Impact on Infinite Asteroids: Analysis of Ejecta Outcomes in Small Body Binary Systems

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IMPACT ON INFINITE ASTEROIDS: ANALYSIS OF EJECTA OUTCOMES IN SMALL BODY BINARY SYSTEMS

by

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A dissertation submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Physics, Planetary Sciences Track in the Department of Physics in the College of Sciences at the University of Central Florida Orlando, Florida

Spring Term
2023

Major Professor: Yanga Fernandez
ABSTRACT

Binary asteroid systems make up roughly 15% of objects occupying near-Earth space, the Main Belt, and trans-Neptunian space. The impact history of asteroids in binary systems represents an interesting aspect of the general problem pertaining to the nature and evolution of surfaces for such objects. Specifically, the post-impact dynamics of ejecta and its relation to surface modification is a challenging question owing, in part, to the unusual gravitational field in a binary system and the subsequent capture and emplacement of debris on either binary component. Observable differences or similarities between the two bodies in the color, reflectance, thermal properties, and grain properties of their respective regoliths could give insight into the system’s past and the circumstances of recent impacts. Here we present simulations of impact scenarios in a wide variety of binary systems in order to generate a large family of prediction models for resurfacing and ejecta covering outcomes due to impacts. In this way, we can address our main science question of how specific binary system parameters influence the evolution of their surfaces. To create a library of ejecta outcomes, we first developed the Rebound Ejecta Dynamics (RED) package (Larson and Sarid 2021), an N-body integrator designed to model post-impact debris dynamics that builds on the existing Rebound software (Rein and Tamayo 2015). This package allows us to vary the many of the important parameters of a binary system, including primary-secondary separation, rotation periods, and mass ratios, as well as impact-related parameters, such as impact surface location, ejecta size and velocity distribution, and ejecta compositions. Our simulations generally use $10^4$ particles and cover one week of simulation time. From our simulations, we calculate the percentage of the system that is resurfaced, the distance that particles travel from the impact site, and the
percentage of particles that impact the surface. These regions of resurfacing can often be observed with different colors or spectral properties than the original surface. We find that there are trends in ejecta end-states as a function of binary system properties (i.e., primary rotation period and system mass ratio) for several common impact scenarios. We analyzed the dominant effect that influences the outcome of each impact event.
For my grandparents, Cliff and Winkie.

Thank you for never doubting my dreams and potential.
I owe much of my success to my parents, Anne and David Larson, my sister, Christine Larson, and my grandparents, Clifford and Winifred Larson. They have truly been there for me since the beginning and encouraged me to embark on this journey into the unknown. They are my cheerleaders, my shoulders to cry on, and my support when I simply cannot go on anymore. Thank you so very much for always being there for me through all my crazy adventures!

Next, I’d like to thank my partner, Brian Zamarripa Roman, and my cat, Renae, for being amazing assistants in finishing this dissertation. Brian cheered me on and held me accountable when we were stuck at home during COVID. Renae made sure to remind me to get up and play every now and then.

Thirdly, thank you to my advisors, Yan Fernandez and Gal Sarid, for pushing me to always be a better researcher. I appreciate their advice and when everything seemed impossible. Additionally, I appreciate the support from my committee members, Dan Britt and Josh Colwell, who challenged me to learn more and approach my work from different perspectives.

Thank you to all of my friends, old and new, for believing in me and encouraging me to keep growing and learning. I’d like to thank the cosplay community, especially Elle and Becky, for showing me that everyone can be a hero to someone.

Finally, I’d like to acknowledge Wonder Woman, She-Ra, and Captain Marvel for proving to me that everyone has the power to make a difference in our world. They may be comic book characters, but their stories restored my faith that my work can make a difference and that I can be a real world planetary defender even without superpowers. A special thank you goes to Wonder Woman for showing me that through patience and perseverance, the truth can always be found.
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CHAPTER 1: INTRODUCTION

Uncovering the dynamical history of a solar system proves exceptionally difficult; however, small clues are scattered throughout the system, waiting to be pieced together. Impact debris and craters act as dynamical snapshots that capture the exact dynamical conditions of a system at the time of impact. Through the numerical analysis of how debris interacts with small body systems, we can determine what types of ejecta outcomes to expect from varying dynamical parameters.

1.1 Impact Modeling

Ejecta dynamics models cover a wide variety of effects and stages following an impact. Here we compare computational models to the Rebound ejecta dynamics model that we developed. To do this, we must compare the models’ computational efficiency as well as the effects implemented by each model. While most models implement a hydrodynamic code to approximate the motion of the particles, some models, like the Rebound ejecta dynamics package proposed here, use an N-body integrator to calculate the motion of individual particles. N-body integrator PKDGRAV (Stadel 2001) can build a wide variety of target body shapes out of individual particles, using artificial viscosity to dampen interactions within the body. Yu et al. (2017) and Schwartz et al. (2016) chose PKDGRAV as well as scaling laws by Holsapple (1993) and Housen and Holsapple (2011) to model the DART impact due to its ability to build individual shape models and to trace individual particles. However, while PKDGRAV and Rebound are both N-body integrators, Rebound performs better than PKDGRAV in terms of computational costs for comparable tasks (Rein and
Yu et al. (2017) use the physical properties derived by Michel et al. (2016) to construct a 3D model of Didymos in PKDGRAV out of individual particles and implement artificial viscosity to limit particle interactions within the body. While an effective method of building a non-spherical body, this method is very computationally intensive and risks developing errors over time should the particles in the target body start to bounce off each other.

Other ejecta dynamics models implement scaling laws to approximate the evolution of an ejecta cloud. Housen et al. (1983) formed a basic model for impact ejecta based on scaling laws as well as momentum and energy coupling. They analyze the differences between the strength regime and gravity regime velocities. In the strength regime, the strength of the target body material maintains the structure and integrity of the body; whereas in the gravity regime, the self gravity of the target body is required to hold the body together. These regimes are determined by the internal structure of the target body which influences the velocity of the ejected material. Table 1 in Housen et al. (1983) outlines the various scaling relations necessary to compute various features and characteristics of the ejecta cloud and resulting crater. Larger ejecta fragments require a larger energy input in order to put them into motion; therefore, the larger fragments will travel slower on lower ballistic trajectories (Melosh 2011). Conversely the smaller ejecta fragments a very low energy input initially in order to be ejected and will therefore follow higher ballistic trajectories at higher velocities. Approximately $1 - 3\%$ of the total excavated mass is ejected through spallation when the shock wave reaches the free surface and rapidly releases pressure, causing loose particles on the surface to be ejected at high velocities (Melosh 2011). Due to the total amount of spalled material being so small in comparison to the total volume of ejected material, we assume this volume to be negligible when constructing our ejecta plume.
Hydrodynamic simulations, specifically designed for the DART experiment, yield consistent $\beta$ predictions (Stickle et al. 2018), within a factor of 3, which makes the demands on the execution and analysis more stringent. However, hydrodynamic simulations do not perform well in the transition from the near-field to the far-field trajectory calculations and do not fully consider effects of a binary system (presence of an additional gravity potential and non-spherical shape of secondary component). In addition, the extent to which these simulation methods can track particle-particle interaction is limited, both spatially and temporally. The early stages of ejecta cloud evolution involve many collisions between particles during which some particles accrete into larger particles while others break apart, changing the particle size and velocity distributions (Housen et al. 1983). Unlike a hydrodynamic code, an N-body code is able to follow particle interactions and adjust for changes in these distributions.

1.2 Impact Experiments

Deep Impact was the first major impact experiment on a small body. While the impact was on the comet 9P/Tempel 1 rather than an asteroid, the observations of the resulting debris provided valuable insight into impact processes on a small, gravitationally bound object. For this experiment, a 364 kg spacecraft impacted the surface of Tempel 1 at a speed of 10.3 km s$^{-1}$ with the intent of learning more about the nucleus of a comet (A’Hearn et al. 2005). The impact created a large plume of lofted debris that obscured the view of the crater (Busko et al. 2007). Modeling efforts by Richardson (2007) estimated the extent of effects such as radiation pressure, coma gas pressure, impact-induced vapor pressure and cometary activity, and ice sublimation on the ejecta created by the impact. From the observed radiation pressure effects, which were small but not neg-
ligible, Richardson (2007) predicted a mean particle size of 6 – 12\(\mu\)m for the slower particles and 0.1 – 2\(\mu\)m for the high velocity particles. Despite the large, opaque cloud of debris, the gas levels of Tempel 1 returned to their original levels within 24 hr of the impact event and dust dissipated to a normal level after 48 hr (Keller et al. 2005, Küppers et al. 2005, Lisse et al. 2006, Schleicher et al. 2006).

The DART experiment was a technology demonstration of an approach to solving the challenge of an asteroid on a collision path with Earth. This approach was designated a “kinetic impact” in which a small mass with large momentum was set to impact the asteroid and deflect its course (Cheng et al. 2016) The stated technology goals of NASA’s DART mission included (Cheng et al. 2016): (I) Measure asteroid deflection to within 10%; (II) Return high resolution images of target prior to impact; (III) Autonomous guidance with proportional navigation to hit the center of a 150 meter target body. From here, the mission investigation team carefully mapped potential non-negligible effects of ejecta dynamics in complex dynamical environments. Determining these non-negligible effects is crucial for analyzing the momentum transfer factors in impact events and is the crux of the kinetic impact approach as an Earth impact mitigation strategy. Careful evaluation of short- and long-term ejecta dynamics was crucial for spacecraft health, optimized operations of the complementary real-time Earth-based observations and analysis of measurements from accompanying space assets (e.g. a proposed cubesat imager to visit and observe the target system several years after the DART experiment).
1.3 Binary Systems

The most recent count of asteroids and TNOs with satellites is approximately 490 as of February 2023 (Warner et al. 2009). This value has increased significantly over the past 10 years due to improvements in observational techniques that allow scientists to detect smaller objects. Studying binary asteroids is crucial to our understanding of solar system formation and asteroid dynamics. Without a satellite or second body, estimations of the bodies’ masses becomes significantly more difficult with greater room for error. An accurate estimation of the bodies’ masses is required to determine porosity, which affects the internal structure and forces that make up bodies and can cause changes in dynamical solutions (Ou et al. 2022).

Several formation mechanisms have been proposed for binary asteroid systems. One of the first suggested mechanisms was that binaries were formed through a collision that disrupted part of the primary body. Durda et al. (2004) proposed two main options for binary formation as a result of collision: Escaping Ejecta Binary systems (EEBs) and SMAshed Target Satellites (SMATS). EEBs are created when ejecta from an impact on a single asteroid is captured and coalesces into a satellite as the ejecta escapes (Durda et al. 2004). SMATS form from a disrupted asteroid that splits into a main body and a smaller satellite (Durda et al. 2004). Numerical models using smooth particle hydrodynamic (SPH) codes showed that re-accumulation and formation of a satellite is a very common outcome after the disruption of an asteroid (Michel et al. 2001, 2002, 2003, 2004, Durda et al. 2004, 2007). Typically EEBs and SMATS can be identified by the slowly rotating primary body with EEBs typically composed of two roughly equal-sized bodies and SMATS being characterized by a much smaller satellite (Durda et al. 2004).
The second binary formation mechanism that has been suggested involves the rotational disruption of an asteroid due to spinning beyond a limit that can sustain the integrity of the body. Bottke et al. (2002) proposed that YORP can rapidly increase the spin rate of asteroids to a critical limit at which the internal forces cannot sustain the asteroid’s shape. Binary systems formed through rotational disruption can be identified by the elongated shape of the primary body, the slowly rotating (often tidally locked) secondary, the highly elliptical secondary orbit, and the near-critical rotation period (3-6 hr) of the primary body (Walsh and Richardson 2006). Margot et al. (2002) suggested that only 16% of known binary systems formed in this manner while the rest are either EEBs or SMATS.

1.4 Knowledge Gaps

While much is known about ejecta dynamics in single body systems and more binary systems are being found with advances in technology, until the recent DART experiment, very little had been investigated regarding impact ejecta dynamics in binary systems. One of the first preliminary DART impact models by Yu et al. (2017) predicted a large cloud of ejecta particles that completely coated both the primary and secondary bodies over the course of two weeks. Schwartz et al. (2016) approached the DART impact from an N-body perspective rather than using an SPH code to determine the velocity of particles ejected at different angles from the crater. Initial SPH models (Jutzi and Michel 2014) suggested that high velocity particles were ejected at a lower ejection angle while slow particles were lofted straight off the surface. Ferrari and Lavagna (2018) discussed the complexity of Didymos as a binary system and suggested that the latitude and longitude of the impact event strongly influences the outcome of the resulting ejecta.
Recent observations and analysis of the DART impact concluded that the Didymos system is composed of two S-type asteroids with very rocky surfaces (Daly et al. 2023). The rockiness of the impact site may have influenced the formation of the crater in ways that were not predicted by initial models (Daly et al. 2023). The impact on Dimorphos was significantly more effective at changing its orbital period than was initially predicted. While the initial goal was to change the orbital period by -7 minutes, the final measured change in period was $-33.0 \pm 1.0$ minutes due potentially to the additional momentum transfer from the ejection of debris (Thomas et al. 2023). Additionally, the DART mission observed the first formation of a tail from an asteroid due to an impact event (Li et al. 2023). Previously, debris tails from asteroids had only been observed on active asteroids where the activity may or may not have originated from an impact event (Jewitt and Hsieh 2022).

The significant changes in momentum caused by the ejecta and the resulting debris tail after the DART impact prove just how little we know about debris interactions with binary small body systems. The Didymos system is the only impact on a binary system for which we have experimental results, but this one system is not representative of the vast variety of binary systems in existence. To gain a more comprehensive understanding of debris interactions with binary systems, we require a numerical investigation into potential impact outcomes in a wide variety of system configurations.

With the use of our Rebound Ejecta Dynamics (RED) package described in chapter 2, we examined ejecta particles interacting in a wide variety of binary systems to determine the influence of rotational effects and the presence of a secondary body on the ejecta outcomes. We categorized trends in ejecta outcomes to study the specific characteristics of ejecta patterns on the surface and how they relate to the effects influencing the particle dynamics.
Over the next several chapters, I will introduce the RED package and its utilities (chp. 2), describe the utility of characterizing and categorizing surface ejecta patterns (chp. 3), and create a library of ejecta outcomes with varying rotation periods and mass ratios between the primary and secondary bodies (chp. 4). Finally, I discuss the implications of these studies and how they can be applied to future investigations.
CHAPTER 2: AN N-BODY APPROACH TO MODELING DEBRIS AND EJECTA OFF SMALL BODIES: IMPLEMENTATION AND APPLICATION

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2.1 Introduction

Ejecta off small bodies is a fundamental consequence of processes that activate and modify surfaces of those small bodies. Recent reviews of small body impact and ejecta processes include those by Scheeres et al. (2002), Housen and Holsapple (2011), and Jutzi et al. (2015). With OSIRIS-Rex’s discovery of frequent particle ejection events from asteroid Bennu (Chesley et al. 2020), and with active impact experiments such as those by Deep Impact (A’Hearn et al. 2005, Richardson 2007) and DART (Cheng et al. 2016), understanding the role of ejecta in small-body evolution has taken on added significant currency. It can be driven by internal processes, like sublimation, rotational fission or thermal fatigue, but is more commonly associated with external forcing applied during impact events. Studies of post-impact debris clouds benefit investigations into ring formation, par-
ticle size distributions, cometary jets, and ejecta distributions. Many numerical models of impact ejecta are based on scaling relations by Housen et al. (1983), Holsapple (1993), Richardson (2007), and Housen and Holsapple (2011). Scaling relations are a result of numerical models combined with experimental results. These relations are then used in other models, in order to include this process without having to calculate it explicitly.

Hydrodynamic models estimate the flow of material in a cloud as a gradient. An example of this method can be found in the Maxwell Z Model (Maxwell 1977), which estimates the flow of material out of a crater through streamlines. Computational models that implement a hydrodynamic approach, including iSALE-2D (Amsden et al. 1980, Wünnewann et al. 2006, Collins et al. 2004, Ivanov et al. 1997, Melosh et al. 1992), iSALE-3D (Elbeshhausen, D. and Wünnewann, K. and Collins, G. S. 2009, Elbeshhausen and Wünnewann 2011), and FLAG (Burton 1991, Caramana et al. 1998), focus primarily on the shock physics and fluid dynamics of the cratering processes. Richardson (2007) updates the analytic models by Housen et al. (1983) to improve on interactions near the crater rim. Richardson (2011) then applies these updated scaling laws to various 2D and 3D impact scenarios to model the flow of material from a crater to escape or back to the surface. The mathematical model by Richardson (2007) sets up an analytical foundation upon which a dynamical framework using tracer particles to follow material flow is built. By using tracer particles to follow material flow using the streamline method, Richardson (2011) analyzes how material is transported out of the impact crater and is deposited on the surface, creating a result similar to what is seen in N-body simulations.

