Modeling And Analysis Of The Eds Maglev System With The Halbach Magnet Array

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MODELING AND ANALYSIS OF THE EDS MAGLEV SYSTEM WITH
THE HALBACH MAGNET ARRAY

by

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A dissertation submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
in the School of Electrical Engineering and Computer Science
in the College of Engineering and Computer Science
at the University of Central Florida
Orlando, Florida

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2007

Major Professor: Chan Ham
The magnetic field analysis based on the wavelet transform is performed. The Halbach array magnetic field analysis has been studied using many methods such as magnetic scalar potential, magnetic vector potential, Fourier analysis and Finite Element Methods. But these analyses cannot identify a transient oscillation at the beginning stage of levitation. The wavelet transform is used for analyzing the transient oscillatory response of an EDS Maglev system. The proposed scheme explains the under-damped dynamics that results from the cradle’s dynamic response to the irregular distribution of the magnetic field. It suggests this EDS Maglev system that responds to a vertical repulsive force could be subject to such instability at the beginning stage of a low levitation height. The proposed method is useful in analyzing instabilities at the beginning stage of levitation height.

A controller for the EDS maglev system with the Halbach array magnet is designed for the beginning stage of levitation and after reaching the defined levitation height. To design a controller for the EDS system, two different stages are suggested. Before the object reaches a stable position and after it has reached a stable position. A stable position can be referred to as a nominal height. The former is the stage I and the latter is the stage II. At the stage I, to achieve a nominal height the robust controller is investigated. At the stage II, both translational and rotational motions are considered for the control design. To maintain system stability, damping control as well as LQR control are performed. The proposed method is helpful to understand system dynamics and achieve system stability.
To my father, my mother, my sister, my wife Jooyoun Song,
and my lovely daughter Hyeseung.
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<th>Description</th>
</tr>
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<tbody>
<tr>
<td>CAD</td>
<td>Computer Aided Design</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree Of Freedom</td>
</tr>
<tr>
<td>EDS</td>
<td>Electro Dynamic Suspension</td>
</tr>
<tr>
<td>EMS</td>
<td>Electro Magnetic Suspension</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FT</td>
<td>Fourier Transform</td>
</tr>
<tr>
<td>LLNL</td>
<td>Lawrence Livermore National Laboratory</td>
</tr>
<tr>
<td>LQR</td>
<td>Linear Quadratic Regulator</td>
</tr>
<tr>
<td>MAGLEV</td>
<td>Magnetic Levitation</td>
</tr>
<tr>
<td>MQS</td>
<td>Magnetoquasistatic</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>NdFeB</td>
<td>Neodymium Iron Boron</td>
</tr>
<tr>
<td>PM</td>
<td>Permanent Magnet</td>
</tr>
<tr>
<td>PMS</td>
<td>Permanent Magnet System</td>
</tr>
<tr>
<td>PMSM</td>
<td>Permanent Magnet Synchronous Motor</td>
</tr>
<tr>
<td>SC</td>
<td>Superconductor</td>
</tr>
<tr>
<td>WT</td>
<td>Wavelet Transform</td>
</tr>
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1. CHAPTER ONE: INTRODUCTION

1.1. Background

"We may perhaps learn to deprive large masses of their gravity and give them absolute levity, for the sake of easy transport." - Benjamin Franklin, 1780

MAGLEV (Magnetic levitation) uses the magnetic field to make an object be suspended in the air without contacting any surface. The ability to support a load without physical contact makes maglev attractive for use in high-speed transportation systems.

High-speed Maglev systems have been designed and tested from the first patent for a magnetic levitation train propelled by linear motors, German Patent 707032, issued in June 1941 to the first commercial application of a high-speed Maglev line is the initial operating segment demonstration line in Shanghai, China that transports people 30 km (18.6 miles) to the airport in just 7 minutes 20 seconds (top speed of 431 km/h or 268 mph, average speed 250 km/h or 150 mph) [1].

Maglev research has been focused on two systems: ElectroDynamic Suspension (EDS) System and ElectroMagnetic Suspension (EMS) System. The EDS system is based on a repulsive force of magnets. The induction current from either permanent magnets or superconducting magnets as the magnetic field source can be used to not only provide a levitation force but also a guidance force as well [2-4].
The EMS system is based on an attractive force of magnets. The attractive force between electromagnets on the vehicle and guideway lifts up the vehicle. This system is inherently unstable so controlling the gap between the lifted object and the magnetic field source is needed to maintain a balance between gravity and the force of attraction. [2-4].

Figure 1.1 illustrates the concept of EMS and EDS briefly.

![EMS and EDS System Concept](image)

**Figure 1.1 EMS and EDS System Concept**

### 1.2. Research Goal and Motivation

The goals of this research are analysis of the Halbach array magnetic field and design of the controller for the EDS maglev system with the Halbach array magnet.

To investigate this goal, these approaches are performed:

i. A Halbach array analysis based on wavelet transforms.

ii. Control design scheme for the stage I, until the levitation height reaches a
nominal height, using nonlinear robust control algorithm

iii. Control design scheme for the stage II, after the levitation height reached a nominal height, using damping and optimized Linear Quadratic Regulator (LQR) control algorithm.

