Nonlinear modeling of concrete-filled FRP tubes using the finite element method

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by

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Fiber reinforced plastic (FRP) has in recent years emerged as an alternative reinforcement for structural concrete. The properties that make FRP appealing to the construction industry include its resistance to electro-chemical corrosion, high strength-to-weight ratio, and electrical insulation. An efficient use of FRP is in the form of concrete-filled tubes, where the tube not only protects the concrete against environmental factors, but also provides confinement and external reinforcement. Since use of fiber composites for confining concrete is relatively new, analytical work in this area is limited to the models that were originally developed for transverse steel reinforcement. However, it has been shown that such models overestimate the strength of FRP-confined concrete, and result in unsafe design. This study is focused on developing a nonlinear finite element model for the analysis of FRP-confined concrete. Solid elements are used for the concrete core, along with a non-associative Drucker-Prager type plasticity, which takes into account the pressure sensitivity of the material. The parameters used to model the concrete are cohesion, angle of internal friction, and the dilatancy angle. The jacket is modeled by linear-elastic membrane shell elements. A parametric program was developed within the ANSYS® software to automatically generate the mesh for various geometric shapes and material properties. A sensitivity analysis was conducted to determine the optimum number of elements required in the mesh. The model was validated against previous experiments at UCF. The stress-strain curves compared
favorably with the test data. The model was also used for square sections to compare their confinement effectiveness with that of circular sections. The analysis, much the same as the experiments, showed stress concentrations around the corners. The stress concentration and the confinement effectiveness both depend on the corner radius of the section. Finally, the model was used to examine the plastic deformations of concrete-filled FRP tubes under cyclic loading, i.e., unloading and reloading in compression.
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TABLE OF CONTENTS

LIST OF TABLES ........................................................................................................... vii
LIST OF FIGURES ......................................................................................................... viii

CHAPTER 1 INTRODUCTION .................................................................................. 1
    Objective of Research ............................................................................................... 3
    Structure of Thesis ................................................................................................... 3

CHAPTER 2 LITERATURE REVIEW ....................................................................... 5
    Steel-Based Confinement Models ............................................................................. 8
    FRP-Based Empirical Models ................................................................................... 9
    Finite Element Analysis and Plasticity Approach .................................................... 10

CHAPTER 3 FINITE ELEMENT MODELING .............................................................. 13
    Plasticity Approach ................................................................................................. 13
    Plasticity of Confined Concrete ............................................................................... 20
    Finite Element Mesh ............................................................................................... 22
    Element Types .......................................................................................................... 25
    Material Properties .................................................................................................. 29
    Parametric Input ....................................................................................................... 29

CHAPTER 4 ANALYSIS OF RESULTS ................................................................... 37
    Post-Processing ....................................................................................................... 38
    Mesh Refinement .................................................................................................... 39
    Experimental Data Base .......................................................................................... 46
    Stress-Strain Response ............................................................................................ 46
    Volumetric Response ............................................................................................... 63
    Dilation Response ................................................................................................... 73
    Shape Effect ............................................................................................................ 75
    Cyclic Loading ........................................................................................................ 89
LIST OF TABLES

Table 4.1 Test Matrix for the Sensitivity Analysis of Circular Sections ......................... .40
Table 4.2 Test Matrix for the Sensitivity Analysis of Square Section ............................ .43
Table 4.3 Jacket Properties in Test Specimen ............................................................... .46
LIST OF FIGURES

Figure 2.1 Stress-Strain Behavior of Multi-linear Kinematic Hardening ....................... 11
Figure 3.1 Stress-Strain Response of Various Plasticity Options .................................... 14
Figure 3.2 Yield Surfaces for (a) Kinematic Hardening and (b) Drucker-Prager .......... 15
Figure 3.3 Hardening Rules: (a) Isotropic Hardening, (b) Kinematic hardening .......... 16
Figure 3.4 Drucker-Prager and Mohr-Colulomb Yield Surfaces .................................... 18
Figure 3.5 DP Criterion: (a) Meridian Plane, \( \theta = 0 \), (b) \( \pi \) Plane (Chen 1982) ........ 18
Figure 3.6 Plastic Volume Expansion Associated with the Yield Surface ................. 19
Figure 3.7 Finite Element Mesh for the Top Quarter of Circular Specimens ............... 23
Figure 3.8 Finite Element Mesh for the Top Quarter of Square Specimens ................. 24
Figure 3.9 Plan View of the Finite Element Mesh for Circular Sections ................. 24
Figure 3.10 Plan View of the Finite Element Mesh for Circular Sections ................. 25
Figure 3.11 SOLID65 Element ...................................................................................... 27
Figure 3.12 SOLID41 Element ...................................................................................... 28
Figure 3.13 Flow Chart for Modeling of Circular Sections ............................................. 34
Figure 3.13 Continued ................................................................................................... 35
Figure 3.13 Continued ................................................................................................... 36
Figure 4.1 Nodal Axial Stresses for the Circular Specimen ............................................ 38
Figure 4.2 Sensitivity of Finite Element Model to Mesh Refinement – Circular Section .. 41
Figure 4.3 Sensitivity of Finite Element Model to Mesh Refinement in the Vertical Direction ........................................................... 44
Figure 4.4 Sensitivity of Finite Element Model to Mesh Refinement – Square Section .. 45
Figure 4.5 Comparison of Various Models with 6-Ply Specimens of Batch A ............... 48
Figure 4.6 Comparison of Various Models with 10-Ply Specimens of Batch A ......... 49
Figure 4.7 Comparison of Various Models with 14-Ply Specimens of Batch A ........... 50
Figure 4.8 Comparison of Various Models with 6-Ply Specimens of Batch B .......... 51
Figure 4.9 Comparison of Various Models with 10-Ply Specimens of Batch B ......... 52
Figure 4.10 Comparison of Various Models with 14-Ply Specimens of Batch B .......... 53
Figure 4.11 Comparison of Various Models with 6-Ply Specimens of Batch C .......... 54
Figure 4.12 Comparison of Various Models with 10-Ply Specimens of Batch C .......... 55
Figure 4.13 Comparison of Various Models with 14-Ply Specimens of Batch C .......... 56
Figure 4.14 Effect of Friction Angle on Stress – Strain Response of FRP – Confined Concrete ................................................................. 58
Figure 4.15 Effect of Cohesion on Stress – Strain Response of FRP – Confined Concrete .......................................................................................... 60
Figure 4.16 Combined Effect of Friction Angle and Cohesion on Stress – Strain Response of FRP – Confined Concrete .................................................. 61
Figure 4.17 Effect of Dilatancy Angle and Cohesion on Stress – Strain Response of FRP – Confined Concrete .............................................................. 62
Figure 4.18 Comparison of Finite Element Model with 6-Ply Specimens of Batch A .... 64
Figure 4.19 Comparison of Finite Element Model with 10-Ply Specimens of Batch A ... 65
Figure 4.20 Comparison of Finite Element Model with 14-Ply Specimens of Batch A ... 66
Figure 4.21 Comparison of Finite Element Model with 6-Ply Specimens of Batch B .... 67
Figure 4.22 Comparison of Finite Element Model with 10-Ply Specimens of Batch B ... 68
Figure 4.23 Comparison of Finite Element Model with 14-Ply Specimens of Batch B ... 69
Figure 4.24 Comparison of Finite Element Model with 6-Ply Specimens of Batch C .... 70
Figure 4.25 Comparison of Finite Element Model with 10-Ply Specimens of Batch C ... 71
Figure 4.26 Comparison of Finite Element Model with 14-Ply Specimens of Batch C ... 72
Figure 4.27 Comparison of Finite Element Model with Volumetric Response of FRP – Confined Concrete ................................................................. 74
Figure 4.28 Comparison of Finite Element Model with Dilation Response of FRP – Confined Concrete ........................................................................... 76
Figure 4.29 Typical Failure Mode of Concrete-Filled Square FRP Tubes [34] .......... 77
Figure 4.30 Axial Stress Contours for the Sharp Edge Square Section .................... 79
Figure 4.31 Enlarged View of Axial Stress Contours at the Corner of Sharp Edge Section ........................................................................................................ 79
Figure 4.33 Axial Stress Contours for the Square Section with 1.5 ......................... 80
Figure 4.34 Axial Stress Contours for the Circular Section ....................................... 81
Figure 4.35 Effect of Corner Radius on Biaxial Stress – Strain Response of FRP – Confined Concrete ................................................................. 83
Figure 4.36 Spatial Variation of Axial Stress – Radial Strain Response for Sharp Edge Square Section ........................................................................................................ 84
Figure 4.37 Spatial Variation of Axial Stress – Radial Strain Response for 0.75 inch Corner Radius ......................................................................................................................................................... 85
Figure 4.38 Spatial Variation of Axial Stress – Radial Strain Response for 1.5 inch Corner Radius ......................................................................................................................................................... 86
Figure 4.39 Spatial Variation of Axial Stress – Radial Strain Response for Circular Section ................................................................................................................................. 87
Figure 4.40 Effect of Corner Radius on Spatial Variation of Radial Strains .......... 88
Figure 4.41 Effect of Jacket Thickness on Confinement with Square Sections .......... 91
Figure 4.42 Comparison of Finite Element Model with Test Results of Pico [34] ........ 92
Figure 4.43 Stress – Strain Response of FRP – Confined Concrete (Model and Tests) Under Cyclic Loading ................................................................. 93
Fiber reinforced plastic (FRP) has in recent years emerged as an alternative reinforcement for structural concrete. The properties that make FRP appealing to the construction industry include its resistance to electro-chemical corrosion, high strength-to-weight ratio, and electrical insulation. While replacing conventional steel reinforcement with FRP rods, grids, gratings, or tendons has received much attention, innovative applications have also been developed to combine FRP with concrete in a hybrid system. Mirmiran and Shahawy [1] proposed hybrid beam-columns in the form of concrete-filled FRP tubes. The tube may be a multi-layer composite shell that consists of at least two plies; an inner ply of axial fibers and an outer ply of hoop fibers. Deskovic and Triantafillou [2] proposed a cost-effective hybrid beam consisting of a glass FRP box beam with a concrete layer on its compression side and a carbon FRP laminate bonded to its tension side. More recently, Hall and Mottram [3] proposed a combined FRP reinforcement and permanent formwork. The system which may be used for slabs or beams consists of concrete cast onto a pultruded FRP panel with two shear studs (as T-up stands). These hybrid systems have three common characteristics; (a) FRP panel or shell acts as permanent formwork for concrete, (b) reinforcement for concrete is provided
externally by FRP, and (c) composite action between concrete and FRP can affect the
capacity and performance of the system.

