Characterizing the Particle Size Distribution in Saturn's Rings Using Cassini UVIS Stellar Occultation Data

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CHARACTERIZING THE PARTICLE SIZE DISTRIBUTION IN SATURN’S RINGS USING CASSINI UVIS STELLAR OCCULTATION DATA

by

STEPHANIE ECKERT
B.S. Furman University, 2015

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Physics in the College of Sciences at the University of Central Florida Orlando, Florida

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ABSTRACT

NASA’s Cassini mission to Saturn revolutionized modern understanding of the planet’s vast and intricate ring system. We use stellar occultation data from Cassini’s UVIS High Speed Photometer (HSP) to characterize the particle size distribution in the rings with two methods. First, we discern the sizes of the smallest particles at ring edges by forward-modeling observed diffraction signatures which appear as spikes in the signal, the shape and amplitude of which depends on the size and abundance of the smallest particles. We then probe the upper end of the size distribution using occultation statistics.

Although the distribution of photon counts in the absence of ring particles follows Poisson statistics for which the variance is equal to the mean, random variations in the sizes and abundance of particles introduce excess variance. Previous studies have interpreted excess variance in stellar occultation data in terms of an effective particle size. The assumption of small particles is invalid in Saturn’s A and B rings where ring particles cluster together into elongated structures called self-gravity wakes. We calculate the statistical moments within spiral density waves, undulating structures excited throughout Saturn’s rings at locations of resonance with satellites.

In our diffraction analysis, we find more detections of diffraction at edges near the outer A and B rings than at edges within the C ring and Cassini Division, consistent with the prediction that edges directly perturbed by satellites have a greater population of sub-cm particles than edges confined by other mechanisms. In our moments analysis, we find that the granola bar model for regularly spaced wakes cannot match the observed statistics of both density wave troughs and peaks with a single set of parameters $S$ and $W$, which may indicate that wakes are more opaque in the wave crests due to compression than they are in the troughs.
I dedicate this work to my parents, Eric and Patricia, who have always supported me.
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LIST OF ACRONYMS AND SYMBOLS

**UVIS**
The Cassini Ultraviolet Imaging Spectrograph.

**VIMS**
The Cassini Visual and Infrared Mapping Spectrometer.

**RSS**
The Cassini Radio Science Subsystem.

**ISS**
The Cassini Imaging Science Subsystem.

**FOV**
Field of view.

a
Particle radius.

$a_{\text{min}}$
Minimum particle size.

$a_{\text{max}}$
Maximum particle size.

I
Intensity of signal in stellar occultation data.

$I_0$
Intensity of the unocculted star.

$n_0$
Surface number density of 10-cm particles in the rings. (#/m$^2$/m)$^2$

q
Power-law index.

λ
Wavelength.

θ
Scattering angle. This is 180° minus the phase angle.

κ
Epicyclic frequency of a particle’s orbit.

B
Ring-opening angle.

ϕ
Azimuthal (clock) angle (360° in the ring plane)

τ
Optical depth.

τ$_N$
Normal optical depth
CHAPTER ONE: INTRODUCTION

1.1 Historical background: the advent of ring science

The first recognition of anything resembling rings around a planet occurred in the early 17th century when Galileo first turned his telescope toward Saturn. During his first observation in 1610, he noted the appearance of strange “lobe”-like structures at the edges of the planet. Around the same time, he discovered the four Galilean satellites orbiting Jupiter and concluded that the lobes at Saturn were also “moons”—he did not realize that he had made the first observation of a planetary ring system. He documented his observations pictorially as shown in Figure 1.1. When Galileo turned his telescope back to Saturn in 1612, he was astonished to find that the mysterious lobes had vanished. In 1616, they returned but appeared more like arms than lobes which he described as “two half ellipses with two little dark triangles in the middle … and contiguous to the middle globe of Saturn”. Planetary rings have captivated astronomers ever since.

An explanation for this disappearing and reappearing phenomenon would not come until 1659 with Christiaan Huygens’ publication of Systema Saturnium. He came up with a Latin anagram in which he postulated that Saturn must be surrounded by “a thin flat ring, nowhere touching and inclined to ecliptic”. Figure 1.2 shows Huygens’ original sketch. Observations of Saturn continued to spur ardent debate about the proper interpretation of the rings for hundreds of years. Fortunately, today’s astronomers have solved this puzzle: the rings are so thin that they become nearly invisible from Earth’s point-of-view when Earth crosses Saturn’s equatorial plane, which occurs about every 15 years.
Figure 1.1. Original sketches of Saturn by Galileo. Top: Galileo’s original sketch of Saturn from his 1610 observation featuring two external lobe structures on either side of the main body. Bottom: Galileo’s subsequent sketch in 1616, where he conjectured that the structures appeared more like “arms” than lobes.

Figure 1.2. Original illustration from Christiaan Huygen’s *Systema Saturnium*. The drawing features a flat disk which encircles but remains completely disjoint from the planet.

The first speculation that the rings might be more complex than a flat, homogeneous disk came in 1660 when Jean Chapelain suggested that Saturn might actually be surrounded by innumerable tiny satellites. Shortly thereafter in 1676, Jean Dominique Cassini discovered a dark region which he interpreted as a gap in the rings. This “gap” would later be identified as a complex collection of rings and smaller gaps itself – the Cassini Division, but the suggestion that
a gap might exist in the rings supported the multitudinous satellites hypothesis. Unfortunately for Chapelain, the accuracy of his conjectures would not be realized in his lifetime. In 1799, Pierre Simon Laplace speculated that Saturn might be surrounded by multiple narrow solid rings and proved the instability of a uniform, solid disk under the gravitational pull of a celestial body. That Saturn’s rings are composed of a vast ensemble of individual particles was not fully understood until 1857, when James Clerk Maxwell won the Adams Prize for proving that Saturn’s rings must be comprised of an indefinite number of tiny, individual particles (Alexander, 1962; Brush et al., 1983). Planetary scientists have been steadfastly working to reveal the size distribution of these particles ever since.

Over the past four centuries, our view of the rings has transformed from that of a smooth, continuous, static disk to that of a vast ensemble of icy particles engaged in a delicate gravitational dance. To date, all of the giant planets in our solar system have been found to harbor ring systems of their own. While each of these ring systems is magnificent in its own right, Saturn’s remains the most vast and intricate. Our work on characterizing the particle size distribution in Saturn’s rings and how particles aggregate in clumps is motivated by the need for a fuller understanding of the many unique structures and phenomena that are driven by ring particle dynamics. Our objective is not only to reveal more about the present-day configuration of features in Saturn’s rings, but also to use our findings to infer clues about their dynamical evolution, particularly at edges and within small scale structures. This evolution appears primarily driven by satellites, with embedded moons and moonlets inducing perturbations that form wakes, clear narrow gaps, and create ‘propeller’ structures, embedded moonlets on the
order of ~100 meters in size which are too small to clear a full gap in the rings, thereby creating a double-lobed pattern (Tiscareno et al., 2008).

Ring particle dynamics are analogous to the mechanisms involved in planet formation, as the stochastic collisions and non-axisymmetric processes that occur in the disk-like ring system of Saturn mimic those that occur in a protoplanetary disk (Esposito, 2014). By constraining the particle size distribution in different regions throughout the ring system and tracking the evolution of particles on short time scales, we hope to answer broader questions concerning the ongoing collisional evolution of the ring system. The formation and destruction of aggregates affects the overall dynamical evolution of the rings, with implications for their age and origin.

1.2 The Cassini Mission

NASA’s Cassini mission launched in October 1997 and has forever transformed our understanding of Saturn and its rings. The Cassini spacecraft, accompanied by the Huygens probe, entered Saturn’s orbit on July 1, 2004. Upon its highly anticipated arrival, it began unravelling intricate and complex structure far beyond what 17th century astronomers had ever imagined. The precursor missions to Cassini which provided the first up-close view of Saturn and its vast ring system were Pioneer 11 in the 1979 and Voyagers 1 and 2 in the 1980 and 1981, respectively. These missions discovered new satellites, detected ambiguous phenomena like spokes in the B ring, and provided the impetus for Cassini as they generated more questions about the nature of planetary rings than answers. Clearly, a flagship scale mission was needed to resolve the key ring science questions.
Cassini would go on to accomplish a number of wide-ranging science objectives, from surveying ring structure and dynamics to determining atmospheric conditions on Saturn’s moon Titan and much more. In 2009, Cassini observed the rings at Saturn equinox. During this time, the ring plane was exactly parallel to incoming light rays from the sun, enabling the detection of numerous small objects through the projection of their shadows on the rings. The shadows extended more than five times the size of the features themselves in some case, revealing objects too small to be resolved in images (Esposito, 2014). Cassini also confirmed that the ring particles were composed almost entirely of water ice, with some contaminants producing a reddish color in the visible part of the spectrum. The red color is indicative of pollution by silicates or carbon-rich organics (Cuzzi et al., 2009). Ring particle composition is not static, however, as the rings are continuously bombarded by high-energy solar photons, interplanetary meteoroids, and ions from Saturn’s magnetosphere. This regular pollution and alteration of the ring particles presents a particular difficulty in determining the rings’ age, which we will discuss in section 1.3.1.

This dissertation is based on analysis of data from Cassini’s Ultraviolet Imaging Spectrograph (UVIS), depicted in Figure 1.3, which was uniquely suited to investigate both the grand scale and fine structure of the rings.

Throughout its 13-years at the ringed planet, UVIS recorded a total of 197 ring stellar occultations (e.g., observations of stars as the line of sight from the spacecraft to the star intersects the ring plane). These observations occurred over a wide range of viewing geometries, providing unique information about ring structure from multiple aspects. The mission was ultimately extended twice and finally concluded in September 2017 when scientists purposefully adjusted Cassini’s trajectory to cause it to crash into Saturn – but not before sending back
impressive amounts of data from incredibly up-close encounters with the rings. From its arrival in 2004 to its final plunge through the atmosphere in 2017, Cassini succeeded in vastly expanding our knowledge of the Saturnian ring system.

1.2.1 Cassini’s Instrument Suite

Cassini’s science payload consisted of twelve instruments. The UVIS instrument is composed of three different channels: the high speed photometer (HSP) channel, which had an extremely short sampling interval that mapped to high spatial resolution in the ring plane, enabling the determination of complex structure; the far ultraviolet (FUV) channel, which imaged and measured UV spectral reflectance of the rings to provide information on the background noise for the HSP observations; and the extreme ultraviolet (EUV) channel, which observed solar occultations of the rings as well as EUV spectral reflectance imaging. Although the photometer was the primary channel for observing stellar occultations of the rings, the FUV channel simultaneously recorded data for some ring stellar occultations. The FUV data has lower spatial resolution than the HSP data, but the imaging characteristics help model the background signal (Colwell et al., 2010). UVIS was accompanied by two other instruments designed to record occultations: VIMS (the Visible and Infrared Mapping Spectrometer) which observed occultations of infrared stars, and the RSS (Radio Science Subsystem) which transmitted coherent radio signals through the rings to receivers on Earth. UVIS had the highest spatial resolution of the three instruments and observed at wavelengths much shorter than the smallest particles in the main rings, with a spectral band pass of 110-190 nm. VIMS performed detailed spectral mapping of the rings, as well as satellite surfaces and atmospheres in addition to its
occultation measurements. The RSS simultaneously transmitted coherent radio signals through the rings at three different wavelengths: 0.94 cm (Ka-band), 3.6 cm (X-band), and 13 cm (S-band). The Imaging Science Subsystem (ISS) took spectacular images the rings at visible, near-infrared, and near-UV wavelengths. The Composite Infrared Spectrometer (CIRS) and radar instrument (RADAR) determined the temperature profile of the rings. A magnetometer (MAG) and the Magnetospheric Imaging Instrument (MIMI) investigated how the rings and dust interact with Saturn’s magnetosphere. The Radio and Plasma Wave Science instrument (RPWS) which was equipped with a Langmuir probe to measure the impact of Saturn’s kilometric radiation on time-dependent ring phenomena such as spokes. The Ion and Neutral Mass Spectrometer (INMS) performed in situ analyses which measured the chemical, elemental, and isotopic composition of the ring environment. The Cosmic Dust Analyzer (CDA) and the Cassini Plasma Spectrometer (CAPS) also analyzed data in situ, which studied the composition of icy dust grains near Saturn.

Figure 1.3. Artist depiction of the Cassini spacecraft alongside the actual UVIS instrument. Left: A three-dimensional digital representation of the Cassini spacecraft with the UVIS instrument highlighted in blue. Right: The UVIS instrument prior to launch (Image credit: Laboratory for Atmospheric and Space Physics at the University of Colorado Boulder).
1.2.2 UVIS

UVIS was equipped with four separate channels (Esposito et al., 2004). The first of these is the HDAC (Hydrogen-Deuterium Absorption Cell) channel, which used a hydrogen cell, a deuterium cell, and electron multiplier photodetector channel to measure hydrogen and deuterium in the Saturn system. The Far Ultraviolet Channel (FUV) observed stellar occultations as well as spectral imaging while the Extreme Ultraviolet Channel (EUV) was used to observe solar occultations in addition to EUV spectral imaging. The data analyzed in this dissertation is from the High-Speed Photometer (HSP) which is ideal for detecting fine structure due to its spatial resolution of less than 10 m in many cases (Colwell et. al., 2010). The data must be calibrated to corrected for systematic offsets by removing non-stellar background signal and accounting for a drift in the instrument’s sensitivity, called the ramp-up effect, over the course of an occultation. Chapter 2 contains a detailed discussion of the data calibration procedure and ramp-up effect. An engineering diagram indicating the various channels is shown in Figure 1.4.

1.3 Unresolved Questions in ring science

Despite hundreds of years of observing Saturn’s rings, there are an unsettling a number of questions that remain unresolved. Among the most provocative of these are: “How old are Saturn’s rings?”, “How did the rings form?”, and “How massive are the rings?” Finally, at the focal point of this dissertation, “How does the local dynamical environment in Saturn’s ring system affect the aggregation and fragmentation of particles and clumps?”
1.3.1 How old are the rings?

The age of Saturn’s rings remains a controversial topic in planetary science. Although the rings are a vast, intricate, seemingly ancient system, they appear paradoxically bright and young. The purity of Saturn’s ring particles ($\gtrsim 95\%$ water ice) is inconsistent with that of other ring systems, whose particles are much more contaminated by carbon-rich organics (Cuzzi et al., 2009). Harris (1984) postulated that rings share a common origin with satellites, satellite debris being the source of ring material. Once the rings form, various processes act to alter them—meteoroid bombardment, interparticle collisions, and sputtering all change the ring particles’ fundamental properties, grinding and darkening them on short cosmological timescales.
The competing physical processes of viscous spreading, gas drag, particle coagulation, and conservation of angular momentum make a robust determination of the age of the rings particularly difficult. Esposito (1986) and Cuzzi and Estrada (1998) derived low age estimates for Saturn’s A ring, Esposito (1986) by viscous spreading and Cuzzi and Estrada (1998) by darkening of material by meteoroid bombardment. Harris (1984) suggested that planetary rings were formed not in conjunction with their parent planets but formed later by the disruption of satellites too massive to be destroyed in early accretionary processes. Colwell and Esposito (1993) found that in the case of narrow rings formed via the disruption of a satellite, the largest fragments become confining moons which naturally clear gaps and “shepherd” the rings. Results of Monte Carlo simulations (see section 1.3.5.2) of 300 evolutionary histories of moon populations at Neptune by collisional cascade after a catastrophic fragmentation event by Colwell and Esposito (1992) are shown in Figure 1.5.

More recently, Cassini’s CDA measured the meteoroid flux at Saturn’s rings in-situ in an effort to determine how long it would take to darken the rings to their current albedo given a pure ice origin. Because the rings are so flat and thin, they have a high surface area to mass ratio. This makes contamination effects of constant bombardment by carbonaceous meteoroids particularly important. However, as the ring material becomes more polluted, ballistic transport redistributes the pollutants through impact ejecta (Cuzzi and Estrada, 1998). Different regions have different levels of susceptibility to contamination, with higher mass regions having more material to “recycle” than less massive ones, resulting in regional variations in composition. Cuzzi and Estrada (1998) used a radiative transfer model to investigate regional color and albedo differences in the rings and found that compositional differences correlate with differences in
surface mass density (and therefore, susceptibility to contamination). The radial redistribution of material by meteoroid collisions is called ballistic transport because the particles follow a ballistic trajectory between disruption and re-accretion (Durisen et al., 1989, 1992, 1996; Cuzzi and Estrada, 1998). (Durisen et al., 1989, 1992, 1996) showed that many ring structures, such as the sharp inner edges of the A and B rings, can form as a natural consequence of meteoroids impacting the rings.

Cosmic recycling occurs when ring particles whose surfaces have been contaminated are broken up, either by collisions or meteoroid impacts, exposing the pure icy components underneath. The cycle continues as impacts create debris which re-accretes into unconsolidated rubble piles, essentially “cleaning” the particles. Saturn’s dense B ring is especially massive, so pollutants here are distributed to an extremely high volume of ring material, further diluting their effects. The process of cosmic recycling continuously constantly replenishes the rings so that while they may appear cosmologically young, the age of the ring system as a whole remains ambiguous (Esposito, 2014).
Figure 1.5. Monte Carlo simulations of the evolution of a collisional cascade after a catastrophic fragmentation event.

The above figure taken from Colwell and Esposito (1992) shows the results of Monte Carlo simulations of 300 evolutionary histories after catastrophic disruption of Neptune’s moons. The simulation initialized with a “seed” moon of 40 km radius at the orbit of Thalassa. Their results indicate that Neptune’s small satellites must have evolved through the successive breakup of large aggregates that began around the birth of the Solar System. The solid line follows the evolution of the mean, asterisks the median, and squares the mode of the size largest surviving remnant over time (Figure from Colwell and Esposito, 1992).

But the apparent youth of the rings is statistically problematic, as the likelihood of an occurrence of the catastrophic impact of a sufficiently sized body to create Saturn’s rings is small (Lissauer et al., 1988; Wing Ip, 1998; Dones, 1991). The timescale for viscous spreading of the rings as the particles experience momentum transfer is quite short on cosmological timescales. As particles collide and exchange momentum, the rings spread radially. Satellites slow this spreading as momentum is transferred through the resonance to the moons (Esposito, 2014). Even including this effect, the lifetime of the current ring system is between just $10^7$ and $10^9$ years. Hsu et al. (2018) described results from direct, in-situ detections of dust grains around
Saturn’s dense rings by Cassini’s CDA. Measurements by the CDA constrained the influx of exogenous mass at Saturn’s rings, which, combined with estimates of the ring mass from gravity data, allowed estimates of the rings age. Surprisingly, these observations are consistent with younger rings (~ 100-150 million years). In further support of young rings, Charnoz et al. (2010) found that a population of moonlets exterior to the main rings are dynamically younger than the formation timescale for regular satellites (≲ 10^7 years) and underdense (~600 kg/m^3). However, Crida et al. (2019) point out that there are still interpretations of the CDA data that allow for older rings and that the "old rings" theory has certainly not been disproven.

1.3.2. What is the origin of Saturn’s rings?

The puzzle of Saturn’s rings’ origin and evolution remains largely a mystery. The three competing theories include that the rings are ancient remnants accreted from the original solar nebula, that they formed via collisional cascade upon the sudden break up of a satellite, or that a grazing comet was split by Saturn’s tidal forces (e.g, Charnoz et al., 2009). Canup (2010) postulated that a Titan-sized, differentiated object may have been dragged into Saturn’s Roche zone where its icy mantle was stripped, and its rocky core fell victim to Saturn’s gravity. A collisional cascade is said to occur if moons are shattered into smaller fragments which are eventually ground to dust (Esposito and Colwell, 2003). It is unclear whether the rings are really as young as they appear, which would imply they formed in an extremely rare event, or if cosmic recycling has continually renewed the system (Esposito, 2014).
1.3.3. How massive are Saturn’s rings?

Of all the ring systems in our Solar System, Saturn’s is by far the most massive. Most of this mass lies within the B ring. Hedman and Nicholson (2016) constrained the total mass of the B ring to between one- and two-thirds the mass of satellite Mimas, which has a radius ~200 km, through a multi-profile wavelet-based analysis of 5 density waves in the B ring. Later, Iess et al. (2019) analyzed data from Cassini’s Grand Finale when it flew between the planet and the rings to measure the gravitational pull on the spacecraft. By separating the contribution from the planet versus the ring, they found that the total mass of Saturn’s rings is about 0.41 times that of satellite Mimas. Given this result, the likelihood of an event causing the disruption of a body of this size occurring within the last few hundred million years is small, consistent with the hypothesis that the rings are primordial.

On the other hand, this relatively small mass for the ring system is difficult to reconcile with bombardment of the rings by micrometeoroids for billions of years which should both darken the rings to a much lower albedo than is observed and cause spreading and structural evolution that is inconsistent with some ring features, such as the inner edges of the A and B rings. Doyle et al. (1989) suggested that the relatively high albedo of Saturn’s A and B rings implies that they have only retained a small fraction of primitive carbonaceous material, indicative of a more recent origin. Cuzzi and Estrada (1998) suggest a neutral extrinsic polluting material such as that observed in interplanetary dust particles (IDPs) is needed to explain the darkening of the rings and estimate an age of just ~100 million years for the rings, less than one 40th of the age of the solar system. More recently, Zhang et al. (2017) calculated the exposure age of the A and B rings and Cassini Division, finding an approximate overall age of $\leq 150$
million years for Saturn’s rings. Recall, however, that compositional studies of the exposure age of the rings may make the rings appear artificially young due to cosmic recycling as described in section 1.3.1.

1.3.4. What is the particle size distribution of the rings?

Ever since we discovered that the rings are actually made up of innumerable icy particles, planetary scientists have been trying to determine their sizes. Unfortunately, we cannot physically interact with particles 1.2 billion kilometers from Earth in the same way that we can with materials on Earth. Even Cassini’s incredibly high-resolution instruments were unable to resolve the rings’ smallest constituents—individual ring particles. Although we cannot directly observe them, we are able to draw inferences about the size distribution of these particles from radio, solar, and stellar occultations as well as the phase function from imaging of tenuous rings. In this dissertation we will use Cassini UVIS stellar occultation data to probe the distribution of clumps and aggregates throughout the rings, particularly within small-scale structures such as self-gravity wakes and density waves, described below.

Particle sizes vary widely throughout the rings, ranging from centimeters to tens of meter-sized boulders. While the orbital velocity of Saturn’s ring particles is a speedy 20 kilometers per second, interparticle collisions occur at extremely low speeds on the order of millimeters per second (Cuzzi et al., 2009). Saturn’s rings are well known to generally follow a truncated inverse power-law size distribution such that
\[ n(a)da = Ca^{-q}da, a_{\text{min}} \leq a \leq a_{\text{max}} \] (1.1)

where \( n(a)da \) represents the number of particles of radius \( a \) within interval \([a, a + da]\) per square meter, \( q \) is the power-law index, \( C \) is a constant, and \( a_{\text{min}} \) and \( a_{\text{max}} \) correspond to the radii of the minimum and maximum particle sizes. The particle size distribution is closely related to the ring optical depth. Normal optical depth, which is what would be observed if observing the rings perpendicular to the ring plane, can be related to the particle size distribution by (e.g. Cuzzi et al. 2009):

\[ \tau_n(\lambda) = \int_{a_{\text{min}}}^{a_{\text{max}}} \pi a^2 Q_{\text{occ}}(a, \lambda, f) \ n(a) \ da \] (1.2)

where \( Q_{\text{occ}} \) is the effective extinction efficiency of particles of size \( a \) in the rings, and \( f \) is the fractional portion of the stellar diffraction pattern covering ring material \((f = 1 \text{ except near a ring edge, Cuzzi et al., 1985})\). By combining multiple observations of optical depth at different wavelengths, Equation \( \tau_n(\lambda) = \int_{a_{\text{min}}}^{a_{\text{max}}} \pi a^2 Q_{\text{occ}}(a, \lambda, f) \ n(a) \ da \) can be inverted to retrieve the size distribution in terms of the particle cross section per unit area, \( n(a)da \).

**1.3.4.1 Particle sizes from post-Voyager stellar occultations**

French and Nicholson (2000) combined the optical depth profile obtained from the Voyager PPS occultation with a ground-based stellar occultation of star 28 Sgr to constrain the power-law index in Saturn’s main rings, finding \( q = 2.75 \) for \( 30 \text{ cm} \leq a \leq 20 \text{ m} \) and \( q = 3 \) for \( 1 \text{ cm} \leq a \leq 20 \text{ m} \). Harbison et al. (2013) performed an analysis of diffracted light in VIMS solar occultations and found evidence for a population of sub-cm particles interior to the
Encke gap. Becker et al. (2016) (hereafter B16) analyzed diffraction signatures at ring edges in the outer A ring and found a steepening of the particle size distribution with increasing distance from Saturn. This also corresponds to edges that are strongly perturbed by satellite resonances which may drive more vigorous collisions between ring particles, leading to the production of more small debris. This is consistent with the results of Jerousek et al. (2016) (hereafter J16), who found a significant fraction of sub-cm particles only in the outer A ring and B1 regions and an increasing abundance of small particles across the Trans-Encke region with decreasing distance from the A ring outer edge. Later, Jerousek et al. (2020) implemented the thin layers model of Zebker et al (1985) to determine particle size distribution parameters throughout in the C ring and Cassini Division. In the background C ring, they found $a_{\text{min}} \sim 4.1$ mm, $a_{\text{max}} \sim 10-15$ m, and $q \sim 3.16$, while in the plateaus they found $a_{\text{min}} \sim 6$ mm, $a_{\text{max}} \sim 5-6$ m, and $q \sim 3.05$; additionally, they found $q \sim 3.0$ in the Cassini Division background and ramp while $q \sim 2.9$ in the Cassini Division’s Triple Band feature.

In this study we seek to build upon these foundational works. In Chapter 4 we will describe how we expanded upon the model of B16 for diffraction at the edges of the outer A ring and Encke gap to apply to narrow gaps and ringlets, expanding the determination of small particle populations at ring edges to 17 total edges. In Chapter 5 we will extract new information about particle sizes from the moments of the stellar occultation data, expanding the analysis of Colwell et al. (2018), who analyzed only the variance at 10 km resolution across the rings from two occultations.
1.3.5. How can the dynamical evolution of rings be modeled?

1.3.5.1 N body simulations

Ring particle dynamics can be simulated using N-body simulations which model the interactions of virtual particles including the particles’ self-gravity. N-body simulations consider the sum of the forces acting on each of these particles and determine each particle’s trajectory through a straightforward application of Newton’s laws. The motions of the particles and outcomes of collisions are calculated at each step of the simulation until it reaches a steady-state. Unfortunately, the computational efficiency of an N-body simulation is critically limited by the number of particles simulated and real planetary rings contain far too many particles to calculate each of their trajectories. They are called N-body simulations because they maximize efficiency by considering only a small number of bodies, in this case a population of particles characteristic of some region of Saturn’s rings. Most modern N-body simulations consider tens of thousands of particles while a few are able to simulate up to a million bodies (Esposito, 2014).

1.3.5.2 Monte Carlo simulations

The dynamical evolution of the rings can also be simulated probabilistically using a Monte Carlo method. Monte Carlo simulations provide a means to track the evolution of planetary rings through a stochastic perspective alternative to the deterministic nature of N-body simulations. This method consists of a series of random draws from a distribution of probabilities that determine the subsequent successive states of the system at each time step. Monte Carlo
simulations draw randomly from a set of conditional probabilities in a Markov chain, a system which moves from one state to the next according to a set of probabilistic rules. State vectors are computed from the expectation values for the Monte Carlo simulation after each discrete step in time. Colwell and Esposito (1992) directly compared Monte Carlo simulations to observations of Neptune’s rings and Elliot and Esposito (2011) applied the Monte Carlo method to estimate the growth of regolith and pollution of the rings by micrometeoroids.

1.4 Structure of Saturn’s rings

We begin our review of the structure of the rings with a layout of the rings’ taxonomy. Saturn’s rings consist of a series of major divisions and subdivisions. Most major rings can be sorted into two categories: dense rings and faint rings. We use these two subcategories to guide our section of the rings’ macrostructure below. The innermost ring is the tenuous D ring, followed by the plateau-studded C ring; the Cassini Division is wedged between the dense B ring and density wave riddled A ring; the narrow F ring is nestled between satellites Prometheus and Pandora; and furthest from the planet extends the dusty G ring and diffuse E ring whose source is the geysers of Enceladus. The main rings are highlighted in Figure 1.6 juxtaposed with an ISS image mosaic. In this section, we will also discuss some of the most interesting small-scale structures that characterize Saturn’s rings, specifically density and bending waves, self-gravity wakes, and propeller objects. Colwell et al. (2009b) provide an overview of ring structure, including naming and locations of the features described below.
Figure 1.6. An optical depth profile of the rings from the $\beta$ Centauri Rev 081 ingress occultation stacked atop a mosaic ISS image of the rings. (Bottom) Image Credit: NASA/JPL-CalTech/SSI.

1.4.1. Dense Rings

1.4.1.1. C ring and Cassini Division

The C ring and Cassini Division share similar characteristics; namely, they have similar optical depths, colors, and albedos, with darker and less red particles than the A and B rings (Cuzzi and Estrada, 1998). The C ring is the innermost of Saturn’s main rings. It has a thin, undulating background in optical depth, punctuated by sharp, narrow (~10-100 km) structures called plateaus. These structures appear as bright, banded features superimposed on the more diffuse background C ring. The optical depth ranges from less than 0.1 in the slightly undulating background to about 0.4 in the plateaus (Colwell et al., 2009b). Some narrower features have
optical depths greater than 1. The C ring also hosts several resonance-driven gaps and ringlets. Within the Bond gap resides a 20 km wide apparently noncircular ringlet designated 1.470Rs by Porco and Nicholson (1987), the inner edge of which Nicholson et al. (2014) confirmed to be in line with the location of the Mimas 3:1 vertical resonance. The Titan -1:0 nodal bending wave also lies within the C ring at 77525 km from the center of Saturn. Induced by the nodal regression rate of the ring particles corresponding to the orbital frequency of Titan, it is the only bending wave observed to propagate outward (Rosen and Lissauer, 1988; Nicholson and Hedman, 2016). Another well-known feature of the C ring is the eccentric ringlet within the Maxwell gap, which many speculated would be maintained by one or multiple ring-moons (Esposito, 2014.) Despite Cassini’s up-close observations of the region, however, no moons have been found there to date and the mechanism maintaining this ringlet remains unknown.

The Cassini Division was first observed in 1675 by Giovanni Domenico Cassini, who noticed a dark region separating the inner and outer halves of Saturn’s rings. Voyager observations confirmed that rather than simply separating the A and B rings, the Cassini Division is actually a ring region itself. The region bears a striking resemblance to the C ring (Cuzzi et al., 1984; Esposito et al., 1984) with a characteristic optical depth of about 0.05 to 0.15, and one feature, the “triple band”, with an optical depth of about 0.3. The Cassini Division, like the C ring, is populated by many gaps, noncircular ringlets, and density waves.

1.4.1.2. The B ring

The B ring is the most opaque, massive, and mysterious of Saturn’s rings. It is partitioned into five sub-regions based on the structure in optical depth: B1, B2, B3, B4 and B5. The
innermost portion of the ring is B1, which extends from the inner edge of the ring at approximately 92000 km to 99000 km. B1 is the most transparent region, with a median optical depth of ~1.1 compared to ~3.6 in the densest portion of the central B ring. B1 also contains a relatively featureless region dubbed the “flat spot”, named for its featureless optical depth profile, which spans the region from roughly 94450 to 95350 km. The only clearly observed density wave within the B ring, the Janus 2:1 wave, also resides within B1 and is generated by an inner Lindblad resonance (e.g., an orbital resonance in which the epicyclic frequency of an orbiting ring particle is an integer multiple of the forcing frequency of the perturbing satellite) at 96427 km. Regions B2 and B3 constitute the central B ring, spanning the region from 99000 km to 104500 km (B2) and 104500 to 110000 km (B3). Optical depth profiles of B2 are essentially bimodal, alternating between $\tau \sim 1.5$ and $\tau \geq 4$, and “square-wave” shaped, in contrast to the more triangular shaped profiles of B3 (Colwell et al., 2009b). The B3 region is the highest optical depth portion of the rings with a median $\tau \sim 3.6$ and sections where $\tau > 5$ even for the highest signal-to-noise occultations (Colwell et al., 2007), although it is the B2 region that is home to the most opaque regions. There are several narrow (~50 km) segments of relatively low $\tau \sim 2$ which punctuate B3 at 300-900 km intervals (Colwell et al., 2009b). At ~110300 km, the optical depth begins decreasing erratically. In contrast with the periodic structure observed in B1 and the bimodal structure observed in B2, this region is highly irregular. The B5 region encompasses the outermost 1000 km of the B ring which is characterized by non-axisymmetric structure due to the non-circular nature of the outer edge of the B ring. The B ring outer edge is confined by the Mimas 2:1 inner Lindblad resonance at ~117555 km (Lissauer and Cuzzi, 1982). Multiple modes tied to Mimas have been observed on the outer edge of the ring resulting in a
complex shape and structure (Spitale and Porco, 2010). As a result, the exact location of the edge varies by up to 160 km (Colwell et al., 2009b). One of Cassini’s noteworthy discoveries was the presence of narrow, streaky structure referred to informally as “straw” in the troughs of some A ring density waves during Saturn orbit insertion (SOI), as well as at the outer edge of the B ring (Porco et al., 2005). Figure 1.7 shows two images from Cassini of resolved straw at the B ring outer edge (left) and in the Janus 6:5 density wave in the A ring (right).

Figure 1.7. Resolved straw-like texture at the outer edge of the B ring. Cassini’s Imaging Science Subsystem (ISS) took the above images of so-called ‘straw’ at the outer edge of the B ring (left; PIA21057: NAASA/JPL/Space Science Institute) and within troughs of the Janus 6:5 density wave (right; PIA21060: NASA/JPL-CalTech/Space Science Institute).

1.4.1.3 A ring

Spiral density waves are prominent features of Saturn’s A ring. These waves are induced by resonances with moons, primarily Prometheus, Pandora, Janus, and Epimetheus, although there is even a wave caused by a resonance with distant Iapetus (Tiscareno et al., 2013).
Perturbations from co-orbital satellites Janus and Epimetheus confine the A ring outer edge, creating a 7-lobed shape along the resonant streamlines (Porco et al., 1984b). Janus and Epimetheus swap orbits once every four years (coincidentally at times corresponding to the Winter Olympics), causing a gravitational pull-and-tug as the moon at the smaller semi-major axis catches up and its semi-major axis increases as the moon at the larger semi-major axis slows down and its semi-major axis decreases, resulting in a distinct horseshoe-shaped orbit. The A ring’s optical depth is highest in the inner portion of the ring, which hosts the strong Pandora 5:4 density wave, which has the longest wave train of any wave in the A ring. Between the Pandora 5:4 and Janus 4:3 resonances, a viscous overstability (Schmidt et al., 2009; Lehmann et al., 2019) produces periodic structure with a wavelength of ~150 m, which has been detected in Cassini VIMS, UVIS, and RSS occultations (Salo et al., 2001; Thomson et al., 2007; Colwell et al., 2007; Colwell et al., 2009b).

Embedded moonlets ~100 meters in size perturb the rings producing a two-lobed pattern dubbed propellers. Larger moons open a full gap around the rings, such as the Encke gap and Keeler gap. The region between the Janus 4:3 resonance at 125240 km and the Encke gap at ~133423 km is considered the central A ring, where propeller structures form in three narrow radial bands (Tiscareno et al., 2008). The outer portion of the ring is punctuated by two gaps, the Encke and Keeler gaps, located ~133000 and 136500 km from Saturn’s center, respectively. Both gaps are cleared by small satellites: the Encke gap by Pan and the Keeler gap by Daphnis. The ~35 km wide Keeler gap is especially narrow and as a result, the moon Daphnis stirs up clear, transient wisps of material in its wake, shown in Figure 1.8, while such a clear pattern is absent in the ~320 km wide Encke gap.
1.4.2 Tenuous Rings

1.4.2.1 D ring

The innermost of Saturn’s rings is the tenuous D ring. Cassini surprised scientists with its observations of the D ring which had changed substantially since Voyager had observed it nearly
30 years prior, showcasing the tendency of the ring structure to vary on short timescales. In one of the largest secular changes ever recorded in Saturn’s rings, Cassini observed that the brightest of three narrow ringlets seen by Voyager (dubbed D72) had both dimmed and broadened from less than 40 km wide to roughly 250 km, and its central brightest feature had migrated inward 200 km (Horányi et al., 2009). Observations by Cassini’s ISS and VIMS over a wide range of viewing configurations revealed a number of fine, complex structures that vary in time and longitude. Because it occupies the closest position to Saturn, the D ring is the source of some material that spirals into Saturn’s atmosphere and was detected by the Cassini INMS and CDA instruments during the Grand Finale orbits that threaded the gap between the D ring and the atmosphere (Waite et al., 2018; Hsu et al. 2018; Tiscareno et al., 2019; Miller et al., 2020). Further adding to the harsh nature of this environment, Krimigis et al. (2005) detected a radiation belt near the region.

