1-1-1996

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Recommended Citation

Shivamoggi, B. K. and Mulser, P., "Dielectric Screening In The Thomas-Fermi Model" (1996). *Faculty Bibliography 1990s*. 1757.

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Dielectric screening in the Thomas-Fermi model

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(Received 11 March 1996; revised manuscript received 21 June 1996)

In this paper, we investigate the dielectric screening process in the Thomas-Fermi model with several variations such as the thermal effects, relativistic effects, motion of the test charge, and the effect of a very strong magnetic field. Comparisons are made with the corresponding results in a classical plasma. Some apparently anomalous results are pointed out and explained. [S1050-2947(96)03712-2]

PACS number(s): 31.15.-p

I. INTRODUCTION

In the Thomas-Fermi method, matter is modeled by a lattice of shielded nuclei immersed in a sea of uniformly distributed electrons. Thus we have a uniform gas of electrons of charge density $-n_0e$ superimposed on a background of positive charge density n_0e . Consider now a positive point charge q placed in a sea of such conduction electrons. It will then attract electrons, creating a surplus of negative charge in its neighborhood, which reduces (or screens) its Coulomb field. The screened Coulomb potential of this point charge as seen by an electron is that due to the given point charge plus a cloud of screening electrons induced by it. Assuming that the given point charge is weak enough to produce only a linear response in the electron gas, the screened Coulomb potential is given self-consistently by

$$\nabla^2\Phi = 4\pi e(n - n_0) - 4\pi q\delta(\mathbf{r}). \quad (1)$$

We will, in the following, investigate the dielectric screening described by Eq. (1), in the following two regimes: (i) In the classical Thomas-Fermi regime we will discuss thermal effects, relativistic effects, combined relativistic and thermal effects, the effect of untrapped particles, and the effect of motion of the test charge; (ii) in the regime of a very strong magnetic field we will discuss the effect of a cold electron gas, thermal effects, relativistic effects, and the combined relativistic and thermal effects.

II. CLASSICAL THOMAS-FERMI REGIME

A. Thermal effects

Using the number density expression given by Feynman, Metropolis, and Teller [1], Eq. (1) becomes

$$\nabla^2\Phi = 4\pi e \left[\int_0^\infty \frac{2 \frac{4\pi p^2 dp}{h^3}}{e^{[p^2/2m - (\mu + e\Phi)](1/KT) + 1}} - \int_0^\infty \frac{2 \frac{4\pi p^2 dp}{h^3}}{e^{[p^2/2m - \mu](1/KT) + 1}} \right] - 4\pi q\delta(\mathbf{r}) \quad (2)$$

or

$$\nabla^2\phi = \frac{16\pi^2 e}{h^3} (2mKT)^{3/2} \left[I_{1/2}\left(\frac{\mu + e\phi}{KT}\right) - I_{1/2}\left(\frac{\mu}{KT}\right) \right] - 4\pi q\delta(\mathbf{r}), \quad (3)$$

where

$$I_p(\eta) \equiv \int_0^\infty \frac{y^p dy}{e^{(y-\eta)+1}}. \quad (4)$$

Let us consider a weakly nondegenerate regime, i.e.,

$$\left| \frac{\mu + e\phi}{KT} \right| \gg 1.$$

In this limit, (4) has the asymptotic expansion

$$I_{1/2}(\eta) \approx \frac{2}{3} \eta^{3/2} \left(1 + \frac{\pi^2}{8} \frac{1}{\eta^2} + \dots \right) \quad \text{for } \eta \text{ large.} \quad (5)$$

Using (5) and linearizing in ϕ , we obtain, from Eq. (3),

$$\nabla^2\phi = \frac{3}{2} \lambda_{F1}^{-2} \phi - 4\pi q\delta(\mathbf{r}), \quad (6)$$

where

$$\lambda_{F1}^{-2} \equiv \frac{4\pi n_0 \left(1 - \frac{\nu}{3} \right) e^2}{\mu}, \quad n_0 \equiv \frac{1}{3\pi^2} \left(\frac{2m\mu}{\hbar^2} \right)^{3/2},$$

$$\nu \equiv \frac{\pi^2}{8} \left(\frac{KT}{\mu} \right)^2 \ll 1.$$

Equation (6) has the solution

$$\phi = \frac{q}{r} e^{-\sqrt{3/2}(r/\lambda_{F1})}. \quad (7)$$

Equation (7) shows that thermal effects increase the screening length, as in a classical plasma [2].

