2023

Evolution of Density and Velocity Perturbations in a Slowly Contracting Universe

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ANALYSIS OF THE EVOLUTION OF DENSITY AND VELOCITY PERTURBATIONS IN A SLOWLY CONTRACTING UNIVERSE

by

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A thesis submitted in partial fulfillment of the requirements
for the degree of Honors Undergraduate Thesis in Physics
in the College of Sciences
and in the Burnett Honors College
at the University of Central Florida

Fall Term
2023

ABSTRACT

One focus of research in cosmology regards the growth of structure in the universe: how we end up with stars, galaxies, galaxy clusters, and large scale structure in a universe that appears homogeneous and isotropic on large scales. Using cosmological perturbation theory, we investigate the evolution of density and velocity perturbations corresponding to a universe that is slowly contracting as proposed in [1], testing with and comparing different values for the equation-of-state parameter. This allows for the comparison of the growth of large scale structure in scenarios including a matter-dominated expanding universe, a dark energy-dominated expanding universe, and now, an ekpyrotic scalar field-dominated contracting universe. These predictions become observationally useful in the context of two point correlation functions to describe clustering. It is valuable to discriminate between various cosmological models to understand both the distant past and the ultimate fate of our universe.

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CHAPTER 1: INTRODUCTION

Cosmology is grounded in the idea that the universe is homogeneous and isotropic on large scales, known as the cosmological principle. This evidently breaks down on smaller scales: we observe inhomogeneities including stars, galaxies, galaxy clusters, and large-scale structure. Here we will focus on the large-scale structure of the universe, which is believed to be significantly influenced by two factors: gravitation and the expansion of the universe.

Nearly a century ago, astronomer Edwin Hubble announced the finding that we know as Hubble's Law. The recessional velocity of galaxies was found to be proportional to their distance from us, indicating the expansion of the universe [3]. It is intuitive that the expansion of space affects the way its contents interact, making the formation of structure a more formidable task. However, this effect is countered by gravity, allowing for density perturbations to grow despite the expansion of the universe. A Newtonian view of gravity is enough to understand how this could happen. As more mass accumulates somewhere in space and density increases, gravitational attraction increases proportionally. Though more complex physics is always at play, it is essentially the compounding of this effect that creates structure.

The formation of structure in the universe is commonly examined as the evolution of mass density for a universe in a matter-dominated phase. It is also assumed that gravitational interactions alone develop inhomogeneities from primordial fluctuations to the present state of the universe. These assumptions are widely regarded as valid, but there is considerable uncertainty surrounding the circumstances that dictate the evolution of both the early universe and the fate of the future universe.

The leading model to describe the very early universe and explain the homogeneity and isotropy noted in the cosmological principle is cosmic inflation. Many alternatives to inflation involve an

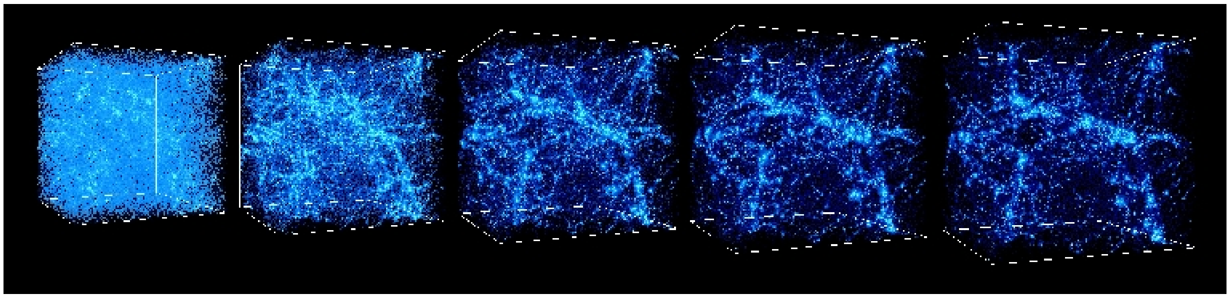

Figure 1.1: Evolution of structure in a 43 Mpc box. The leftmost box corresponds to $z = 9.83$ and the rightmost box corresponds to $z = 0$.

ekpyrotic or contracting phase governed by a scalar field with an equation-of-state parameter which varies significantly from that of matter. Therefore, matter density (and thus structure) in a universe dominated by such a scalar field is expected to differ in its evolution than structure developing in a matter or cosmological constant-dominated state.

Employing cosmological perturbation theory, we investigate the evolution of density and velocity perturbations corresponding to an ekpyrotic contracting phase, comparing different values for the equation-of-state parameter. This allows for the comparison of the growth of large scale structure in scenarios including a matter-dominated expanding universe, a dark energy-dominated expanding universe, and now, a scalar field-dominated contracting universe. We then continue by considering the timescales on which one could discriminate between the leading cosmological model, ΛCDM, and the cyclic universe examined in this work.

CHAPTER 2: MOTIVATION FOR INFLATION

Inflation was first posited in the 1980s to provide a solution to a handful of cosmological "problems": observed features of the universe that remained unexplained by the existing theoretical framework. Three key cosmological problems that inflation was designed to address are the horizon problem, the flatness problem, and the monopole problem. In essence, these problems point out features of the universe that *are*, but not *need* be—to the point of astounding and scientifically suspicious coincidence.

The horizon problem focuses on the troublingly homogeneity and isotropy of the universe on exceedingly large scales. Upon examination of the temperature spectrum of the CMB [4], points farther apart than the horizon distance—too distant for light-speed communication given the age of the universe—appear not just causally connected, but also in thermal equilibrium. Inflation solves the horizon problem with a brief period of rapid expansion causing the horizon size to grow exponentially, perhaps by 30 orders of magnitude. This happens before photon decoupling, so it leaves no electromagnetic signature.