These models are beneficial for determining the flow of material and gradient changes (e.g., temperature profiles, pressure, shock, etc.); however, hydrodynamic models lack the particle nature that can trace individual interactions within a cloud. While it is possible to include tracer particles that simulate a particle distribution, a full N-body integration can calculate the trajectories of individual particles outside of a grid or gradient. This is advantageous since it would allow
for a detailed investigation of how particles of different size and different properties behave after ejection, thus providing us more insight into the specific processes at play. In addition, while hydrodynamic models typically simulate the impact cratering event itself, we are primarily interested in the post-impact debris dynamics. This added flexibility means that N-body code can take as inputs any conditions immediately after the event that produced the ejecta in the first place.

Here we choose to use a new N-body code called Rebound (Rein and Liu 2012) to examine the evolution of ejecta clouds off small bodies. We call our implementation of Rebound the “Rebound Ejecta Dynamics package,” or “RED.” The Rebound python module is an N-body code with both symplectic and non-symplectic integrators as well as a newly developed hybrid integrator (similar to the hybrid integrator used by MERCURY (Chambers and Migliorini 1997), an N-body integrator that excels at large scale orbital evolution calculations) that switches between symplectic and non-symplectic integrators based on the a pre-defined condition of the system. The Rebound integrators have been validated with comparisons to other N-body integrators (Silburt et al. 2016) and the demonstration of a multitude of scenarios in the Rebound simulation archive (Rein and Tamayo 2017). We choose the Python Rebound module over other N-body integrators for this study due to its robustness and ability to carry out higher level computations at a lower performance cost than other N-body integrators currently being used. The two most commonly used N-body integrators for ejecta dynamics are SyMBA (Duncan et al. 1998) and pkdgrav (Stadel 2001), so we are building on earlier work that has successfully used N-body code in this physical context. Another popular N-body package capable of integrating complex systems is SyMBA (Duncan et al. 1998). While SyMBA does not have a specific hybrid integrator, it builds shells broken down into smaller and smaller time steps surrounding each particle in order to break down the interaction potential. This simulates a version of the Wisdom-Holman symplectic integrator (Wisdom and Holman 1991, Sil-
burt et al. 2016). Comparisons between the Rebound, MERCURY, and SyMBA hybrid integrators by Silburt et al. (2016) show that although Rebound took longer to run than SyMBA, it maintained the lowest errors in energy while SyMBA’s energy error grew rapidly. Rein and Tamayo (2017) does a thorough analysis of the Rebound WHFast, IAS15, and HERMES integrators and concludes that overall Rebound performs more accurately and at a lower performance cost.

2.1.1 Rebound benefits

As mentioned, Rebound (Rein and Tamayo 2015) offers the ability to calculate individual collisions between particles due to the N-body nature of the code. Hydrodynamic models struggle to efficiently include the effects of particle collisions since they are not comprised of individual particles.

Additionally, Rebound’s structure allows us to add physical effects as individual functions to better simulate specific ejecta scenarios. This creates a flexibility of RED that allows users to select which effects (see table 2.1) to include in order to simulate variations on a single impact or to simplify simulations to increase computational efficiency. Some scenarios would be greatly slowed by the addition of effects that have little influence on their outcomes; in these cases, the removal of such effects can improve calculation time without sacrificing accuracy. RED not only uses Rebound to build simulations, but also is a package of various effects to be implemented into Rebound for use with other projects outside of the ejecta dynamics application. Ultimately, RED is a Python package of various effects in the solar system that can improve the accuracy of any Rebound simulation; however, here we primarily focus on the implementation of RED and its application as an ejecta dynamics model using the DART impact as a benchmark system.
Table 2.1: List of effects implemented into this study of ejecta dynamics. The effects are listed in the order that they are implemented into the Rebound simulation. See figure 2.1 for a diagram of these effects. *Future work due to need to first establish basic particle activity in a system. Also, particle-particle collisions follow particle-in-a-box style interactions which differs from other effects implemented here.

<table>
<thead>
<tr>
<th>Order</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Develop basic model of particles being ejected</td>
</tr>
<tr>
<td>2</td>
<td>Determine size distribution of particles</td>
</tr>
<tr>
<td>3</td>
<td>Implement non-axisymmetric gravity and rotation</td>
</tr>
<tr>
<td>4</td>
<td>Implement radiation pressure effects (dependent on particle size &amp; material)</td>
</tr>
<tr>
<td>5</td>
<td>Apply to binary/triple systems</td>
</tr>
<tr>
<td>6*</td>
<td>Allow particle-particle interactions</td>
</tr>
</tbody>
</table>

Advances in numerical implementations and computational tools have improved the accuracy of ejecta dynamics models. Schwartz et al. (2016) compares analytic scaling laws, hydrocodes, and N-body integrators in order to determine the most efficient model for excavation of ejecta. This model uses the N-body integrator pkdgrav (Stadel 2001) to trace particle dynamics post-excavation. Many previous models do not consider additional effects such as solar radiation and planetary perturbations. Our approach examines a wide array of effects outlined in table 2.1. Figure 2.1 shows a sketch diagram of how these effects interact with each other and affect the particles’ motions.
Figure 2.1: A diagram of the effects implemented in this study. Arrows represent the directions of forces. Numbers next to the different effects correspond to the order in which the effects are implemented. See table 2.1 for descriptions of each effect.
2.1.2 **DART as a Computational Benchmark**

The AIDA/DART mission (Stickle et al. 2016, Schwartz et al. 2016) to impact a satellite on the binary moon of the Didymos system will help model ejecta distributions in a low gravity vacuum. Due to the close proximity of the Didymos system to the Earth and sun ($a = 1.64$ AU), the ejecta will be influenced by radiation pressure as well as gravitational effects due to perturbing planets and the non-spherical nature of Didymos and its moon. Initial numerical and computational modeling has been performed (Schwartz et al. 2016); however, more extensive models implementing a wider range of effects – as we are doing here with RED – will provide more accurate representations of the expected ejecta plume. These models can be used as predictive tools to determine the trajectories of post-impact debris clouds.

We are applying RED to the DART mission in support of preliminary impact modeling. The Didymos system can be used initially to benchmark RED and refine the added effects. Later, this package will produce a library of possible solutions given a set of initial conditions. Since we will not know the exact initial conditions of the impact until hours before impact, it would be impossible to run a full simulation with the exact set of initial impact conditions. Instead, once the impact conditions are known, we search the library for the solution produced by that particular set of initial conditions. This solution can then be compared to ground and cubesat observations, and we are able to determine the momentum transfer parameter, $\beta$, associated with that particular impact and period change. DART offers the opportunity to examine a real example of an impact on an asteroid to which we can apply RED to learn more about impact conditions on asteroids as well as determine the effects that most heavily influence different types of impacts. The N-body aspect of RED allows us to follow the debris cloud evolution on a small, individual particle scale as well as a larger, system-wide scale.
In this paper we apply the N-body code *Rebound* as we implement it as RED to examine the evolution of ejecta at short timescales no longer than a day or two. First, we describe how we implemented *Rebound* and six physical effects to study ejecta evolution. Then, we demonstrate several scenarios with the application of the physical effects. Finally, we discuss applications of this code and future developments.

2.2 Methodology

In this section we outline the set-up of our system as well as describe the various effects in the following subsections. Specific details of how each effect was implemented using *Rebound* and derivations can be found in Appendix A.

Before adding any additional effects or even setting up the system, we must define the coordinate frames used to describe particle positions and motion. The primary coordinate frame can be thought of as the universal frame. This is essentially the grid upon which everything in our system exists, and *Rebound* automatically initiates this coordinate frame when a simulation is first set up. It also simplifies calculations to input the target body at the origin of this system such that the semi-major axis, \(a\), lies along the x-axis, \(b\) is along the y-axis, and the semi-minor axis, \(c\), is along the z-axis (as seen in 2.2. All subsequent object positions, forces, and velocities are always input to and output from *Rebound* in terms of this universal coordinate frame; however, we are allowed to translate the universal frame into something a bit more comprehensible while calculating how the particles interact with the target body.
Since it is rare for impact scaling laws or observations to have data relative to only the very center of the target, we define a secondary, crater-centric coordinate frame that focuses on the particles’ positions relative to the initial impact site. This is primarily required for non-spherical, non-rotating target bodies since the surface features change over time in these cases and the forces experienced by the particles may change based on the positions of the particles relative to the surface. Similarly, mapping particles’ positions relative to the impact site (especially when the impact site is on a rotating body) has more meaning to us than simply recording particle positions relative to the center of the target body.

To convert from the universal coordinate frame to this crater-centric coordinate frame, rather than moving the target itself, we create the illusion of the target moving by moving the universe around the fixed target. For example, a target rotating around the semi-minor axis, \( c \), requires calculating how much the target would have rotated by that time step, then rotating the positions of all the particles clockwise by the rotation amount. From this position relative to the surface, the particles will experience the same forces as they would have if the target body had rotated counter-clockwise and the particles had remained still. While the simplest solution seems to be to simply move the target body relative to the particles, this is in fact far more complicated and would require rotating and manipulating an entire gravitational potential when all that is required is simply where the particles are relative to the surface. Similarly, and this is most important with binary systems, if the ellipsoidal target is tilted with respect to the binary, both the particle positions as well as the binary’s position must be rotated with respect to the surface at each time step to account for the system being aligned with a universal coordinate frame. Therefore, at each time step it is absolutely essential to make sure the particles are correctly represented relative to the target’s surface.
Figure 2.2: The target body is placed at the origin of the system, which we define as the universal frame. A secondary local coordinate frame defines particle locations relative to the surface of the target body. For an ellipsoid, we initially align the semi-major axis, \( a \), with the universal \( x \)-axis. The ejecta cone is initially set up in a local, crater-centric frame before the particles’ position and velocity vectors are converted to the universal frame to be added into the \textit{Rebound} simulation.
The initial set-up of particles also requires a coordinate frame shift. Since *Rebound* only accepts positions and velocities in terms of the universal frame, we must set up the initial cone in a very local frame in which the particles make up a disk on the x-y plane that covers the entire transient crater. Therefore, to define the size of the initial disk, we use the scaling laws by Richardson (2007) for the volume of a crater to approximate the transient crater radius:

\[
V = \frac{1}{3} \pi r_{tc}^3
\]  

(2.1)

where \(V\) is the transient crater volume and \(r_{tc}\) is the radius of the transient crater. Here we make the same approximation as Richardson (2007) and assume that the crater depth, \(H\), is approximately \(\frac{1}{3}\) the diameter of the crater, which has been shown in impact experiments to be a reasonable approximation (Melosh 1989, Schmidt and Housen 1987). We then calculate the volume of the transient crater by in terms of the material properties of the target (\(t\)) and the impactor (\(i\)) (Richardson 2007):

\[
V = K_1 \left( \frac{m_i}{\rho_i} \right) \left[ \left( \frac{ga}{v_i^2} \right) \left( \frac{\rho_i}{\rho_t} \right)^{-\frac{1}{3}} + \left( \frac{\bar{Y}}{\rho_i v_i^2} \right)^{\frac{2+\mu}{2}} \right] - \frac{2\mu}{2+\mu}
\]  

(2.2)

where \(K_1, \mu,\) and \(\bar{Y}\) are properties of the target material. The first term of this expression defines the gravity regime while the second term defines the strength regime, which is based on the impacted material strength, \(\bar{Y}\). Since we assume a target strength of zero, the second term in this equation will also go to zero. By assuming a set of material properties (given by Holsapple (1993)) and impactor parameters (in this study we chose impactor parameters similar to the DART impact), we set 2.1 and 2.2 equal to each other and solve for the transient crater radius, which we will define as the initial particle disk radius:
This disk lies about 1 cm above the surface to prevent from being counted as particles already landed on the target’s surface.

Additionally, velocities are assigned to each particle based on the Richardson (2007) scaling laws:

\[ v_p(r) = \left[ v_e^2 - C_{vpg}^2 g r - C_{vps}^2 \bar{Y} \rho_t \right]^{\frac{1}{2}} \]  

where \( v_e \) is the effective velocity that does not go to zero at the transient crater radius (Housen et al. 1983), \( \bar{Y} \) is the target material strength, \( \rho_t \) is the density of the target body, and \( C_{vpg} \) and \( C_{vps} \) are constants for the gravity dominant regime and the strength regime respectively. Here we only consider the gravity dominated regime and assume a material strength of zero. Therefore, we only need to focus on defining \( C_{vpg} \) since the term including \( C_{vps} \) goes to zero (Richardson 2007):

\[ C_{vpg} = \frac{\sqrt{2}}{C_{Tg}} \left( \frac{\mu}{\mu + 1} \right) \]  

Note that constant \( C_{Tg} \) can be approximated to be \( K_{Tg} \). Both \( K_{Tg} \) and \( \mu \) are properties of the target material specified by Holsapple (1993); here we assume the target to be made of a material similar to sand so \( K_{Tg} = 0.5 \) (Housen and Holsapple 2011) and \( \mu = 0.41 \) (Holsapple 1993).

The core component of the velocity distribution is the effective velocity equation by Housen et al. (1983), which was later improved upon by Richardson (2007) to include the gravity and strength terms. This component is defined by Richardson (2007) as
\[ v_e(r) = C_{vpg} \sqrt{\frac{g R_g}{R_g}} \left( \frac{r}{R_g} \right)^{-\frac{1}{p}} \] (2.6)

where \( R_g \) is the transient crater radius and \( r \) is the radius at which the particle is initiated (this is between zero and \( R_g \)). We substitute equation 2.6 into equation 2.4 to determine a velocity distribution based on the material properties and impact parameters.

A version of this velocity distribution can be seen in figure 2.3, which depicts the effective velocity at different effective crater radii. The effective velocity and effective radius here are the velocities and radii for an impact similar to what will be seen for the DART impact divided by the highest velocity and the transient crater radius respectively. All particles are ejected at an angle of 45° relative to the local surface plane. While ejection angles tend to vary slightly according to experiments by Cintala et al. (1999), we simply chose to have a uniform ejection angle with varying velocities.

After assigning positions and velocities to the particles in the local crater frame, we translate the vectors into the universal frame to be added to the Rebound simulation. We use a 3D rotation matrix and the impact latitude and longitude to rotate the particle vectors from the crater frame to the universal frame. Since this conversion between the local frame and universal frame typically occurs at least once every time step in order to recalculate forces and additional effects, we include a specific 3D rotation matrix function in the python package for Rebound.
Figure 2.3: Normalized velocity distribution of particles based on radial distance from the center of the crater to the transient crater radius. Particles closer to the center of the ejecta plume exit the surface at a higher velocity than particles towards the outside of the plume. This particular scaling example is calculated based on an impact velocity of 6.6 km s$^{-1}$, which is the projected impact velocity for the DART impact (Cheng et al. 2016). Actual velocities for each simulation are calculated by multiplying this distribution by the escape velocity of the impacted body.
2.2.1 Gravitational Effects

Our initial model assumes all bodies are spherical. However, shape models of asteroids and other small bodies show that small bodies are not typically perfectly spherical; rather, they exhibit more ellipsoidal or “potato” shapes. Particles near the surface of an ellipsoidal body will experience a different gravitational force depending on their location relative to the body. Therefore, we implement an ellipsoidal gravitational potential for the target body as well as any potential secondary components. While other methods for calculating irregular gravitational fields exist (e.g. Werner (1994)), we chose the ellipsoidal gravitational potential as a way to minimize computational requirements while still maintaining a more accurate approximation than a sphere. In future development stages of the RED package, we plan to include more advanced methods of gravitational field mapping so that we can consider more complex shape models and examine the more subtle physical effects that may arise in such situations. We will implement it either through additional numerical modules or ingested look-up tables with interpolation. In general, complex shape models can be based on more detailed analysis and interpretation, such as those derived from data acquired during in-situ robotic missions to small bodies, or ground-based radar observations.

Next, we include the rotation of the target body as an additional velocity vector added to the particles near the target body. In order to produce this effect, we calculate the linear velocity of the impact point on the surface at time zero. For a body with a vertical axis of rotation, this will translate to a velocity vector in the x and y directions, but if the axis of rotation is tilted, a z component will be included in the vector. Similarly to how we set up the initial ejecta distribution, we begin by assuming that the axis of rotation is vertical along the z-axis. Then, we calculate the instantaneous velocity at time zero for the latitude and longitude of the impact site. Finally, the 3D rotation matrix function tilts the instantaneous velocity vector the amount that the rotation axis is tilted. Note: the direction in which the axis is tilted must be specified and included in
the rotation matrix, and is given by some angle between $0^\circ$ and $360^\circ$ in the xy-plane. Due to the nature of how this code is set up, no additional accelerations need to be added to the particles. As the particles leave the surface of the body, they will also have an added velocity due to the instantaneous rotation velocity at the point at which the particles left the body. Once the particles are off the surface, they are only affected by the gravitational field (and any radiation pressure effects) and no longer influenced directly by the rotation of the target body.