The mysterious ring exhibits several interesting phenomena. Hedman et al. (2007a) noted a vertical corrugation of material into a ~30 km wavelength periodic structure between 73200 and 74000 km which may have been caused by an impact that occurred in 1984. In addition, the D ring hosts a sheet of diffuse material near the outer edge that is only visible at low phase angles (< 60°) and appears as a series of narrow ringlets at high phase angles (Horányi et al., 2009). Differences in photometric and spectral properties indicate average particle sizes ranging from 1 to 100 microns (Hedman et al., 2007a). Information about the particle size distribution in the D ring is especially difficult to untangle due to the low signal to noise ratio at such low optical depths (Hedman et al., 2007a). In Chapter 2 of this thesis, we will review a statistical search we performed in search of fine features in Saturn’s D ring using the m-statistic method.
developed used to search for ring features in Voyager 2 stellar occultation data of the rings of Uranus (Colwell and Esposito, 1990).

1.4.2.2. F ring

Saturn’s F ring does not fall under the category of either dense or tenuous rings. Pioneer 11 discovered the F ring during its flyby of Saturn in 1979 (Gehrels et al., 1980) and subsequently confirmed by Voyager (W.H. Smith et al., 1981; B.A. Smith et al., 1982). This ring is narrow and slightly elliptical with an opaque ~50 km wide core (Colwell et al., 2009b). Differential azimuthal structure forms as ring material is twirled into a so-called “braided” pattern (Esposito, 2014). Bosh et al. (2002) analyzed the size distribution of the F ring its differential power-law index to $2 < q < 2.5$. The ring is maintained by nearby moons Prometheus (interior) and Pandora (exterior) and its core is uniquely narrow and opaque.

The F ring also contains a series of irregular clumps which are generally attributed to either meteoroids impacting invisible parent bodies (Showalter, 1998; French et al., 2012) or debris from the collisions of these bodies (Esposito, 2014). Although the ring is composed primarily of micron-sized particles, the observation of “mini-jets” by Murray et al. (2008) indicated the presence of a few large particles as well. A further perplexing aspect of this ring is that it was originally assumed to be confined by the so-called “shepherding” of moons Pandora and Prometheus, but Showalter and Burns’ (1982) analysis of Voyager data showed that the satellite torques did not balance. Additionally, the F ring is notorious for its high-tempo dynamic nature: Voyager observed changes in Saturn’s F ring on a timescale of just days (Esposito, 2014).
Saturn hosts a total of 82 satellites which range in size from hundreds of meters to thousands of kilometers. Two of the most studied are Titan, the largest satellite with a hazy atmosphere, and Enceladus, a geologically active body that shoots plumes of water vapor and small particles from crevasses on its surface. The two main ring edges are shepherded by resonances with moons: the outer A ring edge is confined by the 7:6 Janus/Epimetheus co-orbital resonance while the outer B ring edge is maintained by the strong Mimas 2:1 resonance. Analysis of variations in particle size distribution with moon longitude indicate fragmentation induced by dynamical disturbances which Esposito et al. (2012) characterize using an analogy to a biological predator-prey cycle which is described in section 1.5.3.

The rings are punctuated by a multitude of transparent gaps and opaque ringlets. The Cassini Division is littered with narrow ringlets such as the Herschel and Huygens ringlets. The outer A ring has two famous gaps, the Encke and Keeler gaps, which are opened by ring-moons Pan and Daphnis, respectively. Showalter (1991) identified the presence of Pan, a ravioli-shaped satellite with a particularly pronounced equatorial region, within the Encke gap. We defer further discussion of gaps and ringlets to Chapter 4, in which we describe our results of modeling diffraction signatures at their edges to constrain the sizes of the smallest particles.

1.5 Small scale structures

In addition to investigating macroscale features in the rings, Voyager and Cassini observed evidence (both direct and indirect) of a variety of comparatively small features, such as self-gravity wakes, spokes, and density waves. In this section we give a brief overview of a few of the most noteworthy of these structures.
1.5.1 Density Waves

Perhaps the most visually striking structures in Saturn’s rings are spiral density waves, oscillations in the particle number density in the rings at the locations of resonances with moons. Saturn’s A ring hosts a plethora of such waves, induced by Lindblad resonances. These radially propagating compression waves produce distinctive azimuthal spiral structure akin to the spiral arms of galaxies (Goldreich and Tremaine, 1982). The tightly wrapped arms are made up of alternating compressed and rarefied regions of ring particles. As the disturbance propagates toward the perturbing satellite, energy transfer occurs through interparticle collisions until it is collisionally damped. Angular momentum is conserved in the interaction while mechanical energy is dissipated in collisions, resulting in a gradual orbital decay of ring particle orbits and expansion of the moon’s orbit. This exchange of angular momentum between the particles and the moon at a resonance allows the moon’s mass to slow the collisional (viscous) spreading of the ring. Interparticle gravitation provides a restoring force on the wave which results in a dispersion of the wavelength. Observations of the wave dispersion can thus be used to measure the local ring mass (Esposito et al. 1983, Spilker et al. 2004, Tiscareno et al. 2007, Colwell et al. 2009b, Nicholson and Hedman (2010)).

While density waves appear static from the perturbing moons frame of reference, spiral structure akin to the galactic spiral arms of galaxies (Goldreich and Tremaine, 1982) occurs as particles streamline in an $m$-lobed pattern. These streamlines are a manifestation of an azimuthal gradient in gravitational potential with $m$ maxima which causes a spiral pattern. The physical effect of resonant forcing is an increase in ring particle density in the crests of the wave and a decrease in the troughs. Collisions damp the amplitude of the density oscillations with an
approximately linearly decrease in wave amplitude with distance from the resonance. The radial extent of density waves varies from just a few kilometers at weaker resonances to hundreds of kilometers for the strongest resonances in the A and B rings. Density waves can be thought of as in situ tools with which researchers can probe the surface mass density of the rings.

Cuzzi et al. (1981) reported the first detection of a density wave in Saturn’s rings from Voyager data. Many more density waves were found in Voyager occultation data as Lane et al. (1982) analyzed the PPS stellar occultation, Holberg et al. (1982) the UV stellar occultation, and Marouf et al. (1986) the radio science occultation. Figure 1.9 shows the Prometheus 12:11 density wave and the Mimas 5:3 bending wave in the A ring captured by the Cassini ISS. Figure 1.10 shows two sample occultations of density waves with smoothed optical depth profiles, illustrating the small-scale undulations present in these structures. It is clear that the Mimas 5:3 density wave, which starts with a wavelength of roughly 50 km and extends almost 200 km from the resonance, is morphologically different from the Pandora 5:4 density wave, whose arms are wound much more tightly and extends roughly 300 km out from the resonance.
Figure 1.9. The Prometheus 12:11 density wave and the Mimas 5:3 bending wave in the A ring. This is a narrow-angle camera image of Saturn's rings (~290 meters per pixel) taken after the successful completion of the orbit insertion burn. The image shows the Prometheus 12:11 density wave and the Mimas 5:3 bending wave in the A ring. (Image credit: NASA/JPL/SSI)

Figure 1.10. The Mimas 5:3 and Pandora 5:4 density wave trains.  

a) The UVIS $\beta$ Centauri Rev 78 Egress occultation of the Mimas 5:3 density wave induced by the inner Lindblad resonance at ~125266 km in the outer A ring, boxcar smoothed by 100 points (~ 650 m). The data are smoothed by 100 points to illustrate the undulations of the wave.  

b) The Pandora 5:4 density wave in the UVIS $\beta$ Centauri Rev 78 Egress occultation. The data are smoothed by 150 points (~ 1 km) to illustrate the undulations of the wave.
1.5.1.1 Theory

Following Lin and Shu (1964)’s derivation, the dispersion relation for spiral density waves can be expressed as

\[
(\omega - m\Omega(R))^2 = \kappa^2(R) - 2\pi G \sigma_0 |k|
\]  
(1.3)

Where \(\omega\) is the disturbance frequency, \(\Omega\) is the azimuthal frequency, \(\kappa\) is the epicyclic frequency of the particle, \(|k|\) is the radial wavenumber of the density wave, \(G\) is the gravitational constant, \(\sigma_0\) is the surface mass density, \(R\) is the radial distance from Saturn, and \(m\) is an integer determined by the order of the resonance which also corresponds to the number of spiral arms generated. The wavelength damps with increasing distance from the resonance according to a dimensionless radial parameter (e.g., Tiscareno et al., 2006):

\[
\xi = \left(\frac{D_L r_L}{2 \pi G \sigma_0}\right)^{\frac{1}{2}} \left(\frac{r-r_L}{r_L}\right),
\]  
(1.4)

where \(r_L\) is the Lindblad resonance in kilometers from Saturn, and

\[
D_L = 3(m-1)n^2 + J_2 \left(\frac{R_S}{r_L}\right)^2 \left(\frac{21}{2} - \frac{9}{2}(m-1)\right)n^2,
\]  
(1.5)

where \(R_S = 60330\) km is the radius of Saturn (Kliore et al., 1980), \(J_2 = 1.629071 \times 10^{-6}\) (Jacobsen et al., 2006) is the spherical harmonic coefficient of Saturn’s gravitational field, and \(n\) is the mean motion of a ring particle. A Lindblad resonance is defined as the orbital location at which the radial or epicyclic frequency of a ring particle, \(\kappa\), is equal to an integer multiple of the pattern speed of the perturbing potential in the frame of reference of the ring particle.
1.5.1.2 Density waves as probes of local surface mass density

Spiral density waves can be readily identified in occultation data via power spectral analysis. Because density waves disperse linearly, the slope of their dispersion relation, which depends on the azimuthal and epicyclic frequencies of the particles as well as the order of the resonance and surface mass density of the underlying ring material, can be inverted to obtain the local surface mass density. Using this technique, Spilker et al. (2004) provided a comprehensive survey of surface mass density from dispersions of density waves in the A ring and Tiscareno et al. (2007) found that the surface mass density increases monotonically across the inner to mid-A ring. Spilker et al. (2004) discovered decreasing surface mass density from ~45 \( \frac{g}{cm} \) in the inner A ring to ~20 \( \frac{g}{cm} \) past the Encke gap. Tiscareno et al. (2007) analyzed high resolution Cassini images of density waves which were not detected by Voyager. More recently, Tiscareno and Harris (2017) used the continuous wavelet transform technique, a technique that finds the greatest power at each spatial wavenumber \( k \) at each radial location \( r \) over a one-dimensional radial scan, to map radial variations in surface mass density across the rings in more detail than any previous work. Their results indicated slightly lower estimates of surface mass density in the A ring, especially the central A ring, than had been found in previous publications.

1.5.2. Self-gravity wakes

The existence of self-gravity wakes in Saturn’s rings, ephemeral aggregates of ring particles that form under the competing influences of their mutual self-gravity and Keplerian shear, was known long before the arrival of Cassini in 2004. In Saturn’s dense A and B rings,
these wakes are formed when the local surface mass density of the rings is sufficiently high and the radial epicyclic frequency of the particles sufficiently small that the mutual gravity between ring particles causes them to clump together. As a natural consequence of Keplerian shear, these clumps are canted about 20-25° from the local azimuthal direction (Colwell et al., 2006, 2007; Hedman et al., 2007b) and the length of the shadows cast by these structures thus depends heavily on the viewing geometry. The existence of wakes was first speculated by Camichael (1958) to explain the phenomenon of an observed azimuthal brightness asymmetry—which presents as a partitioning of the ring into quadrants of alternating brightness—and was later confirmed by Lumme et al. (1983). Later, Salo (1992, 1995) and Richardson (1994) conducted numerical simulations that demonstrated self-gravity wakes forming naturally in a self-gravitating ring.

1.5.2.1 Theoretical overview

Julian and Toomre (1966) demonstrated that the wavelength scale of self-gravity wake features in a galactic disk depends on the epicyclic frequency of the particles’ orbits and on the local surface mass density of the rings. The most unstable wavelength to gravitational collapse is given by the Toomre critical wavelength,

$$\lambda_{\text{crit}} = \frac{4\pi^2 G \sigma}{\kappa^2},$$  \hspace{1cm} (1.6)

where $G$ is the gravitational constant, $\sigma$ is the surface mass density, and $\kappa$ is the ring particle’s epicyclic frequency. Later, Salo et al. (2004) used numerical simulations for self-gravity wakes in a Keplerian disk to model their effect on the observed ring reflectance and optical depth in
Saturn’s rings. Typical critical wavelengths are on the order of ~ 50 meters in the A ring, and 50-100 meters in the B ring. Self-gravity wakes cannot form in low optical depth regions like the C ring and Cassini Division because the surface mass density, and correspondingly the wavelength scale for gravitational collapse, is less than or equal to the size of the largest individual ring particles.

We will discuss models of the wakes in the next section. Figure 1.12 and Figure 1.13 describe the angles and geometry referred to here. The orientation of the wakes (e.g. the cant of the wakes in the prograde direction of orbital motion) causes wide variations in the optical depth. When $\varphi - \varphi_{\text{wake}} \lesssim 10^\circ$, the view of the wakes is almost end-on so that the optical depth is minimized, while when $\varphi - \varphi_{\text{wake}} \gtrsim 80^\circ$ the view is nearly perpendicular across the wakes and the optical depth is maximized. Figure 1.11 below is taken from J16 and reveals resolved self-gravity wake structures in a UVIS particle-tracking occultation of the rings, an occultation in which the relative velocity of the spacecraft with respect to the orbital velocity of the ring particles is extremely slow (in this case, ~0.81 km/s).
1.5.2.2 Precedent studies

Colwell et al. (2006) performed an analysis using a ray tracing model that implied that self-gravity wakes are shaped similarly to granola bars, as illustrated in Figure 1.12; that is, highly flattened, nearly opaque bars separated by nearly transparent gaps. The model assumes an bi-modal distribution for Saturn’s A ring consisting of wakes with $\tau \approx \infty$ and gaps with $\tau G \approx 0.1$. The observed optical depth in wake-dominant regions is highly dependent on the shape, spacing, and relative optical depth of the wakes. An alternative to the granola bar model for self-gravity wakes is the “pasta” model of Hedman et al. (2007b), who modeled self-gravity wakes as infinitely long, parallel, opaque cylinders with elliptical cross-sectional areas of characteristic width $W$, height $H$, spacing $\lambda$, and alignment $\phi_{\text{wake}}$ measured relative to the radial direction, shown in Figure 1.13 below. The gaps between the wakes are filled with particles and have finite optical depth as in the granola bar model. They compared their results with optical depth profiles...
from Cassini VIMS occultations. They used an observation of Omicron Ceti at extremely low elevation angle ($B = 3.5^\circ$) to determine wake orientation and found that wakes are canted at $\sim 20 - 25^\circ$ from the direction of orbital motion consistent with theoretical expectations for the cant angle of self-gravity wakes in a Keplerian disk. Tiscareno et al. (2010) confirmed that this simple model reproduces optical depth measurements for ring opening angles $|B|$ larger than about $10^\circ$.

Figure 1.12. Colwell et al.’s (2006) granola bar model for self-gravity wakes.

Figure 1.13. Hedman et al.’s (2007b) cylindrical model for self-gravity wakes.

Later, J16 constrained the sizes of particles between self-gravity wakes using Colwell et al.’s (2006) granola bar model by a combined analysis of UVIS and VIMS occultations. They found a significant population of sub-cm sized particles in the outermost portion of the A ring.
and in region B1 in the inner B ring. In the region between the Encke gap and A ring outer edge, they report an increase in the amount of sub-cm particles with increasing ring plane radius. They speculate that satellite perturbations in this region induce more interparticle collisions which subsequently erode and disaggregate the particles.

Although J16 provided a robust survey of self-gravity wake parameters throughout the rings, they did not characterize the parameters in density waves, which are scattered across the rings. Because density waves are active sites for cyclic particle aggregation and disruption, they are ideal regions to test against Esposito et al. (2012)’s predator-prey model for ring dynamics as described in section 1.5.3. With this in mind, we will undertake a robust analysis of self-gravity wake parameters within density wave troughs from occultation statistics in Chapter 5 of this work, determining best-fitting parameters for a granola bar model for self-gravity wakes using statistical moments of Cassini UVIS stellar occultation data.

1.5.3 A predator-prey analog for ring dynamics

The existence of self-gravity wakes (Colwell et al., 2006, 2007; Hedman et al., 2007; Nicholson and Hedman, 2009) implies that the ring particles have a tendency to aggregate into trailing spiral structures. Lewis and Stewart (2005, 2009) conducted simulations of the formation of temporary aggregates followed by disaggregation. When large objects are included, they magnify this effect as the large “seed” particles shed off their smaller constituents. Esposito et al. (2012) proposed a so-called predator-prey model for ring particle dynamics, a simple model of accretion and fragmentation which postulates that satellite perturbations drive the cyclical growth of large aggregates. These gravitationally accelerate the ring particles to higher relative
velocities which result in more collisions and subsequent fragmentation. The model is named for its analogy to classic predator-prey population models of foxes (predator) and hares (prey); the aggregate mass is the prey, which grows without limit in the absence of the predator, the relative velocity dispersion of the ring particles.

Density waves are excellent test-beds for this theory. Density waves excite the ring particles to higher eccentricities which results in streamline crowding (Lewis and Stewart (2000, 2005)), which may be magnified by the increased number density of the particles in the density wave crests. The crowding damps the particles’ relative velocity and the particles reaggregate.

1.5.4. Spokes

In 1980, Voyager 1 observed dark lanes of radial, wedge-shaped markings in Saturn’s B ring which would later become known as “spokes” (B. A. Smith et al., 1982). Voyager’s observations of the rings backlit by the Sun at high-phase angle maximized the effect diffraction by micron-scale particles, revealing vivid spokes. Because these particles are on the same order of size as the wavelength of incident light, they preferentially scatter in the forward direction, causing the spokes to disappear on approach but brighten when viewed looking back toward the sun. Voyager first observed spokes extending across the B ring for thousands of kilometers and growing rapidly with typical lifetimes on the order of hours. In 1982, B. A. Smith et al. found that one spoke had grown from to 6000 km wide in under 5 minutes (Esposito, 2014).

The spokes are thought to result from electromagnetic effects as the high charge-to-mass ratio of micron-scale particles allows them to be heavily influenced by these forces. In accordance with this hypothesis, Cuzzi and Durisen (1990) postulated that spokes would be more
likely to occur at dawn ring ansa, where meteoroid impacts on the rings are most energetic. In
the beginning of the Cassini mission, Sun was at higher elevation. Consequently, it produced a
powerful photo-electron sheath above the rings which prevented the elevation of small grains
above the rings and Cassini did not detect any spokes. In 2008 however, when the Sun’s lower
elevation weakened this sheath, spokes reappeared. This supported Mitchell et al. (2006)’s
assertion that spokes are seasonal features.

1.5.5. Propeller Objects

Propeller-shaped features in Saturn’s rings occur when objects too small to clear a full
gap but large enough to clear small sections in their immediate vicinity produce perturbations in
the nearby ring particle orbits, downstream of the moon in the orbital sense. This produces a
long, narrow region of alternating high and low optical depth on either side of the moon in the
orbital direction resembling a two-bladed propeller. The structures were first described as
“propeller-like” by Spahn and Sremčević (2000). Tiscareno et al. (2008) later described their
appearance as “two gaps, oriented primarily along the orbital direction, symmetric about the
origin.” Cassini found these intermediately sized moonlets to be on the scale of ~100 m–1 km.
The propellers can be thought of as azimuthally incomplete gaps; more massive moons result in
perturbations that can open a gap all the way around the rings, as in the Encke and Keeler gaps in
the outer A ring. These unique objects are primarily detected in three narrow belts of the A ring
which lie between 126750 and 132000 km (Tiscareno et al., 2008), although there is one clear
propeller feature in UVIS occultation data in the B ring flat spot, and small openings in the C
ring plateaus have been postulated to be due to the same propeller phenomenon (Baillé et al.
Notably, the A ring propeller belts are devoid of density wave disturbances. An image of an observed propeller structure is shown in Figure 1.14 below.

Figure 1.14. Cassini ISS image of propellers in Saturn’s rings. The above image (PIA21433) was taken by the Cassini ISS Narrow Angle camera. The top of the image shows the view of the sunlit rings while the bottom shows unilluminated rings with light filtering from the backlit rings (Image credit: NASA/JPL-CalTech/Space Science Institute).

1.6 Organization of this manuscript

This research is motivated by the need for a fuller understanding of unique structures in Saturn’s ever-evolving dynamic ring system. We partition the remaining chapters of this work as follows. In Chapter 2, we review the unique utility of the HSP in addressing questions about the structure of Saturn’s rings, detailing the calibration and geometric viewing parameters of a UVIS stellar occultation. Chapter 3 explores the fundamental physics of the wave nature of light and
diffraction by ring particles with a detailed explanation of implications for the particle size distribution in the rings. Chapter 4 provides an in-depth review of diffraction signatures at ring edges and our original work to modify the model of B16 for application to narrow gaps and ringlets. In Chapter 5, we examine occultation statistics in small-scale density wave structures, surveying the higher order moments in UVIS data and how we can interpret these moments in terms of the ring particle size distribution. In Chapter 6, we discuss the implications of our results and directions for future research.
Table 1.1. Overview of the taxonomy of Saturn’s major rings. Values are taken from Table 3.1 in Esposito (2014) and Table 5.1 in Jerousek (2018).

<table>
<thead>
<tr>
<th>Region</th>
<th>Radial Extent (km from Saturn)</th>
<th>Characteristic Optical depth</th>
<th>Power law index</th>
<th>$a_{\text{min}}$ (cm)</th>
<th>$a_{\text{max}}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D ring</td>
<td>66000—74000</td>
<td>$10^{-3}$</td>
<td></td>
<td>~$10^{-4}$</td>
<td>~$10^{-4}$</td>
</tr>
<tr>
<td>C ring</td>
<td>74490—91983</td>
<td>$0.05 - 0.5$</td>
<td>3.1</td>
<td>0.35—0.5</td>
<td>2—5.5</td>
</tr>
<tr>
<td>B ring</td>
<td>91983—117516</td>
<td>$\leq 2.5$</td>
<td>2.75</td>
<td>0.5—3.0</td>
<td>1.0—7.0</td>
</tr>
<tr>
<td>Cassini Division</td>
<td>117516—122053</td>
<td>$0.05 - 0.15$</td>
<td>~3.0</td>
<td>0.45—0.5</td>
<td>2.0—4.5</td>
</tr>
<tr>
<td>A ring</td>
<td>122053—136774</td>
<td>0.65</td>
<td>2.75—2.9</td>
<td>0.35—4.0</td>
<td>2.0—12.0</td>
</tr>
<tr>
<td>F ring</td>
<td>140200—140250</td>
<td>$0.1 - 0.5$</td>
<td>2.0—3.0</td>
<td>~$10^{-4}$</td>
<td>~$10^{-3}$</td>
</tr>
<tr>
<td>G ring</td>
<td>166000—175000</td>
<td>$10^{-6}$</td>
<td>1.5—3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E ring</td>
<td>180000—1200000</td>
<td>$10^{-5}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER TWO: STELLAR OCCULTATIONS

2.1 Introduction

Over the course of its thirteen years at Saturn, Cassini’s Ultraviolet Imaging Spectrograph (UVIS) instrument observed an impressive 197 total stellar occultations of Saturn’s rings. Analyses of these data have revealed a multitude of unexpected structures in the rings, including viscous overstability oscillations (Thomson et al., 2007), elongated clumps of agglomerated particles called self-gravity wakes (Colwell et al. 2006, 2007; Hedman et al. 2007a) a number of large particle aggregates within the F ring core identified as kittens by Esposito et al. (2012), and propeller moonlets (Tiscareno et al., 2006a). But stellar occultations have been a particularly useful tool in scrutinizing planetary ring structure since well before Cassini, providing a means of extracting information about the sizes of the particles indirectly through the behavior of electromagnetic radiation as it passes through the rings. In fact, the Kuiper Airborne Observatory discovered Uranus’ ring system in 1977 (Elliot et al., 1977), which was later confirmed by Voyager 2, the Hubble Space Telescope (HST), and other Earth-based observations.

During a stellar occultation, a celestial object of interest—in our case, Saturn’s rings—passes in front of a star from the point of view of an observer. As Saturn’s rings occulted stars from the perspective of Cassini, some of the stellar photons incident on the rings interacted with the local particles by either absorption or scattering. Others passed through and could be detected. Scattered light in this study consists primarily of diffracted light. Reflection may be neglected at UVIS’ ultraviolet wavelengths where the ring particles have a very low albedo.
In this work, we present an extensive analysis of sixty-three different stellar occultations by the HSP from a multitude of viewing geometries in an effort to survey the morphology of structures in the rings from different aspects. We employ two distinct methods to characterize the particle size distribution in Saturn’s rings: first, analyzing the shape of diffraction signatures at the edges of rings, ringlets, and gaps; and second, analyzing statistical moments of the data. Taken together, these two methods provide information on the upper and lower bounds of the size distribution. Occultation statistical moments (specifically variance and skewness) are most sensitive to the presence of large particles in the FOV, while diffraction effects are most significant for small particles.

We begin this chapter with a review of the most impactful precedent studies of stellar occultations of Saturn’s rings followed by a review of Cassini’s high-speed photometer instrument and its unique attributes and drawbacks. Next, we will review the data calibration procedure including the removal of background noise as well as the HSP’s lossy data compression algorithm and its impact on the statistical moments of the data. We also review the key parameters of the viewing geometry of stellar occultations and the benefit of using a multitude of viewing configurations to characterize three-dimensional structures in the rings. Finally, we will review our search for statistically significant features in the D ring using the $m$-statistic method (Colwell and Esposito 1990).

2.2 Precedent studies

There is a long history of observations of stellar occultations of Saturn’s rings prior to Cassini. These include the combined ground-based observations of the occultation of the bright
star 28 Sagitarii (28 Sgr) in 1989 (Hubbard et al., 1993; French et al., 1993; Harrington et al.,
1993; French and Nicholson, 2000), observations performed by the HST (Elliot et al., 1993), as
well as the Voyager 2 PPS occultation of δ Scorpii (Showalter and Nicholson, 1990). A detailed
review of Voyager results concerning planetary rings can be found in Cuzzi et al. (1984).

Showalter and Nicholson (1990) were the first to interpret the statistics of a stellar
occultation of Saturn’s rings in terms of the particle size distribution. They calculated the excess
variance in the Voyager 2 PPS stellar occultation of δ Scorpii by comparing the observed
variance with that expected of a Poisson distribution. The finite sizes of the ring particles
produce a correlation in the probability of absorption or transmission of photons by the ring,
increasing the variance above that from a Poisson distribution where events are completely
uncorrelated. They derived an effective particle size from the extra variance in the data, finding
the smallest weighted particle size in the C ring. They determined $R < 2.8$ meters in the C ring
and $R < 4.5$ meters in the Cassini Division, where $R$ is the particle size retrieved from the excess
variance. This size is weighted toward the largest particles in a power-law size distribution. They
found $5.7 \leq R \leq 8.8$ meters in the inner B ring and $R \sim 9 - 12$ meters in the A ring. Without a
model including the effects of self-gravity wakes, however, a clear interpretation of their results
is possible only in regions of low optical depth such as the C ring and Cassini Division.

Elliot et al. (1993) analyzed the extremely slow occultation of Saturn’s rings recorded by
the Hubble Space Telescope of star GSC 6323-01396. This occultation occurred near a stationary
point which minimized the apparent motion of Saturn relative to the star, allowing the detection
of numerous ring features. Along with Hubbard et al. (1993) and French et al. (1993), they also
solved for the geometry of the 28 Sgr occultation using circular edge features in the rings and
determined a radius scale as well as coordinates for Saturn’s pole with respect to the ring-plane.

French and Nicholson (2000) used the Voyager Photopolarimeter Subsystem (PPS) optical depth profile of δ Scorpii to estimate and subtract the directly transmitted signal from 28 Sgr to obtain high signal-to-noise scattered light profiles. Using a one-dimensional scattering model at three different wavelengths (3.9, 2.1, and 0.9 μm), they fit parameters \( q, a_{\text{min}}, \) and \( a_{\text{max}} \) of the power-law size distributions (see \( n da = C a^{-q} da, a_{\text{min}} \leq a \leq a_{\text{max}} \) \( (1.1) \)) for each of the main rings (A, B and C). Their results were largely in agreement with Zebker et al. (1985) and Showalter and Nicholson (1990). They found a flat and narrow size distribution in the inner A ring and B ring with \( q = 2.75, a_{\text{min}} = 30 \text{ cm and } a_{\text{max}} = 20 \text{ m}, \) twice the size indicated by Voyager RSS results. They also reported that the C ring had the highest number of centimeter-sized particles, with \( a_{\text{min}} = 1 \text{ cm and } q = 3.1. \) Additionally, they noted an increase in centimeter-sized particles with increasing radial distance from Saturn across the A ring (French and Nicholson, 2000). Again, however, their model assumed that all ring particles were isolated from each other and did not account for the presence of self-gravity wakes which are ubiquitous in the A and B rings (but absent from the C ring).

Stellar occultation data are well complemented by radio occultation data (Tyler et al., 1983; Marouf et al., 1983, 1986; Zebker et al. 1985), as both can be used to constrain the particle size distribution in the rings (Cuzzi et al., 2000). Companion papers Tyler et al. (1983), Marouf et al. (1983, 1986), and Zebker et al. (1985) each presented analyses of observations of radio signals through the rings from the Voyager 1 radio occultation experiment at dual microwave wavelengths of 3.6 and 13 cm. Due to the phase coherence of radio signals, the forward-scattered
component of the signal can be inverted to obtain measurements of the ring optical depth. The fact that large particles play the dominant role in diffracting light at microwave wavelengths enabled them to place upper bounds on the size distribution. Phase-shifting the radio signal and reconstructing the optical depth profile to correct for diffraction, they resolved numerous fine scale features such as narrow gaps and ringlets. They found $2 < a_{max} < 5$ m in the C ring, $8 < a_{max} < 11$ m in the A ring, and a sharp cutoff in the size distribution at $a \sim 5$ m. Zebker et al. (1985) found similar results when introducing an N-thin-layers model and accounting for multiple scattering. Their results confirmed that the size distribution of Saturn’s main rings was best described by a power-law size distribution with index $q \sim 3$, consistent with expectations based on collisional laboratory experiments and asteroid observations (Zebker et al., 1985).

2.3 Features of the UVIS HSP

Stellar occultation data obtained by Cassini are particularly useful for studying small-scale structures in Saturn’s rings and have allowed an unparalleled view into previously undetected structures such as narrow radial bands of straw-like texture, clumpy self-gravity wakes, and undulating density waves induced by satellite perturbations. The HSP’s ring stellar occultations are ideal for analyzing fine structures in the rings with a 1–2 millisecond integration period corresponding to a radial resolution on the order of just 10 m (Colwell et al., 2007, 2010), enabling the detection of narrow features on this scale. We will exploit the HSP’s high resolution in this dissertation to analyze dynamic structures such as spiral density waves, which have wavelengths ranging from tens of kilometers to just hundreds of meters. The UVIS HSP has a 6 by 6 mrad aperture with an effective acceptance angle of 3.39 milliradians and a bandpass of
110-190 nm, shorter than previous observations by a factor of ~6 (Becker, 2016). An additional benefit of Cassini’s long and fruitful mission is the sheer number of occultations observed over a wide range of viewing geometries, which are all available now for public access through the Planetary Data System (PDS).

2.3.1 Degradation over time

Although the HSP is a uniquely capable instrument for ring observations, it also has two systematic features which must be considered. The first is that the instrument’s sensitivity to incident photons declines over time. This means that as the mission progressed, fewer and fewer photons were detected from the same target star. The impact of this effect is illustrated in Figure 2.1, which shows two occultations of the same star, α Virginis, during Cassini’s 116th revolution and 232nd revolution. Although the ring region (Encke Gap) and star being observed are the same, the later occultation’s count rate is substantially lower than that of the earlier occultation due to the degradation of the sensitivity of the HSP as the instrument accumulated more and more total counts.
Figure 2.1. Comparison of two occultations of the same star in the Encke gap by the HSP at different times. Two observations of the same star with the same integration period, $\alpha$ Virginis 116 Ingress (black) and $\alpha$ Virginis 232 Egress (red), illustrating the impact of the HSP’s decreasing sensitivity over time. The plot shows counts per millisecond averaged over 50 data points (to highlight the magnitude of the offset) for a 50 kilometer segment of the A ring from 133500-133550 km.

2.3.2 Ramp-up effect

The UVIS HSP instrument also exhibited a systematic ramp-up effect in ring gaps (Colwell et al., 2010). The instrument experienced a gradual increase in sensitivity after the position of the target star behind the rings transitioned from dense material to transparent gaps. Immediately after emerging from behind the rings, the measured signal was noticeably below $I_0$ by an amount that varied from occultation to occultation but was frequently $\sim$10% of $I_0$ (Colwell et al., 2010). This was followed by a non-linear increase in $I$ to within a few percent of $I_0$ after which there is another, more gradual increase until the signal eventually asymptotes off at $I_0$. 
This important instrumental effect significantly impacts observations of our area of interest, namely the ring edges.

The time scale for the ramp-up effect varies between occultations. As a result, the diffraction signal may be within the instrumental ramp-up portion of an occultation. This generally occurs when observing the outer edge of a ring (inner edge of a gap) in egress occultations and the inner edge of a ring (outer edge of a gap) in ingress occultations. Because the HSP sensitivity increases with time, in the ingress occultation the signal decreases with increasing ring plane radius while in the egress occultation it increases with increasing ring plane radius. The effect is shown in Figure 2.2, where we overlay an ingress and egress occultation of the same star, revealing clear disparities in the data at both the outer edge of the A ring and the inner edge of the Encke gap.

![Figure 2.2. The HSP ramp-up effect.](image)

The two α Crucis Rev 100 stellar occultations at the outer A ring region, ingress in black and egress in red, overlaid to illustrate the HSP ramp-up effect. During an egress occultation the radial position of the star increases with time, while during an ingress occultation, the radial distance of the star decreases with time. The ramp-up effect decreases the signal at the outer
edges of gaps in ingress occultations and decreases the signal at the inner edges of gaps in egress occultations.

This effect is particularly relevant in our study of diffraction signatures at ring edges. We accounted for the ramp-up response of the HSP in our analysis of diffraction signatures (see Chapter 4) using the method of B16, considering only ingress occultations for the inner edges of gaps (outer edges of rings) and only egress occultations for the outer edges of gaps (inner edges of rings). These signatures tend to be sharp, with a radial width of ~ 5 km. The time in the gap needed for the response to become linear is approximately 20 milliseconds (B16). Our procedure for removing the trend is then simplified from the polynomial trend-removal method implemented by B16, but our results at A ring outer edge/Encke gap edges remain consistent with theirs.

