Integrating Multiobjective Optimization With The Six Sigma Methodology For Online Process Control

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INTEGRATING MULTIOBJECTIVE OPTIMIZATION WITH THE SIX SIGMA METHODOLOGY FOR ONLINE PROCESS CONTROL

by

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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Industrial Engineering and Management Systems in the College of Engineering and Computer Science at the University of Central Florida Orlando, Florida

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ABSTRACT

Over the past two decades, the Define-Measure-Analyze-Improve-Control (DMAIC) framework of the Six Sigma methodology and a host of statistical tools have been brought to bear on process improvement efforts in today’s businesses. However, a major challenge of implementing the Six Sigma methodology is maintaining the process improvements and providing real-time performance feedback and control after solutions are implemented, especially in the presence of multiple process performance objectives. The consideration of a multiplicity of objectives in business and process improvement is commonplace and, quite frankly, necessary. However, balancing the collection of objectives is challenging as the objectives are inextricably linked, and, oftentimes, in conflict.

Previous studies have reported varied success in enhancing the Six Sigma methodology by integrating optimization methods in order to reduce variability. These studies focus these enhancements primarily within the Improve phase of the Six Sigma methodology, optimizing a single objective. The current research and practice of using the Six Sigma methodology and optimization methods do little to address the real-time feedback and control for online process control in the case of multiple objectives.

This research proposes an innovative integrated Six Sigma multiobjective optimization (SSMO) approach for online process control. It integrates the Six Sigma DMAIC framework with a nature-inspired optimization procedure that iteratively perturbs a set of decision variables providing feedback to the online process, eventually converging to a set of tradeoff process configurations that improves and maintains process stability. For proof of concept, the approach is applied to a general business process model – a well-known inventory management model – that is formally defined and specifies various process costs as objective functions. The proposed
SSMO approach and the business process model are programmed and incorporated into a software platform. Computational experiments are performed using both three sigma (3σ)-based and six sigma (6σ)-based process control, and the results reveal that the proposed SSMO approach performs far better than the traditional approaches in improving the stability of the process. This research investigation shows that the benefits of enhancing the Six Sigma method for multiobjective optimization and for online process control are immense.
To Prophet Muhammad (peace be upon him)

To my beloved father, Hashiem Abualsauod

To my lovely mother, Huda Blisi
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Finally, I would also like to thank my ‘Abualsauod family’: Abdullah, Haitham, Hanadi, Hawazin, and Ahmed for their patience, help and constant support throughout my student life at UCF. You are and always will be my family.
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CHAPTER 1:
INTRODUCTION

1.1 The Six Sigma Methodology

In industrial manufacturing and service environments, it is important to reduce the variation of processes in order to improve the overall quality within an organization. The traditional evaluation of quality level is usually performed based on measuring customers’ needs. This evaluation generally transforms customers’ needs into target values that are compared to the average performance measures of the process (or product). As the deviation between average measures and target values decreases, the quality level of the process (or product) increases. Customers desire consistent, reliable, and predictable processes and products that deliver best-in-class levels of quality. This is what the Six Sigma methodology strives to achieve (Kapur & Feng, 2005).

Over the years, many service and manufacturing organizations have implemented the Six Sigma methodology improving the average yield in these organizations after its implementation (Kumar et al., 2008). For instance, in the 1990s, General Electric (GE) adopts Six Sigma in almost every division of the company, implementing approximately 6,000 process improvement projects. From the implementation of the process improvement project results in the first few years at the company under the Six Sigma methodology, GE announces that the savings from implementing Six Sigma projects reaches about US$150 million. By the end of 1999, GE reports US$3 billion in savings attributed to the Six Sigma improvement projects, (Pyzdek et al., 2009; Snee & Hoerl, 2003). Table 1.1 is a partial listing of the industrial organizations, projects, performance metrics and benefits/savings realized when implementing Six Sigma process improvement projects.
Table 1.1: Reported benefits/savings from implementing the Six Sigma methodology at industrial organizations (summarized from Kwak & Anbari (2006)).

<table>
<thead>
<tr>
<th>Company / Project</th>
<th>Performance Measure</th>
<th>Benefits/Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raytheon / aircraft integration systems</td>
<td>Depot maintenance inspection time</td>
<td>Reduction in maintenance inspection time by 88% (in days)</td>
</tr>
<tr>
<td>General Electric / Railcar leasing business</td>
<td>Turnaround time at repair shops</td>
<td>Turnaround time reduced by 62%</td>
</tr>
<tr>
<td>Allied Signal (Honeywell) / Laminates plant in South Carolina</td>
<td>Capacity, cycle time, inventory, and on-time delivery</td>
<td>Increase in capacity by 50%, reduction in cycle time by 50%, and on-time delivery up 100%</td>
</tr>
<tr>
<td>Allied Signal (Honeywell) / Bendix IQ brake pads</td>
<td>Concept-to-Shipment cycle time</td>
<td>Shipment cycle time reduction of 10 months</td>
</tr>
<tr>
<td>Hughes Aircraft’s Missiles Systems Group / Wave soldering operations</td>
<td>Quality and productivity</td>
<td>Quality up 1000% and productivity up 500%</td>
</tr>
<tr>
<td>Dow Chemical / Rail delivery project</td>
<td>Financial</td>
<td>Savings of $2.45 million</td>
</tr>
<tr>
<td>DuPont / Yerkes plant in New York</td>
<td>Financial</td>
<td>Savings by approximately US$25 million</td>
</tr>
</tbody>
</table>

Since its inception, the Six Sigma methodology has been adopted in both manufacturing and service settings. It has been proven that it is an effective methodology for improving quality, productivity, cost, customer satisfaction, sales, and profitability (Deshmukh & Chavan, 2012; Zhu & Hassan, 2012). Such development is reflected in the rising trends in published studies as shown in Figure 1.1. Many studies address the fundamentals of Six Sigma and its applications from different perspectives such as in semiconductor manufacturing (e.g., Su & Chou, 2008), automotive parts manufacturing (e.g., Krishna et al., 2008), aluminum recycling (e.g., Das & Hughes, 2006), aerospace industry (e.g., Maleyeff & Krayenvenger, 2004), military (e.g., Stefanko, 2009), chemical industry (e.g., Motwani et al., 2004), banking industry (e.g., Immaneni et al., 2007), software industry (e.g., Antony & Fergusson, 2004), general service industry (e.g., Antony, 2006; Ehrlich, 2002; El-Haik & Roy, 2005; George, 2003), supply chain management (e.g., Knowles et al., 2005), healthcare industry (e.g., Taner et al., 2007), education (e.g., Weinstein et al., 2008), and Six Sigma deployment (e.g., Adams et al. 2003).
1.2 The Six Sigma Methodological Frameworks

In implementing the Six Sigma improvement methodology, two frameworks are commonly used: Define-Measure-Analyze-Improve-Control (or, DMAIC) and Define-Measure-Analyze-Design-Verify (or, DMADV). DMAIC focuses on improving existing processes, products, and/or services to meet customer needs, and DMADV focuses on designing new processes, products, and/or services to meet customer needs.

In recent years, many academicians and practitioners believe that it is possible to enhance Six Sigma by integrating it with other improvement approaches in order to achieve higher degree of quality (Nonthaleerak & Hendry, 2006; Thirunavukkarasu et al., 2008). For example, Rodriguez (2008) integrates the Six Sigma methodology with the Balanced Scorecard planning and management tool to improve business performance and customer satisfaction. Ehie & Sheu (2005) integrate the Theory of Constraints with the DMAIC framework. Amer et al. (2008) integrate DMADV with the fuzzy logic modeling to optimize order fulfillment within a supply chain. Miller & Ferrin (2005) integrate Six Sigma with simulation modeling to provide decision-
makers with the amount of improvement that might be possible to achieve the desired quality level and evaluate possible scenarios for improvement.

Brady (2005) and Brady & Allen (2006) point out that enhancing the Six Sigma methodology occurs within the following three levels of decision-making:

1. Micro Level – Involves the use of individual statistical methods that have been pre-defined and pre-selected;
2. Meso Level – Supervisor level of decision-making about method selection and timing; and
3. Macro Level – Deals with the overall quality programs.

Table 1.2 summarizes some of the proposed areas of future research that relate to the Six Sigma methodology and Six Sigma practice as it relates to the three levels of decision-making. According to Allen (2006), research on new statistical micro-level methods for general uses can be highly valuable to Six Sigma practitioners. Furthermore, advances in computing technology and optimization techniques provide unprecedented opportunities for the development of enhanced Six Sigma-based improvement methods.

Table 1.2: Overview of future research in Six Sigma (Allen, 2006).

<table>
<thead>
<tr>
<th>Proposed Area</th>
<th>Decision-Making Level</th>
<th>Possible Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apply quantitative techniques to analyze overall quality performance and management practices</td>
<td>Macro</td>
<td>Enhanced overall quality level adoption and management strategies</td>
</tr>
<tr>
<td>Improved design of project strategies</td>
<td>Meso</td>
<td>Improved training plans and resources</td>
</tr>
<tr>
<td>Apply quantitative techniques</td>
<td>Micro</td>
<td>User-friendly software offering additional method options</td>
</tr>
</tbody>
</table>

1.3 **The Consideration of Multiple Objectives in the Six Sigma Methodology**

A firm’s ability to make the most appropriate critical decisions can translate into a great competitive advantage. Often, these critical decisions involve multiple and conflicting objectives that must be addressed simultaneously. Multiobjective optimization solution approaches aim at
finding “satisfying” solutions when the problem involves more than one objective (Coello Coello, 2006; Deb, 2001). Therefore, the integration of optimization procedures and the Six Sigma methodology could aid in capturing and reducing such variations in the presence of multiple objectives.

As pointed out by Brady & Allen (2006), McManus (2006), Rybarczyk (2005) and Tjahjono et al. (2010), new gaps in Six Sigma research exist. There is, in fact, a need for robust techniques that allow Six Sigma researchers and practitioners to apply the methodology to problems with multiple objectives which can provide multiple compromised solutions that help to improve process control efficiency.

1.4 Objectives of this Research Investigation

To date, few researchers address integrating multiobjective optimization techniques with the Six Sigma methodology, such as using multiobjective optimization for robust Design for Six Sigma (DFSS) in manufacturing (e.g., Baril et al., 2010; Shimoyama et al., 2008), or using multiobjective stochastic modeling to improve existing service processes (e.g., Franca et al., 2010). However, the literature does not provide a holistic framework for implementing the integration of the Six Sigma methodology and multiobjective optimization techniques.

This research aims to explore and develop the statistical and optimization strategies in order to improve the Six Sigma methodology. More specifically, the integration of multiobjective optimization within the Improve and Control phases of the DMAIC framework is explored with the following primary objectives of this investigation.
Objective 1: Rationalize that the Six Sigma quality approach and the multiobjective optimization strategies can be effectively integrated for online process control to enhance decision-making at the micro level.

Objective 2: Build a holistic framework that allows for the robustness of online process performance optimization using an integration of the multiobjective optimization and three-sigma (3σ) quality evaluation, and

Objective 3: Enhance framework to allow online process performance optimization using an integration of the multiobjective optimization and the Six Sigma methodology.

This investigation should contribute quite significantly to the body of knowledge and advance the state-of-the-art in design optimization and the Six Sigma methodology. This research potentially improves the process performance, and in turn, improves process control, and decision-making.

