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Low-intensity laser-electron scattering in the Thomson limit

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(Received 27 August 1993)

Within the context of laser-electron scattering for low intensities of the incoming photon beam, we present a discussion of the leading multiphoton reaction, two-photon absorption by a free electron in a Compton-like process. The two-photon angular distribution is calculated in quantum electrodynamics and compared with existing classical and semiclassical calculations in the Thomson limit. We find that for incident linearly polarized beams all such calculations agree in this limit. Furthermore, the present approach allows us to study the possible effect of the quantum-mechanical correlation of pairs of coincident photons, i.e., the degree of second-order coherence.

PACS number(s): 12.20.Ds, 13.10.+q, 42.50.Wm

The subject of laser-electron scattering remains one of interest since the development of the laser [1]. A basic process studied here corresponds to the absorption of n laser photons by a free electron and the emission of a single photon, $e+n\gamma\rightarrow e+\gamma$. Recent advances in the availability of extremely short, ultrastrong, laser pulses [2] have induced a renewed interest in high intensity laser-electron scattering [3,4]. While the general emphasis of theoretical work has been concentrated in the strong intensities provided by a laser beam, either within a semiclassical [5–7] or entirely classical [8] approach, at the other extreme of low intensity photon beams a fully quantum electrodynamics (QED) approach is of interest because it allows some important checks not thoroughly discussed in the literature so far. In particular, (i) an explicit QED derivation of a multiphoton cross section that takes into account the correlation properties of the incident light beam; (ii) to determine whether or not there is consistency, in the Thomson (TH) limit, between QED, semiclassical [5,6], and classical [9] computations for a multiphoton process initiated by linearly polarized light. It is well known that for Compton scattering all such results agree in this limit.

In this paper, we examine laser-electron scattering under low incident light intensities. We thus limit the present discussion to the two-photon process $e+2\gamma\rightarrow e+\gamma$, as higher-order multiphoton rates are much smaller. Specifically, with respect to (ii), some discrepancy with the classical cross section of Ref. [9] has been reported by some authors [10,11]. We show below how these results can be readily reconciled. Furthermore, theoretical work exists regarding the equivalence of the semiclassical and quantum approach within scalar electrodynamics [12,13] and at the level of the general form of the transition rate [14]. In the present work, we complement these efforts by comparing our QED calculation with the semiclassical method at the amplitude level.

On the other hand, concerning (i), there is a meaningful physical difference between the Compton and the two-photon reactions. The latter, being a nonlinear process, must depend upon the correlation characteristics of the participating beam which, in the language of quantum optics, is given by the degree of second-order coherence $g^{(2)}(0)$. By analogy to the well-known situation of transition rates in the scattering of light by atoms, one expects that the cross section of the two-photon process should also depend upon the degree of coherence. A familiar feature that emerges here is that $g^{(2)}(0)$ may, for some types of beams, lie outside its classical range of values. Such values cannot even in principle be derived from semiclassical or classical calculations. From this point of view the TH limit of the two-photon reaction is not exactly equivalent, as is the case in Compton scattering for all incident beams, to classical electromagnetism.

We work in the laboratory frame, with the electron at rest, and assume that the incident linearly polarized light field is in a single-mode state. It is convenient to define the initial and final photon states of the incident beam as $|n\rangle$ and $|n-2\rangle$, in the number state basis (mode subscripts $\mathbf{k},\lambda$ suppressed). In this way the scattering amplitude takes the form

$$S_{fi}=\langle n-2|a^2|n\rangle(-e)^3\left[\frac{1}{\sqrt{2\omega V}}\right]^2\frac{1}{\sqrt{2\omega' V}}\left[\frac{1}{\sqrt{V}}\right]^2 \times(2\pi)^4\delta^4(p+2k-p'-k')F_{2\gamma} ,$$

where a is the annihilation operator for the incident photons and $F_{2\gamma}$ is the invariant amplitude corresponding to the diagrams of Fig. 1. It is readily given by the standard Feynman-Dyson rules:

$$F_{2\gamma}=\bar{u}'\left[\not{\epsilon}'\frac{\not{p}+2\not{k}+m}{(p+2k)^2-m^2}\not{\epsilon}\frac{\not{p}+\not{k}+m}{(p+k)^2-m^2}\not{\epsilon} +\not{\epsilon}\frac{\not{p}+\not{k}-\not{k}'+m}{(p+k-k')^2-m^2}\not{\epsilon}\frac{\not{p}-\not{k}'+m}{(p-k')^2-m^2}\not{\epsilon}' +\not{\epsilon}\frac{\not{p}+\not{k}-\not{k}'+m}{(p+k-k')^2-m^2}\not{\epsilon}'\frac{\not{p}+\not{k}+m}{(p+k)^2-m^2}\not{\epsilon}\right]u . \tag{1}$$