The flatness problem notes that the density of the universe is incredibly close to the critical density, producing practically no curvature. Specifically, we now observe flatness such that

$$|1-\Omega_0| \leq 0.005 \tag{2.1}$$

[5], where $\Omega_0 = 1$ indicates a perfectly flat universe. Physics does not demand this flatness: Ω could assume any value greater than 0, and its value plays a significant role in the ultimate fate of the universe. That Ω_0 is so nearly one is exceedingly unlikely and demands explanation. The flatness problem becomes more astonishing at times nearing the Big Bang. With a value of Ω_0 so

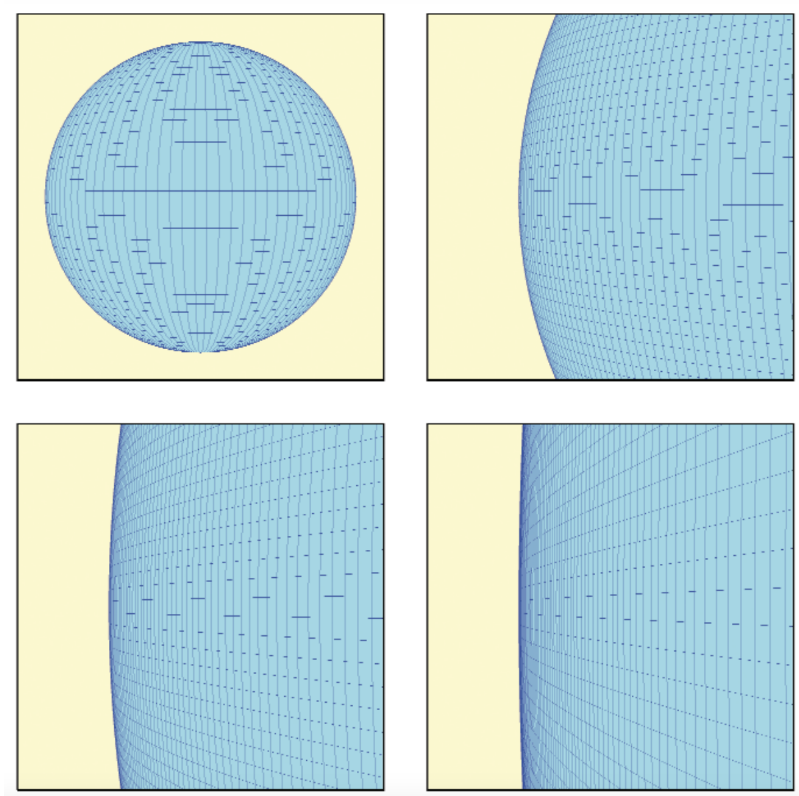

Figure 2.1: Visual representation of the inflation solution to the flatness problem [2].

close to unity at present time, Ω must have been significantly closer to one in the past. At Planck time, Ω_0 is further constrained such that

$$|1-\Omega_p| \leq 2\times 10^{-62} \tag{2.2}$$

[5]. Cosmologists generally agree that a value so close to one is not coincidence. Inflation addresses the flatness problem with the exponential expansion responsible for flattening curved geometric elements.

The monopole problem straddles the bounds of cosmology and particle physics. Grand Unified Theories and our understanding of the Big Bang suggest the plentiful existence of magnetic monopoles, yet we detect none. If magnetic monopoles were created before inflation, their number density would become so small that we could not expect to encounter any now.

CHAPTER 3: ALTERNATIVES TO INFLATION

Inflation is well-justified and remains the consensus view of cosmologists. However, it contains many free parameters that can be chosen somewhat arbitrarily. Inflation has passed a few observational tests, but confirmation remains incomplete. As such, inflation is not the only proposed solution to the aforementioned cosmological problems. Alternate resolutions have arisen in the twenty-first century cyclic and ekpyrotic models [1] [6], which would also meet observational constraints. Motivated by M-Theory, the ekpyrotic universe theory proposes that the Big Bang was the result of the collision of two three-dimensional branes in a fourth dimension, such that the kinetic energy of the colliding branes was converted to elementary particles. This collision process eliminates concerns surrounding the Big Bang's initial singularity, as the temperature is finite. In turn, a finite temperature solves the monopole problem since the massive monopoles cannot be produced in these lower-temperature conditions. The horizon problem is solved because the collision of the two branes is nearly simultaneous everywhere, and the Hubble radius is infinitesimal in comparison to the collision region. The flatness problem is solved by requiring the bulk brane to be flat, initially at rest, and parallel to the boundary branes [6].

More recent cyclic models invoke ekpyrotic elements, but produce an observationally alternative outcome. As the name suggests, the universe evolves in cycles, particularly of expanding and contracting phases. In the cyclic model proposed by Ijjas and Steinhardt [1], the universe undergoes slow contraction, followed by a bounce before it can reach a singularity, then a phase of familiar expansion. This cycle repeats indefinitely. The horizon problem is quickly addressed by instituting a period of contraction and subsequent bounce, allowing for causal connection. The flatness problem is addressed if the period of contraction endures long enough for the Hubble parameter to be reduced by 60 e-folds. Finally, the monopole problem is addressed similarly to the ekpyrotic scenario: monopoles would be too massive to be abundantly created provided sufficiently low reheating temperatures. Initially this cyclic model was believed to allow for a universe with no marked beginning, but this was recently disproved [7].

CHAPTER 4: GRAVITATIONAL INSTABILITY

As previously mentioned, the approximate homogeneity of the universe is only valid on very large scales. Gravity causes instability in the mass distribution in expanding Friedmann-Lemaitre-Robertson-Walker models. This instability naturally extends to contracting universes, where the effects of gravitation and contraction compound to form large scale structures.

Regardless, the large scale homogeneity of our universe indicates that far in the past, the universe was most likely even more homogeneous. This is a similar idea to the quantifiable level of flatness given in 2.2. Understanding the gravitational instability of the universe is key to understanding the formation and evolution of large scale structure, which form from small perturbations in mass density.

Mass density can be expressed as in equation 5.80 in [8], as

$$\rho(\mathbf{x},t) = \rho_b(t)[1+\delta(\mathbf{x},t)] \tag{4.1}$$

where $\rho_b(t)$ is the mean background mass density, which varies as $\rho_b \propto a(t)^{-3}$. The value of $\delta(\mathbf{x},t)$ then describes the fractional departure of the localized mass density from the mean. For the remaining discussion, δ is referred to as the density perturbation, and $|\delta| \ll 1$. Finally, $\mathbf{x}$ denotes the comoving spatial coordinate.

For the following discussion, we will need a few relations that come from these concepts. First, note that ∇ indicates operations with respect to x at a fixed time. Then $\nabla = a\nabla_r$.

