Cavity model analysis of microstrip ring antennas using green's functions

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CAVITY MODEL ANALYSIS OF MICROSTRIP RING ANTENNAS USING GREEN'S FUNCTIONS

by

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B.S. Instituto Tecnológico y de Estudios Superiores de Monterrey, 1990

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering in the Department of Electrical and Computer Engineering in the College of Engineering at the University of Central Florida Orlando, Florida

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ABSTRACT

Microstrip ring antennas have been experimentally tested by several researchers due to their specific radiation characteristics that make them very suitable for many applications, from medical uses to mobile communications.

Ring antennas have been found to have a larger bandwidth compared to other conventional types of microstrip patch antennas. Furthermore, their size is smaller when compared to circular and rectangular resonant structures. The TM$_{1m}$ modes ($m = 2, 4, 6..,$ where $m$ represents radial variations) radiate conical beams in the broadside direction, while TM$_{n1}$ modes ($n = 2, 4, 6..,$ where $n$ represents azimuthal variations) have a radiation pattern with a null in the broadside direction but an omnidirectional azimuth coverage. Dual frequency behavior as well as improved bandwidth can be achieved by using stacked microstrip ring antennas. These are features that render them useful for mobile communications, because by the appropriate selection of the radiation mode, azimuth and elevation beam steering can be obtained. The radiation characteristics and input impedance are studied for both the stacked and the single microstrip ring antennas. A Cavity Model Analysis is performed along with Green's Functions to predict results that are in good agreement with experimental data.
To the memory of my father.

To my mother.

To Conchita.
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INTRODUCTION

Microstrip antennas have been investigated by many researchers due to their small size and easy integrability with microwave integrated circuits, which makes them very attractive when cost and dimensions are important. They do not represent very good radiators by themselves, and that is why a substantial effort has been placed to improve their characteristics, so that they can meet the requirements of today’s high demand of better communication systems. One of these features is the impedance bandwidth, which is very narrow for conventional rectangular and circular patches.

Microstrip ring antennas have a better bandwidth and smaller size than their rectangular and circular counterparts when they are excited with the right resonant modes, and also produce conical beams in their radiation patterns. With those outstanding characteristics, they have the potential to be used for applications where reduction in size and weight and an increase in bandwidth are important, like mobile and satellite communication systems.

This thesis has been divided into four chapters. The first chapter begins with an overview of microstrip antennas, and approximate methods of analysis; then, the basic characteristics of the annular patches are mentioned, as well as the Cavity Model and the Green’s Function method of analysis. The second chapter covers the theoretical study of
single ring microstrip antennas. The behavior of the antenna as a resonant cavity is
analyzed, including the fields inside the cavity, the electric and magnetic stored energies,
and the conductor and dielectric losses. Moreover, the magnetic wall model is studied, in
order to obtain the radiated fields, power transmitted and the input impedance.

The third chapter shows the results obtained with the cavity model developed.
Resonant frequency, radiation patterns, input impedance, bandwidth, directivity, gain and
efficiency are compared with previous reports. The fourth chapter is an extension of the
model for the single element. Stacked microstrip antennas are analyzed. Resonant
frequencies and radiation patterns are obtained and compared with experimental data.

Additional work must be done in order to obtain accurate results for the input
impedance of stacked elements and to include the effect of coupling in the case of single
elements. This would allow to develop an accurate model for the case of concentric rings,
which can be thought as a collection of concentric equivalent magnetic currents.

The disadvantages of the Cavity Model with Green's Functions are outweighed by the
easiness of its implementation. The computational effort is reduced and the approximate
results are in very good agreement to the experimental results.

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CHAPTER 1
CHARACTERISTICS OF MICROSTRIP RING ANTENNAS

Microstrip Ring Antennas

Basic Characteristics

Many projects and experiments have been conducted about rectangular and circular patches. In fact, theories and numerical techniques have been exhaustively modified to model these conventional shapes; and they work very well. Microstrip rings have some features that make them better than conventional patches. A bandwidth significantly broader than that of rectangular and circular patches can be obtained by a proper choice of the dimensions of the ring and the dominant mode of operation [1].

The first work dates back to 1973, a universal mode chart for microstrip ring resonators by Wu and Rosenbaum [2]. After that, researchers have paid attention to their outstanding characteristics. However, few attempts have been conducted to develop efficient methods of analysis for this specific kind of antennas. As a matter of fact, very few papers have been written about stacked annular microstrip antennas and concentric rings [3].
For a ring resonator, the resonant frequency of the lowest order mode can be lower than that of a circular disk of approximately the same size. This can be explained by observing that the average path length traveled by the current in the ring is longer than for a circular disk, for the lowest order mode. Figure 1 gives a picture of this concept [1]. So, ring antennas can be expected to be smaller in size and weight than a circular disk at the same resonant frequency. This is a very valuable characteristic. Table 1 compares various parameters of conventional and annular patches, for a typical configuration at 2 GHz, when radiating modes are excited [4].

Ring resonators can excite many modes, because they involve radial and azimuthal variations. So, TM<sub>m,n</sub> modes can be excited, where "m" represents the azimuth variations and "n", the radial variations. These modes have been investigated previously. The bandwidth of the TM<sub>1,1</sub> mode is very narrow, while that of the TM<sub>1,2</sub> mode is relatively wide. This means that the TM<sub>1,1</sub> mode is good for resonator applications and the TM<sub>1,2</sub> mode for antennas [1].

Actually, TM<sub>1,n</sub> modes (n = 1,3,5..) give poor radiation in the normal direction and have a low radiation efficiency. TM<sub>1,n</sub> modes (n = 2,4,6..) are good radiation modes. In fact, TM<sub>m,1</sub> modes (m = 2,4..) have also been investigated previously to switch modes in concentric rings, exhibiting conical shaped beams and good performance [3]. The excitation of higher order modes involves the generation of surface wave power; however, for microstrip rings, the lower "high-order" modes have shown to be very stable, and useful for applications where broader bandwidth is required. That is why this work will be concentrated in TM<sub>1,n</sub> modes (n = 2,4,..) modes, which can be distinguished from TM<sub>m,1</sub>
Table 1. Comparison of Rectangular, Circular and Annular Microstrip Antennas [4].

Assumed operating frequency of 2 GHz; TM_{11} mode is excited for circular and rectangular patches; \( \varepsilon_r = 2.32 \), height = 0.159 cm. Rectangular patch dimensions: \( W = 5.82 \) cm, \( L = 4.85 \) cm. Circular patch dimensions: \( a = 2.78 \) cm. Annular patch dimensions: for TM_{11} mode, \( a = 1.1 \) cm, \( b = 2.2 \) cm; for TM_{12} mode, \( a = 4.4 \) cm, \( b = 8.8 \) cm. ( * Note: the bandwidth of the TM_{11} and TM_{12} modes for the annular patch is obtained as a variation of one unit from the minimum standing wave ratio).
modes (m = 2, 4, ..) modes in the radiation properties: the former have very good radiation in the normal direction and the last have conical beams with a null in the normal direction.

The main advantages of microstrip ring antennas are listed next [1],[3],[5],[6]:

a) They have a significantly broader bandwidth compared to other printed circuit resonant antennas.

b) For a given operating frequency, their size is smaller than those of corresponding disk antennas; an additional benefit of this is that a more dense array of elements can be built, reducing the grating lobe problem.

c) They can be used as a radiating element at a low frequency band in conjunction with another high frequency element at the center slot to provide a compact dual band antenna.

d) Dual frequency behavior and increased bandwidth can be obtained by using concentric rings, stacked rings, or even stacked concentric rings.

e) Dual frequency behavior of concentric rings can be obtained by exciting higher order modes. Mode switched antennas are a cheap alternative to an expensive phase array antenna with continuous beam steering.

Microstrip Ring Antennas also have some disadvantages. Mainly, an increase in bandwidth is obtained at the expense of a reduction in the gain, compared to a circular disk. However, the reduction is not very significant and the values of gain are good for many applications. Also, some complexity in the analysis is added because it is necessary to include the effect of coupling, even for the simplest methods of analysis, like the Cavity Model. Coupling occurs between the two equivalent magnetic current sources, at the
outer and inner edges. When concentric rings are studied, coupling between the outer edge of the inner ring, and the inner edge of the outer ring is very strong and reduces the resonant frequency expected with a simple cavity model. Coupling affects also the results for input impedance, even for single annular patches [7]. These are important considerations.

Methods of Analysis

The analysis and design of antennas requires the full solution of Maxwell's equations. Microstrip antennas are even more difficult to model. The analysis is complicated by the presence of an inhomogeneous dielectric medium, narrow-band electrical characteristics, and a wide variety of patch, feed, and substrate configurations.

The existing theories to model microstrip elements can be classified as follows [8]-[10]:

1) Reduced Analysis (maintain simplicity at the expense of accuracy or versatility).
2) Full Wave Analysis (maintain accuracy at the expense of computational simplicity).

The reduced analysis theories introduce approximations to simplify the problem. The methods included under this category are:

a) The Cavity Model.

b) The Transmission Line Model.

c) The Multiport Network Model.

The advantages of these simplified methods are many, since they give a physical insight to the operation of the antenna, and predict under certain conditions, very accurate
results with relatively simple calculations. Unfortunately, they have a limited capacity to handle mutual coupling, large arrays, surface wave effects and thicker dielectrics, but there are ways to come up with corrections to the models so that they can achieve better results.

The Cavity Model uses a magnetic wall boundary condition approximation along the edge of the patch. The Transmission Line Model treats the antenna as a transmission line section with lumped loads at the edges. The Multiport Network Model generalizes the Cavity Model.

The Full Wave Analysis Methods account for the dielectric substrate in a rigorous way, assuming that the substrate is infinite in extent in the lateral dimensions and enforcing the proper boundary conditions at the air - dielectric interface. In most of the cases, the exact Green's Function for the dielectric substrate is obtained, because this allows to include in the model the effects of space wave radiation, surface wave modes, dielectric loss and coupling. The numerical techniques included under this category are:

a) Moment Method Solution.

b) Finite Elements Method.

c) Finite Differences Method.

The advantages of Full Wave solutions are the accuracy (specially for input impedance, mutual coupling, radar cross section), completeness (include the effect of surface waves, external coupling and space wave radiation), versatility (arbitrary elements, different feeding techniques, multilayer geometries, anisotropic substrates, arrays). The disadvantage is the computational complexity.
In this thesis, the cavity model was chosen as the method of analysis that could give a physical insight to the properties of the antenna and accurate results. Instead of using the conventional modal expansion which involves a double summation of all the modes of the antenna, the closed form of the Green’s Function for a single patch was obtained.

The Green’s Function approach gives a large degree of flexibility to the analysis. Just by setting the appropriate boundary conditions, one can go to the Cavity Model (perfect magnetic walls at the edges), or the Wall Admittance Model (non-perfect magnetic walls). Green’s Function can also lead to a Method of Moments solution. However, to maintain the idea of explaining the different aspects of the electro-magnetic behavior of the antenna, the Cavity Model was selected as a good approximate solution. In fact, with some corrections to account for the non-homogeneous media and dispersion effects, excellent results have been obtained [4].

For example, the resonant frequency for a single patch was obtained by correcting the dimensions of the outer and inner radius since they are affected by the fringing fields at the edges. The relative permitivity was corrected by the dispersion effects in the dielectric; so, an effective dielectric constant was used. Very accurate results that were matched with experiments were obtained.

In the next chapters, the properties of single and stacked ring antennas will be analyzed with an emphasis in the resonant frequencies and radiation patterns.
Figure 1. Difference in current path lengths between the circular disk and the annular patch for the TM$_{11}$ mode.

Figure 2. Side view of an annular microstrip antenna fed by a coaxial probe feed.
CHAPTER 2
THEORETICAL STUDY OF A SINGLE RING MICROSTRIP ANTENNA

Green’s Functions Analysis

Fundamentals

The Green’s Functions Method of analysis was chosen because of its flexibility and generality in solving the problem of second-order partial differential equations derived from Maxwell’s equations. By applying the appropriate boundary conditions, the form of the solution for most of those equations is an infinite series, provided that the differential equation and its boundary conditions are separable in the coordinate system chosen. Since closed form solutions are often desirable for any engineering problem and Green’s Functions can provide them, this approach was adopted in this thesis [11].

An annular microstrip antenna fed by a coaxial line is comprised of an annular conducting strip, with inner and outer radii a and b, placed on the top surface of a substrate of height “h”, and relative permittivity εr, as shown in Figure 2. The substrate is supported by a ground plane.

The first task in this work is to determine the Green’s Function for the TM_{20} modes with independent z variations. The last assumption was taken considering that the
substrate height \( h \) is much less than the wavelength \( \lambda \), i.e., \( h \ll \lambda \). So, the electric field \( E \), and in the z direction \( E_z \) and the electric current density \( J \), and in the z direction \( J_z \) have no z variations, are constant. If a coaxial probe feed is assumed, this means that only the z component of the electric current exists [11].