Here, $u=u_{p,s}$ and $u'=u_{p',s'}$ are the Dirac spinors, and

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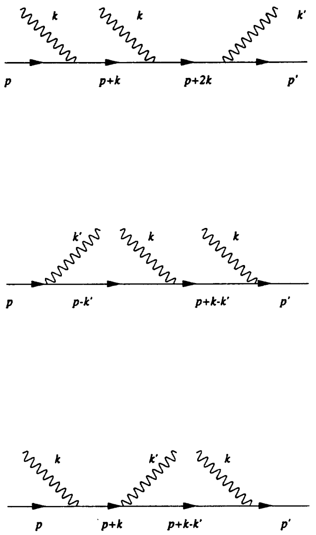

FIG. 1. Lowest-order Feynman diagrams for $e\gamma\gamma\rightarrow e\gamma$.

p,s and p',s' are the momentum, spin state of the incident and scattered electrons, respectively. Similarly, $k,\varepsilon=\varepsilon(\mathbf{k},\lambda)$, $k',\varepsilon'=\varepsilon'(\mathbf{k}',\lambda')$ denote the momentum, polarization vector of the ingoing and outgoing photons, and λ,λ' specify their polarization states.

We next consider the differential transition rate. Within our framework, the definition for the rate can be expressed as

$$d\Gamma_{2\gamma}=e^6\frac{\rho_e}{8m\omega^2}\frac{|\langle n-2|a^2|n\rangle|^2}{V^2}\overline{|F_{2\gamma}|^2}d\mathcal{S}_{\text{Lips}}\ , \tag{2}$$

where the Lorentz invariant phase space is $d\mathcal{S}_{\text{Lips}}=(2\pi)^{-2}\delta^4(p+2k-p'-k')d^3p'd^3k'/(4E'\omega')$, and m,ω (E',ω') are the initial (final) energies of the electron and photons. The notation $\overline{|F_{2\gamma}|^2}$ denotes the usual electron spin sums.

The transition rate in Eq. (2) is proportional to $|\langle n-2|a^2|n\rangle|^2=\langle n|\hat{N}(\hat{N}-1)|n\rangle$, where $\hat{N}=a^+a$. This result applies for incident photons whose initial state is $|n\rangle$ and whose number is therefore specified precisely. For a beam in some general state that corresponds to single-mode mixed states, the above relation is replaced by $\text{Tr}[\hat{\rho}_{\text{beam}}\hat{N}(\hat{N}-1)]=\langle\hat{N}(\hat{N}-1)\rangle$, where $\hat{\rho}_{\text{beam}}$ denotes the beam density matrix operator. The photon expectation value can be reexpressed in terms of the quantum-mechanical degree of second-order coherence, $\langle\hat{N}(\hat{N}-1)\rangle=g^{(2)}(0)\langle\hat{N}\rangle^2$. It turns out that (2) is proportional to $g^{(2)}$ and to the square of the photon density, $\rho_\gamma=\langle\hat{N}\rangle/V$.

A most convenient method in evaluating $\overline{|F_{2\gamma}|^2}$ consists in reexpressing Eq. (1) as $F_{2\gamma}=\bar{u}'(M'_{2\gamma}+M''_{2\gamma})u$, where

$$M'_{2\gamma}=\left[\frac{\varepsilon'\cdot k}{\tau}-\frac{2\varepsilon\cdot k'\varepsilon\cdot\varepsilon'}{\tau'}\right]\frac{\not{k}}{2\tau'}\ , \tag{3a}$$