In concrete-filled FRP tubes, the tube acts as the formwork as well as external
reinforcement for concrete. Furthermore, protecting the concrete core against corrosion
and moisture intrusion is one of the roles of the tube. Previous research at the University
of Central Florida (UCF) has focused on the feasibility of concrete-filled FRP tubes.
Tests have shown that, depending on the degree of confinement, ultimate strengths could
be increased by 2-3 times those of unconfined concrete [33]. The ultimate strains can also
be enhanced in the order of 10-15 times those of plain concrete [33]. The research has
also shown that hybrid FRP-concrete columns with no internal reinforcement are
technically feasible, as the interaction diagrams of such columns corresponded to steel
reinforced beam-columns with over 5% of reinforcement.

Since use of fiber composites for confinement of concrete is relatively new,
theoretical work in this area is limited to the models that were originally developed for
transverse steel reinforcement. However, it has been shown that concrete behaves very
differently when confined by elasto-plastic materials such as steel as compared to linearly
elastic materials such as fiber composites [4]. Applying the same models to FRP-
confined concrete may result in overestimating the strength and unsafe design. In the
absence of reliable models, construction industry may be forced to either avoid the use of
advanced composites, or to incorporate high "factors of safety," making composite
construction less economical. Previous research at UCF had led to development of a
semi-empirical confinement model [5]. However, theoretical approaches such as
plasticity and nonlinear finite element were not explored in the past.
This research will focus on developing a nonlinear finite element model with a non-associative Drucker-Prager type plasticity for FRP-confined concrete. Results will be compared with test data of previous experiments at UCF. Also, effect of cross-sectional shape and cyclic loading will be studied. The finite element analysis will be performed using the ANSYS® software package.

Objective of Research

The objectives of this study are as follows:

1. Investigate the use of a non-associative Drucker-Prager type plasticity in a nonlinear finite element model for FRP-confined concrete under concentric loading. Also, compare the results with test data of previous experiments at UCF.
2. Investigate the modeling capabilities of the finite element program for shape effect and stress concentrations in square tubes with various corner radii, and compare with the experimental results.
3. Use the finite element model to study the effect of loading and unloading of concrete-filled tubes in compression, and compare with the experimental results.

Structure of Thesis

This thesis consists of five chapters. Chapter 2 is a review of previous research in the following areas; modeling of confined concrete using the finite element analysis,
modeling of concrete using the Drucker-Prager plasticity, and characteristics of FRP-confined concrete. Chapter 3 is a detailed explanation of the nonlinear finite element model. Chapter 4 is devoted to the application of the model, comparison with test data, and a parametric study on the effect of various column variables on the confinement with fiber composites. Chapter 5 provides summary and conclusion for this study.
CHAPTER 2

LITERATURE REVIEW

It has been well established that by inhibiting lateral expansion of concrete, i.e., Poisson effect, its compressive strength and ductility improve significantly. The necessary confinement may be developed by internal hoops or spirals in new construction and external hoops or spirals in retrofitting projects. In recent years, other methods of confinement have been examined or re-visited. These include concrete-filled tubes (steel or FRP) for new construction, and jacketing (steel or FRP) of existing columns for retrofitting projects. The behavior of jacketed columns is expected to be similar to that of concrete-filled tubes, as both systems work on the principle of confinement.

Confinement methods could be divided into two categories: passive and active. Most methods of confinement such as transverse reinforcement, steel or FRP jacketing, and concrete-filled (steel or FRP) tubes fall in the category of passive confinement. In this category, the confining pressure is developed once the hoop elongation is imposed on the surrounding member by the expansion of concrete. The mechanics of confinement is, therefore, dependent on two factors; the tendency of concrete to dilate, and the radial stiffness of the confining member to restrain its dilation. This will place the concrete core in radial compression, and the confining member in hoop tension. Consequently, the degree of confinement is a function of the axial strain imposed on the concrete column. On the other hand, radial pressure (e.g., hydrostatic pressure) or circumferential post-
tensioning constitutes an active confinement. Of course, in this case too, the actual level of confinement for ultimate strength becomes a function of the strain energy stored in the confining member.

Fiber-wrapping technology was first used in practice for concrete chimneys in Japan [6]. The concept was then extended to the retrofitting of concrete columns [7]. In the U.S., Hexcel-Fyfe Corp. has installed field demonstration wraps for the California Department of Transportation [8]. They adopted a method called "active wrapping," in which pressurized cement grout is pumped between the original column and the composite wrap. A few of the columns thus wrapped have since failed by fiber fracture, which is now attributed to the wrapping mechanism. This method is replaced with a "passive wrap," i.e., without pressurized grouting. In an effort to minimize the on-site installation time and cost, an approach similar to steel-jacketing was taken by the researchers at the Pennsylvania State University who investigated a system of pre-formed FRP shells. The two half-cylinder shells are joined on site by applying adhesives. Tests at Penn State indicated that such systems fail by separation of the shells along the joint [9]. Similar approaches have also been introduced at the University of Southern California [10]. Another jacketing method includes wrapping thick FRP cables/tapes around concrete columns [9]. Researchers at the University of Arizona have used precured E-glass and polyester straps (or tapes) with 0.03 to 0.04 inch thickness to wrap around existing columns with an epoxy adhesive [11]. Testing quarter-scale columns, they achieved ductilities of up to five times the as-built columns, and with no shear failure up to twice the stroke limit of control columns [12].
Although most studies on FRP-confined concrete have been conducted in the past five years [13], the first attempt at such confinement mechanism was made in late 1970's. Kurt [14] suggested using commercially available plastic pipes (PVC or ABS) filled with concrete. His experimental studies indicated that plastic pipes were more effective than steel pipes in confining concrete. For a slenderness ratio of less than 20, plastic-encased concrete showed a 45° shear failure, both in the concrete core and in the plastic pipe, resulting from the combination of axial compression and hoop tension in the pipe. Since the plastic used by Kurt was weak, the enhancement in column’s strength was not significant. Later, Fardis and Khalili [15a & b] from MIT wrapped bi-directional FRP on 3"x6" and 4"x8" concrete cylinders under uniaxial compression tests, and on 3"x6"x48" beams under third-point bending. They achieved considerable strength and ductility enhancements. In early 1990's, as part of an investigation into the effect of confinement on high-strength concrete, Lahlou et al. [16] tested two 2"x4" glass FRP tubes filled with concrete. However, since the fibers were axially oriented (pultruted), they did not observe any significant enhancement in concrete strength.

Since the early years of development of the fiber-wrapping technology, three distinct modeling techniques have been suggested;

1. Using (and extending) steel-based confinement models;

2. Developing new FRP-based empirical models; and


A brief description of each method follows:
Steel-Based Confinement Models

Of the models for steel-confined concrete, the one that has been repeatedly mentioned and used by far the most is that of Mander et al. [17]. They developed a stress-strain model for concrete subjected to uniaxial compression, and confined by transverse steel reinforcement. The concrete section may contain any type of confining steel; either spiral or circular hoops, or rectangular stirrups with or without supplementary cross-ties. A single equation defines the entire stress-strain curve. The model allows for cyclic loading, and includes the effect of strain-rate. The influence of various types of confinement is taken into account by defining an effective lateral confining stress, which is dependent on the configuration of the transverse and longitudinal reinforcement. An energy balance approach is used to predict the axial compressive strain in concrete corresponding to the first fracture of transverse reinforcement. The method involves equating the strain energy capacity of transverse reinforcement to the strain energy stored in concrete as the result of confinement.

This model was used directly for fiber-wrapped specimens by Saadatmanesh et al. [11]. They generated moment-thrust interaction diagrams based on these results. Later, studies by Mirmiran and Shahawy [4,18] and Nanni and Bradford [9] showed that Mander’s model overestimates the strength while grossly underestimating the ductility of confined concrete.

Variations of steel-based models have also been applied to FRP-confined concrete. Mirmiran and Shahawy [1] adapted Madas and Elnashai [19] for fiber composites. The model attempted to enforce strain compatibility between the jacket and the core. This was done by using a third-degree polynomial suggested by Elwi and
Murray [20]. However, it was later shown that dilation characteristics of FRP-confined concrete are considerably different [18]. Another modification of Mander’s model [17] was suggested by Priestley and Seible [21]. They developed an empirical relationship for the ultimate strain of FRP-confined concrete rather than using Mander’s energy-balance approach. Most recently, Monti and Spoelstra [22] proposed a confinement model for fiber-wrapped circular columns. They used a model similar to Ahmad and Shah [23]. However, they used Mander’s stress-strain relationship [17], and Pantazopoulou’s model [24] for lateral strains and for strain compatibility. They showed their model to compare reasonably well with the data of Picher et al. [25].