The Cavity Model

The cavity model considers the microstrip patch as a cavity resonator. This simple model must be extended to include the feed source. Therefore, the wave equation in the presence of a current source (assuming no impressed magnetic current, \( M_i \)) is [11]:

\[
\nabla^2 \vec{E} + \omega^2 \mu \varepsilon \vec{E} = j \omega \mu J_i + \frac{1}{\varepsilon} \nabla \cdot \vec{q}_{ev} + j \omega \mu \sigma \vec{E} \quad (2.1)
\]

Since \( J_c = \sigma \cdot E \), then:

\[
\nabla^2 \vec{E} + \omega^2 \mu \varepsilon \vec{E} = j \omega \mu \left(J_i + J_c\right) + \frac{1}{\varepsilon} \nabla \cdot \vec{q}_{ev} \quad (2.2)
\]

which by the use of

\[
\nabla \cdot \vec{J}_{ic} = -j \omega \cdot \vec{q}_{ev} \quad (2.3)
\]

yields

\[
\nabla^2 \vec{E} + \omega^2 \mu \varepsilon \vec{E} = j \omega \mu J_{ic} \frac{1}{j \omega \varepsilon d} \nabla \left( \nabla \cdot \vec{J}_{ic} \right) \quad (2.4)
\]

where if we consider that the analysis is carried out by assuming a coaxial probe feed, then the electric current is uniform and directed along \( z \). Therefore, \( J_{icx} = 0 \), \( J_{icy} = 0 \), \( J_{icz} = \text{const.} \)
The electric current can be assumed constant and uniform considering that the substrate height \(( h)\) is smaller than the wavelength \(( \lambda)\), i.e., \(h \ll \lambda\). So, equation (2.4) and the continuity equation become:

\[ \nabla \cdot J_{ic} = 0 \]  

\[ \nabla^2 \mathbf{E} + \omega^2 \varepsilon \mu \mathbf{E} = j \omega \mu J_{ic} \]  

Since only the \(z\) components exist, then:

\[ \nabla^2 E_z + \omega^2 \varepsilon \mu E_z = j \omega \mu J_{icz} \]  

Green’s Function of a Microstrip Ring Resonator

The Green’s Function must satisfy the same partial differential equation that the electric field. Therefore,

\[ \nabla^2 G + \omega^2 \mu \varepsilon G = \delta \left( \mathbf{r} - \mathbf{r}_\text{f} \right) \]  

\[ \delta \left( \mathbf{r} - \mathbf{r}_\text{f} \right) \equiv \frac{1}{\rho} \delta \left( \rho - \rho_\text{f} \right) \cdot \delta \left( \phi - \phi_\text{f} \right) \]

Also, it must satisfy the same boundary conditions. However, in order to give more flexibility to the design, these conditions will not be included at this time. This procedure will give the freedom to modify the model with the appropriate boundary conditions and obtain two different approaches: the first one can consider perfect magnetic walls surrounding the edges of the resonator; the second one can consider a wall admittance.
The closed form solution has been chosen. This can be formulated by choosing functions that satisfy the boundary conditions either along \( \rho = a \) or \( \rho = b \), or at certain values of \( \phi \). Functions that can satisfy the boundary conditions at any value of \( \phi \) are chosen. So, the initial representation of the Green’s function is:

\[
G(\rho, \phi, \rho_f, \phi_f) = \sum_{m=0}^{\infty} g_m(\rho, \rho_f, \phi_f) \cdot \cos(m \cdot \phi)
\]  

Equation (2.8) can be expanded as:

\[
\frac{\partial^2 G}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial G}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 G}{\partial \phi^2} + k_d^2 \cdot G = \delta(\bar{\rho} - \bar{\rho}_f)
\]  

Substituting (2.10) into (2.11) yields:

\[
\sum_{m=0}^{\infty} \left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} - \frac{m^2}{\rho^2} + k_d^2 \right) g_m(\rho, \rho_f, \phi_f) \cdot \cos(m \cdot \phi) = \delta(\bar{\rho} - \bar{\rho}_f)
\]  

Since the expansion functions have to be orthogonal, an orthogonality relationship must be used.

\[
\int_0^{2\pi} \cos(m \cdot \phi) \cdot \cos(n \cdot \phi) \, d\phi = A
\]

\[
A = \pi \text{ for } m = n
\]

\[
A = 0 \text{ otherwise}
\]

Using the above condition, multiplying both sides of equation (2.12) by \( \cos(n \cdot \phi) \), and integrating, and considering the fact that (2.13) delivers a value only for \( m = n \), which makes the summation disappear, the following relationship is obtained. Note that in the integral on the left, \( n \) or \( m \) can be used now:
Therefore, the differential equation for the expansion functions become:

\[
\pi \left[ \frac{d^2 g_m}{d\rho^2} + \frac{1}{\rho} \frac{dg_m}{d\rho} + \left( k_d^2 - \frac{m^2}{\rho^2} \right) g_m \right] = \int_0^{2\pi} \delta(\rho - \rho_f) \cdot \cos(n\phi) \, d\phi 
\] (2.14)

The homogeneous equation is:

\[
\rho \frac{d^2 g_m}{d\rho^2} + \frac{dg_m}{d\rho} + \left( \rho \cdot k_d^2 - \frac{m^2}{\rho} \right) g_m = 0 
\] (2.15)

or in the Sturm-Liouville form:

\[
\frac{d}{d\rho} \left( \rho \frac{dg_m}{d\rho} \right) - \frac{m^2}{\rho} g_m + k_d^2 \rho g_m = 0 
\] (2.16)

where

\[
p(\rho) = \rho \quad r(\rho) = \rho \quad q(\rho) = \frac{m^2}{\rho} \quad \lambda = k_d^2
\] (2.18)

The Sturm-Liouville form is preferred because as it will be seen later, the Wronskian and other parameters are defined in terms of this form of the differential equation.

The solution of either (2.16) or (2.17) is well known, given by a set of Bessel functions. In the following equations, \( J_n(x) \) represents the Bessel function of the first kind of order \( n \), and \( Y_n(x) \) is the Bessel function of the second kind of order \( n \), also called Neumann’s function. The solution is:
Green's functions have five basic properties:

1. They must satisfy the homogeneous differential equation except at \( \rho = \rho_f \).

2. It must be symmetrical with respect to \( \rho \) and \( \rho_f \).

3. It must satisfy certain homogeneous boundary conditions.

4. It must be continuous at \( \rho = \rho_f \).

5. Its derivative has a discontinuity of \( 1/\rho(\rho_f) \) at \( \rho = \rho_f \).

So far, conditions 1 and 3 have already been considered (actually, condition 3 will be seen later when the boundary conditions be included in the model). Conditions 3, 4 and 5 are accomplished by obtaining a function called the Wronskian.

Therefore, applying conditions 4 and 5 it can be shown that the expansion functions of equation (2.10) are:

\[
g_m(\rho, \rho_f) = \frac{g_m(\rho_f)^{<2>}}{p(\rho_f) \cdot W(\rho_f)} \cdot g_m(\rho)^{<1>} \quad \text{for} \ a \leq \rho \leq \rho_f
\]

\[
g_m(\rho, \rho_f) = \frac{g_m(\rho_f)^{<1>}}{p(\rho_f) \cdot W(\rho_f)} \cdot g_m(\rho)^{<2>} \quad \text{for} \ \rho_f \leq \rho \leq b
\]

In this case, the Wronskian is defined as:

\[
W(\rho) = g_m(\rho)^{<1>} \cdot \frac{dg_m(\rho)^{<2>}}{d\rho} - g_m(\rho)^{<2>} \cdot \frac{dg_m(\rho)^{<1>}}{d\rho}
\]
Using the property of Bessel Functions [11]:

\[
J_n(x) \cdot \frac{dY_n(x)}{dx} - Y_n(x) \cdot \frac{dJ_n(x)}{dx} = \frac{2}{\pi \cdot x} \tag{2.22}
\]

Using equations (2.19) and (2.22) into (2.21), it can be shown that the Wronskian is reduced to:

\[
W(\rho_f) = \frac{2 \cdot A_m \cdot C_m}{\pi \cdot \rho_f} \left( \frac{D_m - B_m}{C_m - A_m} \right) \tag{2.23}
\]

Now, let's define a couple of equations that will help to reduce the size of the equations:

\[
f_a(\rho) = J_m(k_d \cdot \rho) + A \cdot Y_m(k_d \cdot \rho)
\]

\[
f_b(\rho) = J_m(k_d \cdot \rho) + B \cdot Y_m(k_d \cdot \rho)
\]

Using equations (2.18), (2.19), (2.23) and (2.24) into (2.20), yields:

\[
g_m(\rho) = \frac{f_b(\rho_f)}{2 \cdot (B - A)} \cdot f_a(\rho) \cdot \cos(m \cdot \phi_f) \quad \text{for } a \leq \rho \leq \rho_f \tag{2.25}
\]

\[
g_m(\rho) = \frac{f_a(\rho_f)}{2 \cdot (B - A)} \cdot f_b(\rho) \cdot \cos(m \cdot \phi_f) \quad \text{for } \rho_f \leq \rho \leq b
\]

where:

\[
B = \frac{D_m}{C_m} \quad \text{and} \quad A = \frac{B_m}{A_m} \tag{2.26}
\]

So, finally, the desired Green's Function for this problem is:
\[ G(\rho, \phi) = \sum_{m=0}^{\infty} \frac{f_b(\rho_f)}{2(B-A)} \cdot f_a(\rho) \cdot \cos(m\cdot\phi_f) \cdot \cos(m\cdot\phi) \quad \text{for } a \leq \rho \leq \rho_f \]

\[ G(\rho, \phi) = \sum_{m=0}^{\infty} \frac{f_a(\rho_f)}{2(B-A)} \cdot f_b(\rho) \cdot \cos(m\cdot\phi_f) \cdot \cos(m\cdot\phi) \quad \text{for } \rho_f \leq \rho \leq b \]

The above equation is very valuable. It represents the Green’s Function for a Single Microstrip Ring Cavity Resonator. So far, no boundary conditions have been applied; the variables B and A will then be obtained by applying the boundary conditions. Actually, equation (2.27) will become the core of this thesis, because as it will be shown in the next pages, it can be used to predict the electric and magnetic fields inside the cavity. Using a cavity model with some modifications to account for fringing fields, very good results can be achieved.
Fields inside the Cavity

Derivation of Electric and Magnetic Fields

Using equations (2.7) and (2.27), the solution for the electric field (E-field) inside the cavity can be derived. It should be pointed out that the E-field is going to be directed along z, due to the reasons stated in equations (2.1) - (2.7). The E-field is \( z \)-independent according to those assumptions; this will become one of the drawbacks of this model to predict results for thick substrates. Otherwise, the model will behave very well. This and other issues will be discussed later.

The \( z \)-directed, electric field inside the cavity is [11]:

\[
E_z (\rho, \phi) = j \cdot \omega \cdot \mu \cdot I_o \cdot G(\rho, \phi) \tag{2.28}
\]

In here, since \( \beta_z \) is equal to zero because of the \( z \) independence, then the wave number becomes \( k_d \), the wave number in the dielectric substrate. If losses are considered, then a loss tangent (\( \tan \delta \)) must be inserted to the relative permittivity. If there are no losses, then \( \tan \delta \) equals zero, i.e [11]:

\[
k_d = \omega \cdot \sqrt{\mu \cdot \varepsilon} \quad \varepsilon = \varepsilon_0 \cdot \varepsilon_r \cdot (1 - j \cdot \tan \delta) \quad \mu = \mu_0 \tag{2.29}
\]

Using the wave equation for the TM \( z \) modes and the vector magnetic potential, it can be shown that the equations for the electric field inside a cavity for those modes are [11]:
If the vector magnetic potential \( A_z \) is considered, and separable for the coordinate system chosen (cylindrical in this case), then the solution for a wave equation of the form of (2.7), where only the \( z \) components are considered, is:

\[
A_z(\rho, \phi, z) = f(\rho) \cdot g(\phi) \cdot h(z)
\]

where

\[
f(\rho) = A_1 \cdot J_m(k_d \rho) + B_1 \cdot Y_m(k_d \rho)
\]

\[
g(\phi) = C_2 \cdot \cos(m\phi) + D_2 \cdot \sin(m\phi)
\]

\[
h(z) = C_3 \cdot \cos(\beta_z z) + D_3 \cdot \sin(\beta_z z)
\]

Then, by applying the condition of uniformity along \( z \) and even modes for the azimuth, the vector magnetic potential becomes:

\[
A_z(\rho, \phi) = A_0 \left[ A_1 \cdot J_m(k_d \rho) + B_1 \cdot Y_m(k_d \rho) \right] \cdot \cos(m\phi)
\]

and the wave number is:

\[
k_d^2 + \beta_z^2 = \beta^2
\]

where by making \( \beta_z \) equal to zero, \( k_d \) becomes the wave number for the TM \( z \) modes.
Using equations (2.30) and (2.33), it can be shown that the vector magnetic potential can also be represented by:

\[ A_z(\rho, \phi) = j \cdot \frac{E_z(\rho, \phi)}{\omega} \]  

(2.34)

Since \( A_z \) is independent of \( z \), then \( E_\rho \) and \( E_\phi \) are zero. Also, by using (2.30) and (2.34) the equations for the magnetic field are:

\[ H_\rho = j \cdot \frac{1}{\omega \cdot \mu_0 \cdot \rho} \frac{\delta E_z}{\delta \phi} \]

\[ H_\phi = -1 \cdot j \cdot \frac{1}{\omega \cdot \mu_0} \frac{\delta E_z}{\delta \phi} \]  

(2.35)

The following variables are defined, and they will be useful to write the final expressions for the fields inside the cavity:

\[ K_m = \frac{\cos (m \phi_f)}{2(B_m - A_m)} \]  

(2.36)

\[ E_{am} = j \cdot \omega \cdot \mu_0 \cdot I_{ms} \cdot K_m \cdot f_b(\rho_f) \]  

(2.37)

\[ E_{bm} = j \cdot \omega \cdot \mu_0 \cdot I_{ms} \cdot K_m \cdot f_a(\rho_f) \]

