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APPLICATION OF NON-LOCAL APPROACHES FOR PREDICTING THE RESPONSE OF V-NOTCH UNDER THERMOMECHANICAL FATIGUE LOADING

by

TRUNG V. NGUYEN

A Thesis submitted in partial fulfillment of the
requirements for the Honors in the Major Program
in Aerospace Engineering in the College
Engineering and Computer Science and in the
Burnett Honors College at the University of Central
Florida
Orlando, Florida

Spring Term 2013

Thesis Chair: Dr. Ali P. Gordon

Abstract

The topic of this thesis is the construction of a formula to approximate stress-strain responses at notches under thermomechanical fatigue (TMF) loading. The understanding of material behavior of the V-notched component which experiences TMF is important to the mechanical industries where V-notched structures are often utilized. In such applications, it is crucial that the designers be able to predict the material behavior; therefore, the purpose of this research is to examine and to model the precise effects a stress concentration will have on a specimen made of a generic Ni-base superalloy. The effects of non-isothermal loading will be studied, and it is the goal of this research to formulate an extension of Neuber's rule appropriate for TMF which is to approximate the temperature range with a single value, T^*. One strategy to extend Neuber's rule, which relies on Finite Element Modeling (FEM), Bilinear Kinetic Hardening Model (BKIN), and test data, will be used to predict the stress-strain behavior at the notch of a thin plate subjected to axial loading. In addition, the CHABOCHE model will be utilized in the FEA to have the highest fidelity to material response at high temperatures. Parametric study of the FEA simulations will be employed to determine the correlation between the Neuber hyperbola, temperature range, stress concentration, the nominal stress, and the temperature cycling. Using the Neuber hyperbola and simplified constitutive model (i.e., bilinear kinematic strain hardening), the stress-strain solutions of the specimen will be calculated and compared to analytical results.

Dedications

For my family whose love and supports help me to achieve many great accomplishments.

Acknowledgments

I would like to express the deepest appreciation to my Thesis Chair, Dr. Ali P. Gordon, who provides technical supports and guides me through my undergraduate study. In addition, Dr. Gordon's inspiration in doing research has motivated me to take my education to the graduate level. Without the help from Dr. Gordon, this research would not be feasible. I would also like to thank the graduate research assistants in the Mechanics of Materials Research Group at UCF. Scott Keller helped me to expand my knowledge in mechanics of material field. Calvin Stewart supported me in writing and developing ANSYS codes.

I would also like to thank my family for their continuous support throughout my study. By encouraging and providing physical supports, they help me to accomplish many great achievements in my life.

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Nomenclature

α	Back Stress
$\epsilon_{elastic}$, $\epsilon_{plastic}$, ϵ_{total}	Elastic, plastic, total strain, respectively
ρ	Notch root radius
σ, σ_y	Engineering stress, yielding stress
γ_i	Recall term for non-linear effect in CHABOCHE model
a	Notch Angel
C_1, C_2	Constants in CHABOCHE model
d	Distance between bottom of plate and tip of v-notch
E_1,E_2	Elastic and plastic modulus, respectively
h	Thickness of the plate
j	Interception of tangent modulus and y-axis
K_t	Stress concentration factor
r_n	Notch radial location
S	Nominal stress at the notch
t	Notch depth

1. Introduction

Industrial gas turbine blades must be designed to operate under high temperature and severe mechanical loads which cycle based on the workload of the machine. These conditions are known as thermomechanical fatigue (TMF). In application, designers usually incorporate small divots to the leading edge of the blade for cooling purpose. In most cases, these features act as stress concentrations where plasticity can localize. Consequently, notches serve as sites for crack initiation and reduce the fatigue life of component (Dowling 1979). It, therefore, is important to capture the maximum stress (σ_{max}), stress range ($\Delta\sigma$), elastic/plastic strain range ($\Delta\epsilon$) localized at the notch root. In recent years, several non-local shakedown methods have been developed to approximate the distribution of stress caused by plastic flow in a zone of the stress concentration. These models are limited to isothermal conditions only. Among them are the methods established by Neuber (Neuber 1961) and Molinka and Glinka (Monlinski K.; Glinka G. n.d.). With the aid of constitutive models, such as the Bilinear Kinematic Hardening (i.e., BKIN) model or the Nonlinear Kinematic Hardening (i.e, CHABOCHE) model, these local approximation methods have excellent prediction of material response at high temperature. The current research addresses extending a non-local method to non-isothermal conditions. This investigation develops a formulation for the equivalent isothermal temperature, T^*, which can be used to predict notch tip response under non-isothermal conditions. This equivalent temperature can be used to calculate the elastic and tangent modulus of BKIN model while compensating for the BKIN's inaccuracy in TMF. Based on these results, this effort develops a method to help engineers approximate the critical stress-strain responds at the notch tip without the use of the finite element method. The second chapter of this research will review material properties of generic and material models that are used in the finite element analysis (i.e., FEA). The third chapter of this thesis will discuss the set up of the FEA and the derivation of formula which yields T^*. Finally, a discussion on the final result will be made in chapter fourth and fifth, and a conclusion can be drawn based on this result in chapter sixth.

2. Literature Review

2.1 Materials

The temperature in the combustion chamber of gas turbine can reach 1300°C before it is blown into the turbine. By coating these blades and using notched structures, the temperature that they have to withstand drops to the range of 750°C - 950°C (Albeirutty H. M.; Alghamdi S. A.; Najjar S. Y. 2004). Because of these extreme temperature and load, Ni base superalloy, which is designed for long term mechanical exposure, are excellent candidates for the material of the turbine blade material. These solids have high strength, and good corrosion/heat resistance; therefore, they are used widely in gas turbine and other machines subjected to fluctuated temperature and moisture environment. For a generic material, the temperature dependence of yield and tensile strengths are shown in Figure 1 (Miskovic Z.; Janovic M.; Gligic M.; Likic B. n.d.).

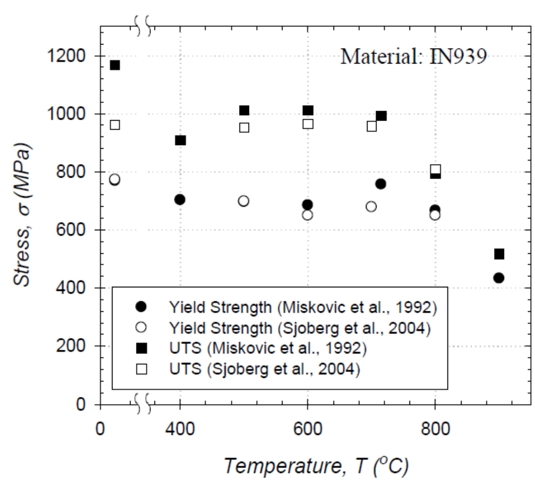
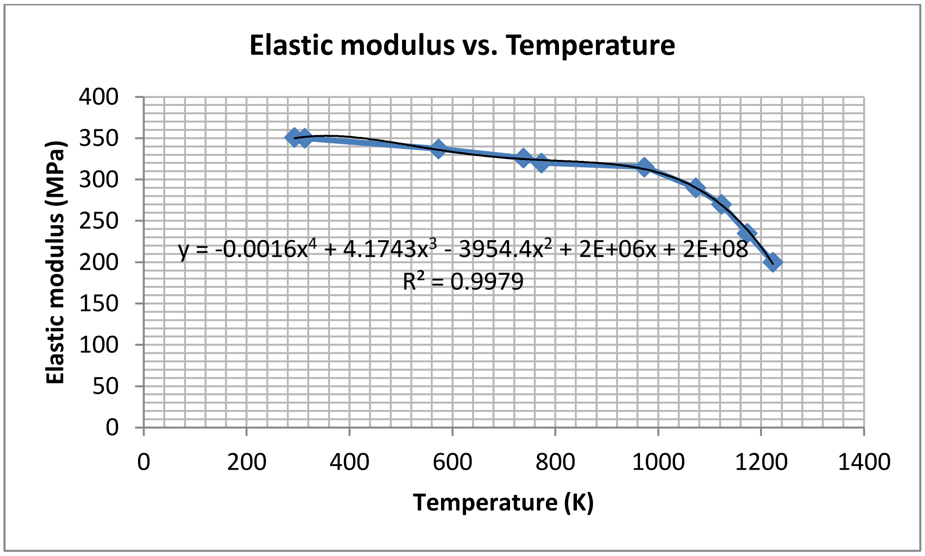


Figure 1: Temperature dependence of yield and ultimate strengths of generic Ni-base superalloy

The temperature dependent of elastic modulus, tangent modulus, and yielding stress are found using history data.



