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An Examination of a Decade of K-5 Mathematics Standards in the United States

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AN EXAMINATION OF A DECADE OF K-5 MATHEMATICS STANDARDS IN THE UNITED STATES

by

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ABSTRACT

This qualitative content analysis research study examined changes made to K-5 state mathematics standards across the United States from 2012 to 2022. This study aimed to answer the research question: *In what ways, if any, do K-5 state mathematics standards differ from the CCSSM?* This was accomplished through four additional sub questions which include: (1) *In what ways, if any, do K-5 state mathematics student process and practice standards differ from the CCSSM?* (2) *In what ways, if any, do K-5 state mathematics standards content domains differ from the CCSSM?* (3) *In what ways, if any, do states describe how learning trajectories are addressed in K-5 state mathematics standards*? and (4) *In what ways, if any, is the relationship between procedural and conceptual learning outcomes represented in K-5 state mathematics standards*? Data collection included state mathematics standards documents and any publicly available relevant supporting documents found on state department of education websites. Of the 21 standard revisions from 15 states considered for the study, revisions from six states were selected for coding. From the coding process, themes were developed regarding patterns in changes that occurred in individual states' standards. The most prominent and common themes of changes included the addition of standards (e.g., personal finance, estimation, patterns, statistics, and probability), the merging of domains, the lack of specific evidence to the inclusion of learning trajectories in the development of revisions, and movement away from a balanced approach to learning outcomes. There were no consistencies in changes across all states that were coded. The results from this study can be used to promote consistency for future considerations for states that are revising their standards or to urge the reconvening of a writing committee for a revision of the Common Core State Standards for Mathematics.

I dedicate this dissertation to my husband, Erik Peter Schmidt, who continuously believes in and supports my life-long goals.

And to my parents, Jeff Todd Seegmiller and Michelle Marie Seegmiller, who always believe that I will achieve all that I set out to accomplish.

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I was blissfully unaware when I decided to embark on my PhD journey, that day-to-day life as we knew it would forever be altered. Working on my PhD during a pandemic presented its own unique challenges and rewards. I am eternally grateful for those who supported me while I worked on this dissertation study and uplifted me during my PhD program. This includes my mentor, dissertation committee members, professors, academic siblings, family, and friends who kept me grounded and focused while providing encouragement, guidance, and support.

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I served with on the FCTM board. It was through our conversations and friendships that I could push myself to hold the bigger picture of connecting teachers, students, and mathematics as a vision in my scholarly and instructional work.

Finally, I would like to thank my first principal, Shirley Williamson. You ignited a passion when you asked me if I would like to attend the NCCTM state mathematics conference in 2013. Our conversations about mathematics education in my beginning years of teaching led me to explore this path in academia. I would not be here without you recognizing what I could not see in myself all those years ago.

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LIST OF ACRONYMS

CCSSM	Common Core State Standards for Mathematics
CCSSO	Council of Chief State School Officers
CGI	Cognitively Guided Instruction
MOU	Memorandum of Understanding
MTP	Mathematics Teaching Practices
NAEP	National Assessment of Educational Progress
NCEE	National Center for Education Excellence
NCEST	National Council on Education Standards and Testing
NCLB	No Child Left Behind
NCTM	National Council of Teachers of Mathematics
NGA	National Governor's Association
NEGP	National Education Goals Panel
NRC	National Research Council
NSF	National Science Foundation
RME	Realistic Mathematics Education
RTTTI	Race to the Top Initiative
SMP	Standards for Mathematical Practice
SMSG	School Mathematics Study Group

CHAPTER ONE: INTRODUCTION

Researcher's Reflexivity

Acknowledging and examining personal and professional motivation for conducting research is an important component of any qualitative study. Thoughtfully considering my own experiences, assumptions, and beliefs within the larger scale of my research was ongoing prior to and during this study. I will refer to myself, the researcher, using first person verbs (I, my, etc.) throughout this study.

I feel impelled to share that I did not fall in love with mathematics until I was an adult, and it was my experiences during my adult years that affected the intentional decisions that were made during this study. It is significant to this study to disclose that my entire decade-long career as an educator in K-12 public schools had been since the release of the Common Core State Standards. This included my coursework as an undergraduate student, where I, as a preservice teacher, was learning the standards as in-service teachers were expecting the upcoming change to standards. This positioned me as an "expert" where veteran teachers were asking for interpretations of the standards and inquiring about alignment with the standards that were being implemented.

It was through my experiences as a K-12 educator that I concluded standards dictate what occurs in classrooms across the country each day. My experiences sparked my interest in educational policies. My progression in both my scholarly and professional career opened opportunities for me to engage in work with mathematics standards. I used my extensive knowledge and interest in standards to serve as a member of the K-5 expert team for the 2019 Florida standard revisions. Each domain was comprised of an expert team to review standards, resulting in five expert teams. I served on all five expert teams, which included: (1) K-5 number and operation in base-ten and counting cardinality, (2) operations and algebraic thinking, (3) geometry, (4) number and operations-fractions, and (5) measurement and data. This role naturally led to me having an extensive awareness of the changes made to Florida mathematics standards compared to other state standards prior to beginning the study. I have also served as a reviewer of instructional materials for mathematics adoption for the state of Florida and a committee member for the National Assessment of Educational Progress pilot study for achievement level descriptors. Each role heavily relied on a vast understanding of mathematics standards and continued to fuel my passion for standards.

Standards in education have also affected my personal life. I have been a military spouse for nearly five years across two duty stations in two different states. As a military spouse, I have seen the academic struggles of military children (and their families) when they transition between states with different educational standards. Not only has this affected those in my military family, but it is also soon going to affect my immediate family. Two of my brother in-laws are current active-duty military members in the Marine Corps and Air Force with multiple young children. One of my brothers-in-law recently received orders to be stationed in England, further complicating the educational opportunities of his children. This situation is a reality for the 1.6 million military children (United States Department of Defense, 2023) who face educational challenges associated with the transient lifestyle of their parents' career.

These culminating experiences led me to my belief that common standards across the United States ensure that those with transient lifestyles receive equitable educational opportunities; the same opportunities that those who live in the richest zip codes receive. This notion goes beyond the 1.6 million military children, and includes those displaced by natural disasters, war, rising housing market costs, and those hardest affected by current levels of inflation. It was through the belief that common standards provide the best educational support for those with transient lifestyles that I applied a macro lens to this study.

Introduction

Five words that will undeniably receive a reaction when mentioned to educators or parents alike are the entire premise of this dissertation study: The Common Core State Standards (CCSS). Current dynamics have highly politicized the implementation of the CCSS, resulting in misinformation among the public, although the standards initially served as a viable solution to many of the issues facing education in the United States. The current status of standards is messy, as a public display of growing disdain has resulted in a mass exodus of political and societal support. States have revised standards because of the exodus of support. As a field, we must thoughtfully consider how states are handling revisions being made that shift educators away from the CCSS and the impact this phenomenon has on ensuring equitable and high-quality mathematics learning opportunities for all children in the United States.

To discuss implications of state adjustments from the CCSS requires information regarding the formation of the CCSS. This study solely focuses on United States K-5 mathematics standards but providing an overview of K-12 education provides context and highlights the significance of this study. This chapter outlines an overview of what I will refer to as “The Common Core Era” in the United States. The chapter will conclude with a focus on the purpose, the research questions, and the significance of this study.

A K-12 education system with a set of national standards in the United States was once a vision of educational leaders (Confrey, 2007), dreamt of over the course of decades. Educational leaders were unsure, but hopeful, that political parties and those within the academic fields would be able to set aside their differences (Rothman, 2011) for this vision to come to fruition.

This vision of national standards faced numerous obstacles including the “math wars” (Schoenfeld, 2004), the politics associated with accountability in the realm of education (Woodward, 2004), and continual, sustained reform movements (Ravitch, 2010) spanning five decades (Greer, 2018). Additionally, state mathematics standards prior to 2009 varied in:

- rigor (Lavenia et al., 2015; Porter et al., 2011),
- depth (National Governors Association & Council of Chief State School Officers [NGA & CCSSO], 2010; National Research Council [NRC], 2001; Schmidt et al., 1997),
- consistency (Cobb & Jackson, 2011; Porter et al., 2011),
- number of standards per grade level, and
- grain size of specificity of learning expectations (Reys, 2006)

The state mathematics standards prior to 2009 also lacked cohesion (Cobb & Jackson, 2011) and coherence (Greer, 2018). This inevitably caused major issues with publishing companies as they attempted to match state standards for adoption purposes, further promoting the “mile wide, inch deep” (NGA & CCSSO, 2010) stigma associated with state standards prior to 2012, or what will be referred to as “pre-Common Core State Standards” throughout this study.

It was evident through calls to action from national educational organizations and agencies that the developed set of national mathematics standards, later to be known as the Common Core State Standards for Mathematics (CCSSM), needed to be well articulated for educators and educational stakeholders alike (Fisher et al., 1993; National Council of Teachers of Mathematics [NCTM], 2000, 2006; NRC, 2001). Transitioning away from vague, often shallow, state mathematics standards was crucial because standards that lack specificity and clarity “are nothing more than vacuous verbiage” (Ravitch, 2010, p. 20), leading to a checklist of standards to teach (Rothman, 2011) with little connection among mathematical ideas. The value of clarity and focus within mathematics standards cannot be lost because they serve as a guide for instruction (Rothman, 2011). Standards are linked to curricular choices made by schools, instructional activities implemented by educators, and assessments created by educators and publishers (Confrey, 2007).

In addition to standards being well articulated, the proposed idea of national standards also needed to demonstrate a focused and coherent relationship among mathematics topics (NCTM, 2000, 2004, 2006). Historically, state mathematics standards had provided the number of courses a student would need to graduate high school without providing guidance on *what* students needed to learn (Rothman, 2011) across grade bands, resulting in a disjointed sequence of instruction. This call for focus and coherence of mathematical content is evident through multiple board-driven National Council of Teachers of Mathematics (NCTM) publications including (1) *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), (2) *Principles and Standards for School Mathematics* (NCTM, 2000), and (3) *Curriculum Focal Points: A Quest for Coherence* (NCTM, 2006). Adding to the urgency of this call, The National Research Council (NRC) (2001), a nonprofit institution that offers scientific research-based advice under a congressional charter, provided guidance to improve mathematics learning for all students.

Calls to Action from National Educational Organizations

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) provided a set of standards as a response to improve the overall quality of mathematics education in grades K-12 driven by increasing power from other nations. Included in the report were both curriculum and evaluation standards that focused on problem-solving, communication, and reasoning (NCTM, 1989) with emphasis on instructional tasks as an aide for connecting mathematics across procedures and domains. The standards were organized by grade bands to guide schools through the creation of corresponding curricula (Rothman, 2011). The recommendations outlined in the report were not necessarily new ideas (Owens, 1988); however, the report called for an entirely different vision in mathematics education from what most students had ever experienced. The report consistently maintained that it was not a complete curriculum guide (NCTM, 1989) but instead a catalyst for change through a vision for mathematics education. Ultimately, the report served as a framework to guide reform in K-12 mathematics teaching and learning during the 1990s through the development of curricula, textbooks, resource materials, and evaluation criteria (Owens, 1988).

Eleven years later, NCTM (2000) published *Principles and Standards for School Mathematics,* which served as an updated version of the 1989 *Curriculum and Evaluation Standards* (Ferrini-Mundy, 2000). The 2000 Standards entailed a set of ten standards that described the mathematics that students should know and be able to do from Prekindergarten to twelfth grade (Rothman, 2011). The ten standards were broken up into five content areas (number and operations, algebra, geometry, measurement, and data analysis and probability) and five mathematical processes (problem-solving, reasoning and proof, communication, connections, and representation) (NCTM, 2000). Included within the standards were grade level expectations by grade band that described what the mathematical content and processes should look like in a classroom. The authors asserted that the publication served as a common language to provide a guide for “focused, sustained efforts to improve students’ school mathematics education” (NCTM, 2000, p. 5), which followed a very similar sentiment found initially in the *Curriculum and Evaluation Standards* (NCTM, 1989).

In their report, the NRC (2001) addressed the issues of public concern about incomplete student understanding of mathematics through discussion of mathematical proficiency. The committee acknowledged that no singular term could represent the all-encompassing aspects needed in mathematics instruction, but that the term *mathematical proficiency* defined what they thought it meant for “anyone to learn mathematics successfully” (NRC, 2001, p. 5). The culmination of their research resulted in the five strands of mathematical proficiency, which are all interwoven and interdependent of one another. The five strands (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) served as a framework for instructional decisions on whole numbers, rational numbers, algebra, measurement, geometry, and statistics and probability (NRC, 2001). Further, the report attested that the teaching and learning of mathematics for mathematical proficiency had considerations beyond the five strands, with interactions among and between students and teachers (NRC, 2001), serving as a factor in their implementation.

Continuing the call of improving both the teaching and learning of mathematics, NCTM published *Curriculum Focal Points* (NCTM, 2006) which asserted that a curriculum entails more than just a collection of activities; it must be focused, coherent, and well-articulated across specific grade levels moving the conversation to greater precision than the grade bands previously used. To ease the impact of varying state mathematics standards across the United States, the document outlined “curriculum focal points across K-8 that provided foundations for further mathematical learning” (p. 5). Each focal point explicitly related back to strands from *Principles and Standards for School Mathematics* (NCTM, 2000), further showing the connections between content and process standards. The outlined focal points served to help educators foster a deep understanding of key mathematical concepts that would grow and connect (Schielack & Seely, 2007) throughout a student’s mathematical career. It also served as a starting point to create dialogue about frameworks for K-8 curricular reform (Fennell, 2007).

The First Set of National Standards

While calls from national organizations attempted to propel mathematics education forward, unfortunately, educational policies inadvertently undermined the imperative work. The No Child Left Behind Act (NCLB) required states to adopt "challenging academic content standards" (Reys, 2006) and establish their own definitions of adequate yearly progress (AYP) (Rothman, 2011). This allowed states to set their own achievement levels while rewarding and penalizing individual schools based on AYP achievement (Greer, 2018). Consequently, this incentivized states to lower assessment standards (Watt, 2011) to appear as though high achievement levels were being attained. Through this process, "teaching to the test" (Greer, 2018; Rothman, 2011) became normalized in the tested subject areas, specifically in mathematics and reading.

As mounting social and political factors continued to raise awareness that student achievement had not improved both nationally and internationally, despite laws and initiatives in place, national education organizations began to call on Congress for action. Education Sector hosted a debate in 2006 on the need for national standards (Rothman, 2011). The National Association of Secondary School Principals included a plea for common standards in English language arts and mathematics (Manzo, 2008). The Nelson A. Rockefeller Institute of Government hosted a symposium to discuss intergovernmental approaches for the improvement of standards and assessments (Watt, 2011). The NRC released reports on the differences between state mathematics standards, and the James B. Hunt Jr., Institute for Educational Leadership and Policy (2008) outlined recommendations for the development of common standards. Finally, in 2009, the National Governor's Association (NGA) and the Council of Chief State School Officers (CCSSO) met to form a National Policy Forum for the CCSS initiative (Watt, 2011).

The National Policy Forum culminated in the NGA and CCSSO announcing the names of the states and territories that had signed a memorandum of understanding (MOU) to participate in the development of common standards in June 2009, which included all states except for Alaska, Missouri, South Carolina, and Texas (Watt, 2011). The lack of agreement by these four states can be attributed to the CCSS not being written at the time of the MOU (Schneider, 2015).

The CCSSM were developed through benchmarking international standards and achievement scores (Schneider, 2015). The standards were created with the intention to increase the academic expectations for all students in the United States while increasing achievement on international assessments (Rothman, 2011) and improving college readiness (NGA & CCSSO, 2011). However, this lofty goal did not consider that removing benchmarked standards from their cultural context could alter their effectiveness. For example, solely examining the mathematics standards of Japan without considering the Japanese cultural context removed important societal beliefs such as valuing group work over individual work, the pressure placed on students to be accepted into prestigious colleges, and the potential career advancement associated with college acceptance (Schneider, 2015).

The CCSSM were informed by the structure of mathematics and three specific areas of educational research, which included, “large-scale comparative studies, research on children’s learning trajectories, and other research on cognition and learning” (Common Core Standards Writing Team, 2013, p. 6). The intentional use of large-scale comparative studies allowed the standards to be organized by topics in a logical manner that were both sequential and hierarchical (Common Core Standards Writing Team, 2013; Schmidt & Houang, 2012), which placed emphasis on learning trajectories. The Common Core Standards Writing Team defines a learning trajectory based on the research of Clements & Sarama (2009), with a learning trajectory having three parts: (1) a mathematical goal, (2) a developmental path to reach that goal, and (3) instructional activities to move along the developmental path. Groups of related standards, referred to as domains, were an organizational feature of the standards. The established domains within the standards (Counting and Cardinality, Operations and Algebraic Thinking, Number and Operations in Base Ten, Geometry, and Measurement) were guided by research on learning progressions and reflect instructional sequences and activities (Common Core Standards Writing Team, 2013).

Focus, coherence, and rigor were the three focal points (Schneider, 2015; Student Achievement Partners, 2013) of the mathematics standards. The intentional incorporation of attention on these three terms were used to package the complexity of the standards and the associated research that was used during development as concise explanations for all educational stakeholders. The emphasis on *focus* referred to prior standards being “a mile-wide and an inch deep” (NGA & CCSSO, 2010) with the CCSSM allowing students to obtain a foundation of conceptual understanding, procedural skill and fluency, and the ability to solve problems outside classrooms (Student Achievement Partners, 2013). *Rigor* referred to the ability to solve problems outside of mathematics classrooms. The mathematics standards demonstrate a balanced combination of procedure and understanding to enable students to have a foundation of flexible mathematical thinking that contributes to engagement in mathematical practices (NGA & CCSSO, 2010).

Despite the challenges associated with the massive undertaking of creating what they hoped would be national standards, the first set of common, voluntary K-12 mathematics and English language arts standards was released in 2010 (NGA & CCSSO, 2010), one year after the memorandum of participation in which 46 states had signed. Adoption of the standards at the state level was voluntary (Larson & Kanold, 2016); however, the federal government encouraged adoption through initiatives such as Race to the Top (RTTTI) funding and waivers from the NCLB Act (Dingman et al., 2013; Jochim & Lavery, 2015; LaVenia et al., 2015).

RTTTI, a federal grant program, included $4.35 billion available in funding to states that met criteria outlined by the United States Department of Education. Those in opposition to the involvement of the federal government in education have referred to this program as the proverbial “dangling of the carrot” to states (Rothman, 2011). The criteria included evidence of (1) strengthening standards and assessments (2) improving data systems (3) enhancing leadership among teachers and school leadership and (4) initiating a process to “turn-around” under-performing schools (Rothman, 2011). The grant was a competition, with three phases (Howell, 2015) in which 46 states completed applications (Shober, 2016). The second element, strengthening standards and assessment, required states to demonstrate adoption of the CCSS by August 2, 2010 (Shober, 2016). Delaware and Tennessee received a collective $600 million during the first round of RTTT, with the remaining funds being distributed during the second round to the District of Columbia, Florida, Georgia, Hawaii, Maryland, Massachusetts, New York, North Carolina, Ohio, and Rhode Island (Schneider, 2015). Arizona, Colorado, Illinois, Kentucky, Louisiana, New Jersey, and Pennsylvania received funding during the third (and final) phase (Howell, 2015). Notably, each state received a varying amount based on the criteria and discretion of the United States Department of Education. Table 1 presents total award amounts by state.

Table 1

RTTI Award Amounts

State	Award Amount (in millions)	Round
Delaware	$120	1
Tennessee	$500	1
District of Columbia	$75	2
Florida	$700	2
Georgia	$400	2
Hawaii	$75	2
Maryland	$250	2
Massachusetts	$250	2
New York	$700	2
North Carolina	$400	2
Ohio	$400	2
Rhode Island	$75	2
Arizona	$25	3
Colorado	$18	3
Illinois	$43	3
Kentucky	$17	3
Louisiana	$17	3
Pennsylvania	$41	3
New Jersey	$38	3

These common, voluntary standards (with this study focusing specifically on mathematics), outline what students should know and be able to do at specific grade levels (NGA & CCSSO, 2010). Additionally, the NGA and CCSSO developed the Standards for Mathematical Practice (SMP) to “describe varieties of expertise that mathematics educators at all levels should seek to develop in their students” (NGA & CCSSO, 2010, p. 6). The SMPs were developed from the NCTM process standards (NCTM, 2006) which included problem-solving, reasoning and proof, communication, connections, and representation and the NRC's (2001) strands of mathematical proficiency (adaptative reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition) to create the eight SMPs embedded in the CCSSM.

A natural wondering about this historic event lends itself to the number of states that ultimately fully adopted and implemented the standards. While the answer should be straightforward, the answer depends on the criteria and resources used to answer the question. Initial adoption occurred from 2010-2011 with some states delaying implementation until the 2017 school year. In total, the CCSSM was originally adopted by 45 states with Alaska, Nebraska, Texas, and Virginia instead using their own state standards for both mathematics and English language arts (EdGate, 2018; Schneider, 2015)). Minnesota partially adopted the CCSSM with just English language arts, and used their own standards for mathematics (EdGate, 2018; Schneider, 2015). Adding further confusion initiated by political involvement, some states quickly reversed their adoption, renamed their standards to include their state name, or modified the standards before the 45th state implemented the CCSSM. The initial bi-partisan support deteriorated quickly with some political figures declaring their states were "no longer the Common Core State Standards", despite their revisions to standards being an exact copy of the CCSSM without the title of "Common Core." This resulted in an ever-changing response being published in the press regarding exactly how many states had fully adopted and/or implemented the CCSSM during the 2012-2022 timeframe.

Statement of the Problem

In this section, the problem associated with this study is outlined by sharing four surrounding issues. First, concerns raised prior to the creation of the CCSSM are now being expressed as part of the reason to transition to new standards. Second, proposed solutions of revisions to the common standards (Confrey, 2007; NCTM, 2000; Rothman, 2011) prior to the implementation of the CCSSM, had largely been ignored. Third, information about state standards from 2012 to present remains unclear. Finally, websites were promoting misinformation further confusing public debate. All these issues directly relate to the problem this study is framed around; that is, standards are transitioning back to a pre-Common Core era where United States K-5 state mathematics standards lacked consistency and cohesion with little documentation of *what* changes occurred. Also not clear is documentation of what standards have been removed from state revisions.

Many of the same concerns that served as an impetus for a set of national standards more than a decade ago remain. For example, *Catalyzing Change* (NCTM, 2020) calls for focus and coherence and the need for a “consistent, systematic, and widespread implementation of college and career readiness standards in the ways in which they were intended” (p. 65). The *Catalyzing Change* series (NCTM 2018; 2020a; 2020b) includes intentional grade band publications in Early Childhood and Elementary, Middle School, and High School mathematics. Additionally, societal frustrations regarding the quality of standards guiding instruction in public education and the achievement levels of students (Loveless, 2016) are also being articulated again, with research (Lee & Wu, 2017) concluding that the CCSSM has improved performance standards among international ranking, but not necessarily performance outcomes on international assessments. Parental concerns about being unable to assist their children with homework

(Garland, 2014; Richards, 2014) contribute to societal frustration. These factors all demonstrate a cyclical relationship of educational changes, as demonstrated in Figure 1.

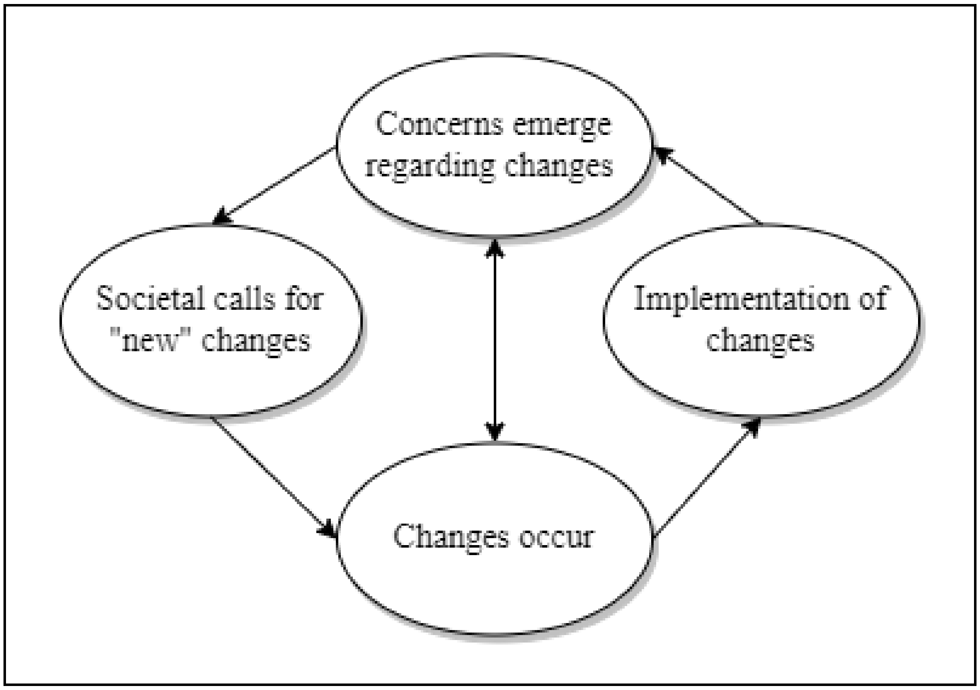

Figure 1

Cycle of Educational Changes

Another issue facing the CCSSM is a solution that has largely remained ignored. Prior to the creation of the CCSSM, a proposed solution was suggested to address the concern regarding the quality of mathematics standards. The proposed solution, termed a “bootstrapping process” (Confrey, 2007), consists of continual improvement through revisiting standards and engaging in necessary revisions in response to specific feedback from educators. By revisiting the standards, the quality of the standards would continually be reconsidered and evaluated. A similar proposal was also suggested by NCTM (2000) as they stated, “any vision of school mathematics teaching and learning needs ongoing examination; it needs to be refined continually in light of the greater understanding achieved through practice, research, and evidence-based critiques” (p. 380).

Despite these suggestions being proposed prior to the creation of the CCSSM, there has neither been a reconvening of the writing team nor any adjustments to the CCSSM at the national level based on feedback since states began adopting the CCSSM. NCTM and the Mathematical Association of America (MAA) released a joint statement (NCTM, 2022) emphasizing the importance of developing proficiency and providing supports for students to be successful in calculus and beyond, which further provides justification for the reconvening of a writing team to revise the CCSSM to ensure those supports are in place.

The mathematics education community suggested (prior to the CCSSM being drafted) that for national standards to be successful, a reconvening of the standards writing team would be necessary. This ignored suggestion can be contributed to a variety of factors that delve into the copyright licensing component of the CCSS (Schneider, 2015). It is public perception that the CCSS were in fact written "by the states, for the states" due to the narrative by political leaders and the media while the standards were being developed. The fine print of the CCSS licensing document states that "NGA Center/CCSSO shall be acknowledged as the sole owners and developers of the Common Core State Standards, and no claims to the contrary shall be made" (Common Core State Standard Initiative [CCSSI], 2022, para. 4; Schneider, 2015, p. 124). The document later identifies that 'impermissible uses" to include "revising, including editing" (Schneider, 2015, p.137), while specifically mentioning abridged or condensed versions of the standards that adjust the meaning of intent of the standards. The wording on the license prohibits users (states in this case) to modify the standards. However, RTTTI allowed for up to 15% of the CCSS to be altered through additions to the standards (Schneider, 2015) to be considered for the initiative.

During this dissertation, the NGA and CCSSO website abruptly disappeared in August and remained down for approximately five months. There was chatter on Twitter that a Common Core State Standards app through the company MasteryConnect existed for those that wanted access to the standards (Danielson, 2022). However, no supporting documents were found on the app; its primary function is strictly for the standards. Additionally, much to my surprise, the app is also only available for download on Apple products, which was an unfortunate circumstance as I could not access the app.

The CCSS website was again accessible in January of 2023, but was a mere shell of what the site had once been. Almost all the supporting documents had disappeared and only the standards documents remained. This substantially impacted direct quotations, especially regarding copyright. Schneider (2015) captured much of what was available within her publication, but an internet archive website has the old Common Core State Standards website intact (for now).

Instead of reconvening for needed adjustments, several individual states, on their own accord, have deviated from the voluntary common standards and made their own revisions to the wording and content of the standards. The CCSS website tracks states that have decided to transition away from the CCSSM and updates a visual map when states no longer have the CCSSM adopted. Currently, the Common Core website (NGA & CCSSO, 2022) signifies states that never adopted the standards (Alaska, Minnesota, Nebraska, Texas, and Virginia) or first adopted but later repealed the standards (Arizona, Indiana, Florida, Oklahoma, and South Carolina) (NGA & CCSSO, 2010). A hover feature, available on the website, exists for each state which indicates when the CCSSM was adopted by that state.

This information leads to the third and fourth issues surrounding the CCSSM currently, with unclear published information confusing public debate ranging from incomplete to inaccurate reporting. Other websites exist that contain maps that depict state-level changes that are not documented on the CCSS website. For example, the EdGate (2018) website indicates states that have rewritten, reversed the adoption of, or renamed their standards. Their website includes 15 additional states that are not published on the Common Core website, providing the perception that there are more states that are disassociating themselves from the CCSSM. EdGate's map (2018) includes: Alabama, Arkansas, Georgia, Kansas, Kentucky, Louisiana, Mississippi, Missouri, Pennsylvania, New Jersey, New York, North Dakota, Rhode Island, Tennessee, and West Virginia. These states have been referred to as "Standards Adapted from the Common Core" (Opfer et al., 2016) in some sources. This phrasing is not reflected on the EdGate website.

At first glance, the map on EdGate's website visually supports the notion that most states are deviating from the CCSSM. However, many of the included states on this map simply renamed their state standards and kept the wording of the CCSSM. This phenomenon, of standards that were by and large the CCSSM without the CCSSM title, will be referred to as *emulative standards* throughout this study. This information is not noted on the CCSS website and further creates confusion among the public regarding which standards their state uses to guide instructional decisions. The College and Career Readiness State Legislation website (National Conference of State Legislatures, 2019) does include a visual of states that have rebranded the CCSSM into their state standards revisions. It also includes information not found on other visual maps pertaining to standards revisions indicating states that have taken judicial action, executive action, or legislative action on standards. Their visual also supports the impression that most states are deviating from the CCSSM. A Google search of "Common Core Mathematics Map" or "Map of Mathematics State Standards" results in dozens of variations of color coding and labels on outlines of the United States, further complicating the answer to which states require the use of the content of the CCSSM.

Additionally, some states have touted that they have eliminated the CCSSM from their state curriculum. However, this typically politically charged statement does not depict the entire picture of the inner workings of state standards. True elimination of the CCSSM would require a complete restructuring of their state standards, without the guidance of literature used in the development of the CCSSM. Statements such as these are harmful as they are not accurate and thereby misguide the public to believe that the CCSSM is no longer an influence on the instructional decisions in classrooms. This statement also creates a perception of transitioning mathematics back to how "it has always been taught." This statement refers to how some parents learned mathematics; with the teacher disseminating information procedurally and students practicing the computational procedures. Finally, a political perception occurs as states that are still associated with the CCSSM could be interpreted as being affiliated with a particular political party even though no political party was involved with their creation. This further divides the public perception within the United States on *what*, *how*, and *when* topics should be taught in mathematics.

While the phenomenon being exhibited by some states of adjusting standards supports Confrey's "boot strapping" suggestion (2007), it also simultaneously moves the public education system back to the same issues as the pre-Common Core era where mathematics standards across states had inconsistencies and lacked cohesion (Jochim & Lavery, 2015; Watt, 2011). Further complicating the issue is that the publicly available information on the timeline of state mathematics standard changes and the specific details of the changes occurring is nebulous. The lack of direct answers in a clear, readable manner necessitates the need for a source documenting the specific changes that have occurred since the adoption of the CCSSM. A singular document that addresses the changes that have occurred with K-5 mathematics standards since the adoption of the CCSSM would better inform all stakeholders and provide synthesized guidance to state educational leaders, which is the primary product of this dissertation. As individual states shift to their own developed mathematics standards, stakeholders in education need to be kept informed of what these shifts entail and how they will impact student learning.

Specifically, little research exists documenting the transition that is currently occurring within United States K-5 state mathematics standards documents despite the importance of this research. While documentation exists regarding the initial perceived weaknesses of the CCSSM, it is important that it is included in this study so it can be in one document with changes that are currently occurring. This information can also provide guidance to educational stakeholders as states consider revisions to their standards. Table 2 outlines some of the concerns with the K-5 CCSSM reported by members of the United States Coalition for World Class Math (Absher, 2014), a group comprised of “mathematically literature parents” who oppose the CCSSM (United States Coalition for World Class Math, 2009).

Table 2

Concerns Regarding CCSSM

Domain	**Concern**
Operations and Algebraic Thinking	Prime factorization not taught Least common denominators/greatest common factors not explicitly taught
Number and Operations in Base Ten	Proficiency with addition and subtraction is not expected until 4th grade Proficiency with the standard algorithm of multiplication is not expected until 5th grade Proficiency with the standard algorithm of division is not expected until 6th grade
Number and Operations-Fractions	Decimal introduction is not until 4th grade Does not include conversations among fractions, decimals, and percents Money is not an introduction to decimals
Geometry	Key geometrical concepts are not taught (i.e. area of triangle, sum of angles in triangle, isosceles and equilateral triangles, or constructions with straightedge and compass)

To consider standards from the metaphorical perspective of a road map, the need for such direction becomes much more apparent as without one, “you are sure to drive in circles and get nowhere” (Ravitch, 2010, p. 236). A calling, made by numerous education researchers, urges other researchers to engage in analysis of curriculum standards that will (a) inform public debate and (b) support education professionals to improve students’ learning opportunities (Tran et al., 2016). This dissertation study aimed to fulfill this call of informing public debate by determining the changes that have occurred within United States K-5 state mathematics standards from 2012-2022.

Purpose and Research Questions

The purpose of this research study is to examine the ways in which United States K-5 state mathematics standards have changed during the past decade (2012-2022). This time frame reveals changes that have occurred since the implementation of the CCSSM. Themes were developed from patterns that emerged from the data. Findings from this study will produce a singular document with associated changes from 2012-2022 clearly articulated within and across states selected for coding. Lastly, this study will provide information regarding the direction K-5 state mathematics standards are headed based on trends and themes that emerge from the data.

Research Questions

The following research question was used to guide this study: *In what ways, if any, do K-5 state mathematics standards differ from the CCSSM?* Four sub questions were used to accomplish the goal of answering the overarching research question. The four sub questions include:

1) *In what ways, if any, do K-5 state mathematics student process and practice standards differ from the CCSSM?*
2) *In what ways, if any, do K-5 state mathematics standards content domains differ from the CCSSM?*
3) *In what ways, if any, do states describe how learning trajectories are addressed in K-5 state mathematics standards?*
4) *In what ways, if any, is the relationship between procedural and conceptual learning outcomes represented in K-5 state mathematics standards?*

Significance of Study

This study can contribute significantly to the field as there are numerous nuances, complexities, and instances of lack of clarity with the current status of United States K-5 state mathematics standards. First, states have created their own standards historically, and this has resulted in less than desirable situations for students who move to different states during their K-12 educational careers. Further complicating the status of United States K-5 state mathematics standards, each state has implemented different standards writing processes which can result in outcomes of varying quality. Some state writing teams include educators in the writing process, while other states relied solely on experts. Some state department of education websites publish the names of each committee member with their specific role to the writing team, while other states vaguely describe the process in a few paragraphs that leave many important details unaddressed. There is not a singular document where all changes are described for educational stakeholders to use as guidance or to inform conversations regarding changes to mathematics standards.

Secondly, what has already been studied includes quality (Cobb & Jackson, 2011), rigor (Carmichael et al., 2010), global analysis (Khaliqi, 2016), and comparisons of pre-2010 state mathematics standards to the CCSSM (Dingman et al., 2013). All the previously mentioned studies were quantitative studies. Currently, there is not a qualitative study examining the differences among United States K-5 state mathematics standards. This confirms a need for this study's design to better inform stakeholders of changes that are occurring from a varying methodological approach.

Finally, the CCSSM was developed with a balanced combination of conceptual understanding and procedural fluency (NGA & CCSSO, 2010), following the recommendations of both NCTM (2006) and the NRC (2001). As states revise their standards, questions should be raised about the emphasis on instructional approaches to teaching mathematics. Developing understanding enables students to engage in the SMPs, as, without this understanding, students may rely more heavily on procedures only (NGA & CCSSO, 2010). Importantly, this study offers insight into trends and themes that emerge from the revisions that are being made to K-5 state mathematics standards through a qualitative lens.

