Spacecraft Loads Prediction via Sensitivity Analysis and Optimization

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SPACECRAFT LOADS PREDICTION
VIA SENSITIVITY ANALYSIS AND OPTIMIZATION

by

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B.S. University of Central Florida, 1978

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Discrepancies between the predicted responses of a finite element analysis (FEA) and reference data from test results arise for many reasons. Some are due to measurement errors, such as inaccurate sensors, noise in the acquisition system or environmental effects. Some are due to analyst errors precipitated by a lack of familiarity with the modeling or solver software. Still others are introduced by uncertainty in the governing physical relations (linear versus non-linear behavior), boundary conditions or the element material/geometrical properties. It is the uncertainty effects introduced by this last group that this study seeks to redress. The objective is the obtainment of model improvements that will reduce errors in predicted versus measured responses. This technique, whereby measured structural data is used to correct finite element model (FEM) errors, has become known as “model updating”.

Model updating modifies any or all of the mass, stiffness, and damping parameters of a FEM until an improved agreement between the FEA data and test data is achieved. Unlike direct methods, producing a mathematical model representing a given state, the goal of FE model updating is to achieve an improved match between model and test data by making physically meaningful changes.

This study replaces measured responses by reference output obtained from a FEA of a small spacecraft. This FEM is referred to as the “Baseline” model. A “Perturbed” model is created from this baseline by making prescribed changes to the various component masses. The degree of mass variation results from the level of confidence existing at a mature stage of the design
cycle. Statistical mean levels of confidence are assigned based on the type of mass of which there are three types:

- Concentrated masses – nonstructural, lumped mass formulation (uncoupled)
- Smeared masses – nonstructural mass over length or area, lumped mass formulation (uncoupled)
- Mass density – volumetric mass, lumped mass formulation (uncoupled)

A methodology is presented that accurately predicts the forces occurring at the interface between the spacecraft and the launch vehicle. The methodology quantifies the relationships between spacecraft mass variations and the interface accelerations in the form of sensitivity coefficients. These coefficients are obtained by performing design sensitivity /optimization analyses while updating the Perturbed model to correlate with the Baseline model.

The interface forces are responses obtained from a frequency response analysis that runs within the optimization analysis. These forces arise due to the imposition of unit white noise applied across a frequency range extending up to 200 hertz, a cut-off frequency encompassing the lift-off energy required to elicit global mass response. The focus is on lift-off as it is characterized by base excitation, which produces the largest interface forces.
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CHAPTER ONE: INTRODUCTION

The objective of this study is to determine how parameters affect the accurate prediction of the interface forces developed between a spacecraft and its launch vehicle. The strategy for achieving this objective involves the development of a model updating methodology based on optimization with sensitivity coefficients.

Model updating is defined as modifying any or all of the mass, stiffness, and damping parameters of a FEM until an improved agreement between the FEA data and test data is achieved. Lacking validation test data, this study uses the Baseline model to generate output, which is then used as random-vibration test data. This is, in effect, replacing the test article with the Baseline model to obtain reference data with which to update the Perturbed model. Although a random-vibration test was assumed, any modal test would suffice.

Structural dynamics model validation is “the process of demonstrating or attaining the condition that the coefficients in a model are sufficiently accurate to enable that model to provide an acceptably correct description of the subject structure’s dynamic behavior” [1].

This process of model updating to achieve model validation is complex: “In practice, the acceptance of a spacecraft computational model is usually based on the evaluation of predicted and measured modal properties such as natural frequencies, mode shapes and effective masses, which are important characteristics that affect the structure’s response to applied forces. In fact natural frequencies affect response amplitude, mode shapes affect how loads distribute in the structure and modal effective masses seen from the launcher interface directly relates to the
forces across the spacecraft-launcher interface” [2]. In this study, these validation criteria of natural frequencies, mode shapes and effective masses are addressed:

- Natural Frequencies – Optimization frequency constraints
- Mode Shapes – Modal assurance criterion (MAC)
- Effective Masses – Correlation between measured and predicted interface forces

Sarafin et al [3] has defined the process of developing spacecraft designs thusly:

- Define Mission Requirements
- Develop Preliminary Design
- Develop Design Details
- Develop Production and Test Plans
- Produce and Test Dedicated Test and Flight Articles
- Perform Final Verification

Attention is focused on the last two bullets by assuming the design cycle has matured to the point where a dedicated test article has been constructed. The test article won’t be used for flight but has critical features that match the flight design and, as part of the final verification cycle, will be subjected to analysis validation tests.

Although strain gages would provide the desired forces directly, accelerometers are specified for use in the test simply because they are easier to install. Since test results are measured data, they are quantitative and known to a high degree of accuracy. Accordingly, acceleration is selected as the convergence criterion for the optimization process and it is specified that the objective is to
be the minimization of the difference between the response accelerations from the Perturbed and Baseline models.

As opposed to quantitative results, qualitative results are inferred from the measured accelerometer data so they aren’t known to the same level of accuracy. These results consist of the interface forces and response frequencies.

The interface forces are predicted by the updated Perturbed FEM. In a real scenario, without the benefit of measured force data to compare against, it would be difficult to assess how accurate the predicted forces are. However, this study shall use the output forces from the Baseline FEA as the comparative metric for assessing the accuracy of the methodology.

The accuracy of the predicted response frequencies is much easier to assess since they’re directly inferred from the accelerometer data via the FFT process. The FFT transforms time domain data into the frequency domain; the results of such a transform are depicted in Figure 1. Inaccuracies are introduced during the time history processing from sources such as noise, sampling frequency, filtering and windowing. Nevertheless, these processes are well understood so that a 2-σ mean level of confidence is ascribed to the predicted response frequencies.

In all references to a “mean”, it is understood that the reference is to the corresponding value of the Baseline model and that the distributions are normal (Gaussian).
As originally intended, the spacecraft was to have been excited using a launch derived forcing function because this event is characterized by vigorous base excursions, which produces the largest bolt loads. This data was not available so biaxial frequency response analyses were performed instead. The authors feel the shift from the time domain to the frequency domain is appropriate for the reasons presented next.

All structures have a preferred response to frequency dependent loading. When a spectrum of input frequencies is applied to a structure, the responses are altered in accordance with the structure’s mass, stiffness and damping. Further, because the input and output are known, one can calculate the output-to-input ratio at any point in the structure and for any degree of freedom (DOF). The ratio of output-to-input is known as a transfer function but when the input is frequency dependent then this ratio is known as the frequency response function (FRF).

What has been done is to conduct a frequency response analysis by imposing a uniform 1-in/sec\(^2\) base acceleration in both horizontal axes. Such uniform loads are sometimes referred to as ‘white
noise, a concession to white light, which consists of all frequencies in the visible spectrum. This type of loading creates equal energy at all response frequencies. The reason for using a unit load is because it yields the response FRFs directly.

So a specific output-to-input relationship of frequency-to-acceleration has been established via modal testing. Also, a way to replicate this relationship analytically is indicated by performing a frequency response analysis on the Baseline model. When the magnitudes of these accelerations are used as target design responses in an optimization analysis, the desired cause-and-effect relationship between mass and interface forces will be established.

The study begins by presenting elements of the model’s architecture. This is followed by the problem definition and the challenge of developing the analytical model. After the model and study objectives are covered, descriptions of the solution strategy and the various types of analyses used are presented. This is followed by a presentation and discussion of the analyses results. The study ends by offering conclusions, making recommendations and discussing topics for future work.
CHAPTER TWO: MODEL ARCHITECTURE

The analytical model is a small spacecraft of modest complexity and is depicted as an assembled view in Figure 2 and as an exploded view in Figure 3. The model was delivered from NASA/KSC as a NASTRAN bulk data file. This file was imported into the UGS/IDEAS software package, which was used as the pre and post processor.

Figure 2 Spacecraft Isometric
Figures 2 and 3 represent the Baseline model. It is treated as a fully updated FEM having been correlated to modal test results. A summary of the elements, grids and masses of this model follows:

- Number of grids = 1460
- Number of BAR elements = 585
- Number of BUSH (elastic) elements = 56
- Number of CONM2 (lumped mass) elements = 78
- Number of CQUADR (quadrilateral plate) elements = 1093
- Number of CTRIAR (triangular plate) elements = 184
- Mass = 4.481821 lbf·sec²/in

Figure 4 depicts the spacecraft without the solar panels. The main body is called the Chassis. It consists of plate elements and 1-D framing members to which a large number of lumped masses are attached; either directly or via rigid elements. Because information relating to the actual cross-sections was not available, each framing member’s depiction is based on the software’s interpretation according to the FEM cross-sectional properties. One can expect that the members do not actually look as they’re shown.

![Figure 4 Spacecraft Chassis](image-url)
Figure 5 depicts the spacecraft with the skin and framing members removed. This inner structure is called the Module and is thought to contain processing and power components. In addition to the Module, there are lumped masses depicted that attach to the Chassis.

Figure 5 Spacecraft Module

Figure 6 illustrates how the Module is supported by 1-D beam elements.

Figure 6 Module Support Structure
CHAPTER THREE: PROBLEM DEFINITION

At this phase of the study only the affects of change of mass are presented. In future phases, the affects of varying the stiffness (as well as the mass) by updating the member’s cross-sectional properties may be considered. For beams, the most common way to do this is to update the properties of area and moments of inertia while for plates it’s the thickness. As with any optimization effort, care must be taken that the updated design variable is physically meaningful; as, for example, that the resulting cross-sectional properties yield a shape that is commercially available.

Interface forces are developed at points were the Chassis bolts to the Adapter. These points were determined to be the 56 locations where elements of the Chassis and Adapter were found to share the same nodes. Coincident nodes were placed at these points and used to decouple the Chassis from the Adapter. They were subsequently re-coupled by attaching springs (BUSH elements) to the coincident nodes as depicted in Figure 7.

It is desirable to impose the interface loads as uniformly as possible to preclude local failure. For example, a bending moment should be introduced as an axial stress distribution according to the expression \( f = \frac{Mc}{I} \) and not as a couple.

To that end, the adapter in this study uses 56 bolts at each end as a means of uniformly distributing the loads.

Figure 7 Adapter to Chassis Interface Bolts
As a means of assigning translational spring stiffnesses, an arbitrary bolt diameter of 9/16 was chosen. The shear stiffnesses were assigned to be an order of magnitude greater than the axial stiffness. The translational DOF of each spring was established in accordance to a cylindrical coordinate system located at the center of the Adapter and in the plane of the bolts.

To obtain robust design sensitivity coefficients requires selecting both the proper response location and response component. The larger the coefficient’s magnitude, the more direct is the causal relationship between that input design variable and the output response it elicits. These responses are the variables in the objective function. Since the minimization of this function is the optimizer’s goal, one can greatly facilitate both the rate and quality of the convergence by maximizing the sensitivity coefficients.

In the effort to identify locations possessing large KE, intuition may lead one to choose X-acceleration components as responses to X-excitation and Y-acceleration components as responses to Y-excitation. However, choosing a location based its KE alone may result in selecting a point on the spacecraft that is responding with large displacements but has little mass and/or stiffness. Such a choice would result in a design response with poor sensitivity with respect to the design variable of mass. For example, one should expect to have a hard time optimizing the mass components of a car based on the displacement at the end of its antenna. It might be done but one should take care not to use a local mode that excited only the antenna. Also, picking X and Y accelerations simply because they’re aligned with the excitation axes ignores the kinematics of the responses and the mechanics of the boundary conditions.
When choosing points on the spacecraft to place the accelerometers, consideration should be given to both the nature of the excitation, the type of support and the desired responses. Also, since mass is the design variable, one must also consider the mass characteristics of the structure.

It is desired to impose base accelerations in order to predict the resulting bolt forces in the interface between the Adapter and Chassis. Two circumstances led to the choice of base excitation as the means of exciting the structure:

- Base excitation moves the entire spacecraft, so it is perfect to study the effects of updating a global design variable like mass.
- The interface forces occur at the base; an optimal elevation between the excitation source and the masses that will be updated.

With respect to sensor location, it is assumed that the tops of the bolts are more accessible than the bottoms, so all the acceleration data results from responses at the tops of the bolts.

With respect to the response component, note that both the X and Y responses are shear components in the bolts and so they only take into account the mass of the spacecraft and not its CG. Also, the shear stiffnesses in the material around each bolt are not constant. For example, in Figure 8, when considering responses in the plane of the Adapter flange, two of the bolts will have shear reactions tangent to the flange, two will react normal to the flange and the rest will have varying components of both. A hypothetical response is shown in Figure 8 for 8 equally spaced bolts. The tangent stiffness is much greater than the normal stiffness in a manner similar to a plate where in-plane stiffness is much greater than the normal stiffness.
Unlike X and Y, the Z responses are axial components in the bolts and so they take into account both the mass of the spacecraft and its CG. As such, they counteract both the X and Y overturning moments. Also, the stiffnesses, in the material around each bolt that react axial forces, are constant. These two factors make the Z-accelerations a better choice of design response than the shear accelerations.

Finally, it is needlessly burdensome to monitor the responses in all of the bolts when a subset, if correctly chosen, will suffice. The selected subset consists of 4 bolts located 90 degrees apart and aligned along the cardinal axes. These bolts are called ‘Sentinel’ bolts and are shown in Figure 9.
Figure 9 Monitored Interface Bolts

To evaluate the interface forces under mass uncertainty, an uncertainty propagation method is required. Fonesca [4] discusses the three most common methods and lists them as the Monte Carlo simulation method, perturbation method and the fuzzy method. These methods create a computational meta-model by transforming the deterministic FEM via modification of design parameters according to probability profiles. Unfortunately, limited specific information on the degree of uncertainty associated with the various parameters is available [5]. With this in mind
and a focus on developing the response prediction methodology, it was decided to assign mean confidence intervals.

In the Introduction section, it was noted that the means are equal to the corresponding Baseline values and the confidence intervals are based on the normal distribution. These intervals are created by assigning minimum and maximum values to each mass that are equal to the product of the mass type mean and the corresponding minimum and maximum levels of confidence. For example, suppose the mean (Baseline) value for a mass is 5 and that the confidence interval is $1-\sigma$. Because the $1-\sigma$ confidence interval is 0.683 to 1.317, the minimum mass value is $5*0.683=3.415$ and the maximum is $5*1.317=6.585$.

The assignment of confidence intervals to each mass type is based on the following mental imagery. Imagine that the lumped masses represent sensors that have yet to be purchased so that they’ve been assigned a $1-\sigma$ mean confidence level. The smeared masses might represent components that are critical to subsystems responsible for attitude control, communications, command and data handling, etc. These subsystems are common to every spacecraft and have been assigned a $2-\sigma$ mean confidence level. The volumetric masses represent the spacecraft’s frame or support structure. Because its design is assumed to be quite mature these masses have been assigned a $3-\sigma$ mean confidence level. All of the mass types and assigned confidence intervals are summarized below:

- **Lumped masses** – $1-\sigma$ (0.683-1.317)
- **Smeared masses (nonstructural mass)** – $2-\sigma$ (0.955-1.045)
- **Volumetric masses (mass density)** – $3-\sigma$ (0.997-1.003)
The adopted approach for providing accurate interface forces is a methodology based on a three-solution sequence with methods as explained below.

The three-solution sequence that comprises the methodology is modal, frequency response and optimization. This part of the methodology is quite general and applicable to any updating effort based on a frequency response approach.

The methods refer to a library of twelve user selectable optimization settings that allow tuning of the analysis to accommodate the solution of a wide range of design variables. These settings define the type of optimizer, the type of approximate model method and the objective methods used for convergence:

- Optimizer
  - DOT
  - MSCADS
- Approximate Model
  - Direct Linearization
  - Mixed Method
- Objective Method
  - User defined synthetic equation
  - NASTRAN supplied Match function
    - Option Beta method
    - Default Least Squares fit method
For an explanation of these terms, refer to the Optimization and Design Sensitivity section under the Solution Strategy section. Selected examples of how some of these options are requested can be found in the Optimization and Design Sensitivity section under Analysis, while more extensive coverage is given in Appendix C.

**Modal Analysis (Solution 103)**

The usual first step in performing a dynamic analysis is determining the natural frequencies and mode shapes of the structure with damping neglected. Typically, these results characterize the basic dynamic behavior of the structure and are an indication of how the structure will respond to dynamic loading. However, in the present context, this analysis, performed on the Baseline model, will yield eigenvalues and eigenvectors that will be treated as experimentally derived so as to provide suitable metrics for the model optimization process.

Seven methods of real eigenvalue extraction are provided in MSC/Nastran. These methods are numerical approaches to solving for natural frequencies and modes shapes. The reason for seven different numerical techniques is because no one method is the best for all problems. While most of the methods can be applied to all problems, the choice is often based on the efficiency of the solution process.