Conservation of mass can be written generally, where $\mathbf{u}$ is a velocity field, as

$$(\frac{\partial \rho}{\partial t}) + \nabla_r \cdot (\rho \mathbf{u}) = 0 \tag{4.2}$$

and rewritten in comoving coordinates as

$$(\frac{\partial}{\partial t} - \frac{\dot{a}}{a}\mathbf{x}\cdot\nabla)[\rho_b(t)(1+\delta)] + \frac{\rho_b}{a}\nabla\cdot[(1+\delta)(\dot{a}\mathbf{x}+\mathbf{v})] = 0 \tag{4.3}$$

Where dot denotes differentiation with respect to time. Using $\rho_b \propto a(t)^{-3}$, we can say $\dot{\rho}_b = -3\rho_b\frac{\dot{a}}{a}$. Then after expanding the derivatives, we get

$$\frac{\partial\delta}{\partial t} + \frac{1}{a}\nabla\cdot[(1+\delta)\mathbf{v}] = 0 \tag{4.4}$$

We will also need the perturbed Poisson equation. The original Poisson equation for the gravitational potential is

$$\nabla_r^2\Phi = 4\pi G\rho. \tag{4.5}$$

This can then be rewritten as

$$-\frac{1}{a}\nabla\cdot\mathbf{g} = \frac{1}{a^2}\nabla^2\Phi = 4\pi G\rho_b(1+\delta) - \Lambda \tag{4.6}$$

The unperturbed part of the equation can be removed by writing

$$\Phi = \phi(\mathbf{x},t) + \frac{2}{3}\pi G\rho_b a^2 x^2 - \frac{1}{6}\Lambda a^2 x^2 \tag{4.7}$$

Which brings the Poisson equation to

$$\nabla^2\phi = 4\pi G\rho_b a^2\delta \tag{4.8}$$

We then combine these equations with the Euler equation:

$$(\frac{\partial\mathbf{u}}{\partial t})_r + (\mathbf{u}\cdot\nabla_r)\mathbf{u} = -\nabla_r\Phi \tag{4.9}$$

the velocity equation:

$$\mathbf{u} = \dot{a}\mathbf{x} + \mathbf{v}(\mathbf{x}, t) \tag{4.10}$$

and 4.7, to get

$$\frac{\partial v}{\partial t} + \frac{\dot{a}}{a}\mathbf{v} + \frac{1}{a}(\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{a}\nabla\phi. \tag{4.11}$$

This equation, along with 4.8 and 4.4 describe how mass fluctuations evolve for the approximate case of a pressureless ideal fluid. All of this assumes that the peculiar velocities and gravitational potential given by ϕ are much less than one, i.e. the Newtonian limit. However, density and velocity fluctuations can be nonlinear. Now, the evolution of δ and v can be calculated using perturbation theory. We will start with first-order, where terms of order $\delta^2, \delta v, v^2$, or higher are dropped. In this limit, 4.4 becomes

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla\cdot v = 0 \tag{4.12}$$

and 4.11 becomes

$$\frac{\partial v}{\partial t} + \frac{\dot{a}}{a}v + \frac{1}{a}\nabla\phi = 0 \tag{4.13}$$

We will then move on second-order, where $\delta^2, \delta v$, and v^2 terms are considered, but no higher.

CHAPTER 5: PERTURBATIVE ANALYSIS

First-order Density Perturbations

The following analysis is motivated by [1] and inspired by [9]. We begin with equations not specific to the contracting scenario.

$$\frac{\partial v}{\partial t} + \frac{\dot{a}}{a}v + \frac{1}{a}(v \cdot \nabla)v = -\frac{1}{a}\nabla\phi \tag{5.1}$$

Where dot denotes differentiation with respect to time. Dividing by a and using

$$\frac{1}{a}\frac{\partial v}{\partial t} = \frac{\partial}{\partial t}(\frac{v}{a}) + v\frac{\dot{a}}{a^2} \tag{5.2}$$

Yields

$$\frac{\partial}{\partial t}(\frac{v}{a}) + \frac{2\dot{a}v}{a^2} + (\frac{v}{a} \cdot \nabla)\frac{v}{a} = -\frac{1}{a^2}\nabla\phi \tag{5.3}$$

We now use $a \propto t^{1/\varepsilon}$, and therefore $\frac{\dot{a}}{a} \propto \frac{1}{\varepsilon t}$, where ε is an equation of state parameter given by $\varepsilon \equiv \frac{3}{2}(1 + \frac{p}{\rho})$. Here ρ is energy density and p is pressure.

The right side of the previous equation is manipulated using the Poisson equation, $\nabla^2\phi = 4\pi G\rho_0 a^2\delta$. New notation is introduced such that

$$\phi = \nabla^{-2} 4\pi G \rho_0 a^2 \delta \tag{5.4}$$

The notation ∇^{-2} will appear regularly in our discussion. After substituting 5.4 into 5.22, we proceed to write the RHS in terms of t. The Friedmann equation for a flat universe states

$$(\frac{\dot{a}}{a})^2 = \frac{8}{3}\pi G \rho_0 \tag{5.5}$$

The above equations are combined in detail in [10] (equation 10.3). This yields

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G \rho_0 \delta \tag{5.6}$$

With the help of the time dependence of a and 5.5, the right hand side of the above can be rewritten:

$$4\pi G \rho_0 = \frac{3}{2}(\frac{\dot{a}}{a})^2 \propto \frac{3}{2}\frac{1}{\varepsilon^2 t^2} \tag{5.7}$$

Which can then be restated as.

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2}{\varepsilon t}\frac{\partial \delta}{\partial t} - \frac{3}{2\varepsilon^2 t^2}\delta = 0 \tag{5.8}$$

We can use the assertive *equal to* rather than *proportional to* using well-known results. The value of ε for matter is $\frac{3}{2}$. When this is substituted in and the partial differential equation is solved, the known result is indeed obtained.

Our second-order partial differential equation for δ can be solved by assuming a solution of the form $\delta \propto t^{\lambda}$. Solving gives

$$\lambda = \pm\sqrt{\frac{5}{2\varepsilon^2} - \frac{1}{\varepsilon} + \frac{1}{4}} - \frac{1}{\varepsilon} + \frac{1}{2} \tag{5.9}$$

As a sanity check, we confirm with a known result. For matter, $\varepsilon = \frac{3}{2}$. Plugging into 5.9 gives $\delta = c_1 t^{\frac{2}{3}} + c_2 t^{-1}$, which is the accepted result.

Alas, we are not interested in a matter dominated universe. In [1] the authors state that for ekpyrotic contraction, $\varepsilon \gg 1$. For $\varepsilon \to \infty$,

$$\delta_{1,\infty} = At + B \tag{5.10}$$

For $\varepsilon = 100$, we have

$$\delta_{1,100} = At^{0.980} + Bt^{0.000153} \tag{5.11}$$

And, for $\varepsilon = 10$, we have

$$\delta_{1,10} = At^{0.818} + Bt^{-0.0183} \tag{5.12}$$

In each of the above, A and B are arbitrary functions of position describing density conditions at some initial time. They are generally taken to be random, Gaussian variables.