Organization of the Dissertation

The organization of the remaining chapters of this dissertation is as follows: Chapter 2 provides an in-depth review of the literature on the history of mathematics standards in the United States including the development of the CCSSM, the history of types of mathematics understanding (procedural and conceptual), the content domains found within the CCSSM, and the history and role of learning trajectories within the development of the CCSSM. It will also provide an overview of the conceptual framework used to guide this study. Chapter 3 outlines the use of qualitative content analysis of documents as the methodology for this study, including a discussion on the research design with exclusion and inclusion criteria. Data sources and analysis of data are discussed. Chapter 4 provides results from six of the states that were selected for coding and themes that were established among the standards revisions. Finally, Chapter 5 includes discussion of conclusions and interpretations of the study.

Summary

Previously conducted studies have focused on the quality of state standards, the level of rigor of state standards, the global competitiveness of United States standards, and the achievement level of standards in conjunction with state assessment results. The goal of this study was to determine what changes have occurred to United States K-5 state mathematics standards during the past decade, or 2012-2022. This study specifically examined (1) the student process and practice standards, (2) the content domains, (3) the presence of learning trajectories, and (4) whether a shift in emphasis related to conceptual or procedural knowledge was an outcome of the standards to tell a more complete, concise picture of what has changed in the United States K-5 state mathematics standards. Incorporating a qualitative methodology into this study ensured that standards documents were examined and described through an in-depth view, reporting on data through unique, descriptive themes from emerging patterns.

Key Terms

Conceptual Understanding: "Comprehension of mathematical concepts, operations, and relations" (NRC, 2001, p. 116).

Crosswalk: A document that is used as a reference tool for both educators and parents to compare changes (New York State Education Department [NYSED], 2019) and similarities among mathematics standards.

Domains: Groups of related standards (NGA & CCSSO, 2010, p. 5).

Learning Trajectories: "Descriptions of children's thinking and learning in a specific mathematical domain, a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of support in children's achievement of specific goals in that mathematical domain" (Clements & Sarama, 2004, p. 83).

Procedural Fluency: "Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (NRC, 2001, p. 116).

Process Standards: Represent ways of developing and applying content knowledge including "problem-solving, reasoning and proof, communication, connections, and representation" (NCTM, 2000, p. 7; NGA & CCSSO, 2010).

Mathematical Proficiencies: Interwoven, interdependent strands that represent different aspects that capture what is necessary to learn mathematics successfully including: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (NRC, 2001).

Standards for Mathematical Practice: **“**Varieties of expertise that mathematics educators at all levels should seek to develop in their students” (NGA & CCSSO, 2010, p. 6), which rely on processes and proficiencies (NCTM, 2000; NRC, 2001).

CHAPTER TWO: REVIEW OF THE LITERATURE

The purpose of this research study is to examine the ways in which the K-5 state mathematics standards have changed during the past decade (2012-2022). The time frame 2012-2022 was chosen for this study because by the year 2012 most states had either already adopted the CCSSM or had decided to not adopt the CCSSM. This study focuses on states with K-5 mathematics standards substantially different from the CCSSM. In this study, emulative standards are defined as standards that were by and large the CCSSM without the CCSSM title. Emulative standards were not considered as part of this study because the lack of changes outside of a name change does not impact the standards. The states of interest included: (1) states that never adopted the CCSSM and (2) states that amended or repealed their prior adoption of the CCSSM in the past decade. This study sought to answer the overarching research question: *In what ways, if any, do K-5 state mathematics standards differ from the CCSSM?* Answering the overarching research question was accomplished by addressing the following sub questions:

1) *In what ways, if any, do K-5 state mathematics student process and practice standards differ from the CCSSM?*
2) *In what ways, if any, do K-5 state mathematics standards content domains differ from the CCSSM?*
3) *In what ways, if any, do states describe how learning trajectories are addressed in K-5 state mathematics standards?*
4) *In what ways, if any, is the relationship between procedural and conceptual learning outcomes represented in K-5 state mathematics standards?*

Chapter 2 addresses the existing literature related to the study purpose, as well as the research question and sub questions. First, the chapter reviews the literature on mathematics standards. This includes an overview of the history of reform movements in mathematics education since the 1950's using the lens of instructional focus on the continuum of conceptual and procedural understanding. This lens provides insight into the current purposes of mathematics standards and serves as a reminder that reflecting on work from the past can guide current work (Malaty, 2006) and future direction. Second, literature pertaining to the research questions and guiding framework are discussed. This includes four main elements: (a) process and practice standards; (b) content domains; (c) learning trajectories; and (d) conceptual and procedural knowledge. Finally, an explanation of the conceptual framework is provided to transition to the methodology shared in Chapter 3.

Literature Search

Numerous databases were used during the review of the literature. These included: Education Resources Information Center (ERIC), ProQuest Research Library, and EBSCO Academic Search Premier. Sources were also obtained from Google Scholar, the University of Central Florida library, and suggestions from members of my dissertation committee. The literature used in this review includes peer-reviewed manuscripts from research journals, peer-reviewed manuscripts from practitioner journals, select dissertations, proceedings from national conferences, documents, information available on state department of education websites, policy reports, and pertinent books.

Key search terms during the literature search are provided in Table 3. Additional limits were provided for each search to strengthen the yielded results. The limit of 2009 was used in conjunction with any searches using the term "Common Core" due to the year the standards were released to the public. Other searches (that were conducted prior to this dissertation that did not use the limit of 2009) yielded results that included speculation regarding a set of national standards and did not enhance the literature for this study.

Table 3

Literature Search

Database	Search Terms	Yielded Results	Additional Limits
ERIC	(common core state standards) AND (learning trajectories) AND (mathematics or math)	72	Peer-Reviewed, Academic Journal; 2009-2022
ERIC	(common core) AND (comparison) AND mathematics	60	Academic Journals, Reports, Dissertations; 2009-2022
ProQuest	(mathematics state standards) AND (common core)	22	Peer Reviewed; 2009-2022
EBSCOhost	(mathematics state standards) AND (common core math standards) AND (differences OR policy)	56	Peer-Reviewed, 2009-2022
Google Scholar	"elementary mathematics" "common core standards" "comparison"	20	2009-2022
UCF Library	"mathematics education reform"	51	Peer-Reviewed Journal, Available Online, Book, Available on Shelf

All yielded results were considered for inclusion in Chapter 2 as part of the literature review. The yielded results produced overlaps and any duplicates among the searches were removed, which significantly lowered the number used in the literature review. After reading the abstract of each yielded result, I then determined if it was a good fit for this chapter. Yielded results that led to further publications through relevant references were also included in this chapter. Further, recommendations by committee members were not included in Table 1 but are incorporated into this chapter as appropriate.

History of Mathematics Standards in the United States

Framing the history of the subject of mathematics and the impacts the subject has on day-to-day living helps frame current trends in mathematics education. It further identifies why changes are being made to state standards while focusing on the objective of learning mathematics.

Education in the 17th century was not a requirement for children, and the focal point of education was drastically different from how education is guided by academic standards today (Gray, 2008). The goal of being able to read the Bible dominated instruction as colonies began to create schoolhouses targeted at wealthier Caucasian populations (Dexter, 1904). A paradigm shift occurred in the 19th century that resulted in education being state-sponsored and serving broader demographics (Gray, 2008). Subsequently, major events unfolded such as the first public school in the United States was formed in 1821 (Dexter, 1904), the nation's first historically Black college and university was established in 1837 (Lovett, 2015), and the national bureau of education was established in 1867 (Dexter, 1904).

As the United States shifted its goal to developing both economically and technologically, the purpose of education shifted from educating some to educating the entire population to create productive members of society. In turn, a select group of subject areas began to become required components of education. Historically, through this progression of incorporation of subjects, mathematics was not always a subject for all learners (Furr, 1996; Larson & Kanold; 2016). Mathematics has historical roots in educating the elite, with a primary focus on educating boys (Furr, 1996), and was not considered a subject to be learned by young children (DeVault & Weaver, 1970; Furr, 1996; Larson & Kanold, 2016). As the subject grew in popularity as a prerequisite for college acceptance, (Furr, 1996) and more citizens needed arithmetic in their daily lives (Larson & Kanold, 2016), changes to the subject were made from a top-down approach beginning in secondary school and eventually trickling into elementary school instruction (Furr, 1996). As mathematics developed as a focal point within the daily instructional school schedule, debates emerged on *what*, *when,* and *how* students should learn various mathematics topics (Larson & Kanold, 2016; Ravitch, 2010).

The Rugged Terrain of Instructional Focus

Whether or not mathematics topics should be taught conceptually or procedurally has been a persistent debate within society (Furr, 1996; Hiebert & Lefevre, 1986; Jones & Coxford, 1970) and is a question that is revisited every few decades (Larson & Kanold, 2016). The first mathematics textbook for children, Nicolas Pike's *Arithmetic* (1788), disseminated mathematics knowledge following what is known as the "traditional" approach, with the teacher telling rules and the students practicing procedures (Jones & Coxford, 1970; Larson & Kanold, 2016). The publication of Warren Colburn's *An Arithmetic on the Plan of Pestalozzi* (1821) challenged this traditional approach and used discovery, reasoning, and understanding as the focus of the acquisition of mathematics knowledge (Furr, 1996; Jones & Coxford, 1970; Larson & Kanold, 2016). From this point forward in history, debates ensued around these approaches to mathematics knowledge. The next section in this chapter will examine the chronological history of these debates and reform movements in mathematics education.

History of Reform Movements

This debate between a focus on conceptual versus procedural learning of mathematics has shifted back and forth from one instructional emphasis to the other (Larson & Kanold, 2016). Debates first centered on whether procedural fluency *or* conceptual understanding needed to be prioritized (Hiebert & Lefevre, 1986). These elements ultimately acted as competing factors for instructional decisions of educators with problem-solving, concept knowledge, and algorithm knowledge being treated as separate entities (Hiebert & Lefevre, 1986; Silver, 1986). Over time, shifts in research and thinking began to emphasize the balance of acquisition of both types of knowledge (NRC, 2001) with that belief currently being accepted in today's educational focus (NCTM, 2020; NGA & CCSSO, 2010). One of the eight mathematics teaching practices (MTP) outlined by NCTM (2014) emphasizes that mathematics instruction should focus on the development of "*both* conceptual understanding and procedural fluency" (p. 42). This MTP is supported in the CCSSM as evidenced through standards that ask learners to be able to connect procedures to underlying concepts.

The terms conceptual understanding and procedural fluency are two of the five strands of mathematical proficiency as outlined by Kilpatrick, Swafford, and Findell (NRC, 2001) in the seminal work, *Adding It Up*. While conceptual understanding and procedural fluency reflect today's terminology, both terms have been referred to by other names throughout history (Hiebert & Lefevre, 1986). Table 4 offers an overview of various terminology historically used to represent conceptual understanding and procedural fluency.

Table 4

Selected Historical Efforts Related to the Debate on Instructional Emphasis

Author(s)	**Year**	**Other Names**	**Conceptual Alignment**	**Procedural Alignment**
Anderson	1983	Declarative and Procedural Knowledge	Declarative knowledge	Procedural knowledge
Tulving	1983	Semantic and Episodic Memory	Episodic memory	Semantic memory
Resnick	1982	Semantics and Syntax	Syntax	Semantics
Gelman & Gallistel	1978	Principles and Skills	Principle	Skills
Skemp	1978	Instrumental and Relational Understanding	Relational understanding	Instrumental understanding
Gagné	1977	Skill Learning; Intellectual skills, cognitive strategy, verbal information	Intellectual skills Cognitive strategy	Verbal Information
Scheffler	1965	Propositional use (knowing that) and procedural use (knowing how to)	Propositional use	Procedural use
Bigge	1964	Memory-level learning and Understanding-level learning	Understanding-level	Memory-level

The following sections elaborate on the research, standards, and policy documents that have guided the debate between conceptual understanding and procedural fluency from the 1950s to the current day. The reform movement began to be debated as early as the 1930's through the 1940's (Walmsley, 2003) but *new math* projects at collegiate institutions began in the 1950's.

New Math Movement (1950s-1960s)

The *new math* movement was a response to major historical events of the time including World War II and the launch of Sputnik by Russia in 1957 (Kline, 1973; Larson & Kanold, 2016; Woodward, 2004). Both events resulted in a societal outpouring of criticism of the public school system (especially the mathematics and science programs) and a demand for excellence in education (Lagemann, 2000; Spring, 1976; Woodward, 2004). The new math movement fostered a common misconception among members of society that reforming mathematics instruction was a new idea, despite evidence that this idea had been discussed as far back as the 1800s (Furr, 1996). The era, coined as the new math movement, was considered the "golden age" in mathematics education due to the enormous increase in federal funding for both research and the preparation of teachers (Kilpatrick, 1992).

During this era, the School Mathematics Study Group (SMSG) was formed with the support of a National Science Foundation (NSF) grant to improve mathematics curriculum (Spring, 1976). The goal of SMSG was to create a program to enhance instruction in secondary and eventually elementary schools (Spring, 1976). The focus on abstract concepts and processes began in high school programs and progressed more slowly in the middle and elementary curriculum (Fey, 1978). Edward G. Begle served as the director of SMSG where his role overseeing writing teams resulted in the creation of "new math" textbooks that focused on abstract reasoning (Begle, 1972; Kilpatrick, 2015). Textbooks they rapidly created were the source that determined both the topics and the timeframe of mathematics instruction and SMSG accomplished its goal through “a new curriculum that was recognizably superior to any in existence” (Spring, 1976, p. 121) as new math was used to teach mathematics better.

Within the *new math* reform, Van Engen became known to some as a pioneer with his involvement in the creation of progressive textbooks (Ellis & Berry, 2005). Professional development included demonstrations of discovery lessons (Fey, 1978). The incorporation of manipulatives into mathematics instruction for conceptual understanding was also a product of new math (Fey, 1978), credited to Catherine Stern (Ellis & Berry, 2005). The movement resulted in shifts in emphasis from the manipulation of numbers and symbols to a focus on the underlying structure of the manipulations (Kidd et al., 1970). Bruner’s work (1960) focused on understanding and discovery as an acquisition of mathematics knowledge.

Ultimately, the new math movement was declared a failure (Fey, 1978; Larson & Kanold, 2016; Woodward, 2004). Critiques cited that the lack of professional development for teachers played a major role in the fall of the new curriculum (Kline, 1973; Woodward, 2004). Kline (1973) offered an analysis that critiqued the decision to put curriculum development ahead of addressing the number of underqualified teachers and felt the NSF funding could have been better used “to improve the mathematical backgrounds of elementary and high school teachers” (pp. 28-29). Another common critique declared that a combination of new content, curriculum structure, and pedagogical styles was too ambitious of a goal to occur all at once (Fey, 1978). Finally, the impact of parent and teacher frustration in reconceptualizing arithmetic contributed to the demise of new math (Glennon, 1973). The critiques represent the frustrations surrounding a hypothetical swing of mathematics education that failed to consider a balanced approach to understanding content. These are all critiques that would be heard again in future reforms.

Back-to-Basics Movement (1970s-1980s)

The overwhelming number of changes in mathematics education in such a quick period, in addition to the perceived shortcomings of the changes, led to a societal push to go “back to the basics” (Ellis & Berry, 2005). Prior arguments for the change of mathematics in schools were ignored and there truly was not a formal concept of what “the basics” entailed (Schoenfeld, 2004). The instructional emphasis was now on procedural fluency as this movement was “diametrically opposite to that taken by the new mathematics” reform (Kline, 1973, p. 173).

Mathematics instruction in the back-to-basics movement focused on skill-based and decontextualized instruction (Ellis & Berry, 2005) with success in the subject being viewed as accuracy and speed (Larson & Kanold, 2016). In defense of asserting this skill-based approach, Kline (1973) argued that less than a majority of students end up attending college, with very few students specializing in the field of mathematics, so mathematics education “should be broad rather than deep” (p. 174). However, Erlwanger’s work (1973) demonstrated how children can initially appear to be doing well in mathematics, but true understanding might be framed around incorrect rules and procedures.

Much of the back-to-basics movement was reactionary to public opinion rather than based on the recommendations of specialists in mathematics or education (Malaty, 2006). This drastic shift naturally left those who believed in the new mathematics reform feeling as though the back-to-basics movement would not help achieve the goal of understanding (Malaty, 2006). Not surprisingly, the next decade led back to a former instructional emphasis.

Standards-Based Reform Movement (1980s)

NCTM (1980) published *An Agenda for Action: Recommendations for School Mathematics of the 1980s* to influence change in mathematics education. The President of NCTM at the time, Shirley Hill, described the state of mathematics education as nearing a "crisis stage" as the curriculum was being narrowed to go back to the basics (McLeod et al., 1996). The publication set forth eight recommendations which included a shift in instructional focus to problem-solving and basic skills being more than just computational fluency (Larson & Kanold, 2016; NCTM, 1980).

A report released by the National Commission on Excellence in Education (NCEE), *A Nation at Risk* (NCEE, 1983), described how the education system was failing students (LaVenia et al., 2015) as "knowledge, learning, information, and skilled intelligence" were deemed the new "raw materials of international commerce" (NCEE, 1983, p. 10). The report cited inadequacies in expectations of students, use of instructional time, and teaching conditions (NCEE, 1983). Most importantly, the report raised concerns regarding the "diluted" curriculum referred to as a "smorgasbord" (NCEE, 1983, p. 21), which prompted societal calls for increased curricular requirements to compete with foreign countries academically (LaVenia et al., 2015). Further, it sparked concern in American society that the launch of Sputnik indicated a lapse in scientific intellectual advancement in America and a lack of economic advancement being portrayed in countries such as Japan, South Korea, and Germany (NCEE, 1983).

In response, the Dwight D. Eisenhower Mathematics and Science Education Program was enacted in 1988 to increase understanding of mathematics and science becoming the largest federal program for preparing elementary and secondary teachers in mathematics and science (Wagner, 1992). In addition, NCTM further elaborated on the views from *An Agenda for Action* through the publication of the *Curriculum and Evaluation Standards for School Mathematics* in 1989 beginning what came to be known as the standards-based reform movement (Ellis & Berry, 2005; Herrera & Owens, 2001; Larson & Kanold, 2016). These standards served as voluntary national content standards and emphasized understanding and reasoning for all students (NCTM, 1989; Schoenfeld, 2004). The fundamental content represented in the standards created a framework for reform in mathematics education (NRC, 2001).

During this same time, the research of Cognitively Guided Instruction (CGI) was one of the first models integrating research on learning with research on teaching (NCTM, 2004). Carpenter's work with colleagues (Carpenter, 1986; Carpenter et al., 1989) progressed the field through the conclusion that learning procedures of a mathematics topic does not ensure that students will acquire conceptual understanding. The focus in classroom instruction on procedures resulted in "impoverished conceptual knowledge and tenuous links between the conceptual knowledge that is learned and related to the procedural knowledge" (Carpenter et al., 1978, p. 130). Carpenter's calls for educators to develop both types of knowledge emphasized the shift in thinking that occurred during this era that types of knowledge cannot be viewed as silos, but instead are an iterative, interwoven process (Carpenter, 1986; Rittle-Johnson et al., 2015).

Standards Backlash and Math Wars (1990s-2000s)

Various political actions, in addition to the reform occurring in mathematics education, led to what would become known as the math wars. In 1991, both the National Council on Education Standards and Testing (NCEST) and the National Education Goals Panel (NEGP) (LaVenia et al., 2015; Watt, 2011were formed. The NCEST recommended national standards and assessments and the NEGP monitored progress toward national goals (LaVenia et al., 2015). The Goals 2000: Educate America Act was passed in 1994, which required states to use national standards to develop and align state standards and assessments (Watt, 2011). Further, the Improving America's School Act, also passed in 1994, required states to create standards and state assessments that aligned with the standards in both mathematics and reading (Confrey, 2007; Watt, 2011).

In response, 41 states used the 1989 NCTM standards to create state standards or curriculum frameworks (Larson & Kanold, 2016; McLeod, 2003). Further projecting the reform, NSF requested proposals for curricula that aligned with the 1989 NCTM standards (Schoenfeld, 2004). As a result, a variety of curriculum materials were produced to support the new standards, but not all were necessarily reflective of the original intentions of the standards (Schoenfeld, 2004). Due to the ambiguity of the standards, interpretations of the standards were vastly different. The variety of curriculum materials naturally prompted societal backlash and what would become known as the "math wars" of the nineties. Some have referenced this as the "new new math" era (Herrera & Owens, 2001); however, this phrase inaccurately depicts the reform movement. The use of the phrase "new new math" further advances the ideas related to the 1950s reform (Herrera & Owens, 2001) that the instructional approaches in mathematics were "new", further promoting public confusion on how long these ideas had been implemented in classrooms.

California was at the forefront of the math wars, with its state standards representing what was considered a progressive step toward change (Schoenfeld, 2004). The state developed materials used to represent the state standards faced similar backlash as the new math reform. Concerns emerged from worried parents that they would be unable to help their children with homework (Schoenfeld, 2004) because there was a decreased emphasis on procedural skills, practice, and memorization (Larson & Kanold, 2016). Groups were formed that opposed the standards-based reform with entire websites dedicated to swaying others to join their cause, and politicians became involved in the debates (Schoenfeld, 2004). Mathematicians spoke out against mathematics educators, furthering the rift in public support. By the late nineties, there were clear lines of opposition in California to the standards that gained a national following.

The 2000s marked a shift in emphasis within research in mathematics education. In 2001, the NRC published *Adding It Up* (2001) as an attempt to bring cohesion to the public debates that had been raging for years. The emphasis on balance and interdependency of five strands (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) of mathematical thinking represented a pivotal moment in mathematics education. Debates that had been on-going for decades focused on either conceptual understanding or procedural fluency as an instructional emphasis. Kilpatrick et al. (NRC, 2001), reiterated that “one of the most serious and persistent problems facing school mathematics in the United States is the tendency to concentrate on one strand of proficiency to the exclusion of the rest” (p. 11).

Despite the debates, research from the late nineties supported the NRC’s position of a balanced approach to mathematical thinking. Rittle-Johnson and Alibali (1999) determined in their studies that the relationship between conceptual and procedural knowledge is not unidirectional and gains in one type of understanding ultimately lead to gains in the other. They warned that despite this relationship, “their influence on one another may not be equivalent” (Rittle-Johnson & Alibali, 1999, p. 188) despite previous research (Rittle-Johnson & Siegler, 1998), stating that there is no fixed order of acquisition of skills versus concepts. Rittle-Johnson’s and Alibali’s research provided further support that a balance of the two knowledge types was necessary within instruction.

Two major NCTM publications from the early 2000's served as guidance for standards: *Principles and Standards for School Mathematics* (NCTM, 2000) and *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics- the Quest for Coherence* (NCTM, 2006). Within *Principles and Standards*, necessary components of high-quality mathematics programs were outlined with the premise that all students should learn mathematics with understanding (NCTM, 2000). Not only did the publication create a vision for school mathematics, but it also offered a way to focus curricula while addressing process standards (problem solving, reasoning and proof, communication, connections, and representation) as important elements of instruction (NCTM, 2000). Further, NCTM published *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* in 2006 to build from *Principles and Standards. Curriculum Focal Points* outlined important mathematical topics for each grade level through deep mathematical understanding offering "opportunities for improving both the teaching and learning of mathematics" (NCTM, 2006, p. 1) at a time when state standards lacked clarity, focus, and consistency. The focus on process standards within these documents served as a complement to the balanced approach of conceptual and procedural understanding.

Political actions at the national level did not support the balance in mathematical knowledge that both the NRC and NCTM were promoting. In 2001, the NCLB Act required each state to regulate students' progress in reading and mathematics in grades 3-8, and once during 10-12 grades, through high-stakes testing (No Child Left Behind, 2022; Rothman, 2011). In addition, NCLB also required states to establish a definition of proficiency and set annual goals to demonstrate that AYP (Larson & Kanold, 2016; Watt, 2011) had been attained. Further, NCLB (2022) mandated schools to report to parents and the public on school performance and teacher quality and increase the qualifications of teachers and paraprofessionals.

This consequently resulted in a plethora of variations in what states deemed as levels of student achievement with assessments aligned poorly to standards. To avoid detrimental consequences outlined in the NCLB act, sometimes unsavory practices were used to meet the demands of AYP (Confrey, 2007; LaVenia et al., 2015) resulting in an instructional focus on procedural skills to attain high scores on standardized assessments (Larson & Kanold, 2016). In 2008, the National Mathematics Advisory Panel recommended that to reflect research, the curriculum should "simultaneously develop conceptual understanding, computational fluency, and problem-solving skills" (National Mathematics Advisory Panel, 2008, p. xix). This declaration signified the end of the math wars (Larson & Kanold, 2016).

<u>The Common Core State Standards (2010s-present)</u>

The effort toward common standards, coordinated through the NGA and CCSSO, was led by state leaders, including governors and state commissioners (LaVenia et al., 2015; NGA & CCSSO, 2010). The goal of the CCSSM was to prepare all students, regardless of their socio-economic status, race, religion, or gender, to be college and career ready at the conclusion of their high school education (Larson & Kanold, 2016; NGA & CCSSO, 2010). In addition, the standards were intended to eliminate inconsistencies across state standards while addressing content at a deeper level and set proficiency standards to equivalent levels of international performance (NGA & CCSSO, 2010). The CCSSM writing team used some of the same seminal mathematics publications (*Principles and Standards for School Mathematics, Adding it Up,* and *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics*) previously used by states, in conjunction with other research, to create a vision for both student learning and goals for K-12 mathematics education (Watt, 2011).

There was initially bipartisan support for the CCSSM, indicating that education was a priority for policymakers. Within a year of publication, 45 states and the District of Columbia had adopted the standards (Jochim & Lavery, 2015; Reys et al., 2013). At the height of the implementation of the CCSSM, researchers flocked to unveil and understand all components of the CCSSM ranging from research pertaining to how states planned to engage educators in professional development to the differences between the CCSSM and the preceding standards. Shortly after the implementation of the standards, a nationwide debate ensued regarding the acceptable levels of governmental control that should occur in the public education system and the effectiveness of the CCSSM (Jochim & Lavery, 2015; Watt, 2011). This widespread criticism led to some states renaming their standards to appear as though they had shed their state of the CCSSM, or that entirely new standards were developed (Lavenia et al, 2015). This phenomenon leads back to the initial concern of inconsistencies and lack of cohesion across state mathematics standards.

The remainder of this chapter will focus on the elements included in the four sub questions for this study. The elements include the process and practice standards, domains in standards, learning trajectories, and conceptual and procedural knowledge. Additionally, the guiding framework used for this study is introduced.

Process and Practice Standards

The CCSSM SMPs place emphasis on describing *how* to do mathematics. They unveil the variety of skills in which students interact with mathematics and can represent mathematics through experiences. These imperative standards are a necessary component in each K-5 mathematics classroom as they are a “significant aspect of learning mathematics” (Koestler et al., 2013, p. vi) and truly should be infused in all topics and grade levels of mathematics, as they are not to be taught as isolated standards but are intended to be embedded elements of mathematics instruction daily. The SMPs further support the development of attaining mathematics knowledge and are based on the process standards recommendations of NCTM (2000).

The process standards, published in *Principles and Standards for School Mathematics* (NCTM, 2000), describe how students are expected to engage in mathematics. These are rooted in the ways that students are expected to learn mathematics. The process standards include: problem solving, reasoning and proof, communication, connections, and representation. Each of these elements played a significant role in the development of the SMPs and were recommendations based heavily on research conducted by expert writing teams through NCTM. They provided further guidance during analysis as a guiding frame to states' practice and process standards that differed from the SMPs.

Domains in Standards

The CCSSM content standards are organized into three elements (NGA & CCSSO, 2010), which are comprised of standards, clusters, and domains. The standards "define what students should understand and be able to do" (p. 5). The clusters are "groups of related standards" (p. 5). Finally, domains are "larger groups of related standards" (p. 5) that can be found in a cluster of grades and represent mathematical concepts (Dacey & Polly, 2012). The K-5 domains in the CCSSM include: (a) counting and cardinality, (b) operations and algebraic thinking, (c) number and operations in base ten, (d) measurement and data, (e) geometry, and (f) number and operations-fractions (NGA & CCSSO, 2010).

An integral aspect of the domains is for students to have opportunities to develop an understanding of connections among the domains. As the domains span various grade levels and include multiple topics, the interconnectedness of each domain should not be lost in the development of student understanding. This notion further supports the connections between conceptual and procedural knowledge (Dacey & Polly, 2012). The standards themselves develop both conceptual and procedural understanding as the goal of proficiency is attained.

Prior to the development of the CCSSM, each individual state developed its own standards (Confrey, et al., 2014), resulting in a wide variety of content and topic coverage. The inconsistencies resulted in consideration by the writing team for the CCSSM on how students learn while sequencing the topics in the domains and standards (NGA & CCSSO, 2010). The CCSSM writing team considered the most important mathematical ideas from high school and worked backward to create standards that supported the development of those mathematical ideas (Maloney et al., 2014). This supported their goal of developing big ideas over long periods of time (or following a learning trajectory for each domain), which provides educators with a better source to support gradual learning over time (Confrey et al., 2014).

Learning Trajectories

Progressions of mathematical topics are the framework for consideration made by the writing team for the CCSSM, as the standards were written for a cumulative growth of increasingly sophisticated knowledge (NGA & CCSSO, 2010). The specific focus of research on large-scale comparative studies provided details on children's developmental milestones associated with development of specific mathematical abilities (Common Core Standards Writing Team, 2013). Confrey et al. (2014) posed concerns that learning trajectories might be treated as a "cure all" associated with the CCSSM and might be considered a fad despite the notion that learning trajectories provide scientific foundations associated with learning of ideas over time. The concerns raised by Confrey et al. (2014) should be considered by all that work with mathematics standards, and especially those associated with states that are revising standards to ensure that students' beginning and end goals in mathematics learning are set through scientific-based evidence. This section shares the varying definitions established within the field of mathematics education, briefly describes trajectory models, and relates learning trajectories to standards and curriculum.

There are varying names that are synonymously used to represent the idea of learning trajectories within the field of mathematics education. This includes, "learning trajectories," "learning progressions," and "learning paths" (Common Core Writing Team, 2013). With the synonymous use of terms, there are varying definitions used as well. Confrey et al. (2022) define a learning trajectory as being "empirically grounded descriptions of how students' reasoning evolves from less to more sophisticated" (p. 90). Battista (2011) signified a difference between trajectories and progressions by stating that, "trajectories include descriptions of instruction, progressions do not" (p. 512). Learning trajectories, as defined by Clements and Sarama (2004), are:

> Descriptions of children's thinking and learning in a specific mathematical domain, and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain. (p. 83)

While this definition is readily accepted today, research on learning trajectories in the past has highlighted linear sequences within their learning trajectories in terms of student achievement of one level moved the students to the next level (Baroody, 1987; Carpenter, et al., 1999; Steffe & Cobb, 1988). Research about students' thinking and ultimately the progress of that thinking occurred well before the terms "learning progressions" or "learning trajectory" were used in mathematics education research (Carpenter & Moser, 1984; Erlwanger, 1973; Gibb, 1956; Piaget, 1970; Vergnaud, 1982; Vergnaud, 1988; Vygotsky, 1978). These ideas stem back to two theories. The first is Piaget's Theory on Constructivist Learning, with constructivism defined as an adaptive learning approach with the belief that humans construct knowledge and meaning from their experiences (Fosnot, 2005). The second is Vygotsky's Theory of Socio-Constructivism, which asserts that individuals actively participate in the creation of their knowledge in social and cultural settings (Schreiber & Valle, 2013).

With roots in constructivism and socio-constructivism (Confrey, 2019), the beginning of learning progressions and trajectories has been cited back to two sources (Clements & Sarama, 2004). The first source is Realistic Mathematics Education (RME) (Gravemeijer, 1994), a domain specific theory of mathematics education developed in the late 1960's (Van den Heuvel-Panhuizen & Drijvers, 2020) that has been credited with roots for learning trajectories. The other source is the seminal work of Simon (1995), where the term "hypothetical learning trajectory" in the mathematics education space was first used.

Trajectory Models

One of the original trajectory models, RME, uses an approach to establish guidelines for the sequence of tasks through the study of students' solutions (Gravemeijer, 1994). This theory is built around contexts to develop mathematics concepts (Confrey, 2019). An element of RME is the notion of "levels" (Confrey, 2019) where children can freely move between levels, whether higher or lower, based on their understanding of a topic (Gravemeijer, 1994).

Another trajectory model came from Simon (1995), who proposed the idea of a "hypothetical learning trajectory" as a reference to the predicted learning path students might encounter. Simon was in a dual role of both being the teacher and researcher at the time and emphasized that the learning trajectory is not known in advance, which is the reason the path is hypothetical. Through his definition, the hypothetical learning trajectory constitutes three parts, "the learning goal that defines the direction, the learning activities, and the hypothetical learning process- a prediction of how the students' thinking, and understanding will evolve in the context of the learning activities" (Simon, 1995, p. 136).

Additionally, the term *hierarchical interactionalism* trajectory is used as a "'natural' developmental progression and helps describe and justify a learning trajectory (Sarama & Clements, 2009, p. 4). This framework illustrates types of knowledge that can develop simultaneously and is best thought about using the metaphor of climbing a wall (Confrey & Toutkoushian, 2019). This metaphor shows particular thinking developing over time with the movement of thinking occurring fluidly in both vertical directions. Further, Clements and Sarama (2004), established that "any constructed learning trajectory is hypothesized to be a productive route, but not necessarily the most productive route for all students" (p. 84).

<u>Progress of Trajectories</u>

Significant strides have been made in the past two decades regarding research on mathematics learning trajectories in specific domains. Learning trajectories are based on theoretical frameworks and years of research with consideration for the grain size of the levels being studied based on the purpose of the trajectory (Confrey, 2019). Unlike in original learning models, today's conception of learning trajectories reinforces that students' progress through a trajectory that is not strictly linear (Confrey et al., 2014).

Confrey et al. (2014), label learning trajectories with the status of having progressed to an instructional tool that can, “scaffold instruction and be enacted through instruction in the classroom” (p. 63), with commonalities across learning trajectories that help show how students’ thinking can progress to more sophisticated levels of understanding and reasoning in mathematical concepts (Maloney et al., 2014). Sarama and Clements (2009) determined that a learning trajectory “has three parts: a specific mathematical goal, a developmental path along which children develop to reach that goal, and a set of instructional activities that help children move along that path” (p. 2). Additionally, learning trajectories can provide support as a language (Confrey et al., 2014) for educators to strengthen mathematical practices.

Relating Learning Trajectories to Standards

Typically, standards define what students should know and be able to do by attaching prescribed time frames (e.g., grade level) to select clusters of topics (NGA & CCSSO, 2010). Before the creation of the CCSSM, standards in some states served as a mere checklist of topics to be taught (Daro et al., 2011; NRC, 2001) without consideration of the interconnectedness of mathematics topics with vertical alignment across grades or horizontal alignment within a grade (Duschl et al., 2011). The standards were used as a checklist within the instructional materials adoption as districts and schools, a practice currently still used by some states to ensure alignment between standards and curriculum. The CCSSM writing team opted to use three areas of research to inform decisions they made about mathematics topics and subsequently associated segments of the learning progressions with grade levels to create the common standards (Daro et al., 2011; NGA & CCSSO, 2010). These areas include: (a) large-scale comparative studies, (b) learning trajectories, and (c) other research on cognition and learning (Zimba, 2014).

The writing team referred to the research used for decisions as both learning trajectories and learning progressions (NGA & CCSSO, 2010; Confrey, 2019), taking into careful consideration the hierarchical logic of mathematical topics when writing the standards (NGA & CCSSO, 2010). To further support stakeholder understanding of the standards, the writing team created progression documents for each domain and grade band. They ultimately opted to call published supporting documents "learning progressions."

Relating Learning Trajectories to Curriculum

From the standpoint of mathematics instruction, at a minimum, educators should know the mathematics within their instructional grade level and the mathematics that precedes and follows their grade level (Dixon et al., 2016; NCTM, 2006). This span of grade-level understanding is referred to as vertical coherence (Confrey et al., 2022). If curriculum, professional development, and preparation of preservice educators cohesively target the understanding of learning progressions of mathematics topics, educators have an avenue for "supporting both the learner who struggles and the learner who needs enrichment" (Dixon et al., 2016, p. 7). The Common Core progression documents support this notion, as each domain provides guidance through the entire grade band and ends the document with a section describing where the domain progression is heading. Karp et al. (2021) reiterate that purposefully designing instruction with tasks that "focus on important big ideas can lead to a much more rigorous approach to the teaching of mathematical topics" (p. 98).