The methods of eigenvalue extraction belong to one or both of the following two groups:

- Transformation methods
- Tracking methods
In the transformation method, the eigenvalue equation is first transformed into a special form from which eigenvalues may easily be extracted. In the tracking method, the eigenvalues are extracted one at a time using an iterative procedure.

The recommended real eigenvalue extraction method in MSC/Nastran is the Lanczos method. The Lanczos method combines the best characteristics of both the tracking and transformation methods. For most models the Lanczos method is the best method to use and has been chosen for this study.

Besides the basic dynamic behavior, another reason to conduct a modal analysis is to make use of several diagnostic tools available in this solution sequence. Two of these tools are:

- **Modal effective mass** – Available for base excitation only. Based on the element’s masses and the structure’s mode shapes. This is a global property that reveals which modes are most energetic by calculating the mass percentage moving in a particular direction per mode:

  \[ \varepsilon^T [m] [\varepsilon] \]

  where \( \varepsilon \equiv \) modal participation factors based on \( \{\Phi\}^T [M] \{\Phi\} \), \( \Phi_r \equiv \) rigid body vector to account for directionality, \( m \equiv \) generalized mass

- **Grid point kinetic energy (KE)** – Based on the KE at a selected grid. This is a local property that provides a measure of which grid DOF is responding most energetically as a function of mode number:

  \[ \phi_e^T m_{ee} \phi_e \]

  where \( \phi_e \equiv \) mass normalized eigenvectors so that total grid KE scales to unity
Frequency response analysis is a method used to compute structural response to steady-state oscillatory excitation. Since lift-off is characterized as a wideband excitation and because frequency response analysis presents a means for imposing a wide spectrum input, the requirement for this type of analysis is implicit.

Two different numerical methods can be used in frequency response analysis. The direct method solves the coupled equations of motion in terms of forcing frequency. The modal method utilizes the mode shapes of the structure to reduce and uncouple the equations of motion (when modal or no damping is used); the solution for a particular forcing frequency is obtained through the summation of the individual modal responses.

The choice of the method depends on the problem. In general, larger models may be solved more efficiently in modal frequency response because the numerical solution is a solution of a smaller system of uncoupled equations. Using the modal approach to solve the uncoupled equations is very efficient, even for very large numbers of excitation frequencies. On the other hand, the major portion of the effort in a modal frequency response analysis is the calculation of the modes, an aspect that is address later. For large systems with a large number of modes, this operation can be as costly as a direct solution. This result is especially true for high-frequency excitation. To capture high frequency response in a modal solution, less accurate, high-frequency modes must be computed. For small models with a few excitation frequencies, the direct method may be the most efficient because it solves the equations without first computing the modes. The
direct method is more accurate than the modal method because the direct method is not concerned with mode truncation.

In this analysis, the modal DOF formulation was chosen for three reasons:

- Because of its efficiency, this solution sequence is preferred over the direct method.
- Widespread usage in the analytical community has led to a robust solution sequence.
- The modal formulation must be used to obtain the modal kinetic energy and the FRF. The FRF obtained from the Baseline model, though analytically derived, will be used as an experimentally obtained FRF. Experimental FRFs and eigenvalues are commonly used in model updating.

Some key features of this solution sequence:

- Damping – A constant 1% modal damping is applied as undamped or very lightly damped structures exhibit large dynamic responses for excitation frequencies near resonant frequencies. A small change in the model (or running it on another computer) may result in large changes in such responses.
- Modal KE – A global property based on the element’s masses, modal coordinates and the frequencies that excite the mode upon which the coordinates are based:

\[
[KE] = \text{ABS } [0.5 \{ \text{diag}(\omega_i^2) \} [M_{hh}] [\phi_h] \ast [\phi_h]], \quad i \equiv \text{freq}, \ h \equiv \text{mode}
\]

Figure 10 Modal KE Formulation
KE reveals how the structure responds under load. By simultaneously ‘bombarding’ the structure with a spectrum of frequencies one may observe things like modal coupling where closely spaced modes are all excited and the resulting KE is due to the sum of their responses. It is crucial to determine the frequency spectrums that energize the most structural mass.

- **FRF** – This is a model based on modal DOF and was alternately subjected to a constant 1 in/sec² acceleration in the X and Y horizontal axes. Unit accelerations were used because the resulting responses are the FRFs. The acceleration was held constant across the frequency bandwidth of 0.1-200 Hz. A constant acceleration imposes a uniform power distribution across all frequencies. This is known as ‘white noise’ and characterizes the dynamic properties of a structure to a broadband field of excitation. The bandwidth begins at 0.1 Hz to preclude rigid body responses and ends at 200 Hz as this was believed to encompass most of the energy content of the launch phase of the mission. It is this phase that generates the largest interface forces.

- Maximum interface accelerations, and the frequencies at which they occur, are obtained. These frequency dependent accelerations are used as convergence criteria for the optimization analyses, details of which will be covered in the next section.

**Design Optimization and Sensitivity Analysis (Solution 200)**

The decision to use optimization is motivated by two objectives; to effectively update the Perturbed spacecraft masses to correlate with the baseline masses and to obtain the interface force sensitivities to changes in mass.
This section defines what optimization and sensitivity are, followed by a metaphorical example. The section concludes with some fundamentals for those not familiar with these subjects. This part is titled “Overview of Fundamentals” and has been excerpted from MSC’s Design Sensitivity and Optimization User’s Guide [6].

**Design Optimization**

This is a broad and active area of technology. However, in the context of structures that can be analyzed with MSC/Nastran, it refers to the search for a structural design that is optimal, with respect to some user defined objective, while varying structural parameters. While performing this search, the design is guided to satisfy operating limits that are imposed on the response of the structure and by further limits on the values the structural parameters can assume.

**Design Sensitivity**

The design optimization capability in MSC/Nastran benefits significantly from being based on a design sensitivity analysis. Design sensitivity analysis computes the rates of change of structural responses with respect to changes in design parameters. These design parameters, or design variables, can be such things as shell thicknesses, beam dimensions, hole radii, material variables and so on. These rates of change are akin to partial derivatives and are called design sensitivity coefficients.

These sensitivities are computed explicitly in MSC/NASTRAN and are extremely valuable as they can be used to predict how a design change will affect an important response. When they are applied with an optimizer, they greatly improve the efficiency of the search since the
algorithm now not only knows the current state of the design but also has an idea of which way to look for an improved design.

As a final note about the value of sensitivity analysis, it is useful to consider what would occur if you did not have this information available. If the design task was to find the value of a single design variable that provided the best performance, one could conceive that performing analyses with several values of this parameter would provide information that could be displayed as a function of the variable and the optimum design could be visually determined. Now consider the case with two design variables. The number of analyses required is expanded and the plot now becomes three dimensional, but it still should be possible to find an optimum in this fashion. But now consider 10 variables and then one hundred and then one thousand. At some point, the most powerful computer would be overwhelmed by the number of analyses required and in making sense of the analysis results. With sensitivity analyses, it is quite reasonable to pose design tasks using a set of design variables that is far beyond that possible without the sensitivity information. Even for a small number of design variables, the presence of design sensitivities makes finding solutions much more efficient than what would be possible without these sensitivities.

**A Simple Example**

A useful way of introducing basic optimization concepts is to visualize a ‘design task’ of finding the lowest spot in a pasture.

Suppose we are standing in a pasture that slopes down to a small pond and would like to find the point of lowest elevation (the pond); this is the ‘objective.’ The location of any point is
quantified in terms of coordinates and these are the ‘design variables.’ Because the pasture is fenced they will force us to restrict our search to within the enclosed region. These fences, or ‘constraints’, act as bounds in the ‘design space’, a region that defines all of the possible positions in the pasture. Only one out of all points in the pasture can be considered an optimum, though. For simplicity, we are neglecting the presence of local minima.

Finding the lowest point is no real problem. All we need to do is have a good look about and note that the pond appears to be at the lowest elevation. We have scanned the pasture, analyzed thousands of possible candidates at a glance, and made an immediate decision. If we were blindfolded, though, our decision-making process would not be as simple, and that is exactly the task a numerical optimizer is faced with.

Being blindfolded, the elevation of a single point can only be determined by us haphazardly moving from point to point and evaluating hundreds or thousands of points. This will take considerable effort. We need a systematic method of searching for an optimal location. There are numerous techniques available to solve such a problem, all of which are classified as numerical optimization algorithms.

Generally, numerical optimization methods seek to determine a direction of travel or ‘search direction’ that moves us down toward the pond as quickly as possible, yet allows us to find an optimum that lies within the fences. A sequence of search directions is usually employed during the overall search procedure.
In this example, one could easily find a search direction, even though blindfolded, by taking small steps from side to side and then forward and back to test for elevation changes. This will allow us to determine which direction will move us down slope the fastest and, based on this estimate, proceed until encountering a fence or the slope starts to climb up again. These small steps establish the local value of the ‘gradient’ of the objective function. This information establishes a probable direction in which to search for a minimum. Numerical optimization algorithms that rely on gradient information are termed ‘gradient-based’. Once we have done the best one can possibly do in this direction, we find another search direction, and again proceed as before. We continue to repeat this procedure until one cannot reduce the objective function any further; that is, we have reached a point at which any move will take us up hill (an unconstrained option) or a point where we can move further downhill only by climbing over a fence (a constrained optimum). The process of reconciling the minimization or maximization of the objective function without violating the constraints is conducted in accordance with the so called Kuhn-Tucker conditions [7].

To quantify the location of a point in the pasture, we might use north-south and east-west coordinates corresponding to the elevation at a given point as the design variables. In design optimization terms, this is a two ‘design variable’ space since two coordinate values are required to uniquely specify a point in the design space. Two design variables are the most one can easily visualize. Considering that an optimizer may have to deal with tens or even hundreds of design variables, the task becomes understandably more complex.
There is another condition that may apply to our exercise that illustrates an additional concept. The fences in the pasture are properly considered constraints, but we may have been told than under no circumstances can one go north further than a specified amount, even though there is no fence there. This constraint that is imposed directly on the design variable is called a ‘side constraint.’

This example has introduced the following concepts:

- **Objective** – In the example, the objective was to find the point of lowest elevation. In this study the objective is to minimize the difference between the measured accelerations at the Sentinel bolts and the predicted accelerations from the FEA.
- **Design Variable** – In the example, design variables were all the points in the pasture. In this study, design variables are all the mass values.
- **Constraints** – In the example, the constraints were the fences. In this study the constraints are modal frequencies.
- **Design Space** – In the example, the design space were all points in the pasture.
- **Side Constraints** – This term was not illustrated in our example but in this study side constraints are bounds placed on the magnitude that each mass value can assume. Recall that each type of mass is assigned a variance, which effectively bounds the allowable mass values.

**Overview of Fundamentals**

Up to this point a general overview of design optimization has been presented and a number of concepts introduced that are specific to MSC/NASTRAN’s implementation of Design Sensitivity
and Optimization. In the sections that follow, data will be presented that was predicated on certain user selectable parameters. In order to understand the motivation in their selection, this section delves a little deeper into the optimization methodology by developing the concepts relevant to the interface features used.

This section can be thought of as a description of the flow chart shown in Figure 11. The figure shows a design loop with a number of blocks. The blocks, and the section in which they are described, is as follows:

- **Initial Design** – The initial design is based on the perturbed masses, whose values have been set to the low end of their ranges. The means of these ranges are the Baseline FEM mass values.

![Figure 11 Implementation of Structural Optimization](image)

**Figure 11 Implementation of Structural Optimization**

- Initial Design – The initial design is based on the perturbed masses, whose values have been set to the low end of their ranges. The means of these ranges are the Baseline FEM mass values.
As previously explained, each mass distribution is assumed Gaussian and, according to mass type, is assigned a 1-\(\sigma\), 2-\(\sigma\) or 3-\(\sigma\) mean confidence level.

- **Structural Analysis** – The analysis function that is to be performed. Recall that a test article was subjected to modal testing and accelerations were measured at the Sentinel bolts. In seeking to replicate these responses, the FEM is updated by instructing the optimizer to perform a frequency response analysis.

- **Constraint Screening** – This refers to the process that is used to identify those constraints that are likely to drive the redesign process. When the sensitivity of a design variable is below a default value, it is eliminated from that design cycle. It is not necessarily eliminated from the analysis since sensitivities may change from one cycle to the next. This is one way that the optimizer is unburdened from performing calculations that result in minimal gains.

- **Sensitivity Analysis** – This feature has already been covered.

- **Optimizer** – MSC/Nastran utilizes a variety of optimization algorithms of which two are used in this study:
  - DOT – Stands for Design Optimization Tools and represents algorithms from VR&D.
  - MSCADS – Stands for MSC Automated Design Synthesis and represents an enhanced suite of optimization algorithms adapted from the public domain version of ADS.

- **Approximate Model** – Nastran uses formal approximations to the finite element analysis and the sensitivity analysis to avoid the high cost associated with repeated finite element analyses during design optimization. As shown in Figure 11, the optimizer interacts with the approximate model when it requires information. Given a set of design variables, the optimizer needs information on the function values (the value of the objective and the values of the retained constraints) and the gradient values (gradient of the objective with respect to
• Direct Linearization – Based on a simple first-order Taylor series expansion in terms of the design variables.

• Mixed Method – Uses a combination of direct and reciprocal approximations, depending on the response type being approximated. For this study, frequency dependent acceleration is the response type. For this type of response, the first-order Taylor series expansions are expressed in terms of reciprocal variables since frequency and acceleration are inversely proportional to mass.

• The Improved Design – This is the point at which the finite element model is updated based on the results from the optimizer so that a new finite element analysis can occur.

• Converged – Design Optimization is an iterative process and therefore, a key feature of the implementation is determining when to stop the iterations. There are two levels at which convergence is tested: the first and lower level is at the optimizer level; the second and higher level is with respect to the overall design cycles. It is the second level that pertains to Figure 11. This topic is too broad to be considered in depth here. Let us simply say that this study is predicated on what’s termed “Hard” convergence which compares the results of the most recent finite element analysis with those from the previous design cycle. Since this test compares the exact results (within the limits of the FEA) from two consecutive analyses, the
conclusions are said to be based on hard evidence. Since this test is conclusive, this is the
default test for determining whether or not to terminate the design-cycle process.
CHAPTER FIVE: ANALYSIS

As a precursor to the analyses, the spacecraft was discretized into nine components, based on an intuitive sense of mass and stiffness, as depicted in Figure 12 and serves a twofold purpose:

- It facilitates the model checkout process of confirming there are no missing members or grids in each of the component structures.

- Once there is confidence that all components are structurally cohesive, each is assigned its own unique mass properties. In general, the more finely substructured the spacecraft, the more control the optimizer has over updating specific parts of the model. For example, if 90% of the spacecraft is compose of 6061-T6 aluminum and then every time the model is updated 90% of the model will be affected, it will be impossible to draw conclusions about which part of the spacecraft is really contributing to changes in the interface forces.

![Figure 12 Spacecraft Components](image)

Figure 12 Spacecraft Components
Modal Analysis (Solution 103)

Solution 103 characterizes both the modal attributes and responses of the spacecraft based on the Lanczos eigenvalue extraction method:

- **Modal Attributes**
  - MAC (Modal Assurance Criterion) – Mode shape correlation between test (Baseline FEM) and predicted (updated Perturbed FEM)
    - MAC – scalar value between 0 and 1 where values near one indicate a high degree of correlation between two mode shapes.

- **Modal Responses**
  - Local Response – Most energetic nodes and directions
    - Sensor placement and orientation – Key on the most energetic grids (placement) and response direction (orientation)
      - Spatially, intuition provides a perspective to sensor location if one recognizes the spacecraft as a cantilever where the inertial load path must pass thru the bolts
      - Sensor orientation is less intuitive
  - Global Response – Which modes are most energetic based on % of mass excited (mass is global design variable for optimization)
    - Find modes that involve most mass
      - Maximize magnitude of sensitivity coefficients (cause-effect)
      - Establish frequencies to use as constraints (feasible domain)

The MAC is the most widely used criterion for vector correlation (mainly because of its simplicity) and consists of the correlation coefficient of vector pairs in two vector sets defined at
the same DOFs. In general one set corresponds to measured mode shapes at a number of sensors while the other set corresponds to the observation of analytical mode shapes. For two vectors that are proportional the MAC equals 1 (perfect correlation). Values above 0.9 are generally considered highly correlated. Values below 0.6 should be considered with much caution as they may or may not indicate correlation. Depending on whether or not damping is addressed, two forms of the MAC are given:

\[
\frac{\sum_{j=1}^{m} \psi_1^*(j) \psi_2(j)}{\sqrt{\sum_{j=1}^{m} \psi_1^*(j) \psi_1(j) \sum_{j=1}^{m} \psi_2^*(j) \psi_2(j)}} = \psi^* \equiv \text{vector of complex conjugate mode shape (damping)}
\]

\[
\frac{\sum_{j=1}^{m} \psi_1^2}{\sqrt{\sum_{j=1}^{m} \psi_1^2 \sum_{j=1}^{m} \psi_2^2}} \quad \text{MACs presented are predicated on real modal analysis (damping neglected)}
\]

**Figure 13 MAC Formulation**

Because mass is a global design variable, there is a need to know which global modes are energetic. Also, since frequency is a constraint for the optimization runs, it is required to identify energetic global modes in order to establish candidate constraint frequencies. Global modes are indicated by the effective mass of the system. The effective mass gives us a measure of how much mass is moving in either the X or Y direction as a function of the mode number.