**FRP-Based Empirical Models**

Perhaps the first attempt at developing FRP-specific models can be credited to Fardis and Khalili [15a & b]. They suggested a hyperbolic equation for the stress-strain relation. However, Nanni and Bradford [9] showed the model to grossly underestimate the ductility of fiber-wrapped columns, while comparing reasonably well for the strength. Ahmad, Khaloo and Irshaid [26] conducted an investigation of the confinement effectiveness of fiberglass spirals as transverse reinforcement for concrete columns. They related the peak stress of confined concrete to the spacing of the spirals. The only FRP-based empirical model, to date, is that of Samaan, Mirmiran and Shahawy [5] which uses a bilinear stress-strain relationship and incorporates the stiffness of the jacket in calculating the lateral strains.
Rochette and Labossière [27] have used an incremental finite element technique to evaluate the response of fiber-wrapped square concrete columns. They modeled concrete as an elastic-perfectly plastic material. The elastic region followed the Hooke’s law, while an associated flow rule was used for the plastic region. The associated flow rule was based on a yield function using the Drucker-Prager criterion, as

\[ f = \sqrt{J_2} + \alpha I_1 = k \]  

(2.1)

where \( f \) is a yield function, \( \alpha \) and \( k \) are positive material constants, \( I_1 \) is the first stress invariant, and \( J_2 \) is the second stress invariant. The two stress invariants are calculated as

\[ I_1 = \sigma_1 + \sigma_2 + \sigma_3 \]  

(2.2)

\[ J_2 = \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1 \]  

(2.3)

where \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are the stresses in the three dimensional space. The Drucker-Prager yield criterion will be discussed further in Chapter 3. Their model compared favorably with the results of their previous uni-axial compression tests [25]. They concluded, however, that a more complex elasto-plastic formulation of concrete behavior is needed to enhance the model for various cross sections, fiber orientations, and load combinations.

Earlier, Karabinis and Kiousis [28] had used the same approach for modeling of steel-confined concrete. However, Mirmiran and Shahawy [4] reported that the model of Karabinis and Kiousis can not predict the strength or ductility of FRP-confined concrete. Mirmiran [29] proposed a method for analyzing confined concrete using the finite element analysis software package of ANSYS®. This package contains a special element for concrete. This element, SOLID65, accounts for both cracking and crushing failure.
modes. Another element used in ANSYS® is SHELL41, which can simulate the linear-elastic behavior of the jacket or tube element. The model was for the top quarter of a concrete-filled FRP cylindrical tube, and consisted of 48 concrete elements and 24 jacket elements, with a total of 84 nodes. In order to have the jacket contribute only as a confining member, the elastic modulus in the direction of the axial load was set to a number close to zero. A rate-independent plasticity approach was used with a perfectly plastic von Mises yield criterion. This type of material behavior is called multi-linear kinematic hardening (MKIN), and uses the Besseling model (the sub-layer or overlay model). An associative flow determines the direction of the plastic straining, and is given by

\[ \{ d\varepsilon^{pl} \} = \lambda \{ \frac{\partial Q}{\partial \sigma} \} \]

(2.4)

where \( \lambda \) is a plastic multiplier, \( Q \) is a function of stress termed the plastic potential, \( \varepsilon^{pl} \) is the elastic strain, and \( \sigma \) is the stress. Details of this approach are explained in Chapter 3.

The stress-strain curve for the multi-linear kinematic hardening is shown in Figure 2.1.

![Figure 2.1 Stress-Strain Behavior of Multi-linear Kinematic Hardening](image)
This method of modeling the cylinders showed good agreement with the concrete-filled steel tubes. However, results for FRP-confined concrete were not acceptable, as the model deviated from the experimental data. The main conclusion from the study [29a] was that von Mises yield criterion by itself is not an acceptable criterion for modeling concrete, since it does not account for the effect of hydrostatic stress. The better approach would be to use the Drucker-Prager plasticity, which is accomplished in this study.
CHAPTER 3

FINITE ELEMENT MODELING

As discussed in Chapter 2, the von Mises criterion is not suitable for modeling of confined concrete. The better approach would be to use a non-associative Drucker-Prager (DP) type plasticity that accounts for the pressure sensitivity of concrete. In this chapter, the plasticity approach is described, and details of the DP model are explained. Then, the various components of the finite element model are outlined.

Plasticity Approach

Material non-linearity or plasticity is path dependent, meaning that plastic deformations and strains are functions of the full history of the applied loads [35]. On the other hand, linear elastic deformations depend only on the final state of stresses. Therefore, any materially non-linear problem is incremental by nature. The final deformation is the sum of incremental deformations following the strain path. Rate-independent plasticity constitutes an irreversible straining that occurs in a material once the yield surface is reached. The ANSYS® software provides the following options to characterize different types of non-linear material behavior:

1. Classical bilinear kinematic hardening;
2. Multi-linear kinematic hardening;
3. Bilinear isotropic hardening;
4. Multi-linear isotropic hardening;
5. Anisotropic material;
6. Drucker-Prager (DP) plasticity; and
7. User specified material behavior.

The stress-strain response of each plasticity option is shown in Figure 3.1

![Figure 3.1 Stress-Strain Response of Various Plasticity Options](image-url)
Plasticity theory provides a mathematical relationship that characterizes the elasto-plastic response of materials. A uni-axial elasto-plastic response is the simplest form of incremental straining, which is represented by two regions of elastic and perfectly plastic. The perfectly plastic behavior implies that no strain hardening exists. There are three ingredients in the rate-independent plasticity theory; the yield criterion, the flow rule, and the hardening rule.

**Yield Criterion:** The yield criterion determines the limit of elastic behavior under any combination of stresses. For multi-axial state of stress, an equivalent stress of $\sigma_e$ is defined as

$$\sigma_e = f(\sigma)$$

where $\sigma$ is the stress tensor, and $f$ is the yield function. Once $\sigma_e$ equals the material yield parameter of $\sigma_y$, the material develops plastic strains. In stress space, this phenomenon is termed as having reached the yield surface. Figure 3.2 shows the yield surfaces for the kinematic hardening and the Drucker-Prager plasticity options in the three-dimensional stress space ($\sigma_1, \sigma_2, \text{and} \sigma_3$). Negative stresses represent compression.

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**Figure 3.2 Yield Surfaces for (a) Kinematic Hardening, and (b) Drucker-Prager**
Flow Rule: The flow rule determines the direction of plastic straining as given earlier by Equation (2.4). The plastic potential, $Q$ is a function of stress, and determines the direction of plastic straining. If $Q$ is the same as the yield function ($f$), the flow rule is termed associative, and the plastic strains will occur in a direction normal to the yield surface. Otherwise, the flow rule is non-associative. The associative or non-associative nature of the flow rule is explained in more detail, later in this chapter.

Hardening Rule: The hardening rule describes the changing of the yield surface with progressive yielding so that the stress states for subsequent yielding can be established. Hardening can take one of the two following forms: isotropic (or work) hardening and kinematic hardening. In isotropic (or work) hardening, the yield surface remains centered about its initial centerline, and expands in size as the plastic strains develop. In kinematic hardening, the yield surface remains constant in size, but the surface moves in the stress space with progressive yielding. The two hardening rules are shown schematically in Figure 3.3.

Figure 3.3 Hardening Rules: (a) Isotropic Hardening, (b) Kinematic hardening
The Drucker-Prager (DP) model is an extension of the von Mises yield criterion. However, it includes the effect of hydrostatic pressure on the shearing resistance of the material. The stress-strain behavior for the DP model was shown earlier in Figure 3.1 (f). The equivalent stress for the DP model is

\[ \sigma_e = 3 \beta \sigma_m + \left[ \frac{1}{2} \{s\}^T [M] \{s\} \right]^{\frac{1}{2}} \]  

(3.2)

where \( \sigma_m \) is the mean or hydrostatic stress, \( \{s\} \) is the deviatoric stress vector (i.e., the difference between the mean stress and the actual stresses), \([M]\) is a special 6 x 6 diagonal matrix, and \( \beta \) is a material constant given by

\[ \beta = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)} \]  

(3.3)

where \( \phi \) is the angle of internal friction. The yield parameter of the material is defined as

\[ \sigma_y = \frac{6c \cos \phi}{\sqrt{3}(3 - \sin \phi)} \]  

(3.4)

where \( c \) is the cohesion value of the material. The failure surface for the DP model in the principal-stress space is a right-circular cone as shown in Figure 3.4. The meridian and cross section on the \( \pi \) plane are shown in Figure 3.5. The material parameters \( c \) and \( \phi \) are chosen such that the failure surface corresponds to the outer aspices of the hexagonal Mohr-Coulomb yield surface. The DP surface can be regarded as a smooth Mohr-Coulomb surface or as an extension of the von Mises surface with pressure dependency.
Figure 3.4 Drucker-Prager and Mohr-Coulomb Yield Surfaces

Figure 3.5 DP Criterion, (a) Meridian Plane, \( \theta = 0 \), (b) \( \pi \) Plane (Chen 1982)
An angle of dilatancy ($\phi_d$) is defined for establishing the flow rule. If $\phi_r = \phi$, then the flow rule is associative, and the plastic straining occurs normal to the yield surface, and there will be a volumetric expansion of the material with plastic strains. However, if $\phi_r < \phi$, the flow rule is non-associative and there will be less volumetric expansion. Clearly, if $\phi_r$ is zero, there will be no volumetric expansion. The associated flow rule and the corresponding volume expansion are shown schematically in Figure 3.6.