Curriculum traditionally falls short in terms of incorporating learning trajectories. This is an important consideration as "what is actually taught in classrooms is strongly influenced by the available textbooks because most educators use textbooks as their primary instructional material" (NRC, 2001, p. 36). Textbooks, including supplemental materials prior to the CCSSM, failed to address progressions of student thinking (NRC, 2001) both with tasks introduced to students and for educator understanding, which negatively impacts student learning. The lack of inclusion of learning trajectories within the curriculum results in fragmented learning (Clements, 2007) that tends to lead to large review sections incorporated in the textbook and a statistically lower chance of proficiency (NRC, 2001). Additionally, textbooks were intended to be manufactured and sold to as many states as possible, which did not allow for alignment between textbooks and each set of state standards (Rothman, 2011). Twenty states participate in textbook adoption at the state level (Education Commission of the States, 2022), with policies varying on textbook alignment criteria and procedures.

This dissertation serves to inform policymakers, curriculum developers, mathematical instructional leaders, teachers, administrators, researchers, and other stakeholders on whether, and the extent to which, revised state standards are reverting back to fragmented, disconnected topics as learning objectives. Learning trajectories are already considered a structure for improving instruction in mathematics (Confrey, et al., 2014; Daro et al., 2011) and can serve as guidance for states that are ultimately transitioning away from the CCSSM.

Conceptual and Procedural Knowledge

The two terms regarding the types of knowledge can be attributed to Hiebert and Lefevre (1986), who defined conceptual knowledge as "knowledge that is rich in relationships, much like a connected web of knowledge where linking relationships connect the siloed facts and pieces of information together" (pp. 3-4). They defined procedural knowledge as the "familiarity with individual symbols and syntactic conventions which consists of rules or procedures" (pp. 7-8). These two types of knowledge cannot be separated (Rittle-Johnson & Alibali, 1999) and influence the development of each other (Byrnes & Wasik, 1991; Rittle-Johnson et al., 2001).

The ongoing societal debate of *what* topics need to be taught in mathematics and *how* the topics should be taught (NRC, 2001) typically falls to one side of the continuum or the other; teach procedurally or teach conceptually (Rittle-Johnson et al., 2001). However, teaching to only one continuum doesn't fully develop the mathematical knowledge of students (Rittle-Johnson et al., 2001) and has been considered one of the most persistent problems in mathematics education (NRC, 2001). Teaching conceptually with the goal of understanding is much more difficult to attain (Von Glasersfeld & Steffe, 1991) than reinforcement of procedural skills. Numerous factors contribute to the decision to teach to one continuum or the other.

The pedagogical beliefs of teachers on *how* to teach include their experiences as students (Schmidt et al., *in-review),* their collegiate education courses (Schmidt et al., *in-review*), access to curriculum resources (Remillard, 2005), and their exposure to professional development (NRC, 2001) on the two types of understanding. Time is another contributing factor as teaching conceptually is attributed to being the more time-consuming of the two types of understanding (Pesek & Kirshner, 2000). Additionally, pressure from administrators to raise test scores results in educators spending more time emphasizing procedural knowledge as pacing guides and curriculum outline a specific number of days content can be taught to stay "on pace" to teach all topics before an end-of-year high-stakes assessment (Pesek & Kirshner, 2000).

Connecting conceptual understanding and procedural fluency is imperative to help students develop meaning for symbols, recall procedures, and use procedures (Hiebert & Lefevre, 1986). "Students are not fully competent in mathematics if either kind of knowledge is deficient or if they both have been acquired but remain separate entities" (Hiebert & Lefevre, 1986; p. 9). Further, if procedures lack connections to conceptual knowledge, the procedures can deteriorate quickly (Hiebert & Lefevre, 1986). An understanding of both concepts and procedures develops a complete understanding of mathematics (Ohlsson & Rees, 1991; Rittle-Johnson et al., 2015).

In the past decade, the synthesis of research provides the current recommendation to build procedural fluency from conceptual understanding (Clements, et al., 2017; NCTM, 2020; NRC, 2001). Larson and Kanold (2016) sum this recommendation up as the best way forward for mathematics education is through the equilibrium position, which “balances the emphasis on procedures and conceptual understanding” (Larson & Kanold, 2016, p. 41). Connecting procedures to underlying concepts allows students to better retrieve and use the procedures (Fuson et al., 2005).

Conceptual Framework

To better understand the shifts occurring in selected state mathematics standards, an adapted version of the proficiencies, processes, and practices from NCTM’s *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations* (2020), will serve as the conceptual framework for this study. The adapted version of the proficiencies, processes, and practices include elements from the following three documents: (1) the NRC’s (2001) *Adding it Up*; (2) the CCSSM SMPs (NGA & CCSSO, 2010); and (3) NCTM’s *Principles and Standards for School Mathematics* (2000). Each document includes pertinent information that guides what needs to be included in mathematics standards. Figure 2 identifies an adapted version of the proficiencies, process, and practices outlined in *Catalyzing Change in Early Childhood and Elementary Mathematics*: *Initiating Critical Conversation* (2020), specifically the components that guide the analysis of this dissertation. Following the figure, a description of how this serves as a guiding framework for each of the four research sub questions of this study is provided.

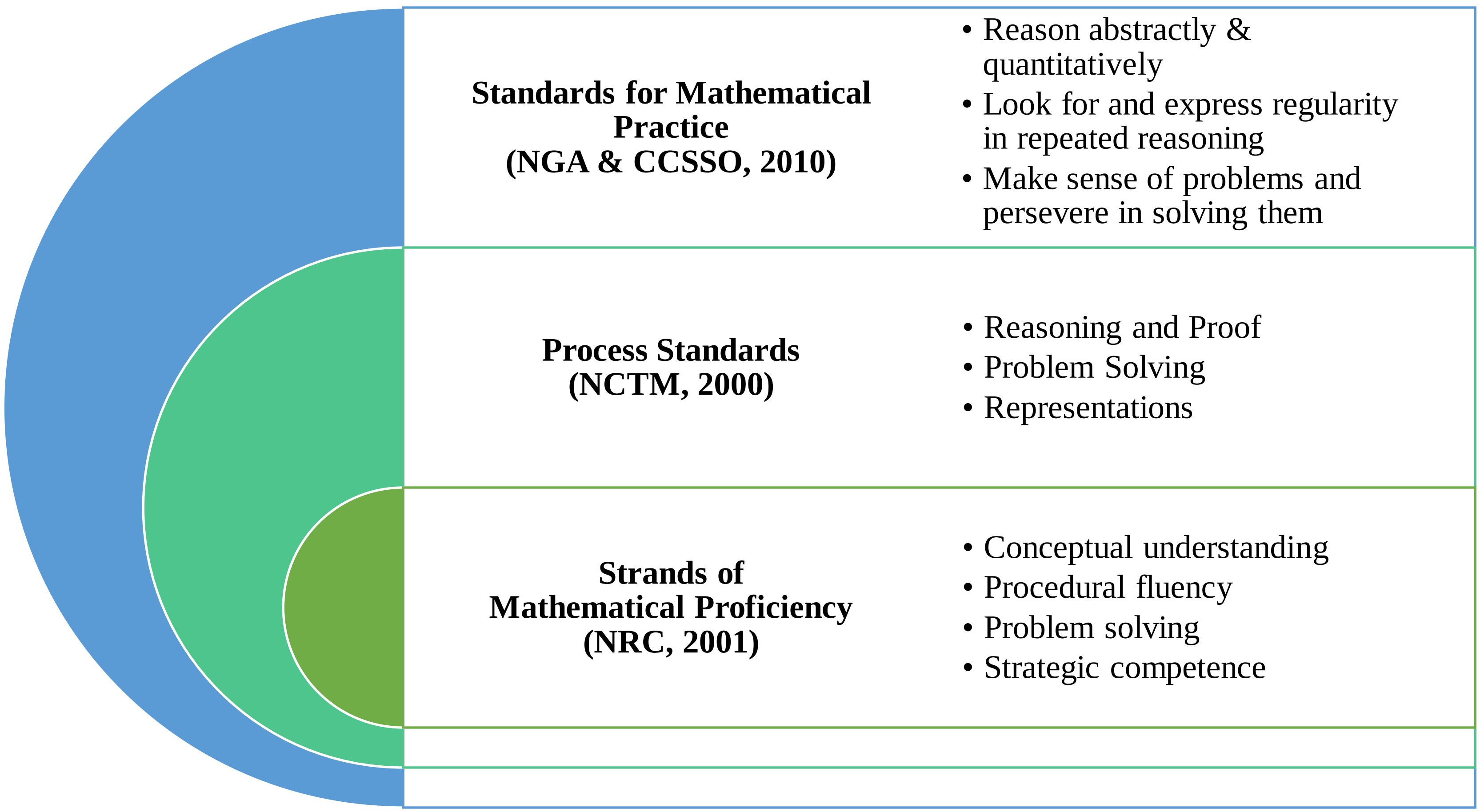

Figure 2

Mathematics Proficiencies, Processes, and Practices Specifically Guiding the Work of This Study

Note. Adapted from *Catalyzing Change in Early Childhood and Elementary Mathematics* (NCTM, 2020).

One of the elements of the framework, the Strands of Mathematical Proficiency, came from the NRC's publication *Adding it Up* (2001), which combines research in cognitive psychology and mathematics education to define what it means for students to learn mathematics successfully. This allowed for *Adding it Up* to play a pivotal role in education reform as the document included research-based recommendations that called for changes in the mathematics curriculum, instructional materials, assessments, classroom practices, teacher preparation, and professional learning opportunities. Three strands of mathematical proficiency (conceptual understanding, procedural fluency, and strategic competence) are all interwoven and interdependent components that guide this study. The editors of *Adding it Up* compiled their extensive experiences in mathematics education with research in cognitive science to complete the comprehensive view of the strands of mathematical proficiency. While the editors warned that mathematical proficiency cannot be achieved by focusing on a singular strand, standards documents in this study were analyzed based on changes in the observable strands (conceptual understanding, procedural fluency, and strategic competence). Unobservable strands (adaptive reasoning and productive disposition) rely on both the selection of tasks and students' observable behaviors while interacting with mathematical tasks, and, therefore, were excluded from the framework for this study.

The development of the overarching research question and the four sub questions for this study were framed around the relationships visualized in Figure 3. Figure 3 shows the relationships between the creation of the CCSSM being informed by learning trajectories, which helps determine if standards are focused on conceptual understanding or procedural fluency. It should be noted that standards can reflect a focus on both conceptual understanding and procedural fluency. Learning trajectories influence the domains that exist within the standards.

Finally, the use of processes and practices both support student understanding of standards and can allow students opportunities to demonstrate conceptual understanding and procedural fluency but does not always do both, which is why the word “or” is used in the figure. Each component of the figure was carefully considered in the development of the research questions used in this study. The methodology of qualitative content analysis was used to examine the changes that have occurred among United States K-5 state mathematics standards.

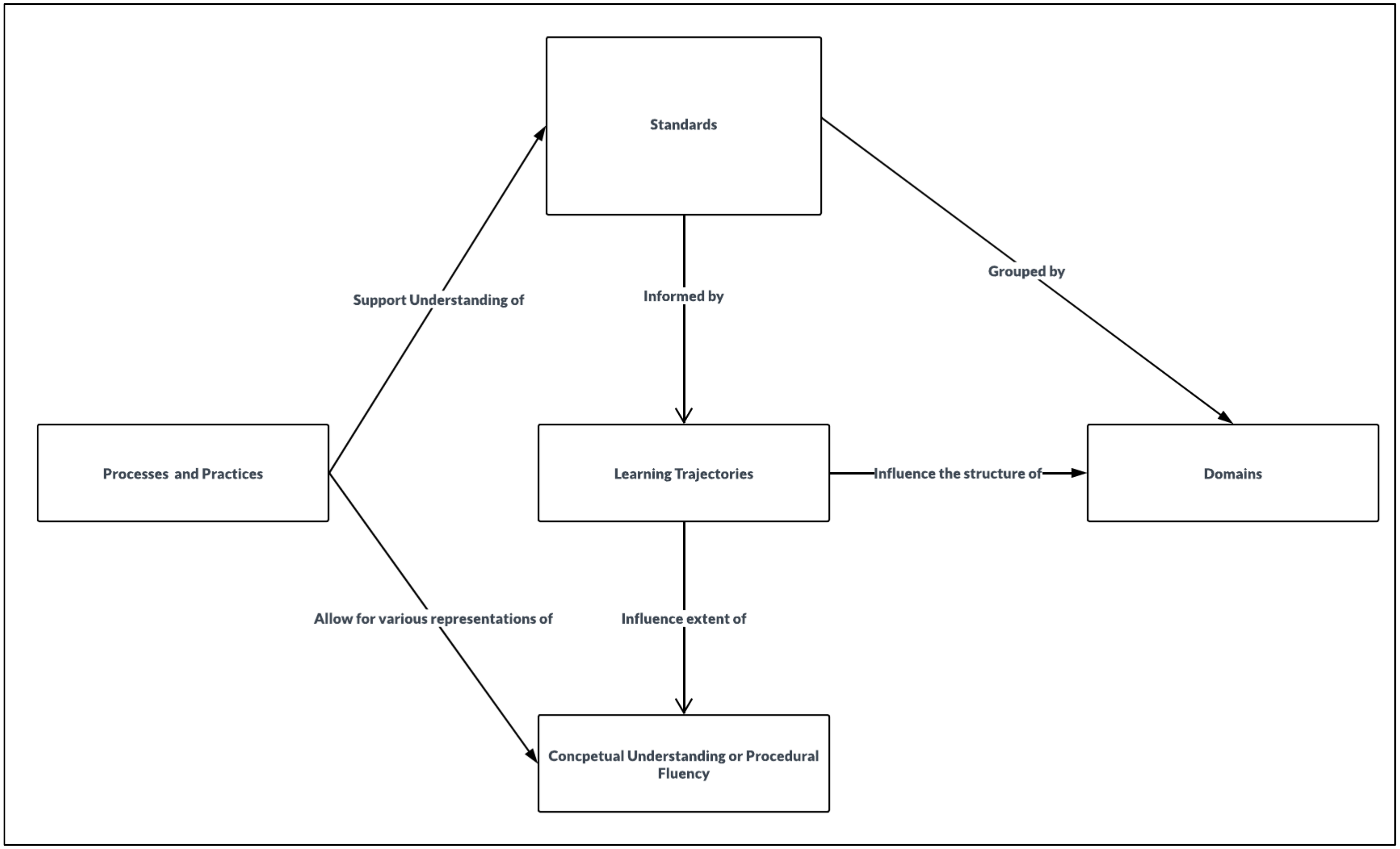

Figure 3

Connections Guiding the Development of Research Questions

Research sub question 1 (*In what ways, if any, do K-5 state mathematics student process and practice standards differ from the CCSSM?*) was formulated based on the SMP component of the guiding framework, where the expertise and knowledge that contributed to the development of the SMPs were used as a baseline to compare to states that have differing practice and process standards. Each CCSSM SMP was examined against any state practice standard document for differences among terminology. There are overlapping elements to each of the proficiency, process, and practice standards, with some elements using differing names but representing the same ideas. Each of the three documents serve as integral, interwoven components to this study. Not all these elements in the documents were focal points of this study.

Research sub question 2 (*In what ways, if any, do K-5 state mathematics standards content domains differ from the CCSSM*?) was based on some of the content standards also found in the *Principles and Standards of School Mathematics* publication (NCTM, 2000), which featured the process standards found in Figure 3. While this was not a component of the table found in *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Issues*, it was addressed in this study similarly to the research sub question 1. Each domain in state standards mathematics document was examined against the domains found in the CCSSM, which were formulated from the *Principles and Standards of School Mathematics* (NCTM, 2000).

As standards documents were being analyzed, each department of education website was also examined for the use of the word "learning trajectory" or "learning progression" as evidence of the use of learning trajectories during the standards revision process. This addressed research sub question 3 (*In what ways, if any, do states describe how learning trajectories are addressed in K-5 state mathematics standards?)*.

Research sub question 4 (*In what ways, if any, is the relationship between procedural and conceptual learning outcomes represented in K-5 state mathematics standards?*) relates to elements from each of the documents in Figure 3. *Conceptual understanding* "refers to an integrated and functional grasp of mathematical ideas" (NRC, 2001, p. 118), which details how students *know* mathematics. The editors of *Adding it Up* described that an indicator of conceptual understanding is when the presence of multiple strategies for a topic are represented (NRC, 2001). This connects with *procedural fluency*, as the knowledge of procedures and when to use them to support the conceptual understanding of a topic (NRC, 2001). *Strategic competence* requires students to "formulate mathematical problems, represent them, and solve them" (NRC, 2001, p. 124), which complements both strands previously mentioned. Evidence of strategic competence was examined within codes developed from changes made to standards by examining the phrases that had changed. The phrases were sorted into the categories of conceptual, procedural, or both, which reflected both conceptual and procedural learning outcomes.

Summary

In conclusion, this literature review discussed the reform movements in mathematics education since the 1950s. Then, literature pertaining to the four research sub questions and guiding frameworks was addressed. This included four elements: (1) process and practice standards; (2) content domains; (3) learning trajectories; and (4) conceptual and procedural knowledge. Each of the four elements were interwoven components of my conceptual framework. They significantly impacted the formation of my overarching research question: *In what ways, if any, do K-5 state mathematics standards differ from the CCSSM?*

Four research sub questions for this study were as follows:

1) *In what ways, if any, do K-5 state mathematics student process and practice standards differ from the CCSSM?*
2) *In what ways, if any, do K-5 state mathematics standards content domains differ from the CCSSM?*
3) *In what ways, if any, do states describe how learning trajectories are addressed in K-5 state mathematics standards?*
4) *In what ways, if any, is the relationship between procedural and conceptual learning outcomes represented in K-5 state mathematics standards?*

Chapter 3 describes the research methodology used in this study. First, a rationale for the use of qualitative research and the chosen methodology is discussed. Details regarding the research design are disclosed including the phases of the analysis of data. Next, ethical issues are reported. Finally, delimitations of the study are disclosed.

CHAPTER THREE: METHODOLOGY

This chapter describes the research methodology used in this study. First, a rationale for the use of qualitative research and the methodology is discussed. Next, the research design including the data sources collected along with the inclusion and exclusion criteria for the sources are shared. Then, the phases of analyzing data are described and ethical issues that were addressed throughout the research phases are reviewed. Finally, delimitations are shared with a summary concluding the chapter.

Restatement of Purpose and Research Questions

The purpose of this research study was to examine how United States K-5 state mathematics standards have changed during the past decade (2012-2022). The time frame 2012-2022 was chosen for this study because by the year 2012 most states had either already adopted the CCSSM or had decided to not adopt the CCSSM. This study focused on states with standards substantially different from the CCSSM. States of interest included states that never adopted the CCSSM and states that have amended or repealed their prior adoption of the mathematics standards in the past decade. A state was excluded from the study if revisions of their standards basically aligned with CCSSM (e.g. a word was added to an existing CCSSM, a clarification was added, the only change was the name of the document) and are referred to in this study as emulative standards.

This study sought to answer the overarching research question: *In what ways, if any, do K-5 state mathematics standard differ from the CCSSM?* Answering the overarching research question was accomplished by addressing the following four sub questions:

1) *In what ways, if any, do K-5 state mathematics student process and practice standards differ from the CCSSM?*

2) *In what ways, if any, do K-5 state mathematics standards content domains differ from the CCSSM?*
3) *In what ways, if any, do states describe how learning trajectories are addressed in K-5 state mathematics standards?*
4) *In what ways, if any, is the relationship between procedural and conceptual learning outcomes represented in K-5 state mathematics standards?*

Rationale for Qualitative Research

Existing research on analyzing mathematics standards prior to or during the initial adoption of the CCSSM can be classified into two categories (Tran et al., 2016). These categories include either rating the quality of standards in comparison to previous state standards or describing the similarities and differences between or within sets of standards. Figure 4 provides organization of these quantitative studies related to standards adoption post-CCSSM.

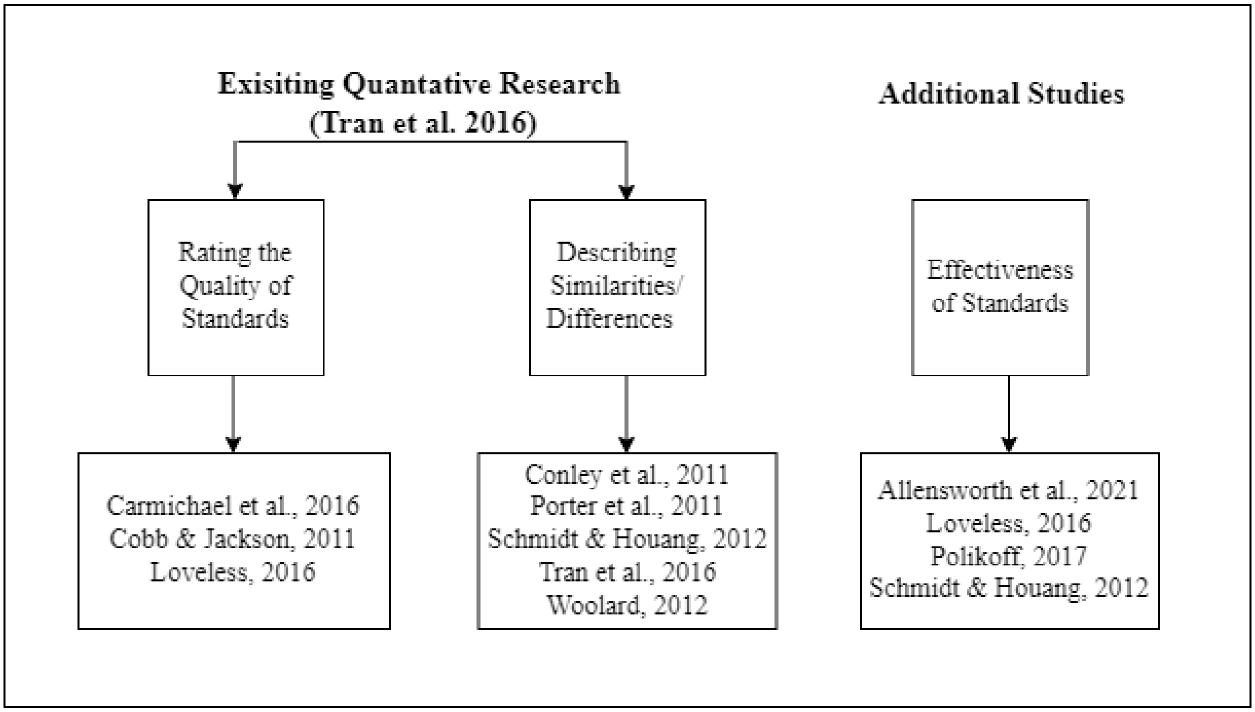

Figure 4

Research Post-CCSSM

Additional studies include the effectiveness of the standards; however, the studies in this category typically examined test scores (Allensworth, et al., 2021; Loveless, 2016; Polikoff, 2017; Schmidt & Houang, 2012). Also, multiple studies have been conducted by the Thomas B. Fordham Institute with their reports relying on quantitative analysis and assigning letter grades (Raimi & Braden,1998; Wilson et al., 2005) or an overall numerical rating (Friedberg et al., 2020; Griffith & McDougald, 2018; Petrilli & Finn, 2000) of the standards.

While these studies have provided valuable information to the field, they have solely relied on quantitative research. The field still needs to be informed of the specific changes as revisions to standards are occurring because the responsibility of educating future generations relies on the best-formulated standards to guide instruction. Qualitative research lends itself to the ability to contribute to the understanding of contexts and situations with human interpretation

(Hsieh & Shannon, 2005; Mayring, 2022). Relying solely on quantitative analysis removes the human factor from research and analysis (Bogdan & Biklen, 1998) which can have dramatic impacts on perceptions of standards. This dissertation study uniquely applied qualitative methodologies to gain in-depth insights into the distinctions regarding the changes that occurred in United States K-5 state mathematics standards from 2012-2022.

This study examined the ways in which practices, domains, learning trajectories, and conceptual and procedural learning outcomes in K-5 state mathematics standards differ from the CCSSM. The goal of this dissertation study is to provide valuable insights to the broad body of mathematics education stakeholders regarding the changes occurring to United States K-5 mathematics state standards. Using qualitative research as an approach provided a varying perspective to the already established research regarding United States K-5 state mathematics standards and provided more specific insights into the differences and nuances of such revised state standards.

Research Methodology

To better understand how standards have changed since 2012, I used qualitative content analysis as the methodology to complete this study. Qualitative content analysis is considered a method of analysis used on text (Mayring, 2022) or images/graphics (Krippendorff, 2004). Content analysis analyzes textual data for themes or patterns while shedding insight into a particular phenomenon (Hsieh & Shannon, 2005; Krippendorff, 2004). The use of content analysis directly addresses the research question and sub questions while allowing the opportunity to develop a deeper understanding of the phenomenon currently occurring in mathematics standards. Krippendorff's (2004) definition of content analysis was used for this study, which defines the methodology as a research technique that makes valid, replicable inferences from text resources.

Also, qualitative content analysis provided the advantage of observing connections through altered wording within documents to the absence of a process or practice. The meanings in the observations were able to be coded (instead of quantified) in this study. Finally, the use of qualitative content analysis provided a detailed description of the changes to United States K-5 state mathematics standards that recognizes the varying word choices across states and presents the changes with context to stakeholders (rather than only numerically).

Research Design

The analysis of the ways in which United States K-5 state mathematics standards changed from 2012-2022 was conducted. State standards documents and supporting documents created during the past decade were examined. The year 2012 was used as the first year of the analysis as states at that point had either adopted, implemented, declined to adopt, or revised the CCSSM. The standards documents and supporting documents were used as the data sources for this study.

Study Inclusion and Exclusion Criteria

The documents under consideration for this study were the K-5 state mathematics standards for each state in the United States. The inclusion criteria for this study were any state that had never adopted the CCSSM and any state that had amended or repealed its state standards since the adoption of the CCSSM. States were excluded from the study if their standards were emulative standards (e.g., revisions were made that only included a name change to the standard, only minor revisions were made to the standards such as a word was added to an existing CCSSM, or a clarification was added).

Criteria for inclusion in this study began with a preliminary search. The preliminary search on the CCSS website (NGA & CCSSO, 2010) indicated that five states (Alaska, Minnesota, Nebraska, Texas, and Virginia) never adopted the CCSSM, with Minnesota never adopting the mathematics standards. Additional searches on the state department of education websites indicated that ten states (Alabama, Arizona, Florida, Indiana, Kentucky, New York, Nevada, Oklahoma, South Carolina, and Tennessee) at one point had adopted the CCSSM but have since repealed the standards and created their own state standards with varying alignment with the CCSSM. These fifteen states and their subsequent documents were of interest for this study. Emulative standards, or states that demonstrated minimal changes or were by and large the CCSSM without the title of CCSSM, were not considered for this study. Table 5 provides an overview of the states considered for this study that never adopted the CCSSM. The table also provides the year of their most recent revision to their standards.

Table 5

States That Never Adopted the CCSSM

State	Current Standards Name	Last Year of Standards Revision
Alaska	Alaska Mathematics Standards	2012
Minnesota	Minnesota K-12 Academic Standards in Mathematics	2021
Nebraska	Nebraska’s College and Career Ready Standards for Mathematics	2015
Texas	Texas Essential Knowledge and Skills	2012
Virginia	Mathematics Standards of Learning for Virginia Public Schools	2016

Table 6 provides an overview of states considered for this study that had once adopted the CCSSM and have since appealed, rescinded, or rewritten the CCSSM.

Table 6

States That Had Once Adopted the CCSSM

State	Current Standards Name	Year CCSSM was Adopted	Year CCSSM was Repealed, Rescinded, or Rewritten
Alabama	Alabama Course of Study	2010	2013
Arizona	Arizona Mathematics Standards	2010	2015
Florida	Benchmarks for Excellent Student Thinking (B.E.S.T.)	2010	2019
Indiana	Indiana Academic Standards	2010	2014
Kentucky	Kentucky Academic Standards	2010	2017
Nevada	Nevada Academic Content Standards in Mathematics	2010	2012
New York	New York State Next Generation Mathematics Learning Standards	2010	2015
Oklahoma	Oklahoma Academic Standards for Mathematics	2010	2014
South Carolina	South Carolina College and Career Ready Standards	2010	2014
Tennessee	Tennessee Academic Standards for Mathematics	2010	2016

Data Sources

Multiple documents from publicly available websites were collected as primary data sources. The documents included: (1) the K-5 Common Core State Standards for Mathematics, (2) the K-5 Standards of Mathematical Practice, (3) current K-5 state mathematics standards for the 15 states included in this study, (4) current K-5 state process and/or practice standards for the 15 states included in this study and (5) supplemental K-5 state standard documents that included additional information for the 15 states. Such supplemental documents were collected based on availability by states, which included any supporting guidance pertaining to the literature used during the revision of standards, the process of writing the adjusted standards, or standards writing team information. All documents were collected from the individual state department of education websites. Each state had different supplemental documents, which resulted in a vast range of information obtained, and is discussed in Chapter 4.

Data Analysis

The analysis of the United States K-5 state mathematics standards selected in this study included an in-depth comparison to the K-5 CCSSM with respect to:

- any differences in the student process and practice standards
- any differences in content domains
- evidence of learning trajectories through the K-5 vertical span
- the relationship between procedural and conceptual learning outcomes within the standards

This included an analysis of standards in states that never adopted the CCSSM. Even if a state did not adopt the CCSSM, the CCSSM was still used as a base-line set of standards. This decision was made due to the criteria set forth by the RTTTI, where no more than 15% of changes to the CCSSM standards were allowed to be considered for the initiative. If a state did not apply for the RTTTI it is disclosed within chapters 4 and 5.

The data analysis for this study can best be described in four phases. The phases were developed in accordance with the advice of Mayring (2015), which entails making decisions that were determined in advance to “how the material is to be approached, which parts are to be analyzed in what sequence, and what conditions must be obtained in order for an encoding to be carried out” (p. 372). Inductive analysis was used as an approach to analyze the documents except for when answering research sub question 4. The use of inductive analysis was chosen as knowledge regarding the phenomenon occurring is fragmented (Lauri & Kyngäs, 2005, as cited in Elo & Kyngäs, 2008). Research sub question 4 (*in what ways, if any, is the relationship between procedural and conceptual learning outcomes represented in K-5 state mathematics standards?*) relied on deductive analysis (Burns & Grove, 2005, as cited in Elo & Kyngäs, 2008). This is due to types of learning outcomes being based on established theories (Maloney et al., 2014).

Figure 5 provides a visual overview of the phases of the data analysis procedures used during the study. The figure describes the process used to consolidate documents from the 15 selected states to those with notable differences addressing the research question and sub questions posed. The four research sub questions were used as the criterion during the first phase of the analysis of all applicable United States K-5 state mathematics standards documents and supporting documents. A state was not selected to proceed beyond the coding process (as shown in Figure 5) if their standards were determined to have less than 15% of changes from the CCSSM.

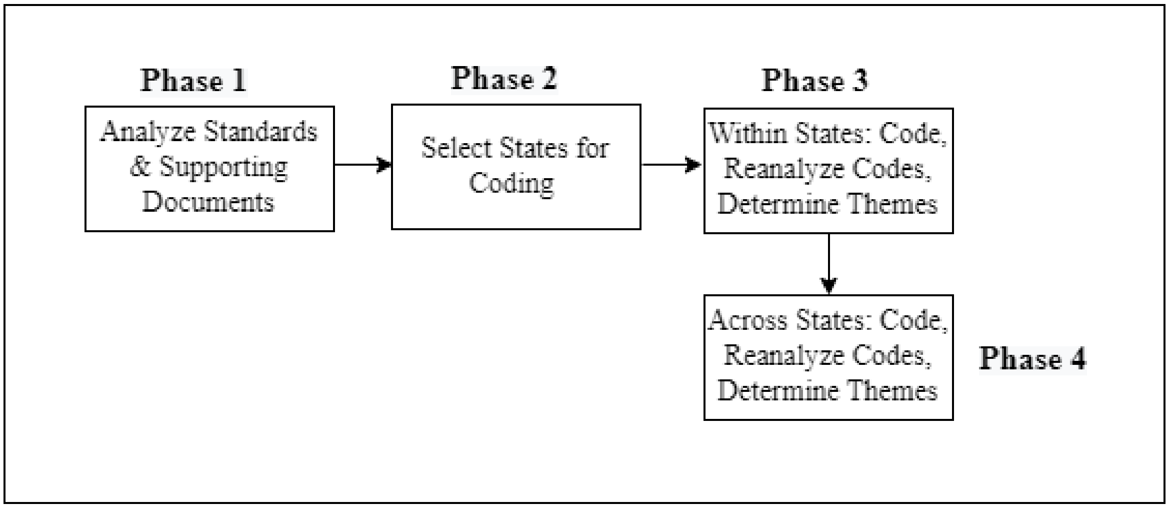

Figure 5

Data Analysis Phases

<u>Phase 1</u>

From phase 1 forward, a basic content analytical process was employed (Hsieh & Shannon, 2005; Mayring, 2022). A basic content analytical process consists of a research question, text material, specific content analytical technique(s), content analytical units, assignment of inductive categories to text, analysis of category distribution, and quality criteria (Mayring, 2022). This approach allows categories to flow from the data (Hsieh & Shannon, 2005). The data analysis process followed the steps outlined in Krippendorff (2004), which include (1) developing a coding scheme from the data, (2) developing categories, and (3) identifying emergent themes relying on a line-by-line coding technique (Mayring, 2022).

Figure 6 provides an overview of the coding process used during phases 2 - 4, with each stage being color-coded. The green stage is the initial stage where documents are analyzed, the orange stage is the second stage where coding and theme development is completed within individual states, and the yellow stage is the final stage of coding where themes are developed across states. The fourth stage (in purple) led to conclusions from the developed themes that include discussion of implications. The focal point of the stages addressed one research question at a time to organize the analysis process.