As it can be seen, \( I_{ms} \) has been included instead of the feed current \( I_o \). Actually, \( I_{ms} \) is the feed current corresponding to the \( TM_m \) mode. It is obtained from the condition of the magnetic field (and hence, it is the modal current) at \( \rho = \rho_f \), the feed point, and extracting the \( m \)-th term in the Fourier expansion of the surface current, \( J_s \) [12].

\[ I_{ms} = \frac{I_o}{\pi \cdot \sigma_m} \quad \sigma_m = 2 \quad \text{if } m = 0 \]

\[ \sigma_m = 1 \quad \text{if } m > 0 \]  

(2.38)

The surface current for the \( m \)-th mode is:
\[ J_{ms} = I_{ms} \frac{\delta(\phi)}{\rho_f} \quad (2.39) \]

So, the electric field inside the cavity is:

\[ E_z(\rho, \phi) = \sum_{m=0}^{\infty} E_{am} f_a(\rho) \cdot \cos(m \cdot \phi) \quad \text{for} \quad a \leq \rho \leq \rho_f \quad (2.40) \]

\[ E_z(\rho, \phi) = \sum_{m=0}^{\infty} E_{bm} f_b(\rho) \cdot \cos(m \cdot \phi) \quad \text{for} \quad \rho_f \leq \rho \leq b \]

The expression for the magnetic field inside the cavity are:

\[ H_\rho(\rho, \phi) = -\frac{j}{\omega \cdot \mu_0 \cdot \rho} \sum_{m=0}^{\infty} m \cdot E_{am} f_a(\rho) \cdot \sin(m \cdot \phi) \quad \text{for} \quad a \leq \rho \leq \rho_f \quad (2.41) \]

\[ H_\rho(\rho, \phi) = -\frac{j}{\omega \cdot \mu_0 \cdot \rho} \sum_{m=0}^{\infty} m \cdot E_{bm} f_b(\rho) \cdot \sin(m \cdot \phi) \quad \text{for} \quad \rho_f \leq \rho \leq b \]

\[ H_\phi(\rho, \phi) = -\frac{j}{\omega \cdot \mu_0} \sum_{m=0}^{\infty} \frac{df_a(\rho)}{d\rho} \cdot \cos(m \cdot \phi) \quad \text{for} \quad a \leq \rho \leq \rho_f \quad (2.42) \]

\[ H_\phi(\rho, \phi) = -\frac{j}{\omega \cdot \mu_0} \sum_{m=0}^{\infty} \frac{df_b(\rho)}{d\rho} \cdot \cos(m \cdot \phi) \quad \text{for} \quad \rho_f \leq \rho \leq b \]

\( E_\rho \) and \( E_\phi \) are equal to zero, and, as it is shown by equation (2.30), \( H_z \) is zero for TM \( z \) modes.
Equations (2.40) through (2.42) are very important. They still don’t include the boundary conditions. In order to include some, let’s take the Cavity Model with perfect magnetic walls. For such a model, the tangential magnetic field must vanish at the edges of the ring [13]. $H_z$ is zero already. The only tangential field left is $H_\phi$, so:

$$H_\phi (\rho = a) = H_\phi (\rho = b) = 0$$ (2.43)

Since $H_\phi$ involves a summation, in order to make it zero, each mode has to be zero. So, it is valid to analyze each mode separately. By applying (2.43) into (2.42), and after some algebraic manipulations, the following equations are obtained:

$$A_m = \frac{d}{d\rho} J_m(k_d a) \frac{d}{d\rho} Y_m(k_d a) \quad B_m = \frac{d}{d\rho} J_m(k_d b) \frac{d}{d\rho} Y_m(k_d b)$$ (2.44)

As it can be seen, equations (2.40)-(2.42), have no constants. In other words, all the variables included are frequency dependent. So, for frequency analysis, all the quantities talked so far are frequency dependent (except for the dimensions, which are considered “constant”; however, as it will be studied later, even the effective dimensions will become frequency dependent to account for dispersion effects).

The above comment is very important because during the development of this thesis, $A_m$ and $B_m$ were initially considered constant, and that was a mistake that was corrected later. Even $A_m$ and $B_m$ are frequency dependent, as it can be seen if we look at $k_d$ appearing into the equation.
It can be shown, by using the continuity property of the Green’s Function and its derivative, that at the resonant frequency (and only at that frequency), the variables $A_m$ and $B_m$ are equal, i.e.:

$$A_m = B_m \quad \text{at resonance} \quad (2.45)$$

So, if the analysis is done only at the resonant frequency, the well-known expression for the E-field is obtained [14]-[16]:

$$E_z(\rho, \phi) = E_0 f_a(\rho) \cdot \cos(m \cdot \phi) \quad (2.46)$$

It is a single expression because since $A_m$ and $B_m$ are equal, then $E_{am}$ and $E_{bm}$ are equal, and called $E_o$ for simplicity; $f_a(\rho)$ and $f_b(\rho)$ are also equal. This is a valid expression between the inner and outer edges (a and b). Once again, valid only at the resonant frequency. If a frequency analysis has to be done, then equation (2.46) is no longer valid and equations (2.40) through (2.42) have to be used along with (2.44) for the Cavity Model with Perfect Magnetic Walls. If a Wall Admittance Model variation is going to be used, then a different set of boundary conditions are used [17].

$$H_{\phi_1} = -Y_{sa} E_{z1} \quad \text{at} \quad \rho = a$$

$$H_{\phi_2} = -Y_{sb} E_{z2} \quad \text{at} \quad \rho = b \quad (2.47)$$

$$E_{z1} = E_{z2} \quad \text{or} \quad H_{\rho_1} = H_{\rho_2} \quad \text{at} \quad \rho = \rho_f$$

$$H_{\phi_2} - H_{\phi_1} = J_s \quad \text{at} \quad \rho = \rho_f$$

In this case, the subscripts 1 and 2 refer to the regions inside the cavity for which $\rho$ is less than or greater than $\rho_f$, respectively; $Y_{sa}$ and $Y_{sb}$ represent the wall admittance at the
edges of the ring. However, for the purpose of this thesis, this model won't be developed, so instead, the conditions of (2.44) will be taken.

If we take a look at equation (2.36), it seems obvious that when \( A_m \) is equal to \( B_m \), i.e. at resonance, the variable \( K_m \) becomes infinity; therefore, since all the fields depend on \( K_m \), they are infinite at resonance. This could be true if we were dealing with an ideal cavity and probes that at the resonant frequency, the resonant modal field is the one with the biggest excitation. If substrate losses are considered, an imaginary part appears in the wave number as can be obtained from equation (2.29), therefore removing the singularities of the fields at the resonant frequency of each mode. Figures 3 and 4 depict typical plots of the Electric Field inside a microstrip ring cavity resonator. The peak seen in Figure 3 occurs at the resonant frequency.

Figure 4 shows that at a specific frequency, the electric field has radial variation, and has many (actually infinite) maxima and minima. The relative maxima and minima represent a resonant condition for a modal field. Therefore, a design procedure can be outlined by using these points. If the inner radius dimension, \( a \), is chosen as well as the operating frequency; then the E-field is plotted versus \( \rho \), and the outer radius dimension can be chosen at a maximum or a minimum of the E-field, for maximum modal field excitation. However, \( b \) can also be chosen at a point different that a peak, for example, close to zero. For some radiation modes the magnetic currents at the inner and outer edges cancel out; thus, selecting the outer ring dimension close to zero will avoid this cancellation and just radiation from the inner magnetic current will be obtained.
Figure 3. Typical plot of the Normalized Electric Field inside a microstrip ring cavity versus radial variation at the frequency of operation. In this case $\varepsilon_r = 2.32$, $a = 2.5$ cm, $b = 5$ cm and the mode of operation is $\text{TM}_{12}$ at 3.77 GHz.

Figure 4. Typical plot of the Normalized Electric Field in dB inside a microstrip ring cavity versus frequency (from Figure 3).
Derivation of Power Losses

Losses in a microstrip antenna arise from three sources: conductor losses in the patch, dielectric substrate losses and surface waves. The first two, are dominant for thin substrate heights. Surface waves appear for thick substrates and when higher order modes are excited, but they can be neglected for thinner structures. So, only conductor and dielectric losses are taken into consideration in the following derivation [4], [18].

The surface current on the bottom part of the patch is:

\[ K = J_{sp} = n \times H \]

Therefore, the components are obtained:

\[ K_\rho = H_\phi \quad K_\phi = -H_\rho \]  \hspace{1cm} (2.49)

The power lost in the conductors depends on the surface current calculated above, as follows:

\[ P_c = 2 \cdot \frac{R_s}{2} \int J_{sp} \cdot \overrightarrow{J_{sp}} \, dS \quad u = k_d \cdot \rho \]  \hspace{1cm} (2.50)

where \( R_s \) is the surface resistance, i.e., a measure of the skin depth effect, and can be predicted with:

\[ R_s = \sqrt{\frac{\pi \cdot f \cdot \mu \_0}{\sigma c}} \]  \hspace{1cm} (2.51)

So, by using (2.49) into (2.50) results in:
From (2.52), doing some algebraic manipulations, the final expression for the conductor losses is obtained:

\[
P_c = R_s \int_a^b \int_0^{2\pi} \left( K_\rho^2 + K_\phi^2 \right) \rho \, d\phi \, d\rho \tag{2.52}
\]

As it was seen in equation (2.50), "u" is used as a change of variable considering that \(f_a\) and \(f_b\) are actually not only function of \(\rho\) but also of \(k_d\), so they can be represented with another notation: \(f_a(k_d, \rho)\) and \(f_b(k_d, \rho)\), and that explains the derivatives in (2.55) and (2.56) with respect to the argument. Figure 5 shows a typical plot, where, as expected, maximum loss occurs for maximum fields, i.e., at resonance. Some convergence problems were found when the computer simulation of (2.54) was done; to correct this problem, an equivalent expression for the derivative of Bessel functions was used:

\[
P_1 = E_{am}^2 \int_a^b \int_{k_d}^\infty k \, d^2 \rho \cdot \left( \frac{m^2}{u^2} \cdot f_a \left( \frac{u}{k_d} \right)^2 + \left( \frac{d}{du} f_a(u) \right)^2 \right) \cdot u \, du \tag{2.54}
\]

\[
P_2 = E_{bm}^2 \int_a^b \int_{k_d}^\infty k \, d^2 \rho \cdot \left( \frac{m^2}{u^2} \cdot f_b \left( \frac{u}{k_d} \right)^2 + \left( \frac{d}{du} f_b(u) \right)^2 \right) \cdot u \, du \tag{2.55}
\]

As it was seen in equation (2.50), "u" is used as a change of variable considering that \(f_a\) and \(f_b\) are actually not only function of \(\rho\) but also of \(k_d\), so they can be represented with another notation: \(f_a(k_d, \rho)\) and \(f_b(k_d, \rho)\), and that explains the derivatives in (2.55) and (2.56) with respect to the argument. Figure 5 shows a typical plot, where, as expected, maximum loss occurs for maximum fields, i.e., at resonance. Some convergence problems were found when the computer simulation of (2.54) was done; to correct this problem, an equivalent expression for the derivative of Bessel functions was used:

\[
\frac{d}{du} f_a(u) = J_{m-1}(u) - \frac{m}{u} J_m(u) + A_m \cdot Y_{m-1}(u) - \frac{m}{u} A_m \cdot Y_m(u) \tag{2.56}
\]
Figure 5. Typical plot of the conductor losses of a microstrip ring antenna for the $\text{TM}_{12}$ mode (from Figure 3).

Figure 6. Typical plot of the dielectric losses of a microstrip ring antenna for the $\text{TM}_{12}$ mode (from Figure 3).
with a similar expression for the derivative of $f_b(u)$, both deducted from the property of
the derivative of a Bessel function.

The power lost in the dielectric can be predicted by means of the following
expression:

$$P_d = \frac{\omega \cdot \varepsilon \cdot \tan \delta}{2} \int \vec{E} \cdot \nabla \vec{E} \, dV$$  \hspace{1cm} (2.57)

Using equation (2.57) and after some algebraic manipulations, the final expression is
obtained:

$$P_d = \frac{\omega \cdot \varepsilon \cdot \tan \delta}{2} \cdot \pi \cdot h \cdot \left( E_{am}^2 \int_a^b f_a(\rho)^2 \cdot \rho \, d\rho + E_{bm}^2 \int_a^b f_b(\rho)^2 \cdot \rho \, d\rho \right)$$  \hspace{1cm} (2.58)

A typical plot of (2.58) is shown in Figure 6. Again, as expected, the maximum
occurs at resonance because that point represents the maximum fields.

Derivation of Electric and Magnetic Energies

The expression for the stored electric energy inside a cavity can be shown to be:

$$W_e = \frac{P_d}{2 \cdot \omega \cdot \tan \delta}$$  \hspace{1cm} (2.59)

and the stored magnetic energy inside a cavity is:

$$W_m = \frac{1}{4} \cdot \mu_0 \cdot h \cdot \frac{P_c}{R_s}$$  \hspace{1cm} (2.60)
The expression for $P_c$ and $P_d$ has to be substituted into equations (2.59) and (2.60) to deduct the energy inside a microstrip ring resonator. Figures 7 and 8 show plots of the electric and magnetic energy for this case. At resonance, the electric and magnetic energies are maximum, and furthermore, they are equal because the fields are maximum.

\[ W_e(f_r) = W_m(f_r) \]  \hspace{1cm} (2.61)

Figure 9 shows a plot of the difference between the magnetic and the electric energies. This difference is important because the reactive power that is generated depends on this value, and then, the reactance of the input impedance is affected. The difference is zero at resonance, so an ideal zero-reactance (i.e, just real input impedance) can be expected at resonance. Once again, this issue will be discussed properly in the next chapter.
Figure 7. Typical plot of the normalized stored electric energy inside a microstrip ring resonator for the TM$_{12}$ mode (from Figure 3).

