The Development of Computational Models for Melting-Solidification Applications Using the Volume-Of-Fluid Method

Brendon Cavainolo
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THE DEVELOPMENT OF COMPUTATIONAL MODELS FOR MELTING-SOLIDIFICATION APPLICATIONS USING THE VOLUME-OF-FLUID METHOD

by

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B.S. UNIVERSITY OF CENTRAL FLORIDA, 2021

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Sciences in the Department of Mechanical and Aerospace Engineering in the College of Engineering and Computer Science at the University of Central Florida Orlando, Florida

Spring Term
2023
ABSTRACT

Fluids-related issues in the Aerospace industry are often multiphase in scope. Numerical modeling, such as computational fluid dynamics, is used to describe these problems, as they are difficult or impossible to describe analytically. This research uses computational fluid dynamics to describe multiphase problems related to melting-solidification and particle impingement. Firstly, a numerical model was established that uses the Volume-of-Fluid method to resolve a melting/solidifying particle. This model was verified against experiments and simplified analytical models, and a mesh independence study was done to ensure the results were independent of the mesh sizing. Next, the model was applied to two separate but related problems. The Artemis program has renewed interest in lunar dust mitigation. It is proposed that lunar regolith partially melts and becomes “sticky” when coming into contact with a jet flame, like a landing rocket. The method above was applied to a lunar regolith particle to show how these “sticky” particles can adhere to surfaces. The direct resolution methodology was also applied to a melted sand particle impinging and infiltrating a yttria-stabilized zirconia thermal barrier coating, as seen in engine turbines. Sand can infiltrate the thermal barrier coating and decrease its effectiveness. The infiltration from a single particle was compared to the infiltration from a stream of melted sand. These three efforts showcase the usefulness of directly resolving small particles using the Volume-of-Fluid method.
ACKNOWLEDGEMENTS

I would like to acknowledge the National Science Foundation Graduate Research Fellowship for providing funding for my education and this research. I would also like to acknowledge the scientists at the German Aerospace Center in Cologne, Germany for allowing me to visit and conduct research using their facilities. I would like to acknowledge my labmates, who have been extremely helpful, and encourage me to continue performing great research by performing great research themselves.
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<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIAA</td>
<td>American Institute of Aeronautics and Astronautics</td>
</tr>
<tr>
<td>ASME</td>
<td>American Society of Mechanical Engineers</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CFL</td>
<td>Courant-Friedrichs-Lewy Stability Condition</td>
</tr>
<tr>
<td>CMAS</td>
<td>Calcium-Magnesium-Alumino-Silicate</td>
</tr>
<tr>
<td>EB-PVD</td>
<td>Electron Beam Physical Vapor Deposition</td>
</tr>
<tr>
<td>FD</td>
<td>Finite-Difference</td>
</tr>
<tr>
<td>FE</td>
<td>Finite-Element</td>
</tr>
<tr>
<td>FV</td>
<td>Finite-Volume</td>
</tr>
<tr>
<td>GRD</td>
<td>Giordano Viscosity Model</td>
</tr>
<tr>
<td>IAC</td>
<td>International Astronautical Congress</td>
</tr>
<tr>
<td>Kn</td>
<td>Knudsen Number</td>
</tr>
<tr>
<td>LMDE</td>
<td>Lunar Module Descent Engine</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>N-S</td>
<td>Navier-Stokes</td>
</tr>
<tr>
<td>Nu</td>
<td>Nusselt Number</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl Number</td>
</tr>
<tr>
<td>Re</td>
<td>Reynold’s Number</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning Electron Microscopy</td>
</tr>
<tr>
<td>St</td>
<td>Stokes’ Number</td>
</tr>
<tr>
<td>TBC</td>
<td>Thermal Barrier Coating</td>
</tr>
<tr>
<td>TEM</td>
<td>Transmission Electron Microscopy</td>
</tr>
<tr>
<td>VOF</td>
<td>Volume-of-Fluid</td>
</tr>
<tr>
<td>We</td>
<td>Weber Number</td>
</tr>
<tr>
<td>YSZ</td>
<td>Yttria-Stabilized Zirconia</td>
</tr>
</tbody>
</table>
CHAPTER ONE:
INTRODUCTION

Background on Computational Fluid Dynamics

This section presents an overview of the fundamental mathematics and physics concepts that are relevant to this thesis, and discusses the organization of this thesis. Computational Fluid Dynamics (CFD) is a method of solving the governing equations of fluid flow using approximate, numerical methods [2]. More specifically, the Navier-Stokes equations (shown in index form below) are simplified based on the user-defined physics of the simulation, and are approximated with a numerical approximation technique, such as finite-difference (FD), finite-volume (FV), or finite-element methods (FE). Next, the user-defined numerical scheme (usually implicit or explicit) and the numerical accuracy (usually first or second order) are employed. Then, the resulting form of the N-S equations are organized into linear systems of equations, and solved [2].

Conservation of Mass
\[
\frac{\partial u_i}{\partial x_i} = 0
\]

Conservation of Momentum
\[
\rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = \rho g_i - \frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j^2}
\]

Conservation of Energy
\[
\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \alpha \frac{\partial T}{\partial x_i} + \frac{\theta}{\rho c_p}
\]

The FV method solves the integral forms of the above equations (derived from Reynold’s Transport Theorem). The first step in finding a FV solution is to discretize a spatial domain into a certain number of cells, and to discretize the temporal domain into timesteps. Boundary conditions and initial conditions are used to create an initial solution on the spatial and temporal domains. At
each cell face, a wall flux is solved for (say, mass or momentum flux), this wall flux is interpolated to then solve for the flux at the cell center. This is done over and over again until a solution is reached (that is, when convergence criteria are satisfied) [2].

Time discretization techniques are often one of two categories: explicit, or implicit. Explicit discretization relies solely on information from the previous time-steo, whereas implicit discretization relies on both information from the previous timestep, and information from the next timestep [2].

Another important consideration in CFD simulations is whether to use a segregated or coupled method to solve the governing equations. The main difference between the two methods is that the coupled method combines all of the governing equations into one large linear system of equations, and solves all of the flow variables simultaneously, and employs density-based schemes. On the other hand, segregated solvers are pressure-based and solve flow variables iteratively using several different systems of equations [3].

Stability in a CFD simulation is generally characterized by Von-Neumann Stability Analysis, which essentially says that the round-off error, $\epsilon$, must take the form $|\epsilon^{n+1}/\epsilon^n| \leq 1$, where $n$ is the timestep. Round-off error is represented by an arbitrary function, and that arbitrary function is approximated using a Fourier Series expansion. The result of this analysis leads to a dimensionless number, called the Courant number, which is a function of the velocity, mesh size, and timestep. The criterion for stability is called the CFL criterion, and generally means that the Courant number must be close to unity for a solution to be stable. It is important to note that this the CFL criteria is generally necessary, but not always sufficient for a stable solution. As solvers get more complex, there are more stability criteria to consider [2].
\[ Co = u \frac{\Delta t}{\Delta x} \sim 1 \]

The work in this thesis relies heavily on numerical methods for multiphase flow, specifically the Eulerian approach to multiphase flow. In an Eulerian multiphase flow, each phase is modeled as continuous, as opposed to a Lagrangian approach where one or multiple phases are modeled as discrete [4]. There are several techniques used in Eulerian multiphase flow problems, and they can generally be categorized in one of two ways: surface-tracking, or volume-tracking [5]. The difference between the two methods can be seen in Figure 1. The surface-tracking method uses marker points and interpolation to track the interface, whereas in volume-tracking methods, there is no explicit market that tracks the interface, and special methods are required [5]. One such way to track the volumetric interface is through the Volume-of-Fluid (VOF) method.

![Figure 1: Difference Between Surface-tracking (left) and Volume-tracking (right) methods in Eulerian Multiphase Flow [5]](image)

The Volume-of-Fluid (VOF) method is a volume method that obtains a single set of flow equations by volume-averaging the governing equations for each fluid. The volume-averaging can best be described in Figure 2. Then, the information for each fluid is tracked with a volume fraction
transport equation seen below [6]. Special treatment of the interface is also possible, such as including the surface tension force, where the curvature of the interface is determined and from there, another term is added to the momentum balance equations [7].

\[
\frac{\partial \gamma}{\partial t} + \frac{\partial U_i \gamma}{\partial d_i} = 0
\]

Issues arise in the VOF method when the above equations are discretized in certain ways. For example, the first order upwind method causes smearing at the interface because of numerical diffusion [5].