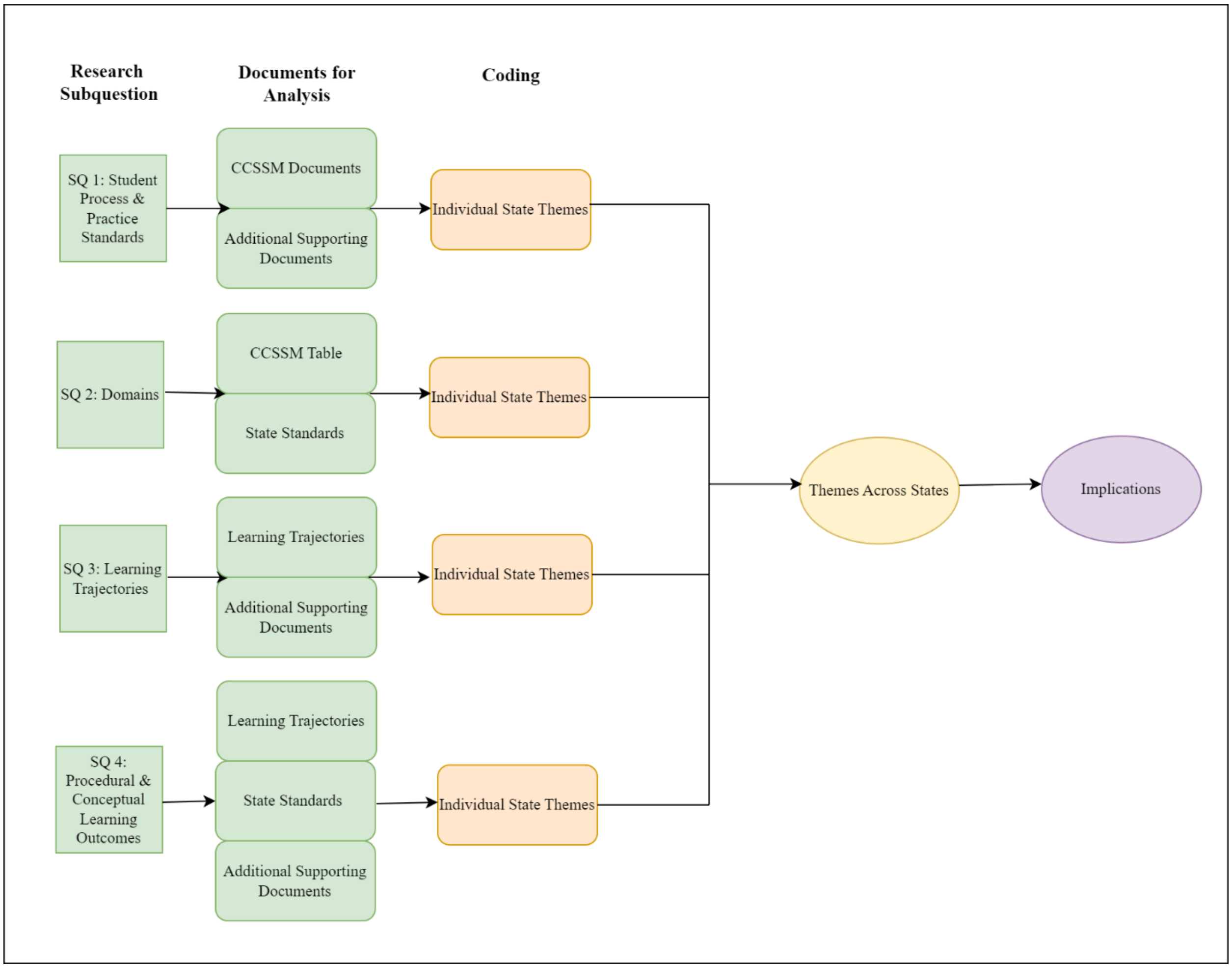

Figure 6

Phase 2-4 Coding Overview

As shown in Figure 6, the initial comparison was completed using at least two data sets at a time. The first data set was the CCSSM, and the second data set was a single state's revised K-5 mathematics standards. If additional state supporting documents were needed to complete the analysis, they served as a third document. Both the CCSSM and K-5 state sets of standards were decomposed into rows within Excel. I examined both sets of standards for differences. Two analytical approaches that I relied on during the initial analysis were 1) the application of color-coding to indicate differences among the sets of standards and 2) analytical notes for thoughts that were recorded in a research notebook (Schreir, 2012). This process was repeated until all applicable United States K-5 state mathematics standards had been compared against the original CCSSM. I then determined what percent of changes had been made to the standards from the CCSSM by counting the number or rows of standards and the number of rows with color-coded changes for a state to be considered for the coding process. If more than 15% of the standards had been changed, the state was selected to move to phase 2 of this study. The 15% of changes was used for this study based on the requirements set forth by the RTTTI, which only allowed up to 15% of changes to be considered in alignment with the CCSS adoption.

Phase 2

Based on the analysis conducted during phase 1, states were selected for the coding process of phase 2. During this phase, I used the K-5 state mathematics standards documents and any additional supporting documents that states had to answer each individual research sub question.

To answer research sub question 1 (*in what ways, if any, do K-5 state mathematics student process and practice standards differ from the CCSSM?*), I examined the student process and practice standards document from an individual state and color-coded differences from the CCSSM SMPs. I then recorded any thoughts in a researcher journal based on themes that might have emerged from observable differences. This process was repeated until all applicable state student processes and practices documents had been examined in conjunction with the CCSSM SMPs.

To answer research sub question 2 (*in what ways, if any, do K-5 state mathematics standards content domains differ than in the CCSSM*?), I created a table in Excel of the CCSSM domains and reexamined K-5 state mathematics standard documents or any additional supporting documents as needed. I noted any differences between an individual state's domains and the CCSSM domains within the table. I then recorded any thoughts in a journal that might have emerged from observable differences. This process was repeated until all domains of the applicable states had been examined in conjunction with the CCSSM Domains.

To answer research sub question 3 (*in what ways, if any, do states describe how learning trajectories are addressed in K-5 state mathematics standards*?), I used learning trajectory documents obtained from each state website involved in the study. If a state indicated specific research or a specific learning trajectory was used in the development of the standards, I noted this through highlighting of the document and recorded the information in a researcher journal. This process was repeated until all state standards had been examined in conjunction with information regarding learning trajectories within the state standards documents.

Lastly, to answer research sub question 4 (*in what ways, if any, is the relationship between procedural and conceptual learning outcomes represented in K-5 state mathematics standards?*), I used the learning trajectories documents, state standards, and additional state supporting documents. Each of the documents were analyzed and changes to the standards were color coded in a table within Excel. I used the definitions and examples of conceptual understanding and procedural fluency found within the conceptual framework of this study to determine which learning outcome was supported. An additional category of "both" was applied to this as well as some changes to standards supported both conceptual understanding and procedural learning outcomes. During this process all thoughts were recorded in a researcher journal.

After the initial round of coding, I sought the advice of expert committee members to ensure that there was an agreement with my coding process, which was used as an approach to enhance the credibility of the study. This also ensured that I had used the same coding rules throughout the coding process, a suggestion on best qualitative practices during coding from Mayring (2022). During the first meeting with committee members, definitions were discussed. Additionally, codes were further refined and categorized to better organize data. During the second meeting with committee members, codes were compared between myself and a committee member. Any discrepancies in our codes and definitions were discussed until an agreed-upon conclusion was reached.

I revisited the coding tables and reanalyzed each coded table a third time making revisions if categories were generalized or too specific (Mayring, 2022). This process was completed for each research sub question in a table. The developed codes were used to help answer each individual research sub question, which is discussed in the section "How Data is Reported and Displayed".

Phase 3

During this phase, I focused on determining patterns within the differences that emerged from each state for each research sub question through the coding of the tables. I applied conceptual mapping (Grbich, 2007) using a table within Excel. Each row in the table represented an observable difference with a code to represent the difference. From this, categories and themes emerged for each state for each research sub question. These themes are shared in Chapter 4.

Phase 4

During this phase, I focused on determining patterns across states for each research sub question. I applied conceptual mapping (Grbich, 2007) using the themes that had emerged from phase 4. Each row in the table represented a theme for an individual state, and those themes were analyzed for emerging trends across multiple states. These themes are shared in Chapter 4. In addition, the themes were related back to each research sub question to contribute to the overall findings. No consistencies were found across all state standards that were selected for coding which resulted in no across-state analysis.

How Data is Reported and Displayed

The data obtained from this study are organized and conveyed in Chapter 4. The data are organized by state and within each state section data are also shared in order of each research sub question. Figure 7 provides an overview of how the data are organized in Chapter 4. Key insights and discussion regarding codes and themes from data analysis is shared in Chapter 5.

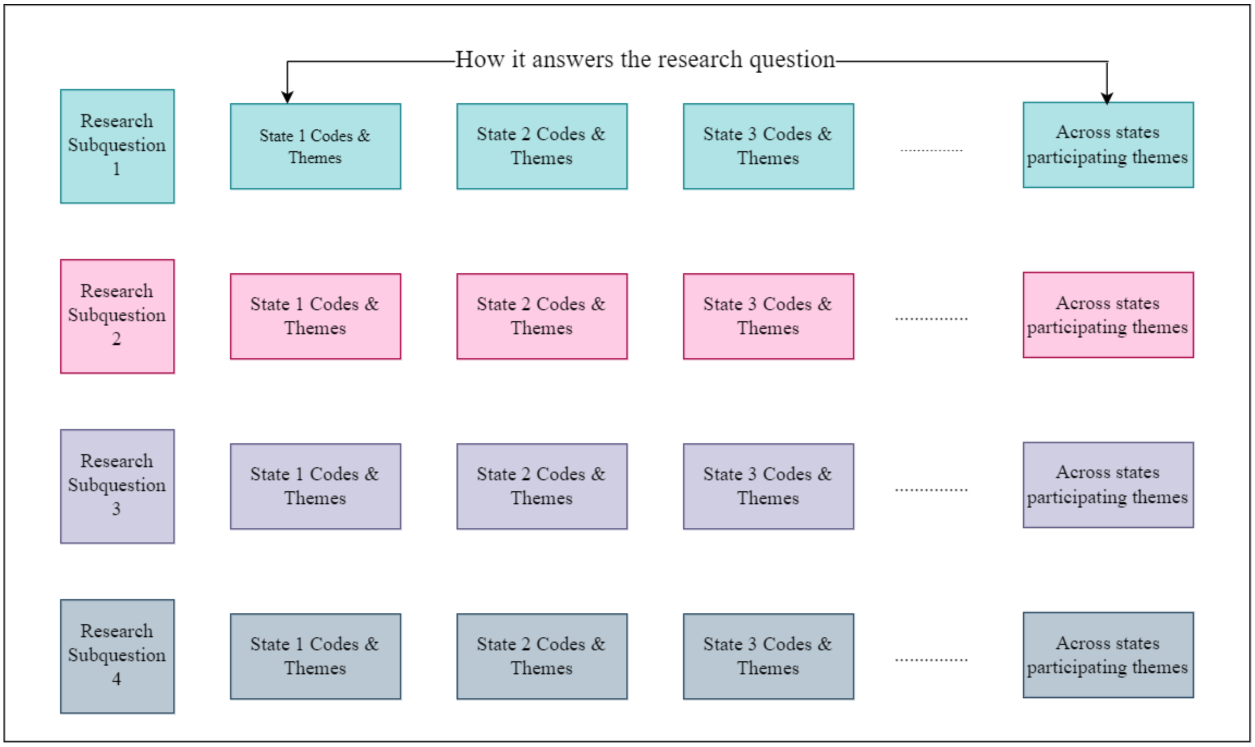

Figure 7

Data Results

Trustworthiness of Data

Conducting qualitative research requires researchers to engage in the responsibility of ensuring that trustworthiness is established at each stage of research (Lincoln & Guba, 1985). Trustworthiness can be established in numerous ways including credibility, transferability, dependability, and confirmability (Creswell & Guetterman, 2019). Within these four categories of trustworthiness, approaches to validity are discussed regarding how they were managed during the duration of the study.

Credibility

Credibility is used within qualitative research to validate findings (Creswell & Guetterman, 2019). The importance of credibility cannot be undervalued as it helps readers determine if the findings and interpretations are accurate (Noble & Smith, 2015). This can be achieved through numerous strategies including meticulous data organization with transparency of decisions, interpretations, and accounting for personal biases (Noble & Smith, 2015) which are discussed in subsequent sections. Using multiple data sources to develop themes and codes can also contribute to a study's credibility (Creswell & Guetterman, 2019). This study included the use of K-5 state standards and supporting documents (e.g., Crosswalks, process and practice standards, overviews, or policy documents) as primary data sources. Each state department of education website ranged in the supporting documents available, which resulted in an array of documents for analysis. Codes and themes were developed within each data source and then compared and reconfigured across all data sources to increase credibility. The codes were also checked by expert committee members to increase reliability and further credibility.

Transferability

Transferability allows for the readers to determine if the results from a study can be generalized or transferred to another context (Bitsch, 2005). One approach used in qualitative research that allows for transferability is thick description (Lincoln & Guba, 1985). To ensure that transferability was achieved, I provided a substantial amount of information regarding each component of the research process, trying to remain as transparent as possible in all decision-making.

In addition to the previously mentioned approaches, I maintained documentation of my thoughts during the coding process using a researcher journal. This approach was used to increase transparency (Linneberg & Korsgaard, 2019). The documentation included decisions made during the coding process in addition to pathways to final coding conclusions. This was continuously reviewed during the data analysis process to maintain consistency with each set of state standards that were analyzed. It was also later shared with select dissertation committee members during peer review to guide discussions and increase dependability.

Dependability

Dependability requires the aspect of consistency (Lincoln & Guba, 1985). To ensure that dependability was achieved, I consulted with expert reviewers on my dissertation committee throughout the coding process. Select members of my dissertation committee examined portions of the coding and analysis process (Janesick, 2015) across two meetings. Each meeting was approximately one hour. During the second meeting, one committee member coded a set of state standards data and cross compared their results with my results. We discussed codes that they had labeled that I had not used, and we came to a consensus on definitions of codes that were too vague or did not have accurate language. This research technique also enhances trustworthiness, credibility, (Janesick, 2015; Lincoln & Guba, 1985) and interrater reliability (Creswell & Guetterman, 2019).

Confirmability

Confirmability requires that I, the researcher, maintain neutrality without the interference of preferences or viewpoints that might not be grounded in the data (Lincoln & Guba, 1985). The approach of reflexivity was used to achieve the confirmability of this study. Reflexivity requires researchers to disclose their biases and experiences to help the reader understand how the researcher's position shaped the study (Creswell & Poth, 2018; Weiner-Levy & Popper-Giveon, 2013). Further, Creswell and Poth (2018) indicate that disclosing researcher bias establishes validity in qualitative studies.

During the opening section of this document, I noted personal experiences that ultimately affected the scope of this study. It is advantageous to disclose those experiences to achieve confirmability within this study. My most pertinent experience includes contributing to the development of the Florida B.E.S.T. standards in 2019 where I served as a member of the K-5 standard revisions expert team for:

1. Number and Operations in Base-Ten and Counting and Cardinality,
2. Geometry,
3. Number and Operations-Fractions, and
4. Measurement and Data

Serving in this role further shaped many of my beliefs towards mathematics standards and insights into the creation of mathematics standards. Notably, this experience adjusted the perspective I held going into the study, which in turn influenced the questions I asked during the study.

Prior to this study, I also had experience adjusting the achievement level descriptors for the National Assessment of Educational Progress (NAEP) through a pilot study committee. This role heavily relied on knowledge of mathematics standards and expectations of what students should be able to do by the end of 4^{th} grade mathematics instruction. Additionally, I also had experience examining curriculum for textbook adoption in the state of Florida during the 2021-2022 year. This opportunity provided further experience with the FL B.E.S.T. standards. Unfortunately, the textbook adoption year was also highly politicized, which contributed to biases that may have been brought to the study. While some political leaders within my state of residence have stated that the CCSS were officially eliminated, I felt it was necessary to determine if this was true as it was my belief that the CCSS truly cannot be eliminated from renditions of successive standards without a complete over-hauling. I also entered this study believing that the attack on social emotional learning within mathematics textbooks was not the best decision for children and that this argument was especially ill-timed as students had lost a year and a half of in-person schooling and even more than ever needed academic and emotional support to transition back to daily schooling activities.

Lastly, I have served as a board member for the Florida Council of Teachers of Mathematics since 2019, which has informed my perception of not just mathematics standards but also mathematics education in general. As an organization, we are constantly trying our best to connect educators, mathematics, and students.

Overall, it was important that I remained aware of any bias pertaining to standards that could have existed throughout the duration of the study. It was especially important during the Serving in this role has brought considerable discussions that have shaped many of my beliefs that functioned as an underlying component of this study. This was especially important to consider during the coding process so that I did not look for a specific result or adjust my study to obtain a specific result. As I presented results from this study, I aimed to only present results without associated opinions.

Delimitations

As with every study, there were delimitations that restricted the questions I could answer or inferences that I could conclude from the findings. First, I did not engage in an in-depth analysis of the political and social factors that influence states and their decisions regarding standards. The sheer breadth and complexity of these two components were beyond the purpose of this study, which was to examine the ways in which state standards have changed during the past decade. Second, the study focused on the time range of 2012-2022 despite mathematics standards, curriculum, and revisions existing outside of this period. Third, the grade range of K-5 standards was the focus of the study, however, using all grade level standards could have contributed to a more complete picture of what is happening to mathematics standards. This was not feasible in the scope of this dissertation research. Limiting the study to grades K-5 also narrowed the overall insight into the learning trajectories and their impact on standards but provides space for a more in-depth analysis in future research.

In conclusion, due to the vast areas that are available to be researched when examining the K-5 state mathematics standards, it is necessary to be transparent about what the study did not accomplish. This investigation did not examine the overall rigor of standards compared to previous state standards, coherence, alignment, or focus of any revised state standards. Nor did this study rate or rank state standards based on perceived quality. The scope of this study strictly focused on the changes in K-5 state mathematics standards from 2012-2022.

Summary

This chapter focused on the methodology that I used in this study. A rationale for the use of qualitative research, chosen methodology, and data collected to complete the study were provided. A detailed description of the procedures and phases of the data analysis was also discussed. Information regarding the trustworthiness of the data was shared as well. In addition, I disclosed personal experiences that could have contributed to my overall bias to the study. This in turn provided readers insight into why the study was positioned in the manner that it was.

In the next chapter, results from the analysis are shared. This includes the patterns and themes that emerged within and across state standards that were analyzed in the study. The chapter is presented in four sections: (1) introduction, (2) analysis of individual states included in the study, (3) analysis across all states included in the study, and (4) conclusion. The reporting of the results followed the outline in Figure 6, are described in detail in chapter 4.

CHAPTER FOUR: RESULTS

Chapter 4 presents the results of the qualitative analysis of the United States K-5 mathematics standards as described in Chapter 3 from 2012 to 2022. I identified themes both within individual state mathematics standards documents and across different state mathematics standards documents. The remainder of this chapter is comprised of the following sections: (1) States not selected for coding, (2) Florida, (3) Georgia, (4) Nebraska, (5) Oklahoma, (6) Texas, (7) Virginia, and (8) Themes across states.

States Not Selected for Coding

After completing a line-by-line comparison of the K-5 mathematics standards in the 15 states considered for this study with the CCSSM, six states progressed to the coding stage. This was determined by states exhibiting more than 15% of changes to the standards. The 15% of changes to the standards was determined by counting the number of rows of standards in the CCSSM Excel sheet and comparing it to the number of rows of standards in each state Excel sheet. A calculation was conducted of the number of cells in the state that were color-coded with changes. If fewer than 15% of changes occurred, I did not select the revision for the coding process. Table 7 provides a list of the states not selected for coding and the year of the mathematics standard revisions.

Table 7

State Standards Not Selected for Coding

State	Year (s) of Mathematics Standard Revisions
Alabama	2016, 2019
Alaska	2012
Arizona	2016 (updated in 2018)
Florida	2014
Georgia	2022
Indiana	2014, 2020
Kentucky	2019
Nebraska	2015
New York	2017
South Carolina	2015
Tennessee	2018

Alabama and Indiana underwent two revision processes during the timeframe of this study, which are indicated by multiple years listed in the column *Year(s) of Mathematics Standard Revisions* in Table 7. Arizona includes a note in the column *Year(s) of Mathematics Standard Revisions* that the standards were updated in 2018. This update was minimal and included a change in wording to three elementary standards and disclosed two copy and paste errors found in the 2016 standards. All states outlined in Table 7 demonstrated changes to their mathematics standards that did not exceed more than the designated 15% outlined by the RTTTI. Despite not being selected for coding, the states in Table 7 had changes that are deemed worth discussion and are explored in the remainder of this section.

It is pertinent to consider additional elements of mathematics standards documents aside from the actual standards themselves. These elements paint a more complete picture of what the department of education in each state would like to convey to the public. These additional elements include preface statements to the standards, the standards writing process including committee members, and position statements (if any). It should be noted that each state offers vastly different information surrounding the additional elements of their standards documents, and each department of education controls what is publicly available regarding the development of the standards writing process.

Regarding changes made to state standards that were not selected for coding, first, some states included prefaces in their mathematics standards documents. The Alabama State Department of Education (2019) noted in their preface that:

> Content standards in this document are minimum and required (*Code of Alabama,* 1975 §16-35-4). They are fundamental and specific, but not exhaustive. In developing local curriculum, school systems may include additional content standards to reflect local philosophies and add implementation guidelines, resources, and activities which are beyond the core of this document. (p. viii)

The Alaska Department of Education and Early Development had a similar preface statement in their standards that expanded on Alabama's statement. Their document stated,

> These standards do not tell teachers how to teach, nor do they attempt to override the unique qualities of each student and classroom. They simply establish a strong foundation of knowledge and skills all students need for success after graduation. It is up to schools and teachers to decide how to put the standards into practice and incorporate other state

> and local standards, including cultural standards. (Alaska Department of Early Education and Child Development. [AKDEECD], 2012, p. 4)

Indiana had a comparable statement in a section which identified what the Academic Standards are not. In this section it was listed that,

> (1) The standards are not curriculum, (2) The standards are not instructional practices, (3) The standards do not necessarily address students who are far below or far above grade-level, and (4) The standards do not cover all aspects of what is necessary for college and career readiness. (Indiana Department of Education [INDOE], 2014b, p. 5)

The CCSSM document took a different approach than Alaska, Alabama, and Indiana by reiterating in their preface that the CCSSM "do not dictate curriculum or teaching methods" (NGA & CCSSO, 2010, p. 5) but the statement did not discuss or insinuate that the standards are a minimum, non-exhaustive set of learning outcomes.

The next change to standards related documents involved states vaguely describing the standards writing process. For example, the Alabama State Department of Education (2019) stated that their standards writing team

> …conducted exhaustive research during the development of this Course of Study, analyzing mathematics standards and curricula from other states, the *2016 Revised Alabama Course of Study: Mathematics*, national reports and recommendations on K-12 mathematics education, the latest NAEP Frameworks, and numerous articles in professional journals and magazines. Many members attended state, regional, and national conventions to update their knowledge of current trends and issues in mathematics education. (p. viii)

The 2016 Alabama mathematics standards preface included a similarly worded declaration. While the provided information is more descriptive than some other states, the specifics are missing on which mathematics conferences and sessions were attended by writing committee members and what articles were being used by writing committee members to provide guidance on the standards writing process.

While some states publish both the names and current occupation of committee members serving on the standards writing committee, other states made vague comments about their writing committee. The Alaska Department of Education and Early Development (2012) alluded to writing committee members by stating that "industry leaders were part of Alaska's standards review" (p. 4). The Tennessee Department of Education (2018) made a similar claim, stating that their standards, "were reviewed and developed by Tennessee teachers for Tennessee schools" (p. 1). The South Carolina Department of Education (2015) produced a statement equally vague by stating that their standards were:

> Collaboratively written by a team of South Carolina classroom teachers, instructional coaches, district leaders, higher education faculty, and educators who specialize in English Language Learners, special education, career and technology education, and assessment who were selected through an application and rubric process by the South Carolina Department of Education. (p. 3)

The South Carolina Department of Education provided the most descriptive outline but did not share the application or rubric process, or the criteria for selection of the writing team.

Similarly, the Indiana Department of Education (2014a) stated that, "Technical Teams were responsible for reviewing the existing Indiana Academic Standards (Common Core State Standards) and providing suggestions for edits and word changes to improve the clarity and progression of the standards" (p. 2). They composed a separate evaluation team with members who had either previously taken part on a Technical Team or an Advisory Team. The Indiana standards document (INDOE, 2014a) stated that:

> The Evaluation Teams were made up of K-12 educators who represented a wide variety of Indiana school districts and over 445 years of combined classroom teaching experience, and higher education subjects matter experts in English/Language Arts and Mathematics, representing Indiana's public and private institutions of higher education. (p. 3)

While this information is helpful to understand the standards writing process, the criteria of participating on a Technical Team or an Advisory Team previously is vague and does not describe the criteria for how members had been selected for participation on these teams.

A commonality among the CCSSM and states not selected for coding is the lack of disclosure as to who (if anyone) received compensation for their time serving on the standards writing committees. Additionally, it was not indicated within documents or websites if reviewers were compensated for their time or if there was a vetting process while selecting reviewers. Further, neither the CCSSM nor state standards documents released information pertaining to demographics of who participated on the writing teams

No specific information regarding the procedures followed describing the selection of the writing committee members or the review committee members was disclosed by the states in Table 7. This included the credentials of writing committee members. The Alabama State Department of Education (2016 & 2019) did provide the occupational role of committee members but did not describe their experiences that qualified them to be members of the writing committee.

Table 8 provides additional information found in standards documents in a comprehensive manner for states not selected for coding. Information included in the table indicates if the following information was included in the state standards document or website: (1) list of committee members, (2) educational positions of committee members at the time of the standards writing process, (3) conceptual frameworks mentioned to inform the writing of the standards, and (4) position statements disclosed. Only two states with three collective standards revisions demonstrated this information within their standards document or on their department of education website.

Table 8

Overview of Additional Standards Information

State	Committee Members	Positions of Committee	Conceptual Framework	Position Statements
Alabama 2019	✓	▪ 10 Teachers (elementary, middle, and high) ▪ 2 Retired Teachers ▪ 7 Specialists, Coaches, Supervisors, Content Directors ▪ 5 Professors, Instructors ▪ 6 Directors ▪ 1 Senior Vice President ▪ 1 Attorney	✓	▪ Access and Equity ▪ Teaching and Learning Mathematics ▪ Mathematics Curriculum ▪ Mathematical Tools and Technology ▪ Assessment of Mathematics Learning ▪ Professional Mathematics Teachers

State	Committee Members	Positions of Committee	Conceptual Framework	Position Statements
Alabama 2016	✓	✓	✓	▪ Equity ▪ Curriculum ▪ Teaching ▪ Learning ▪ Assessment ▪ Technology
Kentucky 2019			Review of standards from AZ, CA, IN, IA, KS, MS, NY, NC, and “other content standards” (Kentucky Department of Education [KYDOE], 2019, p. 6) “Participants brought their own knowledge to the process, along with documents and information from the following Clements, D (2016), Van De Walle, J. Karp, K., & Bay-Williams, J. (2019), & Achieve (2017)” (KYDOE, 2019, p. 7).	

Changes to Emulative Standards

The term emulative standards has been used throughout this document to refer to state standard revisions that were by and large the CCSSM without the title of the CCSSM. Standards were selected for coding in this study if they had more than 15% of changes to the CCSSM. If a set of standards was not selected for coding, the standards still had potential to demonstrate *some* changes, just not more than 15% of changes from the CCSSM. When considering the changes made to standards documents, some sources declare there are in fact *no* changes. For example, the Eagle Forum of Alabama, a 501c4 non-profit social welfare organization, indicated in a blog post, that side-by-side comparisons of the emulative 2019 Alabama Mathematics Standards to the CCSSM have resulted in “no significant difference” (Eagle Forum of Alabama, 2019, p.1). The phrase “no significant difference” should not be considered from using the phrase from a statistical standpoint and should be critiqued thoughtfully as there *are* actual differences between the documents. Intentional effort by state level writing committees was devoted to content topics with any lapse of connections within the CCSSM. Table 9 outlines more specific changes that were evident between the emulative standards not selected for coding during this study and the CCSSM.

Table 9

Emulative Standards Changes

Changes to Standards	Alabama 2019	Alabama 2016	Alaska 2012	Arizona 2016	Florida 2014	Indiana 2020	Indiana 2014	Kentucky 2019	New York 2017	South Carolina 2015	Tennessee 2018
Identification of US currency explicitly addressed		✓	✓	✓	✓	✓		✓	✓	✓	✓
Specific pattern exploration prior to 3rd grade (mixture of extend, repeat, and grow)	✓		✓			✓	✓		✓	✓	
Calendar concepts addressed (day, week, month, and/or year)			✓			✓	✓				
Instructional focus of "major" and "supporting" standards delineated through color-doing or geometric shapes *FL uses the phrase "supporting clusters" with warning statement in standards not to sort major and supporting clusters				✓	✓						✓
Includes coherence of standards section (within								✓	✓		

Changes to Standards	Alabama 2019	Alabama 2016	Alaska 2012	Arizona 2016	Florida 2014	Indiana 2020	Indiana 2014	Kentucky 2019	New York 2017	South Carolina 2015	Tennessee 2018
grade and across different grade levels)											
Example representations added								✓	✓		
Mean, Median, Mode explored						✓	✓				
Included counting back as a standard	✓										✓
Includes CCSSM standard in brackets behind each standard		✓			✓						
Added literacy skills for mathematical proficiency											✓
Includes PK standards									✓		
Denotes additional connections to SMP notes within standards documents									✓		
Note section further explaining definitions in									✓		

Changes to Standards	Alabama 2019	Alabama 2016	Alaska 2012	Arizona 2016	Florida 2014	Indiana 2020	Indiana 2014	Kentucky 2019	New York 2017	South Carolina 2015	Tennessee 2018
narrative connection to standards (such as fluency)											
Compares temperatures of objects							✓				
Added "cognitive complexity" to each standard					✓						
Adjusted clusters				✓							
Decompose numbers into prime factors				✓							
Probability explored earlier than 6th grade	✓										
Adjusted domains of standards	✓										

Note. The standards information in this table is compiled from: Alabama State Department of Education, 2016, 2019; AKDEECD, 2012; Arizona Department of Education, 2016; Florida Department of Education [FLDOE], 2014; INDOE, 2014;2020; KYDOE. 2019; NYSED, 2017; South Carolina Department of Education, 2015; Tennessee Department of Education, 2018.

Standards Selected for Coding

To provide the most comprehensive picture of the changes made to state standards, a timeline is provided in Figure 8. Color coding was used to indicate political affiliations of the state governor at the time of the original CCSSM adoption and at the time of mathematics standards revision. The democratic party is represented by the color blue, the green party is represented by the color green, and the republican party is represented by the color red. States that never adopted the CCSSM are included in the figure and each year of reelections for governor are color-coded to share the political affiliations that opted to continue with their established state mathematics standards.

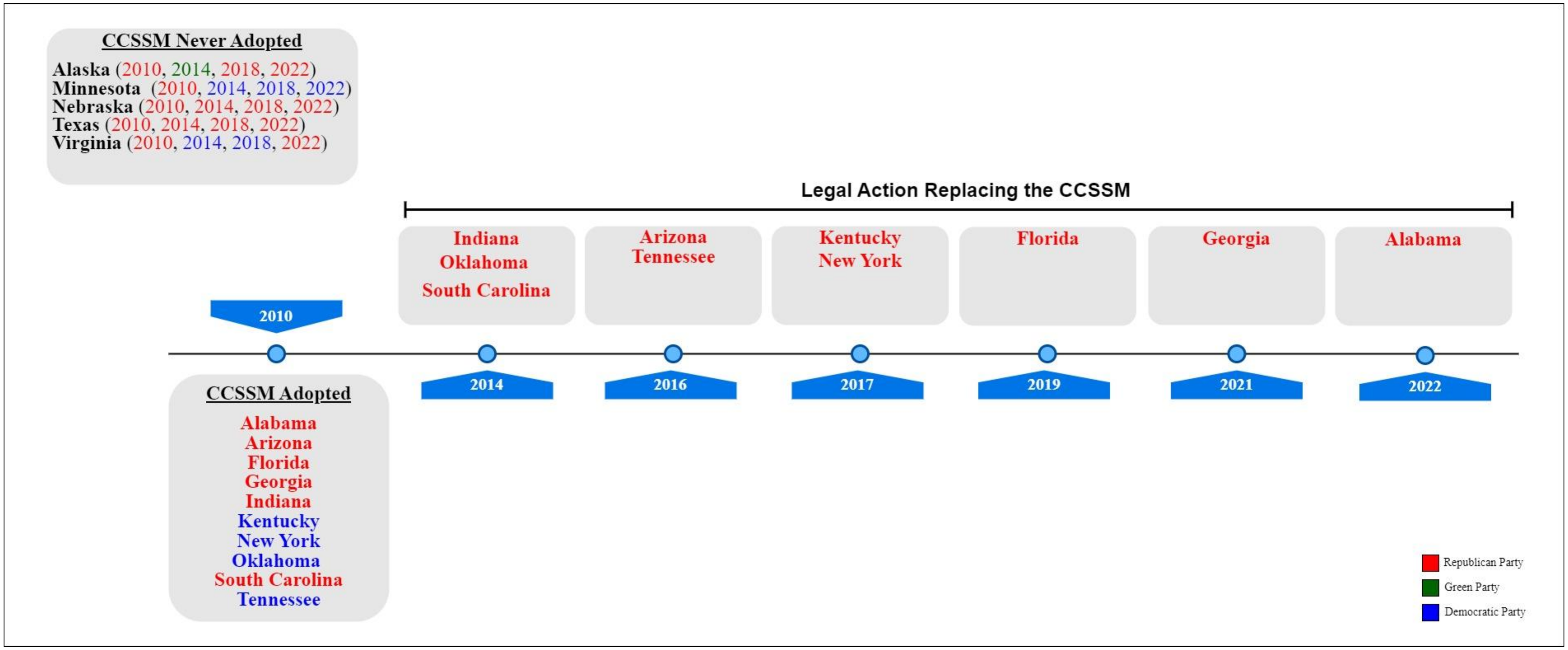

Figure 8

Timeline of Legal Action by States

Figure 8 includes all states considered in this study. The remainder of the chapter will focus on the six states that were selected for coding, which include: Florida 2019, Georgia 2021, Oklahoma 2016, Nebraska 2022, Texas 2014, and Virginia 2016. Minnesota was originally selected for coding; however, their most recent revisions to state mathematics standards have not been approved by the board of education. Their revisions do reflect more than a 15% change to the standards, but they are not finalized currently. The analysis of their state standards will be shared in a future research study. Next, changes made to the state standards that are substantial deviations from the CCSSM are described by state.

Florida 2019

Florida adopted the CCSSM in 2010 and elected to review the standards based on public concern ("What are the Florida", 2015) regarding the success of the new standards. This resulted in the 2014 adoption of the *Mathematics Florida Standards*, also referred to as MAFS, which did not have enough deviations from the CCSSM to be coded for this study. A politicized process unfolded in Florida in which then Governor, Ron DeSantis, issued Executive Order 19-32 to improve education in Florida through the "elimination of Common Core" (Exec Order No. 19-32, 2019, p. 1). The result of the executive order is the Florida 2019 Benchmarks for Excellent Student Thinking Standards, which have been coined by the Florida Department of Education as the B.E.S.T. standards because they are "the B.E.S.T. in the nation" (FLDOE, 2019, p. 1).

To answer the overarching research question of *in what ways, if any, do K-5 state mathematics standards differ from the CCSSM*, a line-by-line comparison of the 2019 Florida B.E.S.T. standards and the CCSSM was conducted within Excel. This enabled me to move from the stage of sampling units to determine the recording units for the study (Krippendorff, 2004). Deviations from the CCSSM were color coded in blue, and any missing verbiage in the original standards were color coded in orange. After the line-by-line comparison was complete, coding of the deviations began.

The most notable addition to the Florida 2019 standards is the inclusion of *benchmark clarification* statements. Benchmark clarification statements were added to "ensure a comprehensive understanding of the intentions of the benchmarks and to increase transparency of expectations" (FLDOE, 2019, p. 2). The K-5 mathematics standards include 196 clarification statements. Interestingly, upper elementary grades received the highest number of clarification statements. In grade order (beginning with Kindergarten), the following numbers indicate the clarification statements per grade level: 15, 22, 16, 48, 59, and 36. Importantly, some clarification statements were repeated multiple times within a mathematical domain. For example, the benchmark clarification “denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, 16 and 100” appeared four times within the fourth grade “fractions” domain. Repetitive benchmark clarifications were not removed during the coding process.

During the coding process, I met with committee members to develop coding definitions and refine codes. The standards, proficiencies, and practices table were relied on for the creation of codes and categories except for the financial literacy category. The financial literacy category was coded using guidance on definitions from the Council for Economic Education and Jump$tart Coalition for Personal Financial Literacy, who joined together to create the National Standards for Personal Financial Education (Council for Economic Education, 2021). In an attempt to synthesize all of the codes for an interpretable format, I relied on inductive category formation (Mayring, 2022) for my original round of codes. I consulted the methodology to support this process. Figure 9 includes the created codes.

Teacher Knowledge

- cognitive demand
- connections
- content knowledge language
- explicit instructional strategies
- fluency
- non-technical jargon
- number relationships
- problem types
- standard/ traditional algorithm
- unit reference

Financial Literacy

- coin identification
- operations with money
- personal finance
- financial organization
- career
- accounting
- economics

Math Specific Concepts

- comparison
- data representation
- estimation
- inequality and operational/algebraic symbols
- patterns
- properties of operations
- properties of shapes
- spatial reasoning
- statistics

Communication of Ideas

- manipulatives
- models
- multiple representations
- SMP

Number Sense

- specificity counting
- subitizing
- place value
- rounding
- foundational fraction sense/reasoning

Specific CCSSM Deviation

- standard placement variation
- missing shape
- range of numbers used differs
- non-expectation clarification
- verb change

Figure 9

Coding Categories and Codes

The changes could be described based on the six main categories that were developed. This includes teacher knowledge pertinent to the mathematics standards, incorporation of financial literacy, changes in mathematics specific concepts, communication of ideas by the students pertaining to the mathematics standards, number sense, and specific deviations from the CCSSM. Collectively, 41 codes were developed within the six categories. The definition of codes listed in Figure 9 are available in Table 10 and Appendix B.