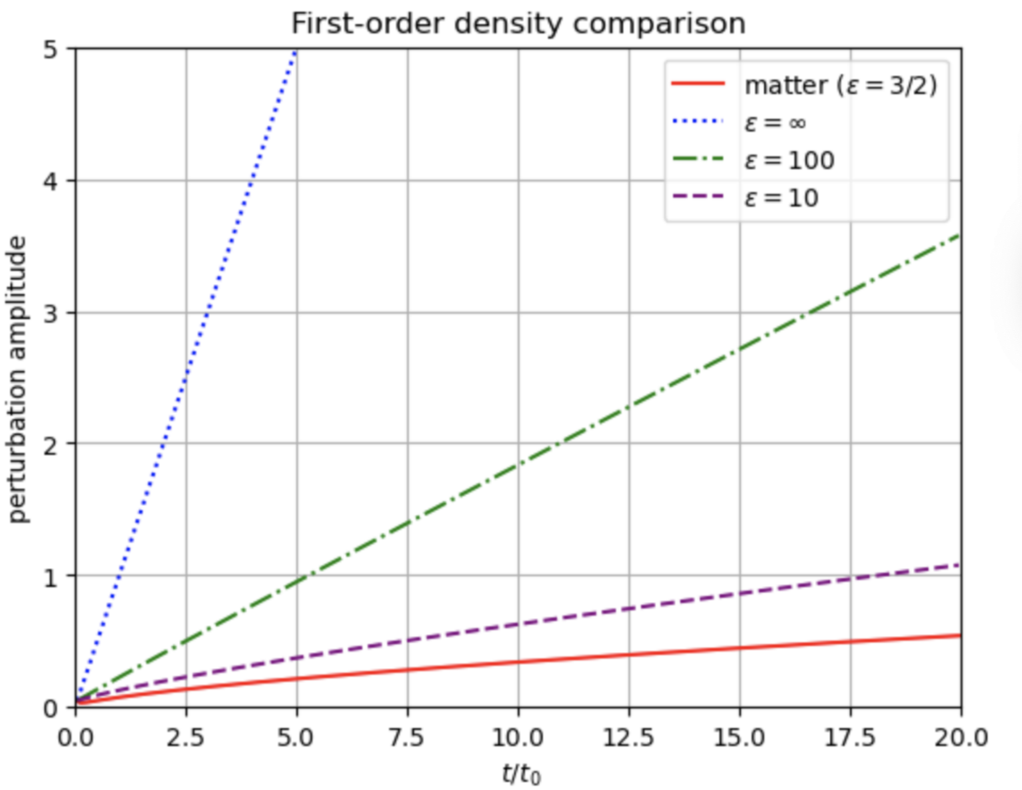

Figure 5.1: Time evolution of first-order density perturbations for the slowly contracting universe proposed by [1].

First-order Velocity Perturbations

Having solved for the first-order density perturbations, obtaining the first-order velocity modes becomes simple. Generally,

$$\frac{v_1}{a} = -\nabla\nabla^{-2}(\dot{\delta}_1) + \mathbf{C}t^{-2/\varepsilon} \tag{5.13}$$

Where **C** is initial rotational velocity. All that is left to be done is take the time derivatives of 5.10, 5.11, and 5.12. Doing so yields

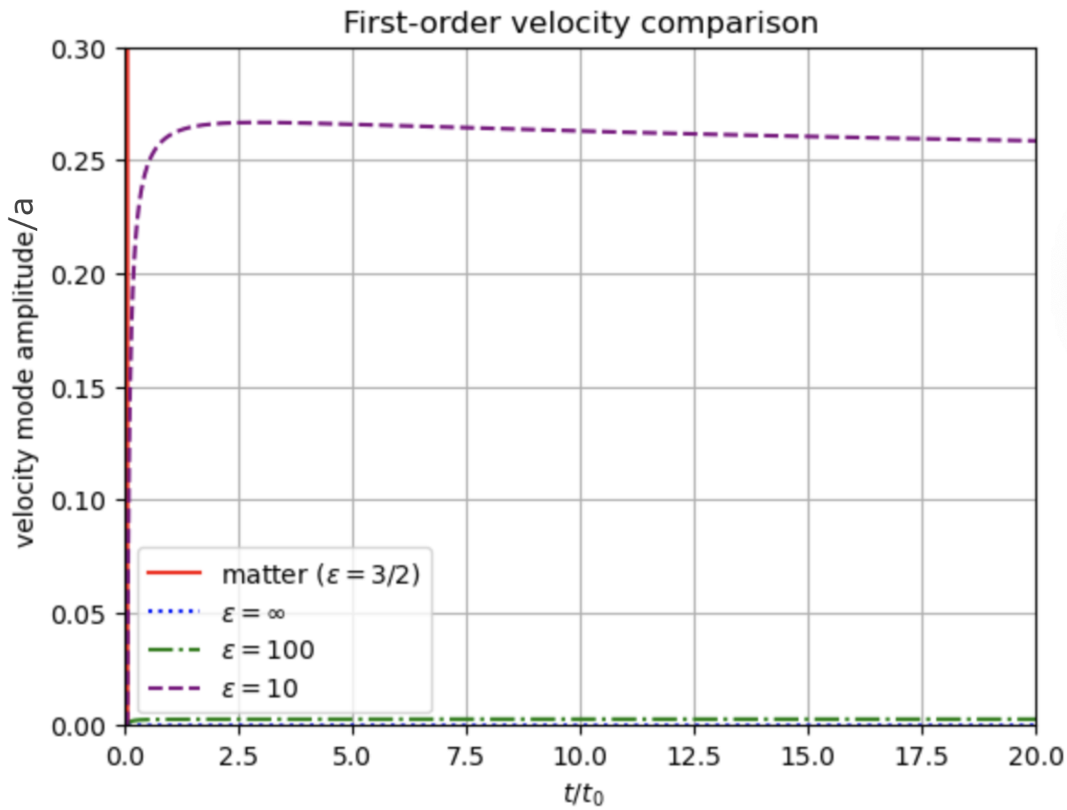

Figure 5.2: Time evolution of first-order velocity perturbations for the slowly contracting universe proposed by [1].

$$\frac{v_{1,\infty}}{a} = -\nabla\nabla^{-2}A + \mathbf{C} \tag{5.14}$$

$$\frac{v_{1,100}}{a} = -\nabla\nabla^{-2}(0.98At^{-0.02} - 0.000153Bt^{-1.000153}) + \mathbf{C}t^{-1/50} \tag{5.15}$$

$$\frac{v_{1,10}}{a} = -\nabla\nabla^{-2}(0.818At^{-0.182} - 0.0183Bt^{-1.0183}) + \mathbf{C}t^{-1/5} \tag{5.16}$$