The motivation of this research is that the Halbach array magnetic field analysis has been studied using many methods such as magnetic scalar potential, magnetic vector potential, Fourier analysis and Finite Element Methods. But these analyses cannot identify a transient oscillation that occurs at the beginning stage of levitation. Wavelet transform is used for analyzing the transient oscillatory response of an EDS Maglev system. The advantage of wavelet transform is that it is localized in both time and frequency, whereas the standard Fourier transform is only localized in frequency.

Levitation is the process by which an object is suspended against gravity, in a stable position, by a force without physical contact [5]. From this definition, two stages can be classified; before the object reaches a stable position and after it has reached a stable position. A stable position is referred to as a nominal height. The former is the stage I and the latter is the stage II.

At the stage I, achieving a nominal height is a key point so a robust controller is suggested. At the stage II, stability is the main target during the system running. To maintain the system stability, damping control and LQR control are proposed.
1.3. Literature Review

Electromagnetic levitation and suspension technique is explained by B.V. Jayawant [2, 3]. This book explains magnetic levitation techniques using permanent magnets and repulsion forces. Various Maglev projects such as German Transrapid, the Japanese MLX, and Swissmetro are considered [12-17]. Foster-Miller built a null flux coil demonstration system with Permanent Magnet (PM) called Maglifter [18, 19]. Maglev technology for space launch system is addressed [20, 21]. The Lawrence Livermore National Laboratory (LLNL) has developed an Inductrack, a demonstration system using Halbach arrays [6-9]. Theoretical derivation of levitation force based on system configuration and experimental analysis is illustrated. D.M. Rote and Y. Cai reviewed Repulsive-Force Maglev suspension systems [10]. It summarizes the results of work reported in the literature over the past 25 years that pertains to understanding those factors that influence the dynamic stability of repulsive-force magnetic levitation suspension systems.

For the Halbach array magnet analysis, single square PM generated magnetic flux density [22], the transfer relation with magnetic scalar potential [23], magnetic vector potential [24] and Fourier series [6, 25] have been used to calculate the magnetic field. Q.Han showed EDS Maglev system based on the Halbach array system in his dissertation [11]. The practical Halbach magnetic field is analyzed using Fourier series analysis and the Finite Element Method. Q.Han, C.Ham analyzed four-and eight-piece Halbach array using Fourier analysis and Finite Element Method [26]. The field analysis results obtained using scalar potential and Fourier series methods are confirmed with FEM results.
For Wavelet analysis, Wavelet transform is introduced and explained from fundamentals [30-32]. Fourier analysis and Wavelet analysis are compared [33, 34]. Gerald Kaiser derived a new representation for solutions of Maxwell’s equations and analyzed electrodynamics based on Wavelet transform [35, 36]. Robertson David C., Camps Octavia I. and Mayer Jeff use Wavelet transform to analyze electromagnetic transients with power system. In their papers, Wavelet transform provides an effective and efficient means of decomposing voltage and current signals of power system transients [37, 38]. V. C. Chancey, G. T. Flowers and C. L. Howard consider the harmonic Wavelet as a tool for extracting transient patterns from measured vibration data [39].

For system stability and control design, Y. Cai and et al. summarized EDS Maglev system stability [10, 41, 42]. In their research, dynamics of EDS becomes unstable above a certain critical speed since it produces negative damping and there are also many uncertainties associated with the track, and disturbances induced from operational environments. Negative and Passive Damping techniques are studied for the EDS maglev system [42-45]. F. C. Moon shows instabilities of Maglev through his model and experiment [46, 47]. Carabelli et al. presented a control system on a PMS repulsive levitation system. It shows that the passive repulsive system is stable along the vertical axis and in pitch and roll, while being unstable in lateral direction and yaw [48, 49]. D. L. Atherton, A. R. Eastham, and K. Sturgess proposed secondary magnetic damping using short-circuited aluminum coils coupled to the linear synchronous motor [50]. I. Boldea shows that an EDS system with active control of magnet currents can provide good ride comfort without a secondary suspension system [51]. LQI optimal control and Robust control of multiple DOF for EDS systems was studied [52, 53]. T. Morizane, N. Kimura, and K. Taniguchi
presented a control design for both propulsion and levitation of an EMS system using Linear Induction Motor [54]. Mei-Yung Chen, Ming-Jyh Wang, and Li-Chen Fu presented an adaptive precision positioning controller for a small EDS maglev [55, 56]. R.Kluka suggested 6-DOF modeling of the Inductrack in his thesis [57]. The dynamic motional characteristics of the vehicle and the control mechanism design are showed. J.Kaloust, C.Ham proposed Nonlinear Robust Control Design for Levitation and Propulsion of a MagLev System in the Presence of Uncertain System Dynamics [58]. The recursive controller is designed using nonlinear state transformation and Lyapunov’s direct method is used in order to guarantee global stability for the nonlinear maglev system.

1.4. Major Contribution

As levitation height between the cradle and the track is mainly a factor of magnetic field intensity, magnetic field analyses at various distances between the magnetic surface and the observation point from the magnetic filed surface, levitation height, are studied. A transient oscillation at the beginning stage of levitation cannot be identified with the Fourier series method and Finite Element Method. In this dissertation, a new approach based on Wavelet transform is suggested to analyze the transient oscillatory response of an EDS Maglev system.

