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The capacitance associated with free-carrier charge storage in the quasineutral region is a primary factor in limiting the switching speed of p/n junction devices. This capacitance has been conventionally modeled using assumptions such as low-level injection, nondegeneracy, complete impurity ionization, and no space-charge region thickness modulation. These assumptions can give rise to a large error in device modeling, particularly for modern devices with very small geometry and high bias conditions. In this article, a comprehensive quasineutral region capacitance model including relevant device physics is developed. Comparisons between the present and conventional models are made, and the effects of using these two different models on the total capacitance of junction diode are also investigated.
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I. INTRODUCTION

The switching speed of a p/n junction device (i.e., diode, bipolar transistor, etc.) is often limited by the time required to supply or remove the minority-carrier charge stored in the quasineutral region. In device modeling and circuit simulation, such charge storage is represented by the quasineutral region capacitance (QNR) C_{QNR}.[1,2] Conventionally, C_{QNR} is modeled using the following four assumptions: (1) low-level injection, (2) no space-charge region thickness modulation, (3) 100% impurity ionization (all impurity dopants are completely ionized), and (4) nondegenerate material. Details of these assumptions will be discussed later. Some studies have been reported in the literature concerning an individual assumption,[3–6] but a comprehensive C_{QNR} model derived without including all these assumptions is not yet available.

This article develops a comprehensive C_{QNR} model including relevant device physics. Issues such as different doping concentrations and applied voltages are addressed. Detailed comparisons between the present and conventional models will be made, and the effects of using these two different models on the total capacitance of junction diode are also investigated.

II. MODEL DEVELOPMENT

In the following, we will focus on the quasineutral base (QNB) region of an n/p junction diode at room temperature, but the approach applies generally to other quasineutral regions of p/n junction devices. The quasineutral base region capacitance C_{QNB} is defined by the change of the minority-carrier charge Q_{QNB} in the p-type QNB with respect to the change of the applied voltage V_A:

$$C_{\mathrm{QNB}} = dQ_{\mathrm{QNB}}/dV_A, \tag{1}$$

where Q_{QNB} is given by

$$Q_{\mathrm{QNB}} = \int_{X_p}^{X_B} (n_0 + \Delta n)\,dx. \tag{2}$$

Here X_B is the position of base metal contact, X_p is the space-charge region (SCR) edge in the base, n_0 is the equilibrium electron concentration and is related to the equilibrium hole concentration p_0 as $n_0 = n_i^2/p_0$ (n_i is the intrinsic free carrier concentration), and Δn is the excess electron concentration in the QNB. In the conventional approach, X_p is assumed constant when taking the derivative with respect to V_A (i.e., no SCR thickness modulation), p_0 is assumed equal to the doping concentration N_A in the base (i.e., 100% impurity ionization), Δn is much smaller than p_0 in the base (i.e., low-level injection), and the base is not heavily doped (i.e., nondegenerate material). Using these assumptions, the conventional QNB capacitance model $C_{\mathrm{QNB,con}}$ is derived as

$$C_{\mathrm{QNB,con}} = (X_B - X_p)\Delta n(X_p)(q/2V_T), \tag{3}$$

where $V_T = kT/q$ (k is the Boltzmann constant and T is the absolute temperature) is the thermal voltage.

None of the above assumptions will be used in the present model development given below.

A. All-level injection

The ambipolar transport equation (ATE) is a useful tool to investigate the moderate- and high-level injection problems in semiconductor devices.[7] It is derived from the electron and hole continuity equations and uses the condition that charge neutrality exists in the quasineutral region even under the nonequilibrium condition.[8] For a constant base doping concentration, the ATE can be expressed as

$$D_a \frac{d^2\Delta n}{dx^2} + \mu_a \xi(x) \frac{d\Delta n}{dx} + G_n - R_n = 0, \tag{4}$$

where G_n and R_n are the electron generation and recombination rates, respectively, ξ is the electric field in the QNB originating from moderate- and high-level injection,

