsive noise with different impulsivity by modifying our iterative bit MAP decoding algorithm.

Finally in chapter VI, a brief summary of the accomplished work is presented followed by the discussion of further research in this area.

1.3 Contributions

The key contributions of this thesis are summarized below:

(1) Parity-concatenated TCM scheme, which functions as a Turbo TCM and gain several advantages over the conventional TTCM, is proposed; (chapter III)

(2) A robust iterative Bit MAP decoding algorithm is developed for the proposed parity-concatenated TCM. The superior performance can be achieved; (chapter III)
(3) The proposed parity-concatenated TCM is applied to OFDM/UWB system, which improves the UWB proposals in IEEE P802.15 WPAN project by offering much higher spectral efficiency. The real world application is suggested; (chapter III)

(4) Performance evaluation for turbo TCM using union bound is explored with a new method for computing the error weight distribution for turbo TCM codes; (chapter III)

(5) The performance of proposed parity-concatenated TCM scheme and iterative decoding algorithm is confirmed by density evolution; (chapter III)

(6) The proposed parity-concatenated TCM is applied to OFDM/HomePlug system, which improves the Home Plug system by offering higher spectral efficiency and better performance. The real world application is also suggested; (chapter IV)

(7) The performance of TTCM coded OFDM system suffering impulsive noise in different channels with different impulsivity is evaluated. The iterative bit MAP algorithm is modified to match the statistical property of the impulsive noise. (chapter V)
1.4 Paper List


(2) Y. Wang, L. Yang, and L. Wei, Turbo TCM Coded OFDM System For Powerline Channel, Turbo-coding 2006, Apr. 3-7, 2006, Munich, Germany.


(5) Y. Wang and L. Wei, High Speed Turbo TCM OFDM Powerline System, prepared for journal paper submission.

(6) L. Yang, Y. Wang, and L. Wei, Turbo TCM Coded OFDM Systems for Impulsive Noise Channel, prepared for journal paper submission.
CHAPTER II

LITERATURE REVIEW

2.1 Trellis Coded Modulation

Power and bandwidth are limited resources in modern communication systems, and efficient exploitation of these resources will invariably increase the complexity of the system. One very successful scheme of achieving significant coding gain over conventional uncoded multilevel modulation without compromising bandwidth efficiency was proposed by Ungerboeck [47] in 1976 and was subsequently termed as trellis-coded modulation (TCM) [40] [41] [42]. TCM schemes employ redundant nonbinary modulation in combination with a finite-state encoder which governs the selection of modulation signals to generate coded signal sequences. In the receiver, the noisy signals are decoded by a maximum-likelihood sequence decoder. A simple 4-state TCM scheme can achieve 3 dB gain over conventional uncoded modulation without band-
width expansion or reduction of the effective information transmission rate. With more complex TCM scheme (multi-dimensional TCM), the coding gain can reach 6 dB. The most practical selection is 4-D WEI TCM scheme [44].

2.1.1 Ungerboeck’s Trellis-coded Modulation

The concept of TCM is to use signal-set expansion to provide redundancy for coding, and to design coding and signal mapping functions jointly so as to maximize the "free distance" (minimum squared Euclidean distance –MSED) between coded signal sequences. This allows the construction of modulation codes whose free distance significantly exceeds the minimum distance between uncoded modulation signals, at the same information rate, bandwidth, and the signal power. Figure 2.1 depicts the general structure of TCM encoder/modulator.

Figure 2.1: General structure of encoder / modulator for trellis-coded modulation.

When $k$ bits are to be transmitted per encoder/modulator operation, $\tilde{k} \leq k$ bits are expanded by a rate $\tilde{k}/(\tilde{k} + 1)$ binary convolutional encoder into $\tilde{k} + 1$ coded bits.
These bits are used to select one of $2^{\hat{k}+1}$ subsets of a redundant bits determined $2^{\hat{k}+1}$-ary signal set. The remaining $k - \hat{k}$ uncoded bits determine which of the $2^{k-\hat{k}}$ signals in this subset is to be transmitted.

![Diagram](image)

Figure 2.2: Partitioning of 8-PSK channel signals into subsets with increasing minimum subset distances ($\Delta_0 < \Delta_1 < \Delta_2$; $E|a_n^2| = 1$).

Maximizing free minimum squared Euclidean distance (MSED) is based on a mapping rule called "mapping by set partitioning". This mapping follows from successive partitioning of a channel-signal set into subsets with increasing minimum distance $\Delta_0 < \Delta_1 < \Delta_2...$ between the signals of these subsets. The partitioning is repeated $\hat{k} + 1$ times until $\Delta_{\hat{k}+1}$ is equal or greater than the desired MSED of the TCM to be designed. This concept is illustrated in Figure 2.2 and 2.3(a) for 8-PSK and 16-QASK modulation respectively, and is applicable to all modulation forms of Figure 2.1.

The encoding process of trellis code can be represented by the trellis diagram.
Figure 2.3(b) shows the trellis representation of a 4-state Ungerboeck code \((h^0, h^1) = (2, 5)\) [40]. The thicker line in Figure 2.3(b) represents an error event. Note that transition from current state to next state actually comprises 4 parallel transitions resulting from 2 uncoded bits.

The soft-decision decoding of the trellis codes is accomplished in two steps: In the first step, called "subset decoding", within each subset of signals assigned to parallel transitions, the signal closest to the received channel output is determined. These signals are stored together with their squared distances from the channel output. In the second step, the Viterbi algorithm [48] is used to find the signal path through the code trellis with the minimum sum of squared distances from the sequence of noisy channel outputs received. Only the signals already chosen by subset decoding are considered.

When the TCM is employed for the transmission over AWGN channel at high SNR, the BER (bit error rate) performance of TCM is mainly determined by the MSED.
$d_{\text{free}}^2$, which is the minimum value of the squared parallel transition distance $\Delta_k^2$ and the coded minimum squared Euclidean distance $\Delta_k^2$, i.e., $d_{\text{free}}^2 = \min(\Delta_k^2, \Delta_{k+1}^2)$. If $\Delta_k^2 < \Delta_{k+1}^2$, we can say the BER caused by $\Delta_k^2$ is dominant and $\Delta_{k+1}^2$ can be ignored.

For the encoder realizations, Figure 2.4 gives two structures. One is called a systematic encoder with feedback and the other is feedback free encoder. The forward and backward connections in the systematic encoder are specified by the parity-check coefficients of the code.

2.1.2 Multi-dimensional TCM

Many powerful multi-dimensional (M-D) trellis codes have been discovered due to a number of potential advantages over the usual 2-D schemes. One of them is the M-D Wei codes [44] [45] which have been the most attractive selection for many applications such as the high-rate voice band modem and the ADSL modem.

Multi-dimensional signals can be transmitted as sequences of constituent one- or
two-dimensional signals. For instance, 2N-D TCM encoder, can be viewed as formed
by N constituent 2-D encoders. If each 2-D signal transmit \( k \) bits, then each 2N-D
signal needs to transmit \( Nk \) bits. The principle of using a redundant signal set
of twice the size needed for uncoded modulation is maintained. Thus, 2N-D TCM
schemes uses \( 2^{Nk+1} \)-ary sets of 2N-D signals. Compared to 2-D TCM scheme, this
results in less signal redundancy in the constituent 2-D signal sets.

Some terminology regarding the M-D set partition needs to be clarified here. A
lattice is partitioned into families, subfamilies and sublattices with strict increasing
MSED. Only the bottom level of a partitioning is referred to as sublattice. This level
will be assigned to the state transition or equivalently, specified by the output of a
trellis code.

In general, the partitioning of a 2N-D lattice may be done as follows. Suppose the
desired MSED of each 2N-D sublattice is \( \Delta_0 \). The first step is to partition its con-
stituent N-D lattices into families, subfamilies and sublattices with increasing MSED.
Each finer partitioning of the N-D lattice increases the MSED by a factor of two, with
the MSED of each N-D sublattice also equal to \( \Delta_0 \). The second step is to form 2N-
D types, each type corresponding to a concatenation of a pair of N-D sublattices.
The MSED of each 2N-D type is also \( \Delta_0 \). Those 2N-D types are grouped into 2N-D
sublattices with the same MSED, based on the N-D subfamilies. If there are M N-
D sublattices in each N-D subfamily, then each 2N-D sublattice comprises M 2N-D
types. M-D lattice partition is based iteratively on a partitioning of the constituent
2D lattices.

![Diagram of 4D lattice partitioning]

**Figure 2.6:** Set partitioning of 4D lattice

To partition the 4D rectangular lattice with the MSED $\Delta_0$, into eight 4D sublattices with MSED $4\Delta_0$, each constituent 2D rectangular lattice with MSED $\Delta_0$ is first partitioned into two 2D families $A \cup B$ and $C \cup D$ with MSED $2\Delta_0$, which are further partitioned into four 2D sublattices $A,B,C,D$ with MSED $4\Delta_0$, as shown in Figure 2.5.
Sixteen 4D types may then be defined, each corresponding to a concatenation of a pair of 2D sublattices, and denoted as \((A,A), (A,B),..., (D,D)\). The MSED of each 4D types is \(4\Delta_0\). 16 4D types can be grouped into 8 4D sublattices, denoted as 0,1,...,7, as shown in Table 2.1. The grouping, while yielding only half as many 4D sublattices as 4D types, is done in such a way which maintains the MSED of each 4D sublattice at \(4\Delta_0\). This kind of grouping simplifies the construction of trellis codes using those sublattices. 8-D and 16 - D lattice partition can be easily extended following the above partitioning rule.

<table>
<thead>
<tr>
<th>Sublattice (subset)</th>
<th>(y_0)</th>
<th>(u_{1a})</th>
<th>(u_{2a})</th>
<th>(u_{3a})</th>
<th>4D Types</th>
<th>(v_0)</th>
<th>(v_1)</th>
<th>(v_{l_{n+1}})</th>
<th>(v_{l_{n+1}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(A, A)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>(B, B)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>(A, B)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>(C, D)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(A, C)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>(C, B)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>(D, A)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>(C, A)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In [44], Wei has constructed several M-D trellis codes with 4-D and 8-D constellation. One of them is the well known 4-D 16-state Wei trellis codes. Figure 2.7 shows the encoder structure for this code.

During encoding process, two 2-D symbols are simultaneously inputted into a 4-D trellis encoder at every two successive time intervals \(n, n+1\). Three bits are encoded through trellis encoder after a differential encoder, while the rest bits in two 2-D
symbols remain uncoded. In the Wei code design, three of those uncoded bits are encoded via a 4-D block encoder which actually implements the shell mapping presented in [49]. Four output bits $Y_0, I_1, I_2$, and $I_2'$ are then converted by a bit converter to produce two groups of coded bits which correspond to a 2-D sub-constellations in one 4-D trellis code comprising two 2-D trellis codes. In the receiver, the VA is applied to decode received 4-D signals. The only difference with decoding the 2-D trellis codes is the calculation of branch metrics for 4-D subsets.

A conventional maximum-likelihood decoding algorithm such as Viterbi algorithm is used as the decoder for TCM codes. First, the decoder must determine the point in each of the M-D subsets which is closest to the received point, and calculate its associated metric (the squared Euclidean distance between the two points). Each received 2N-D point is divided into a pair of N-D points. The closest point in each 2N-D subset and its associated metric are found based on the point in each of the N-D subsets which is closest to the corresponding received N-D point and its associated
metric. The N-D subsets are those used to construct the 2N-D subsets. The foregoing process may be used iteratively to obtain the closest point in each 2N-D subset and its associated metric based on the closest point in each of the basic 2-D subset and its associated metric.

![Flow chart](image)

**Figure 2.8: Viterbi decoding algorithm for 16-state code of Figure 2.7**

Flow chart in Figure 2.8 shows the Viterbi decoding algorithm for a 16-state code mentioned previously. First, for each of the two received 2-D points of a received 4-D point, the decoder determines the closest 2-D point in each of the four 2-D subsets of 192-point 2D constellation of Figure 2.5, and calculates its associated metric. These metric are called 2-D subset metrics. Because there are only 48 2-D points in each of the four 2-D subsets, this step is quite easy, being no more complex than that
required for a 2-D code. Next, the decoder determines the closest 4-D point in each of the 16 4-D types (see Table 2.1) and calculates its associated metric. These metrics are called 4-D type metrics. The 4-D type metric for a 4-D type is obtained by adding the two 2-D subset metrics for the pair of 2-D subsets corresponding to that 4-D type. Finally, the decoder compares the two 4-D type metrics corresponding to two 4-D types within each 4-D subset. The smaller 4-D type metric becomes the 4-D subset metric associated with that 4-D subset, and the 4-D point associated with the smallest 4-D type metric is the closest 4-D type point in that 4-D subset. These 4-D subset metrics are then used to extend the trellis paths and generate final decisions on the transmitted 4-D points in the usual way.

2.1.3 Forney’s Concatenated TCM

As a popular choice in digital communications, the Forney’s concatenated code consists of two separate codes which are combined to form a large code [46]. Generally, Forney’s concatenated coding system includes a moderate-strength trellis inner encoder, a powerful algebraic block outer encoder and a conventional block table-like interleaver, as illustrated in Figure 2.9

In the decoder, firstly a maximum-likelihood (ML) or near-ML decoding algorithm is used to achieve a moderate error rate like $10^{-2} - 10^{-3}$ at a code rate as close to capacity as possible, then a block decoder is applied to drive the error rate down to
as low an error rate as may be described in [50]. With such "separated" decoding scheme, it was shown in [46] that the error rate could be made to decrease exponentially with block length at any rate less than capacity, while decoding complexity increase only polynomially.

2.2 Turbo Codes

Turbo codes, first presented to the coding community in 1993 by Berrou, Glavieux, and Thitimajshima [2], represent the most important breakthrough in coding theory since Ungerboeck introduced trellis codes in 1982 [40]. Whereas Ungerboeck’s work eventually led to coded modulation schemes capable of operation near capacity on band-limited channels [51], the original turbo codes offer near capacity performance for deep space and satellite channels. Many of the structural properties of the turbo codes have now been put on a firm theoretical footing [7] [52] [8] [53] [54] [55], and several innovations on the turbo theme have appeared in [56] [5] [53] [54] [57].

Turbo codes are parallel concatenated convolutional codes(PCCC) whose encoder
is formed by two or more constituent systematic encoder joined through one or more interleavers. A turbo encoder is shown in Figure 2.10, which is formed by parallel concatenation of two recursive systematic convolutional (RSC) encoder separated by a pseudo-random interleaver. The data flow \( u_k \) at time \( k \) goes directly to a first elementary RSC encoder \( C_1 \) and after interleaving, it feeds \( u_n \) at time \( k \) a second elementary RSC encoder \( C_2 \). These two encoders are not necessarily identical. Data \( u_k \) is systematically transmitted as symbol \( x_k \) and redundancy \( y_{1k} \) and \( y_{2k} \) produced by \( C_1 \) and \( C_2 \) may be completely transmitted for an \( R = 1/3 \) encoding or punctured for higher rate. If the coded outputs \( (y_{1k}, y_{2k}) \) of encoder \( C_1 \) and \( C_2 \) are used respectively \( n_1 \) and \( n_2 \) times and so on, the encoder \( C_1 \) rate \( R_1 \) and encoder \( C_2 \) rate \( R_2 \) are equal to

\[
R_1 = \frac{n_1 + n_2}{2n_1 + n_2} \quad R_2 = \frac{n_1 + n_2}{n_1 + 2n_2} \tag{II.1}
\]

The suboptimal iterative decoding structure is modular, and consists of a set of concatenated decoding modules, one for each constituent code, connected through
the same interleavers used at the encoder side. Such suboptimum iterative decoding algorithm offers near-ML performance. Each component decoders are based on a maximum a posteriori (MAP) probability algorithm or a soft output Viterbi algorithm (SOVA) [58] generating a weighted soft estimate of the input sequence. The iterative process performs the information exchange between the component decoders.

![Diagram](image)

Figure 2.11: Principle of the decoder in accordance with a serial concatenated scheme

The suboptimal iterative decoding structure is modular and consists of a set of concatenated decoding modules, one for each constituent code, connected through the same interleavers used at the encoder side. Each decoder performs weighted soft decoding of the input sequence. Bit error probabilities as low as $10^{-6}$ at $E_b/N_0 = -0.6$ dB have been shown by simulation [59] using rates as low as $1/15$. Parallel concatenated convolutional codes yield very large coding gains (10-11 dB) at the expense of a date rate reduction or bandwidth increase.

The basic turbo decoder scheme can be depicted as in Figure 2.11 [2] [60]. Two
elementary decoders (DEC1 and DEC2) are concatenated in a serial format. The first elementary decoder DEC1 is associated with lower rate $R_1$ encoder C1 and yields a soft (weighted) decision. The error burst at the decoder DEC1 output are scattered by the interleaver and the encoder delay $L_1$ is inserted to take the decoder DEC1 delay into account. The redundant information $y_k$ is demultiplexed and sent to decoder DEC1 when $y_k = y_{1k}$ and toward decoder DEC2 when $y_k = y_{2k}$. When the redundant information of a given encoder (C1 or C2) is not emitted, the corresponding decoder input is set to zero.

The first decoder DEC1 deliver a weighted (soft) decision, Logarithm of Likelihood Ratio (LLR) $\Lambda_1(u_k)$ which is associated with each decoded bit $u_k$, to the second decoder DEC2.

$$\Lambda_1(u_k) = \log \frac{P_r\{u_k = 1/\text{observation}\}}{P_r\{u_k = 0/\text{observation}\}}$$  \hspace{1cm} (II.2)

where $P_r\{u_k = 1/\text{observation}\}, i = 0, 1$ is the a posteriori probability (APP) of the data bit $u_k$.