We begin by binning each edge occultation to 0.5 km resolution. We then compute a linear fit to a 15 km region of data in the center of the gap. This accomplishes two goals: (1) allowing sufficient time for us to assume the effect is linear and (2) being far enough from the edge of interest so that the diffraction signal, which may be as wide as 5 km, is not included. To correct the data for the ramp-up effect, we divide the binned data by this linear fit and normalize the result. Figure 2.3 illustrates the ramp-up removal process at the Keeler Gap inner edge for the α Crucis Rev 100 Ingress occultation. We will further discuss the process of ramp-up removal in Chapter 4, where we investigate diffraction signatures at ring edges.
Figure 2.3. Removal of the ramp-up effect at the inner edge of the Keeler gap. The ramp-up removal for the Keeler gap in the α Crucis Rev 100 Ingress occultation. Because the Keeler gap is narrow (~35 km wide), the ramp-up effect is more difficult to remove. To remove the trend, we normalize and divide the original data, shown in dashed black, by a linear fit to the central 15 km of the gap. The result is the trend-removed data, in solid black.

2.4 Data Calibration

2.4.1 UVIS HSP lossy compression algorithm

In order to reduce total data volume, the HSP was equipped with software which implemented the following algorithm to encode data in a 9-bit compression mode if $I > 128$ (e.g., Esposito, 1999):

$$I_{\text{comp}} = \text{FLOOR}\left(\sqrt{2I} + 0.5\right) + 128$$ (2.1)

$$I_{\text{decomp}} = \text{LONG}\left(0.5[I_{\text{comp}} - 128]^2\right)$$ (2.2)

where FLOOR is a function that rounds the data to the next lowest integer value and LONG returns a longword integer value. The data are compressed according to $I_{\text{comp}} =$
FLOOR(√2I + 0.5) + 128 \hspace{1cm} (2.1) \text{ and then decompressed following}\]
\[I_{\text{decomp}} = \text{LONG} \left( 0.5 \left[ I_{\text{comp}} - 128 \right]^2 \right) \hspace{1cm} (2.2). \text{ The compression is lossy because the decompressed data are assigned discrete values and some of the information contained in the original photon count measurement is lost. For example, consider a single integration period that counts 159 photons. After running this through the compression algorithm, the decompressed result is 162. However, suppose the next integration period counts 165 photons—this would also result in a decompressed value of 162. The information that the two original measurements were different is lost. When } I \leq 128, \text{ there is no loss of information in the compression scheme because all the data can be stored in 7 bits. The compression algorithm was designed to have no effect on the measurement and uncertainty in the measurement of the ring transparency. However, it does affect the higher moments of the data, such as the variance, skewness, and kurtosis.}
Figure 2.4. Impact of data compression on occultation statistics. The stacked plots show the statistical moments of the data for hundreds of stellar occultations. In the top plot, the ratio of the variance to the mean is plotted against the mean, while the skewness and kurtosis versus the mean are shown in the middle and bottom plots, respectively. The solid line in the skewness plot shows a linear fit. Note the peak around 128, the cut-off for the compression algorithm. The higher order moments of skewness and kurtosis are affected beyond a count rate of $I \sim 128$ by many factors more than the variance.
2.4.2 Accounting for background signal

The signal recorded during stellar occultations included not just light from the observed star, but also counts from background sources. In the case of the HSP, whose bandpass ranges from 110-190 nm, the two sources of background are reflected sunlight and Lyman-\(\alpha\) emission from interplanetary hydrogen. The source of the background depends upon the observation in question. When Cassini observed Saturn’s rings illuminated by the sun, most of the background was reflected UV sunlight from the rings. When Cassini observed the unlit rings, the background was dominated by Lyman-\(\alpha\) emission (Colwell et al. 2007, 2010). Figure 2.5 shows a particularly opaque region of Saturn’s B ring where the observed signal is essentially pure background.

![Image](image.png)

Figure 2.5. Line-of-sight optical depth profile of a nearly opaque patch of the central B ring. The above profile shows of a 300 km segment of B2. The count rates recorded in dense regions like this are almost purely background signal from Lyman-\(\alpha\) emission or Saturnshine.
2.4.3 Normal optical depth

Observations of stellar occultation events by the HSP can be interpreted as measurements of the transparency, $T$, of the rings. The rings attenuate incident light by a factor of $e^{-\tau}$, where the dimensionless parameter, $\tau$ is the optical depth. The normal optical depth, corrected by a factor for the path length of the line of sight through the rings, can be computed as (e.g. Colwell et al., 2010):

$$\tau_N = \mu \ln \left( \frac{1}{T} \right) = \min \left[ \mu \ln \left( \frac{I_0}{I-b} \right), \tau_{\text{max}} \right],$$

(2.3)

where $I_0$ is the brightness of the observed star, the denominator, $I-b$, is the number of incident photons over the integration period, $I$, minus the background signal, $b$, $\mu = \sin |B|$, and $\tau_{\text{max}}$ is the maximum normal optical depth which can be distinguished from infinity. $B$ is the angle between the ring plane and the line of sight to the star. Systematic uncertainties may occur as a result of interpolating the background and star brightness between points where those quantities can be directly measured from the data.

In addition to systematic uncertainties, there is an inherent statistical uncertainty in our measurements due to the Poissonian nature of stellar occultation data. If our data were Poisson distributed, then the variance would equal the mean so that the standard deviation would be calculated as $\sigma = \sqrt{I}$. Then for a $1\sigma$ confidence interval, the transparency would be limited to a minimum value of $T = \frac{\sqrt{I}}{I_0}$, which yields a maximum value for the optical depth of $\tau_{\text{max}} = \ln \left( \frac{I_0}{\sqrt{I}} \right)$. This value depends on the brightness of the star and the number of data points that are binned together when analyzing the data. Colwell et al. (2010) constrained the optical depth
considering only Poisson error and no systematic uncertainties and found Z-score standard deviation bounds as

\[ \tau_{\pm} = \mu \ln(I_0) - \mu \ln \left\{ I_0 e^{\mu} \mp Z \sqrt{I_0 e^{\mu} + b} \right\} \]  

where \( Z \) is the number of standard deviations for the confidence intervals in optical depth. Binning the data by \( N \) points increases \( I_0 \) (and \( I \)) by a factor of \( N \), leading to an increase in \( \tau_{\text{max}} \) of \( \ln(N) \) and corresponding reductions in the sizes of the upper and lower uncertainty bounds on the optical depth.

### 2.5 Instrument Viewing Geometry

Each of the UVIS HSP ring stellar occultations are defined by a unique set of geometric viewing parameters which include the elevation angle, \( B \), line-of-sight distance, \( D_{\text{LOS}} \) and an azimuthal “clock” angle, \( \phi \). Because Saturn’s rings lie within its equatorial plane, \( B \) is simply measured as the angle from the line of sight vector to the ring plane. It is also sometimes referred to as the ring opening angle for a particular star. The azimuthal angle, \( \phi \), is measured in the ring plane as the counterclockwise angle from the outward radial vector \( \hat{R} \) in the ring plane at the measurement point to the line-of-sight vector to the star projected onto the ring plane. The line-of-sight distance affects the measurement area, \( A \), due to contributions from the scattered signal and diffraction, and different viewing angles provide measurements of the same ring material with different aspects which can be inverted to reveal the distribution of particles and clumps. The geometric parameters of a typical Cassini UVIS stellar occultation are shown in Figure 2.6.
Figure 2.6. Cartoon depicting the geometric parameters of a stellar occultation. Cassini’s line-of-sight is depicted in cyan with the integration area shown as the cyan square where the line-of-sight intersects the ring plane. The elevation angle, $B$, is shown in red and the azimuthal angle, $\varphi$, is shown in green. $\hat{R}$ represents the outward radial reference vector from which $\varphi$ sweeps out an azimuthal swath in the counterclockwise direction.

The geometric parameters for all occultations analyzed in this dissertation (see Table 2.1) are determined by NASA’s Navigation and Ancillary Information Facility (NAIF)’s SPICE toolkit, where data from various solar system missions are stored in kernel files. These kernels include the spacecraft ephemeris data, size and orientation of the target, instrument and spacecraft pointing, as well as algorithms for coordinate matrix transformations. In this work, we use an IDL program titled “Geometer” written by Joshua Colwell (and supplied to the NASA PDS) which outputs these parameters given inputs of the SPICE kernel files, ephemeris time of the observation, target body, and the right ascension, declination and proper motion of the occulted star in the 1991.25 epoch from Hipparcos satellite data.
2.6 D ring features analysis

2.6.1 Introduction

The innermost of Saturn’s rings is the D ring, which lies close to the planet so that the impact of tides and higher order gravity moments is stronger here than anywhere else in the rings (Hedman et al., 2007a). The extreme nature of the D ring environment is further amplified by its proximity to a local radiation belt (Krimigis et al., 2005). These combined conditions make the D ring a unique test-bed for dynamical models.

The D ring is so tenuous that it is nearly transparent, and occultations through this region reveal very few resolvable features. During our analysis of UVIS stellar occultations of the D ring, we corresponded with Matthew Hedman (personal communication, 2017), who had spotted evidence of an extremely narrow, eccentric feature with a precession rate of about 27° per day at approximately 74430 km in data from approximately thirty VIMS and three UVIS occultations. This discussion prompted us to perform a systematic statistical search of UVIS’ inventory of stellar occultations for similar features within the D ring. Figure 2.7 below showcases an example of one prominent potential feature which we observed in five occultations between 74260 and 74265 km.

2.6.2 Theory

We followed the $m$ statistic method of Colwell and Esposito (1990) who analyzed Voyager occultations of the Uranian ring system. Given that a stellar occultation of an area of
free space (e.g., no rings) is a discrete data set guided by Poisson counting statistics, then the probability of detecting a photon count at or below a value $C$ is given by a sum over the Poisson distribution function

$$P(C, \mu) = \sum_{x=0}^{C} \frac{e^{-\lambda} \lambda^x}{x!}$$

(2.5)

where $\lambda$ is the distribution mean and $\mu$ is the sample mean. Colwell and Esposito (1990) defined a parameter $m$ to be a number representing the probability that we would expect to observe an event at or below count $C$ by pure chance throughout a given data set as

$$m(C, \mu) = nP(C, \mu),$$

(2.6)

where $C < \lambda$ and $n$ is the total number of opportunities for the event to occur in the data set. Therefore, the number of opportunities for an unbinned data set is the total number of data points while for binned data it is the total number of bins. In this way, an $m(C, \lambda) = 10$ would indicate that we would expect about 10 measurements with a count rate less than or equal to $C$ within the total number of events in the data set while an $m$ value of 1 would indicate that we would expect just one such event to occur by chance. We focused on events with $m$ values at or below 0.01, or events that would only occur by chance approximately 1% of the time.
2.6.3 Methodology

Figure 2.7 Feature with low $m$ value in the D ring.
The above set of plots show what appears to be a disturbance in the $\beta$ Crucis Rev 98 Ingress occultation in Saturn’s D ring at approximately 74256km. The leftmost plot shows the calculated $m$ value, marked by a red star, which we found to be more than five standard deviations from a Poisson distribution. The top right plot shows the degree of separation of this low $m$ value from all of the other $m$ values in the data set. The bottom right plot shows that the point lies outside of $5\sigma$ standard deviations from the mean of the data set.

We binned data from occultations with bright stars and measurements within the D ring region at six different bin sizes: 5, 10, 20, 50, 100, and 500 points per bin. We conducted our analysis over the entire D ring with a radial extent of 73000 – 74480 kilometers. Because Poisson statistics are determined by the sample mean $\lambda$, we needed to remove the ramp-up effect in the HSP data before we compute the mean to ensure an adequate comparison. To accomplish this, we cut the data in the D ring into separate regions 100 km in length. We then removed the
ramp-up effect for each cut in each occultation at each bin size, took the mean, and calculated the probability and $m$ values for each binned point using that mean. For data segments of this length, the ramp up behavior was linear. We then ran these data cuts through a code called “mstat” which I wrote in IDL that performs the probability and $m$ statistic calculations presented in

$$P(C, \mu) = \sum_{x=0}^{C} \frac{e^{-\lambda} \lambda^x}{x!} \quad (2.5) \quad \text{and} \quad m(C, \mu) = nP(C, \mu),$$

(2.6) for each bin. In total, we found one hundred values of $m \leq 0.01$, indicating 100 detections of an event that we would only expect to occur by chance in $\approx 1\%$ of the time. We then arranged our plots by longitude in an attempt to track the progression of each identified feature over time. We used the longitude value at the radial location of the feature. We then calculated the pattern speed, $\omega$, using the $J_2$ harmonic for Saturn to calculate the longitudinal progression of that feature over time. Finally, we arranged the plots in chronological order to examine the apparent progression of the narrow features over time. See Figure 2.8 for an example of one of these maps.
Figure 2.8. Low $m$ value detections of the same apparent feature in five occultations. The occultations are stacked in chronological order according to longitude. The lowest $m$ value is marked by a red circle at the peak optical depth. There appear to be minor fluctuations in the radial position of the feature over time.

To test our methodology, we created a sample of random data sets using a random generator set to a Poisson distribution in IDL. We then subjected these random data sets to the same analysis procedure with the additional application of a compression/decompression algorithm to mimic the compression/decompression in Cassini UVIS data. We found that the calculated $m$ values for the random data were in line with what we would expect for a Poisson distribution, suggesting that the $m$ statistic test is indeed an accurate method of determining
statistical significance of data following a Poisson distribution. Our results are shown in Figure 2.9.

![Figure 2.9. Comparison of m values in D ring data versus randomly generated test data.](image)

The leftmost plot above shows the minimum \( m \) values calculated for our randomly generated Poisson data set. These results are consistent with expected results for Poisson distributed data, with the lowest \( m \) values of all of the trials clustering around one. \( m = 1 \) here indicates that one such event is expected to occur by chance. On the right, \( m \) values from D ring occultation data by contrast appear to cluster around 0.0001, with a range of \(~0.01-10^{11}\)\). For comparison, each data point represents the lowest \( m \) value in a separate occultation for 100 data sets.

### 2.6.4 Comparison with data outside the F ring

As a final check, we tested our significant detections against stellar occultation data from exterior to the F ring, where we would similarly expect negligible particle interference. Given the absence of ring material in this region, this data can be assumed to follow Poisson counting statistics. We performed the same procedure on this data as on our D ring data and randomly generated data, cutting the occultations into 100 km segments and binning them at the same variety of bin sizes. Surprisingly, our results were similar to our \( m \) value calculations in the D ring, indicating that we cannot accept the statistical significance that our \( m \) values appear to indicate at face value.
2.6.5 Results

We performed a statistical analysis of a hundred apparent features to detect significant deviations from expected values drawn from a Poisson distribution. Although many of the features that we identified appear deceptively sharp, we observed similar detections during a test of occultation data in the Roche Division which is free of ring particles, as shown in Figure 2.10.
Overall, we identified 21 disturbances in the D ring that are significant to greater than 4 standard deviations from a Poisson distribution. Table 2.2 provides an overview of our results. Despite these seemingly significant initial results with low $m$ and high $Z$ values, we found similar detections in our analysis of stellar occultations in the Roche Division, where there are no rings. The failure of this second test suggests that our $m$ statistic calculations are muddled by unmodeled instrumental effects. We speculate that this may be due to the degradation in instrument performance over time. Figure 2.11 below, which shows that data from exterior to the F ring in the earliest Cassini stellar occultations has fewer data points with low $m$ values compared to later occultations, supports this conclusion.

Figure 2.11. Evidence for a systematic decrease in lowest $m$ with increased instrument exposure. The two plots at the top of the figure are from early in Cassini UVIS observations. The $\alpha$ Virginis occultations shown are only from Cassini’s 8th revolution around Saturn, when the
instrument had accumulated fewer total counts. The bottom plot shows a similar number of data points for the β Centauri Rev 85 occultation much later in Cassini’s history. Perhaps the high number of low m values that we calculated are a result of instrument degradation.
Table 2.1 Inventory of all stellar occultations analyzed in this dissertation.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Radial extent (km)</th>
<th>$I_0$ (counts/ms)</th>
<th>$b$ (counts/ms)</th>
<th>$D_{LOS}$ ($R_S$)</th>
<th>$\varphi$ span (°)</th>
<th>$A$ span ($m^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>126Tau008E</td>
<td>70385—141382</td>
<td>7.9</td>
<td>3.12</td>
<td>20.6</td>
<td>-21.1</td>
<td>88.8—130.1</td>
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<td>AlpAra032I</td>
<td>61336—139777</td>
<td>77.8</td>
<td>0.13</td>
<td>11.2</td>
<td>54.4</td>
<td>276.5—280.9</td>
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<tr>
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<td>0.15</td>
<td>11.1</td>
<td>54.4</td>
<td>276.6—280.8</td>
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<td>71.1</td>
<td>0.14</td>
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<td>54.4</td>
<td>221.2—252.0</td>
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<tr>
<td>AlpAra063E</td>
<td>73265—141558</td>
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<td>0.05</td>
<td>7</td>
<td>54.4</td>
<td>95.8—112.3</td>
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<td>11.9</td>
<td>68.2</td>
<td>125.0—181.6</td>
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<td>13.5</td>
<td>68.2</td>
<td>83.5—123.7</td>
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<td>13.9</td>
<td>68.2</td>
<td>124.2—164.5</td>
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<td>17.3</td>
<td>82.2—115.6</td>
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<td>17.3</td>
<td>116.1—149.8</td>
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<td>219.8—266.3</td>
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<td>5.9</td>
<td>17.3</td>
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<td>$\varphi$ span (°)</td>
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<td>$D_{LOS} (R_S)$</td>
<td>$B$ (°)</td>
<td>$\varphi$ span (°)</td>
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<td>38.6</td>
<td>219.0—232.1</td>
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1 $R_S = 60330$ km.

$I_0$ – Unocculted stellar signal (counts/ms)

$B$ – Ring elevation angle (°)

$\varphi$ – Clock angle (°)

$D_{LOS}$ – Line-of-sight distance of Cassini ($R_S$)
Table 2.2. Statistical significance of potential narrow features in Saturn’s D ring.

<table>
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<th>Approximate feature location (km)</th>
<th>Number of detections with ( m \leq 0.01 )</th>
<th>Number of occultations observed in</th>
<th>Lowest ( m ) value</th>
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CHAPTER THREE: DIFFRACTION

3.1 Introduction

In 1905, Albert Einstein first explained the photoelectric effect, discovering that electromagnetic radiation simultaneously behaves as both a particle and a wave. The possession of both wave- and particle-like qualities by a physical entity (in this case, a photon) is commonly referred to as wave-particle duality. In this work, we will exploit both the wave- and particle-like nature of light to tease out information about the particle size distribution in Saturn’s rings. The HSP detector was able to count photons because of their discrete, particulate nature—hence why the data are always integer-valued. At the same time, the incident light beam also behaves like a wave, exhibiting wavelike phenomena such as diffraction. Because the angular size of a given diffraction pattern is determined by the wavelength of incident light and the size of the obstructing particle, the diffracted signal can then be used to constrain and characterize the particle size distribution. In this chapter, we will review the physical process of diffraction. We will begin by reviewing the necessary theoretical background on diffraction theory in section 3.1. In section 3.2, we will discuss the physical mechanisms which contribute to diffraction in Saturn’s rings, describing single and multiple diffraction and how each occurs. In section 3.3, we will review previous studies of the particle size distribution from stellar occultations and explore the relationship between the angular distribution of the scattered signal and particle size. In section 3.4, we will discuss the application of diffraction theory in our models.
3.1.1 Historical Background

The nature of light was a fundamental area of inquiry for seventeenth century physics. Snell’s Law defined the mathematical relationship between the incidence angle of a light beam and its angle of refraction when crossing the boundary of two different media (i.e., air, water, glass, etc.). Fermat’s principle described the behavior of light rays, stating that a ray traveling between two points will always follow the shortest path (e.g., the path of least time). In 1678, Huygens’ principle (also referred to as the Huygens—Fresnel principle) stated that each point along a given wave front could be considered the source of a secondary wavelet that expands in every direction. The surface tangent to that series of secondary wavelets forms a new wave front called an envelope. Much later, Young would show that the pattern of maxima and minima in the shadow of a strand of hair was created by interference between waves encountering both sides of the strand, although he was unable to discern the waves’ point of origin. Fresnel ultimately solved this puzzle using Huygens’ principle coupled with Young’s principle of interference, finding that when the wave front encounters an obstacle such as a hair, the original wave front is partially blocked and does not go on to form the envelope of the new secondary wave front.

Bessel functions also came to be a useful mechanism for describing wave-like physical phenomena at this time, and will be used in this work to model the single-scattered near-forward signal intensity relative to the free-space incident power per unit area. In particular, spherical Bessel functions can be used when integrating over a ring composed of spherical particles with circular symmetry. These functions are solutions to the second order differential equation given by

\[ x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0, \]  \hspace{1cm} (3.1)
where $n$ is the order of the Bessel function, which appears commonly in physical problems. In 1732, Bernoulli used the zeroth order Bessel function as a solution to the problem of an oscillating chain which considers a narrow chain suspended from a fixed point where the origin is the unattached end of the chain in equilibrium position. Bessel himself found astronomical application for the function in 1824, using the functions as coefficients in a series expansion to describe planetary perturbations on the Sun (Idris et al., 2016).

### 3.1.2 Diffraction by Saturn’s rings

We can learn a great deal about the particle size distribution of Saturn’s rings by examining the behavior of light scattered by the rings’ constituent particles. The forward scattering geometry of both radio and stellar ring occultations enables unique information about the particle size distribution to be extracted from these observations. Diffraction occurs when electromagnetic radiation (e.g., UV light) encounters a ring particle and the wave is bent and scattered around it. The extinction and scattering of incident light is dictated by the size, shape, and spatial distribution of ring particles and clumps. We follow van de Hulst (1957) and B16 in assuming only Fraunhofer diffraction, which we will justify in Section 3.2.2.

### 3.2 Theory

#### 3.2.1 Classical Ring Model

The so-called classical model of the interaction of electromagnetic radiation with planetary rings imagines the rings as a classic radiative-transfer problem. That is, consider the rings to be made up of a uniform distribution of loosely packed particles in one extended layer many
particles thick with no clustering. This model assumes that the particles are well separated spatially from one another and large relative to the incident wavelength. For this classical model (e.g., Cuzzi et al., 2009):

$$\tau_n(\lambda) = \int_{a_{\text{min}}}^{a_{\text{max}}} \pi a^2 Q_{\text{occ}}(a, \lambda, f) \, n(a) \, da,$$

where $Q_{\text{occ}}(a, \lambda, f)$ is the effective extinction efficiency of a spherical particle with radius $a$ at radiation wavelength $\lambda$, and $f$ is the fractional area of the diffraction cone containing ring material. We will discuss the parameter $Q_{\text{occ}}(a, \lambda, f)$ in more detail in the section 3.2.3 when we introduce Babinet’s principle. Eq. 3.2 is useful for determining particle size because combined observations at multiple wavelengths can be inverted to obtain $n(a) \, da$ (particles per unit area per size increment). This technique requires measurements over a broad range of wavelengths that are sensitive to the full range of sizes in the particle size distribution. In this dissertation, we exploit the high spatial resolution and short wavelength of observation of the UVIS HSP to analyze diffraction signatures from small particles. Very small particles, invisible to the shortest RSS wavelengths, diffract starlight into the gap adjacent to a ring where the HSP can observe it. This provides a tool to constrain the population of the smallest particles near ring edges where collision velocities may be high due to satellite perturbations.

### 3.2.2 Fraunhofer Diffraction

The incident light beam observed during a stellar occultation can be considered a plane wave front. Each point along this wave front can be modeled as the origin of a new radiating wave (van de Hulst, 1957). When the wave front encounters a particle, a new wave front emerges with a missing section proportional to the geometric shadow size of the particle. By Huygens’ principle, this gives rise to a phase function (e.g., an angular intensity distribution)
that can be modeled with the Fraunhofer diffraction approximation when the observer is far enough away that the diffracted wave is also planar (van de Hulst, 1957). The Fraunhofer diffraction pattern depends only on the size and shape of the obstructing particle and is not affected by the particle composition or surface roughness (van de Hulst, 1957).

We begin by verifying that the minimum line-of-sight distance ($D_{\text{LOS}}$) from the spacecraft to the ring plane is sufficiently large that diffracted light follows a Fraunhofer diffraction pattern. First, we calculate the dimensionless Fresnel number for UVIS HSP stellar occultations, a criterion used to define near and far-field approximations, as

$$F = \frac{a^2}{\lambda D_{\text{LOS}}}, \quad (3.3)$$

where $a$ is the particle radius, $\lambda$ is the wavelength of the incoming light, and $D_{\text{LOS}}$ is the distance of Cassini from the ring plane along the line of sight to the star. For our observations, the smallest $D_{\text{LOS}} \approx 1.5 \times 10^5$ km, and the central bandpass of the UVIS HSP is $\lambda = 0.15 \, \mu$m. We are in the far field regime as long as $F < 1$, which is true for a particle of radius $a < 4.5$ m. Therefore, Fraunhofer diffraction is applicable for particles smaller than 4.5 m. Because the radial scale of a diffraction signature is inversely proportional to the size of the diffracting particle, we can safely ignore cases of $a > 4.5$ m. An illustration of the Fraunhofer diffraction pattern is given in Figure 3.2 below.

3.2.3 Babinet’s principle and the extinction paradox

Babinet’s Principle states that the diffraction pattern caused by light bending around a spherical obstacle is the diffraction pattern for a gap of equal size (Cuzzi et al., 2009), which resembles an Airy disk like the one shown in Figure 3.1. The concept is perhaps best described as a thought experiment, or rather, a set of two thought experiments. Suppose for
the first experiment that a beam of light is incident upon an opaque plane with a transparent hole in the center; we will refer to this beam as wave front A. In the second experiment, a second beam is incident upon one particle of the exact same size and shape as the hole in the previous experiment; we will refer to this beam as wave front B. During the crossing of the obstacle, the sum of the two wave fronts is equal to the total intensity of the original wave front because the disturbance in both experiments is equal in magnitude but opposite in sign. As a result, the amplitude of the diffracted light (which is determined by squaring the intensity) is the same for both cases (van de Hulst, 1957). Babinet’s Principle naturally gives rise to a seemingly counterintuitive paradox which states that the amount of light removed from an incident light beam by the sum of absorption and scattering by a ring particle is equal to twice that particle’s cross-sectional area times the flux.

In the case of an observation of a light beam incident on an isolated ring particle from a very large distance, all energy incident on the particle is either scattered or absorbed with efficiencies $Q_{\text{sca}}$ and $Q_{\text{abs}}$, respectively. Diffraction is a type of scattered light and is thus scattered with efficiency $Q_{\text{sca}}$. The total extinction, $Q_{\text{ext}}$, is equal to the sum of the scattered and absorbed components (e.g., $Q_{\text{ext}} = Q_{\text{sca}} + Q_{\text{abs}}$). The amount of light absorbed depends on the complex index of refraction, which strongly decreases beyond 150 nm wavelengths as water ice becomes less absorptive (Bradley et al., 2010). The diffraction lobes of large particles are much smaller than those of small particles and as a result they are fully captured by the detector. At the midpoint of the UVIS HSP bandpass, the geometric optics regime is justified (e.g., $x = \frac{2\pi a}{\lambda} \gg 1$) for particles larger than about a micron, which is true in the main rings—therefore, $Q_{\text{ext}} = 2$. The physical interpretation of this result is that a large particle
removes an amount of light from the incident beam equal to twice its cross-sectional area (van de Hulst, 1957), also known as the Extinction Paradox.

A stellar occultation of Saturn’s rings collects not only the light diffracted by particles within the FOV but also additional light diffracted into the FOV by other nearby particles. The degree to which this occurs is characterized by parameter $Q_{occ}$, the effective extinction efficiency. When all of the outgoing diffracted light is replaced by incoming diffracted light by other particles in the field-of-view, then $Q_{occ} = 1$. If none of the outgoing diffracted signal is replaced, then $Q_{occ} = 2$. If only a fraction of the diffracted light is replaced by diffraction from neighboring particles, then $1 < Q_{occ} < 2$. Because the rings are highly absorbing at UV wavelengths, reflection and refraction can be ignored.

![Airy Disk](Image source: Introduction to Optics. 3rd ed., Pedrotti et al., Cambridge University Press, 2017.)

Figure 3.1. Airy disk diffraction pattern. Illustration of an airy disk diffraction pattern due to the diffraction of a point source of light. (Image source: Introduction to Optics. 3rd ed., Pedrotti et al., Cambridge University Press, 2017.)
3.3 What contributes to diffraction in the rings?

3.3.1 Single diffraction

The classical ring model described by

\[ P(\theta, a) = \left( \frac{2J_1(k a \sin \theta)}{k a \sin \theta} \right)^2 \]

(3.5) below considers only single particle scattering of the incident photons. Generally, single-particle scattering dominates when \( \tau \lesssim 1 \).

In this work, we follow B16 in considering only single-scattering. For single scattering, the ratio of the intensity of forward-scattered light, \( I_1(\theta, \lambda) \), to the free-space incident power per unit area, \( I_i \), is given by Cuzzi et al. (2009) as

\[
\frac{I_1(\theta, \lambda)}{I_i} = \frac{2}{4\pi} \int_{a_{\min}}^{a_{\max}} a^2 P(\theta, a) n(a) da
\]

where

\[ P(\theta, a) = \left( \frac{2J_1(k a \sin \theta)}{k a \sin \theta} \right)^2 \]

(3.5) is the phase function for Fraunhofer diffraction, \( k = \frac{2\pi}{\lambda} \) is the plane wave number, \( J_1 \) is the Bessel function of the first kind and order 1, \( \theta \) is the scattering angle, and \( \mu = \sin |B| \), where \( B \) is the ring opening angle (e.g., the elevation angle from the line-of-sight to the ring plane), and the function is normalized such that (e.g., French and Nicholson, 2000):

\[
\int P(\theta) d\Omega = 2\pi \int_0^{\pi} P(\theta) \sin \theta d\theta = 4\pi.
\]

An illustration of the phase function is given in Figure 3.2 below.

---

1 We note that an incorrect version of Equation (3.5) also appears as Equation (2) in Eckert et al. (2021). Equation (3.5) in this work is the correct version.
Figure 3.2. Fraunhofer diffraction pattern.
A sample angular intensity function following a Fraunhofer diffraction pattern. The amplitude of
the intensity is maximum at $\theta = 0$ and minimized at the zero-crossings which are on regular
intervals of width equal to the ratio of the particle diameter to the wavelength. The phase
function damps with increasing distance from it central $\theta = 0$.

3.3.2 Multiple Diffraction

In reality the rings are not a monolayer. Particles are distributed vertically according to
the size, as smaller particles attain higher velocities and thus a wider distribution while lager
particles remain on trajectories more closely confined to the ring plane. Evidence of this radial
arrangement is observed in brightness variations seen during Saturn’s opposition, when the Sun
is directly behind the observer, called the opposition effect (Esposito, 2014).

When $\tau > 1$ the single-scattering approximation is insufficient. As the optical depth
increases, incident photons are more likely to be scattered off multiple particles. Zebker et al.
(1985) proposed an $N$-layers model, also called the thin layers model. In this model, $N$ represents
the number of thin particle monolayers. Light may be scattered multiple times by particles in
successive layers. The impact of multiple diffraction on stellar occultations of the rings is a
broadening of the angular intensity distribution of the diffracted light (Zebker et al., 1985). This
widening of the diffraction signal might cause a particle to appear smaller than it would if only single diffraction were considered. In this work we account for single diffraction only, therefore our results for \( d_{\text{min}} \) would tend toward underestimation and should be considered a lower limit.

Consider an \( n^{\text{th}} \) order scattering event that follows a Poisson distribution with rate parameter \( \tau_q \). According to Cuzzi et al. (2009), the observed near-forward scattered signal is

\[
\frac{I_s(\theta, \lambda)}{I_i} = \sum_{n=0}^{\infty} \frac{I_n(\theta, \lambda)}{I_i} = \sum_{n=0}^{\infty} \left[ \frac{1}{n!} \tau_q^n e^{-\tau_q} \right] \left[ \frac{1}{4\pi} \sigma_0 \Phi(\theta) \right]^n,
\]

(3.7)

where

\[
\sigma_0 \Phi(\theta) = \int_{a_c}^{a_e} \frac{\pi a^2 [2 J_1(k a \sin \theta)] n(a) da}{\int_{a_c}^{a_e} \pi a^2 \Omega_{\text{occ}}(a, \lambda, f) n(a) da},
\]

(3.9)

\( \sigma_0 \) is the single scattering albedo (also the probability that light is scattered after each interaction with a ring particle), \( \Phi(\theta) \) is the particle phase function, and \([.]^n\) denotes convolution of the term in brackets with itself \( n \) times. Following Zebker et al. (1985) in substituting the binomial distribution in place of the Poisson distribution, \( \frac{I_s(\theta, \lambda)}{I_i} = \sum_{n=0}^{\infty} \frac{I_n(\theta, \lambda)}{I_i} \)

(3.7) becomes

\[
\frac{I_s(\theta, \lambda)}{I_i} = \sum_{n=1}^{N} \binom{N}{n} p^n (1 - p)^{N-n} \left[ \frac{1}{4\pi} \sigma_0 \Phi(\theta) \right]^n,
\]

(3.10)

where \( p \) is the probability of a single scattering event and \( N \) is the upper limit on the number of scattering events as incident light crosses the ring plane. In this scenario, a photon emerges without hitting any ring particles with probability \( (1 - p)^N = e^{-\tau_q} \).
3.4 Deriving particle size distribution from diffraction in the rings

3.4.1 Predecessor results

In this dissertation we build upon many previous studies that constrained the particle size distribution in Saturn’s rings. Zebker et al. (1985) inverted the near-forward scattered signal from the Voyager Radio Science Subsystem (RSS) in the X- and S-bands (3.6 cm and 13 cm respectively) in the A ring, finding \( q = 3.03 \) in the outer A ring and \( q = 2.93 \) in the inner A ring. French and Nicholson (2000) analyzed the stellar occultation of 28 Sagittarius by the rings using both Voyager photopolarimeter (PPS) data and ground-based observations, enabling them to separate the diffraction signal from the direct signal. They combined data at wavelengths of 0.9, 2.1, and 3.9 \( \mu m \) to constrain the power-law size distribution with \( q = 2.75 \) for particles ranging in size from 30 cm to 20 m in the inner A ring and \( q = 2.9 \) for particles ranging in size from 1 cm to 20 m in the outer A ring. Marouf et al. (1986) applied a similar technique as Zebker et al. (1985) in the X-, S-, and Ka- (0.94 cm) bands to the Cassini RSS ring occultations and found both a sharp cut-off in the outer A ring region of \( a_{\text{min}} \sim 5 \text{ mm} \) for \( q = 3.2 \) and a relative absence of particles smaller than 50 cm or \( q < 2.8 \) in the inner A ring region. Harbison et al. (2013) analyzed solar occultation data from the Cassini Visual and Infrared Mapping Spectrometer (VIMS) instrument and identified a population of sub-millimeter particles interior to the Encke gap, finding that for \( a_{\text{min}} < 0.3 \text{ mm} \) for \( q = 2.75 \) and \( a_{\text{min}} = 0.56^{+0.35}_{-0.16} \text{ mm} \) for \( q = 2.9 \). Finally, B16 implemented a forward-modeling technique of diffraction signatures at ring edges to determine mean values of \( q = 2.9 \) and \( a_{\text{min}} = 15 \text{ mm} \), \( q = 3.1 \) and \( a_{\text{min}} = 9.3 \text{ mm} \), and \( q = 3.2 \) and \( a_{\text{min}} = 4.4 \text{ mm} \) at the inner edge of the Encke gap, outer edge of the Encke gap, and outer edge of the A ring, respectively. We use a modified version
of the model developed by B16 in our work on modeling diffraction signatures at edges, which we will discuss in detail in the next chapter (Chapter 4). Their results were consistent with J16’s finding a significant fraction of sub-cm particles only in the outer A ring and B1 regions and an increasing abundance of small particles across the Trans-Encke region with decreasing distance from the A ring outer edge. Later, Jerousek et al. (2020) implemented the thin layers model of Zebker et al (1985) to determine particle size distribution parameters throughout in the C ring and Cassini Division. In the background C ring, they found $a_{\text{min}} \sim 4.1 \text{ mm}$, $a_{\max} \sim 10-15 \text{ m}$, and $q \sim 3.16$, while in the plateaus they found $a_{\text{min}} \sim 6 \text{ mm}$, $a_{\max} \sim 5-6 \text{ m}$, and $q \sim 3.05$; additionally, they found $q \sim 3.0$ in the Cassini Division background and ramp while $q \sim 2.9$ in the Cassini Division’s Triple Band feature. Table 3.1 provides a comprehensive summary of previous results for particle size distribution constraints for each ring region.