B. Relativistic effects

Using the number density expression given by Vallarta and Rosen [3], Eq. (1) becomes

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$$\nabla^2 \phi = 4\pi e \frac{m^3 c^3}{3\pi^2 \hbar^3} \left[\left\{ \frac{(mc^2 + \mu + e\phi)^2}{m^2 c^4} - 1 \right\}^{3/2} - \left\{ \frac{(mc^2 + \mu)^2}{m^2 c^4} - 1 \right\}^{3/2} \right] - 4\pi q \delta(\mathbf{r}). \quad (8)$$

Let us consider a weakly relativistic regime, i.e.,

$$\left| \frac{\mu + e\phi}{mc^2} \right| \ll 1.$$

In this limit, on linearizing further in ϕ , Eq. (8) leads to

$$\nabla^2 \phi = \frac{3}{2} \lambda_{F_2}^{-2} \phi - 4\pi q \delta(\mathbf{r}), \quad (9)$$

where

$$\lambda_{F_2}^{-2} \equiv \frac{4\pi n_0 \left(1 + \frac{\sigma}{2}\right) e^2}{\mu}, \quad \sigma \equiv \frac{\mu}{mc^2} \ll 1.$$

Equation (9) has the solution

$$\phi = \frac{q}{r} e^{-\sqrt{3/2}(r/\lambda_{F_2})}. \quad (10)$$

Equation (10) shows that the relativistic effects reduce the screening length, which is due to the enhanced concentration of relativistic electrons near the nucleus [4].

C. Combined relativistic and thermal effects

Using the number density expression given by Shiva-moggi [5], Eq. (1) becomes

$$\nabla^2 \phi = 4\pi e \left[\int_0^\infty \frac{2 \frac{4\pi p^2 dp}{h^3}}{e^{[\sqrt{p^2 c^2 + m^2 c^4} - (mc^2 + \mu + e\phi)](1/KT)} + 1} - \int_0^\infty \frac{2 \frac{4\pi p^2 dp}{h^3}}{e^{[\sqrt{p^2 c^2 + m^2 c^4} - (mc^2 + \mu)](1/KT)} + 1} \right] - 4\pi q \delta(\mathbf{r}). \quad (11)$$

Let us consider a weakly nondegenerate regime, i.e.,

$$\left| \frac{mc^2 + \mu + e\phi}{KT} \right| \gg 1.$$

In this limit, using the asymptotic evaluations given in [6], Eq. (11) becomes

$$\begin{aligned} \nabla^2 \phi = 4\pi e \frac{m^3 c^3}{3\pi^2 \hbar^3} & \left[\left\{ \frac{(mc^2 + \mu + e\phi)^2}{m^2 c^4} - 1 \right\}^{3/2} \right. \\ & \times \left\{ 1 + \frac{\pi^2}{2} \left(\frac{KT}{mc^2} \right)^2 \left[\frac{(mc^2 + \mu + e\phi)^2}{m^2 c^4} - 1 \right]^{-2} \right\} \\ & - \left\{ \frac{(mc^2 + \mu)^2}{m^2 c^4} - 1 \right\}^{3/2} \left\{ 1 + \frac{\pi^2}{2} \left(\frac{KT}{mc^2} \right)^2 \right. \\ & \left. \left. \times \left[\frac{(mc^2 + \mu)^2}{m^2 c^4} - 1 \right]^{-2} \right\} \right] - 4\pi q \delta(\mathbf{r}). \quad (12) \end{aligned}$$