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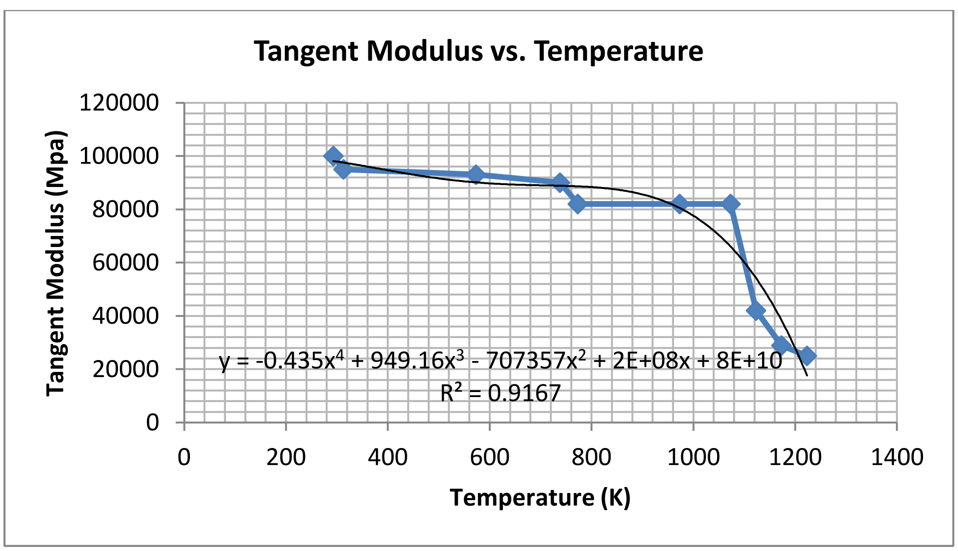

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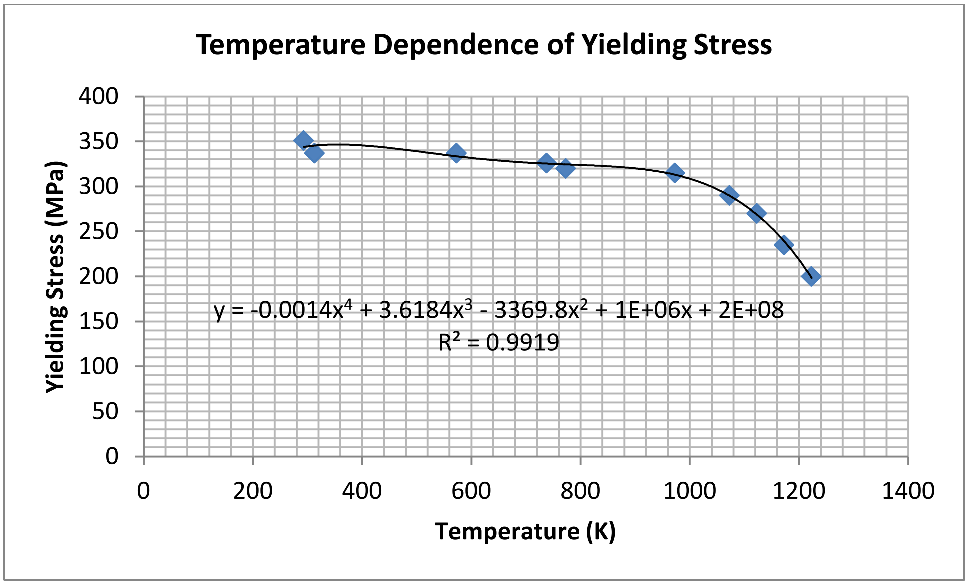


(c)

Figure 2a & 2b & 2c: Temperature dependence of elastic modulus, tangent modulus, and yielding stress of the generic material

by interpolating the data, the equations for elastic and tangent modulus as function s of temperature are

$$E_{elastic} = -0.001T^4 + 4.174T^3 - 3954T^2 + 2 * 10^6 T + 2 * 10^8 \quad (1)$$

$$E_T = -0.435T^4 + 949.1T^3 - 70735T^2 + 2 * 10^8 T + 8 * 10^{10} \quad (2)$$

$$\sigma_{yield} = -0.001T^4 + 3.618T^3 - 3369T^2 + 1 * 10^6 T + 2 * 10^8 \quad (3)$$

The material microstructure consists of γ-solid solution matrix and γ'-intermetallic precipitate phase. A Cuboidal precipitates in the material are bimodally distributed with the matrix phase (Jovanovic M. T.; Miskovic Z.; Lukic B. 1998), and high Cr-content imparts a high oxidation resistance to the material (Nazmy M. Y.; Wuthrich C. 1983). As a result, this generic material has considerable stiffness and creep resistance (Gordon. et al., 2008)

Table 1: IN939 Compositions, wt%

C	Cr	Co	W	Mo	Nb	Ta	Ti	Al	Zr	B	Ni
0.15	22.4	19.0	2.0	…	1.0	1.4	3.7	1.9	0.1	0.01	Bal

2.2 Neuber's Rule

Neuber's rule assumes that stress and strain solutions at the notch root can be expressed as nominal elastic stress and strain response (S and e, respectively) and nominal, theoretical stress concentration factor (K_t). Upon yielding at the notch tip, the stress concentration can be approximated as

$$K_t = \sqrt{K_\sigma K_\varepsilon} \quad (4)$$

by assuming plastic deformation happens at the notch only, the product of stress and strain at the notch is found to be

$$\frac{(K_t S)^2}{E} = \sigma\epsilon \qquad (5)$$

where E is the elastic modulus, and ϵ is the sum of elastic strain and plastic strain. The solution to Neuber'rule is the intersection of the Neuber hyperbola and the tensile curve as in Fig. 3 (Gordon Ali P.; Eric P. Williams; Michael Schulist 2008)

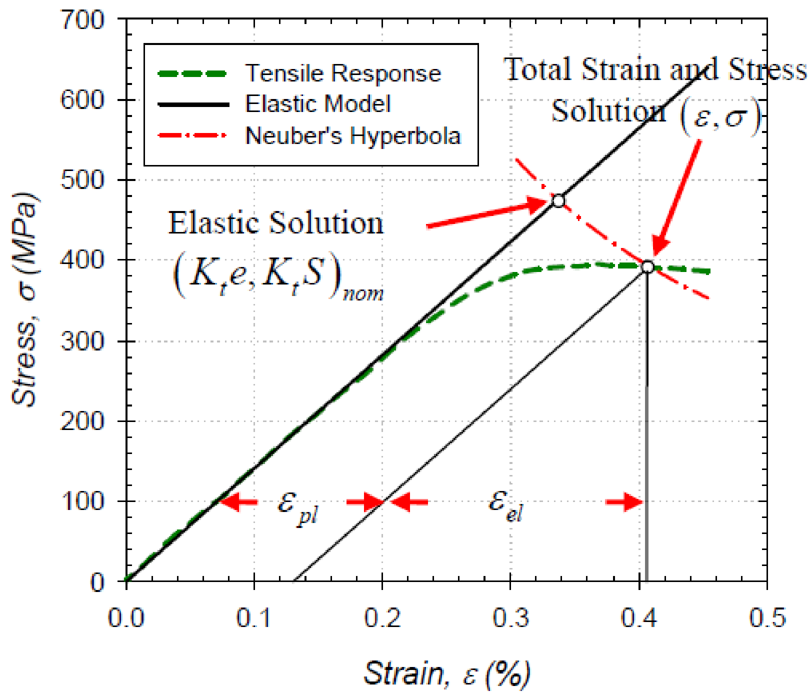


Figure 3: Interception of stress strain curve and σε total

Neuber's rule provides local notch root response on the basic of tensile behavior and the stress concentration factor, k_t. However, it is not applicable for non-isothermal condition.

2.3 Review Modeling

2.3.1 BKIN model

The equation that approximates the stress-strain solution at the notch in this research is developed based on the rate independent Bilinear Kinematics Hardening, BKIN, model. This model assumes the total stress range is twice the yielding stress; thus, it accounts for the Bauschinger Effect (ANSYS 2011).

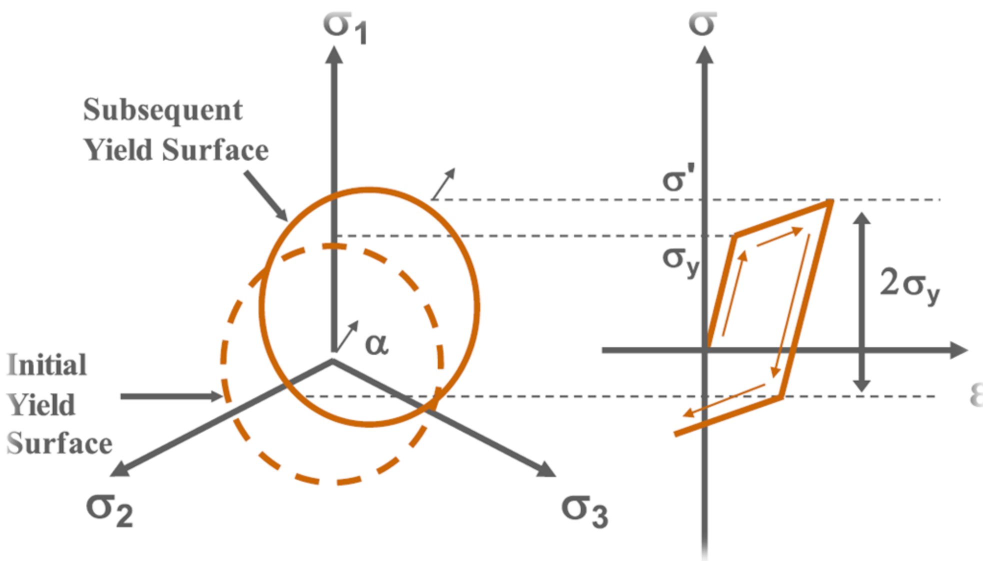


Figure 4: Stress-strain behavior of BKIN

The Von-Mises yield surface of this model is defined by the function

$$F(T) = \sqrt{\frac{3}{2}(\boldsymbol{s} - \boldsymbol{\alpha}):(\boldsymbol{s} - \boldsymbol{\alpha})} - \sigma_{yield}(T) = 0 \qquad (6)$$

where s is the deviatoric stress, $\sigma_{yield}$is the uniacial yield stress, and α is the back stress which is also the location of the center of the yield surface. For BKIN model, the change in back stress is linearly proportional to the change in plastic strain.

$$\Delta\alpha(T) = \frac{2}{3}C(T) * \Delta\epsilon_{pl} \qquad (7)$$

where C is material constant and $\epsilon_{pl}$is plastic strain. The BKIN model suggested that the initial slope of the curve is taken as the elastic modulus of the material, $E_{elastic}$. This fact makes the elastic strain smaller when the temperature and nominal stress are in phase, and it makes the elastic strain larger when the temperature and nominal stress are out of phase. In order to compensate for these deviations, the equivalent temperature, T^*, is expected to be closer to the maximum temperature for the in-phase case, and closer to the minimum temperature for out of phase case. At the yielding stress, the curve continues along the second slope, which is known as the tangent modulus or E_2 (Gordon Ali P.; Eric P. Williams; Michael Schulist 2008). There are only few methods used to estimate this tangent modulus; however, engineers and scientists usually determine it based on their experience and the actual experiment data. These estimations are more likely to contain errors when the temperature and the nominal stress fluctuating with time. Thus it is the goal of this research to derive a formula to calculate the appropriate T^* used to determine the elastic modulus, tangent modulus, and yielding stress of IN939. These moduli and yielding stress will be used to reconstruct the material behavior of the notched specimen.