Second-order Density Perturbations

For the following analysis, we let $B = 0$ as it is the decaying mode for all $\varepsilon < \infty$. Two key equations for this section are:

$$\dot{\delta}_2 + \frac{1}{a}\nabla \cdot v_2 = -\frac{1}{a}\nabla \cdot (v_1 \delta_1) \tag{5.17}$$

$$\frac{\partial}{\partial t}(\frac{v_2}{a}) + \frac{v_2}{a}\frac{2}{\varepsilon t} + \frac{3}{2}\frac{1}{\varepsilon^2 t^2}\nabla\nabla^{-2}\delta_2 = -(\frac{v_1}{a} \cdot \nabla)\frac{v_1}{a} \tag{5.18}$$

Case Where $\varepsilon \to \infty$

Substituting the last sections' findings into 5.17 gives

$$\dot{\delta}_{2,\infty} + \frac{1}{a}\nabla \cdot v_{2,\infty} = -\nabla \cdot [At(-\nabla\nabla^{-2}A + \mathbf{C})] \tag{5.19}$$

We now define some constants that will be used for the rest of our discussion:

$$E_1 = \nabla \cdot (A\nabla\nabla^{-2}A) \tag{5.20}$$

and

$$E_2 = -\nabla \cdot (\mathbf{C}A) \tag{5.21}$$

We use these to rewrite 5.19 as

$$\dot{\delta}_{2,\infty} + \frac{1}{a}\nabla \cdot v_{2,\infty} = (E_1 + E_2)t \equiv Et \tag{5.22}$$

In 5.18, all terms with ε in the denominator vanish, leaving

$$\frac{\partial}{\partial t}(\frac{v_2}{a}) = -(\frac{v_1}{a} \cdot \nabla)\frac{v_1}{a} \tag{5.23}$$

$$= -[(-\nabla\nabla^{-2}A + \mathbf{C}) \cdot \nabla](-\nabla\nabla^{-2}A + \mathbf{C}) \tag{5.24}$$

$$= -(\nabla\nabla^{-2}A) \cdot \nabla(\nabla\nabla^{-2}A) + (\nabla\nabla^{-2}A \cdot \nabla)\mathbf{C} + \mathbf{C} \cdot \nabla(\nabla\nabla^{-2}A) - (\mathbf{C} \cdot \nabla)\mathbf{C} \tag{5.25}$$

We once again define some constants to be used for the rest of our discussion:

$$\mathbf{F_1} = -(\nabla\nabla^{-2}A) \cdot \nabla(\nabla\nabla^{-2}A) = -\frac{1}{2}\nabla(\nabla\nabla^{-2}A \cdot \nabla\nabla^{-2}A) \tag{5.26}$$

$$\mathbf{F_2} = (\nabla\nabla^{-2}A \cdot \nabla)\mathbf{C} + \mathbf{C} \cdot \nabla(\nabla\nabla^{-2}A) \tag{5.27}$$

and

$$\mathbf{F_3} = -(\mathbf{C} \cdot \nabla)\mathbf{C} \tag{5.28}$$

so that

$$\frac{\partial}{\partial t}(\frac{v_2}{a}) = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} \equiv \mathbf{F} \tag{5.29}$$

We now take the divergence of 5.29 and the time derivative of 5.22, combining them to produce a second-order differential equation for $\delta_{2,\infty}$.

$$\nabla \cdot \frac{\partial}{\partial t}(\frac{v_{2,\infty}}{a}) = \nabla \cdot \mathbf{F} \tag{5.30}$$

$$\ddot{\delta}_{2,\infty} + \frac{\partial}{\partial t}\nabla \cdot \frac{v_{2,\infty}}{a} = E \tag{5.31}$$

$$\ddot{\delta}_{2,\infty} + (\nabla \cdot \mathbf{F}) - E = 0 \tag{5.32}$$

which has a solution of the form $\delta_{2,\infty} = H(x)t^2$, where H(x) some function of position. This implies $\dot{\delta}_{2,\infty} = 2H(x)t$ and $\ddot{\delta}_{2,\infty} = 2H(x)$. Plugging into 5.32 to solve for $H(x)$ yields our second-order solution:

$$\delta_{2,\infty} = \frac{1}{2}t^2(E - \nabla \cdot \mathbf{F}) \tag{5.33}$$

Case Where $\varepsilon = 100$

We follow the same approach used in the last section to find the time evolution of second-order density perturbations. Using 5.17, we have

$$\dot{\delta}_{2,100} + \frac{1}{a}\nabla \cdot v_{2,100} = -\frac{1}{a}\nabla \cdot (v_{1,100}\delta_{1,100}) \tag{5.34}$$

$$= -\nabla \cdot (At^{0.98}[\nabla\nabla^{-2}(-0.98At^{-0.02}) + \mathbf{C}t^{-0.02}] \tag{5.35}$$

$$= t^{0.96}(0.98E_1 + E_2) \tag{5.36}$$

where E_1 and E_2 are the same constants defined in 5.20 and 5.21. For algebraic ease, we redefine E for this section as $E \equiv 0.98E_1 + E_2$. This allows us to rewrite 5.17 as

$$\dot{\delta}_{2,100} + \frac{1}{a}\nabla \cdot v_2 = Et^{0.96} \tag{5.37}$$

We now turn to 5.18 and make the appropriate substitutions.

$$\frac{\partial}{\partial t}(\frac{v_{2,100}}{a}) + \frac{v_{2,100}}{a}\frac{1}{50t} + \frac{3}{2}\frac{1}{10000t^2}\nabla\nabla^{-2}\delta_{2,100} = -(\frac{v_{1,100}}{a}\cdot\nabla)\frac{v_{1,100}}{a} \tag{5.38}$$

$$= -[(-\nabla\nabla^{-2}0.98At^{-0.02} + \mathbf{C}t^{-0.02})\cdot\nabla](-\nabla\nabla^{-2}0.98At^{-0.02} + \mathbf{C}t^{-0.02}) \tag{5.39}$$

$$= t^{-0.04}(0.9604\mathbf{F_1} + 0.98\mathbf{F_2} + \mathbf{F_3}) \tag{5.40}$$

where $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_3}$ are the same as defined in 5.26, 5.27, and 5.28. For algebraic ease, we redefine $\mathbf{F} \equiv 0.9604\mathbf{F_1} + 0.98\mathbf{F_2} + \mathbf{F_3}$, giving a more digestible result:

$$\frac{\partial}{\partial t}(\frac{v_{2,100}}{a}) + \frac{v_{2,100}}{a}\frac{1}{50t} + \frac{3}{2}\frac{1}{10000t^2}\nabla\nabla^{-2}\delta_{2,100} = \mathbf{F}t^{-0.04} \tag{5.41}$$