Table 10

Codes with Definitions, Alphabetical

Code	Definition
accounting	Standards address components of accounting terminology/understanding
career	Standards involve language pertinent to the understanding of a career
cognitive demand	Standards adjust cognitive demand
coin identification	Standards require identification of United States currency
comparison	Standards involve students to compare/contrast attributes or elements of a mathematics topic
connections	Standards include equivalence among mathematical ideas
content knowledge language	Standards include language pertinent to teacher content knowledge
data representation	Standards include creating and/or interpreting specific types of graphs
economics	Standards pertaining to production, distribution, and consumption of goods and services
estimation	Standards explicitly address estimation of a quantity or measure

Code	Definition
explicit instructional strategies	Standards address specific instructional strategies
financial organization	Standards include elements to keep finances organized at the personal/business level
fluency	Standards involve recall as a descriptor
foundational fraction sense/reasoning	Standards include equal sharing, partitioning, reasoning to demonstrate fractional understanding
inequality/operational/algebraic symbols	Standards include specific types of symbols or explicitly use equations with symbolic notation
manipulatives	Standards have wording referring to incorporating physical objects that students and/or teacher use
missing shape	Standards are missing shape(s) that are included in the CCSSM
models	Standards address a mathematical representation of a real-world scenario
multiple representations	Standards call for various representations of a mathematics concept
non-expectation clarification	Standards define what is not expected within instruction at that time

Code	Definition
non-technical jargon	Standards incorporate terminology that doesn't reflect precise mathematical terms
number relationships	Standards include mathematical terminology that stems from number relationships (part-part-whole, compose, decompose)
operations with money	Standards specifically call for operations to be used in calculating amounts of currency
patterns	Standards address pattern growth, extensions, identification, creation, rules, and/or transfer
personal finance	Standards pertaining to economics/finances that do not explicitly address the calculation of money
place value	Standards include groupings of tens
problem types	Standards include "practical," "real-world," "applicable," "story," or "picture"
properties of operations	Standards incorporate knowledge of properties of operations
properties of shapes	Standards include understanding of attributes/properties of a geometric shapes
range of numbers used differs	Standards use a different number goal than CCSSM
rounding	Standards require students to round to a specified place value

Code	Definition
SMP	Standards of mathematical practice language is referenced as part of standard
spatial reasoning	Standards rely on students to use spatial reasoning to complete geometric problem
specificity counting	Standards include types of counting (e.g. orally, forward, backward) or skip counting by given intervals (1's, 5's, 10's)
standard placement variation	Exact standard is in different grade in CCSSM (can be moved up or down)
standard/traditional algorithm	Standards require the use of a standard or traditional algorithm
subitizing	Standards require students to recognize number of objects without counting
unit reference	Standards include the use of a unit whether it be day/month/year, temperature, time, measurement, place value, number line usage, or counting
verb change	Standards include a difference in verbs represented in the CCSSM

The most notable deviations from the CCSSM for the Florida B.E.S.T standards based on the coding process includes:

- larger and smaller number expectations in standards,
- inclusion of explicit instructional strategies,
- adjustment to cognitive demand of standards,
- explicit expectations of estimation related to computations,
- increased emphasis on fluency,
- movement of standard algorithm use to an earlier grade,
- focus on developing a sense of understanding of United States currency,
- pattern work earlier in grade bands, and
- the inclusion of clarification statements to delineate the expectations or nonexpectations of a standard.

Given the large amount of data, only the three most frequently occurring codes in the state standards will be shared. In Table 10, the three codes with the most frequent occurrences are further explained. Underneath each header cell of “Code & Number of Occurrences” a bolded phrase for the code and a bolded number is listed indicating how many times that code occurred across the K-5 standards. Italics are added within cells to emphasize a specific change to the standards. Some standards received multiple codes during the coding process and may appear in the table in respective columns. Descriptions of changes not included in the table are in narrative form following Table 11.

Table 11

Florida Standards Changes from Top Three Most Frequent Codes

FLORIDA BENCHMARKS FOR EXCELLENT STUDENT THINKING STANDARDS MATHEMATICS			
Grade Level	**Code & Number of Occurrences**		
	Number Difference **81**	**Explicit Instructional Strategies** **55**	**Cognitive Demand** **50**
Kindergarten	Extended comparing numbers from up to 10 to up to 20	Locate, order, and compare numbers from 0 to 20 using the *number line* and terms less than, equal to or greater than Instruction includes *non-examples* for geometric attributes	Understand that rearranging a group of objects does not change the total number of objects but many change the order of an object in that group Locate and order numbers from 0 to 20 using a number line Moved relate addition facts to subtraction facts from 0 to 10 from first grade Provide different representations for numbers 0 to 10 Identify the attribute of volume of objects Backwards counting within 20

FLORIDA BENCHMARKS FOR EXCELLENT STUDENT THINKING STANDARDS MATHEMATICS			
Grade 1	Added skip counting by 2s Subtraction of single digit number from two digits numbers, instead of within 20	Included equation formats with the sum or difference on either side of the equal sign, which was not explicitly stated but shown through examples Measure from zero on a ruler moved from 2nd grade to 1st grade Partition circles into halves and semicircles, which extends the 1st grade standard of partitioning circles in half	Restate a subtraction problem as a missing addend problem using the inverse relationship between addition and subtraction Count backwards from any given number within 120 by ones Plot and order whole numbers up to 100 using a number line and place value Moved recall addition facts and related subtraction facts with automaticity from 1st grade Skip count by 2s to 20 and by 5s to 100
Grade 2	Round to nearest 10 and 100 moved from 3rd grade to 2nd grade Added specific limitation of data scales to increments of 1s, 5s, and 10s	Recognition of odd and even numbers with specific strategies of skip counting, arrays, and patterns in the ones place outlined	Plot and order numbers up to 1,000 using a number line and place value Identify a number that is 100 more or 100 less than a given three-digit number Create real world situations based on an equation

FLORIDA BENCHMARKS FOR EXCELLENT STUDENT THINKING STANDARDS MATHEMATICS			
Grade 3	Multiplication fluency expectation moved to 12 × 12 from 10 × 10	Explore multiplication with *equal groups, arrays, area models, and equations*	Plot and order numbers up to 10,000 using a number line and place value
	Extended fraction denominators from 2, 3, 4, 6, & 8 to include 5, 10, and 12	Language of *part of a whole, part of a set, a point on a number line, a visual model, or in fractional notation* used in standards	Conceptual understanding of fractions emphasized in expectations
	Added limitation of number lines scaled by 50s, 100s and 1000s for plotting, ordering, and comparing numbers	Fractions with manipulatives or visual models including circle graphs explicitly stated	Plot order and compare fractional numbers using a number line and connection with ruler
	Added standard of reading and writing numbers to 10,000	Select and use appropriate tools to measure the volume of liquid *within a beaker*, temperature, measurement *on a linear scale* with connection to the *number line*	Understand the context of the problem, as well as the quantities within the problem emphasized in expectations
		Fold paper along a line of symmetry	Explain whether an equation involving multiplication or division is true or false
		Data displays can be *represented horizontally and vertically*, circle graphs are limited to *showing total values in each category*	The product or quotient can b eon either side of the equal sign in an equation
			Use ordinal numbers (1^{st}, 2^{nd}, 3^{rd},…) to describe the position of a number within a sequence

FLORIDA BENCHMARKS FOR EXCELLENT STUDENT THINKING STANDARDS MATHEMATICS			
			Instruction develops the understanding that there could be no line of symmetry, exactly one line of symmetry, or more than one line of symmetry
Grade 4	Number and base ten operations limited to up to 1,000,000 instead of no limitation in CCSSM Division with four digit dividends and two digit divisors instead of one digit divisors Added denominator of 16 Benchmark angles of 30°, 45°, 60°, 90°, and 180° used	*Scaled number line* with ones, tenths, and hundredths for comparing numbers Problems include the *unknown on either side of the equal sign* with multiplication and division problems Use of *digital measurements* and *scales* Convert from *smaller to larger* units in a single unit (customary/metric systems)	Plot and order numbers up to 1,000,000 and decimals up to hundredths using a number line and place value Identify the number that is one-tenth more, one-tenth less, one-hundredth more and one-hundredth less than a given number Describe how the numerator and denominator are affected when equivalent fractions are generated Plot and order fractions, including mixed numbers and fractions greater than one on a number line and through reasoning about size
Grade 5	Decimal standards go to thousandths place	*Scaled number line* for comparison of numbers up to thousandths place	When multiplying a given number by a fraction less than 1 or a fraction greater than 1, predict and explain the relative size of the product to the given number without calculating, including problems in which remainders must be interpreted within the context

FLORIDA BENCHMARKS FOR EXCELLENT STUDENT THINKING STANDARDS MATHEMATICS			
	Round decimals to the nearest tenth or hundredth	Use of *models* for multiplication and division of numbers with decimals including estimation, rounding, and place value	Expressions are limited to any combination of the arithmetic operations, including parentheses, with whole numbers, decimals, and fractions
	Denominators limited to numbers within 20 when multiplying fractions by fractions	Use of *two column table* to record inputs and output of numerical patterns	Conversions include length, time, volume, and capacity represented as whole numbers, fractions, and decimals
		Addition and subtraction of fractions with unlike denominators (including fractions greater than 1 and mixed numbers) includes manipulatives, drawings, or the properties of operations	Numerical data with graphs includes fractional and decimal values, with decimal values being restricted to no smaller than the hundredths place and fraction denominators restricted to 1, 2, 3, and 4 but can be greater than one

Note. The standards information in this table is compiled from FLDOE, 2019; NGA & CCSSO, 2010.

There are other notable changes to Florida standards that should be shared that are not reflected in Table 11. Florida standards use the vocabulary of "square angles" or "square corners" in addition to "right angles", specifically in third grade standards. The reference to movement on a number line is defined as a "jump." "Fractions greater than one" is a phrase that is also used throughout fractions standards from third to fifth grades. None of these vocabulary terms were used or addressed in the CCSSM. Below, additional changes are synthesized by grade level.

Kindergarten

In the revised FL standards, students in Kindergarten are introduced to the number line, where they must locate, order, and compare numbers from 0 to 20 using the number line. The CCSSM addresses this in second grade. Students in Kindergarten are also expected to add within 0 to 10 using related subtraction facts with procedural reliability, which is a first-grade standard in CCSSM. Students also are asked to explain why addition or subtraction equations are true, which lends itself to understanding of equality, a first-grade standard in CCSSM. With measurement, Kindergarteners are expected to express the length of an object by laying nonstandard objects end to end, which is a first-grade standard in CCSSM. A clarification benchmark in Kindergarten relies on the development of the understanding of spatial relationships in geometry, which was not explicitly addressed in the Kindergarten CCSSM.

First Grade

First graders are expected to estimate the length of an object to the nearest inch and measure length to the nearest inch or centimeter using a ruler. This is previously a second-grade standard in the CCSSM. A standard included in first grade requires the identification of coins and how many of each coin generates a dollar. Further, students must also know the value of bills and how many bills generate $100. This is a precursor to standards in future grade levels that require solving word problems involving coins and dollars. First grade data standards include the use of tally marks, which are not explicitly addressed in the CCSSM. Data sets do include the use of 2D geometric figures based on their defining attributes. The incorporation of data sets and geometric figures was not explicitly addressed in the CCSSM. Students are also expected to represent data and compare totals of various categories, which was not addressed in CCSSM until second grade.

Second Grade

Second graders are expected to use tally marks, a strategy that was not explicitly stated in the CCSSM. A prior first grade standard of applying the associate and commutative properties of addition was moved to second grade. Perimeter is explored with whole numbers and figures used are limited to regular triangles, rectangles, squares, and pentagons. Lines of symmetry of two-dimensional figures was explored, which was previously a fourth-grade standard. Categorization of two-dimensional figures by attributes was moved from third to second grade. Telling time to the nearest five minutes was moved from third to second grade. Second graders are also expected to round numbers to the nearest 10, which was previously part of a third-grade standard. Finally, plotting and ordering numbers on a number line was an explicit standard that was not addressed in the CCSSM.

Third Grade

Third graders are expected to understand line symmetry and determine if there is no, one, or more than one line of symmetry, which was formally a fourth-grade standard. Third graders must also describe and draw geometric attributes (points, lines, line segments, rays, intersecting lines, perpendicular lines, and parallel lines), another former fourth grade standard in the

CCSSM. Students must be able to determine and explain if a whole number from 0 to 1,000 is even or odd. Fractions greater than one are addressed in third grade, prior to beginning operations with fractions in fourth grade. The standard algorithm of addition and subtraction of whole numbers was moved from fourth grade to third grade. Finally, third graders are expected to read and write numbers with various number forms, compose, decompose, plot, order, and compare numbers on a number line up to 10,000, a standard that was not explicitly addressed until fourth grade in the CCSSM.

Fourth Grade

Fourth graders are expected to use stem-and-leaf plots with fractional values to determine mode, median, or range. A clarification in fourth grade extended benchmark angles to the use of 30°, 45°, 60°, 90°, and 180°. A second-grade currency standard was moved to fourth grade with the inclusion of decimal notation as an expectation. The standard algorithm of two-digit numbers for multiplication was moved from fifth grade to fourth grade.

Fifth Grade

Fifth graders in the state of Florida are now expected to determine mean, mode, median, or range of a numerical data set, which was addressed in sixth grade in CCSSM.

Sub Question Findings

To answer sub question 1, the Common Core SMPs were compared to the Florida Mathematical Thinking and Reasoning Standards (MTRs). The most notable differences include the exclusion of "using appropriate tools strategically" and "reason abstractly and quantitatively" Aside from these two differences, majority of the MTRs could be interpreted within the description of the CCSSM SMPs. Table 12 shows the specific changes to the Florida MTRs with a column devoted to descriptors of the aligned SMP.

Table 12

Comparison of SMPs and MTRs

Common Core State Standards for Mathematical Practice	Florida Mathematical Thinking and Reasoning Standards	Descriptors of Aligned SMP (NGA & CCSSO, 2010)
Make sense of problems and persevere to solve them	Demonstrate understanding by representing problems in multiple ways	The phrase multiple ways not used in SMP's, but "students check their answers to problems using a different method" (p. 6) is mentioned
Reason abstractly and quantitatively		
Construct viable arguments and critique the reasoning of others	Engage in discussions that reflect on the mathematical thinking of self and others	"They justify their conclusions, communicate them to others, and respond to the arguments of others" (pp. 6-7)
Model with mathematics	Apply mathematics to real-world contexts	In SMP's, "solve problems arising in everyday life, society, and workplace" (p. 6)
Use appropriate tools strategically		
Attend to precision	Actively participate in effortful learning both individually and collectively	"Try to communicate precisely to others" (p. 7)

Common Core State Standards for Mathematical Practice	Florida Mathematical Thinking and Reasoning Standards	Descriptors of Aligned SMP (NGA & CCSSO, 2010)
Look for and make use of structure	Use patterns and structure to help understand and connect mathematical concepts	"Look closely to discern a pattern or structure" (p. 8)
Look for and express regularity in repeated reasoning		
	Complete tasks with mathematical fluency	
	Assess the reasonableness of solutions	

To answer sub question 2, the CCSSM Domains were compared to the Florida Strands. Table 13 compares the CCSSM Domains and the Florida Strands with similar domains and strands positioned in the same rows. If no like domains or strands were identified, the particular cell in the row was intentionally left blank. The most notable differences between the domains are that “Counting and Cardinality” and “Base-Ten” were absent. The concepts of “Analysis of Data” and “Probability” were added. The term “Reasoning” was added to both Algebra and Geometry domains, but did not alter the intended outcome of either domain.

Table 13

Domains and Strands Comparison

CCSSM Domains	**Florida Strands**
Counting and Cardinality	
Operations and Algebraic Thinking	Algebraic Reasoning
Number and Operations in Base Ten	Number Sense and Operations
Number and Operations in Fractions	Fractions
Measurement and Data	Measurement Data Analysis and Probability
Geometry	Geometric Reasoning

To answer sub question 3, the Florida B.E.S.T. standards document and the Florida Department of Education website were both consulted. A vague reference was made regarding the materials used during the standards writing process. “The mathematics teacher expert workgroups drew on the work of the National Council of Teachers of Mathematics (NCTM); expectations from national and international assessments such as ACT, SAT, NAEP and TIMSS; comments from public and specialty stakeholders and feedback from national mathematics and standards experts” (FLDOE, 2019, p. 2). A 2020 session update on the background of the creation of the mathematics standards includes a bullet point that declares, “appropriate progression of content within the grade level/course and across grade levels/courses” (FLDOE, 2020, p. 23) was used. Despite this statement, there are no direct research citations regarding consideration of learning trajectories and their impact on the standards’ overall vertical and horizontal coherence.

There is a publicly available document on standards progressions for each grade band (K-5, 6-8, 9-12), with each Florida standard in sequential order. No authors are cited with the creation of this document. There is also a publicly available document with progression of standards by strand. The font size in both documents is difficult to read without enlarging the document well beyond its original size, contributing to limited access of the document. The FLDOE additionally has a fluency expectation chart from grades K-8 that color codes exploration, procedural reliability, procedural fluency, and recall benchmarks in the strands of algebraic reasoning, measurement, and data analysis and probability.

Finally, to answer sub question 4, each standard that demonstrated a difference was coded as a procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes during the coding process using the letters P, C, or B. This process of determining the learning outcome associated with the changes demonstrated by states was completed by examining the blue color-coded word differences in Excel. The color-coded word differences represented changes in state standards from the CCSSM. The color-coded words were then sorted into the categories of procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes.

Figure 10 demonstrates the frequency of learning outcomes made to the standards that demonstrated changes from the CCSSM. No repetitive clarification benchmarks were removed during this process. When considering the learning outcome of standards that demonstrated differences in Florida K-5 mathematics standards, more emphasis was used for procedural outcomes with conceptual learning outcomes receiving the least amount of emphasis.

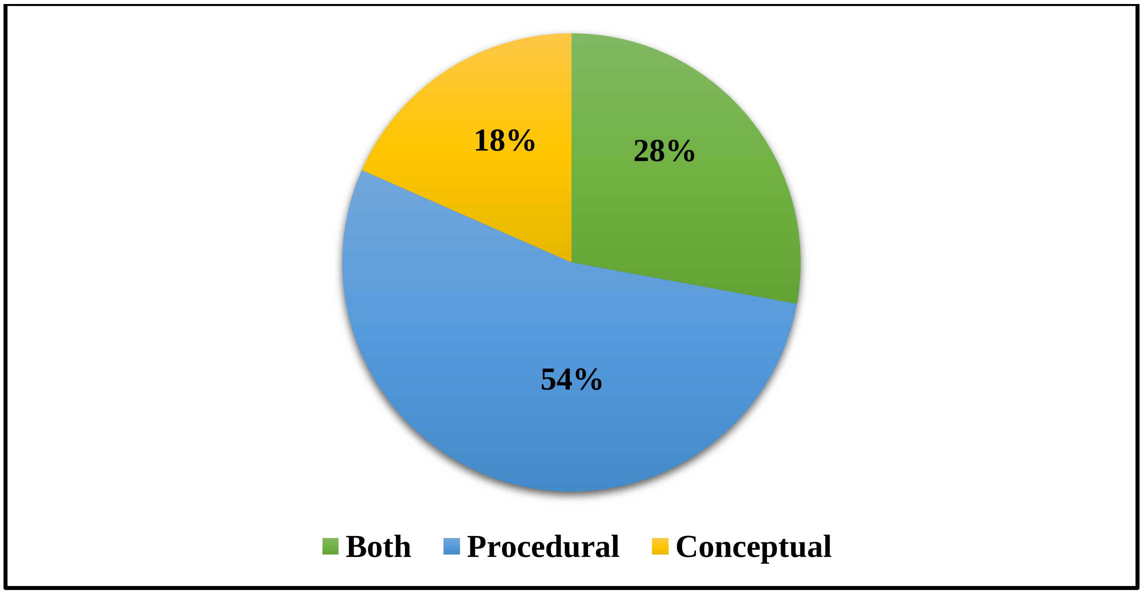

Figure 10

Percentage of Florida Standard Revisions as Having Conceptual or Procedural Learning Outcomes

The 2020 session update document states that there is an "intentional balance of conceptual understanding and procedural fluency" (FLDOE, 2020, p. 24). No explanation was provided on how the balance was determined or calculated.

In conclusion, the 2019 Florida B.E.S.T. standards differed in their mathematics standards, standards of mathematical practice, and emphasis on learning outcomes of the changes from the CCSSM. The writing team members were not listed on the FLDOE website nor was the criteria for selection of the writing team shared. There are multiple training PowerPoints converted to PDFs shared by the FLDOE as well as links to LiveBinder. It was within the LiveBinder link that a 2021 welcome session shared more about the writing process which was deemed a "three-tiered review" by groups labeled as teacher experts, specialty stakeholders, and the public (FLDOE, 2021). The expert groups examined the standards by CCSSM domains with a total of five groups. A timeline was provided but no other information related to the creation of the mathematics standards was listed.

Georgia 2021

Since the adoption of the CCSSM, Georgia has undergone two standards revisions. The first occurred in 2016 and was called the Mathematics Georgia Standards of Excellence. The second revision occurred in 2021 and produced the Georgia K-12 Mathematics Standards. The Georgia K-12 Mathematics Standards include numerous changes that are worth discussion. Most notably, the standards document includes two main sections: (1) expectations and (2) evidence of student learning. The expectations section includes standards and their corresponding codes. The evidence of student learning includes additional information to enhance the expectation categories including: (1) fundamentals, (2) relevance and application, (3) strategies and methods, (4) age/developmentally appropriate, (5) examples, and (6) terminology. Each standard does not have identical associated categories and the number of bullet points associated with each category changes according to standard. The inclusion of these categories establishes clarity and details for educators who are unpacking standards for instructional purposes.

Additionally, each set of grade level standards has a *key content competencies* page. Within this page, teachers are instructed to keep the focus of instruction and assessment of content mastery on the key competencies from the provided list. Each grade level has a varying number of key competencies; however, the range is from seven to nine competencies per grade level.

The standards document includes an overview of the review and revision process which included educator working groups, surveys, an academic review committee, and a citizens review committee. The educator working groups created progression documents that the academic review committee examined and approved. A statement on the use of mathematical strategies and methods to affirm local control is in the standards document. The statement ensures that, “teachers are afforded the flexibility to support the individual needs of their students” (Georgia Department of Education [GADOE], 2021, p. 3) and reiterates that state tests will focus on student understanding of a mathematics concept or skill instead of a specific solution strategy.

The most notable deviations from the CCSSM for the Georgia Mathematics standards based on the coding process include verb changes to the language used within the standards, greater or fewer differences in number expectations in standards, and levels of cognitive demand being either higher or lower. In Table 13, the top three codes with the highest frequencies are further explained. Underneath each header cell of “Code & Number of Occurrences” a bolded phrase for the code and a bolded number is listed indicating how many times that code occurred across the K-5 standards. Italics are added within cells to emphasize a change to the standards. Some standards received multiple codes during the coding process and may appear in the table in respective columns. Descriptions of changes not included in the table are in narrative form following Table 14.

Table 14

Georgia Standards Changes from Top Three Most Frequent Codes

GEORGIA MATHEMATICS STANDARDS			
Grade Level	**Code & Number of Occurrences**		
	Verb Change **56**	**Number Difference** **26**	**Teacher Content Knowledge** **23**
Kindergarten	*Describe* numbers from 11 to 19 added to compose and decompose *Identify* written numbers 0-20 instead of *write* *Sort* and *classify* added to *identify and compare* two-dimensional and three dimensional shapes	Given a number from 1-20, identify one more or one less; not addressed in CCSSM Count backward from 20; not addressed in CCSSM Use two or more shapes to form a larger shape; no specified number of shapes identified in CCSSM	*Explain* the last counted number represents the quantity in a set (cardinality) Describe relative position with positional words Represent addition and subtraction in authentic situations with a variety of representations and strategies
Grade 1	*Explain* that two digits represent tens and ones instead of *understand* *Identify, sort, and classify* added to *build and draw* shapes with defining attributes *Estimate, measure, and record* lengths instead of *order* and *express* lengths of objects	Compare and order up to 100 instead of two two-digit numbers Add and subtract within 20 using properties of operations instead of 100 Add and subtract multiples of 10 within 100 instead of range of 10-90 for subtraction	Develop strategies for addition and subtraction by exploring strings of related problems Recognize the inverse relationship to solve authentic addition and subtraction problems Repeating patterns with a core of up to three elements
Grade 2	*Represent* and *order* whole numbers added to *compare*	Count forward by 25s to 1,000, instead of 5s, 10s, and 100s	Construct simple measuring instruments using unit models

GEORGIA MATHEMATICS STANDARDS			
	Estimate and *measure* elapsed time added to *tell* and *write* time *Find* the value of a group of coins and *determine* combinations of coins added to *solve* problems involving bills and coins *Identify* added to *describe* parts of a whole	Count forward and backwards by ones from any number within 1,000 instead of forward by 5s, 10s, and 100s Elapsed time to the hour and half hour, instead of tell time to the nearest five minutes Patterns with addition and subtraction within 20	Use appropriate graphical displays to solve relevant everyday life problems based on gathered information Solve addition and subtraction problems using part-whole strategies
Grade 3	*Analyze* three-dimensional figures, *identify* and *describe* quadrilaterals as faces of these figures *Investigate* area instead of *find* *Discover* and *explain* how to find area with multiplication instead of multiply side lengths to find area	Round numbers up to 1,000 instead of round whole numbers Apply strategies to addition and subtraction problems up to 10,000 Elapsed time to hour half hour, and quarter hour instead of intervals of minutes Estimate time to the nearest fifteen minutes	Solve practical, relevant problems using part-whole strategies, visual representations and/or concrete models for multiplying and dividing within 100 Describe how multiple copies of a unit fraction form a non-unit fraction, including parts of a whole, parts of a set, points on a number line, distances on a number line, and area models Compare fractions flexibly with a variety of tools and strategies Represent fractions greater than one in multiple ways
Grade 4	*Show* and *extend* understanding to find value of digit when shifted to right or left instead of *recognize*	Read and write whole numbers to the hundred-thousands place instead of multi-digit whole numbers	Add and subtract fluently with place value understanding, properties of operations, and relationships between operations

GEORGIA MATHEMATICS STANDARDS			
	Interpret and *model in addition to solve* multiplication comparison problems *Represent* whole numbers as sum of unit fractions instead of *decompose* *Demonstrate* the concept of equivalent fractions instead of *recognize* and *generate* *Represent, read,* and *write* fractions with denominators of 10 or 100 using decimal notation instead of *use* *Explore* and *investigate* geometric attributes in addition to *draw*	Elapsed time to the nearest minute; third grade requirement in CCSSM Draw angles based on relationship to angle measure of 90 degrees instead of sketch angles of specified measure	Explain the identity property of multiplication as it relates to equivalent fractions Compare fractions by flexibly using a variety of tools and strategies Solve problems involving area and perimeter of composite rectangles
Grade 5	*Interpret* and *evaluate* expressions in addition to *write* *Determine* through exploration and investigate the attributes of geometric figures of a category instead of *understand* *Discover* and *explain* how the volume of a right rectangular prism can be found instead of *find*	Fluently multiply up to three-digit by two-digit whole numbers instead of multi-digit whole numbers Fluently divide whole numbers up to four-digit dividends and two-digit divisors up to 25 instead of no restriction Round decimals to the hundredths place	Compare and order up to three fractions by flexibly using a variety of tools and strategies Solve addition and subtraction of decimal problems with a variety of strategies

Note. All standards information obtained from GADOE, 2021; NGA & CCSSO, 2010.

There are other notable changes to Georgia standards that should be shared that are not reflected in Table 14. Georgia standards use the phrases “practical, mathematical problems”, “authentic problems”, “relevant problems” or “problems relevant to everyday life” instead of the phrase “real-world problems” that was used throughout the CCSSM. Few standards were added extending identification of money to earlier grade levels and patterns began at earlier grade levels. However, many standards were removed during the revision from the CCSSM.

Some standards were either deconstructed from one standard in the CCSSM into multiple standards in the Georgia Mathematics Standards, or multiple CCSSM standards were combined into one standard in the Georgia Mathematics Standards. An example of a deconstructed standard includes the CCSSM 2.G.3, which states,

> Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape (NGA & CCSSO, 2010, p. 20).

Within the Georgia 2021 standards, this statement is deconstructed into two standards, which includes: 1) 2.GSR.7.3 “Partition circles and rectangles into two, three, or four equal shares. Identify and describe equal-sized parts of the whole using fractional names (“halves,” “thirds,” “fourths”, “half of,” “third of,” “quarter of,” etc.)” and 2) 2.GSR.7.4 “Recognize that equal shares of identical wholes may be different shapes within the same whole” (GADOE, 2021, pp.33-34). Below, additional changes and missing standards are synthesized by grade level.

Kindergarten

Students in Kindergarten are introduced to money through the identification of pennies, nickels, and dimes, which pairs with the benchmarks of 5 and 10 within instruction. Students are expected to count backwards, a skill not explicitly stated in the CCSSM. The exploration of patterns, specifically repeating patterns, is also introduced at this grade level. A data standard was also added that requires students to ask and answer questions based on gathered information and observations.

Counting standards that are missing include saying number names in standard order, and demonstrating understanding that each successive number name refers to a quantity that is one larger. The comparison of numbers between 1 and 10 as written numerals is also missing. Making ten from any number one to nine is also missing. A geometry standard is missing that includes correctly naming shapes.

First Grade

First graders are expected to measure elapsed time to the hour via a provided number line. They are also expected to identify all United States coins. Repeating patterns are investigated with identification, creation, and description of growing, shrinking, and repeated patterns based on addition and subtraction of the specific values of 1s, 2s, 5s, and 10s.

Operations and Algebraic thinking standards that were removed include solving word problems with up to three whole numbers using various strategies, understanding subtraction as an unknown-addend problem, and relating counting to addition and subtraction. Number and Operations in Base Ten standards that were removed include thinking of a ten as a bundle of ten ones, the composition of numbers from 11 to 19, knowing the value of the digit in the tens place in multiples of ten, and adding within 100 using a variety of strategies. Missing Measurement and Data standards include the organization, representation, and interpretation of data with up to three categories with specific questions. Geometry standards that are missing include "describe partitional shares using specific words and phrases" and "understand that the partitions are decomposing equal shares to create smaller shares."

Second Grade

Second graders are not expected to identify, describe, and create numerical patterns with addition and subtraction, which includes growing and shrinking patterns involving either operation up to 20. A data standard was added that requires students to ask and answer questions based on gathered information and observations. Students are expected to estimate, and measure elapsed time to the hour and half hour on a timeline. They are also expected to find the value of a group of coins or combinations of coins that equate to less than one hundred cents. Finally, second graders are introduced to line symmetry and expected to identify at least one line of symmetry in everyday objects.

Standards that are missing within second grade include the specific problem types that represent addition and subtraction problems and knowing all sums of two one-digit numbers from memory. Number and Operations in Base Ten standards that are missing include thinking of 100 as a bundle of ten tens, the value of the hundreds place digit in multiples of 100, reading and writing numbers up to 1,000 in various representations, adding up to four two-digit numbers based on place value and properties of operations, and explaining why addition and subtraction strategies work. Measurement and Data standards that are missing include the specific tools to measure length, solving length word problems with addition and subtraction, and drawing picture and bar graphs to solve problems with data sets of up to four categories. Geometry standards that are missing require students to partition a rectangle into same-size square columns and rows and count to determine the total number of squares.

Third Grade

Third graders are expected to read, write, and compare numbers up to 10,000. They are also expected to use the meaning of the equal sign to determine if expressions involving all four operations are equivalent. A standard was added in that students are expected to estimate time to the nearest fifteen minutes from the analysis of an analog clock. Additionally, a fourth-grade standard involving lines, line segments, and angles was moved to third grade standards. Finally, third graders are expected to identify lines of symmetry in polygons.

Standards that are missing from Operations and Algebraic Thinking strands include interpreting products and quotients of whole numbers and understanding division as an unknown-factor problem. A majority of fraction standards are missing compared to the CCSSM, with the Georgia Mathematics Standards only having four fraction standards instead of the nine available in the CCSSM. The missing standards include understanding the equal parts of a fraction, understanding fractions on a number line, understanding the intervals on a number line, understanding the endpoints on a number line, and explaining equivalence of fractions in special cases. Missing Geometry standards include drawing a scaled picture and bar graph to represent several categories of data and solving one or two step problems, and most of the area standards. The CCSSM has nine standards pertaining to area, where the Georgia Mathematics Standards only has three.

Fourth Grade

Fourth grade has a few added standards that are not found in the CCSSM. Students are expected to know that a digit can move to the left or right and the value of the digit changes based on the relationship with multiplication and division. Students are expected to create dot plots to display distribution of measurement data. Finally, students are expected to use input-output rules to describe patterns and relationships in problems.

The standard algorithm of two-digit addition and subtraction is missing from the Number and Base Ten Operations standard. Distinguishing multiplicative comparison from additive comparison, representing problems with equation using a letter for an unknown quantity, and determine if a number is a multiple of a given number are missing from Operations and Algebraic thinking standards. All standards pertaining to multiplying fractions by a whole number are excluded from the Georgia Mathematics Standards. Additionally, the area and perimeter formulas for rectangles are missing as well.

Fifth Grade

Fifth graders in the state of Georgia are now expected to compare and order up to three fractions with different numerators and denominators. The data standard incorporated throughout the K-5 measurement and data reasoning domain expecting students to ask and answer questions about gathered and observed data is also included in this grade level. A fourth-grade standard of solving problems with different units of measurement, distance, mass, weight, volume, and time was moved to this grade level.

Missing standards include solving word problems involving addition and subtraction of fractions with benchmark fractions and number sense, interpreting multiplication as scaling with fractions, finding the area of rectangles with fractional side lengths, and solving word problems with multiplication of fractions and mixed numbers. Writing expressions and interpreting without evaluating is also missing. A few standards pertaining to volume have been omitted as well that includes using unit cubes to represent a side length and measuring volumes by counting cubes. Finally, standards related to the coordinate plane have also been omitted.

<u>Sub Questions Findings</u>

To answer sub question 1, the Common Core SMPs were compared to the Georgia Mathematical Practices. The Georgia K-8 Mathematics Standards did not alter the CCSSM Standards of Mathematical Practices. All 8 SMPs remain the same, however, their standards document does not provide the same detailed information on each SMP, that the CCSSM provides.

To answer sub question 2, the CCSSM Domains were compared to the Georgia Big Ideas. Table 15 compares the CCSSM Domains and the Georgia Big Ideas with similar domains and ideas positioned in the same rows. If no like domains or strands existed, the cell in the row was intentionally left blank. The most notable differences between the domains are that "Counting and Cardinality" and "Operations Base-Ten" were merged into other strands. The concepts of "Data and Statistical Reasoning", "Spatial Reasoning", and "Mathematical Practices and Modeling" were added, as well as the term "Reasoning" to all big ideas.

Table 15

Domains and Big Ideas Comparison

CCSSM Domains	Georgia Big Ideas
Counting and Cardinality	Numerical Reasoning
Operations and Algebraic Thinking	Patterning & Algebraic Reasoning
Number and Operations in Base Ten	Numerical Reasoning
Number and Operations in Fractions	Numerical Reasoning
Measurement and Data	Measurement & Data Reasoning Data & Statistical Reasoning
Geometry	Geometric & Spatial Reasoning Mathematical Practices & Modeling

To answer sub question 3, the Georgia Mathematics Standards document and the Georgia Department of Education website were both consulted. There is a document on the Georgia Department of Education website that provides a visual progression of mathematics expectations, but no references were made as to how the document was created or what resources were used to create the learning progressions. The document did not declare who created the document, however, the footer of each page included the Georgia Department of Education.

Finally, to answer sub question 4, each standard that demonstrated a difference was coded as a procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes during the coding process using the letters B, P, or

C. This process of determining the learning outcome associated with the changes demonstrated by states was completed by examining the blue color-coded word differences in Excel. The color-coded word differences represented changes in state standards from the CCSSM. The color-coded words were then sorted into the categories of procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes.

Figure 11 demonstrates the frequency of learning outcomes made to the standards that deviated from the CCSSM. When considering the learning outcome of standards that demonstrated differences in Georgia K-5 mathematics standards, more emphasis was used for procedural outcomes with conceptual learning outcomes receiving the least amount of emphasis.