1.5 Organization of the Remainder of this Document

The remainder of this document is organized as follows. In Chapter 2, an overview of the Six Sigma methodology is presented, explaining the general methodology and current enhancements to the Six Sigma methodology including to its DMAIC framework. Chapter 3 provides an overview of statistical process control (SPC) as it applies to Six Sigma and the DMAIC framework, and Chapter 4 presents a brief overview of multiobjective optimization. Readers familiar with SPC and multiobjective optimization may proceed directly to Chapter 5,
without loss of continuity, where the proposed Six Sigma multiobjective optimization (SSMO) approach is described.

The performance of the proposed SSMO approach is evaluated. First, Chapter 6 presents a computational study using the approach under a three-sigma \((3\sigma)\) quality level. Then, in Chapter 7, a computational study is conducted using the proposed approach under a six sigma \((6\sigma)\) quality level. This work is concluded in Chapter 8 with a summary of the accomplishments and directions of future research.
CHAPTER 2: REVIEW OF PREVIOUS RELATED LITERATURE

2.1 Introduction

The integration of Six Sigma and multiobjective optimization is the combination of two proven methodologies focused on improving decision-making. Very limited research exists that focuses on the integration of the Six Sigma methodology and optimization strategies. This chapter is divided into five sections. Sections 2.2 and 2.3 provide an overview of the two primary areas of study – Six Sigma and multiobjective optimization. Section 2.4 provides a detailed review of the Six Sigma methodology and multiobjective optimization Six Sigma-based research. Finally, Section 2.5 summarizes the chapter.

2.2 Overview of Optimization Methods

Most of existing real-world problems involve the simultaneous optimization of multiple objectives. In the last two decades, many researchers and practitioners are becoming more interested in the multiobjective field optimization due to the fact that most of decision-making problems involve several measures of process performance, which need to be optimized simultaneously. One of the most common results in optimizing multiple objectives simultaneously is that solutions, in general, are not uniquely determined. This is because most problems that involve conflicting objectives and result in multiple compromise solutions (or, tradeoff), where a decision-maker can select the most desirable solution among the multiple solutions.
Over the years, researchers use several optimization techniques and methods to handle multiobjective optimization problems. These methods categorized based on gradient-based and non-gradient-based methods, as shown in Figure 2.1.

Figure 2.1: Overview of common optimization methods.

2.2.1 Gradient-Based Optimization Methods

Gradient-based search methods can be divided into two different categories: Direct and Indirect. Direct methods determine the exact solution in a fixed number of operations, and indirect methods produce approximations in an undetermined number of operations (Jamil, 2012). Several studies address integrating gradient-based techniques with the Six Sigma methodology, as is discussed in Section 2.4.2. However, as pointed out by Feng (2005) and Koch et al. (2004), the objective in Six Sigma-based optimization usually falls under multimodal problems, and thus the solution reached may only be a local optimum and not a global optimum. To find global optima, one should start gradient-based optimization iteratively from multiple starting points.
2.2.2 Non-Gradient-Based Optimization Methods

With the advancements in computing technology over the past decade, faster and more enhanced non-gradient-based approaches have been developed that are capable of handling complex problems with much less computational expense. Thus, these types of methods are preferred in the cases where decisions are time-sensitive. Figure 2.1 further classifies non-gradient-based methods.

In solving multiobjective optimization problems, many of previous work convert the set of multiple objectives into a single objective leading to a single solution. This technique and other techniques that use single weighted objective function lack of optimizing multiple objectives simultaneously (Deb, 2001). Therefore, researchers and practitioners are motivated to search for alternative techniques that optimize multiple objectives simultaneously and provide a set of multiple compromised (i.e., Pareto optimal) solutions instead of a single solution. Thus, the development of multiobjective optimization solution procedures using Pareto-based methods is preferred. Further information about Pareto-based methods can be found in Chapter 4.

2.3 Overview of the Six Sigma Methodology and Its Frameworks

Six Sigma is a focused method developed based on common proven quality concepts and principles such as the 14 management principles of Deming, Juran, Crosby, and others. Six Sigma also incorporates the use of statistical tools (Pyzdek et al., 2009). In the pursuit of improved quality, two structured frameworks (shown in Figure 2.2) are used for implementing the Six Sigma methodology: (1) Define-Measure-Analyze-Improve-Control (DMAIC) and Define-Measure-Analyze-Design-Verify (DMADV) (Al-Aomar, 2006). While DMADV focuses on designing new processes, products, and services to meet customer needs at the 6σ level,
DMAIC focuses on improving existing processes, products and services (Ferrin & Muthler, 2002).

Figure 2.2: The Six Sigma DMAIC and DMADV frameworks.
2.3.1 Six Sigma Measurement of Process Performance

The Six Sigma process performance can be measured by the two measures: (1) Defect Rate and (2) Sigma Quality Level.

2.3.1.1 Defect Rate

A defect is defined as a nonconformance to customer requirements while delivering a service or product to customers (Crosby, 1995). A defect rate $p$ is the ratio of the number of defective items that are out of specification to the total inspected items. The number of defective items out of one million processed or inspected items is called the parts per million (PPM) defect rate. In some cases, as in service settings, where PPM cannot be used, another performance measure called defects per million opportunities (DPMO) is often used to evaluate process performance. DPMO is the number of defective items that do not meet the required specification out of one million possible opportunities.

A process is considered meeting a $6\sigma$ quality level when its performance achieves 3.4 DPMO under the assumption that the process performance values are normally-distributed. To calculate DPMO, the total number of defects is divided by the total number of opportunities for a defect. Then, that quotient is multiplied by $10^6$. In other words,

$$DPMO = \frac{D}{U \times O} \times 10^6 \quad (2.1)$$

where $D$ is number of defects identified in a collected sample of process performance observations, $U$ is the number of units in the sample and $O$ is the number of opportunities for error per unit.
2.3.1.2 **Sigma Quality Level**

Sigma $\sigma$ is used to represent variability, and a sigma quality level is associated with process variation and specification limits. Specification limits are the design tolerances or performance ranges that customers demand of the products they consume or of the services they are rendered. The sigma quality level for a production or service process is the distance from a process mean to the closer specification limit and is computed as

$$\sigma = \frac{\tau}{2 \times S_p}$$ (2.2)

where $\sigma$ is the sigma level of quality, $S_p$ is the sample standard deviation of the process and $\tau$ is the desired allowable process tolerance.

In practice, it is desired to maintain the mean performance of the process at a target value; however, the process mean varies and drifts over the long-term for various reasons. This means that the process mean shifts from the target value periodically, which causes a change in the defect rate $p$. A process maintains Six Sigma quality level when the process mean aligns with a target value, and the distance from the process mean to each specification limit is $6\sigma$ (Davis et al., 1993; Zeng, 2009). Table 2.1 summarizes how the defect rate changes when the sigma quality level changes with respect to a shift in the process mean (Breyfogle, 2003).
Table 2.1: Defect rates at various sigma quality levels (Breyfogle, 2003).

<table>
<thead>
<tr>
<th>Sigma Quality Level</th>
<th>DPMO (Process Mean Aligned with Target Value)</th>
<th>DPMO (Process Mean with 1.5σ Shift)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1σ</td>
<td>317,311</td>
<td>697,672</td>
</tr>
<tr>
<td>2σ</td>
<td>45,500</td>
<td>308,770</td>
</tr>
<tr>
<td>3σ</td>
<td>2,700</td>
<td>66,811</td>
</tr>
<tr>
<td>4σ</td>
<td>63.4</td>
<td>6,210</td>
</tr>
<tr>
<td>5σ</td>
<td>0.57</td>
<td>233</td>
</tr>
<tr>
<td>6σ</td>
<td>0.002</td>
<td>3.4</td>
</tr>
</tbody>
</table>

The Six Sigma methodology allows for a maximum shift of the process mean that are ±1.5σ from the mean. Table 2.2 provides a brief description of the most common methods used in detecting shifts in mean, along with their strengths and weaknesses (Alexandrov et al., 2012; Cai et al., 2012; Chen, 2011; Fryer & Nicholson, 1999; Rodionov, 2005).
Table 2.2: Common methods for detecting shifts in the process mean (summarized from Alexandrov et al., 2012; Cai et al., 2012; Chen, 2011; Fryer & Nicholson, 1999; Rodionov, 2005).

<table>
<thead>
<tr>
<th>Method</th>
<th>Characteristics</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-test</td>
<td>Based on determining the probability of change point appearance that exceeds a given threshold.</td>
<td>Strong technique that assumes normality and equal variances</td>
<td>Lack of accuracy for data that do not follow normal distribution</td>
</tr>
<tr>
<td>Mann-Kendall</td>
<td>Non-parametric test that ranks the data to obtain change point occurrence.</td>
<td>Easy to use</td>
<td>Not efficient with data that exhibits trend patterns</td>
</tr>
<tr>
<td>Signal-to-Noise Ratio</td>
<td>Uses signal-to-noise ratio which compares a single value with input data in order to determine the confidence level of mean shift occurrence</td>
<td>Fairly easy to use</td>
<td>Similar to Mann-Kendall method, this method lacks accuracy in the case of trends, and it is not efficient for a single change point scenario</td>
</tr>
<tr>
<td>Bootstrap</td>
<td>Based on testing homogeneity in the mean value of the input data to calculate the confidence level of mean shift occurrence</td>
<td>Works with any frequency distribution with no assumption acquired</td>
<td>Require large amount of data to increase test sensitivity</td>
</tr>
<tr>
<td>Regression</td>
<td>Uses forecasts future values based on previously observed values to use it for detecting shifts.</td>
<td>Strong for detecting multiple change points.</td>
<td>Require small amount of data to increase test sensitivity</td>
</tr>
</tbody>
</table>

2.4 Integration of the Six Sigma Methodology with Other Methods

Several existing studies focus on extending the Six Sigma methodology by integrating the methodology with other methodologies and philosophies including computer simulation, the Theory of Constraints, data envelopment analysis and multiobjective optimization.

2.4.1 Six Sigma and Simulation

Hahn et al. (2000) argue that simulation is becoming a mainstay decision analysis process due to its ability to evaluate candidate decisions for difficult business questions. They point out that simulation can be used within the Six Sigma methodology to evaluate improvement
alternatives. Other researchers discuss the integration of Six Sigma and computer simulation in order to improve the a better solutions, such as improving patient experience at hospitals (e.g., Miller & Ferrin 2005), reducing the processing time of the lodge at the Center of Capital One Financial Service Corporation (Seifert 2005), addressing the effects of variation and assessing interaction effects between various subsystems for improving the operational and design issues in a server manufacturing environment (e.g., Ramakrishnan et al., 2008), improving customer satisfaction by reducing waiting time at a communications company. (e.g., Goldman et al., 2003), increasing the probability and reducing process and product development cycle time (e.g., Luce et al., 2005), reducing the waiting time at an emergency department (Mandahawi et al., 2010), designing assembly lines in manufacturing facilities (e.g., Tjahjono et al., 2009). Other researchers explore the fundamental relationships between Six Sigma and simulation (e.g., El-Haik & Al-Aomar, 2006; Ferrin & Muthler, 2002). They summarize the impact of using simulation within the different phases of the DMAIC framework. Although simulation has been successfully integrated with both the Six Sigma DMAIC and DMADV frameworks, using simulation alone lacks optimizing ability, and thus, should be combined with other analysis techniques to become more effective for process design and control.