The equivalent stress parameter $\hat{\sigma}_e^{pl}$ is interpreted as the von Mises equivalent stress at yield at the current hydrostatic stress level, and is defined as

$$\hat{\sigma}_e^{pl} = \sqrt{3}(\sigma_y - 3\beta\sigma_m)$$

(3.5)
Plasticity of Confined Concrete

The characteristic behavior of concrete material includes three distinct regions; the un-cracked elastic stage, crack propagation, and the plastic range. As stated by Chen [35], the following assumptions are often made for modeling of concrete:

1. Concrete behaves like a perfectly plastic material after its maximum load-carrying capacity is exceeded;
2. Its failure surface is taken directly as the fixed surface in the stress space; and
3. The plastic-strain-increment vector is assumed to be normal to the yield surface of the current stress state, i.e., an associative flow rule.

Studies on fiber-wrapped concrete at the University of Sherbrooke [25,27] have suggested the use of an elastic-perfectly plastic material response for concrete. The cohesion \( c \) and the angle of internal friction \( \phi \), given by Mohr’s circle, are related as follows

\[
f'_{co} = \frac{2c \cos \phi}{1 - \sin \phi}
\]  

(3.6)

and

\[
k_1 = \frac{1 + \sin \phi}{1 - \sin \phi}
\]  

(3.7)

where \( f'_{co} \) is the unconfined strength of concrete, and \( k_1 \) is the confinement effectiveness factor. The confinement effectiveness factor was first suggested by Richart et al. [30] in a linear relation as below

\[
f'_{cc} = f'_{co} + k_1 f_r
\]  

(3.8)

where \( f'_{cc} \) is the confined strength of concrete, and \( f_r \) is the confinement pressure given by
\[ f_r = \frac{2f_j t_j}{D} \]  

(3.9)

where \( f_j \) is the jacket strength, \( t_j \) is the jacket thickness, and \( D \) is the core diameter. A value of \( k_1 = 4.1 \) was suggested by Richart et al. [30]. For this value of \( k_1 \), and a concrete strength of 4000 psi, values of \( c \) and \( \phi \) are calculated from Equations (3.6) and (3.7) as 988 psi and 37.43°, respectively. Other researchers have suggested a non-linear confinement effectiveness, implying that as the confinement pressure increases, the strength of confined concrete does not increase at the same rate. Mander et al. [17] proposed the following formula for circular sections confined by steel stirrups or sleeves

\[
f''_{\text{cc}} = f'_{\text{co}} \left[ -1.254 + 2.254 \sqrt{1 + 7.94 \frac{f_r}{f'_{\text{co}}} - 2 \frac{f_r}{f'_{\text{co}}}} \right]
\]  

(3.10)

More recently, Samaan et al. [5] proposed the following relationship for circular sections confined by fiber composites

\[
f'_{\text{cu}} = f'_{\text{co}} + 3.38 f'^{0.7}
\]  

(3.11)

where \( f_{\text{cu}} \) is the ultimate strength of FRP-confined concrete. Once the confined strength of concrete and its confinement effectiveness are determined from any of these models (Richart et al. [30], Mander et al. [17], or Samaan et al. [5]), the material parameters for the DP model can be calculated from Equations (3.6) and (3.7), or as re-arranged below

\[
\phi = \sin^{-1} \left[ \frac{k_1 - 1}{k_1 + 1} \right]
\]  

(3.12)

\[
c = \left( \frac{f'_{\text{co}}}{2k_1} \right) \frac{1 + \sin \phi}{\cos \phi}
\]  

(3.13)
A recent study by Rochette and Labossière [27] suggests the following relationships for $c$ and $\phi$ (after simplifications):

\[
\phi = \sin^{-1}\left[\frac{3}{1 + 1.592332f_{co}'(ksi)}\right]
\]  

\[
c(\text{psi}) = [f_{co}'(\text{psi}) - 1256]\frac{3 - \sin \phi}{6\cos \phi}
\]  

In the present study, the values suggested by Rochette and Labossière [27] are used, while the models of Richart et al. [30], Mander et al. [17], and Samaan et al. [5] are also investigated for comparison. Additionally, as shown in Chapter 4, a sensitivity analysis is conducted to determine the effect of dilatancy angle ($\phi_f$). It is determined that for best results, the dilatancy angle ($\phi_f$) of FRP-confined concrete needs to be set equal to zero, which makes the flow rule non-associative and the corresponding volumetric expansion negligible.

Finite Element Mesh

ANSYS® program, Version 5.3 was used in this study [36]. Two types of cross sections were modeled; circular and square sections. Since geometry and loading (uniaxial compression) are both symmetric, only 1/8 of the specimens (top quarter) is modeled for either type of cross section. The top quarter is divided into slices in the vertical (axial) direction. Also, the cross section is divided into series of elements in the two planar directions. The division of cross-sectional elements in the circular sections results in a fine mesh in the radial and circumferential directions. Figures 3.7 and 3.8 show the three-dimensional mesh for the circular and square specimens, where $D$ is the core diameter of the circular tubes or inside diameter of square tubes, $H$ is the overall...
depth of the specimen, and $\Delta Z$ is the depth of each slice in the axial direction. Figures 3.9 and 3.10 show the plan view of the two types of cross section, where $\Delta R$ is the length of each element in the radial direction, $\Delta \theta$ is the angular width of each element in the circumferential direction, and $R_c$ is the corner radius of square tubes.

![Figure 3.7 Finite Element Mesh for the Top Quarter of Circular Specimens](image)

Figure 3.7 Finite Element Mesh for the Top Quarter of Circular Specimens
Figure 3.8 Finite Element Mesh for the Top Quarter of Square Specimens

Figure 3.9 Plan View of the Finite Element Mesh for Circular Sections
A parametric input file was created to automate the mesh generation as well as material and geometric input. The parametric file is based on FORTRAN for its Boolean logic, and is explained in more detail later in this chapter.

**Element Types**

The concrete core was modeled by the 8-noded (or 6-noded) SOLID65 element. The nodal configuration, geometry, and coordinate system of SOLID65 element is shown in Figure 3.11. The 8-noded elements are brick type, and the 6-noded elements are the wedge elements. The wedge elements, i.e., the *Prism Option* (see Figure 3.11), are used for the first annular division around the center of the circular sections. SOLID65 can...
model concrete with or without reinforcements. The element consists of a single solid material and up to three smeared reinforcing materials in three different orientations. The solid material, i.e., plain concrete, is treated as an isotropic homogeneous material. The rebar specifications in each of the three orientations may include a different material characteristic and a different volumetric ratio. The reinforcing bars can take tension and compression but not shear. Therefore, shear stirrups need to be modeled as such. The rebar capabilities of the element were not utilized in this study.

The concrete element is capable of cracking in tension and crushing in compression. Cracking can occur in any of the three orthogonal directions. The element can also accommodate plastic deformations and creep. The material data for concrete may consist of different tensile and compressive strengths. The shear transfer coefficient allows the user to model the aggregate interlock along the surface of the crack. A complete loss of shear transfer, i.e., a smooth crack surface, is modeled by a zero shear transfer coefficient. On the other hand, a rough crack with no loss of shear transfer has a coefficient of 1.0. This specification can be made for both the closed and open cracks. However, this capability was not used in this study due to the nature of loading in uniaxial compression tests of cylinders. Other capabilities of the element that were not utilized in this study include; creep, large deflections, stress stiffening, birth and death of elements, and adaptive descent of the stress-strain response. The element can be loaded by surface pressure, body forces such as own weight, and nodal loads.
The stress-strain matrix \([D]\) used for the solid element is given by

\[
[D] = \left[ 1 - \sum_{i=1}^{N_r} V_i^R \right] [D^c] + \sum_{i=1}^{N_r} V_i^R [D^r],
\]

where \(N_r\) is the number of reinforcing materials, \(V_i^R\) is the volumetric ratio of the reinforcing material \(i (i = 1, 2, \text{and} 3)\) with respect to total volume of the element, \([D^c]\) is the stress-strain matrix for concrete, and \([D^r]_i\) is the stress-strain matrix for the reinforcement material \(i\). The nodal and element output consists of nodal displacements, average uni-axial elastic strain, average uni-axial plastic strain, and the various stress components.

The jacket is modeled by a 4-noded linear elastic membrane shell element (SHELL41) for fiber composites, and an elasto-plastic shell element (SHELL43) for steel. SHELL41 is a 3-dimensional element with membrane (in-plane) stiffness but no bending (out-of-plane) stiffness. The element has only three translational degrees of
freedom per node, but no rotational degree of freedom is permitted. The element has the following capabilities: variable thickness, orthotropic properties, stress stiffening, large deflection, and a cloth option. The orthotropy of the element can be defined by a material direction angle $\theta$, which may be different from the element coordinate system. The cloth option adds a non-linear feature to the element, which constitutes a wrinkle as the element goes into compression. Wrinkling may be in one or both orthogonal directions. The element can be loaded by surface or edge pressure. The output associated with SHELL41 is similar to that of SOLID65. The element with its nodal configuration, geometry, and coordinate system is shown in Figure 3.12.