$$M''_{2\gamma}=\left[\frac{1}{\tau}-\frac{1}{\tau'}\right]\frac{\not{\varepsilon}'}{4}+\frac{\varepsilon\cdot k'}{4\tau'^2}\not{\varepsilon}\not{\varepsilon}'(\not{p}'-\not{p})+\frac{\varepsilon\cdot k'}{2\tau'}\left[\frac{1}{\tau'}-\frac{1}{\tau}\right]\not{\varepsilon}'\not{\varepsilon}\not{k}\ , \tag{3b}$$

here, $\tau=p\cdot k$, $\tau'=p'\cdot k$, and both the initial and final photons are transversely polarized in the laboratory frame. It will be observed that in the limit of incident photons of extremely small momentum the electron essentially does not recoil, staying practically at rest, then in the laboratory frame $p'\approx p$ or $\not{p}'\approx\not{p}$, and only the first term of $M''_{2\gamma}$ survives but its contribution to $\overline{|F_{2\gamma}|^2}$ vanishes. Thus, the leading term follows immediately from $M'_{2\gamma}$ alone [15],

$$\overline{|F_{2\gamma}|^2}=\frac{1}{m^2}\left\{(\hat{\varepsilon}'\cdot\hat{\mathbf{k}}+4\hat{\varepsilon}\cdot\hat{\varepsilon}'\hat{\varepsilon}\cdot\hat{\mathbf{k}}')^2+O\left[\left[\frac{\omega}{m}\right]^2\right]\right\}. \tag{4}$$

Taking the TH limit to the amplitude (3) before squaring it, certainly provides an effortless way to obtain Eq. (4), simpler than via calculations for intense fields [5,6], or nonrelativistic [16] and numerical computations [11]. An exact trace calculation verifies that (4) is the correct result in the TH limit $\omega/m\ll 1$.

From the previous equations we may calculate the rate for unpolarized final photons to go into solid angle $d\Omega$. For the incident photons we take $\hat{\mathbf{k}}$ along the z axis, $\hat{\varepsilon}$ in the x direction, and the final polarization vectors are chosen as in Ref. [17]. The angular distribution for final photon polarization not detected and incident photons linearly polarized is now straightforwardly evaluated in the TH limit, to $O(\omega/m)$, as

$$\frac{d\sigma_{2\gamma}}{d\Omega}=g^{(2)}(0)\frac{\rho_\gamma\pi\alpha^3}{m^4\omega}16\sin^2\theta[(\cos\theta\cos^2\phi-\tfrac{1}{4})^2+\tfrac{1}{4}\sin^2 2\phi]\ , \tag{5}$$

where $d\Gamma_{2\gamma}/d\Omega=\rho_e\rho_\gamma d\sigma_{2\gamma}/d\Omega$. The normalization term in (5), excluding the degree of coherence $g^{(2)}(0)$, can be reexpressed in terms of the classical parameters for the intensity $\eta^2=4\pi\alpha\rho_\gamma/(m^2\omega)$, and electron radius r_0, as $(\eta r_0/2)^2=\rho_\gamma\pi\alpha^3/(m^4\omega)$. Thus, it is natural to expect that this result ought to agree with classical and semiclassical calculations. This point is taken up next. Notice that the combination $(\eta r_0)^2$ enters naturally here, a fact better illustrated by simple dimensional considerations based on the diagrams of Fig. 1. Therefore, perturbative QED computations like the present one, as well as the low-intensity limit of calculations for high-intensity laser fields, lead to this kind of dependence in the cross section.

Detailed semiclassical calculations for multiphoton Compton scattering have been discussed in the case of intense incident, linearly polarized, laser fields [5,6]. Here the incoming beam is not quantized, it is taken as a plane monochromatic electromagnetic wave. The equivalence of this method in the limit of low intensities with our previous QED calculation, at the level of the cross section and *excluding* the degree of coherence, can be readily verified. (a) The leading term in Eq. (4) agrees with the calculation of Brown and Kibble [18]. (b) Similarly, the angular distribution in Eq. (5) can be obtained from the result of Nikishov and Ritus [19]. For low-intensity fields however this equivalence already occurs at the amplitude level as demonstrated below. This is in contrast with previous arguments [14] of such an equivalence existing only for the cross sections in the case of intense beams.