Let us consider further a weakly relativistic regime, i.e.,

$$\left| \frac{\mu + e\phi}{mc^2} \right| \ll 1.$$

In this limit, on linearizing in ϕ , Eq. (12) leads to

$$\nabla^2 \phi = \frac{3}{2} \lambda_{F_3}^{-2} \phi - 4\pi q \delta(\mathbf{r}), \quad (13)$$

where

$$\lambda_{F_3}^{-2} \equiv \frac{4\pi n_0 \left(1 + \frac{\sigma}{2}\right) \left(1 - \frac{\nu}{3}\right) e^2}{\mu}.$$

Equation (13) has the solution

$$\phi = \frac{q}{r} e^{-\sqrt{3/2}(r/\lambda_{F_3})}. \quad (14)$$

It may be noted that the linearized solution (14) [or (7) or (10), for that matter] is valid provided

$$\Gamma = \frac{e^2 a}{\mu} \ll 1, \quad (15)$$

where a is the interparticle distance

$$a = \left(\frac{4\pi n_0}{3} \right)^{1/3}.$$

In this limit, we have

$$\frac{\lambda_{F_3}^2}{a^2} = \frac{\mu}{4\pi n_0 (1 + \sigma/2) (1 - \nu/3) e^2} = \frac{1/3}{(1 + \sigma/2) (1 - \nu/3) \Gamma} \gg 1, \quad (16)$$

implying that the number of electrons in the Debye sphere is very large, which is very similar to the result for a classical plasma [2].

D. Effect of untrapped particles

Let us assume that the untrapped particles behave classically and are distributed according to

$$f(\mathbf{r}, \mathbf{v}) = \begin{cases} \left(\frac{KT}{2\pi m} \right)^{3/2} e^{-(mv^2/2 - e\phi)/KT}, & \frac{mv^2}{2} \geq |e\phi| \\ 0, & \frac{mv^2}{2} < |e\phi|. \end{cases} \quad (17)$$

Equation (1) then becomes

$$\begin{aligned} \nabla^2 \frac{e\phi}{KT} = & \lambda_1^{-2} \left[\left(1 + \frac{e\phi}{\mu} \right)^{3/2} - 1 \right] + \lambda_2^{-2} \left[e^{e\phi/KT} \right. \\ & \times \left\{ 1 - \operatorname{erf} \left(\sqrt{\frac{e\phi}{KT}} \right) \right\} + \frac{2}{\sqrt{\pi}} \sqrt{\frac{e\phi}{KT}} - 1 \left. \right] \\ & - 4\pi \frac{eq}{KT} \delta(\mathbf{r}), \end{aligned} \quad (18)$$

where

$$\begin{aligned} \lambda_1^{-2} & \equiv \frac{4\pi n_{01} e^2}{KT}, \quad n_{01} \equiv \frac{1}{3\pi^2} \left(\frac{2m\mu}{\hbar^2} \right)^{3/2}, \\ \lambda_2^{-2} & \equiv \frac{4\pi n_{02} e^2}{KT}, \end{aligned}$$

n_{02} being the number density of untrapped particles. On linearizing in ϕ , Eq. (18) leads to

$$\nabla^2 \phi = \lambda^{-2} \phi - 4\pi q \delta(\mathbf{r}), \quad (19)$$

where

$$\begin{aligned} \lambda^{-2} & \equiv \frac{3}{2} \lambda_F^{-2} + \lambda_2^{-2}, \quad \lambda_F^{-2} \equiv \frac{4\pi n_{01} e^2}{\mu}, \\ n_{01} & \equiv \frac{1}{3\pi^2} \left(\frac{2m\mu}{\hbar^2} \right)^{3/2}. \end{aligned}$$

Equation (19) has the solution

$$\phi = \frac{q}{r} e^{-r/\lambda}. \quad (20)$$

E. Effect of motion of a test charge

The dielectric screening process changes considerably if the test charge begins to move, because it takes time to polarize the plasma. For this problem, it is necessary to take the fully quantum-mechanical approach for the electron gas whereby one considers the collective modes in this gas to behave like quasiparticles called plasmons. These plasmons then interact with the electrons and with each other.