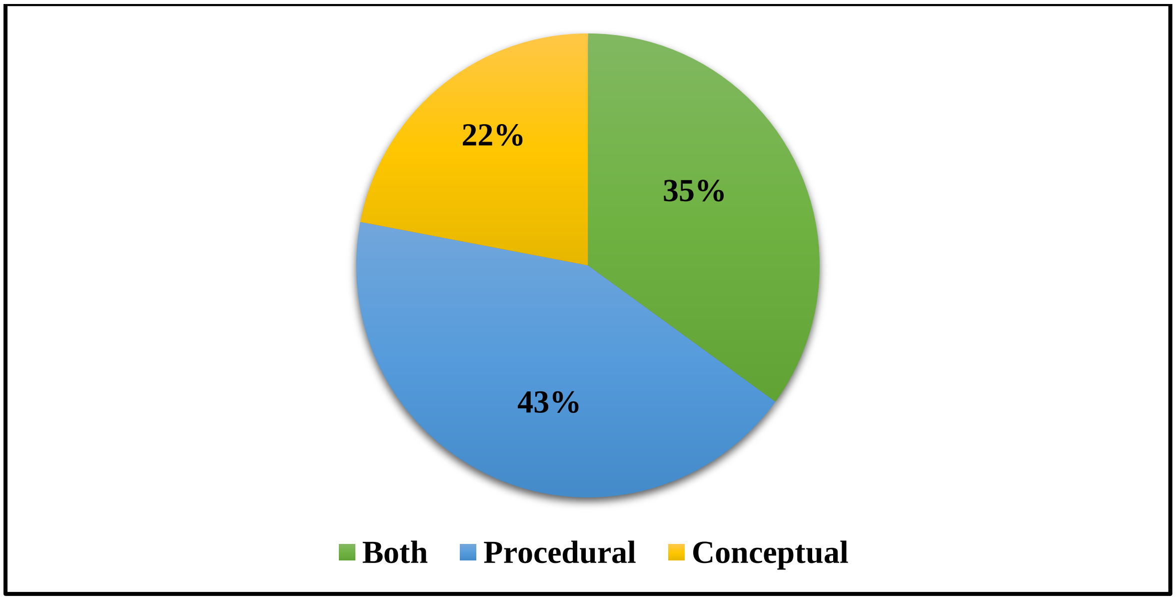

Figure 11

Percentage of Georgia Standard Revisions as Having Conceptual or Procedural Learning Outcomes

In conclusion, majority of the changes that occurred to the Georgia Mathematics Standards include the addition of pattern exploration and data exploration through a single, consistent standard repeated in each grade level. However, many of the changes were verb changes or omission of standards from the CCSSM.

It is important to note that the Georgia Department of Education website states that the 2021 mathematics standards were “Georgia developed” (GADOE, 2023, para. 2) with the mathematics standards document further stating that the standards are, “Georgia-owned and Georgia-grown” (GADOE, 2021, p.1). The website further goes on to assert that the standards are clear and understandable while providing a strong foundation in mathematics and, “present a reasonable amount of content in each year” (GADOE, 2023, para. 2).

A wide range of documents are available on the Georgia Department of Education website. This includes an 81-page explanation of changes and improvements document which listed the standards that were added during the standards revision process, a learning progression document, a mathematical practices document, an early numeracy CGI problem-types (with Carpenter et. al., 2006 cited within the document), a support guide for English learners, links to professional learning opportunities by grade band, and a 241-page mathematics glossary. The glossary allows for readers to submit a term to be added to the glossary. A definition and example(s) are provided for each term. Some examples offer interactive components to demonstrate the actions associated with the term.

Nebraska 2022

Nebraska is one of four states that never adopted the CCSSM, however Nebraska did apply for the RTTTI. During the beginning of the Common Core era, Nebraska continued with implementation of the 2009 Nebraska Mathematics Standards. Nebraska §79.760.01 (Academic Content Standards Act, 2022) requires standards to be reviewed and updated every seven years, which resulted in the 2015 College and Career Ready Standards for Mathematics and subsequently the 2022 College and Career Ready Standards for Mathematics. Both the 2015 and 2022 Nebraska standards reflect a deviation greater than 15% of the CCSSM, but for this study to remain as current as possible, only the 2022 standards were coded.

The Nebraska Department of Education (NDE) website offers a Crosswalk document from their 2015 mathematics standards to the 2022 mathematics standards, a key instructional shifts document, a standards glossary, and a webinar video describing the mathematics standards revision process. Additionally, Nebraska demonstrates commitment to communities getting involved with mathematics by providing PDF versions of “Family Math” and “Do Math” packets on their website that are available in 21st Century Community Learning Centers afterschool program libraries within the state. The website also provides a link to a free summer mathematics learning program sponsored by Quantiles Mathematics (NDE, 2019).

In the webinar video, an overview of the mathematics standards revision process does mention the use of the RAPID model, a nontraditional acronym for input, recommend, agree, decide, and perform (Bain & Company 2023). The webinar presenter explains the RAPID model and define the role of each writing committee member (subject matter expert, post-secondary advisor, team facilitator, and revision team member) but do not mention specific committee members' names (NDE Teaching, Learning, and Assessment, 2022). However, the mathematics standard document itself does offer an acknowledgements section where specific names of team members that were involved in the revision process are included (NDE, 2022). Their education role and school/organization are listed, but their specific roles within the RAPID model are not provided.

To answer the coding research question of *in what ways, if any, do K-5 state mathematics standards differ from the CCSSM*, a line-by-line comparison of the 2022 Nebraska College and Career Ready Standards for Mathematics was conducted within Excel. Deviations from the CCSSM were color coded in blue, and any missing verbiage in standards were color coded in orange.

The most notable deviations in the Nebraska standards from the CCSSM includes the omission of parts of multiple standards. The number of omissions ranged in each grade level. Additionally, standards that were excluded included those found in fourth grade pertaining to multiplication comparisons, work with patterns, and rounding whole numbers. Few standards were added, and language did differ from the CCSSM. Table 16 includes the three codes with the most frequent occurrences following the coding process. In each cell of codes is a bolded number indicating the number of times that code occurred across the K-5 standards.

Table 16

Nebraska Standards Changes from Top Three Most Frequent Codes

NEBRASKA COLLEGE AND CAREER READY STANDARDS FOR MATHEMATICS			
Grade Level	**Code & Number of Occurrences**		
	Verb Change **61**	**Cognitive Demand** **42**	**Number Difference** **14**
Kindergarten	*Explain* addition and subtraction relationships in addition to *represent*	Show the relationship between number and quantities when counting objects	Count verbally backward within 20
	Efficiently, *flexibly*, and *accurately* add and subtract within 5 instead of *fluently*	Provide verbal explanation of number pairs equal to 10	Compare the number of objects in two groups up to 20 (instead of 10 in CCSSM)
	Name two-dimensional and three-dimensional shapes in addition to *identify*	Describe one or more attributes of shapes	
	Create shapes instead of *model*	Identify, sort, classify objects by size, shape, color, and other attributes	
	Identify and *sort* shapes in addition to *classify*	Identify objects that do not belong to a group and explain the reasoning	
Grade 1	*Efficiently*, *flexibly*, and *accurately* add and subtract within 10 instead of *fluently*	Create authentic addition or subtraction problem within 20	Count verbally by ones and tens, instead of starting at any number less than 120 in CCSSM

NEBRASKA COLLEGE AND CAREER READY STANDARDS FOR MATHEMATICS			
	Use instead of *understanding* the meaning of the equal sign *Create* an authentic problem for adding and subtracting within 20 *instead* of solve word problems	Describe parallel or non-parallel lines and sides of shapes Collect data, ask and answer questions about data points using a picture graph; missing *interpret data* from standard	Add and subtract within 10, instead of within 20 in CCSSM
Grade 2	*Create* authentic problem involving addition and subtraction within 100 instead of *use* addition and subtraction within 100 to solve word problems *Identify* a group of objects up to 20 instead of *determine* *Describe* faces of two dimensional and three-dimensional shapes, *Identify* and *count* attributes instead of *recognize* *Divide* instead of *partition* circles and rectangles *Ask* authentic questions to generate data using picture graphs and bar graphs instead of *draw*	Ask authentic questions to generate data and represent data on picture graphs and bar graphs, create and represent data on a line plot Justify comparison of two three-digit numbers Create authentic addition and subtraction problems within 100, with unknowns in all positions Count within 1,000 starting at a variety of multiples of 5, 10, or 100 Analyze data including one-step comparison problems from picture graphs or bar graphs	Add up to three two-digit numbers (instead of add up to four two-digit numbers in CCSSM) Identify a group of objects from 0 to 2- as even or odd (instead of up to 20)
Grade 3	*Partition* two-dimensional figures into unit fractions instead of *understand*	Partition two-dimensional figures and express area of each part as a unit fraction	Add and subtract up to four-digit whole numbers with and without regrouping (instead of within 1,000 in CCSSM)

NEBRASKA COLLEGE AND CAREER READY STANDARDS FOR MATHEMATICS

	Show and *identify* equivalent fractions instead of *understand* *Interpret* in addition to *solve* two-step problems with whole numbers and four operations *Interpret* and *explain* the meaning of multiplication and division instead of *fluently multiply* and *divide* using strategies	Justify and identify fractions that are equivalent to whole numbers Interpret and explain the meaning of multiplication and division through various strategies Identify and use appropriate tools in both customary and metric units to solve authentic problems involving length, weight, mass, liquid volume, and capacity	Solve and write one-step whole number equation to represent authentic problems using the four operations; as opposed to two-step word problems in CCSSM Create picture and bar graphs with more than four categories; as opposed to several categories in CCSSM
	Sort quadrilaterals instead of *understand* *Analyze* data and *make* simple statements using information found in graphs and plots	Analyze data and make statements using information from picture graphs, line plots, and bar graphs	
Grade 4	*Represent* and *justify* comparisons of whole numbers up to 1,000,000 instead of *compare* *Demonstrate* how mixed number is equivalent and how equivalent fractions are generated through multiplication in addition to *explain*	Order fractions with unlike numerators and denominators using number lines, reasoning, and/or equivalence Explain the meaning of adding and subtracting fractions with reasoning strategies	Read, write, and demonstrate equivalent representation up to 1,000,000; no limitation in CCSSM Solve one and two step authentic problems (instead of multi-step in CCSSM) with letter to represent the unknown quantity

NEBRASKA COLLEGE AND CAREER READY STANDARDS FOR MATHEMATICS			
	Explain addition and subtraction of fractions in addition to *understand* *Identify*, *create*, and *describe* specific geometric components instead of *draw*	Determine the reasonableness of products and quotients using estimation and number sense Determine the reasonableness of measurements including time, length, weight, mass, capacity, and angles	
	Draw and *justify* lines of symmetry in addition to *recognize* *Generate* simple conversions within customary and metric systems instead of *know* relative sizes *Generate* and *represent* data instead of *make* a line plot		
Grade 5	Decimal standards go to thousandths place Round decimals to the nearest tenth or hundredth	*Scaled number line* for comparison of numbers up to thousandths place Use of *models* for multiplication and division of numbers with decimals including estimation, rounding, and place value	When multiplying a given number by a fraction less than 1 or a fraction greater than 1, predict and explain the relative size of the product to the given number without calculating, including problems in which remainders must be interpreted within the context Expressions are any combination of the arithmetic operations; including parentheses, with whole numbers, decimals, and fractions

NEBRASKA COLLEGE AND CAREER READY STANDARDS FOR MATHEMATICS			
	Denominators limited to numbers within 20 when multiplying fractions by fractions	Use of *two column table* to record inputs and output of numerical patterns Addition and subtraction of fractions with unlike denominators (including fractions greater than 1 and mixed numbers); includes manipulatives, drawings, or the properties of operations	Conversions include length, time, volume, and capacity represented as whole numbers, fractions, and decimals Numerical data with graphs includes fractional and decimal values; decimal values are no smaller than the hundredths place and fraction denominators restricted to 1, 2, 3, and 4 but can be greater than one

Note. All standards information obtained from NDE, 2022; NGA & CCSSO, 2010.

There are other notable differences between the CCSSM and the Nebraska standards that should be shared that are not reflected in Table 16. Additional changes made to the Nebraska College and Career Ready Mathematics Standards are synthesized by grade level.

Kindergarten

Subitizing has an emphasis in Kindergarten standards. Students are also introduced to coins and clocks in this grade level.

First Grade

Subitizing is emphasized again in first grade. Patterns are introduced with 2s, 5s, and 10s. The relationships of addition and subtraction are explored to solve problems and compose or decompose specific properties of operation. Additionally, rhombi, trapezoids, and hexagons are explored in Geometry. Parallel lines in geometric figures are also introduced. Students are expected to count the value of like collections of coins and relate the like collections to patterns of skip counting.

Second Grade

Subitizing continues to receive an emphasis with collections connected to multiplicative thinking. Skip counting using various multiples is also emphasized. An emphasis is placed on recognition of edges, faces, and vertices of solid shapes. Data standards are added that require the use of picture and bar graphs to represent data.

Third Grade

An emphasis is placed on multiple representations through the use of objects, representations, and various number forms up to 10,000. Students rely on estimations and number sense to determine the reasonableness of sums and differences. Geometry standards include sorting quadrilaterals into categories based on their attributes. Data standards include the collection of data through observations, surveys, and experiments and students are expected to analyze data from picture graphs, line plots, and bar graphs.

Fourth Grade

Fourth graders are expected to explain and demonstrate equivalence of mixed numbers and fractions greater than one with various strategies. They are also expected to use the identity property of multiplication to generate equivalent fractions. An emphasis on estimation and number sense strategies is used for determining the reasonableness of products and quotients.

Fifth Grade

Fifth graders are expected to represent and justify comparisons of whole numbers, fractions, mixed numbers and decimals through the thousandths place. Students are expected to add and subtract fractions and mixed numbers with unlike denominators without simplifying. Again, an emphasis is placed on justifying reasonableness of computations with all number types in this grade level.

While other states in this study have sections describing additional changes at grade levels, the Nebraska 2022 standards are unique due to the sheer quantity of standards absent from their revisions. The removal of entire sections of the CCSSM within a standard ultimately adjusts the cognitive demand of the overall trajectory of acquired mathematics knowledge. Table 17 provides an overview of standards that were not present in the 2022 revisions to the Nebraska standards. The removal of standards was calculated into the overall 15% determination of changes made to standards.

Table 17

Nebraska Absent Standards

Grade level	**Absent Standards**
4th grade	4.OA.1, 4.OA.2, 4.OA.3 Interpret multiplicative comparisons, solve word problems with multiplicative comparisons,
	4.OA.5 Generate number or shape patterns that follow a given rule
	4.NBT.3 Round numbers to any place
	4.NBT.1 A digit in one place represents ten times what it represents in the place to its right
	Part of 4.NBT.4 Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number
	Part of 4.NBT.5 Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
	4.NBT.6 Missing strategies to find whole number quotients, including properties of operations, relationship between multiplication and division, explaining through equations, rectangular arrays, and area models.
	4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions
	Part of 4.NF.2: Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or
	4.NF.3, 4.NF.3.a, 4.NF.3.b Understand a fraction *a/b* with *a* >1 as a sum o fractions *1/b*, understand addition and subtraction of fractions as joining and separating parts referring to the same whole, decompose a fraction into a sum of fractions with the same denominator in more than one way
	4.NF.4.a, 4.NF.4.b, 4.NF.4.c Understand a fraction a/b as a multiple of 1/b, Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. Solve word problems involving

	multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
	4.NF.5, 4.NF.6, 4.NF.7 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100, use decimal notation for fractions with denominators 10 or 100, compare two decimals
	4.MD.1 Record measurement equivalents in a two-column table
	Parts of 4.MD.2 Use the four operations to solve word problems involving distances, liquid volumes, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
	4.MD.5, Part of 4.MD.6, Part of 4.MD.7: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement, Sketch angles of specified measure, When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure..
	Part of 4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figure
5th grade	5.G.3 Understanding of attributes belonging to a category also belong to subcategories of two-dimensional figures
	5.NBT.1 a digit is 10 times as much in a place to the right
	5.NBT.2 Explanation of patterns observed with zeros when multiplying by powers of 10 and placement of decimal when dividing by a power of
	5.OA.2 write expressions and interpret without calculating
	5.OA.3 Generate patterns using rules and identify relationships between corresponding terms

	5.NBT.4 Round decimals to any place
	Parts of 5.NF.3 Interpret a fraction as division of the numerator by the denominator
	5.NF.5 Interpret multiplication as scaling
	5.NF.7a-5.NF.7.c Divide unit fractions by whole number and whole numbers by unit fractions
	5.MD.2 Making a line plot to display fractional measurements, solve problems with 4 operations and fractions involving information from line plot
	5.MD.3.a-5.MD.3.b unit cube understanding and a solid figure can be packed without gaps or overlaps using *n* unit cubes has a volume of *n* cubic units
	5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume
	Part of 5.MD.5.c Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems
	5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation

Note. All standards information obtained from NDE, 2022.

Sub Questions Findings

To answer sub question 1, the Common Core SMPs were compared to the Nebraska Mathematical Practices. The most notable differences include the exclusion of “using appropriate tools strategically” “attend to precision” and “look for and express regularity in repeated reasoning”. Table 18 shows the specific changes to the Nebraska Mathematical Practices. No descriptors were needed to compare the practices due to their alignment.

Table 18

Comparison of SMPs and MPs

Common Core State Standards for Mathematical Practice	Nebraska Mathematical Processes Standards	Descriptors of Aligned SMP
Make sense of problems and persevere to solve them	Make sense of problems and persevere in solving them	
Reason abstractly and quantitatively	Reason quantitatively and abstractly and consider the reasoning of others	
Construct viable arguments and critique the reasoning of others	Explain and justify mathematical ideas sing precise mathematical language in written or oral communication	
Model with mathematics	Create and use representations to organize, record, and communicate mathematical ideas	
Use appropriate tools strategically		
Attend to precision		

Common Core State Standards for Mathematical Practice	Nebraska Mathematical Processes Standards	Descriptors of Aligned SMP
Look for and make use of structure	Analyze mathematical relationships to connect mathematical ideas	
Look for and express regularity in repeated reasoning		

Note. All other comparison tables to SMP's tables have column *Descriptors of Aligned SMP*. Due to alignment, no descriptors were used in Table 17.

To answer sub question 2, the CCSSM Domains were compared to the Nebraska content strands. Table 19 compares the CCSSM Domains and the Nebraska content strands with similar domains and ideas positioned in the same rows. If no like domains or strands existed, the cell in the row was intentionally left blank. The most notable differences between the domains are that "Counting and Cardinality", "Operations in Base-Ten", "Operations with Fractions", and "Measurement" were merged into other content strands. The concepts of "Ratio and Proportions" were added as strands.

Table 19

Domains and Content Strands Comparison

CCSSM Domains	Nebraska Content Strands
Counting and Cardinality	Number
Operations and Algebraic Thinking	Algebra
Number and Operations in Base Ten	Number
Number and Operations in Fractions	Number
Measurement and Data	Data
Geometry	Geometry
	Ratios and Proportions

To answer sub question 3, the Nebraska College and Career Ready Standards document and the Nebraska Department of Education website were both consulted. No information was available that discussed learning trajectories or learning progression in the creation of the revised standards.

Finally, to answer sub question 4, each standard that demonstrated a difference was coded as a procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes during the coding process using the letters P, C, or B. This process of determining the learning outcome associated with the changes demonstrated by states was completed by examining the blue color-coded word differences in Excel. The color-coded word differences represented changes in state standards from the CCSSM. The color-coded words were then sorted into the categories of procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes. Figure 12 demonstrates the frequency of learning outcomes made to the standards that deviated from the CCSSM.

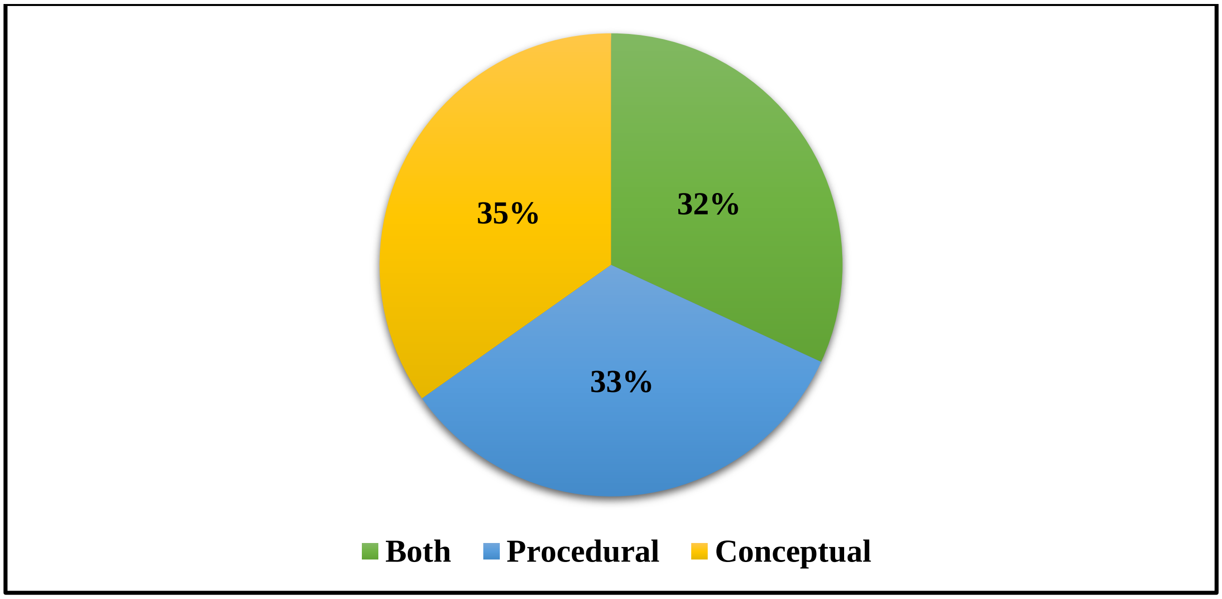

Figure 12

Percentage of Nebraska Standard Revisions as Having Conceptual or Procedural Learning Outcomes

The learning outcomes represent a balance of procedural and conceptual knowledge. Few standards were added to the Nebraska mathematics standards and elements of the CCSSM were removed, resulting in the balanced approach to learning outcomes that align with the balanced approach of the CCSSM.

In conclusion, the changes made to the Nebraska K-5 mathematics standards were the absence of parts or entire standards from the CCSSM. Multiple domains were merged into fewer strands. Fewer SMPs were used in the Nebraska standards. Most notably, Nebraska offers a balanced approach to learning outcomes through their standards revisions.

Oklahoma 2022

Oklahoma adopted the CCSSM in 2010 and repealed the standards in 2014 (Loveless, 2021). This resulted in the 2016 adoption of the Oklahoma Academic Standards Mathematics.

The Oklahoma State Department of Education requires revisions to standards every six years per Oklahoma §70 11-103.6a (Schools Act, 2020), which led to the 2022 Oklahoma Academic Standards Mathematics.

To answer the overarching research question of *in what ways, if any, do K-5 state mathematics standards differ from the CCSSM*, a line-by-line comparison of the 2022 Oklahoma (OAS-M) standards was conducted within Excel. Deviations from the CCSSM were color coded in blue, and any missing verbiage in standards were color coded in orange.

The most notable changes in the OAS-M from the CCSSM includes the addition of pre-K standards, pattern work beginning in Kindergarten, the inclusion of standards related to calendar time (e.g., days, tomorrow, yesterday), fraction standards begin in first grade, currency identification begins in first grade, the inclusion of estimation standards, volume exploration beginning in third grade, and the inclusion of temperature standards. Table 20 includes the three codes with the most frequent occurrences following the coding process. In each cell of codes is a bolded number indicating how many times that code occurred across the K-5 standards.

Table 20

Oklahoma Standards Changes from Top Three Most Frequent Codes

OKLAHOMA ACADEMIC STANDARDS MATHEMATICS			
Grade Level	**Code & Number of Occurrences**		
	Manipulatives **27**	**Unit Reference** **21**	**Cognitive Demand** **20**
Kindergarten	Representations of whole numbers from 0 to 10 represented with manipulatives Compare and order numbers with and without objects from 1 to 10 Compose and decompose numbers up to 10 using objects and pictures Distribute a set of objects into at least two smaller equal sets	Develop time concepts using words such as yesterday, today, tomorrow, morning, afternoon, and night	Draw conclusions from real-objects and picture graphs Compare and order objects according to location and measurable attributes Use smaller shapes to form a larger shape when there is an outline to follow Identify attributes of two-dimensional shapes using informal and formal geometric language interchangeably
Grade 1	Use concrete representations to describe whole numbers between 10 and 100 in terms of tens and ones	Use number relationships to locate the position of a given whole number on an open number line up to 20	Compose and decompose larger shapes using smaller two-dimensional shapes

OKLAHOMA ACADEMIC STANDARDS MATHEMATICS			
	Representations of numbers to 100 includes: numerals, words, addition and subtraction, pictures, tally marks, number lines, and manipulatives Partition a regular polygon using physical models and recognize when those parts are equal		Draw conclusions from picture and bar-type graphs
Grade 2	Read, write, discuss, and represent whole numbers up to 1,000. Representations include: numerals, words, pictures, tally marks, number lines and manipulatives Use concrete models and structured arrangements, such as repeated addition, arrays and ten frames to develop understanding of multiplication	Explain the relationship between length and the numbers on a ruler by using a ruler to measure lengths to the nearest whole unit	Draw conclusions and make predictions from information in a graph
Grade 3	Read, write, discuss, and represent whole numbers up to 100,000. Representations include: numerals, expressions with operations, words, pictures, number lines, and manipulatives	Represent multiplication facts by manipulatives, repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting Recognize unit fractions and use them to compose and decompose fractions	No codes

		Measure the length of objects to the nearest whole centimeter or meter Use an analog thermometer to determine temperature to the nearest degree in Fahrenheit and Celsius	
Grade 4	Use models to order and compare whole and fractions less than and greater than one Decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations Represent tenths and hundredths with concrete models, making connections between fractions and decimals	Compare and order decimals and whole numbers using place value, a number line and models such as grids and base 10 blocks Solve problems involving the conversion of one measure of time to another	Solve multi-step real-world and mathematical problems requiring the use of addition, subtraction, and multiplication of multi-digit whole numbers. Use strategies including: the relationship between operations, the use of appropriate technology, and the context of the problem to assess the reasonableness of results Use an understanding of place value to multiply or divide a number by 10, 100 and 1,000

OKLAHOMA ACADEMIC STANDARDS MATHEMATICS			
Grade 5	Add and subtract fractions, mixed numbers, and decimals using a variety of representations: fraction strips, area models, number lines, fraction rods Represent decimals using a variety of models including: 10 by 10 grids, rational number wheel, base-ten blocks, meter stick Choose and instrument and measure the length of an object to the nearest whole centimeter or 1/16-inch	Find 0.1, 0.01, and 0.001 more than a number. Find 0.1, 0.01, and 0.001 less than a number	Estimate sums and differences of fractions with like and unlike denominators, mixed numbers, and decimals to assess the reasonableness of the results Describe, classify and construct triangles, including equilateral, right, scalene, and isosceles triangles Describe and classify three-dimensional figures including cubes, rectangular prisms, and pyramids by the number of edges, faces or vertices as well as the shapes of faces Create and analyze line and double-bar graphs with whole numbers, fractions, and decimals

Note. The standards information in this table is compiled from OSDE, 2022; NGA & CCSSO, 2010.

Aside from the changes noted in Table 20, there are other notable changes to the Oklahoma standards that should be shared. They are synthesized by grade level below.

Kindergarten

Subitizing has an emphasis in Kindergarten in Oklahoma standards. Finding one more or one less is a standard that was added to this grade level. A focus on conceptual understanding of addition and subtraction to 10 was also added. Identification of coins, including quarters was added to standards. Patterns were also emphasized. Measurement and Data standards include being able to tell time as it relates to daily life and collecting, organizing, and interpreting data.

First Grade

Subitizing continued to be emphasized in standards. Currency standards include determine the value of a collection of coins up to one dollar. Pattern work continued with repeating, growing, and shrinking patterns explored in a variety of contexts. Volume and capacity are explored in Geometry standards. Additionally, students are expected to draw conclusions from picture and bar graphs.

Second Grade

Students are expected to round to the nearest 10 and 100 in second grade. A focus on estimation is incorporated with number and base ten operations of addition and subtraction up to 100. Notably, partitioning in relation to fractional understanding includes length models, set models, and area models. Currency standards and pattern standards are extended into this grade level and build from previous grade level standards. students are also expected to make predictions from information located on a graph.

<u>Third Grade</u>

An emphasis is placed on finding 100,000, 1,000, or 100 more or less than a given number. Students are also expected to compare and order numbers up to 100,000. Students are expected to round up to the nearest ten thousand place. The standard algorithm is expected for multiplication of two-digit by one-digit numbers. Fractional foundation sense is also emphasized in this grade level through the construction of length, set, and area models, composition and decomposition of unit fractions, and the use of models and number lines to order fractions. Pattern standards get extended to all four operations and geometric patterns. Types of angles are also explored in third grade.

<u>Fourth Grade</u>

Fourth graders are expected to demonstrate fluency with both multiplication and division facts up to 12. Standards are added that focus on computational estimation. Standards with decimals require students to compare and order decimals through various strategies, including benchmark decimals. Students are expected to find change within a total cost of $20. Pattern standards are extended to input-output charts and tables and function machines with operations on whole numbers. Volume is also explored in fourth grade. Data representations include frequency tables, timelines, and Venn diagrams with fractional and decimal amounts.

<u>Fifth Grade</u>

Emphasis is placed on estimation again as students are expected to estimate solutions to division problems to assess the reasonableness of their results and estimation is used to find sums and differences of fractions, mixed numbers and decimals. The standard algorithm is expected to be used with addition and subtraction of decimals. Students are also expected to find 0.1, 0.01, and 0.001 more than and less than a given number. A standard is added that requires students to determine if an equation with a variable is true or false. Students are also expected to work with nets of three-dimensional figures. Finally, students are expected to work with central tendency and range of a set of data.

<u>Sub Question Findings</u>

To answer sub question 1, the Common Core SMPs were compared to the Oklahoma Mathematical Actions and Processes document. The most notable differences include the exclusion of 'using appropriate tools strategically", "attend to precision", "look for and make use of structure", and "look for and express regularity in repeated reasoning". Additionally, Oklahoma's Mathematical Actions and Processes focus on procedural fluency and conceptual understanding and added a strand on developing productive mathematical dispositions. Table 21 outlines the specific changes in comparison to the CCSSM. A column is devoted to any strands that align with a description of an SMP. If alignment existed, the cell was intentionally left blank.

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Table 21

Comparison of SMPs and MAPs

Common Core State Standards for Mathematical Practice	Oklahoma Mathematical Actions and Processes	Descriptors of Aligned SMP (NGA & CCSSO, 2010)
Make sense of problems and persevere to solve them	Develop strategies for problem solving Develop a deep and flexible conceptual understanding Develop the ability to make conjectures, model, and generalize	"They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt" (p. 6)
Reason abstractly and quantitatively	Develop mathematical reasoning	
Construct viable arguments and critique the reasoning of others	Develop the ability to communicate mathematically	
Model with mathematics		

Common Core State Standards for Mathematical Practice	Oklahoma Mathematical Actions and Processes	Descriptors of Aligned SMP (NGA & CCSSO, 2010)
Use appropriate tools strategically		
Attend to precision		
Look for and make use of structure		
Look for and express regularity in repeated reasoning		
	Develop a productive mathematical disposition	
	Develop accurate and appropriate procedural fluency	

To answer sub question 2, the Common Core Domains were compared to the Oklahoma Strands. Table 22 compares the Common Core Domains and the Oklahoma Strands. The most notable differences between the domains are that “Counting and Cardinality” was merged and the specific emphasis on “Base-Ten” and “Fractions” were also merged into other strands. Probability was added to the strands. If no like domains or strand existed, the cell in the row was intentionally left blank.

Table 22

Domains and Strands Comparison

CCSSM Domains	**Oklahoma Strands**
Counting and Cardinality	Number and Operations
Operations and Algebraic Thinking	Algebraic Reasoning and Algebra
Number and Operations in Base Ten	Number and Operations
Number and Operations in Fractions	Number and Operations
Measurement and Data	Data and Probability
Geometry	Geometry and Measurement

To answer sub-question 3, the Oklahoma State Department of Education website and standards document were reviewed for indications of learning trajectories. A vague phrase was included that the standards, “are the result of the contributions of hundreds of mathematics educators and mathematicians from across the state of Oklahoma. This document reflects a balanced synthesis of the work of all members of the Oklahoma Academic Standards for Mathematics Writing Committee and feedback from educators, mathematicians, external reviews, and numerous education stakeholders” (Oklahoma State Department of Education [OSDE], 2022b, p. 3). No specific reference was made pertaining to the consideration of learning trajectories in the development of the standards. However, a section on the Oklahoma website labeled “Resources for Administrators” offers a framework for how educators have analyzed standards and objectives with guidance on learning progression and unit analysis. It is located behind an access portal with login credentials, so it is not available to the general public.

Finally, to answer sub question 4, each standard that demonstrated a difference was coded as a procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes during the coding process using the letters P, C, or B. This process of determining the learning outcome associated with the changes demonstrated by states was completed by examining the blue color-coded word differences in Excel. The color-coded word differences represented changes in state standards from the CCSSM. The color-coded words were then sorted into the categories of procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes. Figure 13 provides the percentages of learning outcomes to the revisions to standards.

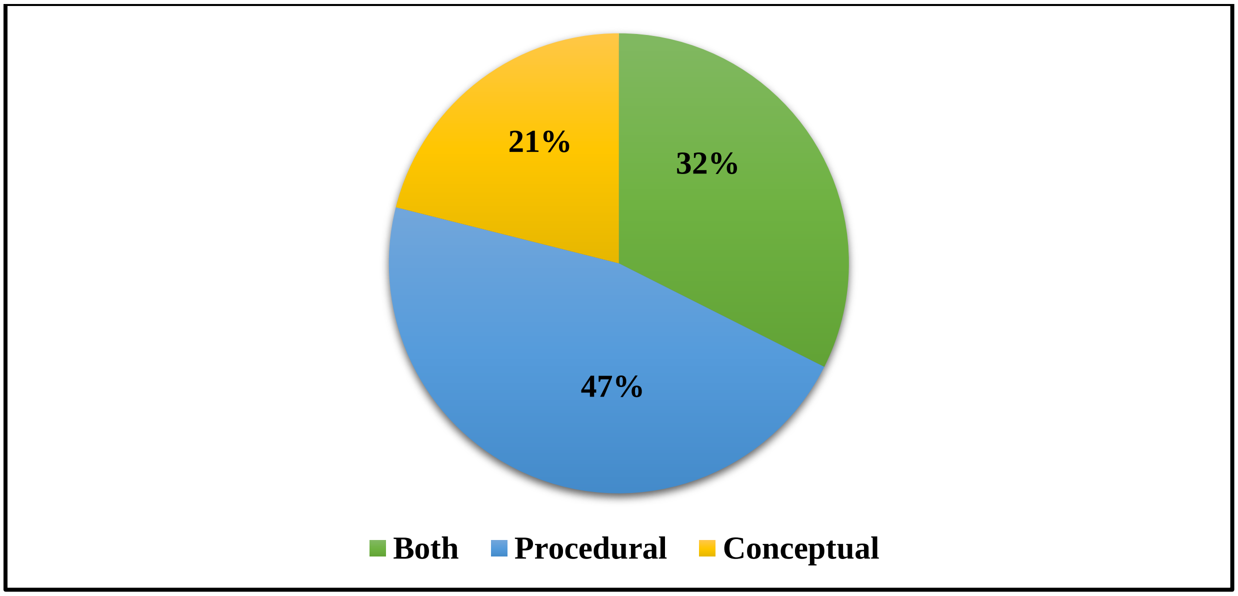

Figure 13:

Percentage of Oklahoma Standard Revisions as Having Conceptual or Procedural Learning Outcomes

When considering the learning outcome of standards that demonstrated differences in Oklahoma K-5 mathematics standards, more emphasis was used for procedural outcomes. Standards that had both procedural and conceptual learning outcomes were relativity the same emphasis. Conceptual learning outcomes received the least amount of emphasis.

In conclusion, the Oklahoma Academic Standards demonstrated changes in their mathematics standards, domains, SMPs, and emphasis on learning outcomes from the CCSSM. A specific focus on procedural fluency was incorporated into the revised SMPS, which was also reflected in the learning outcomes. Changes to the domains include the merging of domains into strands and the addition of probability.