Figure 8. Typical plot of the normalized stored magnetic energy inside a microstrip ring resonator for the TM$_{12}$ mode (from Figure 3).
Figure 9. Typical plot of the normalized difference between the electric and magnetic energies stored inside a microstrip ring resonator (from Figure 3).

Figure 10. Typical plot of the normalized radiated power by a microstrip ring antenna for the TM\textsubscript{12} mode (from Figure 3).
Fields Radiated by a Single Ring

Electric and Magnetic Radiated Fields

To determine the radiated field of a microstrip ring antenna, the electric field inside the cavity obtained at the two apertures formed at the edges is considered to be the source of radiation. In this case, by applying an image theory approximation (assuming an infinite ground plane), an equivalent magnetic current source is obtained [13]:

\[ M_s = -2 \mathbf{n} \times \mathbf{E} \quad \text{for} \quad 0 \leq z_s \leq h \]
\[ M_s = 0 \quad \text{otherwise} \quad (2.62) \]

where the unit vector is in the normal direction to the aperture, i.e., \( \rho \) direction.

Doing the above calculations yields:

\[ M_{s\phi} = 2 \mathbf{E} \cdot f_a(a) \cdot \cos(m \phi_s) \quad \rho = a \quad \text{and} \quad 0 \leq z_s \leq h \quad (2.63) \]
\[ M_{s\phi} = 2 \mathbf{E} \cdot f_b(a) \cdot \cos(m \phi_s) \quad \rho = b \quad \text{and} \quad 0 \leq z_s \leq h \quad (2.64) \]

where the subscript “s” stands for source, the source coordinates; this is important, because the magnetic current is not at the origin, although it has circular symmetry.

Using aperture theory, it can be shown that the electric and magnetic fields can be calculated with the following equations [13]:

34
\[ E_r = 0 \]
\[ E_\theta = -j \cdot k_o \cdot \frac{e}{4 \cdot \pi \cdot r} \cdot (L_\phi + \eta_o \cdot N_\theta) \]
\[ E_\phi = j \cdot k_o \cdot \frac{e}{4 \cdot \pi \cdot r} \cdot (L_\theta - \eta_o \cdot N_\phi) \]
\[ H_r = 0 \]
\[ H_\theta = \frac{E_\phi}{\eta} \]
\[ H_\phi = \frac{E_\theta}{\eta} \quad (2.65) \]

where \( L_\theta, L_\phi, N_\theta \) and \( N_\phi \) depend on the induced magnetic and equivalent current sources on the aperture. Since only the magnetic source is considered, it can be easily deducted that [13]:

\[ N_\theta = N_\phi = 0 \quad (2.66) \]

\[ L_\theta = \int M_\phi \cdot \cos(\theta) \cdot \sin(\phi - \phi_s) \cdot e^{j \cdot k_o \cdot r_s \cdot \cos \psi} \cdot dS_s \quad (2.67) \]

\[ L_\phi = \int M_\phi \cdot \cos(\phi - \phi_s) \cdot e^{j \cdot k_o \cdot r_s \cdot \cos \psi} \cdot dS_s \quad (2.68) \]

where the integration is taken over the aperture. The unknown variables in the previous equations are [13]:

\[ dS_s = \rho_s \cdot d\phi_s \cdot dz_s \]
\[ r_s \cdot \cos \psi = \rho_s \cdot \sin \theta \cdot \cos(\phi - \phi_s) \quad (2.69) \]

Using equations (2.64) through (2.69), the radiated electric and magnetic fields are obtained. For simplicity, the steps are not shown here; even though they are not complex, they require extensive algebraic manipulation and the use of integral tables for Bessel functions. With this in mind, some variables that will allow to express the final equations in a simplified way are defined:
A_1(\theta) = J_{m-1}(k_o \cdot a \cdot \sin\theta) - J_{m+1}(k_o \cdot a \cdot \sin\theta) \quad (2.70)

B_1(\theta) = J_{m-1}(k_o \cdot b \cdot \sin\theta) - J_{m+1}(k_o \cdot b \cdot \sin\theta) \quad (2.71)

A_2(\theta) = J_{m-1}(k_o \cdot a \cdot \sin\theta) + J_{m+1}(k_o \cdot a \cdot \sin\theta) \quad (2.72)

B_2(\theta) = J_{m-1}(k_o \cdot b \cdot \sin\theta) + J_{m+1}(k_o \cdot b \cdot \sin\theta) \quad (2.73)

A_\theta = a \cdot E_{am} \cdot f_a(a) \cdot A_1(\theta) - b \cdot E_{bm} \cdot f_b(b) \cdot B_1(\theta) \quad (2.74)

A_\phi = (a \cdot E_{am} \cdot f_a(a) \cdot A_2(\theta) - b \cdot E_{bm} \cdot f_b(b) \cdot B_2(\theta)) \cdot \cos\theta \quad (2.75)

Therefore, using equations (2.64) - (2.75), yields:

\[ E_\theta = -j \cdot k_o \cdot e^{\frac{-j \cdot k_o \cdot r}{2 \cdot r}} \cdot \sin(m \cdot \phi) \cdot A_\theta \quad (2.76) \]

\[ E_\phi = j \cdot k_o \cdot e^{\frac{-j \cdot k_o \cdot r}{2 \cdot r}} \cdot \sin(m \cdot \phi) \cdot A_\phi \quad (2.77) \]

The importance of equations (2.76) and (2.77) is that radiation patterns will be easily obtained. A closed form representation has been derived. The assumption made is that the magnetic current sources must add up in order to yield radiation. In the next chapter, the issue concerning radiating and non-radiating modes will be discussed; basically, what happens is that for certain modes of operation, the electric fields at the aperture (and consequently the magnetic currents) cancel out, instead of adding up; those modes will have a high quality factor (Q) and the radiation efficiency will be very poor.
Radiated Power

The calculation of the radiated power by the apertures is very important. Although a closed form expression was possible for the radiated fields, unfortunately, power involves a surface integral of an already complex function. So, without numerical methods it is impossible to solve for its value.

The average power radiated is obtained by integrating the radiation intensity over the entire solid angle of $4\pi$ [13]:

$$ P_{\text{rad}} = \int \int U(\theta, \phi) \cdot \sin\theta \, d\theta \, d\phi $$

(2.78)

and the radiation intensity is defined as [13]:

$$ U(\theta, \phi) = \frac{r^2}{2 \cdot \eta_o} \cdot |E(r, \theta, \phi)| = \frac{r^2}{2 \cdot \eta_o} \cdot \left[ \left( |E_\theta| \right)^2 + \left( |E_\phi| \right)^2 \right] $$

(2.79)

So, doing some algebraic calculations with equations (2.76) through (2.79), the power radiated by the two apertures of a microstrip ring antenna is:

$$ P_{\text{rad}} = \frac{(k_o \cdot h)^2}{8 \cdot \eta_o} \cdot \pi \cdot \int_0^{\pi/2} \left( A_\theta^2 + A_\phi^2 \right) \cdot \sin\theta \, d\theta $$

(2.80)

This expression will be useful to calculate the directivity, the input impedance and the bandwidth (and of course, all the quantities related to them); the radiation resistance will be the main contributor for the input impedance, and it will be predicted by means of
equation (2.80). Figure 10 shows a plot of the radiated power; once more, the peak corresponds to the resonant frequency.
CHAPTER 3
SOME RESULTS FOR MICROSTRIP RING ANTENNAS

Resonant Frequency

In the previous chapters, the possibility of exciting different modes in a microstrip ring antenna was mentioned. As it can be deducted from the Green’s Function of a microstrip ring antenna, there is an infinite number of modes, i.e. resonant frequencies, that can be obtained. So, it is very important to know which one is being excited.

A universal mode chart for microstrip ring resonators, based on a radial waveguide mode, was presented by Y. S. Wu and F.J. Rosenbaum [2], relating the resonant frequency to the width of the ring conductor.

The following conclusions can be made from the universal mode chart, which relates $TM_{mn}$ modes (m stands for azimuthal variations and n for radial variations) [2]:

1) For narrow rings ($W / R \to 0$, or more precisely, $W / R < 0.2$), the resonant modes are $TM_{m1}$ modes, where $k_d \cdot R \approx m$. This can be a useful tool for simplified design of those specific modes.

2) As $W / R$ increases, $k_d \cdot R$ becomes less than m for a given mode.

3) When the width reaches half the guided wavelength, higher order $TM_{mn}$ modes start to appear ($m \geq 0$, $n > 1$).
4) As the inner radius reduces \((W/R \to 1)\), the ring becomes closer to a disk resonator.

The task of obtaining the resonant frequency can be done by considering the cavity model. Such a model was developed in chapter two, and the result was represented by equations (2.44) and (2.45), which for clarity purposes will be repeated here:

\[
\frac{d}{dp} \frac{J_m(k \cdot a)}{Y_m(k \cdot a)} = \frac{d}{dp} \frac{J_m(k \cdot b)}{Y_m(k \cdot b)} \quad (3.1)
\]

\[
A_m = B_m \quad \text{at resonance} \quad (3.2)
\]

Combining equations (3.1) and (3.2), the so called characteristic equation for the resonant modes can be obtained[1]:

\[
\frac{d}{dp} J_m(k \cdot a) \cdot \frac{d}{dp} Y_m(k \cdot b) - \frac{d}{dp} J_m(k \cdot b) \cdot \frac{d}{dp} Y_m(k \cdot a) = 0 \quad (3.3)
\]

In equation (3.3) it must be understood that the derivatives of the Bessel functions are evaluated at \(p = a\) and \(p = b\).

If the inner and outer radii \(a\) and \(b\) are known, equation (3.3) can be solved for its zeroes. Each zero represents a different resonant \(TM_{mn}\) mode. Since \(m\) (the azimuthal variations) is part of the equation, then \(n\) (the radial variations) is an implicit variable. Since \(m \geq 0\) and \(n > 1\), therefore for a given value of \(m\), \(n = 1\) corresponds to the first zero, \(n = 2\) to the second zero, and so on. Each zero will yield a different value for the wave number, \(k_a\), and thus, for the resonant frequency as related by equation (2.29). This
Figure 11. Typical plot of the characteristic equation of a microstrip ring resonator. Each zero crossing represents the resonant frequency of a $\text{TM}_{mn}$ mode for constant $m$ ($\text{TM}_{11}$, $\text{TM}_{12}$, $\text{TM}_{13}$, ...). $a = 2.5 \text{ cm}$, $b = 5 \text{ cm}$, $h = 0.159 \text{ cm}$, $\varepsilon_r = 2.32$.

Figure 12. Typical plot of the characteristic equation of a microstrip ring resonator. The $\text{TM}_{11}$ and $\text{TM}_{12}$ modes exist for wave numbers of $k_{d1} = 69.86$ and $k_{d2} = 226.25$ respectively. $a = 1 \text{ cm}$, $b = 2 \text{ cm}$, $h = 0.318 \text{ cm}$, $\varepsilon_r = 2.32$. 
concept is illustrated in Figure 11. In Figure 12, the values of the zero crossings are shown. For each value of m, plots similar to those of Figures 11 or 12 will be obtained; however, the roots of the equation will be different since they will match different modes.

Unfortunately, the only use of equation (3.3) to determine the resonant frequency of a microstrip ring antenna is not accurate enough, so some approximations have to be done. Two different phenomena occur inside the microstrip antenna which will shift the value of the resonant frequency as obtained by (3.3): a dispersion effect, which takes into account the non-TEM nature of the microstrip model by making the effective dielectric constant and impedance functions of frequency; and a fringe field effect, which considers the extension of the fields inside the cavity beyond the boundaries set by the physical dimensions of the patch, thus introducing the concept of “effective dimensions” of the microstrip [4], [19]-[22].