As an optimum decoding algorithm, Viterbi algorithm doesn’t work here (especially for the first decoder DEC1) since it can not yield bit APP. Thus the BCJR [61] algorithm was modified to decode RSC codes [2] [60]. Then the LLR $\Lambda_1(u_k)$ associated with each decoded bit $u_k$ becomes

$$\Lambda_1(u_k) = \log \frac{\sum_m \lambda^1_k(m)}{\sum_m \lambda^0_k(m)}$$  \hspace{1cm} (II.3)
where $\lambda^i_k$ is the joint probability defined by

$$\lambda^i_k = P_r\{u_k = i, S_k = m|R^N_1\} \quad (II.4)$$

$S_k$ is the encoder state with K-tuple, $R^N_1$ is the received codeword sequence. Finally, the decoder can make decision by comparing $\Lambda_1(u_k)$ to a threshold equal to zero

$$\hat{u}_k = 1 \text{ if } \Lambda_1(u_k) > 0$$
$$\hat{u}_k = 0 \text{ if } \Lambda_1(u_k) < 0$$

Following the BCJR algorithm [61], equation (II.3) can be further expanded as

$$\Lambda_1(u_k) = \log \frac{\sum_{m} \sum_{m'} \sum_{j=0}^{1} \gamma_1(R_k, m', m)\alpha_{k-1}^{j}(m')\beta_k(m)}{\sum_{m} \sum_{m'} \sum_{j=0}^{1} \gamma_0(R_k, m', m)\alpha_{k-1}^{j}(m')\beta_k(m)} \quad (II.6)$$

In [2] [60], if the decoder inputs are independent, the LLR $\Lambda_1(u_k)$ can be decomposed into two parts:

$$\Lambda_1(u_k) = \log \frac{p(x_k|u_k = 1)}{p(x_k|u_k = 0)} + \log \frac{\sum_{m} \sum_{m'} \sum_{j=0}^{1} \gamma_1(y_k, m', m)\alpha_{k-1}^{j}(m')\beta_k(m)}{\sum_{m} \sum_{m'} \sum_{j=0}^{1} \gamma_0(y_k, m', m)\alpha_{k-1}^{j}(m')\beta_k(m)} \quad (II.7)$$

Conditionally to $u_k = 1$ (resp. $u_k = 0$), variable $x_k$ are Gaussian with mean 1 (resp. -1) and variance $\sigma^2$, thus the LLR $\Lambda(u_k)$ is still equal to

$$\Lambda_1(u_k) = \frac{2}{\sigma^2} x_k + W_k \quad (II.8)$$

where

$$W_k = \Lambda_1(u_k)|_{x_k=0} = \log \frac{\sum_{m} \sum_{m'} \sum_{j=0}^{1} \gamma_1(y_k, m', m)\alpha_{k-1}^{j}(m')\beta_k(m)}{\sum_{m} \sum_{m'} \sum_{j=0}^{1} \gamma_0(y_k, m', m)\alpha_{k-1}^{j}(m')\beta_k(m)} \quad (II.9)$$
$W_k$ is a function of the redundant information introduced by the encoder and does not depend on the decoder input. It represents the extrinsic information supplied by the decoder.

![Feedback decoder for turbo codes](image)

Figure 2.12: Feedback decoder for turbo codes

Thus a feedback decoder scheme can be used for decoding the two parallel concatenated encoders [2]. Figure 2.12 illustrates the realization of the above idea. Now both decoders can use modified BCJR algorithm, since for the second decoder, we have

$$
\Lambda_2(u_k) = f(\Lambda_1(u_k)) + W_{2k} \quad (II.10)
$$

with

$$
\Lambda_1(u_k) = \frac{2}{\sigma^2} x_k + W_{1k} \quad (II.11)
$$

Due to the presence of interleaving between DEC1 and DEC2, extrinsic information and observation $x_k, y_{1k}$ are weakly correlated. Therefore, $W_{2k}$ and $x_k, y_{1k}$ can be jointly used for carrying out a new decoding of bit $u_k$ with LLRs being rewritten as:
\[
\Lambda_1(u_k) = \frac{2}{\sigma^2} x_k + \frac{2}{\sigma^2} z_k + W_k
\]
\[
\hat{\Lambda}_1(u_n) = \Lambda_1(u_n)_{z_n=0}
\] (II.12)

\[
z_k = W_{2k} = \Lambda_2(u_k)|_{\Lambda_1(u_k)=0}
\]
decision at the decoder would be
\[
\hat{u}_k = [\Lambda_2(u_k)]
\] (II.13)

By increasing the number of iterations in the turbo decoding, the bit error probability as low as \(10^{-5} - 10^{-7}\) can be achieved at a SNR close to the fundamental limits established by Shannon.

### 2.3 Turbo Trellis Coded Modulation

The merge of TCM and PCCC was proposed to achieve simultaneously large encoding gains and high bandwidth efficiency [7] [13] [12] [17] [9]. For Gaussian channels, turbo-coded modulation techniques can be broadly classified into binary schemes and turbo trellis-coded modulation. The first group can be further divided into "pragmatic" scheme with a single component binary turbo code and multilevel binary turbo codes. Turbo trellis-coded modulation schemes can be classified into two cases puncturing either parity symbol or information symbol.
2.3.1 Binary Turbo Coded Modulation

In pragmatic turbo coded modulation design [12], a single binary turbo code of rate $1/n$ is used as the component code. The output of the turbo encoder is then simply mapped onto an $M$-ary modulator. Decoding is done by calculating the log-likelihood function for each encoded binary digit based on the received noisy symbol and the signal subsets in the signal constellation specified by each binary digit. The stream of the bit likelihood values is then passed to the binary turbo decoder which can be based either on MAP or soft output Viterbi algorithms (SOVA). By modifying the puncturing function and modulation signal constellation, it is possible to obtain a large family of turbo coded modulation schemes. However, although this system utilizes a bandwidth efficient modulation scheme, the encoder and modulator are not designed cooperatively as in TCM systems.

Multilevel turbo codes are constructed by using turbo codes as the component codes in [17] [62]. The transmitter for an $M$-ary signal constellation consists of $l = \log_2 M$ parallel binary encoders as shown in Figure 2.13.

![Figure 2.13: Multilevel turbo encoder](image-url)
A message sequence is split into \( l \) blocks. Each message block \( \mathbf{u}_i \) is then encoded by an individual binary turbo encoder. The output digits of the encoders form a binary vector \((v_1, v_2, ..., v_l)\), which is mapped onto an \( M \)-ary signal constellation.

The maximum likelihood decoder operates on the overall code trellis. In general, however, this decoder is too complicated to implement. Alternatively, a suboptimum technique, called multistage decoding [63], can be used, resulting the same asymptotic error performance as the maximum likelihood decoding.

The most significant contribution of Wachsmann and Huber is that they proposed a technique for selecting the component code rates. In this design, the component rate at a particular modulation level, is chosen to be equal to the equivalent binary input channel associated with that level. For infinite code lengths, in theory, as the overall channel capacity equals to the sum of the channel capacities for all levels, this design results in error free decoding. Therefore, they are suitable candidates for component codes in a multilevel scheme. And the good performance leads to the assumption of the negligible error propagation between modulation levels, which enables the multistage decoding. However, for small block size, there could be significant loss in terms of the SNR needed to achieve a certain BER.

### 2.3.2 Symbol interleaved Turbo TCM

In [9] [10], a turbo trellis-coded modulation (TTCM) system was presented in
which two recursive Ungerboeck type trellis codes with rate \( k/(k+1) \) are concatenated in parallel. Figure 2.14 shows the encoder structure comprising of two recursive convolutional encoders linked by a symbol interleaver and followed by a signal mapper.

![Diagram](image)

Figure 2.14: Turbo TCM encoder with parity symbol puncturing

It is noted that the interleaver is constrained to interleave symbols. That is, the ordering of \( k \) information bits arriving at the interleaver at a particular instant remains unchanged. For the component trellis code, some of the input bits may not be encoded. In practical implementations these inputs do not need to be interleaved, but are directly used to select the final point in a signal subset. At the receiver, the values of these bits are estimated by set decoding [40].

The output of the second encoder is de-interleaved. This ensures that the \( k \) information bits which determine the encoded \((k+1)\) binary digits of both the upper and lower encoder at a given time instant are identical. The selector then alternately connects the upper and lower encoder to the channel. Thus, the parity symbols is alternately chosen from the upper and lower encoder. Each information group appears
in the transmitted sequence only once.

In the receiver, the log-MAP algorithm or SOVA decoding algorithms are used to decode the turbo codes except that the symbol probability is used as the extrinsic information rather than the bit probability.

### 2.3.3 Bit interleaved Turbo TCM

As a different type of turbo TCM scheme, parallel concatenation of two recursive trellis codes with puncturing of systematic bits was proposed by Benedetto, Divsalar, Montorsi and Pollara [13]. The basic idea of the scheme is to puncture the output symbols of each trellis encoder and select the puncturing pattern such that the output symbols of the parallel concatenated code contains the input information only once. The simple method to realize above idea is first to select a rate $\frac{b}{b+1}$ constituent code where the outputs are mapped to a $2^{b+1}$-level modulation based on Ungerboeck’s set partitioning [41]. If MPSK modulation is used, for every $b$ bits at the input of the parallel concatenated encoder we transmit two consecutive $2^{b+1}$ PSK signals, one per each encoder output. For the case of using M-QAM modulation, the $b+1$ outputs of the first component encoder are mapped into the $2^{b+1}$ in-phase level (I-channel) of $2^{2b+2}$-QAM signal set, and the $b+1$ outputs of the second component encoder are mapped into the $2^{b+1}$ quadrature level (Q-channel). The throughput of these two system are $b/2$ bits/sec/Hz and $b$ bits/sec/Hz, respectively.
A better solution to parallel concatenated TCM (PCTCM) is to select $b/2$ systematic outputs from the first constituent encoder and puncture the rest of the systematic outputs, but use the parity bit of the $\frac{b}{b+1}$ code. Then do the same to the second constituent code, but select only those systematic bits which were punctured in the first encoder. Two interleavers will be required in this system, the first interleaver permutes the bits selected by the first encoder and the second one interleave those punctured by the first encoder. $2^{1+b/2}$ PSK symbols per encoder can be used for MPSK to achieve throughput of $b/2$. And $2^{1+b/2}$ levels can be used for both I-channel and Q-channel in M-QAM to achieve the throughput of $b$ bits/sec/Hz. A 16QAM turbo trellis-coded modulation encoder is given in Figure 2.15.

![Diagram of Turbo Trellis-Coded Modulation](image)

**Figure 2.15: Turbo trellis-coded modulation, 16 QAM, 2bits/s/Hz**

Decoding this type of turbo TCM is a straightforward application of the iterative symbol-by-symbol MAP algorithm for a binary turbo codes. The only differences
are: 1) the extrinsic information computed for a symbol needs to be converted to a bit level since they are carried out on a bit level; 2) after interleaving/de-interleaving operations, the bit \( a \text{ priori} \) probabilities need to be converted to a symbol level since they will be used in the branch transition probability calculation in the symbol MAP algorithm [64].

### 2.4 Multicarrier Modulation and OFDM

An alternative approach to the design of bandwidth-efficient communication system in the presence of channel distortion is to subdivide the available channel bandwidth into a number of equal-bandwidth subchannels, where the bandwidth of each subchannel is sufficiently narrow so that the frequency response characteristics of the subchannels are nearly ideal. Such a subdivision of the overall bandwidth into smaller subchannels is referred to as multicarrier modulation (MCM). The basic idea of multicarrier modulation is quite simple and follows naturally from the competing desires for high data rates and intersymbol interference (ISI) free channels. In order to have a channel that does not have ISI, the symbol time \( T_{\text{sym}} \) has to be larger - often significantly larger - than the channel delay spread \( T_m \). Typically, it is assumed that \( T_{\text{sym}} \approx 10T_m \) in order to satisfy this ISI-free condition [65].

Multicarrier modulation divides the high-rate transmit bitstreams into \( N \) parallel low-rate substreams, each of which has \( T_{\text{sym}} \gg T_m \), and is hence ISI-free. These
individual substreams can then be sent over $N$ parallel subchannels, maintaining the total desired data rate. Consequently, the data is transmitted by frequency-division multiplexing (FDM). By selecting the symbol rate $1/T_{sym}$ on each of the subchannels to be equal to the separation $\Delta_f$ of adjacent subcarriers, the subcarriers are orthogonal over the symbol interval $T_{sym}$ and independent of the relative phase relationship between subcarriers. In this case, we have orthogonal frequency-division multiplexing (OFDM). The data rate on each subchannel is much less than the total data rate, and so the corresponding subchannel bandwidth is much less than the total system bandwidth. The number of substreams is chosen to insure that each subchannel has a bandwidth less than the coherence bandwidth of the channel, so the subchannels experience relatively flat fading. Thus, the ISI on each subchannel is small. Moreover, in the discrete implementation of OFDM, often called discrete multitone (DMT), the ISI can be completely eliminated through the use of a cyclic prefix. The subchannels in OFDM need not be contiguous, so a large continuous block of spectrum is not needed for high rate multicarrier communications.

Over the past few years, there has been increasing interest in multicarrier modulation for a variety of applications. However, multicarrier modulation is not a new technique. It was first used for military HF radios in the late 1950’s and early 1960’s. Starting around 1990, multicarrier modulation has been used in many diverse wired and wireless applications, such as Digital Audio Broadcasting in Europe, digital subscribe lines (DSL) and newly emerging uses for multicarrier techniques in-
cluding fixed wireless broadband services and mobile wireless broadband known as FLASH-OFDM [65]. One of OFDM’s successes is its adoption as the standard of choice in Wireless Personal Area Networks (WPAN) and Wireless Local Area Network (WLAN) systems (e.g., IEEE P802.15-03 [39], IEEE 802.11a, IEEE 802.11g, Hiper-LAN II).

2.4.1 Data Transmission Using Multicarriers

The simplest form of multicarrier modulation divides the data stream into multiple substreams to be transmitted over different orthogonal subchannels centered at different subcarrier frequencies. Consider a linearly-modulated system with data rate \( R \) and passband bandwidth \( B \). The coherence bandwidth for the channel is assumed to be \( B_c < B \), so the signal experiences frequency-selective fading. The basic premise of multicarrier modulation is to break this wideband system into \( N \) linearly-modulated subsystems in parallel, each with subchannel bandwidth \( B_N = B/N \) and data rate \( R_N = R/N \). For \( N \) sufficiently large, the subchannel bandwidth \( B_N \ll B_c \), which insures relatively flat fading on each subchannel. This can also be seen in time domain: the symbol time \( T_N \) of the modulated signal in each subchannel is proportional to the subchannel bandwidth \( 1/B_N \). So \( B_N \ll B_c \) implies that \( T_N \approx 1/B_N \gg 1/B_c \approx T_m \), where \( T_m \) denotes the delay spread of the channel. Thus, if \( N S T \) is sufficiently large, the symbol time is much bigger than the delay spread, so each subchannel experiences
little ISI degradation.

Figure 2.16 illustrates a multicarrier transmitter. The bit stream is divided into \(N\) substreams via a serial-to-parallel converter. The \(n\)th substream is linearly-modulated (typically via QAM or PSK) relative to the subcarrier frequency \(f_n\) and occupies passband \(B_N\). We assume coherent demodulation of the subcarriers so the subcarrier phase is neglected in our analysis. If we assume raised cosine pulses for \(g(t)\) we get a symbol time \(T_N = (1 + \beta)/B_N\) for each substream, where \(\beta\) is the rolloff factor of the pulse shape. The modulated signals associated with all the subchannels are summed together to form the transmitted signal, given as

\[
s(t) = \sum_{i=0}^{N-1} s_i g(t) \cos(2\pi f_i t + \phi_i),
\]

where \(s_i\) is the complex symbol associated with the \(i\)th subcarrier and \(\phi_i\) is the phase offset of the \(i\)th carrier. For nonoverlapping subchannels we set \(f_i = f_0 + i(B_N), i = 0, \ldots, N - 1\). The substreams then occupy orthogonal subchannels with passband bandwidth \(B_N\), yielding a total passband bandwidth \(NB_N = B\) and data rate \(NR_N \approx R\). Thus, this form of multicarrier modulation does not change the data rate or signal bandwidth relative to the original system, but it almost completely eliminates ISI for \(B_N \ll B_c\).

The receiver for this multicarrier modulation is shown in Figure 2.18. Each substream is passed through a narrowband filter to remove the other substreams, demodulated, and combined via a parallel-to-serial converter to form the original data.
Figure 2.16: Multicarrier transmitter.

Figure 2.17: Transmitted signal.
stream. Note that the \( i \)th subchannel will be affected by flat fading corresponding to a channel gain \( \alpha_i = |H(f_i)| \).

