A Posteriori And Interactive Approaches For Decision-making With Multiple Stochastic Objectives

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A POSTERIORI AND INTERACTIVE APPROACHES FOR DECISION-MAKING WITH MULTIPLE STOCHASTIC OBJECTIVES

by

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ABSTRACT

Computer simulation is a popular method that is often used as a decision support tool in industry to estimate the performance of systems too complex for analytical solutions. It is a tool that assists decision-makers to improve organizational performance and achieve performance objectives in which simulated conditions can be randomly varied so that critical situations can be investigated without real-world risk. Due to the stochastic nature of many of the input process variables in simulation models, the output from the simulation model experiments are random. Thus, experimental runs of computer simulations yield only estimates of the values of performance objectives, where these estimates are themselves random variables.

Most real-world decisions involve the simultaneous optimization of multiple, and often conflicting, objectives. Researchers and practitioners use various approaches to solve these multiobjective problems. Many of the approaches that integrate the simulation models with stochastic multiple objective optimization algorithms have been proposed, many of which use the Pareto-based approaches that generate a finite set of compromise, or tradeoff, solutions. Nevertheless, identification of the most preferred solution can be a daunting task to the decision-maker and is an order of magnitude harder in the presence of stochastic objectives. However, to the best of this researcher's knowledge, there has been no focused efforts and existing work that attempts to reduce the number of tradeoff solutions while considering the stochastic nature of a set of objective functions.

In this research, two approaches that consider multiple stochastic objectives when reducing the set of the tradeoff solutions are designed and proposed. The first proposed approach is an *a posteriori* approach, which uses a given set of Pareto optima as input. The second approach is an interactive-based approach that articulates decision-maker preferences during the optimization process. A detailed description of both approaches is given, and computational studies are conducted to evaluate the efficacy of the two approaches. The computational results show the promise of the proposed approaches, in that each approach effectively reduces the set of compromise solutions to a reasonably manageable size for the decision-maker. This is a significant step beyond current applications of decision-making process in the presence of multiple stochastic objectives and should serve as an effective approach to support decision-making under uncertainty.

To my parents

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CHAPTER 1:
INTRODUCTION

1.1 Overview of Multiobjective Optimization

Succeeding in business, no matter how it is segmented, means winning in the global marketplace. From the executive manager of a large company to managers of small, privately-held companies – and even not-for-profit institutions – managers cannot anticipate success in business without a clear understanding of how critical decisions can be translated into a competitive advantage.

Many real-world problem scenarios tend to use a decision-making process that seeks tradeoff, or compromise, solutions rather than to seeking a single global optimal solution, as these critical decisions often involve multiple, often conflicting, objectives that must be addressed simultaneously. Multiobjective decision problems, unlike single objective decision problems, address a number of objective functions to be minimized and/or maximized. There are many mathematical programming techniques for multiobjective optimization. Most of the recent work focuses on the approximation of the Pareto optimal solution set (Abraham, Jain, & Goldberg, 2005). In other words, instead of identifying a single global solution, multiobjective optimization results in a number of tradeoff (or compromise) solutions for the set of objectives. This set of tradeoff solutions is known as the set of non-dominated Pareto optimal, or Pareto efficient, solutions (Coello, 2006). See Figure 1.1. A Pareto optimal solution is non-dominated if none of the objective functions can be improved without the degradation in one or more of the other objectives (Winston, 2003). Without additional preference information, all solutions in the set of Pareto optima can be considered equally good mathematically.

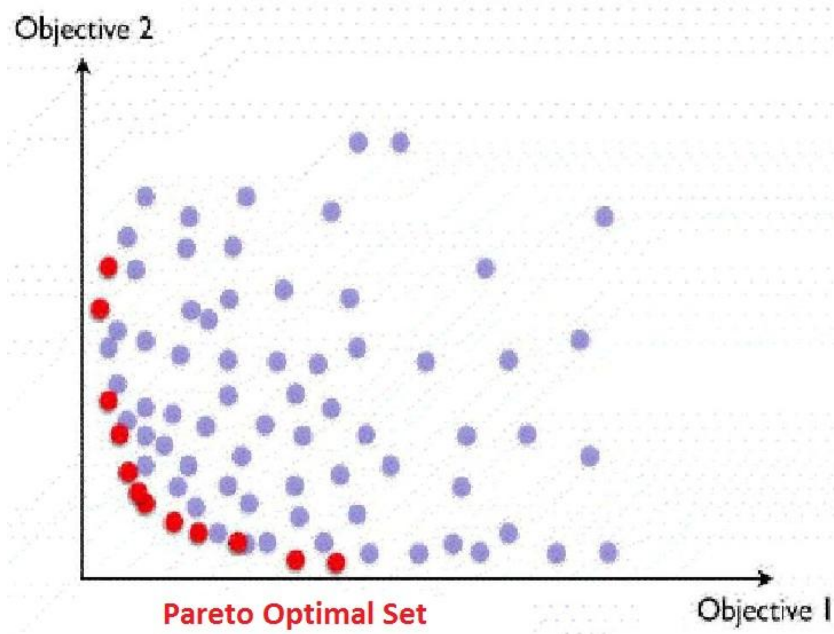

Figure 1.1: Objective space assuming that two objectives are to be minimized. The red points represent the Pareto front, or efficient frontier.

1.1.1 Pareto Optimality Methods

There are several multiobjective optimization approaches that are Pareto-based that generate the approximate Pareto frontier. However, it is only within the last two decades that researchers and practitioners have realized of the potential of using evolutionary algorithms (EAs) in this area, as this family of stochastic optimization metaheuristic search methods can effectively generate a set of Pareto optima (Coello, 2001). These algorithms have proven themselves as general, robust and powerful search mechanisms. Particularly, they possess several characteristics that are desirable for real-world problems involving multiple conflicting objectives, and intractably large and highly complex search spaces (Wang, Zhang, Gao, & Li, 2008). Furthermore, EAs are less susceptible to the shape or continuity of a Pareto front. For example, they can easily deal with discontinuous or concave Pareto fronts.

1.2 Using Simulation Modeling

Simulation is a powerful tool and often the tool of choice that enables decision-makers in research and in business to evaluate and improve organizational performance. The ability to model a physical process on the computer, incorporating the uncertainties and non-stationary that are inherent in virtually all real dynamic systems provides an advantage for analysis and decision-making. Decision-makers frequently use simulation within their organizations to model, evaluate and compare proposed, often complex and mathematically intractable, designs of their systems and processes with the goal of optimizing a particular performance objective (or set of performance objectives). When using computer simulation, the output from a model is stochastic since input probability distributions are used to characterize the stochastic behavior of subcomponents within the simulation model. Usually the model's performance results are reported in terms of means and standard deviations (or, in terms of confidence intervals at some level of significance). The confidence intervals are generated when multiple independent replications are run for a particular simulation evaluation. The confidence intervals represent the precision of the estimate of the true population value of the performance measure, or set of performance measures of interest.

1.3 Challenges of and the Need to Improve Decision-Making

Simulation is used frequently in decision-making, especially within an optimization framework. Simulation optimization typically requires a large number of simulation evaluations due to the stochastic components of simulation models (Syberfeldt, Ng, John, & Moore, 2009). In the presence of multiple objectives, each solution in the set of Pareto optima is an estimate of a non-dominated solution value represented by both a mean and a standard deviation. Therefore, care must be taken when considering dominance among the compromise solutions in the set.

Furthermore, the solution of a multiobjective optimization problem consists of a large set of compromise solutions. From a practical standpoint, the decision-maker needs only one solution. The set of compromise solutions can be extremely large, potentially overwhelming the decision-maker in his/her task of selecting the most appropriate solution. Choosing a candidate solution over the others or reducing the number of candidate solutions to select from is not a simple task. This problem can be challenging when presented with an extraordinarily large set of potential compromise solutions. Therefore, some intelligent means of reducing and organizing the set of solutions in the presence of stochastic objectives is required.

1.4 Research Gaps

There are a limited number of researchers who attempt to generate the set of Pareto optimal solutions while considering the stochastic nature of the objective functions. Other researchers focus on reducing the number of Pareto optimal solutions generated by a Pareto-based solution approach. These include approximating the number of Pareto optimal solutions (e.g., Boonma & Suzuki, 2009; Chen, Han, Liu, Jiang, & Zhao, 2012; Hendriks, Geilen, & Basten, 2011; Trautmann, Mehnen, & Naujoks, 2009), and using clustering analysis to reduce the number of Pareto optimal solutions to a smaller set (e.g., Aguirre, Taboada, Coit, & Wattanapongsakorn, 2011; Aguirre & Taboada, 2011; Noghin, 2011; Zio & Bazzo, 2011).

To the best of this researcher's knowledge, the literature provides a little work for reducing the number of the Pareto optimal solutions while considering the stochastic nature of the objectives. New efforts concerning improvement of decision-making for multiple objective problems and the need to reduce and organize the set of non-dominated solutions in the presence of stochastic objectives may benefit the decision-maker and provide a contribution not only to the practitioner body of knowledge, but also to the research community.

1.5 Objectives of This Research Investigation

This research aims to improve the decision-making under uncertainty and specifically focuses on the multiobjective optimization problem in order to reduce and organize the usually large set of candidate tradeoff solutions in the presence of stochastic objectives. In short, improve the decision-making solution identification and selection process when faced with multiple stochastic objectives. In addition, this research builds a framework that allows reducing and organizing the set of non-dominated solutions while considering the stochastic nature of the objective functions.

It is important to note that the decision-maker should provide preference data to ensure that the set of solutions with which the decision-maker is presented are, first, feasible and, second, suitable. Approaches of the articulation of decision-maker preferences may be done either before (*a priori* methods), during (interactive methods), or after (*a posteriori* methods) the decision-making process, which is typically the optimization of an objective function (or a set of objective functions). The focus in this investigation is *a posteriori* approaches and interactive approaches. In *a posteriori* approaches, the decision-maker selects a solution from a given generated set of tradeoff solutions based on his/her preferences. In interactive approaches, the decision-maker preferences guide the optimization process as the set of tradeoff solutions is being generated. The following are the primary objectives of this research investigation.

Objective 1: Design an *a posteriori* decision-making solution selection process in the presence of multiple stochastic objectives; and

Objective 2: Design an interactive decision-making solution selection process in the presence of multiple stochastic objectives.

1.6 Contributions of this Research Investigation

This investigation contributes quite significantly to the body of knowledge and advances the state-of-the-art in solving multiobjective decision problems. The research addresses the challenging problem of decision-making under uncertainty, especially in the presence of multiple stochastic objectives. It effectively deals with stochastic objectives and reduces the number of the tradeoff, or compromise, solutions effectively.

CHAPTER 2:
REVIEW OF EXISTING RESEARCH LITERATURE

2.1 Introduction

Several multiobjective optimization approaches exist that generate finite sets of Pareto optima, and these sets often contain a very large number of Pareto optimal solutions, which can be overwhelming to the decision-maker in the task of selecting the most appropriate solution to implement. Only a few researchers have proposed methods to generate Pareto optimal solutions while considering the stochastic nature of the objective functions (e.g., Boonma & Suzuki, 2009; Chen et al., 2012; Hendriks et al., 2011; Trautmann et al., 2009). In order to be adequately representative of the possibilities and tradeoffs, the number of the Pareto optimal solutions under stochastic objectives may be too large for decision-makers to practically consider. In this chapter, a review of existing work in reducing and organizing the number of the Pareto optimal solutions for better decision-making is given.

2.2 Multiobjective Optimization

Most real-world decision problems involve the simultaneous optimization of multiple objectives that are to be minimized or maximized. The multiobjective optimization problem, in its general form, considers a solution **x** of a vector of n decision variables (i.e., x_i where $i = 1,\ldots, n$) and m objectives, where $m > 1$. The problem can be generally expressed as follows:

$$\text{Minimize/Maximize} \quad f_m(\mathbf{x}), \qquad m = 1, 2, \ldots, M; \tag{2.1}$$

$$\text{Subject to} \quad g_j(\mathbf{x}) \geq 0, \qquad j = 1, 2, \ldots, J; \tag{2.2}$$

$$h_k(\mathbf{x}) = 0, \qquad k = 1, 2, \ldots, K. \tag{2.3}$$

$$x_i^{(L)}(\mathbf{x}) \leq x_i(\mathbf{x}) \leq x_i^{(U)}(\mathbf{x}), \qquad i = 1, 2, \ldots, n; \tag{2.4}$$

where $g_j(\mathbf{x})$ and $h_k(\mathbf{x})$ are constraints. Additionally, a solution **x** is feasible if it satisfies all of the J and K constraints.

In general, the solutions in multiobjective optimization problems are not uniquely determined. In fact, particularly in the case where two or more objectives conflict, usually many solutions exist that satisfy all relevant objectives; hence the most desirable solution, or at least, the best compromised solution, is selected from among them.

2.2.1 The Decision Space and the Objective Space

The solutions to a multiobjective optimization problem is usually depicted as a decision variable space in the overall search space, as shown in Figure 2.1 (left). It is clear that not all solutions in the rectangular decision space are feasible. Every feasible solution in this space can be mapped to a solution in the feasible objective space shown in Figure 2.1 (right) (Deb, 2001).

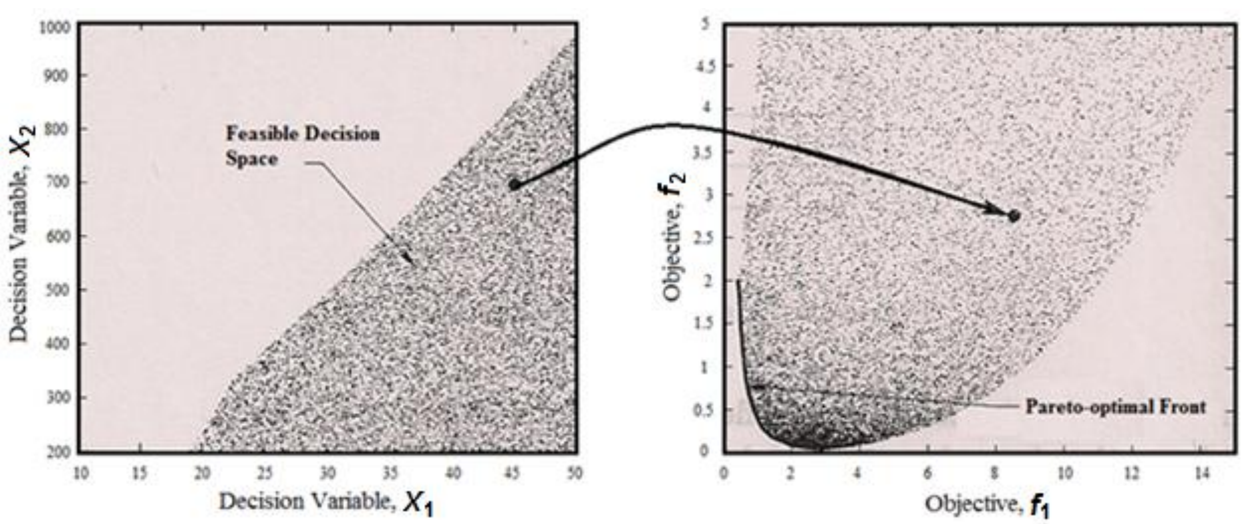

Figure 2.1: The feasible decision variable space (left) & the feasible objective space (right).

In the feasible objective space, all solutions on the curve shown in Figure 2.1 (right) are called Pareto optimal solutions, or non-dominated solution set. The curve formed by joining these solutions is known as Pareto optimal front, or the efficient frontier (Coello, Aguirre, & Zitzler, 2007). It is important to note that the feasible objective space not only contains Pareto optimal non-dominated solutions, but also solutions that are dominated. So, the entire feasible solution search space can be divided into two sets of solutions – Pareto optimal and non-Pareto optimal (Deb, 2001).

2.2.2 Pareto Dominance

The concept of Pareto dominance is of extreme importance in multiobjective optimization, especially when some or all of the objectives are in conflict (Pareto, 1971). In such a case, there is no single point (solution) that yields the best value for all objectives. Instead, the best solutions, often called a Pareto or non-dominated set, are a group of solutions such that selecting any one of them in place of another will always sacrifice quality for at least one objective, while improving at least one other (Guanqi, Wu, Bo, Wenbin, & Cheng, 2012; Le & Landa-Silva, 2007).

A solution **A** to a multiobjective problem is Pareto optimal if no other feasible solution is at least as good as **A** with respect to every objective and strictly better than **A** with respect to at least one objective. On the other hand, a feasible solution **A** dominates a feasible solution **B** to a multiobjective problem if **A** is at least as good as **B** with respect to every objective and is strictly better than **B** with respect to at least one objective. Solution **A** is non-dominated if it is not dominated by any solution, and the Pareto optimal solutions is the set of all non-dominated feasible solutions (Winston, 2003). Figure 2.2 illustrates the concept of Pareto dominance.

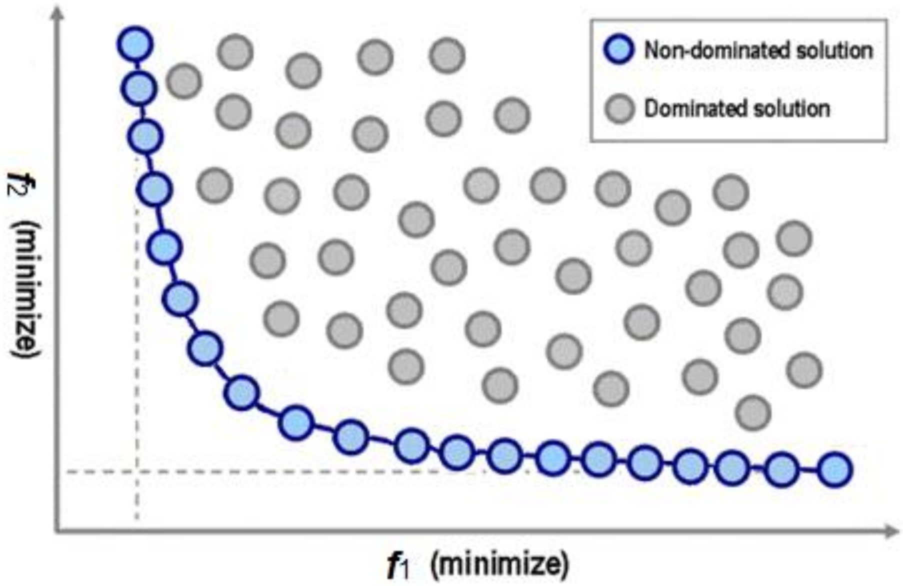

Figure 2.2: The concept of Pareto dominance.

The definitions of Pareto optimality, Pareto dominance, Pareto optimal set, and Pareto frontier are now summarized from Coello (2001).

Definition 2.1 (Pareto Optimality): A vector of decision variables $\vec{x}^* \in$ **F** is Pareto optimal if there does not exist another $\vec{x} \in$ **F** such that $f_i(\vec{x}) < f_i(\vec{x}^*)$ for all $i = 1, \ldots, k$ and $f_j(\vec{x}) < f_i(\vec{x}^*)$ for at least one j (assuming minimization of both f_i and f_j).

F is the set of all feasible solutions of the problem (i.e., where the constraints are satisfied). This definition says that $\vec{x}^*$ is Pareto optimal if there exists no feasible vector of decision variables $\vec{x} \in$ **F** that would decrease some objective without causing a simultaneous increase in at least one other objective.

Definition 2.2 (Pareto Dominance): A vector of decision variables $\vec{a} = (a_1, \ldots, a_k)$ is said to dominate vector $\vec{b} = (b_1, \ldots, b_k)$ (expressed as $\vec{a} \preceq \vec{b}$) if and only if *a* is partially less than *b*, *i.e.*, $\forall i \in \{1, \ldots, k\}, a_i \leq b_i \wedge \exists i \in \{1, \ldots, k\} : a_i < b_i$.

Definition 2.3 (Pareto Optimal Set): The Pareto optimal set (P^*) is defined as:

$$P^* := \{x \in \mathbf{F} \mid \neg \exists\ P' \in \mathbf{F}\ \vec{f}(x') \preceq \vec{f}(\mathbf{x})\} \tag{2.5}$$

Definition 2.4 (Pareto Front): For a given multiobjective problem $\vec{f}(\mathbf{x})$ and Pareto optimal set P^*, the Pareto front (p_F^*) is defined as:

$$p_F^* := \{\vec{u} = \vec{f} = (f_1(\mathbf{x}), \ldots, f_k(\mathbf{x})) \mid x \in P^*\} \tag{2.6}$$

2.3 Overview of Multiobjective Optimization Problems

A number of multiobjective optimization methods have been developed over the years. Recent publications classified the multiobjective optimization problems as non-Pareto-based techniques and Pareto-based techniques (Azadivar & Lee, 1988; Azadivar, 1992; Carson & Maria, 1997; Kalyanmoy Deb, 2001; Fu, 1994; Marler & Arora, 2004; Swisher, Hyden, Jacobson, & Schruben, 2000) as shown in Figure 2.3.

Non-Pareto-based techniques do not incorporate the concept of Pareto optimality and are categorized to classical no-preference methods (i.e., do not assume any information about the importance of the objectives). On the other hand, Pareto-based techniques use non-dominated solution ranking and selection methods to move the population towards the Pareto frontier. It is categorized as nature-inspired metaheuristic algorithms and classical preference-based methods.

The classical preference-based methods are categorized as *a posteriori* methods, *a priori* methods, and interactive methods.

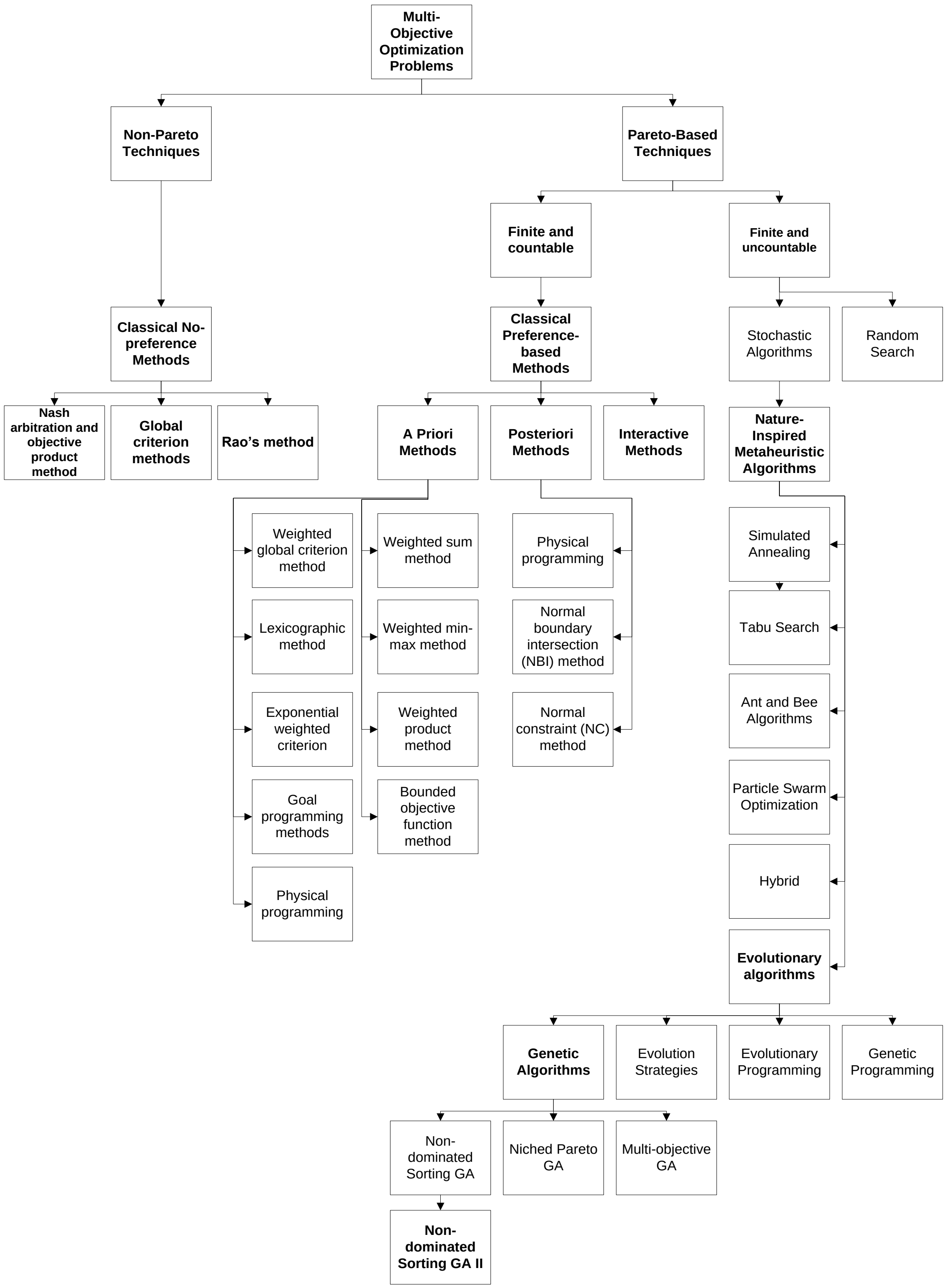

Figure 2.3: Overview of multiobjective optimization problems.