![DIAGRAM]

Figure 2: Cases for volume averaging in the VOF method

**Background on CMAS and TBC**

Calcium-Magnesium-Alumino-Silicate (CMAS) particles are a classification of particles that include sand, and volcanic ash [8]. CMAS compositions can vary greatly around the world, and thus, representative samples are often synthesized. A comparison of a sample of CMAS taken from Icelandic volcanic ash, and two synthesized variants of CMAS is shown in Table 1 [9]. Important properties, such as viscosity, vary greatly from sample to sample. However, models
exist that can predict some of these properties based on the composition, such as the GRD viscosity model [10]. Unfortunately, experiments carried out to measure the viscosity of synthesized CMAS samples shows a large discrepancy between the GRD model and experimental measurements, as seen in Figure 1 [11].

Table 1: Compositions of Synthesized (C1 & C2) and natural CMAS [9]

<table>
<thead>
<tr>
<th>Mole %</th>
<th>CaO</th>
<th>MgO</th>
<th>Al₂O₃</th>
<th>SiO₂</th>
<th>FeO</th>
<th>TiO₂</th>
<th>Na₂O</th>
<th>K₂O</th>
<th>Main phases</th>
<th>Viscosity at 1250 °C (Log Pa·s)</th>
<th>Melting range [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMAS 1</td>
<td>24.7</td>
<td>12.4</td>
<td>11.1</td>
<td>41.7</td>
<td>8.7</td>
<td>1.6</td>
<td>–</td>
<td>–</td>
<td>Pyroxene + Anorthite</td>
<td>–0.25</td>
<td>1230-1250</td>
</tr>
<tr>
<td>CMAS 2</td>
<td>32.4</td>
<td>11.2</td>
<td>9.8</td>
<td>37.3</td>
<td>7.8</td>
<td>1.4</td>
<td>–</td>
<td>–</td>
<td>Pyroxene + Mellilitte</td>
<td>–0.69</td>
<td>1215-1245</td>
</tr>
<tr>
<td>Iceland VA</td>
<td>12.5</td>
<td>6.1</td>
<td>7.4</td>
<td>49.7</td>
<td>17.6</td>
<td>4.3</td>
<td>2</td>
<td>0.4</td>
<td>Amorphous</td>
<td>3.5</td>
<td>1050-1200</td>
</tr>
</tbody>
</table>

Figure 3: CMAS Viscosity comparison between experimental measurements and the GRD Model

Thermal barrier coatings are a category of high-temperature ceramic coating that are extensively used in regulating temperatures on turbine blades, especially in aircraft engines. It is critical to attain higher temperatures in these engines because higher temperatures lead to increases in efficiency [12]. A typical structure of the area around a TBC consists of the actual coating, an
oxide layer, a coat that bonds the TBC to the metallic substrate, and the metallic substrate. This structure can be seen in Figure 4 [13].

The hallmarks of an effective coating include: low thermal conductivity, phase stability, thermal shock resistance, and oxidation resistance, all of which are to protect the metallic substrate from undesirable conditions [14]. Many materials with low conductivity can be used as potential TBCs, including metals, and gemstones [15]. One of the current industry standard materials are forms of yttria-stabilized zirconia (YSZ), as it possesses greater thermal shock resistance than most other ceramics, and are stable at room temperature [16]. A disadvantage of this yttria-stabilized zirconia is that sintering, and phase changes can occur at temperatures in excess of 1200°C [17, 18].

Another important property to consider when manufacturing a TBC is the porosity. Porosity leads to lower densities, which is important because thermal conductivity and specific heat capacity are correlated to density [19]. A common method of manufacturing a popular form of YSZ, one that is 7% yttria (7YSZ), is electron beam physical vapor deposition (EB-PVD). EB-PVD produces porous 7YSZ coatings by creating a columnar microstructure in the top coat [20]. EB-PVD works by taking a target anode and hitting it with an electron beam from a charged tungsten filament. The anode is the material that the desired coating is formed from. As the beam hits the anode, the anode turns into a gaseous phase, and then solid particles precipitate from the gaseous phase onto everything in the vacuum chamber, including metallic substrates [21]. Alternatives to PVD methods include pulsed laser deposition [22], atmospheric plasma spraying [23], and chemical vapor deposition (including laser-induced) [24]. The scope of this work focuses solely on 7YSZ EB-PVD coatings.
Important properties of 7YSZ TBCs include thermal conductivity and specific heat capacity, both of which vary slightly with temperature. A graph showing thermal conductivity of various TBCs, including 7YSZ is shown in Figure 5, which shows that the thermal conductivity hovers around 1.5 W/m-K [25]. Specific heat capacity as a function of temperature for 7YSZ is shown in Figure 6, which shows a gradual increase in specific heat capacity as temperature increases [26].
Figure 5: Thermal conductivity as a function of temperature for various TBCs [25]

<table>
<thead>
<tr>
<th>Temperature [°C]</th>
<th>Thermal conductivity of top coat [W/mK]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7YSZ</td>
</tr>
<tr>
<td>1,000</td>
<td>1.38</td>
</tr>
<tr>
<td>1,100</td>
<td>1.39</td>
</tr>
<tr>
<td>1,200</td>
<td>1.53</td>
</tr>
<tr>
<td>1,300</td>
<td>1.59</td>
</tr>
</tbody>
</table>

Figure 6: Specific heat capacity of several TBCs, including 7YSZ [26]

TBCs must be evaluated nondestructively during their lifecycle to ensure aircraft safety criteria are met [13]. One such test uses time-domain spectroscopy in the terahertz regime to determine surface roughness, thickness, and evolution of internal structures in the TBC [27]. The
terahertz time-domain method has also been used to determine locations of stress-induced, air-filled pores [28]. Another method uses impedance spectroscopy to measure changes in the reaction layer [29]. Other methods include ultrasonic detection [30], reflection-enhanced luminescence [31], and 3D optical topometry [32]. There are some disadvantages to these nondestructive tests, including the harmful effects of X-rays on humans [33, 34].

TBCs can experience a variety of failure mechanisms, ranging from stresses from thermal cycling [35] to the mismatch of properties between the oxide layer and other layers [36]. Another failure mechanism experienced by TBCs, and a focus of this paper, is the sintering caused by infiltration of CMAS [37]. Much of the erosion caused by CMAS in aero-engines is caused by solid CMAS particles that make it past the turbine blade [38, 39]. However, melted CMAS particles are becoming more of a problem as aero-engines achieve hotter temperature for increased efficiencies [40]. Most CMAS particles melt between 1150 – 1250 °C, depending on their composition [41-44]. However, sintering occurs when CMAS comes into contact with the TBC. In one such case, volcanic ash interacted with the YSZ TBC, and caused yttria to leach into the VA from the TBC, and form yttria iron garnet [45]. The CMAS can do this as partially or fully-melted CMAS forms deposits on top of TBCs [46].

Images such as those seen in Figure 7 [47] are achieved through scanning electron microscopy (SEM), along with sample preparation techniques such as sectioning and polishing [48]. SEM is typically used to determine the location of reaction zones and infiltration depth [49]. Transmission electron microscopy (TEM) is used to determine reaction products and phases in the oxide coating [49, 50].
Figure 7: SEM image of an air-plasma sprayed TBC [47]
**CHAPTER TWO: DIRECTLY RESOLVED CMAS PARTICLE**

This chapter is adapted from a paper published at the 2022 American Society of Mechanical Engineering Fluids Engineering Division Summer Meeting. Copyright permission was granted by the publishing organization. See following citation for more information.


**Introduction**

The ingestion of CMAS (Calcium-Magnesium-Alumino-Silicate) particles into aircraft engines is an issue for aircraft safety and resilience. One incident occurred on British Airways Flight 9 in 1982, where all four engines on a Boeing 747 failed during flight through a cloud of volcanic ash (composed of CMAS). In this scenario, the CMAS particles melt due to high temperatures within the engine and resolidify on coolant lines [51]. This process can erode compressor blades, erode/infiltrate thermal barrier coatings, and cause overheating and stall [51]. The threat of CMAS ingestion can also take a financial toll on the commercial aviation industry. For example, the eruption of Eyjafjallajökull in Iceland caused the closure of European airspace for six days, and estimates say the closures cost commercial airlines around $1.7 billion [52]. The potential damage to aircraft and the world economy makes it imperative to understand how CMAS behaves inside aircraft engines.
This work seeks to do the following: 1) Give an overview of necessary background information to understand the processes involved in simulating CMAS, 2) Provide details of methodology in which melting-solidification of CMAS particles is directly resolved, 3) validate the previously mentioned approach using experimental correlations, theoretical values, and mesh studies, 4) Extracting transient information from the resolved CMAS particle, including temperature ratios, non-dimensional numbers, and forces acting on the particle.