2.4.2 Six Sigma and the Theory of Constraints

Theory of Constraints (TOC) is introduced by Eliyahu M. Goldratt during the 1980s (Goldratt et al., 1992). For the purpose of continuous improvement, TOC uses a systematic approach which consists of four steps:

1. Identify a system’s constraints (i.e., bottleneck and non-bottlenecks).
2. Develop a plan to effectively utilize the system’s bottleneck.
3. Subordinate non-bottlenecks that support the bottleneck utilization plan.

4. After resolving a constraint, go back to Step 1.

Similar to the Six Sigma methodology, TOC has been successfully implemented in many settings. Sierra & Malone (2003) integrate the Six Sigma methodology with TOC and propose a new technique in handling management constraints. Furthermore, Ehie & Sheu (2005) propose a framework that combines TOC and the Six Sigma methodology to improve its gear cutting operation at an axle manufacturing company. The framework identifies the system constraints and throughput using TOC and then develops an improvement plan under the Six Sigma methodology. After implementing the proposed framework, the company reduces the inventory level of blades and the estimated total savings by $200,000 per year.

Although the integration of Six Sigma and TOC can provide managers with a continuous improvement platform, the integrated frameworks proposed by Sierra & Malone (2003) and Ehie & Sheu (2005) have limitations in handling multiple objectives. Moreover, the proposed frameworks do not evaluate other possible optimization scenarios which would provide decision-makers with additional information regarding their robustness.

2.4.3 Six Sigma and Data Envelopment Analysis

Data envelopment analysis (DEA) is introduced by Charnes (1978) to measure the scale efficiencies of various public sector firms. It is a linear programming technique and considered to be an effective method for measuring the relative performance of organizational units when the presence of multiple inputs and outputs makes comparisons difficult (Charnes, 1978; Kumar et al., 2007). Feng & Antony (2010) develop an integrated DEA and Six Sigma model and evaluate it through a case study for assessing and improving health service efficiency. The
authors implement DEA within the Six Sigma framework in such a way that DEA is used in each phase of the DMAIC framework. The Six Sigma project is carried out in the Department of Gynecological Oncology at the University of Texas M.D. Anderson Cancer Center. In the case study, physicians’ performance is evaluated by calculating the average efficiency score using the standard deviation of all physicians. Some of the benefits highlighted in the study are the reduction of the percentage of clinical time for inefficient physicians by 22.3% and the decrease in the associated cost by $1,708. This case study realizes most of the gain using DEA within the Six Sigma methodology. However, the study does not perform multivariate analysis of the DEA scores.

Several researchers integrate Six Sigma with other statistical applications and tools such as Pareto analysis (e.g., Thomas & Barton, 2006), histograms (e.g., Miles, 2006), run charts (e.g., Snee, 2004), control charts (Banuelas et al., 2005), hypothesis testing (Henderson & Evans, 2000), failure mode and effects analysis (FMEA) (e.g., Su & Chou, 2008), the gamma distribution (e.g., Hsu et al., 2008), cause-and-effect matrices (e.g., Sokovic et al., 2005), regression analysis (e.g., Kumar et al., 2008), capability analysis (e.g., Maleyeff & Kaminsky, 2002), sampling plans (e.g., Basu, 2004), designs of experiment (e.g., Li et al., 2006; Raisinghani et al., 2005), statistical process control (e.g., Schroeder et al., 2008), TRIZ (e.g., Smith & Phadke, 2005), t-test, chi-squared test, TOC and TRIZ combined (e.g., Shankar, 2010), scatter plots (e.g., Henderson & Evans, 2000), Quality Function Deployment (e.g. Sharma, 2003), artificial intelligence, fuzzy logic, and artificial neural networks (e.g., Patterson et al., 2005). However, few researchers discuss integrating Six Sigma with multiobjective optimization methods in order to improve the outcome of the process improvement and design in a way that
provides flexibility to decision-makers for choosing the best options from a set of alternatives for multiple process performance objectives.

2.4.4 Six Sigma and Multiobjective Optimization

Recent research considers the idea of combining the Six Sigma methodology and multiobjective optimization as a method to improve the outcome of the Improve and Design phases. Chen et al. (2008) propose an approach to address the multiobjective optimization problem by using Taguchi methods (Taguchi, 1995) within the Six Sigma methodology, specifically, within the Improve phase of the DMAIC framework in order to optimize the roundness of holes made by an aging plasma cutting machine. The researchers point out that using the traditional design of experiments techniques, such as $2^k$ factorial design, may increase the time and costs of a quality improvement process compared to the Taguchi methods.

Similarly, several existing works address the application of Response Surface Method (RSM) within the Six Sigma frameworks such as optimizing radial forging operation variables (e.g., Sahoo et al., 2008), reducing the cost of prototype and physical tests for a vehicle design process (e.g., Gu & Yang, 2006), minimizing the mass while observing deformation for a structural element (e.g., Roos et al., 2006), optimizing the performance of the inter-metal dielectric process (e.g., Su et al., 2009), optimizing a vehicle structural design for side impact crashworthiness (e.g., Koch et al., 2004; Vlahinos & Kelkar, 2002), improving the design of powertrain mounting systems (e.g., Wu & Shangguan, 2010), improving the qualified rate of coloring inspection about the Bevel gear in a centrifugal ventilator (e.g., Cao & Xie, 2011), and optimizing the design of cantilever and deep draw forming of a cylindrical cup (e.g., Jun & Juan, 2006). However, Koch (2002) points out that RSM approaches have difficulty in handling
continuous uncertainty profiles, like a normal distribution, and the tradeoff between multiple process design objectives. In regards to these difficulties, multiobjective optimization evolutionary algorithms (MOEAs) have advantages over RSM. Moreover, there is no plan for controlling the proposed improvement after implementation. Shimoyama et al. (2008) argue that integrating MOEA procedures with the Six Sigma methodology aids not only in achieving process improvement but also increases the method’s robustness (i.e., performance sensitivity against errors and uncertainties). Therefore, MOEAs have attracted considerable attention for more practical process designs and improvements.

Within the subject of integrating the Six Sigma methodology with multiobjective optimization, a few researchers use the approach that converts the multiobjective optimization problem to a single-objective problem. Six Sigma case studies in which this approach is used include optimizing the design of liquid packaging pump in order to design a pump that provide a flow rate between specification limits so that minimal defects are produced for the least cost per part (e.g., Luce et al., 2005), improving passenger safety and reducing vehicle cost (e.g., Sun et al., 2011), developing a multiobjective model for project portfolio selection to implement Lean and Six Sigma concepts (e.g., Hu et al., 2008), optimizing the airfoil design for a Mars aircraft (Shimoyama et al., 2007), optimizing a welded beam design in order to find an optimal set of dimensions that can carry a certain load with minimum total fabricating cost (Shimoyama et al., 2005; Shimoyama et al., 2008), improving decision settings for a set of manufacturing operations in order to improve the quality of products and reduce the associated cost (e.g., Azzabi et al., 2009), improving the process of penicillin production (Dassau & Lewin, 2006), minimizing the material flow intra- and interloops and minimization of the maximum amount of intercell flow, considering the limitation of tandem automated guided vehicle work loading (e.g., Shirazi et al.,
2010), minimizing the total cost involved in supply chain processes to ensure high delivery probability within customer-specified delivery windows (e.g., Antony et al., 2006), optimizing the process parameters of the deep drawing operations (e.g., Anand & Shukla, 2007), optimizing the design of a sliding rack to achieve the maximum performance with minimum weight (e.g., Cong et al., 2010), and finding solutions that are reasonably good in terms of optimality and robustness against small perturbations in values (e.g., Ono et al., 2009).

Although converting a multiobjective problem into a single-objective problem is considered to be a common approach to solving multiobjective optimization problems, the desire as highlighted in the literature on Six Sigma-based multiobjective optimization, is the need to optimize all objectives simultaneously in order to provide a set of multiple (i.e., Pareto optimal) solutions to reveal the tradeoff relationship among the multiple objectives. For example, Baril et al. (2010) use a DFSS interactive multiobjective optimization algorithm to generate a set of Pareto optima that maintain a probability of constraint satisfaction. The proposed methodology is applied to vehicle crash worthiness design optimization for side impact. Similarly, Nishida et al. (2008) utilizes particle swarm optimization with a multiobjective genetic algorithm to extract the significant Six Sigma-based design information within acceptable computational costs.

There are many existing studies using multiobjective optimization techniques to improve process control such as optimizing exponentially-weighted moving average (EWMA) parameters to improve detecting process mean shifts and to reduce process control cost (e.g., Aparisi et al., 2010; Epprecht et al., 2010), determining the best sample size for control charts using genetic algorithms (e.g., Kaya, 2009), enhancing testing power of $\bar{X}$ and $R$ control charts by using genetic algorithms to minimize the Type I error (e.g., Bakir & Altunkaynak, 2004), determining the economic design of the $\bar{X}$ control charts with a realistic monitoring error model embedded
(e.g., Shiau et al., 2006), designing a multivariate control scheme consisting of two or three $\bar{X}$ charts using genetic algorithms to optimize the charts parameters (e.g., Aparisi et al., 2010), optimizing the performance of attribute control charts using genetic algorithms (Perez et al., 2010). However, these studies do not consider the Six Sigma DMAIC and DMADV frameworks, nor is the Six Sigma quality level performance measure considered during the optimization process.

2.5 Summary

Although several researchers have integrated statistical techniques to improve the quality of the solutions generated using the Six Sigma methodology, there is limited work that addresses the use of multiobjective optimization techniques for process improvement and under the $6\sigma$ quality level expectation. Further, it can be concluded that the Six Sigma practitioners have yet to take full advantage of multiobjective optimization techniques in the Improve and Control Phases of DMAIC in order to design a robust and economic monitoring process that maintains variability at a $6\sigma$ level of quality.

In Chapter 3, an overview of statistical process control (SPC) as it applies to Six Sigma and the DMAIC framework is presented, followed by a brief overview of multiobjective optimization given in Chapter 4. Readers familiar with SPC and multiobjective optimization may proceed directly to Chapter 5 without loss of continuity, where the proposed Six Sigma multiobjective optimization (SSMO) approach is described.
CHAPTER 3:
OVERVIEW OF STATISTICAL PROCESS CONTROL

3.1 Introduction

Under the analytical decision-making concept, statistical process control (SPC) allows a decision-maker to monitor process variation and to evaluate the performance of a process. The critical decision in process control is deciding whether the variation appears in the process is natural or, requires correction to the process (Thor et al., 2007). The use of SPC to quantify and reduce variation is key to its implementation within the Six Sigma methodology (Breyfogle, 2003; Neave, 1990).

3.2 Statistical Process Control Tools

SPC uses statistically-based methods to evaluate a process or its output to achieve or maintain a state of control. To implement this concept in practice, it is important to know the statistically-based quality tools and their potential uses. A review of the open literature suggests that the most common SPC tools are: (1) Pareto analysis, (2) cause-and-effect (or, fishbone) diagrams, (3) scatter diagrams or plots, (4) check sheets, (5) control charts, (6) run charts, (7) normal probability plots, and (8) histograms. Although control charts are generally viewed at the core of the SPC tools, they are not, by themselves, sufficient for Six Sigma projects (Goh & Xie, 2003; Pyzdek & Keller, 2003). The integration of the most common SPC tools in the five phases of the Six Sigma DMAIC framework is needed.
3.2.1 Pareto Analysis

Pareto analysis is an SPC tool that is most commonly used in the Analyze phase of the DMAIC framework in order to display categories of problems graphically so they can be prioritized. The approach arranges data so that the few vital factors (approximately 20%) that are causing most (approximately 80%) of the problems are revealed. Pareto analysis shows in descending order of importance, impact or contribution the categories of problems, defects or opportunities.