2.3.1 Overview of Nature-Inspired Metaheuristic Algorithms

Many of the more popular metaheuristics are nature-inspired, and almost all metaheuristics algorithms are suitable for global optimization (Rennard, 2007; Yang, 2010). In addition, computer simulation incorporating metaheuristic search algorithms has become an indispensable tool for solving real-world optimization problems (Yang, 2010). A number of researchers provide comprehensive reviews of nature-inspired metaheuristics algorithms and discuss their applicability to general combinatorial optimization problems (e.g., Yang, 2010).

2.3.2 Overview of Evolutionary Algorithms

Evolutionary algorithms (EAs) are based on the Darwinian principle of natural selection and reproduction wherein the probability of selection for reproduction is directly proportional to their rate of survival (i.e., their fitness) in their environment. In other words, individual solutions that are better able to perform tasks in their environment survive and reproduce at higher rate than those that do not perform those tasks as well. The idea of using the principles of natural evolution to solve optimization problems come out after a period of intensive research and experimentation in late 1960s and mid 1970s (Bäck, Hoffmeister, & Schwefel, 1991; Bäck & Schwefel, 1993). Since then, the use of computerized approaches that simulate the evolution process in an attempt to solve combinatorial optimization problems has steadily increased (Khuri, Bäck, & Heitkötter, 1994).

EAs use a population of solutions in each iteration in order to find multiple tradeoff solutions when used in multiobjective optimization. This population of solutions is a sample of points in the solution search space. The ability to find multiple optimal solutions in one single simulation run makes EAs unique in solving multiobjective optimization problems (Deb, 2001).

This class of search procedures include a variety of techniques, such as genetic algorithms, evolutionary programming, evolution strategies, and genetic programming (Bäck, Schwefel, & Informatik, 1996; Syberfeldt et al., 2009), as shown in Figure 2.4.

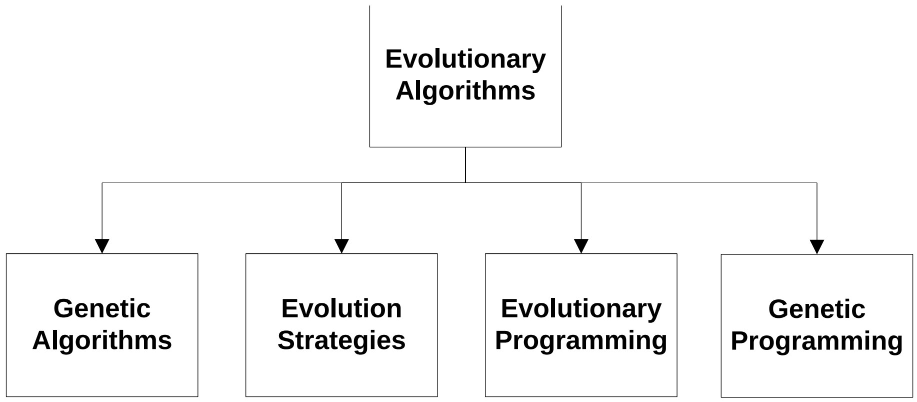

Figure 2.4: Overview of evolutionary algorithms.

2.3.2.1 Genetic Algorithms

Genetic algorithms (GAs) are probably the most well-known evolutionary algorithms that have proven useful in a variety of optimization problems. Its individuals are traditionally represented in binary strings (Tsutsui & Ghosh, 1997; Yang, 2010). GAs are developed by Holland (1992) and his collaborators in the 1960s and 1970s. Figure 2.5 provide an overview of existing GAs that have been developed for multiobjective optimization and are categorized as non-Pareto-based (e.g., Hajela & Lin, 1992; Schaffer, 1984) and Pareto-based algorithms (e.g., Deb, Pratap, Agarwal, & Meyarivan, 2002; Fonseca & Fleming, 1993; Horn, Nafpliotis, & Goldberg, 1994).

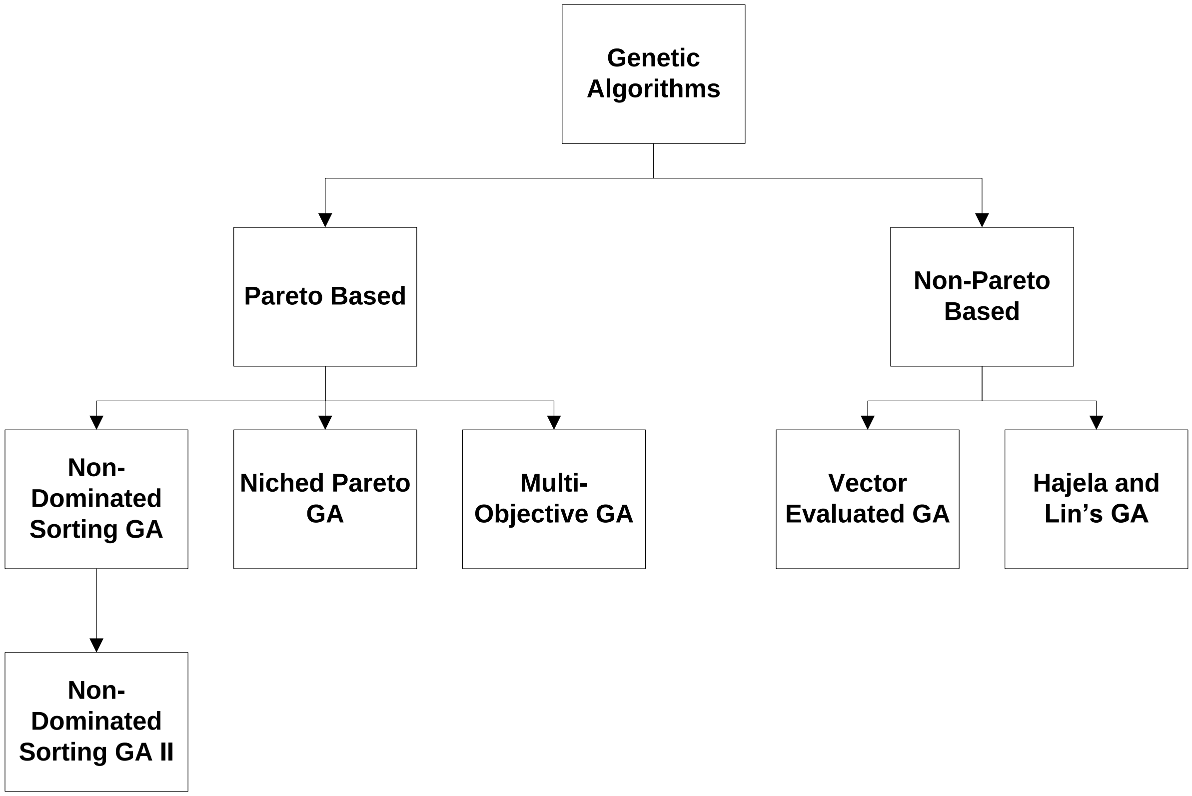

Figure 2.5: Overview of genetic algorithms for multiobjective optimization.

The non-Pareto-based approaches do not directly employ the concept of Pareto dominance, but are able to evolve multiple non-dominated solutions in parallel while the Pareto-based approaches incorporate the concept of Pareto dominance. To find a set of non-dominated solutions approximating the Pareto optimal set, Goldberg (1989) suggests the non-dominated ranking and selection of the best individuals based on their rank. Goldberg's non-dominated ranking procedure assigns Rank 1 to the non-dominated individuals and temporarily removes them from the population, then finds a new set of non-dominated individuals, Rank 2, and so on. The fundamental idea of this procedure is to assign equal probability of reproduction to all non-dominated individuals (Coello, 2001; Goldberg, 1989).

Genetic algorithms, like all procedures in the class of evolutionary algorithms, maintain a population of structures that represent a sample of search points in the space of potential solutions to a given problem. They deal with various types of optimization whether the objective (fitness) function is stationary or non-stationary (change with time), linear or nonlinear, continuous or discontinuous, or with random noise. The core algorithmic procedure includes fitness evaluation, selection and reproduction, which involve crossover and mutation operations (Hajela & Lin, 1992). The non-dominated sorting genetic algorithm II (NSGA II) is a popular Pareto dominance based multiobjective optimization algorithm (Deb, 2001). It is a genetic algorithm searching for an approximation to the Pareto set of a multiobjective optimization problem by the successive computation of a series of generations of solutions (Deb et al., 2002).

2.4 Multiobjective Optimization and Simulation-Based Decision-Making

Most real-world decisions involve the simultaneous, optimization of multiple, and often conflicting, objectives. Due to the "satisficing" of the objectives, often a large set of compromise, or tradeoff, solutions that seek to balance the set of objectives are identified. This set of solutions characterizes the efficient frontier in the objective space from which the decision-maker can select the most preferred solution. The best tradeoff solution is selected according to decision-maker (or, set of decision-maker) preferences and existing and future physical, technological and financial constraints. In order to generate solutions that balance the multiple objectives, researchers and practitioners typically use procedures that generate the set of Pareto optima.

Applications of the optimization of multiple objectives, in research and in practice, typically involve using metaheuristic search procedures in deterministic settings (e.g., Bae, Qiu, & Fox, 2010; Menon, Bates, & Postlethwaite, 2006; Milickovic et al., 2001; Pacheco, Casado, Alegre, & Alvarez, 2008; Pop, Vlad, Chifu, Salomie, & Dinsoreanu, 2011; Tasgetiren, Pan, Bulut, & Suganthan, 2011).

However, the success of these search procedures is not as consistent in noisy environments where the objective functions are stochastic such as when using simulation as the evaluator of the individual objective functions. Evolutionary algorithms (EAs) are generally believed to be able to handle deterministic or stochastic objective functions fairly well since promising areas of the search space are sampled several times (Li, Ji, Wu, & Xue, 2010; Togelius et al., 2010).

Aguirre & Taboada (2011) address the multiobjective optimization problem and propose a two-stage algorithm with: (1) the optimization stage and (2) the post-Pareto analysis stage. The first stage focuses on obtaining a set of non-dominated solutions. An EA-based simulation optimization approach requires a large number of simulation evaluations due to the stochastic components not only of simulation model but also because of the stochastic features of EAs before a satisfactory solution can be found (Syberfeldt et al., 2009). The second stage of decision-making, known as "Pareto Analysis", and it is as important as the optimization stage of finding an approximate set of non-dominated solutions. It involves the selection of one solution from the set of Pareto optima. Thus, choosing a single solution over the others or reducing the number of solutions to select from is not a simple task, and can be overwhelming when presented with an extraordinarily large set of potential compromising solutions.

2.4.1 Multiobjective Optimization and Cluster Analysis

Traditional Pareto analysis approaches produce large sets of non-dominated solutions effectively placing the decision-maker in a challenging position to select one solution over other compromise solutions. Several studies propose ways to reduce the number of Pareto solutions to a reasonable number based on prior information known by the decision-maker. One approach is called the axiomatic approach. The decision-maker is ready to sacrifice some of the values according to his/her preferences on a set of objectives while to improve some of other values according to the preferred sets of the objectives (Noghin, 2011).

Many researchers use clustering analysis to reduce the set of Pareto solutions. Clustering analysis is the task of constructing the *m* groups or clusters of qualitatively or quantitatively similar objects, directly from a set of *n* original objects. The clusters are generally non-overlapping or mutually exclusive (Morse, 1980). Clustering analysis techniques can be used to organize and classify the solutions. Clustering the set of Pareto solutions and then selecting a preferred solution or set of solutions from each cluster to represent the original set of Pareto optima can help a decision-maker in his/her choice of the best solution to implement (Chaudhari, Dharaskar, & Thakare, 2010). Furthermore, a number of numerical studies that compare clustering algorithms to reduce the set of Pareto optimal solutions shows hierarchical clustering algorithms are highly recommended and preferable over the other clustering algorithms such as the direct clustering. Hierarchical algorithms are shown to perform reasonably well, such as the centroid clustering algorithm (Zitzler & Thiele, 1999; Zitzler & Thiele, 1998).

Syberfeldt, Ng, John, & Moore (2010) propose an approach to reduce the set of Pareto optima in the presence of stochastic objectives using an evolutionary algorithm by re-sampling until the solution reaches to a given confidence level. The approach also clusters the set of Pareto optimal solutions in the presence of stochastic objectives. The set of Pareto solutions are clustered based on the difference in their mean values. In addition, Zio & Bazzo (2011) suggest a two-way procedure with providing a number of representative solutions that is presented to the decision-maker. The original set of Pareto solutions are clustered into "families," which are then synthetically represented by a "head-of-the-family" solution. The representative solutions are produced by considering their distance from an ideal solution (which optimizes all objectives simultaneously). In the latter situation, a fuzzy scoring procedure is applied for ranking solution alternatives. Moreover, Deb & Goel (2001) propose an evolutionary algorithm to produce a set of solutions then to check for the set of non-dominated solutions and finally cluster analysis is used to narrow down the set of Pareto optima. In the clustering stage, each solution belongs to a stand-alone cluster and then the distance between each cluster is calculated to find the centroids of each cluster by computing the Euclidean distance between the centroids. This algorithm considers clusters with minimum distance to be merged together into a bigger cluster. Nonetheless, the previous step is continuing until the desired number of clusters is recognized. Lastly, the solution closest in distance to the centroid of a cluster is retained and consider while all the other solutions from each cluster are neglected.

Many have used the dynamic growing self-organizing tree (DGSOT) algorithm to perform post-Pareto analysis. The advantages of this algorithm shows that there is no initial number of clusters needed, optimal number of clusters is effective at each hierarchical level, and misclustered data are rearranged by reassigning data from previous hierarchical levels. Therefore, the decision-maker can better analyze a smaller set of representative solutions instead of the whole Pareto front (Aguirre, Taboada, Coit, & Wattanapongsakorn, 2011; Aguirre & Taboada, 2011). Similarly, Sakata, Faceli, De Souto, & De Carvalho (2010) suggest a selection strategy to reduce the set of Pareto optimal solutions obtained from Pareto-based multiobjective genetic algorithms with an automatically adjustable threshold. The strategy facilitates a better selection of the most evident partitions while no initial setting is required. The strategy presents a better set of solutions and maintains the diversity within the partitions in the reduced set.

However, many of the approaches listed above are powerful to reduce the set of Pareto solutions but unfortunately without considering the uncertainty of the objectives. On the other hand few others considered the uncertainty of the objectives on their approaches to reduce the set of Pareto solutions to a smaller set but not consider the number of solutions at the smaller set or even to prioritize the representative solutions.

2.4.2 Multiobjective Optimization and Decision Analysis

The essential issue with multiobjective decision-making is deciding how best to strike an appropriate balance among a set of objectives such that an increase in value in one objective does not cause a decrease in value in another objective (Haimes, Li, & Tulsiani, 1990). Most of the existing work in the open research literature integrates multiobjective algorithms and decision analysis to approximate and visualize the robust set of Pareto optima such as Krishna & Baskaran (2007), McConaghy, Palmers, Steyaert, & Gielen (2009), and Zhong & Li (2007).

Many researchers have used the popular swing weighting approach among the other multi-criteria decision-making approaches in the presence of multiple objectives. Using swing weights, the decision-maker determines which solution are the most important, the second most important, etc. and also the degree to which each objective is more important than the others. These numbers are then normalized to sum to 1.0 (Clemen & Reilly, 2004; Weber, Eisenführ, & Von Winterfeldt, 1988).

2.5 Summary

In summary, there is little work that addresses how to reduce the set of Pareto solutions in the presence of stochastic objectives. Furthermore, it can be concluded that the multiobjective optimization practitioners have yet to take full advantage of cluster analysis and/or decision analysis to improve the decision-making procedure by reducing the set of Pareto optima effectively in the presence of stochastic objectives. These include approximating the number of Pareto optimal solutions and using clustering analysis to reduce the number of Pareto optimal solutions to a smaller set. To the best of this researcher's knowledge, the literature provides a little work for reducing the number of the Pareto optimal solutions while considering the stochastic nature of the objectives. New efforts concerning improvement of decision-making for multiple objective problems and the need to reduce and organize the set of non-dominated solutions in the presence of stochastic objectives may benefit the decision-maker and provide a contribution not only to the practitioner body of knowledge, but also to the research community.

CHAPTER 3:
A POSTERIORI APPROACH FOR DECISION-MAKING WITH MULTIPLE STOCHASTIC OBJECTIVES

3.1 Introduction

In this chapter, an *a posteriori* approach is presented. This investigation specifically focuses on how to intelligently and effectively reduce the number of candidate compromise solutions while considering the stochastic nature of a set of multiple objectives. The approach effectively articulates the decision-maker preferences *after* the optimization process (i.e., an *a posteriori* analysis). The approach uses statistical analysis and clustering analysis on the Pareto optimal solutions in order to reduce the number of solutions to set of representative solutions that is presented to the decision-maker for final selection.

3.2 Proposed Approach

The proposed *a posteriori* approach to reduce the number of candidate Pareto optimal solutions consists of three sequential general phases – Reduce, Cluster, and Prioritize. Figure 3.1 shows the logic of the *a posteriori* proposed approach.

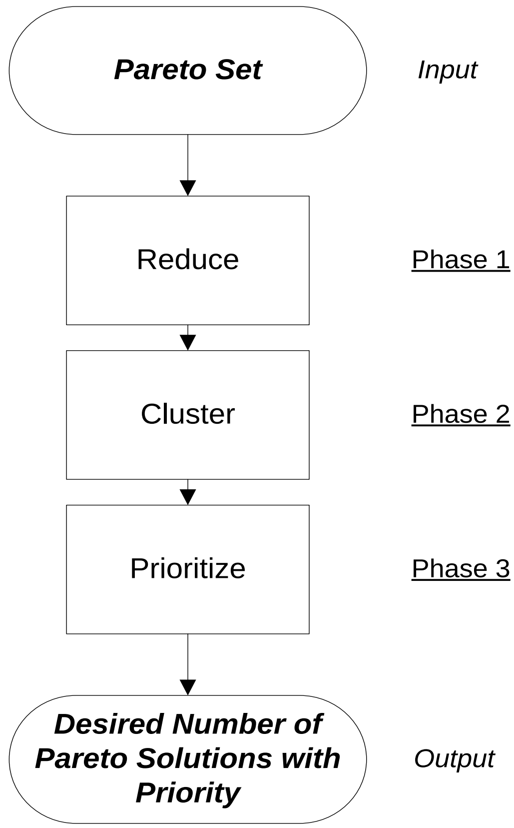

Figure 3.1: General logic flow of the *a posteriori* approach.

The proposed approach begins with a given set **P** of Pareto optima (or, tradeoff solutions) as input. The initial set of Pareto optima that is produced by an integrated simulation optimization computational framework that integrates the multiobjective optimization algorithm and a stochastic computer simulation model, which represents the decision setting and conditions of the problem to be solved, is used to generate the initial set of Pareto optima. The purpose of the multiobjective optimization algorithm is to generate candidate solutions in the form of vectors of values for the *n* decision variables. The purpose of the simulation model is to evaluate the relevant measures of performance that are to be optimized, and the measures of performance are represented by mean and standard deviation values (i.e., confidence intervals at a level of significance).

It is important to note here that, before the first Phase of the approach begins, the user of the proposed approach chooses a variability factor, v (between 0.5 and 1) where v is a normal probability of a stochastic p solution falling within an interval that is within k standard deviations of the mean of the solution values in **P**. In addition, the user of the approach converts all solution values in **P** to minimum or maximum as needed.

3.2.1 Phase 1 – Solution Set Reduction

Phase 1 is illustrated in Figure 3.2. As discussed previously, the proposed approach begins with an initial set of P Pareto optima as input. Figure 3.3 shows an example set **P** of Pareto optimal solutions (assuming a two-objective minimization problem). For each solution in the initial set **P** of Pareto optimal solutions, the lower confidence level (assuming minimization) in each of the m objective dimensions is computed. Note that, in case of maximization, objectives can be converted to minimization by multiplying the objectives by -1, without loss of generality. Using the associated standard deviations, the precision of the initial set **P** of Pareto optimal solution values is represented by the confidence intervals, computed using Eq. 3.1, along each objective space dimension, creating an upper and a lower limit for each Pareto optimal solution. Figure 3.4 shows the lower confidence limit curve that corresponds to each Pareto solution mean in the example Pareto optimal solution set.

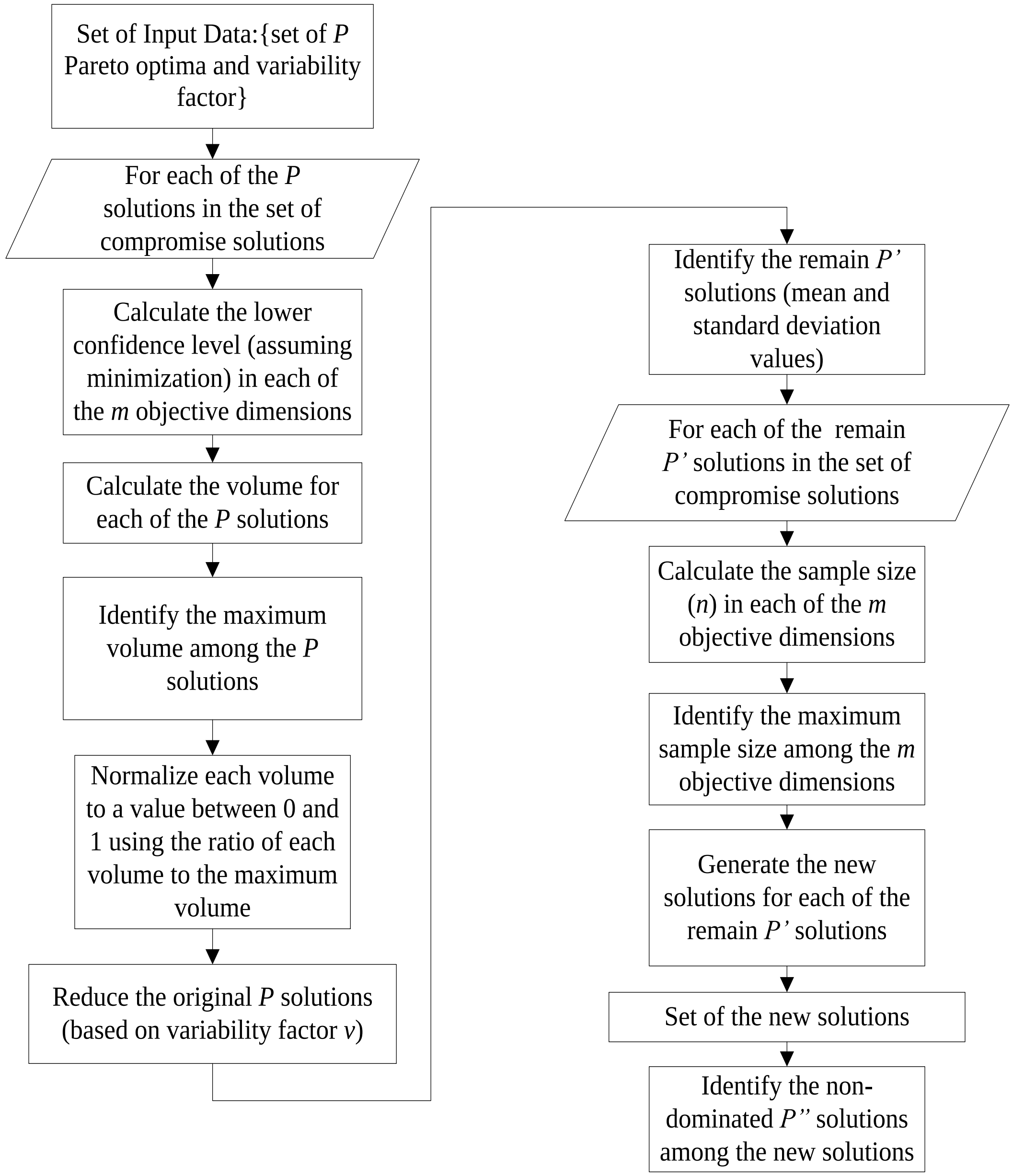

Figure 3.2: Overview of Phase 1 of the proposed *a posteriori* approach.

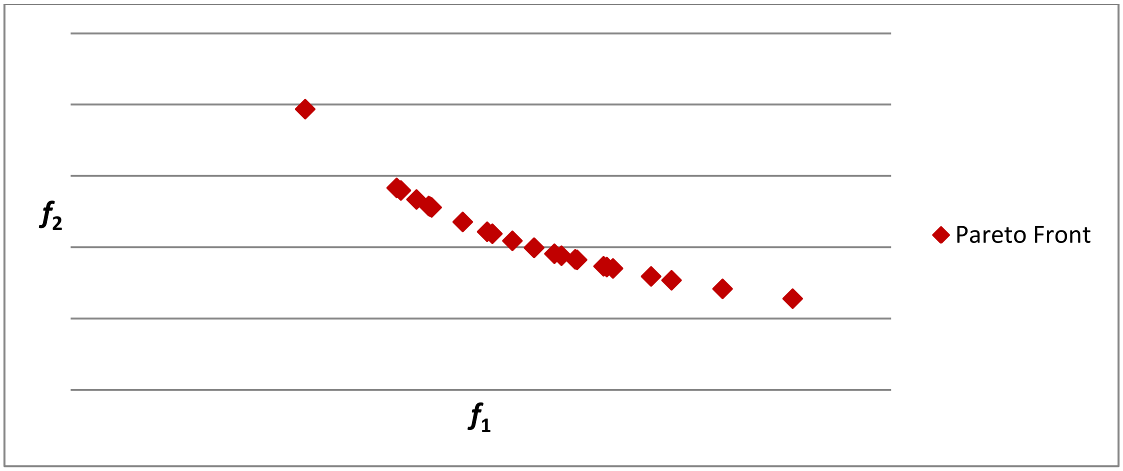

Figure 3.3: Example set of Pareto optimal mean values.