Suppose that for $t < 0$ there is a uniform electron gas described by an equilibrium distribution function $F_0(\mathbf{k})$ and that there is no electric field. At $t = 0$, a test charge q is introduced into the electron gas at $\mathbf{r} = \mathbf{0}$ and is allowed to move with uniform velocity $\mathbf{v}_0$.

The perturbed distribution function then satisfies the Heisenberg equation [7]

$$\begin{aligned} \frac{\partial}{\partial t} F(\mathbf{k}', \mathbf{k}, t) = & \frac{1}{i\hbar} (E_{\mathbf{k}'} - E_{\mathbf{k}}) F(\mathbf{k}', \mathbf{k}, t) \\ & + \frac{ie}{\hbar} \sum_{\mathbf{p}} \{ \hat{\phi}(\mathbf{p} - \mathbf{k}) F(\mathbf{k}', \mathbf{p}, t) \\ & + \hat{\phi}(\mathbf{k}' - \mathbf{p}) F(\mathbf{p}, \mathbf{k}, t) \}, \end{aligned} \quad (21)$$

with

$$\nabla^2 \phi = -4\pi e \sum_{\mathbf{k}} \sum_{\mathbf{p}} F(\mathbf{p}, \mathbf{p} + \mathbf{k}, t) \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{\Omega} - 4\pi q \delta(\mathbf{r} - \mathbf{v}_0 t), \quad (22)$$

where $\hat{\phi}$ is the Fourier transform

$$\hat{\phi}(\mathbf{k}) = \int \frac{d\mathbf{r}}{\Omega} e^{i\mathbf{k} \cdot \mathbf{r}} \phi(\mathbf{r}),$$

Ω being the volume of the system and

$$E_{\mathbf{p}} \equiv \frac{\hbar^2 \mathbf{p}^2}{2m}.$$

We are suppressing the argument t for ϕ .

If we now assume that the electron gas is only slightly perturbed by the test charge, we may linearize about the equilibrium state

$$F(\mathbf{k}', \mathbf{k}, t) = F_0(\mathbf{k}) \delta_{\mathbf{k}, \mathbf{k}'} + F_1(\mathbf{k}', \mathbf{k}, t), \quad (23)$$

where we have on Laplace transforming Eq. (21),

$$F_1(\mathbf{k}', \mathbf{k}) = \frac{ie}{\hbar} \tilde{\phi}(\mathbf{k} - \mathbf{k}') \frac{F_0(\mathbf{k}') - F_0(\mathbf{k})}{s + i\omega(\mathbf{k}', \mathbf{k})}, \quad (24)$$

where

$$\tilde{\phi}(\mathbf{k}) = \int_0^\infty e^{-st} \hat{\phi}(\mathbf{k}) dt$$

and

$$\omega(\mathbf{k}', \mathbf{k}) = \frac{E_{\mathbf{k}'} - E_{\mathbf{k}}}{\hbar}.$$

Using (24), Eq. (22) becomes

$$\begin{aligned} \nabla^2 \phi = & - \sum_{\mathbf{k}} \sum_{\mathbf{p}} \frac{i4\pi e^2}{\hbar \Omega} \tilde{\phi}(-\mathbf{k}) \frac{F_0(\mathbf{p}) - F_0(\mathbf{p} + \mathbf{k})}{s + i\omega(\mathbf{p}, \mathbf{p} + \mathbf{k})} e^{i\mathbf{k} \cdot \mathbf{r}} \\ & - 4\pi q \delta(\mathbf{r} - \mathbf{v}_0 t). \end{aligned} \quad (25)$$

Upon Fourier transforming and taking the electron gas to be completely degenerate, i.e.,

$$F_0(\mathbf{p}) = \begin{cases} 1, & p < p_F \\ 0, & p > p_F \end{cases}, \quad (26)$$

we obtain, from Eq. (25),

$$\tilde{\phi}(\mathbf{k}) = \frac{4\pi q/k^2}{(s + i\mathbf{k} \cdot \mathbf{v}_0) \varepsilon(\mathbf{k}, s)}, \quad (27)$$