Information about each bolt’s local modes is also require in order to determine which node’s DOF is responding most energetically at the local modes that match the global modes and to
determine the best location and orientation for the sensors. Local modes are indicated by the grid point KE. This form of KE provides a measure of which grid DOF is responding most energetically as a function of mode number.

As a prelude to base excitation, the modal model was supported by fully constraining the independent node of a rigid element located at the base. This type of element is often referred to as a “spider” element because of the multiple dependent nodes. The independent node is located at the center of the Adapter and even with the bottom flange. This element is shown in Figure 14 below.

**Figure 14 Spacecraft Base Excitation FEM**
Frequency Response Analysis (Solution 111)

When a large mass is rigidly attached to the base of a much smaller mass and then excited, its response will drive the smaller mass in a manner similar to an earthquake shaking a building. This is called ‘base excitation’ and is the technique used in this study to impose frequency dependent loading.

To determine the target interface accelerations, frequency constraints and whether response coupling is possible, one performs a forced frequency response analysis on the Baseline FEM. This is best accomplished by imposing a ‘white noise’ forcing function and then requesting modal kinetic energy output for all energies greater than 0.001 (default).

White-noise excitation provides uniform power across the input spectrum. Because of this property it elicits a modal response without any frequency bias. This is not its only useful property. For example, one may determine the frequency response function (FRF) at any desired DOF if a unit white-noise excitation is applied. The FRF is the ratio of output to input and this relationship may be depicted by plotting the response of a selected DOF as a function of frequency.

Having defined the input excitation as frequency dependent base acceleration, there’s the need to determine which component of the acceleration response to output. Since it is known that modes 7 and 8 are kinetically energetic, an animation of these modes should reveal which displacement component of the Sentinel bolts is most vigorously excited. That is not necessary however, since
the modal analysis output of the grid KE has already revealed that the desired acceleration component is the Z-axial response.

To excite the spacecraft, the independent node of the spider element is fully constrained except along the axis of excitation. A uniform 1-in/sec$^2$ acceleration is then imposed to the resonating mass over a frequency spectrum of 0.1 to 200 Hz. The beginning of this spectrum is greater than 0 Hz to preclude any rigid body responses.

**Design Sensitivity and Optimization Analysis (Solution 200)**

This section contains snippets from some of the input files used in the optimization phase of this study. The intent is to provide an overview of how the solution strategy was implemented in the code. For a more in depth explanation of the meaning of the input commands, the interested reader is referred to the NASTRAN Quick Reference Guides [8].

To provide results for the optimizer to converge with, an analysis of some sort must run within the optimization process. As a prelude to performing a modal frequency response analysis, the modes and mode shapes are determined first. The request for these analyses is made at the subcase level and is indicated on lines 4 and 9 as depicted in Figure 15.
It may be observed that line 5 has been commented out. This line invokes the MODTRAK feature and is available only for modal analyses (ANALYSIS = MODES). It tracks the modes so that they’re ordered based on their original physical behavior rather than on their frequency order, which is useful in an optimization analysis where mass and/or stiffness characteristics may change. The concern was that modes 7 and 8, being important modes and separated by on 1 Hz, might “mode swap”. This was not observed so that is why the MODTRAK feature was disabled. MODTRAK constructs a cross-orthogonality check between the current design cycle and the previous cycle per the equation: $\phi_i^T M_i \phi_{i-1} = t_i$. In this equation, the $i$ subscript denotes the current design cycle, $i-1$ the previous design cycle and the prime symbol over the $\phi$ signifies mass-normalized eigenvectors.

When there are no changes in the mode shape, order, or mass of the system, then the cross-orthogonality check would yield a diagonal matrix. However, if the mode shapes change due to a mass and/or stiffness variation, then their correlation will be less than 1.0. For this situation, the mode numbers will also change and dominant off-diagonal terms will be present in $t_i$. Where
sufficient correlation exists, the results of this cross-orthogonality check can be used as a basis to determine which modes have switched.

Three fundamental issues, dealing with posing the problem to the optimizer, are the choice of design variables, response variables and how the problem is constrained. These issues are addressed in the following paragraphs. To facilitate the discussion, snippets of the input files are provided as necessary.

The use of masses as the design variables has been discussed along with how their confidence levels have been applied. These confidence levels are the side constraints and operate locally on each design variable. However, there is a global issue relating to the resonator mass that requires a brief discussion.

When the large mass method is used in response analysis, the resonator mass is chosen to be several orders of magnitude larger than the structure. Because the resonator mass is not under the optimizer’s control, this large discrepancy can adversely affect the response sensitivities of the structure’s masses. The solution is to “hide” the resonator by assigning its mass using the CMASS4 bulk data input and placing this mass on a scalar point instead of a grid point. By using a scalar point instead of a grid, the resonator is effectively removed from the model geometry and will not appear in the Grid Point Weight Generator.

Depending on the type of analysis, there are a number of responses that the optimizer will accept. From Figure 15, line 9, a request for NASTRAN to perform a modal frequency analysis, within
the optimization solution, has been made. This allows the user to choose frequency dependent acceleration as a design response by specifying FRACCL on the design response bulk input DRESP1 as shown on lines 3 thru 6 of Figure 16. Also on lines 3 thru 6, note the last three fields of input. This is where the acceleration component, the frequency at which the acceleration is to be extracted and the location where the acceleration occurs are called out. This snippet from the input file is for X-excitation. Compare the frequency and node callouts with those from Table 1, page 74, for X-excitation.

| 1 | $ f 1 d s : 7 i s a c c l c o m p o n e n t f o r g r i d ( 1 : A x , 2 : A y , 3 : A z ) | 8 i s f r e q @ m a x a c c l | 9 i s g r i d I D |
| 2 | $ g r i d A o f b u s h |
| 3 | DRESP1,401,Az200,FRACCL,,,3.36,566,200 |
| 4 | DRESP1,403,Az216,FRACCL,,,3.37,474,216 |
| 5 | DRESP1,405,Az232,FRACCL,,,3.36,566,232 |
| 6 | DRESP1,407,Az248,FRACCL,,,3.37,38,248 |
| 7 | $ x s p e c i f y a n d l a b e l x , y a n d z m a x i m u m o b j e c t i v e i n t e r f a c e a c c e l s ( f r o m B / L o u t p u t ) a s d e s i g n |
| 8 | DTABLE 0Az200 20.47 0Az216 7.30 0Az232 19.06 0Az248 7.55 |

**Figure 16 Declare Design Response Frequency and Magnitude**

Design constraints provide certain restrictions, or limits, to ensure that as the optimization process advances toward achieving the design objective, other design conditions do not become compromised. Constraints define what’s called the *feasibility domain*. In the pasture example recall that the constraints were the fences around the pasture.

The nature of a constraint depends, primarily, on the type of analysis solution. At the present time the available options are *linear static* and *normal modes*. Recall the choice of frequency response as the analysis used for the optimization and that, as a prelude to this analysis, normal modes are extracted; this means the normal modes option is invoked. This option, in the context of a frequency response optimization, makes frequency the appropriate constraint choice for the
optimization. The modal analysis has indicated that the major X-excitation mode is 7 and that the major Y-excitation mode is 8. Because these modes have such large associated effective masses, this makes them and their modal frequencies, useful as constraints.

There are two ways that frequencies are obtained to be used in the excitation of the structure. One way is to depend on the modal extraction process to find these frequencies. A potential problem arises in that, as the masses are updated the modes are also changed. This means that the Perturbed spacecraft is not excited at exactly the same frequencies as the Baseline spacecraft. This is, perhaps, avoidable by the proper selection of the convergence objective. However, as a precaution, apply both modal and explicit excitations frequencies, where the explicit frequencies are from the Baseline modal analysis. Modal frequencies are declared via FREQ4 bulk data input and explicit frequencies are declared via FREQ bulk data input as depicted in Figure 17 below.
The importance of using the frequencies of modes 7 and 8 as constraints has been discussed. How these constraints are requested is depicted in Figure 18. Note that the modes have been shifted up by one order relative to the modal analysis modes. Because the spacecraft must be allowed to move along the axis of excitation, a provision was made to provide inertia relief and this means that mode 1 becomes a rigid body mode.
Figure 10 illustrated that an approximate model is part of the optimization process. As discussed, both the Direct Linearization (APRCOD=1) and Mixed Method (APRCOD=2) approximate models were used in this study. The Direct Linearization method is shown on line 5 in Figure 19.

The OPTCOD parameter permits the specification of the optimization code to be used. Per the “Overview of Fundamentals” section, both the DOT and MSCADS algorithms are used in this study. MSCADS is the default for shape and sizing optimization while the DOT code is available as an option. A request for the OPTCOD=DOT option is shown on line 6, Figure 19.

Figure 18 Declare Frequency Constraints
The optimizer manipulates the design variables, subject to the constraints, to minimize the objective function. This function is a scalar quantity, which operates on the response acceleration at the top of each Sentinel bolt.

Two different objective functions were used for this study; one supplied by MSC and the second being a user defined synthetic function. The interested reader may reference *Innovative Uses of Synthetic Responses in Design Optimization* [9] for additional information related to the use of synthetic responses.

- The MSC function is requested on line 4, field 4 of Figure 20 with the keyword “MATCH”. Additionally, there is a method associated with this function which is called-out on line 4, field 6 with the keyword “BETA”. This function seeks to match the optimized accelerations with the objective accelerations using the beta method. This method is based on a spawned artificial design variable.
• The MSC function is again specified on line 3, field 4 of Figure 21. However, this time the
default “LEAST SQUARES” method is requested by leaving field 6 blank.

```
1 $x$ specify and label $x,y$ and $z$ maximum objective interface accel
2 $x$ (from B/L output) as design responses:
3 $x$ $2$ $3$ $4$ $5$ $6$ $7$ $8$ $9$
4 DRESP2  207  ACCL  MATCH
5          DTABLE  OAz200  OAz216  OAz232  OAz248
6 DRESP1  401  403  405  407
```

Figure 21 Match Function using Least Squares Method

• The user defined synthetic function is requested on line 10, field 4 of Figure 22 with the
keyword “101” which points to line 14 and the design equation with ID=101. This equation
is simply the sum of the absolute values of the differences between the optimized
accelerations and the objective accelerations. It is this equation that is to be minimized. Note
that the values of the variables in the equation are determined by lines 11 and 12 of the
DRESP2 beginning on line 10. This means, from line 11: OAz200, OAz216, OAz232,
OAz248 and from line 12 (by way of the DRESP1 callouts): Az200, Az216, Az232, Az248.

```
1 $x$ iden:  7 is accel component for grid (1=Ax,2=Ay,3=Az) | 8 is freq @ max accel | 9 is grid ID
2 $x$ grid A of bush
3 DRESP1,401,Az200,FRAICAL...3.36,566,200
4 DRESP1,403,Az216,FRAICAL...3.37,474,216
5 DRESP1,405,Az232,FRAICAL...3.36,566,232
6 DRESP1,407,Az248,FRAICAL...3.37,38,248
7 $x$ specify and label $x,y$ and $z$ maximum objective interface accel (from B/L output) as design responses:
8 DTABLE  OAz200  20.47  OAz216  7.30  OAz232  19.60  OAz248  7.55
9 $x$ $2$ $3$ $4$ $5$ $6$ $7$ $8$ $9$
10 DRESP2  207  ACCL  101
11 DTABLE  OAz200  OAz216  OAz232  OAz248
12 DRESP1  401  403  405  407
13 $x$ specify minimization of error function as objective
14 DFEQTN 101 F(OAz200,OAz216,OAz232,OAz200,Az216,Az232,OAz248,
15 OAz248)=ABS(OAz200-OAz248)+ABS(Az216-OAz216)+ABS(Az232-
16 OAz232)+ABS(Az248-OAz248)
```

Figure 22 User Defined Synthetic Function
CHAPTER SIX: RESULTS

In this section are presented the results of the three-solution sequence. While the modal and frequency response analyses provided critical frequency dependent optimization tuning metrics, the updating methodology hinges on the optimization analyses so their output represents the majority of the material presented.

Modal Analysis (Solution 103)

The model was run using modal solution sequence SOL 103 and a request was made for the modes, mode shapes, modal effective mass and grid point KE. The input file, minus grid and element data, is provided in Appendix A.

Mass updating is predicated on the results of a frequency response solution that runs within the optimization analysis. Because these results are used as convergence criteria by the optimizer, information is required relating to modes that excite the most mass and the DOF that responds most vigorously for that mode. The “target” patterns in Figures 23 and 24 depict a near rigid body rotation of the spacecraft for modes 7 and 8. Because these modes excite so much mass, they’ll figure prominently in the optimization analyses. The reasons for choosing the Z-axis of the Sentinel bolts as the response best suited for updating the masses during optimization have been presented; now the figures attest to the appropriateness of this choice as the plots present visual confirmation that only the Z DOF is excited by modes 7 and 8.
Mode 7 characterized by rotation of SC (note concentric disp contours)

Mode 7 excites Z DOF which is the bolt’s longitudinal (axial) axis

Mode 7 occurs @ 36.57 Hz

Figure 23 Mode 7 Attributes

Mode 8 characterized by rotation of SC (note concentric disp contours)

Mode 8 excites Z DOF which is the bolt’s longitudinal (axial) axis

Mode 8 occurs @ 37.87 Hz

Figure 24 Mode 8 Attributes

Plots of the mode number versus modal effective mass, for both X and Y excitation, are presented in Figures 25 and 26. Plots of the mode number versus nodal KE, for each of the four Sentinel bolts, are depicted in Figures 27 thru 30.
Figure 25 X-Translational Modal Effective Masses

Figure 26 Y-Translational Modal Effective Masses
Figure 25 indicates that mode 7 is by far the largest contributor to the total effective mass for X-axis excitation while for Y-axis excitation Figure 26 indicates mode 8. For both excitations, the dominant mode is approximately 75 times greater than the next largest modal contributor. What is not clear is that mode 7 is very close to mode 8 in frequency. In fact, only 1 Hz separates them and, as presented in the frequency response analyses, the close proximity of these modal frequencies will result in response coupling.

![GRID 200](image)

*Figure 27 Top of Bolt 30000 Kinetic Energy*
Figure 28 Top of Bolt 30014 Kinetic Energy

Figure 29 Top of Bolt 30028 Kinetic Energy
Figures 27 thru 30 indicate that only the Z-axis response is excited by both modes 7 and 8 (for an example see Figure 31 below). These modes are axial bolt modes and one shall make use of this information when performing the frequency response analyses. The absence of X response for mode 7 and Y response for mode 8 is not surprising as the bolts are much stiffer in shear.

Figure 30 Top of Bolt 30042 Kinetic Energy

Figure 31 Modal Effective Mass vs Mode 7 (X-excitation)
**Frequency Response Analysis (Solution 111)**

Modal frequency response solution sequence SOL 111 was run and a request was made for modal KE and displacements. Also, the maximum response accelerations and forces in the bolts and the frequencies at which these responses occurred was also requested. The input files for both X and Y-axis excitation, minus grid and element data, is provided in Appendix B.

There are two ways to specify the lowest eigenvalues to be extracted. One way is to choose a frequency that is greater than zero but less than the fundamental frequency and specify it in the eigenvalues extraction bulk input. MSC advises against this method. Because SOL 111 uses augmentation or residual vectors to increase the accuracy of the modal formulation, it has evidently been observed that this method can compromise the residual vectors. Instead, MSC recommends using the parameter “LFREQ” and omitting the specification of the initial frequency from the range of frequencies to be extracted on the eigenvalues extraction input. The LFREQ and eigenvalues extraction bulk inputs are shown on lines 3 and 5 of Figure 32 below, which is excerpted from an input file. When the parameter is used, the first mode is a rigid body mode because of the SUPORT command required for inertia relief. Inertia relief, in the axis of excitation, is required so that the structure is allowed to move in that DOF. Since inertia relief is not required for the modal analysis (SOL 103), one must be aware that the effective mass modes are offset 1 mode relative to the frequency response KE modes.

```
1 BEGIN BULK
2 $ no rigid body modes below 0.1 Hz
3 param.lfreq.0.1
4 $ 3x.range=3x200Hz
5 eigr.1.1.660.
```

Figure 32 LFREQ Bulk Data Input
Plots of the modal kinetic energy versus forcing frequency, for both X and Y excitation, are presented in Figures 33 and 34.