In Maglev system, forces are produced by the interaction of the magnet arrays as they travel across the coils in the track. These forces are used to levitate and position the carriage through its travel. To design a controller for the system, two different stages are suggested.
Before the object reaches a stable position and after reached a stable position. A stable position can be entitled a nominal height. The former is the stage I and the latter is the stage II. At the stage I, achieving a nominal height is the key point to initiate a study of a robust controller. At the stage II, stability is the main target during the system running. To maintain the system stability, damping control and LQR control is investigated.

1.5. Dissertation Format

This dissertation is organized as follows:

Chapter One includes the background, research goals, motivation, literature review and major contribution.

Chapter two introduces magnetic levitation technology, permanent magnetic Halbach array and Lawrence Livermore national laboratory system

Chapter three presents magnetic field analysis theory, magnetic harmonic analysis theory, wavelet analysis, and wavelet analysis of Halbach array

Chapter four discusses modeling of the EDS maglev system and control method.

Chapter five shows the summary of the research and considers future challenges

A list of references is given at the end of dissertation.
2. CHAPTER TWO: MAGNETIC LEVITATION TECHNOLOGY

2.1. Summary of Magnetic Levitation Technology

In Maglev research areas, there are several opportunities and challenges. In this section, current maglev project and maglev technologies are summarized.

2.1.1. Maglev Project

Transrapid constructed the first operational high-speed conventional maglev railway in the world, the Shanghai Maglev Train from downtown Shanghai, China to the new Shanghai airport at Pudong. It was inaugurated in 2002. The highest speed achieved on the Shanghai track has been 501 km/h (311 mph), over a track length of 30 km. Transrapid uses EMS technology. The track will be extended by 160 km before the 2010 World Expo begins in Shanghai.

Japan has a test track in Yamanashi prefecture where test trains JR-Maglev MLX01 have reached 581 km/h (363 mph), faster than wheeled trains. These trains use superconducting magnets, which allow for a larger gap, and repulsive-type "Electro-Dynamic Suspension" (EDS). In comparison Transrapid uses conventional electromagnets and attractive-type "Electro-Magnetic Suspension" (EMS). These "Superconducting Maglev Shinkansen", developed by the Central Japan Railway Co. ("JR Central") and Kawasaki Heavy Industries, are currently the fastest trains in the world, achieving a record speed of 581 km/h on December 2, 2003.

The world's first commercial automated "Urban Maglev" system commenced operation in
March 2005 in Aichi, Japan. This is the nine-station 8.9 km long Tobu-kyuryo Line, otherwise known as Linimo. The line has a minimum operating radius of 75 m and a maximum gradient of 6%. The linear-motor magnetic-levitated train has a top speed of 100 km/h. The line serves the local community as well as the Expo 2005 fair site. The trains were designed by the Chubu HSST Development Corporation, which also operates a test track in Nagoya.

Urban-type maglev patterned after the HSST have been constructed and demonstrated in Korea, and a Korean commercial version Rotem is now under construction in Daejeon and projected to go into operation by April of 2007.

In the U.S., Navy is investigating Maglev to launch aircraft from carriers. It converts stored energy to aircraft kinetic energy. Navy’s $373 million Electromagnetic Aircraft Launch System (EALS) project is in the building phase. Later prototype catapults based on linear synchronous motors will be tested at the naval facility. Pennsylvania plans a 76-km link joining Pittsburgh to its international airport and two other cities. In October 2002, the San Bernardino (Calif.) Associated Governments, approved funds for feasibility and pre-construction studies for a 433-km Anaheim-to-Las Vegas maglev line. Project supporters hope to begin construction in mid-2007 [16]. Maglev for space launch assist system has been researched [6-9, 18-21]. The space shuttle has cost about $4500 per kilogram to overcome Earth’s gravity and enter space orbit. A shuttle mission typically costs more than $400 million per flight. During the past decade NASA has studied ways of assisting the launch of space vehicles to reduce the cost. A launch assist system requires a low maintenance, inexpensive, environmentally clean, safe, and reliable system. NASA is pursuing a launch scheme that accelerates the craft horizontally along the ground using a power source external to the vehicle, thereby eliminating one stage of rockets.
This approach would allow the first stage to be replaced with an alternative power source that is not mounted on the craft, therefore reducing cost and complexity of the launch system. By providing an initial velocity to the space vehicle it is possible to save over 20% of the onboard fuel. Also by lowering the amount of fuel, more payloads can be added, the size of the vehicle can be reduced or a stronger more robust vehicle can be built. Once solid rocket boosters are ignited they can be extinguished by only complete consumption. The launch cannot be aborted once the solid rockets have been ignited, consume and jettison the solid rockets, and land at another location. The Maglev launch assistant system would allow the craft to reach a speed at which all systems could be assessed under load, and the determination to complete or abort the launch could be made while still on the runway. NASA has a program called the Advanced Space Transportation Program (ASTP) to develop technologies in the next 25 years that will improve safety and reliability by a factor of 10,000 while reducing the cost for space access by a factor of 100. Among these technologies is the area of launch assist. Magnetic levitation and propulsion are viewed as a safe, reliable, and inexpensive launch assist for sending payloads into orbit. NASA’s plan is to mature these technologies in the next 25 years to achieve the goal of launching a full sized space vehicle for under $300 a kilogram. NASA has contracted with three companies to initially produce magnetic levitation concepts; Foster–Miller (FM); Lawrence Livermore National Laboratory (LLNL); and PRT Advanced Maglev Systems. Each of these contracts was to show a small demonstration of their concepts at the conclusion of the first phase. Two of the prototypes with a total cost of up to half million dollars are located at the Florida Space Institute (FSI) currently.
2.1.2. Maglev Technology