FIG. 1. Coordinate system specifying the orientation of $\mathbf{r}$ and $\mathbf{v}_0$.

where

$$\varepsilon(\mathbf{k}, s) \equiv 1 + \frac{4k_F}{\pi k^2 a_0} \left[\frac{1}{2} - g(x_+) - g(x_-) \right],$$

$$g(x) \equiv \frac{m^2 \left(x^2 - \frac{\hbar^2 k^2 k_F^2}{m^2} \right)}{4\hbar^2 k^3 k_F} \ln \left(\frac{x + \frac{\hbar k k_F}{m}}{x - \frac{\hbar k k_F}{m}} \right),$$

$$x_{\pm} = is \pm \frac{\hbar k^2}{2m}, \quad a_0 \equiv \frac{\hbar^2}{me^2}.$$

Inverting the Fourier-Laplace transform and taking the limit $t \Rightarrow \infty$, we have, from (27),

$$\phi(\mathbf{r}, t) = \frac{4\pi q}{(2\pi)^3} \int \frac{e^{-i\mathbf{k} \cdot (\mathbf{v}_0 t - \mathbf{r})}}{k^2 \varepsilon(\mathbf{k}, -i\mathbf{k} \cdot \mathbf{v}_0)} d\mathbf{k}, \quad t \Rightarrow \infty. \quad (28)$$

Let us consider the case

$$\left| \frac{\mathbf{k} \cdot \mathbf{v}_0 \pm \frac{\hbar k^2}{2m}}{\frac{\hbar k k_F}{m}} \right| \ll 1, \quad (29)$$

which leads to the elimination of Friedel oscillations [8]. In this limit, we obtain

$$\varepsilon(\mathbf{k}, -i\mathbf{k} \cdot \mathbf{v}_0) \approx 1 + \frac{4k_F}{\pi k^2 a_0} \left[1 - \frac{(\hat{\mathbf{k}} \cdot \mathbf{v}_0)}{\hbar k_F/m} \right]$$

$$= 1 + \frac{\frac{3}{2}}{k^2 \lambda_F^2} \left[1 - \frac{(\hat{\mathbf{k}} \cdot \mathbf{v}_0)}{\hbar k_F/m} \right], \quad (30)$$

where $\hat{\mathbf{k}} \equiv \mathbf{k}/k$. Using (30) and orienting the coordinate system such that $\tilde{\mathbf{r}} \equiv \mathbf{r} - \mathbf{v}_0 t$ lies along the z axis and $\mathbf{v}_0$ lies in the yz -plane (see Fig. 1), (28) leads to

$$\phi(\mathbf{r}, t) = \frac{q}{2\pi^2} \int_0^{2\pi} d\varphi \int_{-1}^1 d\mu \int_0^\infty dk \frac{e^{ik\tilde{r}\mu}}{1 + \frac{\frac{3}{2}}{k^2 \lambda_F^2}}$$

$$+ \frac{q}{2\pi^2} \left(\frac{v_0}{\hbar k_F/m} \right) \frac{2}{3} \lambda_F^2$$

$$\times \int_0^\infty d\varphi \int_{-1}^1 d\mu \int_0^\infty dk \frac{k^2 e^{ik\tilde{r}\mu}}{1 + \frac{2}{3} k^2 \lambda_F^2}$$

$$\times (\mu \cos \gamma + \sqrt{1 - \mu^2} \sin \gamma \sin \varphi),$$

$$t \Rightarrow \infty, \quad (31)$$

where

$$\mu \equiv \cos \theta, \quad \tilde{r} = |\mathbf{r} - \mathbf{v}_0 t|.$$

Expanding the integrand in the second term on the right-hand side in (31) about $k=0$, we obtain

$$\phi(\mathbf{r}, t) = \frac{q}{|\mathbf{r} - \mathbf{v}_0 t|} e^{-\sqrt{3/2}(|\mathbf{r} - \mathbf{v}_0 t|/\lambda_F)} + \frac{1}{|\mathbf{r} - \mathbf{v}_0 t|^3}$$