The BKIN model is used in this research to approximate the stress strain response under non-isothermal conditions because it is simpler than the other models, and it is still accurate when the plastic strain is small. This model assumes that the plastic yielding is linearly proportional to the stress. This assumption is justified because the plastic strain at the notch is small and happens only at the region around the notch tip. The major disadvantage of this model when it is applied to fatigue analysis is that it is historical independent. The model will produce the same cyclic stress-strain response as long as all the conditions, such as load and temperature range, are kept the same. Thus, in order to approximate the TMF response with this model, the nonlinear kinematics hardening, CHABOCHE, will be employed to account for the history dependence of the fatigue test.

2.3.2 CHABOCHE model

The CHABOCHE model is one of the powerful constitute models used to study the plastic behavior of material in fatigue test. This nonlinear kinematic hardening model is rate-independent and able to account for Bauschinger effect. The advantage of this model is that it can be modified to solve for complex behaviors of the materials under various conditions; however, this advantage also increases the complexity of calibrating the material parameters.

The yielding function for the CHABOCHE model is similar to BKIN's

$$F(T) = \sqrt{\frac{3}{2}(\boldsymbol{s}-\boldsymbol{\alpha}):(\boldsymbol{s}-\boldsymbol{\alpha})} - \sigma_{yield}(T) = 0 \qquad (8)$$

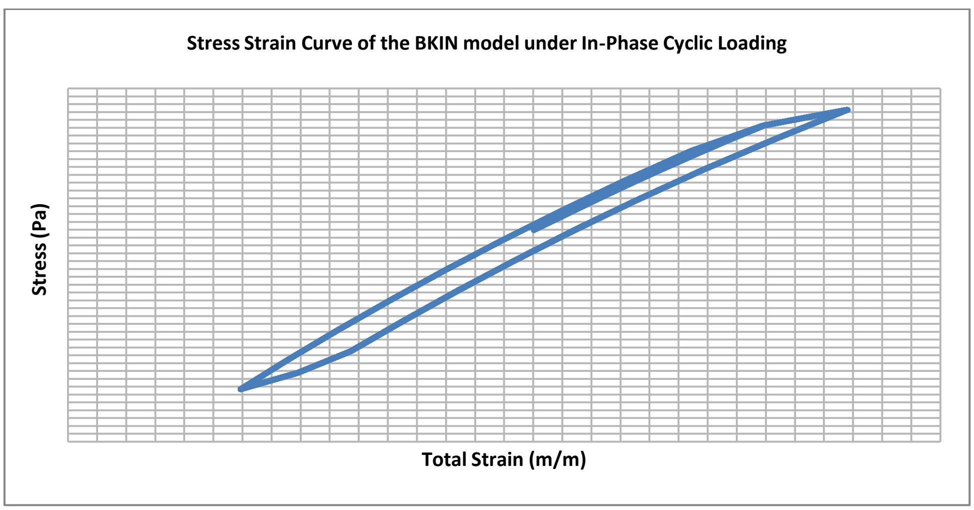
however the evolution law of CHABOCHE model has a nonlinear term

$$\Delta\alpha(T) = \frac{2}{3}C(T) * \Delta\epsilon_{pl} - \gamma_i \alpha_i * \lambda \qquad (9)$$

where λ is accumulated plastic strain, T denotes the temperature, and γ is rate of decrease of hardening modulus. The back stress in CHABOCHE model can be represented as a superposition of multiple kinematic models (Doyle 2011)

$$\{\Delta\alpha\}_i = \frac{2}{3}C_i\{\Delta\epsilon^{pl}\} - \gamma_i\{\alpha_i\}\Delta\epsilon^{pl} + \frac{1}{C_i}\left(\frac{dC_i}{dT}\right)\Delta T\{\alpha\} \qquad (10)$$

γ_i is also called the "recall term" that produces nonlinear effect (Sheldon 2008). Due to the complexity of calibrating the material constant, this Thesis uses the first order CHABOCHE model which contains only, C_1, and γ_1. For the first order of CHABOCHE model (i.e., n = 1), the parameter C_1 describes the tangent modulus of the material, $E_{T.}$ The method used to calibrating material other parameters will be discussed shortly after this introduction of the CHABOCHE model. In addition to the flexibility, the CHABOCHE model is chosen because of three reasons. First, it enables the description of the nonlinearity of stress-strain loops under cyclically stable conditions. Second, this model, similar to BKIN and MKIN model, can be used to simulate monotonic hardening and Bauschinger effect (ANSYS 2011). Lastly, it is able to describe the cyclic material's behavior with asymptotic plastic shakedown. The differences between the material behaviors under cyclic load of the CHABOCHE model and the BKIN model can be observed in Figure 3a and 3b.



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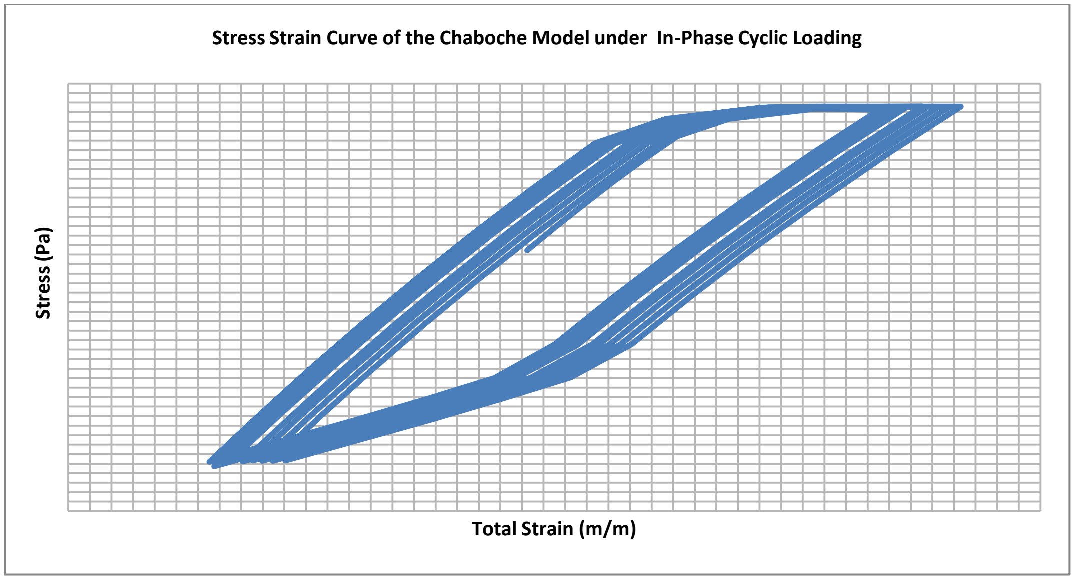


(b)

Figure 5a & 5b: The difference between the material behaviors of CHABOCHE and BKIN model

Fig. 5b shows that CHABOCHE model represents the historical dependent behavior of the material by showing that the stress strain curve is shifted to the right after each cycle. However, the BKIN model, Fig.5a, shows that there is always a specific strain for an amount of nominal stress applied on the specimen, regardless the path the stress takes. Due to the importance of the CHABOCHE model, the next paragraphs will explain in detail the meaning of each parameter in the model and the calibration method used.

Similar to BKIN model, this first order CHABOCHE model is linear kinematic hardening (Sheldon 2008). It is possible to study the similarity between BKIN model and 1st order CHABOCHE model by setting $\gamma = 0$ and plotting them on the same graph as in Fig.6 below.