Translating to the present notation, we quote the result from Ref. [6] for the n-photon absorption amplitude [20]

$$N_{n\gamma}=\bar{u}'\left[A_0\not{\varepsilon}'+A_1\frac{m\eta}{2}\left[\frac{\not{\varepsilon}'\not{k}\not{\varepsilon}}{\tau}+\frac{\not{\varepsilon}\not{k}\not{\varepsilon}'}{\tau'}\right]+A_2\frac{m^2\eta^2}{2}\frac{\varepsilon'\cdot k}{\tau\tau'}\not{k}\right]u\ , \tag{6a}$$

where

$$A_j(n,x,y)=\pi^{-1}\int_0^{\pi}\cos^j\phi\cos(n\phi-x\sin\phi+y\sin2\phi)d\phi,\quad j=0,1,2, \tag{6b}$$

$$x=-m\eta\frac{\varepsilon\cdot p'}{\tau'},\quad y=\frac{m^2\eta^2}{8}\left[\frac{1}{\tau'}-\frac{1}{\tau}\right]. \tag{6c}$$

One can readily check that for $\eta\ll1$ the $n=2$ functions A_j become

$$A_0=x^2/8-y/2,\quad A_1=x/4,\quad A_2=\tfrac{1}{4}\ . \tag{7}$$

The term $\bar{u}'\not{\varepsilon}\not{k}\not{\varepsilon}'u=-2^{-1}\bar{u}'(\not{p}'+\not{k}'-\not{p})\not{\varepsilon}\not{\varepsilon}'u$ in Eq. (6a) can be reexpressed, by virtue of the Dirac equation for $\bar{u}'$ and u and the anticommutation properties of the γ matrices, as $-2^{-1}\bar{u}'[\not{\varepsilon}\not{\varepsilon}'\not{k}'+2(\varepsilon\cdot k')\not{\varepsilon}']u$. Thus together with (7), a more convenient form of (6a) for $n=2$ is

$$N_{2\gamma}=-\left[\frac{m\eta}{2}\right]^2F_{2\gamma}\ , \tag{8a}$$

$$F_{2\gamma}=\bar{u}'\left[\frac{1}{4}\left[\frac{1}{\tau}-\frac{1}{\tau'}\right]\not{\varepsilon}'+\frac{\varepsilon'\cdot k}{2\tau\tau'}\not{k}-\frac{\varepsilon\cdot k'}{\tau'}\left[\frac{\not{\varepsilon}'\not{\varepsilon}\not{k}}{2\tau}+\frac{\not{\varepsilon}\not{\varepsilon}'\not{k}'}{4\tau'}\right]\right]u\ . \tag{8b}$$

This is the QED invariant amplitude found before after reexpressing each term in (1), with the help of elementary spinor algebra, respectively, as $\bar{u}'\not{\varepsilon}'u/4\tau,\bar{u}'(\varepsilon\cdot k'\not{\varepsilon}\not{k}'\not{\varepsilon}'-\tau'\not{\varepsilon}')u/(4\tau'^2)$, and $\bar{u}'(\varepsilon'\cdot k\not{k}-\varepsilon\cdot k'\not{\varepsilon}'\not{\varepsilon}\not{k})u/(2\tau\tau')$.

We conclude that for low photon densities in the incident beam, the semiclassical and QED calculations have exactly the same invariant amplitude up to the factor $\langle n-2|a^2|n\rangle$, which is the origin of the quantum degree of coherence that affects the overall normalization of the cross section. We note that substitution of (7) in the cross section given in Ref. [6] must, therefore, reproduce Eq. (5) without, of course, the quantum-mechanical degree of second-order coherence.

A nonlinear Compton scattering calculation has been carried out before within classical electrodynamics and from the point of view adopted here, i.e., the second harmonic cross section in the TH limit, for unpolarized final photons, and for low-intensity incident laser fields [9]. As emphasized by several authors [10,11,13] the classical result is similar, yet seemingly not equal, to the angular distribution in (5). The apparent difference between these results cannot be attributed, however, to the use of distinct Lorentz frames as previously claimed [10]. For $\eta\ll1$ the momentum of the electron in the wave is essentially the same as the momentum of a free electron, thus the rest frame of the electron in both cases is practically the same.