3.4.2 Relationship between particle size and radial extent of scattered signal

According to B16, at the HSP observation central wavelength of 150 nm, we can expect particles to augment the incident light by up to 6%. The diffracted light is scattered into a cone of angular width equal to the ratio of the wavelength of incident light ($\lambda$) to the particle diameter. As the particle size increases, the width of the cone decreases and the light is scattered over a smaller area. For particles much larger than the wavelength, the cone is so narrow that it is no longer discernable from the incident light beam (Esposito, 2014), in which case, $Q_{\text{occ}} = 1$. If a particle is very large, then by definition its diffraction lobe is very narrow. Diffracted light from large particles is confined to very small scattering angles—however, the shape of the pattern changes on very fine scales. This is illustrated in Figure 3.3, which shows the phase function curves for different particle sizes.
Figure 3.3. $P(\theta)$ versus scattering angle for a series of particle sizes. $J_1$ represents the Bessel Function of the first kind and order 1. The particle sizes $a_{\text{min}}$ are equal to the lower bounds on the minimum particle sizes contributing diffraction to each of the five FOVs used in this study and the upper bound we place on $a_{\text{min}}$ of 500 mm.

3.5 Application to our models

As a result of the incoherent nature of the signal observed by the HSP, diffraction signatures can only be identified in gaps because when the star is behind the rings, the small diffraction signature cannot be distinguished from the direct signal which fluctuates with ring optical depth. For this reason, ring edges are unique locations at which diffracted light can be directly observed despite the incoherent nature of the light detected by the HSP. B16 observed an abrupt increase in the signal that exceeded the light from the star itself in several HSP occultations at the outer edge of the A ring and at the Encke gap edges. Small particles within the FOV diffracted extra light into the instrument in addition to the direct stellar signal, resulting in an increase in the measured signal above the stellar baseline signal. B16 created a forward-model to characterize observed diffraction signatures from ring particles near edges based on the sizes of the diffraction lobes created by particles of different sizes.
In this dissertation we expand on B16’s work, presenting an updated and expanded analysis of HSP occultation measurements at ring edges. We define edge occultations as times at which the HSP FOV (6 by 6 square milliradians) simultaneously encompassed both an edge (of a gap, ring, or ringlet) and the occulted star in the gap adjacent to that edge. The shape and intensity of diffraction signatures depend on the size and abundance of the smallest particles in the rings (Van de Hulst, 1957). In this study, we aim to constrain the sizes of the smallest particles in the instrument FOV within a multitude of gaps, sampling 14 different ring edges across different distinct ring regions for diffraction signatures: the outer edge of the B and A rings, the inner and outer edges of the Encke and Keeler gaps, the inner and outer edges of several narrow ringlets including the Huygens ringlet, Strange ringlet, Herschel ringlet, Laplace ringlet, and ringlet in the Cassini Division, as well as the Titan and Maxwell ringlets in the C ring. Figure 3.4 shows an example of two diffraction signatures at the outer A ring and inner Keeler gap edges.

Figure 3.4. Diffraction signatures at A ring (outer) and Keeler gap (inner) edges. Spikes at ring edges that rise above the signal due to the star alone (Io) can only be caused by diffraction of extra light into the detector by small ring particles, which diffract light over a wider angle and greater spatial extent than large ring particles.
At the UVIS central bandpass of 150 nm, the Fraunhofer phase function drops to zero at very small scattering angles for the cm-scale and larger particles in Saturn’s rings. The model therefore requires high angular resolution at these narrow angles to capture the full detail of the diffraction pattern for larger particle size values. We accomplish this using B16’s method of breaking up the HSP field of view into multiple grids of virtual pixels. It should be noted that all mentions of ‘pixels’ from this point onward are exclusively virtual pixels in our simulated model FOVs. We create a series of five gridded FOVs centered on the target star. The smallest FOV is densely populated by virtual pixels while the largest FOV is populated by less densely spaced pixels to optimize the computational time while accurately capturing the variations in signal due to the Bessel function signature of the diffracted light. The smallest grid has the highest resolution and is used to assess the signal from the largest ring particles, since the extent of the scattering angle is small. The largest virtual FOV grid has the lowest spatial resolution but is broad enough to capture the diffracted signal from the smallest ring particles.

Figure 3.5. Schematic of FOV discretization based on particle size. The above cartoon illustrates the five different FOV sizes used to capture the scattering cone for each particle size regime. The smallest particles scatter light over the largest area (purple circle)
while larger particles subtend more narrow diffraction cones (i.e., the purple circle for \( a_{\text{min}} < 1 \text{mm} \) versus the red circle for \( 50 \text{ mm} < a_{\text{min}} < 500 \text{ mm} \)).

The discretized FOV grids were made for the following particle size ranges: \( a_{\text{min}} \leq 1 \text{ mm}, 1 \text{ mm} \leq a_{\text{min}} \leq 5 \text{ mm}, 5 \text{ mm} \leq a_{\text{min}} \leq 10 \text{ mm}, 10 \text{ mm} \leq a_{\text{min}} \leq 50 \text{ mm}, \) and \( 50 \text{ mm} \leq a_{\text{min}} \leq 500 \text{ mm}. \) B16 chose these size ranges based on the angular extent of the Bessel function, \( J_1, \) as a function of the particle size, \( a. \) We calculate the intensity of the forward-scattered diffracted light by summing the intensity over each pixel within each FOV as the angular separation of the star from the ring edge varies according to

\[
P(\theta, a) = \left( \frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right)^2
\]

(3.5) Then we use these calculations to generate a synthetic diffraction signature at 0.5 km intervals in distance from the edge up to 10 km from the edge. Beyond 10 km, the signal is only calculated at 10 km intervals because the diffracted signal is relatively small. We will discuss the model in detail in the next chapter (Chapter 4)
Table 3.1. Summary of studies of the particle size distribution in various ring regions

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CHAPTER FOUR: MODELING DIFFRACTION SIGNATURES AT
RING EDGES

4.1 Observations of diffraction at ring edges

4.1.1 Diffraction Signatures

The first mechanism by which we characterize the particle size distribution in Saturn’s rings in this dissertation is modeling diffraction signatures we observe at many ring edges. Because data obtained by the UVIS HSP are incoherent, we cannot separate the diffracted component of the signal except in the special circumstance of ring edge observations. Consider an egress occultation of a star through a large gap like Saturn’s Encke gap. When the ramp-up behavior ended and the star traversed the gap, the HSP observed the signal from the star alone, $I_0$. As the star approached the outer edge of the gap in many occultations, however, we observe spikes in the signal that exceed the $I_0$ baseline, indicating a secondary contribution to the signal besides the star itself. B16 used the fact that this additional light must due to diffraction by nearby ring particles to constrain the sizes of the smallest particles at the edges of Saturn’s Encke gap and A ring outer edge. Given that the lobe of the characteristic Fraunhofer diffraction pattern depends on the particle size, the shape and intensity of observed diffraction signatures depend on the size and abundance of the smallest particles that exist in the region. Figure 4.1 shows an example of a particularly sharp diffraction signature at the outer edge of the B ring in the κ Centauri Rev 47 Ingress occultation. In this chapter, we will use a forward modeling method to
probe the lower bounds of the size distribution. In Chapter 5, we will use statistical analysis of the data to constrain the upper limit of the size distribution.

Figure 4.1. Characteristic spike-like diffraction signature at the outer edge of the B ring. Diffraction signatures at ring edges exhibit a characteristic spike-like shape. The above plot shows such a spike for the κ Centauri Rev 47 Ingress occultation at the B ring outer edge. Because this occultation has an integration period of 2 ms, we divide the raw data by 2 to obtain units of counts per millisecond. We scale the y-axis by a factor of $10^{-3}$ to simplify the tick labels.

4.1.2 Regions analyzed

In this chapter we will present our resulting best fit models for detections of diffraction signatures at each of the ring edges studied. While we present the best fit model $a_{\text{min}}$ and $q$ values for each detection, we emphasize that the focus of this work is not so much on the values of $a_{\text{min}}$ and $q$ at these edges as it is on the frequency of observed detections of diffraction across different regions of the rings. A brief summary of the characteristics of each region is given below.
4.1.2.1 C ring

The Maxwell and Titan ringlets are two narrow and eccentric ringlets within the C ring (Porco et al., 1983). The structure of the Maxwell ringlet is complicated by a wave-like structure conjectured to be induced by internal oscillations within the planet (French et al., 2016). A wavelet analysis by French et al. (2016) identified a 2-armed spiral pattern consistent with a density wave driven by an \( m = 2 \) outer Lindblad resonance. Although their N-body simulations of embedded eccentric ringlets are able to reproduce observed structure, they caution that the nature of the relationship between gaps and their resonances is not fully understood.

The Titan ringlet within the Colombo gap is also in a peculiar resonance- e.g., the -1:0 apsidal resonance with satellite Titan. This inner vertical resonance produces a one-armed spiral bending wave which was the first outward propagating bending wave discovered in Saturn’s rings (Rosen and Lissauer, 1988). Resonances drive collisions and clumping among the particles and therefore heavily influence the particle size distribution. Overall, we find a lack of detections at both edges of both ringlets, indicative of a dearth of small particles.

4.1.2.2 Cassini Division

The Cassini Division consists of eight named gaps, three of which contain noncircular ringlets that we will analyze in this work. Within this ring region, we search for detections of diffraction at the inner and outer edges of the Huygens, Strange (also called R6), and Herschel ringlets, as well as the inner edge of the Laplace ringlet. We exclude the outer edge of the Laplace ringlet due to complex structure at the edge that suppresses the diffraction signal there. We find no detections of diffraction at the Huygens ringlet, a bright and eccentric ringlet which lies within the Huygens gap that begins at the outer edge of the B ring. We do find multiple
detections of diffraction at the edges of the Strange ringlet, another eccentric ringlet which lies within the Huygens gap but is narrower and more inclined than the Huygens ringlet and as such is not always visible in UVIS data. We take this into account in calculating our detection frequencies for this edge, only including occultations in which the ringlet is observed as candidates for diffraction signatures. The Herschel ringlet, wider and less dense than the other Cassini Division ringlets and the opaque Laplace ringlet, which is also eccentric but not inclined, both do not precess rigidly and as such do not conform to the dynamic behavior expected for eccentric ringlets (French et al., 2016).

4.1.2.3 B ring outer edge

The B ring outer edge is confined by a 2:1 inner Lindblad resonance with satellite Mimas (Porco et al., 1983). This mean motion resonance causes the radial location of the edge to librate relative to Mimas with an \( m = 2 \) pattern (Spitale and Porco, 2010). Because of this, our observations of the B ring edge vary widely in distance from Saturn, sometimes by hundreds of kilometers.

4.1.2.4 Outer A ring

Saturn’s A ring is punctuated by numerous spiral density waves as well as self-gravity wakes, which we will discuss extensively in Chapter 5. Within this region, we extend B16’s analysis of the outer A ring and Encke gap by adding new detections of diffraction at these edges from Cassini’s final occultations which were not available at the time they published. With our modified forward-modeling technique, we will also report diffraction signature detections at the edges of the narrow Keeler gap, which B16 detected but did not attempt to model.
4.1.3 Challenges

Modeling detected diffraction signals at ring edges is complicated by two factors: (1) detections that do not conform to the expected signature morphology (e.g., smooth and sharply peaked) and (2) detections suppressed by the instrumental ramp-up effect. We will refer to (1) henceforth as “complex edges”. Complex edges are most common in occultations with a low signal-to-noise ratio. While we do attempt to model these edges, they tend to have large error bars that impedes our ability to obtain strong constraints on the particle size distribution.

Due to the HSP’s instrumental ramp-up effect, which we described in detail in section, edge observations are challenging. The ramp-up response is non-linear and unique to every occultation so that it cannot be systematically removed. We address this challenge primarily by avoiding gap inner (outer) edges in Egress (Ingress) occultations and also by modeling the ramp-up in the gap as a linear effect and removing the trend (for an example, see Figure 2.3).

4.2 Detecting diffraction

We follow B16 in using the term $Q_{\text{occ}}(a_c, \lambda, f)$ to define the “effective” extinction efficiency of the rings. The term $Q_{\text{occ}}$ depends on the critical particle size, $a_c$, the size at which all light (including both the direct and diffracted components of the signal) would be captured for a detector with angular FOV size $\theta_{\text{FOV}}$ (e.g., Cuzzi et al., 2009)

$$a_c = \frac{\lambda}{2\theta_{\text{FOV}}} \quad (4.1)$$

The aperture of the HSP is approximately 6 by 6 milliradians squared. Following B16, we can calculate an effective angular size of the instrument’s FOV by modeling the HSP aperture as a circle with $A = \pi(\theta_{\text{HSP}})^2$. Then
\[ \theta_{\text{HSP}} = 3.39 \text{ mrad} \]

is the effective acceptance angle of the HSP. This means that for particles as small as \( a_c = \frac{\lambda}{2\theta_{\text{FOV}}} = \frac{0.15}{2(3.39)} = 22 \) microns, the HSP detector will capture enough diffracted light from nearby particles to entirely replace the light diffracted out by particles in the direct line-of-sight. Therefore, for particles larger than a few tens of microns, the diffraction lobe is fully captured and both the direct and diffracted light are observed so that \( Q_{\text{occ}} = 1 \). For smaller particles, the light diffracted out cannot be fully replaced by the light diffracted in by nearby particles and \( 1 \leq Q_{\text{occ}} \leq 2 \). Because \( Q_{\text{ext}} = 2 \) in the large particle limit, we find the value of \( Q_{\text{occ}} \) by subtracting the fraction of the light that is diffracted out of the field of view by neighboring ring particles \( (f) \) from \( Q_{\text{ext}} \) such that \( Q_{\text{occ}} \approx 2 - f \). For a uniform distribution of ring particles that has as large an angular distribution as the angle by which light is diffracted by ring particles, \( f = 1 \), while for an infinitely narrow ring, \( f \rightarrow 0 \). We approximate \( f \approx 0.5 \) for observations of ring edges because the angular separation of the nearest neighboring ring material from the edge is still larger than the angle that the light is diffracted by so that approximately half the light lost from the FOV due to diffraction is replaced by diffracted light from particles at the ring edge and consequently that \( Q_{\text{occ}} \approx 1.5 \). The dependence of \( Q_{\text{occ}} \) upon the angular size of the FOV and the diffraction lobe is illustrated in Figure 4.2.
Figure 4.2. Diagram from Becker (2016) illustrating the dependence of $Q_{occ}$ upon the angular size of the FOV and the diffraction lobe.

The yellow circle labeled region of diffraction represents the region for which nearby particles of size $a$ would replace the light diffracted out of the FOV by particles of size $a$ along the line-of-sight. The teal box representing a large FOV captures the full diffraction lobe ($Q_{occ} = 2$) while the green box representing a small FOV does not ($1 \leq Q_{occ} \leq 2$).

4.2.1 Detection criteria

We follow the criteria outlined in B16 to determine diffraction signal observability. We select stars which are sufficiently bright to ensure that the increase in signal of the observed diffraction spike above $I_0$ can be distinguished from noise. We also require that the occultation geometry provides a rapid transition from gap to ring by not having a large projection of the vertical thickness of the ring into the gap. We use the method of Jerousek et al. (2011) to quantify the edge resolution by calculating the angle $\alpha$,

$$\tan \alpha = \frac{\tan |B|}{\cos |\varphi|}$$

(4.3)

where $B$ is the angle between the ring plane and the LOS from Cassini to the star and $\varphi$ is the angle between the radial direction at the point where the LOS vector crosses the ring plane and the projection of the LOS vector onto the ring plane. Based on the sizes of particles observed in
the rings and the width of the resulting diffraction lobe we expect diffraction signatures to be
detectable at higher values of $\alpha$ indicating a more direct view of the ring edge. At low values of
$\alpha$, the transition of the signal across the edge spans more points than a typical diffraction signal,
causing the diffracted light to mix with a partially attenuated stellar signal. Our precise criteria
for determining diffraction detectability are described in section 3.4. While we do see diffraction
signatures in occultations that do not meet these criteria, we adopt these relatively strict criteria
to have a baseline of occultation measurements where we can assume that a diffraction signature
would be observed if the particle size distribution extends to particles smaller than one cm. This
enables us to compare frequencies of detections between ring edges to identify differences in the
abundance of small particles between ring regions.

While we do not detect diffraction at every edge with every occultation that meets the
criteria for detectability, we do note a general trend of higher detection frequency at high $\alpha$
values (between 60° and 80°). There are a total of 90 diffraction signature detections (including
those from B16); of these, 70 have $\alpha$ values greater than 60°. For comparison, there are 13
detections for which $\alpha$ lies between 40° and 60° and just 7 detections for which $\alpha$ lies between 0°
and 40°. Figure 4.3 below shows both the candidate occultations and occultations with detections
of diffraction at the Keeler gap inner edge.
Figure 4.3: Detections of diffraction amongst detectable occultations at the Keeler gap inner edge.
The comparison of $\alpha$ versus $I_0$ in our determination of diffraction detectability for all occultations at the inner edge of the Keeler gap. Each occultation with data in this region is represented by a black circle. Of the occultations with available data, we consider only ingress occultations for the inner edge of the Keeler gap to avoid complications caused by the HSP instrumental ramp-up effect, described in Section 3.4. We exclude all occultations to the left of the green line from our cumulative frequency analysis.

We check for diffraction signatures at every edge (not impacted by the HSP ramp-up) in every occultation. Most detections are in occultations that meet the above criteria for $I_0$ and $\alpha$. While there are a few detections in occultations that fall short of these criteria but still exhibit a clear diffraction spike, we exclude these edges from our analysis of frequency of detection but include them in calculating observed particle size distribution parameters. We found a total of 90 detections of diffraction via our automated search. Table 4.1 below summarizes the occultation parameters for all detected diffraction signatures analyzed at all edges in both this study and B16.
Table 4.1. List of diffraction detections.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Edge (inner/outer)</th>
<th>( I_0 ) ( (\text{counts}_m) )</th>
<th>( \alpha ) (°)</th>
<th>( B ) (°)</th>
<th>( D_{\text{LOS}} ) (R₃)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlpAra032 Ingress</td>
<td>A ring (outer)</td>
<td>38.9</td>
<td>82.5</td>
<td>54.4</td>
<td>11.0</td>
<td>0.03</td>
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<td>A ring (outer)</td>
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<td>83.0</td>
<td>54.4</td>
<td>11.0</td>
<td>0.05</td>
</tr>
<tr>
<td>AlpCru092 Ingress*</td>
<td>A ring (outer)</td>
<td>518.3</td>
<td>72.7</td>
<td>68.2</td>
<td>11.5</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>AlpCru100 Ingress*</td>
<td>A ring (outer)</td>
<td>438.5</td>
<td>74.4</td>
<td>68.2</td>
<td>13.2</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>AlpVir210 Ingress</td>
<td>A ring (outer)</td>
<td>138.6</td>
<td>29.4</td>
<td>17.3</td>
<td>31.2</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>AlpVir211 Ingress</td>
<td>A ring (outer)</td>
<td>135.5</td>
<td>31.9</td>
<td>17.3</td>
<td>17.5</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>BetCen077 Ingress</td>
<td>A ring (outer)</td>
<td>588.1</td>
<td>86.0</td>
<td>66.7</td>
<td>8.1</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>BetCen081 Ingress*</td>
<td>A ring (outer)</td>
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<td>83.1</td>
<td>66.7</td>
<td>9.8</td>
<td>&lt; 0.01</td>
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<td>BetCen100 Ingress*</td>
<td>A ring (outer)</td>
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<td>81.0</td>
<td>66.7</td>
<td>9.8</td>
<td>&lt; 0.01</td>
</tr>
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<td>BetCen104 Ingress*</td>
<td>A ring (outer)</td>
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<td>81.8</td>
<td>66.7</td>
<td>13.3</td>
<td>&lt; 0.01</td>
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<tr>
<td>BetCen105 Ingress*</td>
<td>A ring (outer)</td>
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<td>66.9</td>
<td>66.7</td>
<td>17.0</td>
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<td>BetCen109 Ingress*</td>
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<td>BetCen112 Ingress*</td>
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<td>A ring (outer)</td>
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<td>80.6</td>
<td>47.0</td>
<td>12.5</td>
<td>&lt; 0.01</td>
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<tr>
<td>GamAra037 Ingress</td>
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<td>18.3</td>
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<td>48.7</td>
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<td>47.1</td>
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<td>15.9</td>
<td>&lt; 0.01</td>
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<td>&lt; 0.01</td>
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<tr>
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<td>A ring (outer)</td>
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<td>61.8</td>
<td>53.6</td>
<td>15.4</td>
<td>&lt; 0.01</td>
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<td>17.3</td>
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<td>$I_0$ (counts/ms)</td>
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<td>$B$ ($^\circ$)</td>
<td>DLOS (Rs)</td>
<td>p-value</td>
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<td>41.7</td>
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</tr>
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<td>&lt; 0.01</td>
</tr>
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<td>R6 (inner)</td>
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<td>BetCen092 Egress</td>
<td>R6 (inner)</td>
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<td>66.7</td>
<td>8.4</td>
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</tr>
<tr>
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<td>0.05</td>
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<td>R6 (inner)</td>
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<td>42.4</td>
<td>41.7</td>
<td>13.9</td>
<td>0.02</td>
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<tr>
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<td>47.5</td>
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<td>B ring (outer)</td>
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<td>48.6</td>
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<td>LamSco044 Ingress</td>
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<td>63.2</td>
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Occultation  Edge  $I_0$ $\alpha$  $B$  DLOS  p-value

101
<table>
<thead>
<tr>
<th></th>
<th>(inner/outer)</th>
<th>(counts/ms)</th>
<th>(R_s)</th>
<th>p-value</th>
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<tr>
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<td>75.5</td>
<td>66.7</td>
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<td>BetCen102 Ingress</td>
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<td>81.0</td>
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</tr>
<tr>
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<td>Titan (inner)</td>
<td>351.4</td>
<td>77.3</td>
<td>66.7</td>
</tr>
</tbody>
</table>

1 R_s = 60330 km.

I_o – Unocculted stellar signal (counts/ms)
B – Ring elevation angle (°)
D_ISO – Line-of-sight distance of Cassini (R_s)
p-value - the probability of obtaining a slope at least as steep as the ring edge slope by chance from the parent population
* Previously reported in B16

4.2.2 Blind test for diffraction

Although most diffraction at the ring edges can be clearly identified by visual inspection, we further expand on the work of B16 by implementing an objective test for detecting diffraction. In order to be included in our list of detections, we require a 95% confidence that diffraction signatures are real as opposed to random noise. To do this, we require that the maximum point be the edge point and calculate linear fits to the data points adjacent to the edge (presumably containing detection of diffracted light) in segments of 5 and 10 points. We also calculate linear fits to segments of 5 and 10 points from the same occultation in a broad gap or a region beyond the edge of the A ring where there is no ring signal in the data. We first confirm that the distribution of slopes from the data near the ring edge is consistent with coming from the same parent population of slopes that produced the distribution of slopes from the control data (usually the region well outside the A ring). We next compare the slope of the linear fit to the data immediately adjacent to the ring edge with the distribution of slopes from the control region. If the slope of the linear fit of the data immediately adjacent to the edge has a less than 5% chance of being randomly drawn from the control distribution, which consists of 100 slopes of linear fits to data outside of the F ring, we identify this as a statistically significant detection of a
real signal at the 95% confidence level and we include this edge in our list of detections and model it. We check both the slopes of 5- and 10-point linear fits because of differences in the resolution of different occultations. We compute the p-value of each detection as the probability of obtaining a slope at least as steep as the ring edge slope by chance from the parent population. As an additional check, we compare the variance of the population of slopes near the given ring edge to the variance of the population of slopes from a large gap using the f-test. Figure 4.4 shows the linear fits for the α Crucis Rev 100 Ingress occultation at the outer edge of the Keeler gap.

![Linear fits for the α Crucis Rev 100 Ingress occultation at the outer edge of the Keeler gap.](image)

Figure 4.4: Linear fits to 5-point segments at the outer edge of the Keeler gap in the α Crucis Rev 100 Egress occultation. The slope at the edge is steeper than any of the slopes of linear fits to the control region outside of the F ring. This indicates that the probability of drawing a slope at least this steep from this distribution by chance is very low, corresponding to a p-value < 0.01. Asterisks are raw data binned to 0.5 km and normalized to the gap signal level.

Using this blind test for diffraction signal detection we find a total of 9 new detections at the Encke gap edges (2) and A ring outer edge (7) beyond those reported in B16. In addition, we find 10 diffraction signatures modeled in B16 that either do not meet our updated criteria or do not pass our blind test; 1 at the A ring edge, 7 at the Encke gap outer edge, and 2 at the Encke
gap inner edge. In 5 of 10 of these cases, the edges in question were not the max points. We exclude these marginal detections from our cumulative analysis. To account for the relative importance of a sufficiently high signal level and viewing angle and to develop a baseline to which we can compare diffraction detection frequencies between different ring regions, we exclude occultations that lie to the left of the line connecting the points (0 counts/ms, 90 degrees) and (100 counts/ms, 0 degrees) from our cumulative frequency calculations. All but 2 detections for all 15 edges fall in the region greater than or equal to the line. Figure 4.3 illustrates our frequency calculation at the inner edge of the Keeler gap. We emphasize that while many of our calculated best fit values for $a_{\text{min}}$ and $q$ are poorly constrained, the calculation of detection frequency reveals its own unique information about the particle population at these ring edges.

4.3 Analysis

4.3.1 Updates to the B16 model

Some edges when binned at high resolution were complicated by binary star signatures. When binning at higher resolution than 0.5 km, the binary star signature becomes apparent. We don't account for binary stars when binning at 0.5 km because at this resolution the signal from the two stars combines in one grid block of the FOV. At 0.2 and 0.3 km resolution, however, we often see two peaks at the edge, the nearest peak to the edge much lower than the next nearest peak. Despite adjusting the binning resolution several times, we still did not detect any noteworthy diffraction signatures at either of the Titan ringlet edges, as shown in Figure 4.5.
Figure 4.5. $\beta$ Centauri Rev 104 Egress occultation at the inner edge of the Titan ringlet binned at 0.3 km resolution.
The above figure illustrates the binary star signature that becomes apparent when we bin data to higher resolution than 0.5 km. The characteristic of the double star can be seen at the edge, where there is one low peak closest to the edge followed by another, higher peak bit further out. The double hump at the edge indicates that the apparent spacing between the stars in the binary star, in this case $\beta$ Centauri, is wider than the resolution at which we are binning.

The choice of binning resolution made a significant impact on the sharpness of the signal in many occultations of the Herschel ringlet. For example, in the $\zeta$ Pupii Rev 208 Egress occultation, we observed a notably more pronounced spike at the Herschel ringlet inner edge when binning the same data to 0.2 rather than 0.5 km resolution (see Figure 4.6). Ultimately, we chose to bin all of the data at 0.5 km intervals for ease of comparison.
Figure 4.6. Herschel ringlet edge at two different radial resolutions. The left plot is the ζ Pup Rev 208 Egress stellar occultation binned at 0.3 km resolution. In this plot, there is an obvious diffraction spike at the ringlet edge. The right plot shows the same data binned instead to 0.5 km resolution. In this plot, the diffraction signal is not at all clear. Due to this discrepancy, going forward we plan to model both signals and compare the results.

We also adapted the B16 model for narrow gaps. B16 noted the detection of several sharp signals at Keeler gap edges, but determined that the narrowness of the ~35 km gap did not allow sufficient time for the HSP to reach maximum sensitivity, making removal of the ramp-up ineffective. To remedy this issue, we tested a new method—we instead took a linear fit to the first few seconds of binned Encke gap data, which is separated from the Keeler gap by about 300 km, and used the parameters of that fit to remove the ramp up effect in the Keeler gap. We found that error bars using this method were too large to draw meaningful conclusions. After this, we tried simply taking a linear fit to the binned Keeler gap data in the central region of the gap (~ 5 km from each edge), which worked much better. Our conclusion from this was that the HSP ramp-up is sufficiently linear after the first few seconds in the gap, and the effect can be removed by excluding the times of non-linear ramp-up from our fit entirely.
4.3.4 Removal of the HSP ramp-up effect

The UVIS HSP exhibits an instrumental effect that significantly impacts observations of our area of interest, namely ring edges (Colwell et al. 2010). The instrument experiences a gradual increase in sensitivity after the star emerges from behind dense ring material and enters a gap. Immediately after entering a gap or emerging from behind a ring the measured signal is noticeably below \( I_0 \) by an amount that varies from occultation to occultation but is frequently \(~10\%\) of \( I_0 \). That is followed by a non-linear increase in \( I \) until it is within a few per cent of \( I_0 \) after which there is a more gradual increase until the signal asymptotes at \( I_0 \). The time scales for these changes in sensitivity vary between occultations.

As a result, the diffraction signal may be within the instrumental ramp-up portion of an occultation. This generally occurs at the outer edge of a ring (or inner edge of a gap) in egress occultations and the inner edge of a ring (or outer edge of a gap) in ingress occultations. We illustrate the effect by overlaying an ingress and an egress occultation of the same star in Figure 4.7, which reveals clear disparities in the data at both the outer edge of the A ring and the inner edge of the Keeler gap. In each occultation the HSP sensitivity increases with distance from the ring edge. In the ingress occultation the signal decreases with increasing ring plane radius, while in the egress occultation the signal increases with increasing ring plane radius.
Figure 4.7: Illustration of the HSP ramp-up in Ingress versus Egress occultations. The two α Crucis Rev 100 stellar occultations at the outer A ring region, ingress in black and egress in gray, overlaid to illustrate the HSP ramp-up effect. During an egress occultation the radial position of the star increases with time, while during an ingress occultation, the radial distance of the star decreases with time. The ramp-up effect decreases the signal at the outer edges of gaps in ingress occultations and decreases the signal at the inner edges of gaps in egress occultations.

This effect is particularly relevant to our study of diffraction at ring edges. We account for the ramp-up response of the HSP by considering only ingress occultations for the inner edges of gaps (outer edges of rings) and only egress occultations for the outer edges of gaps (inner edges of rings) for the response to become linear. This allows sufficient time for the response to become linear.

To address the HSP ramp-up effect we bin each occultation to 0.5 km resolution and compute a linear fit to a region of data both far enough inside the gap that the nonlinear initial ramp-up effect has ended and sufficiently far from the edge that the diffraction spike is not included. We divide the binned data by the linear fit and normalize the result. Figure 4.8 illustrates our ramp-up removal process at the Keeler Gap inner edge for the α Crucis Rev 100 ingress occultation.
Figure 4.8: Illustration of the removal of the HSP ramp-up effect in the Keeler gap. The figure above shows the ramp-up removal technique applied to the Keeler gap in the α Crucis Rev 100 Ingress occultation. Because the Keeler gap is narrow (~35 km wide), the ramp-up effect is more difficult to remove. To remove the trend, we normalize and divide the original data, shown in dashed gray, by a linear fit to the central 15 km of the gap. The result is the trend-removed data, shown in black. Also plotted is the difference between the normalized values pre- and post- ramp-up removal, scaled by 5 and offset by 1.3 to illustrate the trend.

4.3.5 Implementation of model

4.3.5.1 Reconstruction of spacecraft viewing geometry

NASA’s Navigation and Ancillary Information Facility (NAIF) SPICE toolkit, publicly available at https://naif.jpl.nasa.gov/naif/naif/. The toolkit consists of a library of application program interfaces (APIs) and underlying subroutines and functions provided as source code. Ancillary data are data that enable scientists and engineers to determine where a spacecraft was located at a time of interest, how the spacecraft and instruments were oriented, the location, size, shape, and orientation of the target of observation, and what events may have occurred on the spacecraft or on the ground during the time of interest. SPICE is used to organize and package these ancillary data in kernels. The SPICE abbreviation stands for Spacecraft, Planet, Instrument,
C-matrix, and Events. Each of these corresponds to a type of kernel. A kernel is a file containing ancillary data that is used to determine observation geometry, which can be used to analyze mission data.

Figure 4.9. Illustration of vectors used in computing spacecraft orientation for a Cassini observation of Phoebe with the NAIF SPICE toolkit. A diagram of the remote-sensing activity involving spacecraft orientation and reference frames. The red vectors indicate the line-of-sight from Cassini to Phoebe and Cassini to Earth, as well as the nominal boresight of the high gain antenna on Cassini. The line of sight vectors are computed using cspice spkezr, which returns the position and velocity of the target relative to the observer, followed by cspice sxform, which returns the state transformation matrix from one frame to another at the specified epoch. The angular separation is computed using cspice vsep, which returns the separation angle between two line-of-sight vectors. (Image Credit: NASA’s Navigation and Ancillary Information Facility diagram for ‘xform’ exercise.)

4.3.5.2 Construction of the field of view

We use an updated version of the forward modeling technique developed by B16 which generates synthetic diffraction signatures for various combinations of $a_{min}$ and $q$ using HSP stellar occultation data binned to 0.5 km. For the large particle sizes in Saturn’s rings (compared to the UVIS wavelength), the Fraunhofer phase function (Eq. 4) varies over a narrow range of
scattering angles close to zero. The model therefore requires higher angular resolution to capture the full detail of the diffraction pattern for larger particle size values. The scattering angle is defined as \( \theta = \frac{2\pi a}{\lambda} \) where \( \lambda = 0.15 \mu m \) is the wavelength of incident light. We break up the HSP field of view into multiple grids of virtual pixels. The grids are centered on star and are designed to optimize the computational time while accurately capturing the variations in signal due to the Bessel function signature of the diffracted light. The smallest grid has the highest resolution and is used to assess the signal from the largest ring particles, since the extent of the scattering angle is small but the Bessel function pattern varies with significantly as a function of angular distance from the star. The largest virtual FOV grid has the lowest spatial resolution but is broad enough to capture the diffracted signal from the smallest ring particles.

4.3.5.2 Calculating I/F

The discretized field-of-view (hereafter FOV) grids were made for the following particle size ranges: \( a_{\min} \leq 1 \) mm, \( 1 \) mm \( \leq a_{\min} \leq 5 \) mm, \( 5 \) mm \( \leq a_{\min} \leq 10 \) mm, \( 10 \) mm \( \leq a_{\min} \leq 50 \) mm, and \( 50 \) mm \( \leq a_{\min} \leq 500 \) mm. B16 chose these size ranges based on the angular extent of the Bessel function, \( J_1 \), as a function of \( a \) (e.g., \( P(\theta, a) = \left( \frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right)^2 \)). See B16 for more details on the FOV grids. We calculate the intensity of forward-scattered diffracted light for each virtual pixel across the FOV of the instrument as the angular separation of the star from the ring edge varies. We calculate the intensity of the scattered light for each virtual pixel in each virtual FOV at 0.5 km intervals corresponding to each binned data point up to 10 km from the edge. Beyond 10 km the signal is only calculated at 10 km intervals where the diffraction signal is relatively small.
4.3.5.3 Determination of the best fit

We determine the best fit model signal for each detection by calculating the $a_{\text{min}}$ and $q$ values which yield the minimum reduced $\chi^2$ value ($\chi^\text{r}_v^2$). We compare the model to each data point up to 10 km for most edges, however at the Herschel and Strange ringlet edges, where the diffraction signatures are characteristically narrower, we compare the model to each data point up to just 5 km. Beyond 10 km (or 5 km for the Herschel and Strange ringlets), we bin both the model and data by 10 points before calculating the residuals. For some best fits, $\chi^\text{r}_v^2$ falls below 1. This is a consequence of our model having only two free parameters, $a_{\text{min}}$ and $q$, and many more data points, resulting in an over-constrained model. We also occasionally find high best fit reduced $\chi^\text{r}_v^2$ values (> 1.5). This could indicate an underestimation of the error bars on the data because we do not account for underlying variations in ring structure in our uncertainty calculations. We define confidence levels using a $\chi^2$ distribution with $n-2$ degrees of freedom, where $n$ is the number of data points, such that the probability that a random value drawn from that distribution is greater than the cut-off equals the user input value. We use the inverse probabilities associated with integer multiples of traditional $\sigma$ values to determine bound values for the $\chi^\text{r}_v^2$ for three confidence intervals. Typically, we use the $2\sigma$ bound to determine the range of our error bars. For occultations in which the best fit corresponds to reduced $\chi^\text{r}_v^2 > 1.5$, we use the $3\sigma$ bound and for occultations in which the best fit corresponds to reduced $\chi^\text{r}_v^2 < 0.8$ we use the $1\sigma$ bound. We determine error bars for $a_{\text{min}}$ and $q$ separately using traditional $\sigma$ values by assuming the best fit $a_{\text{min}}$ to bound $q$ and assuming the best fit $q$ to bound $a_{\text{min}}$. Figure 4.10 shows a visual representation of our error bar determination for the $\alpha$ Crucis Rev 100 Ingress occultation at the inner edge of the Keeler gap. The dotted lines show the bounds of the error.
bars corresponding to 1σ in red, 2σ in blue, and 3σ in pink over computed values for $\chi^2_v$ for each $a_{\text{min}}$ value fit assuming the best fit $q$ and the errors for $q$ using the best-fit value of $a_{\text{min}}$ (see Figure 4.10 below). For the 2σ confidence level, the lower bound on $a_{\text{min}}$ is $\approx 5$ mm while the upper bound on $a_{\text{min}}$ is unconstrained. For $q$, the 2σ confidence bounds are 2.6-3.4.