![Figure 33 Forcing Frequency vs X-Translational K.E.](image-url)

**Figure 33 Forcing Frequency vs X-Translational K.E.**
Figure 34 Forcing Frequency vs Y-Translational K.E.

As previously mentioned, the frequency responses of modes 7 and 8 are coupled; now two preceding figures offer corroboration. For X-axis excitation, mode 7 response frequencies span 36-38 Hz while mode 8 spans 37.192-38 Hz. This demonstrates that mode 7 response frequencies include all of mode 8. For Y-axis excitation, mode 7 response frequencies span 36.2-37 Hz while mode 8 spans 36.2-39 Hz. This demonstrates that mode 8 response frequencies include all of mode 7.
It’s informative to compare modal analysis Figures 25 and 26 to frequency response analyses Figures 33 and 34. From Figure 25 the 4 modes with the largest ordered effective X-mass are 7, 8, 17 and 21. From Figure 33 the 4 modes with the highest ordered KE are 7, 8, 17 and 21. So the modal effective mass modes match the modal KE modes as depicted below.

Figure 35 Modal Effective Mass vs Modal KE (X-excitation)

From Figure 26 the 4 modes with the largest ordered effective Y-mass are 7, 8, 20 and 80. From Figure 34 the 4 modes with the highest ordered KE are 1, 7, 8 and 20. In this instance, the modal effective mass modes don’t match the modal KE modes as depicted in Figure 36. Why do these modes switch dominance as the axis of excitation moves from X to Y? The answer to this question requires an understanding of what these modes look like, so let us return to this question after the presenting the eigenvector plots.
Figure 36 Modal Effective Mass vs Modal KE (Y-excitation)

Figure 37 Y-excitation / Mode 1 Response ($\Delta$=in, F=Hz)
Figure 38 X-excitation / Mode 7 Response (no panels, $\Delta=$in, $F=$Hz)

Figure 39 Y-excitation / Mode 8 Response (no panels, $\Delta=$in, $F=$Hz)
Figure 40 X-excitation / Mode 17 Response (Δ=in, F=Hz)

Figure 41 Y-excitation / Mode 20 Response (Δ=in, F=Hz)
Figure 42 X-excitation / Mode 21 Response ($\Delta$=in, $F$=Hz)

Figure 37 leads us back to the question of why mode 1 replaced mode 80. A look at this figure shows that the solar panels, which are lightweight components, have translated almost 6 inches. So although low in effective mass, they possess high KE due to their large displacements. As the two sets of panels are different, this behavior is not observed in the X direction.

Figures 38 and 39 have the solar panels removed so that it may be observed that the Chassis is in a near state of rigid body rotation. An indication that rotation is present is the near symmetric “target” like pattern of the displacement contours. For pure centroidal rotation, about the interface bolts centroid, the resulting contour displacements would be displayed as a series of concentric circles about a dot.
Figure 38 makes clear that the mode 7 response to X-excitation is Y-rotation while the converse is true for mode 8, Figure 39. It is the orthogonality of these eigenvectors that promotes the shift in dominance of one modal response over the other as the axis of excitation changes between X and Y. This shift in dominance is depicted in Figure 43.

Figure 43 Mode 7 vs Mode 8 Dominance

Orthogonal responses can couple in a nonsymmetrical structure. However, the close proximity of these modal frequencies is an indication of the mass and stiffness symmetry of the spacecraft. As evidenced by the NASTRAN Grid Point Weight Generator in Table 4, there is very little bias in the CG of the spacecraft. That is to be expected since spin stabilization is one method of control. The coupling mechanism may be attributable to the flexible appendages. Regardless of the mechanism, response coupling may be observed by progressing thru the X-excitation frequencies, from 36 to 38 Hz, in Figure 43. Doing so, one would notice the axis of rotation
begins to shift from Y toward X as one pass 37 Hz. This shift becomes more pronounced, as the forcing frequency approaches 37.6 Hz, the modal frequency of mode 8.

Modes 17, 20 and 21 are depicted in Figures 40 thru 42 respectively. These modes excite some of the panels and are best understood with the benefit of an animated display. The frequencies associated with these modes will be used as constraints in the optimization analyses but figure much less prominently than modes 7 and 8.

Having obtained the required information, two important optimization frequency parameters can now be determined. The first parameter relates to the frequencies to use in the frequency response analysis to run within the optimization solution sequence. Explicit “target” frequencies need to be chosen because, as the masses change during optimization updating, so too will the modal frequencies. The second parameter relates to the constraint frequencies.

The requirement for explicitly defined target frequencies will be explained more fully in the optimization section. In the mean time, note that finding the target frequencies is a simple matter of identifying the Baseline modal frequencies up to 200 Hz, the analysis cutoff frequency. Specifying these discrete frequencies in the run file, via the “FREQ” bulk data input, causes the responses at these frequencies to be calculated during the optimization run. As a visual comparison between discrete and spectral response accelerations, Figures 44 thru 47 are presented below for frequencies between 36 and 59 Hz. This frequency range is shown because it represents the interval of maximum modal KE for X-excitation. Only the response at each Sentinel bolt due to X-excitations is depicted but Y-excitations are similar.
Figure 44 X-Excitation – Spectral vs Discrete Response for Grid 200

Figure 45 X-Excitation – Spectral vs Discrete Response for Grid 216
Figure 46 X-Excitation – Spectral vs Discrete Response for Grid 232

Figure 47 X-Excitation – Spectral vs Discrete Response for Grid 248
Observe the excellent correlation between the discrete and the spectral traces over the frequency range of 36-38 Hz; where the most important modes, 7 and 8, occur. Also, note how the acceleration magnitudes of grids 200 and 232 are almost 3 times greater than grids 216 and 248. This is as expected since they lie along the axis of excitation. Also, the fact that grids 216 and 248 are not zero reaffirms the previous conclusion that the displacements due to modes 7 and 8 are non-centroidal and thus possess a translational component. Since grids 216 and 248 lie along the axis of rotation, they would experience no accelerations for pure rotation.

Regarding the constraint frequencies, selecting them is a bit more involved. Recall that there are only 4 modes with appreciable KE so that these modes are good candidates. Also, since optimization uses design sensitivity, it is desirable to update the masses based on the maximum accelerations at the Sentinel bolts. If the maximum bolt accelerations occur somewhere in the spectrum of frequencies found in Figures 33 and 34, then using these modes as constraints should be a good choice.

It has been demonstrated that modes 7 and 8 are common to both X and Y excitations and that the modal analysis indicates that the axial Z-axial acceleration of the Sentinel bolts responds most vigorously to these modes. With this in mind, a rerun could be performed and request made for frequency dependent Z-axial acceleration and force at each of the four Sentinel grid points. If the proper component is selected, then the maximum responses should occur at frequencies close to those of modes 7 and 8. As part of the requested information, a table of maximum responses is included in the output. Excerpts from both output tables of the X and Y analyses are provided below.
Table 1 Maximum Z-axis Response Summary

<table>
<thead>
<tr>
<th>RESPONSE COMPONENT</th>
<th>EXCITATION</th>
<th>ID</th>
<th>RESPONSE MAGNITUDE</th>
<th>FREQUENCY (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z-AXIS (bolt axial) ACCELERATION (in / sec²)</td>
<td>X</td>
<td>GRID 200</td>
<td>20.465</td>
<td>36.566</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GRID 216</td>
<td>7.305</td>
<td>37.474</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GRID 232</td>
<td>19.004</td>
<td>36.566</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GRID 248</td>
<td>7.552</td>
<td>37.380</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>GRID 200</td>
<td>8.191</td>
<td>37.380</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GRID 216</td>
<td>21.331</td>
<td>37.568</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GRID 232</td>
<td>7.375</td>
<td>37.380</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GRID 248</td>
<td>21.417</td>
<td>37.568</td>
</tr>
</tbody>
</table>

For X-excitation, Table 1 shows that the maximum accelerations for the Sentinel bolts corresponding to grids 200 and 232 occur at the modal frequency for mode 7. For the Y-excitation, the Sentinel bolts corresponding to grids 216 and 248 occur at the modal frequency for mode 8. This is compelling evidence of the strong coupling between the Z-axial acceleration and modes 7 and 8, so these modes must be used as constraints. The other modes, indicated in Figures 33 and 34, will also be used to increase the frequency correlation. If convergence problems arise, they will be omitted but modes 7 and 8 must be retained. An example of how to use this information to construct a constraint is shown in Figure 48 for X-excitation, mode 8.
To establish constraints for optimization we relate modes to frequency spectrums.

Example: Per slide 5, level of confidence for random-vibration test is 2-σ (.955 – 1.045). Mode 7 modal frequency is 36.57 Hz. ∴ constrained to 34.92 - 38.21 Hz when the forcing freq is 36 - 38 Hz.

Figure 48 Constraint Selection Example

Some might wonder why it is that not all of the acceleration response frequencies match their force response counterparts. Remember that only the response at the top of each Sentinel bolt is output since it’s presupposed that’s where the accelerometers are attached. Without accelerometers on both ends of a bolt it’s impossible to categorically conclude how much force is generated.

Is it a coincidence that the frequencies where high system KE occurs so closely correlates to the bolt’s frequencies? Actually, this is expected because of the FEMs boundary conditions.

The Chassis is bolted to the Adapter. The interaction between a bolt and the clamped material is quite complicated. The fidelity of the model is coarse as it was intended to convey mass magnitude, mass dispersion and stiffness. Models used for stress purposes are more finely...
detailed since they must depict the stress gradients. These gradients may be quite pronounced, which is especially true of contact stresses. Since bolted connections possess contact stresses, the fidelity of the model is inadequate to give an accurate representation of the true stress state. Also, bolted connections demonstrate nonlinear stiffnesses because the clamped material represents two surfaces in contact with one another. These surfaces are referred to as “faying” surfaces since they can move relative to one another.

When a bolted joint is in compression, the joint stiffness is initially due to the clamped material. As the compressive load increases, the bearing area increases and so does the stiffness. When the load is reversed, the faying surfaces begin to unload but almost this entire load ends up being applied to the clamped material. This is because the volume of clamped material is much greater than the bolt volume so it is much stiffer. It is not until the surfaces are unloaded that tensile force is imposed on the bolt which means the joint stiffness is much reduced.

Modeling this behavior requires gap elements that change their stiffnesses as they open and close. Such complex behavior can be modeled in the time domain by performing a nonlinear transient analysis but is not available in the frequency domain.

So this is why the model does not take into account the true behavior of a bolted joint. The surface-to-surface contact between the Chassis and Adapter is not modeled. Instead, these components are joined together only at the bolts. Areas of the Chassis floor and the Adapter flange adjacent to the bolts will deform somewhat but because the bolts are much more compliant, most of the relative movement between the spacecraft and Adapter, both tensile and
compressive, is resisted mainly by these fasteners. This explains the high degree of correlation between the KE and bolts frequencies. This also explains why the range of these frequencies is bounded by modes 7 and 8. These modes represent an almost pure rigid body rotation of the entire spacecraft. This is a lot of mass moving together and the principal resistive forces are due to the axial displacements of the bolts.

**Design Sensitivity and Optimization Analysis (Solution 200)**

The model was run using design sensitivity and optimization solution sequence SOL 200 and a request was made for frequency dependent Z-acceleration (accelerance or inertance FRF) at the top of each Sentinel bolt, frequency dependent axial force in each Sentinel bolt and the sensitivity coefficients for frequency, acceleration and the objective function. The input files, minus grid and element data, are provided in Appendix C.

NOTE: For reasons yet to be determined, when two runs of a same model were compared, one run might converge while the rerun didn’t or converged at a different number of design cycles. Lacking repeatability of the results presents a problem when trying to determine which choice of parameter settings performed best. In an effort to compensate for this bias all input files were run twice. MSC/NASTRAN development has determined this is a machine dependent problem which is being investigated.

A simplified explanation of this solution sequence for a design cycle follows:

- Update masses subject to each mass type’s variance
• Check that updated masses don’t violate side constraints established according to each mass type’s level of confidence relative to the mean or Baseline value:
  • Smeared mass ≡ 1-σ ⇒ 68.3% – 131.7% mean confidence
  • Concentrated mass ≡ 2-σ ⇒ 95.5% – 104.5% mean confidence
  • Mass density ≡ 3-σ ⇒ 99.7% – 100.3% mean confidence

• Perform modal & freq response analysis @ each update cycle
  • Obtain response frequencies and mode shapes with 1% damping
    • Modes constrained by freq ranges based on response frequency spectrums per Baseline FEA (simulated test data)
  • Obtain response output
    • Acceleration & force FRFs – Responses as a function of excitation frequencies
      • Local response – Acceleration & force @ 4 Sentinel bolts
  • Minimize the objective equation subject to convergence criteria
    • Convergence criteria – Objective equation is the absolute sum of the difference of the Baseline and Perturbed accelerations at each Sentinel bolt
      • Obtain response output
        • Baseline acceleration – Maximum acceleration and the frequency at which this maximum occurs
        • Perturbed acceleration – Response acceleration at the frequency at which the maximum Baseline acceleration occurs

There are numerous metrics that can be used to suggest a preferred or “best performing” methodology. One way is to determine how effectively each of the design variables converged to
their baseline values. This is a bottom line assessment since it involves the mass types (density, smeared, lumped) directly. Another way is to compare the design responses of frequency and acceleration with their baseline values. This might be termed a level one abstraction since the comparison of response quantities arises as the result of updating the design variables. Since there is nothing to suggest one of these approaches over the other and in the interest of completeness, both results will be presented in this section. Let us call the bottom line approach “BL” and the level one approach “L1”.

Let us look at the BL approach first. One might like to plot all the variables at each design cycle and compare them to their target values. However, because there are 143 mass design variables, it would be confusing to try and compare all of them; especially since there are three mass types and each has its own target value. However, one can plot the average of each mass type versus design cycle but the sensitivity coefficient of each design variable must be accounted for. In the following equation “n” is equal to the number of design variables for each mass type, “x” is the value of the design mass variable and “|DS|” is the absolute value of the sensitivity coefficient. The equation appears below and must be recomputed for each of the three mass types.

$\bar{X}_{norm} = \frac{\sum_{i=1}^{n} x_i |DS_i|}{\sum_{i=1}^{n} |DS_i|}$

*Figure 49 Normalized Parameters Formulation*
Sensitivity coefficients are calculated for all the response types. That means there are sensitivities for each of the 4 bolt accelerations, the 4 constraint frequencies and the objective function. The question arises as to which type of sensitivity coefficient to use.

Sensitivities are required for the accelerations and frequencies since they must converge to the target values. However, while convergence for these responses is a necessary condition to meet the design objective, they are not in themselves the objective. The optimization objective is to minimize the objective function. This function operates on the aforementioned responses in a manner defined by an MSC or user supplied equation. Basing this equation on the objective function sensitivities provides the global objective metric and so it’s the best relative measure of which method performed best.

In all the optimization runs, a naming convention was adopted for each input file predicated on the parameter settings. All the optimization based tables and plots use this same convention. The resulting names have the following meanings (brackets connote options):

- \([x, y][d, s][d, m][a1, a2][es, mb, ml][,.1]\)
  - \([x, y]\) connotes “x” or “y”-axis excitation
  - \([d, s]\) connotes “discrete” or “spectral” (discrete is not presented in this section but is referenced in Figs. 35, 36, 37 and 38)
  - \([d, m]\) connotes “DOT” or “MSCADS” optimizer
  - \([a1, a2]\) connotes “direct linearization” or “mixed method” approximate model
  - \([es, mb, ml]\) connotes user defined “equation synthetic” or NASTRAN supplied “match” function using option “beta” method or default (least squares fit) method.
Plots based on the Beta method are not presented because this method is based on minimizing a spawned design variable that makes comparisons with the other methods inappropriate. In the remaining plots, there is no significance to whether a trace is solid, dashed or dotted except for visual clarity.

In Figures 50 thru 55, the target value for all mass coefficients is unity (1). These values represent the ratio between the masses of the updated-Perturbed and Baseline models. Because each of the mass types has a different variance, the statistical dispersion has been included, as part of each heading and the ordinate range has been adjusted accordingly.

![Figure 50 X-Density Top Performer-Candidates](image-url)
Figure 51 X-Lumped Top Performer-Candidates

Figure 52 X-Smeared Top Performer-Candidates
Figure 53 Y-Density Top Performer-Candidates

Figure 54 Y-Lumped Top Performer-Candidates
Let us begin the discussion of Figures 50 thru 55 by commenting on the values at the end of the first design cycle. Recall that the initial value of each mass type was arbitrarily set equal to the minimum value of the statistical range. The difference between this point (i.e. 0.683 per the heading in Figure 55) and the first point of each trace represents the optimizer’s first move in the search direction. The process by which the optimizer decides how much to move during cycle 1 is discussed in Appendix D of the MSC/Design Sensitivity and Optimization User’s Guide. It’s an involved process, taking into account not only the minimization of the objective function but also how much the design variables can be changed before violating the constraints. Regardless of the process, it’s interesting to note that the cycle 1 values of the 2-σ and 3-σ traces lie close to the maximum of their respective ranges. These mass types have small variances so this is not surprising. Conversely, the cycle 1 values of the 1-σ traces (Figures 55 & 55) lie close to the midpoint of their respective ranges. Because these masses have large variances, this means an
initial leap from 0.683 to approximately 1! This immediate jump to a value so close to the target of 1 is interpreted as an indication of the close coupling between the design variables and the chosen component (Z axis) of acceleration.