Finally, including other bodies orbiting the target body introduces an additional gravitational acceleration on the particles. Orbital parameters, mass, and radius of the binary are input in relation to the target body; therefore, if two bodies of similar mass orbit their center of mass, we will observe this system from the perspective of one of the bodies rather than from the perspective of the center of mass. When this secondary body is added to the system, it is placed in orbit around the target body.

2.2.2 Effects Dependent on Size Distribution

In addition to gravitational effects, we include effects dependent on particle size, such as radiation pressure. We use a power law distribution similar to O’Brien and Greenberg (2003) to determine the radius of each particle before they are added to the simulation:

\[
dN = C r_p^{-p} dr_p
\] (2.7)
Figure 2.4: The ratio of acceleration due to radiation pressure to acceleration due to gravity for varying particle sizes being ejected from a 100m radius circular asteroid. Each line represents the ratio at a certain distance from the surface of the body, the bottom line representing 0m from the surface and the top line indicating 400m from the surface.

where $C$ is a constant, $N$ is the number of occurrences of a particle with radius $r_p$ and $p$ is the power of the distribution. In our simulations we chose to use $p = 3$, which is consistent with other size distributions where $2 < p < 4$ (O’Brien and Greenberg 2003). By integrating equation 2.7 between chosen minimum and maximum radii, we determine the number of particles for each radius. Since equation 2.7 only provides the continuous solution, a certain resolution between the minimum and maximum radii must be selected. The resolution describes the number of desired radii between the minimum and maximum radii, providing an incremental solution that can be applied to the particles.
themselves. For these simulations, we use a range of radii $10^{-4}$ cm $< r_p < 1$ cm and a resolution of 100. Each particle is then assigned to a bin corresponding to a certain radius. Equation 2.7 defines how many particles can be assigned to each bin. We chose to use 100 bins for these simulations because for a simulation of $10^4$ particles, using more bins makes it more difficult to distinguish between distinct particle size groups and the effects on these groups, and using fewer bins leads to too low a resolution to see distinct effects.

While the particle sizes do not have a large effect on gravitational forces, radiation pressure from the sun exerts a slight but perceivable force on smaller ejecta particles than the larger particles. Figure 2.4 shows the influence of the radiation pressure force relative to the gravitational acceleration felt by particles of varying radii. This effect is relative to the distance from the sun as well as the radius of the particle:

$$a = \frac{3\epsilon K_{sc}}{4r_p\rho c}$$

(2.8)

where $K_{sc}$ is the solar constant calculated at the target body’s distance from the sun, $r_p$ is the particle radius, $c$ is the speed of light, and $\epsilon$ is an absorption factor. For our purposes, we assign $\epsilon$ to be 1.5 as an average value for all particles. Derivations for equation 2.8 can be found in Appendix A. We also include a shadowing effect so that as particles pass "behind" a large body in the system, the radiation pressure goes zero. For this addition, we assign a cylinder with the radius of the object and assume that this cylinder goes to infinity from the object in the opposite direction of the sun. Any particle within this cylinder does not experience any additional radiation pressure forces.
Table 2.2: General parameters for Model A, a binary asteroid system modeled loosely based on the Didymos system. Model A1 represents the primary component and Model A2 represents the secondary component of the binary system. The orbital data for A2 are in reference to its orbit around A1 while the A1 data describe the system’s orbit about the sun. Data presented below courtesy of the JPL Horizons Database.

<table>
<thead>
<tr>
<th>System</th>
<th>Model A1</th>
<th>Model A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass [kg]</td>
<td>$4.56 \times 10^{11}$</td>
<td>$3 \times 10^9$</td>
</tr>
<tr>
<td>Radius [m]</td>
<td>400</td>
<td>75</td>
</tr>
<tr>
<td>Binary</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Orbital Radius</td>
<td>1.64 AU</td>
<td>1.2 km</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.38</td>
<td>-</td>
</tr>
<tr>
<td>Inclination</td>
<td>$3.41^\circ$</td>
<td>-</td>
</tr>
<tr>
<td>Lon. of Asc. Node (Ω)</td>
<td>73.21°</td>
<td>-</td>
</tr>
<tr>
<td>Arg. of Periapsis (ω)</td>
<td>319.30°</td>
<td>-</td>
</tr>
<tr>
<td>Esc. Vel. ($v_{esc}$) [m s$^{-1}$]</td>
<td>0.39</td>
<td>.07</td>
</tr>
</tbody>
</table>

The Cartesian unit vector of this acceleration is determined based on the unit vector from the sun to each individual particle. Finally the acceleration components can be added to the acceleration components for each of the particles. This force creates a drag on the particles that is proportional to the size of the particle. Similarly, particles beyond 2.5 AU from the sun feel little to no radiation effects from the sun. Therefore, in simulations that take place beyond 2.5 AU can neglect all radiation effects.

2.3 Results

To test the effects described in the previous section, we simulate a cloud interacting with a binary asteroid system (Model A) similar to the Didymos system. Information about the system parameters can be referenced in table 2.2. This section discusses the test scenarios used to demonstrate each of the additional effects. Initial set-ups for all simulations are described in table 2.3 where each column corresponds to a figure highlighting the progression of that simulation.
Table 2.3: All particles are initiated with the same basic ejecta cone parameters; however, we vary the additional effects applied to each simulation. Figure 2.5 represents the basic case with no additional effects. Figure 2.6 is the case in which the target is ellipsoidal with an axis ratio similar to the estimated ratio for Dimorphos. To demonstrate a larger extent of the ellipsoid’s gravitational influence, we chose a longitude of $45^\circ$ so that the ejecta would not lie directly on an ellipsoid axis. Figure 2.7 contains a target body that rotates around a vertical axis once every 6 hours. Figure 2.8 contains a binary system where the smaller object, A2, is the target and the larger, A1, orbits around the target. Finally, figure 2.10 employs a particle size distribution so that smaller particles are influenced by radiation pressure.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>2.5</th>
<th>2.6</th>
<th>2.7</th>
<th>2.8</th>
<th>2.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitude ($\theta'$)</td>
<td>$90^\circ$</td>
<td>$45^\circ$</td>
<td>$90^\circ$</td>
<td>$90^\circ$</td>
<td>$90^\circ$</td>
</tr>
<tr>
<td>Latitude ($\phi'$)</td>
<td>$0^\circ$</td>
<td>$0^\circ$</td>
<td>$0^\circ$</td>
<td>$0^\circ$</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>Target Axis Ratio</td>
<td>-</td>
<td>$1.82:1.36:1$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Rotation Per. [hr]</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Binary?</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Particle Radii [m]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$10^{-4}$ to 1</td>
</tr>
</tbody>
</table>

2.3.1 Gravitational Effects

First, we start with the simplest case involving no additional effects to ensure that the basic ejection of material is operating correctly before complicating the simulations with more complex effects. As shown in figure 2.5, the target body is spherical, so ejecta particles experience a consistent gravitational acceleration from a single body regardless of the initial longitude and latitude of the impact event. High velocity particles near the center of the impact are rapidly lofted far from the surface. The lower velocity particles towards the outside of the crater land close to the edge of the crater while leaving the impact site free of debris. Overall, we observe that 65.37% of particles land on the target body throughout the course of the simulation. Even with only a portion of all $10^4$ particles landing back on the body, the entire target body becomes covered in debris, primarily due to the relatively small target body size combined with the high impact velocity of 6.6 km s$^{-1}$ which allows particles to loft farther from the body. This simulation establishes a baseline against which all subsequent simulations with additional effects can be compared.
Figure 2.5: Initial test of basic ejecta plume with no additional effects on Model A2. Here we ignore any binary effects by removing A1 from the simulation. The first nine panels show snapshots of the x-y projection while the latter nine panels are snapshots of the x-z projection. Specific details regarding the exact initial parameters of this simulation can be found in table 2.1.
While in many situations it is realistic to use a spherical target body, often smaller bodies are more ellipsoidal; therefore, it is more reasonable to implement the ellipsoidal gravitational potential described in section 3.2.1 as an initial step towards more advanced gravitational potential implementations. In figure 2.6, we simulate the drift of particles due to the varying gravitational potential where the initial particle distribution is $45^\circ$ from the major axis of the ellipsoid and lies along the equator of the ellipsoid. The longer axis of the ellipsoid has a higher gravitational potential than the other axes of the ellipsoid and can be seen here, causing the cloud to drift towards the lobes with higher potential; however, this effect is slight and only noticeable close to the target surface. Farther from the body, the ellipsoidal potential appears closer to the point mass potential created by the spherical target body. In this scenario, 64.71% of the $10^4$ particles landed on the surface. Since this is similar to the amount of particles that landed in the no effects scenario, we determine that the ellipsoidal gravitational potential does not greatly affect particle trajectories in the long-term or the far-field, but the final distribution of landed particles may change due to surface variations.

In addition to an ellipsoidal potential, rotation of the target body alters near-field ejecta dynamics. As particles exit the surface, they retain the linear velocity caused by the rotation of A2 at the instant they left the surface, as shown in figure 2.7. High velocity particles are lofted at a much faster rate than the rotation velocity, so the ejection velocity component dominates over the rotational component. Slower particles experience a larger influence from the rotational component and therefore appear to continue rotating along with the body above the surface. While rotation does not largely affect the quantity of particles that fall back on the surface (in this scenario 64.26% of all particles landed on A2), the distribution of particles is greatly altered due to material “drifting”
Figure 2.6: A simulation of a cloud off of an ellipsoidal A2. Changing the ejection location on an ellipsoidal body changes the results due to particles being introduced to a completely new gravity field. Here particles are ejected from the equator at a longitude of 45°. The first nine panels show snapshots of the x-y projection while the latter nine panels are snapshots of the x-z projection. Specific details regarding the exact initial parameters of this simulation can be found in table 2.1.
off of the original impact site. In simulations with A2 as the target body, once again the majority of the surface is covered in debris; however, larger bodies or less energetic impact scenarios may lead to a final ejecta distribution that looks similar to the particle distribution caused by an oblique impact on a non-rotating object. Additionally, combining rotation with the ellipsoidal gravitational potential creates a gravitational potential that varies over time at a given point in space.

We designed the Model A system as a binary asteroid system in order to test the effects of a secondary component on an ejecta plume. Post-impact debris clouds in binary systems not only experience a gravitational influence from the target body, but they also are influenced by the secondary component. Particles that pass beyond the Roche limit of the target body leave the gravitational influence of the target body and may be influenced by the gravity of the secondary object, as seen in figure 2.8. In binary systems, the orientation of the impact with respect to the secondary object plays a crucial role in the final distribution of particles. Figure 2.8 demonstrates an impact on A2 90° from the position of body A1, where A1 orbits counterclockwise towards the impact site. Object A1 sweeps up the particles from the ejecta cone and pulls the particles around A2. Similar interactions occurred for the other three binary scenarios in which we altered the location of the impact site relative to A1, which lies on the positive x-axis. As seen in table 2.4 and figure 2.9 (a graphical representation of table 2.4), roughly the same amount of particles land on A2 regardless of the starting position; however, more particles land on A1 if the starting position near the start of the orbit (between 0° and 90°). Particles in the 0° and the 90° longitude scenarios eject particles directly onto A1 or immediately into the orbit of A1 to be swept up and do not have as much to disperse or fall back on A2 as in the 180° and 270° scenarios. Based on figure 2.9, most of the material landing on A1 has already done so by the time particles begin landing on A2, so there are less particles that have the chance of falling onto A2. Comparing to the no effects simulation, we note that the presence of a binary component as well as the location of the impact event relative to the binary component significantly affect the evolution of the ejecta plume.
Figure 2.7: Here target body A2 is spherical and rotates once every 6 hours about a vertical axis. Particles are given an additional rotational velocity incorporated into the ejection velocity at the initial set-up. The first nine panels show snapshots of the x-y projection while the latter nine panels are snapshots of the x-z projection. Specific details regarding the exact initial parameters of this simulation can be found in table 2.1.
Figure 2.8: Both A1 (denoted by large circle with hash marks) and A2 (denoted by solid circle) are assumed to be spherical with the ejecta exiting A2 $90^\circ$ from A1. The first nine panels show snapshots of the x-y projection while the latter nine panels are snapshots of the x-z projection. Specific details regarding the exact initial parameters of this simulation can be found in table 2.1.
Table 2.4: Percentages of particles that landed on A1 and A2 in the binary system simulations depicted in 2.8 (for the $90^\circ$ case) and 2.9. The impact site angle on A2 is defined by looking at A2 from above (cross-section of the xy-plane) where the positive x-axis is $0^\circ$ and the positive y-axis is $90^\circ$. For each scenario A1 begins at $0^\circ$ and orbits counterclockwise.

<table>
<thead>
<tr>
<th>Impact Site on A2</th>
<th>% on A1</th>
<th>% on A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>7.41</td>
<td>64.36</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>7.78</td>
<td>64.80</td>
</tr>
<tr>
<td>$180^\circ$</td>
<td>1.68</td>
<td>63.57</td>
</tr>
<tr>
<td>$270^\circ$</td>
<td>0.60</td>
<td>64.95</td>
</tr>
</tbody>
</table>

Figure 2.9: A certain percentage of particles in a binary system fall on the orbiting body (A1) while the rest fall back on the target body (A2). The darker line depicts the percentage of particles that fall back onto A2 at each time step, and the lighter line represents the percentage of particles that fall back on A1. The starting locations occur at the following longitudes: $0^\circ$ (top left), $90^\circ$ (top right), $180^\circ$ (bottom left), and $270^\circ$ (bottom right).
2.3.2 Effects Dependent on Size Distribution

Radiation pressure mostly affects the smaller particles in an ejecta plume; similarly, the target body must have an orbit close enough to the sun to experience the solar radiation pressure. Our Model B asteroid system can be classified as a Near Earth Object (NEO), orbiting the sun at approximately 1.6 AU, close enough to the sun to experience a fairly strong radiation pressure in comparison to objects with orbits outside 2.5 AU. Figure 2.10 provides an example of the particle drift due to radiation pressure pushing particles away from the sun. Here the particles are initially ejected on the side of A2 opposite the sun. Over time the particles drift slightly to align with this vector moving away from the sun. Particles behind A2 fall into its shadow where they experience no radiation pressure, so at first the radiation effects are not seen. However, as the particles leave the shadow, the ejecta cone narrows and drifts away from A2. While 62.58% of particles fall back on the surface (just slightly under the amount that fall back in the no effects scenario), only the shadowed side of A2 accumulates debris compared to the entire surface when there are no effects. Ejecta that normally would have drifted around the entire surface are prevented from even crossing over to the sun-ward side of the target. Similar to in the binary system case, the position of the impact site relative to the sun influences the plume evolution and final particle distribution.

The next step we intend to take in the development of this package is to include particle-particle interactions. While not as important in the later stages of ejecta plume evolution, inter-particle collisions play a crucial role in defining the particle size and velocity distributions during the early lofting stages of evolution. We anticipate that depending on the initial velocity regimes defined, the overall distribution will either experience an increase in particle size due to accretion (thus inhibiting radiation pressure effects) or particle size will decrease due to the break up of other particles during collisions (thus increasing radiation pressure effects). Further development of this function is necessary in able to demonstrate accuracy of these anticipated results.
Figure 2.10: The particle size distribution implemented here only plays a role in the particle trajectories when coupled with solar radiation pressure. We assume A2 to be spherical. The arrow labeled "Sun" points away from the sun, denoting the acceleration vector that the particles experience due to radiation pressure. The first nine panels show snapshots of the x-y projection while the latter nine panels are snapshots of the x-z projection. Specific details regarding the exact initial parameters of this simulation can be found in table 2.3.
2.4 Discussion and Future Work

Applications for the RED package range from modeling of ejecta to cometary outbursts to surface activity. As discussed previously, we are applying this model to the preliminary modeling of the DART impact. Since the exact impact parameters are not yet known and will not be known for certain until just before the impact, we are assembling a library of possible outcome solutions based on different impact conditions. Due to the shorter required computational time that Rebound provides, we are able to produce a large number of simulation outcomes (approximately 200 ejecta scenarios) much faster than other N-body codes can. Each simulation will have some assigned momentum transfer parameter, $\beta$, that corresponds to a set of initial conditions. Observations of the impact provide a final distribution from which we can reverse engineer a set of initial conditions. We can determine the corresponding $\beta$ associated with an impact of similar conditions by comparing the observations to our library of simulations.