$$\times \left[\frac{q}{\pi} \left(\frac{v_0}{\hbar k_F/m} \right) \frac{2}{3} \lambda_F^2 \cos \gamma \int_{-1}^1 d\mu \int_0^\infty d\eta \right.$$

$$\left. \times e^{i\eta\mu} \eta^2 \mu \right] + \dots, \quad t \Rightarrow \infty, \quad (32)$$

where

$$\eta = k\tilde{r}.$$

According to (32), we have, for a slowly moving test charge, the Debye potential for small distances from the test charge and an inverse third power of distance potential for very large distances. These results are very similar to the corresponding results for a classical plasma [9,10].

III. EFFECT OF A VERY STRONG MAGNETIC FIELD

A. Cold electron gas

For an atom in a very strong magnetic field, the magnetic confinement of the electrons perpendicular to the magnetic field will dominate the Coulomb interaction with the nucleus. Thus the electrons remain in their ground state and tend to move in their cylindrical shells with the axes directed along the magnetic field and precess around the nucleus. The oscillatory motion of the electrons along the magnetic field is confined by the Coulomb field.

Using the number density expression given by Kadomtsev [11], Eq. (1) becomes

$$\nabla^2 \phi = 4\pi e \left\{ \frac{eB\sqrt{2m}}{2\pi^2 \hbar^2 c} [(\mu + e\phi)^{1/2} - \mu^{1/2}] \right\} - 4\pi q \delta(\mathbf{r}), \quad (33)$$

where $\mathbf{B}$ is the magnetic field. Linearizing in ϕ , Eq. (33) leads to

$$\nabla^2 \phi = \lambda_{F_4}^{-2} \phi - 4\pi q \delta(\mathbf{r}), \quad (34)$$

where

$$\lambda_{F_4}^{-2} \equiv \frac{4\pi\epsilon\tilde{n}_0 e^2}{\mu}, \quad \tilde{n}_0 \equiv \frac{1}{4\pi^2} \sqrt{\frac{2m\mu}{\hbar^2}},$$

$$\epsilon \equiv \left(\frac{\hbar e B}{m^2 c^3} \right) \frac{m c^2}{e^2/a_0} \gg 1.$$

Now, if we assume that the cylindrical shells in which the electrons move are distributed uniformly in a large sphere, the ground state of the atom may be taken to be spherically symmetric. Equation (34) then has the solution

$$\phi = \frac{q}{r} e^{-r/\lambda_{F_4}}. \quad (35)$$

Observe that the screening region is very small since λ_{F_4} is very small. Actually, λ_{F_4} is so small that

$$\frac{\lambda_{F_4}}{a} = \frac{1}{3\epsilon\Gamma} = O(1) \quad (36)$$

even though the coupling parameter $\Gamma \ll 1$ [see (15)], since $\epsilon \gg 1$. Thus, in the strongly magnetized regime, the Debye sphere would contain only very few particles, despite the weakly coupled nature of the regime.

B. Thermal effects

We assume nondegeneracy in the electron behavior only with respect to the slow longitudinal motions. Using the number density expression given by Shivamoggi and Schram [12], Eq. (1) becomes

$$\nabla^2 \phi = 4\pi e \left[\frac{eB}{2\pi^2 \hbar^2 c} \left\{ \int_0^\infty \frac{dp}{e^{[p^2/2m - (\mu + e\phi)](1/KT)} + 1} - \int_0^\infty \frac{dp}{e^{[p^2/2m - \mu](1/KT)} + 1} \right\} \right] - 4\pi q \delta(\mathbf{r}) \quad (37)$$

or

$$\nabla^2 \phi = \frac{e^2 B}{\pi^2 \hbar^2 c} (2mKT)^{1/2} \left[I_{-1/2} \left(\frac{\mu + e\phi}{KT} \right) - I_{-1/2} \left(\frac{\mu}{KT} \right) \right] - 4\pi q \delta(\mathbf{r}), \quad (38)$$

where $I_p(\eta)$ is defined in (4). Considering a weakly nondegenerate regime and noting the asymptotic expansion

$$I_{-1/2}(\eta) \approx 2\eta^{1/2} \left(1 - \frac{\pi^2}{24} \frac{1}{\eta^2} + \dots \right) \quad \text{for } \eta \text{ large}, \quad (39)$$