$$D_a = \frac{D_n D_p (p+n)}{nD_n + pD_p} \simeq \frac{D_n D_p [p_0 + 2\Delta n(x)]}{p_0 D_p + (D_n + D_p)\Delta n(x)} \tag{5}$$

is the ambipolar diffusion coefficient, and

$$\mu_a = \frac{\mu_n \mu_p (p-n)}{p\mu_p + n\mu_n} \simeq \frac{\mu_n \mu_p p_0}{\mu_p p_0 + (\mu_n + \mu_p)\Delta n(x)} \tag{6}$$

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is the ambipolar mobility. In Eqs. (4)–(6), D_n and D_p are the electron and hole diffusion coefficients, μ_n and μ_p are the electron and hole mobilities, and n and p are the nonequilibrium electron and hole concentrations, respectively. Note that ξ reduces to zero, and D_a and μ_a reduce to D_n and μ_n, respectively, if low injection ($p_0 \gg \Delta n$) prevails in the QNB. Also note that $p_0 = N_A$ if 100% ionization is assumed. The electric field in (4) is a retarding field which hinders the electron transport in the QNB and is a function of the total current density J passing through the diode.[3] The simplest approach, which is used conventionally but not in the present study, is to assume $J=0$. For the case of a bipolar junction transistor (BJT), ξ is a function of the majority current density J_p passing through the QNB, and an empirical J_p model was developed in Ref. 9 to analyze the BJT under high-level injection.

Two iterative procedures are required to solve the ATE.[3] The first one is used to calculate $n(x)$ in the QNB based on an assumed ξ, and the second is needed to calculate the correct ξ after $n(x)$ is solved. The correct $n(x)$ in the QNB can be obtained after several iterations, which can then be used in Eqs. (1) and (2) to calculate the QNB capacitance for all levels of free-carrier injection.

Qualitatively, the inclusion of a high-level mechanism will not affect C_{QNB} at low voltages but will decrease C_{QNB} at high voltages. This is due to the fact that the number of minority carriers in the QNB at high voltages is reduced by the high-level injection effect.[2]

B. Incomplete ionization

A model for the equilibrium majority carrier density p_0 in the QNB is needed. It is assumed conventionally that the impurity atoms are completely ionized (i.e., 100% ionization, or $p_0 = N_A$) at room temperature based on the hypothesis that the thermal energy is sufficiently large to break the hole from the p-type impurity nucleus. This assumption is valid only if the doping concentration is relatively low (i.e., less than 10^{17} cm^{-3}) and it becomes questionable as the doping concentration is increased.[4] For example, at room temperature, the impurity ionization percentages (i.e., p_0/N_A) are about 80% and 40% for $N_A = 10^{18}$ and 10^{19} cm^{-3}, respectively.[4] The incomplete ionization model in the p-type QNB can be derived from the condition of charge neutrality:

$$\rho = 0 = q(p_0 - n_0 - N_A^-), \tag{7}$$

where ρ is the space charge density and N_A^- is the ionized acceptor impurity density. Based on Boltzmann statistics (i.e., for nondegenerate material), the equilibrium electron and hole concentrations in (7) can be expressed in terms of the Fermi energy E_F:

$$n_0 = N_C \exp[(E_F - E_C)/kT], \tag{8}$$

$$p_0 = N_V \exp[(E_V - E_F)/kT], \tag{9}$$

$$N_A^- = N_A / \{1 + g_A \exp[(E_A - E_F)/kT]\}. \tag{10}$$

Here E_C and E_V are the conduction and valence band edges, N_C and N_V are the effective densities of states for electrons and holes, k is the Boltzmann constant, T is the absolute temperature, E_A is the acceptor energy, and g_A is the degeneracy factor for E_A. Combining the above equations, one can numerically solve E_F, and thus p_0 and n_0, for any particular N_A.