We once again proceed by taking the divergence and a time derivative, now of 5.41 and 5.37, respectively.

$$\nabla\cdot\frac{\partial}{\partial t}(\frac{v_{2,100}}{a}) + \nabla\cdot\frac{v_{2,100}}{a}\frac{1}{50t} + \nabla\cdot\frac{3}{2}\frac{1}{10000t^2}\nabla\nabla^{-2}\delta_{2,100} = \nabla\cdot\mathbf{F}t^{-0.04} \tag{5.42}$$

$$\ddot{\delta}_{2,100} + \nabla \cdot \frac{\partial}{\partial t}(\frac{v_{2,100}}{a}) = 0.96Et^{-0.04} \tag{5.43}$$

Note that the first term of 5.42 is equal to the second term in 5.43, allowing us to combine them.

$$0.96Et^{-0.04} - \ddot{\delta}_{2,100} + \frac{1}{50t}\nabla \cdot \frac{v_{2,100}}{a} + \frac{3}{20000t^2}\nabla\nabla^{-2}\delta_{2,100} = \nabla \cdot \mathbf{F}t^{-0.04} \tag{5.44}$$

The third term of the above equation is a multiple of the second term of 5.37, so we can say

$$\frac{1}{50t}\nabla \cdot \frac{v_{2,100}}{a} = \frac{1}{50t}(Et^{0.96} - \dot{\delta}_{2,100}) \tag{5.45}$$

Rearranging and combining like terms gives our second-order differential equation:

$$\ddot{\delta}_{2,100} + \frac{1}{50t}\dot{\delta}_{2,100} - \frac{3}{20000t^2}\delta_{2,100} - t^{-0.04}(0.98E - \nabla \cdot \mathbf{F}) = 0 \tag{5.46}$$

which has a solution of the from $\delta_{2,100} = J(x)t^{1.96}$, implying $\dot{\delta}_{2,100} = 1.96J(x)t^{1.96}$ and $\ddot{\delta}_{2,100} = 1.881J(x)t^{-0.04}$, which we plug in to 5.46.

$$t^{-0.04}(1.92065J(x) - 0.98E + \nabla \cdot \mathbf{F}) = 0 \tag{5.47}$$

The above can be solved for $J(x)$, which completes the second-order solution for density perturbations,

$$\delta_{2,100} = 0.521(0.98E - \nabla \cdot \mathbf{F})t^{1.96} \tag{5.48}$$

Case Where $\varepsilon = 10$

Once again, we follow the same general procedure. Using 5.17, we have

$$\dot{\delta}_{2,10} + \frac{1}{a}\nabla \cdot v_{2,10} = -\frac{1}{a}\nabla \cdot (v_{1,10}\delta_{1,10}) \tag{5.49}$$

$$= -\nabla \cdot (At^{0.818}[\nabla\nabla^{-2}(-0.818At^{-0.182}) + \mathbf{C}t^{-0.2}] \tag{5.50}$$

$$\dot{\delta}_{2,10} + \frac{1}{a}\nabla \cdot v_{2,10} = 0.818E_1t^{0.636} + E_2t^{0.618} \tag{5.51}$$

where E_1 and E_2 are the same constants defined in 5.20 and 5.21. We now turn to 5.18 and make the appropriate substitutions.

$$\frac{\partial}{\partial t}(\frac{v_{2,10}}{a}) + \frac{v_{2,10}}{a}\frac{1}{5t} + \frac{3}{2}\frac{1}{100t^2}\nabla\nabla^{-2}\delta_{2,10} = -(\frac{v_{1,10}}{a}\cdot\nabla)\frac{v_{1,10}}{a} \tag{5.52}$$

$$= -[(-\nabla\nabla^{-2}0.818At^{-0.182} + \mathbf{C}t^{-0.2})\cdot\nabla](-\nabla\nabla^{-2}0.818At^{-0.182} + \mathbf{C}t^{-0.2}) \tag{5.53}$$

$$= 0.669t^{-0.364}\mathbf{F_1} + 0.818t^{-0.382}\mathbf{F_2} + t^{-0.4}\mathbf{F_3} \tag{5.54}$$

where $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_3}$ are the same as defined in 5.26, 5.27, and 5.28.

We once again proceed by taking the divergence and a time derivative, now of 5.54 and 5.51, respectively.

$$\nabla \cdot \frac{\partial}{\partial t}(\frac{v_{2,10}}{a}) + \nabla \cdot \frac{v_{2,10}}{a}\frac{1}{5t} + \nabla \cdot \frac{3}{2}\frac{1}{100t^2}\nabla\nabla^{-2}\delta_{2,10} = \nabla \cdot R.H.S. \tag{5.55}$$

$$\ddot{\delta}_{2,10} + \nabla \cdot \frac{\partial}{\partial t}(\frac{v_{2,10}}{a}) = 0.52E_1 t^{-0.364} + 0.618E_2 t^{-0.382} \tag{5.56}$$

Note that the first term of 5.55 is equal to the second term in 5.56, allowing us to combine them.

$$0.52E_1 t^{-0.364} + 0.618E_2 t^{-0.382} - \ddot{\delta}_{2,10} + \frac{1}{5t}(0.818E_1 t^{0.636} + E_2 t^{0.618} - \dot{\delta}_{2,10}) + \frac{3}{200t^2}\delta_{2,10} = \nabla \cdot R.H.S \tag{5.57}$$

Continuing, we find

$$\ddot{\delta}_{2,10} + \frac{1}{5t}\dot{\delta}_{2,10} - \frac{3}{200t^2}\delta_{2,10} = t^{-0.364}(0.6836E_1 - 0.669\nabla \cdot \mathbf{F_1} + t^{-0.382}(0.818E_2 - 0.828\nabla \cdot \mathbf{F_2}) - t^{-0.4}(\nabla \cdot \mathbf{F_3}) \tag{5.58}$$