The confidence intervals are computed using

$$\bar{x} \pm t_{\frac{\alpha}{2},n-1}\left(\frac{s}{\sqrt{n}}\right), \quad (3.1)$$

where $\bar{x}$ is the mean objective value from the n replications, s is the standard deviation of the objective value from the n replications, α is the level of significance, and $t_{\alpha/2,n-1}$ is the upper $\alpha/2$ critical value for the t-distribution with n-1 degrees of freedom.

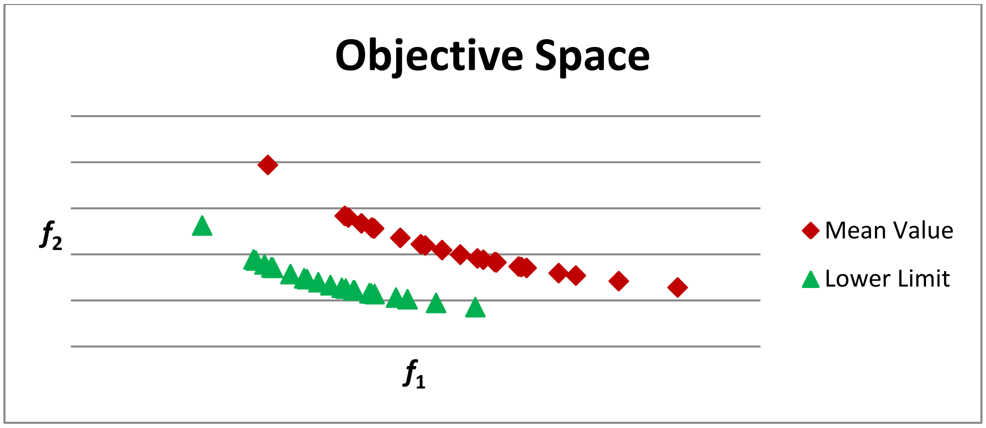

Figure 3.4: Set of Pareto optimal mean values and their lower confidence interval limit (assuming a minimization problem) for the example.

Next, the volume for each of the solutions in **P** with respect to each solution's half-length for the *m* objectives is calculated as it serves as a guidance criterion for finding a reduced and a good approximations to the Pareto front (e.g., Beume, Fonseca, Lopez-Ibanez, Paquete, & Vahrenhold, 2009; Le & Landa-Silva, 2007). The volumes represent the stochastic boundary for each of the solutions in **P**. Then, the maximum volume among the solutions in **P** is identified, and each volume is normalized to a value between 0 and 1 using the ratio of each volume to the maximum volume.

The original set **P** of Pareto optimal solutions is, then, reduced based on the variability factor *v* pre-specified by the user, as all the volume percentages are compared to *v* (Anderson, 1986). For example, consider if, for a particular solution, the normalized volume is greater than *v*. Then, the solution is ignored from the original set **P** and is not considered further. However, if the solution's normalized volume is less than or equal to *v*, then that solution is considered further in the analysis. The variability factor value chosen by the analyst can be varied, and the most appropriate value of *v* can be determined experimentally. The reduced set of original Pareto optimal solutions is then represented by **P**'.

Now, for each of the remaining tradeoff solutions in **P**', the sample size *n* in each of the *m* objective dimensions is calculated. The sample size is a representative portion for each original stochastic solution of the remaining original solutions now in **P**' (Garza & Williamson, 2001). The sample size *n* is computed using

$$n = (\frac{Z_{\alpha/2}\,\sigma}{H})^2. \qquad (3.2)$$

where $Z_{\alpha/2}$ is the upper $\alpha/2$ critical value for the normal distribution, σ is the standard deviation of the objective value, α is the level of significance, and H is the half-width of the confidence interval.

The maximum sample size *N* among the *m* objectives is identified and is considered further in the analysis for each solution in **P**'. After that, each of the remaining solutions is replicated and replaced by *N*-time of solutions. These *N*-time solutions for each solution in **P**' are generated by using the normal probability distribution for the random variate generation (Law, 2007). The newly-generated solutions fall between the Pareto curve produced by the original solutions in **P** and the curve of the corresponding lower confidence limits, as shown in Figure 3.5.

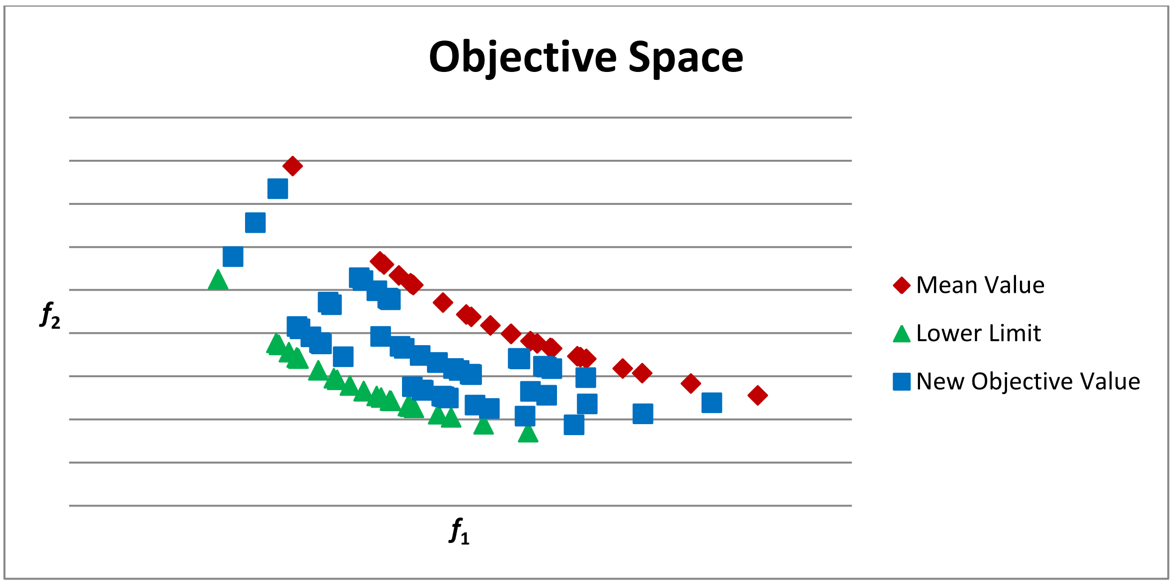

Figure 3.5: Set of Pareto optimal solutions and their lower limits and new solutions for the example.

Then, the non-dominated **P**'' solutions among the new set of solutions are identified. Figure 3.6 shows an example of dominated and the set **P**" of non-dominated solutions. The non-dominated solutions are considered for Phase 2. The logic of Phase 1 is shown using pseudocode in Figure 3.7.

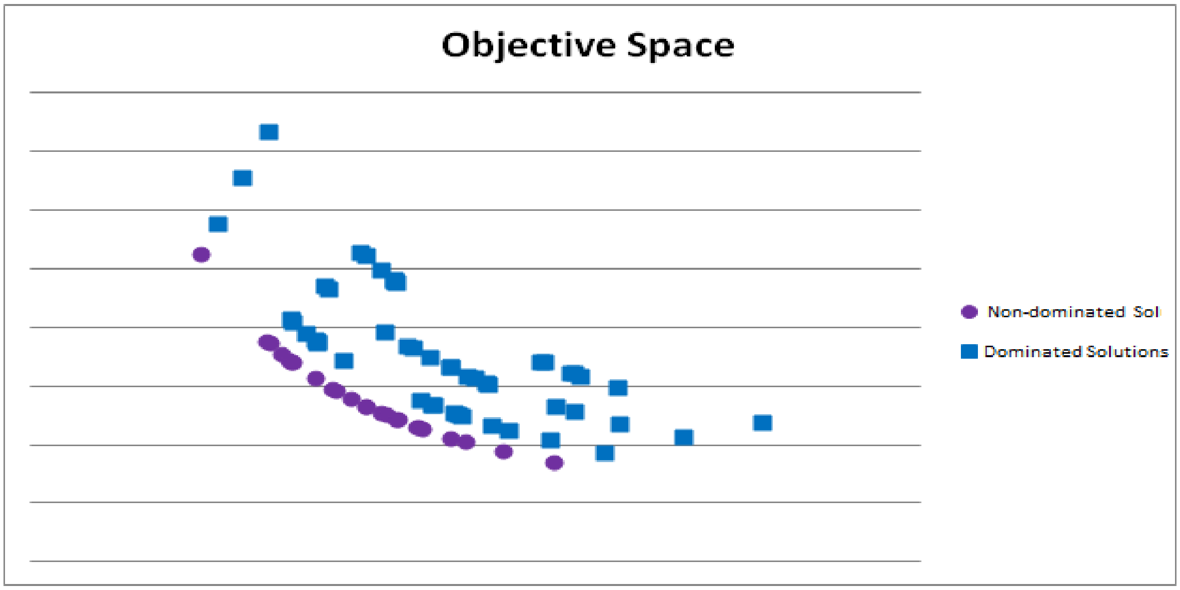

Figure 3.6: The set of dominated and non-dominated solutions for the example.

Pareto frontier **P** = {1, 2, …, p}
Convert the objective value into minimum (if needed)
For ($i = 1 \leq |\mathbf{P}|$)
    Lower confidence level (LCL_i)
    Volume (V_i)
    Maximum volume (MV_i)
    Volume as percentage of the maximum one (PV_i)
Read variability factor v
For ($i = 1 \leq |\mathbf{P}|$)
    If ($PV_i \leq v$)
    P'= {1, 2, …, p'}
    Sample size (n_i)
Maximum sample size (M_n)
For ($i = 1 \leq M_n \times |\mathbf{P}'|$)
    New objective values **F**= {1, 2, …, $M_n \times p$'}
    Non-dominated objective value **P**'' = {1, 2, …, p''}
Report the output (**P**'')

Figure 3.7: Pseudocode for Phase 1.

3.2.2 Phase 2 – Clustering

The non-dominated solutions in set **P**'', and the desired number of clusters c are the input for Phase 2, which is briefly illustrated in Figure 3.8. Clustering analysis is applied to the solutions in set **P**" identified in Phase 1. The centroid linkage method, which is an agglomerative clustering approach, is a widely-used approach for analyzing large datasets (Zitzler & Thiele, 1999; Zitzler & Thiele, 1998), and it is used here.

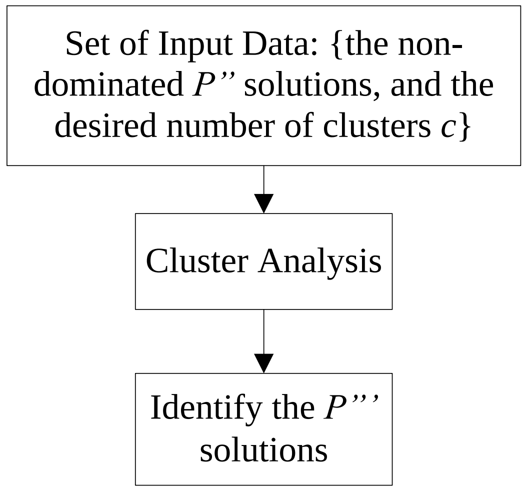

Figure 3.8: Overview of Phase 2 of the proposed *a posteriori* approach.

In the centroid linkage method, a distance matrix between the data points is constructed. The centroid linkage method uses the squared Euclidean distance as the distance measure between two data points (i.e., tradeoff solutions) in the objective space. It calculates the distance between two clusters as the sum of distances between cluster means. Then, it involves merging clusters with the most similar mean vectors. In the centroid method, the centroid of a merged cluster is a weighted combination of the centroids of the two individual clusters, whereas the weights are proportional to the sizes of the clusters. This particular clustering approach requires the number of desired clusters to be pre-specified by the model analyst (Everitt, Landau, & Leese, 2001). The final set **P**''' of solutions is identified according to the pre-specified number of clusters chosen by the analyst. The centroid is calculated for each of the final desired clusters, and then the closest point (solution) in distance to the centroid is considered for prioritization in next Phase. The final set **P**''' of solutions is presented in the form of a dendrogram to illustrate the arrangement of the clusters produced by hierarchical clustering approach. The dendrogram, or tree diagram, is a mathematical and pictorial representation of the complete clustering procedure, which illustrates the process and the partitions produced at each stage as shown in Figure 3.9. The logic of Phase 2 is shown using pseudocode in Figure 3.10.

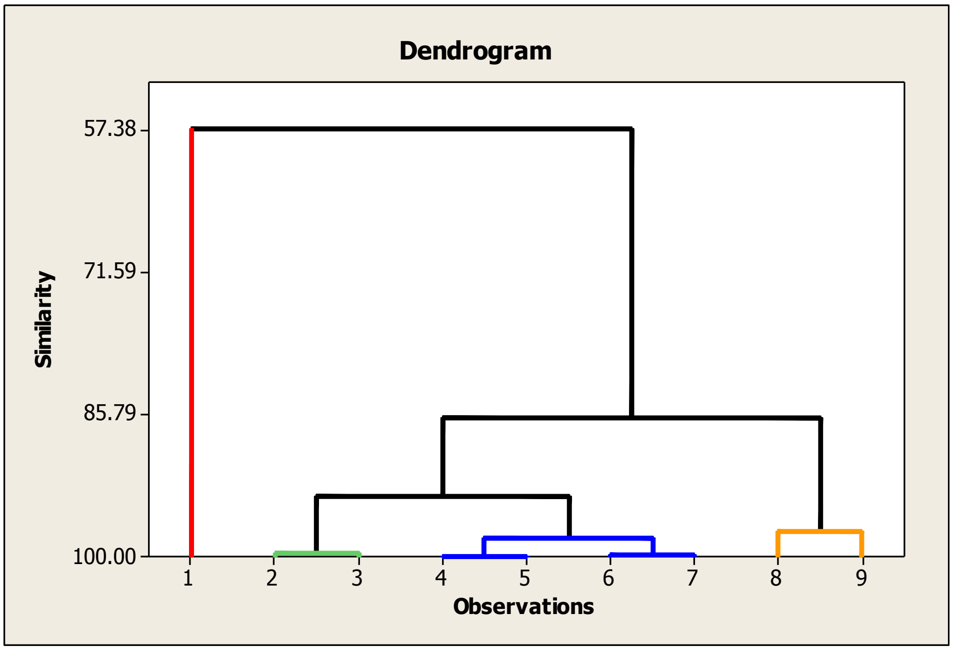

Figure 3.9: Example of a dendrogram.

```
Read P'' = {1, 2, …, p''}
Read desired number of clusters c
        Cluster analysis for P'' with c
Report the output P''' = {1, 2, …, c}
```

Figure 3.10: Pseudocode for Phase 2.

3.2.3 Phase 3 - Prioritization

The set **P**''' is the input for Phase 3, which is briefly illustrated in Figure 3.11. In general, evaluating and prioritizing large set of candidate solutions is a particularly difficult task for decision-makers. Nonetheless, multiobjective decision-making approaches are usually used to select the most proper solution among the other available solutions (Noghin, 2011).

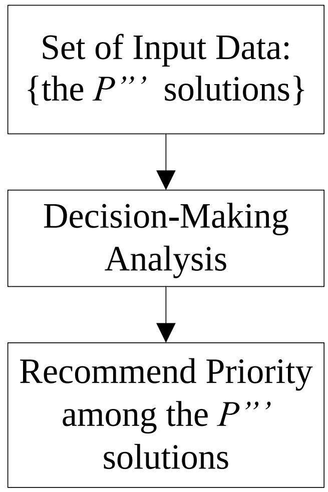

Figure 3.11: Overview of Phase 3 of the proposed *a posteriori* approach.

In this Phase, prioritization of the representative set of solutions in **P**''', which are identified in Phase 2, is performed. Many researchers have used the swing weighting approach, among other multi-criteria decision-making approaches in the presence of multiple objectives. In general when using swing weights, the decision-maker determines the representative solutions that are the most important, the second most important, etc. as well as the relative degree of importance. These numbers are then normalized to sum to 1.0 (Clemen & Reilly, 2004; Weber et al., 1988). The swing weighting approach is used for Phase 3 in order to prioritize the set of representative solutions.

In this Phase, the decision-maker preferences on objectives are identified. For example, consider that a two-objective problem with a lower value of Objective 2 is the most desired, and then a lower value of Objective 1 is the second most desired. Second, a swing weight assessment is performed of the set of objectives of the problem. Table 3.1 summarizes the assessment of the two-objective example problem. The first row indicates the worst possible outcome, or the outcome that is at the worst level on each of the attributes (solutions). Each of the succeeding rows “swings” one of the attributes from worst to best.

Table 3.1: Swing weight assessment for the two-objective example problem.

Attribute Swing from Worst to Best	Consequence to Compare	Rank	Rating	Weight
(Benchmark)	100, 50	3	0	0.00 = 0/140
O_1: Objective 1	10, 50	2	40	0.29 = 40/140
O_2: Objective 2	100, 5	1	100	0.71 = 100/140
		Total	140	1.00

Then, the objectives are rank ordered. For instance, for this example, there are three hypothetical set of solutions to compare, and it is safe to assume that the benchmark solution – the one that is worse on all objectives – is ranked third (worse) overall. The others are compared to determine which ranks first (best), and second. The ratings of the objectives are based on decision-maker preferences. The rating for the Benchmark objective is 0 and the rating for the most preferred objective is 100. The rating for the other objectives must fall between 0 and 100. With these assessments of the objectives, the table is completed and weights can be calculated. The weights are the normalized ratings that sum to 1.0.

Next, the overall utility for each representative Pareto optimal solution in set **P**”’ is calculated. For example, the utilities for the alternatives in **P**”’ are shown in Table 3.2, which are calculated using Eq. 3.3 to Eq. 3.6.

Table 3.2: The feasible alternatives (solutions).

Representative Solution	Objective 1	Objective 2
1	100	5
2	75	30
3	45	50
4	10	25

$$U\ (100, 5) = O_1(0) \quad + O_2(1) \qquad = 0.714 \tag{3.3}$$
$$U\ (75, 30) = O_1(0.13) + O_2(0.17) \qquad = 0.157 \tag{3.4}$$
$$U\ (45, 50) = O_1(0.22) + O_2(0.10) \qquad = 0.135 \tag{3.5}$$
$$U\ (10, 25) = O_1(1) \quad + O_2(0) \qquad = 0.286 \tag{3.6}$$

Finally, with the utilities calculated, priority among the representative tradeoff solutions can be determined, as shown in Table 3.3. Figure 3.12 graphically shows the probability to have:

- All objectives worst that not in favor of the priority by the decision-maker,
- All objectives best, and
- Some objectives are best and other is worst.

Phase 3 steps can schematically be represented as the pseudocode shown in Figure 3.13.

Table 3.3: The feasible alternatives (solutions) with priority.

Representative Solution	Objective 1	Objective 2	Utility	Priority
1	100	5	0.714	1
2	75	30	0.157	3
3	45	50	0.135	4
4	10	25	0.286	2

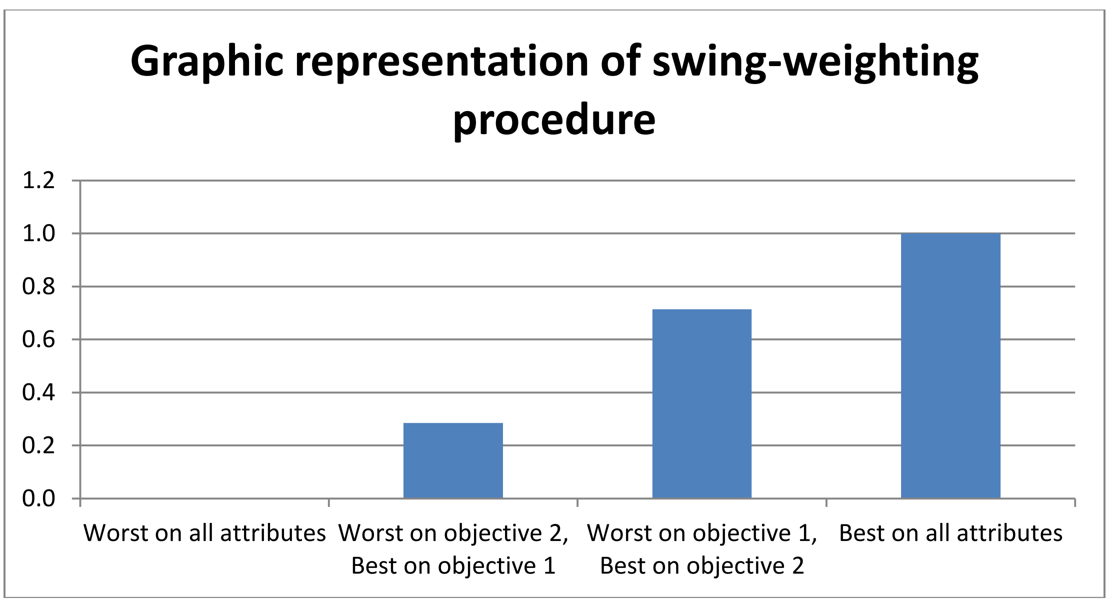

Figure 3.12: Graphic representation of swing weight procedure.

```
Read P''' = {1, 2, …, c}
        Decision-making analysis for P'''
Recommend priority among P'''
```

Figure 3.13: Pseudocode for Phase 3.

3.3 Summary

The *a posteriori* approach presented in this chapter effectively articulates the decision-maker preferences after the optimization process in the presence of multiple stochastic objectives. The *a posteriori* approach allows reducing and organizing the set of non-dominated solutions considering the stochastic nature of the objective functions.

CHAPTER 4:
COMPUTATIONAL STUDY: SOLVING THE (*s*, *S*) INVENTORY PROBLEM BY THE A POSTERIORI APPROACH

4.1 Introduction

This chapter applies the proposed *a posteriori* approach to a well-known inventory problem. A numerical simulation model of the inventory problem is integrated with a multiobjective evolutionary algorithm. The non-dominated sorting genetic algorithm II (NSGA II) is used to optimize the decision variables and generate the set of Pareto optimal solutions. The *a posteriori* proposed approach begins with this set of tradeoff solutions as input.

First, in this chapter, the details of the inventory case study problem are presented in Section 4.2. Then, in Section 4.3, the computational results after applying the proposed approach is presented and discussed. Next, Section 4.4 summarizes the results from the empirical analysis of identifying the most appropriate variability factor v. Then, Section 4.5 presents the computational results when only clustering analysis is applied to the set **P** of original Pareto optimal solutions. Finally, Section 4.6 shows the computational results when a simulation optimization approach is applied to the case study problem. Section 4.7 summarizes the chapter.

4.2 Case Study: The (*s*, *S*) Inventory Problem

The (s, S) inventory problem involves a random demand distribution and the goal of identifying a reorder point s and order-up-to point S that for the demand distribution that optimizes (i.e., balances) inventory costs. The little s and the big S in this inventory problem are the decision variables.

For the sake of this case study, it is assumed that a company sells a single product and would like to determine how many units it should have in inventory for each of the next n

months, where n is a fixed input parameter. The time between demand realizations are independent and identically distributed (IID) exponential random variables. The value of the demand realizations are assumed IID random variables, independent of when the demand occurs with

$$D = \begin{cases} 1 & w.p. \quad 1/6 \\ 2 & w.p. \quad 1/3 \\ 3 & w.p. \quad 1/3 \\ 4 & w.p. \quad 1/6 \end{cases} \tag{4.1}$$

where w.p. is read “with probability.”

At the beginning of each month, the company reviews the inventory level and decides how many items to order from its supplier. The first decision variable s is the minimum level reached by the inventory is the minimum order level. The second decision variable S is the maximum level of the inventory. If the company orders Q items, it incurs a cost of $K+iQ$, where K is the fixed ordering cost and i is the incremental cost per item ordered. If $Q = 0$, no cost is incurred. When an order of quantity Q is placed, the time required for it to arrive (called the delivery lag or lead time) is a random variable. The company uses a stationary (s, S) inventory policy to decide how much to order, i.e,

$$Q = \begin{cases} S - I & if\ I < s \\ 0 & if\ I \geq s \end{cases} \tag{4.2}$$

where I is the inventory level at the beginning of the month.

When a demand realization occurs, it is satisfied immediately if the inventory level is at least as large as the demand. If the demand exceeds the inventory level, the excess of demand over supply is backlogged and satisfied by future deliveries. When an order arrives, it is first used to eliminate as much of the backlog (if any) as possible. The remainder of the order (if any) is added to the inventory (Hopp & Spearman, 2011).

The company is interested in minimizing $H(s, S)$, the average holding cost per period, and $B(s, S)$, the average shortage cost per period.

$$H(s,S) = h \frac{\int_0^n I^+(t)dt}{n} \tag{4.3}$$

$$B(s,S) = P \frac{\int_0^n I^-(t)dt}{n} \tag{4.4}$$

4.3 Application of the Proposed *A Posteriori* Approach to the Case Study

The simulation framework for the (s, S) inventory with backlogging model integration with the NSGA II MOEA is illustrated in Figure 4.1. In this case example, a two-objective, two-variable minimization problem is considered. The average holding cost per period and the average shortage cost per period are the objectives. Suppose that, for this problem, four representative solutions are desired. The *a posteriori* approach begins with a given set **P** of Pareto optimal solutions as input. The user of the approach chooses a variability factor, v (between 0.5 and 1). Recall that v is a normal probability of the tradeoff solutions falling within the interval within k standard deviations of the set **P** of Pareto optimal solution values (means).