In a future companion paper, we will introduce and benchmark the RED package’s particle-particle interaction suite. This function will calculate relative velocities of colliding particles. The velocity regime in which the collision resides determines the type of collision that occurs. This can be generally distributed into low, medium and high velocity regime, where the particles can bounce off in an elastic manner, disrupt and re-accumulate, or have a hybrid behavior, which is probably sensitive to specific collision conditions (impact angle, mass ratio, location in the gravitational potential of the asteroid system).
Additionally, this package can be applied to a wide range of other dynamical systems, such as cometary and Centaur outbursts. While this package does not specifically include dynamical effects from the gas, additional forces may be added to imitate the influence of the gas on the particles. Analyzing simulation results in the form of light curves allows for comparisons to outburst observations. Similarly, due to the bit-wise integration scheme built into Rebound, it is possible to input observed light curves then trace the outburst back in time to extract the initial conditions leading up to the outburst.

An open-source beta version of the Rebound ejecta dynamics package introduced in this paper has been released on GitHub.
CHAPTER 3: CATEGORIZATION OF SPATIAL AND TEMPORAL EJECTA OUTCOMES IN BINARY SYSTEMS BASED ON VARIATIONS OF THE DIDYMOS SYSTEM

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Submitted for publication to The Planetary Science Journal.

3.1 Introduction

In the recent past, the number of known binary asteroid systems has grown significantly due to improvements in observational technology and to an expanded network of asteroid monitoring stations. Current assessments suggest that about 235 Main Belt asteroids (MBA’s) and 93 Near Earth asteroids (NEA’s) are multiple body systems,¹ and approximately 12% of observed NEA’s between 2017 and 2019 were found to be binary asteroid systems (Virkki et al. 2022). Many more binary systems are hypothesized to exist (Margot et al. 2015) but simply may not be observable due to the separation of the system or the size of the satellite. These systems may hold keys to uncovering the dynamical history of our solar system since their formation mechanisms and their longevity are tied to various properties. Impact cratering records on small bodies provide some

¹For a catalog of asteroids with satellites, see the JPL ephemerides or https://www.johnstonsarchive.net/astro/asteroidmoons.html.
context for how binary systems formed and evolved over time. Similar to how the lunar cratering record informs us of events such as the Late Heavy Bombardment (Wetherill 1975), craters on small bodies tell a history of small body dynamics. For example, Greenberg et al. (1996) dated the surface of asteroid 243 Ida based on the size frequency distribution to be either a very old surface or a very young surface. The orbit of its moon, Dactyl, and an analysis of its cratering record indicates that both bodies formed around the same time from the same body (Greenberg et al. 1996). Additionally, Singer et al. (2019) and Spencer et al. (2020) examined the size frequency distribution of craters on Kuiper Belt object (KBO) (486958) Arrokoth to uncover the geologic history of KBO’s. Due to the primordial nature of the contact binary, Arrokoth, the cratering record on this body uncovers insight into the composition, origin, and evolution of primordial KBO’s (McKinnon et al. 2020). While Arrokoth provides information about ejecta evolution around a contact binary in the Kuiper Belt, most ejecta and cratering record studies in the main belt only examine evolution in single body systems. Holsapple (2022) collects a survey of observed main belt asteroid spin rates and their relation to cratering records in the main belt. Most impacts result in little to no significant influence on the system; however, on occasion an impactor will contribute enough kinetic energy to cause mass loss and/or changes in the spin rate. These outcomes are dictated by the ejecta velocity and mass as well as the strength of the impacted body.

The recent successful Double Asteroid Redirection Test (DART) (Cheng et al. 2016) allowed for the first time an in-situ investigation of the effects of a hyper-velocity impact on a binary asteroid system (the Didymos system made up of Didymos and Dimorphos). Leading up to the impact, extensive modeling examined potential outcomes based on the observed system as well as hypothetical changes to the system. Previously, ejecta dynamics missions only examined single body systems such as the Deep Impact experiments (A’Hearn et al. 2005, Richardson 2007) and the OSIRIS-REx investigation on Bennu, which discovered particle ejection from the surface (Chesley et al. 2020). Improvements in impact modeling capabilities and technology made by Scheeres
et al. (2002), Housen and Holsapple (2011), and Jutzi et al. (2015) have improved mission planning for retrieval missions and studies of resurfacing, cometary jets, surface activity, and ring formation. The Hayabusa2 spacecraft revealed the structure of asteroid (162173) Ryugu via an impact experiment coupled with modeling efforts. Based on the dynamics of the ejecta produced by the impact, Kadono et al. (2020) estimated the grain sizes, tensile strength of boulders, and hypothesized the subsurface structure of Ryugu. While these experiments have been immensely valuable for uncovering the dynamical history of asteroids, these studies only examined single body systems, and so far the DART investigation is the only experiment to study ejecta in a binary asteroid system. Therefore, a computational study into ejecta dynamics in a broader range of binary systems is worthwhile.

The Rebound Ejecta Dynamics package (RED) is a Python package created by Larson and Sarid (2021) using the N-body integrator Rebound (Rein and Liu 2012) to simulate post-impact debris clouds in small body systems. This package includes effects such as particle size distributions, solar radiation pressure, ellipsoidal gravitational potential, shape model gravitational potential, multiple system bodies, and rotational effects (Larson and Sarid 2021). Here we explore combinations of a subset of these effects to create variations on the Didymos system to simulate various binary asteroid system configurations. By varying system set-ups, we will produce unique ejecta surface distributions and cumulative distribution functions (CDFs) that determine the rate at which particles land on the surface. We note that for the current study we take the source of the ejecta to be the primary body, unlike in the DART event which had an impact on the smaller secondary. We predict that similar effects will produce similar trends in ejecta distributions and CDFs such that impact conditions can be predicted based on an image of an ejecta blanket. We expect combina-
ations of effect influences that produce unique distributions as well as cases in which certain effects dominate over others. Through sorting and categorization combined with statistical analysis and a large data set, we determine the association of each effect with types of ejecta blanket patterns. Using this information, one should be able to determine the history of an asteroid based on ejecta distribution patterns that do not match the current system configuration.

Our primary objective is to examine the interactions of ejecta with a binary asteroid system and to determine what information regarding the system conditions can be derived from the spacial and temporal patterns of ejecta that re-impact the primary and secondary bodies. Secondary objectives for this study include mapping surface patterns of ejecta under various conditions as well as the rate of resurfacing and to analyze the relationships between various parameters and the types of distributions that can be produced. We accomplish these objectives by applying the RED package to variations on the Didymos system to examine the effects of a binary asteroid system on ejecta dynamics. First, we describe the five physical parameters that we vary in our system and the statistical techniques used to analyze variations in ejecta distributions. Then, we analyze the simulated ejecta distributions and provide provenance maps of ejecta outcomes. Finally, we discuss the implications of our results as well as future investigations.

3.2 Methods

In this section we outline the set-up of the phase space we intend to examine. Specific details of the full phase space including all simulation parameters can be found in table 3.3. Our phase space covers a total of 72 simulations, along 5 parameters: 3 shapes, 3 locations of ejecta’s source, 2 separations between the components, 2 mass ratios of the components, and 2 target-body rotation periods. We describe each of these parameters below. While our specific, future goal is to eventually understand the particulars of the Didymos system and its post-DART impact environment, our
goal here is to instead sample a wider variety of parameter space that includes but is not limited to Didymos’s particular situation. Therefore our choices for sampling parameter space, described below, will not be limited to what we know about the Didymos system. We note that unlike in the DART experiment which impacted the secondary body, we are examining an impact on the primary body. All simulations include a binary asteroid system as well as radiation pressure due to the system’s close proximity to the sun (within 2 AU).

The limiting values for these simulations are by no means a full representation of the entire population of asteroids or small body binary systems; to cover such a phase space would require at the very least hundreds of simulations. We chose our system variations based on some general trends within the asteroid population. In the following subsections, we outline the parameters we chose to investigate and how these parameters compare to the general asteroid population.

3.2.1 Variations in Gravitational Effects

We examined the effects of the shape of the target body on the ejecta. We focus our efforts on examining differences between spherical and ellipsoidal target bodies, but also tested one more-realistic shape model case. Other variations in shape, such as craters and other larger surface features, require a more computationally-intensive calculation of gravitational acceleration calculated from the target body’s shape model. We include several tests with the shape model implementation for comparison to the ellipsoidal and spherical methods as a demonstration of how surface features affect gravity fields.
Larger asteroids such as Vesta (Thomas et al. 1996) and Pallas (Larson et al. 1983) are large enough to maintain roughly spherical structures with Hygeia (Barucci et al. 2002, Vernazza et al. 2020) as one of the smallest spherical bodies in the Main Belt. Very small objects (< 200m diameter) are observable via radio telescope, but are often too small to be able to produce an accurate shape model; the general shape of the object may be observable, but surface features are indistinguishable (Benner et al. 2015). In these cases, a simple ellipsoidal approximation is sufficient. Asteroids with a diameter smaller than 25km have a median axis ratio of $a/b = 1.6$ while larger asteroids with a diameter larger than 50km tend to be more spherical with an average axis ratio of $a/b = 1.23$ based on data collected by DAMIT (Cibulková et al. 2016, Durech et al. 2010). However, Cibulková et al. (2016) notes that these calculations are biased due to a greater number of elongated asteroids collected in the DAMIT database. McNeill et al. (2016) calculated that the average axis ratio for asteroids with diameters less than 8km is $a/b = 1.18$ based on an analysis of PanSTARRS data. Our chosen axis ratio of $a/b = 1.3$ is close enough to the average values to reasonably represent a standard asteroid.

Our first test case is using a simple spherical body. Both the primary and secondary are spherical with radii of 398m for the primary body and 369.2m or 85.7m for the secondary (depending on the primary to secondary mass ratio being tested in the simulation; see Section 2.4 below).

The next set of simulations is tested with ellipsoidal bodies. We base our primary and secondary axes ratios on the pre-encounter measurements that have been taken for Didymos and Dimorphos (estimated to be $1.3 > 1.2 > 1$ (Naidu et al. 2020)). Our target body is based roughly on Didymos with axes $a = 430\text{m}$, $b = 400\text{m}$, and $c = 370\text{m}$. For the cases when the primary mass is roughly equal to the secondary mass, the secondary has axes $a = 480\text{m}$, $b = 370\text{m}$, and $c = 308\text{m}$. When assuming a much larger primary body than secondary body, the secondary has axes $a = 110\text{m}$, $b = 85.7\text{m}$, and $c = 70.8\text{m}$. Details of the exact implementation of the ellipsoidal gravitational potential can be found in Larson and Sarid (2021).
Finally, we implement the realistic shape models determined by radar observations. Since Dimorphos did not have a well constrained shape model before the encounter, we estimate the secondary body’s shape as an ellipsoid with the same axes as in the ellipsoid simulations. For the primary, we use the shape model created by Naidu et al. (2020) as an input file. To calculate the mass distribution throughout the body, we implement the mascon (mass concentration) layer model by Venditti (2013). We assume constant density throughout the interior of each object. The gravity field is determined by summing the gravitational acceleration caused by each mascon on a given point (in this case, each particle or body in the system). This net acceleration is applied to every particle or body at each time step as the system configuration changes.

3.2.2 Variations in Impact Location

It is useful in general to investigate how impact location on a body affects ejecta development. For instance, an impact event on one of the main lobes will experience a higher gravitational acceleration towards the impact site causing the ejecta to remain closer to the crater; however, an impact off of the primary semi-major axis between the lobes may result in the spreading of ejecta due to larger gravitational influences from either side of the body. Similarly, an impact event on the equator will evolve more rapidly into a ring about the body than if the impact were closer to the axis of rotation.

Since impacts can occur at any location on a body, as seen by the numerous craters at all longitudes and latitudes (Benner et al. 2015), but may produce different dynamical solutions depending on the impact location, we selected three locations (two on the equator and one off the equator) as a representative sample of what can generally be expected at different longitudes and latitudes. For each of our three target body shapes, we examine impact locations at (0°N, 0°E), (0°N, 90°E),
and (60°N, 0°E). For simulations with the spherical body, testing impact locations at the same latitude at various longitudes is redundant; therefore, we include the two equatorial impact sites for completeness. For the ellipsoidal and realistic shape model cases, the spin axis goes through the body’s shortest axis. A longitude of 0°E corresponds to the longitude of the largest axis, and a longitude of 90°E corresponds to the direction perpendicular to these other two axes.

3.2.3 Variations in Binary Separation

The separation of the primary and secondary bodies factors into how much ejecta interacts with the binary system. Here we vary the distance between the bodies to determine the degree to which this parameter affects the system. The separation is determined by some percentage of the Hill Sphere of the primary body in the system since systems become dynamically unstable beyond this limit (Ou et al. 2022). For our particular system, we estimate a Hill Sphere of 3.2km based on the mass of the primary and the mass of the sun (Chebotarev 1965). Using a percentage of the Hill Sphere allows for our results to be scaled to different sizes of systems rather than being applicable to only one specific system configuration. We set the closest separation at 25% of the Hill Sphere radius (0.8km) and the farthest separation at 80% of the Hill Sphere radius (2.56km). A large number of asteroids are contact binaries (0% Hill Sphere) and can be modeled using a shape model (if it exists) or two very closely orbiting ellipsoids; however, here we focus on non-contact binaries, so we save the contact binary case for a future study.
In addition to the binary separation, varying the mass ratio between the primary and secondary
\((m_p : m_s)\) influences how much the secondary interacts with the ejecta. Pravec et al. (2010)
observed that binary asteroids typically have a mass ratio less than 0.2, meaning that the secondary
body is less than 20% the mass of the primary body. Mass ratios greater than 50% (both bodies
are relatively similar in mass) are rare, but have been observed (Pravec et al. 2010). Therefore, we
test two extreme scenarios: \(m_s\) is 80% of \(m_p\) for the roughly similar mass case and \(m_s\) is 1% of
\(m_p\) for the case in which the secondary is much smaller than the primary. With a primary mass of
\(m_p = 5.27 \times 10^{11}\) kg, we set the secondary to have a mass of \(m_s = 4.22 \times 10^{11}\) kg for the roughly
similar mass case and \(m_s = 5.27 \times 10^9\) kg for the smaller mass case. The latter case in which the
secondary is much smaller than the primary is most similar to the Didymos system which has a
primary mass of \(1.3 \times 10^{11}\) kg and a secondary mass of \(1.2 \times 10^9\) kg based on an assumed density
of 2170 kg/m³ (Naidu et al. 2020).

3.2.5 Variations in Target Body Rotation Period

Here we test two variations on target body rotation period. Bodies larger than approximately 100m
in diameter typically do not have rotation periods faster than 2.2hr due to the instability of the
larger bodies’ structures at higher rotation rates (Stephens et al. 2010, Warner et al. 2011, Stephens
and Warner 2018). While some asteroids can have rotation periods as slow as 24+hr, Larson and
Sarid (2021) modeled a range of rotation periods from 2hr to 24hr and found that periods slower
than 12hr did not noticeably change the ejecta outcomes. Therefore, we set our upper limit for the rotation period to be 12hr. Our fast rotation case has a target body rotation period of 2 hours while in the slow rotation case, the target body rotates once every 12 hours. Slower rotation speeds in systems tend to have less of a flattening effect on cloud particles; therefore, we do not expect ejecta particles to spread out around the equator as rapidly as in the rapid rotation case.

3.2.6 Analysis Techniques

We analyze the 72 total simulations through provenance maps (spatial data) and cumulative distribution functions (temporal data). Provenance maps are created simply by recording the latitude and longitude of each landed particle location and plotting these points on a map of the surface with the impact site indicated by a star. This acts as a representation of what one could observe around a crater long after all the dust has cleared from the system; it is essentially a snapshot of the ejecta blanket. While provenance maps are good for determining where the ejecta goes, they tell us nothing about the rate at which particles landed on the surface. The cumulative distribution function (CDF) records the cumulative number of particles that have landed on the target body or the secondary body at each time step. We interpret the CDFs by plotting the total number of landed particles versus time in which the CDF slope indicates the rate of landing particles (steep slope indicates high rate of particles landing, shallow slope indicates few particles landing at each time step).