Although this simple type of multicarrier modulation is easy to understand, it has several significant shortcomings. First, in a realistic implementation, subchannels will occupy a large bandwidth than under ideal raised pulse shaping since the pulse shape must be time-limited. Let \( \varepsilon/T_N \) denote the additional bandwidth required sue to time-limiting of these pulse shapes. The subchannels must then be separated by \( (1 + \beta + \varepsilon)/T_N \), and since the multicarrier system has \( N \) subchannels, the bandwidth penalty for time limiting is \( \varepsilon N/T_N \). In particular, the total required bandwidth for nonoverlapping subchannels is

\[
B = \frac{N(1 + \beta + \varepsilon)}{T_N} \tag{II.15}
\]

Thus, this form of multicarrier modulation can be spectrally inefficient. Additionally, near-ideal (and hence expensive) low pass filters will be required to maintain the orthogonality of the subcarriers at the receiver. Perhaps most importantly, this scheme requires \( N \) independent modulators and demodulators, which entails significant expense, size, and power consumption. Section 2.4.3 presents the discrete implementation of multicarrier modulation, which eliminates the need for multiple modulators and demodulators.
2.4.2 Mitigation of Subcarrier Fading

The advantage of multicarrier modulation is that each subchannel is relatively narrowband, which mitigates the effects of delay spread. However, each subchannel experiences flat-fading, which can cause large BERs on some of the subchannels. In particular, if the transmit power on subcarrier $i$ is $P_i$, and the fading on that subcarrier is $\alpha_i$, then the received SNR is $Q_i = \alpha_i^2 P_i/(N_0 B_N)$, where $B_N$ is the bandwidth of each subchannel. If $\alpha_i$ is small then the received SNR on the $i$th subchannel is quite low, which can lead to high BER on that subchannel. Moreover, in wireless channels the $\alpha_i$’s will vary over time according to a given fading distribution, resulting in the same performance degradation associated with flat fading for single carrier system. Since flat fading can seriously degrade performance in each subchannel, it is important
to compensate for flat fading in the subchannels. There are several techniques for
doing this, including coding with interleaving over time and frequency, frequency
equalization, precoding, and adaptive loading and etc. Moreover, in rapidly changing
channels it is difficult to estimate the channel at the receiver and feed this information
back to the transmitter. Without channel information at the transmitter, precoding
and adaptive loading cannot be done, so only coding with interleaving is effective at
fading mitigation, which will be discussed shortly [65].

Coding with Interleaving over Time and Frequency

The basic idea in coding with interleaving over time and frequency is to encode
data into codewords, interleave the resulting coded bits over both time and frequency,
and then transmit the coded bits over different subchannels such that the coded bits
within a given codeword all experience independent fading. If most of the subchan-
nels have a high SNR, the codeword will have most coded bits received correctly, and
the errors associated with the few bad subchannels can be corrected. Coding across
subchannels basically exploits the frequency diversity inherent to a multicarrier sys-
tem to correct errors. This technique only works well if there is sufficient frequency
diversity across the total system bandwidth, which will significantly reduce the ef-
fect of coding. Most coding for OFDM assumes channel information in the decoder.
Channel estimates are typically obtained by a two dimensional pilot symbol trans-
mission over both time and frequency.
Note that coding with frequency/time interleaving takes advantage of the fact that data on all the subcarriers is associated with the same user, and can therefore be jointly processed. The other techniques for fading mitigation discussed in subsections are all basically flat fading compensation techniques, which apply equally to multicarrier systems as well as narrowband flat fading single carrier systems.

**Frequency Equalization**

In frequency equalization the flat fading $\alpha_i$ on the $i$th subchannel is basically inverted in the receiver. Specifically, the received signal is multiplied by $1/\alpha_i$, which gives a resultant signal power $\alpha_i^2 P_i/\alpha_i^2 = P_i$. While this removes the impact of flat fading on the signal, it enhances the noise. Specifically, the incoming noise signal is also multiplied by $1/\alpha_i$, so the noise power becomes $N_0 B N/\alpha_i^2$ and the resultant SNR on the $i$th subchannel after frequency equalization is the same as before equalization. Therefore, frequency equalization does not really change the performance degradation association with subcarrier flat fading. Other techniques regarding flat fading can also be found in [65].

**2.4.3 Discrete Implementation of Multicarrier**

Although multicarrier modulation was invented in the 1950’s, its requirement for separate modulators and demodulators on each subchannel was far too complex for the most system implementations at the time. However, the development of simple
and cheap implementation of the discrete Fourier transform (DFT) and the inverse DFT (IDFT) twenty years later, combined with the realization that multicarrier modulation can be implemented with these algorithms, ignited its widespread use. In this section, we will illustrate OFDM, which implements multicarrier modulation using DFT and IDFT.

The DFT and Its Properties

Let \( x[n], 0 \leq n \leq N - 1 \), denote a discrete time sequence. The \( N \)-point DFT of \( x[n] \) is defined as

\[
DFT\{x[n]\} = X[i] \triangleq \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi ni}{N}}, 0 \leq i \leq N - 1. \tag{II.16}
\]

where \( X[i] \) characterizes the frequency content of the time samples \( x[n] \) associated with the original signal \( x(t) \). The sequence \( x[n] \) can be recovered from its DFT using IDFT:

\[
IDFT\{X[i]\} = x[n] \triangleq \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{j\frac{2\pi ni}{N}}, 0 \leq i \leq N - 1. \tag{II.17}
\]

The DFT and its inverse are typically performed in hardware using fast Fourier transform (FFT) and inverse FFT (IFFT).

When an input data stream \( x[n] \) is sent through a linear time-invariant discrete-time channel \( h[n] \), the output \( y[n] \) is the discrete-time convolution of the input and the channel impulse response:

\[
y[n] = h[n] * x[n] = x[n] * h[n] = \sum_{k} h[k] x[n - k]. \tag{II.18}
\]
The $N$-point circular convolution of $x[n]$ and $h[n]$ is defined as

$$y[n] = h[n] \otimes x[n] = x[n] \otimes h[n] = \sum_k h[k] x[n-k]_N.$$

(II.19)

where $[n-k]_N$ denotes $[n-k]$ modulo $N$. In other words, $x[n-k]_N$ is a periodic version of $x[n-k]$ with period $N$. It is easily verified that $y[n]$ given by II.20 is also periodic with period $N$. From the definition of the DFT, circular convolution in time leads to multiplication in frequency:

$$DFT\{y[n] = h[n] \otimes x[n]\} = X[i] H[i], 0 \leq i \leq N - 1.$$

(II.20)

If the channel and input are circularly convoluted then if $h[n]$ is known at the receiver, the original data sequence $x[n]$ can be recovered by taking the IDFT of $Y[i]/H[i], 0 \leq i \leq N - 1$. Unfortunately, the channel output is not a circular convolution but a linear convolution. However, the linear convolution between the channel input and impulse response can be turned into a circular convolution by adding a specific prefix to the input called a cyclic prefix.

The Cyclic Prefix

Consider a channel input sequence $x[n] = x[0], \ldots, x[N-1]$ of length $N$ and a discrete-time channel with finite impulse response (FIR) $h[n] = h[0], \ldots, h[\mu]$ of length $\mu + 1 = T_m/T_s$, where $T_m$ is the channel delay spread and $T_s$ the sampling time associated with the discrete time sequence. The cyclic prefix for $x[n]$ is defined as $\{x[N-\mu], \ldots, x[N-1]\}$: it consists of the last $L$ values of the $x[n]$ sequence. For each
input sequence of length $N$, these last $\mu$ samples are appended to the beginning of the sequence. This yields a new sequence $\tilde{x}[n], -\mu \leq n \leq N - 1$, of length $N + \mu$, where $\tilde{x}[-\mu], \ldots, \tilde{x}[N - 1], x[0], \ldots, x[N - 1]$, as shown in Figure 2.19. Note that with this definition, $\tilde{x}[n] = x[n]_N$ for $-\mu \leq n \leq N - 1$, which implies that $\tilde{x}[n - k] = x[n - k]_N$ for $-\mu \leq n - k \leq N - 1$.

![Figure 2.19: Cyclic prefix of length $\mu$.](image)

Suppose $\tilde{x}$ is input to a discrete-time channel with impulse response $h[n]$. The channel output $y[n], 0 \leq n \leq N - 1$ is then

$$
\mathbf{H} = \tilde{x}[n] \ast h[n] \\
= \sum_{k=0}^{\mu-1} h[k] \tilde{x}[n-k] \\
= \sum_{k=0}^{\mu-1} h[k] x[n-k]_N \\
= x[n] \otimes h[n],
$$

where the third equality follows from the fact that for $0 \leq k \leq \mu - 1, \tilde{x}[n-k] = x[n-k]_N$ for $0 \leq n \leq N - 1$. Thus, by appending a cyclic prefix to the channel input, the linear convolution associated with the channel impulse response $y[n]$ for
$0 \leq n \leq N-1$, becomes a circular convolution. Taking the DFT of the channel output in the absence of noise then yields

$$Y[i] = DFT\{y[n] = x[n] \otimes h[n]\} = X[i]H[i], 0 \leq i \leq N-1. \quad \text{(II.21)}$$

and the input sequence $x[n], 0 \leq n \leq N-1,$ can be recovered from the channel output $y[n]$ for $0 \leq n \leq N-1,$ for known $h[n]$ by

$$x[n] = IDFT\{Y[i]/H[i]\} = IDFT\{DFT\{y[n]\}/DFT\{h[n]\}\}. \quad \text{(II.22)}$$

Note that $y[n], -\mu \leq n \leq N-1,$ has length $N + \mu,$ yet from (II.22) the first $\mu$ samples $y[-\mu], \ldots, y[-1]$ are not needed to recover $x[n], 0 \leq n \leq N-1,$ due to the redundancy associated with the cyclic prefix. Moreover, if we assume that the input $x[n]$ is divided into data blocks of size $N$ with a cyclic prefix appended to each block to form $\tilde{x}[n],$ then the first $\mu$ samples of $y[n] = h[n] * \tilde{x}[n]$ in a given block are corrupted by InterBlock Interference (IBI) associated with the last $\mu$ samples of $x[n]$ in the priori block, as illustrated in Figure 2.20. The cyclic prefix serves to eliminate IBI between the data blocks since the first $\mu$ samples of the channel output affected by this IBI can be discarded without any loss relative to the original information sequence. In continuous time this is equivalent to using a guard band of duration $T_m$ (the channel delay spread) after every block of $N$ symbols of duration $NT_{sym}$ to eliminate the IBI between these data blocks.

The benefits of adding a cyclic prefix come at a cost. Since $\mu$ symbols are added to the input data blocks, there is an overhead of $\mu/N,$ resulting in a data rate reduction.
The transmit power associated with sending the cyclic prefix is also wasted since this prefix consists of redundant data. It is clear from Figure 2.20 that any prefix of length $\mu$ appended to input blocks of size $N$ eliminates IBI between data blocks if the first $\mu$ samples of the block are discarded. In particular, the prefix can consist of all zero symbols, in which case although the data rate is still reduced, no power is used in transmitting the prefix. The tradeoffs associated with the cyclic prefix versus this all-zero prefix will be discussed in Chap III.

The above analysis motivates the design of OFDM. In OFDM, the input data is divided into blocks of size $Z$ referred to as an OFDM symbol. A cyclic prefix is added to each OFDM symbol to induce circular convolution of the input and channel impulse response. At the receiver, the output samples affected by IBI between OFDM symbols are removed. The DFT of the remaining samples are used to recover the original input sequence. Details of the OFDM system design will be given in next section.

**Orthogonal Frequency Division Multiplexing (OFDM)**

The OFDM implementation of multicarrier modulation is shown in Figure 2.21. The input data stream is modulated by a QAM modulator, resulting a complex sym-
bol $X[0], X[1], \ldots, X[N-1]$. This symbol stream is passed through a serial-to-parallel converter, whose output is a set of $N$ parallel QAM symbols $X[0], X[1], \ldots, X[N-1]$ corresponding to the symbols transmitted over each of the subcarriers. Thus, the $N$ symbols output from the serial-to-parallel converter are the discrete frequency components of the OFDM modulator output $s(t)$. In order to generate $s(t)$, these frequency components are converted into time samples by performing an inverse DFT on these $N$ symbols, which is efficiently implemented using the IFFT algorithm. The IFFT yields the OFDM symbol consisting of the sequence $x[n] = x[0], \ldots, x[N-1]$ of length $N$, where

$$x[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X[i] e^{j2\pi ni/N}, 0 \leq n \leq N - 1. \quad (II.23)$$

This sequence corresponds to samples of the multicarrier signal: i.e. the multicarrier signal consists of linearly modulated subchannels, and right hand side of (II.23) corresponds to samples of a sum of QAM symbols $X[i]$ each modulated by carrier frequency $e^{j2\pi ni/T}, i = 0, \ldots, N - 1$. The cyclic prefix is then added to the OFDM symbol, and the resulting time samples $\tilde{x}[n] = \tilde{x}[-\mu], \ldots, \tilde{x}[N-1] = x[N-\mu], \ldots, x[0], \ldots, x[N-1]$ are ordered by the parallel-to-serial converter and pass through a D/A converter, resulting in baseband OFDM signal $\tilde{x}(t)$, which is then upconverted to frequency $f_0$.

The transmitted signal is fitted by the channel impulse response $h(t)$ and corrupted by additive noise, so that the received signal is $y(t) = \tilde{x}(t) * h(t) + n(t)$. This signal is downconverted to baseband and filtered to remove the high frequency
components. The A/D converter samples the resulting signal to obtain \( y[n] = \tilde{x}(n) \ast h(n) + v(n), -\mu \leq n \leq N - 1 \). The prefix of \( y[n] \) consisting of the first \( \mu \) samples is then removed. This results in \( N \) times samples whose DFT in the absence of noise is \( Y[i] = H[i]X[i] \). These time samples are serial-to-parallel converted and passed through an FFT, which results in scaled versions of the original symbols \( H[i]X[i] \), where \( H[i] = H[f_i] \) is the flat-fading channel gain associated with the \( i \)th subchannel. The FFT output is parallel-to-serial converted and passed through a QAM demodulator to recover the original data.

The OFDM system effectively decompose the wideband channel into a set of narrowband orthogonal subchannels with a different QAM symbol sent over each subchannel. Knowledge of the channel gains \( H[i], i = 0, \ldots, N - 1 \) is not needed for this decomposition, in the same way that a continuous time channel with frequency response \( H[f] \) can be divided into orthogonal subchannels without knowledge of \( H[f] \) by splitting the total signal bandwidth into nonoverlapping subbands. The demodulator can use the channel gains to recover the original QAM symbols by dividing out these gains: \( X[i] = Y[i]/H[i] \). This process is called frequency equalization. However, frequency equalization leads to noise enhancement, since the noise in the \( i \)th subchannel is also scaled by \( 1/H[i] \).
Figure 2.21: OFDM with IFFT/FFT implementation.
CHAPTER III
TURBO TCM CODED OFDM SYSTEM FOR UWB CHANNELS

Ultra-wideband (UWB) radio is a fast emerging technology with uniquely attractive features inviting major advances in wireless communications, networking, radar, imaging, and positioning system. By its rule-making proposal in 2002, the FCC essentially unleashed new bandwidth of (3.6-10.1 GHz) at the noise floor, where UWB radios overlapping coexistent RF systems can operate using low-power ultra-short information bearing pulses. This leads to a rapidly growing research efforts targeting a host of UWB applications, such as short-range high-speed access to internet, covert communication links, localization at centimeter-meter level accuracy, high-resolution ground-penetration radar, through-wall imaging, precision navigation and asset tracking, just to name a few. UWB characterizes transmission system with instantaneous
spectral occupancy in excess of 500 MHz. Such systems rely on ultra-short waveforms that can be free of sine-wave carriers and do not require IF processing because they can operate at baseband.

It is essential for a wireless system to deal with the existence of multiple propagation paths (multipath) exhibiting different delays, resulting from objects in the environment causing multiple reflections on the way to the receiver. The large bandwidth of UWB waveforms significantly increases the ability of the receiver to resolve the different reflections in the channel. Two basic solutions for inter-symbol interference (ISI) caused by multi-path channels are equalization and orthogonal frequency-division multiplexing (OFDM) [31].

OFDM has been a promising solution for efficiently capturing multipath energy in highly dispersive UWB channels and delivering high data rate transmission. One of OFDM's successes is its adoption as the standard of choice in Wireless Personal Area Networks (WPAN) and Wireless Local Area Network (WLAN) systems (e.g., IEEE P802.15-03 [39], IEEE 802.11a, IEEE 802.11g, Hiper-LAN II). Convolutional encoded OFDM has been introduced in the proposed standard to combat flat fading experienced in each subcarrier [66] [67]. The incoming information bits are channel coded prior to serial-to-parallel conversion and carefully interleaved. This procedure splits the information to be transmitted over a large number of subcarriers, and at the same time, provides a link between bits transmitted on those separated subcarriers of the signal spectrum in such a way that information conveyed by faded subcarriers
can be reconstructed through the coding link to the information conveyed by well-received subcarriers.

One of UWB proposals in the IEEE P802.15 WPAN project is to use a multi-band orthogonal frequency-division multiplexing (OFDM) system and punctured convolutional codes for UWB channels supporting data rate up to 480Mb/s. In this section we examine the possibility of improving the proposed system using Turbo TCM with QAM constellation for higher data rate transmission. We construct a punctured parity-concatenated trellis codes in which a TCM code is used as the inner code and a simple parity-check code is used as the outer code. Then, the bit performance is examined when applied to the OFDM systems in the UWB channel environments. The study shows that the system can offer data rate of 640Mbps via 16QAM modulation and 1.2 Gbps via 64QAM modulation. The code performance is confirmed by density evolution.
Figure 3.1: Block diagram of coded OFDM system.
3.1 OFDM System For UWB Channel

The block diagram of the functions included in the coded OFDM system is presented in figure 3.1. On the transmitter side, source information bits are first encoded and then mapped onto a higher sized constellation, such as QPSK, 16QAM or 64QAM. Then, the streams of mapped complex numbers are grouped to modulate subcarriers in OFDM frequency band. FFT and inverse FFT (IFFT) are used for a simple implementation [66]. IFFT is performed to construct so-called “time domain” OFDM symbols, as we mentioned in chapter II. In order to enable a very simple equalization scheme in the frequency domain, classic multicarrier systems insert at the transmitter, after IFFT modulation, a time-domain redundant Cyclic Prefix (CP) of length larger than the FIR channel memory. At the receiver side, the reverse order operations are performed to recover the source information. CP is discarded to avoid inter-block interference (IBI) and each truncated block is FFT processed - an operation converting the frequency-selective channel output into parallel flat-faded independent subchannel outputs each corresponding to a different subcarrier. Unless zero, flat fads are removed by dividing each subchannel output with a simple gain equal to the channel transfer function values at the corresponding subcarrier.

