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Magnetic Reconnection Induced By Perturbation On Boundaries

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\[ I_1 = \int_0^1 r J_0^2(p_0 r) \, dr, \quad I_2 = \int_0^1 r J_0^2(p_0 r) \, dr, \]
\[ I_3 = \int_0^1 r J_0(p_0 r) J_1(p_0 r) \, dr, \]
\[ I_4 = \int_0^1 J_0(p_0 r) J_1(p_0 r) J_1(p_0 r) \, dr. \]  \tag{14}

For discussion we consider the mode \( s = 1 \) for which \( p_{0r} = 2.4048 \). We propose a possible practical situation with \( N_0 \sim 10^9 \text{ cm}^{-3}, \quad T_e \sim 1.5 \text{ eV}, \quad T_i \leq 0.2 \text{ eV}, \quad H \sim 5 \times 10^3 \text{ Oe}, \quad a \sim 10 \text{ cm}. \) However, we numerically evaluate the coefficients \( \delta_1, \delta_2 \) for a wide range of values of plasma density, temperature, and the strength of the external magnetic field that satisfy the assumptions made earlier, i.e., the plasma pressure is small compared to the magnetic pressure and the characteristic time is longer than the ion cyclotron period. As long as \( \rho_{10}/c_s, a/\Omega \) remain \( \leq 0.1, \delta_1 \) is found to depend weakly on plasma parameters as well as on the strength of the external magnetic field. Its value is mainly determined by finite geometry and lies between 0.715 and 0.735. Under such a practical situation, which is supposed to be the usual case, we find it is always permissible to approximate Eq. (13) by the following:

\[ \frac{\partial \alpha}{\partial \tau} + \delta_1 \frac{\partial \alpha}{\partial \xi} + \frac{\alpha}{2a} \frac{\partial^2 \alpha}{\partial \xi^2} = 0, \]  \tag{15}

where \( \delta_1 \) lies between 0.715 and 0.735 and for the experimental setup proposed above it has the value 0.720.

In an unbounded medium the nonlinear behavior of ion-acoustic waves propagating along an essentially infinite magnetic field may be described by the following well-known KdV equation:

\[ \frac{\partial \alpha}{\partial \tau} + \alpha \frac{\partial \alpha}{\partial \xi} + \frac{\alpha^2}{2L} \frac{\partial^3 \alpha}{\partial \xi^3} = 0, \]  \tag{16}

where \( L \) is the characteristic scale length of field quantity variations.

Comparing (15) and (16) we find that the main effect introduced by finite geometry enters through the coefficient of the nonlinear term.

The stationary solution to the KdV equation (13) in a frame moving with a velocity \( c = c_0 + A/c_0 \) is given by

\[ \alpha = \frac{3A}{c_0 \delta_1} \text{sech}^2 \sqrt{\frac{A}{4c_0 \delta_2}} \left( \xi - \frac{A}{c_0} \tau \right). \]  \tag{17}

We define the Mach number as \( M = c/c_0 \). It is now clear from Eq. (17) that, for a fixed amplitude, \( M \rightarrow 1 \) in bounded plasma in the practical situations mentioned above is smaller by a factor \( \delta_1 \) \((< 1)\) than those in unbounded plasma.

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Magnetic reconnection induced by perturbation on boundaries

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The magnetic field reconnection is considered in a plasma induced by perturbing the boundaries of a slab of incompressible plasma with a magnetic neutral surface inside. It is assumed that the boundaries of the plasma slab are perturbed at both fast and slow rates compared with the hydromagnetic evolution rate, and the ensuing adjustments in the plasma and the magnetic field threading through it are investigated.