Next, we employ a closed card sorting technique (Spencer 2009, Pampoukidou and Katsanos 2021) to determine trends within the spatial data and the temporal data. Initially, we removed all identifiable data from the provenance maps and the CDF’s and the authors visually compared the patterns of each data set to determine observable trends. By grouping similar patterns together for both the provenance maps and the CDF’s, we identified five types of spacial ejecta distributions and eight.
temporal distributions. Other categorizations of these data are possible, including subcategories, but we found these particular categorizations to be most characteristic of the main patterns created by these simulations. Future studies may identify a more extensive categorization system, but our goal in this study is to set a baseline of possible categorizations and how they relate to observable data. The provenance map categories are as follows with examples of each category shown in Figure 3.1:

- **evenly spread** - particles cover the entire surface of the target body even if a higher density of particles exists around the impact site

- **local** - particles are clustered almost exclusively around the impact site and tend to not spread beyond $50^\circ$ from the impact site

- **equator** - a high density of particles is stretched across the equator spanning at least $90^\circ$ longitude and a very low density of particles at latitudes exceeding $30^\circ$ from the equator

- **hole opposite impact site** - a high density of particles centers around the impact site while $180^\circ$ from the impact site exists very few to no particles in a circular or ovular shape

- **cluster opposite impact site** - contains high density of particles around impact site as well as a high density of particles approximately $180^\circ$ from the impact site; other particles may be scattered elsewhere across the surface as long as there are two high density regions on opposite sides of the body

The CDF categories are as follows with examples of each category shown in Figure 3.2:

- **basic** - a standard-looking power law distribution representing a steady rate of particles landing on the surface; the most common category
Figure 3.1: Examples of each provenance map category showing ejecta on the target object (the red star signifies impact location): (a) evenly spread, (b) local, (c) equator, (d) hole opposite impact site, and (e) cluster opposite impact site. The circles in (d) and (e) respectively highlight the absence of particles and the cluster of particles that distinguish these categories.
• **flat basic** - similar to basic, but with a steeper initial slope suggesting that particles resurface at a much higher rate initially; some particles may land on the secondary body as well

• **extra long** - looks very similar to the basic category, however, particles land over a much longer timescale; very similar to the gentle slope category

• **gentle slope** - similar to extra long with the exception of a much shallower initial slope representing a much slower initial rate of particle resurfacing than in the extra long category

• **100%** - all particles land on the target body with none on the secondary; similar shape to flat basic, but with all particles landing on the target body

• **lots initially** - a more unrealistic scenario in which a high percentage of landed particles impacts the target body at the first time step and over time the rest of the landed particles accumulate on the target body

• **varying** - follows a very different distribution and often spans a longer timescale with particles landing on each body in stages

• **questionable** - represents a fairly unrealistic scenario in which a high percentage of landed particles immediately at the first time step while less than 20% of the remaining landed particles landed on the secondary body over the next couple time steps

To confirm the validity of our categorization, we asked five people (two graduate students in the planetary science program at University of Central Florida and three undergraduates in non-science majors) to sort first the provenance map note cards and then the CDF note cards into the designated categories based on patterns observed in the plots. By using some participants unfamiliar with planetary science, we ensured that the participants were sorting exclusively based on the shape of the ejecta distribution rather than introducing preconceived notions of surface processing and
Figure 3.2: Examples of each CDF category: (a) basic, (b) flat basic, (c) extra long, (d) gentle slope, (e) 100%, (f) lots initially, (g) varying, and (h) questionable. Note the different time scales along the x-axis. The dashed line on (c), (d), and (g) represents the timescale that the other plots span.
speculating on which effects may be in play. We again removed identifying information from the
data and explained the defining characteristics of each category to the participants. Figures 3.1
and 3.2 were used as examples of each category for participants to reference while sorting each
provenance map and CDF into categories. After sorting the CDFs and provenance maps into their
respective categories, we counted the number of occurrences that a provenance map/CDF sorted
into a particular category had a particular effect (e.g., the number of local provenance maps that
corresponded to a simulation with a 2 hour rotation period). We compiled 11 tables of counts: each
of the five effects vs the provenance map categories, each of the five effects vs the CDF categories,
and finally the provenance map categories vs CDF categories.

To compare the association between these parameters, we implement the Cramer’s V Test (Cramer
1946), a variation on the $\chi^2$ test that measures the association between two variables on a scale
of 0 (not at all associated) to 1 (completely associated). Below is an example of the Cramer’s V
calculation as applied to the mass ratio and provenance map data shown in table 3.1. First, we sort
each category into two piles: the $m_p = m_s$ cases and the $m_p > m_s$ cases. Next, the number of
cases in each pile is counted and recorded.

To calculate a $\chi^2$ value, we first must calculate an expected value. Since we are testing the as-
sociation of the categories to the effect and not the association of the affect to the category, we
use the total number of simulations sorted into each category ($tot_{cat}$) for the expected value ($ex$)
calculation:

$$ex = \frac{tot_{cat} \times count}{tot_{all}}$$

(3.1)
When testing for association with $\chi^2$, the null hypothesis states that there is no association between variables (i.e., observed and expected values are similar). In this case, a higher $\chi^2$ value leads to the rejection of the null hypothesis, implying the possibility of association between variables.

Here $count$ refers to the values in table 3.1 and $tot_{all}$ refers to the total number of simulations considered ($n = 71$ in this case since simulation number 25 did not experience particles landing on the target body and therefore is considered an outlier). Table 3.2 contains calculations of expected values as calculated using equation 3.1 and table 3.1. We use these expected values and our observed values ($obs$) to determine the $\chi^2$ value:

$$\chi^2 = \sum \frac{(obs - ex)^2}{ex} \quad (3.2)$$
Table 3.1: An example of the mass ratio categorization data. Here we display the number of simulations with either roughly equal primary and secondary masses or a greater primary mass is sorted into each of the provenance map categories.

<table>
<thead>
<tr>
<th></th>
<th>Equator</th>
<th>Local</th>
<th>Hole Opposite</th>
<th>Debris Opposite</th>
<th>Evenly Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_p = m_s$</td>
<td>6</td>
<td>14</td>
<td>9</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>$m_p &gt; m_s$</td>
<td>6</td>
<td>2</td>
<td>9</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 3.2: Expected values for the mass ratio provenance map categorization data are calculated using the data in Table 3.1 and equation 3.1. These values and those in Table 3.1 are used for $\chi^2$ calculations.

<table>
<thead>
<tr>
<th></th>
<th>Equator</th>
<th>Local</th>
<th>Hole Opposite</th>
<th>Debris Opposite</th>
<th>Evenly Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_p = m_s$</td>
<td>5.92</td>
<td>7.89</td>
<td>8.87</td>
<td>5.42</td>
<td>6.90</td>
</tr>
<tr>
<td>$m_p &gt; m_s$</td>
<td>6.08</td>
<td>8.11</td>
<td>9.13</td>
<td>5.58</td>
<td>7.10</td>
</tr>
</tbody>
</table>

For this data set, we calculate a $\chi^2$ of 17.5 with one degree of freedom ($df$, the smaller number of the two variable categories minus one). Figure 3.3 displays all $\chi^2$ calculated for our data. Note that most $\chi^2$ values are generally larger than what would be desired for a goodness-of-fit test. If we set a null hypothesis that there is no relationship between categories and effects (essentially, sorting is purely by chance), we can confidently reject that null hypothesis for this scenario. However, this does not mean that there is in fact a relationship between the categories and effects; it simply means that we know sorting was not by chance. The Cramer’s V test takes this test to the next level to determine if there is indeed an association between categories and effects. The Cramer’s V coefficient is then calculated as

$$C_{CV} = \sqrt{\left(\frac{\chi^2}{n}\right) \frac{1}{df}}$$

or in expanded terms,
\[ C_{CV} = \sqrt{\frac{\chi^2_n}{\min (c - 1, r - 1)}} \] (3.4)

where \( n \) is the total sample size, \( c \) is the number of columns in the data table and \( r \) is the number of rows in the data table. Similar to the \( \chi^2 \) test, the limits of association vary depending on the degrees of freedom; however, we only have either one or two degrees of freedom in our comparisons, and we found that the limits of association for these two degrees of freedom were similar enough that we could apply the levels for one degree of freedom to all cases without large changes in results and interpretation.

\( \chi^2 \) tests are very useful for both goodness of fit tests and association tests; however, this test can only accept or reject a null hypothesis and cannot necessarily prove a hypothesis. It is possible to use \( \chi^2 \) as a test of association and set a null hypothesis stating that the variables in question are not associated (sorting is random). To reject the null hypothesis at a significance level of \( p < 0.05 \) and to suggest an association between variables, a large \( \chi^2 \) statistic is required. The Cramer’s V coefficient takes the \( \chi^2 \) test to the next level to prove that two variables are associated rather than simply rejecting the idea that the variables are not associated. The \( \min (c - 1, r - 1) \) component of the coefficient calculation represents the degrees of freedom (df), which is used to help interpret the coefficients. Since we are examining the effects which have only two or three variations, the df will typically be 1 (mass ratio, rotation period, separation) or 2 (target shape, impact location). For the association test between provenance maps and CDFs the df is 4. We interpret our results based on a common scale in which

- \( 0.8 < C_{CV} < 1.0 \) signifies a very strong association
- \( 0.6 < C_{CV} < 0.8 \) signifies a strong association
- \( 0.4 < C_{CV} < 0.6 \) signifies a relatively strong association
• $0.2 < C_{CV} < 0.4$ signifies moderate association

• $0.1 < C_{CV} < 0.2$ signifies weak association

• $0.0 < C_{CV} < 0.1$ signifies negligible association

(Lee 2016). The closer the Cramer’s V coefficient is to 1, the higher the association is between variables. Similarly, the closer the Cramer’s V coefficient is to 0, the lower the association is between variables. We consider a moderate association to mean that while an effect has a visible influence on an ejecta outcome in some cases, it is very likely that that effect can be dominated by another stronger effect. Some effects are much more difficult to overcome by other influences; therefore, that particular outcome will have a high association with that effect. Other effects will dominate in very specific scenarios but will otherwise be secondary to stronger influences; in these cases, we can see a moderate association. Low to no association occurs when other effects almost always dominate over the effect under examination.

In the next section, we will apply these analysis techniques to the set of 72 simulations described above.

Table 3.3: The parameters of all 72 simulations are displayed here. The observed shape model (Obs.) in simulations 49-72 refers to the radar imagery of Didymos compiled by Naidu et al. (2020). The location coordinates are given as (degrees latitude, degrees longitude).

<table>
<thead>
<tr>
<th>Num.</th>
<th>Shape</th>
<th>Location</th>
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<th>Separation</th>
<th>Rotation</th>
<th>Map Cat.</th>
<th>CDF Cat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sphere</td>
<td>(0°N, 0°E)</td>
<td>$m_p = m_s$</td>
<td>25%$R_{Hill}$</td>
<td>2 hr</td>
<td>Equator</td>
<td>Questionable</td>
</tr>
<tr>
<td>2</td>
<td>Sphere</td>
<td>(0°N, 0°E)</td>
<td>$m_p = m_s$</td>
<td>25%$R_{Hill}$</td>
<td>12 hr</td>
<td>Equator</td>
<td>Questionable</td>
</tr>
<tr>
<td>3</td>
<td>Sphere</td>
<td>(0°N, 0°E)</td>
<td>$m_p = m_s$</td>
<td>80%$R_{Hill}$</td>
<td>2 hr</td>
<td>Local</td>
<td>Questionable</td>
</tr>
<tr>
<td>4</td>
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<td>$m_p = m_s$</td>
<td>80%$R_{Hill}$</td>
<td>12 hr</td>
<td>Hole Opp.</td>
<td>Lots Initially</td>
</tr>
<tr>
<td>5</td>
<td>Sphere</td>
<td>(0°N, 0°E)</td>
<td>$m_p &gt; m_s$</td>
<td>25%$R_{Hill}$</td>
<td>2 hr</td>
<td>Evenly Spread</td>
<td>Questionable</td>
</tr>
</tbody>
</table>
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<th>CDF Cat.</th>
</tr>
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<td>Hole Opp.</td>
<td>Lots Initially</td>
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<tr>
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<td>$m_p &gt; m_s$</td>
<td>80%$R_{Hill}$</td>
<td>2 hr</td>
<td>Evenly Spread</td>
<td>Questionable</td>
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<tr>
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<td>Cluster Opp.</td>
<td>Lots Initially</td>
</tr>
<tr>
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<td>Local</td>
<td>Varying</td>
</tr>
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<td>Local</td>
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<td>Basic</td>
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<td>Equator</td>
<td>Gentle Slope</td>
</tr>
<tr>
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<td>Gentle Slope</td>
</tr>
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<td>Hole Opp.</td>
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<td>Local</td>
<td>Lots Initially</td>
</tr>
<tr>
<td>18</td>
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<td>25%$R_{Hill}$</td>
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<tr>
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<td>Hole Opp.</td>
<td>Lots Initially</td>
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<tr>
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<td>Hole Opp.</td>
<td>Basic</td>
</tr>
<tr>
<td>23</td>
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<td>80%$R_{Hill}$</td>
<td>2 hr</td>
<td>Hole Opp.</td>
<td>Lots Initially</td>
</tr>
<tr>
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<td>$m_p &gt; m_s$</td>
<td>80%$R_{Hill}$</td>
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<td>Hole Opp.</td>
<td>Basic</td>
</tr>
<tr>
<td>25</td>
<td>Ellip.</td>
<td>(0°N, 0°E)</td>
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<td>2 hr</td>
<td>—</td>
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<tr>
<td>26</td>
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<td>Equator</td>
<td>Basic</td>
</tr>
<tr>
<td>27</td>
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<tr>
<td>28</td>
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<td>80%$R_{Hill}$</td>
<td>12 hr</td>
<td>Hole Opp.</td>
<td>Basic</td>
</tr>
</tbody>
</table>
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<tbody>
<tr>
<td>29</td>
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<td>Local</td>
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</tr>
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<td>32</td>
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<td>12 hr</td>
<td>Cluster Opp.</td>
<td>Basic</td>
</tr>
<tr>
<td>33</td>
<td>Ellip.</td>
<td>(0°N, 90°E)</td>
<td>$m_p = m_s$</td>
<td>25%$R_{Hill}$</td>
<td>2 hr</td>
<td>Local</td>
<td>Varying</td>
</tr>
<tr>
<td>34</td>
<td>Ellip.</td>
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<td>$m_p = m_s$</td>
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<td>12 hr</td>
<td>Local</td>
<td>Basic</td>
</tr>
<tr>
<td>35</td>
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<td>$m_p = m_s$</td>
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<td>Equator</td>
<td>Gentle Slope</td>
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<td>36</td>
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<td>(0°N, 90°E)</td>
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<td>12 hr</td>
<td>Evenly Spread</td>
<td>Basic</td>
</tr>
<tr>
<td>37</td>
<td>Ellip.</td>
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<td>$m_p &gt; m_s$</td>
<td>25%$R_{Hill}$</td>
<td>2 hr</td>
<td>Equator</td>
<td>Varying</td>
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<tr>
<td>38</td>
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<td>25%$R_{Hill}$</td>
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<td>Cluster Opp.</td>
<td>Basic</td>
</tr>
<tr>
<td>39</td>
<td>Ellip.</td>
<td>(0°N, 90°E)</td>
<td>$m_p &gt; m_s$</td>
<td>80%$R_{Hill}$</td>
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<td>Equator</td>
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<td>80%$R_{Hill}$</td>
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<td>Cluster Opp.</td>
<td>Basic</td>
</tr>
<tr>
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<td>Flat Basic</td>
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<td>47</td>
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<td>$m_p &gt; m_s$</td>
<td>80%$R_{Hill}$</td>
<td>2 hr</td>
<td>Evenly Spread</td>
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<td>48</td>
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<td>80%$R_{Hill}$</td>
<td>12 hr</td>
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<td>Basic</td>
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<td>49</td>
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<td>Evenly Spread</td>
<td>Questionable</td>
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<th>Rotation</th>
<th>Map Cat.</th>
<th>CDF Cat.</th>
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<td>Lots Initially</td>
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<tr>
<td>53</td>
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<td>Evenly Spread</td>
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<tr>
<td>54</td>
<td>Obs.</td>
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<td>25% $R_{Hill}$</td>
<td>12 hr</td>
<td>Cluster Opp.</td>
<td>Lots Initially</td>
</tr>
<tr>
<td>55</td>
<td>Obs.</td>
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<td>56</td>
<td>Obs.</td>
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<td>12 hr</td>
<td>Cluster Opp.</td>
<td>Lots Initially</td>
</tr>
<tr>
<td>57</td>
<td>Obs.</td>
<td>(0°N, 90°E)</td>
<td>$m_p = m_s$</td>
<td>25% $R_{Hill}$</td>
<td>2 hr</td>
<td>Local</td>
<td>Questionable</td>
</tr>
<tr>
<td>58</td>
<td>Obs.</td>
<td>(0°N, 90°E)</td>
<td>$m_p = m_s$</td>
<td>25% $R_{Hill}$</td>
<td>12 hr</td>
<td>Local</td>
<td>Basic</td>
</tr>
<tr>
<td>59</td>
<td>Obs.</td>
<td>(0°N, 90°E)</td>
<td>$m_p = m_s$</td>
<td>80% $R_{Hill}$</td>
<td>2 hr</td>
<td>Equator</td>
<td>Extra Long</td>
</tr>
<tr>
<td>60</td>
<td>Obs.</td>
<td>(0°N, 90°E)</td>
<td>$m_p = m_s$</td>
<td>80% $R_{Hill}$</td>
<td>12 hr</td>
<td>Cluster Opp.</td>
<td>Basic</td>
</tr>
<tr>
<td>61</td>
<td>Obs.</td>
<td>(0°N, 90°E)</td>
<td>$m_p &gt; m_s$</td>
<td>25% $R_{Hill}$</td>
<td>2 hr</td>
<td>Equator</td>
<td>Gentle Slope</td>
</tr>
<tr>
<td>62</td>
<td>Obs.</td>
<td>(0°N, 90°E)</td>
<td>$m_p &gt; m_s$</td>
<td>25% $R_{Hill}$</td>
<td>12 hr</td>
<td>Cluster Opp.</td>
<td>Basic</td>
</tr>
<tr>
<td>63</td>
<td>Obs.</td>
<td>(0°N, 90°E)</td>
<td>$m_p &gt; m_s$</td>
<td>80% $R_{Hill}$</td>
<td>2 hr</td>
<td>Equator</td>
<td>Extra Long</td>
</tr>
<tr>
<td>64</td>
<td>Obs.</td>
<td>(0°N, 90°E)</td>
<td>$m_p &gt; m_s$</td>
<td>80% $R_{Hill}$</td>
<td>12 hr</td>
<td>Cluster Opp.</td>
<td>Basic</td>
</tr>
<tr>
<td>65</td>
<td>Obs.</td>
<td>(60°N, 0°E)</td>
<td>$m_p = m_s$</td>
<td>25% $R_{Hill}$</td>
<td>2 hr</td>
<td>Hole Opp.</td>
<td>Lots Initially</td>
</tr>
<tr>
<td>66</td>
<td>Obs.</td>
<td>(60°N, 0°E)</td>
<td>$m_p = m_s$</td>
<td>25% $R_{Hill}$</td>
<td>12 hr</td>
<td>Local</td>
<td>Basic</td>
</tr>
<tr>
<td>67</td>
<td>Obs.</td>
<td>(60°N, 0°E)</td>
<td>$m_p = m_s$</td>
<td>80% $R_{Hill}$</td>
<td>2 hr</td>
<td>Evenly Spread</td>
<td>Lots Initially</td>
</tr>
<tr>
<td>68</td>
<td>Obs.</td>
<td>(60°N, 0°E)</td>
<td>$m_p = m_s$</td>
<td>80% $R_{Hill}$</td>
<td>12 hr</td>
<td>Hole Opp.</td>
<td>Basic</td>
</tr>
<tr>
<td>69</td>
<td>Obs.</td>
<td>(60°N, 0°E)</td>
<td>$m_p &gt; m_s$</td>
<td>25% $R_{Hill}$</td>
<td>2 hr</td>
<td>Hole Opp.</td>
<td>Lots Initially</td>
</tr>
<tr>
<td>70</td>
<td>Obs.</td>
<td>(60°N, 0°E)</td>
<td>$m_p &gt; m_s$</td>
<td>25% $R_{Hill}$</td>
<td>12 hr</td>
<td>Hole Opp.</td>
<td>Basic</td>
</tr>
<tr>
<td>71</td>
<td>Obs.</td>
<td>(60°N, 0°E)</td>
<td>$m_p &gt; m_s$</td>
<td>80% $R_{Hill}$</td>
<td>2 hr</td>
<td>Evenly Spread</td>
<td>Lots Initially</td>
</tr>
<tr>
<td>72</td>
<td>Obs.</td>
<td>(60°N, 0°E)</td>
<td>$m_p &gt; m_s$</td>
<td>80% $R_{Hill}$</td>
<td>12 hr</td>
<td>Evenly Spread</td>
<td>Basic</td>
</tr>
</tbody>
</table>
3.3 Results