Earnshaw's theorem states that a collection of point charges cannot be maintained in a stable stationary equilibrium configuration solely by the electrostatic interaction of the charges. This was first proved by Samuel Earnshaw in 1842 [59]. It is usually referenced to magnetic fields, but originally applied to electrostatic fields, and, in fact, applies to any classical inverse-square law force or combination of forces (such as magnetic, electric, and gravitational fields).

Braunbeck carried out a similar analysis specifically for unvarying magnetic and electric fields [60], and deduced that suspension or levitation is not possible in such fields when all materials present have relative permeability $\mu_r > 1$ or relative permittivity $\varepsilon_r > 1$, but that it is possible when materials of $\mu_r < 1$ or $\varepsilon_r < 1$ are introduced. It is impossible for stable suspension or levitation without diamagnetic materials ($\mu_r < 1$) or superconducting materials ($\mu_r = 0$) [2, 3].

Research into the technology of Maglev has focused on two core concepts; electromagnetic suspension (EMS) and electromagnetic suspension (EDS). The EMS is based on a magnet’s attraction to a ferromagnetic material. The EMS systems are inherently unstable so some form of feedback is necessary to stably levitate. Controlling the gap between the suspended object and the magnetic field source maintains a balance between gravity and the force of attraction. The EDS system is based on the induction currents using either permanent magnets or superconducting magnets as the magnetic field source. Therefore, the EDS system has two magnetic fields, one magnetic field induced in the conductor and the other a static magnetic
field, repulsing one another. The coil or magnetic field source could be in motion, relative to one another, to induce the necessary current in the coils for levitation to occur. The EDS system is inherently stable and does not need a complicated feedback control system compared with the EMS system. Although the EDS is inherently stable, the damping force is not large enough to suppress the vibrations excited by guideway irregularities and other disturbances. The dynamics of EDS become unstable above a certain critical speed since it produces negative damping that destabilizes the system [40, 41]. At high speed the damping forces are weak, and, at low speed the levitation force is weak and break force is strong. Thus, levitation is feasible only at high speed. PM can be used in repulsive systems to levitate vehicles. The permanent Halbach array is the best candidate for the PM levitation system.

2.2. The Permanent Magnet Halbach Array

The Halbach array is an array of permanent magnets with magnetic field orientations as shown in figure 2.1. This array has the effect of polarizing the magnetic field on one side of the array with no magnetic field on the opposite side. The magnetic field also has the property of being periodic with an associated wavelength, $\lambda$. Klaus Halbach provided the theoretical framework for the design and optimization of this type of permanent magnet array for use in particle accelerators [6-9].

When an electric current is passed through a piece of wire a magnetic field is produced. This is because magnetic fields are due to electric charges in motion.
2.3. Lawrence Livermore National Laboratory Track

By extending the current technology, it becomes attractive to apply maglev as part of the launch assist system for future spacecraft. The vehicle will be mounted to a horizontal maglev track and accelerated to a predetermined speed, about 1,000 km/h. It then will be released from the track and fly as a normal airplane to reach space altitude [6-9].

The reference system used for this research is the repulsive-force maglev track which is called Inductrack developed by the Lawrence Livermore National Laboratory shown in figure 2.2.
This track is located at the Florida Space Institute, University of Central Florida. This system uses the concept of passive magnetic levitation developed during research on passive magnetic bearing systems. The concept behind magnetic levitation is outlined in U.S. Patent 5,722,326 by Richard F. Post. In the Inductrack, a Halbach array produces a sinusoidal variation of magnetic field at a constant distance between the bottom of the array and coil blocks in the track. The carriage uses five magnets in its array for Halbach array configuration as shown in figure 2.1. Relative motion between these magnets and the coils in the track induces a current in the track coils and simultaneously these currents interact with the horizontal magnetic field from the arrays. The Force generated from current filed and magnetic field interaction can levitate the carriage above the track.
Figure 2.3 Halbach Array Field Concentrations

Figure 2.4 shows that three halbach array magnets mounted on the carriage, one is the top position and the others are located at each side. These are mounted on each side at a $45^\circ$ angle. These enhance the stability of the carriage in both the vertical and lateral directions.