Background

Melting and solidification in the context of CMAS particles demands a number of relevant dimensionless parameters. In droplet physics, the Weber number ($W_e$) is often used to predict the breakup mechanism of a droplet. The $W_e$ is defined as the ratio of the inertial forces acting on the droplet to the surface tension forces experienced by the droplet [53]. Breakup mechanisms that liquid droplets can experience include vibrational breakup, bag breakup, bag-and-stamen breakup, sheet stripping, and catastrophic breakup [53]. A summary of experimental correlations between breakup mode and Weber number is shown in Table 2. Due to high surface tension forces, the liquid droplet observed in this study stays within the vibrational breakup regime and is unlikely to break up.

Table 2: Experimental correlations for droplet breakup mode and weber number [53]

<table>
<thead>
<tr>
<th>Breakup Mode</th>
<th>Weber No. Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibrational</td>
<td>$W_e \leq 12$</td>
</tr>
<tr>
<td>Bag</td>
<td>$12 &lt; W_e \leq 50$</td>
</tr>
<tr>
<td>Bag-and-Stamen</td>
<td>$50 &lt; W_e \leq 100$</td>
</tr>
<tr>
<td>Sheet Stripping</td>
<td>$100 &lt; W_e \leq 350$</td>
</tr>
<tr>
<td>Catastrophic Breakup</td>
<td>$350 &lt; W_e$</td>
</tr>
</tbody>
</table>
The Ohnesorge Number \((Oh)\) is defined as the ratio of viscous forces on a droplet to the inertial and surface tension forces seen by that droplet. When the \(Oh\) is less than 0.1, is an indicator that the Weber number is an accurate predictor of the droplet’s breakup mechanism [54]. Reynold’s number \((Re)\) is a widely used value in fluid mechanics that relates the viscous forces to inertial forces and has many different applications. Here, the slip \(Re\) the relevant aerodynamic parameter, which is the \(Re\) associated with the difference of velocity between the CMAS particle \((v_p)\) and the surrounding flow field \((v_g)\). The thermal Stokes number \((St_{th})\) is another parameter of interest for this study, as it will be used for validation, and is defined as the ratio of particle thermal response time to fluid response time [1]. The \(St_{th}\) is also a function of the Nusselt Number \((Nu)\), which is the ratio of convective to conductive heat transfer across the boundary of the particle [1]. The Prandtl number \((Pr)\) is the ratio between the momentum diffusivity to the thermal diffusivity. The equations used to determine each dimensionless parameter are summarized below.

\[
We = \frac{\rho v^2 D}{\sigma}
\]
\[
Oh = \frac{\mu}{\sqrt{\rho \sigma D}}
\]
\[
Re_{slip} = \frac{\rho (v_f - v_p) D}{\mu}
\]
\[
St_{th} = \frac{\rho_p C_p d_p^2 u_c}{6 \kappa_f N u^2 + C}
\]
\[
Nu = \frac{h d_p}{k_f} = 2.0 + 0.6 Re_{slip}^{0.5} Pr^{0.33}
\]
\[
Pr = \frac{c_p \mu}{\kappa_f}
\]
Current literature has explored several important aspects of CMAS interaction with aircraft engines. One such study includes a computational model exploring CMAS particle fan-blade interaction [55]. This study’s goal was to find a distribution of particle sizes that are likely to make it past the fan stage of an aircraft engine [55].

Another important study investigated how the impact of CMAS particle size and chemical composition would affect the particle’s time to equilibrate on the nozzle guide vane. The particle’s properties were summarized using the thermal Stokes number. The ratio of particle impact temperature \( T_{p,imp} \) to the temperature of the fluid \( T_f \) was measured. The plot is shown in Figure 8. The plot shows that the temperature of particles with a \( St_{th} \) below unity tend to equilibrate with the temperature of the surrounding fluid in a fraction of the surrounding fluid’s response time. \( St_{th} \) larger than unity led to particles that take much longer than the surrounding fluid’s response time to equilibrate with the surrounding fluid. This correlation was used as validation in the results section of this paper.

![Figure 8: Validation data for temperature ratio v. Thermal stokes number [1]](image-url)
Another study of interest investigated binary collision of liquid CMAS particles in conditions similar to those seen in a gas turbine [56]. This work sought to investigate CMAS particle coalescence within turbines. An important note of this study is that it also employs a direct resolution methodology to simulate CMAS. However, it does not include particle melting/solidification modeling.

Methods

The simulation of a CMAS particle was accomplished in Star-CCM+ (v.16.02.009), a commercial CFD solver from Siemens PLM Software. The particle simulation was conducted with quarter-symmetry where the domain’s length was 80x larger than the particle’s radius and the width and height was 20x larger than the radius. A diagram of the domain can be seen in Figure 9.

![Figure 9: Shows a diagram of the domain with dimensions.](image)

Mesh

There are several main components when it comes to meshing the particle’s domain. Firstly, for the finest mesh, a trimmed-cell mesh was used where the base size was 40% of the particle’s diameter (Note: this base size was changed to different sizes for the mesh study). The base size was used as the maximum cell size in the far-field. A free-surface adaptive mesh criterion
was implemented that refines the mesh in the area around the interface between the particle and the surrounding fluid. The mesh was refined to the point where the CMAS particle and its interactions with the laminar flow are fully resolved. A diagram of the mesh can be seen in Figure 10.

![Figure 10: Shows a diagram of the mesh where the green arrow indicates the refinement of the far-field, the red square indicates the area around the droplet, and the blue arrow indicates the refinement of the free-surface adaptive mesh](image)

**Computational Model of the Physical System**

One drawback of the proposed, direct CMAS-particle-resolving method is the very small timesteps required to resolve the CMAS particle accurately. So, adaptive time-stepping combined with sub-iterations was used to ensure this happens quickly and accurately. The Courant number $\left(\frac{u\Delta t}{\Delta x}\right)$ for the adaptive-time-stepping was 1.0 and an average Courant number of around 0.5. The physical models used are also 2nd order accurate in time and space.

The particle and its domain were resolved using a laminar flow model for simplicity. Even though conditions in engines are often turbulent to promote mixing, resolving the turbulence in
the context of this study is not relevant since the main goal is to reproduce the trend seen in Figure 8.

For the multiphase physics, the Eulerian Multiphase Volume-of-Fluid method was used to simulate the interactions between the two Eulerian phases, air, and CMAS. The melting-solidification model was also enabled to quantify the degree of melting and resulting deformation of the CMAS particle. The melting-solidification model also includes a convenient flow-stop option that allows solid CMAS to be held in place while outer portions of the particle begin to melt and break off, ensuring the CMAS particle remains in the domain. Equations 7-10 show the governing equations of the Eulerian Multiphase VOF model. In these equations, the subscript i denotes a particular phase, \( a_i = V_i/V \) is the volume fraction of a particular phase, and \( S_{a_i} \) is a source term defined by the initialization of the phases. Equations show the governing processes of the melting-solidification model. Here, \( a^*_s \) is the relative solid volume fraction, and \( f(T^*) \) is assumed to be linear (i.e., \( f(T^*) = 1 - T^* \)).

**Boundary Conditions**

Boundary conditions are relatively simple for this study. A velocity inlet was used where the velocity was set to 100 m/s. The pressure outlet condition was used for the exit condition, and the pressure was set to ambient pressure. All other boundaries are set as symmetry planes to account for the quarter symmetry.

**Benchmark and Mesh Independence Study**

The validation study was conducted by fixing the particle’s properties to those seen in Table 3. An important note here is that the paper whose data is being used for validation analyzes
particles with varying $Pr$ but discovered that $Pr$ only slightly affects the outcome. In this study, the Prandtl number is fixed. The simulation was allowed to run until the particle had fully melted. The temperature ratio described in Figure 8 was plotted against the inverse of time, making the x-axis similar to Figure 8’s x-axis. The same curve was generated for 5 different mesh sizes to ensure the solution was independent of the mesh.