The effective radius for a circular microstrip disk has been calculated by Chew and Kong [1], [16], and has been successfully used by Bhattacharyya [12],[23] to predict the fringing field effect for the annular ring. Since the ring patch has two edges, the fringe field effect on the inner radius is a reduction in its size, i.e., a smaller effective inner radius. On the other hand, a larger effective outer radius is obtained. In general, since the substrate height is small compared to the wavelength, it can be assumed that the fringing field is extended up to a distance smaller than “h” from the edges and zero beyond that (so, a magnetic wall model can be set at that region) [24]. The effective dimensions will determine how far the fringe field will extend its influence.
The effective inner radius can be calculated as follows [17],[25]:

\[
\Delta C_a = \ln\left(\frac{a}{2 \cdot h}\right) + 1.41 \cdot \varepsilon_r + 1.7726 + \frac{h}{a} \left(0.286 \cdot \varepsilon_r + 1.65\right) \tag{3.4}
\]

\[
a_{\text{eff}} = a \cdot \sqrt{1 - \frac{2 \cdot h}{\pi \cdot a \cdot \varepsilon_r} \cdot \Delta C_a} \tag{3.5}
\]

And for the effective outer radius, the equations are:

\[
\Delta C_b = \ln\left(\frac{b}{2 \cdot h}\right) + 1.41 \cdot \varepsilon_r + 1.7726 + \frac{h}{b} \left(0.286 \cdot \varepsilon_r + 1.65\right) \tag{3.6}
\]

\[
b_{\text{eff}} = b \cdot \sqrt{1 + \frac{2 \cdot h}{\pi \cdot b \cdot \varepsilon_r} \cdot \Delta C_b} \tag{3.7}
\]

In order to consider the dispersion effects due to the difference of the quasi-static model for a microstrip and the real non-TEM nature of its fields, some assumptions have to be made. Wolff and Knoppik [26] made a study of the microstrip ring resonator and dispersion measurement on microstrip lines; although that wasn’t the main part of their study, they found out that the curvature of the ring influences the resonant frequency. Large resonators do not suffer from this effect too much; but if short resonators are used to make dispersion measurements, the curvature of the ring needs to be taken into account (the terms large and short come up after comparing the width with the mean radius of the ring; a large ring has a large mean radius compared to its width). As a matter of fact, this study suggests the possibility of extending the quasi-static analysis of a microstrip line to a microstrip ring with good results. Some researchers have shown it valid [7], [26]-[28].
The dispersion effect is included in the model by assuming a frequency-dependent effective dielectric constant, rather than the uniform value assumed by the quasi-static model. An accurate model in closed form has been presented by Kirschning and Jansen [27] for the effective dielectric constant of a single microstrip line which is valid with high accuracy up to mm-wave frequencies. This model was used for the microstrip ring resonator in this thesis with excellent results. The equations are \((f) \text{ is in GHz and } h \text{ is in cm}) [27]:

\[
\varepsilon_{\text{eff}}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{\text{eff}(0)}}{1 + P(f)} \tag{3.8}
\]

\[
P(f) = P_1 \cdot P_2 \left[ \left( 0.1844 + P_3 \cdot P_4 \right) \cdot 10 \cdot f \cdot h \right]^{1.5763} \tag{3.9}
\]

\[
P_1 = 0.27488 + \left[ 0.6315 + \frac{0.525}{\left( 1 + 0.157 \cdot f \cdot h \right)^{20}} \right] \cdot u \tag{3.10}
\]

\[
P_2 = 0.33622 \cdot \left( 1 - e^{-0.03442 \varepsilon_r} \right) \tag{3.11}
\]

\[
P_3 = 0.0363 \cdot e^{-4.6 \cdot u} \cdot \left[ 1 - e^{-\left( \frac{f \cdot h}{3.87} \right)^{4.97}} \right] \tag{3.12}
\]

\[
P_4 = 1 + 2.751 \cdot \left[ 1 - e^{-\left( \frac{\varepsilon_r}{15.916} \right)^8} \right] \tag{3.13}
\]

\[
\varepsilon_{\text{eff}(0)} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \cdot \frac{1}{\sqrt{1 + 12 \cdot \frac{h}{b - a}}} \tag{3.14}
\]
Table 2 shows the excellent results obtained simply by correcting the resonant frequency obtained using the cavity model, with the fringe field and dispersion effects.

Results are compared to experimental data and maintain an accuracy within 1%. The effective dimensions are calculated first using equations (3.4) to (3.7), then the effective dielectric constant is obtained using equations (3.8) through (3.14). Finally, the resonant frequency is obtained by finding the roots of equation (3.3).
Table 2. Resonant Frequency of a Single Ring Microstrip Antenna.

Resonant frequency calculations using the modified cavity model to account for fringing fields and dispersion effects, compared to experimental data from different references.
Radiated Fields

The equations for the radiated field by a magnetic current source around the edges of a microstrip ring antenna were already derived in Chapter 2, using a magnetic wall approximation and image theory to account for the assumed infinite ground plane. The real value of these expressions is that they express in closed form the behavior of the radiated fields (although they include Bessel functions).

Figures 13 through 18 [3], [14], [16] show the excellent agreement between the closed form expressions and experimental data for the radiation patterns. The following statements can be concluded on the most practical and useful modes of these antennas:

1) The TM$_{1n}$ modes ($n \geq 1$) have maximum radiation in the broadside ($\theta = 90^\circ$) direction (called broadside modes).

2) The TM$_{m1}$ modes ($m \neq 1$) have a null in the broadside direction and have a maximum on a different elevation angle $\theta$ (called non-broadside modes).

3) For either TM$_{1n}$ or TM$_{m1}$ modes, the E-plane pattern ($\phi = 90^\circ$) has a null at elevation angles of +90 or -90 degrees. The H-plane patterns ($\phi = 0^\circ$) don't have nulls at +90 or -90 degrees.

The above statements can be acknowledged by looking at Figures 13 through 18, which show the most important modes of operation for microstrip ring antennas: TM$_{11}$, TM$_{12}$, TM$_{21}$, and TM$_{41}$. Each mode has its own advantages and disadvantages, so a good design must compromise between those features. A closing discussion on the main features of each of these modes will be given at the end of this chapter. But it can be
Figure 13. Comparison of the Cavity Model results in dB versus experimental data [3] for the H-plane radiation pattern of a ring antenna working in the TM$_{21}$ mode at 5.7 GHz.

Figure 14. Comparison of the Cavity Model results in dB versus experimental data [3] for the H-plane radiation pattern of a ring antenna working in the TM$_{41}$ mode at 5.7 GHz.
Figure 15. Comparison of theoretical and experimental data [16] in dB for the H-plane radiation pattern of a ring antenna working in the TM_{11} mode at 2.25 Ghz.

Figure 16. Comparison of theoretical and experimental data [16] in dB for the E-plane radiation pattern of a ring antenna working in the TM_{11} mode at 2.25 Ghz.
Figure 17. Comparison of theoretical and experimental data [14] in dB for the H-plane radiation pattern of a ring antenna working in the TM$_{12}$ mode at 8.46 Ghz.

Figure 18. Comparison of theoretical and experimental data [14] in dB for the E-plane radiation pattern of a ring antenna working in the TM$_{12}$ mode at 8.46 Ghz.
advanced that the TM$_{12}$ modes are the best for antenna applications, with better impedance bandwidth than its circular or rectangular counterparts, good gain, radiation in the broadside direction and excellent efficiency but with the drawback of an increased size compared to a circular patch.

Three dimensional plots of the TM$_{12}$ and TM$_{21}$ modes can be seen on Figures 19 and 20. Figure 19 shows very clearly the non-broadside nature of the TM$_{21}$ mode with its null at a zero degree elevation angle. Figure 20 shows the TM$_{12}$ mode, with its main lobe in the broadside direction and two sidelobes at different angles.
Figure 19. Three dimensional plot of the radiation pattern of a microstrip ring antenna working in the TM$_{21}$ mode at 5.63 Ghz (from Figure 13).
Radiation Pattern of the TM12 mode

Figure 20. Three dimensional plot of the radiation pattern of a microstrip ring antenna working in the TM$_{12}$ mode at 3.77 Ghz (from Figure 3).
Discussion on Bandwidth, Directivity and Efficiency

Once the power expressions (conductor, dielectric and surface wave losses, as well as the radiated power) and the electric and magnetic stored energies inside the cavity have been derived, a quality factor for the microstrip ring resonator can be obtained.

The definition of the Quality Factor is [11]:

\[
Q_x = 2\pi f \frac{W_T}{P_x}
\]

(3.15)

where \( P_x \) represents either the conductor, dielectric or surface wave power losses, or the radiated power, and \( W_T \) represents the total stored energy in the resonant cavity (electric plus magnetic stored energy). The quality factor of the resonant cavity for each one of the power expressions involved is obtained by evaluating equation (3.15) at the resonant frequency.

Assuming that the microstrip resonator resembles the behavior of a RLC circuit, a 'total' quality factor can be obtained as [18]:

\[
\frac{1}{Q_T} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_{sw}} + \frac{1}{Q_{rad}}
\]

(3.16)

where \( Q_c, Q_d, Q_{sw} \) and \( Q_{rad} \) represent the quality factors due to the conductor, dielectric, surface wave and radiated power expressions. In most microstrip antennas, since the radiated power will be dominant over the conductor, dielectric and surface wave losses, \( Q_{rad} \) will also be usually the dominant one, with the smaller value among the three. Since for thin substrates ( \( h << \lambda \) ) the surface wave power is very small, \( Q_{sw} \) becomes
very large and thus the term in the total quality factor expression involving $Q_{sw}$ can be neglected. And even the conductor and dielectric losses are smaller than the radiation power, so in some cases, $Q_T \approx Q_{rad}$. However, this is not always the case for microstrip ring structures. Some modes, like the TM$_{11}$ mode have a very narrow bandwidth, resulting in a very high quality factor; this means that those modes have a low radiation efficiency (i.e., large losses compared to the radiated power) and are better for ring resonator applications. Therefore, although $Q_T$ will be closer to $Q_{rad}$, it will be smaller. Of course, their bandwidth can always be increased (and thus the quality factor lowered) by increasing the substrate height (with the corresponding limit imposed by the generation of surface waves power), but compared to other modes like the TM$_{12}$ mode they don't have such a nice appeal for antenna applications [16],[24].

Using the definitions of the total energy at resonance (using the fact that the electric and magnetic energies are the same at resonance, so the total energy can be obtained as twice the electric or twice the magnetic energy) and the conductor and dielectric power losses at resonance, the following expressions can be easily obtained:

\[
P_d(f_r) = 2 \cdot \pi \cdot f_r \cdot W \cdot T \cdot \tan \delta
\]

\[
Q_d = \frac{1}{\tan \delta}
\]  

\[
P_c(f_r) = 2 \sqrt{\frac{\pi \cdot f_r \cdot W \cdot T}{\mu_0 \cdot \sigma_c \cdot h}}
\]

\[
Q_c = h \sqrt{\pi \cdot f_r \cdot \mu_0 \cdot \sigma_c}
\]
The bandwidth can be defined in several ways. In this case, the definition of impedance bandwidth has been taken. The bandwidth is defined here as that band of frequencies where the input SWR (Standing Wave Ratio) is less than a specified value, usually 2:1, assuming that a unity SWR is obtained at the design frequency. The bandwidth may then be expressed in terms of $Q_T$ and the maximum allowable VSWR as follows [10]:

$$BW = \frac{VSWR_{\text{max}} - 1}{Q_T \sqrt{VSWR_{\text{max}}}} \quad (3.21)$$

The bandwidth for circularly polarized microstrip patches must be defined in terms of the band of frequencies over which the axial ratio of the radiated energy is within certain limits, typically 3 dB. This bandwidth is usually much less than the previously defined impedance bandwidth. It has been shown that the bandwidth of single feed circularly polarized antennas is very limited [10]. For the purposes of this thesis, the impedance bandwidth will be used as a default.

The antenna efficiency can be obtained as the ratio of the radiated power to the input power (represented by $P_T$, which can be obtained by adding the losses to the radiation power). In equation form [29]:

$$\eta = \frac{P_{\text{rad}}}{P_T} \quad (3.22)$$

From the above equation, doing a simple substitution of the definitions of the quality factors for the radiated and total power expressions, the following definition is obtained:
Using the above equations, the antenna loss can be defined as [29]:

\[
\text{Antenna Loss} = 10 \cdot \log \left( \frac{1}{\eta} \right) \text{ dB}
\]  

(3.24)

The calculation of the directivity involves the maximum radiation intensity and the radiated power, as follows:

\[
D = 4 \cdot \pi \cdot \frac{U_{\text{max}}}{P_{\text{rad}}}
\]  

(3.25)

For non-broadside modes, the azimuth and elevation coordinates of the direction of the maximum radiation intensity can be easily obtained with the assistance of the three dimensional plot of the radiation pattern.

The gain is obtained using equations (3.19) and (3.21) as:

\[
\text{Gain} = \eta \cdot \text{Directivity}
\]  

(3.26)

and it can be expressed in decibels as 10·log(Gain).

The radiation characteristics of single microstrip ring patches have already been investigated. Sultan [29],[30] studied TM_{11} modes for an outer-edge fed element and concluded that different structures can have the same performance at the critical frequency which satisfies \( h \cdot k_{o} = \text{constant} \), where \( k_{o} \) is the wave number in free space. So, for different substrate heights (and even different ring widths) at their corresponding frequencies to make constant the height - wave number relationship, the quality factor, bandwidth, gain
and efficiency can be the same. He also showed that at high frequencies, narrower ring structures have smaller quality factor and thus larger bandwidth when compared to wider ring structures operating at the same frequency; and at lower frequencies, the narrower ring antennas have larger quality factor and thus smaller bandwidth. Of course, the bandwidth depends on the substrate height; increasing the substrate thickness the bandwidth is increased (and thus the quality factor reduced). Sultan [29] made plots of bandwidth versus frequency at different substrate heights and different ring widths and of the gain versus frequency to support the above arguments. It was concluded that the concept of constant gain-bandwidth product is followed.
The solution of the input impedance of a microstrip antenna using a Cavity Model approximation is a good challenge. It has been shown that the Green’s Function of an ideal cavity will lead to infinite electromagnetic fields at the resonant frequency; therefore, the predicted input impedance would be infinite also. An improved cavity model must consider the effect of the loss tangent of the dielectric material on the resonant frequency, by using the wave number defined as \( k_d = 2\pi f \sqrt{\mu_\epsilon (1-j \tan \delta)} \).

For an ideal cavity, the electric field inside the cavity is purely imaginary as it can be deducted from equation (2.28). This implies that the input impedance is purely reactive and eventually infinite at the resonant frequency because of the reasons stated above. In reality, because of the radiation, conductor, dielectric and surface wave losses, the magnitude of the input impedance is finite but can be assumed to be much larger than the other modes at the specific resonant frequency. Thus, at resonance, the input impedance is a real number.