![Figure 3.12 SHELL41 Element](image)

Note - x and y are in the plane of the element
x is along side I-J if $\theta = 0$

The steel jacket is modeled by the SHELL43 element. The SHELL43 element is capable of modeling linear, warped, and moderately-thick shells. The element has a total of six degrees of freedom at each node; three translational, and three rotational. The element has both in-plane (membrane) and out-of-plane (bending) stiffnesses. The shape...
functions for the in-plane directions are linear. The out-of-plane shape functions follow a mixed interpolation of tensorial components. This element has plasticity, stress stiffening, large deflection, and large strain capabilities. Its nodal configuration, geometry, coordinate system, and output are similar to SHELL41.

Material Properties

Modulus of elasticity of concrete is calculated automatically from its unconfined strength using the following equation

\[ E_c = 47586 \sqrt{f_c} \] (3.17)

as proposed by Ahmad and Shah [23], unless the user prefers direct input or ACI formula as below

\[ E_c = 57,000 \sqrt{f_c} \] (3.18)

The \( c \) and \( \phi \) values are calculated using Equations (3.14) and (3.15), unless the user prefers one of the models by Richart et al. [30], Mander et al. [17], Samaan et al. [5], or a direct input. The user inputs the dilation as a percentage (0 to 100). The default dilatancy angle is zero, unless the user prefers partial or full associative flow rule. Shell material is assumed linear elastic for FRP, and bilinear kinematic for steel. In order to limit the effect of jacket to confinement only, its elastic modulus in the axial direction was set close to zero.

Parametric Input

As stated earlier in this chapter, the ANSYS® finite element program allows the user to input the information by one of the following two methods:
1.) Direct interactive input through Graphical User Interface (GUI); or
2.) Input file.

The second method makes it easier to enter data and commands that are the same for most case studies, and only change those that vary from one case study to another. The ANSYS® Parametric Design Language (APDL) allows the use of variables, and prompts the user for entering the desired value for each variable. It further allows the user to select a pre-set or default value for each variable. The APDL processes all parametric input through the various equations very similar to FORTRAN or any other programming language.

Separate input files were developed for the circular and square specimens. Below, an explanation of each parametric input file is presented:

*Circular Sections*: Figure 3.13 shows the flow chart of the APDL input file for the circular sections. The following steps can be recognized from the flow chart:

Step 1. Geometric Input: The user is prompted to enter the core diameter (D) and specimen height (H). Also, the user inputs the number of elements in each of the radial, angular (circumferential) and depth directions.

Step 2. Concrete Properties: The user inputs the compressive strength of concrete ($f_c$). Also, the user selects one of the following three options for calculating the modulus of elasticity of concrete ($E_c$):

(a) Ahmad and Shah’s Method: Equation (3.17)

(b) ACI Method: Equation (3.18)

(c) Manual Input
Step 3. Confinement Characteristics: The user identifies whether the concrete specimen is jacketed (by steel or FRP materials) or not, and if not jacketed, whether or not it is subjected to a lateral pressure. In each case, the user is prompted to enter either thickness and material properties of the jacket or the value of the applied hydrostatic pressure. If the specimen is jacketed, the confining pressure is calculated from the thickness and material properties of the jacket. The user then selects a method, from among the following five, to establish the Drucker-Prager parameters for the concrete core; i.e., \( c \) and \( \phi \):

(a) Richart et al. [30]

(b) Samaan et al. [5]

(c) Mander et al. [17]

(d) Rochette and Labossière [27]

(e) Manual input

For each case, the program automatically calculates the DP parameters or prompts the user for the desired values. The user is also prompted to enter the percent dilatancy of the concrete core (default is zero).

Step 4. Element Types: Proper element types are assigned to concrete, and steel or FRP shell (if one is chosen by the user).

Step 5. Mesh Generation: The entire mesh for the top quarter of the specimen is generated automatically using DO-Loop capabilities of the APDL. The following steps are taken:

(a) The origin of the coordinate system is set as the first node at the center of concrete core.
(b) The nodes on the XY plane are generated as concentric circles with radii equal to the fraction of the core radius.

(c) The nodes in the Z direction are generated by duplicating the same pattern as that of the XY plane.

(d) The solid elements for the concrete core are identified and assembled in the first slice on the XY plane.

(e) The shell elements for the jacket (if one exists) are identified on the boundary of the first slice on the XY plane.

(f) The solid and shell (if any) elements along the Z directions are duplicated for the remaining slices using the same pattern as that of the first slice on the XY plane.

Note that the first annular series of solid concrete elements follow the *Prism Option* as shown in Figure 3.11.

Step 6. Boundary Conditions: Due to symmetry, only the top quarter of the cylinder is modeled. Accordingly, on each of the three planes of symmetry, nodal degrees of freedom perpendicular to the plane of symmetry are fixed. Also, general boundary conditions for the stability of the model are established.

Step 7. Displacement Control Analysis: The user is prompted to enter the expected range of axial strains, from which the maximum deflection for the model is estimated. This deflection is first divided into the user-defined number of time steps or increments, and then applied to all nodes on the top surface of the model. To simplify the application of displacements, all nodes on the top surface are tied to each other to move
together in the Z direction so as to simulate the rigid loading plate of the compression testing machine.

Step 8. Lateral Pressure: If the cylinder is subjected to lateral pressure, a surface pressure is applied incrementally on the exterior surface of the concrete core.

Step 9. Post-Processing: Once the analysis is completed, various components of the stress and strain tensor for the concrete elements and the shell elements (if any) are selected to develop a time-history analysis in the form of tables, charts and graphs. The information can also be imported to any spreadsheet program. Also, the stress and displacement contours can be developed interactively within the ANSYS® program. The post-processing component is explained in more detail in Chapter 4.

Square Sections: The input file for the square sections is very similar to that described for the circular sections. Therefore, only those steps that are somewhat different are explained below:

Step 1. Geometric Input: The user enters the inside dimension of the tube (D), specimen height (H), and corner radius (Re). The user also inputs the number of elements in each of the radial, angular (circumferential) and depth directions, as well as the number of elements in the corner (or exterior) segment of the model.

Step 4. Mesh Generation: The mesh for the top quarter of the specimen is automatically generated for the following segments, each with a user-defined spacing:

(a) Interior segment: \( \left( D - \frac{1}{2} R_c \right) \cdot \left( D - \frac{1}{2} R_c \right) \)

(b) Exterior segments: \( \left( D - \frac{1}{2} R_c \right) \cdot R_c \) and \( R_c \cdot \left( D - \frac{1}{2} R_c \right) \)

(c) Corner segments: \( R_c \), modeled the same as the circular section.
START
Enter Core Diameter (D), and Specimen Height (H)
Enter Concrete Strength (fc')
Enter Number of Spacings in the Radial, Angular, and Depth Directions

Manual Entry for Concrete Modulus (Ec) ? Yes
No

Use ACI formula ? (or Ahmad & Shah) Yes
No
Ec=47,586(fc')^1/2
Ec=57,000(fc')^1/2

Is Concrete Jacketed ? Yes
No

Is Lateral Pressure Applied ? Yes
Enter Applied Pressure (fr)
No

Is Jacket FRP ? (or steel) Yes
No (Steel)

Enter Jacket Modulus (Es) and Yield Strength (fy)
Enter Jacket Modulus (Ej) and Ultimate Strength (fj)

Calculate Confining Pressure (fr) from (3.9)

Figure 3.13 Flow Chart for Modeling of Circular Sections
Enter Confinement Model:
(1) Richart, (2) Samaan, (3) Mander, (4) Rochette, (5) Parametric
[Default is Parametric Input]

Use Richart's Model?

Use Samaan's model?

Use Mander's model?

Use Rochette and Labossière's model?

Calculate $f_{cu}$ from (3.11)
\[ k_1 = \frac{(f_{cu} - f_{co})}{f_{r}} \]
Calculate $c$ and $\phi$ from (3.12) & (3.13)

Calculate $f_{cu}$ from (3.10)
\[ k_1 = \frac{(f_{cu} - f_{co})}{f_{r}} \]
Calculate $c$ and $\phi$ from (3.12) & (3.13)

Calculate $c$ and $\phi$ from (3.14) & (3.15)

Establish Boundary Conditions on Plane of Symmetry

Enter Expected Range of Axial Strains

Establish Displacement Control Analysis

Is Lateral Pressure Applied?

Establish Load Increments

Figure 3.13 Continued
Figure 3.13 Continued
CHAPTER 4

ANALYSIS OF RESULTS

This chapter contains the results of the finite element analysis performed in this study. First, the methods of post-processing the results are described. There are two main types of post-processing in the ANSYS® program; general and time-history post-processing. The latter provides a step-by-step variation of any desired variable such as stress or strain at various nodes or elements in the model. The former post-processor provides plotting and listing capabilities for the results at any specific time step (usually the last time step). The results include deformations and contour plots of stresses and strains (both plastic and elastic). Next, a sensitivity analysis is performed to optimize the size of the finite element mesh, and to determine the proper extent of refinement required for each type of cross section. Then, the stress-strain, volumetric and dilation responses of circular sections are compared with test results. A discussion of the parameters used in the DP plasticity are also presented. The effect of the corner radius in the square section is discussed next. The analysis shows how the stress concentrations vary with the change in corner radius. Finally, an analysis of cyclic loading is performed to show the capabilities of the model to map out the response of confined concrete under repeated loading and unloading.
Post-Processing

The ANSYS® program contains two methods by which the results of an analysis can be viewed or retrieved; the General Postprocessor (POST1) and the Time-History Postprocessor (POST26). The general post-processor allows the user to review the analysis results over the entire model, or selected portions of the model for a specifically defined combination of loads at a single time step. POST1 is mainly used for viewing the final results of a model graphically. Through the graphical user interface (GUI) of the program, the user can select the element or nodal solution of plastic and elastic stresses and strains, and deformed and undeformed shape that will be displayed. As an example, Figure 4.1 shows the nodal solution for stresses in the z direction of a circular specimen. The figure also shows the deformed (solid lines) and undeformed shape (dashed lines) for the circular cross-section. The varying colors depict the changes in stress. This graphical feature will be used later in this chapter to show the increase in stress that occurs with the square cross-section model.