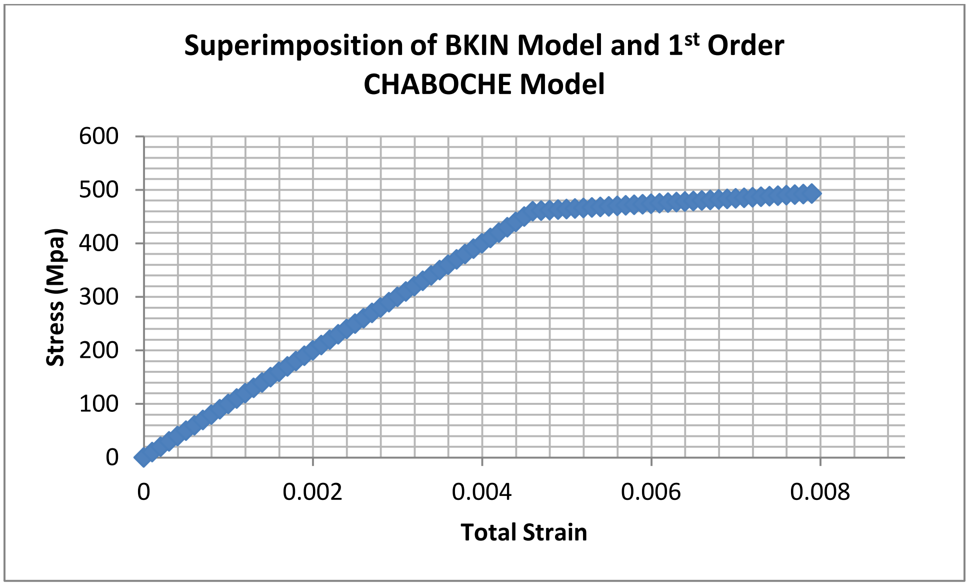


Figure 6: Relationship between BKIN model and CHABOCHE model

Figure 6 shows that those two plots are superimposing in both elastic and plastic region (Sheldon 2008). Even though those two plots are similar, E_T in BKIN model is based on the total strain while E_T in CHABOCHE model is based on equivalent plastic strain (Sheldon 2008). Using a stress value $\sigma > \sigma_{yield}$, the relationship between C_1 and E_T can be expressed as

$$E_T = \frac{\sigma - \sigma_{yield}}{\frac{\sigma - \sigma_{yield}}{C_1} + \frac{\sigma}{E_{elastic}} - \frac{\sigma_{yield}}{E_{elastic}}} \quad (11)$$

in addition, the second parameter, γ_1, controls the rate at which the hardening modulus decreases with increasing plastic strain (Sheldon 2008). Fig. 7shows the change of plastic behavior as $\gamma_1$is varied and other parameters are kept constant.

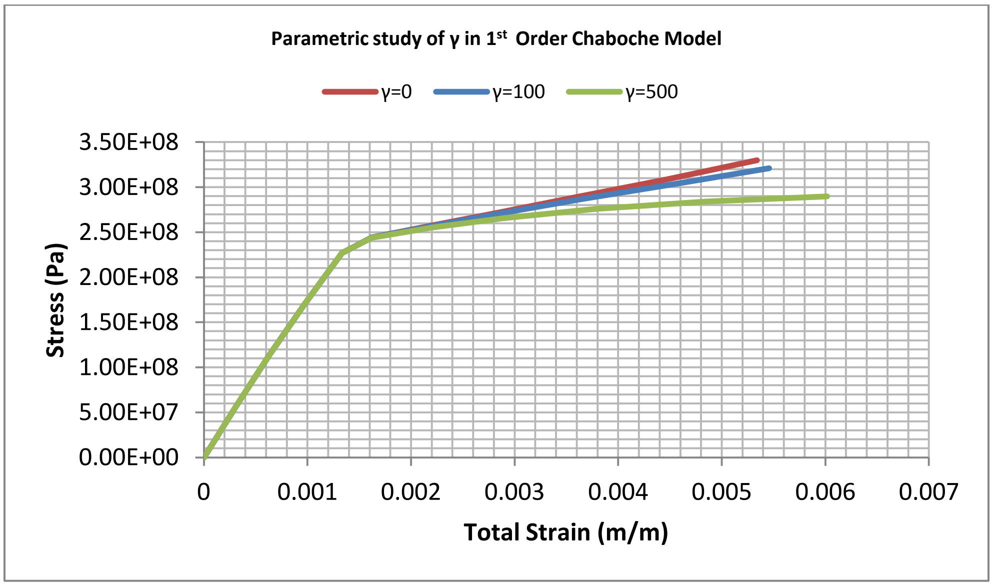


Figure 7: CHABOCHE model with varying γ

Equation 10 indicates that the back stress increment, $\{\dot{\alpha}\}$, will be lowered as plastic strain increased (Sheldon 2008). Figure 8 shows the comparison between the BKIN model and the CHABOCHE model with nonzero γ_i (Sheldon 2008).

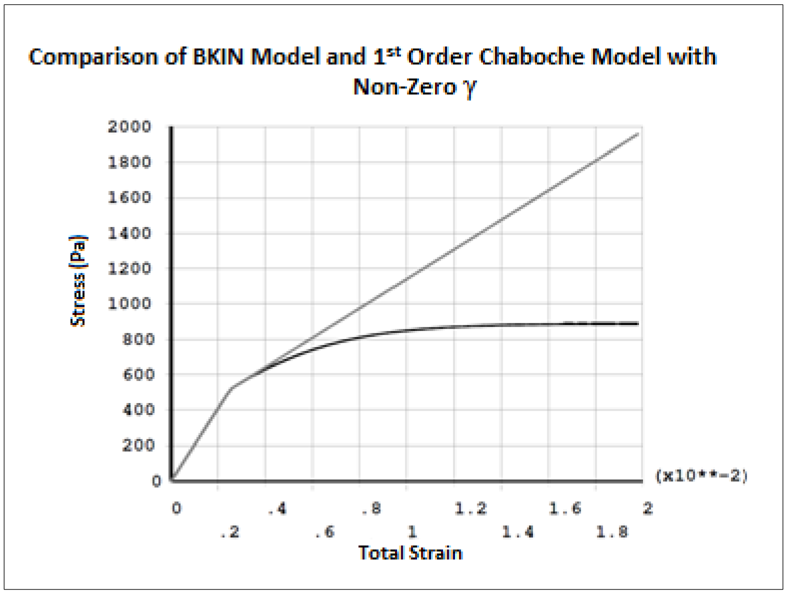
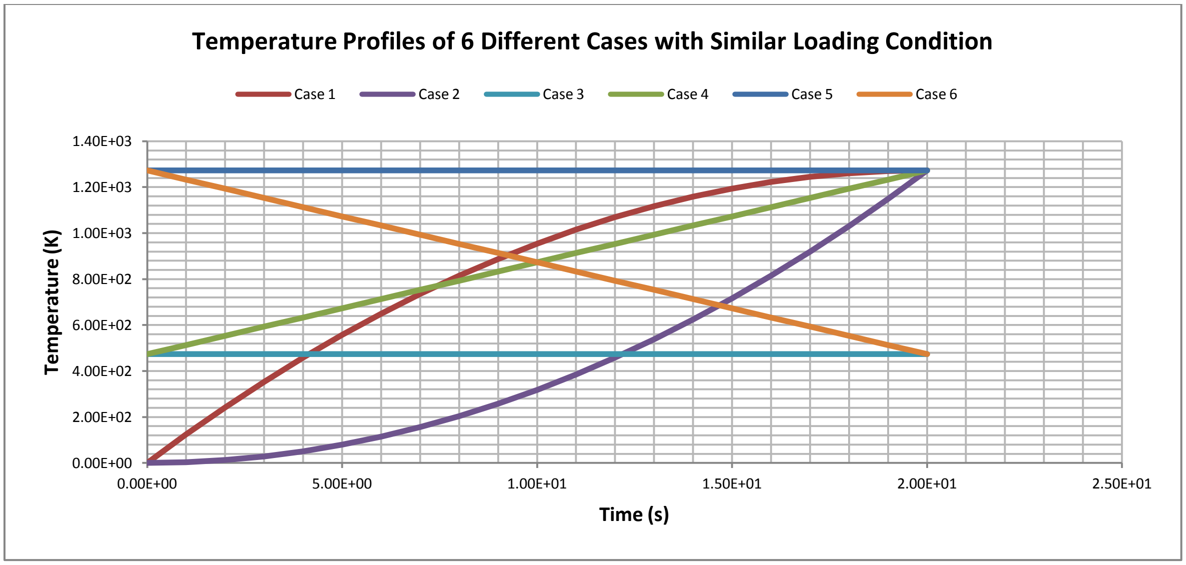


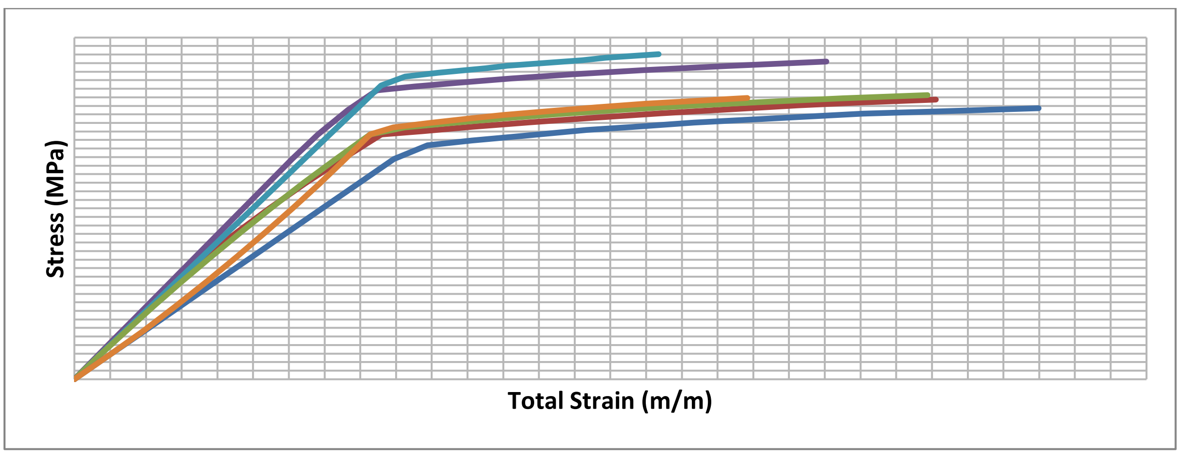
Figure 8: BKIN model and CHABOCHE with nonzero γ

Notice that the initial slope of the two models are the same at yielding stress, then the slope of CHABOCHE model decreases to zero as total strain goes to infinity.