Gathering Results

The spatially varying properties of the CMAS particle are tracked via the center of mass of the particle. So, for example, in $Re_{slip}$, $v_p$ is the velocity of the particle’s center of mass. Tracking the center of mass is necessary for extracting results for this direct resolution process. Once the position data is extracted, first and second-order finite-differences are performed to construct the particle’s velocity and acceleration history. From the velocity and acceleration history, the time-varying variables of interest can be calculated as well, such as $Re_{slip}$ and drag. The particle’s temperature cannot be monitored in the previously described way due to the particle not being a lumped system. Instead, both the particle’s average temperature and the average temperature of an isosurface encompassing the particle are tracked. The particle length scale is the initial diameter of the particle, which, in all cases, is 100 $\mu$m. This length scale was used in calculations of $Re_{slip}$ and $We$, meaning that an assumption was made that the particle’s deviations from a sphere are minor throughout the time simulated. Hence, dimensionless parameters in this study assume a constant spherical shape of the particle.
Table 3: CMAS properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>2690</td>
</tr>
<tr>
<td>Specific Heat (J/kg-K)</td>
<td>800</td>
</tr>
<tr>
<td>Thermal Conductivity (W/m-K)</td>
<td>92.35</td>
</tr>
<tr>
<td>Latent Heat of Fusion (J/kg-K)</td>
<td>144</td>
</tr>
<tr>
<td>Dynamic Viscosity (Pa-s)</td>
<td>11</td>
</tr>
<tr>
<td>Solidus Temperature (K)</td>
<td>1400</td>
</tr>
<tr>
<td>Liquidus Temperature (K)</td>
<td>1420</td>
</tr>
<tr>
<td>Surface Tension Coefficient (N/m)</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Results And Discussion

Validation and Mesh Studies

The validation and mesh (in progress) study results are summarized in Figure 11. It is shown that the particle follows a similar pattern as in Figure 8. Figure 11 corresponds to points in Figure 8 whose $S_{th}$ are greater than 10.

The simulations show that as the CMAS particle rises in temperature, the outer layers of the particle begin to melt. As the outer layers melt, the airflow begins to strip the liquid CMAS from the particle, and the liquid then leaves the domain at the pressure outlet. This behavior is seen in Figure 12, where the white layer surrounding the particle shows a drop in solid volume fraction, indicative of melting. The CMAS particle takes around 0.6 ms to melt in the conditions presented here, as seen in Figure 15.
A note here is that these simulations run very slowly compared to the time required for the particle to melt. Recommended timesteps from the different solvers range from $1 \times 10^{-8}$ s to $2.5 \times 10^{-7}$ s, while the droplet takes around 0.6 ms to melt entirely. The simulation would need to be run for a more extended period for the particle to achieve the entire curve seen in Figure 8 (i.e., for the melted liquid particle to approach the temperature of the surrounding fluid). Some data on the curve in Figure 11 was omitted due to noise caused by temperature correction limitations in the initial timesteps of the simulation. Figure 11 also shows that the simulation has reached monotonic convergence, as the approach the converged character. It is seen that a cell size of $4 \times 10^{-5}$ m and $5 \times 10^{-5}$ m both show extremely similar validation curves. The $4 \times 10^{-5}$ m case will be the one used for graphs of data proceeding Figure 11.

Figure 11: Validation Data For Temperature Ratio V. Thermal Stokes Number [1]
Variables of Interest

Using the method described above, the CMAS center of mass was a starting point for gathering information about several dimensionless parameters. The Weber Number of the particle was graphed as a function of time in Figure 14. The Weber number is always lower than 12, so either no breakup or vibrational breakup [53] would occur in the particle once it has fully melted. However, it is unlikely that any breakup will occur due to the high surface tension seen in Table 3. The dimensionless parameters that depend on velocity have similarly shaped curves, as seen in Figure 13. The \( Re \) and \( We \) decrease over time because they rely on the slip velocity. Slip velocity decreases from the flow speed as the droplet accelerates after it is fully melted, causing the flow-stop condition for solid cells to disappear.

If it is assumed that drag is the only force acting on the particle, then Newton’s 1st Law, along with the second-order finite-difference of position, can be used to create a drag v. time curve (Figure 16). Interestingly, the time-averaged drag value extracted from this data is \( 1.5268 \times 10^{-6} \) N. A theoretical drag value calculated from Stokes’ Law for drag on spheres at low \( Re \) is \( 1.706 \times 10^{-6} \) N. Comparing these two values show only a 10% difference. The relative closeness of these
values provided confidence in the aerodynamics prediction of the direct resolution approach. Plugging in the correlation retrieved from Stokes’ Law to the general equation for drag coefficient yields an approximate form of the drag coefficient for spheres at low Reynolds numbers. This correlation gives time-varying drag coefficient seen in Figure 17, and a time-averaged drag coefficient for the particle of 0.0356.

\[
F_{D,Stokes} = 6\pi \mu R U_p \\
C_{d,Stokes} = \frac{F_D}{0.5\rho U^2 A} = \frac{24}{Re_{slip}}
\]  

Figure 13: Shows reynold’s number as a function of time
Figure 14: Shows the particle’s Weber number as a function of time

Figure 15: Shows the ratio of solid cmas to the overall volume of cmas as a function of time

Figure 16: Shows the time-varying drag on the CMAS particle using Newton’s 2\textsuperscript{nd} Law.
Figure 17: Shows time-varying drag coefficient of the cmas particle based on stokes’ law

**Conclusion**

A direct resolution approach from the VOF method was used to simulate a melting CMAS particle. The resultant time-varying temperature ratio (\(T_p/T_g\)) was compared to experimental data [1] that plotted the same temperature ratio against the thermal Stokes number. This validation study shows that the direct resolution approach is valid at least for \(St_{th}\) greater than 10. Comparison to the remainder of the curve requires extended simulation runtime and a larger flow field domain. Plots of Reynold’s number and Weber number were extracted from the simulation by finite-differencing the particle’s center of mass’s position data. Time-varying drag was also plotted in this method, and time-averaged drag compared to Stokes’ Law’s theoretical value. The drag comparison yielded results of similar magnitude, which further validates this method.
Future Efforts

With the direct resolution approach of simulating a melting CMAS particle developed, the method can better understand various applications. One such application is a melting CMAS particle interacting with various airfoils to understand their interactions with turbine blades. This proposed effort would involve coupling the method presented in this paper with either an Eulerian dispersed multiphase flow field or a Lagrangian phase of CMAS particles over an airfoil. Streamline data from the CMAS particles would be extracted and converted to time-varying velocity data would be fed into the direct resolution simulation. From here, particle breakup, sticking probabilities, and more can be calculated to predict the particle’s interaction with the airfoil.

This method of simulating CMAS can also be used in thermal barrier coating infiltration studies, where capillary effects drive the melting CMAS into feathered microstructures. Effects of heat transfer and shear force from the droplet on the thermal barrier coating would be quantified and design recommendations could be made.

Other future efforts could include applying this method to other types of particles that experience melting, including water droplets and lunar dust particles. Similar studies to the one described above could be done for these particle types.

Lastly, the numerical methods involved in VOF melting-solidification can be moved to open-source codes, such as OpenFOAM, to allow for better pipelining between coupled simulation methods.
CHAPTER THREE: LUNAR REGOLITH MELTING, SOLIDIFICATION, AND IMPACT

The following chapter is adapted from a paper submitted to the 73rd International Astronautical Congress. The concept of the work described here takes the now validated methodology from Chapter 4 above, and applies it to a problem that involves a lunar regolith particle encountering a plume jet, melting, and impacting a wall at a high velocity. Copyright permission was retained by the author. See the citation for further information.


Introduction

Lunar dust particles are problematic for many reasons. These particles have a tendency to adhere to surfaces, which can create issues from tracking particles inside habitats [57]. This property also causes particles to degrade mechanical components [58]. Mitigating the impact of lunar dust particles is going to be a challenge for future lunar exploration missions, such as the Artemis program.

This work seeks to understand dynamic and thermal properties of lunar dust particles as they interact with rocket plumes. What makes this work unique is the explicit focus on the melting-solidification of the particles, and how that melting affects the particle’s interactions with other structures. Firstly, a review of the relevant literature is conducted. Then the numerical methodology is described. Next, results from the simulations are discussed. Lastly, conclusions
are made based on the results. A combination of analytical models and CFD will be used to understand this problem.

**Literature Review**

Methods of simulating rocket plumes in low-atmosphere environments are very well-documented, and usually involve the coupling of a Navier-Stokes based CFD solver, and a DSMC solver, such as in Figure 18.