As will be shown later, taking into account incomplete ionization will increase C_{QNB} for a wide range of V_A. This is because the majority carrier density is reduced by the incomplete ionization, which then results in an increase in the minority carrier storage in the QNB.

C. Degenerate material

For a semiconductor with a relatively high doping concentration (i.e., higher than 10^{17} cm^{-3}), the Pauli exclusion principle applies, and the nondegeneracy assumption used conventionally becomes invalid. When the degeneracy is considered, the Boltzmann statistics need to be replaced by the Fermi statistics. In particular, Eqs. (8) and (9) need to be changed to

$$n_0 = N_C F_{1/2}[(E_F - E_C)/kT], \tag{11}$$

$$p_0 = N_V F_{1/2}[(E_V - E_F)/kT], \tag{12}$$

where $F_{1/2}$ is the Fermi–Dirac integral of the order of 1/2. The intrinsic free-carrier concentration n_i is also affected by the degeneracy because $n_i = (n_0 p_0)^{0.5}$.

The degeneracy mechanism decreases the minority carrier density in the QNB and consequently decreases C_{QNB}.

D. SCR thickness modulation

Based on the assumption that the electrons and holes are depleted in the SCR, the bias-dependent SCR edge X_p is given by[2]

$$X_p = \{2 n_0 \epsilon (V_{\mathrm{bi}} - V_A) / [q p_0 (n_0 + p_0)]\}^{0.5}, \tag{13}$$

where ϵ is the dielectric permittivity and $V_{\mathrm{bi}} = V_T \ln(n_0 p_0 / n_i^2)$ is the junction built-in potential. Note that n_0 and p_0, rather than N_D (n-type doping concentration in emitter) and N_A, were used in the above equation to include the effects of incomplete ionization. It can be seen in (13) that X_p depends weakly on V_A (i.e., $X_p \propto V_A^{0.5}$). As a result, in modeling the quasineutral base capacitance, X_p is conventionally treated as a constant when taking a derivative with respect to V_A (i.e., no space-charge region thickness modulation). Using this approach, the derivation of dQ_{QNB}/dV_A becomes less complicated, and a simple expression shown in (3) can be obtained. Such an assumption will not be used in the present study, and the bias dependency of X_p will be accounted for in the capacitance model.

It is apparent from the device physics that C_{QNB} will be increased if the effect of SCR thickness modulation is incorporated into the model, because the modulation decreases the SCR thickness and consequently increases the QNB region.

E. Comprehensive C_{QNB} model

The procedure for calculating the QNB capacitance from the present comprehensive C_{QNB} model is as follows. First, the equilibrium electron and hole concentrations (n_0 and p_0) in the QNB are calculated including the effects of Fermi

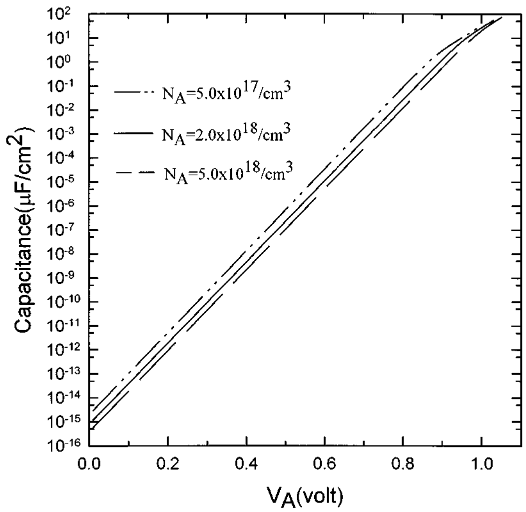

FIG. 1. Quasineutral base capacitance of an n/p junction diode calculated from the present model for three different base doping concentrations.