This research will use the first order CHABOCHE model because of its simplicity in calibrating the model constants. This research also uses most of the material properties in previous researches to calibrate these constants. The yielding stress, σ_{yield}, and C_1 at specific temperature are found by using the historical data in BKIN model. Those parameters are the same as the yielding stress and tangent modulus at specific temperature in BKIN model. The values of $\gamma_1$are found by curve fitting the actual data using ANSYS. Also, the elastic modulus, which is a function of temperature, will be used with the CHABOCHE model to simulate the elastic behavior of the specimen. Due to the use of the CHABOCHE model and the equation of elastic modulus, the specimen is expected to be hardened or softened when the temperature decreased or increased, respectively. These effects can be observed in Fig. 9a & 9b which show the material behaviors of a smooth specimen under various temperature ranges and similar monotonic load condition.



(a)



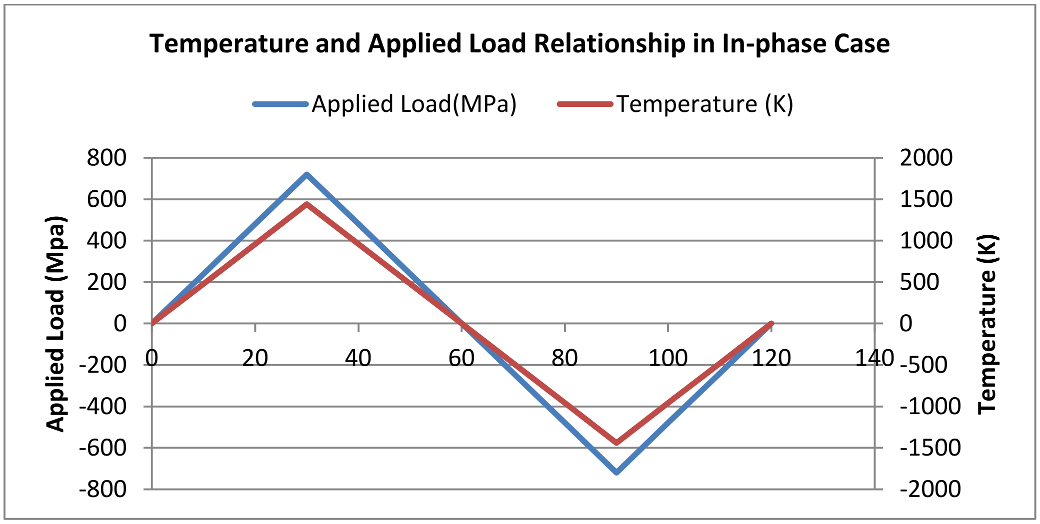
(b)

Figure 9a & 9b: Stress- Strain Behaviors of 6 Temperature Profiles using CHABOCHE Model

Considering case 1, 2, 4, and 5, the stress strain curve is lowest in case 5, where the temperature is highest. This fact indicates that the material will undergo more deformation for the same stress level. Case 2 always has the lowest temperature in comparison to 1, 4, and 5 so that its material is the stiffest. As a result, case 2 has highest stress strain curve and lowest plastic strain. Case 1 and 4 are similar so their stress strain curves are expected to reassemble each other. In addition, since the temperature in case 3 is linearly decreasing from 1273.15K to 473.15K and the temperature in case 6 is constant at 473.15K, case 6 is supported to be stiffest. The stress strain curve of case 6 is, therefore, higher than case 3. The conclusions derived from Figure 9a & 9b confirm that the accuracy of the CHABOCHE used in this research.

2.4 Thermomechanical Fatigue

Thermomechanical fatigue (i.e. TMF) is the condition where the specimen undergoes cyclic load and temperature. This condition reduces the lifespan of components in many high temperature and pressure applications such as turbine blades. Due to the difficulty in simulating the thermal stress cycling, many early works used isothermal fatigue tests at various temperatures and loads to approximate TMF condition. Thus, these works did not capture the damage micromechanisms under fluctuated temperature (Changan Cai, Peter K. Liaw, Mingliang Ye, Jie Yu 1999). The fatigue failure can be divided into 2 categories: High-cycle fatigue (HCF) and Low-cycle fatigue (LCF). HCF is associated with small load so that the fatigue life exceeds 10^4 cycles. LCF uses sufficiently large load that results in the fatigue life less than 10^4. Besides the magnitude of the load, TMF can be differentiated by the phase between the temperature and the applied load. This Thesis will consider two extreme cases which are in-phase and out-phase case. The relation between temperature and applied load of those cases are illustrated in Figure 10a & 10b



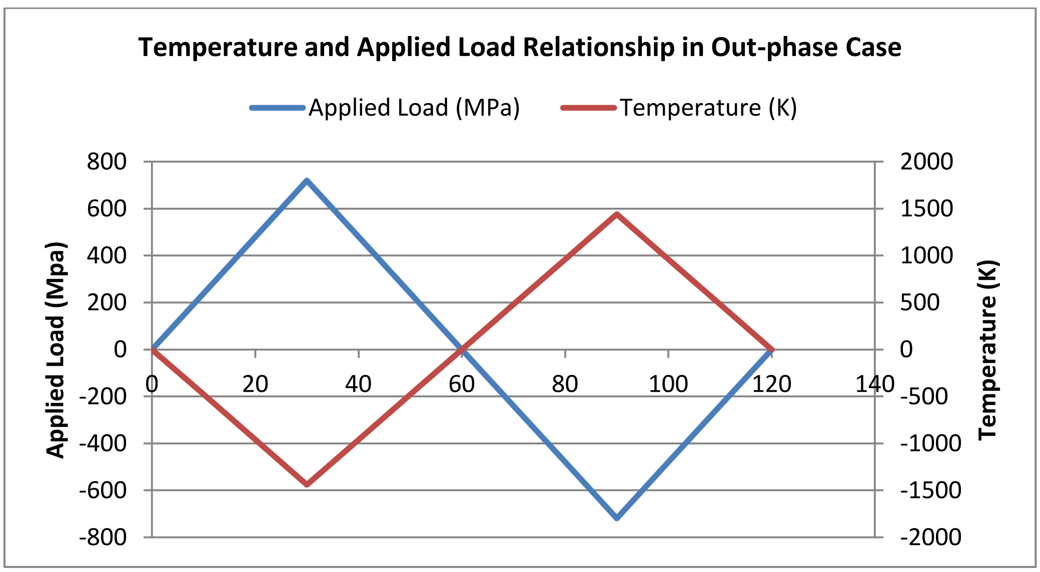


Figure 10 a & 10b: Relationship between applied load and temperature in in-phase and out-phase case

In the in-phase case, the temperature and applied load will reach their highest values at the same time, thus the phase angle will be 0^{o}. On the other hand, the out-phase case will have the phase angle of 180^{o} because the highest stress will occur at the lowest temperature.

2.5 Hypotheses

Even though Neuber's rule is not applicable for TMF, the elastic modulus in equation 5 is temperature dependent. Therefore, it is possible to find an equivalent temperature, T^{*}, that can improve the accuracy of Neuber's rule in non-isothermal condition. Rewrite equation 5 in terms of elastic modulus as

$$E(T^{*}) = \frac{k_t^2 * S^2}{\sigma\epsilon} \qquad (12)$$

By using parametric study, the equation of $\sigma\epsilon$ can be deduced; thus, equation 12 will yield T^{*}. The goal of this Thesis is to use BKIN model to approximate the material behavior at the notch root; thus, BKIN model should intercept Neuber's hyperbola at the highest stress as in Fig. 11

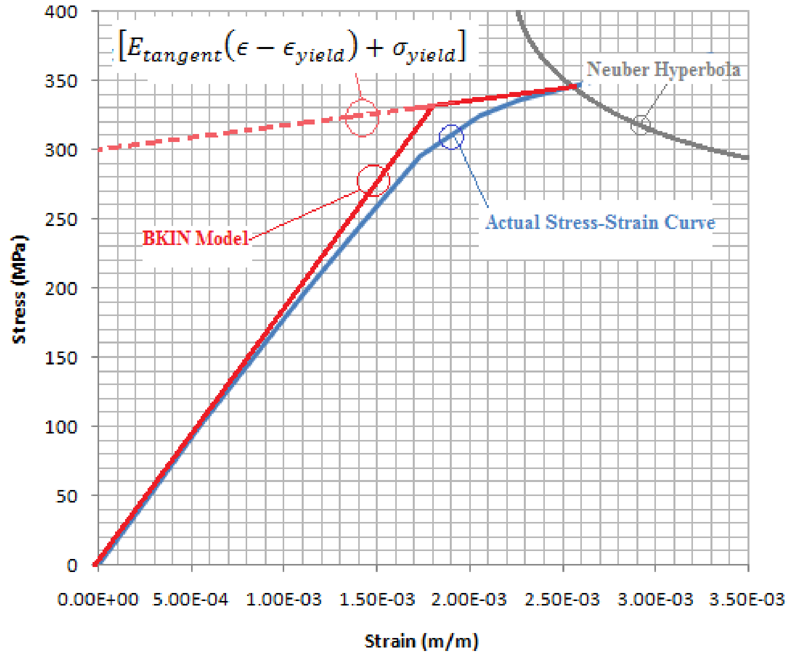


Figure 11: BKIN model used to approximate the material behavior

Additionally, equation 12 can be rewritten to find the elastic solution as

$$\epsilon = \frac{k_t S}{E(T^*)} \qquad (13)$$

Since T^* makes equation12 valid, it will be able to make the equation 13 valid as well. In other word, T^* can be used to calculate the equivalent elastic modulus and elastic strain of the material. Neuber hyperbola can also be combined with BKIN model to get

$$\sigma\epsilon = \begin{cases} E_{elastic}(T).\epsilon^2 & ,\epsilon \leq \epsilon_{yield} \\ \epsilon[E_{tangent}(\epsilon - \epsilon_{yield}) + \sigma_{yield}] & ,\epsilon \geq \epsilon_{yield} \end{cases} \qquad (14)$$

σ_{yield} in equation 14 can be found by plotting yielding stress as function of temperature versus applied stress as function of temperature. The intersection of those plots will indicate both yielding stress and temperature at yielding.