![Figure 18: Description of N-S/DSMC Coupling [59]](image)

Here, the three regions (continuous, transition/rarefied, and free molecular) can be described by their Kn. Kn is a dimensionless number defined as the ratio between a molecule’s mean free path and a characteristic length scale. For a Boltzmann gas, Kn can be written as seen below.

\[
Kn = \frac{k_b T}{\sqrt{2\pi d^2 p_L}}
\]
There are different ranges of Kn associated with different flow regimes, summarized in Table 4. In continuum flow, the assumptions of the N-S equations are valid and thus, Navier-Stokes-based CFD solvers can be used to solve for this, such as those present in commercial software such as Star-CCM+. In Slip flow, the fluid can still be treated as a continuum, but the no-slip condition is no longer valid. In transitional and free-molecular flow, the continuum assumption is no longer valid, so a different type of solver must be used. One such solver is DSMC. The DSMC method involves using probabilistic modelling to solve the Boltzmann Equation, seen below [60], where force refers to external forces on the particles, diff refers to particle diffusion, and coll refers to particle collisions. While commercial software often does not have DSMC solvers built-in, open-source tools, such as OpenFOAM’s dsmeFoam+ [61] can be used.

\[
\frac{df}{dt} = (\frac{\partial f}{\partial t})_{\text{force}} + (\frac{\partial f}{\partial t})_{\text{diff}} + (\frac{\partial f}{\partial t})_{\text{coll}}
\]

<table>
<thead>
<tr>
<th>Range</th>
<th>Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kn &lt; 0.1</td>
<td>Continuum Flow</td>
</tr>
<tr>
<td>0.01 &lt; Kn &lt; 0.1</td>
<td>Slip Flow</td>
</tr>
<tr>
<td>0.1 &lt; Kn &lt; 10</td>
<td>Transitional/Rarefied Flow</td>
</tr>
<tr>
<td>Kn &gt; 10</td>
<td>Free-molecular Flow</td>
</tr>
</tbody>
</table>
For the multiphase aspect of this problem, regolith particles are often simulated using an Eulerian-Lagrangian method. In this method, particles are simulated as a dispersed phase, driven by the continuous phase. This is done because simulating a number of particles on the order of $10^7$ is often not feasible from a computational resource perspective. This is especially true when the user intends to simulate particles with a distribution of diameters [62]. Lunar regolith particles exist in diameters between 0.046 mm and 0.11 mm [63], thus, a Eulerian-Lagrangian simulation strikes a better balance between realism and computational cost than an Eulerian simulation would. However, if a simulation of a single particle is desired, an Eulerian-Eulerian simulation is much more reasonable.

The Eulerian-Lagrangian multiphase method has been used for several jet plume simulations, including one in a simulated Martian environment. Here, velocity history of the Lagrangian particles were extracted to estimate the destructive impact of the regolith particles [64]. An important note for this study however is that the continuum assumption is valid on Mars, so the method was not coupled to a DSMC solver. The method was also not coupled to an Eulerian-Eulerian method that focuses on a single regolith particle.

Another study took this method one step forward by using a DSMC method to simulate the jet plume [65]. A single particle is also simulated using a Lagrangian approach with one-way coupling with the DSMC solvers [65]. An empirical correlation was used to estimate the drag force and drag coefficient on the Lagrangian particles [66] (seen below) and the dynamics of the particles were solved via Newton’s 2nd Law.
\[
C_D = \begin{cases} 
24.0Re^{-1}, & Re < 2 \\
18.5Re^{-1}, & 2 \leq Re < 500 \\
0.44, & Re \geq 500 
\end{cases}
\]

\[
F_{D,i} = m_p \frac{3\rho C_D}{4\rho p d_p} |u_{i,t} - u_{p,i}| (u_{i,t} - u_{p,i})
\]

Here, \( \text{Re} \) is given by \( Re_p = \frac{\rho |u_{i} - u_{p}| d_p}{AT^\beta} \), where \( A \) and \( \beta \) are empirical coefficients, and \( T \) is the particle temperature [66].

The shear seen by the ground, velocity, pressure, and density fields were used to demonstrate the destructive properties of the flow field. These values were reported in terms of radial distance from the centerline of the plume, and the altitude of the lander [66]. From this work, it is seen that lighter particles (particles whose weight do not overcome the drag being exerted on the particle) experience straight line, whereas larger particles experience parabolic trajectories [66]. Smaller particles could reach speeds up to 755 m/s. Since these particles are leaving the plume region, they experience no resistance and can damage structures and instruments [66].

Another study was conducted that coupled a source flow to a DSMC solver as a replacement for simulating the continuum region [67]. This was done in the context of a flat plate in the wake of a cylinder, and to analyze the effect of non-negligible pressures in an object’s wake if the object is immersed in a plume, expanding into vacuum conditions [67]. The source flow model was used to estimate boundary conditions for the DSMC solver. This was done to avoid having to simulate the high-density regions of the flow using the DSMC solver, because simulating
higher densities with DSMC is more computationally expensive. Eq. 4-8 show the equations used in the source flow model.

\[
\rho(r, \theta) = \frac{2A p_0}{V^2} \left( \frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}} \left( \frac{r_e}{r} \right)^2 f(\theta)
\]

\[
f(\theta) = \cos \left( \frac{\pi \theta}{2 \theta_L} \right)^{\frac{\gamma + 0.41}{\gamma - 1}}
\]

\[
\theta_L = \frac{\pi}{2} \left( \frac{\gamma + 1}{\sqrt{\gamma - 1}} - 1 \right)
\]

\[
V = \sqrt{\left( \frac{2\gamma}{\gamma - 1} \right) \frac{k T_0}{m}}
\]

\[
A = \frac{1}{\int_{0}^{\theta_L} \sin \theta f(\theta) d\theta}
\]

**Methodology**

For this study, the source flow model described in Section 1.1 is employed, coupled to an Eulerian-Eulerian simulation of a single, fully-resolved lunar regolith particle.

**Source Flow Model**

The source flow model was written in Python 3 and applies Eq. 4-8 to a 2-dimensional grid. Where possible, the properties for the Lunar-Module Descent Engine (LMDE) were used as inputs for the source flow model [57]. These properties are summarized in Table 5.
Table 5: LMDE Properties

<table>
<thead>
<tr>
<th>Property (Units)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chamber Pressure (kPa)</td>
<td>712.9</td>
</tr>
<tr>
<td>Nozzle Exit Temperature (K)</td>
<td>1533</td>
</tr>
<tr>
<td>Nozzle Exit Radius (m)</td>
<td>0.8001</td>
</tr>
<tr>
<td>Molecular mass (kg/mol)</td>
<td>3.40</td>
</tr>
</tbody>
</table>

From here, Eq. 7 is used to calculate a limiting velocity. The limiting velocity is essentially how fast a single molecule of gas could be travelling from the plume. An angle, $\theta$, is calculated for every grid point relative to the position of the jet plume. $\theta_L$ is the limiting theta, or the maximum value $\theta$ can have. From here, density values can be calculated from Eq. 4 for the 2D grid. The average density on the 2D grid’s outlet was calculated, and this, along with velocity data, will be used for the initial and boundary conditions for the Eulerian-Eulerian VOF simulation.

**VOF Simulations**

The simulation of a lunar regolith particle was accomplished in Star-CCM+ (v.16.02.009), a commercial CFD solver from Siemens PLM Software. The simulation was conducted in quarter-symmetry. The domain’s length was 80x larger than the particle’s radius and the width and height was 20x larger than the radius. A diagram of the domain if it was in full 3D can be seen in Figure 19. A summary of the boundary conditions is shown in Table 6.
Figure 19: Example diagram of the computational domain used for the VOF simulation.

Table 6: List of Boundary Conditions

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Type</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Velocity Inlet</td>
<td>1702 m/s</td>
</tr>
<tr>
<td>2</td>
<td>Wall</td>
<td>Adiabatic</td>
</tr>
<tr>
<td>Unlabeled</td>
<td>Pressure Outlet</td>
<td>455 Pa</td>
</tr>
</tbody>
</table>

The base size for the mesh was 40 percent of the regolith particle’s diameter. This meshing size was set as the maximum cell size in the domain. From here, a free-surface adaptive mesh was used to refine the interface between the lunar regolith and the surrounding gas. The mesh was refined to the point where the regolith particle and its interactions with the laminar flow are fully resolved. A diagram of this mesh can be seen in Figure 20.
A disadvantage of this direct regolith-particle-resolving method is the very small timesteps required to resolve the regolith particle accurately. To overcome this, adaptive time-stepping and sub-iterations were used to ensure the unsteady simulation remains stable. The time-stepping was controlled by the Courant number \( \frac{u \Delta t}{\Delta x} \), where the maximum Courant number was set to 1.0, and the average Courant number was set to 0.5. The physical models used for the simulation were 2\(^{nd}\) order accurate in time and space.

The particle and its domain were resolved using a laminar flow model. This was done because the \( \text{Re} \left( Re = \frac{\rho UL}{\mu} \right) \) is < 1000 for the flow, given the low density of gas from the source flow model.

For the multiphase physics, the Eulerian Multiphase VOF method was used to simulate the interactions between the two Eulerian phases, the lunar regolith, and the surrounding gas (\( N_2 \) in this case). A melting-solidification model was enabled to allow the particle to melt in the flow. The VOF Method for modeling, and in some cases, fully resolving multiphase flows has proven
useful in several applications. Firstly, a proof of concept application was developed, directly resolving a melting CMAS particle by validating the Thermal Stokes Number against experimental data. This showed that the VOF method can be used for these directly resolved melting-solidification cases.