Note that each plot’s ordinate is divided into 6 segments with 3 segments above unity and 3 segments below. Most of the traces tend to converge in the adjacent segments above and below unity, which demonstrates that the general performances of all the methods were quite acceptable.

To establish which method performed best, let us eliminate the 3 methods whose final converged values are maximums or minimums. Such a criterion omits the 3 methods that vary most from the mean of unity. This leaves 5 candidates for each mass type. Let us then compare these candidates across mass types to check for commonality. Thus, any of the 5 methods that are included in each of the mass types remains a candidate. These survivors are plotted next.
Figure 56 X-Density Top Performers

Figure 57 X-Lumped Top Performers
Figure 58 X-Smeared Top Performers

Figure 59 Y-Density Top Performers
Figure 60 Y-Lumped Top Performers

Figure 61 Y-Smeared Top Performers
Only the blue trace is depicted in each of the Figures 56 thru 61. Observe that the convergence values of all the traces are similar, as are the number of design cycles. Based on these plots, it seems reasonable to conclude that the ma2es method is the best performer using the baseline or BL criterion.

Whereas the BL criterion was based on the assessment of how well the design variables of the Perturbed model correlate with those of the Baseline model, the L1 criterion compares the design responses of frequency and acceleration with their target values. The results of this criterion are best depicted in tabular format. Appendix D contains the tables from all the runs and includes the beta method that was missing from the BL criterion. These tables also contain information relating to the force in the bolts.

The tables that follow have been excerpted from those in Appendix D. They present only the best run from each method (recall that each method was run twice). The values shown represent the ratio of the average of the four updated maximum bolt accelerations to the corresponding Baseline model average.
Table 2 X-Excitation Acceleration Top Performer-Candidates

<table>
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<th>xsda1es</th>
<th>xsda1mb</th>
<th>xsda1ml.1</th>
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<tbody>
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</tbody>
</table>

The gold cells signify the top contenders (closest to unity) while the green cells signify the next best contenders. Since the gold cells represent two different methods while a single method is desired, these methods are eliminated. Conversely, the green cells represent the same method.

Table 3 Y-Excitation Acceleration Top Performer-Candidates

<table>
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<tr>
<td>0.99998</td>
<td>1.02235</td>
<td>1.00305</td>
</tr>
</tbody>
</table>

The gold cells signify the top contenders (closest to unity) while the green cells signify the next best contenders. Since the gold cells represent two different methods while a single method is desired, these methods are eliminated. Conversely, the green cells represent the same method.
(ma2es) so it becomes the best performer. Also, note that this is the same method chosen by the BL criterion.

Both the BL and L1 criteria represent objective, mathematical based processes by which to select the best performing method. However, they lack a visual perspective that comes with examining the frequency versus acceleration plots of all the methods. For that, plots are present next that depict the area around the maximum response. The intent is to provide a close-up of the traces so some of the information has been cropped out. For a more expansive view of the traces, see Appendix E where plots representing all the methods and extending over the full frequency range are included. Plots depicting all methods have a heavy, light blue trace labeled “S111” and a heavy dark blue trace labeled “sma2es”. The “S111” trace represents the spectral frequency response analysis (Sol 111), which is the baseline and the “sma2es” trace represents the best performing method. Plots depicting only this best performer and the baseline traces are also included, but only for Y-Excitation where the traces are so closely clustered comparisons are difficult.
Figure 62 X-Excitation Accelerations @ Bolt 30000 – Baseline vs All Methods

Figure 63 X-Excitation Accelerations @ Bolt 30014 – Baseline vs All Methods
Figure 64 X-Excitation Accelerations @ Bolt 30028 – Baseline vs All Methods

Figure 65 X-Excitation Accelerations @ Bolt 30042 – Baseline vs All Methods
Figure 66 Y-Excitation Accelerations @ Bolt 30000 – Baseline vs All Methods

Figure 67 Y-Excitation Accelerations @ Bolt 30000 – Baseline vs Best Method
Figure 68 Y-Excitation Accelerations @ Bolt 30014 – Baseline vs All Methods

Figure 69 Y-Excitation Accelerations @ Bolt 30014 – Baseline vs Best Method
Figure 70 Y-Excitation Accelerations @ Bolt 30028 – Baseline vs All Methods

Figure 71 Y-Excitation Accelerations @ Bolt 30028 – Baseline vs Best Method
Figure 72 Y-Excitation Accelerations @ Bolt 30042 – Baseline vs All Methods

Figure 73 Y-Excitation Accelerations @ Bolt 30042 – Baseline vs Best Method
Most of the methods performed so respectively that, had the BL and L1 criteria checks not been performed, it would be virtually impossible to pick a “best” method based solely on visual comparisons between the traces in Figures 62 thru 73.

Having selected the best performer, one can use its output to determine how accurate the predicted interface forces are. This is done by presenting interface force plots of both the Baseline model and the updated model, which is based on the sma2es parameter settings. For comparison purposes, these plots contain traces of both updated model runs.

Before continuing, a few of points are made:

- When comparing force magnitudes bear in mind that the ordinate scale is not constant.
- Optimization objective is to minimize the difference between updated and baseline responses.
- Optimized responses are frequency dependent and frequency is constrained relative to a 2-\(\sigma\) mean level of confidence.

Figure 74 helps explain the last two bullets and why magnitudes are better correlated than frequencies.
Recall that a 2-σ dispersion is applied to random-vibration test freqs. When these test freqs are used as optimization constraints, the 2-σ criterion creates a design space in which the freqs at which the max responses occur will vary with respect to the corresponding test freqs.

Accel is response predicted from optimization. Objective is min of diff bwn updated response and test data.

Once optimization completed, predicted forces are obtained based on the updated masses.

Figure 74 Baseline vs Updated Responses

Figure 75 X-Excitation Forces @ Bolt 30000 – Baseline vs Best Method
Figure 76 X-Excitation Forces @ Bolt 30014 – Baseline vs Best Method

Figure 77 X-Excitation Forces @ Bolt 30028 – Baseline vs Best Method
Figure 78 X-Excitation Forces @ Bolt 30042 – Baseline vs Best Method

Figure 79 Y-Excitation Forces @ Bolt 30000 – Baseline vs Best Method
Figure 80 Y-Excitation Forces @ Bolt 30014 – Baseline vs Best Method

Figure 81 Y-Excitation Forces @ Bolt 30028 – Baseline vs Best Method
Figures 75 thru 82 demonstrate that both the X and Y-excitation plots depict predicted responses that compare very well with the measured responses. In particular, observe that the maximum Y-excitation predicted force responses are virtually identical with respect to the measured responses. As a point of fact, go to Appendix D and look at the table labeled “Ysma2es”. The column labeled “frcMAG” shows that the maximum ratio of predicted to measured force responses is 1.001. The X-excitation predicted responses, though less accurate, display excellent correlation as evidenced by the table labeled “Xsma2es”. Here, it may be noted that the maximum force ratio is 1.006. This means the maximum error of the maximum predicted interlace forces, for both X and Y-excitations, is only 0.6%.
Frequency comparisons are straightforward as all the plots have the same frequency resolution (0.15 Hz). As with the magnitudes, the plots indicate that the Y-excitation frequencies are most closely correlated to the measured frequencies. Again, the X-excitation plots exhibit a lower degree of correlation, yet are well within the 2-σ mean confidence constraint envelope indicated in Figure 74. As before, one may obtain the maximum error by going to Appendix D. This time, look for the table labeled “Xsma2es” and the column labeled “FREQ”, which is across from the column labeled “frcMAG”. The maximum frequency ratio is 0.997 or a maximum error of 0.3%.

As the nature of the repeatability issue is not understood, one can only speculate as to why the X-excitation and rerun traces lie on top of one another when this is not the case for the Y-excitation plots. Still, given the plots depict that the Y-excitation correlates better with the measured responses than the X-excitation; one might suppose that the situation should be reversed.

Since mass is the global design variable, it is reasonable to compare how accurately the best performer predicted both the spacecraft’s total mass and its center of gravity. For that, Table 5 is presented below. It contains information for the Baseline model as well as both updated Perturbed models.
Table 4 Baseline vs Perturbed Model Mass (lbm, inches)

<table>
<thead>
<tr>
<th>DIRECTION</th>
<th>MASS AXIS SYSTEM (S)</th>
<th>MASS</th>
<th>X-C.G.</th>
<th>Y-C.G.</th>
<th>Z-C.G.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>X</td>
<td>4.481821E+00</td>
<td>1.793592E-19</td>
<td>2.005182E-01</td>
<td>2.525077E+01</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>4.481821E+00</td>
<td>-2.604055E-01</td>
<td>-2.610395E-19</td>
<td>2.525077E+01</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>4.481821E+00</td>
<td>-2.604055E-01</td>
<td>2.005182E-01</td>
<td>1.016903E-19</td>
</tr>
<tr>
<td>X-Perturbed</td>
<td>X</td>
<td>4.501578E+00</td>
<td>-1.544136E-18</td>
<td>1.549397E-01</td>
<td>2.519367E+01</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>4.501578E+00</td>
<td>-2.037500E-01</td>
<td>1.156977E-18</td>
<td>2.519367E+01</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>4.501578E+00</td>
<td>-2.037500E-01</td>
<td>1.549397E-01</td>
<td>6.036676E-19</td>
</tr>
<tr>
<td>Y-Perturbed</td>
<td>X</td>
<td>4.509991E+00</td>
<td>4.817615E-20</td>
<td>1.900345E-01</td>
<td>2.527329E+01</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>4.509991E+00</td>
<td>-2.411147E-01</td>
<td>-5.299377E-19</td>
<td>2.527329E+01</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>4.509991E+00</td>
<td>-2.411147E-01</td>
<td>1.900345E-01</td>
<td>8.686570E-19</td>
</tr>
</tbody>
</table>

Table 4 indicates that the X-Perturbed model overestimated the true mass by 0.44% while the Y-Perturbed model overestimated by 0.43%. Note that all of the diagonal center of gravity elements are numeric zero, as they should be. The X-Perturbed model underestimates the Y center of gravity by 22.73% while the Y-Perturbed model underestimates the X center of gravity by 7.395%. It is not clear why the X-Perturbed model performs so poorly with respect to predicting the Y center of gravity. It may have to do with a local stiffness asymmetry, possibly with the flexible solar panels or the appendages. The apparent greater flexibility of solrpn12, with respect to solrpn1 (see Figure 12), has been documented. The X-Perturbed model underestimates the Z center of gravity by 0.226% while the Y-Perturbed model overestimates by 0.089%. This is consistent with the more accurately predicted Y responses as compared to the X responses.

To end this section, plots of the sensitivity coefficients versus the mass design variables are presented next. These coefficients are calculated at each design cycle and for all design responses. For this study, these responses are the constraint frequencies, the frequency dependent...
accelerations and the objective function. Because the coefficients change for each design cycle, the plots that follow depict objective function sensitivity coefficients for the first design cycle.

Figure 83 X-Excitation Sensitivity Coefficients vs Design Variables

Figure 84 Y-Excitation Sensitivity Coefficients vs Design Variables
Because much of the spacecraft is modeled with lumped and smeared masses, Figures 83 and 84 show only four mass density variables plotted. As it happens, only four of the nine components depicted in Figure 12 possess mass density and they are all of the same type aluminum. The symmetry between the X and Y coefficients echoes the symmetry of the spacecraft.

With respect to the coefficients, the largest belong to the lumped mass design variables. Evidently, concentrating all these masses at the ends of their supports has a pronounced effect on the sentinel bolts’ accelerations. However, if there is interest in establishing the particular masses that figure most prominently in eliciting the maximum bolts’ accelerations, then plot the coefficients as a function of that mass type as depicted in the figure below.

![X-SEN COEFFS vs DESIGN VARS (Design Cycle 1)](image)

Figure 85 X-Excitation Sensitivity Coefficients vs Lumped Masses
CHAPTER SEVEN: CONCLUSIONS

Gradient-based algorithms use sensitivity coefficients to find the direction of steepest descent. In doing so, the optimizer marginalizes those design variables possessing small sensitivities from analytical consideration. The upside is not only an increased rate of convergence but the information provided by the sensitivity analysis is worthy, in its own right, of the analysts consideration.

Depending on your objectives and they’ll probably change during the various phases of the design cycle, sensitivity analysis provides a systematic approach to determining where model changes should be made for the best chance at attaining the desired response. If one is in a ‘flat’ design space, then large changes in a design variable will not significantly affect the response variable so that fine-tuning is not warranted. By adding “smarts” to the analytical design process, the time saving implications, particularly for the early design phases of a project, may be significant.

To maximize the sensitivity benefits requires a close coupling between the design variables and responses. The process for achieving this coupling is set forth in the methodology presented in this study. The quality of convergence is demonstrated in Tables 2 and 3 where the sma2es method (best performer) indicates X-excitation convergence at 6 cycles and Y-excitation convergence at 10 cycles. This rapid convergence, along with the strong design response correlation between Baseline and Perturbed acceleration FRFs, indicates a robust methodology.
Beyond this study, in a broader context of problem solving, the methodology is enhanced by the methods presented in Appendix C. These methods are a toolbox of alternative approaches to achieving convergence or fine tuning the accuracy of the design variables to engineer the desired solution.

Due to the number of optimization settings, the input file naming terminology is necessarily cryptic. With regard to the best performer, the selection of the sma2es method means:

- [s] connotes “spectral”. This means to apply the full spectrum of forcing frequencies throughout the range of interest (0-200 Hz).
- [m] connotes the “MSCADS” optimizer.
- [a2] connotes the “mixed method” approximate model. For frequency dependent accelerations, the approximate model is predicated on first-order Taylor series expansions that are expressed in terms of reciprocal variables. It is perhaps no surprise that this parameter worked best since frequency and acceleration are inversely proportional to mass.
- [es] connotes user defined “equation synthetic” as depicted in Figure 22.

Having successfully updated the model, Figures 72 thru 79 demonstrate the excellent correlation between the predicted interface forces and the actual interface forces. This degree of correlation, based on acceleration instead of force as the design response, validates the selection of accelerometers. Because these sensors were chosen due to their ease of installation, compared to strain gages, the added benefit is a simplification of the test setup.

While acknowledging that the non-repeatability issue has introduced some bias to the results, it is nonetheless felt that the chosen methodology results in an updated model that accurately
correlates predicted Sentinel bolt accelerations with measured accelerations. This methodology includes a means to determine both the appropriate location and the degree of freedom to instrument and this is key, for if the proper choices are made then large sensitivities result and thus a successful optimization. That this is so, has been demonstrated by the difficulty in picking a “best” performer. Almost all of the runs resulted in excellent correlation between predicted and measured responses.
CHAPTER EIGHT: RECOMMENDATIONS

The simplification of the bolted joints was necessitated by a desire to avoid a nonlinear analysis and the requirement to pursue a modal solution approach. The inaccuracy of the assumed linear behavior is minimized if the bolt preloads are not exceeded. It is a consequence of the many interface bolts used that this is almost assuredly so.

However, if the difference between predicted and measure responses are significant, then consider using spacers to provide a small separation between the spacecraft and Adapter. These spacers would only be required during the modal tests and would be replaced after data acquisition. As it is desirable that the spacers have the same tensile/compressive stiffnesses, then something like a serrated washer should do. The serrations would provide shear slip resistance during base excitation tests. This would be a simple, temporary modification would provide boundary conditions closely resembling the FEM so that the predicted responses should more closely correlate with the measured responses.
CHAPTER NINE: FUTURE WORK

Although Perturbed and Baseline models exhibit good correlation, more parameter-updating analyses are required to determine if the broad application capability of the methodology exists.

Among the 3 parameters ([M], [K], [B]) damping is generally the most difficult to gage. However, it is almost always applied uniformly across the entire model as a discrete value so that assessing model responses to changes in the damping magnitude are straightforward. Unlike damping, mass and stiffness vary across the model and are often represented by many values. This array of values makes them ideal design variables for a gradient based optimization process.

Having demonstrated the ability to accurately predict how interface forces are affected by changes in mass, it is suggested that the next study phase focus on stiffness.