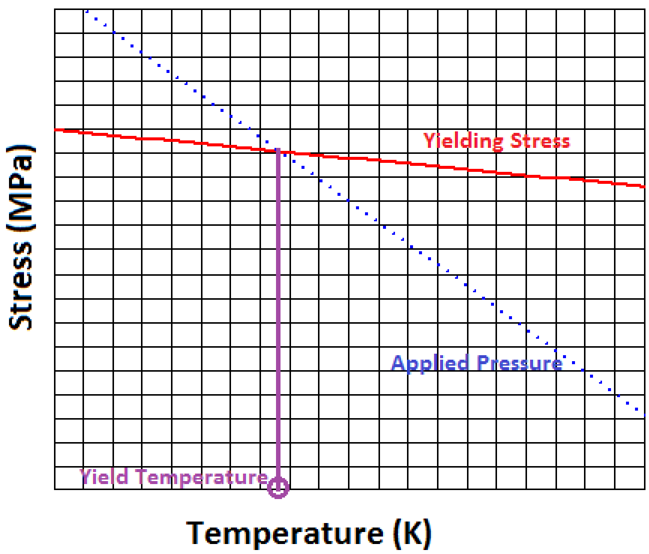


Figure 12: Applied Pressure intercepts Yielding Stress at Yield Temperature

T^* can be used to calculate tangent modulus in the right hand side of equation 13. The left hand side of equation 14 contains both stress and strain. As a result, they needed to be decoupled, and the plastic strain will be calculated separately.

3. Numerical Simulation

3.1 Specimen Design

Due to symmetry, only ¼ of the test specimen will be modeled using ANSYS software. Simple V-notch structure has been used to simulate the stress strain responses of IN939 under cycling load and temperature. Material modeling will be set up using single, solid, and 8-notch elements with the thickness of 2mm. The parametric study on the mesh size was carried out to determine the best mesh size for the simulation. The pressure on the top of the specimen and temperature will linearly increase so that they meet at their maximum values in the in-phase case; oppositely, the maximum pressure will occur with the minimum temperature in the out-phase case. The stress and strain at the notch are expected to be at maximum values during the in phase case and at minimum values during the out of phase case due to the hardening of material under low temperature. Also, pressure and temperature are selected so that plastic deformation only happens around the notch to mimic the real turbine's blade. Fixed supports will be applied on two sides of the specimen, and the load will be applied on specimen's top in type of pressure as seen in Fig. 14 below.

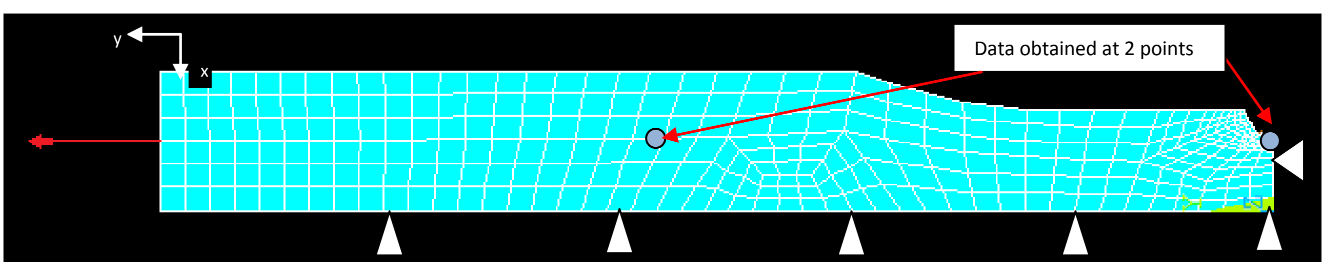


Figure 13 Specimen Constraints

The elasticity behavior of the model is controlled by a polynomial equation, which is obtained by curve fitting stress strain response of the historical data. In order to improve the model's accuracy, the plastic behavior is modeled with a built-in CHABOCHE model. The stress at 2 different points on the specimen will be collected to verify the material properties and boundary conditions. Figure 15 shows the distribution of the stress at the notch. Notice that there are two opposite regions in the picture, one is the maximum stress region and one is the minimum stress region.

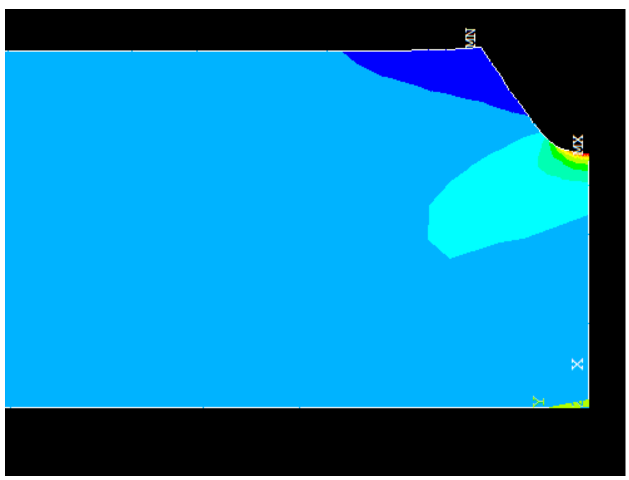


Figure 14: Von-Mises stress distribution

3.2 Formula Development

According to Neuber's rule, the max stress can be written as

$$\sigma\epsilon = f(K_t, S, T_{max}, T_{min}, \Phi) \quad (15)$$

In this equation, Φ is the phase angle. It is 0^{o} for the in-phase case and 180^{o} for the out-phase case. During TMF, the elastic modulus of the material changes as the temperature fluctuated; thus, Neuber's hypothesis, $\sigma\epsilon$, also changes accordingly. By using ANSYS, the stress-strain solutions at the notch can be determined. As a result, the right side of Neuber's rule can be written as

$$\frac{(K_t S)^2}{E(T^*)} = f(K_t, S, T_{max}, T_{min}, \Phi) \quad (16)$$

By knowing the equation of E as a function of temperature, equation (16) can be used to find the equivalent temperature, T^*. In order to confirm that the Nuber's hypothesis is a function of the parameters in the right hand side of equation (16), Eureqa Formulize program will be used to find the equations of Neuber's hypothesis as functions of the combinations of those parameters. Then these equations will be used to calculate the Calculated $\sigma*\varepsilon$. These new data will be plotted against the actual values from ANSYS as in Fig. 16. Since all data points of the combination of T_{max}, T_{min}, S, Φ, and K_t are lying on the line with the slope of 1, the parametric study verifies that Neuber's hypothesis is a function of these parameters.

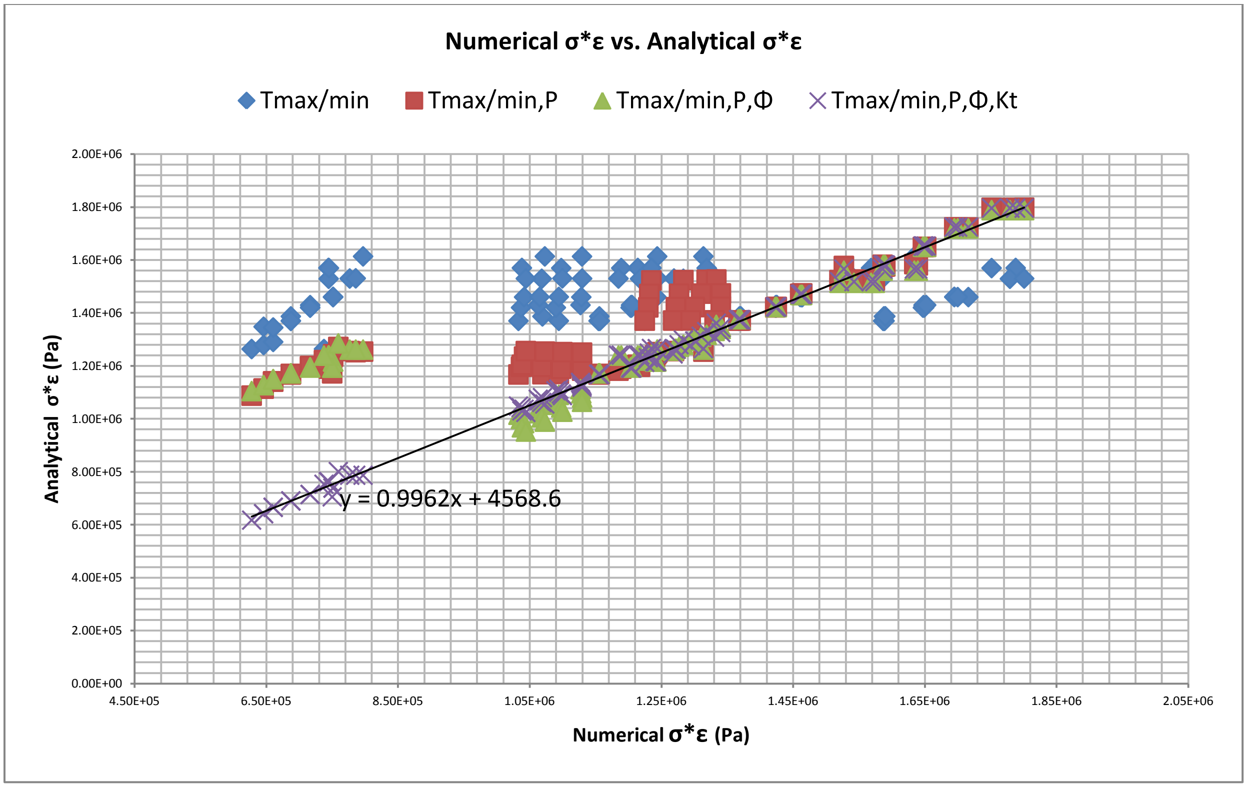


Figure 15: Parametric study on the combination of parameters in the right hand side of equation (12)