Next, the same method was applied to a lunar regolith particle. This time, however, the flow was coupled to a source-flow model to account for rarefied flow conditions. Also, the melting particle experienced an impact with an adiabatic wall. Results from this study showed how a partially melted lunar regolith particle can stick to surfaces, which can be problematic for lunar missions, especially when accumulation is accounted for.

Lastly, the VOF Method was used to resolve a capillary-driven flow, the infiltration of molten CMAS into a 7YSZ EB-PVD thermal barrier coating. These simulations were characterized by the infiltration depth of the CMAS as a function of time. The simulated infiltration was several orders of magnitude faster than the experimentally measured infiltration. Possible explanations include the lack of a solidification or phase change model in the simulation, or that the modeled TBC is not sufficient for accurately resolving the infiltration. Future work here includes the addition of solidification and phase change models, and performing a similar simulation on a domain consisting of a nano-CT scan of an actual TBC.

The overall success of the models developed using the VOF Method shows its robustness at handling a wide range of multiphase flows. The VOF Method is not without its faults, however. The directly resolved CMAS particle experienced stability issues at certain mesh sizes, and had a very long run time. While the stability issues can be readily fixed by changing the mesh, the long runtime persisted through the lunar regolith and CMAS infiltration cases. Future work could
involve further optimization of the SIMPLE-C algorithm to run on GPUs, allowing faster overall runtimes.
APPENDIX A: COMMONLY USED EQUATIONS shows the governing equations that Star-CCM+ uses for the Eulerian VOF model. In these equations, the subscript \( i \) denotes a particular phase, \( a_i = V_i/V \) is the volume fraction of a particular phase, and \( S_{a_i} \) is a source term defined by the initialization of the phases. Lastly, the governing processes of the melting-solidification model is described. Where, \( a_s^\ast \) is the solid volume fraction of the cell, and \( f(T^\ast) \) is assumed to be linear (i.e., \( f(T^\ast) = 1 - T^\ast \)).

The properties of the lunar regolith that were used are based on various sources. However, where possible, the properties will be taken from a single Apollo landing site. Firstly, the thermal conductivity of Lunar Regolith has been determined to vary based on temperature [68].

![Figure 21: Thermal conductivity as a function of regolith temperature [68]](image)

From the same study, the specific heat capacity of lunar regolith was also investigated and its relationship with temperature is shown in Figure 22 [68]. Note that these values were calculated based on a latent heat of fusion of 1429 J/kg-K [68].
Figure 22: Specific-heat capacity for lunar regolith as a function of temperature [68].

The density of lunar regolith can vary greatly depending on soil depth, and landing site [69]. However, values generally range between 300 kg/m$^3$ and 2000 kg/m$^3$ [69].

Unfortunately, values for dynamic viscosity must come from lunar regolith simulant, rather than regolith itself. Values for regolith simulant are readily available due to the interest in using regolith to build lunar structures with additive manufacturing [70]. Viscosity values were calculated based on the shear rate, and it was found that a dynamic viscosity between 8 Pa-s and 9 Pa-s for various shear rates [70]. Thus, an average value of 8.5 Pa-s will be used as the dynamic viscosity of the melted regolith.

Lastly, melting temperature is needed for the melting-solidification model. Since the regolith is a combination of several materials, the melting temperature is a range between 1373 K to 1653 K [68].

Some assumptions must be made to simplify some parts of the interface between the source flow model and the VOF simulation. Importantly, the source flow model assumes that the
temperature of the field is the same as the chamber temperature of the nozzle. So, this will have to be the starting temperature of the VOF simulation as well [67].

Results and Discussion

In this section, results from the source flow model will be shown and discussed. The implementation of the source flow model’s data into the VOF model is also discussed. Lastly, data and flow visualization from the VOF model will be discussed.

The source flow model was created using a field assumed to extend 2.5 m in the +x-direction, and 0.5 m in the -y and +y directions. The 2D grid was created such that the domain was discretized into 40 points in each direction, resulting in a 40x40 grid to resolve the 2.5 m x 1.0 m domain. The results from this calculation were output in the form of two graphs, and some text data. The first graph seen in Figure 23. is a contour plot of the domain’s density. The values in the graph provide sensible densities around the nozzle (at 0,0) is higher than in the far-field. The density data of interest exists here at x=2.5 m. Since the density varies radially from the nozzle, the density will take different values along x=10 m. So, the average density along the x=10m line will be used as the density for the VOF simulation.

With the average density on this boundary being ~0.017 kg/m³, and the properties of nitrogen being known, the ideal gas law can be used to find an approximate pressure of 455 Pa. The domain of the VOF Simulation was initialized at this pressure.
Figure 23: Contour plot of exhaust gas density. Nozzle location at (0,0).

The limiting velocity was found to be 1701 m/s from the source flow model. This velocity was used as the initial condition for the gas surrounding the regolith particle in the VOF simulation.

Figure 24 shows the initialization of the VOF domain. The solid volume fraction of the particle is tracked via a resampled volume that assigns 3D voxels to mesh cells, and colors them based on a scalar field. Since the combustion chamber temperature is between the liquidus and solidus temperatures, the particle is initialized with a solid volume fraction of around 0.36.

Figure 24: Visualization of the VOF initialization.
As the simulation is started, the regolith particle begins to accelerate due to the speed of the surrounding gas. The particle’s temperature also begins to equilibrate with the flow. The ratio of the particle’s average temperature to the average temperature of the surrounding gas is plotted in Figure 25. The temperature ratio in Figure 25 starts so high because the gas starts at a cool ambient temperature and then heats up from the boundary condition, simulating the actual temperature increase from a descent vehicle. The deformation of the regolith particle after just 5 time-steps can be seen in Figure 26. Here, the liquid outer layer of the regolith can be seen deforming around the solid core. This is important because it shows that the liquid outer layer is not entirely stripped from the particle, making it more likely to stick to surfaces.

![Figure 25: Plot of Temperature Ratio (i.e., Mean regolith temperature normalized by the mean gas temperature) versus time on a regolith particle.](image-url)
The ratio of the total volume of solid regolith to the total volume of regolith is also plotted against time in Figure 27. Here it can be seen that the particle remains partially liquid throughout the entire simulation, meaning that it can interact and stick to the wall at the end of the domain, rather than just bouncing off.
A visualization of the particle sticking to the wall is seen in Figure 28. Here, the white surface represents the outer most layer of the lunar regolith. The colors inside the white surface represent the solid volume fraction of the regolith. Lastly, the red plane represents the temperature of the gas surrounding the particle, and blue-to-yellow represents the velocity of the gas surrounding the particle. Note that this is from a head-on-collision.

Figure 28: Visualization of Regolith Particle Sticking to Wall

From the position data of the regolith particle’s center of mass shown in Figure 29, there is a correlation between the spike around 30 μs in Figure 27, and the point at which the regolith particle sticks to the wall. This finding is interesting because it shows that a wall at a lower temperature than the regolith particle will cause a sudden solidification in the regolith particle. However, as quickly as the particle resolidifies, it returns nearly to the same solid volume percentage shortly after contacting the wall. Note that this was the case if the wall was set to an adiabatic condition. That is, the wall’s temperature is extracted from the interior of the domain.
Importantly though, this shows that the overall melting of the particle can be controlled by the thermal properties of the wall it contacts. Another important finding here is that if the velocity condition for the inlet is turned off, and the simulation is allowed to run ambiently after the particle has made contact with the wall, it no longer deforms or is affected significantly by gravity. This shows that the particle will not move once the engine from a descent vehicle is no longer being fired.

![Graph](image)

**Figure 29: Position (m) vs. time (s) of regolith's center of mass**

**Conclusions**

An understanding of the melting-solidification of lunar regolith particles between a jet plume and a structure was desired. It was hypothesized that a rocket plume causes a lunar-regolith particle to melt to a degree such that a liquid outer-layer would form around the particle. The liquid outer-layer would cause the particle to stick to surfaces and accumulate on those surfaces over time.
A source flow model was coupled to an Eulerian VOF simulation in order to model the rarefied flow experienced by the particle, and to resolve the melting-solidification of the particle, as well as it’s interactions with a wall. Where possible, values relating to the LMDE were used in the source flow model, and properties of lunar regolith were taken from the same landing sites. When not possible, however, these properties were taken from different landing sites or from regolith simulant.