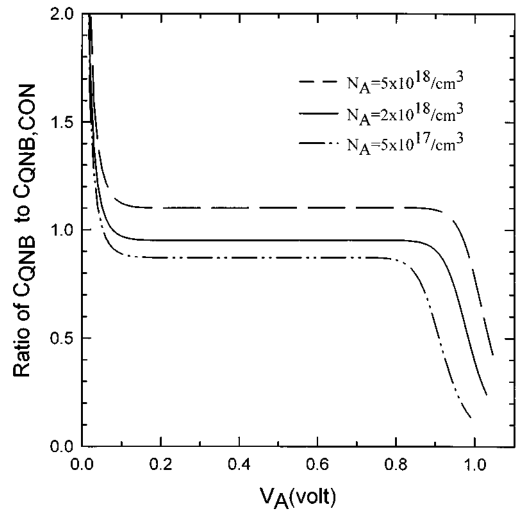

FIG. 2. Ratio of the present comprehensive model (C_{QNB}) to the conventional model ($C_{\mathrm{QNB,con}}$) calculated for three different base doping concentrations.

statistics and incomplete ionization discussed in Secs. II B and II C. These values are then put into the ATE equation to solve for the minority-carrier distribution $n(x)$ in the QNB under all levels of free-carrier injection. This is followed by the integration of $n(x)$ from X_p to X_B, which yields Q_{QNB}. Finally, C_{QNB} can be obtained from taking a derivative of Q_{QNB} with respect to V_A, which is done numerically using $[Q_{\mathrm{QNB}}(V_A^2)-Q_{\mathrm{QNB}}(V_A^1)]/(V_A^2-V_A^1)$, where V_A^2 and V_A^1 are two voltages with a very small interval.

III. RESULTS AND DISCUSSIONS

We will consider an n/p junction diode with a typical makeup of 5×10^{19} cm^{-3} emitter doping concentration, 0.15 μm emitter layer thickness, 0.1 μm base layer thickness, and three different base doping concentrations varying from 5×10^{17} to 5×10^{18} cm^{-3}. Figure 1 shows C_{QNB} vs V_A characteristics calculated from the present model. The capacitance is reduced as N_A is increased. This is because a smaller number of the minority free carriers is injected into the QNB as N_A is increased. Also, in the high bias conditions, C_{QNB} tends to level off at a smaller voltage as N_A is decreased. This is because the level-off phenomenon is caused by the effect of high-level injection included in the present model, and high-level injection is more prominent if the doping concentration is decreased.

Figure 2 compares the present comprehensive model C_{QNB} and the conventional model $C_{\mathrm{QNB,con}}$. The results indicate that C_{QNB} can be larger or smaller than $C_{\mathrm{QNB,con}}$ depending on N_A and V_A. Clearly, the neglect of high-level injection (i.e., $V_A>0.8$ V) in the conventional model considerably overestimates the capacitance (i.e., $C_{\mathrm{QNB}}/C_{\mathrm{QNB,con}}<1$). Also, the conventional model can underestimate and overestimate the capacitance when the doping concentrations are relatively high and low, respectively. Overall, the disagreement between the present and conventional models is relatively small, except for large voltages where high-level injection prevails. Note that very large ratios are found near the zero bias voltage. This is due to the fact that, at zero voltage, the present model yields a very small QNB capacitance, whereas the conventional model yields a zero capacitance. Such a nonzero capacitance, however small, divided by zero gives rise to the very large ratios shown in Fig. 2. At $V_A=0$, $\Delta n=0$, and $Q_{\mathrm{QNB}}=n_0(X_B-X_p)$ [see Eq. (2)]. Thus, $C_{\mathrm{QNB}}=dQ_{\mathrm{QNB}}/dV_A$ is zero only if X_p is independent of V_A, an assumption used in the conventional model but not in the present model. The effects of individual physical mechanism on the capacitance are examined in the following figures.