4.3.1 Generation of the Set of Pareto Solutions

The simulation optimization integrated framework is comprised of the NSGA II multiobjective evolutionary algorithm component and the inventory simulation component. The algorithm iteratively generates decision variables (s, S). Evaluation of the decision variables are performed by the inventory simulation model. The NSGA II optimization algorithm generates pairs of the two decision variables known as the inventory (*s*, *S*) policy. These decision values are passed to the inventory simulation model to generate and replicate the objective function values (i.e., *H* (*s*, *S*): inventory holding cost per month and *B* (*s*, *S*): inventory shortage cost per month). The inventory simulation model returns the mean of the objective function values and corresponding standard deviation values to NSGA II. NSGA II generates and passes the new decision variable values to the inventory simulation model in order to compute the mean objective function values and corresponding standard deviation values. NSGA II then reports the set of Pareto (i.e., reports the mean of the objective function values and the corresponding standard deviation values, and the associated decision variable values).

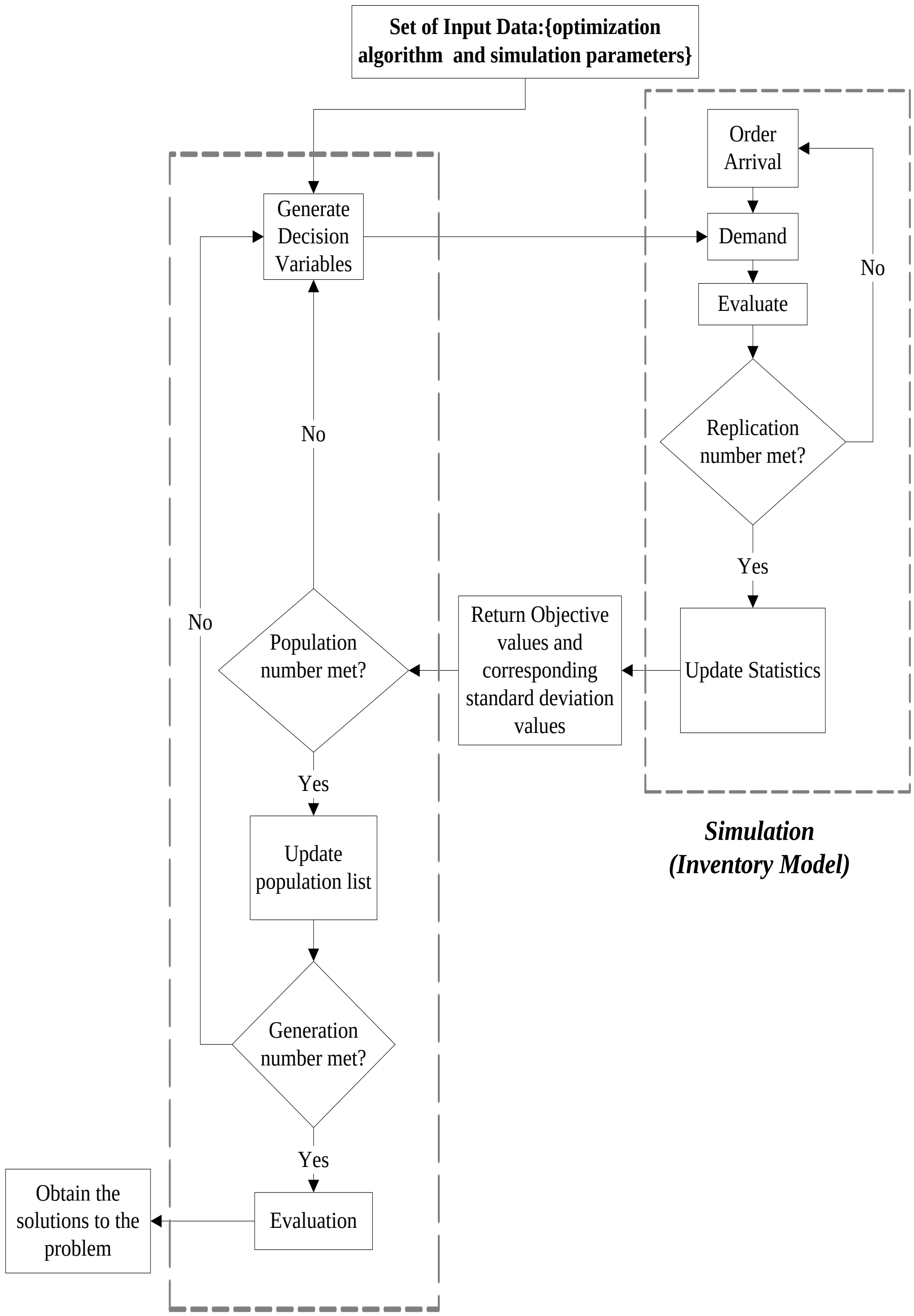

Figure 4.1: Overview of the simulation optimization framework for the (*s*, *S*) inventory with backlogging model.

4.3.2 Parameter Settings for the Simulation Model and NSGA II

Various decision parameter values are set for the inventory simulation model and NSGA II. This section summarizes the parameters settings and range of initial values of the parameters. The parameter values specified for the NSGA II and the inventory simulation model are summarized in Table 4.1 and Table 4.2 respectively.

Table 4.1: Decision and search control parameter values for NSGA II.

Parameter	Value
Population size	100
No. of generations	100
No. of objective functions	2
No. of constraints	0
No. of real variables	2
Lower & Upper limits of the 1st real-coded variable	20, 60
Lower & Upper limits of the 2nd real-coded variable	40, 100
The cross-over probability	1.0
The mutation probability for real-coded vectors	0.5
Distribution Index for real-coded crossover	20
Distribution Index for real-coded mutation	5
No. of binary-coded variable	0

Table 4.2: Parameter values for the (s, S) inventory simulation model.

Parameter	Value
No. of replications	100
Initial inventory level	60
No. of months	120
Mean of inter-demand	0.1
Setup cost	$32.00
Incremental cost	$3.00
Holding cost	$1.00
Shortage cost	$5.00
Minimum delivery lag (month)	0.5
Maximum delivery lag (month)	1.0

The variability factor value v is varied to identify its appropriate setting with experimental values 65%, 75%, and 85%. In addition, the input values and parameters for Phase 2 are shown in Table 4.3. Recall that the input values for Phase 3 are the output of Phase 2.

Table 4.3: Parameter values for Phase 2.

Parameter	Method/Value
Linkage method	Centroid
Distance measure	Squared Euclidean
Number of clusters	4

Suppose that, for this problem, four representative solutions are desired. The *a posteriori* approach begins with a given set **P** of Pareto optima as input, and the user chooses a variability factor, v (between 0.5 and 1). Figure 4.2 shows the original decision space and Figure 4.3 shows the original Pareto optimal front generated by using a simulation multiobjective optimization approach that uses multiobjective evolutionary algorithms and discrete-event simulation. Each point on the curve (as shown in Figure 4.3) is generated after running n = 100 independent simulation replications. For each solution along the curve, the confidence interval along each dimension in objective space is computed.

Figure 4.2: Decision space for the decision variables *s* and *S*.

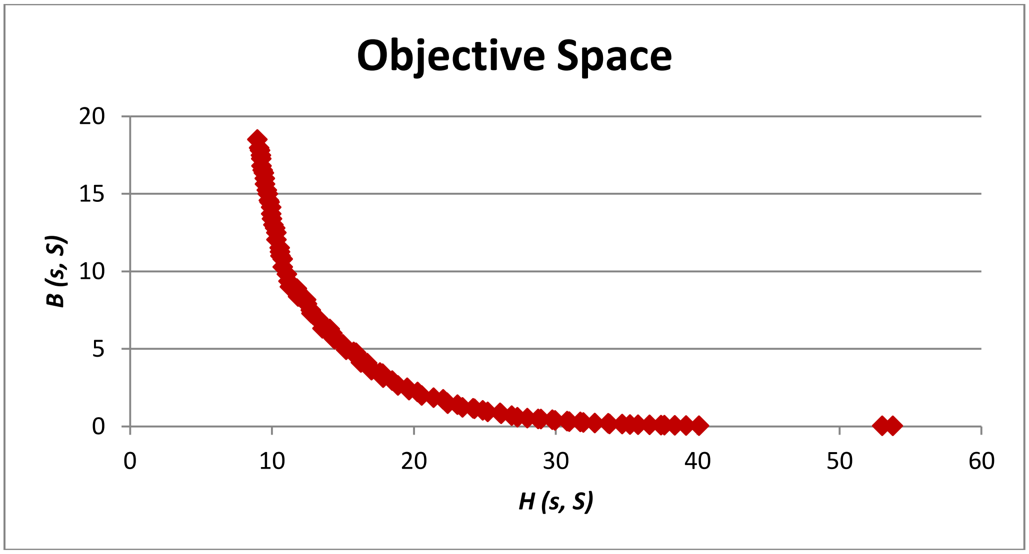

Figure 4.3: Objective space for the original mean objective functions (100 Pareto optimal solutions).

Phase 1 of the *a posteriori* approach starts with computing the lower confidence limit curve (shown in green in Figure 4.4) for each Pareto point (shown in red in Figure 4.4). Here, for illustration, a level of significance α = 10% is assumed. Also, the lower confidence limit is computed since this is a minimization problem.

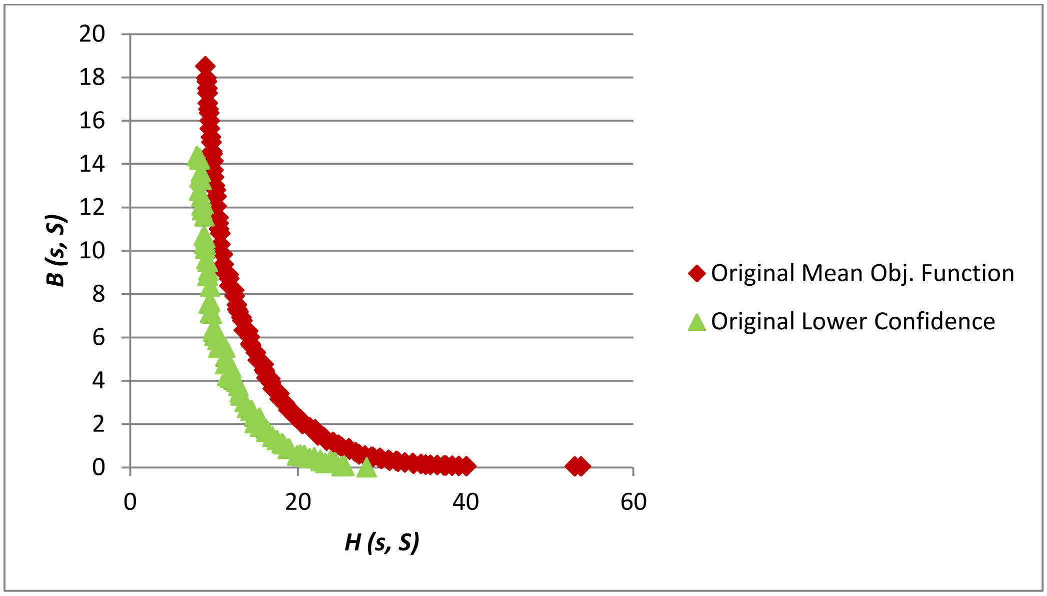

Figure 4.4: Original mean objective function values and the original lower confidence values.

Then, for each of the solutions in set **P,** the area for each point is computed. Each area value is normalized to a value between 0 and 1 using the ratio of each area to the maximum area value. Next, the number of original **P** solutions (based on the variability factor *v*) is reduced. For illustration, *v* = 0.65.

For each of the remaining **P**' solutions in the set of compromise solutions, the sample size in each of the *m* objective dimensions is calculated, noting the maximum sample size. Then, new solutions for each of the remaining original **P**' solutions are generated. The new solutions are bounded between original Pareto optimal front and the original lower confidence curve, as shown in Figure 4.5. Afterward, the reduced set of non-dominated solutions (say, **P**') among the new solutions is identified, as shown in Figure 4.6.

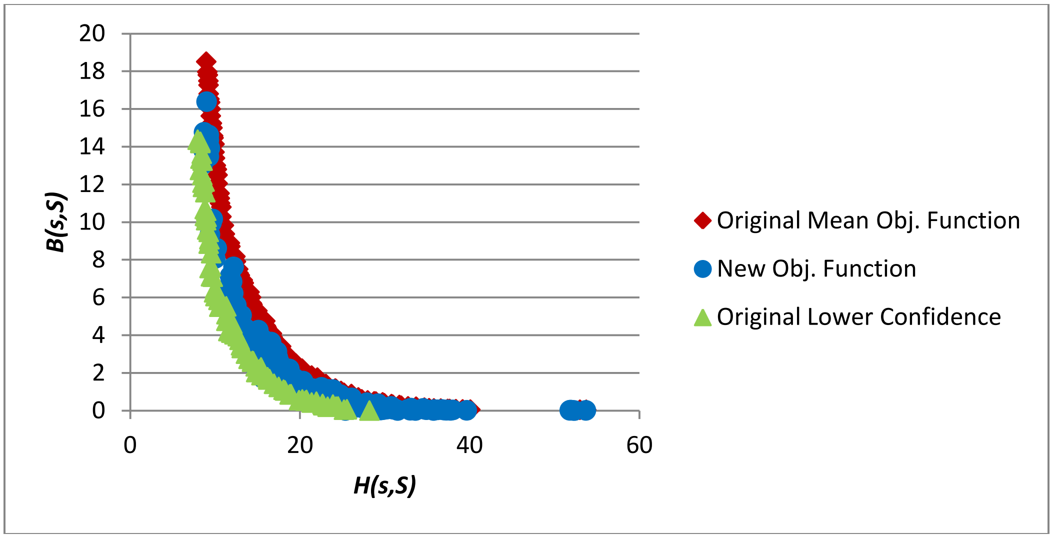

Figure 4.5: The new **P**' solutions (in blue).

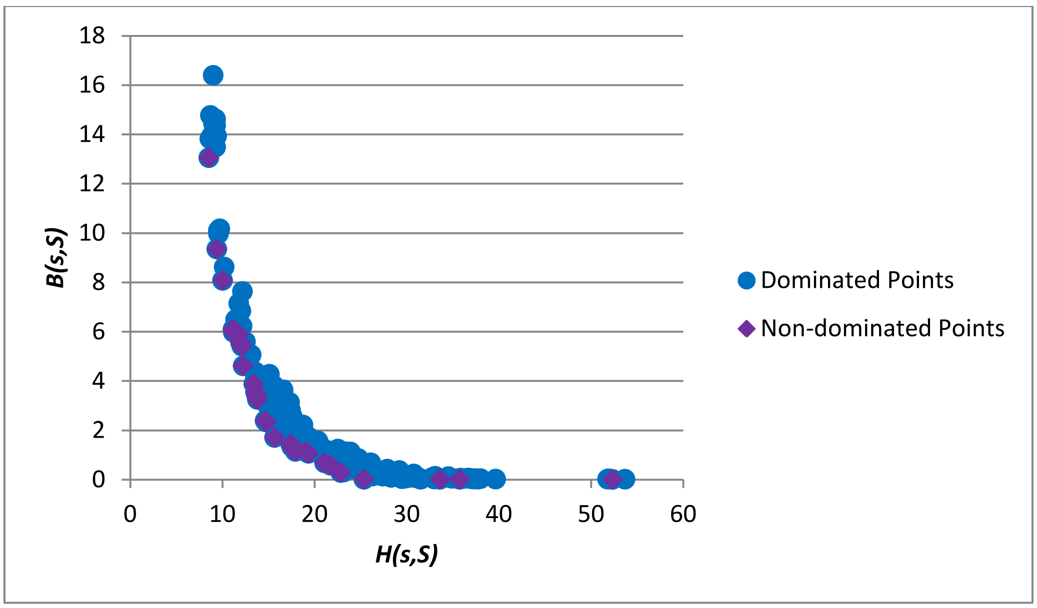

Figure 4.6: The new dominated and non-dominated **P**' solutions.

Phase 2 involves and applies the centroid linkage hierarchical clustering to the set of **P**' non-dominated solutions to group the reduced set **P**" of solutions. Figure 4.7 shows the non-dominated **P**' solutions assuming four representative clusters are desired, and Figure 4.8 shows the dendrogram. The centroid is calculated for each of the four clusters, and then the closest point (solution) in distance to the centroid is considered. These solutions are shown in Table 4.4 and in Figure 4.9.

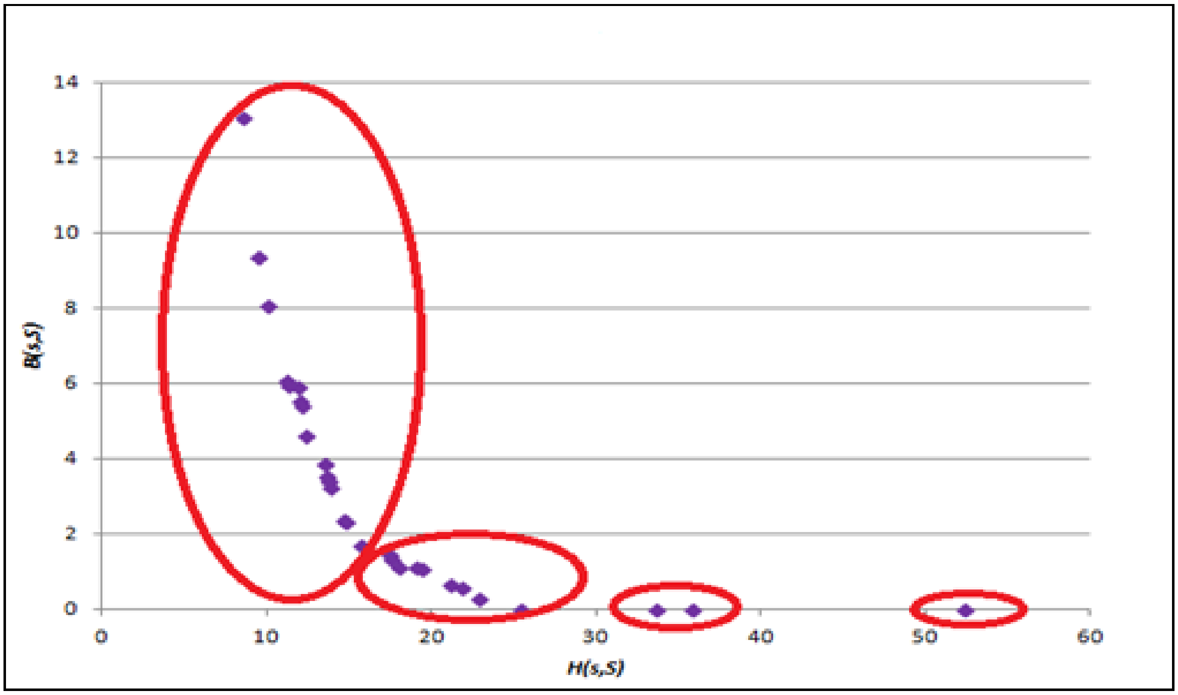

Figure 4.7: The set **P***'* of non-dominated solutions, assuming four clusters.

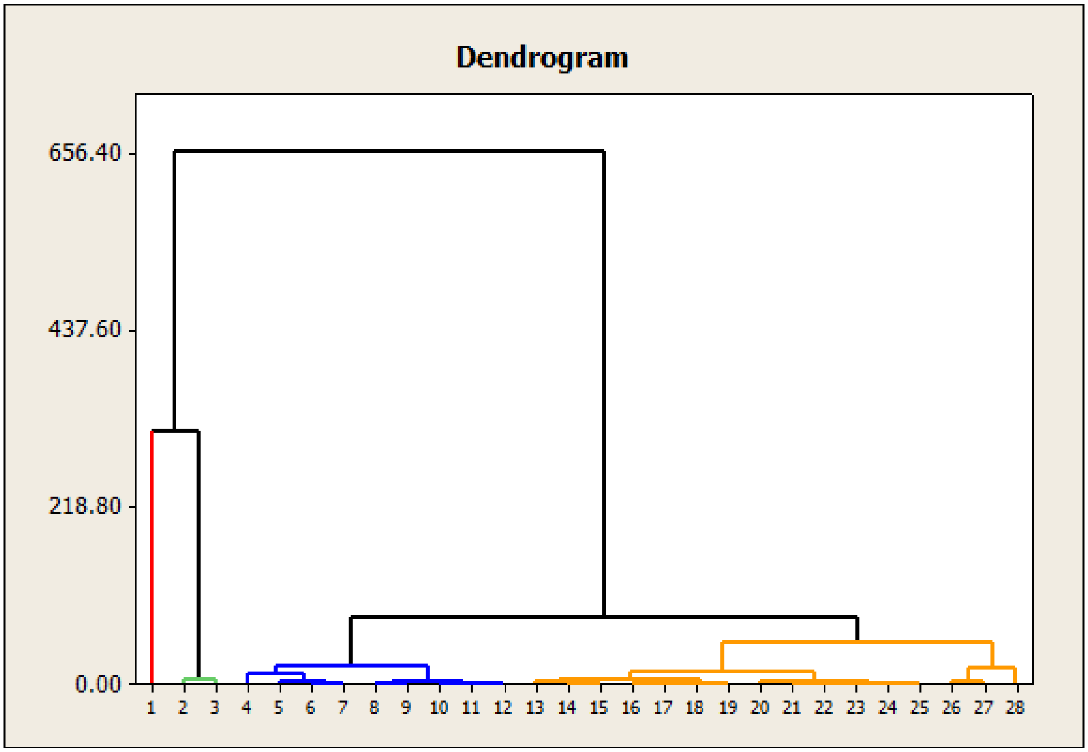

Figure 4.8: The dendrogram assuming four clusters.

Table 4.4: The non-dominated and feasible solutions for the problem.

Representative Solution	*H(s,S)*	*B(s,S)*
1	$52.345	$0.006
2	$35.757	$0.010
3	$21.097	$0.679
4	$12.118	$5.411

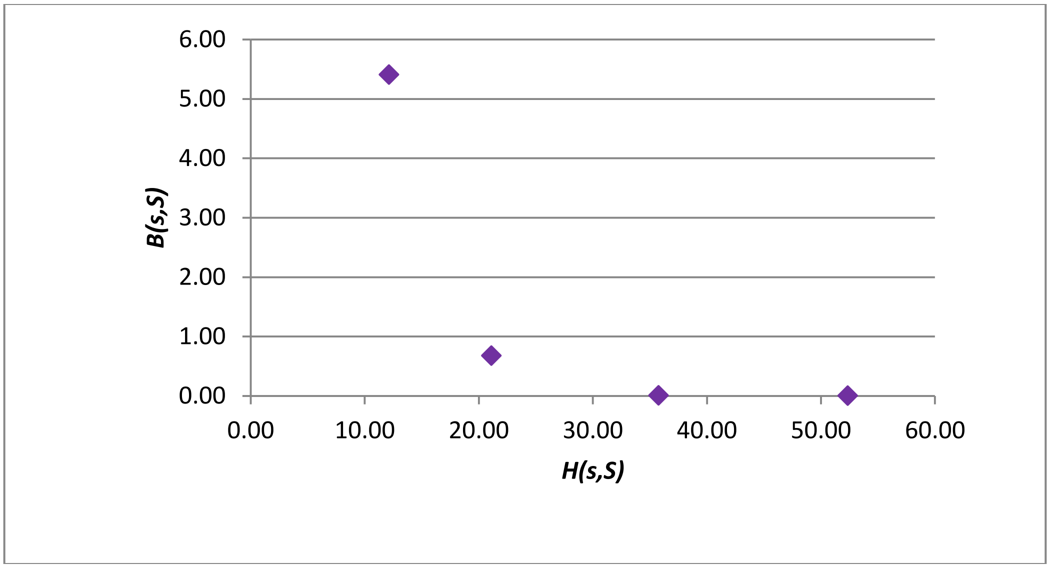

Figure 4.9: The non-dominated and feasible solutions for the problem.

Phase 3 of the *a posteriori* approach prioritizes the representative solutions identified in Phase 2. The swing-weighting approach is used in the *a posteriori* approach. Considering the current problem, assuming a lower value of $B(s, S)$ is desired first, and then a lower value of $H(s, S)$ is desired second. Table 4.5 shows the prioritized solutions using the swing weighting approach. Table 4.6 shows the assessment of the swing weights.

Table 4.5: The feasible solutions with priority.

Priority	***H (s, S)***	***B (s, S)***
1	52.345	0.006
2	35.757	0.010
4	21.097	0.679
3	12.118	5.411

Table 4.6: The assessment of swing weights.