Instead of inserting the CP, it was proposed recently in [68] to pad Zeros (a null signal) at the end of each IFFT modulated block. This new modulation, so termed Zero-padding OFDM (ZP-OFDM), introduces the same amount of redundancy as CP-
OFDM and thus results in the same bit rate loss. Interestingly, ZP-OFDM assures channel-irrespective retrieval of the transmitted symbol blocks even when a channel zero is located on a subcarrier which is not possible possible with CP-OFDM. The price paid by ZP-OFDM is increased receiver complexity (the single FFT requied by CP-OFDM is replaced by FIR filtering). We will focus on CP-OFDM in this chapter to describe the OFDM system. The details for ZP-OFDM and the equalization difference between CP-OFDM and ZP-OFDM will be explained in section 4.2.5.

The FCC specifies that a system must occupy a minimum of 500 MHz bandwidth in order to be classified as an UWB system. The P802.15-03 project defined an unique numbering system for all channels having a spacing of 528MHz and lying within the band 3.1 - 10.6 GHz [39]. According to [69], a 128-point FFT with cyclic prefix length of 60.6ns outperforms a 64-point FFT with a prefix length of 54.9ns by approximately 0.9dB. Therefore, we focus on an OFDM system with a 128-point FFT and 528MHz operating bandwidth.

### 3.1.1 16QAM Turbo TCM Encoder Structure

In chapter II, three turbo TCM coding scheme were discussed. Simulation results show that TTCM proposed by Benedetto [13] outperforms the other two schemes. There are two bit interleavers and two constituent encoders involved in Benedetto’s TTCM scheme. The first interleaver permutes the bits selected by the first constituent
encoder and second one interleaves those bits punctured by the first constituent encoder. For M-QAM, there are $2^{1+b/2}$ levels in both I channel and Q channel, therefore achieve a throughput of $b$ bits/sec/Hz.

We found a simple way to describe the same TTCM code as Benedetto’s. We adopt a punctured concatenation structure in which a TCM code is used as the inner code and a simple parity-check code is used as outer code. By correctly select the interleaver size and pattern, this scheme functions exactly as Benedetto’s TTCM, but saves half of interleavers and constituent encoders. We will name it Parity-concatenated TCM from now on.

Figure 3.2 presents the 16QAM parity-concatenated TCM encoder structure which functions as a 16QAM TTCM encoder [70] [71]. This is equivalent to describing the turbo codes as a repeater (that is the simplest parity check code), interleaver, and one component code [72]. Two bit streams ($u_1$ and $u_2$) are provided at the input of the TCM encoder, one is the original source information bit streams ($u_1$), and the other ($u_2$) is the interleaved version corresponding to the parity checks of the first one except being interleaved. TCM encoder has rate of 2/2, which combines only the original systematic bit (from $u_1$ stream) and the parity-check bit as the encoder outputs. Then, two consecutive clock cycle outputs (or two outputs after further interleaving) will be mapped onto 16QAM constellation, one for in-phase component and the other for quadrature component. If we make the interleaving size of the interleaver before TCM encoder to be half of the information block size, the function
of this concatenated structure is exactly the same as that of 16QAM TTCM shown in figure 2.15.
Figure 3.2: Parity-concatenated TCM encoder, 16QAM
Figure 3.3: Expansion from Benedetto’s TTCM to parity-concatenated TCM
Figure 3.3 illustrate the merge process from standard turbo TCM to parity-concatenated TCM for a short block code [71]. Figure 3.3(a) is the 16QAM block diagram of TTCM encoder with short block inputs $u_1 = u_1^3u_1^2u_1^1u_1^0$ and $u_2 = u_2^3u_2^2u_2^1u_2^0$, whereas $u_1^0$ and $u_2^0$ are the LSBs and $u_1^3$ and $u_2^3$ are the MSBs. Assume after interleaving, two input sequences to the second constituent encoder are $u_1^2u_1^0u_1^3u_1^1$, then 4 output coded sequences would be $u_1^3u_1^2u_1^1u_1^0$, $v_0^3v_0^2v_0^1v_0^0$, $u_2^3u_2^2u_2^1u_2^0$ and $v_0^3v_0^2v_0^1v_0^0$.

Figure 3.3(b) is a simplified coding scheme of figure 3.3(a). Input sequence $u_1$ is the original 4 bits input $u_1^3u_1^2u_1^1u_1^0$ followed by sequence $u_1^1u_1^0u_1^3u_1^1$ which is the interleaved version of original $u_2$ in figure 3.3(a). While Input sequence $u_2$ consists of original 4 bits input $u_2^3u_2^2u_2^1u_2^0$ followed by sequence $u_1^3u_1^2u_1^1u_1^0$ which is the interleaved version of original $u_1$ in figure 3.3(a). With only one constituent encoder as in figure 3.3(a), we will have output sequences $u_2^1u_2^0u_2^3u_2^2u_2^1u_2^0$ and $v_0^3v_0^2v_0^1v_0^0$. The only difference between coding results of figure 3.3(a) and (b) lies in partial parity check bits. In figure 3.3 (a), both constituent encoders start from zero state. If we set the encoder state of figure 3.3(b) to be zero after first 4 steps, then the output parity-check sequence in figure 3.3(b) will have exactly same values as those in figure 3.3(a) except in different order. So we can use encoder in figure 3.3(b) to reproduce encoding results from that of figure 3.3(a). The merge from encoder in figure 3.3(b) to that of figure 3.3(c) is straight forward when we set the interleaver size and pattern as shown in 3.3(c). Then two encoders in figure 3.3(c) and 3.3(b) are equivalent to
each other. Since $u_2$ is an interleaved version of $u_1$, it can be recognized as the corresponding parity-checks of $u_1$. Therefore, the encoder structure in figure 3.3(c) can be constructed through concatenation of an outer parity-check code and inner TCM code. The resulted performance is equivalent to a Benedetto’s turbo TCM encoder. However, it saves one constituent encoder and half of the interleavers compared with Benedetto’s TTCM structure.

There are three advantages when comparing this 16QAM parity-concatenated TCM with standard 16QAM TTCM:

(a) We need to consider less interleavers: only one interleaver for this 16QAM case instead of two as in standard TTCM;

(b) We save one constituent encoder. Both (a) and (b) will be a big advantage regarding the real world implementation of the encoder;

(c) It will be very easy to extend the outer simple parity-check codes to a more complicated structure for variety parity-concatenated codes.

When this coding scheme is applied to the OFDM system over UWB channel, the coded bit stream is interleaved prior to modulation in order to provide robustness against burst errors. The bit interleaving operation is performed in two stages: symbol interleaving followed by OFDM tone interleaving. The symbol interleaver permutes the bits across OFDM symbols to exploit frequency diversity across sub-bands, while the tone interleaver permutes the bits across the data tones within an OFDM symbol.
to exploit frequency diversity across tones and providing robustness against narrow-band interference.

We constrain our symbol interleaver for 16QAM case to a regular block interleaver of size $N_{pack} \times \text{number of encoder output bits}$, where $N_{pack}$ is the input information packet length and the number of encoder output bits is 2. The coded bits will be read in column-wise and read out row-wise. The output of the symbol block interleaver is then passed through a tone block interleaver of size $N_{OFDM} \times \text{tone numbers in one OFDM symbol}$, where $N_{OFDM}$ is the OFDM symbol numbers for one packet and the tone number is 100 for the considered OFDM system. Still the coded bits will be read in column-wise and read out row-wise.

The encoding scheme for 64QAM TTCM will be elaborated in chapter IV.

### 3.1.2 16QAM Gray Mapping

There are three types of mapping techniques often used in TCM modulation: Ungerboeck’s mapping by set partition (alternately named natural mapping), reordered mapping and Gray code mapping. In Table 3.1, signal levels or cosets and the corresponding binary labels are shown for these three mappings. To better understand the reordered mapping, consider an 8PSK constellation which has eight cosets $c_0, c_1, c_2, \ldots, c_7$. Partition the cosets into two groups $c_0, c_2, c_4, c_6$ and $c_1, c_3, c_5, c_7$. (In the binary labels of the cosets, LSB=0 represents the first group and LSB=1 repre-
Table 3.1: Mappings for each dimension of 16QAM

<table>
<thead>
<tr>
<th>Signal levels</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural mapping</td>
<td>00</td>
<td>01</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Reordered mapping</td>
<td>00</td>
<td>01</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Gray code mapping</td>
<td>00</td>
<td>01</td>
<td>11</td>
<td>10</td>
</tr>
</tbody>
</table>

sents the second group). Swap the last two cosets in each groups to obtain the groups $c_0, c_2, c_6, c_4$ and $c_1, c_3, c_7, c_5$. Then recompose the eight cosets into the reordered cosets $c_0, c_1, c_2, c_3, c_6, c_7, c_4, c_5$. For example if $b_2, b_1, b_0$ represents a binary label for natural mapping, where $b_2$ is the MSB and $b_0$ is the LSB, then the reordered mapping is given by $b_2, (b_2 + b_1), b_0$. While for Gray code mapping we have $b_2, (b_2 + b_1), (b_1 + b_0)$.

The 16QAM gray mapping constellations are given in figure 3.4.
Figure 3.4: 16QAM constellation
3.1.3 OFDM Modulation

This section defines the processing that takes as input the mapped complex numbers coming out of turbo TCM encoder and performs the IFFT which modulates the constellation points onto the carrier waveforms in discrete time. The stream of complex numbers is divided into groups of 100 complex numbers. We denote these complex numbers \( c_{n,k} \), which corresponds to subcarrier \( n \) of OFDM symbol \( k \), as follows:

\[
c_{n,k} = d_{n+100\times k}, \quad n = 0, 1, \ldots, N_{SYM} - 1
\]

where \( N_{SYM} \) denotes the number of OFDM symbols in the PHY frame body. An OFDM symbol \( r_{\text{data},k}(t) \) is defined as

\[
r_{\text{data},k}(t) = \sum_{n=0}^{N_{SD}} c_{n,k} e^{j2\pi M(n)\Delta_F(t-T_{CP})} + p_{\text{mod}(k,127)} \sum_{n=-N_{ST}/2}^{N_{ST}/2} P_n e^{j2\pi n\Delta_F(t-T_{CP})}
\]

where \( N_{SD} \) is the number of data subcarriers, \( N_{ST} \) is the number of total subcarriers used, and the function \( M(n) \) defines a mapping from the indices 0 to 99 to the logical frequency offset indices -56 to 56, excluding the locations reserved for the pilot
subcarriers, guard subcarriers and the DC subcarriers as described below:

\[
M(n) = \begin{cases} 
  n - 56 & n = 0 \\
  n - 55 & 1 \leq n \leq 9 \\
  n - 54 & 10 \leq n \leq 18 \\
  n - 53 & 19 \leq n \leq 27 \\
  n - 52 & 28 \leq n \leq 36 \\
  n - 51 & 37 \leq n \leq 45 \\
  n - 50 & 46 \leq n \leq 49 \\
  n - 49 & 50 \leq n \leq 53 \\
  n - 48 & 54 \leq n \leq 62 \\
  n - 47 & 63 \leq n \leq 71 \\
  n - 46 & 72 \leq n \leq 8 \\
  n - 45 & 81 \leq n \leq 89 \\
  n - 44 & 90 \leq n \leq 98 \\
  n - 43 & n = 99 
\end{cases}
\]

The subcarrier frequency allocations is shown in figure 3.5. \(c_n\) represents the data tones, \(P_n\) represents the pilot tones, and \(GI_n\) represents the guard tones. In each OFDM symbol, twelve of the subcarriers are dedicated to pilot signals in order to make coherent detection robust against frequency offsets and phase noise in implementation. These subcarriers shall be put in subcarriers -55, -45, -35, -25, -15, -5, 5, 15, 25, 35, 45, and 55.
Figure 3.5: Subcarrier frequency allocation
In each OFDM symbol ten subcarriers are dedicated to guard subcarriers or guard tones. The guard subcarriers can be used for various purposes, including relaxing the specs on transmitted and receive filters. They shall be located in subcarriers -61, -60, ..., -57, and 57, 58, ..., 61.

In a discrete-time implementation, a set of data points (100 complex numbers from mapping) plus pilot signals and guard tones will be mapped to the IFFT inputs 1 to 61 and 67 to 127. The rest of the inputs, 62 to 66 and 0, are all set to zero. 128 time samples (IFFT interval) will be obtained after using 128-point IFFT operation. The last 32 time samples of the IFFT interval are prepadded to the beginning of the IFFT output to work as cyclic prefix and a guard interval of length 5 is added at the end of the IFFT interval to create the OFDM symbol of 165 time samples.

Let \( C_n \) denotes the complex number vector corresponding to subcarrier \( n \) of \( i \)th OFDM symbol, which includes \( i \)th \( M \times 1 \) information block \( s_{iM} \). Then all of the OFDM symbols \( \tilde{s}_{iM} \) can be constructed using an IFFT through the expression below:

\[
\tilde{s}_{iM}(t + T_{CP}) = \begin{cases} 
\sum_{-N_{ST}/2}^{N_{ST}/2} C_n e^{j2\pi n\Delta_f t}, t \in [0, T_{FFT}] \\
0, & \text{elsewhere}
\end{cases} 
\]  

(III.1)

where the parameters \( \Delta_f \) (528MHz/128=4.125 MHz) and \( N_{ST} \) are defined as the subcarrier frequency spacing and the number of total subcarriers used, respectively.

The resulting waveform has a duration of \( T_{FFT} = 1/\Delta_f \) (242.42ns). A zero-padding cyclic prefix \( (T_{CP} = 32/528MHz = 60.61ns) \) is used in OFDM to mitigate the effect
of multipath. A guard interval \( T_{GI} = 5/528\text{MHz} = 9.47\text{ns} \) ensures that only a single RF transmitter and RF receiver chain are needed for all channel environments and data rates and there is sufficient time for the transmitter and receiver to switch if used in multiband OFDM [69]. \( T_{FFT} \), \( T_{CP} \) and \( T_{GI} \) make up the OFDM symbol period \( T_{sys} \), which is 312.5ns in this case. Then according to the proposed UWB PHY standard [39], 16QAM modulated OFDM system will support data rate of 640 Mbps and 64QAM OFDM system will support data rate of 1.2 Gbps.

### 3.1.4 UWB Channel

Rayleigh fading channel model has been used extensively to model channels for first generation cellular and many other narrow-band wireless systems due to the unresolvable multipath reflections at the receiver. The received envelope can be modelled as a Rayleigh random variable. While for UWB systems, the large bandwidth significantly increase the ability of the receiver to resolve the different reflections in the channel. There are two basic techniques for UWB channel sounding—Time Domain Sounding Technique and Frequency Domain Sounding Technique [73]. And accordingly there are two kinds of models to characterize the UWB channel. One way to describe UWB channel is its time-variant impulse response \( h(t, \tau) \), which can be expressed as

\[
h(t, \tau) = \sum_{n=1}^{N(t)} \alpha_n(t) \delta(t - \tau_n(t))e^{j\theta_n(t)}
\]  

(III.2)
where the parameters of the $n$th path $\alpha_n, \tau_n, \theta_n$, and $N$ are amplitude, delay, phase, and number of relevant multipath components, respectively. There have been literature containing a substantial amount of material regarding UWB indoor propagation models [22] [23] [74] [75] [76] [77], among which IEEE 802.15.3a standard body selected the model in [74] after being properly parameterized for best fit to the certain channel characteristics described in [78].

Another approach to characterize the UWB channel is to use the frequency domain autoregressive (AR) model, which is introduced for UWB channel modeling in [25]. The frequency response of a UWB channel at each point $H(f_n)$ is modelled by an AR process

$$H(f_n, x) - \sum_{i=1}^{p} b_i H(f_{n-i}, x) = V(f_n). \quad (III.3)$$

where $H(f_n, x)$ is the $n$-th sample of the complex frequency response at location $x$, $V(f_n)$ is complex white noise, the complex constants $b_i$ are the parameters of the model, and $p$ is the order of the model. Based on the frequency domain measurements in the 4.3GHz to 5.6GHz frequency band, a second order ($p = 2$) AR model is reported to be sufficient for characterization of the UWB indoor channel [25]. We will use a frequency-domain autoregressive (AR) model [25] since it is generative and has far fewer parameters than the time domain method. As a result, the simulation model can be constructed and the simulation can be performed easily. For a UWB model realization with the T-R separation of LOS 10m, the estimated complex constants $b_i$
could be:

\[ b_1 = -1.6524 + 0.8088i \]
\[ b_2 = 0.5463 + 0.7381i \]

Figure 3.6 and 3.7 present example UWB channel models obtained from [25]. Most of the channels are within a 6 dB variation (see Figure 3.6). A small percentage of the channels exhibit a variation larger than 6 dB (see Figure 3.7) that requires higher SNR to achieve a good performance.
Figure 3.6: Example frequency response of a good UWB channel.
Figure 3.7: Example frequency response of a bad UWB channel.
3.1.5 *CP-OFDM Equalization*

The OFDM symbol blocks will experience IBI when propagating through UWB channels because the underlying channel’s impulse response combines contributions from more than one transmitted block at the receiver. To account for IBI, OFDM systems rely on the so-called cyclic prefix (CP) which consists of redundant symbols replicated at the beginning of each transmitted block. To eliminate IBI, the redundant part of each block is chosen greater than the channel length and is discarded at receiver in a fashion identical to that used in the overlap-save (OLS) method of block convolution. That means by inserting redundant part in the form of CP, we were able to achieve IBI free reception. Further more, when it comes to equalization, such redundancy pays off. Each truncated block at the receiver end is FFT processed – an operation converting the frequency-selective channel into parallel flat-faded independent subchannels, each corresponding to a different subcarrier. Unless zero, flat fades are removed by dividing each subchannel’s output with channel transfer function at the corresponding subcarrier. At the expense of bandwidth overexpansion, coded OFDM ameliorates performance losses incurred by channel having nulls on the transmitted subcarriers [79]. CP and ZP methods are equivalent to each other which relies implicitly on the well-know OLS method as opposed to OLA. Details regarding the ZP-OFDM equalization will be covered in section 4.2.5.
Figure 3.8: Discrete-time block equivalent model of CP-OFDM.
OFDM signal block propagation through UWB channels can be modelled as a FIR filter with the channel impulse response column vector \( \mathbf{h} = [h_0 h_1 \cdots h_{M-1}]^T \) and additive white Gaussian noise (AWGN) \( \tilde{n}_n(i) \) of variance \( \sigma_n^2 \) [79]. Let \( \mathbf{F}_M \) denote the FFT matrix with \((m,k)\)th entry \( e^{-j2\pi mk/M}/\sqrt{M} \). Then, the IFFT matrix can be denoted as \( \mathbf{F}_M^{-1} = \mathbf{F}_M^H \) with \((m,k)\)th entry \( e^{j2\pi mk/M}/\sqrt{M} \) to yield the so-called time domain block vector \( \tilde{s}_M^i = \mathbf{F}_M^H \tilde{s}_M^i \), where \((\cdot)^H\) denotes conjugate transposition. Then in order to remove IBI, a cyclic prefix (CP) will be added onto time-domain block vector as shown in figure 3.8.