Figure 3 shows the ratios of C_{QNB} to $C_{\mathrm{QNB,lo}}$, where $C_{\mathrm{QNB,lo}}$ is the same as the comprehensive C_{QNB} model except that the effect of high-level injection is not included. It can be seen that the capacitance is reduced when high-level injection is considered, and the effect of high-level injection on C_{QNB} is more prominent as N_A is reduced. The reduced capacitance at high-level injection is caused by the fact that a

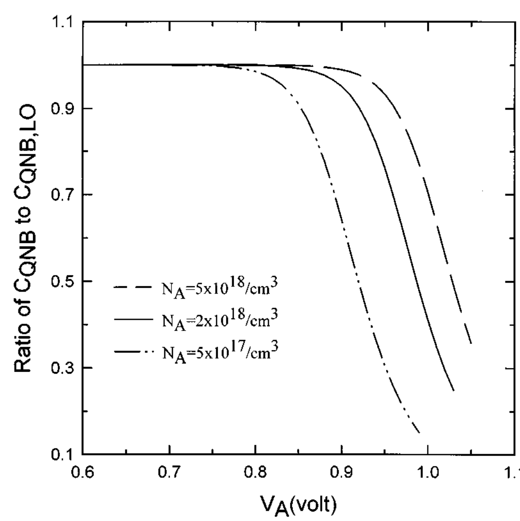

FIG. 3. Ratio of the present model (C_{QNB}) to the model ($C_{\mathrm{QNB,lo}}$) without including the effect of high-level injection.

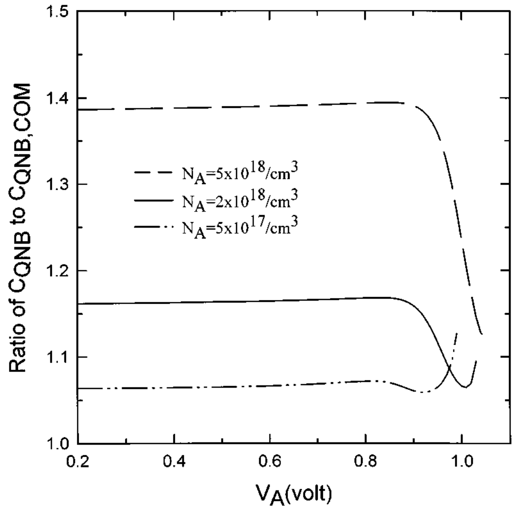

FIG. 4. Ratio of the present model (C_{QNB}) to the model ($C_{\mathrm{QNB,com}}$) without including the effect of incomplete impurity ionization.

retarding electric field is created by high-level injection (see Sec. II A), which then hinders the free-carrier transport in the QNB and consequently reduces the free-carrier charge storage in the QNB.

Figure 4 shows the ratios of C_{QNB} to $C_{\mathrm{QNB,com}}$, where $C_{\mathrm{QNB,com}}$ is the same as the C_{QNB} model except that the effect of incomplete ionization is not included. At small and medium voltages, the importance of incomplete ionization of the capacitance increases with increasing doping concentration, and the trend is reversed when V_A approaches the junction built-in potential V_{bi}. The latter arises because V_{bi}, which is proportional to $n_0 p_0$, is reduced if incomplete ionization is taken into account.

Figure 5 shows the ratios of C_{QNB} to $C_{\mathrm{QNB,non}}$, where $C_{\mathrm{QNB,non}}$ is the same as the C_{QNB} model except that the effect of degeneracy is not included. The results indicate that the effect of degeneracy reduces the capacitance and is less significant at high voltages.