A second-order differential equation with a solution of the from $\delta_{2,10} = H(x)t^{1.636} + I(x)t^{1.618} + J(x)t^{1.6}$. We now find $\dot{\delta}_{2,10}$ and $\ddot{\delta}_{2,10}$ and plug the results back into 5.58 to solve for $H(x)$, $I(x)$, and $J(x)$. We find

$$H(x) = 0.5055E_1 - 0.4947\nabla \cdot \mathbf{F_1} \tag{5.59}$$

$$I(x) = 0.625(E_2 - \nabla \cdot \mathbf{F_2}) \tag{5.60}$$

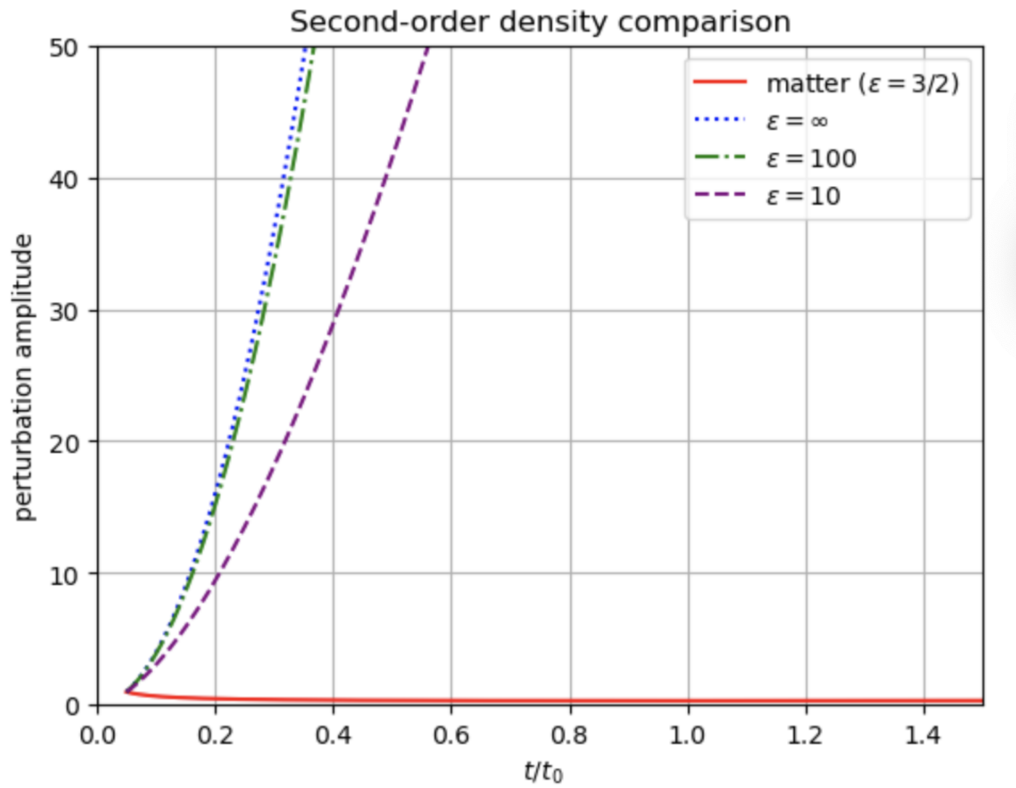

Figure 5.3: Time evolution of second-order density perturbations for the slowly contracting universe proposed by [1].

and

$$J(x) = -0.7905\nabla \cdot \mathbf{F_3} \tag{5.61}$$

Plugging these back in completes the second-order solution for density perturbations.

$$\delta_{2,10} = t^{1.636}(0.5055E_1 - 0.4947\nabla \cdot \mathbf{F_1}) + t^{1.618}[0.625(E_2 - \nabla \cdot \mathbf{F_2})] + t^{1.6}(-0.7905\nabla \cdot \mathbf{F_3}) \tag{5.62}$$

Second-order Velocity Perturbations

Case Where $\varepsilon \to \infty$

To solve for the second-order velocity modes, we start by taking the time derivative of 5.33 and plug it into 5.22.

$$\dot{\delta}_{2,\infty} = t(E - \nabla \cdot \mathbf{F}) \quad (5.63)$$

$$\frac{v_{2,\infty}}{a} = \nabla\nabla^{-2}[Et - t(E - \nabla \cdot \mathbf{F})] = \nabla\nabla^{-2}(\nabla \cdot \mathbf{F})t + D.F. \quad (5.64)$$

D. F. is the divergence-free part and is found by taking the curl of 5.29:

$$\frac{\partial}{\partial t}\left(\frac{v^t_{2,\infty}}{a}\right) = \mathbf{F^t_2} + \mathbf{F^t_3} \quad (5.65)$$

Note that $\nabla \times \mathbf{F_1} = 0$. The superscript t indicates the transverse component. Clearly,

$$\frac{v^t_{2,\infty}}{a} = (\mathbf{F^t_2} + \mathbf{F^t_3})t \quad (5.66)$$

Which gives the following solution for the second-order velocity modes:

$$\frac{v_{2,\infty}}{a} = \nabla\nabla^{-2}(\nabla \cdot \mathbf{F})t + (\mathbf{F^t_2} + \mathbf{F^t_3})t \quad (5.67)$$

Case Where $\varepsilon = 100$

For $\varepsilon = 100$, we follow the same procedure as $\varepsilon \to \infty$. We begin by taking the time derivative of the second-order density solution, and plugging it into 5.37.

$$\dot{\delta}_{2,100} = 1.02(0.98E - \nabla \cdot \mathbf{F})t^{0.96} \tag{5.68}$$

$$\frac{v_{2,100}}{a} = \nabla\nabla^{-2}[Et^{0.96} - 1.02(0.98E - \nabla \cdot \mathbf{F})t^{0.96}] \tag{5.69}$$

$$= \nabla\nabla^{-2}t^{0.96}(0.0004E + 1.02\nabla \cdot \mathbf{F}) + D.F. \tag{5.70}$$

where D. F. is once again the divergence-free part, found by taking the curl of 5.41.

$$\frac{\partial}{\partial t}(\frac{v^t_{2,100}}{a}) + \frac{v^t_{2,100}}{a}\frac{1}{50t} = (\mathbf{F^t_2} + \mathbf{F^t_3})t^{-0.04} \tag{5.71}$$

As before, $\nabla \times \mathbf{F_1} = 0$ and the superscript t indicates the transverse component. It is straightforward to conclude the divergence-free part is

$$\frac{v^t_{2,100}}{a} = 1.02(\mathbf{F^t_2} + \mathbf{F^t_3})t^{0.96} \tag{5.72}$$

plugging this in for $D.F.$ in 5.70 gives the full second-order solution.

$$\frac{v_{2,100}}{a} = \nabla\nabla^{-2}t^{0.96}(0.0004E + 1.02\nabla\cdot\mathbf{F}) + 1.02(\mathbf{F_2^t} + \mathbf{F_3^t})t^{0.96} \tag{5.73}$$