3.2.2 Cause-and-Effect Analysis

Cause-and-effect analysis is another technique that is used by Six Sigma practitioners in the DMAIC Analyze phase to identify, display, and organize possible sources of variation for a specific problem or quality characteristic. Organizing the possible causes is usually performed based on categories such as people involved with the process, methods of how the process is performed, machines or resources used in the process, measurements and data generated from the process, and the environment where the process held. The analysis helps to identify the possible sources (or root causes) of the problem in what is commonly called the Fishbone diagram. This diagram visually displays the relationship between sources of variation and the effect that is being examined by putting the major categories of causes on major branches connecting to the backbone, and various sub-causes are attached to the branches. Similar to the Analyze phase, the Control phase uses SPC tools such as control charts to monitor variations while the process is operating over time.
3.3 Control Charts

Control charts have been successfully implemented in different settings such as manufacturing, service, healthcare, and many others (e.g., Di Mascio, 2002; Shang, 2011; Steiner & MacKay, 2005). These tools, which graphically display the variation of a process over time, are considered to be at the core of statistical process control and are used frequently in the Control phase of Six Sigma DMAIC framework.

3.3.1 Control Chart Characteristics

Statistical control charts depend on two main characteristics: (1) process variation and (2) control charts parameters.

3.3.1.1 Process Variation

Virtually every process has variation, and there are two types of variation that affect the quality characteristics of the process outcome in control charts. The first type is called special cause variation, also referred to as assignable cause variation. This type of variation results from causes that are not normally present in the process, and they can be traced, identified, and eliminated. The second type of variation is called common cause variation, which results from numerous, ever-present differences in the process. Control charts assist in identifying these two types of variation while monitoring process behavior.

3.3.1.2 Control Chart Components

There are different types of control charts. However, a control charts consist of the following six components:
1. Data. The set of observations collected before plotting the control charts.

2. Centerline (CL). Represents the mean value of the all collected data.

3. Plotting Areas. The upper and lower areas of the CL where values are plotted.

4. Vertical, or y-Axis. Represents the magnitude of the data collected.

5. Horizontal, or x-Axis. Displays the chronological order in which the data are collected.

6. Control Limits. The Upper Control Limit (UCL) and the Lower Control limit (LCL) are set at a distance based on the desired number of standard deviations (e.g., $3\sigma$, $4.5\sigma$, $6\sigma$, etc.) above and below the CL.

3.3.2 Process Capability and Specifications Limits

Process capability represents the ability of a process to achieve its purpose as managed by a decision-maker and the process functionality. In order to calculate process capability, specification limits need to be set first. Specification limits define the range of requirements for a product or service that are to be met. The value of the process capability index is calculated using Eq. 3.1, i.e.,

$$Cp = \frac{TL = (USL - LSL)}{6\sigma}$$

where $Cp$ is the process capability index, $TL$ is the specification tolerance, $USL$ is the upper specification limit, $LSL$ is the lower specification limit, and $\sigma$ is the standard deviation. This $Cp$ value is the percentage of product or service meeting the specification limits. For instance, a process capability of 1.0 means that 99.7% of a product (or service) is within the desired specification limits. If a process capability is less than 1.0, then the process quality is considered low, and most companies aim to achieve higher process capability index values. In order to
understand the relationship between control limits and specification limits, the following is a description of the possible cases that may occur (Montgomery, 2007).

Case 1:
When the process is considered capable of meeting the specifications, the natural deviation limits are less than the specification limits. A process that performs at this capability level may show a certain level of process variation and a mean shift.

Case 2:
When the process is not capable of meeting specifications, the natural deviation limits are larger than the specification limits. At this point, a process produces high levels of variability and usually the outcomes from the process (products or services) do not meet the desired requirements.

Case 3:
The last case occurs when the process is centered and capable. This takes place when natural deviation limits and specification limits are equal. The challenge in this situation is that the occurrence of a mean shift or process variability results in nonconformity in the process outcomes.

3.3.3 Types of Control Charts
There are several types of control charts. These types depend on two important factors: (1) subgroups of the data and (2) the type of data. Subgroups are samples of observations from
the total number of observations of historical process performance data and are used when it is impractical or too expensive to collect data on every unit of product or transaction of service in the process. In constructing control charts, subgroups should be homogeneous so that special causes can be recognized.

Control charts are commonly used in the Control phase of the Six Sigma methodology. Table 3.1 summarizes the most common charts that are used for process control based on the data type that is graphed – either variable data or attribute data. Among these control charts, Exponentially-Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) are currently the more popular control charts used for detecting process mean shifts, and they play a key role in this research investigation.

**Table 3.1: Common control charts.**

<table>
<thead>
<tr>
<th>Chart</th>
<th>Method of Measurement</th>
<th>Data Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x}$ and R</td>
<td>Based on actual observations within one subgroup</td>
<td>Variable</td>
</tr>
<tr>
<td>$\bar{x}$ and S</td>
<td>Based on actual observations within one subgroup</td>
<td>Variable</td>
</tr>
<tr>
<td>Individual</td>
<td>Based on actual observations for one observation</td>
<td>Variable</td>
</tr>
<tr>
<td>$p$</td>
<td>Based on the fraction of nonconforming observations within one subgroup</td>
<td>Attribute</td>
</tr>
<tr>
<td>$Np$</td>
<td>Based on the number of nonconforming observations within one subgroup</td>
<td>Attribute</td>
</tr>
<tr>
<td>$C$</td>
<td>Based on the number of nonconforming observations within one subgroup</td>
<td>Attribute</td>
</tr>
<tr>
<td>$u$</td>
<td>Based on the nonconformance per unit within one subgroup</td>
<td>Attribute</td>
</tr>
<tr>
<td>EWMA</td>
<td>Based on the Exponentially-Weighted Moving Average within one subgroup</td>
<td>Attribute or Variable</td>
</tr>
<tr>
<td>CUSUM</td>
<td>Based on the cumulative sum within one subgroup</td>
<td>Attribute or Variable</td>
</tr>
<tr>
<td>Real-Time</td>
<td>Based on a sliding window within one subgroup</td>
<td>Attribute or Variable</td>
</tr>
</tbody>
</table>

3.3.3.1 **Exponentially-Weighted Moving Average Control Chart**

Exponentially-Weighted Moving Average control charts (EWMA) is introduced by Roberts (1959) for monitoring a process mean. It averages the process data so it gives less
weight to data as they removed from the current measurement. Then, the data from the original observations are ordered in sequence using Eq. 3.2, i.e.,

\[ z_t = \lambda \bar{x}_t + (1 - \lambda)z_{t-1} \]  

(3.2)

where \( z_t \) is the estimate of the process mean, \( \lambda \) is a weight constant assigned to the original observation, where \( 0 < \lambda < 1 \), \( t \) is observation time, \( \bar{x}_t \) is the sample mean from time period \( t \), and \( z_t \) is the plotted test statistic. Furthermore, the control limits for EWMA control charts are computed as

\[
\frac{UCL}{LCL} = \mu_0 \pm \omega \sigma \sqrt{\frac{\lambda}{(2 - \lambda)[1 - (1 - \lambda)^{2i}]}}
\]  

(3.3)

where \( \omega \) is the width factor that divides the charts based on the desired sigma level (e.g., \( 3\sigma \), \( 4.5\sigma \), \( 6\sigma \)).

3.3.3.2 Cumulative Sum (CUSUM) Control Chart

Page (1954) develops the Cumulative sum (CUSUM) control charts to detect process mean shifts over a number of collected observations for both attribute data and variable data. It incorporates all information in the sequence of sample observation values by using cumulative sum function that computes the deviations of the collected observations from a target value. The construction of CUSUM control charts is based on the Maximum Likelihood Estimation (MLE) and the cumulative sum functions which are given by

\[
max_{\bar{\mu}} \left\{ \prod_{i=1}^{m} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(x_i - \bar{\mu})^2} \right\}
\]  

(3.4)

\[
min_{\bar{\mu}} \left\{ \sum_{i=1}^{m} \log \left\{ \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(x_i - \bar{\mu})^2} \right\} \right\}
\]  

(3.5)
\[ S_m = \sum_{i=1}^{m} (x_i - \bar{\mu}) \]  

where, \( \max \bar{\mu} \) and \( \min \bar{\mu} \) are the estimates of maximum and minimum mean value \( \bar{\mu} \) over collected observations \( (x_i) \). After estimating the maximum and minimum value of the mean, a target mean \( (\bar{\mu}) \) is computed using the cumulative sum function \( S_m \) (as given in Eq. 3.6) in order to, evaluate \( \bar{\mu} \) of collected observations for meeting or exceeding the maximum and minimum estimated mean value. Similar to EWMA, CUSUM control limits are associated with width factor \( (\omega) \) and the control limits derived by

\[
\begin{align*}
UCL &= \mu_0 + \omega \sigma \\
LCL &= \mu_0 - \omega \sigma
\end{align*}
\] 

where \( \mu_0 \) is the average of collected observations, \( \omega \) is the width factor that is used to divide the charts based on the desired sigma level, and \( \sigma \) is the standard deviation from the collected data.

3.3.4 Rules for Testing Control Charts for Process Mean Shifts

Control charts, in general, are associated with a number of assignable causes that are caused by process variation. These assignable causes usually appear in the control charts in the form of unnatural patterns. The common challenge in recognizing unnatural patterns is that most of the patterns appear with similar features (Bissell, 1994). In order to recognize the type of control chart patterns, researchers categorize the control charts patterns into six common categories, as shown in Figure 3.6. They are: upward shift, downward shift, upward trend, downward trend, cycle and normal.
There are several techniques (or, more commonly, rules) that have been used for testing for mean shifts, including the Nelson Rules, Western Electric Company Rules, Juran Rules, Hughes Rules, Duncan Rules, Gitlow Rules, and Westgard Rules. However, Western Electric Company (WECO) Rules and Nelson Rules are the most common rules for testing and detecting process behavior and that can be implemented in most types of control charts.

Most of the techniques that are used for testing divide control charts into ±3σ zones with centerline in the middle as shown in Figure 3.7. Researchers state that, in order to increase test sensitivity, more than one technique can be used for identifying process patterns depending on the nature of the process and the type of control chart been used (Agarwal & Baker, 2010; Al-Ghanim & Ludeman, 1997; Gauri & Chakraborty, 2006; Guh, 2005).
3.3.4.1 Western Electric Company (WECO) Rules

The Western Electric Company (WECO) Rules assume that the control chart is divided into $3\sigma$ levels on both sides of the centerline, as shown in Figure 3.7. They identify unnatural patterns in a process when any of the following observations occur:

- One observation found outside the $3\sigma$ limit from the centerline;
- Two consecutive observations out of three are greater than the $2\sigma$ limit;
- Four consecutive observations out of five are greater than the $1\sigma$ limit; and
- Eight consecutive observations greater or less than the centerline value.