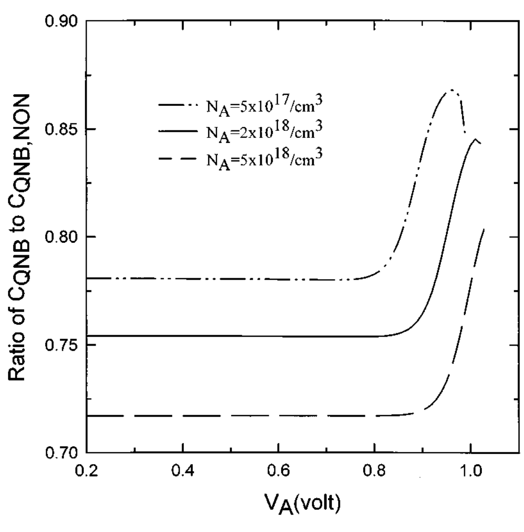

FIG. 5. Ratio of the present model (C_{QNB}) to the model ($C_{\mathrm{QNB,non}}$) without including the effect of degeneracy.

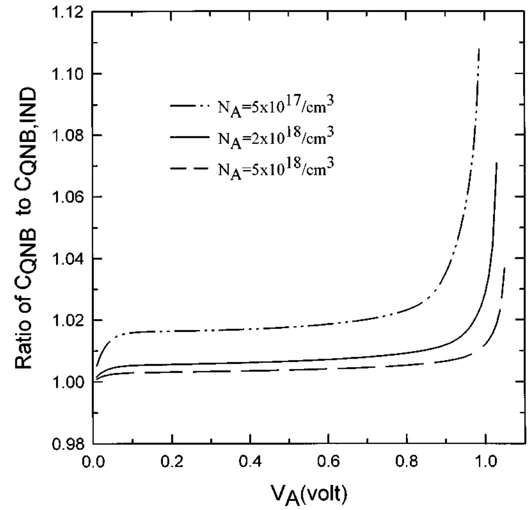

FIG. 6. Ratio of the present model (C_{QNB}) to the model ($C_{\mathrm{QNB,ind}}$) without including the effect of space-charge region thickness modulation.

Figure 6 shows the ratios of C_{QNB} to $C_{\mathrm{QNB,ind}}$, where $C_{\mathrm{QNB,ind}}$ is the same as the C_{QNB} model except that the effect of SCR edge modulation is not included. Such an effect is more important if the doping concentration is decreased due to the fact that a lower base doping concentration increases the SCR thickness in the base and consequently increases the SCR edge modulation effect. In addition, such an effect is shown to have a very small influence on the accuracy of C_{QNB} except for the case where V_A approaches the junction built-in potential.

The preceding results have pointed out clearly that incomplete ionization and degeneracy have opposite effects on C_{QNB} and thus tend to compensate each other when both effects are included in the model. Thus, it can be suggested that, for more accurate C_{QNB} modeling, one should either include or neglect both incomplete ionization and degeneracy effects. On the other hand, the effect of SCR edge modulation is not so critical in C_{QNB} modeling as long as the base doping concentration is relatively high.

Figure 7 plots all three capacitances in a typical n/p junction diode, including the quasineutral emitter capacitance C_{QNE}, space-charge region capacitance C_{SCR}, and the total capacitance C_{tot}. While C_{QNB} and C_{QNE} are calculated from the present comprehensive model, C_{SCR} is obtained using the conventional depletion capacitance model.[2] It can be seen that C_{QNE} and C_{QNB} become the dominant components when V_A is increased beyond 0.8 V.

Finally, the effects of using the conventional and present models on C_{tot} are investigated. Figure 8 compares the total capacitances of the n/p junction diode obtained from the present and conventional models. Clearly, the discrepancy of C_{tot} calculated using the two models is more noticeable at high voltages (i.e., more than 0.8 V) where the quasineutral region capacitance is the dominant component for the total capacitance. At lower voltages, the slightly larger capacitance obtained from the conventional model stems from the use of incomplete ionization assumption in the SCR depletion capacitance model. The difference of C_{tot} obtained from