Figure 4.1 Nodal Axial Stresses for the Circular Specimen
POST26 or time-history post-processor enables the user to review the results at specific points in the model as a function of time, frequency, etc. It has many capabilities, including graphic displays and tabular listings. The output can also be imported to programs such as Excel® for enhanced graphics. An integral part of POST26 are the time-step functions which set the time for each load step and further specify the number of sub-steps within each load step. These functions are used in the input file to simulate the compression tests under an MTS machine. To achieve this, all nodes on the top surface (i.e., at the loading plate) were tied together to simulate a uniform compression. The time steps are created automatically based on the user-specified ultimate axial strain of confined concrete (see the flowchart and previous explanation of the input file). Other POST26 functions help plot or print the specified parameters at a certain node or element. These features are used to produce the results which are shown throughout this chapter.

**Mesh Refinement**

The mesh refinement or sensitivity analysis was performed by using a constant value of $\phi$, $c$, and the dilatancy angle, and by changing the number of elements in the model. Elements were added until little effect was noted on the stress-strain response of the model. Both circular and square sections were taken to be 6” x 12”.

*Circular Section:* The model was first analyzed with only 12 elements. Results are plotted in Figure 4.2. Since the analysis was only to determine the effect the number of elements had on the results, one set of material properties was used for all models. This included using the following values for the DP model for confined concrete; $\phi =$
37.4 degrees, \( c = 987.73 \text{ psi} \), and the percentage of dilatancy was set to zero. This data was manually entered in the program. Also, an FRP jacket was selected with a shell thickness of 0.0568 inches, an ultimate strength of 76,000 psi, and a modulus of elasticity of 5,400,000 psi. The compressive strength of concrete core was set as 4,299 psi. The model was then analyzed with an increased number of elements in the z direction, followed by an increased number of elements in the radial direction. Figure 4.2 shows the stress-strain response for the various models until there was little or no change in the results. After the 180-element model was analyzed, further increase in the number of elements did not significantly change the results as shown in the figure. These models were analyzed with 5 slices in the z direction. The same model was also run with only 2 slices in the Z direction, i.e., using only 72 elements. Table 4.1 shows the number of spacings (or divisions) in each of the three orthogonal directions (radial, angular, and depth) for each model. The table also shows the sensitivity of axial stresses to the mesh refinement.

Table 4.1 Test Matrix for the Sensitivity Analysis of Circular Sections

<table>
<thead>
<tr>
<th>F.E. Model Number</th>
<th>Number of Spacings</th>
<th>Total Number of Elements</th>
<th>Axial Stress (ksi) at 0.03 strain</th>
<th>Percent Improvement over the coarsest model*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Radial Direction</td>
<td>Angular Direction</td>
<td>Depth Direction</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>C2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>C3</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>C4</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>C5</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>75</td>
</tr>
<tr>
<td>C6</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>150</td>
</tr>
<tr>
<td>C7</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>225</td>
</tr>
<tr>
<td>C8</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>900</td>
</tr>
<tr>
<td>C9</td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>60</td>
</tr>
</tbody>
</table>

* The coarsest model is the one with the least number of elements.
Figure 4.2 Sensitivity of Finite Element Model to Mesh Refinement - Circular Section

Note overlapping of curves occurred due to closely similar results.
It was concluded that the model C9 contained the needed number of elements to accurately model the FRP tubes, and that additional elements would only increase the computation cost without improving the accuracy. In order to see the effect of number of elements in the depth direction, model C9 was analyzed with 1 through 5 elements in the Z direction. Figure 4.3 shows the sensitivity of the model to mesh refinement in the vertical direction. Clearly, due to uniform axial stresses, number of elements in the vertical direction does not affect the results. Further study would be needed to determine the effect of number of elements in axial direction for long and slender tubes.

Square Section: Sensitivity analysis of this section was conducted similar to the circular section. The same material properties and FRP jacket were used. The corner radius was set to 0.1 inches. Once again the model was first analyzed with only 44 elements, and then the number of elements was increased until the effect of mesh refinement was negligible. For the square section, refinement of the model can take a variety of forms. Therefore, additional runs were needed to determine the proper number of elements in the model. Figure 4.4 shows the stress-strain response of various models. Table 4.2 shows the description of each model as well as the sensitivity of axial stresses to the mesh refinement. It was concluded that Model S4 provides the best results with the least computational cost. As the corner radius increases, the number of divisions for the exterior elements will also need to be increased for a more accurate analysis.
Table 4.2 Test Matrix for the Sensitivity Analysis of Square Sections

<table>
<thead>
<tr>
<th>FE Model Number</th>
<th>Number of Spacings</th>
<th>Total Number of Elements</th>
<th>Axial Stress (ksi) at 0.03 Strain</th>
<th>Percent Improvement over the coarsest model*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interior Segments</td>
<td>Exterior Segments</td>
<td>Corner (Angular)</td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>44</td>
</tr>
<tr>
<td>S2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>66</td>
</tr>
<tr>
<td>S3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>92</td>
</tr>
<tr>
<td>S4</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>122</td>
</tr>
<tr>
<td>S5</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>156</td>
</tr>
<tr>
<td>S6</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>231</td>
</tr>
<tr>
<td>S7</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>605</td>
</tr>
<tr>
<td>S8</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>425</td>
</tr>
</tbody>
</table>

* The coarsest model is the one with least number of elements.
Figure 4.3 Sensitivity of Finite Element Model to Mesh Refinement in the Vertical Direction
Figure 4.4 Sensitivity of Finite Element Model to Mesh Refinement - Square Section

Note overlapping of curves occurred due to closely similar results.
Experimental Data Base

Scherer [33] tested a total of 30 6” x 12” concrete-FRP cylinders. Three batches of concrete and three tube thickness’ were used in the study. The jacket properties are shown in Table 4.3 [33].

Table 4.3 Jacket Properties in Test Specimen

<table>
<thead>
<tr>
<th>Shell No.</th>
<th>Number of Plies</th>
<th>Jacket Thickness (in.)</th>
<th>Hoop Strength of Jacket (psi)</th>
<th>Modulus of Elasticity (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>0.0568</td>
<td>76,000</td>
<td>5,400,000</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>0.0868</td>
<td>84,000</td>
<td>5,850,000</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>0.1168</td>
<td>93,000</td>
<td>5,910,000</td>
</tr>
</tbody>
</table>

The shell numbers refer to the different numbers of plies used in the study. Each ply represents a layer of filament winding. The cylinders were filled with three concrete mix designs A, B, and C with unconfined concrete strengths of 4,476 psi, 4,299 psi, and 4,637 psi, respectively. The specimens were labeled with designations such as DA11. The second letter refers to the batch number. The first numeral refers to the shell number, and last character indicates the sample number. Note that of each jacket thickness and for each batch of concrete, 2 to 3 samples were tested for repeatability verification. More details on tested specimens can be found in [33] and [29]. In the next several sections, experimental results are compared with the predicted response from the finite element model.

Stress-Strain Response

Figures 4.5 to 4.13 show the comparison between the predicted stress-strain response from the finite element model and the experimental data for the test specimens
of Scherer [33]. The axial stress is plotted against axial and radial strains, where radial (tensile) strains are considered negative. Each figure corresponds to a specific concrete batch and jacket thickness. In each figure, test results for the 2 or 3 samples of the same concrete strength and jacket thickness are shown. Also shown are the predicted response curves for each of the four models by Richart et al. [30], Samaan et al. [5], Mander et al. [17], and Rochette and Labossière [27]. These models were used to determine the Drucker-Prager (DP) parameters, i.e., c and $\phi$. The models were described in detail earlier in Chapter 3. The prediction curves were all developed by assuming a non-associative plasticity with a zero dilatancy angle. The effect of dilatancy angle is discussed in more detail later in this section.

While the results for each method varied considerably, there are certain similarities between the prediction curves. Both axial and radial strain curves appear to be bi-linear, regardless of the method used to calculate the DP parameters. The first slope generally corresponds to the modulus of elasticity of the unconfined concrete core. The transition point between the two slopes corresponds to the peak strength of unconfined concrete. The second slope depends mainly on the stiffness of the jacket, as the concrete core is expected to have been severely damaged around the transition point. The transition point signifies a pseudo-plastic behavior of the FRP-confined concrete. The jacket provides the necessary hoop tension to keep the concrete core in place, and to retard (or curtail) its tendency to dilate. Regardless of the method used, the transition in
Figure 4.5 Comparison of Various Models with 6-Ply Specimens of Batch A
Figure 4.6 Comparison of Various Models with 10-Ply Specimens of Batch A
Figure 4.7 Comparison of Various Models with 14-Ply Specimens of Batch A
Figure 4.8 Comparison of Various Models with 6-Ply Specimens of Batch B
Figure 4.9 Comparison of Various Models with 10-Ply Specimens of Batch B
Figure 4.10 Comparison of Various Models with 14-Ply Specimens of Batch B
Figure 4.11 Comparison of Various Models with 6-Ply Specimens of Batch C
Figure 4.12 Comparison of Various Models with 10-Ply Specimens of Batch C
Figure 4.13 Comparison of Various Models with 14-Ply Specimens of Batch C
the finite element curves is not as smooth as that seen in the test data. This is mainly attributed to the elastic-perfectly-plastic assumption of the Drucker-Prager model.