More work is required to obtain a well defined statistical integration of the design variables across the design space. Although the masses were assigned variations based on well reasoned assumptions an informed consideration was not possible due to the unfamiliarity of the spacecrafts architecture and construction.
APPENDIX A – INPUT FOR MODAL ANALYSIS
EXECUTIVE CONTROL

ID TB $ TIME 600 $ SOL 103 $ DIAG 5,6,8,56 $ GEOMCHECK NONE $ CEND $

CASE CONTROL

Based on Xtrnl SEs: 0=adptr,10=apndg1,20=apndg2,30=apndg3,40=chassis, 50=module,60=modulsuprt,70=solrpnl1,80=solrpnl2

TITLE = NORMAL MODES
SUBTITLE = CALC MODAL EFF MASS
ECHO = none
MPC=300000
$ request part fact
SET 1 = 200,216,232,248
GPKE(punch,thresh=0.001) = 1
DISP(PLOT)=ALL
$ request Modal Effective Mass (based on part fact)
MEFFMASS(print,fracsum)
SPC=1 $ one constraint point
METHOD=1
BEGIN BULK $ BULK DATA $ no rigid body modes
param,lfreq,0.1
$ 3*range=3*200Hz
$ PARAM,CHECKOUT,YES
PARAM,GRDPNT,0
param,post,-2

$ MODAL MODEL

$ resonating mass = 10,000,000 * spacecraft mass
SPOINT,199999
CMASS4,15827,1.E+8,199999

$ Join SPOINT to GRID
MPC       300000  199999       0      1. 99999       2     -1.

$ fix base to extract all modes in all axes. this will result in no
$ rigid bodies so will not track w/ SOL111 because SUPORT creates
$ rigid body mode 1
SPC            1   99999  123456     0.0

$ SUPORT,99999,2

include mass.bdf
ENDDATA
ID TB
TIME 600
SOL 111
$ DIAG 5,6,8,56
GEOMCHECK NONE
CEND

Based on Xtrnl SEs: 0=adptr,10=apndg1,20=apndg2,30=apndg3,40=chassis,
50=module,60=modulsupt,70=solrpnl1,80=solrpnl2

Response to X-excitation: Z grid accel and Z(axial) bolt force
TITLE = X-TRAN UNIT ACCEL EXCITATION
$ Z grid accel is opti design response but want to predict axial bolt force
SUBTITLE = X GRD ACL/Z BLT FRC

ECHO = none
TFL=200000
MPC=300000

$ resonating mass set commented out for this run which is elastic only
$ w/o rigid body modes (eigen extraction f>0) the constant (non oscillatory=>f=0)
$ accel applied to mass can't be plotted
$ SET 1 = 99999,199999
**** Monitored Spring Nodes ****
SET 1 = 200,216,232,248,30000,30014,30028,30042
**** Monitored Springs ****
SET 2 = 30000,30014,30028,30042
$ monitored interface accelerations, paired nodes from springs
$ label : 30000
$ nodes 1 - 2 : 200, 30000
$ ---------------------------------------------------------------
$ label : 30014
$ nodes 1 - 2 : 216, 30014
$ ---------------------------------------------------------------
$ label : 30028
$ nodes     1 - 2          : 232, 30028
$ --------------------------------------------------------------
$ label                    : 30042
$ nodes     1 - 2          : 248, 30042
$ --------------------------------------------------------------
$
$ SET 6 = 1 thru 100
$ modal KE for modes 1-100 (magnitude and phase to 200 Hz)
MODALKE(punch,phase,esort=descend,thresh=.001)=6
$ request magnitude and phase
$    DISP(phase,plot,print)=1
$    DISP(phase)=1
$    VELO(PLOT)=1
$    DISP(PLOT)=ALL
ELFORCE(phase,punch,print)=2
ACCE(phase,punch,print)=1
DISP(plot)=1
SPC=1 $ one constraint point
METHOD=1
SUBCASE 1
    LABEL=ESTABLISH X-TRAN FRF TO 200 Hz
    FREQ=1
    SDAMP=1
    DLOAD=11
$    SKIPON
$------------------------- PLOT
OUTPUT(XYPLOT)
CScale=1.8
XPAPER=26.0
YPAPER=20.0
XGRID LINES=YES
YGRID LINES=YES
XDIVISION=10
XTITLE=                             FREQUENCY(HERTZ)
$---------------Z GRID ACCEL ---------------
YTITLE=        RESP ACCEL - (IN/SEC2)
TCURVE=               GRID 200(Z)
XYPLOT ACCE RESP / 200(T3)
TCURVE=               GRID 30000(Z)
XYPLOT ACCE RESP / 30000(T3)
TCURVE=               GRID 216(Z)
XYPLOT ACCE RESP / 216(T3)
TCURVE=               GRID 30014(Z)
XYPLOT ACCE RESP / 30014(T3)
TCURVE=               GRID 232(Z)
XYPLOT ACCE RESP / 232(T3)

108
TCURVE= GRID 30028(Z)
XYPLOT ACCE RESP / 30028(T3)

TCURVE= GRID 248(Z)
XYPLOT ACCE RESP / 248(T3)

TCURVE= GRID 30042(Z)
XYPLOT ACCE RESP / 30042(T3)

$---------------Z BOLT FORCE ---------------
$spring extension/contraction
YTLOG = YES
XTGRID = YES
YTGRID = YES
XBGRID = YES
YBGRID = YES
YTTITLE=I/F RSP FRC - MAG
YBTITLE=I/F RSP FRC - PHASE

TCURVE=BUSH(Z) 30000
XYPLOT ELFORCE RESP / 30000(4,10)

TCURVE=BUSH(Z) 30014
XYPLOT ELFORCE RESP / 30014(4,10)

TCURVE=BUSH(Z) 30028
XYPLOT ELFORCE RESP / 30028(4,10)

TCURVE=BUSH(Z) 30042
XYPLOT ELFORCE RESP / 30042(4,10)

BEGIN BULK
$
$*
$*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$*
$* BULK DATA
$*
$*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$*
$ no rigid body modes below 0.1 Hz
param,lfreq,0.1
$ 3*range=3*200Hz
eigrl,1,,600.
$ PARAM,CHECKOUT,YES
PARAM,GRDPNT,0
$ param,post,-2
$---------------------------------------------------------------------
$ X-AXIS FREQUENCY MODEL: UNIT ACCELERATION
$---------------------------------------------------------------------
$*
$ resonating mass = 10,000,000 * spacecraft mass
SPOINT,199999
CMASS4,15827,1.E+8,199999
$\star$
$\star$ Join SPOINT to GRID
$\star$ allow movement along X-axis
$\star$ inertia relief along X-axis
$\star$ 1% critical
$\star$ define solution frequencies
$\star$ modal freq 2-200hz as soln pts
$\star$ freq4 defines excitation freq using a spread about each normal mode
$\star$ half-power band width around peak response is 2*critdamp*fN where critdamp=1% so set FSPD=critdamp
$\star$ 9 points around 1/2-power
$\star$ 9 points around 1/2-power
$\star$ 9 points around 1/2-power
$\star$ 9 points around 1/2-power
$\star$ uniform freq spread as soln pts - start @ 2Hz then add 198*1.==198Hz so stop @ 200Hz
$\star$ uniform freq spread as soln pts - start @ 2Hz then add 198*1.==198Hz so stop @ 200Hz
$\star$ apply constant (across all freq) unit acceleration at resonator
$\star$ scale force to resonator mass ==> dynamic load to spoint 199999 = m*g = 1.0E+8*1.0in/sec^2
$\star$ scale force to resonator mass ==> dynamic load to spoint 199999 = m*g = 1.0E+8*1.0in/sec^2
$\star$ scale cos input by unit accel ==> C(f)=1.
$\star$ scale cos input by unit accel ==> C(f)=1.
$\star$TABLED1IDXAXIS YAXIS

110
TABLED1 1
+TB1
+TB1 .0  1.  600.  1.  ENDT
$**************************************************************************
include mass.bdf
ENDDATA
APPENDIX C – INPUT FOR Y-EXCITATION FREQUENCY RESPONSE ANALYSIS
*EXECUTIVE CONTROL*

* CASE CONTROL

* Based on Xtrnl SEs: 0=adptr,10=apndg1,20=apndg2,30=apndg3,40=chassis, 50=module,60=modulsuprt,70=solrpnl1,80=solrpnl2

TITLE = FREQ RESP TO Y-TRAN UNIT ACCEL
SUBTITLE = VIRTUAL MASS BASE-RESONATOR
ECHO = none

TFL=200000
MPC=300000

$ resonating mass set commented out for this run which is elastic only  
$ w/o rigid body modes (eigen extraction f>0) the constant (non oscillatory=>f=0)  
$ accel applied to mass can't be plotted  
$ SET 1 = 99999,199999  
$ **** Monitored Spring Nodes ****  
SET 1 = 200,216,232,248,30000,30014,30028,30042  
$ **** Monitored Springs ****  
SET 2 = 30000,30014,30028,30042  
$ monitored interface accelerations, paired nodes from springs  
$ label : 30000  
$ nodes 1 - 2 : 200, 30000

-----------------------------------------------  
$ label : 30014  
$ nodes 1 - 2 : 216, 30014

-----------------------------------------------  
$ label : 30028  
$ nodes 1 - 2 : 232, 30028

-----------------------------------------------  
$ label : 30042
nodes  1 -  2          : 248, 30042

SET 6 = 1 thru 100

modal KE for modes 1-100 (magnitude and phase to 200 Hz)
MODALKE(punch,phase,esort=descend,thresh=.001)=6

request magnitude and phase
DISP(phase,plot,print)=1

VELO(PLOT)=1

DISP(PLOT)=ALL

ELFORCE(phase,punch,print)=2

ACCE(phase,punch,print)=1

DISP(plot)=1

SPC=1  $ one constraint point

METHOD=1

SUBCASE 1

LABEL=ESTABLISH Y-TRAN FRF TO 200 Hz

FREQ=1

SDAMP=1

DLOAD=11

$ SKIPON

OUTPUT(XYPLOT)

CScale=1.8

XPAPER=26.0

YPAPER=20.0

XGRID LINES=YES

YGRID LINES=YES

XDIVISION=10

XTITLE=                             FREQUENCY(HERTZ)

$--------------------------------------

YTITLE=        RESP ACCEL - (IN/SEC2)

TCURVE=               GRID 200(Z)

XYPLOT ACCE RESP / 200(T3)

TCURVE=               GRID 30000(Z)

XYPLOT ACCE RESP / 30000(T3)

TCURVE=               GRID 216(Z)

XYPLOT ACCE RESP / 216(T3)

TCURVE=               GRID 30014(Z)

XYPLOT ACCE RESP / 30014(T3)

TCURVE=               GRID 232(Z)

XYPLOT ACCE RESP / 232(T3)

TCURVE=               GRID 30028(Z)

XYPLOT ACCE RESP / 30028(T3)

TCURVE=               GRID 248(Z)
XYPLOT ACCE RESP / 248(T3)
TCURVE= GRID 30042(Z)
XYPLOT ACCE RESP / 30042(T3)
$--------------------------------------$
$spring extension/contraction$
YTLOG = YES
XTGRID = YES
YTGRID = YES
XBGRID = YES
YBGRID = YES
YTTITLE=I/F RSP FRC - MAG
YBTITLE=I/F RSP FRC - PHASE
TCURVE=BUSH(Z) 30000
XYPLOT ELFORCE RESP / 30000(4,10)
TCURVE=BUSH(Z) 30014
XYPLOT ELFORCE RESP / 30014(4,10)
TCURVE=BUSH(Z) 30028
XYPLOT ELFORCE RESP / 30028(4,10)
TCURVE=BUSH(Z) 30042
XYPLOT ELFORCE RESP / 30042(4,10)
BEGIN BULK
$
$*
$*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$*
$* BULK DATA
$*
$*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$*$
$ no rigid body modes below 0.1 Hz
param,lfreq,0.1
$ 3*range=3*200Hz
eigrl,1,,600.
$ PARAM,CHECKOUT,YES
PARAM,GRDPNT,0
param,post,-2
$-----------------------------------------------
$ Y-AXIS FREQUENCY MODEL: UNIT ACCELERATION
$-----------------------------------------------
$*
$ resonating mass = 10,000,000 * spacecraft mass
SPOINT,199999
CMASS4,15827,1.E+8,199999
$*
$ Join SPOINT to GRID
$ 300000 199999 0 1. 99999 2 -1.

$ allow movement along Y-axis
SPC 1 99999 13456 0.0
$ inertia relief along Y-axis
SUPORT,99999,2

$----------------------

$ 1% critical
TABDMP1,1,CRIT,,,,,+,+TBD1
+TBD1,0.,01,300.,01,ENDT
$ define solution frequencies
$ modal freq 2-200hz as soln pts
$ freq4 defines excitation freq using a spread about each normal mode
$ half-power band width around peak response is 2*critdamp*fN where critdamp=1% so set
FSPD=critdamp
$  *
$  *
$   9 points around 1/2-power
$  *
$  *
$ $|--1/2 pwr--|
$  +
$  +
$  +
$  +
$FREQ4 SID F1 F2 FSPD NFM
FREQ4 1 2. 200. .01 9

$ even spaced spectral freqs as soln pts - start @ 2Hz then add 198*1.=198Hz so stop @ 200Hz
$FREQ1 SID F1 DF NDF
FREQ1 1 2. 1. 198
$ apply a constant (across all freq) unit acceleration at resonator
$ scale force to resonator mass ===> dynamic load to spoint 199999 = m*g = 1.0E+8*1.0in/sec^2
  = 1.0E+8
$ P = A * C(f) * COS(wt) ===> A=1.0E+8LBF, C(f)=1.0 0-300Hz
$ excitation - spatial
$DAREA SID SPOINT DOF A(LBF)
DAREA 5 199999 0 1.E+8
$ excitation - temporal
$RLOAD1 SID DAREA DELAY DPHASE TC TD
RLOAD1 11 5 1
$ scale cos input by unit accel ===> C(f)=1.
$TABLED1IDXAXIS YAXIS
+TB1
+TB1 .0 1. 300. 1. ENDT
include mass.bdf
ENDDATA
NOTE: XSDA1ES.DAT (this file) is designated as the template file & shown in its entirety. For the remaining runs, only the input that differs with this template is presented.

$*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$*
EXECUTIVE CONTROL
$*
$*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
*
ID TB $ 
TIME 600
SOL 200 $ optimization
DIAG 5,6,8,56
GEOMCHECK NONE
CEND
$*
$*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$*
CASE CONTROL
$*
$*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$*
$ Based on Xtrnl SEs: 0=adptr,10=apndg1,20=apndg2,30=apndg3,40=chassis,
$ 50=module,60=modulsuprt,70=solrpnl1,80=solrpnl2
$ 
TITLE = Mass Optimization
SUBTITLE = Eigenvalues Constraints
ECHO = NONE
$ TFL=200000
MPC=300000
SPC=1 $ one constraint point
METHOD=1
$ output design sensitivity coeff
DSAPRT(FORM,START=1,BY=1,END=LAST)=ALL
$ output design sensitivity coeff and exit w/o opti
$ DSAPRT(FORM,START=1,BY=1,END=SENS)=ALL
SUBCASE 1
DESSUB=300 $ freq constraint
LABEL=Modal
ANALYSIS = MODES $ eigen
$ MODTRAK = 10
SUBCASE 2
LABEL=RUN TO CORRELATE W/ B/L I/F ACCEL
desobj(min) = 207
ANALYSIS = MFREQ $ modal freq
FREQ=1
SDAMP=1
DLOAD=11
$ resonating mass set commented out for this run which is elastic only
$ w/o rigid body modes (eigen extraction f>0) the constant (non oscillatory=>f=0)
$ accel applied to mass can't be plotted
$  SET 1 = 99999,199999
$**** Monitored Spring Nodes ****
SET 1 = 200,216,232,248,30000,30014,30028,30042
$**** Monitored Springs ****
SET 2 = 30000,30014,30028,30042
$ monitored interface accelerations, paired nodes from springs
$  label                      : 30000
$   nodes     1 -    2          : 200, 30000
$---------------------------------------------------------------
$  label                      : 30014
$   nodes     1 -    2          : 216, 30014
$---------------------------------------------------------------
$  label                      : 30028
$   nodes     1 -    2          : 232, 30028
$---------------------------------------------------------------
$  label                      : 30042
$   nodes     1 -    2          : 248, 30042
$---------------------------------------------------------------
$ ELFORCE(phase,punch,print)=2
ACCE(phase,punch,print)=1
$---------------------------- PLOT ----------------------------
OUTPUT(XYPLOT)
CSCALE=1.8
XPAPER=26.0
YPAPER=20.0
XGRID LINES=YES
YGRID LINES=YES
XDIVISION=10
XTITLE=                             FREQUENCY(HERTZ)
$--------------------------------------
YTITLE=        RESP ACCEL - (IN/SEC2)
TCURVE=               GRID 200(Z)
XYPLOT ACCE RESP / 200(T3)
TCURVE=               GRID 30000(Z)
XYPLOT ACCE RESP / 30000(T3)
TCURVE=               GRID 216(Z)
XYPLOT ACCE RESP / 216(T3)
TCURVE=               GRID 30014(Z)
XYPLOT ACCE RESP / 30014(T3)
START OF TEXT