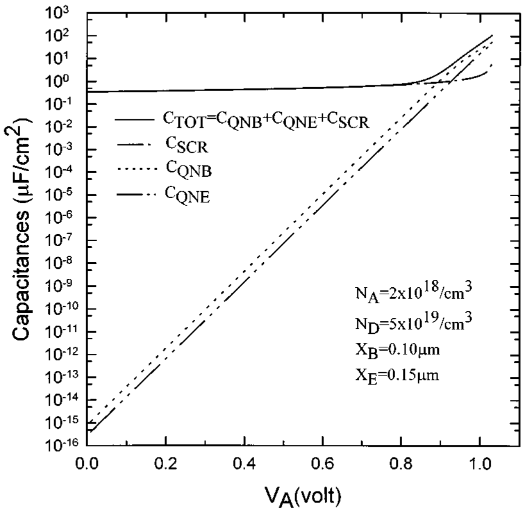

FIG. 7. The calculated total and individual capacitances in the n/p junction diode, where N_A and N_D denote the base and emitter doping concentrations, and X_B and X_E denote the base and emitter layer thicknesses, respectively.

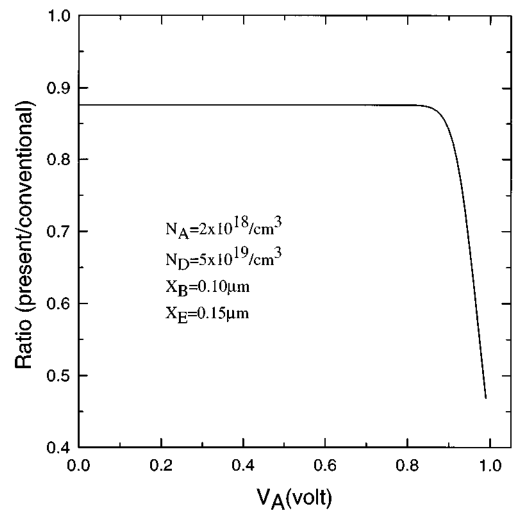

FIG. 9. Ratio of total capacitance calculated from the present model to those calculated from the conventional model.

the two different models is more clearly illustrated in Fig. 9, which plots the ratio of C_{tot} calculated from the present model to those calculated from the conventional model.

IV. CONCLUSIONS

The quasineutral region capacitance is a main factor in limiting the switching speed of p/n junction semiconductor devices. A comprehensive model for such a capacitance including all relevant device physics was presented. Specifically, the physics included were all-level free-carrier injection, incomplete impurity ionization, degeneracy, and space-charge region thickness modulation, all of which were neglected in the conventional quasineutral region capacitance model. It was found that high-level injection gives rise to a large discrepancy between the present model and the conventional model when the device is subject to a high bias condition. The effects of incomplete impurity ionization and degeneracy have opposite trends and thus tend to compensate each other when included in the capacitance model. In addition, it was shown that the effect of space-charge region thickness modulation is not important for modern devices with relatively high doping concentrations. The present study has important implications on the development of more accurate models for simulation of integrated circuits involving p/n junction semiconductor devices such as bipolar transistors and metal–oxide–semiconductor field effect transistors.

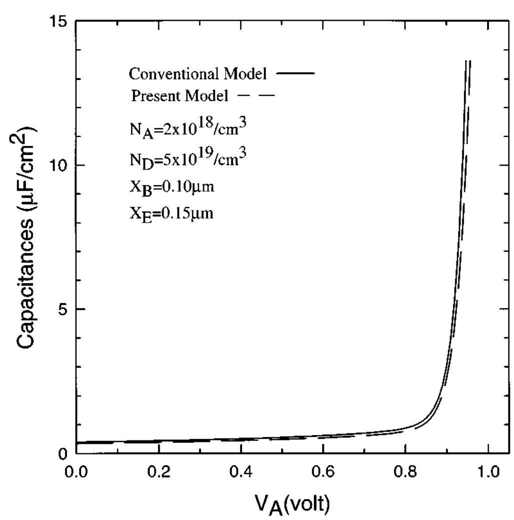

FIG. 8. Comparison of the total capacitance of the n/p junction diode calculated from the present model and the conventional model.

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