Results from the source flow model show low-density, high-speed flow 2.5 m away from the nozzle. These results were utilized as input conditions for the VOF simulation. The VOF simulation shows that the regolith particle initialized at the chamber temperature of the LMDE already starts off partially melted. As the flow solution is generated, the hypothesized liquid outer-layer forms around the particle and deforms. Eventually the particle hits the wall and undergoes deformation up until a certain point where it does not deform any further. The particle also undergoes a brief period of solidification as it impacts the adiabatic wall. The particle quickly returns to the stage of melting it was in before impacting the wall. However, this shows that thermal properties of the wall can impact the state of the droplet. This could potentially help create solutions for the accumulation of these particles on lunar structures.

Further work on these ideas will include particle impacts at multiple angles to see if certain angles lend themselves more to sticking or ricochet of particles. Also, different heat transfer conditions will be explored on the impacted wall to look for effective ways of mitigating particle accumulation. Some examples of different wall conditions include constant temperature, constant heat flux density, convection on external side of the wall, and then all of these conditions, but with a conduction zone inside the wall and using conjugate heat transfer.
CHAPTER FOUR: CMAS INFILTRATION OF A THERMAL BARRIER COATING

The ingestion of CMAS (Calcium-Magnesium-Alumino-Silicate) particles into aircraft engines is an issue for aircraft safety and resilience. One incident occurred on British Airways Flight 9 in 1982, where all four engines on a Boeing 747 failed during flight through a cloud of volcanic ash (composed of CMAS). In this scenario, the CMAS particles melt due to high temperatures within the engine and resolidify on coolant lines [51]. This process can erode compressor blades, erode/infiltrate thermal barrier coatings, and cause overheating and stall [51]. Helicopters operating in desert climates are at an even greater risk for CMAS attack because of how often they operate in sandy conditions [71]. The threat of CMAS ingestion can also take a financial toll on the commercial aviation industry. For example, the eruption of Eyjafjallajökull in Iceland caused the closure of European airspace for six days, and estimates say the closures cost commercial airlines around $1.7 billion [52]. The potential damage to aircraft and the world economy makes it imperative to understand how CMAS behaves inside aircraft engines.

Thermal Barrier Coatings (TBCs) are ceramic coatings present in aircraft engines, usually on the high-pressure turbine blades [72]. TBCs serve to insulate components from prolonged heat exposure, often reducing temperatures from 1700 K down to around 1200 K [73]. High thrust applications demand higher temperature turbines, so it is important to continue developing and understanding coatings that can more effectively mitigate heat transfer in engines. TBC systems can be constructed using various methods and materials. One such method is to use EB-PVD ZrO₂ based coatings, as they have better aerodynamic performance and strain compliance than and APS TBC [74]. The top coat, which is the layer of interest for CMAS infiltration. Figure 30 shows the
columnar microstructure of an EB-PVD coating. Melted CMAS infiltrates the inter-columnar gaps, and as the CMAS solidifies when the engine is not in use, the thermal expansion causes deformation in the TBC microstructure, which causes the desirable thermal insulation properties to degrade.

There are also two analytical approaches to modeling CMAS infiltration in a TBC, these are the open pipe and concentric pipe models [11]. The open pipe model assumes that CMAS infiltrates the inter-columnar gap with a certain permeability given by

$$k_o = \frac{(r_{\text{eff}} \phi)^2}{8\tau(1 - \phi)^2}.$$  \hfill (1)

Here, $k_o$ is the permeability which relates to the ease of infiltration. Permeability is dependent on the $\tau$, which is the tortuosity of the medium, and $\phi$ is the overall pore fraction of the medium. The concentric pipe model modifies the open pipe model by changing the number saying that the gaps on either side of the column are the gaps between concentric pipes. The modified permeability, $k_c$, is given as

$$k_c = \frac{\psi b^2}{8\tau^2} \left[ 1 + \frac{a^2}{b^2} + \left( 1 - \frac{a^2}{b^2} \right) \frac{1}{\ln b} \right].$$  \hfill (2)

Here, $k_c$ is more specific to the various TBC shapes [11]. $\tau$ and $\phi$ are the same as in the open-pipe model. The only major difference is that this model relies on $a$, the radius of the outer pipe, and $b$, the radius of the inner pipe. These relationships drive the infiltration relevant to TBCs. Figure 31 shows the differences between the two pipe models. The pipe models also have their own correlations for quantities such as infiltration time [11, 75]. While these pipe models are useful for modeling overall properties of CMAS infiltration of a TBC, they fail to capture all of the kinetics, such as the infiltration effects in the feather gaps. They also fail to include any kind of
model for heat transfer, or thermal expansion. These are the shortcomings that can be assessed using simulations.

This paper proposes the use of a finite-volume method (more specifically, the Eulerian Volume-of-Fluid Model), to model the infiltration of CMAS into an idealized TBC. Previous simulation-based infiltration models have used an FEM approach [73], but this approach comes with several challenges, including difficulties with properly resolving surface tension forces. Overall the approach has potential to push the understanding of infiltration processes.

Figure 30: Various morphologies of EB-PVD TBCs under magnification [74]. The top panel depicts the TBC surface, while the lower panel shows a side view from a cut TBC.
Current literature has explored several important aspects of CMAS interaction with aircraft engines. One such study includes a computational model exploring CMAS particle fan-blade interaction [55]. This study’s goal was to find a distribution of particle sizes that are likely to make it past the fan stage of an aircraft engine [55].

Another important study investigated how the impact of CMAS particle size and chemical composition would affect the particle’s time to equilibrate on the nozzle guide vane. The particle’s properties were summarized using the thermal Stokes number. The ratio of particle impact temperature ($T_{p,imp}$) to the temperature of the fluid ($T_f$) was measured. The study showed that the temperature of particles with a $St_{th}$ below unity equilibrate with the temperature of the surrounding fluid in a fraction of the surrounding fluid’s response time. $St_{th}$ larger than unity led to particles that take much longer than the surrounding fluid’s response time to equilibrate with the surrounding fluid. This correlation was used as a validation methodology for this method [76].

This work seeks to do the following: 1) Give an overview of necessary background information to understand the processes involved in simulating CMAS, 2) Provide details of a
methodology in which CMAS infiltration into a TBC is directly resolved 3) Perform a design study on a parametrized TBC model to understand which dimensions lead to desirable infiltration properties, such as longer infiltration times, or less heat transfer to the TBC.

Methodology

The first step in simulating the infiltration of CMAS was to ensure the VOF method is a valid approach for capturing the melting-solidification of a CMAS particle. The methodology for validating this approach is discussed in previous work [76], and it is shown that melting particles between the solidus and liquidus temperatures of CMAS can be accurately modeled with the VOF method.

For the multiphase physics, the Eulerian Multiphase Volume-of-Fluid method was used to simulate the interactions between the two Eulerian phases, air, and CMAS. Modeling surface tension properly is critical as the infiltration is driven mostly by capillary forces. The governing equations for the VOF method in Star-CCM+ [62] are given in the context of a finite volume formulation as the conservation of mass, conservation of momentum, and conservation of energy. These equations couple to the Eulerian Multiphase VOF model and can be seen in APPENDIX A: COMMONLY USED EQUATIONS.

In these equations, the subscript \( i \) denotes a particular phase, \( a_i = Vi/V \) is the volume fraction of a particular phase, and \( S_{a_i} \) is a source term defined by the initialization of the phases. The VOF method implements a technique called High-Resolution Interface Capturing (HRIC). HRIC resolves the interface between the primary and secondary phases such that other properties, like surface tension and slip velocity, can be modeled as well [62]. A drawback of using the VOF method for the infiltration is the very small timesteps required to resolve the CMAS infiltration.
So, adaptive time-stepping combined with sub-iterations was used to balance computational speed and accuracy. A free-surface condition was enforced so that the Courant number \( \frac{u\Delta t}{\Delta x} \) at the interface was around 1.0. Such a criterion is often demanded to limit the interfacial motion to only a cell.

The model was initially set up as a 2D geometry, but a 3D geometry was created later to ensure that effects attributed to the nature of 2D problems would not impede the simulation. A depiction of the mesh for the 2D geometry is provided in Figure 32. The mesh is an unstructured mesh composed of (mostly) isotropic tetrahedrals. Due to the extremely slow infiltration speeds expected, boundary layers are not expected to contribute to the overall flow physics, so no prism layers are necessary around the walls. The boundary conditions for the 2D simulation were very simple. The top of the domain is a stagnation inlet with a total pressure set to 10x the reference pressure (1 atm), and the bottom of the domain was set as a pressure outlet with a pressure much smaller than the reference pressure. The pressure difference between the inlet and outlet was set up so that a pressure gradient is maintained throughout the domain similar to experiments conducted by Naraparaju et al. [11]. The sides of the domain are set as pressure outlets as well, but with a periodic interface.
The mesh of the 3D geometry was constructed with the same principles of the 2D mesh, however, it is generally much coarser to decrease simulation times. In 3D, it is also possible to use free-surface adaptive mesh refinement (AMR) to better resolve the CMAS-air interface. The effect of this AMR will be quantified in the results to see if it’s benefit outweighs its cost of simulation time and solution stability. The 2D geometry and mesh can be seen in Figure 32. The parametrization of the modeled TBC is summarized in Figure 33. The 3D geometry and mesh can be seen in Figure 34. The 3D boundary conditions are the same as in 2D. The wall contact angle of the CMAS phase is expected to be somewhere between 40 and 60 degrees based on experiments [11]. These values are temperature dependent though. For simplicity a constant wall contact angle of 40 degrees was chosen.