Figure 3.8 depicts the baseband discrete-time block diagram of the CP-OFDM system [79] [80] [81]. If we denote the signal vector \( s_M^i \) and \( \tilde{s}_M^i \) as \( [s_M^i(0)s_M^i(1) \cdots s_M^i(M-1)]^T \) and \( [\tilde{s}_M^i(0)\tilde{s}_M^i(1) \cdots \tilde{s}_M^i(M-1)]^T \) respectively, then adding a CP of length \( D \) at the beginning of vector \( \tilde{s}_M^i \) results a redundant block \( \tilde{s}_M^i = [\tilde{s}_M^i(M-D) \cdot \cdots \tilde{s}_M^i(M-1)\tilde{s}_M^i(0)\tilde{s}_M^i(1) \cdots \tilde{s}_M^i(M-1)]^T \) which will be sent sequentially through the channel. The total number of time-domain samples per transmitted block is, thus, \( P = M + D \). Consider the \( M \times D \) matrix \( \mathbf{F}_C^i \) formed by the last \( D \) columns of \( \mathbf{F}_M \). Defining \( \mathbf{F}_C^i = [\mathbf{F}_C^i, \mathbf{F}_M] \) as the \( P \times M \) matrix corresponding to the combined multicarrier modulation and CP insertion, the block of symbols to be transmitted can simply expressed as \( \tilde{s}_C^i = \mathbf{F}_C^i s_M^i \).

With \((\cdot)^T\) denoting transposition, the frequency-selective propagation will be modelled as a FIR filter with channel impulse response column vector \( \mathbf{h} = [h_0 \cdots h_{M-1}]^T \) and additive white gaussian noise (AWGN) \( \tilde{n}_n^i \) of variance \( \sigma_n^2 \). In practice, we select
\[ M \geq D \geq L, \text{ where } L \text{ is the channel order (i.e., } h_i = 0, \forall i > L). \text{ Then the } i\text{th received symbol block is given by} \]
\[
\tilde{x}_{cp}^i = H F_{cp}s_M^i + H_{IBI}F_{cp}s_{M-1}^i + \tilde{n}_p^i \quad \text{(III.4)}
\]
where \( H \) is the \( P \times P \) lower triangular Toeplitz filtering matrix and \( H_{IBI} \) is the \( P \times P \) upper triangular Toeplitz filtering matrix, which capture IBI, as follows [68]:

\[
H = \begin{pmatrix}
    h_0 & 0 & \cdots & 0 & 0 \\
    h_1 & h_0 & \cdots & 0 & 0 \\
    \vdots & & \ddots & & \vdots \\
    h_{L-1} & h_{L-2} & \cdots & 0 & 0 \\
    0 & h_{L-1} & \cdots & 0 & 0 \\
    \vdots & & & \ddots & \vdots \\
    0 & 0 & \cdots & h_0 & 0 \\
    0 & 0 & \cdots & h_1 & h_0
\end{pmatrix}_{P \times P}
\]

\[
H_{IBI} = \begin{pmatrix}
    0 & \cdots & 0 & h_L & \cdots & h_1 \\
    0 & \cdots & 0 & 0 & \cdots & h_2 \\
    \vdots & & & & \ddots & \vdots \\
    0 & \cdots & 0 & 0 & \cdots & h_L \\
    0 & \cdots & 0 & 0 & \cdots & 0 \\
    \vdots & & & & & \ddots \\
    0 & \cdots & 0 & 0 & \cdots & 0
\end{pmatrix}_{P \times P}
\]
\[ \tilde{n}_i^T = [\tilde{n}^iP \cdots \tilde{n}^{iP+P-1}] \]

denotes the AWGN vector.

Equalization of CP-OFDM transmission relies on the well-known property that every circulant matrix can be diagonalized by post- (pre-) multiplication by (I)FFT matrices [80]. After removing the CP at the receiver as indicated in figure 3.8, since the channel order satisfies \( L \leq D \), equation III.4 reduces to

\[
\tilde{x}_M^i = C_M(h)F_M^H s_M^i + \tilde{n}_M^i
\]  

(III.5)

where \( C_M(h) \) is \( M \times M \) circulant matrix

\[
C_M(h) = \begin{pmatrix}
    h_0 & 0 & \ldots & h_L & \ldots & h_1 \\
    h_1 & h_0 & \ldots & 0 & \ldots & h_2 \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    h_L & h_{L-1} & \ldots & 0 & \ldots & 0 \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    0 & 0 & \ldots & h_L & \ldots & h_0
\end{pmatrix}_{M \times M}
\]

and \( \tilde{n}_M^i = [\tilde{n}^{iP+D} \cdots \tilde{n}^{iP+P-1}]^T \). The circulant matrix \( C_M(h) \) can be diagonalized by \( M \times M \) FFT matrix, which leads to

\[
X_M^i = F_M C_M(h) F_M^H s_M^i + F_M \tilde{n}_M^i \\
= \text{diag}(H_0 \cdots H_{M-1}) s_M^i + F_M \tilde{n}_M^i \\
= D_M(\tilde{h}_M) s_M^i + n_M^i
\]  

(III.6)

where \( \tilde{h}_M = [H_0 \cdots H_{M-1}]^T = \sqrt{M} F_M h \), with \( H_k = H(2\pi k/M) = \sum_{l=0}^{L} h_l e^{-j2\pi kl/M} \)
denoting the channel transfer function on the kth subcarrier, \( D_M(\tilde{h}_M) \) standing for
the $M \times M$ diagonal matrix with $\hat{h}_M$ on its diagonal. $n^{\prime}_M = F_M \tilde{n}^i + M$.

This CP-OFDM property derives from the fast convolution algorithm based on OLS algorithm for block convolution. It also makes it easy to dealing with ISI channels by simply take into account the scalar channel attenuation, e.g., when computing the branch metric in trellis based decoding algorithm. However, it has the obvious drawback that the symbol transmitted on the $k$th subcarrier can not be recovered if it is hit by a channel zero ($H_k = 0$). The equalization scheme will be referred as CP-OFDM-OLS. We implemented CP-OFDM in 16QAM TTCM coded OFDM system for UWB channel [70] [71].

3.2 Modified Iterative Bit MAP Decoding

The MAP (Maximum Aposteriori Probability) algorithm in iterative decoding calculates the Logarithm of Likelihood Ratio (LLR), $\Lambda(u_b)$, associated with each decoded bit $u_b$ at time $k$ through equation (III.7) [2]:

$$\Lambda(u_b) = \log \frac{P_r\{u_b = 1|\text{observation}\}}{P_r\{u_b = 0|\text{observation}\}}$$

(III.7)

where $P_r\{u_b = i|\text{observation}\}, i = 0, 1$ is the a posteriori probability (APP) of the data bit $u_b$. The APP of a decoded data bit $u_b$ can be derived from the joint probability $\lambda^i_k(m)$ defined by

$$\lambda^i_k(S_k) = P_r\{u_b = i, S_k|y_k\}$$

(III.8)

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where $S_k$ represents the encoder state at time $k$ and $y_k$ is the received channel symbol.

Thus, the APP of a decoded data bit $u_b$ is equal to

$$P_r\{u_b = i | y_k\} = \sum_{S_k} \lambda^i_k(S_k), i = 0, 1$$  \hspace{1cm} (III.9)

From relations (III.7) and (III.9), the LLR $\Lambda(u_b)$ associated with a decoded bit $u_b$ can be written as

$$\Lambda(u_b) = \log \frac{\sum_{S_k} \lambda^1_k(S_k)}{\sum_{S_k} \lambda^0_k(S_k)}$$  \hspace{1cm} (III.10)

Finally the decoder can make a decision by comparing $\Lambda(u_b)$ to a threshold equal to zero

$$\tilde{u}_b = 1 \text{ if } \Lambda(u_b) > 0$$  
$$\tilde{u}_b = 0 \text{ if } \Lambda(u_b) < 0$$

The joint probability $\lambda^i_k(S_k)$ can be rewritten using Bayes rule

$$\lambda^i_k(S_k) = \frac{P_r\{u_b = i, S_k, y^k, y^N_{k+1}\}}{P_r\{y^k, y^N_{k+1}\}}$$

$$= \frac{P_r\{u_b = i, S_k, y^k\}}{P_r\{y^k\}} \cdot \frac{P_r\{y^N_{k+1} | u_b = i, S_k, y^k\}}{P_r\{y^N_{k+1} | y^k\}}$$

in which we assume the information symbol sequence $\{u_k\}$ is made up of $N_u$ independent input symbols $u_k$ with $K$ input bits ($i.e. u_b, b = 1 \ldots K$) in each $u_k$ and take into account that events after time $k$ are not influenced by observations $y^k_1$ and symbol $u_k$ if encoder state $S_k$ is known. For easy computation of the probability
\( \lambda^i_k(S_k) \), probability functions \( \alpha_k(S_k) \), \( \beta_k(S_k) \) and \( \gamma_i(y_k, S_{k-1}, S_k) \) are introduced as follows [61]:

\[
\alpha_k(S_k) = \frac{P_r\{u_b = i, S_k, y_1^k\} P_r\{u_b = i, S_k|y_1^k\}}{P_r\{y_1^k\}} \\
\beta_k(S_k) = \frac{P_r\{y_{k+1}^N|S_k\}}{P_r\{y_{k+1}^N|y_1^k\}} \\
\gamma_i(y_k, S_{k-1}, S_k) = P_r\{u_b = i, y_k^k, S_k|S_{k-1}\}
\]

Then \( \lambda^i_k(S_k) \) can be simplified as:

\[
\lambda^i_k(S_k) = \alpha_k(S_k)\beta_k(S_k) \quad (III.11)
\]

The probabilities \( \alpha_k(S_k) \) and \( \beta_k(S_k) \) can be recursively calculated from probability \( \gamma_i(y_k, S_{k-1}, S_k) \) through

\[
\alpha_k(S_k) = \frac{\sum_{S_{k-1}} \sum_{j=0}^{1} \gamma_i(y_k, S_{k-1}, S_k) \alpha^j_{k-1}(S_{k-1})}{\sum_{S_k} \sum_{S_{k-1}} \sum_{j=0}^{1} \gamma_i(y_k, S_{k-1}, S_k) \alpha^j_{k-1}(S_{k-1})} \\
\beta_k(S_k) = \frac{\sum_{S_{k+1}} \sum_{j=0}^{1} \gamma_i(y_{k+1}, S_k, S_{k+1}) \beta^j_{k+1}(S_{k+1})}{\sum_{S_{k+1}} \sum_{S_k} \sum_{j=0}^{1} \gamma_i(y_{k+1}, S_k, S_{k+1}) \alpha^j_k(S_k)}
\]

and \( \gamma_i(y_k, S_{k-1}, S_k) \) can be determined from transition probabilities of the encoder trellis and the channel, which is given by

\[
\gamma_i(y_k, S_{k-1}, S_k) = p(y_k|u_b = i, S_{k-1}, S_k) \\
\times q(u_b = i|S_{k-1}, S_k) \\
\times \pi(S_k|S_{k-1}) \quad (III.12)
\]
\( p(\cdot | \cdot) \) is the channel transition probability, \( q(\cdot | \cdot) \) is either 1 or 0 depending on whether the \( i \)th bit is associated with transition from \( S_{k-1} \) to \( S_k \) or not, and \( \pi(\cdot | \cdot) \) is the state transition probability that uses the extrinsic information of information \( u_k \).

Using LLR \( \Lambda(u_b) \) definition (III.10) and relations among \( \lambda^i_k, \alpha_k, \beta_k \) and \( \gamma \) we obtain

\[
\Lambda(u_b) = \log \frac{\sum_{S_k} \sum_{S_{k-1}} \gamma_1(y_k, S_{k-1}, S_k) \alpha_{k-1}(S_{k-1}) \beta_k(S_k)}{\sum_{S_k} \sum_{S_{k-1}} \gamma_0(y_k, S_{k-1}, S_k) \alpha_{k-1}(S_{k-1}) \beta_k(S_k)} \tag{III.13}
\]

It was proved in [2] that the LLR \( \Lambda(u_b) \) associated with each decoded bit \( u_b \) is the sum of the LLR of \( u_b \) at the decoder input and of another information called extrinsic information generated by the decoder.

Divsalar [59] for the first time described an iterative decoding scheme for \( q \) parallel concatenated convolutional codes based on approximating the optimum bit decision rule by considering the combination of interleaver and the trellis encoder as a block encoder. The scheme is based on solving a set of nonlinear equations given by \( (q = 2 \) is used here to illustrate the concept, [82] [59])

\[
\hat{L}_{1b} = \log \frac{\sum_{u_{:u_b=1}} P(y_1|u) \prod_{j \neq b} e^{u_j \hat{L}_{2j}}}{\sum_{u_{:u_b=0}} P(y_1|u) \prod_{j \neq b} e^{u_j \hat{L}_{2j}}}
\]

\[
\hat{L}_{2b} = \log \frac{\sum_{u_{:u_b=1}} P(y_2|u) \prod_{j \neq b} e^{u_j \hat{L}_{1j}}}{\sum_{u_{:u_b=0}} P(y_2|u) \prod_{j \neq b} e^{u_j \hat{L}_{1j}}}
\]

for \( b = 1, 2, ..., K \) representing \( b \) input bits per constituent encoder, where \( \hat{L}_{1j} \) are the extrinsic information and \( y_q \) are the received observation vectors corresponding to the \( q \)th trellis code. The final decision is then based on \( L_b = \hat{L}_{1b} + \hat{L}_{2b} \), which passed through a hard limiter with zero threshold.
The above set of nonlinear equations are derived from the optimum bit decision rule
\[
L_b = \log \frac{\sum_{u: u_b = 1} P(y_1 | u)P(y_2 | u)}{\sum_{u: u_b = 0} P(y_1 | u)P(y_2 | u)} \tag{III.14}
\]
using the following approximation
\[
P(u | y_1) \approx \prod_{b=1}^{N} \frac{e^{u_b \hat{L}_{1b}}}{1 + e^{\hat{L}_{1b}}}, \quad P(u | y_2) \approx \prod_{b=1}^{N} \frac{e^{u_b \hat{L}_{2b}}}{1 + e^{\hat{L}_{2b}}} \tag{III.15}
\]
The nonlinear equations in equation (III.14) can be solved by using an iterative procedure
\[
\hat{L}^{(m+1)}_{1b} = \log \frac{\sum_{u: u_b = 1} P(y_1 | u) \prod_{j \neq b} e^{u_j \hat{L}_{2j}}}{\sum_{u: u_b = 0} P(y_1 | u) \prod_{j \neq b} e^{u_j \hat{L}_{2j}}} \tag{III.16}
\]
on \text{for } b = 1, 2, ..., K. Similar recursions hold for \(\hat{L}^{(m+1)}_{2b}\). The recursion starts with the initial condition \(\hat{L}_{1}^{(0)} = \hat{L}_{2}^{(0)} = 0\). The LLR of a symbol \(u\) given the observation \(y\) is calculated first using the symbol MAP algorithm
\[
\lambda(u) = \log \frac{P(u | y)}{P(0 | y)} \tag{III.17}
\]
where \(0\) corresponds to the all-zero symbol. The symbol MAP algorithm [61] can be used to calculate Eq. (III.17), as shown in Figure 3.9 [82]. Then the LLR of the \(b\)th bit within the symbol can be obtained by
\[
L_b = \log \frac{\sum_{u: u_b = 1} e^{\lambda(u)}}{\sum_{u: u_b = 0} e^{\lambda(u)}} \tag{III.18}
\]
The symbol a priori probabilities needed in the symbol MAP algorithm, which will be used in branch transition probability calculation, can be obtained by
\[
P(u = (u_1, u_2, ..., u_K)) = \prod_{b=1}^{K} \frac{e^{u_b \hat{L}_b}}{1 + e^{\hat{L}_b}} \tag{III.19}
\]
with the assumption that the extrinsic bit reliabilities coming from the other decoder are independent.