By including a complex wave number in the calculation of the electric field, what happens is that the electric field is no longer a purely imaginary number [31]. And also, the constants that canceled out at the resonant frequency in the Green’s Function of the ideal cavity \((A_m \text{ and } B_m)\), are no longer equal at resonance. The difference of the real parts of \( A_m \text{ and } B_m \) is still zero at resonance; however, there’s an imaginary part that appears due to the loss tangent, yielding a real part in the electric field and limiting its magnitude at the resonant frequency.
With the above heuristic arguments, it is pretended to introduce the importance that the concept of an effective loss tangent plays in an improved Cavity Model in order to calculate the input impedance. Without this concept it would be impossible to continue. As a matter of fact, the input impedance depends strongly on the effective loss tangent. Increasing the loss tangent of the dielectric material reduces the input impedance. So, because of the sensitivity of the input impedance on the loss tangent, it is very important to specify its value as accurately as possible in order to have a good design tool.

The input impedance depends on the kind of feeding technique that is being used. The simplest analysis corresponds to the case of a coaxial probe feed or a microstrip line feed. Both methods are modeled similarly and are the object of this study for the microstrip ring antenna.

The input impedance can be obtained from the supplied complex power. Using the conservation of energy equation, taking volume integrals and applying the divergence theorem [11] it can be shown that the complex power supplied by the feed is:

\[ P_s(f) = P_{rad}(f) + P_c(f) + P_d(f) + j \cdot 2 \cdot \omega \cdot \left( W_h(f) - W_e(f) \right) \]  

(3.27)

In other words, the supplied power must be equal to the power radiated and lost in the antenna plus the reactive power resulting from the exchange of magnetic and electric energies.

Using circuit theory it can be shown that:

\[ Z_{in} = \frac{1}{2} \cdot \frac{\frac{1}{V \cdot V}}{P_{rad}(f) + P_c(f) + P_d(f) - j \cdot 2 \cdot \omega \cdot \left( W_h(f) - W_e(f) \right)} \]  

(3.28)
The voltage applied between the patch and the ground plane at the feed connection point can be obtained from the following equation [4]:

\[
V = \int_0^h E_z(\rho_o, \phi_o) \, dz = -h \cdot E_z(\rho_o, \phi_o)
\]  

(3.29)

where the assumption of a uniform electric field along \( z \) has been made.

For the case of a microstrip feed line, it has been shown [4] that the impressed electric current is zero, and then the input admittance can be expressed as:

\[
\frac{V_{in}}{I_{in}} = \frac{1}{W} \int_0^W \frac{H(\rho_1, \phi_1) \cdot V(L)}{|V_{in}|} \, dL
\]

(3.30)

where \( W \) is the width of the microstrip conductor, \( V_{in} \) is the voltage between the patch and the ground plane at the feed line connection point, \( L \) denotes the line length along the patch edge and \((\rho_1, \phi_1)\) are the coordinates of the microstrip line feed. For narrow strips, \( V(L) \) is essentially \( V_{in} \). And if \( h \) is small, the input current is [4]:

\[
I_{in} = W \cdot H(\rho_1, \phi_1)
\]

(3.31)

and the input admittance becomes:

\[
Y_{in} = \frac{I_{in}}{V_{in}}
\]

(3.32)

Equation (3.28) can be rewritten in a more familiar form, as:
where the total power losses are expressed as:

\[ P_{T(f)} = P_{rad}(f) + P_{c}(f) + P_{d}(f) \]  \hspace{1cm} (3.34)

Rearranging equation (3.30) yields:

\[
Z_{in}(f) = \frac{1}{2 \cdot P_{T(f)}} \left( \frac{1}{(|V(f)|)^2} - j \cdot 4 \cdot \frac{\omega}{(|V(f)|)^2} \cdot (W_{h(f)} - W_{e}(f)) \right)
\]  \hspace{1cm} (3.35)

The form of the last equation is very similar to the form of the input impedance of a parallel RLC circuit:

\[
Z_{in} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C + \frac{1}{j \cdot \omega \cdot L}}
\]  \hspace{1cm} (3.36)

The difference between equations (3.35) and (3.36) is that all the variables in the first one are frequency dependent, while R, C, L are constants over the bandwidth. However, it would be a good point of comparison to make (3.35) look like (3.33), although preserving the frequency dependence of (3.35). So, a frequency-dependent capacitance can be found by equating:

\[
j \cdot \omega \cdot C(f) = \frac{j \cdot 4 \cdot \omega \cdot W_{e}(f)}{(|V(f)|)^2}
\]  \hspace{1cm} (3.37)

and a frequency-dependent inductance can be found by equating:
\[
\frac{1}{j \cdot \omega \cdot L(f)} = \frac{4 \cdot \omega \cdot W_{h}(f)}{j \cdot (|V(f)|)^2}
\] (3.38)

Therefore, from equations (3.37) and (3.38) it is obtained:

\[
C(f) = 4 \cdot \frac{W_{e}(f)}{(|V(f)|)^2}
\] (3.39)

\[
L(f) = \frac{(|V(f)|)^2}{4 \cdot \omega^2 \cdot W_{h}(f)}
\] (3.40)

and from equation (3.35):

\[
R(f) = \frac{(|V(f)|)^2}{2 \cdot P \cdot T(f)}
\] (3.41)

The values at resonance of (3.39) through (3.41) is very important, and then it is obtained in the following expressions [4]:

\[
R = R(f_r) = \frac{(|V(f_r)|)^2}{2 \cdot P \cdot T(f_r)}
\] (3.42)

\[
C = C(f_r) = 4 \cdot \frac{W_{e}(f_r)}{(|V(f_r)|)^2}
\] (3.43)

\[
L = L(f_r) = \frac{(|V(f_r)|)^2}{4 \cdot \omega^2 \cdot W_{h}(f_r)}
\] (3.44)

Using the fact that at resonance the stored electric and magnetic energies are the same and with the aid of the definition of the total quality factor:
\[ W_{e}(f_r) = W_{h}(f_r) = \frac{1}{2} W_{T}(f_r) \]  

\[ Q_{T} = \omega \cdot r \cdot \frac{W_{T}(f_r)}{P_{T}(f_r)} \]  

thus the following expressions are obtained for the 'inductance' and 'capacitance' at resonance:

\[ C = \frac{Q_{T}}{2 \cdot \pi \cdot f_{r} \cdot R} \]  

\[ L = \frac{R}{2 \cdot \pi \cdot f_{r} \cdot Q_{T}} \]

Assuming that \( R, C, \) and \( L \) remain constant through the bandwidth, then equations (3.42), (3.47) and (3.48) can be used along with (3.36) to obtain an approximate solution of the input impedance, when the patch is being fed by a coaxial probe. More accurate results would be expected using equations (3.28) or (3.33), where all the variables are frequency-dependent.

At this point is when the importance of the effective loss tangent appears. In order to improve the Cavity Model to obtain the electric field and the resonant frequency of the microstrip ring cavity, it was necessary to make corrections to the dimensions to account for the fringe fields, and to the relative permittivity, to account for dispersion effects. In the same way, the loss tangent must be corrected to predict more accurately the losses that occur in the dielectric material [4],[18].
So, in order to obtain accurate results in the calculation of the input impedance, equations (3.33) or (3.36) must be used with the value of the effective loss tangent. This value is obtained by observing equation (3.18), which indicates that the quality factor of the dielectric losses is inversely proportional to the loss tangent of the dielectric material.

Using a similar approach, an effective loss tangent can be derived taking into consideration all the losses (conductor, dielectric and surface waves) and defining it as the inverse of the total quality factor (excluding the radiated power). Thus, the expression for the effective loss tangent becomes [4], [18]:

$$\delta_{\text{eff}} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_{sw}}$$ (3.49)

Summarizing, the input impedance of a microstrip ring antenna fed by a coaxial probe using the Cavity Model can be obtained by obtaining first the effective loss tangent of the dielectric substrate, using equation (3.49) combined with (3.15) to obtain $Q_{sw}$, $Q_c$, and $Q_d$. The fields inside the cavity, radiated fields, power losses and radiated power must be recalculated with the new value of $\delta_{\text{eff}}$ obtained in the previous step (as well as any other variable that is being studied like quality factors, bandwidth, etc.).

Excellent results have been obtained using this technique and are shown in Figures 21 through 24 [32],[33], which show a comparison of the theoretical data resulting with the Improved Cavity Model and experimental data.

Some heuristic conclusions can be obtained from the plots:
Figure 21. Comparison of the theoretical and experimental [32] input impedance of a ring antenna working in the TM_{11} mode with a = 2.5 cm, b = 5 cm, \( \varepsilon_r = 2.32, \) \( h=0.159 \) cm and \( \tan \delta = 0.0012 \) fed with a coaxial probe at 2.7 cm from the center ( \( \tan \delta_{eff} = 0.0026 \) ).
Figure 22. Comparison of the theoretical and experimental [32] input impedance of a ring antenna working in the TM$_{21}$ mode with $a = 2.5$ cm, $b = 5$ cm, $\varepsilon_r = 2.32$, $h = 0.159$ cm and $\tan \delta = 0.0012$ fed with a coaxial probe at 2.7 cm from the center ( $\tan \delta_{\text{eff}} = 0.0013$ ).
Figure 23. Comparison of the theoretical and experimental [32] input impedance of a ring antenna working in the TM$_{31}$ mode with $a = 2.5$ cm, $b = 5$ cm, $\varepsilon_r = 2.32$, $h=0.159$ cm and $\tan\delta = 0.0012$ fed with a coaxial probe at 2.7 cm from the center ( $\tan\delta_{\text{eff}} = 0.00286$ ).
Figure 24. Comparison of the theoretical and experimental [33] input impedance of a ring antenna working in the TM$_{12}$ mode with $a = 3$ cm, $b = 6$ cm, $\varepsilon_r = 2.47$, $h=0.159$ cm and $\tan\delta = 0.0005$ fed with a coaxial probe at 3.45 cm from the center ( $\tan\delta_{\text{eff}} = 0.0005$ ).
1) The input resistance is minimum if the ring is fed at the inner edge and maximum if the ring is fed at the outer edge; in agreement with the behavior of the electric field.

2) For the TM 11 mode, the input impedance doesn’t vary too much with the feed position; this is also in agreement with the electric field inside the cavity, which is almost constant along the ring for this specific mode.

3) The TM 11 mode usually has a high input impedance at resonance (compared to 50 ohms, which is the standard), with typical values around 500 ohms.

4) The TM 12 mode has a typical input impedance of 50 to 60 ohms at resonance.

5) For the TM 21 mode, it varies around 250 ohms.

6) And for the TM 31 mode, it is typically around 80 ohms.

The conclusions stated above do not pretend to be exhaustive; it is only desired to give a good sense of direction in the design procedure and the choice of the right mode of excitation for a microstrip ring antenna.

Conclusions

With all the concepts developed in this chapter, some conclusions can be stated. It is apparent that the TM 12 mode is the best for antenna applications. It also seems that for the case of one radial variation (n = 1) the input impedance decreases as the azimuthal variations (m) are increased (m = 1,2,3...). A qualitative discussion is given next. The ideas given about the input impedance refer to the case of a coaxial probe feed.
The TM$_{11}$ mode seems to be a mode that can offer a narrow bandwidth and a high input impedance at resonance with radiation in the broadside direction. Its size is usually smaller than that of a circular or rectangular patch with the same resonant frequency. Although the bandwidth can be improved with an increase in the substrate thickness, in general this mode is not good for antenna applications that require wider bandwidth and input impedances close to 50 ohms to make less complex matching networks.

The TM$_{12}$ mode has a radiation pattern with a main lobe in the broadside direction and two small sidelobes in different directions. The input impedance is close to 50 Ohms and the typical bandwidth is around 4 %, which is very good for a single patch. The typical gain is about 7 dB. So, this is an excellent mode for an antenna. However, there’s one disadvantage: the size is bigger than a corresponding circular patch with the same resonant frequency; actually, its size can be about twice as big as a circular patch; this can be very important for applications that require reduced sizes.

The TM$_{21}$ and TM$_{41}$ modes have been studied [3] and they can be considered as a compromise between the small size obtained with the TM$_{11}$ mode and the good antenna performance of the TM$_{12}$ mode. However, they do not radiate in the broadside direction, a feature that can make them useless for some applications, but very useful for others, like satellite mobile communications. The TM$_{31}$ mode is also a non-broadside mode with similar features to the TM$_{11}$ mode although its typical input impedance is about 80 ohms.
CHAPTER 4

THEORETICAL STUDY OF STACKED MICROSTRIP RING ANTENNAS

Extension of the Cavity Model

In the following pages, the Cavity Model developed in the previous chapters for a single annular microstrip antenna will be extended and tested on stacked microstrip ring antennas. It has been attempted to obtain the Green’s Function of the stacked structure, which is shown in Figure 27. However, there is a difficulty caused by the presence of the lower patch that arises when an attempt is made to apply the boundary conditions between the substrate and the superstrate. Moreover, the cavity model assumes no ‘z’ variations of the electric field inside the cavity, making it impossible to use any boundary conditions that depend on z. So, in order to analyze the stacked geometry using a cavity model, it has to be considered as two coupled cavities [32].

The motivation of the design of a two layer structure is to create two closely tuned resonances that together can cover a frequency band much broader than a single resonance element, with similar radiation characteristics and gain. The dual-frequency behavior of the stacked antenna allows good performance over a wider band of frequencies. In order to overcome all the difficulties found to obtain the Green’s Function of this kind of structure,
a big effort has been placed in the solution of the resonant frequencies of the lower and upper patches. The idea is to accurately predict the two resonances of the antenna; next, the electric fields generated inside the upper and lower cavities are added together, using the correct effective values of the dielectric constant, inner and outer radii, loss tangent and resonant frequency. Thus, the effect of coupling between the upper and lower patch is included in the parameters used.