Figure 32: 2D geometry with CMAS (left), zoomed out 2D mesh, and 2D mesh zoomed in around wall region
Figure 33: Summary of Parameterized Dimensions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pyramidal Angle</td>
<td>$P_B$</td>
</tr>
<tr>
<td>Feather Inclination</td>
<td>$F_a$</td>
</tr>
<tr>
<td>Feather arm thickness</td>
<td>$F_t$</td>
</tr>
<tr>
<td>Void between feathers</td>
<td>$F_v$</td>
</tr>
<tr>
<td>Length of column</td>
<td>$C_L$</td>
</tr>
<tr>
<td>Diameter of column</td>
<td>$C_D$</td>
</tr>
<tr>
<td>Length of feather arm</td>
<td>$B$</td>
</tr>
<tr>
<td>Inter-columnar gap</td>
<td>$C_g$</td>
</tr>
</tbody>
</table>

Figure 34: Isometric View of 3D geometry with CMAS (left) and 3D mesh cross-section (right)
Preliminary Results

In the 2D simulation, initial time-steps on the order of $10^{-10}$ s were required to satisfy the convective CFL condition. However, the adaptive time-stepping allowed the time-steps to reach an order of $10^{-7}$ s. The infiltration in the 2D simulation is generally stable, though slow. There are two very important issues with this 2D simulation. Firstly, the feathers receive little to no CMAS infiltration. It is known from experiments that this should not be the case. This is happening mainly because the air phase is set to a constant density, and the 2D nature of the simulation causes air to get trapped in the feather arms by the infiltrating CMAS. Once the CMAS has passed the feather gap, the air can no longer leave, so the CMAS cannot infiltrate it. This can be fixed a few different ways: 1) moving everything to a 3D geometry, 2) allowing the walls to be porous to air, allowing the air to escape through the walls and thus CMAS will infiltrate, or 3) incorporating the Ideal Gas Law. Moving forward, options 1 and 2 will be considered, as the Ideal Gas Law introduces more instabilities once enabled.

The next important issue is the overall infiltration time. The infiltration time for the most viscous case is supposed to be around half the time shown in Figure 35 [73]. This means that the CMAS should have reached the very last feather arm in around 500 μs. This is likely happening as a result of the capillary forces and viscous forces involved being too close to one another, making the overall infiltration slower. Steps to take to improve this would be to increase the pressure gradient. The previously mentioned step of allowing the TBC wall to be porous to air will also assist with this.
Using what is known from this 2D simulation, new 2D and 3D simulations were created. The walls of the TBC in these simulations were allowed to be porous to air. Ideally, these new simulations will allow parametric design studies to begin.

Further characterization studies were conducted to provide insight on the sensitivity of parameters such as CMAS viscosity on the overall infiltration kinetics. Infiltration time of the experimental viscosity measurement [11], and the GRD viscosity model [10] were compared in and it was found that the experimental measurement lead to longer infiltration time, which makes sense as the experimental viscosity was generally higher over the temperature range seen in Figure 3.
Figure 36: Infiltration Depth v. Time for the GRD Viscosity Model and the Experimental Viscosity Measurement
CHAPTER SEVEN: CONCLUSIONS AND FINAL REMARKS

The VOF Method for modeling, and in some cases, fully resolving multiphase flows has proven useful in several applications. Firstly, a proof of concept application was developed, directly resolving a melting CMAS particle by validating the Thermal Stokes Number against experimental data. This showed that the VOF method can be used for these directly resolved melting-solidification cases.

Next, the same method was applied to a lunar regolith particle. This time, however, the flow was coupled to a source-flow model to account for rarefied flow conditions. Also, the melting particle experienced an impact with an adiabatic wall. Results from this study showed how a partially melted lunar regolith particle can stick to surfaces, which can be problematic for lunar missions, especially when accumulation is accounted for.

Lastly, the VOF Method was used to resolve a capillary-driven flow, the infiltration of molten CMAS into a 7YSZ EB-PVD thermal barrier coating. These simulations were characterized by the infiltration depth of the CMAS as a function of time. The simulated infiltration was several orders of magnitude faster than the experimentally measured infiltration. Possible explanations include the lack of a solidification or phase change model in the simulation, or that the modeled TBC is not sufficient for accurately resolving the infiltration. Future work here includes the addition of solidification and phase change models, and performing a similar simulation on a domain consisting of a nano-CT scan of an actual TBC.

The overall success of the models developed using the VOF Method shows its robustness at handling a wide range of multiphase flows. The VOF Method is not without its faults, however. The directly resolved CMAS particle experienced stability issues at certain mesh sizes, and had a
very long run time. While the stability issues can be readily fixed by changing the mesh, the long runtime persisted through the lunar regolith and CMAS infiltration cases. Future work could involve further optimization of the SIMPLE-C algorithm to run on GPUs, allowing faster overall runtimes.
APPENDIX A: COMMONLY USED EQUATIONS
Volume Fraction Transport Equation
\[
\frac{\partial}{\partial t} \int_V \alpha_i dV + \oint_A \alpha_i \mathbf{v} \cdot dA = \int_V \left( S_{\alpha_i} - \frac{\alpha_i D \rho_i}{\rho_i} \right) dV - \int_V \frac{1}{\rho_i} \nabla \cdot (\alpha_i \rho_i \mathbf{v}_{d,i}) dV
\]

VOF Continuity Equation
\[
\frac{\partial}{\partial t} \int_V \rho dV + \oint_A \rho \mathbf{v} \cdot dA = \int_V (S(dV, S = \sum_i S_{\alpha_i} \rho_i)
\]

VOF Momentum Equation
\[
\frac{\partial}{\partial t} \int_V \rho \mathbf{v} dV + \oint_A \rho \mathbf{v} \times \mathbf{v} \cdot dA = \oint_A \rho \mathbf{l} \cdot dA + \oint_A T \cdot dA + \int_V \rho g dV + \oint_A f_\mathbf{b} \cdot dA - \sum_i \int_A \alpha_i \rho_i \mathbf{v}_{d,i} \times \mathbf{v}_{d,i} dA
\]

VOF Energy Equation
\[
\frac{\partial}{\partial t} \int_V \rho E dV + \oint_A \left[ \rho H \mathbf{v} + p + \alpha_i \rho_i H_i \mathbf{v}_{d,i} \right] \cdot dA = -\oint_A \dot{q}'' \cdot dA + \oint_A T \cdot v dA + \int_V S_\mathbf{e} dV + \oint_A f_\mathbf{b} dV
\]

VOF Solid Volume Fraction
\[
\alpha_s^* = \begin{cases} 
1 & \text{if } T^* < 0 \\
\tilde{f}(T^*) & \text{if } 0 < T^* < 1 \\
0 & \text{if } 1 < T^*
\end{cases}
\]
Chapter 2

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Chapter 3

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REFERENCES

[1]. Bojdo, N., and Filippone, A. "A Simple Model to Assess the Role of Dust Composition and Size on Deposition in Rotorcraft Engines," *Aerospace* Vol. 6, No. 4, 2019, p. 44.


[25]. Bobzin, K., Bagcivan, N., Brögelmann, T., and Yildirim, B. "Influence of temperature on phase stability and thermal conductivity of single- and double-ceramic-layer EB–PVD TBC top coats consisting of 7YSZ, Gd2Zr2O7 and La2Zr2O7," Surface and Coatings Technology Vol. 237, 2013, pp. 56-64. doi: https://doi.org/10.1016/j.surfcoat.2013.08.013

[26]. Han, J., Zou, Y., Wu, D., and Zhang, Y. "Investigating the thermal, mechanical and thermal cyclic properties of plasma-sprayed Al2O3-7YSZ/7YSZ double ceramic layer TBCs," Journal of the European Ceramic Society Vol. 43, No. 9, 2023, pp. 4124-4135. doi: https://doi.org/10.1016/j.jeurceramsoc.2023.03.004


[62]. Star-ccm+. "Siemens PLM Software". Munich, Germany, Ver. 17.02.004.