Figure 2.4 General Track Arrangements

As the levitation forces develop, the forces from the left and right magnet arrays are equal in the lateral direction. In the vertical direction these arrays cause a downward force that counteracts the levitation force. This causes the carriage to be less likely to lift, enhancing
stability. Dynamics of the maglev system are complex and inherently unstable. One of the focuses of this dissertation is that the exploration of the dynamic motion of the maglev system’s carriage [57].
3. CHAPTER THREE: HALBACH ARRAY MAGNETIC FIELD
MODELING AND ANALYSIS

3.1. Magnetic Field Analysis Theory

In this chapter, Magnetic field analysis theory is summarized.

A vector \( \vec{a} \) can be represented symbolically in terms of three mutually perpendicular vectors addition follows:

\[
\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}
\]

where \( \vec{i} \), \( \vec{j} \) and \( \vec{k} \) are unit vectors in the \( x \), \( y \), \( z \) directions, \( a_x \), \( a_y \) and \( a_z \) are the magnitude projections of the vector \( \vec{a} \) on the \( x \), \( y \) and \( z \) axes respectively.

The scalar product of the vectors \( \vec{a} \) and \( \vec{b} \) is denoted by \( \vec{a} \cdot \vec{b} \) and the magnitude is by definition

\[
\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z
\]

The scalar product can also be represented as follows

\[
\vec{a} \cdot \vec{b} = ab \cos \theta
\]

where, \( a \) and \( b \) are the magnitudes of the vectors \( \vec{a} \) and \( \vec{b} \) respectively, and \( \theta \) is the angle between the two vectors. Figure 3.1 shows the scalar product of vectors \( \vec{a} \) and \( \vec{b} \).
The vector product of two vectors \( \vec{a} \) and \( \vec{b} \) is denoted by \( \vec{a} \times \vec{b} \) and defined as

\[
\vec{a} \times \vec{b} = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
a_x & b_y & a_z \\
b_x & b_y & b_z
\end{vmatrix}
\]

\[
= \vec{i}(a_yb_z - a_zb_y) + \vec{j}(a_zb_x - a_xb_z) + \vec{k}(a_xb_y - a_yb_x)
\]

The vector product is a vector. The magnitude can be expressed by

\[
\vec{a} \times \vec{b} = ab \sin \theta
\]

The direction of the vector \( \vec{a} \times \vec{b} \) is perpendicular to the plane of the vectors \( \vec{a} \) and \( \vec{b} \) which is given by the right hand rule convention.
The gradient of a scalar function $S$ is a vector its magnitude is the directional derivative at the point and direction is the direction of the directional derivative at the point.

Consider a scalar $S$, the value of which is dependent upon its position in space.

$$S = S(x, y, z)$$

The ascendant of $S$ is defined as

$$\mathbf{i} \frac{\partial S}{\partial x} + \mathbf{j} \frac{\partial S}{\partial y} + \mathbf{k} \frac{\partial S}{\partial z}$$

This represents a vector which has a normal direction to the surface at a given point $x, y, z$ and points in the direction of ascending values of $S$. The differential operation indicated above is given a special symbol $\nabla$, defined by

$$\nabla \equiv \mathbf{i} \frac{\partial S}{\partial x} + \mathbf{j} \frac{\partial S}{\partial y} + \mathbf{k} \frac{\partial S}{\partial z}$$

The quantity will be referred to as the gradient of $S$ (or grad $S$): i.e.,

$$\text{Grad } S \equiv \nabla S$$

If the vector $\vec{a}$ is defined at each point $x, y, z$ in a given region, then we say that a field of $\vec{a}$ exists.

$$\vec{a} = \vec{a}(x, y, z)$$

$$= \mathbf{i} a_x(x, y, z) + \mathbf{j} a_y(x, y, z) + \mathbf{k} a_z(x, y, z)$$

which implies three functions of space.

The divergence of a vector is the limit of its surface integral per unit volume as the volume enclosed by the surface goes to zero. The divergence of such a vector field is defined as
\( \text{Div } \vec{a} \equiv \nabla \cdot \vec{a} \)

the term on the right being an abbreviation of

\[
\frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}
\]

Div \( \vec{a} \) is a scalar.

\[
\nabla \cdot \vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}
\]

The integral of the divergence of a vector \( \vec{a} \) over a volume \( V \) is equal to the surface integral of the normal component of the vector over the surface bounding \( S \).

\[
\int_{V} \nabla \cdot \vec{a} \, dV = \int_{S} \vec{n} \cdot \vec{a} \, dS
\]

The curl of a vector is the limit of the ratio of the integral of its cross product with the outward drawn normal, over a closed surface, to the volume enclosed by the surface as the volume goes to zero.

The line integral of a vector around a closed curve is equal to the integral of the normal component of its curl over any surface bounded by the curve.

\[
\oint_{C} \vec{a} \cdot dl = \int_{S} \text{Curl } \vec{a} \cdot \vec{n} \, da
\]

The curl of a vector field \( \vec{a}(x,y,z) \) is defined by

\[
\nabla \times \vec{a} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix} = i \left( \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z} \right) + j \left( \frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x} \right) + k \left( \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y} \right)
\]
Thus, the curl of $\vec{a}$ is a vector having the three components in Cartesian coordinates.