The Green's Function obtained for the single microstrip ring resonator in equation (2.27) will be used for the stacked geometry. A Green's Function is obtained for the lower antenna and another for the upper one; the difference lies in the effective dielectric constants and the patch dimensions stated above. The resulting total electric field inside the cavities can be calculated by adding together the contributions of the field in the superstrate and in the substrate. This assumption can be supported by the fact that the cavity model assumes no 'z' variations of the electric field because of the uniform current fed by the coaxial probe and the small height of the dielectric material compared to the wavelength.

In general, one expression will be obtained for the upper cavity and one for the lower cavity, for the following variables: Green's Functions, electric and magnetic fields inside the cavities, conductor and dielectric losses. However, the radiated electric and magnetic fields, radiated power and input impedance must be concentrated in one expression to account for the effect of the two magnetic current sources. A fairly good approximation
has been obtained using this simple procedure. Although it is not very accurate, the method gives a good physical insight and sense to each one of the variables.

**Resonant Frequency**

The resonant frequencies of the stacked microstrip ring resonator can be deducted using an approach similar to that of single microstrip rings. The problem has to be separated into two different parts: the lower and the upper cavity.

**Lower Patch Resonant Frequency**

The lower patch resonant frequency can be obtained by using the same approach that has been followed throughout this study: extend the work already done for microstrip lines and use it for a microstrip ring. The effect of the curvature of the ring has been shown to be minimal [26].

The analysis of the bottom cavity can be done by thinking of it as a microstrip antenna covered by a dielectric material, neglecting the effects of the upper patch by assuming that the fields are concentrated in the region between the lower patch and the ground plane.

When a microstrip is covered by a dielectric material, its characteristics like impedance, phase velocity, losses, change with the dielectric constant, loss tangent and thickness of the dielectric material. Several methods for the solution of a two dimensional boundary value problem involving two different media are known, for example, the conformal mapping method, the integral equation method, the relaxation method and the
variational method. The analytical treatment of multiple boundaries is much easier in the variational method than in any other numerical method [34],[35].

In the variational method, an approximate charge distribution of the strip conductor is assumed and the resulting formulas for capacitance can be expressed in closed form [34],[35]. The effective dielectric constant of a TEM transmission line is:

\[ \varepsilon_{\text{eff}} = \frac{C_d}{C_0} \quad (4.1) \]

where \( C_d \) and \( C_0 \) are the capacitances of the transmission line structure with and without dielectric, respectively, and \( \varepsilon_{\text{eff}} \) is the effective dielectric constant that considers the effect of the fringing fields in the substrate, the superstrate and the free space.

A quasi-TEM mode is assumed. Then, the expressions for the capacitance are obtained using the variational method. The variational expression for the line capacitance in the \( \beta \) coordinate system is [34],[35]:

\[ \frac{1}{C} = \frac{1}{2 \pi Q^2} \int_{-\infty}^{\infty} f(\beta) \cdot \phi(\beta, h) \, d\beta \quad (4.2) \]

where \( \phi \) and \( f \) are the Fourier Transforms of the potential and charge distribution functions respectively, \( \beta \) is the Fourier transform variable, \( h \) is the substrate thickness, \( d \) is the dielectric cover thickness (superstrate) and \( Q \) denotes the total charge on the strip conductor and is given by [34],[35]:

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The Fourier Transform of the charge distribution is defined as:

\[ Q = \int_{\infty}^{\infty} f(x) \, dx \quad (4.3) \]

\[ f(\beta) = \int_{\infty}^{\infty} f(x) \cdot e^{j \cdot \beta \cdot x} \, dx \quad (4.4) \]

where \( f(x) \) represents the charge distribution on the strip conductor. The charge distribution has been assumed as follows [34],[35]:

\[ f(x) = 1 + \left( \frac{2 \cdot x}{W} \right)^{3} \quad \text{for} \quad -\frac{W}{2} \leq x \leq \frac{W}{2} \]

\[ f(x) = 0 \quad \text{elsewhere} \quad (4.5) \]

Using equations (4.3) through (4.5) it can be shown that [34],[35]:

\[ \frac{\overline{f(\beta)}}{Q} = 1.6 \cdot \sin \left( \frac{\beta \cdot W}{2} \right) + \frac{2.4}{\left( \frac{\beta \cdot W}{2} \right)^{2}} \left[ \cos \left( \frac{\beta \cdot W}{2} \right) - 2 \cdot \frac{\sin \left( \frac{\beta \cdot W}{4} \right)}{\left( \frac{\beta \cdot W}{4} \right)^{2}} \right] \quad (4.6) \]

For the case of a strip conductor, the variational expression for the line capacitance can be expressed as (after including the potential distribution function) [36]:

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where $\varepsilon_{r1}$, $\varepsilon_{r2}$, are the relative permittivities of the substrate and superstrate respectively, $\varepsilon_0$ is the free space permittivity and corresponds to $\varepsilon_{r3} = 1$. Therefore, using equation (4.6) into (3.68) results in the integral that will be used to obtain the capacitance per unit length that will be further used to obtain the effective dielectric constant of (4.1).

The expressions shown above were developed for the case of a microstrip line covered by one finite dielectric cover and one infinite dielectric cover (in this case, the air, which is taken into account by making $\varepsilon_{r3} = 1$).

These results have been extended and used successfully to predict the effective dielectric constant of a microstrip ring. Equation (4.7) involves an integral with an infinite upper limit; in Figure 25, it is shown that it is not necessary because the integrand decays rapidly. Figure 26 shows that with an upper limit of 10 in the integral of (4.7) very accurate results are obtained.

The design procedure for the lower patch resonance is the following, where a normalization factor of $1/(\pi \varepsilon_0 Q^2)$ has been used throughout all the procedure:

1) Use equation (4.7) to obtain $C_a$ normalized.
2) Assume that no dielectric materials are present (i.e., just air) and recalculate equation (4.7) to obtain \( C_0 \) normalized.

3) Obtain \( \varepsilon_{\text{eff}} \) from equation (4.1) and the values obtained in steps 1 and 2.

4) Correct the dimensions of the lower ring, using equation (3.4) through (3.7) but making the substitution of \( \varepsilon_r \) by \( \varepsilon_{\text{eff}} \).

5) Use equation (3.3), the characteristic equation of a microstrip ring antenna, to calculate the wave number.

6) Calculate the resonant frequency from:

\[
 f_r = \frac{k_d}{2 \cdot \pi \cdot \sqrt{\mu \cdot \varepsilon_{\text{eff}}}} \quad (4.8)
\]

Excellent results have been achieved in predicting the lower patch resonant frequency using the six-steps procedure outlined, with an estimated error within 0.5 %.

It is found that the main effect of the dielectric layer is to reduce the resonant frequency when compared to the same microstrip ring without a dielectric cover. As it will be seen later, it also tends to decrease the resonant resistance, accompanied by an increase in the impedance bandwidth.

**Upper Patch Resonant Frequency**

A similar approach can be followed to obtain the resonance frequency of the upper patch. However, the upper patch has to be analyzed like an uncovered microstrip antenna
Figure 25. Plot of the integrand of equation (4.7), for $a_1 = a_2 = 2.5 \text{ cm}$, $b_1 = b_2 = 5 \text{ cm}$, $d = h = 0.159 \text{ cm}$, $\varepsilon_{r1} = \varepsilon_{r2} = 2.32$.

Figure 26. Plot of the normalized inverse line capacitance as a function of the upper limit of the integral in equation (4.7). It is shown that the integral does not need to be evaluated to infinity because in this case, convergence is achieved for $x = 10$. 
Unfortunately, the lower patch does not offer a large ground plane and this will introduce errors in the model.

The effects of coupling are very important, especially in the interface region between the substrate and the superstrate. Their main effect is to change the effective dimensions of the upper ring and thus, increase the resonant frequency compared to the value obtained for the same microstrip ring above a ground plane. Since the fields are concentrated around the edges of the rings, then there is a strong interaction of the fringing fields of the upper and lower patch. Thus, the effective inner and outer radii will be smaller that the effective radii of a microstrip ring above a ground plane. The effect of the reduction of the effective dimensions is to increase the resonant frequency, as expected.

Based on the above discussion, an empirical procedure to determine the resonant frequency of the upper patch has been derived, as follows:

1) Obtain $\varepsilon_{ef}$ from equations (3.8) through (3.14). This is the effective dielectric constant of a single microstrip ring antenna including dispersion effects.

2) Correct the dimensions of the inner and outer radii as follows:

$$a_{\text{eff}} = a \cdot \frac{1 - \frac{2 \cdot h}{\pi \cdot a \cdot \varepsilon_{ef}}}{m \cdot \Delta C_a}$$

$$b_{\text{eff}} = b \cdot \frac{1 - \frac{2 \cdot h}{\pi \cdot b \cdot \varepsilon_{ef}}}{\Delta C_b}$$

where $\Delta C_a$ and $\Delta C_b$ are obtained from equations (3.4) and (3.6) by making the substitution of the effective dielectric constant into them. The only difference between equation (4.9) -
(4.10) and (3.5), (3.7) is the factor m (the number of azimuthal variations of the resonant mode) for $\Delta C_a$ in (4.9) and the minus sign in (4.10) to reduce instead of increase the outer radius dimension.

3) Use equation (3.3), the characteristic equation of a microstrip ring antenna, to calculate the wave number.

4) Calculate the resonance frequency using equation (4.8) but substituting the value of the effective dielectric constant obtained in step 1. The results agree within 0.7% compared to experimental data.

An alternative approach that also yielded good results was to modify step 2 and equations (4.9) and (4.10) as follows:

\[
\begin{align*}
\alpha_{\text{eff}} &= \frac{\sqrt{1 - \frac{2h}{\pi \alpha d \varepsilon}} \cdot m \cdot \Delta C_a}{(4.11)} \\
\beta_{\text{eff}} &= \frac{\sqrt{1 - \frac{2h}{\pi \beta d \varepsilon}} \cdot \Delta C_b}{(4.12)}
\end{align*}
\]

where $\Delta C_a$ and $\Delta C_b$ are obtained from equations (3.4) and (3.6) by making the substitution of $\varepsilon_{eo}$ into them. The resonant frequency is also obtained using $\varepsilon_{eo}$. Excellent agreement within 1.7% error was achieved.

In Table 3 some of the results obtained using these techniques are summarized and compared with experimental data for the TM_{11}, TM_{21} and TM_{31} modes.
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<tbody>
<tr>
<td>TM$_{11}$</td>
<td>0.864</td>
<td>0.919</td>
<td>0.8693</td>
<td>0.92008</td>
<td>0.9247</td>
<td>0.6</td>
<td>0.1</td>
<td>0.6</td>
</tr>
<tr>
<td>TM$_{21}$</td>
<td>1.704</td>
<td>1.858</td>
<td>1.7086</td>
<td>1.8545</td>
<td>1.869</td>
<td>0.3</td>
<td>-0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>TM$_{31}$</td>
<td>2.49</td>
<td>2.722</td>
<td>2.4994</td>
<td>2.7397</td>
<td>2.7677</td>
<td>0.4</td>
<td>0.7</td>
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Table 3. Resonant Frequencies for a Stacked Microstrip Ring Antenna.

Comparison of experimental data [32] with calculated values obtained using the modified cavity model to account for the coupling between the upper and lower cavity; $a_1 = a_2 = 2.5$ cm, $b_1 = b_2 = 5$ cm, $\varepsilon_r = 2.32$ and $h_1 = h_2 = 0.159$ cm. (Note 1: the resonant frequency was obtained using the effective dielectric constant to account for dispersion effects. Note 2: the resonant frequency was obtained using $\varepsilon_\infty$).
Radiated Fields

The electric and magnetic fields radiated by a stacked microstrip antenna can be predicted using the cavity model by adding the contribution of the equivalent magnetic currents of the upper and lower patches.

So, although each cavity is analyzed separately to obtain its fields and parameters, the radiated fields are combined into one single equation that takes into account the contribution of each cavity to the radiated power.

It has been shown [38] using a Hankel Transform Domain Analysis that the radiation patterns for $E_\theta$ and $E_\phi$ are almost exactly the same as those of a single annular-ring patch antenna. This statement can be supported using the cavity model outlined by observing that at either the upper or lower resonance, the contribution to the radiated field of the non-resonant cavity (i.e., the one that is not resonant at the frequency that is being studied) is minimal, because its fields inside the cavity are very small.

With the above ideas in mind, the next steps are followed in order to obtain the radiation pattern of a stacked microstrip ring antenna:

1) For the lower patch, obtain the Green’s Function given by (2.27). The fields inside the cavity are given by equations (2.40) and (2.41). The conductor and dielectric power losses are obtained using equations (2.53) and (2.58) respectively; the magnetic and electric energies stored are given by (2.59) and (2.60). The equivalent magnetic current source is obtained using (2.64). In other words, the cavity is analyzed as if it was standing alone; however, the correct values of effective dielectric constant, resonant frequency and
Figure 27. Side view of a stacked microstrip ring antenna fed by a coaxial probe feed through a hole on the lower patch.

Figure 28. Comparison of theoretical and experimental [38] data for the E-plane radiation pattern of a stacked microstrip ring antenna working in the TM$_{11}$ mode at 0.864 GHz.
effective dimensions and loss tangent have to be used according to the procedure given in the previous section.