Case Where $\varepsilon = 10$

We once again follow the same general procedure. We start by taking the time derivative of 5.62 and plugging it into 5.51. $H(x)$, $I(x)$, and $J(x)$ are given in 5.59, 5.60, and 5.61.

$$\dot{\delta}_{2,10} = 1.636t^{0.636}H(x) + 1.618t^{0.618}I(x) + 1.6t^{0.6}J(x) \tag{5.74}$$

$$\frac{v_{2,10}}{a} = \nabla\nabla^{-2}[(-0.008998E_1 + 0.809\nabla\cdot\mathbf{F_1})t^{0.636} + (-0.01125E_2 + 1.01125\nabla\cdot\mathbf{F_2})t^{0.618} + 1.2648t^{0.6}\nabla\cdot\mathbf{F_3}] + D.F. \tag{5.75}$$

where D. F. is once again the divergence-free part, which we solve for by taking the curl of 5.54.

$$\frac{\partial}{\partial t}\left(\frac{v_{2,10}^t}{a}\right) + \frac{1}{5t}\frac{v_{2,10}^t}{a} = 0.818t^{-0.382}\mathbf{F_2^t} + t^{-0.4}\mathbf{F_3^t} \tag{5.76}$$

$\nabla\times\mathbf{F_1}$ is still 0, and the superscript t indicates the transverse component. It is straightforward to conclude the divergence-free part is

$$\frac{v_{2,10}^t}{a} = \mathbf{F_2^t}t^{0.618} + 1.25\mathbf{F_3^t}t^{0.6} \tag{5.77}$$

Plugging this back into 5.75 gives our final solution:

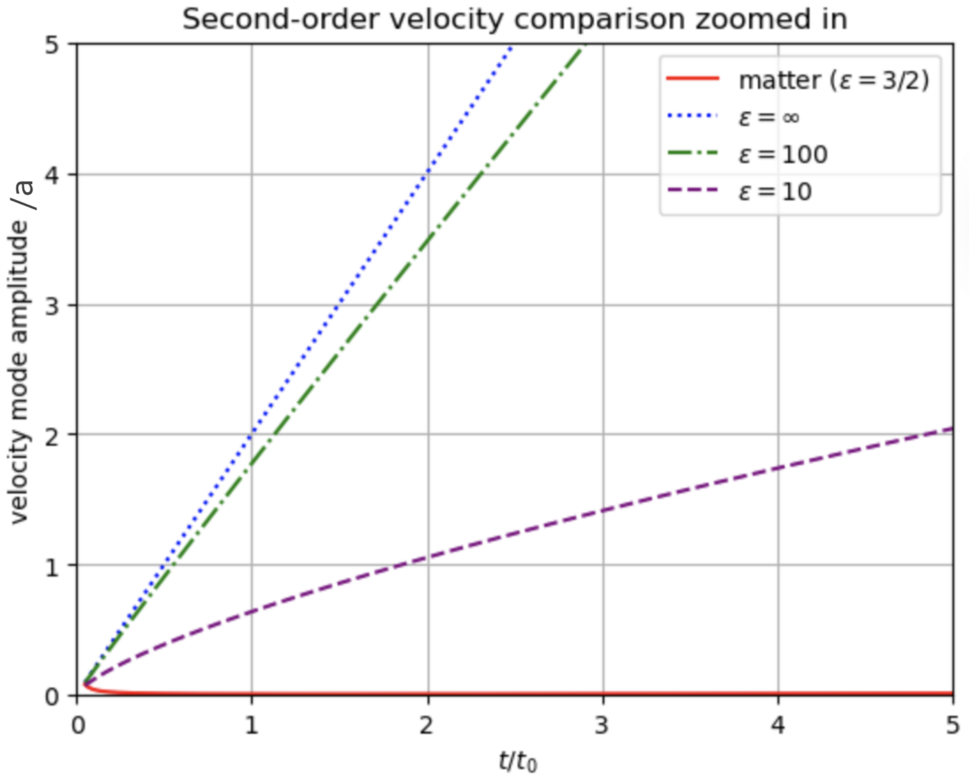

Figure 5.4: Time evolution of second-order velocity perturbations for the slowly contracting universe proposed by [1].

$$\frac{v_{2,10}}{a} = \nabla\nabla^{-2}[(-0.008998E_1 + 0.809\nabla\cdot\mathbf{F_1})t^{0.636} + (-0.01125E_2 + 1.01125\nabla\cdot\mathbf{F_2})t^{0.618}$$

$$+ 1.2648\mathrm{t}^{0.6}\nabla\cdot\mathbf{F_3}] + \mathbf{F_2^t}t^{0.618} + 1.25\mathbf{F_3^t}t^{0.6} \tag{5.78}$$

CHAPTER 6: TWO-POINT CORRELATION FUNCTIONS

The two-point correlation function gives the probability that two galaxies will be within a fixed distance of each other. It thus gives the evolution of clustering, which is observable. However, this evolution varies for different regimes: linear, quasilinear, and nonlinear. These different evolutions are given in equations 16.72 of [11]. Linear is given by

$$\xi(t) \propto [\delta(t)]^2 \tag{6.1}$$

Quasilinear is described by

$$\xi(t) \propto [\delta(t)]^{(6-2\gamma)(1+\alpha)/3} \tag{6.2}$$

And nonlinear is

$$\xi(t) \propto [a(t)]^{3-\gamma}. \tag{6.3}$$

Of course, δ gives the growth law for density perturbations. γ is accepted to be about 1.7, and α is taken to be approximately 4.

The linear regime corresponds to the largest scales, so that is the equation we wish to use. By plugging in our results for the density perturbations, we can predict how the values of the correlation function will evolve with time. This is an observable, and in the future, could be used to discriminate between these various models of early universe cosmology. Squaring the results from the perturbative analysis section is trivial, but through the passage of enough time, could be a useful way to distinguish between different models of the early universe.

CHAPTER 7: CONCLUSIONS

Qualitatively, our results are unsurprising. However, the precise quantitative results may prove useful. At an unknown time, they could be used to discriminate between cosmological models of the early universe, which also determine the ultimate fate of the universe.

Next steps include accounting for statistical properties. A, B, and C from the Perturbative Analysis section are Gaussian random variables, but were treated as unity for the purpose of generating plots. Once statistical properties are accounted for, we could estimate timescales on which the two-point correlation functions will differ significantly enough from the ΛCDM model, providing a more comprehensive result.

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