There are seemingly two major differences between the finite element prediction curves; (a) location of the transition point, and (b) second slope of the response curve. On the other hand, what distinguishes the prediction curves is only the value of DP parameters, \( c \) and \( \phi \). Therefore, it appears that the DP parameters have a pronounced effect on the characteristics of the response curve. The direct effect of these parameters will be studied later in this section. Of the four models used to estimate the values of DP parameters, the models of Richart et al. [30] and Mander et al. [17] both consistently overestimate the strength and stiffness of the confined concrete. In fact, it is interesting to note that both models result in the same DP parameters and response curves for the range of confinement pressure provided in the test specimens. The model of Samaan et al. [5] provides a much better correlation with the experimental results, especially for the transition point. However, the model of Rochette and Labossière [27] appears to match the test data most favorably. As indicated earlier in Chapter 3, Rochette and Labossière [27] applied their model to fiber-wrapped concrete, and reported accurate predictions. Therefore, the present study confirms that the same model can be used for un-bonded concrete-filled FRP tubes.

**Sensitivity Analysis:** To better understand the effect of DP parameters \( \phi \), \( c \), and the dilatancy angle on the stress-strain response of FRP-confined concrete, a sensitivity analysis of these parameters was carried out individually. Figure 4.14 shows the effect of internal friction angle \( \phi \) on the stress-strain response. In order to isolate the effect of \( \phi \),
Figure 4.14 Effect of Friction Angle on Stress-Strain Response of FRP-Confined Concrete
the value of cohesion \( c \) and the dilatancy angle were both set equal to zero. The analysis was conducted for a 14-ply jacket with concrete strength of 4,637 psi, i.e., that of Batch C. It should be noted that since cohesion value is set to zero, the transition point is at the origin of the coordinate system. It is clear that the angle of friction directly affects the second slope of the response.

Figure 4.15 shows the effect of cohesion \( c \) on the stress-strain response. In order to isolate the effect of \( c \), the values of the internal friction angle and the dilatancy angle are both set equal to zero. The analysis was conducted for the same jacket thickness and concrete strength as that used for the sensitivity analysis of friction angle. The figure clearly shows that the cohesion directly affects the transition point of the response.

Figure 4.16 shows the combined effect of friction angle and cohesion (\( \phi \) and \( c \)) on the stress-strain response of FRP-confined concrete. The dilatancy angle was set equal to zero for this comparison. The analysis was conducted for the same jacket thickness and concrete strength as that used for the individual sensitivity analyses of friction angle and cohesion. From Figures 4.14 to 4.16 it can be concluded that while the second slope of the response depends mainly on the friction angle, the location of the transition point is a function of both cohesion and friction angle.

Figure 4.17 shows the effect of dilatancy angle on the stress-strain response of FRP-confined concrete. The analysis was conducted for a 6-ply jacket with concrete strength of 4,299 psi, i.e., that of Batch B. The friction angle and cohesion were set equal to 28 degrees and 1200 psi, respectively. These values will be shown later to favorably match the test results. In the same figure, the test data for Specimens DB11, DB12, and DB13 are shown for comparison. The value of dilatancy angle was varied between zero
Figure 4.15 Effect of Cohesion on Stress-strain Response of FRP-Confined Concrete
Figure 4.16 Combined Effect of Friction Angle and Cohesion on Stress-Strain Response of FRP-Confined Concrete
Figure 4.17 Effect of Dilatancy Angle on Stress-Strain Response on FRP-Confined Concrete
for non-associative plasticity and up to 28 degrees (i.e., equal to the friction angle) for associative plasticity. It is clear that the dilatancy angle partially affects the second slope of the response. From this figure, and by comparing the response with test results for all specimens (not shown here for brevity), it became apparent that the response of FRP-confined concrete can best be modeled by non-associative plasticity. Therefore, the dilatancy angle was selected as zero for the remainder of this study.

**Calibration of the Model:** Based upon the above sensitivity analysis, a detailed calibration study was conducted to establish the values of the DP parameters that best fit the experimental data. The analysis showed that best results can be expected by setting the cohesion, internal friction angle, and the dilatancy angle to 1200 psi, 28 degrees, and zero degrees, respectively. Although the compressive strength of concrete for different batches varied from 4,299 psi to 4,637 psi, this small range did not warrant changing the $\phi$ and $c$ parameters for each batch of concrete. Figures 4.18 to 4.26 show comparison between the test data and the finite element prediction curves with the above selected DP parameters. Generally, a good agreement with test results can be noted for all jacket thickness’ and concrete batches.

**Volumetric Response**

Although the Drucker-Prager model is shown to closely match the stress-strain results for the FRP-confined concrete, its applicability need to be further examined with regard to the volumetric response. This is particularly important because studies by Mirmiran and Shahawy [5] have shown a unique characteristic for the volumetric response of FRP-confined concrete. The volumetric response is calculated as
Figure 4.18 Comparison of Finite Element Model with 6-Ply Specimens of Batch A
Figure 4.19 Comparison of Finite Element Model with 10-Ply Specimens of Batch A
Figure 4.20 Comparison of Finite Element Model with 14-Ply Specimens of Batch A
Figure 4.21 Comparison of Finite Element Model with 6-Ply Specimens of Batch B
Figure 4.22 Comparison of Finite Element Model with 10-Ply Specimens of Batch B
Figure 4.23 Comparison of Finite Element Model with 14-Ply Specimens of Batch B
Figure 4.24 Comparison of Finite Element Model with 6-Ply Specimens of Batch C
Figure 4.25 Comparison of Finite Element Model with 10-Ply Specimens of Batch C
Figure 4.26 Comparison of Finite Element Model with 14-Ply Specimens of Batch C
\[ \frac{\Delta V}{V} = \varepsilon_v = \varepsilon_c + 2\varepsilon_r \]  

(4.1)

where \( \varepsilon_v \) = volumetric strain, \( \varepsilon_c \) = axial strain, and \( \varepsilon_r \) = lateral strain. The sign convention is negative for lateral strains (tensile strains). Volume reduction (compaction) is positive, whereas volume expansion (dilation) is negative. Figure 4.27 shows the volumetric curves for the 6-ply specimens of Batch A, along with the predicted response from the finite element analysis. The analysis was performed with the same DP parameters that were used for the stress-strain curves. The reversal of the volumetric expansion that is clear from the experimental curves is unique to FRP-confined concrete [4]. The FE model, however, only shows a volumetric compaction up to the point of failure, very similar to the response of steel-confined concrete. Therefore, it appears that the Drucker-Prager plasticity can not fully capture all facets of the response of FRP-confined concrete, and further improvements on the modeling of the yielding surfaces may be necessary.

**Dilation Response**

Another unique behavior of FRP-confined concrete is in its dilation response. In order to shed more light on the difference between the Drucker-Prager plasticity and that seen in the response of FRP-confined concrete, the dilation curves are compared. The dilation rate is calculated as

\[ \mu = \frac{d\varepsilon_r}{d\varepsilon_c} = \frac{\varepsilon_r(i) - \varepsilon_r(i-1)}{\varepsilon_c(i) - \varepsilon_c(i-1)} \]  

(4.2)

where \( \varepsilon_r \) and \( \varepsilon_c \) represent the lateral and axial strains, respectively, and the subscripts \( (i) \) and \( (i-1) \) refer to the two consecutive time steps. This relationship can be readily
Figure 4.27 Comparison of Finite Element Model with Volumetric Response of FRP-Confined Concrete
determined from the tabulated information obtained for both the FE model and the tested specimens. Figure 4.28 shows the dilation curves for the 6-ply specimens of Batch A, along with the predicted response from the finite element analysis. The analysis was performed with the same DP parameters that were used for the stress-strain curves. The results clearly show why the DP model fails to capture the volumetric response of FRP-confined concrete. As shown in the figure, the predicted dilation curve matches fairly well with both the initial slope and the proximity of the transition point (with respect to the axial strains). However, the form of plasticity is clearly different. Whereas the actual response of FRP-confined concrete shows a peak in the dilation rate, the DP model, as expected, follows an elastic-perfectly-plastic response. The key in modeling of FRP-confined concrete, therefore, seems to be in representing the yielding surface of the material. Although the DP model can be calibrated fairly well for predicting the axial stress-strain curves, it does not properly establish the true expansion tendencies of the FRP-confined concrete.

**Shape Effect**

An experimental investigation into the effect of cross-sectional shape on the confinement with fiber composites was performed by Pico [34]. He tested a total of 9 concrete-filled FRP tubes with square sections. All tubes were 6'' x 6'' x 12'', and with the same concrete batch with a compressive strength of 5,890 psi. Three tube thicknesses were used; 6, 10, and 14 plies, similar to the circular sections. Figures 4.29 shows the typical failure mode of concrete-filled square FRP tubes. The stress concentrations
Figure 4.28 Comparison of Finite Element Model with Dilation Response of FRP-Confined Concrete
around the edges are apparent from the with patches and the jacket fracture. Pico [34] reported that the corner radius of square sections greatly affects the response.