Laplacian operator is defined by

$$\nabla \cdot \nabla = \nabla^2$$

$$\nabla^2 S = \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2}$$

The curl of the gradient of any scalar field is zero

$$\nabla \times (\nabla S) = 0$$

The divergence of any curl is zero

$$\nabla \cdot (\nabla \times \vec{a}) = 0$$

$$\nabla \times (\nabla \times \vec{a}) = \nabla (\nabla \cdot \vec{a}) - \nabla^2 \vec{a}$$

The base for electromagnetic analysis is the four Maxwell equations, which were derived from earlier BiotSavart law, Faraday’s law and Gauss’s law. In differential form these equations are given by

$$\nabla \times \vec{E} = -\frac{\partial \mu_0 \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\varepsilon_0 \vec{E}}{\partial t}$$

$$\nabla \cdot \varepsilon_0 \vec{E} = \rho$$

$$\nabla \cdot \mu_0 \vec{H} = 0$$

where $\vec{E}$ is the electric field intensity, $\vec{J} = \sigma \vec{E}$ electric current density, $\rho$ is the charge density,
\( \vec{H} \) is the magnetic field intensity, \( \vec{M} \) is magnetization vector, for permanent magnet
\( \vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} \), \( \vec{B} = \mu_0 \mu_r \vec{H} \) is the magnet flux density, \( \mu_0 \) is the magnetic permeability of free space, \( \mu_r \) is the relative magnetic permeability.

Since the divergence of any curl is zero, magnetoquasistatic (MQS) approximation,
\[
\nabla \times \vec{H} = J
\]
\[
\nabla \cdot (\mu_0 \vec{H}) = 0
\]
\[
\mu_0 \vec{H} = \nabla \times \vec{a}
\]

and in MQS systems, for convenience we make \( \nabla \cdot \vec{a} = 0 \). The only other requirement placed on \( \vec{a} \) is that
\[
\nabla \times (\mu_0 \vec{H}) = \nabla \times \nabla \times \vec{a} = \mu_0 J
\]

Using the identity
\[
\nabla \times (\nabla \times \vec{a}) = \nabla (\nabla \cdot \vec{a}) - \nabla^2 \vec{a}
\]

the Poisson’s equation is given by
\[
\nabla^2 \vec{a} = -\mu_0 J
\]

The vector potential \( \vec{a} \) describes magnetic fields that possess curl wherever there is a current density \( J \). The curl of the magnetic induction is zero wherever the current density is zero. When this is the case, the magnetic induction in such regions can be written as the gradient of a scalar potential
\[ \nabla \times \vec{H} = 0 \]
\[ \vec{H} = -\nabla \varphi \]
\[ \nabla \cdot (\vec{B}) = 0 \]

However, the divergence of \( B \) is also zero,
\[ \nabla \cdot B = -\mu_0 \nabla^2 \varphi = 0 \]

where \( \varphi \) is called the magnetic scalar potential, satisfies Laplace’s equation.

### 3.2. Magnetic Field Analysis of Halbach Array

The Halbach array is used in various fields: particle accelerators, magnet bearings, linear motors, PMSM and Maglev designs. A summary of the analysis and applications of Halbach array are given in [61, 62]. The ideal linear Halbach array has sine and cosine magnetization in the vertical and horizontal directions respectively. It has no magnetic field on one side and an enhanced, pure sinusoidal magnetic field on another side [63]. Figure 3.3 illustrates an ideal Halbach array. But the ideal Halbach array is impractical to fabricate, so an array of rectangular or square permanent magnets is used to build a practical (non ideal) Halbach arrays.
Figure 3.3 Ideal Halbach Array Configuration (a), Practical (b)

Figure 3.4 shows the geometry of a permanent magnet sheet with a thickness of $d_t$.

Figure 3.4 The Geometry of a Magnet Sheet and Coordinate Definitions

For the ideal Halbach array, the magnetizations are given by

$$m_x = m_0 \sin (kx)$$

$$m_z = m_0 \cos (kx)$$

(3.1)

For magnet field with no transport currents
\[ \nabla \times \vec{H} = 0 \]  
\[ \vec{H} = -\nabla \varphi \]  \hspace{1cm} (3.2)

For a permanent magnet
\[ \vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} \]

\[ \nabla \cdot \vec{B} = 0, \nabla \cdot \vec{H} = -\nabla \cdot \vec{M} \]  \hspace{1cm} (3.3)

\[ \nabla^2 \varphi = \nabla \cdot \vec{M} \]

where \( \vec{H} \) is the magnetic field intensity, \( \vec{B} \) is the magnetic flux density, \( \mu_0 \) is the magnetic permeability of free space, \( \vec{M} \) is the magnetization vector, and \( \varphi \) is called the magnetic scalar potential.

The flux density of the normal component is also continuous. The potentials are given by Equation 3.4 [63].

\[ \varphi_{\text{cancelled}} = 0 \]

\[ \varphi_{\text{inside}} = \frac{m_0}{k} (e^{kz} - 1) \cos (kx) \]  \hspace{1cm} (3.4)

\[ \varphi_{\text{enhanced}} = \frac{m_0}{k} (1 - e^{kd}) e^{kz} \cos (kx) \]

With \( \vec{H} = -\nabla \varphi \), the magnetic field intensities are given by
\[ \hat{H}_{\text{cancelled}} = 0 \]

\[ \hat{H}_{\text{inside}} = -(i \frac{\partial S}{\partial x} + j \frac{\partial S}{\partial y} + k \frac{\partial S}{\partial z}) \phi \]  \hfill (3.5)

\[ \hat{H}_{\text{enhanced}} = -(i \frac{\partial S}{\partial x} + k \frac{\partial S}{\partial z}) \phi \]

\[ = -i m_0 (1 - e^{k_x}) e^{k_z} \sin (kx) - k m_0 (1 - e^{k_x}) e^{k_z} \cos (kx). \]

Suppose there is another ideal array with a different spatial period and wave number.