Here we report the most notable results from our 72 simulations and analyze the association between effects and observations through the Cramer’s V test. From our sorting with the five participants, we were able to determine which simulations belonged in each of our categories. In 30 of the 72 cases, all participants sorted the simulations into the same categories. Participants disagreed slightly on 32 out of the 72 cases such that one or two of the participants sorted the simulation into a different category than the other participants. Only 10 cases had a more significant disagreement in which the simulations were sorted into into three different categories. To determine which categories to use for each simulation, we selected the most commonly sorted category since all simulations had at least three participants agreeing on one category.

The Cramer’s V test that we use on our sorting cannot directly tell us which effect produces which type of ejecta distribution, but it can confirm that our sorting method is not merely random and that the categories we determined can often be associated with certain parameters in the system. The calculated Cramer’s V coefficients for each effect are recorded in fig. 3.5, where blue bars represent an effect’s association with the CDFs and red bars represent an effect’s association with the ejecta distribution. The highest association can be seen between the rotation period and CDFs with the second strongest association seen between the impact location and CDFs. Most associations between the effects and either CDF or ejecta distribution are considered moderate associations. The mass ratio has the lowest association with CDFs and a fairly strong relationship to the final ejecta distribution. A test of association between the provenance maps and CDFs yielded a Cramer’s V coefficient of 0.5, meaning that a relatively strong association does exist between the spatial distribution of particles and the rate at which they impacted the surface.
While gravitational potential of a body may differ across a surface due to variations in density and/or shape of the body (e.g., additional lobes, an equatorial ridge, etc.), the most noticeable influence of these variations on the ejecta distributions can be seen near the surface and in the short term. The spherical target body created very similar provenance maps to the ellipsoidal target body impacts. The primary difference in provenance maps can be seen in the shape model target body simulations, which often exhibited a region opposite the impact site that experienced no resurfacing while the spherical and ellipsoidal target bodies exhibited more globally spread ejecta patterns. The CDFs for the shape model simulations and the spherical body simulations were similar for the most part, potentially due to the roughly spherical shape of Didymos that causes particles to land on the surface at a similar rate to a spherical body. Ellipsoidal body CDFs appear to be different from the spherical and shape model cases due to particles being swept up by the lobes on either side of the body. These results may vary systems with drastically different shape models for the target bodies, but for the most part, the target body shape played only a small role in influencing the ejecta distributions. Based on our Cramer’s V analysis, Figure 3.5 shows a weak association between ejecta distribution and target body shape (Cramer’s V Coefficient of roughly 0.3) while a moderate association exists between the CDF and the target body shape (Cramer’s V Coefficient of roughly 0.45).

In a few cases the separation between the primary and secondary had a significant impact on the outcome of the ejecta dynamics. When the separation is very close and the two bodies have approximately the same mass, the influence from the secondary body can sometimes overpower other effects to control the ejecta outcome, as seen in fig. 3.4. In this case, the only difference between these two simulations is the system separation, with fig. 3.4a (the close separation) showing a more localized distribution as the particles are influenced more by the secondary and fig. 3.4b (far separation) resulting in the particles spread along the equator due to the stronger influence of the rotation period. Since the separation only has a significant effect on the ejecta outcome if the
Figure 3.4: An example of a case in which the close separation \((25\% R_{Hill})\) shown in (a) dominates over the far separation \((80\% R_{Hill})\) shown in (b). Both simulations have the same parameters with varying system separation (spherical bodies, \(m_1 = m_2\), impact at \((0^\circ \text{N}, 90^\circ \text{E})\), 2 hr rotation period).

(a) A more localized ejecta distribution strongly influenced by the presence of the secondary body.
(b) Ejecta is spread along the equator due to the high rotation period.

masses are similar, we do not see any real changes in the CDF, i.e. we cannot say that the rate of resurfacing had been affected by the separation. Indeed, our analysis of the Cramer's V coefficient confirms that only a low to moderate association exists between the system separation and both the provenance map (coefficient of 0.36) as well as the CDF (coefficient of roughly 0.31). This association is most likely moderate primarily due to the cases in which both bodies have similar masses and therefore, have a stronger influence on the system. Otherwise, the separation alone is quite difficult to observe in both provenance maps and CDFs.
The mass ratio between the primary and secondary bodies determines how much of a presence the secondary body exerts in the system. The influence of the secondary becomes most noticeable when both bodies have roughly the same mass. In these cases, the distribution of particles becomes more global as the secondary pulls particles along its orbit and coats the entire surface as the secondary completes a full orbit of the target body. When the secondary’s mass is much smaller, its influence is not as significant and therefore, the final ejecta distribution will look more similar to a single asteroid system impact. The rate at which particles land is not significantly altered by simply the change in the mass ratio; rather, a combination of effects in combination with the mass ratio can alter the CDF. These results can also be seen in the Cramer’s V coefficients: the coefficient for the association between ejecta distribution and mass ratio is 0.5 (moderate to high association) and the coefficient for association between the CDF and mass ratio is 0.15 (very low association). From this we determine that it is possible to detect influences from a mass ratio in the observed provenance maps, but we cannot determine much regarding a system’s mass ratio from the CDF of an impact.

Variations in impact location change how particles experience the gravitational potential of the system. Outcomes for varying impact location are very dependent on other effects influencing the system. In cases with close separation between the bodies and equal masses, particles are highly influenced by the secondary body and impact the secondary’s surface soon after the impact event, particularly when the the impact site is directly beneath the secondary. Further separation and/or a smaller secondary body reduces the gravitational influence significantly and creates ejecta distributions and CDFs similar to a single body system. Particles ejected from a higher latitude are heavily influenced by the target body rotation period. The higher rotational frequency, and therefore increased angular momentum of the particles, causes the particles to drift towards the equatorial region of the system. In these cases, we observe provenance maps with globally distributed particles as well as particles spread exclusively across the equator, which is also the orbital
plane of the secondary. The rate at which particles land on the surface creates CDFs that rely on
the entire configuration of the system. When particles are ejected from a crater directly beneath
the secondary body, the CDFs reflect a higher rate of particles landing on the secondary body. The
Cramer’s V coefficient (roughly 0.42) for association between impact location and ejecta distribu-
tions reflects moderate association while the association between location and CDFs is very strong
with a coefficient of roughly 0.66.

Altering the rotation period produces the most noticeable changes in the ejecta distributions and
CDFs. The 12 hour rotation period is slow enough that results mimic the non-rotating bodies,
producing provenance maps with very localized distributions. In contrast, the 2 hour rotation
period creates a very rapidly rotating body that adds an additional rotational velocity to the ejecta
trajectories. For the fast rotations, most particles become spread across the equator of the target
body, even for high latitude impacts. Particles that drift around the secondary body eventually
form small ring-like structures that collapse onto the body around the secondary’s equator. Since
the 12 hour rotation period is most similar to the non-rotating case, the CDFs for the slow rotators
typically look most similar to the basic CDF shape and are primarily influenced by other effects.
The 2 hour rotation period CDFs differ from the 12 hour periods in that they tend to cover a
longer time period or have a much gentler slope (i.e., the particles take longer to land on the
surface). Based on the Cramer’s V test of association (shown in fig. 3.5), we found a very high
association between the rotation period and the CDF shape with a moderate to high association
between rotation period and the ejecta distribution seen in the provenance map.
Figure 3.5: A histogram of Cramer’s V coefficients to test association between each varied effect vs CDFs (blue) and each effect vs ejecta distributions (red). High levels of association can be seen between CDFs and rotation period as well as CDFs and impact location. Lowest association is found between the CDFs and mass ratio. The rest experience varying levels of association between the effects and CDF or ejecta distribution.

3.4 Discussion and Future Work

Distinguishing which effects dominate in a binary system requires many observations and simulations to examine and validate data. Our examination of 72 variations on a binary system offer insight into what information can be extracted from crater images. While the results presented here demonstrate an association between the chosen parameters and the categories into which the ejecta distributions were sorted, we cannot conclude that a particular ejecta distribution was dominated solely by any one specific effect. These simulations combine and layer different dimensions of system parameters; therefore, simulation outcomes are unique representations of that specific parameter set with the potential for one or two effects to very heavily dominate over others to produce a certain distribution. Our method of blind sorting with outside members reduced any bias
that could be created due to previous knowledge of impact cratering, but also led to occasional minor discrepancies in the categories into which simulations were sorted. Typically, sorting discrepancies arose when several dominant parameters influenced the system, causing the resulting ejecta distributions and CDF’s to maintain characteristics of more than one category. Additionally, our categorization scheme was created through the blind sorting process, but is not fully representative of all possible ejecta outcomes in a binary asteroid system. For a full representation of ejecta outcomes, we would need to simulate hundreds more systems with even more parameter variations. This study merely provides a baseline of expectations for investigations into ejecta dynamics outcomes in binary asteroid systems in which the primary body is impacted.

We observed that rotation period greatly influences the rate at which particles land on the surface as well as where the resurfacing occurs. Very few effects are able to single-handedly overcome the influence of a high rotation speed. A high enough gravitational potential in the system caused by the presence of a secondary body can overcome rotational effects, though this is rare and requires a combination of a close separation and a high secondary body mass. Close separation effects and mass ratio effects produce similar ejecta distributions due to the fact that both influence the gravity field in the system in similar ways. In observations of binary systems, changes in mass ratio and system separation may be difficult to see in distributions of ejecta. However, changes in the axis of rotation can be seen in observations of impacts at high latitudes. During impacts at high latitudes, especially in cases with high rotation speeds, particles drift towards the equatorial plane of the system and will land more along the equator of the body or be spread between the impact site and the equator. Finding a crater along the equator with a large spread of particles on one side of the crater, but differing from the down-range ejecta distribution of an oblique impact, could suggest that the target body rotation axis tilted since the time of the impact.
Additionally, a strong enough association exists between the ejecta distribution and the CDF (or rate of resurfacing) that it may be possible to extract information regarding the rate at which ejecta landed on the surface based on a crater image. Through these methods of association, we can recreate the conditions of these impact events in order to understand more about the dynamics in and around binary asteroid systems. Discrepancies between the current system configuration and the conditions predicted based on the impact record suggest dynamical evolution of the binary system. For example, observing an impact near the equator of a body with ejecta distributed all along the equator implies that the target body should have a high rotation speed; however, if we observe the target body to be rotating very slowly, we can infer that during the time between when the impact occurred and the time of observation something caused the target body to slow its rotation.

While the Didymos system provides a good standard for a binary system and allows for validation of hypotheses with the DART impact, the number of variations on this system is limited and cannot possibly cover the entire phase space of binary system configurations. In a future study, we will further examine the evolution of ejecta in binary systems by creating a library of system variations that will allow for predictions to be made regarding a system. Using this information, scientists will be able to predict an impact outcome based on the configuration of a system. In this way, observers can determine if changes have been made in the system over time based on observations of ejecta outcomes.
CHAPTER 4: THE FATE OF EJECTA IN NEA BINARY SYSTEMS: DEPENDENCY ON ROTATION PERIODS AND MASS RATIOS

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Excerpts of this text were published in 2023 LPSC abstract #2956.

4.1 Introduction

Surveys of the NEA population have revealed that over 15% of NEAs are binary systems (Pravec et al. 2006). Still, much is unknown about the dynamics and formation of these systems. Many models (e.g. Pravec and Harris (2007), Scheeres et al. (2002)) suggest the formation of binary asteroid systems through the process of rotational fission. During this process, a single asteroid is spun up beyond the critical rotation period of 2.2 hr (Richardson et al. 2002), causing the rotational forces to overcome the internal strength of the asteroid, breaking the asteroid into several components. Binary systems formed in this manner tend to have a rapidly rotating primary body (a residual of the formation mechanism) and a tidally locked, often more slowly rotating secondary body (Scheeres 2007).
The recent impact into the Didymos system for the Double Asteroid Redirection Test (DART) investigation (Cheng et al. 2016) provides an example of debris dynamics in a binary asteroid system; however, the Didymos system is not wholly representative of all binary NEA systems. The wide variety of NEA binary architectures and dynamical states suggests that impact ejecta can evolve in many ways. Therefore, we propose the creation of a library of impact outcomes for a variety of binary systems. Here we focus primarily on variations to the mass ratio between the two bodies and to the primary rotation period. In this abstract, we discuss the simulation configurations and analysis methods. Then, we present the results of our simulations and discuss implications of these outcomes.

4.2 Methods

To create a library of possible simulation outcomes, a large set of simulations are required. To make the problem more tractable, we selected seven different primary rotation periods and 10 system mass ratios, for a total of 70 simulations run using the Rebound Ejecta Dynamics (RED) package (Larson and Sarid 2021).