![Figure 4.29 Typical Failure Mode of Concrete-Filled Square FRP Tubes [34]](image)

Figure 4.29 Typical Failure Mode of Concrete-Filled Square FRP Tubes [34]

The present study attempts to model the same behavior by the finite element method. The FE model will be used to perform a parametric study of the corner radius. The modeling process was explained previously in Chapter 3. The study was conducted on a 6” x 6” x 12” concrete-filled square FRP tube with 6, 10, and 14 plies. The same concrete strength as that of Batch B was used in the analysis. Therefore, the DP parameters of $\phi$, $c$, and dilatancy angle were set to 28 degrees, 1,200 psi, and zero, respectively. Due to symmetry, only the top quarter of the specimen was modeled. The effect of the corner radius was first studied for the 6-ply specimen. Four different radii
were considered; 0, 0.75, 1.5, and 3 inches, which span the entire range between a sharp edge square section and a circular section. The sharp edge section was actually modeled by a very small radius of 0.1 inch. The shape effect parameter was defined as $(2R/D)$, where $R =$ corner radius, and $D =$ inside dimension of the square tube. The shape effect parameter for the cases studied ranged between 0 and 1.

Figure 4.30 shows the contour plot of nodal axial stresses in the sharp edge square section. The figure illustrates the concentration of axial stresses in the corner of the square section. It should be noted here that for all circular and square sections, as explained earlier in this chapter, a very low modulus of elasticity was assumed for the shell in the vertical direction. This would ensure that the jacket only acted as a hoop tension band, rather than axial reinforcement for the concrete core. As a result of this assumption, the stress contours show negligible axial stresses for the shell itself. The figure, however, shows a uniform axial stress everywhere except for the corner, where an area of low utilization (red zone) and an area of high stress concentration are visible. An enlarged view of the axial stress contours around the corner is shown in Figure 4.31. The dark blue zone signifies the area of high stress concentration. At the nodes away from the corner, stresses are considerably lower. The stress gradient is also high, as the corner radius is very small (almost zero).

Figures 4.32 and 4.33 show the axial stress contours for the square sections with 0.75” and 1.5” corner radii, respectively. The shape effect parameter for these sections is 0.25 and 0.5, respectively. Figure 4.34 shows the stress contours for the circular section.
Figure 4.30 Axial Stress Contours for the Sharp Edge Square Section

Figure 4.31 Enlarged View of Axial Stress Contours at the Corner of Sharp Edge Section
Figure 4.32 Axial Stress Contours for the Square Section with 0.75” Corner Radius

Figure 4.33 Axial Stress Contours for the Square Section with 1.5” Corner Radius
These figures collectively point to three important qualitative conclusions:

1. Stress concentration around the edge of square sections is a function of the corner radius. As the corner radius increases, the stress concentration decreases, and the stresses become more uniform.

2. Stress gradient (change of stresses over a distance) is much higher for sharper corner radii. As the section becomes closer to a circle, the gradient approaches zero.

3. By comparing the deformed shape of the tube (shown by solid lines) and the original shape of the cross section (dashed lines), it becomes clear that the walls of the tube bulge outward near the side of the cross-section and away from the corners. This is primarily due to the low flexural rigidity of the walls as compared to their axial rigidity at the corner.
These qualitative observations are similar to those made by Pico [34] in his experimental work. A quantitative analysis was also carried out using the time-history post-processor. Figure 4.35 shows the biaxial stress-strain response of the 6-ply square FRP tube with concrete of Batch B, and four different corner radii, as explained before. The stress-strain response is plotted for the concrete element at the origin of the coordinate system. As shown, the circular section has higher strength and stiffness than the square sections. For sharper edge sections, strength and stiffness of the section both decrease.

Figure 4.36 to 4.39 show the spatial variation of axial stress-radial strain curves for each of the selected radii. In each figure, the stress-strain response at three locations are shown as follows:

1. Concrete element at the center of the section (origin of the coordinate system);
2. Concrete element at the corner of the section; and
3. Concrete element at the mid-span of the wall, i.e., side element.

The figures show that the corner element is stressed much higher than any other point in the section. On the other hand, the side element is least stressed among the ones compared. As the corner radius increases, the difference between the center element and the corner element is decreased markedly. However, since stresses in the side element are function of the flexural span of the wall (i.e., D-2R), they do not change considerably. Uniformity of the stress-strain response in circular sections is paramount from Figure 4.39. The same conclusions can be made from Figure 4.40, which quantifies the effect of
Figure 4.35 Effect of Corner Radius on Biaxial Stress-Strain Response of FRP-Confined Concrete
Figure 4.36 Spatial Variation of Axial Stress-Radial Strain Response for Sharp Edge Square Section
Figure 4.37 Spatial Variation of Axial Stress-Radial Strain Response for 0.75" Corner Radius
Figure 4.38 Spatial Variation of Axial Stress-Radial Strain Response for 1.5" Corner Radius
Figure 4.39 Spatial Variation of Axial Stress-Radial Strain Response for Circular Section
Figure 4.40 Effect of Corner Radius on Spatial Variation of Radial Strains
corner radius on the radial strains at various locations within the cross section. It is clear that corner radius affects the stress concentration as well as stress gradient.

In order to show the effect of jacket thickness on the stress-strain response of square sections, the case of sharp edge square section was studied with three different jacket lay-ups; 6, 10, and 14 plies. Figure 4.41 shows the axial stress-strain response at the center element of each section. The figure confirms the experimental results of Pico [34], in that the jacket thickness does not significantly affect the maximum stresses. Finally, a comparison was made with the test results of Pico [34]. In Figure 4.42, the FE prediction curves are shown as solid lines, whereas the experimental curves are shown as dashed lines. Note that the descending post-peak branch is not captured by the model. It is expected that the corner radius may affect the DP parameters, and further studies are needed to address this issue.

Cyclic Loading

In order to see the applicability of the Drucker-Prager model and the finite element analysis for cyclic loading of FRP-confined concrete, a simple case study was conducted on a circular section. Figure 4.43 shows a typical stress-strain response of the FRP-confined concrete cylinder subjected to five cycles of loading and unloading. The input file was modified to incorporate a user-defined deflection limit at which the specimen would be unloaded. The user will also specify the number of cycles that the specimen would be subjected to loading and unloading. From the figure, the following conclusions can be made:
1. The finite element analysis and the Drucker-Prager plasticity model can be effectively used to apply multiple cycles of loading and unloading to FRP-confined concrete.

2. Since the DP model corresponds to an elastic-perfectly-plastic material, the cyclic response does not show any stiffness or strength degradation. The true behavior of FRP-confined concrete, however, is different, and some degradations occur [4].
Figure 4.41 Effect of Jacket Thickness on Confinement with Square Sections
Figure 4.42 Comparison of Finite Element Model with Test Results of Pico [34]
Figure 4.43 Stress-Strain Response of FRP-Confined Concrete (Model and Tests) Under Cyclic Loading
CHAPTER 5

SUMMARY AND CONCLUSIONS

An efficient use of fiber reinforced plastic (FRP) material is in the form of concrete-filled tubes, where the tube not only protects the concrete against environmental factors, but also acts as the pour form, confinement, and external reinforcement. This study was focused on developing a nonlinear finite element model for the analysis of FRP-confined concrete. Solid elements were used for the concrete core, along with a non-associative Drucker-Prager (DP) plasticity, which takes into account the pressure sensitivity of the material. The parameters used to model the concrete included cohesion, angle of internal friction, and the dilatancy angle. The jacket was modeled by linear-elastic membrane shell elements. A parametric program was developed inside ANSYS® software to automatically generate the mesh for various geometric shapes and material properties. A sensitivity analysis was conducted to determine the optimum number of elements required in the mesh.

The DP parameters were calculated using four different models; Richart et al. [30], Samaan et al. [5], Mander et al. [17], and Rochette and Labossière [27]. Of these models, the ones by Richart et al. [30] and Mander et al. [17] consistently overestimated the strength and stiffness of the confined concrete. Samaan et al. [5] provided a much better correlation with test data, especially for the transition point. However, the model of
Rochette and Labossière [27] clearly matched the test data most favorably. Sensitivity analysis of the DP parameters showed that while the second slope of the response depends mainly on the friction angle, the location of the transitions point is a function of both cohesion and friction angle. The analysis also showed that the response of FRP-confined concrete can best be modeled by a non-associative plasticity, i.e., a zero dilatancy angle. Calibration of the model against the test data of Scherer [33] showed that best results can be expected by setting the cohesion, internal friction angle, and the dilatancy angle to 1,200 psi, 28 degrees, and zero degrees, respectively, for the range of concrete strengths in the experiments.

Study of the volumetric and dilation response of FRP-confined concrete further showed that although the Drucker-Prager plasticity can be applied fairly well for predicting the axial stress-strain response, it does not properly establish the true expansion tendencies of the FRP-confined concrete, because the DP model corresponds to an elastic-perfectly-plastic material.

The finite element model was also used to perform a study on the effect of square versus circular sections. It was concluded that stress concentration around the edges of square sections is a function of the corner radius. As the corner radius increases, the stress concentration decreases, and the stresses become more uniform. Also, stress gradient (change of stresses over a distance) is much higher for sharper corner radii. As the section becomes closer to a circle, the stress gradient approaches zero. Spatial variation of stresses further indicates that corner elements are stressed much higher than any other point in the section. On the other hand, the side elements are least stressed among the ones compared. As the corner radius increases, the differences between the
center element and the corner element are decreased markedly. However, since stresses in the side element are a function of the flexural span of the wall (i.e., D-2R), they do not change considerably.

Cyclic analysis of FRP-confined concrete showed that the finite element analysis and the Drucker-Prager plasticity model can be effectively used to apply multiple cycles of loading and unloading to FRP-confined concrete. However, since the DP model corresponds to an elastic-perfectly-plastic material, the cyclic response does not show any stiffness or strength degradation. The true behavior of FRP-confined concrete, however, is different, and some degradations occur.
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