2) Repeat step 1 but for the upper patch.

3) From steps 1 and 2, a set of radiated electric and magnetic fields is obtained for the lower patch and another for the upper patch, as given by equation (2.65). The $\theta$ and $\phi$ components are obtained from (2.70) through (2.77). However, since only one equation is desired, the final expression for the radiated fields is obtained by adding the corresponding components to yield a total field. Therefore:

$$
\begin{align*}
E_r &= 0 \\
E_\theta &= E_{\theta 1} + E_{\theta 2} \\
E_\phi &= E_{\phi 1} + E_{\phi 2} \\
H_r &= 0 \\
H_\theta &= H_{\theta 1} + H_{\theta 2} \\
H_\phi &= H_{\phi 1} + H_{\phi 2}
\end{align*}
$$

(4.13)

where the subindex 1 corresponds to the lower patch and the subindex 2 to the upper patch.

The results predicted using (4.13) are compared to the data obtained by Zhibo Fan and Kai-Fong Lee [38] in their study using Hankel Transform Domain Analysis. Figures 28, 29 and 30 show the comparison for the E-plane pattern for the TM$_{11}$, TM$_{21}$, and TM$_{31}$ modes. Excellent agreement has been achieved. It is also concluded that the fields are very similar to those of the single ring microstrip antenna. Thus, these fields can be used to obtain the parameters necessary to calculate the input impedance and that will become the topic of the next section.
Figure 29. Comparison of theoretical and experimental [38] data for the H-plane radiation pattern of a stacked microstrip ring antenna working in the TM$_{21}$ mode at 1.704 GHz.

Figure 30. Comparison of theoretical and experimental [38] data for the E-plane radiation pattern of a stacked microstrip ring antenna working in the TM$_{31}$ mode at 2.489 GHz.
**Input Impedance**

The input impedance of the stacked configuration cannot be accurately predicted using the cavity model, because of all the coupling that exists between the upper and lower patch, that affect the magnetic wall boundaries and the fields inside the upper and lower resonant cavities. However, using the model developed in the previous sections, an approximation can be obtained. As a matter of fact, a very good idea about the behavior of the impedance and reactance can be predicted, which can be useful in the direction that one must follow to design a good radiator.

The stacked antenna that is going to be studied is shown in Figure 27. The inner conductor of the coaxial probe passes through a clearance hole in the lower ring and is electrically connected to the upper ring. The lower ring is therefore coupled only through the fringing field. The two rings must be aligned accurately and tightly clamped together. The overall structure can be viewed as two coupled ring cavities [32], [39]-[43].

From (3.74), the radiation intensity and radiated power can be calculated using equations (2.78) and (2.79). Equations (2.74), (2.75) and (2.80) can be modified to obtain the final expression for the radiated power:

\[
P_{\text{rad}} = \left( \frac{k_0(f)}{8 \cdot \eta_0} \right)^2 \cdot \pi \cdot \int_0^{\pi} \left[ (A_{\theta T})^2 + (B_{\theta T})^2 \cdot \cos^2(\theta)^2 \right] \cdot \sin(\theta) \, d\theta
\]

\[A_{\theta T} = A_1(\theta) \cdot h_1 + A_2(\theta) \cdot h_2\]

\[B_{\theta T} = B_1(\theta) \cdot h_1 + B_2(\theta) \cdot h_2\]
Once again, like in the equations that follow, the subindex 1 has been used for the lower cavity and the subindex 2 for the upper cavity.

The total conductor and dielectric losses, and total electric and magnetic energies stored in the antenna are:

\[ P_c(f) = P_{c1}(f) + P_{c2}(f) \]  
\[ P_d(f) = P_{d1}(f) + P_{d2}(f) \]  
\[ W_e(f) = W_{e1}(f) + W_{e2}(f) \]  
\[ W_h(f) = W_{h1}(f) + W_{h2}(f) \]  

(4.15)  
(4.16)  
(4.17)  
(4.18)

From equation (3.29), the voltage applied between the upper and the lower patch (\( V_2 \)) and between the lower patch and the ground plane (\( V_1 \)) at the feed connection point can be obtained as:

\[ V_1 = h_1 \cdot E_{z1}(\rho_o, \phi_o) \]  
\[ V_2 = h_2 \cdot E_{z2}(\rho_o, \phi_o) \]  

(4.19)  
(4.20)

Using this procedure, equations (3.28) or (3.35) can be used together with (4.13) through (4.20) to calculate the contribution to the input impedance of the lower patch and of the upper patch, separately. Since the cavities are being studied uncoupled (although some corrections have been made to account for the coupling), the total input impedance of the antenna can be assumed to be the series connection of the impedances of the upper and lower cavities (thanks to the fact that the lower patch acts as a ground plane for the upper patch, so in fact there is an electrical connection).
Therefore, the input impedance of the stacked microstrip ring antenna can be expressed as follows:

\[
Z_{\text{in}}(f) = Z_{\text{in1}}(f) + Z_{\text{in2}}(f)
\]  

(4.21)

The input reflection coefficient and standing wave ratios can be obtained from their definitions, as follows:

\[
\Gamma_{\text{in}}(f) = 20 \cdot \log \left( \frac{Z_{\text{in1}}(f) - 50}{Z_{\text{in1}}(f) + 50} \right) + 20 \cdot \log \left( \frac{Z_{\text{in2}}(f) - 50}{Z_{\text{in2}}(f) + 50} \right)
\]

(4.22)

\[
\text{SWR}(f) = \left| \frac{\Gamma_{\text{in}}(f)}{10^{\frac{\Gamma_{\text{in}}(f)}{20}}} \right| + \left| \frac{\Gamma_{\text{in}}(f)}{10^{\frac{\Gamma_{\text{in}}(f)}{20}}} \right|
\]

(4.23)

Just as in the case of the single microstrip antenna, a correction for the loss tangent must be considered to achieve good results. This correction is different for the bottom or top dielectric material. Since the upper patch is analyzed like an uncovered antenna, the effective loss tangent can be calculated using equation (3.49) but adding the contribution of the radiated power to the quality factor \(Q_{\text{rad}}\):

\[
\tan \delta_{\text{eff2}} = \frac{1}{Q_{\text{rad}}} + \frac{1}{Q_{\text{c}}} + \frac{1}{Q_{\text{d}}} + \frac{1}{Q_{\text{sw}}}
\]

(4.24)

Since the lower patch is analyzed like a dielectric covered microstrip antenna, some cautions must be taken. Zhibo Fan and Kai-Fong Lee [33] studied the effects of a dielectric cover on a microstrip antenna; it was found that the main effect of the dielectric layer is to reduce the resonant frequency and the resonant resistance, accompanied by an
increase in the impedance bandwidth. Besides, the use of a dielectric material in the upper patch with a higher dielectric constant shifts down the resonant frequency with a slightly smaller resonant resistance, and an increase in the upper patch height yields a smaller resonant resistance as well as wider impedance bandwidth, both cases when compared to the situation of a dielectric cover with the same height and dielectric constant as the lower dielectric material. Using this concepts, and the fact that an increase of the effective loss tangent reduces the resonant resistance, an empirical way to predict the loss tangent for the lower cavity has been studied. The loss tangent of the lower patch can be obtained as follows:

\[
\tan\delta_{\text{eff}1} = \frac{\tan\delta_{\text{eff}}}{\frac{h_2}{h_1}} \tag{4.25}
\]

where \( h_2 \) and \( h_1 \) are the upper and lower dielectric material thicknesses, and \( \tan\delta_{\text{eff}} \) is the effective loss tangent of the dielectric covered microstrip ring antenna, obtained from a similar equation to (4.24).

Summarizing, the input impedance of a stacked microstrip ring antenna fed by a coaxial probe can be obtained by calculating first the effective loss tangent using equations (4.24) and (4.25). The value of \( \tan\delta_{\text{eff}2} \) (upper patch) requires only one iteration; however, a second iteration is necessary to obtain \( \tan\delta_{\text{eff}1} \) (lower patch). The fields inside the cavities, radiated fields, power losses and radiated power must be recalculated with the new values of effective loss tangents obtained in the previous step (as well as any other variable that is being studied, like quality factors, bandwidth, etc.).
The results obtained using (4.21) along with (4.24) and (4.25) are shown in Figures 31, 32 and 33, and are compared to experimental data [32] for the TM_{11}, TM_{21} and TM_{31} modes. Good approximate values are obtained at resonance.

Although accuracy is not one of the virtues of this method, some conclusions can be stated by looking at the curves of the input impedance. Figures 34 through 37 show the dependence of the input resistance to the height and relative permittivity of the substrate and superstrate. In Figure 34 the substrate dielectric constant is maintained constant while varying the superstrate dielectric constant. The parameters used are \( a_1 = a_2 = 3 \text{ cm}, b_1 = b_2 = 6 \text{ cm}, h_1 = 0.159 \text{ cm}, \varepsilon_1 = 2.47 \text{ and } \tan\delta_1 = \tan\delta_2 = 0.005 \), with a feed at 3.45 cm from the center. For \( R_{in} \), \( h_2 = 0.159 \text{ cm and } \varepsilon_{r2} = 2.47 \); for \( R_{in1} \), \( h_2 = 0.318 \text{ cm and } \varepsilon_{r2} = 4.94 \); for \( R_{in2} \), \( h_2 = 0.318 \text{ cm and } \varepsilon_{r2} = 1.24 \). It can be noticed that the lower cavity input resistance has slight variations compared to the variations that occur in the upper cavity resonant frequency and input resistance. In Figure 35, the superstrate dielectric constant is fixed and the substrate dielectric constant is varied, where \( a_1 = a_2 = 3 \text{ cm}, b_1 = b_2 = 6 \text{ cm}, h_1 = h_2 = 0.159 \text{ cm}, \varepsilon_2 = 2.47 \text{ and } \tan\delta_1 = \tan\delta_2 = 0.005 \), with a feed at 3.45 cm from the center. For \( R_{in} \), \( \varepsilon_{r1} = 2.47 \); for \( R_{in1} \), \( \varepsilon_{r1} = 4.94 \); for \( R_{in2} \), \( \varepsilon_{r1} = 1.24 \). The upper cavity resonant frequency has minimal variations compared to the variations that occur in the lower cavity resonant frequency and input resistance.

In Figure 36, the height of the lower ring is fixed and the height of the upper ring is varied, and \( a_1 = a_2 = 3 \text{ cm}, b_1 = b_2 = 6 \text{ cm}, h_1 = 0.159 \text{ cm}, \tan\delta_1 = \tan\delta_2 = 0.0005 \), feed positioned at 3.45 cm from the center. For \( R_{in} \), \( h_2 = 0.159 \text{ cm}; \) for \( R_{in1} \), \( h_2 = 0.318 \text{ cm} \)
and for Rin2, $h_2 = 0.0795$ cm. The most evident effect is a bandwidth increase in the upper patch input impedance, with slight variations in the resonant frequency. And finally, in Figure 37 the height of the upper ring is fixed while varying the height of the lower ring, where $a_1 = a_2 = 3$ cm, $b_1 = b_2 = 6$ cm, $h_2 = 0.159$ cm, $\tan\delta_1 = \tan\delta_2 = 0.0005$, and the feed positioned at 3.45 cm from the center. For Rin, $h_1 = 0.159$ cm; for Rin1, $h_1 = 0.318$ cm and for Rin2, $h_1 = 0.0795$ cm. In this case, the input impedance as well as the resonant frequency have noticeable variations, especially for the lower patch.
Figure 31. Comparison of the theoretical and experimental [32] input impedance of a stacked microstrip ring antenna working in the TM_{11} mode with a_1 = a_2 = 2.5 cm, b_1 = b_2 = 5 cm, \varepsilon_{r1} = \varepsilon_{r2} = 2.32, h_1 = h_2 = 0.159 cm and tan\delta = 0.0012 fed with a coaxial probe at 2.7 cm from the center (tan \delta_{eff1} = 0.00887, tan \delta_{eff2} = 0.00472).
Figure 32. Comparison of the theoretical and experimental [32] input impedance of a stacked microstrip ring antenna working in the TM$_{21}$ mode with $a_1 = a_2 = 2.5$ cm, $b_1 = b_2 = 5$ cm, $\varepsilon_{r1} = \varepsilon_{r2} = 2.32$, $h_1 = h_2 = 0.159$ cm and $\tan\delta = 0.0012$ fed with a coaxial probe at 2.7 cm from the center ( $\tan\delta_{eff1} = 0.00705$, $\tan\delta_{eff2} = 0.00472$ ).
Figure 33. Comparison of the theoretical and experimental [32] input impedance of a stacked microstrip ring antenna working in the TM$_{31}$ mode with $a_1 = a_2 = 2.5$ cm, $b_1 = b_2 = 5$ cm, $\varepsilon_r_1 = \varepsilon_r_2 = 2.32$, $h_1 = h_2 = 0.159$ cm and $\tan\delta = 0.0012$ fed with a coaxial probe at 2.7 cm from the center ( $\tan\delta_{eff_1} = 0.00634$, $\tan\delta_{eff_2} = 0.00606$ ).
Figure 34. Variation of the input resistance of a stacked microstrip ring antenna with the superstrate relative permitivity (see text).

Figure 35. Variation of the input resistance of a stacked microstrip ring antenna with the substrate relative permitivity (see text).
Figure 36. Variation of the input resistance of a stacked microstrip ring antenna with the height of the superstrate (see text).

Figure 37. Variation of the input resistance of a stacked microstrip ring antenna with the height of the substrate (see text).
LIST OF REFERENCES