Within the present notation the classical cross section obtained in Ref. [9] is

$$\frac{d\sigma_{2\gamma}^{\text{Cl}}}{d\Omega}=\left[\frac{r_0\eta}{2}\right]^2(\sin^2\theta+4\sin^22\tilde{\phi}-8\cos^2\tilde{\phi}\cos\theta)\ . \tag{9}$$

Here, $\tilde{\phi}$ is defined as $\cos\tilde{\phi}=\hat{\varepsilon}\cdot\hat{\mathbf{k}}'$ [9] then it is simply related to θ,ϕ according to $\cos\tilde{\phi}=\sin\theta\cos\phi$. The angular term of (5) is thus recovered under this substitution in (9). We conclude that in the TH limit, i.e., for an electron initially at rest, classical, semiclassical, and QED computations agree exactly as far as the angular distribution of the emitted photon is concerned. The results in Ref. [11], however, disagree with ours in this respect [21].

We illustrate the prediction of (5) for the particular situation of a laser field, then $g^{(2)}=1$, which is equal to the lower limit of the classically allowed region. Figure 2 shows our result for such an incident laser beam with $\lambda=100$ nm, and $\eta=5\times10^{-6}$. For comparison, we also depict in this figure the well known Thomson cross section (inset). In the two-photon rate there is of course complete conversion of the incoming photon frequency to the second harmonic, at twice the initial frequency, giving a clear experimental signature for this reaction. It should be noted that the magnitude of the two-photon cross section for this particular value of η, though very small, is comparable to existing QED cross section measurements in high-energy physics. Other realistic values for η may be considered, presumably ranged from one to three orders-of-magnitude larger than the one chosen here [22]. This would enhance the cross section in Fig. 2 up to six orders of magnitude by virtue of its nonlinear dependence on the intensity.

It is also noteworthy that the angular distribution of the 2γ cross section exhibits a distinct and more interesting structure than the Thomson cross section as illustrated in Fig. 2: (i) the angular distribution of Compton scattered photons in the Thomson limit is forward-backward symmetric in the polar angle for all values of the azimuthal angle ϕ, this familiar feature results in a quite undramatic angular distribution; (ii) on the other hand, the two-photon rate shows a sizable polar asymmetry for all angles except at $\phi=\pi/2$. [23], the minimum and the two prominent peaks at $\phi=0$ are all ϕ dependent, as the azimuthal angle increases this structure merges into a single peak; (iii) in particular, note the ϕ dependence of both the position of each maximum as well as its corresponding magnitude, this is in contrast to the case of the fixed forward-backward maximum in the Thomson rate which have the same constant magnitude for all ϕ; (iv) Eq. (5) predicts the appearance of unpolarized scattered photons