![Figure 4.10](image)

4.3.5.4 Errors and Uncertainties

We follow the method of B16 in computing error bars by taking the standard deviation of the unocculted signal, excluding the diffraction spike. B16 used only the signal more than 30 km from the target ring edge to calculate the standard deviation. For many of the edges in our analysis, however, the gaps are narrow and the region more than 30 km from the edge is obstructed by more ring material. In these cases, we still exclude the diffraction peak, but instead compute the standard deviation of the unocculted signal nearer to the edge. In the 30 km wide Keeler Gap, we use the central 15 km of data (the same region that we used to remove the HSP
ramp-up effect) to minimize diffracted light from either edge. At the outer edge of the Strange and Herschel ringlets we use data 5 km or more from the edge. This is a valid approximation for most diffraction signatures at these narrow ringlets because the diffraction signatures detected are narrow and sharp, indicating that there is little contribution to the signal from diffracted light more than a few km from the edge.

4.4 Results

Below, we present our resulting best fit models for detections of diffraction signatures at each of the ring edges studied. While we present the best fit model \( a_{min} \) and \( q \) values for each detection, we emphasize that the focus of this work is not so much on the values of \( a_{min} \) and \( q \) at these edges as it is on the frequency of observed detections of diffraction across different regions of the rings.

4.4.1 C ring

4.4.1.1 Maxwell ringlet

The Maxwell ringlet is a dense, non-circular ringlet embedded within the C ring, a wide but tenuous ring internal to the B ring. This ringlet is particularly interesting because it exhibits an embedded wavelet (Baillié et al. 2012), although to date no moons have been discovered in the gap. We detect diffraction at the inner edge of the Maxwell ringlet in only 2 of the 23 detectable occultations. Interestingly, the best fit models to these two signatures are quite different. At the \( \beta \) Centauri Rev 092 Egress occultation we find a best-fitting \( a_{min} \) of 10 mm and a best-fitting \( q \) of 3.5, while at the \( \zeta \) Pupii Rev 208 Egress occultation we find a best-fitting \( a_{min} \) of
90 mm and a best-fitting $q$ of 3.1. This disparity may imply azimuthal variations in $a_{\text{min}}$ at outer edge of the Maxwell gap. Figure 4.11 shows the detection overlain with three model diffraction signatures for the $\zeta$ Pupii Rev 208 Egress occultation.

![Image](image_url)

Figure 4.11: The $\zeta$ Pup Rev 208 Egress occultation at the Maxwell ringlet inner edge. (Left) Three model curves are overlain on the data. The best fit $a_{\text{min}}$ and $q$, for which $a_{\text{min}} = 90$ mm and $q = 3.1$, is shown in green. The two other fits shown, $a_{\text{min}} = 40$ mm and $q = 3.1$, and $a_{\text{min}} = 20$ mm and $q = 3.1$, in blue and pink respectively, are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 1σ confidence interval.

Table 4.2: Detections at the Maxwell ringlet inner edge.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best Fit $a_{\text{min}}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BetCen092E</td>
<td>$10^{+19}_{-5}$</td>
<td>$3.5^{+0.5}_{-0.5}$</td>
<td>464.9</td>
<td>75.5</td>
</tr>
<tr>
<td>ZetPup208E</td>
<td>$90^{+9}_{-8.0}$</td>
<td>$3.1^{+0.4}_{-0.4}$</td>
<td>61.3</td>
<td>85.0</td>
</tr>
</tbody>
</table>

↓ - Upper limit

We detect diffraction at the outer edge of the Maxwell ringlet in only 1 of 36 detectable occultations. This low detection rate suggests a lower number density of particles of mm-scale or smaller in this region compared other regions in this study. At our singular detection, the $\beta$ Centauri Rev 102 Ingress occultation, we find a best-fitting $a_{\text{min}}$ of 20 mm and a best-fitting $q$ of 3.5. Figure 4.12 shows the diffraction signature overlain with three models for comparison for the $\beta$ Centauri Rev 102 Ingress occultation.
Figure 4.12: The β Centauri Rev 102 Ingress occultation at the Maxwell ringlet outer edge. (Left) Three model curves are overlain on the data. The best fit $a_{\text{min}}$ and $q$, for which $a_{\text{min}} = 20$ mm and $q = 3.5$, is shown in green. The two other fits shown, $a_{\text{min}} = 20$ mm and $q = 2.8$ and $a_{\text{min}} = 5$ mm and $q = 3.5$, in blue and pink respectively, are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 1σ confidence interval.

Table 4.3: Detection at the Maxwell ringlet outer edge.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>$a_{\text{min}}$ (mm)</th>
<th>$q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BetCen102I</td>
<td>$20^{+79}_{-15}$</td>
<td>3.5$^{+1.0}_{-1.0}$</td>
<td>370.5</td>
<td>81.0</td>
</tr>
</tbody>
</table>

↓ - Upper limit

4.4.1.2 Titan ringlet

The narrow Titan ringlet is also located within the C ring. It is governed by an orbital resonance with Titan and is slightly elliptical. We do not detect any diffraction signatures in the 35 occultations that meet our criteria for detection at the outer edge of the Titan ringlet. As in the Huygens ringlet, the absence of diffraction detections suggests a dearth of mm-sized particles in this region. We detect diffraction at the inner edge of the Titan ringlet in only 2 of the 17 detectable occultations. We find a best-fitting $a_{\text{min}}$ of 10 mm and a best-fitting $q$ of 3.5 for both the β Centauri Rev 077 and β Centauri Rev 104 Egress occultations. Figure 4.13 shows the diffraction signature overlain with three models for comparison for the β Centauri Rev 077 Egress occultation at the Titan ringlet inner edge.
Figure 4.13: The β Centauri Rev 077 Egress occultation at the Titan ringlet inner edge. (Left) Three model curves are overlain on the data. The best fit $a_{\text{min}}$ and $q$, for which $a_{\text{min}} = 10$ mm and $q = 3.5$, is shown in green. The two other fits shown, $a_{\text{min}} = 1$ mm and $q = 3.0$ and $a_{\text{min}} = 1$ mm and $q = 3.3$, in blue and pink respectively, are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 1σ confidence interval.

Table 4.4: Detections at the Titan ringlet inner edge.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $a_{\text{min}}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BetCen077E</td>
<td>$10^{+10}_{-5}$</td>
<td>$3.5^{+1.0}_{-1.0}$</td>
<td>606.0</td>
<td>71.0</td>
</tr>
<tr>
<td>BetCen104E</td>
<td>$10^{+10}_{-5}$</td>
<td>$3.5^{+1.0}_{-0.3}$</td>
<td>351.4</td>
<td>77.3</td>
</tr>
</tbody>
</table>

↓ - Upper limit

4.4.2 Cassini Division

The Cassini Division between the A and B rings is characterized by a sheet of low optical depth material ($\tau < 0.1$) interrupted by several gaps that are dynamically linked to the outer edge of the B ring (Hedman and Nicholson, 2016). There are narrow ringlets of large optical depth (the Huygens ringlet and R6) as well as a moderate optical depth plateau-like feature (the triple-band feature). We do not look for diffraction signatures at the edges of gaps in the Cassini Division which are either low optical depth or not sufficiently radially sharp. Below we present analysis of the 4 narrower, high-optical-depth, ringlets within the Cassini Division: the Laplace ringlet, Herschel ringlet, R6 (the so-called ‘Strange ringlet’), and the Huygens ringlet.
4.4.2.1 Huygens ringlet

The dense, eccentric Huygens ringlet lies within the Huygens gap. We detect diffraction at only 3 of 33 detectable occultations at the inner edge. Among our detections, we find an average best-fitting $a_{min}$ of 17 mm and an average $q$ of 3.4. We find $a_{min}$ values ranging from 10 to 20 mm and $q$ values ranging from 3.3 to 3.5. Figure 4.14 shows our analysis for the $\zeta$ Puppii Rev 208 Egress detection.

![Figure 4.14: The $\zeta$ Puppii Rev 208 Egress occultation at the Huygens ringlet inner edge.](image)

(Left) Three model curves overlain on the data. The best fit model curve, for which $a_{min} = 20$ mm and $q = 3.5$, is represented by a green cross. The two other fits shown, $a_{min} = 1$ mm and $q = 3.2$ and $a_{min} = 5$ mm and $q = 3.2$, shown in blue and pink respectively, yield high $\chi^2$ values and are poor fits to the data. (Right) All models, with the exception of low $a_{min}$ and high $q$ combinations, are within the innermost contour (the 1σ confidence interval) for this detection.

Table 4.5: Detections at the inner edge of the Huygens ringlet

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $a_{min}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BetCen077E</td>
<td>$10^{+89}_{-5}$</td>
<td>$3.4^{+0.1}_{-0.9}$</td>
<td>606.0</td>
<td>74.4</td>
</tr>
<tr>
<td>GamAra037E</td>
<td>$20^{+79}_{-19}$</td>
<td>$3.5^{+1.0}_{-1.0}$</td>
<td>27.2</td>
<td>72.3</td>
</tr>
<tr>
<td>ZetPup208E</td>
<td>$20^{+79}_{-10}$</td>
<td>$3.3^{+0.2}_{-0.8}$</td>
<td>61.3</td>
<td>85.0</td>
</tr>
</tbody>
</table>

- Upper limit

We do not detect a diffraction signature in any of the 46 occultations that meet our criteria for the outer edge. Figure 4.15 shows the $\lambda$ Scorpii Rev 044 Ingress occultation as a
typical example of a non-detection at this edge. The absence of any clear diffraction signatures at this edge suggests an absence of particles of mm-scale or smaller in this region, perhaps due to tightly bound shepherding of the particles by mean motion resonances.

Figure 4.15: The $\lambda$ Scorpii Rev 044 Ingress occultation at the Huygens ringlet. The magnitude of the unocculted stellar signal is plotted over the binned data in red. The outer edge of the Huygens ringlet is located at the 117845 km mark. The absence of a spike above the average stellar signal near this edge indicates an absence of small particles near this region which could diffract additional light into the field of view of the instrument.

4.4.2.1 Strange ringlet

The Strange ringlet lies slightly exterior the Huygens ringlet. Due to its inclination and eccentricity, the occultation shadow of the Strange ringlet is sometimes projected onto other ring material preventing an analysis of the edges for diffraction signatures. We include only occultations in which we observe the Strange ringlet within the Huygens gap in our count of potential detections. We observe a noteworthy diffraction signature in 6 of 30 detectable occultations at the inner edge of the Strange ringlet. Among our detections, we find an average best-fitting $a_{\text{min}}$ of 25 mm and an average $q$ of 3.3. We find $a_{\text{min}}$ values ranging from 5 to 50 mm and $q$ values ranging from 2.9 to 3.5. Figure 4.16 shows our analysis for the $\zeta$ Pupii Rev 208 Egress detection.
Figure 4.16: The ζ Puppii Rev 208 Egress occultation at the so-called ‘Strange’ ringlet inner edge. (Left) Three model curves overlain on the data. The best fit model curve, for which \( a_{\text{min}} = 90 \) mm and \( q = 3.5 \), is represented by a green cross. The two other fits shown, \( a_{\text{min}} = 90 \) mm and \( q = 2.8 \) and \( a_{\text{min}} = 40 \) mm and \( q = 3.5 \), shown in blue and pink respectively, yield high \( \chi^2 \) values and are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 1σ confidence interval.

Table 4.6: Detections at the inner edge of the Strange ringlet (R6)

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit ( a_{\text{min}} ) (mm)</th>
<th>Best fit ( q )</th>
<th>( I_0 ) (counts/ms)</th>
<th>( \alpha ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LamSco029E</td>
<td>( 50^{+49}_{-45} )</td>
<td>( 2.9^{+0.6}_{-0.4} )</td>
<td>287.0</td>
<td>42.4</td>
</tr>
<tr>
<td>EpsLup036E</td>
<td>( 50^{+49}_{-45} )</td>
<td>( 3.5^{+1.0}_{-1.0} )</td>
<td>30.8</td>
<td>60.7</td>
</tr>
<tr>
<td>GamAra037E</td>
<td>( 20^{+15}_{-15} )</td>
<td>( 3.5^{+1.0}_{-1.0} )</td>
<td>27.2</td>
<td>72.3</td>
</tr>
<tr>
<td>BetCen077E</td>
<td>( 1^{+49} )</td>
<td>( 3.0^{+0.5}_{-0.5} )</td>
<td>660.6</td>
<td>74.4</td>
</tr>
<tr>
<td>BetCen092E</td>
<td>( 20^{+29}_{-15} )</td>
<td>( 3.2^{+0.3}_{-0.7} )</td>
<td>464.9</td>
<td>76.7</td>
</tr>
<tr>
<td>ZetPup208E</td>
<td>( 90^{+9}_{-40} )</td>
<td>( 3.5^{+0.5}_{-0.5} )</td>
<td>61.3</td>
<td>85.0</td>
</tr>
</tbody>
</table>

\( \uparrow \) - Lower limit
\( \downarrow \) - Upper limit

At the outer edge of the Strange ringlet, we detect diffraction in 4 of 46 occultations. We find an average best fit \( a_{\text{min}} \) of 43 mm and \( q \) of 2.9. We find a broad range of both best fit \( a_{\text{min}} \) values from 20-50 mm and \( q \) values from 2.7-3.0. Figure 4.17 shows our analysis for the \( \alpha \) Virginis Rev 173 Ingress detection.
Figure 4.17: The α Virginis Rev 173 Ingress occultation at the outer edge of the Strange ringlet. (Left) Three model curves overlain on the data. The best fit model curve, for which $a_{\text{min}} = 50 \text{ mm}$ and $q = 2.7$, is represented by a green cross. The two other fits shown, $a_{\text{min}} = 50 \text{ mm}$ and $q = 3.0$ and $a_{\text{min}} = 20 \text{ mm}$ and $q =2.8$, shown in blue and pink respectively, yield high $\chi^2$ values and are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 1σ confidence interval. Errors for this occultation are reported to 1σ due to the low $\chi^2$ value of the best fit.

Table 4.7: Detections at the outer edge of the Strange ringlet (R6).

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $a_{\text{min}}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BetCen104I</td>
<td>$20^{+20}_{-10}$</td>
<td>$3.0^{+0.1}_{-0.2}$</td>
<td>358.4</td>
<td>67.9</td>
</tr>
<tr>
<td>LamSco114I</td>
<td>$50^{+49}_{-40}$</td>
<td>$2.7^{+0.4}_{-0.2}$</td>
<td>88.5</td>
<td>66.3</td>
</tr>
<tr>
<td>AlpVir173I</td>
<td>$50^{+49}_{-30}$</td>
<td>$2.7^{+0.1}_{-0.2}$</td>
<td>127.4</td>
<td>22.7</td>
</tr>
<tr>
<td>ZetPup208I</td>
<td>$50^{+30}_{-50}$</td>
<td>$3.0^{+0.1}_{-0.2}$</td>
<td>53.8</td>
<td>47.5</td>
</tr>
</tbody>
</table>

4.4.2.2 Herschel ringlet

We observe a noteworthy diffraction signature in only 3 of 33 detectable occultations at the inner edge of the Herschel ringlet. Among our three detections at the inner edge, we find an average best-fitting $a_{\text{min}}$ of 17 mm and an average best-fitting $q$ of 3.5. We find best-fitting $a_{\text{min}}$ values ranging from 10 to 20 mm and unusually high $q$ values ranging from 3.4 to 3.5. Figure 4.18 shows our analysis for the β Centauri Rev 105 Egress detection.
Figure 4.18: The β Centauri Rev 105 Egress occultation at the inner edge of the Herschel ringlet. (Left) Three model curves overlain on the data. The best fit model curve, for which $a_{\text{min}} = 50$ mm and $q = 3.2$, is represented by a green cross. The two other fits shown, $a_{\text{min}} = 1$ mm and $q = 3.0$ and $a_{\text{min}} = 1$ mm and $q = 3.3$, shown in blue and pink respectively, yield high $\chi^2$ values and are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 2σ confidence interval.

Table 4.8: Detections at the Herschel ringlet inner edge

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $a_{\text{min}}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlpAra063E</td>
<td>$10^{+8}_{-5}$</td>
<td>$3.5^{+1.0}_{-1.0}$</td>
<td>29.1</td>
<td>77.0</td>
</tr>
<tr>
<td>BetCen104E</td>
<td>$20^{+79}_{-5}$</td>
<td>$3.5^{+1.0}_{-1.0}$</td>
<td>351.4</td>
<td>86.3</td>
</tr>
<tr>
<td>BetCen105E</td>
<td>$50^{+49}_{-30}$</td>
<td>$3.2^{+0.3}_{-0.7}$</td>
<td>300.5</td>
<td>86.3</td>
</tr>
</tbody>
</table>

↑ - Lower limit  
↓ - Upper limit

At the outer edge of the Herschel ringlet, we detect diffraction in just 2 of 39 potential occultations. At the $\alpha$ Virginis Rev 173 Ingress occultation we find a best-fitting $a_{\text{min}}$ of 20 mm and a best-fitting $q$ of 2.9, while at the $\zeta$ Pupii Rev 208 Ingress occultation we find a best-fitting $a_{\text{min}}$ of 30 mm and a best-fitting $q$ of 2.9. Figure 4.19 shows the diffraction signature overlain with three models for comparison for the $\zeta$ Pupii Rev 208 Ingress detection.
Figure 4.19: The ζ Puppi Rev 208 Ingress occultation at the Herschel ringlet outer edge. (Left) Three model curves overlain on the data. The best fit model curve, for which $a_{\text{min}} = 30$ mm and $q = 2.9$, is represented by a green cross. The two other fits shown, $a_{\text{min}} = 30$ mm and $q = 3.5$ and $a_{\text{min}} = 30$ mm and $q = 2.5$, shown in blue and pink respectively, yield high $\chi^2$ values and are poor fits to the data. This figure also illustrates the effect of changing only the value of the $q$ parameter, which corresponds to the sharpness of the signature. (Right) Models with parameters within the innermost contour are within the 1σ confidence interval.

Table 4.9: Detections at the Herschel ringlet outer edge.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $a_{\text{min}}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlpVir134I</td>
<td>$20^{+79}_{-15}$</td>
<td>$2.9^{+0.6}_{-0.4}$</td>
<td>158.1</td>
<td>62.7</td>
</tr>
<tr>
<td>ZetPup208I</td>
<td>$30^{+69}_{-10}$</td>
<td>$2.9^{+0.3}_{-0.4}$</td>
<td>61.3</td>
<td>47.5</td>
</tr>
</tbody>
</table>

4.4.2.3 Laplace ringlet

Due to the extremely narrow nature of the gap external to the Laplace ringlet, we are unable to model diffraction signatures at the outer edge of the ringlet. We detect diffraction at the inner edge of the Laplace ringlet in 3 of 33 detectable occultations. Among our detections, we find an average best-fitting $a_{\text{min}}$ of 25 mm and an average $q$ of 3.1. We find best-fitting $a_{\text{min}}$ values ranging from 5 to 50 mm and $q$ values ranging from 2.6 to 3.5. Figure 4.20 shows the α Virginis Rev 211 Egress occultation at the Laplace ringlet inner edge.
Figure 4.20: The α Virginis 211 Egress occultation at the inner edge of the Laplace ringlet. (Left) Three model curves overlain on the data. The best fit model curve, for which $a_{\text{min}} = 50$ mm and $q = 2.6$, is represented by a green cross. The two other fits shown, $a_{\text{min}} = 50$ mm and $q = 2.8$ and $a_{\text{min}} = 20$ mm and $q = 2.6$, shown in blue and pink respectively, yield high $\chi^2$ values and are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 2σ confidence interval.

Table 4.10: Detections at the inner edge of the Laplace ringlet.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $a_{\text{min}}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BetCen104E</td>
<td>$20^{+39}_{-5}$</td>
<td>$3.5^{+0.2}_{-0.6}$</td>
<td>351.4</td>
<td>86.6</td>
</tr>
<tr>
<td>AlpVir211E</td>
<td>$50^{+49}_{-20}$</td>
<td>$2.6^{+0.1}_{-0.1}$</td>
<td>134.2</td>
<td>19.1</td>
</tr>
<tr>
<td>ZetCen246E</td>
<td>$5^{+94}_{-1}$</td>
<td>$3.3^{+0.2}_{-0.7}$</td>
<td>23.2</td>
<td>84.6</td>
</tr>
</tbody>
</table>

† - Lower limit

4.4.3 B ring outer edge

At the outer edge of the B ring we detect a diffraction signature in 8 of 45 detectable occultations. We find best-fitting $a_{\text{min}}$ values ranging from 1-50 mm with an average of 16 mm and $q$ values ranging from 2.8-3.5 with an average of 3.2. The average best fit $a_{\text{min}}$ at the B ring outer edge is 5 mm greater than the average best fit $a_{\text{min}}$ of 9 mm at the A ring outer edge. The proximity of these averages suggests fundamental similarities either in the particle properties or the dynamical environments at these two edges. Figure 4.21 shows the case of the κ Centauri Rev 42 Ingress occultation, in which we find our highest best fit $a_{\text{min}}$ value at the B ring outer
edge. By contrast, we find a best fit $a_{\text{min}}$ of just 1 mm at the $\zeta$ Centauri Rev 060 Ingress occultation, indicating azimuthal variations in $a_{\text{min}}$ at the outer edge of the B ring, which may be attributed to the influence of self-gravity wakes in this region. Self-gravity wakes are known to cause the reflectance asymmetries observed in both the A and B rings by Camichel (1958), Gehrels and Esposito (1981), Thompson et al. (1981), Dones and Porco (1989), Dones et al. (1993), Dunn et al. (2004), and Nicholson et al., (2005).

Figure 4.21: The $\kappa$ Centauri Rev 42 Ingress occultation at the B ring outer edge. (Left) Three model curves are overlain on the data. The best fit $a_{\text{min}}$ and $q$, for which $a_{\text{min}} = 50$ mm and $q = 3.2$, is shown in green. The two other fits shown, $a_{\text{min}} = 20$ mm and $q = 3.2$ and $a_{\text{min}} = 50$ mm and $q = 2.6$, in blue and pink respectively, are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 1σ confidence interval.

Table 4.11: Detections at the outer edge of the B ring.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best Fit $a_{\text{min}}$ (mm)</th>
<th>Best Fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlpAra035I</td>
<td>$10^{+89}_{-5}$</td>
<td>$3.5^{+1.0}_{-1.0}$</td>
<td>35.5</td>
<td>65.4</td>
</tr>
<tr>
<td>KapCen036I</td>
<td>$30^{+69}_{-25}$</td>
<td>$3.5^{+1.0}_{-1.0}$</td>
<td>44.8</td>
<td>67.6</td>
</tr>
<tr>
<td>KapCen042I</td>
<td>$50^{+49}_{-25}$</td>
<td>$3.2^{+0.3}_{-0.6}$</td>
<td>41.4</td>
<td>49.1</td>
</tr>
<tr>
<td>LamSco044I</td>
<td>$10^{+20}_{-5}$</td>
<td>$2.8^{+0.2}_{-0.2}$</td>
<td>250.5</td>
<td>52.2</td>
</tr>
<tr>
<td>ZetCen060I</td>
<td>$1^{+2}_{+4}$</td>
<td>$2.9^{+0.2}_{-0.2}$</td>
<td>107.7</td>
<td>63.2</td>
</tr>
<tr>
<td>BetCen075I</td>
<td>$10^{+49}_{-5}$</td>
<td>$3.5^{+1.0}_{-0.9}$</td>
<td>596.2</td>
<td>88.2</td>
</tr>
<tr>
<td>BetCen081I</td>
<td>$10^{+29}_{-5}$</td>
<td>$3.5^{+1.0}_{-0.7}$</td>
<td>548.2</td>
<td>83.1</td>
</tr>
<tr>
<td>BetCen104I</td>
<td>$1^{+29}_{+29}$</td>
<td>$2.7^{+0.2}_{-0.2}$</td>
<td>358.4</td>
<td>68.0</td>
</tr>
</tbody>
</table>

↑ - Lower limit  
↓ - Upper limit
4.4.4 Outer A ring

4.4.4.1 Encke gap edges

The 300 km Encke gap within the A ring hosts the small satellite Pan, which acts to clear the gap. B16 performed an analysis of diffraction signals at both edges of the Encke gap for occultations up to Cassini’s 114th rev, which occurred on July 14, 2009. We briefly expand on B16’s analysis, adding one new detection at each edge which passed our rigorous blind test for diffraction.

We report one new detection of diffraction at the ζ Centauri Rev 060 Ingress occultation that meets our updated detection criteria. The best fit synthetic signal, $a_{\text{min}} = 50$ mm and $q = 3.5$, is somewhat inconsistent with B16’s findings of a best fit $a_{\text{min}}$ ranging from 5–30 mm and $q$ ranging from 2.7–3.2. The $2\sigma$ contour, however, is quite broad, resulting in large error bars for this detection. Figure 4.22 shows the data overlain with three synthetic diffraction signatures for comparison for the ζ Centauri Rev 060 Ingress occultation.

Figure 4.22: The ζ Centauri Rev 060 Ingress occultation at the inner edge of the Encke gap.
(Left) Three model curves are overlain on the data. The best fit $a_{\text{min}}$ and $q$, for which $a_{\text{min}} = 50$ mm and $q = 3.5$, is shown in green. The two other fits shown, $a_{\text{min}} = 50$ mm and $q = 2.5$ and $a_{\text{min}} = 10$ mm and $q = 3.5$, in blue and pink respectively, are poor fits to the data. (Right) In this case, the best fit model is the only model within the innermost contour representing the 1σ confidence interval. We report errors for this detection to 2σ.

Table 4.12: New* detections at the inner edge of the Encke gap.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $a_{\text{min}}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZetCen060I</td>
<td>$50^{+5}_{-5}$</td>
<td>$3.5^{+1.0}_{-0.9}$</td>
<td>107.7</td>
<td>61.8</td>
</tr>
</tbody>
</table>

* Prior detections reported in B16.

We report a new detection, the $\lambda$ Scorpii Rev 029 Egress occultation, that meets our updated detection criteria. The best fit, $a_{\text{min}} = 10$ mm and $q = 2.8$, is largely consistent with B16’s findings of an average best fit of $a_{\text{min}} = 9.3$ mm and $q = 3.1$ at the Encke gap outer edge, with ranges of $a_{\text{min}} = 3$–30 mm and $q = 2.9$–3.5. Figure 4.23 shows the data overlain with three synthetic diffraction signatures for comparison for the $\lambda$ Scorpii Rev 029 Egress occultation. We note that the “gap” in the 1σ $\chi^2_v$ values on the contour plot is due to the discretization of the tested values (i.e. we only tested $a_{\text{min}}$ values corresponding to the discrete values of 1, 5, 10, 20, 50, and 100 mm) and not indicative of the true nature of the particle-size distribution. In a similar way, tested $q$ values are discretized and range from $2.5 \leq q \leq 3.5$ (in accordance with constraints imposed by Brilliantov et al. 2015).
Figure 4.23: The $\lambda$ Scorpii Rev 029 Egress occultation at the outer edge of the Encke gap. (Left) Three model curves are overlain on the data. The best fit $a_{\text{min}}$ and $q$, for which $a_{\text{min}} = 10$ mm and $q = 2.8$, is shown in green. The two other fits shown, $a_{\text{min}} = 50$ mm and $q = 2.8$ and $a_{\text{min}} = 5$ mm and $q = 2.9$, in blue and pink respectively, are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 2σ confidence interval. Note that the gap in light coloring between $40 \leq a_{\text{min}} \leq 50$ mm is due to the discretization of tested $a_{\text{min}}$ values and not reflective of the nature of the particle size distribution.

Table 4.13: New* detections at the outer edge of the Encke gap.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $a_{\text{min}}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LamSco029E</td>
<td>$10^{+10}_{-5}$</td>
<td>$2.8^{+0.1}_{-0.1}$</td>
<td>287.0</td>
<td>41.7</td>
</tr>
</tbody>
</table>

* Prior detections reported in B16.

4.4.4.2 Keeler gap edges

We analyze both edges of the Keeler gap near the outer edge of the A ring, subject to the limitations described above due to the HSP instrumental ramp-up behavior. Similar to the Encke Gap and its inhabitant moonlet Pan, the Keeler Gap is host to Daphnis, which stirs up longitudinal waves periodically at the gap edges.

We detect a diffraction signature at the inner edge of the Keeler Gap in 10 of 44 detectable occultations. We observe a wide range in best fit values of $a_{\text{min}}$ between 1-100 mm and $q$ from 2.7-3.5. The mean values are $a_{\text{min}} = 22$ mm, which exceeds our expectation for the Keeler gap given its proximity to the outer edge of the A ring, and $q = 3.2$, which is consistent with
B16’s results near this region. Figure 4.24 shows our analysis for the β Centauri Rev 077 Ingress occultation at the Keeler Gap inner edge.

![Figure 4.24: The β Centauri Rev 077 Ingress occultation at the Keeler Gap inner edge.](image)

(Left) Three model curves overlain on the data. The best fit $a_{\text{min}}$ and $q$, for which $a_{\text{min}} = 6$ mm and $q = 3.4$, is shown in green. The two other fits shown, $a_{\text{min}} = 2$ mm and $q = 3.4$ and $a_{\text{min}} = 6$ mm and $q = 2.7$, shown in blue and pink respectively, are shown to be poor fits to the data.

(Right) Models with parameters within the innermost contour are within the 1σ confidence interval. There are no model fits with $a_{\text{min}}$ greater than 30 mm that fall within the 3σ region for this edge.

Table 4.14: Detections at the inner edge of the Keeler gap

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $a_{\text{min}}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KapCen042I</td>
<td>$6_{-5}^{+4}$</td>
<td>$2.9_{-0.1}^{+0.1}$</td>
<td>41.4</td>
<td>49.1</td>
</tr>
<tr>
<td>BetLup057I</td>
<td>$10_{-10}^{+8}$</td>
<td>$3.3_{-0.2}^{+0.4}$</td>
<td>76.3</td>
<td>60.7</td>
</tr>
<tr>
<td>BetCen077I</td>
<td>$6_{-3}^{+2}$</td>
<td>$3.4_{-0.6}^{+0.1}$</td>
<td>588.1</td>
<td>86.0</td>
</tr>
<tr>
<td>BetCen081I</td>
<td>$1_{-9}^{+99}$</td>
<td>$2.7_{-0.2}^{+0.1}$</td>
<td>548.2</td>
<td>83.1</td>
</tr>
<tr>
<td>BetCen089I</td>
<td>$10_{-10}^{+9}$</td>
<td>$3.1_{-0.6}^{+0.4}$</td>
<td>501.2</td>
<td>80.5</td>
</tr>
<tr>
<td>AlpCru100I</td>
<td>$5_{-5}^{+5}$</td>
<td>$2.9_{-0.2}^{+0.1}$</td>
<td>438.5</td>
<td>74.3</td>
</tr>
<tr>
<td>BetCen102I</td>
<td>$11_{-11}^{+1}$</td>
<td>$3.1_{-0.3}^{+0.3}$</td>
<td>370.5</td>
<td>81.8</td>
</tr>
<tr>
<td>BetCen104I</td>
<td>$50_{-40}^{+49}$</td>
<td>$3.5_{-0.8}^{+0.1}$</td>
<td>358.4</td>
<td>66.9</td>
</tr>
<tr>
<td>ZetCen112I</td>
<td>$20_{-10}^{+10}$</td>
<td>$3.4_{-0.9}^{+0.1}$</td>
<td>37.5</td>
<td>68.3</td>
</tr>
<tr>
<td>AlpVir210I</td>
<td>$100_{-50}^{+1}$</td>
<td>$3.3_{-0.8}^{+0.1}$</td>
<td>138.6</td>
<td>29.3</td>
</tr>
</tbody>
</table>

↑ - Lower limit
↓ - Upper limit

We detect a diffraction signature at the outer edge of the Keeler Gap in 12 of 30 detectable occultations. We observe a range in best-fitting values of $a_{\text{min}}$ between 5-50 mm and $q$ from 2.6-3.5. The mean values, $a_{\text{min}} = 14$ mm and $q = 3.3$, are largely consistent with our results.
at the outer edge of the A ring and B16’s results in the same region. Figure 4.25 shows a prominent diffraction signature in the α Crucis Rev 100 Egress occultation at the Keeler gap outer edge.

Figure 4.25: The α Crucis Rev 100 Egress occultation at the Keeler gap outer edge. (Left) Three model curves are overlain on the data. The best fit \( a_{\text{min}} \) and \( q \), for which \( a_{\text{min}} = 14 \) mm and \( q = 3.4 \), is shown in green. The two other fits shown, \( a_{\text{min}} = 14 \) mm and \( q = 2.7 \) and \( a_{\text{min}} = 6 \) mm and \( q = 3.4 \), in blue and pink respectively, are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 1σ confidence interval. The 3σ region excludes \( a_{\text{min}} \) values greater than 50 mm for this occultation.

Table 4.15: Detections at the outer edge of the Keeler gap.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best Fit ( a_{\text{min}} ) (mm)</th>
<th>Best Fit ( q )</th>
<th>( I_0 ) (counts/ms)</th>
<th>( \alpha ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>126Tau008E</td>
<td>( 20^{+79}_{-10} )</td>
<td>( 3.3^{+0.2}_{-0.8} )</td>
<td>3.</td>
<td>89.7</td>
</tr>
<tr>
<td>GamLup030E</td>
<td>( 10^{+29}_{-1} )</td>
<td>( 3.4^{+0.1}_{-0.6} )</td>
<td>80.1</td>
<td>77.3</td>
</tr>
<tr>
<td>AlpVir034E</td>
<td>( 10^{+15}_{-1} )</td>
<td>( 2.6^{+0.1}_{-0.1} )</td>
<td>520.7</td>
<td>18.4</td>
</tr>
<tr>
<td>GamAra037E</td>
<td>( 10^{+99}_{-1} )</td>
<td>( 3.4^{+0.1}_{-0.9} )</td>
<td>27.2</td>
<td>74.3</td>
</tr>
<tr>
<td>ZetCen062E</td>
<td>( 5^{+10}_{-1} )</td>
<td>( 3.1^{+0.2}_{-0.4} )</td>
<td>106.5</td>
<td>74.2</td>
</tr>
<tr>
<td>BetCen064E</td>
<td>( 20^{+79}_{-19.5} )</td>
<td>( 3.3^{+0.2}_{-0.8} )</td>
<td>628.8</td>
<td>88.2</td>
</tr>
<tr>
<td>BetCen077E</td>
<td>( 10^{+5}_{-2} )</td>
<td>( 3.5^{+0.5}_{-1} )</td>
<td>606.0</td>
<td>75.5</td>
</tr>
<tr>
<td>BetCen078E</td>
<td>( 5^{+25}_{-1} )</td>
<td>( 3.2^{+0.3}_{-0.5} )</td>
<td>564.2</td>
<td>75.6</td>
</tr>
<tr>
<td>AlpCru100E</td>
<td>( 14^{+16}_{-1} )</td>
<td>( 3.4^{+0.1}_{-0.6} )</td>
<td>431.7</td>
<td>89.8</td>
</tr>
<tr>
<td>GamCas100E</td>
<td>( 5^{+94}_{-1} )</td>
<td>( 3.4^{+0.1}_{-0.9} )</td>
<td>56.0</td>
<td>80.1</td>
</tr>
<tr>
<td>BetCen105E</td>
<td>( 10^{+29}_{-5} )</td>
<td>( 3.5^{+0.7}_{-1} )</td>
<td>300.5</td>
<td>88.4</td>
</tr>
<tr>
<td>AlpVir232E</td>
<td>( 50^{+49}_{-30} )</td>
<td>( 2.9^{+0.5}_{-1} )</td>
<td>132.1</td>
<td>87.8</td>
</tr>
</tbody>
</table>

\( \uparrow \) - Lower limit
\( \downarrow \) - Upper limit
4.4.4.3 A ring outer edge

B16 performed an analysis of diffraction signatures at the outer edge of the A ring for occultations up to Cassini’s 114th rev on July 14, 2009. We detect a diffraction signal at the A ring outer edge in seven additional occultations, two of which became available after the publication of B16, α Virginis 210 Ingress an α Virginis 211 Ingress. Figure 4.26 and Figure 4.27 show the diffraction signature overlain with three models for comparison for the α Virginis 210 and the α Virginis 211 Ingress occultations, respectively, which indicate an unusual best fit $q$ of only 2.6. The red dashed lines illustrate the error bars for the α Virginis 210 Ingress occultation on $a_{\min}$ and $q$. Overall, we find a mean best fit $a_{\min}$ of 7 mm and a mean best fit $q$ of 3.1 for the 7 new occultations presented in this work, with best-fitting $a_{\min}$ values ranging from 1−10 mm and best-fitting $q$ values ranging from 2.6−3.5. These results are largely consistent with B16’s findings of an average best fit of $a_{\min} = 4.4$ mm and $q = 3.2$, with ranges of $a_{\min}$ ranging from 1−10 mm and $q$ ranging from 2.8−3.5. Combining B16 and our results for occultations that pass our test for detectability, we find the mean best fit $a_{\min}$ and $q$ values at the outer edge of the A ring are 5 mm and 3.2 respectively. Our results for the A ring outer edge are listed in Table 4.16.