Attribute Swing from Worst to Best	Consequence to Compare		Rank	Rate	Weight
(Benchmark)	52.345	5.411	3.00	0.00	0.00
H (s, S)	12.118	5.411	2.00	75.00	0.43
B (s, S)	52.345	0.006	1.00	100.00	0.57
			Total	175.00	1.00

The overall utility for the four representative Pareto optimal solutions is determined as shown in Eqs. 4.5-4.8. The value of the corresponding weight or the relative utility shows how the prioritized solutions are identified. Eqs 4.5 and 4.8 shows how the weight values shown on Table 4.6 are calculated for $H(s, S)$ and $B(s, S)$.

$$U\,(52.345, 0.006) = H\,(0) \quad + B\,(1) \quad = 0.57 \tag{4.5}$$

$$U\,(35.757, 0.010) = H\,(0.34) \quad + B\,(0.58) \quad = 0.48 \tag{4.6}$$

$$U\,(21.097, 0.679) = H\,(0.57) \quad + B\,(0.01) \quad = 0.25 \tag{4.7}$$

$$U\,(12.118, 5.411) = H\,(1) \quad + B\,(0) \quad = 0.43 \tag{4.8}$$

4.4 Selection of the Appropriate Variability Factor Values *v* – An Empirical Analysis

In the application of the proposed *a posteriori* approach, a reasonable variability factor of v = 65% is used. However, the most appropriate value of the variability factor must be identified. Therefore, the variability factor value is varied using the experimental values of v = 65%, v = 75%, and v = 85%. Table 4.7 and Figure 4.10 show the feasible solutions with priority for the different variability factor values under the *a posteriori* approach. In this empirical analysis, four representative solutions are desired, and the swing weight approach is used in the comparison.

Table 4.7: The feasible solutions with priority for the different variability factor setting.

Priority	*v* = 65%		*v* = 75%		*v* = 85%	
	***H*(*s*, *S*)**	***B*(*s*, *S*)**	***H*(*s*, *S*)**	***B*(*s*, *S*)**	***H*(*s*, *S*)**	***B*(*s*, *S*)**
1	52.345	0.006	31.960	0.001	29.161	0.012
2	35.757	0.010	9.403	9.348	8.644	11.475
3	12.118	5.411	14.285	2.925	13.494	3.541
4	21.097	0.679	22.966	0.119	23.454	0.182

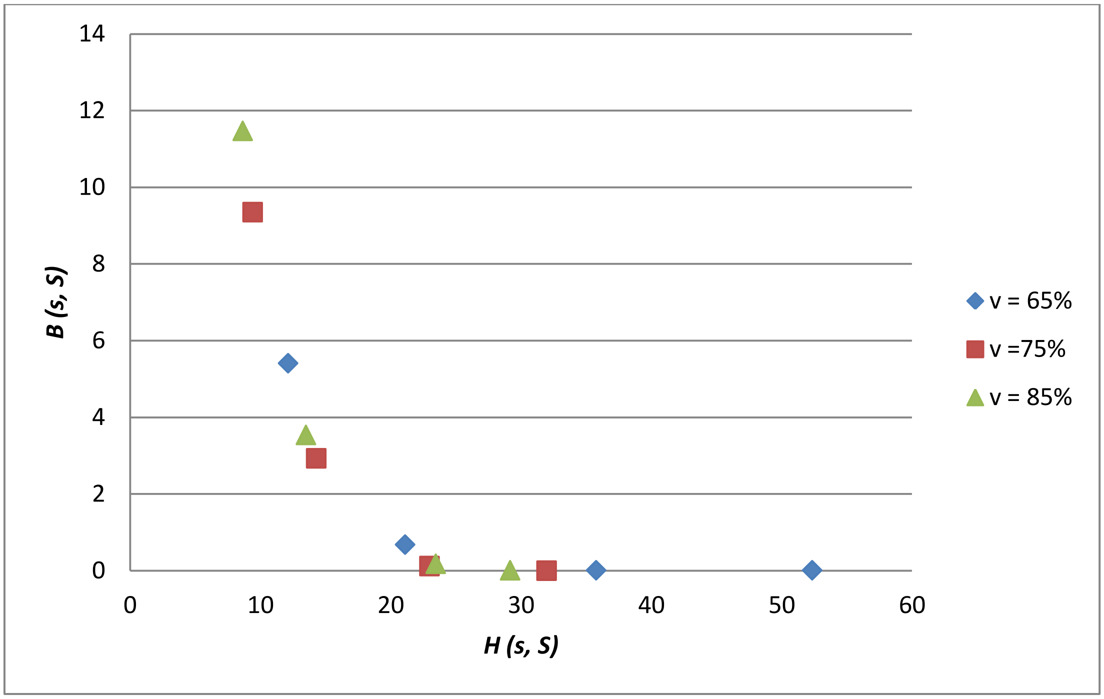

Figure 4.10: The feasible solutions with priority for the different variability factor setting.

Table 4.7 and Figure 4.10 show the feasible solutions with priority by variability factor of 65% performs well when considering the current inventory problem, assuming a lower value of *B*(*s*, *S*) is desired first, and then a lower value of *H*(*s*, *S*) is desired second. The average value of 1.527 for *B*(*s*, *S*) with *v* = 65% is lowest compared to the other value for *B*(*s*, *S*) with different *v* settings of 3.098 and 3.830.

4.5 Application of Clustering Analysis Only to the Case Study

The modified centroid linkage method is applied to the full original set of Pareto optimal solutions shown in Figure 4.3. Recall, that the centroid linkage method uses the squared Euclidean distance as the distance measure between two data points (i.e., solutions) in the objective space. Figure 4.11 shows the original set of Pareto assuming four clusters, and Figure 4.12 shows the original set of Pareto dendrogram using four clusters. The centroid is calculated for each of the resulting clusters, and then the closest point (solution) in distance to the centroid is considered. These solutions are shown in Table 4.8 and represented in Figure 4.13.

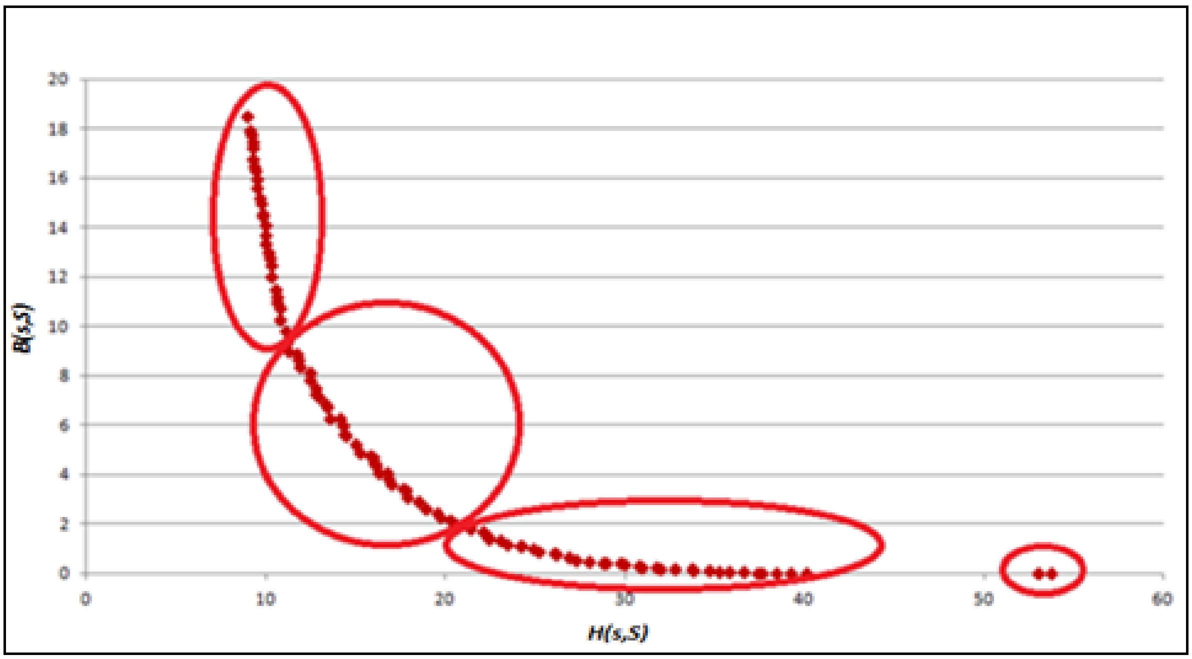

Figure 4.11: The original set of Pareto assuming four clusters.

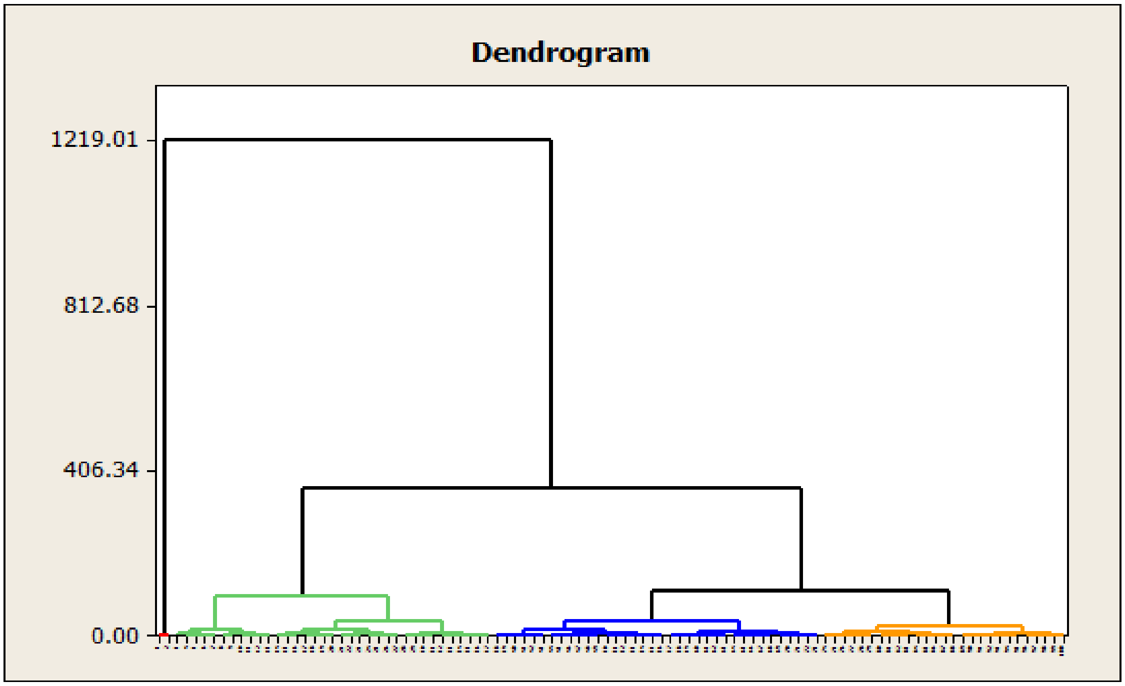

Figure 4.12: The original set of Pareto dendrogram assuming four clusters.

Table 4.8: The original non-dominated and feasible solutions for the problem.

#	***H* (s, S)**	***B* (s, S)**
1	53.000	0.047
2	29.754	0.439
3	14.991	5.298
4	9.938	14.138

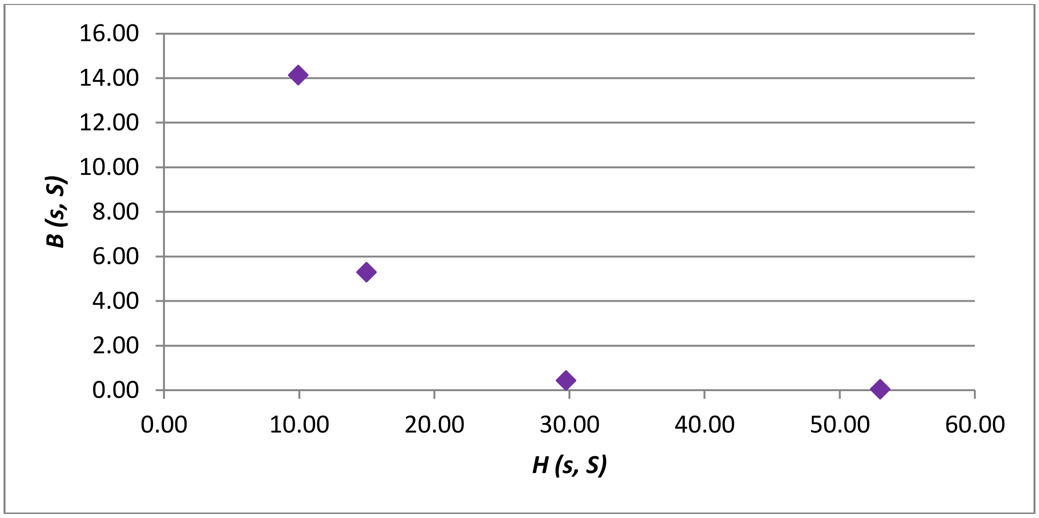

Figure 4.13: The original non-dominated and feasible solutions for the problem.

In assessing the performance of the *a posteriori* approach versus clustering algorithm only using the modified centroid hierarchical algorithm, the performance measures of the clustering analysis internal evaluations are used (Table 4.9). Based on the assessment, the *a posteriori* approach performs far better than using the clustering algorithm only. The Dunn Index is 0.76 for the *a posteriori* approach, and the Davies-Bouldin Index is 0.27 for *a posteriori* approach. Considering the Dunn Index, a higher index value is desired, and considering the Davies-Bouldin Index, a lower index value is desired.

Table 4.9: Summary of the internal evaluation index scores.

	Dunn Index	**Davies–Bouldin Index**
Proposed Approach	0.76	0.27
Clustering Algorithm	0.54	0.42

4.6 Application of Simulation Optimization to the Case Study

In this section, a comparison of the proposed *a posteriori* approach (with v = 65%) and a simulation optimization framework using NSGA II are performed. The parameters used for the simulation model is similar to that shown in Table 4.2. However, for the simulation optimization approach, the population size is four since the desired number of representative solutions is four for this case study. Table 4.10 and Figure 4.14 show the feasible solutions for the two different approaches – the proposed *a posteriori* approach and the simulation optimization approach using NSGA II.

Table 4.10: The feasible solutions for the different approaches.

***A Posteriori* Approach with *v* = 65%**		**Simulation Optimization Approach Using NSGA II (Population Size of 4)**	
***H*(*s*, *S*)**	***B*(*s*, *S*)**	***H*(*s*, *S*)**	***B*(*s*, *S*)**
52.345	0.006	40.721	0.050
35.757	0.010	23.164	1.375
21.097	0.679	11.219	9.618
12.118	5.411	8.966	18.925

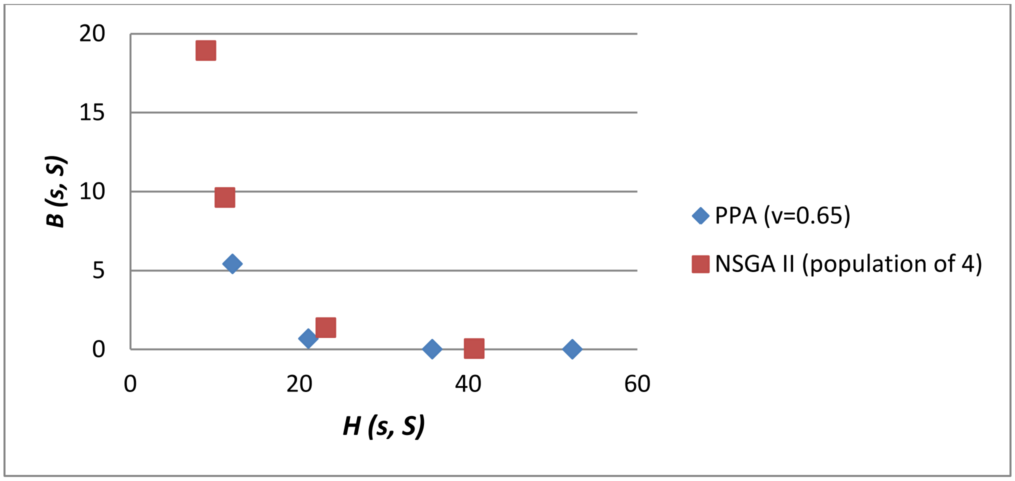

Figure 4.14: The feasible solutions for the different approaches.

Table 4.10 and Figure 4.14 show the feasible solutions for the different approaches whereas the *a posteriori* approach with *v* = 65% performs better, assuming a lower value of *B*(*s*, *S*) is desired first, and then a lower value of *H*(*s*, *S*) is desired second. The average value of 1.527 for *B*(*s*, *S*) with *a posteriori* approach is lowest compared to the simulation optimization approach value for *B*(*s*, *S*) of 7.492. In addition, the results of the *a posteriori* approach and the simulation optimization approach show similar spread in the representative solutions along the Pareto frontier.

4.7 Summary

The objective of this study is the improvement of the decision-making selection process in the presence of stochastic objectives. The three-phased *a posteriori* approach reduces a large set of tradeoff solutions to a manageable number of representative solutions while considering the stochastic nature of the objective functions. Prioritization in support of the representative solutions is considered to assist the decision-maker in identifying the most appropriate solution. The *a posteriori* approach does not consider decision-maker preferences *a priori* except when identifying the final number of representative solutions.

The *a posteriori* approach is appropriate to use for either deterministic or stochastic set of objectives and the availability of the set of Pareto optima solutions is required since the approach is applied after the optimization process.

The results discussed herein show the promise of the *a posteriori* approach. The *a posteriori* approach compared to the cluster analysis approach and compared to a simulation optimization approach (assuming a population size of four) show better results for the interest of decision-maker.

CHAPTER 5:
AN ENHANCED A POSTERIORI APPROACH FOR DECISION-MAKING WITH MULTIPLE STOCHASTIC OBJECTIVES

5.1 Introduction

In this chapter, an enhanced *a posteriori* approach is presented. The enhanced *a posteriori* approach consists of two phases. First, a complete set of Pareto optima is reduced while considering the stochastic nature of the objectives. Second, prioritizing the reduced set of Pareto optima after the optimization process for the decision-maker is performed.

5.2 Proposed Approach

Figure 5.1 shows the logic of the enhanced *a posteriori* approach. The logic is similar to that of the proposed approach described in Chapter 3 sans the “Cluster” step. In other words, the enhanced *a posteriori* approach begins with a given set of **P** Pareto optima as input. The reduction of the candidate set of compromise solutions is performed while considering the statistical precision of the performance measures and preferences on objectives by the decision-maker. Second, the reduced set of solutions is prioritized to assist the decision-maker in identifying the most appropriate compromise solution.

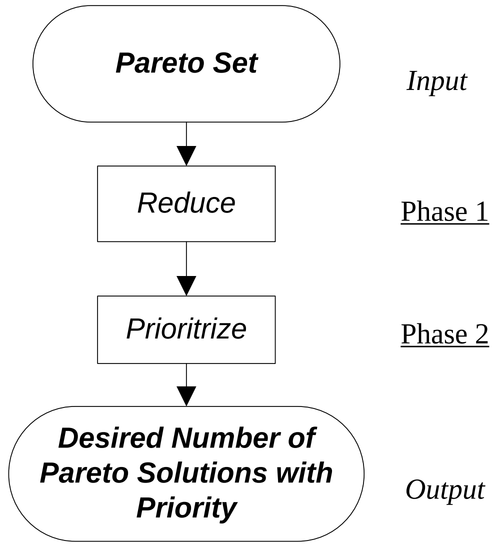

Figure 5.1: General logic flow of the enhanced *a posteriori* approach.

5.2.1 Phase 1 – Reduction

At the beginning, the values of the *m* objectives of interest are generated by applying an appropriate optimization algorithm to the problem and generating a set of Pareto optima. Each Pareto optimal solution is represented by a mean value and a standard deviation value in each of the objective space dimensions. The original set **P** of Pareto optima is the input for Phase 1, which is briefly illustrated in Figure 5.2.

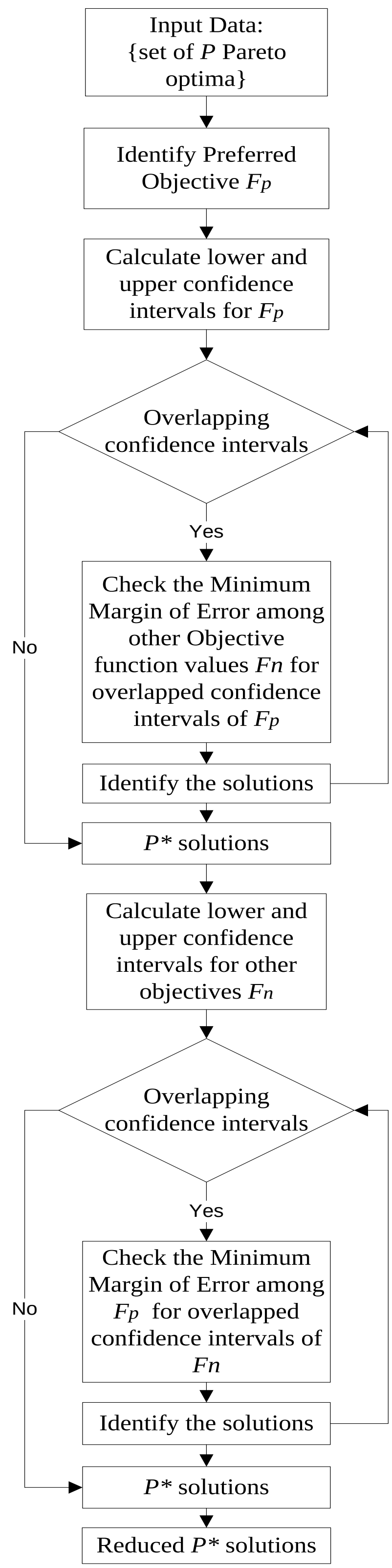

Figure 5.2: Overview of the enhanced *a posteriori* approach Phase 1.

Figure 5.3 shows the set **P** of Pareto optima (assuming a two-objective minimization problem). Using the associated standard deviations, the precision of the set of **P** Pareto optima mean values are represented by the confidence intervals along each objective space dimension, creating an upper and a lower limit for each Pareto optima solution.

First, for each of the solutions in the set **P** of compromise solutions, the confidence interval (assuming minimization) in each of the *m* objective dimension is calculated. In case of maximization, the objectives can be converted to minimization objectives by multiplying the objective values by -1, without loss of generality. Additionally, the preferred objective $\mathbf{F_P}$ (assume Objective 2) is identified by the decision-maker.

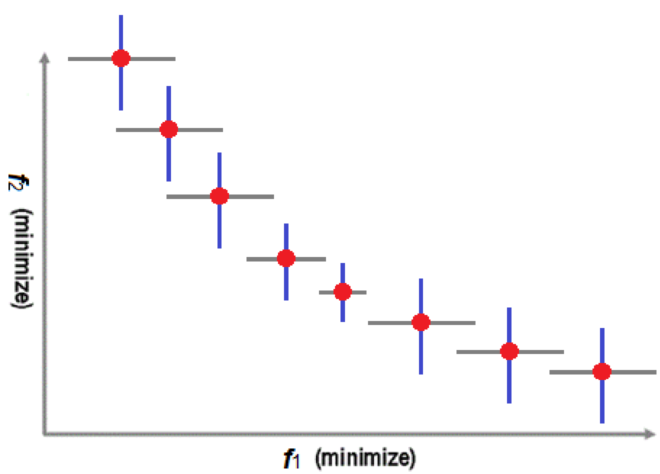

Figure 5.3: Initial set of Pareto optima with confidence intervals along the objective space dimensions.

Second, the set of $\mathbf{F_P}$ solutions is sorted from largest to smallest, and then the overlapping confidence intervals among the whole set are identified as shown in Figure 5.4. Third, the marginal error values associated to all the objectives but the preferred one are calculated. One solution is selected among each set of overlapping confidence intervals for the set of $\mathbf{F_P}$ solutions by identifying the smallest marginal error value associated to the other objectives but the preferred objective (Hart, Michie, & Cooke, 2007; Mendenhall & Sincich, 2012; Willén, 1976).

In case of more than one confidence interval with the smallest marginal error value, more than one solution is selected among each set of overlapping confidence intervals for the set of $\mathbf{F_P}$ solutions. Fourth, the second and the third steps are repeated until there are no more overlapping confidence intervals in the set of $\mathbf{F_P}$ solutions.

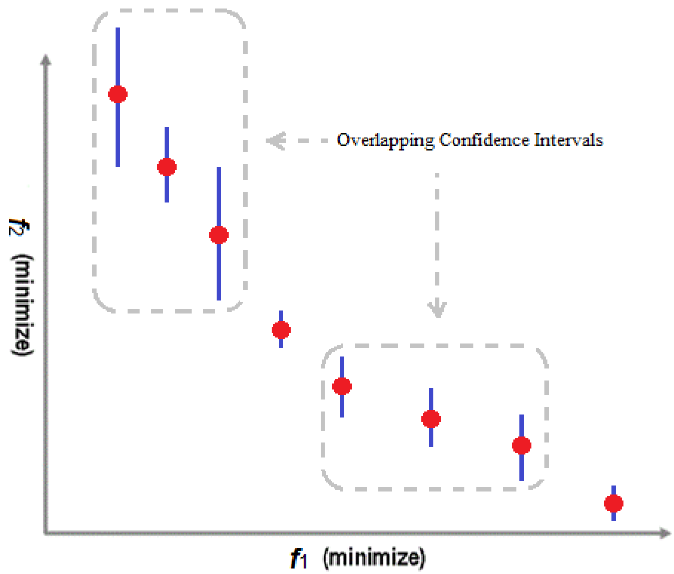

Figure 5.4: Overlapping confidence intervals for Objective 2.

Fifth, the set of $\mathbf{F_n}$ solutions (except the preferred objective) is sorted now from largest to smallest and then the overlapping confidence intervals among the whole set are identified as shown in Figure 5.5.