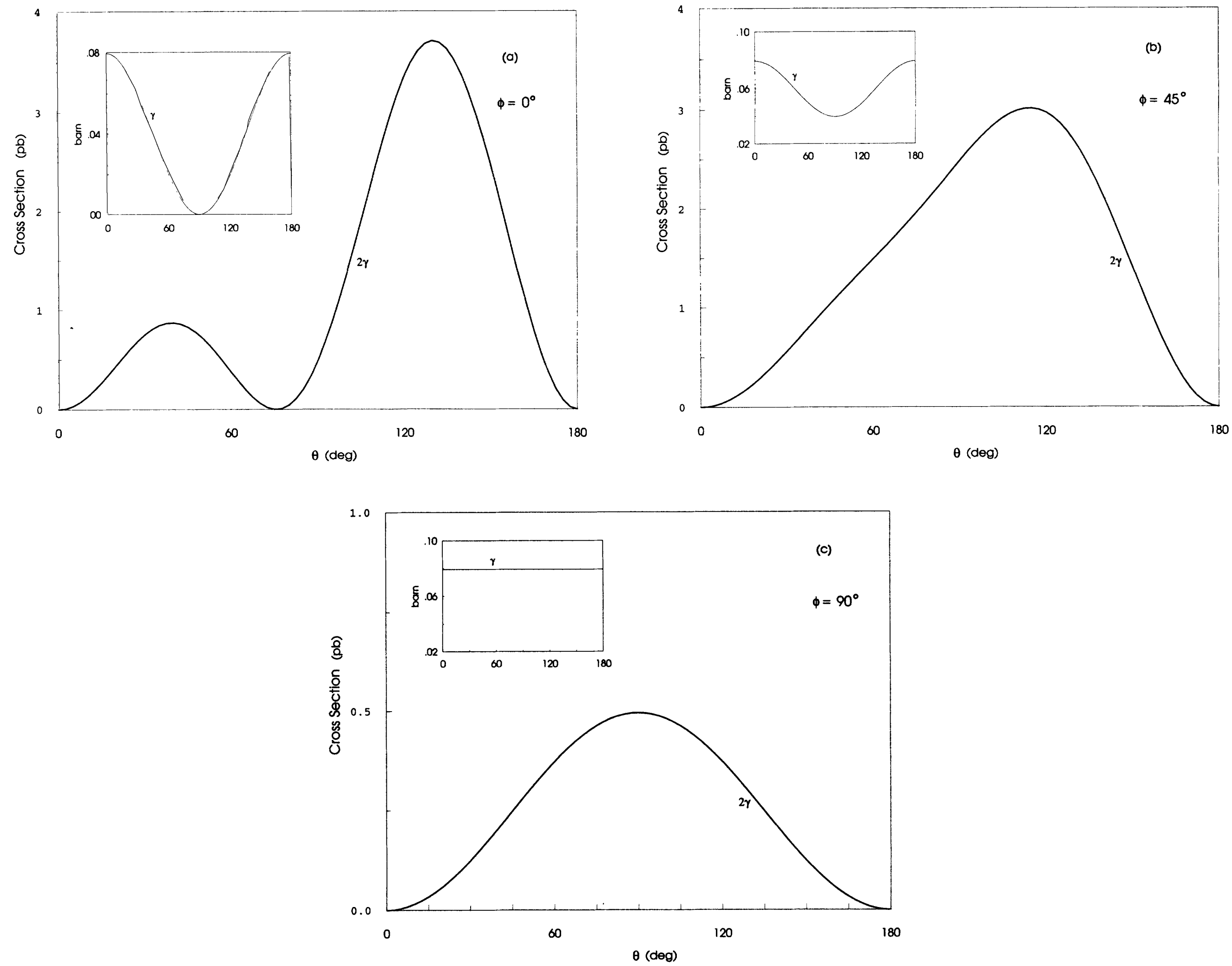

FIG. 2. Angular distribution for unpolarized final photons at three different values of the azimuthal angle, (a) $\phi=0°$, (b) $\phi=45°$, (c) $\phi=90°$. The vertical axis is $d\sigma/d\Omega$. Two-photon cross section 2γ. Compton cross section (inset) γ. The incident beam has $g^{(2)}=1$, $\lambda=100$ ηm, and $\eta=5\times10^{-6}$.

in the direction $\theta=\pi/2$, where the Thomson rate becomes fully polarized.

For two-photon and Compton scattering induced by a linearly polarized beam the angular distribution, in the TH limit, is thus classical in origin. An analogous situation exists for these processes induced by a circularly polarized wave [24]. We note that classical and semiclassical treatments that would take into account the statistical fluctuations of the wave field should lead to an expression identical to (5), proportional to the degree of second-order coherence. However, in contrast with the classical and semiclassical cases where $\infty \geq g^{(2)}(0) \geq 1$, there is an exclusive quantum scope of values $1 > g^{(2)}(0) \geq 0$ in the present QED situation of an incident beam represented by a single-mode statistical mixture [25]. As a result, only the latter establishes a link to possible nonclassical effects. We conclude that at least in principle the two-photon reaction may probe the quantum correlation characteristics of the incident photon beam in the Thomson limit. This seems to be a significant physical difference with the Compton process, and with the results of classical and semiclassical calculations of two-photon absorption.

Consider, for example, the case of a photon-antibunched beam where the cross section is reduced below the one for incident coherent fields, since $g^{(2)}(0)<1$ for nonclassical light. A crude order of magnitude estimate of this effect might be better appreciated by comparing it with the radiative corrections to order α^3 for Compton scattering, which for unpolarized light were calculated by Brown and Feynman [26]. In the Thomson limit the Brown-Feynman cross section takes the form $d\sigma_{\mathrm{BF}}/d\Omega=(d\sigma_\gamma/d\Omega)[1+(\alpha/\pi)\delta+O(\alpha^2)]$, where δ includes both virtual and bremsstrahlung corrections. Explicitly, the angular dependent part of δ, i.e., the term independent of the known energy resolution ΔE, is of order $(\omega/m)^2\ln(\omega/m)\sim6\times10^{-9}$ (for the beam in Fig. 2), which is a very small contribution and, in fact, vanishes in the zero energy limit. Note that the first correction to Eq. (5) coming from the matrix element (4) is also of $O[(\omega/m)^2]$. On the other hand, we have for any arbitrary single-mode beam the bound $1/\langle\hat{N}\rangle \geq 1-g^{(2)}$ [24], which is also expected to be very small, unless the mean number of photons is not extremely large as under the present assumption of a low-intensity beam. A rough qualitative estimate for the beam in Fig. 2 insinuates that $1-g^{(2)}$ could be several orders of magnitude larger than the radiative correction figure for Compton scattering [27]. Nevertheless, the effect of nonclassical light on the two-photon reaction still remains very small to have detectable effects at present, but this may eventually change.