Figure 4.26: The α Virginis Rev 210 Ingress occultation at the A ring outer edge.
(Left) Three model curves overlain on the data. The best fit $a_{\text{min}}$ and $q$, for which $a_{\text{min}} = 5$ mm and $q = 2.6$, is shown in green. The two other fits shown, $a_{\text{min}} = 20$ mm and $q = 2.5$ and $a_{\text{min}} = 20$ mm and $q = 3.2$, in blue and pink respectively, are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 1σ confidence interval.

Figure 4.27: The α Virginis Rev 211 Ingress occultation at the A ring outer edge. (Left) Three model curves overlain on the data. The best fit $a_{\text{min}}$ and $q$, for which $a_{\text{min}} = 1$ mm and $q = 2.6$, is shown in green. The two other fits shown, $a_{\text{min}} = 1$ mm and $q = 2.8$ and $a_{\text{min}} = 1$ mm and $q = 2.7$, in blue and pink respectively, are poor fits to the data. (Right) Models with parameters within the innermost contour are within the 1σ confidence interval. Note that the 1σ contour is restricted to $a_{\text{min}} \leq 5$ mm and $q$ values $\leq 2.7$.

Table 4.16: New* detections at the outer edge of the A ring.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $a_{\text{min}}$ (mm)</th>
<th>Best fit $q$</th>
<th>$I_0$ (counts/ms)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlpAra032I</td>
<td>$10_{-5}^{+40}$</td>
<td>$3.5_{-0.8}^{+1}$</td>
<td>38.9</td>
<td>82.5</td>
</tr>
<tr>
<td>KapCen042I</td>
<td>$10_{-5}^{+90}$</td>
<td>$3.1_{-0.4}^{+0.6}$</td>
<td>41.4</td>
<td>49.1</td>
</tr>
<tr>
<td>DeLup057I</td>
<td>$10_{-5}^{+50}$</td>
<td>$3.5_{-0.8}^{+1}$</td>
<td>48.6</td>
<td>80.6</td>
</tr>
<tr>
<td>BetCen077I</td>
<td>$5_{-1}^{+35}$</td>
<td>$3.2_{-0.8}^{+0.3}$</td>
<td>588.1</td>
<td>86.0</td>
</tr>
<tr>
<td>BetCen085I</td>
<td>$5_{-1}^{+5}$</td>
<td>$3.4_{-0.2}^{+0.1}$</td>
<td>533.8</td>
<td>81.0</td>
</tr>
<tr>
<td>AlpVir210I</td>
<td>$5_{-4.5}^{+10}$</td>
<td>$2.6_{-0.1}^{+0.1}$</td>
<td>138.6</td>
<td>29.4</td>
</tr>
<tr>
<td>AlpVir211I</td>
<td>$1_{-1}^{+4}$</td>
<td>$2.6_{-0.1}^{+0.4}$</td>
<td>135.5</td>
<td>31.9</td>
</tr>
</tbody>
</table>

* Prior detections reported in B16.

↑ - Lower limit
↓ - Upper limit
4.5 Discussion

4.5.1 Detection frequency

We have developed and applied an objective test for detecting diffraction at the edges of rings, ringlets, and gaps in Saturn’s rings in Cassini UVIS stellar occultation data. We scanned for diffraction signals in each occultation at all 17 edges in this study, controlled for observational selection effects, and calculated the best fit $a_{\text{min}}$ and $q$ values for each detection. Table 4.17 summarizes the rate of detection at each edge based on the number of opportunities (brightness and geometry criteria met) to observe a diffraction signal, in decreasing order of the distance of the edge from the planet.

Table 4.17: Frequency of diffraction signal detection.

<table>
<thead>
<tr>
<th>Edge</th>
<th>Detectable $^1$</th>
<th>Total Number of Detections $^2$</th>
<th>Frequency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Ring Outer*</td>
<td>46</td>
<td>21</td>
<td>45</td>
</tr>
<tr>
<td>Encke Inner*</td>
<td>45</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>Encke Outer*</td>
<td>33</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Keeler Inner</td>
<td>44</td>
<td>10</td>
<td>22</td>
</tr>
<tr>
<td>Keeler Outer</td>
<td>30</td>
<td>11</td>
<td>36</td>
</tr>
<tr>
<td>B Ring Outer</td>
<td>45</td>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>Strange (R6) Inner</td>
<td>33</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>Strange (R6) Outer</td>
<td>46</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Huygens Inner</td>
<td>33</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Huygens Outer</td>
<td>46</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Laplace Inner</td>
<td>33</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Herschel Inner</td>
<td>33</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Herschel Outer</td>
<td>39</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Maxwell Inner</td>
<td>23</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Maxwell Outer</td>
<td>36</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Titan Inner</td>
<td>17</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>Titan Outer</td>
<td>35</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*A ring and Encke gap results include detections reported in B16 that pass our detection criteria (described in Section 3.4). 1. Lists the total number of occultation cuts across the edge that meet the star brightness and occultation geometry criteria described in the text. 2. Detections include only detections from the set of detectable occultations.
We observe a systematic decrease in the rate of detected diffraction signatures with increasing proximity to Saturn. This is consistent with the overall decrease in minimum particle size as a function of radial distance from Saturn described in Tyler et al. (1983), Zebker et al. (1985), and French and Nicholson (2000). We also analyzed diffraction signature detections at the A ring outer edge for 7 new occultations not included in B16. We find one new detection at each of the Encke gap edges; the ζ Centauri Rev 060 Ingress occultation at the inner edge and the λ Scorpii Rev 029 Egress occultation at the outer edge. Surprisingly, we did not find any detections of diffraction signatures at either edge of the Encke gap in later Cassini orbits.

4.5.2 Test for correlation of detection frequency with perturbing satellite longitudes

The two most prominent gaps in the outer region of the A ring, the Encke and Keeler gaps, are cleared by the moons orbiting within them, Pan and Daphnis, respectively. To study the relative effect of small gap satellites such as Pan on the particle size distribution, B16 analyzed the correlation of Pan’s longitude relative to the longitude of the detected diffraction signatures in the Encke gap. At the outer Encke gap edge, they found that the only two best-fitting $a_{min}$ values greater than 10 mm as well as non-detections were correlated with recent encounters with Pan. At the inner edge of the Encke gap, however, they found no correlation between $a_{min}$ and Pan. They concluded that while density waves have a stronger effect on collisions in this region, the correlation with Pan at the Encke Gap outer edge is evidence of the predator-prey influence of satellites (Esposito et al. 2012) in which moon forcing triggers the temporary aggregation of particles resulting in streamline crowding.

We carried out the same analysis for the Keeler gap, which was not analyzed in B16.
We find no correlation between the best-fitting $a_{\text{min}}$ values at the Keeler Gap edges and their longitude relative to Daphnis. A possible explanation for this may be that, as a result of its orbiting much closer to the gap edges, Daphnis deflects particle streamlines such that the wakes are damped over much shorter time scales than wakes at the edges of the Encke gap, making its effect increasingly difficult to observe at most viewing geometries. Our results in the outer A ring are consistent with the results of J16, who found an increasing number of sub-cm sized particles with decreasing distance from the A ring outer edge. We also investigated the longitudes of perturbing moons Mimas and Janus relative to the outer B ring and A ring edges, respectively. We found no correction between the best-fitting $a_{\text{min}}$ values at these edges and their longitudes relative to perturbing moons.

**4.5.3 Physical implications**

We find the greatest number of diffraction detections, which indicate the smallest particles, at the edges of the B ring, A ring, and Encke and Keeler gaps, with very few detections at the edges of rings that are not in resonance with a moon. Because we expect mean ring particle radius to be smaller with more energetic collisions between ring particles (Bodrova et al. 2012), our results suggest higher collision velocities at the edges directly perturbed by moons. This is consistent with predictions of the predator-prey model where satellite perturbations drive cyclical growth of large aggregates that in turn gravitationally accelerate the background ring particle population to higher collision velocities (Esposito et al. 2012).

The largest best fits to $a_{\text{min}}$ at the Keeler gap edges are considerably larger than our results for the nearby outer A ring edge, which may be attributed to collisional activity among particles as the moonlet Daphnis stirs up waves at the edges of the gap. Our results imply that this stirring
influences the particle size distribution in the region and induces more clumping. We note that we observe our three highest detection frequency rates at the A ring outer, Keeler gap outer, and Keeler gap inner edges, respectively. Detection frequency at the Encke gap is also relatively high, with a detection at 15% of detectable occultations for both edges. Our results support the hypothesis that regions perturbed by satellite resonances are subject to a predator-prey model of aggregation and fragmentation.

At the outer edge of the B ring, we find a relatively high detection frequency of 18%, which is consistent with our expectation given that the outer edge of the B ring is confined by resonances with Mimas in a similar way that the A ring outer edge is confined by resonances with Janus and Epimetheus. Our result for the κ Centauri Rev 42 Ingress occultation contrasts starkly with our other best fits for this edge, which could indicate azimuthal variations in $a_{\text{min}}$ at the outer edge of the B ring due to the presence of self-gravity wakes. We do not find any evidence for a significant population of particles of sub-millimeter size. This indicates that the collisions in these regions are not energetic enough to overcome contact forces between dust and larger aggregates (Albers and Spahn, 2006).

We find the lowest frequency of diffraction signature detection in the Cassini Division and C ring, which host a number of narrow ringlets. This scarcity of detections indicates a lesser population of sub-cm particles in these regions in comparison with the outer A and B ring regions, which are subject to the predator-prey influence of satellites (Esposito et al. 2012). These null results suggest that the transient population of free-floating particles is particularly small in these regions.
5.1 Occultation Statistics

The distribution of photon counts from a star measured by an ideal photometer is described by Poisson statistics. Fortuitously, these properties can be used to obtain information about the particle size distribution in Saturn’s rings. In Chapter 4, we described a forward-modeling approach to estimating the parameters $a_{\text{min}}$ and $q$ of the particle size distribution at ring edges. An alternative way to constrain the size distribution is through examining the statistical moments of the data, which is the focus of this chapter.

Much information about the particle size distribution of Saturn’s rings can be gleaned from a simple analysis of the statistics of stellar occultations. In this chapter, we will review the methodology of Showalter and Nicholson (1990) (hereafter SN90) and Colwell et al. (2018) (hereafter C18), who used the statistics of stellar occultation data (the former from Voyager 2 and the latter from Cassini) to infer information about the particle size distribution in Saturn’s rings. We will review their model and its limiting assumptions which are invalid in regions populated by large clumps such as self-gravity wakes. C18 did not explore variations in particle properties within local structures, such as density waves. Images and models (Esposito et al., 2012) suggest formation and destruction of clumps within density waves. For an overview of self-gravity wakes see section 1.5.1 and for density waves see section 1.5.2. We examine the troughs and peaks of density waves to look for variations in particle properties there. Further, we extend their methodology to self-gravity wake dominated ring regions by implementing a direct moments-based approach for the moments of the distribution of counts from first principles. We begin by applying the basic moments model to three small density waves in the Cassini Division where the techniques of SN90 and C18 are applicable. Then we will build upon the moments
approach to account for self-gravity wakes in the A and B rings based on the parameters of Colwell et al.’s (2006) granola bar model. We report our results for 9 density waves: 3 Cassini Division waves where there are no self-gravity wakes (Prometheus 9:7, Pan 6:5, and Atlas 5:4) and 6 waves in regions dominated by self-gravity wakes (Janus 2:1 in the inner B ring, Pandora 5:4 and Janus 4:3 in the inner A ring, and Janus 5:4, Mimas 5:3 and Janus 6:5 in the outer A ring).

5.1.1 The Poisson distribution

The Poisson distribution, named after French mathematician Siméon Denis Poisson, is a statistical distribution for a random variable $k$ defined by a mean rate $\lambda$ when the probability of the occurrence of each event is independent of the previous event. The probability of obtaining $k$ events in a fixed time interval where the events occur at a constant mean rate $\lambda$ is given by

$$p(k; \lambda) = \frac{e^{-\lambda\lambda^k}}{k!},$$

where $k$ is an integer. The Poisson distribution is the standard approach to modeling event count data (i.e., photons hitting a detector). In the case of stellar occultation observations by the UVIS HSP, $k$ is the number of photons detected in a single integration period and $\lambda$ is the mean stellar signal in counts per integration period.

Perhaps the most well-known property of the Poisson distribution is that the variance is equal to the mean:

$$\sigma(k) = \sqrt{\lambda}.$$  

(5.2)

In actuality, the HSP raw data measurements are the sum of two Poisson processes. Consider an observation of a star by Cassini with no ring material along the line of sight. In addition to counting the incident stellar photons (which are incident on the detector with mean rate $\lambda_\phi$), the
HSP would also be counting background photons (at a rate $\lambda_B$) due to both the Lyman-\(\alpha\) emission of interplanetary hydrogen and scattered solar photons which are also Poisson distributed. When the rings are present, there is additional scattered sunlight off the rings, increasing $\lambda_B$. Because the sum of two Poisson random variables is also Poisson distributed the total mean signal in the absence of attenuation due to the rings is

$$\lambda_T = \lambda_S + \lambda_B.$$  \hspace{1cm} (5.3)

If we were to observe a star by itself, our measurements would follow this distribution. However, Saturn’s rings obscure the incident light so that the portion of the integration area that is not blocked by particles is no longer constant and attenuates the light by a factor of $e^{-\tau}$. In this scenario, the fraction of the integration area occupied by particles is a random variable and the probability of two photons being transmitted through, or absorbed by, the rings is no longer independent, violating Poisson statistics. Due to the finite sizes of the ring particles, if a photon is absorbed by a ring particle, the next photon’s probability of absorption or transmission is correlated with the fate of the other photon. Ring regions populated by smaller sized particles, like the C ring, behave more like a Poisson distribution than the A ring with its population of large self-gravity wakes, as illustrated in Figure 5.1.
Figure 5.1. Statistical scatter of the Cassini UVIS HSP stellar occultation \(\beta\) Centauri Rev 085 Ingress in two relatively featureless segments of the C ring (left, 82100-82200 km) and A ring (right, 126000-126100 km).
The solid green line shows the mean count rate \(\mu\), the blue dashed line represents \(\mu \pm 1\sigma\), and the red dash-dot line \(\mu \pm \sqrt{\mu}\). It is clear that the data are well-described by Poisson statistics in the C ring, where \(\mu \pm \sqrt{\mu}\) and \(\mu \pm 1\sigma\) are nearly the same value, while in the A ring \(\mu \pm 1\sigma\) has a much greater spread than \(\mu \pm \sqrt{\mu}\), indicating that Poisson statistics are violated.

5.1.2 Role of stochastic processes in ring particle dynamics

The dynamical evolution of Saturn’s rings is governed by a combination of deterministic and random (or stochastic) processes. One example of a deterministic approach to modeling ring particle dynamics is through numerical simulations (described in section 1.3.5.1) of a theoretical ring system which replicate the forces (collisional and gravitational) acting on each ring particle in a simulation with millions of particles (e.g., Salo (1995), Lewis and Stewart (2000, 2009), Salo (2001), Salo et al. (2004), Schmidt et al. (2009)). We can also gain an understanding of the physical characteristics (size distribution, presence of clumps and gaps) of the ring particles by matching the observed characteristics of the occultation data with simulated occultations of different ring particle configurations. Detailed N-body simulations may not produce distributions that match the observed data, leaving one to wonder what the underlying distribution is. Random or arbitrary distributions of ring particles can be used to find configurations that are consistent
with the data and then provide insight into the underlying physical processes that can lead to such configurations.

Stochastic processes are a sequence of events that adhere to probabilistic laws. Large collisional events occur sporadically, and although these events are rare, they reset the state of the system. The importance of a small number of major catastrophic events in the history of a planetary ring system may explain why the ring systems around the giant planets in our solar system all appear so different. One way to simulate planetary ring evolution probabilistically is using Monte Carlo simulations (see section 1.3.5.2 for a review of Monte Carlo simulations of the rings). In this section, we will investigate the statistics of stellar occultations of Saturn’s rings through a probabilistic lens.

At a given optical depth a larger excess variance indicates the presence of larger particles. In geometric terms, large particles increase the variance of the data because doubling the particle’s radius quadruples its cross-sectional area (assuming \(\pi r^2\) for spherical particles), thereby quadrupling the portion of the integration area obstructed. The largest ring particles have cross-sections of up to a hundred square meters and self-gravity wakes are even larger. In cases where the ring particles and wakes are comparable in size to the instrument FOV, stochastic variation in the fraction of the integration area blocked by these particles results in correlations in the blocking of photons and introduces extra variance in the data. A schematic representation of the difference between correlated and uncorrelated photons incident on a hypothetical integration area is shown in Figure 5.2.
Figure 5.2. A schematic representation of the difference between strongly correlated (left) and weakly correlated (right) photon events on a hypothetical integration area. Consider the above cartoon a representation of particles (black circles) in the HSP field-of-view over a hypothetical slab of ring material. Two separate integration areas are represented by the red squares. In the leftmost square, multiple photons (yellow dots) are incident on the largest particles and the events are strongly correlated. In the rightmost square, most of the photons hit either no particles or different particles; these events are only weakly correlated.

Although the simple model of ring particles as spheres of uniform size employed by SN90 and C18 has great utility in regions of low optical depth and stands as the framework for particle size distribution statistical analysis, it has many limitations. Perhaps the strictest condition for meaningful interpretation of the model is that the optical depth must be sufficiently low. When this condition is violated, the sizes of the clumps are large enough that the correlation of photons becomes significant. The fact that this condition is unmet for a large portion of the rings implies a need for a different approach, which we will introduce in section 5.1.4.

5.1.3 Precedent studies

Because the sizes of individual ring particles are much smaller than even the finest resolution Cassini instruments, they cannot be directly observed. Despite this, occultation statistics can be used to deduce clues about the size distribution. Before the Voyager missions, ground-based observations of the thermal emission, radar cross section, eclipse-induced cooling, and opposition brightening of the rings at visual wavelengths provided a baseline mean particle
size distribution (Aumann and Kieffer, 1973; Kawata and Irvine, 1974; Cuzzi and Pollack, 1978; Cuzzi et al., 1980). Their results indicated that the particle size distribution of Saturn’s rings was well-described by a simple power-law,

\[ n(r)dr \propto r^{-q}dr, \]  

where \( q \approx 3 \), and indicated a broad size distribution for the main rings with particles ranging from centimeters to meters in size.

Voyager radio occultations provided optical depth measurements at two wavelengths (3.6 and 13 cm), allowing the size distribution to be inferred from differential optical depth measurements (see \( \tau_n(\lambda) = \int_{a_{\text{min}}}^{a_{\text{max}}} \pi a^2 Q_{\text{occ}}(a, \lambda, f) n(a) \, da \) \( (1.2) \)). The number distribution of the particles was also constrained by comparing occultation optical depth profiles at different wavelengths using

\[ \tau(\lambda) \approx \int_{\lambda/3}^{\infty} (\pi r^2) n(r) \, dr. \]  

(5.5)

The lower integration limit of \( \lambda/3 \) arises from the fact that particles much less than the wavelength of the observation have negligible interaction with the incident light (Hansen and Travis, 1974). A more precise version of Eq. (5.5) includes the wavelength-dependent extinction coefficient, \( Q(r, \lambda) \).

Voyager measured the optical depth across the rings at three different wavelength ranges: ultraviolet, analyzed by Lane et al. (1982) and Sandel et al. (1982); visible, analyzed by Cuzzi et al. (1984); and microwave, analyzed by Tyler et al. (1983). Later, Marouf et al. (1983) and Zebker et al. (1985) constrained the upper end of the size distribution by inverting particle scattering cross sections from Voyager radio occultations.
5.1.3.1 Showalter and Nicholson (1990)

SN90 were the first to interpret the statistical moments of a stellar occultation of Saturn’s rings in terms of the particle size distribution in their analysis of the Voyager 2 photopolarimeter (PPS) stellar occultation of δ Scorpii. They determined that random variations in the finite sizes of the ring particles cause an excess variance in the data beyond the statistical variance expected of a Poisson distribution. They used Poisson statistics to derive an expression for the excess variance in the data which they used to constrain the upper end of the particle size distribution throughout the rings, with the exception of the central B ring, which was effectively opaque to the δ Scorpii Voyager occultation. Rather than determining each of the size parameters, they calculated an effective, cross-section weighted, particle radius:

\[
R_{\text{eff}} = \frac{\int_{a_{\text{min}}}^{a_{\text{max}}} a^4 n(a) da}{\sqrt{\int_{a_{\text{min}}}^{a_{\text{max}}} a^2 n(a) da}}.
\]

(5.6)

SN90 found smallest particle sizes in the C ring, where they determined \(R_{\text{eff}} < 2.8\) m. In the background C ring and C ring ramp, they found \(R_{\text{eff}} \sim 1.2 - 2.4\) m. They reported similarly small particle sizes in the Cassini Division, where they placed an upper bound of \(R_{\text{eff}} < 4.5\) m. In the optically thicker rings, they found larger effective particle cross sections: \(5.7 \leq R_{\text{eff}} \leq 8.8\) m in the inner B ring and \(R_{\text{eff}} \sim 9 - 12\) m in the A ring. The large \(R_{\text{eff}}\) values they found in the A ring far exceeded previous estimates from observations by the Voyager radio science subsystem (RSS) such that the ratio of the results was \(\frac{R_{\text{eff}}(\text{PPS})}{R_{\text{eff}}(\text{RSS})} = 7 \pm 0.6\).

Long before the publication of SN90, numerous observations had identified a distinct azimuthal brightness asymmetry in the A ring which was found to be caused by self-gravity wakes (Camichel, 1958; Colombo et al., 1976; Lumme and Irvine, 1976; Lumme et al., 1977; Gehrels and Esposito, 1981; Thompson et al., 1981; Dones and Porco, 1989). Citing these
observations, SN90 suggested that the inconsistency of their results with radio occultation data might be caused by the presence of these wake structures which invalidate the foundational assumptions of their approach.

Although the simple model of the ring particles as spheres has great utility in regions of low optical depth and stands as the framework for particle size distribution statistical analysis, it also has many limitations. First, it assumes all particles to be spheres of cross-sectional area $\pi R^2$; second, it assumes that the particles are much smaller than the integration area. Therefore, it does not account for wakes or clumps which are clearly observed in regions like the A and B rings and are both non-spherical and comparable in size to the integration area. Further, it is only valid for data that accurately approximate a Poisson distribution. Large clumps and self-gravity wakes may obscure enough of the integration area that multiple photons are blocked by the same clump and the photon events become correlated, violating Poisson statistics. Additionally, the model assumes an integer number of particles and as such it does not account for particles which may intersect the edges of the integration area.

Since SN90, we have learned a great deal more about the structure and orientation of self-gravity wakes throughout Saturn’s rings. Colwell et al. (2006, 2007) presented a granola bar model for these wakes, Hedman et al. (2007b) developed a similar model with tubes with elliptical cross sections instead of rectangular cross sections, and Jerousek et al. (2016) (hereafter J16) derived estimates for self-gravity wake parameters across macroscale structures in the rings, to name a few. To date, however, no study has been performed to evaluate these parameters within the spiral density waves which are known to crowd the highly perturbed A ring. In this chapter, we seek to bridge this gap in our knowledge of the particle size distribution of Saturn’s
rings, specifically focusing on the distribution within spiral density waves which coexist with self-gravity wakes.

5.1.3.2 Colwell et al. (2018)

Toward the end of their paper, SN90 predicted that a more detailed analysis would be performed with Cassini’s higher resolution and signal-to-noise data. The Cassini UVIS HSP has several important advantages over the Voyager 2 PPS experiment which we utilize in this analysis. First, the HSP observed far more stellar occultations; that is, a total of 197 stellar occultations of the rings from a wide range of viewing configurations compared to the single occultation of δ Scorpii observed by the PPS. Additionally, the HSP’s faster integration period of 1-2 milliseconds enabled an approximate spatial resolution of roughly 10 meters, a factor of 10 finer than the resolution of the PPS experiment, which had a 10-millisecond integration period and a radial resolution of ~100 meters. Finally, because the HSP was more sensitive than the PPS it was able to observe stars with a higher signal-to-noise ratio, which is why we see peak signals of several $10^5$ counts/s in the UVIS HSP data compared to a maximum signal of ~ 4000 counts/s in the Voyager PPS occultation. In our analysis we follow C18 in using these unique advantages to perform a more detailed statistical survey of the rings.

C18 employed the same methodology as SN90 to compute an analytic expression for an effective particle size from the statistical moments of Cassini UVIS stellar occultation data across the entire rings at 10 km resolution for two occultations: β Centauri Rev 077 Ingress and α Virginis Rev 008 Egress. We will review the derivation of this expression in section 5.2.3. Overall, they found a smaller effective particle size in the C ring plateaus than they did in the background C ring. They also reported a positive correlation between $R_{\text{eff}}$ and optical depth in
the background C ring. In the Cassini Division, which has a similar optical depth range to the C ring, they observed different trends in $R_{\text{eff}}$ with optical depth. They also noted a sharp jump in $R_{\text{eff}}$ on either side of the Cassini Division ramp but a smooth gradient in $R_{\text{eff}}$ across the C ring ramp.

In addition, their analysis indicated sharp transitions in effective particle size near locations of resonance with some of Saturn’s moons. They attributed these jumps to the fact that their binning resolution of 10 kilometers was insufficient to detect variations at the scale required for density wave oscillations—one of the motivations for this study.

5.1.4 A new approach

In this dissertation we will expand on the analyses of both SN90 and C18 by applying a modified method of moments from first principles to interpret the sizes of particles/clumps from excess variance in stellar occultation data. We will implement this technique on density waves in the Cassini division, inner B ring, and A ring. Our work is unique in that we analyze the statistics of stellar occultation data of Saturn’s rings at higher radial resolution than any previous work and simultaneously account for both density waves and self-gravity wakes in the A ring and inner B ring. We also implement an original algorithm for partitioning density waves into individual peaks and troughs and remove secular trends from the data, allowing a unique interpretation of the statistical moments. We compute the second, third, and fourth order statistical moments of the data (e.g., excess variance, skewness, and kurtosis) for 35 stellar occultations of Saturn’s rings for 9 spiral density waves in different regions of the rings. In this chapter, we will perform an in-depth analysis of density waves from occultation statistics by computing the variance, skewness, and kurtosis within the peaks and troughs of each wave for a multitude of occultations and comparing and contrasting the results.
5.2 Accounting for structures in Saturn’s rings

Cassini detected structures in Saturn’s rings on all observed spatial scales. Perhaps the most visually striking of these structures are spiral density waves, oscillations in the particle number density in the rings at resonance locations caused by perturbations by local satellites. Driven by the exchange of angular momentum between the satellites and ring particles, numerous spiral density waves punctuate Saturn’s A ring, producing distinctive azimuthal structure akin to the arms of spiral galaxies (Goldreich and Tremaine, 1982). The tightly wrapped arms of the wave are created by alternating compressed and rarefied ring particles. As the disturbance propagates toward the perturbing satellite, energy and angular momentum transfer occurs through interparticle collisions which act to damp the wave. The radial extent of undulations in optical depth within density waves ranges from just tens of kilometers in Cassini Division waves to hundreds of kilometers in A ring waves, and their wavelengths range from on the order of hundreds of meters up to tens of kilometers. Here we take advantage of the high spatial resolution of the HSP data for analyzing fine ring structures such as spiral density waves to calculate the statistics within the wave peaks and troughs separately.

5.2.1 Statistical moments

We will begin with a brief summary of each of the statistical moments we analyze and how they may inform us about the particle size distribution in Saturn’s rings. The first statistical moment of a distribution of data is its mean, \( \lambda \). In the case of a random variable \( X \), the mean is defined as the expectation value of \( X \) such that \( E[X] = \lambda \). The second order moment of the data is the variance, \( \sigma_X^2 \), which describes the spread of the data about the mean. In terms of expectation values, the variance can be expressed as

\[
\text{Var}[X] = E[X^2] - E[X]^2 = \sigma_X^2. 
\] (5.7)
The variance is high when the data are widely spread and low when most measurements are close to the mean value, as illustrated in Figure 5.3. When we talk about excess variance, we mean variance in excess of that given in $\text{Var}[X] = E[X^2] - E[X]^2 = \sigma_X^2$.  

(5.7). This excess variance can be caused by the finite sizes of ring particles blocking the starlight (SN90, C18).

![Visual representation of statistical variance](image)

Figure 5.3. Visual representation of statistical variance. Consider a plot of the cumulative frequency of draws from a hypothetical data set. The x axis consists of all possible outcomes and the y axis is the cumulative sum of those outcomes in the data set. Data with high variance stretch more evenly across the possible outcomes while data with low variance peak sharply at the central outcome.

Skewness measures the asymmetry of a distribution of data about its mean. Data that follow a normal distribution are symmetric such that the mean, median, and mode are all equal. Positively skewed data have a mean greater than both the mode and median, while negatively skewed data have a mean lower than both the median and mode. For a Poisson distribution, the skewness is given by

$$\gamma[X] = \frac{1}{\sqrt{\lambda}},$$

(5.8) where $\lambda$ is the mean. In terms of expectation values, the skewness of a random variable $X$ can be expressed in terms of the first three moments of a random variable $X$ as
\[
\gamma[X] = \frac{E[X^3] - 3E[X]\sigma_X^2 - E[X]^3}{\sigma_X^3},
\]
which simplifies to
\[
\gamma[X] = \frac{E[X^3] - 3\lambda \sigma_X^2 - \lambda^3}{\sigma_X^3}.
\]

Figure 5.4 illustrates the difference between distributions with positive, negative, and zero skewness.

In terms of Cassini UVIS stellar occultations, data that are positively skewed have a higher frequency of high photon counts relative to the mean, while data that are negatively skewed have a higher frequency of low photon counts relative to the mean. Data collected over a section of ring material that is highly skewed can be indicative of either gaps or clumps in that region of the rings. Gaps, which appear as high transparency outliers in the distribution, cause a positive skewness in the data while clumps cause a negative skewness by introducing more low transparency (e.g., low photon count rate) outliers.

Figure 5.4. Visual representation of statistical skewness.
Consider a plot of the cumulative frequency of draws from a hypothetical data set. The x axis consists of all possible outcomes and the y axis is the cumulative sum of those outcomes in the data set. When the tail on the right-hand side is longer, the data are positively skewed. When the tail on the left-hand side is longer, the data are negatively skewed. In statistical terms, when the
skewness equals zero, mean = median = mode; when the skewness is positive, mean > median > mode; and when the skewness is negative, mean < median < mode.

Kurtosis is the fourth moment of a statistical distribution which measures the relative prevalence of the extreme values of the distribution, often described in terms of the fatness of the tails. For a Poisson distribution,

\[ \kappa[X] = \kappa' + 3 = \frac{1}{\lambda}, \]  

(5.11)

where \( \kappa' \) is the excess kurtosis. In terms of expectation values, the kurtosis can be expressed in terms of the first four moments of random variable \( X \) as

\[ \kappa[X] = \frac{E[X^4] - 4E[X]^2 + 6E[X]E[X^3] - 3E[X]^2}{\sigma_X^4}, \]  

(5.12)

which simplifies to

\[ \kappa[X] = \frac{E[X^4] - 4\lambda E[X]^2 + 6\lambda^2 \sigma_X^2 - 3\lambda^4}{\sigma_X^4}. \]  

(5.13)

Because the Kurtosis is highly sensitive to extreme values, the presence of both ghosts and clumps strongly affects it. Kurtosis over the whole rings for all of the HSP stellar occultations is highly correlated with skewness, making it particularly difficult to parse any additional information about the underlying ring structure from kurtosis that is not already evident from the skewness. Figure 5.5 illustrates the difference between distributions with positive (leptokurtic), negative (platykurtic), and zero (mesokurtic) excess kurtosis. Leptokurtic distributions have a high number of outliers consistent with a high probability of extreme events while platykurtic distributions are the reverse.
Figure 5.5. Visual representation of statistical kurtosis. Consider a plot of the cumulative frequency of draws from a hypothetical data set. The x axis consists of all possible outcomes and the y axis is the cumulative sum of those outcomes in the data set. The red curve representing a distribution with positive kurtosis is the most peaked while also having thicker tails than the normal distribution (which is shown in blue). The green curve representing a distribution with negative kurtosis is flatter and wider with thinner tails.

5.2.2 Statistics across Saturn’s rings

The general trends of the statistics observed in different regions of Saturn’s rings are shown for sample occultation β Centauri Rev 089 Ingress occultation in Figure 5.6 below, which plots the ratios of the statistical moments across Saturn’s rings for one UVIS occultation at 100 km resolution. Like C18, we find that the ratio of the variance to the mean is nearly always greater than 1, and is highest in optically thick regions like the A and B rings.
Figure 5.6. Statistical moments of the β Cen 089 Ingress occultation across the whole rings.
(a) Optical depth profile of Saturn’s main rings from the β Centauri Rev 089 Ingress occultation. Raw data are plotted in gray while smoothed data are overlaid as a solid black line. (b) The ratio variance over the mean for each 100 km segment of ring material. There are just a few locations at which the ratio equals one, implying that the statistics throughout the rings are largely non-Poissonian. (c) The ratio of the variance to the skewness for each 100 km segment of ring material. (d) The ratio of the skewness to the kurtosis for each 100 km segment of ring material. These two moments are nearly the same across the rings with the exception of the optically thick B ring.
5.2.3 Single-sized particle model

Rather than constraining the parameters of the underlying size distribution, C18 sought to find an effective particle length scale, $R_{\text{eff}}$, across the rings. We follow their methodology in relating excess variance to a single effective particle size in the density waves in the Cassini Division, where it is safe to assume $\delta \tau \ll 1$. For a ring particle of cross-sectional area $\pi R^2$, C18 defined the normalized shadow area, $\delta$, as

$$\delta = \frac{\pi R^2}{\mu A}, \quad (5.14)$$

where $\mu = \sin |B|$ and $A$ is the integration area, which ranges from hundreds to millions of square meters depending on the occultation. The $\mu A$ component in the denominator is a normalization factor that accounts for changes in viewing geometry and integration area from one occultation to another.