TCURVE= GRID 232(Z)
XYPLOT ACCE RESP / 232(T3)
TCURVE= GRID 30028(Z)
XYPLOT ACCE RESP / 30028(T3)
TCURVE= GRID 248(Z)
XYPLOT ACCE RESP / 248(T3)
TCURVE= GRID 30042(Z)
XYPLOT ACCE RESP / 30042(T3)

$--------------------------------------$

$ spring extension/contraction
YTLOG = YES
XTGRID = YES
YTGRID = YES
XBGRID = YES
YBGRID = YES
YTTITLE=I/F RSP FRC - MAG
YBTITLE=I/F RSP FRC - PHASE
TCURVE=BUSH(Z) 30000
XYPLOT ELFORCE RESP / 30000(4,10)
TCURVE=BUSH(Z) 30014
XYPLOT ELFORCE RESP / 30014(4,10)
TCURVE=BUSH(Z) 30028
XYPLOT ELFORCE RESP / 30028(4,10)
TCURVE=BUSH(Z) 30042
XYPLOT ELFORCE RESP / 30042(4,10)

$--------------------------------------$

BEGIN BULK

$ no rigid body modes below 0.1 Hz
param,lfreq,0.1
$ 3*range of interest=3*200Hz
eigrl,1,,600.
$ PARAM,CHECKOUT,YES
PARAM,GRDPNT,0
$ param,post,-2

$---------------------------------------------------------------------

$ X-AXIS FREQUENCY MODEL: UNIT ACCELERATION
$---------------------------------------------------------------------

END OF TEXT
$ resonating mass = 10,000,000 spacecraft mass=4.481821
SPOINT,199999
CMASS4,15827,1.E+8,199999
$*
$ Join SPOINT to GRID
$-------2-------3 dep 4-------5-------6--indp-7-------8-------
MPC       300000  199999       0      1.   99999       1     -1.
$*
$ allow movement along X-axis
SPC            1   99999   23456     0.0
$ inertia relief along X-axis
SUPORT,99999,1
$----------------------
$ 1% critical
TABDMP1,1,CRIT,,,,,+TBD1
+TBD1.,0.,01,600.,01,ENDT
$*
$ define solution frequencies
$**********************************MODAL***********************************
* $ modal freq 2-200hz as soln pts
$ freq4 defines excitation freq using a spread about each normal mode
$ half-power band width around peak response is 2*critdamp*fN where critdamp=1% so set
FSPD=critdamp
$   *
$   *
$   *   9 points around 1/2-power
$   *
$   *
$   *
$   |
$   *
$   *
$   *
$   *
$FREQ4   SID F1 F2 FSPD NFM
FREQ4 1   2. 200. .01 9
$**********************************EVEN*************************************
* $ even spaced spectral freqs as soln pts - start @ 2Hz then add 198*1.=198Hz so stop @ 200Hz
$FREQ1   SID F1 DF NDF
$ FREQ1 1   2. 1. 198
$**********************************DISCRETE***********************************
* $ define solution frequencies
$ discrete excitation pts based on SOL103/SOL111 0-200 Hz eigenvalues. the only difference
between
SOL103 freqs and SOL200 is SOL200 mode 1 = 0. since SOL200 uses SUPORT card then have rigid body

and this causes all freqs to shift down by 1 mode wrt SOL103 (ref the f06 eigen tables)

FREQ SID F1  F2  F3  F4  F5  F6  F7
                37.568 42.688 47.080 47.091 47.840 47.840 47.840
                50.557 54.721 58.588 59.244 62.797 62.797 62.797
                63.924 66.176 69.412 70.294 70.294 70.294 70.294
                90.766 98.076 102.764 104.822 106.063 109.733 111.045
                111.222 111.855 113.055 113.120 114.821 115.953 116.635
                118.155 119.914 123.766 126.938 127.244 132.899 134.822
                137.040 138.157 140.565 141.592 146.447 146.668 147.133
                158.342 158.394 158.446 159.179 160.633 164.063 169.368
                184.224 184.410 187.274 188.193 190.985 192.790 193.767
                195.472 197.864 199.549

apply a constant (across all freq) unit acceleration at resonator

scale force to resonator mass ==> dynamic load to spoint 199999 = m*g = 1.0E+8*1.0in/sec^2 = 1.0E+8

P = A * C(f) * COS(wt) ==> A=1.0E+8LBF, C(f)=1.0 0-300Hz

scale cos input by unit accel ==> C(f)=1.

TABLED1IDXAXIS YAXIS

+TB1 .0 1. 600. 1. ENDT

DESVAR,201,ro6061,.997,.997,1.003
DVMREL1,201,MAT1,6061,RHO,,,,,+,
+201,0.000259
DESVAR,202,ro206061,.997,.997,1.003
DVMREL1,202,MAT1,206061,RHO,,,,,+
DESVAR, 217, cm20333, 955, 955, 1.045
DVCREL1, 217, CONM2, 20333, M,,,,,
+ 217, 0.043486
DESVAR, 218, cm20361, 955, 955, 1.045
DVCREL1, 218, CONM2, 20361, M,,,,,
+ 218, 0.022999
DESVAR, 219, cm20369, 955, 955, 1.045
DVCREL1, 219, CONM2, 20369, M,,,,,
+ 219, 0.022999
DESVAR, 220, cm20401, 955, 955, 1.045
DVCREL1, 220, CONM2, 20401, M,,,,,
+ 220, 0.040689
DESVAR, 221, cm20404, 955, 955, 1.045
DVCREL1, 221, CONM2, 20404, M,,,,,
+ 221, 0.008832
DESVAR, 222, cm20407, 955, 955, 1.045
DVCREL1, 222, CONM2, 20407, M,,,,
+ 222, 0.008832
DESVAR, 223, cm20412, 955, 955, 1.045
DVCREL1, 223, CONM2, 20412, M,,,,
+ 223, 0.031080
DESVAR, 224, cm20433, 955, 955, 1.045
DVCREL1, 224, CONM2, 20433, M,,,,
+ 224, 0.008832
DESVAR, 225, cm20436, 955, 955, 1.045
DVCREL1, 225, CONM2, 20436, M,,,,
+ 225, 0.008832
DESVAR, 226, cm20439, 955, 955, 1.045
DVCREL1, 226, CONM2, 20439, M,,,,
+ 226, 0.008832
DESVAR, 227, cm20444, 955, 955, 1.045
DVCREL1, 227, CONM2, 20444, M,,,,
+ 227, 0.031080
DESVAR, 228, cm20604, 955, 955, 1.045
DVCREL1, 228, CONM2, 20604, M,,,,
+ 228, 0.074333
DESVAR, 229, cm20620, 955, 955, 1.045
DVCREL1, 229, CONM2, 20620, M,,,,
+ 229, 0.002253
DESVAR, 230, cm20625, 955, 955, 1.045
DVCREL1, 230, CONM2, 20625, M,,,,
+ 230, 0.169671
DESVAR, 231, cm20641, 955, 955, 1.045
DVCREL1, 231, CONM2, 20641, M,,,,
+ 231, 0.006812
DESVAR, 232, cm20646, 955, 955, 1.045
DVCREL1,232,CONM2,20646,M,,,+,232,0.016990
DESVAR,233,cm20650,.955,.955,1.045
DVCREL1,233,CONM2,20650,M,,,+,233,0.027195
DESVAR,234,cm20652,.955,.955,1.045
DVCREL1,234,CONM2,20652,M,,,+,234,0.002253
DESVAR,235,cm20668,.955,.955,1.045
DVCREL1,235,CONM2,20668,M,,,+,235,0.031313
DESVAR,236,cm20673,.955,.955,1.045
DVCREL1,236,CONM2,20673,M,,,+,236,0.013209
DESVAR,237,cm20675,.955,.955,1.045
DVCREL1,237,CONM2,20675,M,,,+,237,0.013209
DESVAR,238,cm20684,.955,.955,1.045
DVCREL1,238,CONM2,20684,M,,,+,238,0.034965
DESVAR,239,cm20693,.955,.955,1.045
DVCREL1,239,CONM2,20693,M,,,+,239,0.049081
DESVAR,240,cm20713,.955,.955,1.045
DVCREL1,240,CONM2,20713,M,,,+,240,0.003393
DESVAR,241,cm20714,.955,.955,1.045
DVCREL1,241,CONM2,20714,M,,,+,241,0.002953
DESVAR,242,cm20715,.955,.955,1.045
DVCREL1,242,CONM2,20715,M,,,+,242,0.003393
DESVAR,243,cm20722,.955,.955,1.045
DVCREL1,243,CONM2,20722,M,,,+,243,0.034965
DESVAR,244,cm20729,.955,.955,1.045
DVCREL1,244,CONM2,20729,M,,,+,244,0.028050
DESVAR,245,cm20734,.955,.955,1.045
DVCREL1,245,CONM2,20734,M,,,+,245,0.002953
DESVAR,246,cm20750,.955,.955,1.045
DVCREL1,246,CONM2,20750,M,,,+,246,0.002953
DESVAR,247,cm20778,.955,.955,1.045
DVCREL1,247,CONM2,20778,M,,,+
DESVAR, 248, cm20904, 0.955, 0.955, 1.045
DVCREL1, 248, CONM2, 20904, M, +
+ 248, 0.035975
DESVAR, 249, cm20910, 0.955, 0.955, 1.045
DVCREL1, 249, CONM2, 20910, M, +
+ 249, 0.014841
DESVAR, 250, cm20955, 0.955, 0.955, 1.045
DVCREL1, 250, CONM2, 20955, M, +
+ 250, 0.029785
DESVAR, 251, cm20974, 0.955, 0.955, 1.045
DVCREL1, 251, CONM2, 20974, M, +
+ 251, 0.031987
DESVAR, 252, cm20976, 0.955, 0.955, 1.045
DVCREL1, 252, CONM2, 20976, M, +
+ 252, 0.185600
DESVAR, 253, cm20982, 0.955, 0.955, 1.045
DVCREL1, 253, CONM2, 20982, M, +
+ 253, 0.020306
DESVAR, 254, cm20986, 0.955, 0.955, 1.045
DVCREL1, 254, CONM2, 20986, M, +
+ 254, 0.137840
DESVAR, 255, cm20996, 0.955, 0.955, 1.045
DVCREL1, 255, CONM2, 20996, M, +
+ 255, 0.086454
DESVAR, 256, cm21025, 0.955, 0.955, 1.045
DVCREL1, 256, CONM2, 21025, M, +
+ 256, 0.023414
DESVAR, 257, cm21029, 0.955, 0.955, 1.045
DVCREL1, 257, CONM2, 21029, M, +
+ 257, 0.014711
DESVAR, 258, cm21036, 0.955, 0.955, 1.045
DVCREL1, 258, CONM2, 21036, M, +
+ 258, 0.016550
DESVAR, 259, cm21133, 0.955, 0.955, 1.045
DVCREL1, 259, CONM2, 21133, M, +
+ 259, 0.066330
DESVAR, 260, cm21827, 0.955, 0.955, 1.045
DVCREL1, 260, CONM2, 21827, M, +
+ 260, 0.002253
DESVAR, 261, cm21830, 0.955, 0.955, 1.045
DVCREL1, 261, CONM2, 21830, M, +
+ 261, 0.002253
DESVAR, 262, cm21862, 0.955, 0.955, 1.045
DVCREL1, 262, CONM2, 21862, M, +
+ 262, 0.002253
DVCREL1,278,CONM2,22015,M,,,,,+
+.,278,0.004507
DESVAR,279,cm22025,.955,.955,1.045
DVCREL1,279,CONM2,22025,M,,,,,+
+.,279,0.001295
DESVAR,280,cm22113,.955,.955,1.045
DVCREL1,280,CONM2,22113,M,,,,,+
+.,280,0.004507
DESVAR,281,cm22115,.955,.955,1.045
DVCREL1,281,CONM2,22115,M,,,,,+
+.,281,0.004507
DESVAR,282,cm22125,.955,.955,1.045
DVCREL1,282,CONM2,22125,M,,,,,+
+.,282,0.001295
$*
$* design var = nonstructural mass (PBAR-->field 8)
$ 1-sigma Gaussian
DESVAR,283,pb107000,.683,.683,1.317
DVPREL1,283,PBAR,107000,NSM,,,,,+
+.,283,0.000188
DESVAR,284,pb307000,.683,.683,1.317
DVPREL1,284,PBAR,307000,NSM,,,,,+
+.,284,0.000188
DESVAR,285,pb401400,.683,.683,1.317
DVPREL1,285,PBAR,401400,NSM,,,,,+
+.,285,0.000179
DESVAR,286,pb401500,.683,.683,1.317
DVPREL1,286,PBAR,401500,NSM,,,,,+
+.,286,0.000190
DESVAR,287,pb401510,.683,.683,1.317
DVPREL1,287,PBAR,401510,NSM,,,,,+
+.,287,0.000172
DESVAR,288,pb401600,.683,.683,1.317
DVPREL1,288,PBAR,401600,NSM,,,,,+
+.,288,0.000132
DESVAR,289,pb702300,.683,.683,1.317
DVPREL1,289,PBAR,702300,NSM,,,,,+
+.,289,0.000181
DESVAR,290,pb702301,.683,.683,1.317
DVPREL1,290,PBAR,702301,NSM,,,,,+
+.,290,0.000153
DESVAR,291,pb702400,.683,.683,1.317
DVPREL1,291,PBAR,702400,NSM,,,,,+
+.,291,0.000357
DESVAR,292,pb702401,.683,.683,1.317
DVPREL1,292,PBAR,702401,NSM,,,,,+

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+292,0.000357
DESVAR,293,pb702402,683,683,1.317
DVPREL1,293,PBAR,702402,NSM,,,,,+ 
+293,0.000357
DESVAR,294,pb702405,683,683,1.317
DVPREL1,294,PBAR,702405,NSM,,,,,+ 
+294,0.000068
DESVAR,295,pb702406,683,683,1.317
DVPREL1,295,PBAR,702406,NSM,,,,,+ 
+295,0.000068
DESVAR,296,pb702407,683,683,1.317
DVPREL1,296,PBAR,702407,NSM,,,,,+ 
+296,0.000068
DESVAR,297,pb702408,683,683,1.317
DVPREL1,297,PBAR,702408,NSM,,,,,+ 
+297,0.000068
DESVAR,298,pb702410,683,683,1.317
DVPREL1,298,PBAR,702410,NSM,,,,,+ 
+298,0.000067
DESVAR,299,pb702411,683,683,1.317
DVPREL1,299,PBAR,702411,NSM,,,,,+ 
+299,0.000067
DESVAR,300,pb702412,683,683,1.317
DVPREL1,300,PBAR,702412,NSM,,,,,+ 
+300,0.000067
DESVAR,301,pb702413,683,683,1.317
DVPREL1,301,PBAR,702413,NSM,,,,,+ 
+301,0.000067
DESVAR,302,pb802400,683,683,1.317
DVPREL1,302,PBAR,802400,NSM,,,,,+ 
+302,0.000357
DESVAR,303,pb802401,683,683,1.317
DVPREL1,303,PBAR,802401,NSM,,,,,+ 
+303,0.000357
DESVAR,304,pb802402,683,683,1.317
DVPREL1,304,PBAR,802402,NSM,,,,,+ 
+304,0.000357
DESVAR,305,pb802405,683,683,1.317
DVPREL1,305,PBAR,802405,NSM,,,,,+ 
+305,0.000068
DESVAR,306,pb802406,683,683,1.317
DVPREL1,306,PBAR,802406,NSM,,,,,+ 
+306,0.000068
DESVAR,307,pb802407,683,683,1.317
DVPREL1,307,PBAR,802407,NSM,,,,,+ 
+307,0.000068
DESVAR,308,pb802408,.683,.683,1.317
DVPREL1,308,PBAR,802408,NSM,,,,,+ 
+308,0.000068
DESVAR,309,pb802410,.683,.683,1.317
DVPREL1,309,PBAR,802410,NSM,,,,,+ 
+309,0.000067
DESVAR,310,pb802411,.683,.683,1.317
DVPREL1,310,PBAR,802411,NSM,,,,,+ 
+310,0.000067
DESVAR,311,pb802412,.683,.683,1.317
DVPREL1,311,PBAR,802412,NSM,,,,,+ 
+311,0.000067
DESVAR,312,pb802413,.683,.683,1.317
DVPREL1,312,PBAR,802413,NSM,,,,,+ 
+312,0.000067