Therefore the equation of the Neuber's hypothesis is

$$\sigma\epsilon_{total} = 6.24 * 10^5 \mathrm{K_t} + 3.41 * 10^3 \Phi + 0.0178 * \mathrm{S} + 1.73\mathrm{T_{min}}\Phi + 3.7 * 10^{-6}\mathrm{S T_{max}} - 3.95 * 10^6 - 2.51 * 10^{-5}\mathrm{S}\Phi - 2.26\mathrm{T_{max}}\Phi \quad (17)$$

Note that this equation does not work under elastic condition because all parametric study data include both elastic and plastic strain. However, this equation can be extended to account for elastic condition by adding more data into the parametric study. Equation 17 also good for temperature from 473.15 K to 1273.15 K, applied load from 0MPa to 130MPa, k_t from 2.8 to 4.2, and phase angel equal 0^o or 180^o.

Based on equation (16) and (17), the equation for the equivalent temperature can be found as

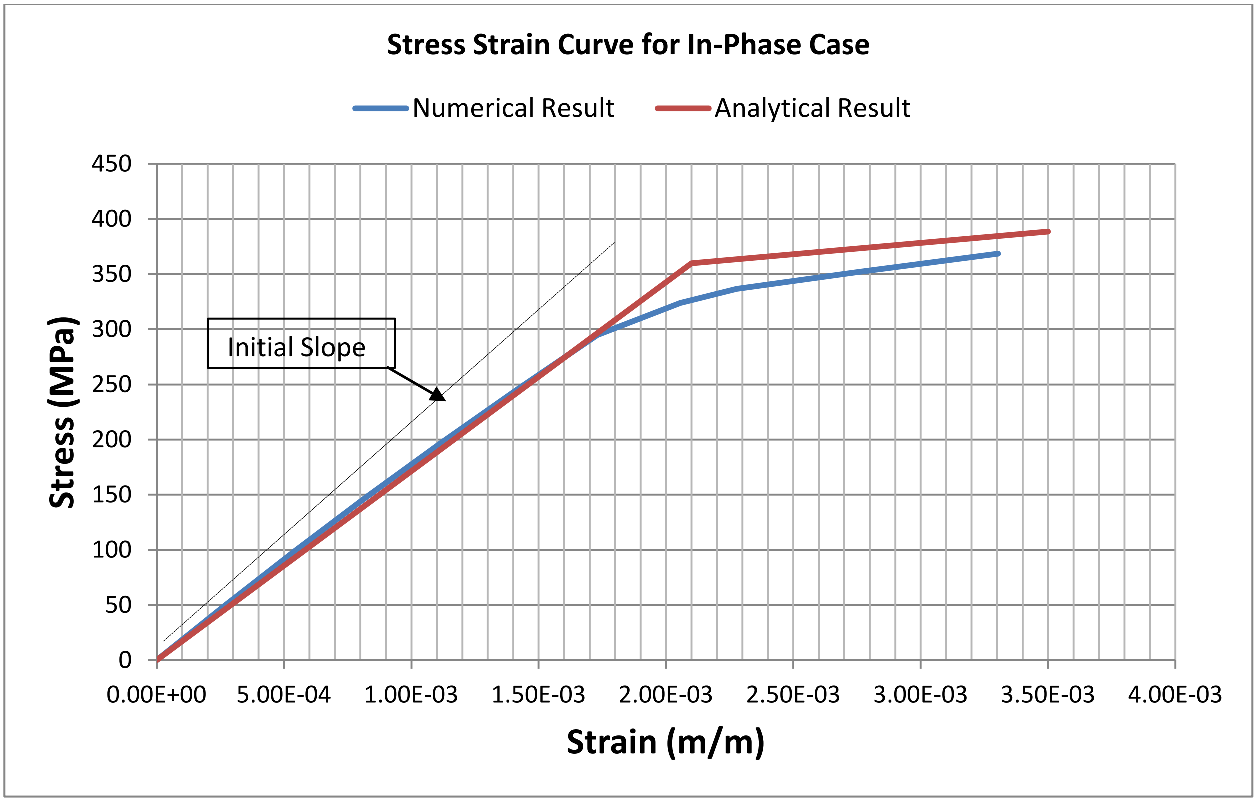
$$T^* = 1766.3 - 1.12 * \sqrt{\frac{4.19*10^{19}+1.19*10^{12}S+2.2*10^{7}\Phi-3.98\mathrm{K_t}S^2+2.4*10^{8}ST_{max}-1.6*10^{9}S\Phi+1.16*10^{14}T_{min}\Phi-1.5*10^{14}T_{max}\Phi-2.6*10^{20}}{-(6.24*10^{12}K_t+1.78*10^{5}S+3.41*10^{10}*\Phi+37\mathrm{ST_{max}}-251\mathrm{S}\Phi+1.73*10^{7}\mathrm{T_{min}}\Phi-2.26*10^{7}*\mathrm{T_{max}}\Phi-3.9*10^{13}}} \quad (18)$$

This equivalent temperature then can be used to find the elastic modulus, tangent modulus, and yielding stress of the studying material. Note that these materials properties are good only for the notched region. However, the equation can be used to calculate the material properties of a normal specimen if the value of K_t is set to 1.

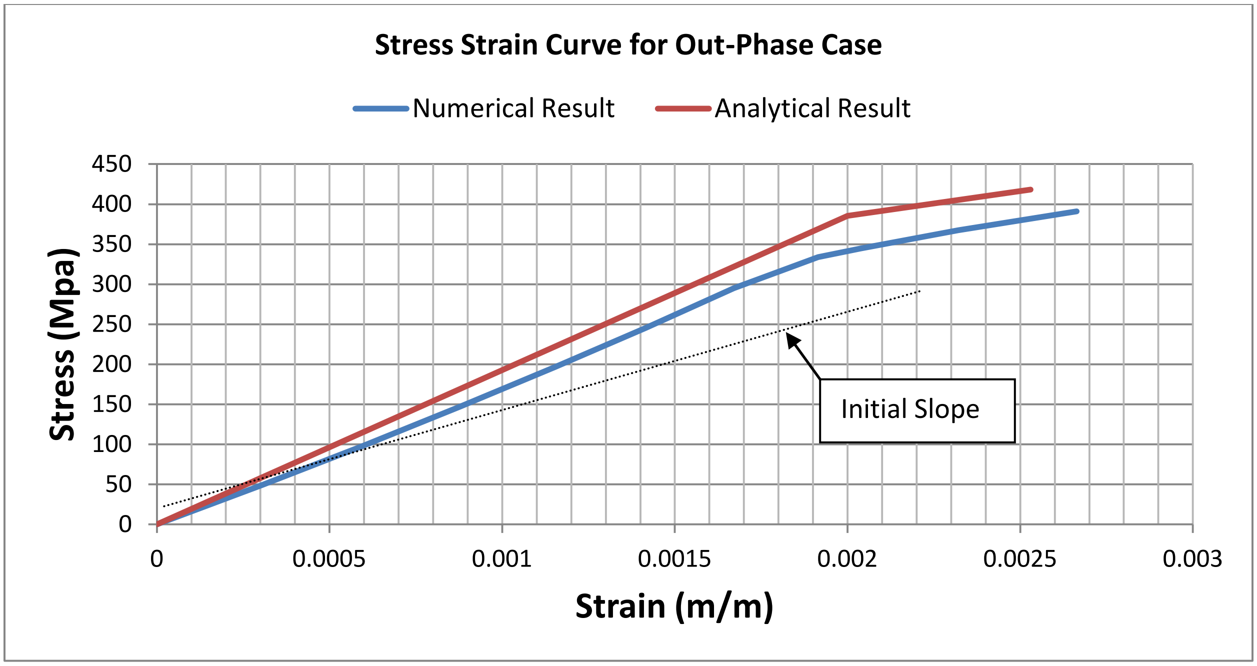
Besides calculating T^*, calculating the maximum stress at the notch is also necessary to determine the material behavior at the notch. In order to accomplish this goal, the stress and elastic/plastic strain in the left hand side of equation 15 has to be separated discuss in Chapter 2.5.

4. Results and Discussion

By applying equation 18 to calculate T^* for some cases, it appears that the equivalent temperatures are slightly higher than the average temperature in the in-phase case and close to the minimum temperature in the out-phase case. Then, the equivalent temperatures will be used to calculate the elastic modulus and the tangent modulus in order to approximate the material behavior of the notched specimen. Figure 17a and 17b show the results of two cases that undergo same loading, 0MPa to 100MPa, and temperature range, 673.15K to 1073.15K, except that the upper one is in-phase and the lower one is out-xphase.