The method outlined by C18 assumes that the positions of particles within $A$ are uncorrelated and that the number of particles within the integration area over a single integration period, $N_R$, follow a Poisson distribution such that:

$$N_R = \frac{-\tau_N \mu}{\ln (1 - \delta)}, \quad (5.15)$$

Following that, the expectation value of the ring transparency is

$$\langle T \rangle = \exp(-N_R \delta) = \exp\left(\frac{\delta \tau_N \mu}{\ln (1 - \delta)}\right), \quad (5.16)$$

and

$$\langle T^2 \rangle = \exp\left(\frac{\tau_N \mu}{\ln (1 - \delta)} [2\delta - \delta^2]\right). \quad (5.17)$$

From here, the excess variance can be computed in a straightforward way from the expectation values of the ring transparency as

$$E = \langle T^2 \rangle - \langle T \rangle^2 \quad (5.18)$$
\[ = \exp\left(\frac{\tau N / \mu}{\ln(1 - \delta)} \left[2\delta - \delta^2\right]\right) - \exp\left(\frac{2\delta \tau N / \mu}{\ln(1 - \delta)}\right). \]

(5.19)

Finally, by using a Taylor series expansion on the first exponential term and ignoring terms with higher order \( \delta \), they found that the approximate excess variance produced by a hypothetical particle of radius \( R \) could be expressed as

\[ E \approx \tau \delta e^{-2\tau}. \]

(5.20)

Importantly, this equation is valid only for an integer number of spherical particles of the same size in low optical depth regions such as the C ring where \( \tau \delta \ll 1 \).

In regions dominated by self-gravity wakes (e.g., Saturn’s A and B rings), we propose the implementation of a different approach. In Section 5.4.2.1, we will test a direct computation of excess variance from expectation values of the transparency against results from SN90’s and C18’s analytic equations in the C ring and find it to be consistent with the method of moments to within a factor of \( \delta \). Notably, we are able to do this without the use of the conditional variance formula, cumbersome moment generating functions, or logarithms.

### 5.3 Occultation statistics within spiral density waves

Key to our methodology is separating the individual peaks and troughs of density waves to remove secular trends in the data before performing statistical analysis. In this section, we will give a brief overview of the waves of interest and the morphology of the regions in which they reside. Next, we will describe the procedure by which we prepared the data for analysis, including the semi-automated routine we use to partition each wave into individual peaks and troughs. Finally, we will review our criteria for selecting occultations to analyze.
5.3.1 Observations

We investigate spiral density waves from three different regions of the rings: the Cassini Division (3 waves), the inner B ring (1 wave), and the A ring (5 waves). The waves we analyze are as follows: Prometheus 9:7, Pan 6:5, and Atlas 5:4 in the Cassini Division; Janus 2:1 in the B ring; and Pandora 5:4, Janus 4:3, Janus 5:4, Mimas 5:3, and Janus 6:5 in the A ring. Figure 5.7 shows an optical depth profile of the entire rings with dashed vertical lines identifying the location of each analyzed. Figure 5.8, Figure 5.9, Figure 5.10, Figure 5.11, Figure 5.12, Figure 5.13, Figure 5.14, Figure 5.15, and Figure 5.16 each zoom in on each analyzed wave in the β Centauri Rev 64 Egress occultation to show the optical depth profiles characteristic of each wave.

![Figure 5.7. Density wave resonance locations.](image)
The β Centauri Rev 64 Egress occultation line-of-sight optical depth (τ_{LOS}) profile of Saturn's rings extending from 85000 km in the C ring to 140000 km between the A and F ring. Each of the 9 studied resonance locations is denoted by a colored vertical line.
Figure 5.8. Janus 2:1 density wave. Line-of-sight optical depth profile of the Janus 2:1 density wave in the β Centauri Rev 64 Egress occultation. The gray is unsmoothed while the black is boxcar smoothed by 100 points. The non-linear nature of this strong density wave is shown by the trend of peaks and troughs growing increasingly narrow with distance from the resonance.

Figure 5.9. Prometheus 9:7 density wave. Line-of-sight optical depth profile of the Prometheus 9:7 density wave in the β Centauri Rev 64 Egress occultation. The gray is unsmoothed while the black is boxcar smoothed by 10 points.
Figure 5.10. Pan 6:5 density wave.
Line-of-sight optical depth profile of the Pan 6:5 density wave in the β Centauri Rev 64 Egress occultation. The gray is unsmoothed while the black is boxcar smoothed by 10 points.

Figure 5.11. Atlas 5:4 density wave.
Line-of-sight optical depth profile of the Atlas 5:4 density wave in the β Centauri Rev 64 Egress occultation. The gray is unsmoothed while the black is boxcar smoothed by 10 points.
Figure 5.12. Pandora 5:4 density wave.
Line-of-sight optical depth profile of the Pandora 5:4 density wave in the $\beta$ Centauri Rev 64 Egress occultation. The gray is unsmoothed while the black is boxcar smoothed by 100 points.

Figure 5.13. Janus 4:3 density wave.
Line-of-sight optical depth profile of the Janus 4:3 density wave in the $\beta$ Centauri Rev 64 Egress occultation. The gray is unsmoothed while the black is boxcar smoothed by 100 points.
5.3.2 Procedure for wave partitioning

In this section we detail our technique for separating density waves into individual peaks and troughs. First, we boxcar average the optical depth profile of each density wave by a number of points, $M$, equal to the ratio:
\[ M = \frac{n}{l} \]  

(5.21)

where \( n \) is the total number of raw data points in the wave train and \( l \) is the radial extent of the wave in kilometers. This effectively averages the optical depth to 1 km binned resolution, which we will call \( \tau_b \). By definition, peaks naturally have a higher density of particles and therefore occur at higher optical depths than the troughs of the same wave. Because of this, we begin by calculate the mean optical depth of the whole wave, \( \tau_m \), and use that to separate the data into two segments; first, potential peak locations where \( \tau_b > 1.1 \tau_m \); and second, potential trough locations where \( \tau_b < 0.9 \tau_m \). Because we seek to isolate the peaks from the troughs, we omit from our analysis the transitional parts of the wave (e.g., where \( 0.9 \tau_m \leq \tau \leq 1.1 \tau_m \)) which appear as steep gradients in the optical depth. An illustration of this is shown in Figure 5.17 below. Then we implement the TS_DIFF algorithm in IDL, which recursively computes the forward difference, \( d \), between elements of a vector of time-series data \( k \) times (in the case of this analysis, \( k = 2 \)).

![Figure 5.17](image)

Figure 5.17. Illustration of the first step of our procedure for partitioning peaks/troughs. Optical depth profile of the Janus 2:1 density wave from the \( \beta \) Centauri Rev 077 Egress occultation smoothed by \( M = 140 \) points. The mean optical depth, \( \tau_m \), is shown in red, data for which \( \tau \geq 1.1 \tau_m \) is shown in blue, and data for which \( \tau \leq 0.9 \tau_m \) is shown in green.
We mark boundaries as the locations at which $|d| \geq 100$ for inner B ring and A ring waves and $|d| \geq 20$ for Cassini Division waves. The reason for the factor of 5 difference in the cut off value for $|d|$ is that the Cassini Division waves are only tens of kilometers (~20 km wide) while the inner B ring and A ring waves extend for hundreds of kilometers. We thus obtain a list of radial boundaries, $B$, such that

$$B = [b_1, b_2, ..., b_N],$$

where $i = 1, 2, ..., N$ are the indices of the listed elements. The elements of the list with odd indices are inner boundaries while the elements with even indices are outer boundaries. Because each peak/trough must have both an inner and outer boundary, we require that $N$ must be even. When $N$ is odd, we remove the last (e.g., highest radial value) boundary from the list which is justified because the further away from the resonance, the more powerful the damping effect.

Still, it is possible that the procedure up to this point may fail to capture every undulation of the wave, especially further down the wave train (e.g., further from Saturn) where peaks and troughs grow increasingly narrow (see this effect illustrated in Figure 5.18 below). Additionally, waves with complex morphologies like the Janus 2:1 density wave may have troughs at the end of the train which are at higher optical depths than peaks at the beginning of the train (see Figure 5.8). To deal with these cases, our IDL routine compares the wave’s optical depth profile and the initial radial boundaries and plots them simultaneously to identify any peaks/troughs that contain more than one undulation. When these occur, the user to manually inputs the approximate radial location of the inner boundary. Then the program isolates that peak/trough and performs a second differencing scan on it using the TS_DIFF routine with a factor of 2 higher sensitivity to undulations (e.g., $|d| \geq 50$ for inner B ring and A ring waves and $|d| \geq 10$ for Cassini Division waves). Because of this, we say that our procedure is semi-automated.
We perform a final check of the radial boundaries obtained to ensure that none of the designated ‘peak’ locations overlap with any of the ‘trough’ locations and vice versa. If overlap does occur, we correct for it by setting the outer boundary of the first peak/trough equal to the initial inner boundary of the next trough/peak and in turn setting the inner boundary of the next peak/trough equal to the initial outer boundary of the first peak/trough, leaving a transitional space in between. Finally, we require for our statistical analysis that the number of data points within each peak/trough be at least 50. The inner and outer radial boundaries for each peak and trough are unique to each occultation in this study.

Figure 5.18. Application of partitioning technique for peaks/troughs in the β Centauri Rev 077 Egress occultation of the Pandora 5:4 density wave. (Left) The β Centauri Rev 077 Egress occultation at the Pandora 5:4 density separated into ‘potential’ peak and trough locations in which the dashed lines represent the radial boundaries of each peak/trough. (Right) The result of the application of the technique to the same segment of data with final peaks shown in blue and troughs shown in lime.

5.3.3 Removal of secular trends

In order to parse excess variance caused by small structures in the rings as opposed to the variance caused by the undulations in optical depth which naturally result from density waves, it is necessary to remove secular patterns in the data before calculating the statistical moments. After we delineate peaks from troughs, we return to the raw data and remove secular trends for
each by dividing that subsection of the raw data by a 2nd order polynomial fit to the raw data.

Finally, we calculate the statistical moments of these trend-removed sections of raw data using the moment function in IDL. The trend-removal technique is demonstrated in Figure 5.19.

![Figure 5.19. Illustration of our technique for removing secular trends in peaks and troughs. (Left) The second peak in the Janus 5:4 density wave in the α Crucis Rev 100 Ingress occultation. (Right) The second trough in the Janus 5:4 density wave in the α Crucis Rev 100 Ingress occultation. We remove secular trends in both peaks and troughs by subtracting a parabolic fit (dashed line) to the raw data (black) and adding back the mean. The data post-trend removal (light gray) indicate that only random fluctuations remain.](image)

**5.3.4 Criteria for occultations used**

Our only requirements for this analysis of occultations of density waves are a sufficiently bright star and an adequate number of data points to perform a meaningful analysis. We require an unocculted star brightness $I_0$ greater than or equal to 50 counts per millisecond in order to exclude particularly dim stars from our analysis as well as a minimum of 50 raw data points per single peak/trough. A summary of these occultations and their parameters is given in Table 5.1.
5.4 Transparency as a random variate – a moments-based approach to stellar occultation statistics

5.4.1 Theory

This section, as well as upcoming section 5.4.3, were completed in partnership with Larry Esposito (personal communication) and will be published in an upcoming paper (Esposito et al., 2022, in prep). A photometer measurement of a stellar occultation is a Poisson counting process. The sum of two Poisson processes, in terms of our transparency calculation \( I \) and \( b \), is also Poisson distributed. We exploit this property to calculate the moments of the transparency directly by summing over a Poisson distribution. Consider a random variable \( X \), which represents the total number of events that occur within a single integration period.

\[
X = \sum_A \bar{Q}_i \bar{T}_i
\]  

(5.23)

where \( \bar{Q}_i \) represents the measured count rate, which is Poisson distributed with mean \( \lambda = I_0 \), and \( \bar{T}_i \) represents the ring transparency as seen by the \( i \)th photon. Let transparency \( \bar{T} \) be a random variate as follows:

\[
T(x, y) = [0, 1]
\]  

(5.24)

so that \( T(x, y) = 0 \) in shadow and \( T(x, y) = 0 \) unobstructed. Let the passage of an incident photon through the integration area \( A \) to be an event at time \( t_i \) at location \((x_i, y_i)\) within \( A \). Then \( \bar{T}_i \) and \( \bar{Q}_i \) are independent because the transparency does not depend on the number, time or location of the events. From this point it is relatively straightforward to calculate the normalized excess variance as \( \text{var}(\bar{T}) = E[\bar{T}^2] - (E[\bar{T}])^2 \). We calculate the expected value of the transparency as

\[
E[T] = \frac{1}{A} \int_A T(x, y) \, dx \, dy = e^{-\tau_N} \mu.
\]  

(5.25)

This result is the factor by which the rings attenuate the stellar signal, which is equivalent to the fraction of the integration area not covered by particle shadows. The variance of \( \bar{X} \) is then calculated as

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\[
\text{var}(\bar{X}) = E[\bar{X}^2] - E[\bar{X}]^2 = E\left[\left(\sum_i \bar{X}_i - E[\bar{X}]\right)^2\right] = E[(\sum_i (\bar{X}_i - \lambda_i))^2]
\] (5.26)

where \(\lambda_i = E[\bar{X}_i]\). Then,

\[
\text{var}(\bar{X}) = E\left[\sum_i (\bar{X}_i - \lambda_i)^2 + \sum_{i \neq j} (\bar{X}_i - \lambda_i)(\bar{X}_j - \lambda_j)\right]
\] (5.27)

\[
= \sum_i \text{var}[\bar{X}_i] + \sum_{i \neq j} \text{cov}[\bar{X}_i, \bar{X}_j]
\] (5.28)

\[
= I_0 E[\bar{T}] + I_0^2 \text{var}(\bar{T}),
\] (5.29)

where \(\text{var}(\bar{T})\) represents the non-Poissonian excess variance. So, the total variance for random variate \(\bar{X}\) is actually the sum of variance due to Poisson statistics and the additional excess variance caused by ring structure.

We extend this technique to calculate the moments of order \(m\) (where \(m = 1\) for variance, \(m = 2\) for skewness, and \(m = 3\) for kurtosis) of the observed ring transparency as follows:

\[
E[T^m] = \sum_n \exp(-N_G) \frac{N_G^n}{n!} T_n^m
\] (5.30)

where \(n\) is the number of particles in \(A\) which are Poisson distributed with mean \(N_G\) and \(\bar{T} = \exp(-n\delta)\). After simplification for \(m = 1\), we find

\[
E[\bar{T}] = \exp\left(N_G(e^{-\delta} - 1)\right).
\] (5.31)

Substituting \(E[\bar{T}] = e^{-\tau_N/\mu}\) and solving for \(N_G\), we obtain

\[
N_G = \frac{-\tau_N/\mu}{e^{-\delta - 1}}.
\] (5.32)

Therefore Equation \(E[T^m] = \sum_n \exp(-N_G) \frac{N_G^n}{n!} T_n^m\) (5.30) becomes

\[
E[T^m] = \exp\left[\frac{-\tau_N/\mu(e^{-m\delta} - 1)}{e^{-\delta - 1}}\right].
\] (5.33)

Using the binomial theorem, \(\frac{(X^n - 1)}{(X - 1)} = 1 + X + X^2 + \cdots + X^{n-1}\), therefore

\[
E[\bar{T}^m] = \exp\left[\frac{-\tau_N}{\mu} \left(1 + e^{-\delta} + e^{-2\delta} + \cdots + e^{-(m-1)\delta}\right)\right].
\] (5.34)
We note that this result agrees with C18’s Equation (16). Ultimately, we arrive at:

$$\sigma_T^2 = \text{var}(\bar{T}) = E[\bar{T}^2] - E[\bar{T}]^2 = \exp\left(\frac{-\tau_N(1-e^{-2\delta})}{e^{-\delta} - 1}\right) - \exp\left(-\frac{2\tau_N}{\mu}\right).$$  (5.35)

In a similar way, we can calculate the skewness using the moments of the transparency as well. By definition,

$$\gamma[X] = \left(\frac{1}{\sigma_X^3}\right) \sum_{i} \sum_{j} \sum_{k} (\bar{X}_i - E[\bar{T}])(\bar{X}_j - E[\bar{T}])(\bar{X}_k - E[\bar{T}]).$$  (5.36)

This may be rewritten as

$$\gamma[X] = \frac{1}{\sigma_X^3} \left[ \sum_{i} (\bar{X}_i - E[\bar{T}])^3 + \sum_{i \neq j \neq k} (\bar{X}_i - E[\bar{T}]) (\bar{X}_j - E[\bar{T}]) (\bar{X}_k - E[\bar{T}]) \right].$$  (5.37)

After simplification, the expression becomes

$$\gamma[X] = \frac{I_0 E[T]}{\sigma^3_X} + \frac{l_3^2 \gamma_T \sigma_T^3}{\sigma^3_X},$$  (5.38)

where the first term represents the Poisson skewness and the $\gamma_T$ and $\sigma_T$ variables in the second term of the calculation represent the skewness and standard deviation (respectively) of the transparency, e.g. the part of the total skewness that is not contributed by natural Poisson statistics. Because

$$\gamma[T] = \frac{E[\bar{T}^3] - 3E[\bar{T}]E[\bar{T}]^2 + 2E[\bar{T}]^3}{\sigma_T^3} = \frac{E[T^3] - 3E[T]\sigma_T^2 - E[T]^3}{\sigma_T^3},$$  (5.39)

and, by Equation $E[T^m] = \exp\left(\frac{-\tau_N}{\mu} (e^{-m\delta} - 1)\right)$.

$$E[T^3] = \exp\left[\frac{-\tau_N}{\mu} (1 + e^{-\delta} + e^{-2\delta})\right].$$  (5.40)

Therefore,
\[
\gamma[T] = \frac{\exp\left[-\frac{TN}{\mu} \left(1 + e^{-\delta} + e^{-2\delta}\right)\right]}{\exp\left[-\frac{TN}{\mu} \left(e^{-\delta_{-1}} - 1\right)\right]} - \left(\frac{TN}{\mu}\right)^3 \quad (5.41)
\]

which simplifies to

\[
\gamma[T] = \frac{1}{\sigma_T^2} \left( \exp \left( -3 \frac{TN}{\mu} \right) \left[ -\delta^2 TN + 3\delta^2 \left( \frac{TN}{\mu} \right)^2 \right] - \left( \frac{-\delta^3 TN}{3 \mu} \right) - 6\delta^3 \left( \frac{TN}{\mu} \right)^2 \right) \quad (5.42)
\]

Finally, plugging the approximation of \( \sigma_T \sim E[T] \sqrt{\frac{\delta TN}{\mu}} \) into \( \gamma[X] = \frac{i_0 E[T]}{\sigma_X^3} + \frac{i_0^2 \gamma_T \sigma_T^3}{\sigma_X^3} \), (5.38), we find

\[
\gamma[X] \sim \frac{i_0 E[T]}{\sigma_X^3} - \sqrt{\frac{\delta TN}{\mu}} + 3 \sqrt{\frac{\delta TN}{\mu}} - \frac{1}{3} \sqrt{\frac{\delta^3 TN}{\mu}} - 6 \frac{\delta^3 TN}{\mu}. \quad (5.43)
\]

The kurtosis can be expressed as two terms as well,

\[
K_X = \frac{K_P \sigma_T^4}{\sigma_X^4} + \frac{i_0^2 \gamma_T \sigma_T^4}{\sigma_X^4}, \quad (5.44)
\]

where the first term represents the Poisson kurtosis and the \( K_T \) and \( \sigma_T \) variables in the second term of the calculation represent the kurtosis and standard deviation (respectively) of the transparency, e.g. the part of the total kurtosis that is not contributed by natural Poisson statistics.

Because

\[
K_T = \frac{E[T^4] - 4E[T]E[T^3] + 6E[T]^2E[T^2] - 3E[T]^4}{\sigma_T^4} = \frac{E[T^4] - 4\lambda E[T^3] + 6\lambda^2 \sigma_T^2 - 3\lambda^4}{\sigma_T^4}, \quad (5.45)
\]

and, by Equation \( E[T^m] = \exp\left[\frac{-TN/\mu \left(e^{-m\delta_{-1}}\right)}{e^{-\delta_{-1}}} \right] \). \quad \text{(5.33),}

\[
E[T^4] = \exp\left[\frac{-TN/\mu \left(1 + e^{-\delta} + e^{-2\delta} + e^{-3\delta}\right)}{\sigma_T^4} \right]. \quad (5.46)
\]

Therefore
\[ K[T] = \frac{\exp\left[\frac{-\tau N}{\mu} (1 + e^{-\delta} + e^{-2\delta} + e^{-3\delta})\right]}{\sigma_T^4} - 4\mu \exp\left[\frac{-\tau N}{\mu} (1 + e^{-\delta} + e^{-2\delta})\right] + 6\lambda^2 \sigma_T^2 - 3\lambda^4. \]  

Finally, plugging this into \( K[X] = \frac{K_p \sigma_T^4}{\sigma_X^4} + \frac{I_0^4 K_T \sigma_T^4}{\sigma_X^4} \) yields

\[ K[X] = \frac{\{I_0^4 E[T] + 3I_0^2 E[T]^2\}}{\sigma_X^4} + \frac{I_0^4}{\sigma_X^4} \left\{ \exp\left[\frac{-\tau N}{\mu} (1 + e^{-\delta} + e^{-2\delta} + e^{-3\delta})\right] - 4E[T] \exp\left[\frac{-\tau N}{\mu}\right] (1 + e^{-\delta} + e^{-2\delta} + e^{-3\delta})] + 6E[T]^2 \exp\left[\frac{\tau N}{\mu} (1 + e^{-\delta})\right] \right\}. \]  

### 5.4.2 Testing the moments approach against previous results in the C ring and Cassini Division

We performed multiple tests to validate our new approach using a direct comparison to SN90 and C18’s results in regions where \( \delta \tau \ll 1 \) as well as comparison of our results for best-fit \( S/W \) values from known \( \lambda_{Toomre} \) and previous best-fit \( S/W \) results from J16. We find that the moments-based approach to occultation statistics yields agreement with SN90’s and C18’s results consistent to order \( \delta \), which gives us confidence in our method. Figure 5.20 illustrates the excellent agreement among all three approaches (SN90, C18, and moments) for a sample case of \( R = 1 \).
5.4.2.1 Why do we need a moments-based approach?

Unlike the C ring and Cassini Division, the particle size distribution in Saturn’s A and B rings is dominated by elongated, aggregate structures called self-gravity wakes. The method of moments is a more direct application of statistical principles to stellar occultation data analysis that eliminates a variety of limiting assumptions of SN90 while agreeing with their results to the first order in regions where $\delta \tau \ll 1$ and Poisson statistics are valid.

5.4.3 Extension to account for self-gravity wakes

We now present a moments-based approach modified to account for the presence of self-gravity wakes in Saturn’s A and B rings using the parameters of Colwell et al. (2006)’s granola bar model (GBM) for self-gravity wakes. For a review of self-gravity wakes in Saturn’s rings, please refer back to Section 1.5.1. In regions dominated by self-gravity wakes, the transparency is

Figure 5.20. Comparison of $E(\tau)$ curves using BetCen077I occultation parameters based on the variance formulas from SN90, C18, and the method of moments for $R = 1$. The red inset box is highly zoomed to show that the discrepancy between the formulas is substantially small as to be essentially negligible.
\[ T = T_g \left( \frac{S}{S+W} \right), \] (5.49)

where \( T_g \) is the transparency of the gap, \( W \) parameter represents the wake width while the \( S \) parameter denotes the width of the separation between wakes based on the granola bar model for self-gravity wakes (Colwell et al., 2006), and \( S/(S+W) \) is the fractional area of the field-of-view covered by gaps. We approximate \( L \sim \sqrt{A} \) because the wakes stretch across the full extent of the integration area. We assume that the number of self-gravity wake centers within the integration area is Poisson-distributed with rate parameter \( \lambda = \frac{L}{S+W} \) such that

\[ E[\bar{T}^m] = T_g^m \exp \left( \frac{L}{S+W} \left[ \exp \left( \frac{-W}{L} \right) - 1 \right] \right). \] (5.50)

We can then calculate the excess variance as

\[ \sigma_T^2 = E[\bar{T}^2] - E[\bar{T}]^2 = T_g^2 \left\{ \exp \left( \frac{L}{S+W} \left[ \exp \left( \frac{-2W}{L} \right) - 1 \right] \right) - \exp \left( \frac{2L}{S+W} \left[ e^{-W/L} - 1 \right] \right) \right\}. \] (5.51)

We now apply \( E[\bar{T}^m] = T_g^m \exp \left( \frac{L}{S+W} \left[ \exp \left( \frac{-mW}{L} \right) - 1 \right] \right) \) (5.50) to obtain the normalized excess variance based on a granola bar model, \( \text{NEV}_{GBM} \):

\[ \text{NEV}_{GBM} = E[\bar{T}^2] - E[\bar{T}]^2 = E[\bar{T}]^2 \left\{ \exp \left( \frac{L}{S+W} \left[ e^{-2W/L} - 1 \right] \right) - 1 \right\}. \] (5.52)

Taking the limit as \( \frac{W}{L} \ll 1 \), we find that

\[ \sigma_T^2 = E[T]^2 \left\{ \exp \left( \frac{W^2}{L(S+W)} \right) - 1 \right\} \] (5.53)
which matches the second term from Esposito’s autocorrelation approach. Similarly, for
skewness based on a granola bar model, $\Gamma_{X, \text{GBM}}$, we find

$$\Gamma_{X, \text{GBM}} = \frac{l_0 E[T]}{\sigma_X^3} + \frac{\bar{E}[T]^3}{\sigma_X^3} \left\{ \frac{\exp \left[ \frac{L}{S+W} \left( e^{-\frac{3W}{L}} - 1 \right) \right]}{\exp \left[ \frac{3L}{S+W} \left( e^{-\frac{W}{L}} - 1 \right) \right]} - \frac{3 \exp \left[ \frac{L}{S+W} \left( e^{-\frac{2W}{L}} - 1 \right) \right]}{\exp \left[ \frac{2L}{S+W} \left( e^{-\frac{W}{L}} - 1 \right) \right]} \right\} + 2 \right\}; \quad (5.54)$$

and finally, for kurtosis, or $K_{X, \text{GBM}}$, we find

$$K_{X, \text{GBM}} = \frac{l_0 E[T]}{\sigma_X^4} + 3 \frac{l_0^2 E[T]^2}{\sigma_X^4} + \frac{\bar{E}[T]^3}{\sigma_X^4} \left\{ \frac{\exp \left[ \frac{L}{S+W} \left( e^{-\frac{4W}{L}} - 1 \right) \right]}{\exp \left[ \frac{4L}{S+W} \left( e^{-\frac{W}{L}} - 1 \right) \right]} - \frac{4 \exp \left[ \frac{L}{S+W} \left( e^{-\frac{3W}{L}} - 1 \right) \right]}{\exp \left[ \frac{3L}{S+W} \left( e^{-\frac{W}{L}} - 1 \right) \right]} + \frac{6 \exp \left[ \frac{L}{S+W} \left( e^{-\frac{2W}{L}} - 1 \right) \right]}{\exp \left[ \frac{2L}{S+W} \left( e^{-\frac{W}{L}} - 1 \right) \right]} \right\} - 3 \}. \quad (5.55)$$

5.4.3.1 Consistency of the model with previous results

We have already confirmed the model’s applicability to data in regions where $\delta \tau \ll 1$. In extending the moments approach to self-gravity wake dominated regions, we perform a series of simple tests on occultation data in regions where self-gravity wakes are present but density wakes are not. To demonstrate the consistency of our results using this method with previous work, we compare our best-fit models in a region of the A ring where self-gravity wakes are prominent, but density waves are absent. We use $\sigma$ estimates from Spilker et al. (2004) and Tiscareno et al. (2007) to fix the value of $S+W$ to be equal to the Toomre least-stable wavelength as described in section 1.5.2.1. Best fitting $R_{\text{eff}}$ values Figure 5.21 illustrates one such comparison; in this same region, J16 found $S/W$ values around 0.5. Requiring that $S/W = 0.5$, we find strong agreement between our model and the data. Overall, we find that our results from the method of moments including self-gravity wakes are largely consistent with estimates from J16, giving us confidence that our model can be used to interpret wake morphology.
In this section we present the results of a moments-based statistical analysis of Cassini UVIS HSP stellar occultation data. As the section is organized according to increasing ring plane radius, we begin by reviewing our findings for the three Cassini Division density waves, for which we apply the spherical particle model. Then, we discuss density waves in regions dominated by self-gravity wake structures, from the inner B ring to the outer A ring, and apply the granola bar model for self-gravity wakes to the moments model and implement a modified technique.

5.5.1 Comparison to statistics in adjacent, featureless regions

We compare the higher order moments in density waves with nearby, relatively flat, featureless (wave-free) regions of the same radial extent for comparison purposes. Because
excess variance depends on optical depth, we compare points from the featureless regions with points from the troughs in the same range of optical depth. Our findings indicate that the normalized excess variance of density wave troughs is statistically consistent with that of nearby featureless regions. See Table 5.2 for a summary of our results for each wave. Normalized excess variance curves for all five density waves and their respective adjacent 100 km regions behave similarly, suggesting similar $R_{\text{eff}}$ both within the density wave peaks and troughs and within regions adjacent to the waves.

![Normalized excess variance curves for all five density waves and their respective adjacent 100 km regions behave similarly, suggesting similar $R_{\text{eff}}$ both within the density wave peaks and troughs and within regions adjacent to the waves.]

Figure 5.22. Comparison of statistics in troughs of density waves with nearby wave-free regions. Normalized excess variance ($E$) for the troughs (open circles) of (Left) the Janus 2:1 density wave and (Right) the Janus 5:4 density wave as a function of line-of-sight optical depth ($\tau$) for the $\beta$ Centauri Rev 78 Egress occultation compared to $E$ for regions interior-adjacent (closed circles), ranging from 96100-96200 km and 130550-130650 km, respectively.

### 5.5.2 Cassini Division density waves

In this section we present a survey of our results for the statistical moments within 3 weak density waves which all lie within a 1000 km stretch of the Cassini Division: Prometheus 9:7 (ILR ~ 118067 km), Pan 6:5 (ILR ~ 118453 km), and Atlas 5:4 (118830 km). Because the mean optical depth in the Cassini Division is only ~0.1, SN90’s and C18’s models are valid here and we can directly compare our findings with theirs. In particular, we focus on computed $R_{\text{eff}}$
values from C18’s analysis of $\alpha$ Virginis Rev 008 Ingress and $\beta$ Centauri Rev 077 Ingress, evaluating more than 30 additional stellar occultations at a wide range of viewing geometries.

We use the method of moments to compute an effective particle size, $R_{\text{eff}}$, within narrow periodic structures such as density waves and self-gravity wakes. We summarize our results in
Table 5.4. Below we show the best fit $R_{\text{eff}}$ values within the $\beta$ Centauri Rev 105 Egress occultation for the Prometheus 9:7 density wave (~118066 km), the Pan 6:5 density wave (~118453 km), and the Atlas 5:4 density wave (~118830 km). For all three results, it can be seen that $0.8 \leq R_{\text{eff}} \pm 2\sigma \leq 2.2$ m. These are consistent with the results of C18, who found $R_{\text{eff}}$~1.5 m around the Prometheus 9:7 and $R_{\text{eff}}$~2 m in the Pan 6:5 and Atlas 5:4 density waves. Our results are also consistent with C18’s conclusion that $R_{\text{eff}}$ increasing gradually across the Cassini Division up to ~120000 km.

Figure 5.23. Best fit effective particle size $R_{\text{eff}}$ for the troughs of the Prometheus 9:7 density wave in the $\beta$ Centauri Rev 105 Egress occultation. Each black X (diamond) represents the calculated normalized excess variance of a single, trend-removed trough (peak). The best fitting $R_{\text{eff}}$ from C18’s formula is shown in black while the best fitting $R_{\text{eff}}$ from moments is shown in blue. The dotted and dashed lines represent the best fit $\pm 2\sigma$, respectively. Note that although the peaks are shown, we only calculate the best fit from the troughs.
Figure 5.24. Best fit effective particle size $R_{\text{eff}}$ for the troughs of the Pan 6:5 density wave in the $\beta$ Centauri Rev 105 Egress occultation.

Each black X (diamond) represents the calculated normalized excess variance of a single, trend-removed trough (peak). The best fitting $R_{\text{eff}}$ from C18’s formula is shown in black while the best fitting $R_{\text{eff}}$ from moments is shown in blue. The dotted and dashed lines represent the best fit $\pm 2\sigma$, respectively. Note that although the peaks are shown, we only calculate the best fit from the troughs.

Figure 5.25. Best fit effective particle size $R_{\text{eff}}$ for the Atlas 5:4 density wave for the Cassini Division in the $\beta$ Centauri Rev 105 Egress occultation.

Each black X (diamond) represents the calculated normalized excess variance of a single, trend-removed trough (peak). The best fitting $R_{\text{eff}}$ from C18’s formula is shown in black while the best fitting $R_{\text{eff}}$ from moments is shown in blue. The dotted and dashed lines represent the best fit $\pm 2\sigma$, respectively. Note that although the peaks are shown, we only calculate the best fit from the troughs.
5.5.3 Wake dominant regions density wave results

Now we will move from the relatively low optical depth Cassini Division waves to waves in regions dominated by opaque self-gravity wake structures. To calculate a best-fit $S/W$, we require that $\lambda_{\text{Toomre}} = S + W$ and test all possible combinations of $S$ and $W$ for which this condition is true. For all 35 occultations analyzed, we compute the $\chi^2$ of each possible set of parameters and identify the best-fit as the one for the $\chi^2$ is minimum. We report only fits for which $0.5 < \chi^2 < 5$, so that our data are not over or under-constrained. In this way we constrain the parameters of self-gravity wakes in density waves throughout the rings.

5.5.3.1 The inner B ring: Janus 2:1

Significant azimuthal asymmetry exists in the inner B ring as well as the A ring. The Janus 2:1 orbital resonance generates the only density wave clearly observed in Saturn’s B ring as the rest of the locations in the B ring where density waves might occur are dominated by complex structure. Located in the B1 region of the inner B ring, where C18 noted a considerable spread in $E(\tau)$. They postulated that the observed slight downward concavity in $E(\tau)$ in the region may be caused by greater particle size variation or the relative abundance of self-gravity wakes. The damping of the Janus 2:1 wave is particularly weak such that the wave train is much longer than the trains of density waves in the A ring, propagating up to $\sim$500 km outward from the resonance. Additionally, Janus 2:1 wave spans a broad gradient in optical depth, which allows a more thorough determination of excess variance as a function of optical depth.

The surface mass density, $\sigma_0$, across the B ring varies widely from 40-140 $g/cm^2$ (Hedman and Nicholson, 2016). Holberg et al. (1982) and Esposito et al. (1983) estimated the surface mass density within the Janus 2:1 density wave to be $\sigma_0 \sim 70$ $g/cm^2$. Although the non-linear nature of the
Janus 2:1 density wave would imply that the surface density cannot be assumed to be constant, Hedman and Nicholson (2016) later confirmed their results, so we use a constant value of $\sigma_0 = 70 \, \frac{g}{cm^2}$ to estimate the Toomre critical wavelength, which yields $\lambda_{\text{Toomre}} = 43$ m.

Figure 5.26. Best-fit self-gravity wake parameters $S$ and $W$ from the moments model for the $\lambda$ Scorpii Rev 029 Egress occultation of the Janus 2:1 density wave in the inner B ring. (Left) Normalized excess variance for each trough ($\tau \lesssim 2$) and peak ($\tau \gtrsim 2$). We fit two separate models for peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 43$ m, the best-fit parameters for the troughs indicate a gap-to-integration area fraction of $\sim 60\%$. (Right) A grid of the calculated $\chi^2$ for several tested combinations of $S$ and $W$ with the self-gravity wakes application of the moments model for occultation statistics. Contour levels are based on 1-3$\sigma$ values. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum $\sigma_0$ including error bars.

Figure 5.27. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the $\lambda$ Scorpii Rev 029 Egress occultation of the Janus 2:1 density wave.
We fit two separate models for the peaks (blue) and troughs (green). We define the best fit $S$ and $W$ parameters to be those for which the $\chi^2$ of the model is minimized. Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 43$ m, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of $\sim 80\%$.