$*
$* design var = nonstructural mass (PSHELL-->field 9)
$ 1-sigma Gaussian
DESVAR,313,ps4000,.683,.683,1.317
DVPREL1,313,PSHELL,4000,NSM,,,,,+ 
+313,0.000007
DESVAR,314,ps4300,.683,.683,1.317
DVPREL1,314,PSHELL,4300,NSM,,,,,+ 
+314,0.000007
DESVAR,315,ps4400,.683,.683,1.317
DVPREL1,315,PSHELL,4400,NSM,,,,,+ 
+315,0.000007
DESVAR,316,ps400300,.683,.683,1.317
DVPREL1,316,PSHELL,400300,NSM,,,,,+ 
+316,0.000107
DESVAR,317,ps400310,.683,.683,1.317
DVPREL1,317,PSHELL,400310,NSM,,,,,+ 
+317,0.000139
DESVAR,318,ps400600,.683,.683,1.317
DVPREL1,318,PSHELL,400600,NSM,,,,,+ 
+318,0.000070
DESVAR,319,ps400601,.683,.683,1.317
DVPREL1,319,PSHELL,400601,NSM,,,,,+ 
+319,0.000056
DESVAR,320,ps400602,.683,.683,1.317
DVPREL1,320,PSHELL,400602,NSM,,,,,+ 
+320,0.000062
DESVAR,321,ps400603,.683,.683,1.317
DVPREL1,321,PSHELL,400603,NSM,,,,,+ 
+321,0.000056
DESVAR,322,ps400604,.683,.683,1.317

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DVPREL1,322,PSHELL,400604,NSM,,,,,+
 +,322,0.000067
DESVAR,323,ps400605,.683,.683,1.317
DVPREL1,323,PSHELL,400605,NSM,,,,,+
 +,323,0.000067
DESVAR,324,ps400606,.683,.683,1.317
DVPREL1,324,PSHELL,400606,NSM,,,,,+
 +,324,0.000068
DESVAR,325,ps400607,.683,.683,1.317
DVPREL1,325,PSHELL,400607,NSM,,,,,+
 +,325,0.000059
DESVAR,326,ps400610,.683,.683,1.317
DVPREL1,326,PSHELL,400610,NSM,,,,,+
 +,326,0.000093
DESVAR,327,ps400615,.683,.683,1.317
DVPREL1,327,PSHELL,400615,NSM,,,,,+
 +,327,0.000090
DESVAR,328,ps400616,.683,.683,1.317
DVPREL1,328,PSHELL,400616,NSM,,,,,+
 +,328,0.000092
DESVAR,329,ps400620,.683,.683,1.317
DVPREL1,329,PSHELL,400620,NSM,,,,,+
 +,329,0.000189
DESVAR,330,ps400621,.683,.683,1.317
DVPREL1,330,PSHELL,400621,NSM,,,,,+
 +,330,0.000284
DESVAR,331,ps400622,.683,.683,1.317
DVPREL1,331,PSHELL,400622,NSM,,,,,+
 +,331,0.000184
DESVAR,332,ps400623,.683,.683,1.317
DVPREL1,332,PSHELL,400623,NSM,,,,,+
 +,332,0.000116
DESVAR,333,ps400624,.683,.683,1.317
DVPREL1,333,PSHELL,400624,NSM,,,,,+
 +,333,0.000189
DESVAR,334,ps400625,.683,.683,1.317
DVPREL1,334,PSHELL,400625,NSM,,,,,+
 +,334,0.000173
DESVAR,335,ps400626,.683,.683,1.317
DVPREL1,335,PSHELL,400626,NSM,,,,,+
 +,335,0.000190
DESVAR,336,ps400627,.683,.683,1.317
DVPREL1,336,PSHELL,400627,NSM,,,,,+
 +,336,0.000120
DESVAR,337,ps400630,.683,.683,1.317
DVPREL1,337,PSHELL,400630,NSM,,,,,+
DESVAR,337,ps400635,683,683,1.317
DVPREL1,338,PSHELL,400635,NSM,,,,,+ 
+338,0.000196
DESVAR,339,ps401000,683,683,1.317
DVPREL1,339,PSHELL,401000,NSM,,,,,+ 
+339,0.000086
DESVAR,340,ps401100,683,683,1.317
DVPREL1,340,PSHELL,401100,NSM,,,,,+ 
+340,0.000110
DESVAR,341,ps702002,683,683,1.317
DVPREL1,341,PSHELL,702002,NSM,,,,,+ 
+341,0.000027
DESVAR,342,ps802000,683,683,1.317
DVPREL1,342,PSHELL,802000,NSM,,,,,+ 
+342,0.000025
DESVAR,343,ps802001,683,683,1.317
DVPREL1,343,PSHELL,802001,NSM,,,,,+ 
+343,0.000024

$*
$ f1ds: 7 is accl component for grid (1=Ax,2=Ay,3=Az) | 8 is freq @ max accl | 9 is grid ID
$ grid A of bush
DRESP1,401,Az200,FRACCL,,,3,36.566,200
DRESP1,403,Az216,FRACCL,,,3,37.474,216
DRESP1,405,Az232,FRACCL,,,3,36.566,232
DRESP1,407,Az248,FRACCL,,,3,37.38,248

$*
$ specify and label x,y and z maximum objective interface accels (from B/L output) as design responses:
DTABLE    OAz200   20.47  OAz216    7.30  OAz232   19.00  OAz248    7.55

$ 2 3 4 5 6 7 8 9
DRESP2       207    ACCL     101
DTABLE  OAz200  OAz216  OAz232  OAz248
DRESP1     401     403     405     407

$*
$ specify minimization of error function as objective
DEQATN 101 F(OAz200,OAz216,OAz232,OAz248,Az200,Az216,Az232, 
Az248) = ABS(Az200-OAz200)+ABS(Az216-OAz216)+ABS(Az232-
OAz232)+ABS(Az248-OAz248)

$
$**********define the normal modes design contraints**********
$* assume modes are well known from testing (2 sigma ==> 0.955 - 1.045)
$ use 2 sigma spread to define rsp freq rng (f1ds 5 & 6)
$
$ Mode 8 has greatest modal mass. From sol 111 B/L run know freq range that elicits 
mode 8 response is: 36.000 - 38.000
f8=36.5656
DRESP1,415,F8,FREQ,,,8
$ when forcing freq is between flds 6 & 7, constrained response between flds 4 & 5
$ 2 3 4 5 6 7 8
DCONSTR 300 415 34.92 38.21 36.00 38.00
$
$ Mode 9 coupled w/ mode 8. From sol 111 B/L run know frcng freq range that elicits mode 9 response is: 37.1919 - 38.000
$ f9=37.5676
DRESP1,416,F9,FREQ,,9
$ when forcing freq is between flds 6 & 7, constrained response between flds 4 & 5
$ 2 3 4 5 6 7 8
DCONSTR 300 416 35.88 39.26 37.19 38.00
$
$ Mode 18. From sol 111 B/L run know frcng freq range that elicits mode 18 response is: 51.796 - 52.847
$ f18=52.3193
DRESP1,417,F18,FREQ,,18
$ when forcing freq is between flds 6 & 7, constrained response between flds 4 & 5
$ 2 3 4 5 6 7 8
DCONSTR 300 417 49.96 54.67 51.80 52.85
$
$ Mode 22. From sol 111 B/L run know frcng freq range that elicits mode 21 response is: 58.2955 - 58.8814
$ f22=58.5885
DRESP1,418,F22,FREQ,,22
$ when forcing freq is between flds 6 & 7, constrained response between flds 4 & 5
$ 2 3 4 5 6 7 8
DCONSTR 300 418 55.95 61.22 58.30 58.88
$ DSCREEN FREQ -9999.
$+++++++++++++++++++++++++++++++++++++++++++++++++++
$ relative convergence criteria (CONV1 default 0.001)and absolute conv criteria (CONV2 default 1.e-20)
DOPTPRM,DESMAX,30,P1,1,P2,15,CONV1,0.001
,CONV2,1.0E-20,CTMIN,.003,IPRINT,1,ITRMOP,2
,DELX,.5,CONVDV,.001,CONVPR,.001,APRCOD,1
,OPTCOD,DOT
$++++++++++++++++++++++++++++++++++++++++++++++++++++
include 'mass.bdf'
ENDDATA
APPENDIX E – INPUT DIFFERENCES FOR SENSITIVITY ANALYSIS AND OPTIMIZATION
NOTE: XSDA1ES.DAT is designated as the template file & shown in its entirety. For the remaining runs, only the input that differs with this template is presented.

Figure 86 Xsda1es vs Xsda1mb

Figure 87 Xsda1es vs Xsda1ml
Figure 88 Xsda1es vs Xsda2es

Figure 89 Xsda1es vs Xsda2mb

Figure 90 Xsda1es vs Xsda2ml

137
Figure 91 Xsda1es vs Xsma1es

Figure 92 Xsda1es vs Xsma1mb

Figure 93 Xsda1es vs Xsma1ml
Figure 94 Xsda1es vs Xsma2es

Figure 95 Xsda1es vs Xsma2mb

Figure 96 Xsda1es vs Xsma2ml
APPENDIX F – CORRELATION TABLES OF BASELINE VERSUS UPDATED MODELS FOR X-EXCITATION
**TABLE 5 X-CORRELATION TABLES 1 THRU 3**

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143
### Table 8 X-Correlation Tables 10 thru 12

#### xsma2es

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**Average MAG Corel wrt S111 = 1.002**

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**Average MAG Corel wrt S111 = 1.000**

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**Average MAG Corel wrt S111 = 1.000**

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**Average MAG Corel wrt S111 = 1.016**

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**Average MAG Corel wrt S111 = 1.012**

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**Average MAG Corel wrt S111 = 1.005**

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**Average MAG Corel wrt S111 = 1.001**
APPENDIX G – CORRELATION TABLES OF BASELINE VERSUS UPDATED MODELS FOR Y-EXCITATION


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Average MAG Corel wrt S111 = 1.000

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Average MAG Corel wrt S111 = 1.010

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Average MAG Corel wrt S111 = 1.010
Table 11 Y-Correlation Tables 7 thru 9

|                        | GRID | FREQ | \( f \) | \( \epsilon \) | \( aclMAG \) | \( \epsilon \) | \( mag \) | GRID | FREQ | \( f \) | \( \epsilon \) | \( aclMAG \) | \( \epsilon \) | \( mag \) |
|------------------------|------|------|---------|-------------|-------------|-------------|--------|------|------|---------|-------------|-------------|--------|
| **ysma1es**            |      |      |         |             |             |             |        |      |      |         |             |             |        |
| CONVERGENCE @ CYCLE=9  |      |      |         |             |             |             |        |      |      |         |             |             |        |
|                        | 216  | 37.568 | 1.000  | 21.330      | 1.000       | 216         | 37.568 | 1.000 | 21.334 | 1.000   | 216         | 37.568     | 1.000  |
|                        | 248  | 37.568 | 1.000  | 21.411      | 1.000       | 248         | 37.568 | 1.000 | 21.418 | 1.000   | 248         | 37.568     | 1.000  |
| **Average MAG Corel wrt S111 =** | 1.000 |      |         |             |             |             |        |      |      |         |             |             |        |
| **ysma1mb**            |      |      |         |             |             |             |        |      |      |         |             |             |        |
| CONVERGENCE @ CYCLE=9  |      |      |         |             |             |             |        |      |      |         |             |             |        |
| GRID                   | 30000 | 37.365 | 1.000  | 1.843       | 1.000       | 30000       | 37.365 | 1.000 | 1.843 | 1.000   | 30000       | 37.365     | 1.000  |
|                        | 30014 | 37.568 | 1.000  | 4.700       | 1.001       | 30014       | 37.568 | 1.000 | 4.701 | 1.001   | 30014       | 37.568     | 1.000  |
|                        | 30028 | 36.679 | 0.998  | 1.606       | 0.997       | 30028       | 36.679 | 0.998 | 1.607 | 0.997   | 30028       | 36.679     | 0.998  |
|                        | 30042 | 37.568 | 1.000  | 4.788       | 1.000       | 30042       | 37.568 | 1.000 | 4.790 | 1.000   | 30042       | 37.568     | 1.000  |
| **Average MAG Corel wrt S111 =** | 0.999 |      |         |             |             |             |        |      |      |         |             |             |        |
| **ysma1ml**            |      |      |         |             |             |             |        |      |      |         |             |             |        |
| CONVERGENCE @ CYCLE=14 |      |      |         |             |             |             |        |      |      |         |             |             |        |
| GRID                   | 200  | 37.476 | 1.003  | 8.251       | 1.007       | 200         | 37.476 | 1.003 | 8.271 | 1.007   | 200         | 37.476     | 1.003  |
|                        | 216  | 37.570 | 1.000  | 21.326      | 1.000       | 216         | 37.570 | 1.000 | 21.344 | 0.997   | 216         | 37.570     | 1.000  |
|                        | 232  | 37.476 | 1.003  | 7.406       | 1.004       | 232         | 37.476 | 1.003 | 7.407 | 1.000   | 232         | 37.476     | 1.003  |
|                        | 248  | 37.570 | 1.000  | 21.423      | 1.001       | 248         | 37.570 | 1.000 | 21.427 | 1.000   | 248         | 37.570     | 1.000  |
| **Average MAG Corel wrt S111 =** | 1.002 |      |         |             |             |             |        |      |      |         |             |             |        |
| **ysma1ml.1**          |      |      |         |             |             |             |        |      |      |         |             |             |        |
| CONVERGENCE @ CYCLE=12 |      |      |         |             |             |             |        |      |      |         |             |             |        |
| GRID                   | 30000 | 37.365 | 1.000  | 1.846       | 1.001       | 30000       | 37.365 | 1.000 | 1.851 | 1.004   | 30000       | 37.365     | 1.000  |
|                        | 30014 | 37.568 | 1.000  | 4.681       | 0.997       | 30014       | 37.568 | 1.000 | 4.685 | 0.998   | 30014       | 37.568     | 1.000  |
|                        | 30028 | 36.657 | 0.995  | 1.620       | 1.006       | 30028       | 36.657 | 0.995 | 1.620 | 1.006   | 30028       | 36.657     | 0.995  |
|                        | 30042 | 37.568 | 1.000  | 4.779       | 0.996       | 30042       | 37.568 | 1.000 | 4.772 | 0.996   | 30042       | 37.568     | 1.000  |
| **Average MAG Corel wrt S111 =** | 0.997 |      |         |             |             |             |        |      |      |         |             |             |        |

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Table 12 Y-Correlation Tables 10 thru 12

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APPENDIX H – Z-RESPONSE PLOTS OF ACCELERATION AND FORCE FOR X-EXCITATION
Figure 97 X-Excitation Full Acceleration Response / Bolt 30000
Figure 98 X-Excitation Peak Acceleration Response / Bolt 30000
Figure 99 X-Excitation Full Acceleration Response / Bolt 30014
Figure 100 X-Excitation Peak Acceleration Response / Bolt 30014
Figure 101 X-Excitation Full Acceleration Response / Bolt 30028
Figure 102 X-Excitation Peak Acceleration Response / Bolt 30028
Figure 103 X-Excitation Full Acceleration Response / Bolt 30042
Figure 104 X-Excitation Peak Acceleration Response / Bolt 30042
Figure 105 X-Excitation Full Force Response / Bolt 30000
Figure 106 X-Excitation Peak Force Response / Bolt 30000
Figure 107 X-Excitation Full Force Response / Bolt 30014
Figure 108 X-Excitation Peak Force Response / Bolt 30014
Figure 109 X-Excitation Full Force Response / Bolt 30028
Figure 110 X-Excitation Peak Force Response / Bolt 30028
Figure 111 X-Excitation Full Force Response / Bolt 30042
Figure 112 X-Excitation Peak Force Response / Bolt 30042
APPENDIX I – Z-RESPONSE PLOTS OF ACCELERATION AND FORCE FOR Y-EXCITATION
Figure 113 Y-Excitation Full Acceleration Response / Bolt 30000
Figure 114 Y-Excitation Peak Acceleration Response / Bolt 30000
Figure 115 Y-Excitation Full Acceleration Response / Bolt 30014
Figure 116 Y-Excitation Peak Acceleration Response / Bolt 30014
Figure 117 Y-Excitation Full Acceleration Response / Bolt 30028
Figure 118 Y-Excitation Peak Acceleration Response / Bolt 30028
Figure 119 Y-Excitation Full Acceleration Response / Bolt 30042
Figure 120 Y-Excitation Peak Acceleration Response / Bolt 30042
Figure 121 Y-Excitation Full Force Response / Bolt 30000
Figure 122 Y-Force Peak Response / Bolt 30000
Figure 123 Y-Excitation Full Force Response / Bolt 30014
Figure 124 Y-Force Peak Response / Bolt 30014
Figure 125 Y-Excitation Full Force Response / Bolt 30028
Figure 126 Y-Force Peak Response / Bolt 30028
Figure 127 Y-Excitation Full Force Response / Bolt 30042
Figure 128 Y-Force Peak Response / Bolt 30042
REFERENCES


9 Johnson, E., H., Innovative Uses of Synthetic Responses in Design Optimization, The MacNeal-Schwendler Corporation, Los Angeles, California