The writing team members were not listed on the Oklahoma State Department of Education website, nor was information regarding the revision process disclosed. It is worthy of discussion that the Oklahoma State Department of Education website does offer a range of support for educators and administrators with direct links for both. Links to a supplemental online based mathematics program, Imagine Math (OSDE, 2022a), are available on the Oklahoma State Department of Education website. Additionally, a calendar of professional learning opportunities and events is available. These resources address the overarching research question as the CCSSM included supporting documents available online during the beginning years of implementation of the standards.

Texas 2014

Texas was one of four states that did not adopt the CCSSM (Schneider, 2015) and it was one of four states that did not apply for the RTTT initiative for either round (Howell, 2015). The latter is important to consider because an element of the application process was either adopting the CCSSM or aligning at least 85% of state standards with the CCSSM. Having not adopted the CCSSM or applied for the RTTT initiative, Texas had no requirements to align their state mathematics standards to the CCSSM. This resulted in the Texas Essential Knowledge and Skills for Mathematics, also known as TEKS. These standards were part of the inclusion criteria of this study and represent a deviation from the CCSSM. They were therefore included in this study and subsequently demonstrated enough deviations to be considered for the coding phase.

The Texas Education Agency (TEA) website displayed a list of standards writing committee members and their occupational positions at the time of the standards writing process. The website also included a list of expert reviewers who were appointed by the State Board of Education. Their comments to standards draft documents are publicly available. Only three of the seven, or 42.8%, of expert reviewers had jobs in the education field within the state of Texas.

The remaining expert reviewers were professors from R1 institutions across the country or a research consultant for a company based outside the state of Texas.

Lack of alignment to the CCSSM for the RTTT is reflected in the structure of the TEKS, which are formatted much differently from other state standards. Texas standards include specific statutory authority statements and reads similarly to a legal document than a document to advise teachers and families on the learning goals of desired grade levels. The introduction to the TEKS begins with a description of how the TEKS (Texas State Board of Education [TXSBOE], 2014) help Texas students prepare for, “the challenges they will face in the 21st century” (p. 1) with the inclusion of statistics, probability, and finance standards that focus on computational thinking and mathematical fluency.

To answer the overarching research question, a line-by-line comparison of the 2014 Texas standards was conducted within Excel. Deviations from the CCSSM were color coded in blue, and any missing verbiage in standards were color coded in orange.

The most notable differences in the Texas standards from the CCSSM includes currency identification beginning in Kindergarten, the addition of personal financial literacy standards, the use of formal and informal geometric language, the inclusion of divisibility rules, the exploration of perfect squares, and the addition of stem and leaf plots into instruction. Table 23 includes the three codes with the most frequently occurring codes following the coding process. In each cell of codes is a bolded number indicating how many times that code occurred across the K-5 standards. Some standards received multiple codes during the coding process and may appear in the table in respective columns.

Table 23

Texas Standards Changes from Top Three Most Frequent Codes

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS FOR MATHEMATICS			
Grade Level	**Code & Number of Occurrences**		
	Explicit Instructional Strategies **36**	**Personal Finance** **24**	**Manipulatives** **21**
Kindergarten	Model the action of joining to represent addition and the action of separating to represent subtraction	Differentiate between money received as income and money received as gifts Identify ways to earn income	Explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences Count forward and backward to at least 20 with and without objects
Grade 1	Order whole numbers up to 120 using place value and open number lines	Distinguish between spending and saving Consider charitable giving	No codes
Grade 2	No codes	Identify examples of borrowing money and distinguish between responsible and irresponsible borrowing	Determine whether a number up to 40 is even or odd using pairings of objects to represent the number

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS FOR MATHEMATICS			
		Distinguish between a deposit and a withdrawal	Model, create, and describe multiplication situations in which equivalent sets of concrete objects are joined
		Calculate how money saved can accumulate into a larger amount over time	Model, create, and describe division situations in which a set of concrete objects is separated into equivalent sets
Grade 3	Compose and decompose numbers up to 100,000 as a sum of ten thousands, thousands, hundreds, tens, and ones using objects, pictorial models, and numbers, (including expanded notation)	Identify decisions involving income, spending, saving, credit, and charitable giving	Represent fractions greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 using objects and pictorial models; this includes strip diagrams and number lines
	Represent a number on a number line between two consecutive multiples of 10; 100; 1,000; or 10,000 and use words to describe relative size of numbers in order to round whole numbers	List reasons to save and explain the benefit of a savings plan, including for college	
	Represent multiplication facts using a variety of approaches including: repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting	Describe the relationship between the availability or scarcity of resources and how that impacts cost	
Grade 4	Represent numbers to 1 million with expanded notation and numerals	Distinguish between fixed and variable expenses	Represent decimals, including tenths and hundredths, using concrete and visual models and money

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS FOR MATHEMATICS			
	Represent decimals, including tenths and hundredths, using concrete and visual models and money Multiply using strategies; mental math, partial products, and the commutative, associative, and distributive properties Represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity	Describe how to allocate a weekly allowance among spending; saving, including for college; and sharing Compare the advantages and disadvantages of various savings options	Represent decimals, including tenths and hundredths, using concrete and visual models and money
Grade 5	Represent multiplication of decimals with products to the hundredths using objects and pictorial models, including area models Represent and solve addition and subtraction of fractions with unequal denominators referring to the same whole using objects and pictorial models and properties of operations	Balance a simple budget, describe actions when expenses exceed a budget Define types of income	Use concrete objects and pictorial models to develop the formulas for the volume of a rectangular prism, including the special form for a cube Represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models

Note. The standards information in this table is compiled from TXSBOE, 2012; NGA & CCSSO, 2010.

There are other notable additions to Texas standards that should be shared. Texas included personal financial literacy standards into their mathematics standards as well as incorporated currency standards into earlier grade levels. Below, additional differences are synthesized by grade level and include various data representations not included in the CCSSM.

Kindergarten

Students in Kindergarten are expected to understand the relative position and magnitude of whole numbers. Additionally, students must be able to count forwards and backwards. Students must be able to identify coins, including quarters. The personal finance standards are the largest addition which includes identifying income, differentiating between money for income and money for gifts, understanding skills necessary for jobs, and distinguishing between wants and needs.

First Grade

First graders are expected to subitize numbers. They are also expected to describe relationships among United States coins. Tally marks and T-charts are used to collect, sort, and organize data in addition to picture and bar graphs. Students must also be able to define money as earned income, distinguish between spending and saving, and understand charitable giving.

Second Grade

Second graders are expected to work with numbers up to 1,200 by comparing and ordering numbers and considering what is one less or one more than a provided number up to 1,200. The standards also incorporate fractional understanding up to eighths. Algorithms of addition and subtraction can be used. Hands on tiling and area are incorporated into second grade. Finally, second graders are expected to understand the difference between borrowing money responsibly and irresponsibly, understand lending and know the benefits of lending, know the difference between a deposit and withdrawal, and understand the difference between producers and consumers.

Third Grade

Third graders are expected to use compatible numbers to estimate solutions with addition and subtraction problems. They are also expected to find the value of a collection of coins and bills. Additionally, strategies are outlined for representation of multiplication facts and multiplying a two-digit number by a one-digit number. Additionally, the additive property of area and multiplication related to rows and columns in a rectangle are used to measure area. Data standards include frequency tables, dot plots, picture graphs, and bar graphs for up to two-step problems pertaining to displayed data. Finally, third graders must be able to explain the relationship between availability and scarcity of resources, understand the connection between human capital and labor/income, describe why it is important to save for college, and understand interest in terms of paying back a loan.

Fourth Grade

Reasonableness continues to be a focal point as fourth graders are expected to evaluate the reasonableness of sums and differences of fractions using various benchmarks. The standard algorithm is expected to be used when adding and subtracting decimals to the hundredths place. Perfect squares of up to 15 by 15 are explored in conjunction with multi-digit multiplication. Students are expected to round to the nearest 10, 100, or 1,000. Estimation of angles in degrees to the nearest whole number is also an area of emphasis. Various data representations from previous grade levels continue to be explored with whole numbers, fractions, and decimals. Finally, students are expected to understand fixed and variable expenses, calculate profit, understand various saving options, and describe the purposes of financial institutions.

Fifth Grade

Fifth graders are expected to use estimation to find solutions for problems involving all operations. They are also expected to use the standard algorithm to divide four-digit dividends by two-digit divisors. A standard was added for students to add and subtraction positive rational numbers fluently. Pattern work includes the rule $y=ax$ or $y = x + a$ and graph the pattern. Data work involve stem and leaf plots and dot plots with fractions and decimals and discrete paired data on scatterplots. Finally, fifth graders are expected to define various taxes including income, payroll, sales, and property. They must also explain the differences between gross and net income and be able to balance a simple budget.

Sub Question Findings

To answer sub question 1, the Common Core SMPs were compared to the Texas Mathematical Process Standards. The most notable differences include the exclusion of "look for and make use of structure," "look for and express regularity in repeated reasoning," "attend to precision," and "reason abstractly and quantitatively". Variations of the SMPs were added, which could be directly aligned. Table 24 shows the specific changes to the Texas Mathematical Process Standards with a column devoted to descriptors of the aligned SMP.

Table 24

Comparison of SMPs and MPSs

Common Core State Standards for Mathematical Practice	Texas Mathematical Process Standards	Descriptors of Aligned SMP (NGA & CCSSO, 2010)
Make sense of problems and persevere to solve them	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution Analyze mathematical relationships to connect and communicate mathematical ideas	"Looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt" (p. 6).
Reason abstractly and quantitatively		
Construct viable arguments and critique the reasoning of others	Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication	"Justify their conclusions, communicate them to others, and respond to the arguments of others" (pp. 6-7).

Common Core State Standards for Mathematical Practice	Texas Mathematical Process Standards	Descriptors of Aligned SMP (NGA & CCSSO, 2010)
Model with mathematics	Apply mathematics to problems arising in everyday life, society, and the workplace Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	"Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace"(p. 7)
Use appropriate tools strategically	Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems Create and use representations to organize, record, and communicate mathematical ideas	"Consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software" (p. 7)
Attend to precision		
Look for and make use of structure		

Common Core State Standards for Mathematical Practice	Texas Mathematical Process Standards	Descriptors of Aligned SMP (NGA & CCSSO, 2010)
Look for and express regularity in repeated reasoning		

Additionally, the introduction segment in TEKS also includes the process standards in paragraph format with each sentence beginning with "Students will" and a statement afterwards. The introduction of the process standards also describes the connection to the NRC's *Adding It Up* publication and how students become fluent in mathematics while listing the primary focal points of the grade level. It ends with a position stating that, "Statements that contain the word 'including' reference content that must be mastered, while those containing the phrase 'such as' are intended as possible illustrative examples" (TXSBOE, 2012, p. 2). These differences were not reflected in Table 23 but are worthy of disclosing due to the change they both demonstrated from the CCSSM.

To answer sub question 2, the Common Core Domains were compared to the Texas Content Strands. The most notable differences include the merging of "Counting and Cardinality" and "Number and Operations in Fractions." An addition to the domains includes "Personal Financial Literacy". Table 25 compares the similarities and differences among the Common Core Domains and the TEKS Content Strands

Table 25

Domains and Strands Comparison

Common Core Domains	**Texas Content Strands**
Counting and Cardinality	Number and Operations
Operations and Algebraic Thinking	Algebraic Reasoning
Number and Operations in Base Ten	Number and Operations
Number and Operations in Fractions	Number and Operations
Measurement and Data	Data Analysis

Common Core Domains	Texas Content Strands
Geometry	Geometry and Measurement
	Personal Financial Literacy

To answer sub-question 3, the TEA website and standards document were reviewed for indications of learning trajectories. There was no information provided regarding the documents used to guide the standards writing process and no specific reference was made regarding learning trajectories.

Finally, to answer sub question 4, each standard that demonstrated a difference were coded as a procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes during the coding process using the letters P, C, or B. This process of determining the learning outcome associated with the changes demonstrated by states was completed by examining the blue color-coded word differences in Excel. The color-coded word differences represented changes in state standards from the CCSSM. The color-coded words were then sorted into the categories of procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes.

Figure 14 demonstrates the frequency of learning outcomes made to the standards that deviated from the CCSSM. When considering the learning outcomes of standards that demonstrated differences in Texas K-5 mathematics standards, much more emphasis was used for procedural outcomes. Conceptual learning outcomes received the least amount of emphasis with 13% of the overall percent of emphasis.

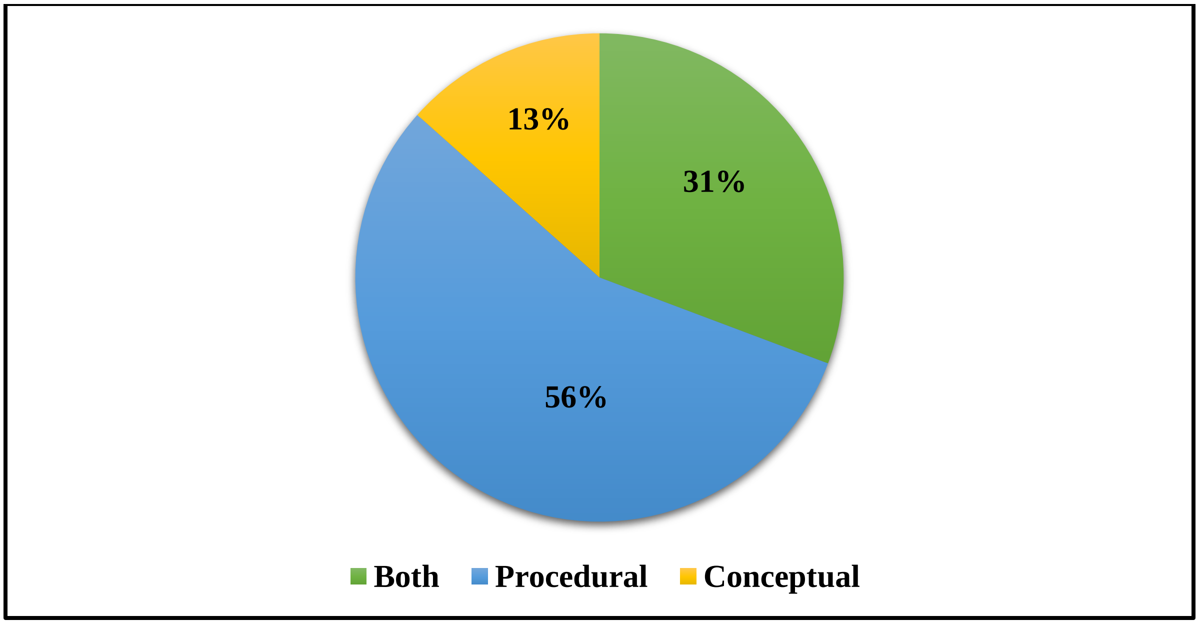

Figure 14

Percentage of Texas Standard Revisions as Having Conceptual or Procedural Learning Outcomes

In conclusion, TEKS deviated from the CCSSM. There was notably the absence of a few SMPs. Another notable deviation included standards and the domain of personal financial literacy. There was also a merging of multiple domains into fewer strands. A lack of balance between procedural and conceptual learning outcomes was also evident.

It is worth noting that the TEA website includes standards proposals and color-coded recommendations dating back to 2011. Additionally, the website lists members of review committees, including their position and district/organization at the time of the writing of the standards. Information pertaining to nominated expert reviewers is provided, along with their reviews of the standards.

Virginia 2016

Virginia was one of four states that opted to not adopt the CCSS from the outset (Schneider, 2015). Virginia did apply for the first round of RTTT but did not win (Howell, 2015), resulting in not being selected to receive RTTTI funding. In Virginia, standards are reviewed every seven years, per *Code of Virginia*, Section 22.1-253.13:1-B (Virginia Board of Education, 2022) which puts the 2009 state standards outside the scope of this study. For purposes of this study, only the 2016 Virginia Mathematics Standards of Learning, referred to as SOLs, were analyzed.

The Virginia Department of Education website offers standards documents, curriculum frameworks, Crosswalk documents (as referred to as summary of revisions) from the last set of adopted mathematics standards in 2009, narrated Crosswalk presentations and test blueprints for applicable grade levels (3^{rd}-5^{th}). These documents are mentioned because they were used during the data collection portion of this study. They also provide further insight into the overarching research question.

To answer the overarching research question of *in what ways, if any, do K-5 state mathematics standards differ from the CCSSM,* a line-by-line comparison of the 2016 Virginia SOLs and the CCSSM was conducted within Excel. Deviations from the CCSSM were color coded in blue, and any missing verbiage in standards were color coded in orange. After the line-by-line comparison was complete, coding of the deviations began.

The most notable additions to the Virginia SOLs include the addition of fraction standards beginning in Kindergarten; the incorporation of currency standards beginning in Kindergarten, the inclusion of the calendar and temperature; the addition of patterns beginning in Kindergarten; the addition of probability standards beginning in second grade; and the inclusion of translations, reflections, rotations, measures of center, circumference, diameter, radius, and chord in fifth grade standards. Table 26 includes the three codes with the most frequent occurrences during the coding process. Each cell of codes will have a bolded number indicating how many times that code occurred across the K-5 standards.

Table 26

Virginia Standards Changes from Top Three Most Frequent Codes

VIRGINIA MATHEMATICS STANDARDS OF LEARNING			
Grade Level	**Code & Number of Occurrences**		
	Number Difference **30**	**Problem Types** **22**	**Verb Change** **21**
Kindergarten	Sort and classify objects according to *one* attribute	Model and solve single step story and picture problems	Identify, describe, extend, create, and transfer repeating patterns
	Investigate and describe part-whole relationship for numbers up to *10*, with fluency up to *5*	Investigate fractions by representing and solving practical problems involving equal sharing with two sharers	Read and interpret data in object graphs, picture graphs, and tables
	Count forward by tens to 100		Collect, organize, and represent data
	Count backward orally by ones when given a number between 1 and 10		*Investigate* the passage of time by *reading* and *interpreting* a calendar
Grade 1	Count forward orally and write in and out of sequence to *110*	Create and solve single step story and picture problems	Identify, trace, describe, and sort plane figures (triangles, squares, rectangles, and circles) according to number of sides, vertices, and angles

VIRGINIA MATHEMATICS STANDARDS OF LEARNING			
	Count backward orally by ones between *1 and 30* Count forward orally by *twos* and *fives* to *110* Order *three* or fewer sets from least to greatest and greatest to least		Collect, organize, and represent various forms of data using tables, picture graphs, and object graphs Read and interpret data displayed in tables, picture graphs, and object graphs Identify, describe, extend, create, and transfer growing and repeating patterns
Grade 2	Count to 99, instead of 1,000 in CCSSM Compare and order between 0 and 999, instead of use two three-digit numbers in CCSSM Count forward by twos, fives, and tens to 120, starting at various multiples of 2, 5, or 10 Round two-digit numbers to the nearest ten, instead of to tens and hundreds places in 3rd grade CCSSM Compare the unit fractions for halves, fourths, eighths, thirds, and sixths, with models	Use of practical problems for solving addition and subtraction single-step problem with whole numbers up to 20	Create and solve single-step and two-step practical problems involving addition and subtraction Count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less Collect, organize, and represent data in pictographs and bar graphs Identify, describe, compare, and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms) Identify and create figures with at least one line of symmetry

VIRGINIA MATHEMATICS STANDARDS OF LEARNING			
	Measure weight to the nearest pound		
Grade 3	Elapsed times of one-hour increments within a 12-hour period, instead of. intervals of minutes in CCSSM Denominators of 12 or less instead of . denominators 2, 3, 4, 6, and 8 in CCSSM Solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less, instead of within 100 in CCSSM Demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10, instead of fluently multiply and divide within 100 in CCSSM Create and solve single-step and multistep practical problems involving sums or differences of two whole numbers, each 9,999 or less, instead of no defined number in CCSSM	Solve practical problems related to elapsed time in one-hour increments within a 12-hour period Solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less Solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less	*Identify* and *describe* congruent and noncongruent figures instead of u*nderstand* in CCSSM *Create* and solve single-step and multistep practical problems involving sums or differences of two whole numbers, each 9,999 or less

VIRGINIA MATHEMATICS STANDARDS OF LEARNING			
	Round to the nearest thousand, instead of ten and hundred places in CCSSM		
Grade 4	Read, write, and identify the place and value of a digit in a nine-digit whole number Round to the nearest thousand, ten thousand, and hundred thousand, instead of any place in CCSSM Round decimals to the nearest whole number Read, write, represent and identify decimals through thousandths, instead of hundredths place in CCSSM	Create a model or practical problem to represent a given probability Practical problems related to time, length, weight/mass, and liquid volume in United States customary units Solve single-step and multistep practical problems involving addition and subtraction with decimals, fractions, and mixed numbers	Represent decimals, including tenths and hundredths, using concrete and visual models and money No codes
Grade 5	No codes	Solve single-step practical problems involving division of decimals, fractions, and mixed numbers	*Identify* and *describe* characteristics of prime and composite numbers instead of *determine* if a number if prime or composite

Note. The standards information in this table is compiled from VDOE, 2016; NGA & CCSSO, 2010.

There are other notable deviations from the CCSSM that should be shared. Most notably, Virginia included personal financial literacy standards into their mathematics standards as well as incorporated currency standards into earlier grade levels. Below, additional changes are synthesized by grade level and included various data representations not included in the CCSSM.

Kindergarten

Students in Kindergarten are expected to count orally backwards and forwards. Fraction standards are also explored with equal sharing among two people. Currency standards are included with identification of coins, including quarters. Skip counting by 25 is not a skill explicitly addressed in Kindergarten standards in the CCSSM. The understanding of a calendar is a standard that was also added. Temperature, volume, and time are also explored. Pattern standards were added, expecting students to identify, describe, extend, create, and transfer repeating patterns. Finally, students are asked to collect, organize, and interpret data in picture graphs and tables.

First Grade

First graders are expected to count orally forward and backwards. Currency standards expect students to determine the value of a collection of coins up to 100 cents. Calendar work is extended to the expectation to read and interpret the calendar. Pattern work is extended into first grade with the same standard from Kindergarten.

Second Grade

Counting expectations extend to second grade where students are expected to count forward and backwards by various multiples. Ordinal numbers and positions are also expected through the twentieth position. Foundational fraction sense includes set, region, and length models with eighths and sixths added to the standards. Estimation skills are reinforced through addition and subtraction standards. Calendar, currency, pattern, and temperature standards are extended and build from previous grade level standards. Students are also expected to draw and identify lines of symmetry.

Third Grade

Third graders are expected to demonstrate fluency with multiplication facts, which include 0, 1, 2, 5, and 10. Currency, pattern, and temperature standards extend and build on prior grad level standards. students are expected to describe and identify congruent and noncongruent figures. Students are also expected to describe probability and possible outcomes of a single event.

Fourth Grade

Decimal standards in fourth grade extend to the thousandths place and students are expected to round decimals to the nearest whole number. Students are expected to find least common multiples and greatest common factors. Probability standards are extended to representing probability between 0 and 1 and determining the likelihood of a simple event.

Fifth Grade

Fifth graders are expected to identify and describe diameter, radius, chord, and circumference of a circle. Students are also expected to know the sum of interior angles in a triangle and apply transformations. Probability standards are extended to the fundamental counting principle. Stem and leaf plots are explored as well as measures of center and spread of a data set. Variables are also explored with one operation.

Sub Questions Findings

To answer sub question 1, the Common Core SMPs were compared to the Virginia Process Goals for Students. The most notable differences include the exclusion of “model with mathematics,” “use appropriate tools strategically,” “attend to precision,” “look for and make use of structure,” and “look for and express regularity in repeated reasoning.” An additional process goal that was not captured in the SMPs is the “mathematical connections.” Table 27 shows the specific changes to the Virginia Process Goals for Students. No additional column was needed for alignment of SMPS to Virginia Process goals. Specific cells were intentionally left blank if no alignment existed.

Table 27

Comparison of SMPs and Process Goals

Common Core State Standards for Mathematical Practice	Virginia Process Goals for Students	Descriptors of Aligned SMP (NGA & CCSSO, 2010)
Make sense of problems and persevere to solve them	Mathematical problem solving	
Reason abstractly and quantitatively	Mathematical reasoning	
Construct viable arguments and critique the reasoning of others	Mathematical communication	
Model with mathematics	Mathematical representations	
Use appropriate tools strategically	Mathematical representations	
Attend to precision		
Look for and make use of structure		
Look for and express regularity in repeated reasoning		

Common Core State Standards for Mathematical Practice	Virginia Process Goals for Students	Descriptors of Aligned SMP (NGA & CCSSO, 2010)
	Mathematical connections	

To answer sub question 2, the Common Core Domains were compared to the Virginia Content Strands. The most notable differences include the merging of "Counting and Cardinality," "Number and Operations in Fractions," and "Data." Probability and Statistics were additional added Domains. Table 28 compares the similarities and differences among the Common Core Domains and the Virginia Content Strands. Cells were intentionally left blank if no alignment existed between the two sets of data.

Table 28

Domains and Content Strands Comparison

Common Core Domains	Virginia Content Strands
Counting and Cardinality	Number and Number Sense
Operations and Algebraic Thinking	Computation and Estimation
	Patterns, Functions, and Algebra
Number and Operations in Base Ten	Number and Number Sense
Number and Operations in Fractions	Number and Number Sense
	Computation and Estimation
Measurement and Data	Measurement and Geometry
Geometry	Measurement and Geometry
	Probability and Statistics

To answer sub question 3, the Virginia Department of Education website and standards document were reviewed for indications of learning trajectories. A more specific description was provided regarding the documents used to guide the standards writing process; however, no specific reference was made regarding learning trajectories. The standards document preface stated:

> The 2016 Mathematics Standards of Learning identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. Information from the College Board, ACT, the National Assessment of Educational Progress (NAEP) Framework, the *Curriculum Focal Points* from the National Council of Teachers of Mathematics (NCTM),

> *Principles and Standards for School Mathematics* from NCTM, *Focus in High School Mathematics: Reasoning and Sense Making* from NCTM, the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report from the American Statistical Association, and the Report of the President's National Mathematics Advisory Panel were considered in identifying mathematics content necessary for success for all students in postsecondary pursuits. (VDE, 2016, p. iii)

Finally, to answer sub question 4, each standard that demonstrated a difference were coded as a procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes during the coding process using the letters P, C, or B. This process of determining the learning outcome associated with the changes demonstrated by states was completed by examining the blue color-coded word differences in Excel. The color-coded word differences represented changes in state standards from the CCSSM. The color-coded words were then sorted into the categories of procedural learning outcome, conceptual learning outcome, or a mixture of both procedural and conceptual learning outcomes.

Figure 15 demonstrates the frequency of learning outcomes made to the standards that demonstrated deviations from the CCSSM. When considering the learning outcomes of standards that demonstrated differences in Virginia K-5 mathematics standards, more emphasis was used for procedural outcomes with a similar emphasis on standards that resulted in both outcomes. Conceptual learning outcomes received the least amount of emphasis.

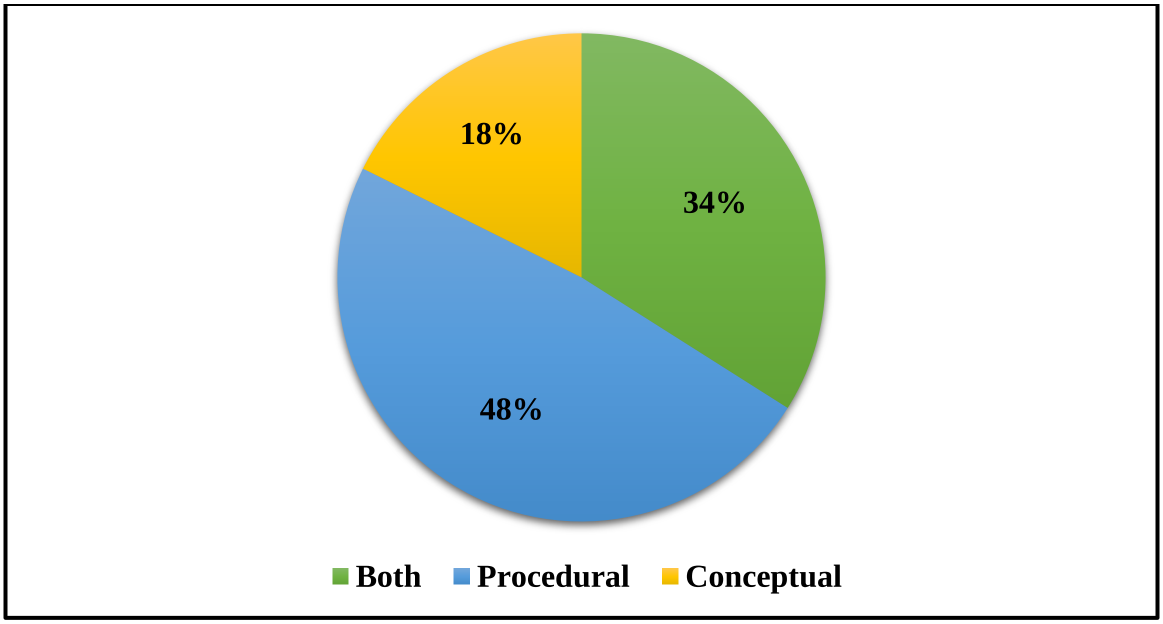

Figure 15

Percentage of Virginia Standard Revisions as Having Conceptual or Procedural Learning Outcomes

In conclusion, the 2016 Virginia SOLs expanded beyond the CCSSM and had a similar structure to the TIMSS assessment frameworks. TIMSS frameworks (Mullis & Martin, 2017) focus heavily on numbers and less heavily on measurement, geometry, and data. It is worth noting that the standards document includes an equity statement, in which the following quote was cited to NCTM: “Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement” (VDOE, 2012, p. vi).

The Virginia DOE further supports their position statement by discussing student engagement as an essential component of equity in mathematics, which is further supported on their DOE website. Links are provided for families and communities, professional learning opportunities, assessment resources, and instructional resources demonstrating a commitment to the involvement of educators, students, and community members in increasing mathematical literacy and numeracy of Virginia students.

Across State Themes

Overall, there were no consistencies across all state standards revisions. Interestingly, one trend that was noted across multiple states was the inclusion of a phrase similar to, “written for [insert state citizenship] by [insert state] educators” on either the state education website or the actual standards document. This occurred in Florida, Georgia, Nebraska, and Oklahoma. The inclusion of this statement, in my opinion, was in direct retaliation of the perceived federal overreach commonly cited by those in opposition to the Common Core State Standards. Federal overreach has been a common argument against the adoption of the Common Core (Rothman, 2011; Schneider, 2015).

An inconsistent theme that emerged across all coded states was changes to verbs used within standards. The change to a verb used within standards at times did adjust the levels of cognitive demand, which emerged during the coding process. The same verbs were never used consistently in the same standard across states, but each state altered the verbs used from the CCSSM. Table 29 lists the verb and adverb changes used across the coded states. The regular font words were used at least once somewhere in the K-5 CCSSM but were used in different places in the coded state standards.

Table 29

Adverb/Verb Usage Across Coded States

Not in Common Core	In Common Core
Accurately	Ask
Applies	Build
Choose	Classify
Collect	Compare
Combine	Compose
Complete	Count
Construct	Create
Contrast	Decompose
Demonstrate	Describe
Discuss	Determine
Divide*	Develop
Duplicate	Draw
Efficiently	Estimate
Explore	Evaluate
Flexibly	Explain
Fold	Express
Form	Extend
Predict	Find
Quantify	Generate
Recall	Identify
Sharing	Interpret
Summarize	Justify
Trace	Measure
Transfer	Model
	Partition

Not in Common Core	In Common Core
	Organize
	Read
	Recognize
	Relate
	Represent
	Show
	Sketch
	Solve
	Sort
	Understand
	Use
	Write

Note. * Divide is used as a verb in standards changes, where students are asked to divide a shape. The CCSSM uses divide within standards as operational language.

From Table 29, the term form is used within standards changes as a verb. Within the CCSSM the word form is used in past tense to recognize angles are formed whenever rays are combined. Form is also used in the CCSSM to describe forms of numbers but is not used as a verb.

The merging of domains was not consistent across all states selected for coding but there are a few domains that multiple states opted to merge. One domain that was merged across all states coded for this study was “Counting and Cardinality.” Another domain that was merged by most states was “Number and Operations in Fractions.” States that merged this domain include Georgia, Nebraska, Oklahoma, Texas, and Virginia. Additionally, domains pertaining to “Probability” or “Statistics” was added by Florida, Georgia, Oklahoma, and Virginia. These are important changes to document as they contribute to mathematics standards understanding.

Many of the states that were selected for coding removed similar SMPs. “Use appropriate tools strategically” was removed by Florida, Nebraska, Oklahoma, and Virginia. “Attend to precision” was removed by Nebraska, Oklahoma, Texas, and Virginia. Finally, “Look for and express regularity in repeated reasoning” was removed by Nebraska, Oklahoma, Texas, and Virginia. Removal of each of these SMPs will have implications, whether positive or negative.

Chapter 4 presented the findings of this study. First changes made by states that were not selected for coding were shared. Then findings from states that were selected for coding were shared in alphabetical order by states and in order by answer to each research sub question. Finally, commonalities across states were shared.

Chapter 5 will include a discussion of the findings from this study. This will include a look at the historical relevance of the CCSSM and its role within standards-based reform. Then dilemmas of the current era of standards will be discussed as well as implications of the study, recommendations from the study, strengths of the study, limitations of the study, suggestions for future research, and a summary.

CHAPTER FIVE: DISCUSSION

The purpose of this research study was to examine the ways in which United States K-5 state mathematics standards have changed during the past decade (2012-2022) using the methodology of qualitative content analysis. This time frame reveals changes that have occurred since the implementation of the CCSSM, which proved to be six states with changes that exceed the parameter of 15% of changes. Of the six states with changes that exceed the parameter of 15% of changes, three states had never adopted the CCSSM between 2012 and 2022. Previous studies have discerned the quantitative differences between state standards prior to or during the implementation of the Common Core State Standards, but these valuable studies have not captured the findings qualitatively nor have they captured the most recent changes.

Originally when I began this study I viewed standards as a linear process, with our transition from individual state standards to the CCSSM being a step towards our next set of focal points. Through this study, I realized that as a nation, we are currently in a cycle with standards. First, some states never adopted the CCSSM and used their own state standards. Second, some states have kept the name "Common Core Standards for Mathematics." Third, some states are removing the label of "CCSSM" and rebranding or renaming their state standards to disassociate from the CCSSM. Finally, some Governors and state leaders are taking legal action to replace the CCSSM with their own state developed standards. Table 30 provides an overview of the state standards in the described standards cycle.

Table 30

Standards Cycle

Never Adopted CCSSM	Kept CCSSM as Title of Standards	Rebranding or Renaming CCSSM	Legal Action to Replace the CCSSM
Alaska	Delaware	Arkansas	Alabama
Nebraska	Hawaii	California	Arizona
Texas	New Mexico	Colorado	Florida
Virginia	Vermont	Connecticut	Georgia
Minnesota*	Washington	Idaho	Indiana
		Illinois	Kentucky
		Iowa	New York
		Kansas	Oklahoma
		Louisiana	South Carolina
		Maine	Tennessee
		Maryland	
		Massachusetts	
		Michigan	
		Mississippi	
		Missouri	
		Montana	
		Nevada	
		New Hampshire	
		New Jersey	

Never Adopted CCSSM	Kept CCSSM as Title of Standards	Rebranding or Renaming CCSSM	Legal Action to Replace the CCSSM
		North Carolina	
		North Dakota	
		Ohio	
		Oregon	
		Pennsylvania	
		Rhode Island	
		South Dakota	
		Utah	
		West Virginia	
		Wisconsin	
		Wyoming	

Note. Minnesota received an asterisk as it adopted the CCSS for ELA but not Mathematics.

The era of the CCSSM is unprecedented in K-12 curricular policy in the United States (Greer, 2018). It represents a historic effort (Gojak, 2013) to unify almost all states to adopt the same mathematics standards. Despite the initial unification, the perceived backlash and public disassociation with the CCSSM displays similar discomfort that the new math movement of the 1950s. Educational reform has historically been surrounded by contextual, societal, and political pressures (Klein, 2003), which includes the activity movement of the 1930s to incorporate progressivism elements into education, including the integration of subjects. Four main interest groups were in conflict during the 1930s (Stanic, 1986) with little agreement on the ideas that should be taught and how they should be taught in schools. This lack of agreement sounds familiar as other educational movements experienced a similar phenomenon.