In our case, we apply the turbo iterative MAP decoding scheme in [2] [61] [82] [83], and make certain modifications to fit our concatenated encoder structure. Since our parity-concatenated encoder structure consists of a TCM inner code and simplest parity-check outer code functioning as repeaters, we only need one bit MAP decoder for the inner code decoding. The outer code decoding can be interpreted as extrinsic information exchange. Therefore, the standard iterative decoder for TTCM can be modified into figure 3.10.

The bit MAP decoder computes the \textit{a posteriori probabilities} \( P(u_b|y, \hat{u}) \) (\( y \) is the received channel symbol and \( \hat{u} \) is the result from previous iteration), or equivalently the log-likelihood ratio \( \Lambda(u_b) = \log(P(u_b = 1|y, \hat{u})/P(u_b = 0|y, \hat{u})) \). Then, the extrinsic information \( L_e(u_b)_{out} \) is extracted from \( L_e(u_b)_{out} = \Lambda(u_b) - L_c(u_b) - L_e(u_b)_{in} \) to avoid information being used repeatedly. It will be supplied to the parity-check decoder. The outer parity-check decoder updates the \( L_e(u_b)_{out} \) into \( L_e(u_b)_{in} \) according to parity check constraints between information bits and supplies it to the bit MAP decoder for the next iteration. \( L_e(u_b)_{in} \) is the extrinsic information, which is used as a priori probability for branch metric computation in MAP decoding process. \( L_c(u_b) \) is the channel reliability for each \( u_b \).
Figure 3.9: Iterative (turbo) decoder structure for two trellis codes

Figure 3.10: Block diagram of the iterative decoder.
Since half of the systematic bits from the inner TCM encoder are punctured, it seems that we can only get channel transition probability for the remaining half of the information bits and parity check bits. However, the punctured information bits are the parity checks of those systematic bits at the encoder outputs except being interleaved. So we can always find the channel transition probability for the punctured information bits through the un-punctured part. The extrinsic information value associated with $\pi(\cdot/\cdot)$ in (III.12) is given as the logarithm format:

$$L_e(u_b) = \log \frac{P(u_b = 1)}{P(u_b = 0)} \quad (III.20)$$

If $q(u_b = 1/S_{k-1}, S_k) = 1$, then

$$\pi(S_k/S_{k-1}) = \frac{e^{L_e(u_b)}}{1 + e^{L_e(u_b)}} \quad (III.21)$$

otherwise

$$\pi(S_k/S_{k-1}) = \frac{1}{1 + e^{L_e(u_b)}} \quad (III.22)$$

### 3.3 System Performance Analysis

#### 3.3.1 Density Evolution for TTCM

Convergence analysis of iterative decoding algorithms for turbo codes has received much attention recently due to its useful application to predicting code performance, its ability to provide insights into the encoder structure, and its usefulness in helping
with the code design. Turbo trellis-coded modulation conjoins signal mapping techniques, such as Ungerboeck’s signal space partition, with turbo coding, to achieve significant coding gains without increasing bandwidth. However, the need for signal mapping makes the encoder structure more complex to design and analyze than binary turbo codes. Hence, the convergence analysis is a very important tool for design and comparison between TTCM schemes. Several models have been proposed to analyze the convergence of iterative decoders. In particular, the extrinsic information transfer (EXIT) method has created a lot of interest.

The density evolution method has been used to confirm the simulation. We approximate the extrinsic information as a Gaussian variable whose mean is equal to half of the variance. In each iteration, we compute the average mean of the extrinsic information and then regenerate the extrinsic information as an independent Gaussian variable. Thus, the dependence between the extrinsic information bits has been wiped out. This is the main difference between density evolution and simulation. Since TCM is typically irregular, density evolution using the all zero sequence may be biased. So we need to consider both 0-bit and 1-bit as input which could bring negative mean according to the definition of extrinsic information in (III.20). We examine the mean of extrinsic information using tens of thousands of randomly generated bit sequences and make it always positive regardless of bit sequences by weighting through the sign of the bit. Such mean can be easily traced by two decoding...
trajectories in the density evolution chart, i.e.,

$$\mu_{L_e} = \bar{L_e}(u_b)(2u_b - 1) \quad (\text{III.23})$$

where overbar denotes the average. For UWB channels, we then average it over more than 2000 UWB channels.

Procedure of density evolution can be summarized as follows:

(1) Before the first iteration starts, all the extrinsic information is set to be zero.

(2) We divide each decoding process into two halves: one half-iteration for TCM followed by another half-iteration for parity check codes. For each half-iteration we can calculate the updated extrinsic information through decoding. Using tens of thousands of simulation we can get the mean of the densities of those updated extrinsic information using (III.23).

(3) Further, we assume the density to be Gaussian with the mean computed in (III.23) and the variance equal to twice of the mean based on density symmetry condition [86]. Then, we regenerate the extrinsic information as independent Gaussian variable for the next half-iteration.

(4) During each half-iteration, SNR is estimated as half of the mean of extrinsic information. SNR before and after each half-iteration then can be tracked in the density evolution chart as in this paper.
Density evolution can be used to determine the threshold, which is the minimum SNR for the decoder to converge assuming infinite block length. In density evolution chart, as long as the SNR is above the threshold, these two constituent transfer curves will never intersect, which means convergence in the limiting case. In figure 3.11, we show density evolution for OFDM systems using 16QAM on Gaussian and UWB fading channels. For Gaussian channels, we find the threshold is $2.6dB$ and show the EXIT chart for $Eb/No = 2.8dB$. On UWB channels, we found that if we take average over all 2000 channels, then EXIT chart shows the clear case of convergence (see curves with solid squares in figure 3.11). However, if we run EXIT over each individual channel instance, then some channel instances require much large SNRs to allow iterative decoding to converge to correct codewords. For example, at $Eb/No = 5.5dB$, about 2% of the channels are difficult to converge (see curves with crosses in figure 3.11). We call them “bad” channels. When $Eb/No$ is small, the percentage of worst channels increases significantly. For example, when $Eb/No = 4.5dB$, about 20% of channels are bad. Good performance can only be achieved unless the interleave can fully randomize the extrinsic information over all channels. If the bits of a packet are interleaved over a number of channels containing significant amount of “bad” channels, then the performance will be much poorer. This is the main reason that the packet error rate curve for UWB could not drop sharply as those on AWGN channels.
Figure 3.11: Density evolution for 16QAM/OFDM on AWGN and UWB channels.
Figure 3.12 presents the density evolution analysis for OFDM system using 64QAM on Gaussian and UWB fading channels. For Gaussian channels, we find the threshold is $3.7 dB$ and show the EXIT chart for $Eb/No = 4.2 dB$. For UWB channels, when we set $Eb/No = 9.2 dB$, about 2% of the channels are bad.
Figure 3.12: Density evolution for 64QAM/OFDM on AWGN and UWB channels.
3.3.2 Bound Performance for TTCM

There are a considerable amount of research has addressed the bound performance evaluation of different codes [48] [53] [88]- [102], which cover the code type from block codes, convolutional codes, turbo codes, TCM codes, and concatenated codes. Different approaches to evaluate the performance of TCM codes or turbo codes have been suggested in [53] [92]- [102]. Duman and Salehi in [93] provide the performance analysis for 16QAM turbo coded modulation system. All of the above turbo type code performance evaluation is based on conventional turbo structure and then finds the average performance bounds (averaged over all possible interleavers). Our encoder functions as a Turbo TCM as shown in figure 2.15 for the 16QAM case, but due to the multiple input streams and the punctured systematic bits, it’s hard to use the evaluation method proposed in [93]. Here we try to explore the exhaustive enumeration of TTCM codewords to confirm the code performance.

For maximum likelihood decoding and transmission over an AWGN channel, the upper and lower union bounds for bit error rate $P_b$ at high signal-to-noise ratios can be written as [94] [7] [93] [89] [90]:

$$P_b \leq \sum_{d_i=d_{\text{min}}}^{\infty} \frac{B_i}{N} A_i Q \left( \sqrt{\frac{d_i^2}{2N_0}} \right) \quad \text{(III.24)}$$

$$P_b \geq \frac{B_{d_{\text{min}}}}{N} A_{d_{\text{min}}} Q \left( \sqrt{\frac{d_{d_{\text{min}}}^2}{2N_0}} \right) \quad \text{(III.25)}$$

where $A_i$ is the numbers of error paths with the Euclidean distance $d_i$, $B_i$ is the average
number of bit errors occurring on paths with $d_i$, and $A_{d_{\text{min}}}$ and $B_{d_{\text{min}}}$ correspond to $A_i$ and $B_i$ when $d_i = d_{\text{min}}$. $N$ is the input package length or interleaver length, $N_0$ is the single-sided spectral noise density of the AWGN channel.

The most difficult task in calculating the union bounds for turbo TCM is to find $A_i$, $B_i$ and $d_i$ of the error path. More precisely the most difficult part is to find the error path since we do not have a simple trellis structure as in [92] to traverse because there are two constituent encoders connected by two interleavers at the inputs. So we need to consider a hyper-trellis similar to [7] to examine the full dynamics of the turbo TCM code.

Turbo TCM code is irregular. We examine error events of $N$ steps. Each error path should be labelled by the input bits and output parity check bit, which is the IRWEF in each step, such as $A^{C_k}(w_1, w_2, z)$, where $w_1, w_2$ are the weights of $u_1$ and $u_2$, $z$ is the weight of parity check bit, and $C_k$ identifies the 1st or 2nd constituent encoder. Since the two encoders are identical, we only need to work this out for one encoder and obtain the other accordingly. Then, we combine two error paths, one from $C_1$ and another from $C_2$, which have the same value in cross summation of $w_1$ and $w_2$, where cross summation ($\sum w_1$) of $C_1$ should be equal to $\sum w_2$ of $C_2$ and $\sum w_2$ of $C_1$ should be equal to $\sum w_1$ of $C_2$ since the positions of interleaved $u_1$ and $u_2$ are changed before the second encoder. After mapping the IRWEF of the combined error path, we can easily find the squared Euclidean distance of the error path from the transmitted path.
Two points requiring attention are: (1) The error paths from two constituent encoders do not have to diverge the starting state from the first step and merge back the ended state in the last step simultaneously when being combined, since the final error path requires only one different output at each step to differ from the correct path, as long as two constituent error paths have same cross summation of $w_1, w_2$. (2) Since there are two interleavers at the inputs, assuming the interleavers are uniform, the probability of the resulted error path will be

$$A_i(w_1, w_2, d_i) = \frac{A^{C_1}(w_1, w_2, d_i) A^{C_2}(w_2, w_1, d_i)}{N}\left(\begin{array}{c}N \\ w_1 \end{array}\right)\left(\begin{array}{c}N \\ w_2 \end{array}\right)$$

(III.26)

where $A^{C_1}(w_1, w_2, d_i)$ and $A^{C_2}(w_2, w_1, d_i)$ are the number of the error paths from encoder $C_1$ and $C_2$ corresponding to the squared Euclidean distance $d_i$ of the combined error path with information weight $w_1$ and $w_2$. So the resulted average error bits on path $(w_1, w_2, d_i)$ is $B_i = w_1 + w_2$.

For arbitrary inputs, we can first find the correct transmission path, and then only record combined paths which are different from the correct one. The bound performance for interleaver size of 10 is given in figure 3.13. We note the consistency of the bound with the simulation results.
Figure 3.13: Bounds on BER for systems with $N = 10$. 
3.4 Numerical Results

The large bandwidth of UWB waveforms significantly increases the ability of the receiver to resolve the different reflections in the channel. OFDM is one of two basic solutions for inter-symbol interference (ISI) removal and efficiently capturing multipath energy in highly dispersive UWB channels and delivering high data rate transmission. The IEEE 802.15 task group came up with a high data rate WPAN with data rate from 55 Mbps to 480 Mbps using punctured convolutional coded OFDM modulation [39]. We are examining the possibility of improving the proposed system using Turbo TCM with QAM constellation for higher data rate transmission. The study shows that the system can offer much higher spectral efficiency, for example, 1.2 Gbps, which is 2.5 times higher than the current proposed system. Results have been confirmed by density evolution in 3.3.1.

The performance of the proposed coding/decoding scheme is evaluated and applied to the OFDM systems for UWB channels. A similar simulation has been done over AWGN channels for performance comparison. System level simulations were performed to estimate the bit error rate (BER) and packet error rate (PER) performance. Table 1 shows a list of key OFDM parameters used in our simulations. The system is assumed to be perfectly synchronized.
Table 3.2: Coded OFDM system parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Info. Data Rate</td>
<td>640Mbps / 1.2Gbps</td>
</tr>
<tr>
<td>Constellation</td>
<td>16QAM / 64QAM</td>
</tr>
<tr>
<td>16-state TCM code</td>
<td>(23,35,27) / (23,35,33,37,31)</td>
</tr>
<tr>
<td>FFT size</td>
<td>128</td>
</tr>
<tr>
<td>Data Tones</td>
<td>100</td>
</tr>
<tr>
<td>System Bandwidth</td>
<td>528MHz</td>
</tr>
<tr>
<td>Subcarrier Frequency Spacing</td>
<td>4.125MHz</td>
</tr>
<tr>
<td>IFFT/FFT Period</td>
<td>242.42ns</td>
</tr>
<tr>
<td>Cyclic Prefix Duration</td>
<td>60.61ns</td>
</tr>
<tr>
<td>Guard Interval Duration</td>
<td>9.47ns</td>
</tr>
<tr>
<td>Symbol Interval</td>
<td>312.5ns</td>
</tr>
<tr>
<td>Time-domain Spreading</td>
<td>Yes</td>
</tr>
<tr>
<td>Multi-path Tolerance</td>
<td>60.6ns</td>
</tr>
<tr>
<td>UWB Channel Model</td>
<td>AR model</td>
</tr>
<tr>
<td>OFDM Equalization</td>
<td>CP-OFDM / ZP-OFDM</td>
</tr>
</tbody>
</table>
3.4.1 640Mbps OFDM System Over UWB Channel

A 16-state TCM code with octal notation (23,35,27) is chosen with 16QAM modulation. The resultant data rate for OFDM/UWB system is 640Mbps. Simulation results are averaged over 2000 packets with a payload of 1k bytes. There are 2000 different UWB channel realizations involved in the simulation.

Figure 3.14 shows the BER performance of the coded 16QAM OFDM system and uncoded OFDM system in both UWB and AWGN channels as a function of $E_b/N_0$. Uncoded modulation scheme is QPSK in order to keep same system coding rate. For UWB channels, the Line of Sight (LOS) distance between the transmitter and receiver is 10m. To measure BER at each point, we simulated up to $1.64 \times 10^7$ bits, which is $2000 \times 41 \text{ OFDM symbols/packet} \times 100 \text{ QAM symbols/OFDM symbol} \times 2 \text{ bits/QAM symbol}$. The coded OFDM curve shows a big performance improvement over uncoded OFDM, especially on UWB channels. Furthermore, a BER of $8 \times 10^{-6}$ is obtained at $E_b/N_0 = 6.7dB$.

Figure 3.15 describes the PER performance of the 640Mbps coded OFDM system and uncoded case over UWB and AWGN channels. The low PER of 0.036 is obtained at $E_b/N_0 = 6.7dB$ for coded OFDM over 10m UWB channels.
Figure 3.14: BER of OFDM/16QAM over UWB and AWGN channel.
Figure 3.15: PER of OFDM/16QAM over UWB and AWGN channel.
3.4.2 1.2Gbps OFDM System Over UWB Channel

A 16-state TCM code with octal notation (23,35,33,37,31) is chosen for 64QAM modulation. The resultant data rate for OFDM/UWB is 1.2Gbps, which is 2.5 times of the data rate for current OFDM/UWB system. All simulation results are averaged over 2000 packets with a payload of 2k bytes for 1.2Gbps system. Similarly, there are 2000 different UWB channel realizations were involved in the simulation.

The BER performance for 64QAM coded OFDM system and 16QAM uncoded OFDM system is illustrated in figure 3.16. Again uncoded modulation scheme is lower than coded modulation scheme to keep the same system coding rate. There are $3.28 \times 10^7$ (2000 packets $\times$ 41 OFDM symbols/packet $\times$ 100 QAM symbols/OFDM symbol $\times$ 4 bits/QAM symbol) random bits simulated to measure the BER. The LOS distance of the UWB channel is 10m. The simulation results indicate a BER of $2.3 \times 10^{-5}$ at $E_b/N_0 = 10.7dB$ for 1.2Gbps coded OFDM system over UWB channels. Figure 3.17 presents the PER performance for the same situation, reporting a low PER of 0.011 at $E_b/N_0 = 10.7dB$ for 1.2Gbps coded OFDM over UWB channels.
Figure 3.16: BER of OFDM/64QAM over UWB and AWGN channel.
Figure 3.17: PER of OFDM/64QAM over UWB and AWGN channel.
4.1 Introduction of Powerline Communications

Powerline communications stands for the use of power supply grid for communication purpose. Power line network has very extensive infrastructure in nearly each building. Because of that fact the use of this network for transmission of data in addition to power supply has gained a lot of attention. Since power line was devised for transmission of power at 50-60 Hz and at most 400 Hz, the use this medium for data transmission, at high frequencies, presents some technically challenging problems.
Besides large attenuation, power line is one of the most electrically contaminated environments, which makes communication extremely difficult. Furthermore the restrictions imposed on the use of various frequency bands in the power line spectrum limit the achievable data rates.