3.3.4.2 Nelson Rules

Nelson Rules, which are an enhancement of the WECO Rules, identify unnatural patterns in the process when any of the following observations occur:

- One observation is greater than the $3\sigma$ limit;
- Nine (or more) consecutive observations are on one side of the centerline;
- Six (or more) consecutive observations are an increasing (or decreasing) trend;
- Fourteen (or more) consecutive observations are in a cycle pattern;
- Two (or three) consecutive observations are greater than the $2\sigma$ limit;

Figure 3.7: Control charts testing zones.
• Four (or five) consecutive observations are greater than the $1\sigma$ limit; and

• Fifteen consecutive observations are within the $1\sigma$ limit.
CHAPTER 4: OVERVIEW OF MULTIOBJECTIVE OPTIMIZATION

4.1 Introduction

Most real-world problems involve addressing multiple, often conflicting, objectives. In this case, a single unique solution is not possible to satisfy the set of objectives simultaneously; therefore, there is a need for methods that are capable of handling multiple objectives and provide solutions that satisfy all objectives simultaneously (Zitzler, 1999).

The general multiobjective problem consists of vector $x$ of $n$ decision variables ($i.e., x_i$ where $i = 1, \ldots, n$) and $m$ objectives, where $m > 1$. The multiobjective optimization problem can be generally expressed as

$$\min (\max) z_i = f_i(x), \ i = 1, \ldots, m,$$

where a solution $x$ is a $n$-dimensional vector of the decision variables that can be continuous or discrete, or both. Eq. 4.1 is subject to $w$ inequality constraints

$$g_j(x) \leq 0, \ j = 1, \ldots, w,$$

and $k$ equality constraints

$$h_l(x) = 0, \ l = 1, \ldots, K.$$

As previously discussed, there are several techniques for solving multiobjective problems; however, the traditional techniques convert the multiple objective problem into a single objective problem using a vector of user-defined weights (Coello, 1999; Jones et al., 2002). As a result, researchers and practitioners search for alternative techniques that able to optimize multiple objective problems simultaneously and provide a set of Pareto optimal (i.e., compromise, or tradeoff) solutions instead of a single solution (Horn et al., 1994). Therefore, applying Pareto-based methods are preferable in multiobjective optimization.
4.2 Multiobjective Optimization via Evolutionary Algorithms (EAs)

Evolutionary algorithms (EAs) are popular methods for generating the set of Pareto optima for multiobjective optimization problems. The concept of EAs is derived from the Darwinian evolution theory and uses principles of biological evolution (Fogel, 1997; Zitzler et al., 2000). Several EAs for multiobjective optimization have been proposed such as Vector Evaluated Genetic Algorithm (VEGA), Multiobjective Genetic Algorithm (MOGA), Non-dominated Sorting Genetic Algorithm (NSGA) and Niched Pareto Genetic Algorithm (NPGA). These algorithms and many others have been widely used and successfully implemented on multiobjective optimization problems. The elitist Non-dominated Sorting Genetic Algorithm II (NSGA II) is one of the more common EAs (Deb et al., 2002). It, too, is considered in this research investigation.

4.3 Non-dominated Sorting Genetic Algorithm II (NSGA II)

NSGA II is introduced by Srinivas & Deb (1994) to promote a faster and enhanced optimization algorithm. Thus, this method is convenient for solving multiobjective problems where time to solve a problem is limited, such as in online process control problems.

The logic of NSGA II follows a repeated cycle, as shown in Figure 4.1. First, the population $M$ is initialized, either randomly or heuristically. Then, evaluation of the solutions in the population is performed. Next, the selection and reproduction of the best set of solutions from the population are performed. Finally, genetic manipulation of the best set of solutions is performed in order to generate the next population.
NSGA II differs from other methods in the solution evaluation procedure used which is based on ranking/fitness assignment. This procedure is performed in two steps: (1) nondominated ranking and (2) crowding distance assignment as shown in Figure 4.2 (a) and (b), respectively. In the first step, each solution is labeled with a dominance status (or, rank). Then, all the individuals that share the same rank value align together to form a layer called a front. The second step takes place to ensure a better spread of the individuals across the front. The
second step determines the average side distance of the cuboid for every \( s^{th} \) solution. Then, the average distance values of all individuals are used to sort the solutions along a front in a descending order. This process repeated and applied on all other individuals until every solution is assigned to a specific front.

![Figure 4.2: Ranking/fitness assignment of NSGA II.](image)

Subsequently, the selection and reproduction step is performed based on the values of the solutions using binary tournament selection method that reproduce the population. The objective of this step is to discard poor performing solutions and select the better solutions in order to create the mating pool. A crossover operator and a mutation operator are used to generate new solutions.

There are problem-dependent parameter values that must be determined. The first parameter is the crossover probability \( P_c \) which is the frequency of exchanging information between selected solution pairs. The value of \( P_c \) depends on the type of problem; however, it is recommended that \( P_c \) values between 0.80 and 1.0 help to intensify the search of the search space (De Jong, 1975).
The second parameter is the mutation operator $P_m$ which is the frequency of introducing diversity to the population. Similar to the first parameter, $P_m$ value depends on the type of problem; however, usually it is recommended that the $P_m$ value falls between 0.005 and 0.20 in order to prevent the search from being trapped at a local optimum (Srinivas & Patnaik, 1994). Finally, there are two more important parameters: population size $M$ and number of generations $G$ (Deb et al., 2002; Veldhuizen & Lamont, 2000).
CHAPTER 5: A SIX SIGMA MULTIOBJECTIVE OPTIMIZATION (SSMO) APPROACH FOR ONLINE PROCESS CONTROL

5.1 Introduction

In this chapter, the methodology for this research investigation is presented. A model is developed to enhance the DMAIC Six Sigma framework during the Improve and Control phases. The overall goal is to provide online process control and feedback during the Improve and Control phases when multiple objectives are present.

Recall that the specific objectives of this research are to: (1) demonstrate the effectiveness of integrating the Six Sigma quality approach with multiobjective optimization strategies, (2) build a holistic framework for online process control optimization and decision-making using an integration of a multiobjective optimization procedure and the three-sigma quality evaluation; and (3) enhance framework to allow online process performance optimization using an integration of the multiobjective optimization and the Six Sigma methodology.

5.2 Overview of the Proposed SSMO Approach

Figure 5.1 is a general flow chart of the proposed Six Sigma multiobjective optimization (SSMO) approach. The logic of the proposed SSMO approach begins after a number of preceding steps, which provide input to the proposed approach. The preceding steps start with the Define phase of the Six Sigma DMAIC methodology, in which the CTQs from the customers are defined. The next step, the Measure phase, includes the evaluation of the process in order to evaluate its current performance. The last step prior to the proposed SSMO approach is the Analyze phase, where root cause analyses are performed and a process performance improvement plan is identified. The process performance objectives and process control
variables and variable values defined in the preceding steps are used as input to the proposed SSMO approach for online process monitoring and control.

![Flowchart](image.png)

Figure 5.1: The general flowchart of the proposed SSMO approach for online process control for the DMAIC framework.

Once the online monitoring starts, the proposed approach detects any unwanted process behavior in real-time, such as unnatural patterns and shifts in mean process behavior. If an unwanted event is detected, a process optimization routine is triggered that attempts to balance the set of process performance objectives simultaneously providing the process decision-maker with a set of compromise (or, tradeoff) solutions, i.e., a set of Pareto optimal solutions that characterizes the Pareto optimal frontier for the set of \( m \) objectives. These solutions include new values for the process control variables and are employed in the Improve phase. After the Improve phase is updated with the new process control variable values, the online monitoring starts again until unwanted process behavior occurs again. Figure 5.2 shows the proposed SSMO approach for online process control in more detail.
5.2.1 Inputs to the Proposed Framework

As shown in Figure 5.2, the input settings for the proposed approach are based on the initial implementation of Define, Measure, Analyze, and Improve phases of the Six Sigma DMAIC framework.

5.2.1.1 Problem Formulation

The project goals and customers’ needs are formulated into an optimization problem with multiple objectives (e.g., minimize total waiting time and minimize total cost). The problem formulation identifies all possible constraints and process decision variables associated with the objectives.
5.2.1.2 Unwanted Process Behavior and Mean Shift Threshold

Statistical control charts are popular tools of SPC and are used frequently in the Control phase of the DMAIC methodology, as discussed in Chapter 3, and control charts are used in the Control phase of the proposed SSMO approach. Unwanted process behavior refers to the type of unnatural patterns that appear in control charts caused by unnatural variations from observed data of process. Threshold refers to the value of the mean shift that invokes the process optimization procedure.

The unwanted process behavior threshold needs to be established for the proposed approach. The purpose of this step is to define the control chart so that it detects unnatural patterns based on the objective function values, such as upward or downward shifts, upward or downward trends, and a cyclical pattern. As discussed in Chapter 3, there are several techniques for detecting unnatural patterns in control charts such as Nelson Rules, Western Electric Company (WECO) Rules, Juran Rules, Hughes Rules, Duncan Rules, Gitlow Rules, and Westgard Rules. Several existing studies (e.g., Al-Ghanim & Ludeman, 1997; Guh et al., 1999) show that the use of more than one technique increases the sensitivity to detect and identify unnatural patterns in control charts. This study uses a hybrid method based on two common techniques – the Nelson Rules and the WECO Rules (Cheng & Hubele, 1996; Gauri & Chakraborty, 2009; Heuzeroth et al., 2003). The online optimization of the objective functions occurs after at least one of the defined conditions used by the hybrid method is met.

The threshold for the mean shift is used to trigger the online optimization of the objective functions. It is important that the process mean is kept at or near the target value to maintain the desired quality level and to reduce variation (Duffuaa & Ben-Daya, 1995; Linderman et al., 2003). Since the mean performance value of the process varies or drifts over time, a test for a
shift in the mean value is continuously computed, monitored and compared to the Centerline (CL), Upper Control Limit (UCL) and Lower Control Limit (LCL) control chart parameters. Furthermore, the Six Sigma methodology allows the mean to shift ±1.5σ about the mean if a process operates under a 4.5 sigma level (Linderman et al., 2003). There are several statistical methods used to detect shifts in a mean as discussed in Chapter 3. Based on the previous studies by Guh et al. (1999) and Jing Yang (2009), the threshold of a mean shift can be identified when one of the following conditions exists for any process performance value (or, set of performance values):

1. The value of the mean shift exceeds half of the specification limits (i.e., greater than or equal to either the UCL minus the CL, or the CL minus the LCL),
2. The percentage of shift in the mean value equals or exceeds 50% of process CL, and
3. The probability of occurrence of a mean shift equals or exceeds 50% of process CL.

This research investigation uses the Bootstrap method, which does not require fundamental assumptions about the distribution of the data – frequency or type (Alexandrov et al., 2012; Rodionov, 2005); however, any method from Table 2.2 can be used in the proposed model to detect mean shifts. The Bootstrap method uses the percentage of shift in mean value to identify threshold of mean shifts, and the theoretical formulation of the Bootstrap is

\[
CL\% = 100 \frac{X}{N} \%
\]  \hspace{1cm} (5.1)

\[
S_{i}^{\text{diff}} = S_{i}^{\text{max}} - S_{i}^{\text{min}}
\]  \hspace{1cm} (5.2)

\[
S_{i} = S_{i-1} + (x_{i} - \bar{x}) \text{ for } i = 0, \ldots, n
\]  \hspace{1cm} (5.3)

where \( CL\% \) represents the percentage of mean shifts occurrence in the observed objective function values, \( N \) is the number of bootstrap samples performed, \( X \) is the number of bootstraps
for which \( S_{d_{\text{iff}}}^l < S_{d_{\text{iff}}} \) where, \( S_{d_{\text{iff}}}^l \) is the difference of the reorder order for the observed objective function values, \( S_{d_{\text{iff}}} \) is the difference of the original order for the observed objective function values, \( S_{\text{max}} \) is the maximum value calculated from \( S_i \) where, \( S_i \) is the cumulative sums of the straps, \( S_{\text{min}} \) is the minimum value calculated from \( S_i \), and \( \bar{x} \) is the average of the observed objective function values.