Define \( k_n = \frac{2 \pi n}{l} = nk \). The new array has the following magnetization values [11],

\[ m_{xn} = m_{no} \sin(k_n x) \]

\[ m_{zn} = m_{no} \cos(k_n x) \]  \hfill (3.6)

For this new array, the magnetic potentials are given by

\[ \phi_{\text{cancelled}} = 0 \]

\[ \phi_{\text{inside}} = \frac{m_{n0}}{k_n} (e^{k_n z} - 1) \cos(k_n x) \]  \hfill (3.7)

\[ \phi_{\text{enhanced}} = \frac{m_{n0}}{k_n} (1 - e^{k_n d}) e^{k_n z} \cos(k_n x) \]

and the magnetic fields are given by
\[ \vec{H}_{\text{cancelled}} = 0 \]

\[ \vec{H}_{\text{inside}} = -\left( i \frac{\partial S}{\partial x} + j \frac{\partial S}{\partial y} + k \frac{\partial S}{\partial z} \right) \varphi \]

\[ \vec{H}_{\text{enhanced}} = -\left( i \frac{\partial S}{\partial x} + k \frac{\partial S}{\partial z} \right) \varphi = -i m_{00}(1 - e^{k_{x}x}) e^{k_{z}z} \sin(k_{n}x) \]

\[ -k m_{00}(1 - e^{k_{x}x}) e^{k_{z}z} \cos(k_{n}x) \]

If there is a pair magnetization of \( m_{x} \) and \( m_{z} \), these can be decomposed into the sine and cosine pairs as follows,

\[ m_{x} = \sum_{i} m_{x_{i}} = \sum_{i} m_{0} f_{n} \sin(k_{n}x) = \sum_{i} m_{0} f_{n} \sin(nkx) \]

\[ m_{z} = \sum_{i} m_{z_{i}} = \sum_{i} m_{0} f_{n} \cos(k_{n}x) = \sum_{i} m_{0} f_{n} \cos(nkx) \]  

(3.9)

3.3. Halbach Array Magnetic Field Modeling

In this section, a model constructed of individual blocks of neodymium-iron-boron permanent magnet material placed in a planar Halbach array configuration is described and modeled. The two dimensional model is geometrically developed in FEMLAB [64-66] and solved using finite element analysis method.

FEMLAB is the software package used to model the permanent magnet Halbach array. Within FEMLAB the Halbach array’s geometry can be described and modeled via a Computer...
Aided Design (CAD). Regions are created from simple geometries such as rectangles, circles and lines. The properties of the regions, such as permeability, magnetization and magnetization direction, and boundary conditions of region, can then be defined. An outer boundary that surrounds the geometry to be modeled operates, for our purposes, as if it were an infinite volume where the magnetic field potential goes to zero as the distance from the field source increases.

The demonstrated Halbach array is constructed of individual neodymium-iron-boron (NdFeB) magnet bars with square cross-section. There will be four magnets per wavelength of the array. However, the array to be modeled will consist of five bars of NdFeB magnets. The length of the magnet bar is along the z direction and will not factor in the described simulations.

An arbitrary wavelength, $\lambda$, of 5 cm is chosen as our model’s starting point. Since the number of magnets per wavelength is 4, the thickness of our magnet bars is 1.25 cm each. The bar is assumed to be of square cross-section. In FEMLAB, the constant $M_{\text{PERM}}$ is defined as the $M$ calculated from $B_r$, value in this simulation is 1.41 Tesla [T]. Remanence is the magnetization left behind in a medium after an external magnetic field is removed. The bar cross-sections are defined, via FEMLAB’s CAD like interface, as 1.25 cm by 1.25 cm square. Figure 3.5 and Table 3.1 show the configuration for the modeling of Halbach array configuration.

![Figure 3.5 Magnetization Orientations and Associated Cross-Section Regions](image)
Table 3.1 Magnetization Orientations and Values for Each Region

<table>
<thead>
<tr>
<th>Region</th>
<th>Magnetization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$M_{\text{PERM}}$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>$(-)M_{\text{PERM}}$</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

Another region is defined to surround the Halbach array and serves as the outer boundary region with the condition that the field goes to zero as the distance from the array increases. That is, the magnetic potential goes to zero. This condition limits the size of our problem and gives proper conditions to solve the problem. In FEMLAB, this condition is called magnetic insulation.