High-speed data communication over low-tension power lines has recently gained lot of attention. This is fueled by the unparalleled growth of the Internet, which has created accelerating demand for digital telecommunications. High bandwidth digital devices are designed to exploit this market. More specifically, these devices use the existing power line infrastructure within the apartment, office or school building for providing a local area network (LAN) to interconnect various digital devices. It has to be noted that the existing infrastructure for communications like telephone line, Cable TV has very few outlets inside the buildings. By use of gateways between these and Power line LANs a variety of services can be offered to customers. Some of the applications, from in-the-home applications to to-the-home applications, include high-speed Internet access, multimedia, smart appliances/remote control, home automation and security; data back up, telecommunications, entertainment and IP-telephony. Powerline communications allows you to plug in, and simply connect.

High bandwidth digital devices for communication on power line use the frequency band between 1 MHz and 30 MHz. In contrast to low bandwidth digital devices, no regulatory standards have been developed for this region of the spectrum. Devices using this unlicensed band need to be compliant with the radiation emission limits
imposed by the regulatory bodies. It should be noted that internationally agreed, distress, broadcast, citizen band and amateur radio frequencies also occupy this portion of the spectrum. Hence, the technologies being developed for high-speed digital communication over power line should have the ability to mask certain frequency bands for future compatibility. In the section that follows gives a brief overview of power line channel characteristics in the frequency band between 1 MHz and 30 MHz.

Since the power line is not designed for communication purpose, the channel exhibits unfavorable transmission properties, such as frequency-selective, narrowband interference, impulse noise and attenuation increase with length and frequency. High bandwidth digital devices communicating on power line devices need to use powerful error correction coding along with appropriate modulation techniques to improve these impairments. The choice of modulation scheme is dependent on the nature of physical medium on which it has to operate. Modulation scheme for use on power line should have the following desirable properties:

1. Ability to overcome non-linear channel characteristics: Power line has a very non-linear channel characteristics. This would make equalization very complex and expensive, if not impossible, for data rates above 10 Mbps with single carrier modulation. The modulation technique for use on power line should have the ability to overcome such non-linearities without the need for a highly complicated equalization;
2. Ability to overcome multipath spread: Impedance mismatch on power lines results in echo signal causing delay spread of the order of 1ms. The modulation technique for use on power line should have the inherent ability to overcome such multipath effect;

3. Ability to adjust dynamically: Power line channel characteristics change dynamically as the load on the power supply varies. The modulation technique for use on power line should have the ability to track such changes without involving large overhead or complexity;

4. Ability to mask certain frequencies: Power line communications equipment use unlicensed frequency band. However it is likely that in the near future various regulatory rules could be developed for this frequency bands also. Hence it is highly desirable to have a modulation technique that could selectively mask certain frequency bands. This property would help in future compatibility and marketability of the product globally.

A modulation scheme that has all these desirable properties is Orthogonal Frequency Division Multiplexing (OFDM). OFDM is generally view as a collection of transmission techniques. When applied in wireless environment it is called OFDM. However in a wired environment the term Discrete Multi Tone (DMT) is more commonly used. OFDM is currently used in the European Digital Audio Broadcast (DAB) standards. Several DAB systems proposed for North America are also based on
OFDM. OFDM under the name DMT has also attracted a great deal of attention as an efficient technology for high-speed transmission on the existing telephone networks (e.g. Asymmetric Digital Subscriber Loop or ADSL).

## 4.2 OFDM System For Power Line Channel

Here we will re-state some advantages of OFDM:

1. Very good at mitigating the effects of time-dispersion;

2. Very good at mitigating the effect of in-band narrowband interference;

3. High bandwidth efficiency and scalable to high data rates

4. Flexible and can be made adaptive; different modulation schemes for subcarriers, bit loading, adaptable bandwidth/data rates possible

5. It makes the Inter Carrier Interference (ICI) zero even in the presence of time dispersion by maintaining orthogonality. It also acts like a guard interval removing Inter Symbol Interference (ISI);

6. It Does not require channel equalization.

All of above mentioned merits make OFDM a good modulation technique in powerline communications. HomePlug networking specifications are the globally recognized standards for high-speed powerline networking. We will investigate the performance
of TTCM coded OFDM system over powerline channels based on HomePlug 1.0 standard.

4.2.1 64QAM Parity-concatenated TCM Encoder

Figure 4.1 describes the simplified turbo TCM encoder for 64QAM modulation [103] [71]. There will be 4 bit streams ($u_1, u_2, u_3$ and $u_4$) into the encoder. However, among those 4 bit streams, two streams ($u_3$ and $u_4$) will be the interleaved versions of the original information input streams ($u_1$ and $u_2$) respectively. Then two consecutive clock cycle outputs will be mapped onto 64QAM constellation via Gray mapping. For comparison, the standard 64QAM TTCM is given in figure 4.2. Obviously parity-concatenated TCM structure for 64QAM case saves 2 interleavers and one constituent encoder.

Again when this coding scheme is applied to the OFDM system over UWB channel, the coded bit stream is interleaved prior to modulation in order to provide robustness against burst errors. In order to improve bit error rate (BER) performance, a more complicated bit interleaver is built for 64QAM. It is a row/column block interleaver with 20 columns and 200 rows. The row number is determined as 2 times the number of usable carriers per OFDM symbol, which is 100 data carriers in OFDM/UWB system.
Figure 4.1: Parity-concatenated TCM encoder, 64QAM
Figure 4.2: Turbo TCM encoder, 64QAM
The interleaver function can be described mathematically as follows. Let \( D \) be the number of bits to be interleaved (\( D = \text{(number of carriers)} \times \text{(Bits per carrier)} \times \text{(number of OFDM symbols)} \)). In this 64QAM OFDM system, \( D \) can be a maximum of 24000 bits (= \( 100 \times 6 \times 40 \)). Then define \( \text{LIM} = D/6 \), which is 4000 in this case, \( W = \) number of columns = 20, and \( S = 8 \) denoting a shift constant. Denote by \( V_{in} \) the non-interleaved input vector and by \( V_{out} \) the interleaved output vector. The function \( k[i] \) below describe the one-to-one mapping between the index \( k[i] \) of \( V_{in} \) and index of \( i \) of \( V_{out} \), such that \( V_{out}(i) = V_{in}(k[i]) \).

\[
k[i] = \text{mod}(W \times (i + S \times \text{floor}(W \times i / \text{LIM}))) - (\text{LIM} - 1) \times \text{floor}(W \times i / \text{LIM}), \text{LIM}), \ i = 0 \ldots, \text{LIM} - 1
\]

(IV.1)

where \( \text{mod}(x,y) \) returns the remainder on dividing \( x \) by \( y \) with the result having the same sign as \( x \). Since we use 64QAM modulation here, the mapping function \( k[i] \) shall be applied 6 times, each time to \( \text{LIM} \) bits in \( V_{in} \). Then 6 bits from 6 length \( \text{LIM} \) output vectors each time shall be combined to map one point in 64QAM constellation.

Alternatively, the interleaver procedure can be described by the way the data is written into and read out of an "interleaver matrix". This is illustrated below through figure 4.3. According to WPAN standard, maximum of 40 OFDM symbols are contained in one Physical Layer (PHY) transmission block. So the interleaver matrix is \( 200 \times 20 \) bits. The number of rows used is equal to 2 times the number of data carriers in one OFDM symbol. The non-interleaved data is written into this
matrix row-wise, starting in row zero (going from left to right), as illustrated in figure 4.3.

Data is read out of the matrix of figure 4.3 column-wise, starting at a given bits, going down the column, and wrapping around to the top (if necessary). Between reading each column a shift of 8 (S parameter defined in equation IV.1) row positions is applied: the first column is read starting in row 0, the second column is read starting in row 8, the third column is read starting in row 16, and so on. Figure 4.4 illustrates how the first two columns of the interleaver matrix (of figure 4.3) are read out. Accordingly there will be 6 matrices as depicted in figure 4.3 holding 6 parts of input elements. The elements of these 6 matrices are read out in the same order as described above, producing 6 equal length vectors. Then combining 6 vectors by using one element of each vector to produce 6 bits which will be mapped to a 64QAM constellation point.
Figure 4.3: Bit interleaver

<table>
<thead>
<tr>
<th>ROW 0:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROW 1:</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>39</td>
</tr>
<tr>
<td>ROW 2:</td>
<td>40</td>
<td>41</td>
<td>42</td>
<td>43</td>
<td>59</td>
</tr>
<tr>
<td>ROW 3:</td>
<td>60</td>
<td>61</td>
<td>62</td>
<td>63</td>
<td>79</td>
</tr>
<tr>
<td>ROW 4:</td>
<td>80</td>
<td>81</td>
<td>82</td>
<td>83</td>
<td>99</td>
</tr>
<tr>
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<td>100</td>
<td>101</td>
<td>102</td>
<td>103</td>
<td>119</td>
</tr>
<tr>
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<td>121</td>
<td>122</td>
<td>123</td>
<td>139</td>
</tr>
<tr>
<td>ROW 7:</td>
<td>140</td>
<td>141</td>
<td>142</td>
<td>143</td>
<td>159</td>
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<td>162</td>
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<tr>
<td>·</td>
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<td>ROW 199:</td>
<td>3980</td>
<td>3981</td>
<td>3982</td>
<td>3983</td>
<td>3999</td>
</tr>
</tbody>
</table>
Figure 4.4: Interleaved data on first 4 symbols
Table 4.1: Mappings for each dimension of 64QAM

<table>
<thead>
<tr>
<th>Signal levels or Cosets</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tbody>
<tr>
<td>Natural mapping</td>
<td>000</td>
<td>001</td>
<td>010</td>
<td>011</td>
<td>100</td>
<td>101</td>
<td>110</td>
<td>111</td>
</tr>
<tr>
<td>Reordered mapping</td>
<td>000</td>
<td>001</td>
<td>010</td>
<td>011</td>
<td>110</td>
<td>111</td>
<td>100</td>
<td>101</td>
</tr>
<tr>
<td>Gray code mapping</td>
<td>000</td>
<td>001</td>
<td>011</td>
<td>010</td>
<td>110</td>
<td>111</td>
<td>101</td>
<td>100</td>
</tr>
</tbody>
</table>

4.2.2 64QAM Gray Mapping

The mapping rules for 64QAM is similar as 16QAM described in chapter III. 64QAM gray mapping and constellation are given in table 4.1 and figure 4.5.

4.2.3 OFDM Modulation

The discrete-time implemented OFDM system model for powerline channel is same as that described in figure 3.1. The bit streams is encoded through 64QAM parity-concatenated TCM encoder. Again the output coded bits will be interleaved prior to modulation in order to provide robustness against burst errors. It is a row/column block interleaver with 20 columns and 168 rows. The row number is determined as 2 times the number of usable carriers per OFDM symbol, which is 84 data carriers in OFDM/Powerline system.

The interleaver function is same as 64QAM/OFDM/UWB system, except that the
Figure 4.5: 64QAM constellation
parameter \( D \), which is the number of bits to be interleaved, equals to \((\text{number of carriers}) \times (\text{Bits per carrier}) \times (\text{number of OFDM symbols}) = 84 \times 4 \times 40 = 13440\). Then accordingly \( \text{LIM} = D/6 \), which is 2240 in this case, \( W \) = number of columns = 20, and \( S = 8 \) denoting a shift constant between reading each column from row/column block interleaver. The relationship between \( V_{\text{out}} \) and \( V_{\text{in}} \) keeps fixed. The mapping function \( k[i] \) shall still be applied 6 times, each time to LIM bits in \( V_{\text{in}} \). Then 6 bits from 6 length LIM output vectors each time shall be combined to map one point in 64QAM constellation.

The OFDM system specified in HomePlug 1.0 places 128 evenly spaced carriers into the frequency band from DC to 25MHz. Of these carriers, 84 are used (numbers 23 to 106, or approximately 4.49MHz to 20.7MHz) to carry information. The timing of the OFDM time-domain signal, based on 50MHz system clock, is determined as follows: A set of mapped data points are modulated onto subcarrier waveforms using 256-point IFFT resulting 256 time samples (IFFT interval). 84 data complex numbers from 64QAM TTCM encoder after gray mapping will be mapped onto 256-point IFFT inputs 22, 23, 24, \( \cdots \), 105. Subcarriers 0, \( \cdots \), 21 and 106, \( \cdots \), 128 are Nulls. Subcarriers 129, \( \cdots \), 255 are the conjugate mirrors of subcarriers 127, 126, \( \cdots \), 1. Then last 172 time samples are inserted in a guard interval at the front of IFFT interval, to create a cyclic extended OFDM symbol of 428 time samples. We replace the cyclic prefix into zero-padding of same number to obtain better equalization performance. Then the IFFT duration \( (T_{\text{FFT}} = 1/\Delta_f = 5.12\mu s) \) and cyclic prefix duration \( (T_{\text{CP}} = \)
3.44\(\mu s\) make up the OFDM symbol period \(T_{sys}\), which is 8.56\(\mu s\). The specification is summarized in Table 4.2.3
Table 4.2: HomePlug 1.0 OFDM Specifications

<table>
<thead>
<tr>
<th>Parameter type</th>
<th>HomePlug 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Bandwidth</td>
<td>16.4 MHz (4.3 ∼ 20.8)</td>
</tr>
<tr>
<td>Number of Data Tones</td>
<td>84</td>
</tr>
<tr>
<td>Sampling Rate</td>
<td>50MHz</td>
</tr>
<tr>
<td>Sub-Carrier BW</td>
<td>195.3 KHz</td>
</tr>
<tr>
<td>FFT size</td>
<td>256</td>
</tr>
<tr>
<td>IFFT/FFT Period</td>
<td>5.12µs</td>
</tr>
<tr>
<td>Cyclic Prefix</td>
<td>172(3.44µs)</td>
</tr>
<tr>
<td>OFDM Symbol Interval</td>
<td>8.56µs</td>
</tr>
<tr>
<td>Channel Model</td>
<td>Real Measured</td>
</tr>
<tr>
<td>Channel Distance</td>
<td>60 feet</td>
</tr>
<tr>
<td>Modulation Constellation</td>
<td>64QAM</td>
</tr>
<tr>
<td>16-state TCM code</td>
<td>(23,35,33,37,31)</td>
</tr>
<tr>
<td>Info. Data Rate</td>
<td>39Mbps</td>
</tr>
</tbody>
</table>
4.2.4  Power Line Channel

Power line channel was measured by passing a narrow pulse (approximately 20ns) into the test bed including (transmitter, power line channel and receiver) and obtaining the impulse response at the receiver. Sampling rate is 100MHz. Figure 4.6 and 4.7 show the 25MHz channel property for 60 feet power line. The frequency response is in 3dB variation.
Figure 4.6: Impulse response of power line channel.
Figure 4.7: Frequency response of power line channel.
4.2.5 **ZP-OFDM Equalization**

As we mentioned in section 3.1.5, the OFDM symbol blocks will experience IBI when propagating through UWB channels because the underlying channel’s impulse response combines contributions from more than one transmitted block at the receiver. To account for IBI, OFDM systems rely on the so-called cyclic prefix (CP) which consists of redundant symbols replicated at the beginning of each transmitted block. Another method to eliminate the IBI is Zero-padding (ZP) which are trailing zeros padded at the end of each transmitted block. The length of trailing zeros can be exactly same as the length of CP in CP-OFDM, which is chosen greater than the channel length. ZP-OFDM is equivalent to CP-OFDM in a sense that overlap-add (OLA) is equivalent to overlap-save (OLS) in block convolution.

Figure 4.8 depicts the baseband discrete-time block diagram of a ZP-OFDM system [79] [80] [81]. The only difference between CP-OFDM and ZP-OFDM is that the CP is replaced by $D$ trailing zeros that are appended at the end of block $\tilde{s}_M$ to yield the $P \times 1$ transmitted vector. This is equivalent to extend $M \times M$ matrix $F_M^H$ to $P \times M$ matrix $F_{zp} = [F_M \ 0]^H$ based upon the relationship between $\tilde{s}_M$ and $\tilde{s}_M^i$. The resultant redundant block $\tilde{s}_{zp}$ will have $P = M + D$ samples, which can be denoted as $\tilde{s}_{zp} = [\tilde{s}_M(0)\tilde{s}_M(1) \cdots \tilde{s}_M(M - 1)0 \cdots 0]^T = F_{zp}s_M$. Then, the expression of the $ith$ received symbol block is given by
Figure 4.8: Discrete-time block equivalent model of ZP-OFDM.
\[ \tilde{x}_{zp}^i = HF_{zp}s_M^i + H_{IBI}F_{zp}s_M^{i-1} + \tilde{n}_P^i \]  

(IV.2)

where \( H \) is the \( P \times P \) lower triangular Toeplitz filtering matrix with first column \( [h_0 \cdots h_L 0 \cdots 0]^T \) and \( H_{IBI} \) is the \( P \times P \) upper triangular Toeplitz filtering matrix with first row \( [0 \cdots 0h_L \cdots h_1] \) as defined in section 3.1.5. The IBI in this case is eliminated due to the all-zero \( D \times M \) matrix \( 0 \) in \( F_{zp} \) which cause \( H_{IBI}F_{zp} = 0 \). \( \tilde{n}_P^i \) denotes the AWGN vector.