I would like to thank the Center for Research and Education in Optics and Lasers and the Free Electron Laser group for support during this work.

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[1] For a general reference to the early literature, see J. H. Eberly, in *Progress in Optics,* edited by E. Wolf (North-Holland, Amsterdam, 1969), Vol VII, p. 359.

[2] For a recent review, see M. C. Richardson, in *Solid State Lasers II,* edited by G. Dube, SPIE Proc. Vol. 1410 (SPIE, Bellingham, WA, 1991), p. 15.

[3] T. J. Englert and E. A. Rinehart, Phys. Rev. A **28**, 1539 (1983); K. T. McDonald, Princeton Report No. DOE/ER/3072-38, 1986.

[4] K. T. McDonald, In *Laser Acceleration of Particles,* edited by C. Joshi and T. Katsouleas, AIP Conf. Proc. No. 130 (New York, 1985), p. 23.

[5] L. S. Brown and T. W. B. Kibble, Phys. Rev. **133**, A705 (1964).

[6] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. **46**, 776 (1964) [Sov. Phys. JETP **19**, 529 (1964)].

[7] I. I. Goldman, Phys. Lett. **8**, 103 (1964).

[8] S. P. Goreslavskii *et al.*, Laser Phys. **3**, 421 (1993).

[9] Vachaspati, Phys. Rev. **128**, 664 (1962); **130**, 2598(E) (1963).

[10] See [5], Appendix C.

[11] A. K. Puntajer and C. Leubner, J. Appl. Phys. **67**, 1606 (1990).

[12] Z. Fried and J. H. Eberly, Phys. Rev. **136**, B871 (1964).

[13] T. W. B. Kibble, Phys. Rev. **138**, B740 (1965).

[14] L. M. Frantz, Phys. Rev. **139**, B1326 (1965).

[15] Spinors are normalized to $\bar{u}u=2m$.

[16] F. Ehlotzky, J. Phys. B **20**, 2619 (1987).

[17] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), p. 680.

[18] See [5], Eq. (3.44).

[19] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. **47**, 1130 (1964) [Sov. Phys. JETP **20**, 757 (1965)].

[20] In our case of transversely polarized photons in the laboratory frame, $\varepsilon\cdot p=0$, $\varepsilon'\cdot p=0$, and the quantity ea^{μ} in Ref. [6] becomes $m\eta\varepsilon^{\mu}$ here.

[21] This is probably due to the use of a nonzero initial electron momentum in the numerical computation of Ref. [11].

[22] A comparison of Eq. (5) and the corresponding result of Ref. [6] for the case of intense incident lasers shows that for $\eta\approx 6\times 10^{-2}$ the two-photon cross sections agree by $<1\%$.

[23] Our angular distribution is identical to the classical result (9) which is not the case in [11], however, there is qualitative agreement regarding this asymmetry with the conclusion of this reference.

[24] N. Narozhnyi, A. Nikishov, and V. Ritus, Zh. Eksp. Teor. Fiz. **47**, 930 (1964) [Sov. Phys. JETP **20**, 622 (1965)].

[25] See, e.g., R. Loudon, *The Quantum Theory of Light* (Oxford University Press, Oxford, 1984); M. D. Reid and D. F. Walls, Phys. Rev. A **34**, 1260 (1986).

[26] L. M. Brown and R. P. Feynman, Phys. Rev. **85**, 231 (1952).

[27] For example, assuming a volume of the beam larger (by over an order of magnitude) than the one estimated for a greatly tight focused ultrashort laser pulse.