It is now generally recognized that in geometries that are not perfectly symmetric, the corresponding magnetostatic equilibria will not have smooth magnetic surfaces, but, under the action of boundary perturbations resonating with the field lines on one of the rational surfaces, will lead to the formation and breakup of current sheets into magnetic islands on these rational surfaces. In order to bring out the physical features of this problem, Kulpsrud and Hahm\(^1\) considered a particularly simple model problem that is tractable analytically. Kulpsrud and Hahm\(^1\) studied the magnetic reconnection in a plasma induced by perturbing the boundaries of a slab of incompressible plasma with a magnetic null surface inside. The boundaries were perturbed at a rate that was slow compared with the hydromagnetic evolution rate but fast compared with the resistive diffusion rate. Consequently to this perturbation, a new state of equilibrium came into being in which the magnetic field assumed a topology similar to the one before but a current sheet developed at the magnetic null surface. This current sheet brought in resistive effects, which then caused the equilibrium state to evolve into another equilibrium. In the process, a magnetic field reconnection occurred and the current sheet disappeared.

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Equation (2) shows that the boundary perturbation has a fast variation characterized by the fast time $t$, and a slow variation characterized by the slow time $\tilde{t} = \tilde{\varepsilon} t$, $\tilde{\varepsilon}$ being the ratio of the Alfvén time scale $\tau_\Lambda$ to the tearing mode time scale $\tau$, and $\tilde{\varepsilon} \ll 1$.

Let the velocity of the plasma flow and the magnetic field consequent to this perturbation be given by

$$\mathbf{v} = \nabla \phi \times \mathbf{i}_x, \quad \mathbf{B} = \nabla \psi \times \mathbf{i}_z.$$  \hspace{1cm} (3)

We then obtain, from the equations of plasma motion and magnetic field transport,

$$\rho \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = - \mathbf{i}_x \cdot [\nabla \psi \times \nabla (\nabla^2 \psi)],$$  \hspace{1cm} (4)

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{B} = \eta \nabla^2 \mathbf{B},$$  \hspace{1cm} (5)

where $\rho$ is the mass density and $\eta$ is the resistivity of the plasma.

Let $T$ be the time scale characterizing the fast variation of the boundary perturbation $\xi(t, \tilde{t}, \tilde{t})$, and let $\tau$ be that for the slow variation of the latter (which is also taken to be the tearing-mode time scale). Let us write

$$\psi(x, y, t, \tilde{t}) = (B_0/2a)x^2 + \psi_1(x, \tilde{t}) \cos ky,$$

$$\phi(x, y, t, \tilde{t}) = (1/kB_0T) \phi_1(x, \tilde{t}) \sin ky.$$ \hspace{1cm} (6)

Equations (4) and (5) then give the linearized equations

$$\frac{\partial \psi_1}{\partial t} + \epsilon \frac{\partial \psi_1}{\partial \tilde{t}} + \frac{x}{a} \phi_1 = \frac{T}{\tau} \tau_\Lambda^{-1} \left( \psi''_1 - k^2 \psi_1 \right),$$ \hspace{1cm} (7)

$$\frac{\tau_\Lambda^{-1}}{T^2} \left( \frac{\partial}{\partial \tilde{t}} \left( \psi''_1 - k^2 \psi_1 \right) + \epsilon \frac{\partial}{\partial t} \left( \phi''_1 - k^2 \phi_1 \right) \right) = \frac{x}{a} \left( \psi''_1 - k^2 \psi_1 \right),$$ \hspace{1cm} (8)

where we have nondimensionalized distances using a reference length $L$, and

$$\tau_R = \frac{L^2}{\eta}, \quad \tau_\Lambda = \frac{\sqrt{\rho}}{kB_0}, \quad \epsilon = \frac{\tilde{\varepsilon} T}{\tau},$$

with primes denoting the differentiation with respect to $x$.

Next, the boundary conditions

$$x = \pm (a + \xi): \quad \psi = \text{const}, \quad \phi = \pm \left( \frac{\partial \xi}{\partial t} + \tilde{\varepsilon} \frac{\partial \xi}{\partial \tilde{t}} \right)$$ \hspace{1cm} (9)

give

$$x = \pm a: \quad \psi_1 = \mp B_0 \delta, \quad \phi_1 = \pm B_0 (\delta_1 + \delta_2).$$ \hspace{1cm} (10)

We have

$$\tau_\Lambda/T \gg 1, \quad \tau_R/T \gg 1.$$ \hspace{1cm} (11a)

Let us assume that

$$\tau_\Lambda^2 \epsilon/T^2 \ll 1.$$ \hspace{1cm} (11b)

We then write

$$\phi_1 = \phi_{1b} + \epsilon \phi_{1s}, \quad \psi_1 = \psi_{1b} + \epsilon \psi_{1s},$$ \hspace{1cm} (12)

where the subscripts $b$ and $s$ refer to the components that may be identified as those produced, respectively, by the fast variation and the slow variation of the boundary perturbation.