Stress Strain Curve for In-Phase Case
Numerical Result
Analytical Result
Initial Slope
Stress (MPa)
450
400
350
300
250
200
150
100
50
0
0.00E+00
5.00E-04
1.00E-03
1.50E-03
2.00E-03
2.50E-03
3.00E-03
3.50E-03
4.00E-03
Strain (m/m)

(a)



(b)

Figure 16a & 16b : Analytical stress-strain curve and ANSYS stress-strain curve for in-phase and out-phase cases

In general, the T^*s compensate for change in temperature in booth in-phase and out-phase case. The black dot lines represent the initial slopes of the stress-strain curves. In order words, the dot lines show the elastic behavior of the material in BKIN model without the correction of T^*. For the in-phase case, T^* compensates for the decrease in elastic modulus by staying close to the maximum temperature. Therefore, the analytical and numerical results in the elastic region are very close to each other. In the out-phase case, T^* stays close to the minimum temperature. Thus, it keeps the analytical result from deviating from the numerical results. The stress strain curve in the in-phase case, Fig.17a, also shows that the T^* has decreased the slope of the elastic curve. This fact increases the total strain of the specimen. The opposite happens to the out-phase case, causing the total strain to decrease.

5. Conclusions

In conclusion, a formula to approximate T^* and a method to calculate the stress strain respond at the V-notch's tip have been developed. T^* can be used in BKIN model to estimate both the elastic and plastic modulus for IN939 notched specimen. The results suggest that T^* is able to compensate for the inaccurate caused by temperature's changing in BKIN model. In addition, the method works with other material and notch type given that different parametric study is carried out for each specific case. Material behavior behind the notch's tip can also be calculated by employing other method such as Xu-Thompson-Topper's formula which requires the stress strain solution at the notch's tip. This research's result will give engineer the ability to quickly approximate the material behavior at the notch's tip on the gas turbine's blade without the use of FEA program.

6. Future Work

Equation 16 was developed with assumption that plastic deformation occurs at the notch root; thus it is not applicable if the notch undergo elastic deformation. The future plan is to increase number of data in the parametric study of Neuber's rule so that it can account for cases where the material undergoes only elastic deformation. In addition, future simulation can be carried out with higher order for CHABOCHE model to closely simulate service conditions. Besides improving the accuracy of the model, future simulations are also planned to evaluate the quality of $E_{tangent}(T^{*})$.

Appendix

Appendix

ANSYS code

```
Finish
/Clear
/PREP7
!***************************************************************************
**
!---Input parameters:
Finish
/PREP7
!---Geometric:
RAD_NTCH=.037*0.0254          ! Root radius of notch                [m]
ANG_NTCH=60                                ! Angle of notch
        [deg]
DIA_NTCH=.251*0.0254          ! Diameter of specimen at notch     [m]
DIA_RED=.360*0.0254                ! Reduced diameter of specimen      [m]
RAD_SHLD=1.0*0.0254                ! Radius of reduction shoulder      [m]
DIA_GRIP=.5*0.0254                 ! Diameter of specimen grip         [m]
LEN_GRIP=1.25*0.0254          ! Lenght of specimen grip           [m]
LEN_BAR=4*0.0254              ! Total length of specimen          [m]

!***************************************************************************
**
!---Parameters derived from geometric relationships
*AFUN, DEG
l1=LEN_BAR/2
l2=LEN_GRIP
d1=DIA_GRIP/2
d2=DIA_RED/2
r1=RAD_SHLD
r2=RAD_NTCH
t=DIA_NTCH/2
a=ANG_NTCH/2
x1=d2+r1-d1
y1=sqrt((r1*r1)-(x1*x1))
x2=sin(a)*r2
y2=cos(a)*r2
x3=(y2/tan(a))-(r2-x2)-t
y3=tan(a)*(d2+x3)
```

```
!*****************************************************************************
**
!---Specimen Geometry:
!---Keypoints
k,  1,   0.0,    0.0
k,  2,   0.0,    l1
k,  3,   d1,     l1
k,  4,   d1,     l1-l2
k,  5,   d2,     l1-l2-y1
k,  6,   d2+r1,  l1-l2-y1
k,  7,   d2,     y3
k,  8,  t+r2-x2, y2
k,  9,    t,     0.0
k, 10,  t+r2,    0.0

! Lines
L,  1,  2                                   ! Line 1
L,  2,  3                                   ! Line 2
L,  3,  4                                   ! Line 3
Larc,  4, 5, 6, r1                     ! Line 4
L,  5,  7                                   ! Line 5
L,  7,  8                                   ! Line 6
Larc, 8, 9, 10, r2                     ! Line 7
L,  9,  1                                   ! Line 8

! Areas
AL, all
ksel,all

!*****************************************************************************
**
!---Element Type and Material Number
ET,1,PLANE183,,,3
R,1,0.002
MAT,1

!*****************************************************************************
**
!---Define Properties of Material 1
!---Elastic Properties
MPTEMP,1,293.15,573.15,773.15,1073.15,1173.15,1223.15
```

```
MPDATA,EX,1,1,2.117e11,1.941e11,1.808e11, 1.578e11,1.49e11,1.442e11
MPDATA,PRXY,1,1,0.36976264,0.41036665,0.4428308,0.48523527,0.49529229,0.49921871
MPDATA,DENS,1,1,8.36576e9,8.27988e9,8.221e9,8.13653e9,8.1094e9,8.09603e9
!
!---CHABOCHE Properties
TB,CHABOCHE,1,10,1                !Activate CHABOCHE data table
!
TBTEMP,293.15
TBDATA,1,3.51e8,100000e6,100      !TBDATA,mat,Yeild Stress, C1,G1
!
TBTEMP,373.15
TBDATA,1,3.37e8,93000e6,200       !TBDATA,mat,Yeild Stress, C1,G1
!
TBTEMP,573.15
TBDATA,1,3.37e8,93000e6,400       !TBDATA,mat,Yeild Stress, C1,G1
!
TBTEMP,738.15
TBDATA,1,3.26e8,90000e6,600       !TBDATA,mat,Yeild Stress, C1,G1
!
TBTEMP,773.15
TBDATA,1,3.20e8,82000e6,800       !TBDATA,mat,Yeild Stress, C1,G1
!
TBTEMP,973.15
TBDATA,1,3.15e8,82000e6,1000      !TBDATA,mat,Yeild Stress, C1,G1
!
TBTEMP,1073.15
TBDATA,1,2.90e8,82000e6,1200      !TBDATA,mat,Yeild Stress, C1,G1
!
TBTEMP,1123.15
TBDATA,1,2.70e8,42000e6,1400      !TBDATA,mat,Yeild Stress, C1,G1
!
TBTEMP,1173.15
TBDATA,1,2.350e8,28900e6,1600     !TBDATA,mat,Yeild Stress, C1,G1
!
TBTEMP,1223.15
TBDATA,1,2.00e8,25000e6,1800      !TBDATA,mat,Yeild Stress, C1,G1
!*************************************************************************
!---Mesh Area
! Mesh Area
AMESH,ALL,,,                                              ! Mesh the Speciment
```

```
!---Refine the mesh
AREFINE,1,,,3,1,OFF,ON                                      ! Refine Mesh for the area
!NREFINE,70,,,4,1
FINISH

!!***************************************************************************
*****
!Machanical Cycling Parameter
load_ini=1000 !1000
load_fin=1300 !1500
load_inc=100.0

tempMAX_ini=473.15              !473.15
tempMAX_fin=1273.15             !1073.15
tempMAX_inc=100.0

tempMIN_ini=673.15         !673.15
tempMIN_fin=473.15         !473.15
tempMIN_inc=-100.0

*DO,load,load_ini,load_fin,load_inc ! Changing load_max [N]
*DO,temp_max,tempMAX_ini,tempMAX_fin,tempMAX_inc       ! Changing temp max [K]
*DO,temp_min,tempMIN_ini,tempMIN_fin,tempMIN_inc ! Changing them min [K]

!Solution
/Solution
!Specify the analysis type
ANTYPE,TRANS,,2,1,,
TRNOPT,FULL,,,,,
nropt,auto                                  ! Uses Newton-Raphson
lnsrch,auto                                 ! Auto line searching for NR
!****************************************************************************
! Constraint
DL,8,1,UY,0,1                                                        !Line8: Zero
Displacement in x,y direction
DL,1,1,UX,0,1                                                        !Line1: Zero
Displacement in x,y direction
loadmax=load/0.0000128                                       !Max Load [N]
loadmin=-load/0.0000128                                      !Min Load [N]
frq_load=0.25
```

```
numcyc=10
SubStep=10
T_load=1/frq_load
!***************************************************************************
BFUNIF,TEMP, temp_min                          !Initial Temperature
*DO,i,0,numcyc,1
SFL,2,PRES,-loadmax
BFA,1,TEMP,temp_max
KBC,0
TIME,(T_load*(1/4)+i*T_load)*10                !Time at the end of this load step
NSUBST,SubStep,,,OFF
LSWRITE,i
OUTRES,ALL,ALL

Solve

SFL,2,PRES,-loadmin
BFA,1,TEMP,temp_min
KBC,0
TIME,(T_load*(3/4)+i*T_load)*10                !Time at the end of this loadtep
NSUBST,SubStep,,,OFF
LSWRITE,i+1
OUTRES,ALL,All
Solve
*ENDDO

OUTRES,ALL,All                                                  !Output ALL
properties for ESOL
FINISH
!****************************************************************************
!Post Solution
/POST26
/NUMVAR,1000
/FORMAT,,E                                                      !Set up
Decimal notation
/FORMAT,,,17,9                                                  !Setup sace
vetween values
/OUTPUT,C:\notch\Notch_Kt=_%load%_%-load%_%temp_max%_%temp_min%,txt
ESOL,2,36,68,EPEL,Y
ESOL,3,36,68,EPPL,Y
ESOL,4,36,68,S,Y
```

```
ESOL,5,205,949,BFE,TEMP
ESOL,6,30,259,S,Y
PRVAR,2,3,4,5,6
*ENDDO
*ENDDO
*ENDDO
FINISH
```

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