Figure 5.28. Best-fit self-gravity wake parameters $S$ and $W$ from the moments model for the $\beta$ Scorpii Rev 104 Egress occultation of the Janus 2:1 density wave in the inner B ring. (Left) Normalized excess variance for each trough ($\tau \leq 1.8$) and peak ($\tau \geq 1.8$). We fit two separate models for peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 43$ m, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of $\sim 40\%$. (Right) A grid of the calculated $\chi^2$ for several tested combinations of $S$ and $W$ with the self-gravity wakes application of the moments model for occultation statistics. Contours are based on 1-3$\sigma$ values. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum $\sigma_0$ including error bars.

Figure 5.29. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the $\beta$ Centauri Rev 104 Egress occultation of the Janus 2:1 density wave. We calculate two separate best fit skewness values for the peaks (blue) and troughs (green). We define the best fit parameters to be those for which the reduced chi-squared is minimized. Given
the fixed value of $\lambda_{\text{Toomre}} = S + W = 43$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~ 80%, twice as high as we found for the excess variance.

Our results indicate that self-gravity wakes are most tightly bound in the peaks of the Janus 2:1 wave, where our average best fit $S/W$ values are the lowest of the peaks of all the density waves in this study. We find an average best-fit $S/W$ in the peaks from variance of 1.3 and from skewness of 0.9. In the troughs of the wave, we find an average best-fit $S/W$ of 2.4 for variance and 2.5 for skewness. Our results are consistent with Colwell et al. (2007)’s prediction of flatter and more tightly packed wakes in the B ring than the A ring.

5.5.3.2 Inner A ring

Consistent with J16’s results, we find the greatest variation in our results in the inner A ring region. Here we analyze the Pandora 5:4 and Janus 4:3 density waves, between which there exists a well-known viscous overstability (Lehmann et al. (2017), Lehmann et al. (2019), Lewis and Stewart (2005)). We find very different best-fitting $S/W$ values for the two waves. Additionally, in both waves we observed an inverse correlation of $S/W$ with $\mu$, contrary to our expectation that $S/W$ should not vary with $\mu$.

5.5.3.2.1 Pandora 5:4

The strong Pandora 5:4 resonance, which generates the longest wave train in Saturn’s rings after the Janus 2:1 resonance, lies at the inner edge of the A ring at ~122313 km. Like the Janus 2:1, it is a non-linear density wave and therefore less likely to conform to expected statistics. Interestingly, we find the highest average gap ratio $\left(\frac{S}{S+W}\right)$ in this region (~ 70%) as
well as the highest average $S/W$. We compute a Pearson’s correlation coefficient of $r = -0.82$ for $S/W$ versus $\mu$.

Figure 5.30. Best-fit self-gravity wake parameters $S$ and $W$ from the moments model. (Left) Normalized excess variance for each trough ($\tau < 1.5$) and peak ($\tau > 1.5$) in the $\alpha$ Crucis Rev 092 Ingress occultation of the Pandora 5:4 density wave in the inner A ring. The model with the lowest reduced chi-squared value is also plotted for both peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 36$, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~70%. (Right) $\chi^2_\nu$ contour grid of combinations of self-gravity wake parameters $S$ and $W$. Contours are based on 1-3$\sigma$ values. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum surface mass density values including error bars ($28^{+5}_{-5}$) reported by Tiscareno et al. (2007).

Figure 5.31. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the $\alpha$ Crucis Rev 092 Ingress occultation of the Pandora 5:4 density wave. We calculate two separate best fit skewness values for the peaks (shown in blue) and troughs (shown in green). We define the best fit parameters to be those for which the reduced chi-
squared is minimized. Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 36$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~80%.

Figure 5.32. Best-fit self-gravity wake parameters $S$ and $W$ from the moments model. (Left) Normalized excess variance for each trough ($\tau < 1.5$) and peak ($\tau > 1.5$) in the $\beta$ Centauri Rev 064 Egress occultation of the Pandora 5:4 density wave in the inner A ring. The model with the lowest reduced chi-squared value is also plotted for both peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 36$, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~60%. (Right) $\chi^2_\nu$ grid of tested combinations of self-gravity wake parameters $S$ and $W$ with the self-gravity wakes application of the moments model for occultation statistics. Contours are based on 1-3$\sigma$ values. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum surface mass density values including error bars ($28^{+5}_{-2}$) reported by Tiscareno et al. (2007).

Figure 5.33. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the $\beta$ Centauri Rev 064 Egress occultation of the Pandora 5:4 density wave. We calculate two separate best fit skewness values for the peaks (shown in blue) and troughs (shown in green). We define the best fit parameters to be those for which the reduced chi-
squared is minimized. Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 36$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~85%.

5.5.3.2.2 Janus 4:3

The Janus 4:3 resonance lies at ~125266 km and generates the strong Janus 4:3 density wave, which extends ~400 kilometers from the resonance. Overall we find an average best fitting $S/W$ of 2.2 and gap ratio of 63% from 31 occultations.

Figure 5.34. Best-fit self-gravity wake parameters to the Janus 4:3 density wave in the α Virginis Rev 173 Egress occultation. (Left) The black diamonds represent the normalized excess variance calculated for each trough ($\tau < 1.3$) and peak ($\tau > 1.3$). We define the best fit parameters to be those for which the reduced chi-squared is minimized; this is also plotted for both peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 68$ m, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~60%. (Right) $\chi^2_{\nu}$ grid of tested combinations of self-gravity wake parameters $S$ and $W$. The contours are based on 1-3σ confidence levels. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum surface mass density values including error bars ($49.6_{-9.5}^{+9.5}$) reported by Spilker et al. (2004).
Figure 5.35. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the $\alpha$ Virginis Rev 173 Egress occultation of the Janus 4:3 density wave. We calculate two separate best fit skewness values for the peaks (shown in blue) and troughs (shown in green). We define the best fit parameters to be those for which the reduced chi-squared is minimized. Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 68$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~60%, consistent with results from the excess variance.

Figure 5.36. Best-fit self-gravity wake parameters to the Janus 4:3 density wave in the $\beta$ Centauri Rev 105 Egress occultation. (Left) The black diamonds represent the normalized excess variance calculated for each trough ($\tau < 0.85$) and peak ($\tau > 0.85$). We define the best fit parameters to be those for which the reduced chi-squared is minimized; this is also plotted for both peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 68$ m, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~60%. (Right) $\chi^2_{\nu}$ grid of tested combinations of self-gravity wake parameters $S$ and $W$. The contours are based on 1-3$\sigma$ confidence levels. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum surface mass density values including error bars ($49.6^{+9.5}_{-15.3}$) reported by Spilker et al. (2004).
Figure 5.37. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the $\beta$ Centauri Rev 105 Egress occultation of the Janus 4:3 density wave. We calculate two separate best fit skewness values for the peaks (shown in blue) and troughs (shown in green). We define the best fit parameters to be those for which the reduced chi-squared is minimized. Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 68$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of $\sim 80\%$.

5.5.3.3 Central A ring

The azimuthal brightness asymmetry of the rings peaks in the central A ring and tapers off at the inner and outer edges. In this region, wakes are most coherent as they are in balance with Keplerian shear. We find very similar best-fitting $S/W$ values for the two waves. We also observed a slight inverse correlation of $S/W$ with $\mu$ for both waves, with Pearson’s correlation coefficients $r = -0.48$ for the wave and $r = -0.31$ for the Mimas wave.

5.5.3.3.1 Janus 5:4

As the Janus 4:3 density wave dampens, the central A ring begins. The first density wave we analyze in this region is generated by the Janus 5:4 resonance at $\sim 130700$ km. Bright “haloes” were seen in this region by Cassini ISS.
Figure 5.38. Best-fit self-gravity wake parameters to the Janus 4:3 density wave in the α Virginis Rev 008 Egress occultation. (Left) The black diamonds represent the normalized excess variance calculated for each trough (τ < 1.5) and peak (τ > 1.5). We define the best fit parameters to be those for which the reduced chi-squared is minimized; this is also plotted for both peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 55$ m, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of $\sim 75\%$. (Right) $\chi^2$ grid of tested combinations of self-gravity wake parameters $S$ and $W$. The contours are based on 1-3σ confidence levels. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum surface mass density values including error bars ($35.3 \pm 3.4$) reported by Spilker et al. (2004).

Figure 5.39. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the α Virginis Rev 008 Egress occultation of the Janus 5:4 density wave. We calculate two separate best fit skewness values for the peaks (shown in blue) and troughs (shown in green). We define the best fit parameters to be those for which the reduced chi-squared is minimized. Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 55$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of $\sim 85\%$.
Figure 5.40. Best-fit self-gravity wake parameters to the Janus 4:3 density wave in the \( \gamma \) Lupii Rev 032 Egress occultation. (Left) The black diamonds represent the normalized excess variance calculated for each trough \((\tau < 1)\) and peak \((\tau > 1)\). We define the best fit parameters to be those for which the reduced chi-squared is minimized; this is also plotted for both peaks (blue) and troughs (green). Given the fixed value of \( \lambda_{\text{Toomre}} = S + W = 55 \) m, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of \(~60\%\). (Right) \( \chi^2 \) grid of tested combinations of self-gravity wake parameters \( S \) and \( W \). The contours are based on 1-3\( \sigma \) confidence levels. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum surface mass density values including error bars \((35.3 \pm 3.4)\) reported by Spilker et al. (2004).

Figure 5.41. Best-fitting self-gravity wake parameters \( S \) and \( W \) to the skewness based on the moments model in the \( \gamma \) Lupii Rev 032 Egress occultation of the Janus 5:4 density wave. We calculate two separate best fit skewness values for the peaks (shown in blue) and troughs (shown in green). We define the best fit parameters to be those for which the reduced chi-squared is minimized. Given the fixed value of \( \lambda_{\text{Toomre}} = S + W = 55 \) meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of \(~65\%\).
5.5.3.3.2 Mimas 5:3

The next density wave we analyze in the central A ring is the Mimas 5:3 density wave, which begins at ~ 130700 km. We note that the Mimas 5:3 resonance, which lies at ~127765 km, also excites the most prominent bending wave in Saturn’s ring system.

![Figure 5.42. Best-fit self-gravity wake parameters to the Mimas 5:3 density wave in the α Virginis Rev 232 Egress occultation.](image1)

(Left) The black diamonds represent the normalized excess variance calculated for each trough (τ < 1.5) and peak (τ > 1.5). We define the best fit parameters to be those for which the reduced chi-squared is minimized; this is also plotted for both peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 46$ m, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~70%. (Right) $\chi^2$ grid of tested combinations of self-gravity wake parameters $S$ and $W$. The contours are based on 1-3σ confidence levels. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum surface mass density values including error bars (28.3 ± 5.7) reported by Spilker et al. (2004).

![Figure 5.43. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the α Virginis Rev 232 Egress occultation of the Mimas 5:3 density wave.](image2)
We calculate two separate best fit skewness values for the peaks (shown in blue) and troughs (shown in green). We define the best fit parameters to be those for which the reduced chi-squared is minimized. Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 46$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of $\sim 65\%$.

Figure 5.44. Best-fit self-gravity wake parameters to the Mimas 5:3 density wave in the $\gamma$ Lupii Rev 032 Egress occultation. (Left) The black diamonds represent the normalized excess variance calculated for each trough ($\tau \lesssim 0.75$) and peak ($\tau \gtrsim 0.75$). We define the best fit parameters to be those for which the reduced chi-squared is minimized; this is also plotted for both peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 46$ m, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of $\sim 65\%$. (Right) $\chi^2_{\nu}$ grid of tested combinations of self-gravity wake parameters $S$ and $W$. The contours are based on 1-3$\sigma$ confidence levels. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum surface mass density values including error bars ($28.3 \pm 5.7$) reported by Spilker et al. (2004).

Figure 5.45. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the $\gamma$ Lupii Rev 032 Egress occultation of the Mimas 5:3 density wave. We calculate two separate best fit skewness values for the peaks (shown in blue) and troughs (shown in green). We define the best fit parameters to be those for which the reduced chi-
squared is minimized. Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 46$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~ 60%.

5.5.3.4 The outer A ring: Janus 6:5

Beyond the Encke gap lies the outer A ring and we analyze the Janus 6:5 density wave in this region. This is the only density wave we analyze which lies exterior to the Encke gap.

Overall, we find an average $S/W$ of and an average gap ratio of 69% for 22 occultations.

Figure 5.46. Best-fit self-gravity wake parameters $S$ and $W$ from the moments model. (Left) Normalized excess variance for each trough ($\tau < 2.4$) and peak ($\tau > 2.4$) in the $\alpha$ Crucis Rev 100 Egress occultation of the Janus 6:5 density wave in the outer A ring. The model with the lowest reduced chi-squared value is also plotted for both peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 64$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of ~65%. (Right) $\chi^2$ grid of tested combinations of self-gravity wake parameters $S$ and $W$ with the self-gravity wakes application of the moments model for occultation statistics. Contours are based on 1-3$\sigma$ values. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum surface mass density values including error bars (37.8 ± 1.4) reported by Spilker et al. (2004).
Figure 5.47. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the $\alpha$ Crucis Rev 100 Egress occultation of the Janus 6:5 density wave. We calculate two separate best-fit skewness values for the peaks (blue) and troughs (green). We define the best fit parameters to be those for which the reduced chi-squared is minimized. Given the fixed value of $\lambda_{\rm Toomre} = S + W = 64$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of $\sim 85\%$.

Figure 5.48. Best-fit self-gravity wake parameters $S$ and $W$ from the moments model. (Left) Normalized excess variance for each trough ($\tau < 1$) and peak ($\tau > 1$) in the $\beta$ Centauri Rev 105 Egress occultation of the Janus 6:5 density wave in the outer A ring. The model with the lowest reduced chi-squared value is also plotted for both peaks (blue) and troughs (green). Given the fixed value of $\lambda_{\rm Toomre} = S + W = 64$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of $\sim 70\%$. (Right) $\chi^2$ grid of tested combinations of self-gravity wake parameters $S$ and $W$ with the self-gravity wakes application of the moments model for occultation statistics. Contours are based on $1$-$3\sigma$ values. Upper (dot-dash) and lower (triple-dot-dash) bounds are based on the maximum and minimum surface mass density values including error bars ($37.8 \pm 1.4$) reported by Spilker et al. (2004).
Figure 5.49. Best-fitting self-gravity wake parameters $S$ and $W$ to the skewness based on the moments model in the $\beta$ Centauri Rev 105 Egress occultation of the Janus 6:5 density wave. We calculate two separate best fit skewness values for the peaks (blue) and troughs (green). We define the best fit parameters to be those for which the reduced chi-squared is minimized. Given the fixed value of $\lambda_{\text{Toomre}} = S + W = 64$ meters, the best-fit parameters for the troughs of the wave indicate a gap-to-integration area fraction of $\sim 80\%$.

5.5.4 Phase dependence of density wave statistics compared to predator prey simulations

We have also performed a preliminary investigation of the phase dependence of each of the moments of the data through personal communications with Larry Esposito to compare our results with the predator-prey model for ring dynamics. We start by assuming a granola bar model with $H/W \ll 1$ and $\tau_{\text{wake}} \sim \infty$. We also assume that the wakes are incompressible such that they remain unaffected in the horizontal dimension by the passage of a density wave (e.g., $W =$ constant). The ring particles in the gap do respond to the passage of the wave and are conserved so that the optical depth of the gap changes as $S$ changes while $H$ and $W$ do not.

Figure 5.50 below illustrates the phase dependence of the normalized excess variance in the Janus 5:4 density wave in the $\beta$ Centauri Rev 105 Ingress occultation. The normalized excess variance observed in the first quarter of the forcing cycle is consistent with wake growth and lags consistent with the lag predicted by the predator-prey cycle. Additionally, the fact that the no wake growth model is symmetric about phase $180^\circ$ while the moments are not suggests that the
wake structure just before the wave crest passes over is different from just after the crest passes over.

Figure 5.50. Phase dependence of the normalized excess variance in the Janus 5:4 density wave in the $\beta$ Centauri Rev 105 Ingress occultation. (Left) Optical depth profile of the Janus 5:4 density wave in the $\beta$ Centauri Rev 105 Ingress occultation with the computed normalized excess variance in the peaks (red diamonds) and troughs (blue diamonds). (Right) The calculated normalized excess variance of the Janus 5:4 density wave versus the phase in degrees. The over-plotted green asterisks were calculated by Larry Esposito assuming no wake growth and conservation of gap particles (e.g., no accretion or erosion) based on the following parameters: $S/W \sim 2$, $\tau_{\text{gap}} \sim 0.2$ from Figure 7 in J16, $\lambda_{\text{Toomre}} = S + W \sim 60$ m, and $W = 20$.

5.5.5. Statistical comparison of waves analyzed

This section will compare and contrast our best-fit self-gravity wake parameter results at different density waves with one another. Additionally, we will infer information about the inner B ring, inner A ring, central A ring and outer A ring regions via an analysis of statistical moments. The foremost conclusion we draw from our results is that a granola bar model is necessary to describe the statistics observed in the A and B rings where self-gravity wakes are ubiquitous, while the spherical particle model is more accurate in the C ring. Additionally, we find that statistics within the troughs are consistent with those of nearby featureless regions. Our conclusions for the moments individually are described below.
5.5.5.1 Excess Variance

There is a discernable difference in the behavior of $E(\tau)$ for the strong Janus 2:1 density wave in the inner B ring as well as for the Janus 4:3 and Pandora 5:4 density waves in the inner A ring, indicating differences in particle aggregate properties across the inner A ring. As shown in Figure , the Pandora 5:4 density wave clearly lies on a curve of $E(\tau)$ which diverges from the $E(\tau)$ curves of the rest of the A ring waves, with a consistently lower normalized excess variance within the same range of optical depth.

![Normalized excess variance](image)

Figure 5.51. Normalized excess variance versus line-of-sight optical depth for waves in self-gravity dominated regions in the $\beta$ Centauri Rev 064 Egress (left) and $\beta$ Centauri Rev 081 Ingress occultation (right).

In the above plots, the Pandora 5:4 density wave clearly lies on a curve of $E(\tau)$ which diverges from the $E(\tau)$ curves of the rest of the A ring waves, with a consistently lower normalized excess variance within the same range of optical depth.

We also find that our results from moments for the granola bar model of self-gravity wakes as regularly spaced rectangular bars cannot match the statistics of both troughs and peaks with a single set of parameters $S$ and $W$ for the same density wave, which implies that $S$ and $W$ vary between the peaks and troughs. The predator-prey model (Esposito et al., 2012) would suggest that as a result of the wakes becoming more compressed in the crests and more rarefied in the troughs, the separation between the granola bars is likewise compressed as a function of
optical depth. The essentially transparent region separating the two granola bars is more easily compressed while the highly opaque bars are less easily compressed.

Notably, we find a strong dependence of $S/W$ on $\mu$. We speculate that this might be caused by vertical splashing caused by increased viscous stirring in these regions, resulting in an increased line-of-sight path through the ring material. The dependence is strongest for the Janus 2:1 and Pandora 5:4 density waves.

Figure 5.52. Best-fitting $S/W$ from normalized excess variance for occultations in which $\varphi - \varphi_{\text{wake}} > 10^\circ$ for the Janus 2:1 density wave. We find a wide range of $S/W$ values in this wave with an average of $S/W \sim 2.4$ and a Pearson’s correlation coefficient of $r = -0.90$. We exclude from this plot occultation for which the line-of-sight to the star is nearly perpendicular to the wakes (e.g., $\varphi - \varphi_{\text{wake}} > 10^\circ$). The negative linear correlation contradicts our prediction that the excess variance in regions covered by nearly flat, opaque self-gravity wakes should not depend on $\mu$. 
Figure 5.53. Best-fitting $S/W$ from normalized excess variance for occultations in which $\varphi - \varphi_{wake} > 10^\circ$ for the Pandora 5:4 and Janus 4:3 density waves.

(Left) We find a wide range of $S/W$ values in the Pandora 5:4 density wave with an average of $S/W \sim 3.2$ and a Pearson’s correlation coefficient of $r = -0.82$. (Right) We find a more reasonable range of $S/W$ values in the Janus 4:3 density wave with an average of $S/W \sim 2.0$ and a Pearson’s correlation coefficient of $r = -0.41$. The negative linear correlations contradict our prediction that the excess variance in regions covered by nearly flat, opaque self-gravity wakes should not depend on $\mu$.

Figure 5.54. Best-fitting $S/W$ from normalized excess variance for occultations in which $\varphi - \varphi_{wake} > 10^\circ$ for the Janus 5:4 and Mimas 5:3 density waves.

(Left) We find $S/W$ values ranging from less than 0.5 up to 3.5 in the Janus 5:4 density wave with an average of $S/W \sim 2.1$ and a Pearson’s correlation coefficient of $r = -0.48$. (Right) We find $S/W$ values ranging from ~0.5 to 3 in the Mimas 5:3 density wave with an average of $S/W \sim 2.2$ and a Pearson’s correlation coefficient of $r = -0.31$. The negative linear correlations contradict our prediction that the excess variance in regions covered by nearly flat, opaque self-gravity wakes should not depend on $\mu$.  

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Figure 5.55. Best-fitting $S/W$ from normalized excess variance for occultations in which $\varphi - \varphi_{\text{wake}} > 10^\circ$ for the Janus 6:5 density wave. We find a wide range of $S/W$ values in this wave with an average of $S/W \sim 2.5$ and a Pearson’s correlation coefficient of $r = -0.75$. The negative linear correlation contradicts our prediction that the excess variance in regions covered by nearly flat, opaque self-gravity wakes should not depend on $\mu$.

5.5.5.2 Skewness

The skewness of stellar occultation data has been suggested to reveal the presence of ghosts, or gaps in the rings. If the data are positively skewed, there are more gaps (positive outliers of the distribution) and if they are negatively skewed, there are more clumps (negative outliers of the distribution).
Figure 5.56. Skewness as a function of line-of-sight optical depth for the Janus 2:1 density wave and the A ring density waves in the α Crucis Rev 092 Ingress (left) and α Virginis Rev 210 Ingress (right) occultations. (Left) Given that in this occultation $\varphi \sim 150^\circ$ for the A ring waves and $\varphi_{\text{wake}} \sim 70^\circ$ (J16) so that $\phi - \phi_{\text{wake}} \sim 150 - 70^\circ = 80^\circ$. The clear general trend is that skewness increases (nearly) linearly with increasing optical depth which matches our expectations based on Monte Carlo simulations. (Right) Given that in this occultation $\varphi \sim 50^\circ$ for the A ring waves and $\varphi_{\text{wake}} \sim 70^\circ$ (J16) so that $\phi - \phi_{\text{wake}} \sim 50 - 70^\circ = -20^\circ$. The clear general trend is that skewness increases (nearly) linearly with increasing optical depth which matches our expectations based on Monte Carlo simulations.

Figure 5.57. Skewness as a function of line-of-sight optical depth for the Janus 2:1 density wave and the A ring density waves in the α Virginis Rev 210 Ingress occultation. Given that in this occultation $\varphi \sim 50^\circ$ for the A ring waves and $\varphi_{\text{wake}} \sim 70^\circ$ (J16) so that $\phi - \phi_{\text{wake}} \sim 130 - 70^\circ = 60^\circ$. The Janus 2:1 wave in red spans the broadest range in optical depth. The troughs of this wave ($-\tau \leq 2$) have a consistently lower skewness than the A ring density waves.
The ratio of $S$ to $\tau$ in some density wave peaks is significantly higher than the ratio in the troughs of the same wave, indicating greater asymmetry of the clump/gap population in the peaks. Skewness measures the asymmetry of the distribution, indicative of either large clumps or small gaps in the rings nicknamed "ghosts" (Baillié et al., 2011). The presence of a few small gaps (high transparency outliers in the distribution) may lead to a positive skewness. The presence of too many gaps, however, increases the symmetry of the distribution as the gaps are no longer outliers.

Figure 5.58. Average ratios of skewness ($S$) to optical depth ($\tau$) within peaks (diamonds) and troughs (crosses) in density waves in Saturn’s A ring within the $\beta$ Centauri Rev 102 Ingress occultation with $1\sigma$ error bars.

With the exception of the Pandora 5:4 density wave, the peak and trough ratios are separated by greater than a $1\sigma$ with peak ratios consistently higher than the trough ratios. These results indicate that the peaks contain more positive outliers to the distribution than the troughs, consistent with the presence of more gaps in the peaks.
Figure 5.59. Ratio of skewness to optical depth in the β Centauri Rev 077 Ingress and Egress occultations for various density waves. 

(Left) Average ratio of skewness (S) to optical depth (τ) of all density waves studied in this analysis as well as the C ring and Cassini Division ramps within the β Centauri Rev 077 Ingress occultation with 1σ error bars. (Right) Average ratio of skewness (S) to optical depth (τ) of the same regions within the β Centauri Rev 077 Egress occultation with 1σ error bars. We note that the behavior of peaks and troughs varies widely in the Cassini Division density waves, with many dipping extensively into negative values which never occurs for either the Jnaus 2:1 or the A ring density waves.

5.5.5.3 Kurtosis

Kurtosis is the fourth moment which measures the fatness (extreme values) of the tails of the distribution. While skewness provides information about gaps and clumps, the presence of too many gaps can increase the symmetry of the distribution such that the gap measurements are no longer outliers and the skewness approaches 0. In these limited cases, which occur primarily in the C ring plateaus, kurtosis may reveal gaps or clumps that skewness cannot detect.
Figure 5.60. Kurtosis and skewness in the $\beta$ Centauri Rev 092 Egress occultation across the main rings. We observe the same general behavior in both statistics. The correlation of kurtosis with skewness across the rings for all occultations is \~0.998, which indicates that there is little new information to be learned from the kurtosis that cannot be gained from analysis of the skewness.

Figure 5.61. Optical depth profile of a section of the C ring from 86000-90000 km boxcar smoothed by 100 points and stacked on top of a plot of skewness (black) and kurtosis (blue) for 10000 point bins of raw data at the same region. Locations where $S$~0 are indicated by red open circles and annotated by text with the corresponding radial locations of features in the region. Note that many of these occur within/near plateau structures. In two cases, one within P7 (~86600 km) and another near the outer edge of P8 (~88600 km), the skewness is nearly zero while kurtosis is slightly negative. This could indicate the presence of clumps.
Table 5.1. Inventory of density waves analyzed with Toomre most-unstable wavelengths calculated using surface mass densities from Spilker et al. (2004) and Tiscareno et al. (2013).

<table>
<thead>
<tr>
<th>Density Wave</th>
<th>Radial extent (km)</th>
<th>Neighboring region radial extent (km)</th>
<th>Toomre most unstable wavelength (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Janus 2:1</td>
<td>96200 – 96600</td>
<td>94800 - 95200</td>
<td>43</td>
</tr>
<tr>
<td>Pandora 5:4</td>
<td>122345 - 122600</td>
<td>122200 – 122300</td>
<td>36</td>
</tr>
<tr>
<td>Janus 4:3</td>
<td>125240 – 125560</td>
<td>125100 – 125200</td>
<td>68</td>
</tr>
<tr>
<td>Janus 5:4</td>
<td>130690 - 130860</td>
<td>130550 – 130650</td>
<td>55</td>
</tr>
<tr>
<td>Mimas 5:3</td>
<td>132320 - 132450</td>
<td>132080 – 132180</td>
<td>46</td>
</tr>
<tr>
<td>Janus 6:5</td>
<td>134230 - 134400</td>
<td>134100 – 134200</td>
<td>64</td>
</tr>
</tbody>
</table>
Table 5.2. List of occultations analyzed for each density wave with geometric parameters.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Waves</th>
<th>$I_0$ (counts per ms)</th>
<th>$\phi$ (°) range</th>
<th>$B$ (°)</th>
<th>$D_{\text{LOS}}$ (R$_{\text{e}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlpVir008 Egress</td>
<td>Pandora 5:4, Janus 4:3, Janus 5:4, Mimas 5:3, Janus 6:5</td>
<td>532</td>
<td>82-116</td>
<td>17.3</td>
<td>3.9</td>
</tr>
<tr>
<td>AlpVir008 Ingress</td>
<td>Pandora 5:4, Janus 4:3, Janus 5:4, Mimas 5:3, Janus 6:5</td>
<td>503</td>
<td>116-150</td>
<td>17.3</td>
<td>3.3</td>
</tr>
<tr>
<td>AlpVir211 Egress</td>
<td>Pan 6:5, Atlas 5:4, Prometheus 9:7, Pandora 5:4, Janus 4:3</td>
<td>121</td>
<td>202-235</td>
<td>17.3</td>
<td>15.4</td>
</tr>
<tr>
<td>Occultation</td>
<td>Waves</td>
<td>$I_0$ (counts per ms)</td>
<td>$\phi$ (°) range</td>
<td>$B$ (°)</td>
<td>$D_{LOS}$ (R$_s$)</td>
</tr>
<tr>
<td>------------------</td>
<td>--------------------------------------------</td>
<td>-----------------------</td>
<td>-------------------</td>
<td>---------</td>
<td>-------------------</td>
</tr>
<tr>
<td>Mission</td>
<td>Range</td>
<td>(Rₚ)</td>
<td>Unocculted stellar signal (counts/ms)</td>
<td>Ring elevation angle (°)</td>
<td>Clock angle (°)</td>
</tr>
<tr>
<td>-------------</td>
<td>---------------</td>
<td>------</td>
<td>-------------------------------------</td>
<td>-------------------------</td>
<td>-----------------</td>
</tr>
</tbody>
</table>

1 Rₚ = 60330 km.
Iₒ – Unocculted stellar signal (counts/ms)
B – Ring elevation angle (°)
φ – Clock angle (°)
D₁ₒₛ – Line-of-sight distance of Cassini (Rₚ)
Table 5.3. List of best fit $R_{\text{eff}}$ for Cassini Division density waves with 2σ error bars.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Best fit $R_{\text{effective}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prometheus 9:7</td>
</tr>
<tr>
<td>AlpCru092 Ingress</td>
<td>2.1 ± 0.5</td>
</tr>
<tr>
<td>AlpCru100 Egress</td>
<td>2.4 ± 0.3</td>
</tr>
<tr>
<td>AlpCru100 Ingress</td>
<td>2.3 ± 0.3</td>
</tr>
<tr>
<td>BetCen064 Egress</td>
<td>3.6 ± 0.6</td>
</tr>
<tr>
<td>BetCen075 Ingress</td>
<td>2.3 ± 0.5</td>
</tr>
<tr>
<td>BetCen077 Egress</td>
<td>2.5 ± 0.4</td>
</tr>
<tr>
<td>BetCen077 Ingress</td>
<td>2.2 ± 0.4</td>
</tr>
<tr>
<td>BetCen078 Egress</td>
<td>3.2 ± 0.3</td>
</tr>
<tr>
<td>BetCen081 Ingress</td>
<td>1.8 ± 1.0</td>
</tr>
<tr>
<td>BetCen085 Ingress</td>
<td>N/A</td>
</tr>
<tr>
<td>BetCen089 Ingress</td>
<td>2.0 ± 0.3</td>
</tr>
<tr>
<td>BetCen092 Egress</td>
<td>2.0 ± 0.8</td>
</tr>
<tr>
<td>BetCen096 Ingress</td>
<td>1.5 ± 0.5</td>
</tr>
<tr>
<td>BetCen102 Ingress</td>
<td>1.3 ± 0.5</td>
</tr>
<tr>
<td>BetCen104 Egress</td>
<td>1.2 ± 0.5</td>
</tr>
<tr>
<td>BetCen104 Ingress</td>
<td>1.5 ± 0.3</td>
</tr>
<tr>
<td>BetCen105 Egress</td>
<td>1.4 ± 0.6</td>
</tr>
<tr>
<td>BetCen105 Ingress</td>
<td>1.3 ± 0.6</td>
</tr>
<tr>
<td>LamSco029 Egress</td>
<td>1.1 ± 0.6</td>
</tr>
<tr>
<td>LamSco044 Ingress</td>
<td>0.9 ± 0.5</td>
</tr>
</tbody>
</table>
Table 5.4. Best-fit $R_{\text{eff}}$ in the troughs of Cassini Division density waves from moments.

<table>
<thead>
<tr>
<th>Density Wave</th>
<th>Radial extent (km)</th>
<th>Average best fit $R$ (# of occultations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prometheus 9:7</td>
<td>~118066—118078</td>
<td>2.1 (22)</td>
</tr>
<tr>
<td>Pan 6:5</td>
<td>~118453—118469</td>
<td>2.4 (29)</td>
</tr>
<tr>
<td>Atlas 5:4</td>
<td>~118830—118848</td>
<td>4.1 (33)</td>
</tr>
</tbody>
</table>

Table 5.5. Best-fit parameters describing self-gravity wake morphology in the troughs of numerous density waves from moments with $\chi^2 < 3$.

<table>
<thead>
<tr>
<th>Density Wave</th>
<th>Average best fit $S/W$ from variance (# of occultations with $\chi^2 &lt; 5$)</th>
<th>Average best fit $S/(S+W)$ from variance (# of occultations with $\chi^2 &lt; 5$)</th>
<th>Average best fit $S/W$ from skewness (# of occultations with $\chi^2 &lt; 5$)</th>
<th>Average best fit $S/(S+W)$ from skewness (# of occultations with $\chi^2 &lt; 5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Janus 2:1</td>
<td>2.4 (26)</td>
<td>0.63 (26)</td>
<td>2.1 (22)</td>
<td>0.59 (22)</td>
</tr>
<tr>
<td>Pandora 5:4</td>
<td>3.2 (33)</td>
<td>0.71 (33)</td>
<td>2.4 (29)</td>
<td>0.60 (29)</td>
</tr>
<tr>
<td>Janus 4:3</td>
<td>2.2 (31)</td>
<td>0.63 (31)</td>
<td>4.1 (33)</td>
<td>0.77 (33)</td>
</tr>
<tr>
<td>Janus 5:4</td>
<td>2.1 (31)</td>
<td>0.67 (31)</td>
<td>3.5 (33)</td>
<td>0.75 (33)</td>
</tr>
<tr>
<td>Mimas 5:3</td>
<td>2.2 (28)</td>
<td>0.67 (28)</td>
<td>3.2 (32)</td>
<td>0.73 (32)</td>
</tr>
<tr>
<td>Janus 6:5</td>
<td>2.5 (22)</td>
<td>0.69 (22)</td>
<td>3.7 (30)</td>
<td>0.74 (30)</td>
</tr>
</tbody>
</table>
CHAPTER SIX: DISCUSSION

There are several avenues of potential research for future graduate students to pursue in studying Saturn’s rings. For example, investigating differences in the particle size distribution within the C ring plateaus versus the background C ring using occultation statistics, particularly skewness which appears to indicate the presence of gaps. One way to do this would be to compare the observed excess variance, skewness, and kurtosis with results from Monte Carlo simulations of Saturn’s rings. Another idea to consider is adding multiple-scattering processes, which significantly impact observations at high slant optical depths, to a forward-model of diffraction signatures observed at many sharp ring edges. Along a similar vein, a modification to our occultation statistics model to account for the non-linear behavior of many strong density waves may be worth pursuing. Additionally, because the Encke and Keeler gaps lie in close proximity to numerous strong and weak density waves within Saturn’s A ring, there might be a unique opportunity for a future graduate student to combine techniques to further determine the nature of the size distribution in these regions. Besides diffraction signatures and occultation statistics, another way to discern the particle size distribution in the rings is by comparing data from observations at different wavelengths as J16 and Jerousek et al. (2020) did using Cassini UVIS, VIMS, and RSS data. Although the topic has already been explored, there is such a wealth of data obtained by Cassini that numerous novel analyses can certainly be performed.

While Pioneer, Voyager, and Cassini, the exploration of the distribution of particles in Saturn’s rings is only just beginning. New observations of Saturn’s rings are anticipated with the beginning of the long-awaited era of the James Webb Space Telescope (JWST). As a starting point, Santos-Sans et al. (2016) have already identified a number of potential stellar occultation
events by rings and other minor solar system bodies that the JWST could observe which have high scientific merit, including an extension of our inventory of non-circular and time-variable features and observations with the NIRCAM F300M filter which has a central wavelength near the water ice and methane absorption bands.
LIST OF REFERENCES


composition and longitudinal fractionation of Saturn ring rain material. *Icarus*, 339, 113595.