Then we obtain, from Eqs. (7), (8), and (10),

$$\frac{\partial \psi_{1b}}{\partial t} + \frac{x}{a} \phi_{1b} = 0,$$ \hspace{1cm} (13)
\[ \phi_{i}'' - k^2 \phi_{i} = 0, \]
\[ x = a: \quad \phi_{i} = B_{0} \delta, \quad \psi_{i} = -B_{0} \delta, \]
where
\[ \delta = \delta - \bar{\delta}, \quad \bar{\delta} = \frac{1}{T} \int_{0}^{T} \delta \, dt, \]
from which we have
\[ \phi_{i} = B_{0} \delta, \quad \sinh \frac{kx}{\sinh ka}, \quad \psi_{i} = -B_{0} \delta \frac{x \sinh \frac{kx}{a}}{\sinh ka}. \] (16)

Equations (16) show that \( B_{i} \) is continuous at \( x = 0 \) so that there is no current sheet at \( x = 0 \). Further, \( \psi_{i} \) has a topology similar to that of \( \psi_{0} \) so that a magnetic reconnection also has not occurred. These results are to be expected because on the fast time scale the current-sheet formation and the magnetic reconnection processes have no time to occur.

Next, we have for the processes on the slow time scale, from Eqs. (7), (8), and (10),
\[ \psi_{1i}'' - k^2 \psi_{1i} = 0, \]
\[ \frac{\partial \psi_{1i}}{\partial t} + \frac{x}{a} \phi_{1i} = 0, \]
\[ x = a: \quad \phi_{1i} = B_{0} \delta, \quad \psi_{1i} = -B_{0} \delta, \]
from which we have
\[ \psi_{1i} = \psi_{1i}(0,t) \left( \frac{\cosh \frac{kx}{\tanh ka}}{\sinh \frac{kx}{\sinh ka}} - B_{0} \delta \frac{\sinh \frac{kx}{a}}{\sinh ka} \right), \]
\[ \phi_{1i} = -\frac{a \psi_{1i}(0,t)}{x} \left( \cosh \frac{kx}{\tanh ka} - \frac{\sinh \frac{kx}{\sinh ka}}{x} \right) - \frac{B_{0} \delta}{x} \sinh \frac{kx}{\sinh ka}. \] (20)

Following the procedure of Kulshred and Hahm\(^2\) we may obtain
\[ \psi_{1}(0,t) \sim \begin{cases} t^{5/4}, & \text{for small } t, \\ -B_{0} \delta/(\cosh ka), & \text{for large } t. \end{cases} \] (21)

Using (12), (16), (20), and (21), we have
\[ \psi_{1}(x,t,\bar{t}) \]
\[ = \begin{cases} -\frac{B_{0} \delta}{a} \frac{x \sinh \frac{kx}{\sinh ka}}{\sinh ka} - \frac{B_{0} \delta}{a} \frac{\sinh \frac{kx}{\sinh ka}}{\cosh ka}, & \text{for small } t, \\ -\frac{B_{0} \delta}{a} \frac{x \sinh \frac{kx}{\sinh ka}}{\sinh ka} - \frac{B_{0} \delta}{a} \frac{\cosh \frac{kx}{\cosh ka}}{\cosh ka}, & \text{for large } t. \end{cases} \] (22)

Equation (22) shows that for a generalized boundary perturbation having both fast and slow variations, nothing happens on the fast time scale. However, on the slow time scale, a current sheet develops at small times, magnetic reconnection occurs for large times, and the current sheet disappears. Even on a slow time scale, the fast-varying part of the boundary perturbation has a negligible effect on the reconnection because it goes to zero rapidly as \( x \to 0 \) (like \( x^2 \)).