Sixth, the marginal error values associated to the preferred objective is calculated. One solution is selected among each set of overlapping confidence intervals for the set of $\mathbf{F_n}$ solutions by identifying the smallest marginal error value associated to the preferred objective. In case of more than one smallest marginal error value is equal, more than one solution are selected among each set of overlapping confidence intervals for the set of $\mathbf{F_n}$ solutions.

Seventh, the fifth and then the sixth steps are repeated again until there are no more overlapping confidence intervals in the set of $\mathbf{F_n}$ solutions.

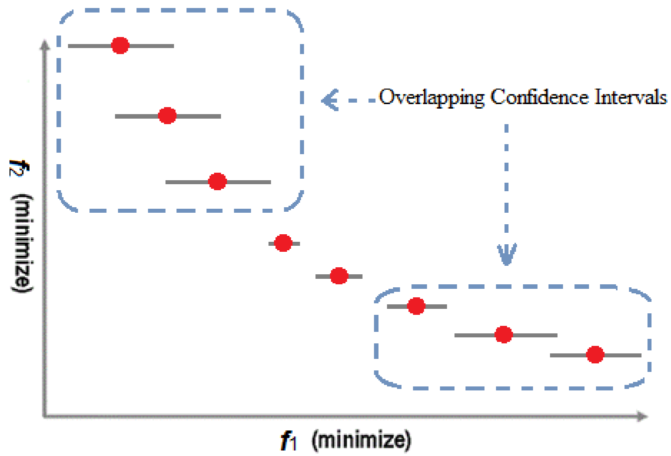

Figure 5.5: Overlapping confidence intervals for Objective 1.

Eighth, the fifth, the sixth, and the seventh steps are repeated for each objective but the preferred one (if there are more than two objectives to study for the problem). Now, the reduced set of Pareto $\mathbf{P}^*$ is considered for Phase 2, which is the set of solutions without overlapping confidence intervals that are recognized after the optimization process. Phase 1 steps can schematically be represented as the pseudocode shown in Figure 5.6.

```
Objective Function F_n,   n = {1, 2, …, n}
Preferred Objective F_p
Pareto frontier P = {1, 2, …, P}

For (i = 1 ≤ |F_p|)
        Lower confidence level (LCL_i)
        Upper confidence level (UCL_i)
        If (Overlapping confidence intervals)
                Minimum Margin of Error among other objective function values F_n
                Identify the solutions
        P* = {identified solutions from previous step and solutions without overlapping
              confidence intervals}
End For
Repeat "For" loop with P* until all solutions are without overlapping confidence intervals

For (i = 1 ≤ |F_n|)
        Lower confidence level (LCL_i)
        Upper confidence level (UCL_i)
        If (Overlapping confidence intervals)
                Minimum Margin of Error among other Objective function values F_p
                Identify the solutions
        P* = {identified solutions from previous step and solutions without overlapping
              confidence intervals}
End For
Repeat "For" loop with P* until all solutions are without overlapping confidence intervals

Report the output (Reduced P*)
```

Figure 5.6: Pseudocode for Phase 1.

5.2.2 Phase 2 – Prioritization

Phase 2 of the enhanced *a posteriori* proposed approach is similar to Phase 3 of the original *a posteriori* proposed approach shown in Chapter 3. The reduced set of $\mathbf{P}^*$ solutions is the input for Phase 2, which is a stage that is performed after the optimization process. In general, evaluating and prioritizing large set of candidate solutions is a particularly difficult task for decision-makers. Nonetheless, multiobjective decision-making approaches are used to select the most proper solution among the other available solutions (Noghin, 2011). Figure 5.7 illustrates Phase 2 of the enhanced *a posteriori* approach.

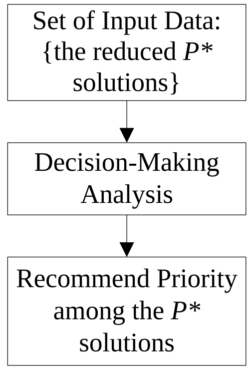

Figure 5.7: Overview of the enhanced *a posteriori* approach Phase 2.

In this Phase, prioritization of the representative $\mathbf{P}^*$ solutions identified in Phase 1 using the popular swing weight approach. Using swing weights, the decision-maker determines which solutions are the most important, second most important, etc. and also by how many times it is more important. These numbers are then normalized to sum to 1.0. The swing weight approach is considered for this Phase.

First, preferences on objectives are identified by the decision-maker (assume a two-objective problem with a lower value of Objective 2 is desired first, and then a lower value of Objective 1 is desired second).

Second, create a table like the one in Table 5.1 for the problem. The first row indicates the worst possible outcome, or the outcome that is at the worst level on each of the attributes (solutions). Each of the succeeding rows “swings” one of the attributes from worst to best. With the table constructed, the assessment can begin.

Table 5.1: Swing-weight assessment.

Attribute Swing from Worst to Best	Consequence to Compare	Rank	Rating	Weight
(Benchmark)	100, 50	3	0	0.00 = 0/140
O_1: Objective 1	10, 50	2	40	0.29 = 40/140
O_2: Objective 2	100, 5	1	100	0.71 = 100/140
		Total	140	1.00

Third, the outcomes are rank ordered. It gives the option to prioritize the objectives. For example, "3" is placed in the "Rank" column for the first row in Table 5.1. There are three hypothetical set of solutions to compare, and it is safe to assume that the benchmark solution – the one that is worse on all objectives – is rank third (worse) overall. The others are compared to determine which ranks first (best), and second.

Fourth, fill in the "Rate" column in the table. Two of the ratings are predetermined; the rating for the Benchmark solution is 0 and the rating for the top-ranked solution is 100. The rating for the other must fall between 0 and 100. With these assessments, the table is completed and weights are calculated. The weights are the normalized ratings and they add up to 1.0.

Fifth, with the weights determined, the overall utility for different alternatives or outcomes is calculated. For example, the utilities for the alternatives (reduced set of $\mathbf{P}^*$ solutions) shown in Table 5.2 are calculated and shown in Equation 5.1-5.4.

Table 5.2: The feasible alternatives (solutions).

Representative Solution	Objective 1	Objective 2
1	100	5
2	75	30
3	45	50
4	10	25

$$U\,(100, 5) = O_1(0) + O_2(1) = 0.714 \quad (5.1)$$
$$U\,(75, 30) = O_1(0.13) + O_2(0.17) = 0.157 \quad (5.2)$$
$$U\,(45, 50) = O_1(0.22) + O_2(0.10) = 0.135 \quad (5.3)$$
$$U\,(10, 25) = O_1(1) + O_2(0) = 0.286 \quad (5.4)$$

Sixth, with the utilities calculated, priority among the alternatives (solutions) is considered as shown in Table 5.3. Figure 5.8 graphically shows the probability to have:

- All objectives worst that not in favor of the priority by the decision-maker,
- All objectives best, and
- Some objectives are best and other is worst.

Phase 2 steps can schematically be represented as the pseudocode shown in Figure 5.9.

Table 5.3: The feasible alternatives (solutions) with priority.

Representative Solution	**Objective 1**	**Objective 2**	**Utility**	**Priority**
1	100	5	0.714	1
2	75	30	0.157	3
3	45	50	0.135	4
4	10	25	0.286	2

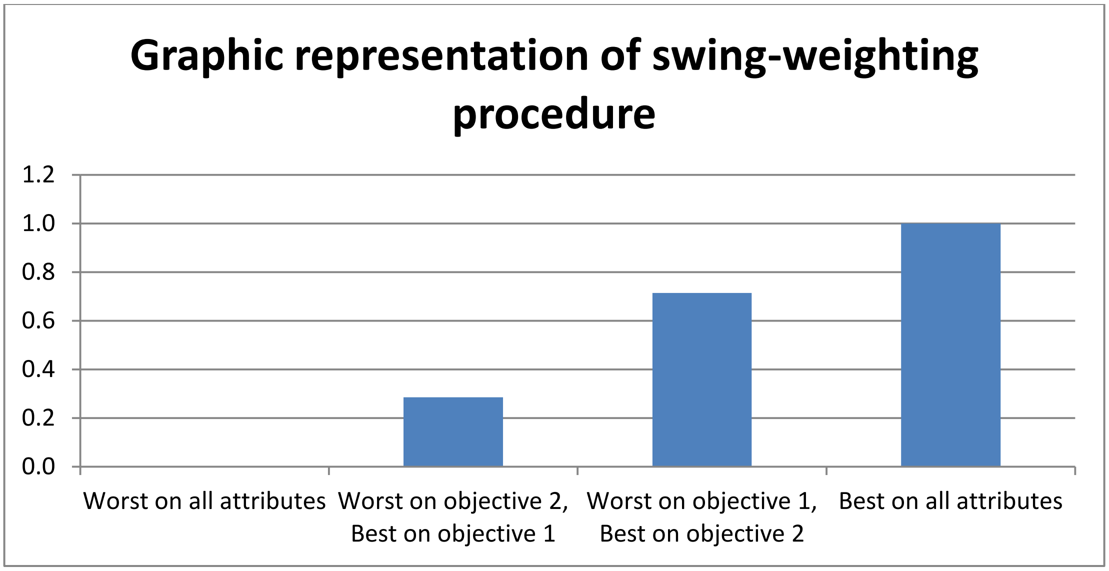

Figure 5.8: Graphic representation of swing-weighting procedure.

```
Read Reduced Set of P* = {1, 2, …, c}
        Decision-making analysis for P*
Recommend priority among P*
```

Figure 5.9: Pseudocode for Phase 2.

5.3 Summary

The enhanced *a posteriori* approach effectively articulates the decision-maker preferences after the optimization process and is intended to design a decision-making solution selection process in the presence of multiple stochastic objectives. The enhanced *a posteriori* approach allows reducing and organizing the set of non-dominated solutions considering the stochastic nature of the objective functions. The enhanced approach consists fewer phases and compared to the original *a posteriori* approached described in Chapter 3. This is why the enhanced *a posteriori* approach is constructed.

CHAPTER 6:
COMPUTIONAL STUDY: SOLVING THE (*s*, *S*) INVENTORY PROBLEM BY THE ENHANCED A POSTERIORI APPROACH

6.1 Introduction

This chapter applies the enhanced proposed *a posteriori* approach to a well-known inventory problem. A numerical simulation model of the inventory problem is integrated with a multiobjective evolutionary algorithm. The non-dominated sorting genetic algorithm II (NSGA II) is used to optimize the decision variables and generate the set of Pareto optimal solutions. The enhanced *a posteriori* proposed approach begins with this set of tradeoff solutions as input. The detail of the inventory case study problem is presented in Section 4.2 in Chapter 4.

First, the computational results after applying the proposed approach are presented and discussed in Section 6.2. Next, Section 6.3 shows the computational results when a simulation optimization approach is applied to the case study problem. Finally, Section 6.4 summarizes the chapter.

6.2 Application of the Proposed Enhanced *A Posteriori* Approach to the Case Study

The simulation framework for the (*s*, *S*) inventory with backlogging model integration with the famous NSGA II is considered for this case study. In this study, a two-objective, two-variable minimization problem is considered. The average holding cost per month and the average shortage cost per month are the objectives. The enhanced *a posteriori* approach begins after the optimization process with a given set of **P** Pareto optima solutions and reduces them. It is applied to enhance the decision-making process.

6.2.1 Generation of the Set of Pareto Solutions

The simulation optimization integrated framework is comprised of the NSGA II multiobjective evolutionary algorithm component and the inventory simulation component. The algorithm iteratively generates decision variables (*s*, *S*). Evaluation of the decision variables are performed by the inventory simulation model. The NSGA II optimization algorithm generates pairs of the two decision variables known as the inventory (*s*, *S*) policy. These decision values are passed to the inventory simulation model to generate and replicate the objective function values (i.e., *H* (*s*, *S*): inventory holding cost per month and *B* (*s*, *S*): inventory shortage cost per month). The inventory simulation model returns the mean of the objective function values and corresponding standard deviation values to NSGA II. NSGA II generates and passes the new decision variable values to the inventory simulation model in order to compute the mean objective function values and corresponding standard deviation values. NSGA II then reports the set of Pareto (i.e., reports the mean of the objective function values and the corresponding standard deviation values, and the associated decision variable values).

6.2.2 Parameter Settings for the Simulation Model and NSGA II

Various input values are used for the inventory simulation model and NSGA II. The parameter values specified for the NSGA II and the inventory simulation model are shown in Table 4.1 and in Table 4.2 respectively in Chapter 4. The parameter used is chosen by the analyst for the purpose of running and evaluating the enhanced *a posteriori* approach.

The enhanced *a posteriori* approach begins after the optimization process with a given set of **P** Pareto optima solutions as input. The approach consists of two phases. Phase one is built by using the Microsoft excel (i.e., to reduce the original set of Pareto solutions). Phase two uses the Microsoft excel for the decision analysis (i.e., swing weighting approach). Figure 6.1 shows the original decision space and Figure 6.2 shows the original Pareto optimal front generated by using a simulation multiobjective optimization approach that uses multiobjective evolutionary algorithms and discrete-event simulation. Each point on the curve (as shown in Figure 6.2) is generated after running n = 100 independent simulation replications. As such, the points (solutions) along the Pareto frontier are the mean objective values across the replications, and each has an associated standard deviation along each dimension in the objective space.

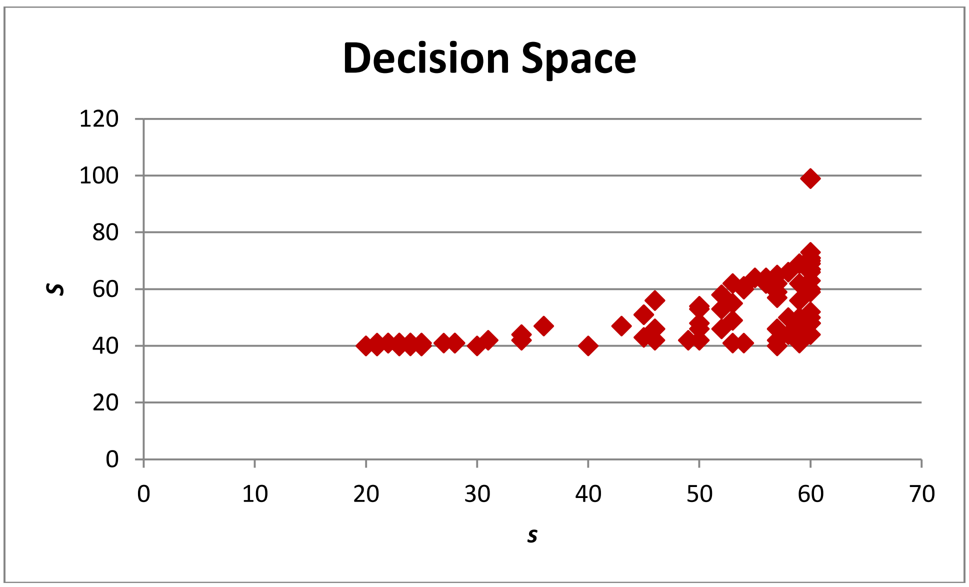

Figure 6.1: Decision space for the decision variables.

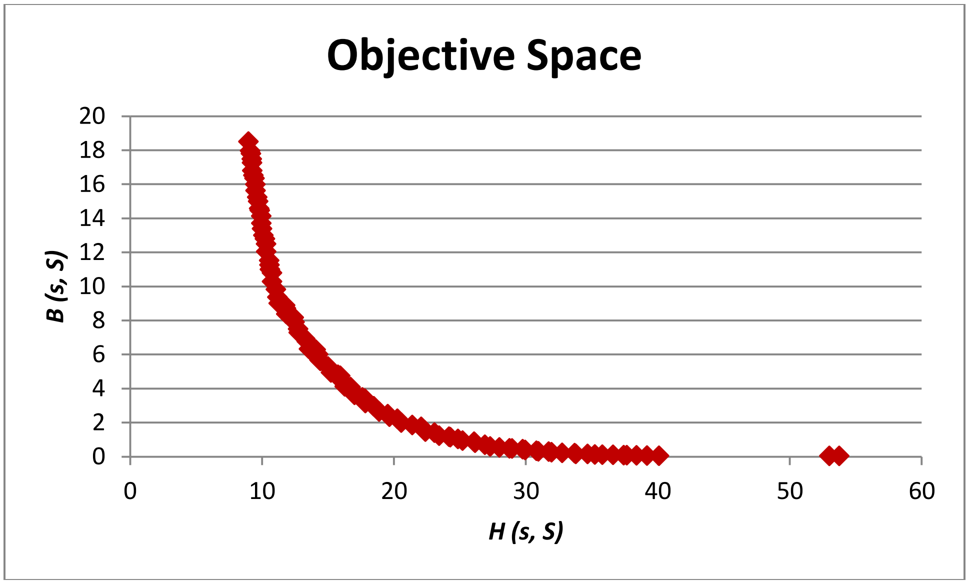

Figure 6.2: Objective space for the original mean objective functions (100 solutions).

Using the standard deviations, the precision of the mean objective values of the Pareto points (solutions) is represented by the confidence interval along each dimension computed using

$$\bar{x} \pm t_{\frac{\alpha}{2},n-1}\left(\frac{s}{\sqrt{n}}\right), \tag{6.1}$$

where $\bar{x}$ is the mean objective value from the n replications, s is the standard deviation of the objective value from the n replications, α is the level of significance, and $t_{\alpha/2,n-1}$ is the upper $\alpha/2$ critical value for the t-distribution with n-1 degrees of freedom.

Phase 1 of the enhanced *a posteriori* approach starts after the optimization process with computing the upper and lower confidence limit for each Pareto point (solution) using Eq. 6.1 (Mendenhall & Sincich, 2012). Here, for illustration, a level of significance α = 10% is assumed. In addition, the preferred objective *B (s, S)* (assume objective 2) is identified by the decision-maker.

First, the set of *B (s, S)* solutions is sorted from largest to smallest and then the overlapping confidence intervals among the whole set are identified. On the other hand, the marginal error values associated to *H (s, S)* is calculated. One solution is selected among each set of overlapping confidence intervals for the set of *B (s, S)* solutions by identifying the smallest marginal error value associated to *H (s, S)*. The first iteration reduced the original set of Pareto **P** (100 solutions) to 36 solutions as shown in Figure 6.3.

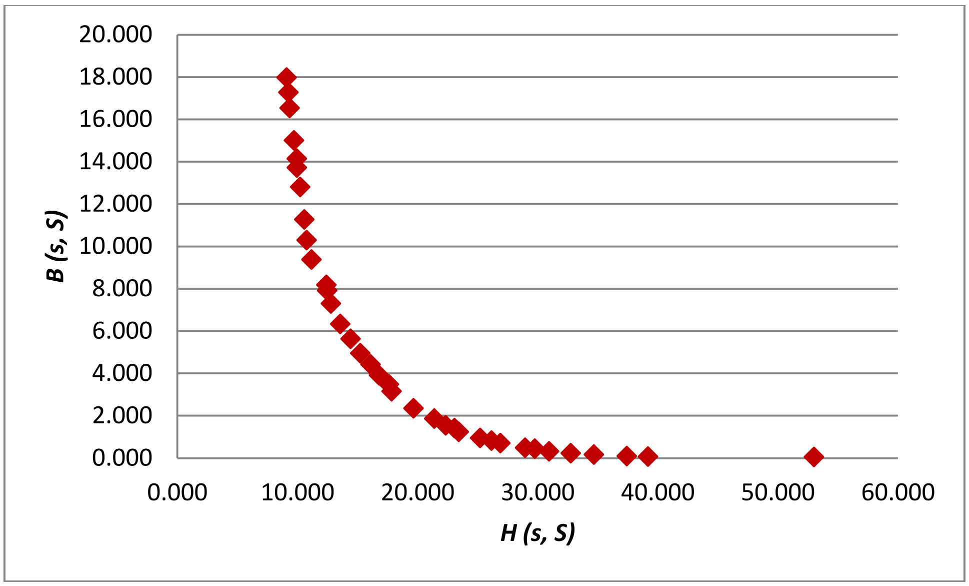

Figure 6.3: Graph of the set of Pareto optima (i.e., 36 compromise solutions).

Second, the previous step (first step) is repeated again to make sure that there are no more overlapping confidence intervals for *B (s, S)* solutions. The second iteration reduced the previous set of Pareto (36 solutions) to 29 solutions as shown in Figure 6.4.

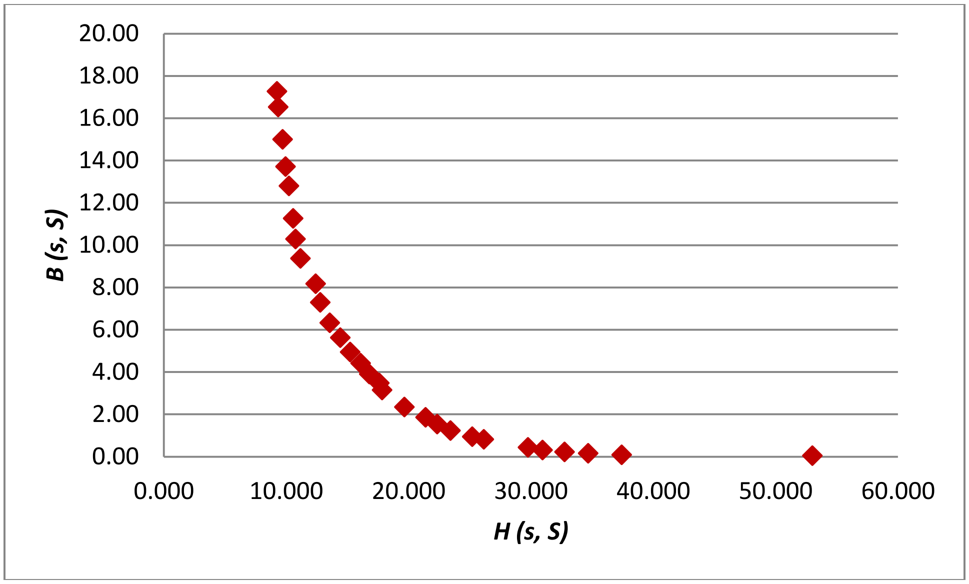

Figure 6.4: Graph of the set of Pareto optima (i.e., 29 compromise solutions).

Third, the previous step (first step) is repeated again to make sure that there are no more overlapping confidence intervals for *B (s, S)* solutions. It is founded that there are no more overlapping confidence intervals for the *B (s, S)* solutions.

Fourth, the previous step (first step) is repeated again but now for *H (s, S)* solutions to check for overlapping confidence intervals within *H (s, S)* solutions. The third iteration reduced the previous set of Pareto (29 solutions) to 26 solutions as shown in Figure 6.5.

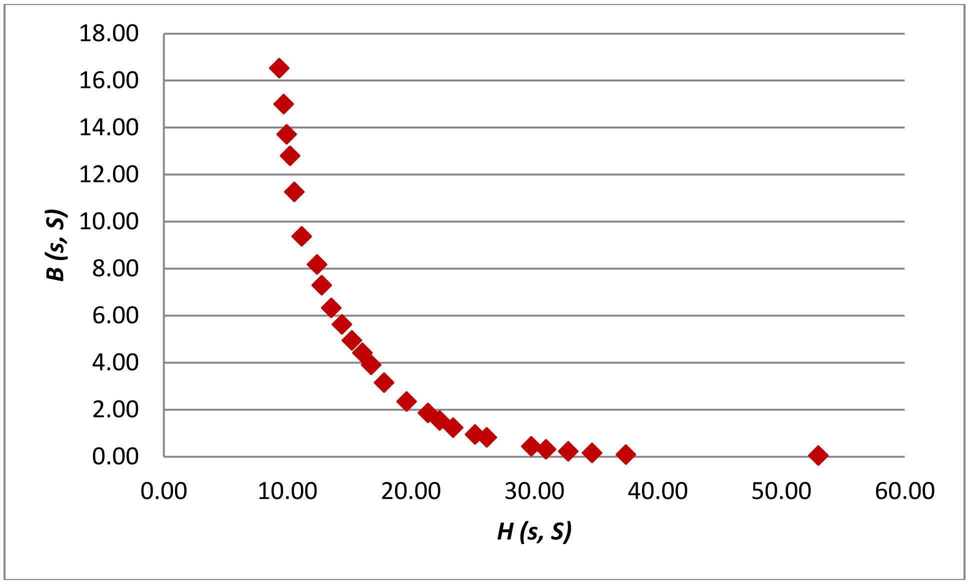

Figure 6.5: Graph of the set of Pareto optima (i.e., 26 compromise solutions).

Fifth, the previous step is repeated again until there are no more overlapping confidence intervals for the $H(s, S)$ solutions. It is found that there are no more overlapping confidence intervals for the $H(s, S)$ solutions. Therefore, the final reduced set of Pareto $\mathbf{P}^*$ is considered for Phase 2, which is the set of solutions without overlapping confidence intervals that are identified after the optimization process, as shown in Figure 6.5 and Table 6.1.

Table 6.1: The reduced set of Pareto optimal solutions (i.e., 26 compromise solutions).

Solution	$H(s, S)$	$B(s, S)$
1	9.339	16.531
2	9.700	15.002
3	9.941	13.715
4	10.217	12.801
5	10.561	11.265
6	11.159	9.372
7	12.401	8.176
8	12.778	7.294
9	13.553	6.329
10	14.415	5.627
11	15.222	4.946
12	16.077	4.418
13	16.785	3.906
14	17.830	3.150
15	19.654	2.347
16	21.384	1.861
17	22.333	1.540
18	23.422	1.233
19	25.198	0.944
20	26.146	0.820
21	29.754	0.439
22	30.944	0.313
23	32.745	0.225
24	34.673	0.160
25	37.414	0.088
26	53.000	0.047

Phase 2 of the enhanced *a posteriori* approach prioritizes the representative solutions identified in Phase 1 using the swing weight approach. Considering the current problem, assuming a lower value of $B(s, S)$ is desired first, and then a lower value of $H(s, S)$ is desired second. Table 6.2 shows the prioritized solutions using the swing weight approach. Table 6.3 shows the assessment of the swing weights.