5.2.1.3 Optimization Algorithm and Parameters

The optimization algorithm and parameters are used to optimize the objective functions when an unnatural condition is detected or a shift in the mean process objective function value occurs. EAs are used in the proposed SSMO approach. Furthermore, as discussed in Chapter 4, there are several advantages of using MOEAs over classical optimization approaches for solving multiobjective optimization problems such as the popularity of these methods and limited assumptions are needed on the objective functions (Łukasiewycz et al., 2008). Specifically, the elitist Non-dominated Sorting Genetic Algorithm II (NSGA II) is used.

5.2.2 Online Control Monitoring and Feedback

After implementing the improvement plan in the Improve phase, it is critical to keep process in control (Agarwal & Baker, 2010). Online process control aims to maintain improvement by generating control charts of the set of objective function values in real-time. Currently, most statistical process control (SPC) tools use observed data to monitor a process and identifies changes in process performance without prescribing control actions (Reneau, 2000; Sun & Matsui, 2008). Therefore, this step is designed not only to monitor processes under a Six Sigma improvement plan, but it also combines feedback control using SPC tools, i.e., control
charts. By providing simultaneous online control for each objective function values, a Six Sigma improvement plan becomes available that can treat multiobjective problems in an automated, predictable, and repeatable approach. When online control charts detect unwanted process behaviors and mean shifts appear in the objective function values, the Optimize returns the process to an in-control state. Furthermore, the online process control provides to decision-makers a Six Sigma-based process control chart that shows when the process reaches a Six Sigma level of quality. In this research investigation, an attempt is made to develop a Six Sigma (6σ)-based control chart to monitor process, including multiobjective problems.

5.2.3 Automated Optimization of Process Control Settings

The process optimization procedure is invoked when an unnatural condition is detected or a shift in the mean process objective function value occurs. The objective of the Optimize process is to aid decision-maker in selecting the new values of decision variables that return the objective functions in-control. When there are several possibly contradicting objectives that need to be optimized simultaneously, a single optimal solution may not applicable for decision-maker to satisfy the tradeoff between objectives but rather a whole set of possible solutions is needed for equivalent (Zitzler, 1999) . This will provide decision-maker with a range of solutions that each one of them represent a good solution for implementation.

Although there are several ways to approach a multiobjective optimization problem, most work focuses on the approximation of the Pareto set. A number of heuristic search procedures such as genetic algorithms, tabu search procedures, ant colony optimization heuristics, etc., could be used to generate the Pareto set (Abraham & Jain, 2005). As the working procedure of these algorithms is characterized by a population of solution candidates and the reproduction process,
the combination of existing solutions enables the generation of new solutions. This procedure enables finding several members of the Pareto optimal set in a single run instead of performing a series of separate runs. Therefore, whenever the control charts detect an unnatural condition or a shift in a mean process objective function value, any of the EAs used to generate a set of Pareto optimal in order to select and implement the new decision variable values that return the process to an in-control state. As discussed earlier, although any heuristic search procedure can be used in the proposed Optimize phase, a multiobjective GA is selected as an example due to the popularity and wide use.

5.3 **Summary**

The objective of this research investigation is to present a framework that has the ability to provide an online feedback control in the Improve phase for multiple objectives when a process is out of control and generate set of decision variable values that assist the process decision-maker to sustain improvement after Six Sigma implementation. It uses control charts to monitor process performance via the real-time computation of a set of process performance objectives. Then, the proposed framework uses a search heuristic approach to generate a set of compromise solutions, i.e., Pareto optimal solutions. In the beginning of the proposed methodology, a set of input settings are required and are defined with the Define-Measure-Analyze-Improve phases of DMAIC methodology. Also, the threshold for out-control-events is pre-specified before the online process monitoring under the proposed framework commences.
CHAPTER 6:
MULTIOBJECTIVE OPTIMIZATION FOR ONLINE PROCESS CONTROL – 3σ-BASED QUALITY LEVEL

6.1 Introduction

This chapter demonstrates the proposed online process control approach using a case study, which is a well-known and well-studied inventory management problem. The objective function formulation, initial conditions, and parameters for the Improve phase are chosen from literature, whereas the objective functions, initial conditions, and parameters for the Improve phase are determined using previous studies.

In this chapter, 3σ-based control charts are used for online process control and tests based on a study by Guh (2005), which uses a hybrid method that integrates both the Nelson Rules and the Western Electric Rules to detect out-of-control events and patterns in 3σ-based control charts. Furthermore, the Cumulative Sum function in the proposed approach utilizes a Bootstrap method to detect shifts in the mean process control objective values. Finally, the multiobjective evolutionary algorithm, Non-dominated Sorting Genetic Algorithm II (NSGA II), is used to optimize the process control variables. It is important to note that the impetus and eventual success of this research investigation is not necessarily predicated upon using these specific heuristic search procedures.

6.2 Case Study Description

It is important to achieve satisfactory levels of customer service while keeping inventory costs within reasonable bounds. There is a need to provide decision-makers an inventory management process with a mechanism for correcting the differences between demand and on-hand and/or received replenishment inventory. Two inventory-related general objective functions
are defined based on the literature. These are: (1) minimizing average holding cost and (2) minimizing unit ordering cost (Jacobs et al., 2011; Teng, 2002). The decision variable is the order quantity $Q$.

There are several well-known and perhaps unrealistic assumptions for this particular inventory model, such as:

- Only one product type is involved;
- Demand $D$ is constant; The purchase price of a unit of the product that makes up the order quantity $Q$ is fixed, i.e., there are no quantity discounts or price breaks for bulk purchases;
- Each order of quantity $Q$ is received in a single delivery and inventory replenishment is made instantaneously;
- Inventory replenishment lead time (delivery or manufacturing) is fixed;
- Continuous review of inventory is conducted; and
- No inventory shortages are allowed.

However, for this case study, the assumptions are sufficient as the tradeoff relationship between inventory average holding costs and ordering costs per unit is most important and relevant here. It is important to note that the assumption of a constant demand $D$ is relaxed such that each realization of demand $D$ is follows a random probability distribution.

A large $Q$ reduces inventory ordering frequency but requires holding a large amount of inventory in order to meet demand $D$. This large amount of on-hand inventory increases inventory average holding costs. On the other hand, a small $Q$ reduces the average amount of inventory but increases ordering frequency and, as a result, the ordering cost per unit. Therefore,
modeling the tradeoff relationship between average holding cost and unit ordering cost makes this problem a useful case study when attempting to effectively balance more than one objective.

The two objective functions considered for this investigation are treated as equally important during the online control and optimization process, and they are:

1. Minimize average holding cost. Average holding cost \( (H) \) is one of the important components of inventory management process, and it refers to the cost of storing a commodity over a period of time. According to several research studies (e.g., Stevenson & Hojati (2002)), the theoretical definition of the average holding cost is:

\[
H = c_h \frac{Q}{2}
\]  
(6.1)

where \( c_h \) is the unit holding cost.

2. Minimize ordering cost per unit. Unit ordering cost \( (O) \) refers to the cost of ordering a single unit when on-hand inventory is not meeting demand.

\[
O = \frac{c_o}{Q}
\]  
(6.2)

where \( c_o \) is the fixed ordering cost for a single order quantity \( Q \).

Both costs functions are considered conflicting objective functions, and a function of \( Q \), the decision variable in this case.

### 6.3 Computational Experiment

For this research investigation, the initial ranges of values for the decision variable \( Q \) are identified from empirical results published in the existing inventory literature. The ranges and values of the decision variable \( Q \), demand \( D \), which follows a uniform distribution, and the relevant costs are listed in Tables 6.1, 6.2, and 6.3 respectively. Specifically, these ranges are
approximated based on experimental results based on the studies by Donaldson (1977) and Jacobs et al. (2011).

Table 6.1: Range of possible decision variable values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order Quantity $Q$ (units)</td>
<td>5</td>
<td>130</td>
</tr>
</tbody>
</table>

Table 6.2: Range of input variable values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Demand Quantity $D$ (units) (uniformly-distributed)</td>
<td>0</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 6.3: Cost values for inventory cost problem (in dollars).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Holding Cost $c_h$</td>
<td>$15</td>
</tr>
<tr>
<td>Fixed Ordering cost per Single Order $c_o$</td>
<td>$220</td>
</tr>
</tbody>
</table>

6.3.1 Online Control Settings for the Case Study

As discussed in CHAPTER 3, there are two types of variation that affect the quality of process control charts. The first type is special cause variation, also called assignable cause variation. The second type of variation is common cause variation, which results from numerous, ever-present differences in the process. Control charts help in identifying these two types of variation while monitoring process behavior. Two types of control charts are used to detect online process variation and mean shifts – an EWMA control chart and a CUSUM control chart. This case study assumes a 3σ-level of quality. Based on the information found in previous case study by Jacobs et al. (2011), Table 6.4 and Table 6.5 list these parameters for an EWMA control chart and a CUSUM control chart, respectively based on 109 daily observations in objective
functions unit ordering cost and average holding cost. Negative values of the lower control limits are round to 0 since that is the minimum cost value for both objective functions.

Table 6.4: Initial EWMA 3σ control chart parameter values.

<table>
<thead>
<tr>
<th>Control Chart Parameter</th>
<th>Average Holding Cost (H)</th>
<th>Unit Ordering Cost (O)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centerline (CL)</td>
<td>$224.00</td>
<td>$4.17</td>
</tr>
<tr>
<td>Upper Control Limit (UCL)</td>
<td>$288.92</td>
<td>$9.23</td>
</tr>
<tr>
<td>Lower Control Limit (LCL)</td>
<td>$160.39</td>
<td>-$0.48</td>
</tr>
</tbody>
</table>

For daily observation \( i = 1 \)

<table>
<thead>
<tr>
<th>Control Chart Parameter</th>
<th>Average Holding Cost (H)</th>
<th>Unit Ordering Cost (O)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centerline (CL)</td>
<td>$224.00</td>
<td>$4.17</td>
</tr>
<tr>
<td>Upper Control Limit (UCL)</td>
<td>$318.69</td>
<td>$11.48</td>
</tr>
<tr>
<td>Lower Control Limit (LCL)</td>
<td>$130.62</td>
<td>-$2.73</td>
</tr>
</tbody>
</table>

For daily observation \( i = 109 \)

<table>
<thead>
<tr>
<th>Control Chart Parameter</th>
<th>Average Holding Cost (H)</th>
<th>Unit Ordering Cost (O)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centerline (CL)</td>
<td>$224.00</td>
<td>$4.17</td>
</tr>
<tr>
<td>Upper Control Limit (UCL)</td>
<td>$570.88</td>
<td>$30.33</td>
</tr>
<tr>
<td>Lower Control Limit (LCL)</td>
<td>-$121.00</td>
<td>-$22.00</td>
</tr>
</tbody>
</table>

Table 6.5: Initial CUSUM 3σ control chart parameter values.