The parallels between the Common Core State Standards and the new math movement of the 1950s-1960s are striking. If the context of the new math era was removed, it would be difficult to discern if the work is describing the new mathematics reform or the CCSSM. Below, the similarities are outlined through direct quotations in Table 31. Emphasis has been added to phrases in the table with bold text. Additionally, topics have been added in a column that signify issues with both the new math movement and the Common Core State Standards.

Table 31

Similarities of CCSSM Movement (2010) to New Math Reform Era of the 1960s

Topic	Quotations Regarding New Math Reform Era
Goal of standards	"The main goal of the authors of the "new math" programs was to present mathematics as a logical structure to children who could then **develop an understanding** and appreciation for mathematical principles" (Walmsley, 2003, p. 1).
Prior focus of standards	"The conventional **curriculum relied on drill and practice with little emphasis on understanding**" (Walmsley, 2003, p. 4).
Lack of instructional training	"Unfortunately many of the materials were **used by teachers with little training** in 'new mathematics' concepts" (Walmsley, 2003, p.4).
Societal frustration	"**Teachers and parents become frustrated with understanding and explaining the content** of the 'new mathematics' curricula" (Walmsley, 2003, p. 4).
Intended results	"Overall, the **general public began to lose interest** in the 'modern mathematics' movement **as it was not providing the results they had hoped**" (Walmsley, 2003, p. 4).
Lack of preparation	"Furthermore, many **elementary teachers** who were forced to teach the 'new math' in their schools did so with an incredibly large void of necessary mathematical knowledge. Much of **their own training in mathematics** quite possible was minimal and **very traditional**. Also, many elementary school teachers do no favor mathematics, and introducing more abstract concepts only caused confusion for both the teacher and his or her students" (Walmsley, 2003, p. 81).
Why behind the math	(In reference to the new mathematics movement) "All students were supposed to be **able to answer "why"** to mathematics questions rather than just being able to do mathematics" (Walmsley, 2003, p. 85).

Topic	Quotations Regarding New Math Reform Era
Lack of autonomy	"Often the administrators of a building or district would make the decision about which "new math" program, if any, would be used in the school; leaving the teachers out of decisions affecting themselves directly. **Without ownership in the decision**, many teachers did not put forth the effort required for a successful program" (Walmsley, 2003, p. 84).

A few arguments for the creation of the CCSSM included a lack of cohesion with prior state standards and promoting higher levels of thinking across subject areas (Schneider, 2015). Arguments against the CCSSM cite that the implementation did not adequately prepare teachers to teach the standards (Gewertz, 2014), families struggled to understand homework (Loveless, 2021), there is no proof the CCSSM raised student achievement levels (Loveless, 2021) and the understanding of the "why" behind simple mathematics was frustrating (Whittaker, 2015). Chillingly, Walmsley (2003) concludes that the new mathematics reform is, "Known to most as a failed movement, and this opinion has hindered the country's ability to learn from history and past mathematics reform" (p. 5). This conclusion can again easily remove the context of "new mathematics reform" from the phrase and be replaced with "the Common Core State Standards," which some have already dubbed as a "failure" (Loveless, 2021; Whittaker, 2015).

While the CCSSM might have been declared a "failure" by some, it is important to note that six state revisions demonstrate changes that exceed the parameter of 15% of changes. This low number of states signifies that the CCSSM is still embedded in revisions of state standards, despite the rebranding or repackaging of some state standards. What has resulted in revisions of state standards since the implementation of the CCSSM is more of a convergence of ideas instead of an absolute ending to the CCSSM.

The remainder of this chapter outlines dilemmas related to the Common Core State Standards. Then, implications from the study are explored for two separate audiences: policy makers and researchers. Recommendations are outlined including the reconvening or reassignment of the CCSSM writing team, extending standards and domains, and additional suggestions. Finally, strengths of this study, limitations of this study, future research,, and a conclusion are presented.

Dilemmas Related to the Common Core State Standards

One dilemma in our current post-Common Core era is that political leaders in some states have been quick to relinquish support of the CCSSM and proclaim that their state standards are, "no longer the Common Core State Standards." In many cases, their state's newly adopted standards have deep roots stemming from the CCSSM. This is a dilemma because it confuses public perception on mathematics standards, as to no longer be considered the CCSSM, the elements that were emphasized within the CCSSM would need to be removed, which would be nearly impossible considering the research and roots of mathematics curriculum that went into their development. Findings from this study show that despite the differences to the standards, traces of the CCSSM still exist within the standards.

Another dilemma associated with the CCSSM pertains to the public distrust associated with the intentions behind previously conducted studies. Schneider (2015) questioned the qualifications of those at the Fordham Institute, a nonprofit organization, and whether those evaluating standards have the qualifications to evaluate standards. Schneider also felt as though the evaluations completed by the Fordham Institute, resulting in letter grades to individual state standards, places pressure on states to consider their overall image to the public when developing standards. This also, "put the Fordham Foundation/Institute in the news, and it afforded Fordham clout before the media and in the public eye" (Schneider, 2015, p. 59). The relationships and connections to the CCSSM of those employed at the Fordham Institute have further complicated the intentions behind the studies. I would like to note that it is not my intention to bring the institute into question or agree or disagree with Schneider's comments. It is worth noting that the Fordham Institute has largely dominated in the review of state standards since the late 1900s, as this is a major component of their research and thus it must be mentioned here.

A final dilemma to be discussed pertains to the cultural expectations of the United States and their role in the movement away from the CCSSM. We live in a society where instantaneous results are expected in most circumstances. This cultural expectation became evident in the quick-natured withdrawal of support for the CCSSM from governors, additional political figures, and members of society. By 2015, some political figures had already deemed the CCSSM a failure. Other countries, including Brazil, Canada, China, France, Germany, India, Netherlands, United Kingdom, Singapore, and South Korea have established nationalized standards (Schmidt et al., 2009). Brazil and France are the only two countries with national standards that have their students' performance rank below the United States according to PISA rankings (Schleicher, 2018). Further consideration of the cultural contribution of instantaneous results might alter the push for individual state revisions to standards as we learn lessons from other countries who are excelling in mathematics education.

Implications

Findings from this study indicated that some states included movement of standards from middle grades to elementary. This included measures of center, parts of a circle, and probability, among other standards. This trend, while currently limited to a small number of states, does have implications. Elementary teachers are in the unique position of teaching multiple subject areas, which is in direct opposition to secondary teachers who specialize in one content area. The movement of more advanced mathematics concepts from middle grades to elementary could prove to be challenging for implementation in elementary classrooms and teacher preparation programs as this could further heighten elementary teacher anxiety for teaching mathematics.

This study also found that states with more than 15% of changes to their standards are focusing more heavily on procedural learning outcomes and evidence of learning trajectories is lacking, which is in direct opposition of the focal points of the CCSSM. Rigor, a balance between conceptual understanding, procedural fluency, and application (Student Achievement Partners, 2013), was one of three foci of the CCSSM. Coherence within the CCSSM focused on progressions across grades (Student Achievement Partners, 2013) with an emphasis on learning trajectories to develop the progressions. As states transition away from the core focal points of the CCSSM, educational stakeholders must consider the ramifications.

Additionally, this study found that of the states that were selected for coding due to the number of changes made to their revisions, many of the states merged or removed domains pertaining to fractions. By removing or merging the domain of fractions into the domain of "numbers," the emphasis on foundational fraction knowledge no longer exists. This change could prove to have unintended consequences as intentional emphasis is not placed on the development of the understanding of fractions with fewer standards devoted to their instructional emphasis. With these findings considered, the remainder of this section will discuss implications stemming from these results for two separate audiences. This includes policy makers and researchers. Each audience individually plays important roles in the development and implementation of mathematics standards.

Much to my surprise, there was a lack of consistency in findings across states that were selected for coding. While there was a few merging of domains that occurred with many of the states selected for coding, revisions across states varied widely and were not consistent. The cohesion across states and their standards was cited as one of many reasons for the development of the CCSSM (Jochim & Lavery, 2015; Schneider, 2015; Watt, 2011). This study provides evidence that we are transitioning back to a pre-Common Core era where mathematics standards once again lack consistency or cohesion.

Implications for Policy Makers

The results from this study indicate that a small number of states, 6 of 50, have greater than 15% changes from the CCSSM. This information is important for policymakers at both the state and national level to consider. Four implications for policy makers that stemmed from this study are (1) the importance of historical lessons, (2) deeply considering the financial impact of decisions, (3) disconnecting political affiliation from suggestions, and (4) connecting research to policy.

First, we are more than a decade out from the first set of voluntary national mathematics standards, where the country experienced alignment among the greatest number of states in the history of our education in the United States with 45 states adopting common standards (Greer, 2018; Schneider, 2015). The vast disparity once exhibited among state standards (Porter et al., 2011) and achievement levels of students on international assessments (Schneider, 2015) is partially what initiated the development of the CCSSM. But the question from this is, *Did we learn anything from the creation and implementation of the Common Core*? If so, why are states not directly addressing the shortcomings that they noticed within the standards as they develop new mathematics state standards? As a country, we are currently stuck in a cycle instead of linear movement to advance mathematics standards. Instead of moving collectively as a nation towards improved standards, some states are reverting back to individual standards that lack consistency in topics covered, agreement on vocabulary used, and pacing and sequencing of topics.

Second, we are a culture and society that tends to expect instantaneous results, which makes it difficult when changing instructional approaches to a subject area and implementing new standards because change takes time and considerable funding. It was projected that to align state and local educational systems to the Common Core State Standards in both mathematics and English language arts, that it would collectively cost the nation nearly $17 billion over a course of seven years (Murphy & Regenstein, 2012). However, according to *The Federalist*, a conservative journal, the Common Core State Standards cost the nation close to $80 billion (Pullmann, 2016). The harsh reality is that not all states even waited seven years before declaring the Common Core a failure. With an investment this costly to implement the CCSS, it seems as though the investment would come with patience to truly determine if the standards were contributing to deeper conceptual understanding of mathematics topics.

Third, I urge policymakers to consider how to best disconnect politics from suggestions that impact children. It is quite easy to be dismissive of a differing opinion, especially if that opinion is associated with an opposing political party. However, there is danger in presuming that affiliation automatically leads to incorrectness. What truly should be answered with any posed solution to problems associated with education is the question, “Is this decision what is best for children?” and further “What does the research say to support this decision?” By practicing and modeling this open-mindedness and commitment to our future leaders of the country by setting aside differences, a path may be carved for a cultural commitment to advancing education within our country.

As our country continues to diversify, decisions regarding equitable instruction need to be considered. Reflecting on the successes and shortcomings of the CCSSM can launch the next set of mathematics standards to achieve greater outcomes than its predecessor(s). Advice from *Catalyzing Change in Early Childhood and Elementary Mathematics* (NCTM, 2020) could be used to consider how we can broaden the purposes of mathematics through mathematics standards and subsequently instruction. This would also implore us to consider how we might provide the most equitable structures in mathematics classrooms through standards and their implementation and interpretation.

Fourth, what should be considered by policymakers is that researchers are bound to be unbiased and must engage in ethical reporting and practices. This unbiased approach creates environments where what is happening is being documented, analyzed, and synthesized to advance each individual area of interest in education. Increasing grant funding for education researchers specifically related to standards development could encourage future standards revisions. Additionally, selecting a team of researchers to approach the problem from varying lenses can create a more complete picture of what is occurring, and subsequent solutions can be posed. Researchers are tools in the puzzle that should be utilized to advance understanding of problems that are occurring.

In summary, only a small number of states have truly changed their standards. But at what cost? The packing and rebranding of the revisions that have been made could prove to be costly reactions at the price of the educational attainment of our future generations of learners. Thoughtful consideration needs to be given to any revisions to standards or policies pertaining to standards by policymakers.

Implications for Researchers

This study used a methodology not typically employed when analyzing mathematics standards, resulting in different conclusions. Three implications that came from this study include: (1) language use, (2) the importance of documents outside of standards themselves, and (3) the incorporation of varying methodologies into mathematics education studies.

First, it was evident throughout this study that various states used language differently. This was demonstrated both within mathematics standards and within available mathematics glossary documents on state department of education websites. While the use of varying language is not a new issue within mathematics education, our goal as a field should be unity of language, as language connects our conjectures, solutions, and discussions in mathematics classrooms.

One example of varying language usage within the study involved data representations where some states used the language “line plots” and other states used the language “dot plots”. Another language difference included geometric terms. One state referred to right angles as “square corners” and “square angles,” which is a less precise use of the terminology. Vocabulary is important for the success of students in mathematics (Riccomini et. al, 2015), especially as students move to different states. Using informal language could further confuse some students if they question how a square could be located within another shape, such as a triangle. It could also frustrate other practitioners who may not agree with informal language in relation to standards instruction. Including parents in the use of vocabulary in mathematics classrooms through easily accessible documentation would further include as many educational stakeholders as possible and help to bridge some of the current disconnect in our field. It would also contribute to the alleviation of parental concerns regarding homework assistance.

Another language difference pertains to the use of learning progressions and learning trajectories. Some states created learning progression documents to help teachers understand the horizontal and vertical alignment of standards. Others referenced learning trajectories, which was a major element of the creation of the CCSSM. There is a danger in synonymously using learning progressions and learning trajectories as each have different meanings but tend to get used interchangeably. As researchers continue to engage in meaningful work related to standards, language usage, learning progressions, and learning trajectories, qualitative research could be a consideration as a methodology.

The language differences further contribute to our division on how best to teach and to learn mathematics. As states adjust vocabulary, it is important to consider the language national mathematics organizations use in their publications. It is important to rely on research of vocabulary acquisition and mathematics instructional strategies as daily decisions are made regarding the implementation of standards. Consistency in the use of language across classrooms, districts, and states will continue to offer a united front for our field and for our learners.

Second, this study examined additional documents outside of standards documents. Other publicly available documents contributed to the findings of this study, which enhanced the results. As other researchers engage in this important work all publicly available standards documents should be used as data. Interpretations of standards are removed when supporting documents from one source provide clarifications. Some state department of education websites offered crosswalk documents and additional resource documents which even included suggested instructional strategies to achieve the objective of a standard. This clarity supports teachers as they work to implement standards within their lessons and units and should be further analyzed by researchers to enhance future guidance of standards.

Third, exploring the use of a methodology that had not been used for standards research with this set of data enhanced studies in a way that is more accessible to educational shareholders. While it was an extremely time-consuming process, it did produce results that examined mathematics standards from a lens much different from interpreting statistical results. Statistical alignment of standards documents is helpful, but considering qualitatively how documents are similar or different allows language to be analyzed. The varying use of language is an area that we can continue to try to connect within the field through future research.

Further, research can provide guidance for mathematics teacher educators and teachers. The implications for educators from this study vary greatly depending on the state in which the educator resides. As more states transition from the CCSSM to their individual state standards, it will be ever important for educators to familiarize themselves with the specific changes to their grade level and to the preceding and succeeding grade levels. This can be best done by attending professional development offered by the state or diving into crosswalk documents (if available) created by the state or district with your grade-level team.

Revisions to standards also expands on the content knowledge that preservice teachers must know prior to entering the field of education, directly impacting collegiate level courses and assessments for licensure. If licensure assessments do not align with state standards, new teachers are at a disadvantage and subsequently their students are as well in terms of being mathematically proficient. If states follow the licensure assessment, teachers will be better equipped to understand and teach the changes to state mathematics standards. However, this could mean that during their college preparation a deeper dive into more mathematical content areas will be necessary.

As states are revising standards, removing strands or topics from the CCSSM can have lasting impacts overall on student understanding of mathematics. For example, Florida eliminated the SMP of "use appropriate tools strategically". While specific tools were included within their standards, this gives the false perception that only the stated tool can be used by students to demonstrate understanding of the mathematical topic. If other states were to revise their standards, they should seriously consider the impacts of both teacher instructional strategies and student learning outcomes.

Another removal of standards is evident in the case of Nebraska. Nebraska's standards do not mention use of standard or traditional algorithms as a strategy that students must learn within the K-5 standards. Conceptual understanding of mathematical topics was a focal point in the K-5 CCSSM, but procedural knowledge was incorporated as well. As states engage in the revision process, they should consider the implications of not explicitly incorporating procedural knowledge into number and operation domains. There is room for research on the effects of the elimination of standards in mathematics as more states revise their standards.

In summary, this study provided insight into areas of research that could be further explored by researchers. This study demonstrated varying language use across revisions that could prove to be consequential to learners. Supporting documents outside of standards themselves should be considered as valuable references by researchers in this field. Finally, the incorporation of varying methodologies into mathematics education studies will further contribute to our understanding of teaching and learning mathematics.

Recommendations

The deviations from the original CCSSM document offered a vast array of changes that should be further considered. Exploration of a revision team to the CCSSM should be of highest priority to the NGA and CCSSO, professional organizations, or the United States Department of Education. Extending standards to encompass more grade levels within domains should be considered by a revision team. Further, capturing precursor domains for future mathematics learning in grades 6-8 should also be considered.

Reconvening of CCSSM Writing Committee

Prior to the CCSSM being formed, Confrey (2007) suggested that in order for national standards to work, there needed to be a reconvening of the writing team as needed to continue to improve the proposed standards. This suggestion has not come to fruition and states have been left to individually tackle standard revisions. The theme of declaring state superiority emerged with some of the states included in this study. Instead of a single state saying they were written for [insert state name] by [insert state citizenship], the CCSSM (and possible subsequent revisions) was written for the states, by the states. This is a much different phrase with a more united sentiment than when individual states boast superiority that their state standards can provide. It is also quite presumptuous that states tout their standards are the "greatest" standards through the renaming of the standards without any research to prove the accuracy.

Reassignment of CCSSM Writing Committee

While the suggestion of Confrey (2007) to reconvene the writing team could have enhanced the CCSSM, the solution does not consider the voices that were and were not represented during the writing of the original CCSSM. For example, of the mathematics standards writing committee, 17 of the 18 members (or 94.44%) who held positions as editor, co-chairs, writers, or reviewers, were Caucasian. If CCSSM revisions do in fact occur, serious consideration to a diverse representation of scholars and experts across demographics, geographics, expertise, and educational role is necessary. Within the considerations, careful thought should also be taken to the kinds of expertise included in the writing of the standards.

Reassigning a team with experts in the subject of mathematics, pedagogy, child development, and the social context of school would follow the recommendations of Schwab (1969). The reassignment of Common Core writing team members could follow the suggestion to revise standards as revisions are needed, with no set limitations on number of years between revision meetings. This would allow for most states to continue to use consistent mathematics standards, which would be most ideal for our transient society. The financial impact of revising the CCSSM on educational budgets nationally and at the state and local levels would need to be carefully constructed and maintained.

Additional Suggestions for Writing Committees

While it was expected, this study demonstrated a wide range of transparency regarding the standards writing process. No state divulged all the decisions that were made, which understandably is a daunting task that some would not want to read. However, current practices of limited transparency during the writing process serves to be problematic in numerous ways.

First, we must consider whose voices are being represented. What voices are not at the table? Were committees formed by asking for volunteers or were committees formed through a more intentional process? Were committee members compensated for their time? Compensation and demographics of committee members alters who the standards are written for and whose voices are being uplifted to determine what mathematics students should be able to do. Some state department of education websites boasted the number of classroom years that a committee had, without defining what constitutes classroom teaching experience or subject matter expert criteria, which can be a slippery slope if probed further. Informing stakeholders of the process for the formation of committees helps create credibility and trust between all parties involved.

It should also be disclosed if any companies or individuals have vested interests in the writing committee formation or decision-making process. Conversations regarding what companies, if any, are sponsoring breaks and/or lunches during the standards writing meetings should also be addressed. Organizations typically have someone who takes notes during committee or board meetings for transparency to members about the actions of those in the room making decisions and why the decision was being made. Similarly, these actions and discussions should be documented for the public, who has vested interest in the decisions that happen behind closed doors. If the processes and decisions were made more transparent, then perhaps there would be less public confusion regarding standards. The final decisions made and the resulting justification behind the decision should be made publicly available for all educational stakeholders.

Additionally, transparency and clarity are needed beyond statements such as those found in the TEKS introduction where they divulge that, “Statements containing the word ‘including’ reference content that must be mastered, while those containing the phrase ‘such as’ are intended as possible illustrative examples” (TXSBOE, 2012, p. 2). This statement is confusing for all educational stakeholders. Is the mastery of phrases with “including” specifically directed at state-level testing? Why are the “such as” elements not also needed for mastery? For example, in grade 1, standard 1.8.A (TXSBOE, 2012) requires students to collect, sort, and organize data in up to three categories using models/representations but those models and representations are left in a “such as” phrase of “such as tally marks or T-charts”. This would imply a lack of consistency in instruction across the state, districts in the state, or even within a school building while this one standard is being taught. As educational stakeholders, we should be able to agree on the representations that students are expected to understand and “master” before graduating from public schools.

Further, statements found on state department of education webpages or state standards documents themselves stating that the standards are the minimum to be taught is quite confounding. If that’s the case, why are the adopted standards actually adopted? More guidance should be provided on what expectations look like outside of the provided standards. Does this imply that higher grade level standards should be taught, or does it mean that a teacher can incorporate any mathematics standards that they wish into a school year to attain higher than “minimum standards”?

Extend Standards and Domains

Changes to standards documented in this study such as the expansion of understanding American currency, beginning pattern work in Kindergarten, explicitly addressing estimation as a skill throughout various grade levels, and exploring multiple data representations should be considered for adoption by other states if revisions to standards are occurring in the near future. All of the mentioned changes to standards build on the CCSSM and enhance their implementation as students experience the mathematics earlier in their educational career. This further prepares them for the already established standards in later grade levels.

An extension of domains should be considered for future mathematics standards revisions. There were no consistencies across states that were coded for this study, but each incorporated various mathematical topics into domains that are not present in the CCSSM. This includes data and statistics, patterns, spatial reasoning, and financial literacy. The CCSSM did have standards that incorporated patterns and reinforced data and spatial reasoning through descriptions of the SMPs. However, the CCSSM did not incorporate financial literacy outside of a second-grade standard that required students to solve word problems with bills and coins.

The states in this study that incorporated financial literacy standards into revised standards allow students to arrive at adulthood better prepared to handle financial decisions, through reasoning and deep understanding of financial elements, which breached further mathematical knowledge than how to compute purchases with bank account balances. Equipping students with financial literacy knowledge can be a powerful tool to eradicating personal debt, as students would be made aware of how to engage in finances. Alarmingly, 30% of undergraduates incur a student loan with public university students averaging a debt of $31,410 by their graduation (Hanson, 2023). If students kept to the advice of financial institution SoFi (McCormack, 2022), then they would borrow only 20% of their discretionary income. Even in the wealthiest paying state, District of Columbia (National Education Association, 2022), a teacher would only be able to take out a loan less than $10,000 to align with this borrowing "rule of thumb".

Borrowing more than $10,000 for a bachelor's degree, which some students must do, restrains entire segments of generations from attaining higher wealth status. Federal student loans are predatory (Schwartz, 2017) and feed off the middle class and poor as a misguided solution to obtaining a higher education degree. By incorporating financial literacy standards early on in education, states are equipping students with knowledge to combat this injustice. Ultimately this knowledge has the power to alter entire family lineage regarding finances and has the potential to do more than a standardized assessment can provide.

While some might argue that financial literacy standards serve a better role in a high school business course or social studies standards, incorporating these standards into mathematics offers an additional applicable read-world experience for students. Additionally, by incorporating financial literacy standards earlier into students' lives, relationships with money have the potential to be drastically altered. For example, Brown and colleagues (2014) found positive effects on credit behaviors of young adults that had participated in financial programs during their K-12 public education careers. The incorporation of financial literacy should be highly considered by states as earning money and making the earned money work for you are two completely different skill sets that are not a current focal point in instruction in the CCSSM.

Additionally, the decision by states to remove the SMP of "use appropriate tools strategically" should be carefully considered for future mathematics standards revisions as it holds ramifications to the incorporation of technology. The description of this SMP requires students to consider tools available to solve a mathematics problem and explicitly lists, "pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software." (NGA & CCSSO, 2010, p.7). Technological tools are not necessarily the focus of the description, but as demonstrated during the COVID-19 pandemic, are helpful aides in mathematics classrooms. Variances in states, districts, and schools funding adjust the amount and quality of technology that is incorporated into mathematics classrooms, which can explain why the CCSSM writing team did not provide specific examples of technology for grade levels. It could also be that the CCSSM did not want to appear biased in their suggestions. Despite specific examples not being incorporated, the mention of technology still needs to be present in revisions of mathematics standards, as the accountability era in education demonstrated that teachers feel pressure to teach only the standards.

Strengths

This study included strengths to provide clarity regarding K-5 mathematics standards to the collective pool of educational stakeholders. Documented changes to state standards from 2012-2022 exist in one document, making information readily available and synthesized to those who seek to understand the changes. This incorporated a new element to the analysis of changes among standards. The study provided an in-depth description of the documents available by each state which conveys to the public which states are committed to transparency regarding decisions made by their department of education.

The first strength directly correlates to considering the impact of the efforts spent by states disassociating themselves from the Common Core State Standards. While this study involved massive amounts of data, only 6 of 50 states (or 12%) in the entire country demonstrated changes that were substantially different from the Common Core State Standards. These changes included adjustments to the standards but did not necessarily indicate a complete transition from the Common Core State Standards. This nominal percentage of 12% of states speaks volumes to the amount of time and resources that states have dedicated to proving that their state does not follow the Common Core State Standards, which proves inefficacious. State budget information pertaining to amount of money and time allocated to the standards revisions process was not easily accessible for the states selected for coding in this study, which further creates an urgency for transparency.

Another strength of this study is that it opens dialogue, including in the field of mathematics education and among policymakers. Some research topics have waves of popularity, and at the height of the release of the Common Core State Standards, researchers flocked to uncover all that they could regarding the standards. However, in the past decade the number of studies pertaining to the Common Core State Standards has declined. This study helped regain traction in terms of conversation regarding the changes occurring to state mathematics standards.

A final strength of this study was that changes to state standards were examined qualitatively. Individual state standards have been examined by the Thomas Fordham Institute but a singular document examining the changes to state standards qualitatively in the past decade has not been established prior to this study, thus the methodology employed is unlike previously used methodologies to analyze mathematics standards.

Limitations

This study was restricted by several limitations. First, much of the development and decision processes about standards revisions are not documented and thus could not be analyzed for this study. Second, states vary in the amount that is shared publicly on their department of education websites, which limits the types and amount of data that could be obtained for the study. Changing policies within states and changing political parties in office also influenced what information was publicly available on the state's department of education websites, which could not be controlled in terms of data collection. Third, this study does not represent all states that have revised their standards because, during the duration of this study, there were states that were in the process of revising their standards, but their revisions were not publicly available before the data analysis stage. This study also had the limitation of examining only K-5 mathematics standards.

Finally, researcher bias is a potential limitation in qualitative research (Creswell & Poth, 2018). My experiences as a K-8 mathematics teacher in both North Carolina and Florida shaped my perspectives that ultimately guided this study. As disclosed in chapter one, my experiences on multiple expert teams for the Florida B.E.S.T. mathematics standards, textbook adoption committee, and NAEP pilot study also influenced the design of this research study. Acknowledging research bias in my study was imperative, as readers can better trust me as the researcher and determine on their own accord if they perceive the study as biased.

Future Research

Students who started Kindergarten in 2012 (the year CCSSM began implementation in some states) will graduate in spring 2025. As we approach the graduation of the first group of students who solely learned the CCSSM, research can help better inform the impacts of the CCSSM. The students themselves can be a source of information regardless of if they attend a four-year institution, as the NGA and CCSSO envisioned standards to act as a goal for "college and career readiness for all students" (NGA & CCSSO, 2010, p. 4). Areas that could be examined are levels of anxiety related to mathematics in CCSSM era students, the number of CCSSM era students who need to take non-credit bearing college mathematics courses while working on a degree, the solution paths CCSSM era students feel most comfortable using when solving operational problems, the mathematical identity of the CCSSM era students, and whether standards are providing access to high-quality, equitable instruction and learning experiences. Each of these research ideas will be multi-faceted and have numerous layers (including the impact of the COVID-19 pandemic) to address the core issues but are valuable insights into the implementation and effects of standards.

Future research can provide a more complete picture of trends that are impacting mathematics across the country at all grade levels by addressing the changes that have occurred in grades 6-12 mathematics. A further phenomenon that was not exhibited in grades K-5, which has been a well-known issue, is the reorganization of mathematics classrooms that remove tracking (NCTM, 2018). Changes in states that opted to remove tracking should have the greatest deviations from the CCSSM, which continued to support the traditional high school mathematics pathway.

The February 2022 NCTM Board of Directors meeting summary provided a note that a discussion and consideration of the need for a framework of standards for high school mathematics education (NCTM, 2022) was discussed. Further, in June 2022 a budget was approved for the meeting of a task force and in November 2022 a report was received by the writing group developing a new initiative for guidance on secondary mathematics (NCTM, 2022). Disseminating changes to high school mathematics standards across the country can help inform this writing task team and the writing task team can provide guidance for other states who are considering revising their standards.

Additionally, studies could be conducted on the interpretation and implementation of mathematics standards in classrooms. Standards are a strong foundation to a mathematics classroom, but *how* the standards are interpreted and implemented drastically alters the outcome of the intended standards. Some states have had numerous revisions of standards within the past decade and teachers may not be aware of the changes that have been made. Another factor in the conversation of interpretation and implementation also rests in the quality and amount of professional development teachers receive when a new set of standards is adopted and implemented.

Further, the amount of time devoted to mathematics can also be researched. States vary in policies and laws pertaining to the instructional minutes spent on various subject areas. Typically, instructional time devoted to mathematics is either not included in the policy or law or it is a lesser amount than instructional time devoted to reading. This phenomenon can drastically alter the pacing of instruction for students and the intended outcomes of standards. Further, the trend of the states that exhibited more than 15% of changes from the CCSSM in regard to emphasis on procedural learning outcomes could further be explored as the balanced approach to learning outcomes was largely ignored. The debate on *how* to teach mathematics and what to emphasize is not new (NRC, 2001), but the progress of the CCSSM in regard to balancing learning outcomes should be considered in future standards revisions.

Finally, future research projects can provide a more in-depth analysis of the alignment of revised standards to learning trajectories used for the Common Core State Standards. An underwhelming number of states selected for coding in this study mentioned learning trajectories. The inclusion of specific learning trajectories by the CCSSM established coherence (Student Achievement Partners, 2013) in the standards and evidence of these trajectories in revisions to standards should be examined with more depth.

Summary

In conclusion, this study sought to answer the question, "*In what ways, if any, do K-5 state mathematics standards differ from the CCSSM*?*"* which was accomplished. However, to sum up the changes made to standards from 2012-2022 briefly is quite difficult, as a range of changes occurred with no consistencies across states. There was a range of CCSSM standards that were omitted on state standards revisions. There was also a range of standards that were added. Some of the most frequent additions to standards included pattern work at earlier grade levels, the inclusion of currency standards, the addition of multiple data representations that were not explicitly stated in the CCSSM, and an explicit focus on estimation with operations. However, these conclusions should not be generalized to all mathematics standard revisions.

The question that we as learners and future learners are left with is, where do we go from here? Loveless (2021) urges that replacing the CCSSM with a different set of standards would not make a difference in the achievement levels of students. He proposed instead that resources be used to improve education which includes "new, effective instructional strategies and curricula that boost learning" (Loveless, 2021, p. 170). His proposal truly calls for a revisioning of the school structure, which has remained largely unchanged with students still being taught in standardized and industrialized ways. At the core of this revisioning still needs to be consistent educational standards for *all* students, despite the number of moves a student might experience throughout their educational career. If states work together to improve mathematics standards based on feedback from the implementation of the CCSSM, the vision of improving education for all students can continue to be at the forefront of instruction as other structural changes are made to the education system.

In conclusion, it is necessary that as we transition forward in our post Common Core era, we keep children at the forefront of our decisions. In the words of former President, John F. Kennedy, "Let us not see the Republican answer or the Democratic answer, but the right answer. Let us not seek to fix the blame for the past" (Kennedy, 1958, para. 35) . As leaders in mathematics education, we must always align our actions and decisions to the right answer addressing, "What is the best decision for *all* our children?" Children are the future, and their educational needs should be met for the prosperity of our country and the world.

APPENDIX A:

INSTITUTIONAL REVIEW BOARD APPROVAL LETTER

Institutional Review Board
FWA00000351
IRB00001138, IRB00012110
Office of Research
12201 Research Parkway
Orlando, FL 32826-3246

UNIVERSITY OF CENTRAL FLORIDA

NOT HUMAN RESEARCH DETERMINATION

August 29, 2022

Dear Ashley Schmidt:

On 8/29/2022, the IRB reviewed the following protocol:

Type of Review:	Initial Study
Title of Study:	An Examination of a Decade of Mathematics Standards in the United States
Investigator:	Ashley Schmidt
IRB ID:	STUDY00004648
Funding:	None
Grant ID:	None
Documents Reviewed:	• HRP-251- FORM - Faculty Advisor Scientific-Scholarly Review fillable form[27334].pdf, Category: Faculty Research Approval; • HRP-250-FORM- Request for NHSR.docx, Category: IRB Protocol

The IRB determined that the proposed activity is not research involving human subjects as defined by DHHS and FDA regulations.

IRB review and approval by this organization is not required. This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made and there are questions about whether these activities are research involving human in which the organization is engaged, please submit a new request to the IRB for a determination. You can create a modification by clicking **Create Modification / CR** within the study.

If you have any questions, please contact the UCF IRB at 407-823-2901 or irb@ucf.edu. Please include your project title and IRB number in all correspondence with this office.

Sincerely,

Jonathan Coker
Designated Reviewer

APPENDIX B:

CODES WITH DEFINITIONS; ALPHABETICAL

Code	**Definition**
accounting	Standards address components of accounting terminology/understanding
career	Standards involve language pertinent to the understanding of a career
cognitive demand	Standards adjust cognitive demand
coin identification	Standards require identification of United States currency
comparison	Standards involve students to compare/contrast attributes or elements of a mathematics topic
connections	Standards include equivalence among mathematical ideas
content knowledge language	Standards include language pertinent to teacher content knowledge
data representation	Standards include creating and/or interpreting specific types of graphs
economics	Standards pertaining to production, distribution, and consumption of goods and services
estimation	Standards explicitly address estimation of a quantity or measure

Code	**Definition**
explicit instructional strategies	Standards address specific instructional strategies
financial organization	Standards include elements to keep finances organized at the personal/business level
fluency	Standards involve recall as a descriptor
foundational fraction sense/reasoning	Standards include equal sharing, partitioning, reasoning to demonstrate fractional understanding
inequality/operational/algebraic symbols	Standards include specific types of symbols or explicitly use equations with symbolic notation
manipulatives	Standards have wording referring to incorporating physical objects that students and/or teacher use
missing shape	Standards are missing shape(s) that are included in the CCSSM
models	Standards address a mathematical representation of a real-world scenario
multiple representations	Standards call for various representations of a mathematics concept
non-expectation clarification	Standards define what is not expected within instruction at that time

Code	Definition
non-technical jargon	Standards incorporate terminology that doesn't reflect precise mathematical terms
number relationships	Standards include mathematical terminology that stems from number relationships (part-part-whole, compose, decompose)
operations with money	Standards specifically call for operations to be used in calculating amounts of currency
patterns	Standards address pattern growth, extensions, identification, creation, rules, and/or transfer
personal finance	Standards pertaining to economics/finances that do not explicitly address the calculation of money
place value	Standards include groupings of tens
problem types	Standards include "practical," "real-world," "applicable," "story," or "picture"
properties of operations	Standards incorporate knowledge of properties of operations
properties of shapes	Standards include understanding of attributes/properties of a geometric shapes
range of numbers used differs	Standards use a different number goal than CCSSM

Code	Definition
rounding	Standards require students to round to a specified place value
SMP	Standards of mathematical practice language is referenced as part of standard
spatial reasoning	Standards rely on students to use spatial reasoning to complete geometric problem
specificity counting	Standards include types of counting (e.g. orally, forward, backward) or skip counting by given intervals (1's, 5's, 10's)
standard placement variation	Exact standard is in different grade in CCSSM (can be moved up or down)
standard/traditional algorithm	Standards require the use of a standard or traditional algorithm
subitizing	Standards require students to recognize number of objects without counting
unit reference	Standards include the use of a unit whether it be day/month/year, temperature, time, measurement, place value, number line usage, or counting
verb change	Standards include a difference in verbs represented in the CCSSM

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