Table 6.2: The feasible solutions with priority.

Priority	***H (s, S)***	***B (s, S)***
1	9.339	16.531
3	9.700	15.002
5	9.941	13.715
6	10.217	12.801
7	10.561	11.265
8	11.159	9.372
9	12.401	8.176
10	12.778	7.294
11	13.553	6.329
13	14.415	5.627
14	15.222	4.946
15	16.077	4.418
16	16.785	3.906
18	17.830	3.150
19	19.654	2.347
21	21.384	1.861
22	22.333	1.540
24	23.422	1.233
25	25.198	0.944
26	26.146	0.820
23	29.754	0.439
20	30.944	0.313
17	32.745	0.225
12	34.673	0.160
4	37.414	0.088
2	53.000	0.047

Table 6.3: The assessment of swing weights.

Attribute Swung from Worst to Best	**Consequence to Compare**		**Rank**	**Rate**	**Weight**
(Benchmark)	53.000	16.531	3	0	0.0000
H (s, S)	9.339	16.531	2	75	0.4286
B (s, S)	53.000	0.047	1	100	0.5714
			Total	175	1

The overall utility for different feasible solutions is determined as shown in Eqs. 6.2-6.27. The value of the corresponding weight or the relative utility shows how the prioritized solutions are identified. Eqs 6.2 through 6.27 shows the swing weight calculations.

$$U(9.339, 16.531) = H(0) + B(1) = 0.5714 \quad (6.2)$$

$$U(9.700, 15.002) = H(0.96) + B(0.00) = 0.4144 \quad (6.3)$$

$$U(9.941, 13.715) = H(0.94) + B(0.00) = 0.4046 \quad (6.4)$$

$$U(10.217, 12.801) = H(0.91) + B(0.00) = 0.3938 \quad (6.5)$$

$$U(10.561, 11.265) = H(0.88) + B(0.00) = 0.3814 \quad (6.6)$$

$$U(11.159, 9.372) = H(0.84) + B(0.00) = 0.3615 \quad (6.7)$$

$$U(12.401, 9.372) = H(0.75) + B(0.01) = 0.3260 \quad (6.8)$$

$$U(12.778, 7.294) = H(0.73) + B(0.01) = 0.3169 \quad (6.9)$$

$$U(13.553, 6.329) = H(0.69) + B(0.01) = 0.2995 \quad (6.10)$$

$$U(14.415, 5.627) = H(0.65) + B(0.01) = 0.2824 \quad (6.11)$$

$$U(15.222, 4.946) = H(0.61) + B(0.01) = 0.2683 \quad (6.12)$$

$$U(16.077, 4.418) = H(0.58) + B(0.01) = 0.2683 \quad (6.13)$$

$$U(16.785, 3.906) = H(0.56) + B(0.01) = 0.2453 \quad (6.14)$$

$$U(17.830, 3.150) = H(0.52) + B(0.01) = 0.2330 \quad (6.15)$$

$$U(19.654, 2.347) = H(0.48) + B(0.02) = 0.2150 \quad (6.16)$$

$$U(21.384, 1.861) = H(0.44) + B(0.03) = 0.2015 \quad (6.17)$$

$$U(22.333, 1.540) = H(0.42) + B(0.03) = 0.1965 \quad (6.18)$$

$$U(23.422, 1.233) = H(0.40) + B(0.04) = 0.1925 \quad (6.19)$$

$$U(25.198, 0.944) = H(0.37) + B(0.05) = 0.1871 \quad (6.20)$$

$$U(26.146, 0.820) = H(0.36) + B(0.06) = 0.1856 \quad (6.21)$$

$$U(29.754, 0.439) = H(0.31) + B(0.11) = 0.1953 \quad (6.22)$$

$$U(30.944, 0.313) = H(0.30) + B(0.15) = 0.2146 \quad (6.23)$$

$$U(32.745, 0.225) = H(0.29) + B(0.21) = 0.2409 \quad (6.24)$$

$$U(34.673, 0.160) = H(0.27) + B(0.29) = 0.2826 \quad (6.25)$$

$$U(37.414, 0.088) = H(0.25) + B(0.53) = 0.4106 \quad (6.26)$$

$$U(53.000, 0.047) = H(1) + B(0) = 0.4286 \quad (6.27)$$

6.3 Application of Simulation Optimization to the Case Study

In this section a comparison between the results of the problem generated by using the enhanced *a posteriori* approach, the original *a posteriori* approach (with v = 65%) and the simulation framework for the (s, S) inventory with backlogging model integration with the NSGA II are illustrated. The parameters used for the simulation model is similar to that shown in Table 4.2 in Chapter 4. However, for the simulation optimization approach, the population size is four since the desired number of representative solutions is four for this case study. Tables 6.4 and Figure 6.6 show the feasible solutions for the different approaches.

Table 6.4: The feasible solutions for the different approaches.

Original *A Posteriori* Approach with *v* = 65%		**Enhanced *A Posteriori* Approach**		**Simulation Optimization Approach (Population Size of 4)**	
H (s, S)	***B (s, S)***	***H (s, S)***	***B (s, S)***	***H (s, S)***	***B (s, S)***
52.345	0.006	53.000	0.047	40.721	0.050
35.757	0.010	37.414	0.088	23.164	1.375
21.097	0.679	9.700	15.002	11.219	9.618
12.118	5.411	9.339	16.531	8.966	18.925

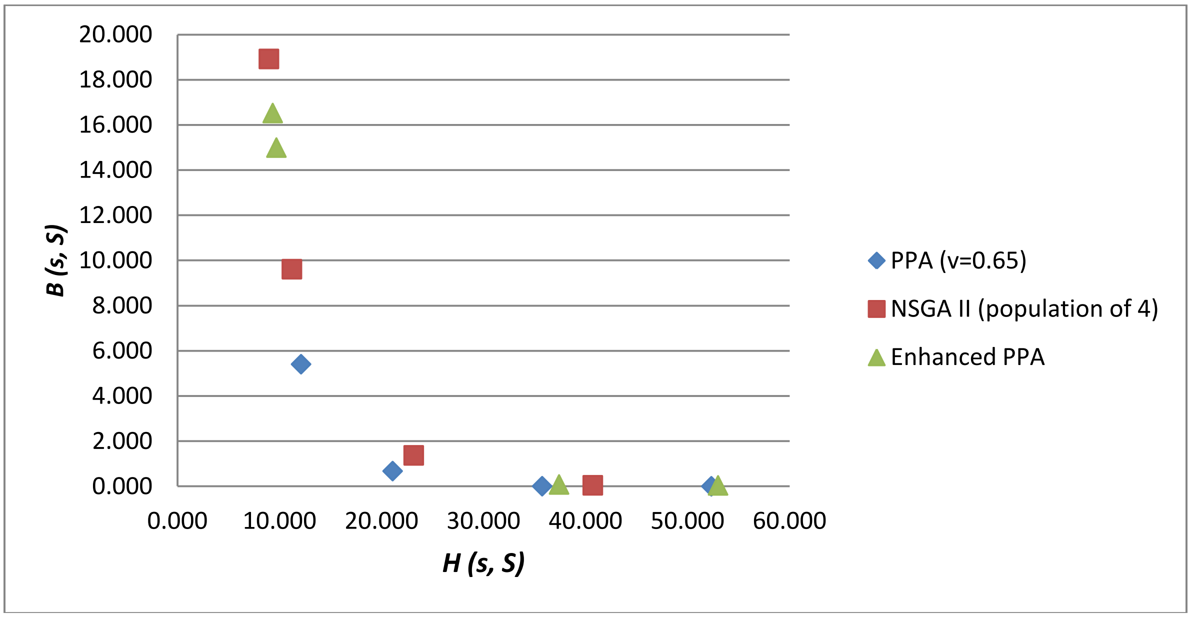

Figure 6.6: The feasible solutions for the different approaches.

Table 6.4 and Figure 6.6 show the feasible solutions for the different approaches, whereas the enhanced *a posteriori* approach is performing reasonably well when considering the current problem, assuming a lower value of $B(s, S)$ is desired first, and then a lower value of $H(s, S)$ is desired second. The results with the enhanced *a posteriori* approach show better spread for the representative solutions compared to the results with the original *a posteriori* approach. In addition, the average value of 27.363 for $H(s, S)$ with enhanced *a posteriori* approach is improved compared to the original *a posteriori* approach value for $H(s, S)$ of 30.329. The enhanced *a posteriori* approach shows faster and less complex analysis compared to the original *a posteriori* approach.

6.4 Summary

The objective of this study is the improvement of the decision-making selection process in the presence of stochastic objectives. With the enhanced *a posteriori* approach, preference information is applied by the decision-maker after the optimization process, and the enhanced *a posteriori* approach is appropriate to use for stochastic set of objectives and the availability of the set of Pareto optima solutions is required since the approach is applied after the optimization process. The results discussed herein show the promise of the enhanced *a posteriori* approach. The enhanced *a posteriori* approach compared to the other approaches show better results in general.

CHAPTER 7:
AN INTERACTIVE APPROACH FOR DECISION-MAKING WITH MULTIPLE STOCHASTIC OBJECTIVES AND COMPUTIONAL STUDY

7.1 Introduction

In this chapter, an interactive proposed approach for this research investigation is presented. This investigation specifically focuses on how to intelligently and effectively reduce the number of the candidate of compromise solutions while considering the stochastic nature of the objective functions. The interactive approach effectively articulates the decision-maker preferences during the optimization process (an interactive) and intends to design a decision-making solution selection process in the presence of multiple stochastic objectives. The interactive approach uses statistical analysis on the Pareto optimal solutions in order to reduce the number of solutions to a set of representative solutions that is presented to the decision-maker for final selection.

The interactive approach begins during the optimization process with articulated information that guide the optimization process to generate a bias set of Pareto optima considering the decision-maker preferences. A computational model that integrates multiobjective optimization and inventory simulation model that represents the problem to be solved is one way to produce the set of noisy Pareto optima. The computational model represented by simulation is constructed to compute a set of stochastic measures of performance, which represent the measures that are to be optimized. The interactive approach begins during the optimization process and then a two phases are considered after the set of Pareto optima is generated. First, reduction of a complete set of Pareto optima is performed considering the variation in the output. Second, prioritizing the reduced number of compromised solutions for the decision-maker is performed. The detail of the inventory case study problem is presented in Section 4.2 in Chapter 4.

First, in this chapter, the details of the interactive proposed approach are presented in Section 7.2. Then, in Section 7.3, the computational results after applying the proposed approach is presented and discussed. Next, Section 7.4 shows the computational results when a simulation optimization approach is applied to the case study problem. Finally, Section 7.5 summarizes the chapter.

7.2 Proposed Approach

In this interactive proposed methodology, an innovative approach that begins during the optimization process and then effectively reduces and prioritizes the set of Pareto solutions while considering the stochastic nature of *m* objective functions is developed. Figure 7.1 shows the logic flow of the interactive approach.

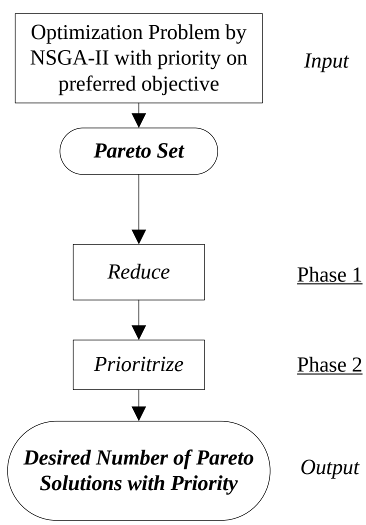

Figure 7.1: General logic flow of the interactive approach.

At the beginning, the interactive approach start during the optimization process and then a bias set of Pareto optima is generated considering the decision-maker preferences. After that, the reduction of the candidate set of compromise solutions (set of Pareto optima) is performed while considering the statistical precision of the performance measures under study and preferences on objectives by the decision-maker. Next, the reduced set of solutions is prioritized to assist the decision-maker in identifying the most appropriate compromise solution. The two phases considered after the optimization process are similar to the one described in section 5.2.

7.2.1 The interactive approach and the optimization process

The interactive approach uses the preference information progressively during the optimization process. Many researchers have made an effort to integrate the decision-maker preferences while solving for the optimization problems, which is embedded in the optimization algorithm to lead a decision maker (DM) to the most preferred solution of her or his choice (Deb et al., 2002; Deb, Sinha, Korhonen, & Wallenius, Oct.; He & Gao, 2009; Konak, Coit, & Smith, 2006; Nojima & Ishibuchi, 2009, 2010).

The interactive approach integrates the decision-maker preferences with NSGA II while solving for the optimization problem as shown in Figure 7.2. The interactive approach with NSGA II steps can schematically be represented as the pseudocode shown in Figure 7.3. The approach prioritizes the preferred objective by decision-maker among the other objectives during the optimization process. In the selection operation step in NSGA II, the individual with minimum preferred objective value is considered among the other individuals in the same population to generate the new population. At the very end, the set of Pareto optima solutions is generated as biased to the preferred objective.

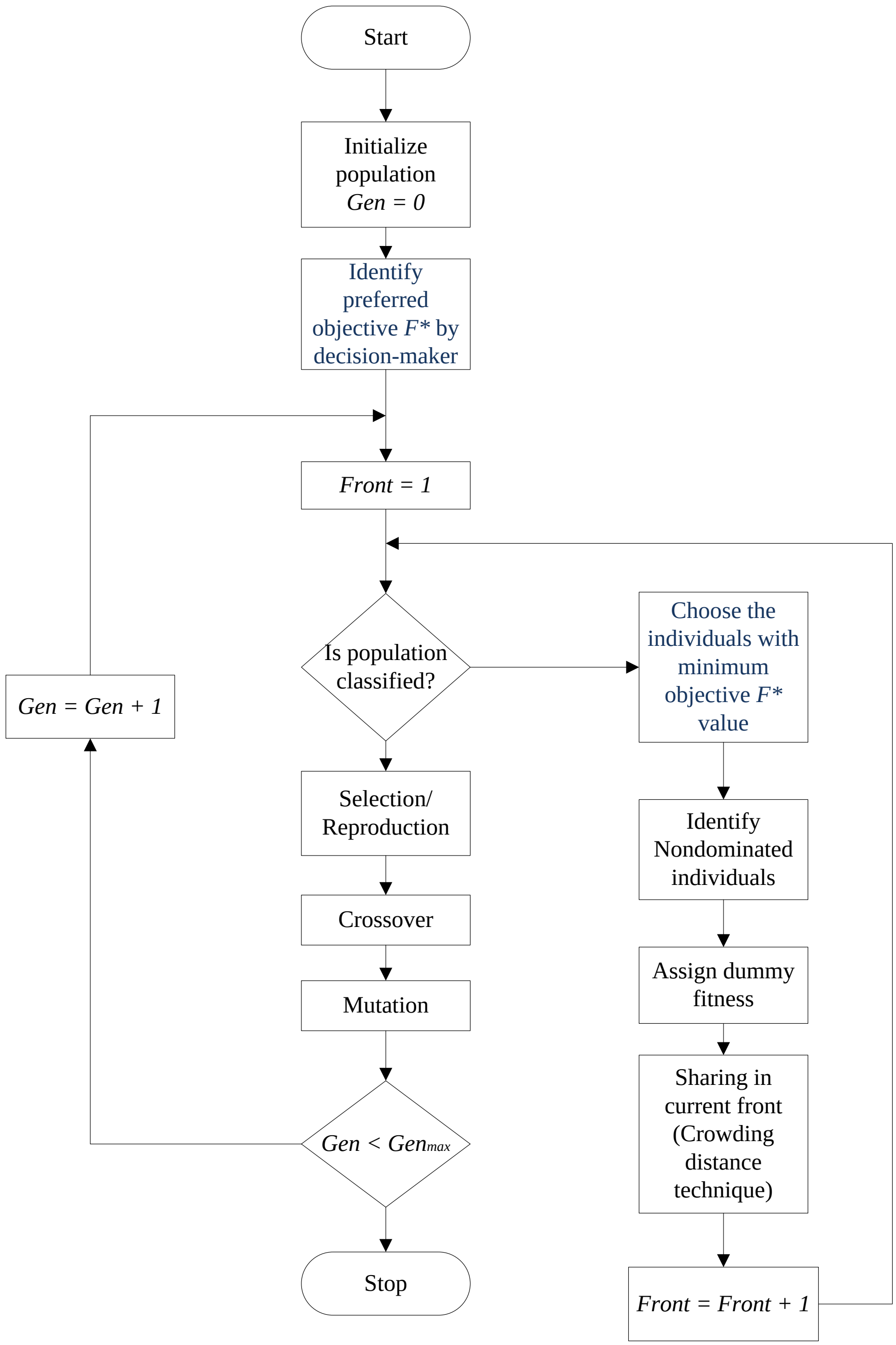

Figure 7.2: The interactive approach with NSGA II.

```
Initialize population
Identify preferred objective
        Generate random population – size M
        Evaluate objective values
        Assign rank based on Pareto dominance
        Generate child population
                Tournament selection
                        Select individuals with minimum preferred objective value
                Recombination and mutation
For i = 1 to G
        With parent and child population
                Assign rank based on Pareto dominance
                Generate sets of non-dominated fronts
                Loop by adding solutions to next generation starting from "first" until M individuals
                found
                Determine crowding distance between points on each front
        Select points (elitist) on the lower front (with lower rank) and are outside a crowding distance
        Create next generation
                Tournament selection
                        Select individuals with minimum preferred objective value
                Recombination and mutation
        Increment generation index
End loop
```

Figure 7.3: Pseudocode for the interactive approach with NSGA II.

7.3 Application of the Proposed Interactive Approach to the Case Study

The simulation framework for the (*s*, *S*) inventory with backlogging model integration with the famous NSGA II is considered for this case study. The interactive approach integrates the decision-maker preferences with NSGA II while solving for the optimization problem is shown in Figure 7.2. In this study, a two-objective, two-variable minimization problem is considered. The average holding cost per month and the average shortage cost per month are the objectives. The interactive approach begins during the optimization process to produce a set of **P** Pareto optima solutions biased to the preferred objective identified by the decision-maker and then to reduces them to a smaller set of solutions. It is applied to enhance the decision-making process. The case study is described in detail in Section 4.2 in Chapter 4.

7.3.1 Generation of the Set of Pareto Solutions

The interactive approach begins during the optimization process and then a biased set of Pareto optima to the preferred objective is generated. The interactive approach integrates the decision-maker preferences with NSGA II while solving for the optimization problem is shown in Figure 7.2.

The simulation optimization integrated framework is comprised of the NSGA II multiobjective evolutionary algorithm component (Figure 7.2) and the inventory simulation component. The algorithm iteratively generates decision variables (*s*, *S*). Evaluation of the decision variables are performed by the inventory simulation model. The NSGA II optimization algorithm generates pairs of the two decision variables known as the inventory (*s*, *S*) policy. These decision values are passed to the inventory simulation model to generate and replicate the objective function values (i.e., *H* (*s*, *S*): inventory holding cost per month and *B* (*s*, *S*): inventory shortage cost per month). The inventory simulation model returns the mean of the objective function values and corresponding standard deviation values to NSGA II. NSGA II generates and passes the new decision variable values to the inventory simulation model in order to compute the mean objective function values and corresponding standard deviation values. NSGA II then reports the set of Pareto (i.e., reports the mean of the objective function values and the corresponding standard deviation values, and the associated decision variable values).

7.3.2 Parameter Settings for the Simulation Model and NSGA II

Various input values are used for the inventory simulation model and NSGA II. The parameter values specified for the NSGA II and the inventory simulation model are shown in

Table 4.1 and in Table 4.2 respectively in Chapter 4. The parameter used is chosen by the analyst for the purpose of running and evaluating the interactive approach.

The interactive approach begins during the optimization process and then a biased set of Pareto optima to the preferred objective is generated. After that, reduction of the candidate set of compromise solutions is performed while considering the statistical precision of the performance measures under study and preferences on objectives by the decision-maker. Next, the reduced set of solutions is prioritized to assist the decision-maker in identifying the most appropriate compromise solution. The biased set of Pareto optima is obtained by the simulation optimization algorithm. The two phases considered after the optimization process are illustrated in section 5.2. The interactive approach is built by using the C++ computer language (i.e., to prioritize the preferred objective among the other objectives during the optimization process).

The interactive approach begins during the optimization process with preferred objective specified by the decision-maker. Assume that, a lower value of *B (s, S)* "objective 2" is desired by the decision-maker. Figure 7.4 shows the original decision space and Figure 7.5 shows the original Pareto optimal front generated by using a simulation multiobjective optimization approach that uses multiobjective evolutionary algorithms and discrete-event simulation. The generated set of Pareto optima is biased to the preferred objective *B (s, S)* as desired by the decision-maker. Each point (solution) on the curve (as shown in Figure 7.5) is generated after running n = 100 independent simulation replications. As such, the points (solutions) along the Pareto frontier are the mean objective values across the replications, and each has an associated standard deviation along each dimension in the objective space.

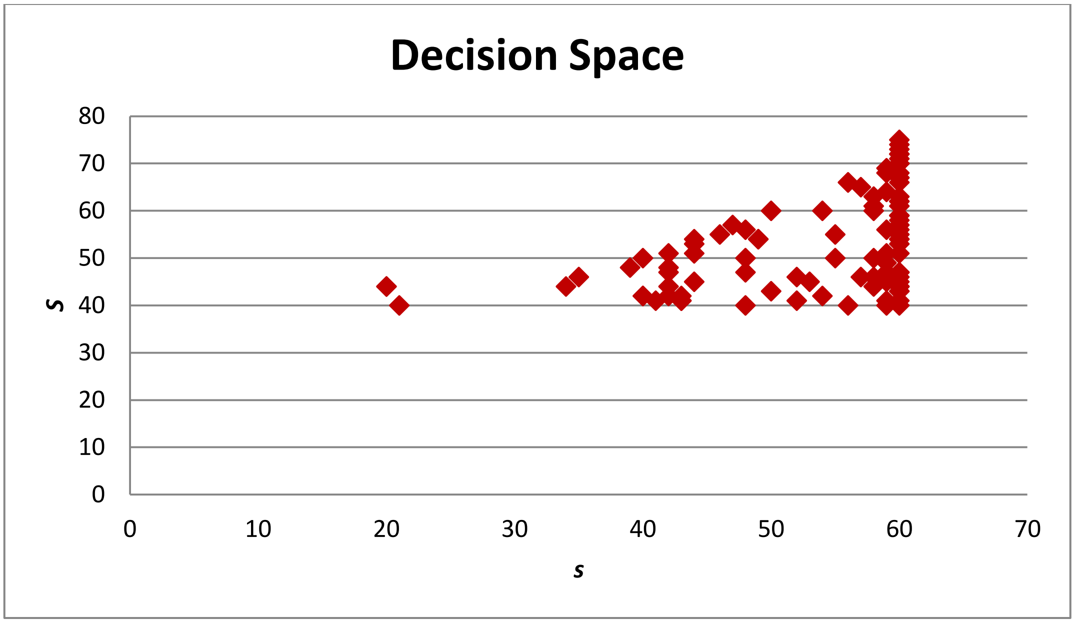

Figure 7.4: Decision space for the decision variables.

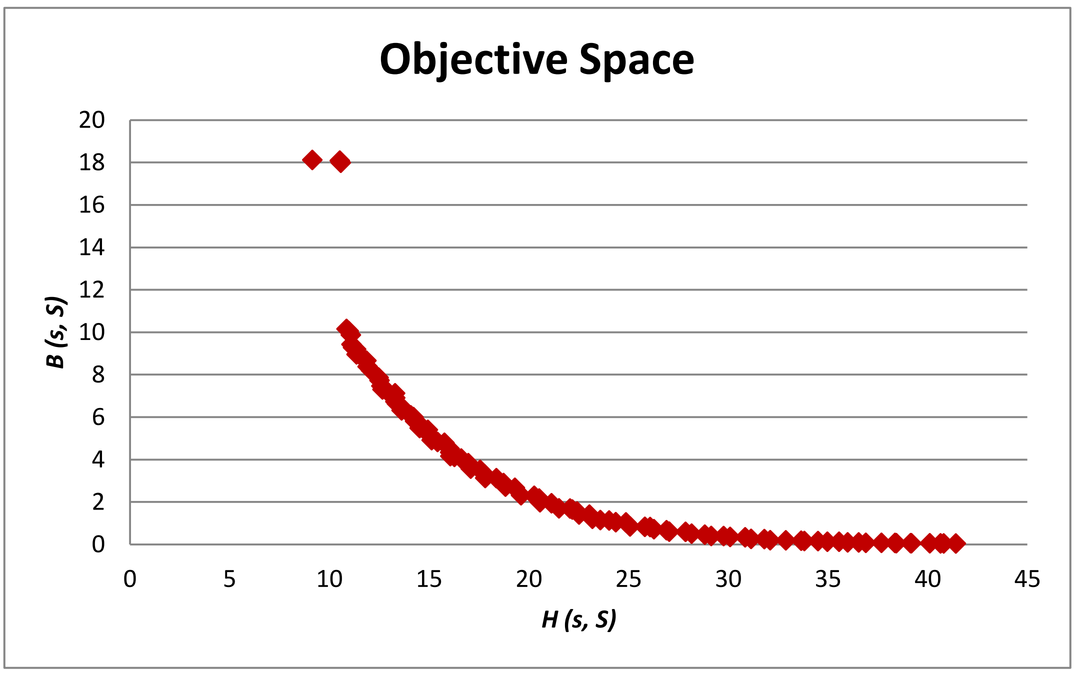

Figure 7.5: Objective space for the original mean objective functions (100 solutions).