Additionally, in implementing the Control phase of the Six Sigma DMAIC framework, change point tests determine whether a change in the mean or standard deviation has taken place. For EWMA control charts, control limits are calculated for each point, which detects shifts that may occur over time. On the other hand, CUSUM control charts detect change points based on a cumulative sum function, such as through the Bootstrap method. The Bootstrap method is widely used to detect shifts that occur over time and, since no assumptions are required about the frequency distribution of the objective functions. Therefore, in this investigation, the Bootstrap method is selected to detect shifts in the mean values over time for both the average holding cost \( H \) and the unit ordering cost \( O \).
6.3.2 Optimize Settings for the Case Study

For proof-of-concept, NSGA II is used to optimize the multiple objectives by varying the process control variable $Q$. The optimization parameter settings of NSGA II have been selected based on a previous studies by Belgasmi et al. (2008) and Chiong et al. (2011), who attempt to optimize a similarly-formulated inventory management process. The set of parameter values for this computational study are summarized in Table 6.6.

Table 6.6: Optimization parameters range and values for inventory cost problem.

<table>
<thead>
<tr>
<th>Optimization Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population Size ($M$)</td>
<td>100</td>
</tr>
<tr>
<td>Generations ($G$)</td>
<td>100</td>
</tr>
<tr>
<td>Crossover Probability ($P_c$)</td>
<td>85%</td>
</tr>
<tr>
<td>Mutation Probability ($P_m$)</td>
<td>5%</td>
</tr>
</tbody>
</table>

6.3.3 $3\sigma$ Online Process Control

The objective of this research is enhancing process control decision-making through the use of online process monitoring and multiobjective optimization. In this section, it is first assumed that no optimization is used to control the process. Then, implementation of the proposed SSMO online process control approach with optimization is presented.

6.3.3.1 Online Process Control with No Optimization

Figure 6.1 and Figure 6.2 show EWMA online process monitoring and CUSUM online process monitoring, respectively, after 109 daily observations for the two objectives – average holding cost and ordering cost per unit. It can be seen that the first 40 observations show both inventory management objective functions are in control and no shifts in mean occur. Then, it can be seen that, after 43 observations, out-of-control observations are detected when the unit
ordering cost exceeds the UCL. However, under the no-optimization assumption, the $Q$ values are not updated when out-of-control or mean shift events are detected.

Figure 6.1: Results of EWMA $3\sigma$-based online control chart observations for inventory cost problem without using optimization.
Figure 6.2: Results of CUSUM 3σ-based online control chart observations for inventory cost problem without using optimization.

Additionally, other results can be found in the observations between 33 and 48, in Figure 6.2 where, a shift in the mean value occurs due to an increasing trend in the ordering cost per unit. Furthermore, Figure 6.3 shows a graphical representation of the Bootstrap CUSUM function when a shift in the mean value occurs due to an increasing trend in average holding cost between 33 and 48 observation. Where, the cumulative sum straps for the original order of the unit ordering cost objective function values $S_i$ drawn in black and compared with the other cumulative sum straps after data reordered. It shows that, at a 92% confidence level, there is a shift in the mean value for the unit ordering cost objective.
Finally, it can be seen from the results that the Control phase of the proposed approach monitors and detects online mean shifts and out-of-control events that occur as seen in the multiple objective function values. Furthermore, the proposed approach generates a graph of the Bootstrap CUSUM function and confidence level of shifts in the mean values. Thus, the implementation of the proposed approach could aid a decision-maker to simultaneously detect out-of-control events and mean shifts while the process is monitoring multiple objective functions in real-time.
6.3.3.2 Online Process Control with Optimization

This section shows the implementation of the proposed SSMO approach with optimization. The implementation at this section uses the same input settings discussed earlier. As shown in Figure 6.4 and Figure 6.5, respectively, an overall representation of online process control using EWMA and CUSUM control charts for the daily observations when optimization is implemented in order to update the order quantity $Q$ when out-of-control and mean shift events are detected.

![EWMA control chart for the inventory cost problem using optimization.](image)

Figure 6.4: Results of EWMA $3\sigma$-based control chart online observations for inventory cost problem using optimization.
Figure 6.5: Results of CUSUM 3σ-based control chart online observations for inventory cost problem using optimization.

After an outlier event is detected when the unit ordering cost value exceeds the UCL, the optimization routine is invoked to generate a set of Pareto optima (i.e., tradeoff solutions). At this point, the Pareto optimal frontier is provided to decision-maker so that the decision-maker can update the value of $Q$, as shown in Figure 6.6. Although the decision-maker can select any point from the Pareto front, this research investigation assumes that the order quantity $Q$ value is selected based on the minimum total cost of the two objective functions. For instance, the result in Figure 6.6 shows that the minimum average total cost found to be $187 at $Q = 23$ units compared to other solutions, such as $300 at $Q = 47$ units and $193 at $Q = 15$ units. The reason for using the minimum difference between the two objective functions is to achieve the highest cost reduction for both objective functions. An investigation to identify the best approach to select the tradeoff solution to update the decision variables is left for future study.
Then, after selecting the new decision variable value, the Improve phase of the SSMO optimization framework updates the value of $Q$ to the selected value. Then, online process monitoring commences again until a shift in the mean value or out-of-control event is detected at which time simultaneous optimization of the set of the objectives occurs. This cycle continues until the last observation of both objective functions.

### 6.3.4 Discussion of Results

Table 6.7 summarizes the results from a process monitoring no-optimization scenario and from the proposed SSMO approach. A comparison of the overall performance for EWMA- and CUSUM-based control charts results in the case of no optimization. A DPMO counter is used as a process performance measure in order to provide a real-time feedback for the process. A defect is defined as any detection of unnatural patterns, mean shift, and out-of-control events that
appear in the unit ordering cost trends and/or average holding cost trends while the process is running. Eq. 2.1 is used to calculate the DPMO at the online observations, where \( O = 3 \), which is the number of opportunities of detecting a defect. In particular, \( O \) is the three categories of defects, which are unnatural patterns, mean shift, and out-of-control events. \( U \) is the number of observed units for both objective functions. The results show that EWMA-based control chart is more stable in monitoring the objective functions by almost 12,000 DPMO less than the CUSUM-based control chart. This difference is due to the exponentially-weighted function that underlies the EWMA control chart in that it exponentially smooths the variability of the observed values.

Table 6.7: Summary of results of the 3\( \sigma \) quality level implementation for the inventory cost problem.

<table>
<thead>
<tr>
<th></th>
<th>EWMA 3( \sigma ) with No Optimization</th>
<th>CUSUM 3( \sigma ) with No Optimization</th>
<th>EWMA 3( \sigma ) with Optimization</th>
<th>CUSUM 3( \sigma ) with Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPMO</td>
<td>6,116</td>
<td>18,348</td>
<td>6,116</td>
<td>9,174</td>
</tr>
<tr>
<td>Order Quantity, ( Q )</td>
<td>50 units</td>
<td>Varies: 50 at initial run, 23 at day 43, and 71 at day 85</td>
<td>Varies: 50 initial run, 23 at day 27, 60 at day 63, and 71 at day 90</td>
<td></td>
</tr>
<tr>
<td>Average holding cost</td>
<td>$249</td>
<td>$208</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Ordering Cost</td>
<td>$28</td>
<td>$26</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Next, the overall performance for EWMA control charts and CUSUM control charts when optimization results are implemented is compared. The results show an overall reduction in both control charts. CUSUM shows significant improvement after implementing optimization by reducing the DPMO from approximately 18,000 to approximately 9,000. On the other hand,
EWMA generates the same DPMO level compared to the case when optimization is not implemented.

6.4 Summary

In this chapter, the proposed integration of multiobjective optimization and the DMAIC framework is used with $3\sigma$-based control charts. The integration provides real-time feedback from the Control phase to the Improve phase of the Six Sigma DMAIC framework. This feedback reduces the average total cost of the inventory problem as well as reduces the DPMO of the online process control. The next two chapters expand the proposed SSMO methodology implementation and experimental design investigation. Chapter 7 starts with performing online process monitoring and control based on the Six Sigma methodology, which has not been conducted in the current research literature. Chapter 8 includes an expanded experimental study to compare the results of $3\sigma$-based online process monitoring and control and $6\sigma$-based online process monitoring and control.
CHAPTER 7: MULTIOBJECTIVE OPTIMIZATION FOR ONLINE PROCESS CONTROL – 6σ-BASED QUALITY LEVEL

7.1 Introduction

In this chapter, the proposed model is modified to allow online process control to monitor and optimize the process based on 6σ quality evaluation.

7.2 Online Control Settings for the Case Study

The control settings of the control charts used in Chapter 6 are modified so that they are based on a 6σ quality level. The selection of the control settings are based on existing studies by Azzabi et al. (2009) and Radhakrishnan & Balamurugan (2010). In constructing the control charts, the determination of the distance between control limits is based on a width factor $w$. The width factor of the CUSUM control charts and the EWMA control charts is computed from a normal $Z$-value, which measures the distance in standard deviations from the mean.

Traditional process control charts used in the Six Sigma methodology are based on a quality level of 3σ, which yields 93.32% conformance. However, in order to modify the quality level of the control charts, the $Z$-value must be modified accordingly. Table 7.1, which is derived from the Normal distribution table, shows the values of different quality levels and the yield associated with each one. Thus, the calculation of $w$ for the control charts is modified based on the $Z$-value for a 6σ level (i.e., $w_{6\sigma} = \frac{6}{4.67}$) instead of a 3σ level (i.e., $w_{3\sigma} = \frac{3}{1.833}$). Based on the modified width factor $w$, Table 7.2 and Table 7.3 list the parameters for the EWMA control chart and the CUSUM control chart, respectively.
Table 7.1: List of quality levels and corresponded Z values (Breyfogle, 2003).

<table>
<thead>
<tr>
<th>Quality Level</th>
<th>Yield</th>
<th>Z-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \sigma$</td>
<td>93.32%</td>
<td>1.833</td>
</tr>
<tr>
<td>$4 \sigma$</td>
<td>99.38%</td>
<td>2.737</td>
</tr>
<tr>
<td>$6 \sigma$</td>
<td>99.999997%</td>
<td>4.671</td>
</tr>
</tbody>
</table>

Table 7.2: Initial EWMA 6$\sigma$-based control parameters values.

<table>
<thead>
<tr>
<th>Control Chart Parameter</th>
<th>Average Holding Cost ($H$)</th>
<th>Unit Ordering Cost ($O$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centerline (CL)</td>
<td>$224.00$</td>
<td>$4.17$</td>
</tr>
<tr>
<td>Upper Control Limit (UCL)</td>
<td>$258.43$</td>
<td>$7.50$</td>
</tr>
<tr>
<td>Lower Control Limit (LCL)</td>
<td>$190.89$</td>
<td>$1.25$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Control Chart Parameter</th>
<th>Average Holding Cost ($H$)</th>
<th>Unit Ordering Cost ($O$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centerline (CL)</td>
<td>$224.00$</td>
<td>$4.17$</td>
</tr>
<tr>
<td>Upper Control Limit (UCL)</td>
<td>$274.07$</td>
<td>$8.95$</td>
</tr>
<tr>
<td>Lower Control Limit (LCL)</td>
<td>$175.25$</td>
<td>$-0.20$</td>
</tr>
</tbody>
</table>

Table 7.3: Initial CUSUM 6$\sigma$-based control parameters values.