4.2.1 General System Configuration

The impact is initiated on the primary (larger) body at (22.5°N, 90°E), meaning the impact site is just above the primary’s equator and 90° along the secondary’s prograde orbit. Ejecta particles range in size from 1 μm to 1 m along a power distribution with a total of $10^4$ particles. Simulations are set to span approximately a week of simulation time with an integration time step of 30 s.
We initialize a binary system orbiting the sun at a distance of 1.5 AU, so we include solar radiation pressure acting on the particles. Both bodies in the system have a density of 2000 kg m$^{-3}$ (Scheeres et al. 2015) and are approximated to be spherical bodies, despite most asteroids being roughly prolate ellipsoids. In chapter 3 (Larson et al. 2023), we determined that for small asteroids, the shape matters most near the surface, otherwise the gravity can be approximated as a sphere or ellipsoid. The primary body has a fixed radius of 1km and a mass with a magnitude of $10^{13}$ kg (characteristic of many small binary systems) (Pravec et al. 2012). Since the impact is on the primary body, we vary the rotation period of this body and maintain a consistent rotation period for the secondary. Since we assume a tidally locked secondary, we calculate a rotation period of 18 hr, which is similar to observed binary NEAs (Pravec et al. 2012). The secondary body maintains an orbital distance of twice the diameter of the primary body, or 4 km (Pravec and Harris 2007, Johnston 2019).

### 4.2.2 Varying Parameters

We create 10 different mass ratios ranging logarithmically from $m_s = 10^{-3} m_p$ to $m_s = m_p$, where $m_s$ is the mass of the secondary and $m_p$ is the mass of the primary. Therefore, the secondary body has a mass with a magnitude ranging from $10^{10}$ kg to $10^{13}$ kg. As a result, we calculate secondary radii from 100 m to 1 km at fairly regular increments: $10^{-3}$, $2.2 \times 10^{-3}$, $5 \times 10^{-3}$, $10^{-2}$, $2.2 \times 10^{-2}$, $5 \times 10^{-2}$, $10^{-1}$, $2.2 \times 10^{-1}$, $5 \times 10^{-1}$, and 1.

For each mass ratio, we test seven different primary body rotation periods. We set the fastest rotation at 2.3 hr, slightly slower than the critical rotation period of 2.2 hr (Richardson et al. 2002). The slowest rotation period is set to 12 hr, since Larson and Sarid (2021) found that rotation slower than 12 hr does not produce significantly different ejecta outcomes. We increase the rotation period linearly by 1-2 hr so that we test rotation periods of 2.3 hr, 3 hr, 4 hr, 6 hr, 8 hr, 10 hr, and 12 hr.
4.2.3 Characterization of Impact Outcomes

To simplify the analysis of the simulation outcomes and to more easily compare outcomes, we define several metrics to describe the placement of ejecta. The main metrics we focus on are the percentage of particles that land on the primary and secondary, the percentage of each body’s surface that is covered by ejecta, and the distance of particles from the impact site on the primary. We calculate the percentage of particles that land on each surface simply by recording the total number of particles that landed at the end of each simulation and dividing this by the total number of particles input into each simulation ($10^4$). To calculate the percentage of each surface that was covered by ejecta, we first divide the surface into a grid with $2.5^\circ \times 2.5^\circ$ boxes. This grid was selected because smaller grid boxes risked larger particles covering more than one grid box and anything larger would have provided an inaccurate representation of surface coverage by individual particles. We then determine which grid boxes contain at least one particle and calculate the surface area covered by these grid boxes, making sure to account for the curvature of the body. This area is divided by the total surface area of the spherical body to find the percentage of the surface that is covered. The distance that particles land from the impact site is calculated not by using the distance formula, which provides a direct linear distance between two points in space, but by calculating the distance between the particle and impact site along the curved surface of the body. Using the distance formula would calculate a distance through the body rather than measuring how much distance along the surface a particle traveled. This distance is calculated for each landed particle by the following equation:

$$d = 2R \arcsin \left( \sqrt{\sin^2 \left( \frac{\Phi_p - \Phi_i}{2} \right) + \cos(\Phi_i) \cos(\Phi_p) \sin^2 \left( \frac{\lambda_p - \lambda_i}{2} \right)} \right)$$

(4.1)
where $R$ is the radius of the body, $\Phi_p$ and $\Phi_i$ are the latitudes of the particle and impact site respectively, and $\lambda_p$ and $\lambda_i$ are the longitudes of the particle and impact site respectively (Kells et al. 1940). We record the minimum and maximum distances at which particles land as well as the average distance for each simulation.

Each metric provides a single value associated with a given mass ratio and rotation period with which we can create heat maps associated with each metric. We interpolate the potential outcomes for system configurations that are in between our tested parameters using Python’s linear interpolation function. Using these interpolations, other researchers can look up dynamical outcomes for a given configuration of mass ratio and rotation period by finding the corresponding point on the heat maps. Additional comparisons of outcomes can be compared using provenance maps and cumulative distribution functions (CDFs) describing the rate of resurfacing.

### 4.3 Results

Through our simulations, we have discovered that the rotation period of the target body has the strongest influence on ejecta outcomes. In a high rotation system, particles tend to oscillate about the equator, creating a distribution of particles around the equatorial region of the primary. Particles do not depend on the longitude of the impact site to experience the effects of the rapidly rotating primary. Impact latitude influences the particles in cases of high latitude impacts. The rapid rotation favors the particles flattening into a ring-like formation and landing near the equator so particles from higher latitudes will migrate towards the equator before landing.
While rotation effects tend to dominate, there are exceptions in which the mass ratio has a stronger influence on the ejecta outcome. This most often occurs when the mass ratio is roughly 1 as shown in fig. 4.1. Figures 4.1a and 4.1b both have 2.3 hr rotation periods but with varying mass ratios. In fig. 4.1a, we notice that particles were limited in their evolution around the equator by the large mass of the secondary. Similarly, for fig. 4.1c and 4.1d which have a 12 hr rotation period, we observe particles spreading farther from the impact site when \( m_s = 10^{-3} m_p \) (fig. 4.1d) because particles are less bound by the secondary influence. Particles in fig. 4.1c remain local to the impact site because they are more strongly influenced by the secondary that is initiated at (0°N, 0°E).

These trends are evident in the particle distances, percentage of particles that land on the primary, and the percentage of surface coverage. Figure 4.2 shows that the average distance that particles land from the impact site decreases when the primary rotation period is greater than 4 hr. Additionally, particles do not land as far from the impact site as the mass ratio approaches 1 since more particles are lost from the system in cases with a greater mass ratio. A similar trend can be seen in fig. 4.3, which shows the minimum distance from the impact site that particles land. Particles land closer to the impact site at rotation periods greater than 4 hr; however, there is no obvious trend between the minimum particle distance and the mass ratio. Therefore, we determine that the rotation period plays a larger roll in determining the minimum and average distances from the impact site. This trend tends to break down when we analyze the maximum distance at which particles land as shown in fig. 4.4. Particles are limited in how far they can land from the impact site by the size of the body, so the largest distance possible for particles to travel is half the circumference of the body. Particles wrap around the body for rotation periods between 2.2 hr and 4 hr and the maximum distance decreases more gradually with increasing rotation period. Additionally, the maximum distance has a strong dependency on the mass ratio such that when \( m_s > 10^{-1} m_p \), particles travel greater distances even at greater rotation periods because of the additional influence from the secondary body pulling particles farther from the impact site.
Figure 4.1: Provenance map examples of the different impact outcomes with varying mass ratios and rotation periods. The red star represents the impact site. (a) The primary has a rotation period of 2.3 hr and a mass ratio of 1 ($m_s = m_p$). (b) The primary has a rotation period of 2.3 hr, but in this scenario the mass ratio is $10^{-3}$ such that $m_s = 10^{-3}m_p$. (c) Here we test a 12 hr rotation period with a mass ratio of 1 ($m_s = m_p$). (d) Finally, we simulate a 12 hr rotation period with a mass ratio of $10^{-3}$ ($m_s = 10^{-3}m_p$).

The percentage of particles that land on a body is very dependent on the impact conditions. Larger impacts (i.e., caused by a large or high velocity impactor) have produce a large amount of ejecta that is imparted with a lot of kinetic energy, so that particles are more likely to be ejected from a system or remain lofted for longer periods of time (Melosh 1984). However, as shown in fig. 4.5, the general trends of percentage of particles that land on the primary should scale such that
Figure 4.2: A representation of average landed particle distances from the impact site for each mass ratio and rotation period combination tested. Average distances for other combinations of mass ratios and rotation periods can be predicted by interpolating between our simulated results. Average distances vary on a scale from 300 m to 1300 m.

regardless of the impact conditions, a system with a fast rotating primary and a mass ratio near unity \( (m_s = m_p) \) will have a higher chance of ejecting particles than a system with the same rotation period and impact conditions but with a small mass ratio \( (m_s < 10^{-1} m_p) \). While in most cases, nearly 100% of particles landed on the primary, in cases of very fast rotation and especially a large mass ratio, a portion of particles \( (< 10\%) \) impacted on the secondary and in some very extreme cases a large portion of particles did not impact either body \( (< 50\%) \). This trend is further accentuated in fig. 4.6 when observing the percentage of the primary body’s surface that is covered by particles. The higher rotation periods tend to see a much greater percentage of the surface being
Figure 4.3: A representation of minimum landed particle distances from the impact site for each mass ratio and rotation period combination tested. Minimum distances for other combinations of mass ratios and rotation periods can be predicted by interpolating between our simulated results. Minimum distances vary on a scale from 0 m to 350 m.

covered by particles due to debris drifting into a disk-like orbit before re-impacting the surface. Similarly, as the mass ratio approaches unity, a larger portion of the surface is covered by particles due to the gravitational influence of the secondary body keeping the debris in orbit for a longer time. However, this trend has a limit in which the influence of the secondary body becomes too powerful and prevents particles from landing on the primary. Particles are either ejected from the system or find orbits about the system, causing a decrease in the percentage of the primary surface that is covered when $10^{-1}m_p < m_s < m_p$. 

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Figure 4.4: A representation of maximum landed particle distances from the impact site for each mass ratio and rotation period combination tested. Maximum distances for other combinations of mass ratios and rotation periods can be predicted by interpolating between our simulated results. Maximum distances vary on a scale from 900 m to 3100 m.
Figure 4.5: A representation of the percentage of particles that landed on the primary body for each mass ratio and rotation period combination tested. Percentages of landed particles for other combinations of mass ratios and rotation periods can be predicted by interpolating between our simulated results. Percentages vary on a scale from 50% to 100%.

4.4 Discussion

The library of impact outcomes presented here offers a unique glimpse into the effects of rotation and mass ratio on post-impact debris dynamics. This library can be used in conjunction with observations to determine the outcome of an impact in a system with an observed set of parameters. Similarly, these results can help determine the history of a binary system based on observations of an ejecta blanket. The ejecta outcome was produced under a given set of conditions that can be determined from this data set. Discrepancies between the observed system conditions and the simulated impact conditions could indicate a change in the system’s dynamical history.
Figure 4.6: A representation of the percentage of the primary surface covered by particles for each mass ratio and rotation period combination tested. Percentages of surface coverage for other combinations of mass ratios and rotation periods can be predicted by interpolating between our simulated results. Percentages vary on a scale from 9% to 35%.

While the results presented here offer an exceptional glimpse into possible outcomes for impact scenarios in binary systems, only two of the influencing effects are examined in this study. To gain a more comprehensive understanding of debris outcomes, we must expand the library to encompass a larger selection of binary asteroid configurations. Future studies should include effects such as system separation, impact location, and impact type.
Investigations into ejecta outcomes in binary systems are still relatively new to the field of impact sciences due to the more recent discoveries of so many binary systems within the past 10 years. The work presented here starts to analyze some of the most basic binary system impact scenarios, from the creation of an N-body ejecta dynamics package to the start of an ejecta outcome library. The RED package provides an open source, Python compatible N-body approach to dynamical modeling of impact ejecta. The inclusion of effects such as radiation pressure, a particle size-frequency distribution, binary components, gravitational potential calculations based on shape models and ellipsoidal harmonics to simulate complex target bodies, and rotational effects, increases the applicability of the package to a wide variety of small body systems at a lower performance cost than other N-body ejecta dynamics codes. Using this tool, we simulated 72 binary systems with varying mass ratios, separations, rotation periods, target body shapes, and impact locations and determined that rotational effects as well as location of impact dominate most ejecta outcomes. However, in equal mass binary systems the secondary body’s gravity dominates the ejecta outcomes. We expanded on this study to determine the limit at which the outcomes transition from being rotation dominated to being dominated by the secondary. Rotation periods faster than 4 hr and a mass ratio greater than 0.5 experience significantly less resurfacing due to particles getting trapped in the system. Additionally, primaries with rotation periods faster than 6 hr experience at least 20% surface coverage and have ejecta spread around most of the equator rather than forming a localized blanket around the impact site as is the case with much slower rotating primaries. Overall, cases with a primary rotation period slower than 6 hr experience a dominant influence from the secondary’s gravitational potential, the strength of which is directly related to the mass ratio of the secondary to primary.
5.1 RED Package

While the current version of the Rebound Ejecta Dynamics package contains a very useful set of additional effects, there are still several additions that would ensure even more accurate simulations and the ability to model an even greater variety of systems. Currently, only ejecta from direct impacts can be simulated, but many impacts occur at more oblique angles, causing particles to be ejected in a down-range trajectory. Implementing oblique impacts into the RED package would greatly increase the variety and complexity of the impact scenarios modeled with this package. Additionally, interactions between particles have the potential to change the outcome of ejecta trajectories, which can influence resurfacing patterns. One of the benefits to creating an ejecta dynamics package using an N-body integrator is the ability to simulate individual particle trajectories. As such, RED has the potential to facilitate collisions between particles that can lead to changes in particle size distributions due to the aggregation of particles and particle trajectories due to particles bouncing off of each other. The outcomes of collisions between particles relies heavily on the porosity and velocity of each particle involved.

5.2 Ejecta Outcome Library

Expanding the library of ejecta outcomes is crucial for understanding ejecta dynamics and resurfacing in binary systems. Combinations of effects such as variations in system separation, impact parameters, and target body composition and shape all play strategic roles in impact outcomes. As we observe more asteroid systems, we can start to include more complex effects, such as tumbling asteroids rather than just spinning. Determining other categorization schemes than those outlined in chapter 3 (Larson et al. 2023) can improve how we discuss and analyze ejecta outcomes. Certain effects and combinations of effects produce signature outcomes that, through the
categorization and comparison of simulation outcomes, can be more easily recognizable on small body surfaces. Many more simulations with a wider variety of system parameters as well as further statistical analysis are required to determine the specific resurfacing patterns caused by each effect; hence, the need for a more extensive library of ejecta outcomes than what was presented in chapter 4.

The expansion of the outcome library will allow us to compare even more of our simulations to observed systems, which may help us determine the evolution of binary systems. Recreating observed impact outcomes on small bodies can determine the effects in play at the time of impact. Discrepancies between the current day parameters and the parameters determined by the outcome library indicates the possibility of the system evolving since the time of impact. Observing and analyzing the remains of impact debris can give us insight into the dynamical evolution of small body binary systems.

5.3 Planetary Defense Applications

While the results presented here do not specifically apply to the Didymos system, this project was completed alongside the planning and execution of the DART investigation (Cheng et al. 2016), and many of our simulated binary systems started as variations of the Didymos system. Further investigations of binary systems will greatly assist in creating impact mitigation strategies. Should a hazardous object pose a real threat to Earth, understanding debris dynamics in a complex gravitational system with multiple bodies will allow us to model the appropriate impact required to deflect the object. Such a mitigation strategy risks resulting debris impacting Earth; therefore, modeling of impacts in a variety of multiple body systems can predict the safest method of asteroid deflection.
Most investigations into potentially hazardous objects (PHOs) focus on objects potentially impa-
tecting Earth; however, large enough impacts on the moon still pose a threat to Earth. Lunar and Martian meteorites prove that debris from impacts on the moon and Mars can travel to Earth and survive entering the atmosphere. Typically objects with a minimum absolute magnitude of 22.0 that pass within 0.05 AU of Earth are considered PHOs. Since the moon orbits the earth at $2 \times 10^{-3}$ AU, the limit for what is considered hazardous reaches far outside the earth-moon system. Although significantly larger than the binary asteroid systems considered in our studies, within the Hill Sphere of the earth-moon system, the mass ratio ($M_{\text{Moon}} = 10^{-2} M_{\text{Earth}}$) rather than the sizes of individual bodies dictates the dynamical outcomes of the ejecta. While Earth’s atmosphere protects the surface from most significant impact damage, the Chelyabinsk meteor was approximated to be 18m in diameter and impacted at $19.16 \pm 0.15$ km s$^{-1}$, causing a massive atmospheric blast that damaged a significant portion of the city and injured over one thousand people (Popova et al. 2013). Although highly unlikely that a boulder larger than 10m would be ejected at a high enough velocity to leave the moon’s gravitational influence to impact the earth in any significant way, a large enough impact on the moon does have the potential to bombard the earth with post-impact debris. Understanding how debris interacts with binary systems even on a planetary level can further inform us of what types of objects and trajectories should be considered hazardous.