We partition \( H \) into two parts: \( H = [H_0, H_{zp}] \), where \( H_0 \) represents its first \( M \) columns and \( H_{zp} \) its last \( D \) columns. Then, the received \( P \times 1 \) vector becomes

\[ \tilde{x}_{zp}^i = HF_{zp}s_M^i + \tilde{n}_P^i = H_0F_M^Hs_M^i + \tilde{n}_P^i \]  

(IV.3)

since last \( D \) rows of \( F_{zp} \) are all zeros. We then split the signal part in \( \tilde{x}_{zp}^i \) in (IV.3) into its upper \( M \times 1 \) part \( \tilde{x}_u^i = H_u s_M^i \) and its lower \( D \times 1 \) part \( \tilde{x}_l^i = H_l s_M^i \), where \( H_u \) (or \( H_l \)) denotes the corresponding \( M \times M \) (or \( D \times M \)) partition of \( H_0 \) as follows:

\[
H_u = \begin{pmatrix}
  h_0 & 0 & \cdots & 0 & 0 \\
  h_1 & h_0 & \cdots & 0 & 0 \\
  \vdots \\
  0 & 0 & \cdots & h_0 & 0 \\
  0 & 0 & \cdots & h_1 & h_0
\end{pmatrix}_{M \times M}
\]
\[ H_I = \begin{pmatrix} 
0 & \cdots & 0 & h_L & \cdots & h_1 \\
0 & \cdots & 0 & 0 & \cdots & h_2 \\
\vdots \\
0 & \cdots & 0 & 0 & \cdots & h_L \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 
\end{pmatrix}_{D \times M} \]

Padding \( M - D \) zeros in \( \tilde{x}_i \) and adding the resulting vector to \( \tilde{x}_u \), we get

\[
\tilde{x}_M^i = \tilde{x}_u^i + \begin{bmatrix} \tilde{x}_l^i \\
o_{(M-D) \times 1} \end{bmatrix} = \left( H_u + \begin{bmatrix} H_l \\
o_{(M-D) \times M} \end{bmatrix} \right) \tilde{s}_M^i = C_M(h)\tilde{s}_M^i. \tag{IV.4}
\]

where \( C_M(h) \) is a \( M \times M \) circulant matrix with first row \( C_M(h) = \text{Circ}_M(h_0 \begin{bmatrix} \cdots & 0 & h_L & \cdots & h_1 \end{bmatrix}) \) defined in section 3.1.5. The noise will be slightly colored due to overlapping and addition (OLA) operation. Then, using FFT to perform demodulation and obtain the received signal in the frequency domain. The procedure is same as the last step in section 3.1.5. This equalization scheme will be referred as CP-OFDM-OLS.

Another ZP-OFDM based equalization scheme is using the \( P \times P \) FFT matrix \( F_P \)
with entries $\exp(-j2\pi mk/P/\sqrt{P})$ to diagonalize the channel circulant matrix, which is illustrated in the lower part of figure 4.8. Due to the $D$ trailing zeros of ZP-OFDM, the last $D$ columns of $H$ do not affect the received block. Thus, the Toeplitz matrix $H$ can be seen as a $P \times P$ circulant matrix $C_P(h) = \text{Circ}_P(h_0, 0 \cdots 0 h_L \cdot h_1)$. Then we can rewrite equation IV.3 as

$$\tilde{x}_{zp}^i = HF_{zp}s_M^i + \tilde{n}_{zp}^i = C_P(h)F_{zp} + \tilde{n}_{zp}^i$$

Then we can do the diagonalization as follows:

$$F_PHF_{zp} = F_P C_P(h)F_{zp} = F_P C_P(h)F_P^*F_PF_{zp} = D_P(\tilde{h}_P)F_PF_{zp}$$

where $\tilde{h}_P = [H(0) \cdots H(2\pi/P) \cdots H(2\pi(P-1)/P)]^T$, $D_P(\tilde{h}_P)$ is the $P \times P$ diagonal matrix with diagonal $\tilde{h}_P$.

Because the channel $H(z)$ is order of $L$, $D_P(\tilde{h}_P)$ can have at most $L$ zero-diagonal entries. However, unlike CP-OFDM, the remaining (at least $P - L$) nonzero entries guarantee zero forcing recovery of $s_M^i$ in ZP-OFDM, regardless of the underlying $L$th-order FIR channel nulls [79]. The equalization scheme will be referred as ZP-OFDM-FAST. We use this ZP-OFDM equalization scheme in our 64QAM TTCM coded OFDM system performance evaluation [103] [71] [111].
4.3 Numerical Results

In order to improve the transmission data rate of the current HomePlug1.0 [104] system, we select 16-state 64QAM TCM code as in chapter III to evaluate the OFDM system performance. The channel is measured 60 feet powerline channel. The resultant data rate is 39Mbps, which is 3 times of the current HomePlug1.0 system (13Mbps). All simulation results are averaged over 500 packets with a payload of 1.7k bytes.

The BER performance of the system is illustrated in Figure 4.9. There are $6.7 \times 10^6$ (500 packets $\times$ 40 OFDM symbols/packet $\times$ 84 QAM symbols/OFDM symbol $\times$ 4 bits/QAM symbol) random bits simulated to measure the BER. We obtain BER of $1.8 \times 10^{-5}$ at $E_b/N_0 = 9.7dB$ for 39Mbps coded OFDM system over power line. Figure 4.10 gives the PER performance for the same situation, reporting a low PER of 0.028 at $E_b/N_0 = 9.7dB$. 
Figure 4.9: BER of OFDM/64QAM over power line and AWGN channel.
Figure 4.10: PER of OFDM/64QAM over power line and AWGN channel.
CHAPTER V

TURBO TCM CODED OFDM SYSTEM FOR IMPULSIVE NOISE CHANNEL

In the real wireless communications systems besides AWGN there are impulsive man-made noise from ignition of automobile or other sources such as power line which affect the performance of the system. The impulsive noise is an additive disturbance that arises primarily from the switching electric equipment. Therefore, bursty or isolated errors are usually generated by an impulsive noise affecting consecutive symbols in trellis based decoding algorithms, such as Viterbi and MAP algorithm, because such decoder relies on the history of the symbol sequence [105]. For OFDM system,
the longer OFDM symbol duration provides an advantage in a presence of impulse
noise, because impulsive noise energy is spread out among simultaneously transmis-
ted OFDM sub-carriers. However, it has been recently recognized that this advantage
turns into a disadvantage if the impulsive noise energy exceeds certain threshold [106].
Further more, the statistical characteristics of the impulsive noise are much different
from those of Gaussian noise. Therefore, the performance of OFDM systems endur-
ing impulsive noise needs to be evaluated for shedding some light on building robust
decoding algorithm against impulsive noise.

We have studied the performance of conventional iterative bit MAP decoder which
is designed for gaussian noise in previous chapters. In this chapter, we will investi-
gate the effect of impulsive noise on the performance of the Turbo TCM coded OFDM
system. The conventional iterative bit MAP decoding algorithm is modified to catch
up the corresponding impulsive noise statistical characteristics. The bit error rate
(BER) performance of the TTCM coded OFDM systems over both AWGN channel
and UWB channel with impulsive noise is evaluated through simulation.

5.1 System and Channel Model

We are considering the OFDM system presented in figure 3.1, in which OFDM
data tones are coded through parity-concatenated TCM with 64QAM constellation
modulation. Trailing Zeros are appended after IFFT modulation, where the system
will be referred as ZP-OFDM. In each OFDM symbol interval symbols \( \{S_k\} \) are transformed by means of IFFT and digital-to-analog conversion to the baseband OFDM signal as

\[
s(t) = \sum_{n=0}^{N-1} S_k e^{(j2\pi kn\Delta f)t}, t \in [0, T_{FFT}],
\]

where \( \Delta f \) and \( N \) are again defined as the subcarrier frequency spacing and the number of total subcarriers used, respectively. \( T_{FFT} \) is the OFDM symbol interval. After the inverse FFT at the transmitter, cyclic prefix or zero-padding prefix is inserted to avoid interblock interference (IBI).

The received signal (in time domain) after down-conversion, analog-to-digital conversion, cyclic prefix removal, and synchronization can be represented as

\[
r_k = \sum_{l=1}^{L} h_l s_{k-l} + w_k + i_k, k = 0, 1, \ldots, N - 1,
\]

where \( s_k = s(kT_{FFT}/N) \), \( h_l \) is the channel impulse response, \( L \) is the order of channel impulse response, \( w_k \) is the additive white Gaussian noise (AWGN) with zero mean and \( i_k \) is the impulse noise. For memoryless channels, \( h_l = 1 \) for \( l = 1, \ldots, L \).

There have been many literature discussing the effect of impulsive noise [105] [106] [107] [108] [109]. Here, we are considering a set of impulsive noise which can be modelled, as in [109], as

\[
p(n_k; A_k) = p(n_k; \sigma^2_k, \eta_k^C) = \frac{\eta_k^C}{2\sigma \sqrt{\gamma(\eta_k^C)\Gamma(\frac{1}{\eta_k^C})}} e^{-\frac{|n_k|^2_{\eta_k^C}}{\eta_k^C}}
\]
where the parameter $A_k = (\sigma^2_k, \eta^C_k)$, $n_k$ is the noise added on transmitted symbol, $\Gamma(.)$ denotes the Gama function, $\sigma^2_k$ is the variance of the noise and $a(\eta^C_k) = \frac{\Gamma\left(\frac{1}{\eta^C_k}\right)}{\Gamma\left(\frac{3}{\eta^C_k}\right)}$. When $\eta^C_k = 2$, $p(n)$ is the Gaussian distribution function. When $\eta^C_k = 1$, $p(n)$ becomes the Laplace distribution function. And when $\eta^C_k = 0.5$, $p(n)$ is the Sqrt noise mentioned in [109].

When OFDM system is applied to UWB channel, the impulsive noise mentioned above will cause severe degradation of the system performance.

5.2 Modified Iterative Bit MAP Decoder

We still apply the turbo iterative decoding scheme as in Chapter III and IV, and make certain modifications to match the statistical characteristics of the channel impulsive noise. The branch metric in the basic MAP algorithm is modified according to the PDF of the impulsive noise [110]. The iterative decoding scheme is kept same as before.

Under Gaussian noise environment, the channel transition probability is based on Gaussian PDF and the Euclidian distance between the received signal and the candidates of the transmitted signal, which can be described by V.3:

$$p = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(y_{kI} - x_{iI})^2 + (y_{kQ} - x_{iQ})^2}{2\delta^2}\right)$$  (V.3)

where I and Q represent in-phase and quadrature component of the $k$th sample in received sequence $y^N_1$. $x_i$ indicates the $i$th candidate of transmitted sequence $x^N_1$.
So the conventional MAP decoding is optimized for Gaussian noise by selecting the symbol which has minimum Euclidean distance from transmitted one.

Above analysis tells that the noise PDF is also used to derive the branch metrics for optimal trellis-based decoding algorithms like MAP. For channels with impulsive noise, the channel noise can not be approximated through Gaussian PDF any more. Therefore, the channel transition probability should be modified accordingly and take into account the statistical distribution of the channel noise. The modified channel probability is given in equation V.4:

$$p = \frac{\eta_k^D}{2\sigma_k \sqrt{a(\eta_k^D) \Gamma(\frac{1}{\eta_k^D})}} e^{-\frac{|n_k|\eta_k^D}{a(\eta_k^D) \eta_k^D}}$$ (V.4)

where $n_k = \sqrt{(y_{kl} - x_{il})^2 + (y_{kQ} - x_{iQ})^2}$ denoting the Euclidean distance between the received symbol and the candidate of transmitted symbol. Similarly, if $\eta_k^D = 2$, the channel probability $p$ is a Gaussian distribution function. Then this MAP is exactly same as the one optimized for Gaussian channel. If $\eta_k^D = 1$, $p$ is a Laplace distribution function. And if $\eta_k^D = 0.5$, $p$ is the Sqrt noise mentioned in [109]. Obviously, the optimal decoder requires that $\eta_k^D = \eta_k^C$. Otherwise, due to channel variation or estimation error, an additional ”mismatched error” will occur to increase the total error probability.

### 5.3 Numerical Results

Simulations of the parity-concatenated TCM coded OFDM system with 64QAM
modulation through different channels with different channel impulsive noise were carried out. 16-state TCM code with octal notation (23,35,33,37,31) is selected and system level simulation were performed to measure the BER performance. The resulted data rate is 1Gbps. The system is assumed to be perfectly synchronized. All simulation results are averaged over 2000 packets with a payload of 2k bytes.

The BER performance of the coded 64QAM OFDM system over AWGN ($\eta = 2.0$) channel and impulsive noise channels ($\eta = 0.5, 1.0$) is evaluated. There are $3.28 \times 10^7$ random bits simulated to measure the BER. Figure 5.1 shows the effect of more impulsivity of noise on the performance. When the $\eta$ becomes large, the performance tends to AWGN.

Figure. 5.2 illustrates the performance for coded 64QAM OFDM system over UWB channel with Gaussian noise and impulsive noise of different $\eta$ parameters. The figure indicates the same effect of impulsive noise on the system performance: the smaller the $\eta$, the sharper the impulsive noise and the severer the performance degradation. Again when $\eta$ becomes large, the performance tends to that with AWGN noise.
Figure 5.1: BER of OFDM/64QAM over memoryless channel with different impulsive noise.
Figure 5.2: BER of OFDM/64QAM over UWB channel with different impulsive noise.
5.4 Summary

The BER performance of Turbo TCM coded OFDM system under AWGN noise and impulsive noises were presented. The simulation results have shown that the performance of OFDM system in the impulsive noise environment depends on the impulsivity of the noise and the decoding algorithm has to take the noise impulsivity into account for optimal decoding. Therefore, we modify the iterative bit MAP algorithm used for Gaussian noise to match the impulsive channel noise statistical characteristics.
6.1 Summary of Results

A brief summary of accomplished work is given in this chapter with an emphasis on the contributions to the subjects of Turbo TCM and OFDM systems.

In this thesis, we constructed a punctured parity-concatenated TCM encoder in which a TCM code is used as the inner code and a simple parity-check code is used as the outer code. It functions as a turbo TCM, which may gain a big advantage in the real world implementation due to the savings of constituent encoder and interleavers, and has potential for offering much higher spectral efficiency when used in OFDM systems. The simple outer parity-check code can be easily extended to more complicated parity-concatenated TCM for coding rate diversity.

Based on the iterative bit MAP decoder for standard binary turbo codes, corre-
sponding iterative decoding algorithm is extended for our parity-concatenated TCM codes. We show several essential requirements to extract the extrinsic information from each iteration, which is required to be independent and non-repeatable, and provide to next iteration as a priori probability for branch metric computation in MAP decoding process.

One of UWB proposals in the IEEE P802.15 WPAN project is to use a multi-band orthogonal frequency-division multiplexing (OFDM) system and punctured convolutional codes for UWB channels supporting data rate up to 480Mb/s. In this paper we examine the possibility of improving the proposed system using Turbo TCM with QAM constellation for higher data rate transmission. We applied our punctured parity-concatenated trellis codes, in which a TCM code is used as the inner code and a simple parity-check code is used as the outer code, to the current OFDM/UWB system. The study shows that the system can offer much higher spectral efficiency, for example, 1.2 Gbps, which is 2.5 times higher than the current proposed system. We show several essential requirements to achieve high rate such as frequency and time diversity, multi-level error protection.

Convergence analysis of iterative decoding algorithms is a very important tool to predict code performance, its ability to provide insights into the encoder structure, and its usefulness in helping with the code design. In this dissertation, we use Gaussian approximation to track the density of extrinsic information in iterative turbo decoders. We model the Gaussian density based on the empirically determined mean
and variance as independent parameters. The method is applied to both AWGN channel and UWB channel for OFDM system and confirms the system performance simulation result.

There are many different approaches to evaluate the performance of TCM codes or turbo codes. Most of the turbo type code performance evaluation is based on conventional turbo structure and then finds the average performance bounds (averaged over all possible interleavers). Since our encoder functions as a Turbo TCM but due to the multiple input streams and punctured information bits, it’s hard to use the evaluation method proposed previously. We try to explore the exhaustive enumeration of TTCM codewords to confirm the code performance. Short block code is evaluated using this method and the consistency between the evaluation and simulation results is obtained.

The same coding scheme can also be applied to the OFDM system for HomePlug powerline channels since OFDM is selected as the modulation scheme in HomePlug standards. Similar simulations are done to OFDM/Powerline system and obtain better bit error rate (BER) and packet error rate (PER) performance. The work has shown that we can deliver data rate of 39Mbps comparing to 13Mbps data rate of current HomePlug1.0 systems.

Another big issue in the real wireless communications systems is the fact that there are impulsive man-made noise from ignition of automobile or other sources such as power line which affect the performance of the system besides AWGN noise. We
investigate the effect of impulsive noise on the performance of the Turbo TCM coded OFDM system and come up with a modified iterative bit MAP decoding algorithm to catch up the corresponding impulsive noise statistical characteristics. The bit error rate (BER) performance of the TTCM coded OFDM systems over both AWGN channel and UWB channel with impulsive noise is evaluated through simulation. Our work has shown that the optimal decoder requires a matched probability distribution function to the channel pdf of the additive impulsive noise. Otherwise, due to channel variation or estimation error, an additional ”mismatched error” will occur to increase the total error probability.

6.2 Further Research

The future research can be extended on following areas:

(1) The parity-concatenated TCM encoder structure described in this dissertation can be further constructed into more complicated coding schemes. Since currently the encoder is built as a concatenation of a simplest parity-check outer code and TCM inner code. More complicated outer codes can be constructed to provide strong error correction ability and higher spectral efficiency;

(2) The iterative bit MAP decoding algorithm can be further optimized for better performance, such as enabling more iteration and avoiding overflow problem;
(3) Based on the results from this dissertation, larger QAM constellation size, such as 256-pint or 1024-point, and multidimensional TCM structure can be explored for even higher data rate transmission through OFDM/UWB and OFDM/HomePlug systems.

(4) Based on the analysis of the effect of impulsive noise on OFDM system in this dissertation, more evaluations of mixed noise conditions can be conducted in the future. Powerful coding scheme for OFDM system and corresponding robust decoding algorithms for un-predicted impulsive noise type in the channel are very necessary in today’s real communication system in which higher spectral efficiency with higher data rate transmission is highly desired [112].
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