<table>
<thead>
<tr>
<th>Control Chart Parameter</th>
<th>Average holding cost ($H$)</th>
<th>Unit ordering cost ($O$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centerline (CL)</td>
<td>224.00</td>
<td>4.17</td>
</tr>
<tr>
<td>Upper Control Limit (UCL)</td>
<td>392.06</td>
<td>16.81</td>
</tr>
<tr>
<td>Lower Control Limit (LCL)</td>
<td>58.00</td>
<td>-8.00</td>
</tr>
</tbody>
</table>

7.3 6$\sigma$ Online Process Control

This section summarizes the implementation of the enhanced proposed approach applied to the inventory management problem under the 6$\sigma$ process monitoring and control. Furthermore, the experiments are performed using 6$\sigma$-based control parameters settings that are discussed earlier in this chapter.

7.3.1 Online Process Control with No Optimization

Initial observations of the objective functions when online process control started can be found in Figure 7.1 and Figure 7.2 which show 6$\sigma$-based online process monitoring using
EWMA and CUSUM control charts respectively for 109 daily observations objective functions unit ordering cost and average holding cost. Initial observations show that both objective functions are in control and no shift in the mean performance occurs during the process run. At this part of the implementation, it is assumed that no changes are made by the decision-maker, and the $Q$ value is not updated when out-of-control or mean shift events are detected.

Figure 7.1: Results of EWMA 6σ-based online control chart observations for inventory cost problem without using optimization.
Figure 7.2: Results of CUSUM $6\sigma$-based online control chart observations for inventory cost problem without using optimization.

An example of detecting out-of-control observations is shown in Figure 7.2. After 34 observations, the unit ordering cost exceeds its associated UCL in the CUSUM control chart. Additional results can be found in the observations between 45 and 62, where a mean shift is detected. It shows a shift in the mean value due to increasing trend in unit ordering cost. Furthermore, Figure 7.3 shows the cumulative sum straps for the original order of the unit ordering cost objective function values $S_t$, which are drawn in black and compared with the other cumulative sum straps after data reordered. It shows that, at a 98% confidence level, there is a shift in the mean value for the unit ordering cost.
Similar to the results from the previous chapter, the proposed model, successfully detects mean shift and out-of-control-events through online process control using Six Sigma-based control charts. Thus, the implementation of the enhanced proposed model could aid a decision-maker to monitor process performance under the Six Sigma-based process control. In addition, it allows decision-makers to detect out-of-control events and mean shift in real-time for multiple
objectives. Next section demonstrates implementation of the proposed model when optimization results are implemented to update decision variable value.

### 7.3.2 Online Process Control with Optimization

This section shows the implementation of the proposed model with the assumption of implementing the optimization results on the decision variable value. The implementation at this section uses the same input settings discussed earlier in this chapter. As shown in Figure 7.4 and Figure 7.5, an overall representation of online process control for final daily observations when optimization results are implemented in order to update the $Q$ value when out-of-control and mean shift events are detected.

![Figure 7.4: Results of EWMA 6σ-based online control chart observations for the inventory cost problem using optimization.](image-url)
Figure 7.5: Results of CUSUM \(6\sigma\)-based online control chart observations for the inventory cost problem using optimization.

An example of selecting a new decision variable value is examined when out-of-control event detected in unit ordering cost objective function. After 34 observations when the unit ordering cost per unit exceeds UCL, the optimization process is triggered. At this point, the Pareto optima front is generated and the decision-maker updates the value of \(Q\) as shown in Figure 7.6. Thus, the \(Q\) value is selected based on the minimum total cost of the two objective functions where, \(Q = 31\) units. Then, after selecting the new decision variable value, the Improve phase updates the value of \(Q\). Once the decision variable is updated, the online process control performs until a shift in mean value or out-of-control event is detected. This loop continues until the last observations occur for all objective functions.
Additionally, after the decision variable value is updated, both objective functions remained in control until a mean shift is detected in the ordering cost. Based on the minimum total cost, a value of $Q = 72$ units is selected to update the decision variable in order to stabilize the process again. In the following section, results from both control charts are examined when no changes are made to update the decision variable value and when optimization is implemented.

7.4 Discussion of Results

In this section, the impact of the SSMO approach for Six Sigma-based online process control is investigated. The investigation is conducted by a comparison between EWMA- and CUSUM-based control charts results using DPMO as performance measure. Results are examined when two different types of 6σ-based control charts in the case of implementing and
neglecting optimization results. Furthermore, the influence of optimization parameters on convergence performance of the optimization procedure is studied in this section.

Initial examination starts with a comparison of the overall performance for EWMA- and CUSUM-based control charts results in the case of no implementation of optimization. The DPMO counter is integrated with the proposed model to provide a real-time performance measure for the inventory management process. Similar to Chapter 6, a defect is defined as any detection of unnatural patterns, mean shift, and out-of-control events appeared in unit ordering cost or average holding cost objective function while the online process control is running. DPMO used to measure the quality level of the inventory process from online observations where, where $O = 3$ which is the number of opportunities of detecting a defect. In particular $O$ are the three categories of defects which are unnatural patterns, mean shift, and out-of-control events. $U$ is the number of observed units for both objective functions. The results analysis that EWMA shows more stability in monitoring the objective functions by almost 25,000 DPMO less than CUSUM-based control chart for the total observations.

Additionally, the proposed SSMO approach is examined by comparing the overall performance for EWMA and CUSUM based control charts when optimization results are implemented. The results show overall reduction in both control charts. CUSUM shows significant improvement after implementing optimization by reducing the DPMO from 58,103 to 15,290. Similarly, EWMA shows reduction in DPMO level from 24,464 to 58,103 when the optimization results are implemented. Table 7.4 summarizes the analysis results from the Six Sigma monitoring approach and proceeding without updating decision variable value.
Table 7.4: Results summary of $6\sigma$-based implantation for inventory cost problem.

<table>
<thead>
<tr>
<th></th>
<th>EWMA $6\sigma$ with No Optimization</th>
<th>CUSUM $6\sigma$ with No Optimization</th>
<th>EWMA $6\sigma$ with Optimization</th>
<th>CUSUM $6\sigma$ with Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPMO</td>
<td>24,464</td>
<td>58,103</td>
<td>6,116</td>
<td>15,290</td>
</tr>
<tr>
<td>Order Quantity, $Q$</td>
<td>Fixed: 50 units</td>
<td>Varies: 50 at initial run, 31 at day 43, and 72 at day 85</td>
<td>Varies: 50 at initial run, 34 at day 32, 81 at day 61, 137 at day 76, and 42 at day 83</td>
<td></td>
</tr>
<tr>
<td>Average holding cost</td>
<td>$249</td>
<td>$242</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Unit Ordering Cost</td>
<td>$28</td>
<td>$26</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7.5 Summary

The integration of multiobjective optimization and the Six Sigma methodology is successfully applied to the inventory process using $6\sigma$-based control charts. The proposed SSMO approach produces in real-time improved process performance during online process control. The results show a reduction in process variation as well as minimization of unit ordering cost and average holding cost. Finally, the results are examined using the Six Sigma methodology performance metric DPMO in order to show the effectiveness of SSMO approach.
CHAPTER 8:  
SUMMARY AND FUTURE RESEARCH DIRECTIONS

8.1 Summary of Research

This research investigation attempts to bridge the gap between multiobjective optimization and the Six Sigma methodology to automate feedback from the Control phase to the Improve phase of DMAIC framework in the case of multiobjective problem. The integration of multiobjective optimization with the Six Sigma methodology to improve online process control requires the blending of domain knowledge in the areas of statistical process control and multiobjective optimization. Furthermore, improving the quality level of a process when multiple objectives are present adds significantly more complexity to the decisions and implementation plans for process improvement. Motivated by the need for enhancing the Six Sigma methodology to improve process control when multiple objectives exist, this research investigation proposes and successfully constructs and demonstrates an SSMO approach which enhances the DMAIC framework. A popular inventory management problem is used as the test case for the proposed SSMO approach.

Results from the implementation of the proposed SSMO approach show the effectiveness of integrating multiobjective optimization methods with the Six Sigma methodology in enhancing decision-making at the micro level. The integration provides automated real-time feedback to maintain the improvement after implementation of DMAIC framework. In particular, the SSMO approach maintains improvement by implementing three phases after the Analyze phase of DMAIC framework. The first phase of SSMO approach Improve sets the initial values for decision variables based on the results of the multiobjective optimization
results. The second phase Control uses online control charts for monitoring multiobjective functions values based on the desired quality evaluation level. The objective functions considered to meet quality evaluation level when none of the following defects found: unnatural patterns, out-of-control events, and mean shifts are detected by the online control charts. Then, third phase Optimize starts when control charts observations at the Control phase are not meeting the desired quality evaluation level. This phase provides an automated feedback to the Improve phase in order to update decision variable values which maintain objective functions within quality evaluation level. The Optimize phase is performed by using NSGA II to generate a set of multiple compromised solutions that allow decision-maker to update the Improve phase settings.

A case study based on a common inventory problem that contains two conflicting objective functions $H$ and $O$ associated with one decision variable $Q$. Furthermore, the input data for the inventory problem randomly-generated for 109 daily demand observations. The proposed SSMO approach is implemented on the inventory problem using four scenarios, where the first and second scenarios use $3\sigma$-based quality evaluation level without using optimization and with using optimization respectively, and the third and fourth scenarios used $6\sigma$-based quality evaluation level without using optimization and with using optimization, respectively. Furthermore, two control charts – EWMA and CUSUM – are used to monitor the set of objective function values. The results from implementing the SSMO approach on 109 daily demand observations shows a reduction in DPMO, $H$, and $O$ for both the quality evaluation levels $3\sigma$ and $6\sigma$. Thus, from this research investigation, the integration of multiobjective optimization with the Six Sigma methodology shows promise. It shows to be effective in reducing DPMO with respect to online process control in the presence of multiple objectives.
8.2 Future Research Directions

From the results presented and the conclusion drawn from this research investigation, there is sufficient motivation for the following extensions of this research investigation.

8.2.1 Utilize Simulation to Forecast Future Performance

The first extension of this investigation includes integrating simulation techniques in order to forecast the impact of selecting all possible scenarios from the Pareto optimal frontier to the objective functions and process quality level. The main impact of integrating simulation methods with the proposed model is that it allows a decision-maker to forecast future process performance based on simulated events, which will reduce the risk of false decisions.

8.2.2 Expand the Use of Control Charts

The proposed SSMO approach is successfully implemented using two different control charts EWMA and CUSUM; however, it is not limited only to these charts. Further exploration for other control charts based on different sigma levels can be explored. In particular, the next step is to apply the framework using different control charts, and sensitivity analysis among control charts in terms of effectively detecting mean shift, unnatural patterns, and out-of-control events.

8.2.3 Improving the Selection of Decision Variables

Additionally, exploring the best technique to select a solution from the set of Pareto optima in order to update the decision variables should prove fruitful. Furthermore, currently there are several techniques such as clustering and data mining that have been developed to
efficiently identify the best item among a set of items. These techniques can be used to reduce size of the Pareto optimal set and simplify decision-making for the decision-maker.
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