---

1 Parameters defined in the JPL Small-Body Database
5.4 Understanding Solar System Origins through Impact Modeling

Examining the cratering history of a body tells us about the conditions that a body experienced throughout its history. However, many bodies in our solar system experience a variety of weathering processes that can modify the cratering record. The New Horizons flyby target Arrokoth orbits the sun at approximately 44 AU as a “cold classical” member of the Kuiper Belt and therefore has been preserved as a relatively non-weathered primordial body (McKinnon et al. 2020, Steckloff et al. 2021). New Horizons noted a large number of crater-like pits on Arrokoth’s surface, as well as a large crater (∼7 km on a ∼35 km long body) on its smaller lobe. While there has been some degradation of the pits over time (Schenk et al. 2021), the features presumably originate from impacts. These impacts dredged up material from the interior with slow moving ejecta coating the surface in minimally processed material. Thus these features set Arrokoth up as a prime target for uncovering clues about the composition and formation mechanisms in the primordial solar system.

The surface of Arrokoth shows regions of high albedo primarily in the lower-lying regions (such as in the craters and around the neck region) and areas of low albedo at higher altitudes (Stern et al. 2019). These regions also correspond to areas of less red surface material in the lower-lying regions and “ultra red” material in the higher regions (Spencer et al. 2020). Grundy et al. (2020) have found traces of methanol, water, and ammonia through spectral analysis of the surface. Modeling done by McKinnon et al. (2020) investigates the formation of Arrokoth through the collapse of a pebble cloud, causing both lobes to be composed of very spectrally similar materials. This formation mechanism is thought to form planetesimals in the wider primordial solar system, making Arrokoth a prime target for studying the conditions of the early solar system. Due to the
recent flyby in January 2019 and the long downlink times (the last of the observations made it back to Earth in late 2020) (Spencer et al. 2020), further investigations into Arrokoth’s history, both through data analysis as well as modeling, are essential and on the forefront of determining more about our solar system’s origins. Understanding the interior properties of such a primordial body would be particularly insightful.

Additionally, Arrokoth represents a very different type of binary system with a unique dynamical history: contact binaries. While for this project we examined only systems in which bodies were separated by at least 25% of the Hill Radius, contact binaries are connected on the surface at some point, so they often appear to be one body composed of two ellipsoidal lobes. Modeling the dynamics of debris in such a system proves difficult due to the complex surface morphology of a contact binary. While RED does have the capability to model gravitational potential using an input shape model, contact binaries can also be approximated as two smaller bodies orbiting each other with surfaces touching at one point. Analyzing the ejected debris from impacts on contact binaries can help us to determine the dynamical history and formation of contact binaries. The debris patterns between the lobes and across the surface indicate the dynamical conditions at the time of impact, allowing us to form a timeline of events in that region of the solar system.

Another application of the ejecta outcome library is to examine the placement of ejecta around the equator as a method of producing an equatorial ridge. An abundance of material around the equator to form an equatorial ridge is quite common amongst asteroids and binaries in particular (Margot et al. 2015, Walsh et al. 2015). Previously, equatorial ridges were thought to have formed through rapid rotation causing material from higher latitudes to slope towards the equator (Ostro et al. 2006, Walsh et al. 2008, Harris et al. 2009). Many of our higher rotation simulations discussed in chapter 4 produced ejecta patterns that centered primarily around the equator of the primary in a similar pattern to some equatorial ridges. It may be possible in some binary systems to form an equatorial ridge from repeated deposition of impact debris along the equator. In cases where the
ridge was formed through material sloping, the surface should appear to be uniform in age with a uniform crater size-frequency distribution. However, if the equatorial ridge formed through impact processes, the ridge would appear much younger than the rest of the surface and would have a much smaller crater size-frequency distribution due to older craters being covered by impact debris along the equator. Evidence for this method of ridge formation can be seen on asteroid (162173) Ryugu, which has a much bluer equatorial ridge than the rest of the surface (Morota et al. 2020). Ikeya and Hirata (2021) modeled different crater distributions on Ryugu and determined that ejecta deposition is a plausible mechanism for equatorial ridge formation. This study of Ryugu in addition to our models of impacts on rapidly rotating asteroids prove that impact ejecta may play a significant role in the formation of some equatorial ridges on asteroids.

5.4.1 Small Body Rings

Over the past decade, several bodies in the outer solar system have been observed to have debris rings. Chariklo, a $124 \pm 9$ km diameter Centaur object (Fornasier et al. 2013) orbiting the sun at 15.8 AU, was the first small body to be observed with rings (Braga-Ribas et al. 2014). The origin of these rings is still unknown, although several formation mechanisms have been hypothesized. Hyodo et al. (2016) proposed that the rings formed through tidal disruption as Chariklo made a close encounter with one of the giant planets. Other notable small bodies with observed rings are the trans-Neptunian object Haumea (Ortiz et al. 2017) and the Centaur Chiron (Ortiz et al. 2015, Ruprecht et al. 2015).
Recently, Morgado et al. (2023) discovered that the Kuiper Belt dwarf planet Quaoar is the third small body in our solar system to have an observed ring system. Much is still unknown about these rings and their dynamical history. These rings pose a peculiar challenge in dynamical modeling in that they are far outside the classical Roche Limit of Quaoar (Morgado et al. 2023). Typically, rings form within or around the Roche Limit due to the lack of gravitational influence beyond a body’s limit (Morgado et al. 2023). The additional gravity of Quaoar’s moon, Weywot, may contribute to the stability of the rings so far from the primary body. Collisions between ring particles also have the potential to increase the stability of a ring (Morgado et al. 2023), allowing the rings to exist outside of the Roche Limit. Simulations of the ring debris around the binary system will help to uncover the formation mechanism and stability of these rings.

Most likely, many other small bodies have rings, but we are simply unable to observe them at the moment. As observational technology improves, we are able to see smaller objects with higher resolution, improving our chances of discovering more small bodies with rings. Simulations of impacts on small bodies can help us predict objects and systems that are more likely than others to have a ring formed from ejecta trapped in orbit. A library of impact outcomes that also present cases in which particles form rings can assist in defining which properties and system configurations are most likely to form a ring. These predictions narrow down the field of possible ring candidates so that observers have a higher chance of finding rings around smaller bodies that we may not have considered to observe previously.
5.5 Conclusion

Numerical models allow us to perform experiments and test scenarios that we may never be able to experience in real life. While the DART mission and Deep Impact provide valuable data about impacts on small bodies, through numerical modeling we can expand upon these two experiments to investigate new system configurations and apply our knowledge of debris dynamics to new small body systems. The numerical solutions presented here are by no means a complete representation of every binary system; rather, they act as a stepping stone for future investigations and observations. Through comparison of ejecta blanket images to the outcomes presented here, future scientists can start to determine the dynamics of the system at the time of impact. A full library of impact outcomes beyond what is presented here will act as a map to untangle the dynamical history of small body systems.
APPENDIX A: MATHEMATICAL DERIVATIONS OF EFFECTS
Derivations of specific equations for each effect can be found here. Conceptual details and assumptions regarding values used in this study can be found in section 2.2. Similarly, specific implementations of effects using *Rebound* can be found in Appendix B.

A.1 Radiation Pressure Derivation

Here we derive equation 2.8 that is used to describe the acceleration due to radiation pressure. We start with the total solar energy absorbed by a particle:

\[ E_{total} = \epsilon K_{sc} \pi r^2 \]  

where \( \epsilon \) is an absorption factor (here we assume \( \epsilon = 1.5 \)), \( K_{sc} \) is the solar constant calculated at the target body’s distance from the sun, and \( r \) is the particle radius. The solar constant is defined such that \( K_{sc} = 1 \) at 1 AU; however, since the amount of solar radiation decreases farther from the sun, we calculate \( K_{sc} \) based on the target body’s location relative to the sun. For systems close to 1 AU it is acceptable to approximate the solar constant as \( K_{sc} \).

The power put out as a result of the solar radiation is described by

\[ P_{rad} = mac \]  

where \( m \) is the mass of the particle, \( a \) is the acceleration that the particle experiences and \( c \) is the speed of light. We set this equal to the total solar energy and solve for the acceleration, \( a \).

\[ a = \frac{\epsilon K_{sc} \pi r^2}{mc} \]  

(A.3)
To simplify this expression, we can solve for the particle mass, \( m = \frac{4}{3} \pi r^3 \rho \), based on some \( \rho \) that corresponds to the particle composition:

\[
a = \frac{3\epsilon K_sc \pi r^2}{4\pi r^3 \rho c}
\]  

(A.4)

Finally, simplifying this expression provides us with an acceleration expression in which the particle radius is the only varying quantity:

\[
a = \frac{3\epsilon K_sc}{4r \rho c}
\]  

(A.5)

### A.2 Ellipsoidal Gravitational Acceleration Derivation

In this section, we derive the expression for the gravitational acceleration applied to particles around an ellipsoidal body. To do this we examine the derivation by Hu (2017), starting with the basic definition of an ellipsoid:

\[
\frac{x'^2}{\lambda^2} + \frac{y'^2}{\lambda^2 - h^2} + \frac{z'^2}{\lambda^2 - k^2} = 1
\]  

(A.6)

where \( \lambda \) represents the first solution of equation A.6 and \( b = \sqrt{\lambda^2 - h^2} \) and \( c = \sqrt{\lambda^2 - k^2} \) are the semi-minor axes of the ellipsoid. Using this equation, we can calculate the semi-major axis of a reference ellipsoid proportional to the shape of the ellipsoidal body on which a particle is positioned. If the ellipsoidal body is denoted as ellipsoid 1 and the reference ellipsoid as ellipsoid 2, then \( b_2 = \left( \frac{a_2}{a_1} \right) b_1 \) and \( c_2 = \left( \frac{a_2}{a_1} \right) c_1 \). Given that \( a_2 = \lambda \), \( b_2 = \sqrt{\lambda^2 - h^2} \), and \( c_2 = \sqrt{\lambda^2 - k^2} \), then
\[
\frac{1}{a_2^2} x'^2 + \frac{a_1^2}{a_2^2 b_1^2} y'^2 + \frac{a_1^2}{a_2^2 c_1^2} z'^2 = 1 \quad \text{(A.7)}
\]

and

\[
a_2 = \sqrt{x'^2 + \frac{a_1^2}{b_1^2} y'^2 + \frac{a_1^2}{c_1^2} z'^2} \quad \text{(A.8)}
\]

Equation A.8 describes the semi-major axis of the reference ellipsoid on which a point \((x', y', z')\) rests in the local body coordinate frame. At this point we calculate the gravitational acceleration experienced at that point based on the gravitational potential. We define this potential using ellipsoidal harmonics to approximate the ellipsoidal nature of the body:

\[
V = \sum_{n=0}^{\infty} \sum_{m=1}^{2n+1} c_{nm} \frac{F_{nm}(\lambda_1)}{F_{nm}(a)} E_{nm}(\lambda_2) E_{nm}(\lambda_3) \quad \text{(A.9)}
\]

where \(c_{nm} = GM\) (\(M\) is the mass of the large body), \(E_{nm}\) is a Lamé function of the first kind, and \(F_{nm}\) is a Lamé function of the second kind. For this approximation, we expand the Lamé functions of the first kind to the first order where \(E_{nm}(\lambda_2), E_{nm}(\lambda_3) = 1\). Therefore, the following can also be derived from the definition of a Lamé function of the second kind given by Byerly (1893):

\[
F_{nm}(x) = (2m + 1) \int_{\lambda}^{\infty} \frac{dx}{\sqrt{(x^2 - h^2)^3}} \quad \text{(A.10)}
\]

Simplifying equation A.9 using equation A.10 results in the following expansion (Byerly 1893):
\[ V = M \int_{\lambda}^{\infty} \frac{dr}{\sqrt{(r^2 - h^2)(r^2 - k^2)}} \]  

(A.11)

where \( M \) is the mass of the target body, and \( r \) represents the semi-major axis of the reference ellipsoid given in equation A.7. Due to the dependency on the local particle position, the gradient is calculated in relation to the gradient of the semi-major axis.

\[
\begin{align*}
\frac{dV}{dx'} &= \frac{M}{\sqrt{(a_2^2-h^2)(a_2^2-k^2)}} \frac{da_2}{dx'} \\
\frac{dV}{dy'} &= \frac{M}{\sqrt{(a_2^2-h^2)(a_2^2-k^2)}} \frac{da_2}{dy'} \\
\frac{dV}{dz'} &= \frac{M}{\sqrt{(a_2^2-h^2)(a_2^2-k^2)}} \frac{da_2}{dz'}
\end{align*}
\]  

(A.12)

where

\[
\begin{align*}
\frac{da_2}{dx'} &= x' \left( x'^2 + \frac{a_1^2}{b_1^2} y'^2 + \frac{a_1^2}{c_1^2} z'^2 \right)^{-\frac{1}{2}} \\
\frac{da_2}{dy'} &= \frac{a_1^2}{b_1^2} y' \left( x'^2 + \frac{a_1^2}{b_1^2} y'^2 + \frac{a_1^2}{c_1^2} z'^2 \right)^{-\frac{1}{2}} \\
\frac{da_2}{dz'} &= \frac{a_1^2}{c_1^2} z' \left( x'^2 + \frac{a_1^2}{b_1^2} y'^2 + \frac{a_1^2}{c_1^2} z'^2 \right)^{-\frac{1}{2}}
\end{align*}
\]  

(A.13)

This acceleration is applied to each of the particles at their local locations. For testing purposes, we can examine this acceleration expression for a spherical body where \( h, k = 0 \) and \( a_1 = b_1 = c_1 \). Simplifying this expression assuming a spherical body does indeed produce the equation for the acceleration produced by a spherical body.
A.3 Definition of Coordinate Systems

In order to ensure the acceleration vectors are applied correctly in a system that moves in several directions (bodies orbiting, rotation along a tilted axis, particle interactions, etc.), we define two coordinate systems: a body-centric coordinate system (referred to hereafter as the local frame) and an ecliptic coordinate system (referred to hereafter as the global frame). While the general concepts of how these frames are related and how to switch between frames is referred to in section 2.2, here we describe the mathematical relations used to convert between the two frames.
APPENDIX B: COMPUTATIONAL KINKS AND PERFORMANCE SPECIFIC TO \textit{REBOUND}
B.1 Rebound Specific Implementation

While the Python version of Rebound is designed for straightforward implementation, developing such a small scale system involves several implementation tricks. An extensive description of specific implementations will be published with the Python package documentation.

For the implementation of ellipsoidal gravity, the default gravity built into the integrator must be eliminated. While it is possible to set the gravitational potential to ”none” in Rebound, this is not recommended since the integration scheme requires a mass and gravitational potential in order to perturb particles properly. Since the particles experience a net acceleration due to all contributing forces in the system, we can counteract the default spherical gravitational acceleration on the particles by simply adding in an equal acceleration acting in the opposite direction to the default acceleration. After the spherical acceleration is eliminated, the ellipsoidal gravitational acceleration can be added using the add forces routine built into Rebound.

B.2 Performance Evaluation

Since we chose Rebound due to its improved computational performance, we benchmark the computational performance of the ejecta dynamics package by recording the CPU and clock run-time requirements for each simulation. Figure B.1 depicts the increase in CPU requirements due to increasing number of particles. We determine that the maximum number of particles we are able to run in a single simulation is roughly $10^5$. Beyond this particle count, additional computational resources are required. A simulation of this magnitude requires approximately one week to run on one node composed of 24 cores. For simulations involving more than $10^5$ particles it is possible to run multiple simulations with $10^5$ particles and combine results. However, this method only works in simulations that do not involve particle-particle interactions. Due to the computational
Figure B.1: CPU run-time requirements for a basic Model A simulation with a varying number of particles and no additional effects.

performance level of the ejecta dynamics package in Rebound, we are able to produce libraries of possible solutions for impacts given varying initial conditions. We are able to produce approximately 20 simulations per week given available resources on the STOKES computational cluster at UCF; therefore, a solution library of 200 solutions with varying initial conditions will take roughly 8-10 weeks to run, accounting for variations in cluster availability and maintenance.


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