Using the standard deviations, the precision of the mean objective values of the Pareto points (solutions) is represented by the confidence interval along each dimension computed using

$$\bar{x} \pm t_{\frac{\alpha}{2},n-1}\left(\frac{s}{\sqrt{n}}\right), \qquad (7.1)$$

where $\bar{x}$ is the mean objective value from the *n* replications, *s* is the standard deviation of the objective value from the *n* replications, α is the level of significance, and $t_{\alpha/2,n\text{-}1}$ is the upper $\alpha/2$ critical value for the *t*-distribution with *n*-1 degrees of freedom.

Thus the two phases, after the optimization process, are previously described in section 5.2 and they are considered for this problem. Phase 1 starts after the optimization process with computing the upper and lower confidence limit for each Pareto point using Eq. 7.1 (Mendenhall & Sincich, 2012). Here, for illustration, a level of significance α = 10% is assumed. In addition, the preferred objective *B (s, S)* (assume objective 2) is identified by the decision-maker.

First, the set of *B (s, S)* solutions is sorted from largest to smallest and then the overlapping confidence intervals among the whole set are identified. On the other hand, the marginal error values associated to *H (s, S)* is calculated. One solution is selected among each set of overlapping confidence intervals for the set of *B (s, S)* solutions by identifying the smallest marginal error value associated to *H (s, S)*. The first iteration reduced the original set of Pareto (100 solutions) to 31 solutions as shown in Figure 7.6.

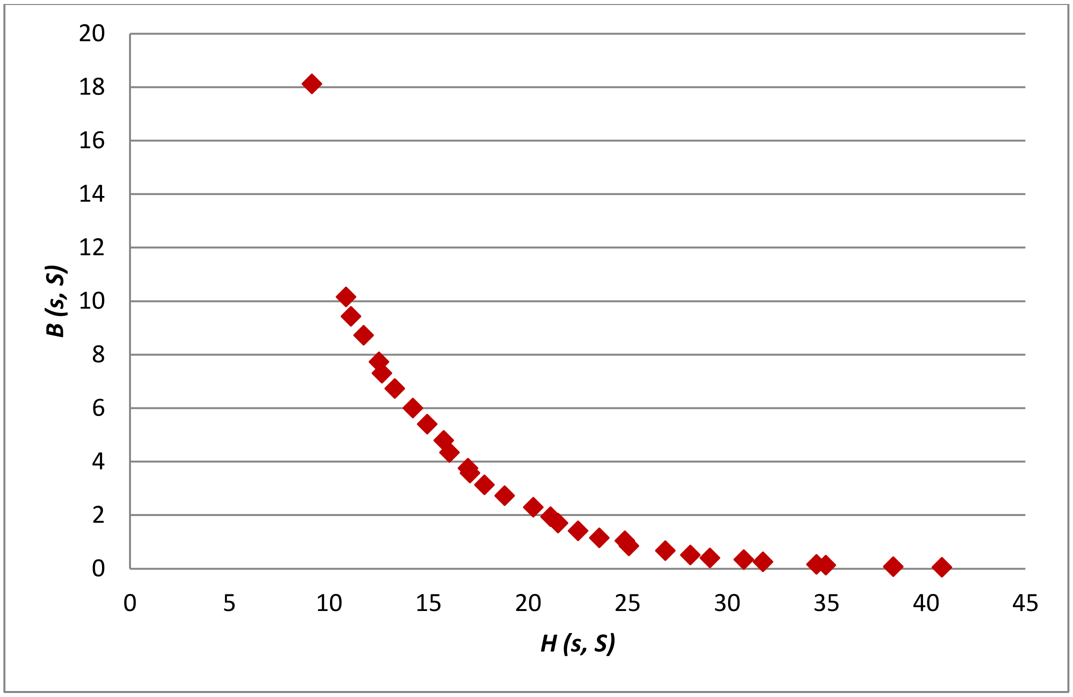

Figure 7.6: Graph of the set of Pareto optima (i.e., 31 compromise solutions).

Second, the previous step (first step) is repeated again to make sure that there are no more overlapping confidence intervals for *B (s, S)* solutions. The second iteration reduced the previous set of Pareto (31 solutions) to 25 solutions as shown in Figure 7.7.

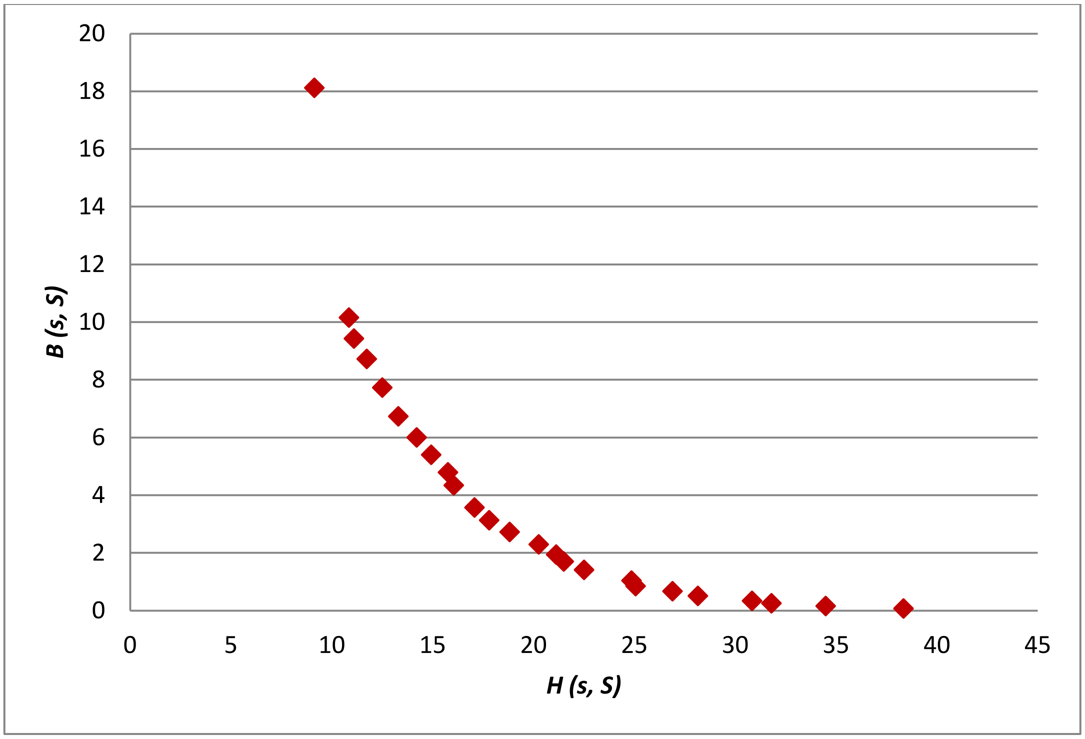

Figure 7.7: Graph of the set of Pareto optima (i.e., 25 compromise solutions).

Third, the previous step (first step) is repeated again to make sure that there are no more overlapping confidence intervals for *B (s, S)* solutions. It is founded that there are no more overlapping confidence intervals for the *B (s, S)* solutions.

Fourth, the previous step (first step) is repeated again but now for *H (s, S)* solutions to check for overlapping confidence intervals within *H (s, S)* solutions. The third iteration reduced the previous set of Pareto (25 solutions) to 24 solutions as shown in Figure 7.8.

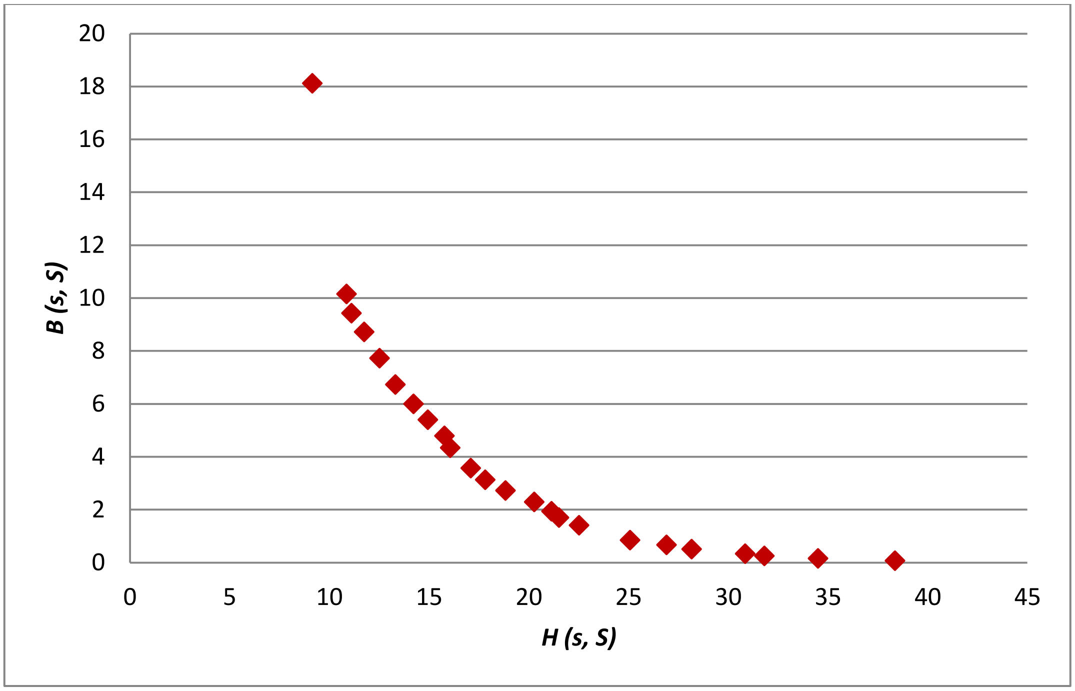

Figure 7.8: Graph of the set of Pareto optima (i.e., 24 compromise solutions).

Fifth, the previous step (first step) is repeated again to make sure that there are no more overlapping confidence intervals for *H (s, S)* solutions. It is founded that there are no more overlapping confidence intervals for the *H (s, S)* solutions. Therefore, the final reduced set of Pareto $\mathbf{P}^*$ is considered for Phase 2, which is the set of solutions without overlapping confidence intervals that are recognized after the optimization process as shown in Figure 7.8 and Table 7.1.

Table 7.1: The reduced set of Pareto (24 solutions).

Solution	***H (s, S)***	***B (s, S)***
1	9.138	18.121
2	10.853	10.158
3	11.100	9.431
4	11.737	8.724
5	12.508	7.731
6	13.303	6.734
7	14.212	6.003
8	14.931	5.403
9	15.763	4.794
10	16.051	4.344
11	17.081	3.574
12	17.807	3.133
13	18.824	2.727
14	20.264	2.296
15	21.131	1.943
16	21.506	1.704
17	22.513	1.414
18	25.068	0.851
19	26.898	0.673
20	28.156	0.511
21	30.842	0.340
22	31.804	0.255
23	34.493	0.160
24	38.353	0.074

Phase 2 prioritizes the representative solutions identified in Phase 1. Many researchers have used the popular swing weighting approach among the other multi-criteria decision-making approaches in the presence of multiple objectives. Using swing weights, the decision-maker determines which solutions are the most important, second most important, etc. and also by how many times it is more important. These numbers are then normalized to sum to 1.0 (Clemen & Reilly, 2004; Weber et al., 1988). The swing-weighting approach is used for Phase 2. Considering the current problem, assuming a lower value of *B (s, S)* is desired first, and then a lower value of *H (s, S)* is desired second. Table 7.2 shows the prioritized solutions using the swing weighting approach. Table 7.3 shows the assessment of the swing weights.

Table 7.2: The feasible solutions with priority.

Priority	***H (s, S)***	***B (s, S)***
1	9.138	18.121
4	10.853	10.158
5	11.100	9.431
6	11.737	8.724
7	12.508	7.731
8	13.303	6.734
10	14.212	6.003
11	14.931	5.403
12	15.763	4.794
13	16.051	4.344
15	17.081	3.574
16	17.807	3.133
17	18.824	2.727
19	20.264	2.296
21	21.131	1.943
22	21.506	1.704
24	22.513	1.414
23	25.068	0.851
20	26.898	0.673
18	28.156	0.511
14	30.842	0.340
9	31.804	0.255
3	34.493	0.160
2	38.353	0.074

Table 7.3: The assessment of swing weights.

Attribute Swung from Worst to Best	Consequence to Compare		Rank	Rate	Weight
(Benchmark)	38.353	18.121	3	0	0
H (s, S)	9.138	18.121	2	75	0.4286
B (s, S)	38.353	0.074	1	100	0.5714
			Total	175	1

The overall utility for different feasible solutions is determined as shown in Eqs. 7.2-7.25. The value of the corresponding weight or the relative utility shows how the prioritized solutions are identified. Eqs 7.2 and 7.25 shows how the weight values shown on table 7.3 are calculated for *H (s, S)* and *B (s, S)*.

$$U\,(9.138,\ 18.121) = H\,(0.00) + B\,(1.00) = 0.5714 \quad (7.2)$$

$$U\,(10.853,\ 10.158) = H\,(0.84) + B\,(0.01) = 0.3650 \quad (7.3)$$

$$U\,(11.100,\ 9.431) = H\,(0.82) + B\,(0.01) = 0.3573 \quad (7.4)$$

$$U\,(11.737,\ 8.724) = H\,(0.78) + B\,(0.01) = 0.3385 \quad (7.5)$$

$$U\,(12.508,\ 7.731) = H\,(0.73) + B\,(0.01) = 0.3186 \quad (7.6)$$

$$U\,(13.303,\ 6.734) = H\,(0.69) + B\,(0.01) = 0.3007 \quad (7.7)$$

$$U\,(14.212,\ 6.003) = H\,(0.64) + B\,(0.01) = 0.2826 \quad (7.8)$$

$$U\,(14.931,\ 5.403) = H\,(0.61) + B\,(0.01) = 0.2701 \quad (7.9)$$

$$U\,(15.763,\ 4.794) = H\,(0.58) + B\,(0.02) = 0.2573 \quad (7.10)$$

$$U\,(16.051,\ 4.344) = H\,(0.57) + B\,(0.02) = 0.2537 \quad (7.11)$$

$$U\,(17.081,\ 3.574) = H\,(0.53) + B\,(0.02) = 0.2411 \quad (7.12)$$

$$U\,(17.807,\ 3.133) = H\,(0.51) + B\,(0.02) = 0.2334 \quad (7.13)$$

$$U\,(18.824,\ 2.727) = H\,(0.49) + B\,(0.03) = 0.2236 \quad (7.14)$$

$$U\,(20.264, 2.296) = H\,(0.45) + B\,(0.03) = 0.2117 \tag{7.15}$$

$$U\,(21.131, 1.943) = H\,(0.43) + B\,(0.04) = 0.2071 \tag{7.16}$$

$$U\,(21.506, 1.704) = H\,(0.42) + B\,(0.04) = 0.2069 \tag{7.17}$$

$$U\,(22.513, 1.414) = H\,(0.41) + B\,(0.05) = 0.2039 \tag{7.18}$$

$$U\,(25.068, 0.851) = H\,(0.36) + B\,(0.09) = 0.2059 \tag{7.19}$$

$$U\,(26.898, 0.673) = H\,(0.34) + B\,(0.11) = 0.2084 \tag{7.20}$$

$$U\,(28.156, 0.511) = H\,(0.32) + B\,(0.14) = 0.2218 \tag{7.21}$$

$$U\,(30.842, 0.340) = H\,(0.30) + B\,(0.22) = 0.2515 \tag{7.22}$$

$$U\,(31.804, 0.255) = H\,(0.29) + B\,(0.29) = 0.2888 \tag{7.23}$$

$$U\,(34.493, 0.160) = H\,(0.26) + B\,(0.46) = 0.3775 \tag{7.24}$$

$$U\,(38.353, 0.074) = H\,(1.00) + B\,(0.00) = 0.4286 \tag{7.25}$$

7.4 Application of Simulation Optimization to the Case Study

In this section a comparison between the results of the problem generated by using the interactive approach, the enhanced *a posteriori* approach, the original *a posteriori* approach (with v = 65%) and the simulation framework for the (s, S) inventory with backlogging model integration with the NSGA II are illustrated. The parameters used for the simulation model is similar to that shown in Table 4.2 in Chapter 4. However, for the simulation optimization approach, the population size is four since the desired number of representative solutions is four for this case study. Table 7.4 and Figure 7.9 show the feasible solutions for the different approaches.

Table 7.4: The feasible solutions for the different approaches.

Original A Posteriori Approach with *v* = 65%		**Enhanced A Posteriori Approach**		**Interactive Approach**		**Simulation Optimization Approach with Population Size of 4**	
H (s, S)	***B (s, S)***	***H (s, S)***	***B (s, S)***	***H (s, S)***	***B (s, S)***	***H (s, S)***	***B (s, S)***
52.345	0.006	53.000	0.047	38.353	0.074	40.721	0.050
35.757	0.010	37.414	0.088	34.493	0.160	23.164	1.375
21.097	0.679	9.700	15.002	10.853	10.158	11.219	9.618
12.118	5.411	9.339	16.531	9.138	18.121	8.966	18.925

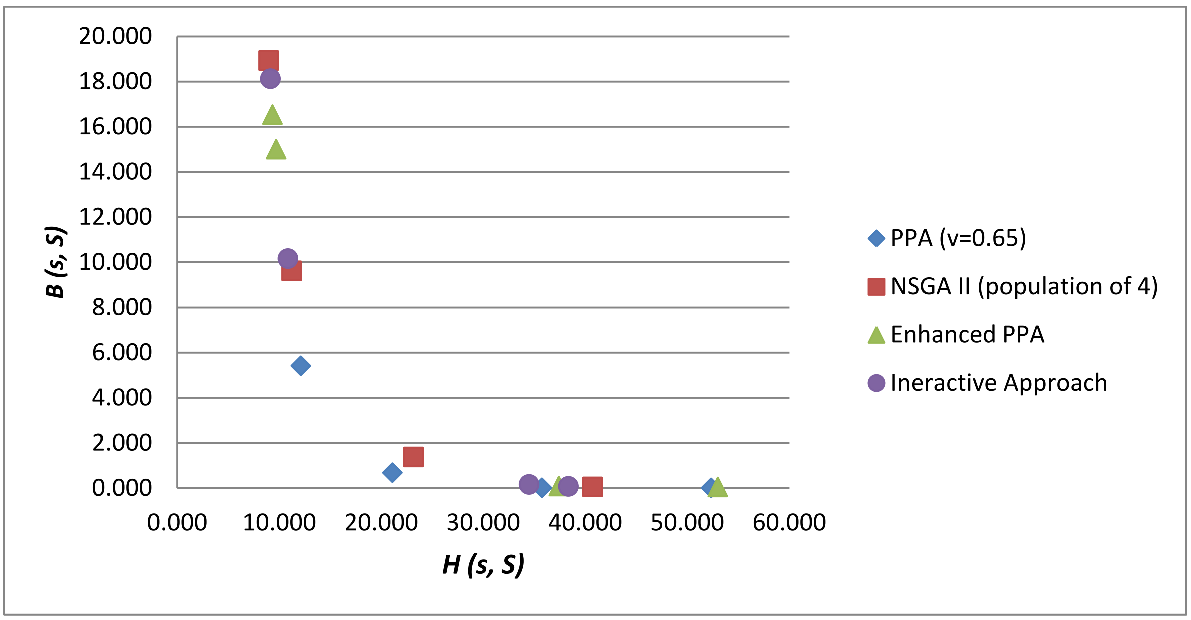

Figure 7.9: The feasible solutions for the different approaches.

Table 7.4 and Figure 7.9 above show the feasible solutions for the different approaches whereas the interactive approach is performing a pretty well when considering the current problem, assuming a lower value of *B (s, S)* is desired first, and then a lower value of *H (s, S)* is desired second. The average value of 7.128 for *B (s, S)* with interactive approach is improved compared to the enhanced *a posteriori* approach value of 7.917 for *B (s, S)* and compared to the simulation optimization approach value of 7.492 for *B (s, S)*. In addition, the results with the interactive approach and the original *a posteriori* approach show approximately equal spread solutions. The interactive approach deals fairly well with stochastic objectives settings only. The interactive approach considers the decision-maker preferences during the optimization process while the original *a posteriori* approach and the enhanced *a posteriori* approach do not consider decision-maker preferences *a priori*.

7.5 Summary

The objective of this study is the improvement of the decision-making selection process in the presence of stochastic objectives. The interactive approach begins during the optimization process and effectively reduces and prioritizes the set of Pareto solutions while considering the stochastic nature of *m* objective functions. The interactive approach effectively articulates the decision-maker preferences during the optimization process and intends to design a decision-making solution selection process in the presence of multiple stochastic objectives. The approach reduces a large set of tradeoff solutions to a manageable number of representative solutions while considering the stochastic nature of the objective functions.

The interactive approach is appropriate to use for stochastic set of objectives and the availability of the set of Pareto optima solutions is not required since the approach is applied during the optimization process.

The results discussed herein show the promise of the interactive approach. The interactive approach compared to the other approaches show pretty good results in general for the interest of decision-maker.

CHAPTER 8:
SUMMARY AND FUTURE RESEARCH DIRECTIONS

8.1 Research Summary

This research aims to improve the decision-making process under uncertainty and specifically focuses on reducing and organizing the set of candidate compromise solutions in the presence of stochastic objectives. The research investigation is a modest attempt to bridge gap of reducing the non-dominated set of solutions while considering the stochastic nature of the objective functions. In Chapter 2, a review of existing work in reducing and organizing the number of the Pareto optimal (tradeoff) solutions for better decision-making is given. Chapter 3 presents the framework of the *a posteriori* approach that effectively articulates the decision-maker preferences after the optimization process. Chapter 4 shows computational results for a common (*s, S*) inventory problem. A numerical simulation model of the inventory problem integrated with a multiobjective evolutionary algorithm (MOEA) is considered. NSGA II is used to optimize the design variables and generate the set of compromise solutions. Chapter 5 presents an enhanced framework of the *a posteriori* approach. The approach uses statistical analysis on the Pareto optimal solutions in order to reduce the number of solutions to set of representative solutions that is presented to the decision-maker for final selection. Chapter 6 presents useful results for a (*s, S*) inventory problem also. The enhanced *a posteriori* approach begins after the optimization process with the original set of tradeoff solutions and reduces them. It is applied to enhance the decision-making process. Chapter 7 presents the framework of the interactive approach, whereas the enhanced *a posteriori* approach is extended and incorporated in an interactive optimization framework. In addition, it presents useful results for a (*s, S*) inventory problem by using the interactive approach.

In summary, objective of research is the improvement of decision-making selection process in the presence of stochastic objectives. The results show promise of the proposed approaches. Nevertheless, the *a posteriori* approaches do not consider decision-maker preferences a *priori* except when identifying the final number of representative solutions. Conversely, the interactive approach considers decision-maker preferences during the optimization process.

The *a posteriori* and the interactive approaches are aimed to help the decision-maker to reduce the number of compromise solutions while considering the uncertainty objectives of real life problem specially when modeling them on computer simulation. The *a posteriori* approaches focus on the post Pareto analysis while the reduction of the compromise solutions took place after the original Pareto front is discovered after the optimization process. On the other hand, the interactive approach begins during the optimization process while considering the stochastic nature of the objective functions and then a biased set of Pareto optima to the preferred objective is generated. The interactive approach integrates the decision-maker preferences with NSGA II while solving for the optimization problem.

The *a posteriori* approach described in Chapter 3 is appropriate to use for either deterministic or stochastic set of objectives and the availability of the set of Pareto optima solutions is required since the approach is applied after the optimization process. The enhanced *a posteriori* approach described in Chapter 5 is appropriate to use for stochastic set of objectives and the availability of the set of Pareto optima solutions is required since the approach is applied after the optimization process. The interactive approach described in Chapter 7 is appropriate to use for stochastic set of objectives and the availability of the set of Pareto optima solutions is not required since the approach is applied during the optimization process. Thus, the strength of the approaches is the use of variability on stochastic problems.

8.2 Future Research Directions

The research investigation presented and the summary has laid sufficient foundation for possible extension of this investigation for future research. The future research seeks potentially to improve the decision-making procedure, and effectively reduces the set of Pareto optima in the presence of stochastic objectives. Some of the potential future works are as follows:

- Apply the concept and the approaches to other disciplines such as civil, electrical, materials engineering and other technologies.
- Enhance the proposed approaches to solve *m*-objective optimization problem.
- Investigate and propose a prioritization approach for the set of Pareto optima without the decision-maker preferences.

In brief, new efforts concerning improvement of decision-making for multiple objective problems and the need to reduce and organize the non-dominated set of solutions in the presence of stochastic objectives may benefit the decision-maker and provide a contribution not only to the practitioners' body of knowledge, but also to the researchers. Pareto analysis is potential concept to integrate within the body of simulation optimization algorithms in the presence multiple objectives to facilitate the decision making process.

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