Eq. (38) becomes, on linearizing further in ϕ ,

$$\nabla^2 \phi = \lambda_{F_5}^{-2} \phi - 4\pi q \delta(\mathbf{r}), \quad (40)$$

where

$$\lambda_{F_5}^{-2} \equiv \frac{4\pi\epsilon(1+\nu)\tilde{n}_0 e^2}{\mu}.$$

Equation (40) has the solution

$$\phi = \frac{q}{r} e^{-r/\lambda_{F_5}}. \quad (41)$$

Observe that, in the strongly magnetized regime, the thermal effects *decrease* the screening length, contrary to the classical unmagnetized case. This result, though apparently anomalous, is purely a geometrical effect and can be traced to the presence of nondegeneracy only in the longitudinal direction (see the Appendix).

C. Relativistic effects

Using the number density expression given by Shivamoggi and Mulser [13], Eq. (1) becomes

$$\nabla^2 \phi = 4\pi e \left\{ \frac{meB}{2\pi^2 \hbar^2} \left[\left(\frac{(mc^2 + \mu + e\phi)^2}{m^2 c^4} - 1 \right)^{1/2} - \left(\frac{(mc^2 + \mu)^2}{m^2 c^4} - 1 \right)^{1/2} \right] \right\} - 4\pi q \delta(\mathbf{r}). \quad (42)$$

Considering a weakly relativistic regime and linearizing further in ϕ , Eq. (42) leads to

$$\nabla^2 \phi = \lambda_{F_6}^{-2} \phi - 4\pi q \delta(\mathbf{r}), \quad (43)$$

where

$$\lambda_{F_6}^{-2} \equiv \frac{4\pi\epsilon(1 + \frac{3}{4}\sigma)\tilde{n}_0 e^2}{\mu}.$$

Equation (43) has the solution

$$\phi = \frac{q}{r} e^{-r/\lambda_{F_6}}. \quad (44)$$

Observe that the relativistic effects reduce the screening length, as with the classical unmagnetized case.

D. Combined relativistic and thermal effects

Using the number density expression given by Shivamoggi [5], Eq. (1) becomes

$$\nabla^2 \phi = 4\pi e \left[\frac{eB}{2\pi^2 \hbar^2 c} \times \left\{ \int_0^\infty \frac{dp}{e^{[\sqrt{p^2 c^2 + m^2 c^4} - (mc^2 + \mu + e\phi)](1/KT)} + 1} - \int_0^\infty \frac{dp}{e^{[\sqrt{p^2 c^2 + m^2 c^4} - (mc^2 + \mu)](1/KT)} + 1} \right\} \right] - 4\pi q \delta(\mathbf{r}). \quad (45)$$

Considering a weakly nondegenerate regime and using the asymptotic evaluations of Shivamoggi [5], Eq. (45) becomes

FIG. 2. Thermal effects in the Fermi-Dirac distribution.

$$\begin{aligned} \nabla^2 \phi = 4\pi e \left(\frac{meB}{2\pi^2 \hbar^2} \left\{ \left[\frac{(mc^2 + \mu + e\phi)^2}{m^2 c^4} - 1 \right]^{1/2} \right. \right. \\ \times \left[1 - \frac{\pi^2}{6} \left(\frac{KT}{mc^2} \right)^2 \left(\frac{(mc^2 + \mu + e\phi)^2}{m^2 c^4} - 1 \right)^{-2} \right] \\ \left. \left. - \left(\frac{(mc^2 + \mu)^2}{m^2 c^4} - 1 \right)^{1/2} \left[1 - \frac{\pi^2}{6} \left(\frac{KT}{mc^2} \right)^2 \right. \right. \right. \right. \\ \left. \left. \times \left(\frac{(mc^2 + \mu)^2}{m^2 c^4} - 1 \right)^{-2} \right] \right\} \right) - 4\pi q \delta(\mathbf{r}). \quad (46) \end{aligned}$$

Considering further a weakly relativistic regime and linearizing in ϕ , Eq. (46) leads to

$$\nabla^2 \phi = \lambda_{F_7}^{-2} \phi - 4\pi q \delta(\mathbf{r}), \quad (47)$$

where

$$\lambda_{F_7}^{-2} = \frac{4\pi e(1 + \frac{3}{4}\sigma)(1 + \nu)\bar{n}_0 e^2}{\mu}.$$

Equation (47) has the solution

$$\phi = \frac{q}{r} e^{-r/\lambda_{F_7}}. \quad (48)$$

IV. CONCLUSION

In this paper, we have investigated the dielectric screening process in the Thomas-Fermi model with several variations such as the thermal effects, relativistic effects, motion of the test charge, and effect of a very strong magnetic field. We have found that the screening process for even a moving test charge in the Thomas-Fermi model is very similar to that in a classical plasma. The screening process in the strongly magnetized case exhibits some apparently anomalous features that, however, on careful examination can be understood completely.

APPENDIX

In order to see the decrease of the screening length due to thermal effects in the strongly magnetized regime, consider the number density of the particles obeying the Fermi-Dirac statistics in the one-dimensional and three-dimensional cases in the momentum space

$$n = \begin{cases} \int_0^\infty \frac{dp}{e^{[p^2/2m - (e\phi + \mu)]/(KT)} + 1} & \text{for the one-dimensional case} \\ \int_0^\infty \frac{4\pi p^2 dp}{e^{[p^2/2m - (e\phi + \mu)]/(KT)} + 1} & \text{for the three-dimensional case} \end{cases} \quad (A1a)$$

or

$$n = \begin{cases} (2mKT)^{1/2} I_{-1/2} \left(\frac{e\phi + \mu}{KT} \right) & \text{for the one-dimensional case} \\ 4\pi (2mKT)^{3/2} I_{1/2} \left(\frac{e\phi + \mu}{KT} \right) & \text{for the three-dimensional case} \end{cases} \quad (A1b)$$

where $I_p(\eta)$ is defined in (4). In the weakly nondegenerate regime, i.e.,

$$\left| \frac{e\phi + \mu}{KT} \right| \gg 1,$$

(A1) becomes

$$n = \begin{cases} 2(2mKT)^{1/2} \left(\frac{e\phi + \mu}{KT} \right)^{1/2} \left[1 - \frac{\pi^2}{24} \left(\frac{KT}{e\phi + \mu} \right)^2 + \dots \right] & \text{for the one-dimensional case} \\ \frac{8\pi}{3} (2mKT)^{3/2} \left(\frac{e\phi + \mu}{KT} \right)^{3/2} \left[1 + \frac{\pi^2}{8} \left(\frac{KT}{e\phi + \mu} \right)^2 + \dots \right] & \text{for the three-dimensional case,} \end{cases} \quad (A2)$$

which describes the decrease (increase) of the screening length due to thermal effects in the one-dimensional (three-dimensional) case in the momentum space.

In order to understand the nature of thermal corrections in the one- and three-dimensional cases in the momentum space, one may look at the Fermi-Dirac distribution function

$$f = \frac{1}{e^{(\varepsilon - \mu)/KT} + 1}, \quad (A3)$$

as sketched in Fig. 2, and note that the contribution from region A outweighs that from region B for the one-dimensional case and vice versa for the three-dimensional case because, in terms of the total energy ε , (A1) can be rewritten as

$$n = \begin{cases} \sqrt{\frac{m}{2}} \int_0^\infty \frac{\varepsilon^{-1/2} d\varepsilon}{e^{(\varepsilon-\mu)/KT} + 1} & \text{for the one-dimensional case} \\ 4\pi\sqrt{2m^3} \int_0^\infty \frac{\varepsilon^{1/2} d\varepsilon}{e^{(\varepsilon-\mu)/KT} + 1} & \text{for the three-dimensional case.} \end{cases} \quad (\text{A4})$$

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