Figure 3.6 is a contour plot that shows the magnetic potential in the z direction. This plot also reveals the magnetic field profile of the Halbach array. Sinusoidal patterning indicative of the Halbach array is also shown. Figure 3.7 is a surface plot showing the relative density of magnetic flux across the boundary of the Halbach array. Note that the magnetic flux is concentrated primarily on the lower surface of the array. Figure 3.8 shows an enhanced field that is lower surface of the array and a cancelled field upper surface of the array.
Figure 3.6 Two Dimensional Contour Plot of the Halbach Array

Figure 3.7 Two Dimensional Surface Plot of the Magnetic Flux Density of the Halbach Array.
To show the magnetic field shape of the four piece Halbach magnet array, respect to various center magnet width cases are illustrated. Enhanced and cancelled side results are shown and summarized. From this study, when we decrease the center width the shape will be similar the sine wave but the value is decreasing. From Figure 3.9 to Figure 3.12, suggested cases are modeled and summarized in Figure 3.14, Figure 3.15 and Table 3.2.
Figure 3.9 Case 1, Center Magnet Width is 50%
X-axis Array (width): 0.625, 1.25, 0.625, 1.25, 0.625, Y (height): 1.25

Figure 3.10 Case 2, Center Magnet Width is 25%
X-axis Array (width): 0.625, 1.25, 0.3125, 1.25, 0.625, Y (height): 1.25
Figure 3.11 Case 3, Center Magnet Width is 10%  
X-axis Array (width): 0.625, 1.25, 0.125, 1.25, 0.625, Y (height): 1.25

Figure 3.12 Case 4, Consider Width is 13 and Height 1 (unit is not considered)  
X-axis Array (width): 2, 3, 3, 3, 2, Y (height): 1
Figure 3.13 Enhanced Side Field

Figure 3.14 Cancelled Side Field
Table 3.2 Configuration Used in Halbach Array Modeling

<table>
<thead>
<tr>
<th>case</th>
<th>width</th>
<th>height</th>
<th>Max value, Enhanced</th>
<th>Max value Cancelled</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.625, 1.25, 0.625, 1.25, 0.625</td>
<td>1.25</td>
<td>429584.4</td>
<td>121528.5</td>
</tr>
<tr>
<td>2</td>
<td>0.625, 1.25, 0.3125, 1.25, 0.625</td>
<td>1.25</td>
<td>395622.9</td>
<td>125442.5</td>
</tr>
<tr>
<td>3</td>
<td>0.625, 1.25, 0.125, 1.25, 0.625</td>
<td>1.25</td>
<td>362752.34</td>
<td>191729.6</td>
</tr>
<tr>
<td>4</td>
<td>2, 3, 3, 3, 2</td>
<td>1</td>
<td>279744.4</td>
<td>151572.2</td>
</tr>
<tr>
<td>org</td>
<td>1.25, 1.25, 1.25, 1.25, 1.25</td>
<td>1.25</td>
<td>492109.9</td>
<td>176698.5</td>
</tr>
</tbody>
</table>

3.4. Wavelet Analysis

In this section, Wavelet is introduced and summarized [30, 31]. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. For many decades, scientists have wanted more appropriate functions than the sines and cosines which comprise the bases of Fourier analysis. A basis function varies in scale by chopping up the same function or data space using different scale sizes. For example, imagine we have a signal over the domain from 0 to 1. We can divide the signal with two step functions that range from 0 to 1/2 and 1/2 to 1. Then we can divide the original signal again using four step functions from 0 to 1/4, 1/4 to 1/2, 1/2 to 3/4, and 3/4 to 1. And so on. Each set of representations
code the original signal with a particular resolution or scale.

Fourier's representation of functions as a superposition of sines and cosines has become ubiquitous for both the analytic and numerical solution of differential equations and for the analysis and treatment of communication signals. Fourier and Wavelet analysis have some very strong links. The Fourier transform (FT) is able to analyze a signal in the time domain for its frequency content. The transform works by first translating a function in the time domain into a function in the frequency domain. The signal can then be analyzed for its frequency content because the Fourier coefficients of the transformed function represent the contribution of each sine and cosine function at each frequency.

Similarities between FT and Wavelet transform (WT) are as follows. For the FT, this domain contains basis functions that are sines and cosines. For the WT, this domain contains more complicated basis functions called Wavelets, mother Wavelets, or analyzing Wavelets. Another similarity is that the basis functions are localized in frequency. The most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are localized in space. Fourier sine and cosine functions are not. This localization feature, along with Wavelets' localization of frequency, makes many functions and operators using Wavelets "sparse" when transformed into the wavelet domain. One thing to remember is that Wavelet transforms do not have a single set of basis functions like the Fourier transform, which utilizes just the sine and cosine functions. Instead, Wavelet transforms have an infinite set of possible basis functions. Thus Wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis.

The WT provides the time-frequency representation. To show this, two different versions
of the same signal are generated, namely, the portion of the signal corresponding to 0-500 Hz (low pass portion) and corresponding to 500-1000 Hz (high pass portion). Then, we take either portion (usually low pass portion) or both, and do the same thing again. This operation is called decomposition.

Assuming that we have taken the lowpass portion, we now have 3 sets of data, each corresponding to the same signal at frequencies 0-250 Hz, 250-500 Hz, 500-1000 Hz. Then we take the lowpass portion again and pass it through low and high pass filters; we now have 4 sets of signals corresponding to 0-125 Hz, 125-250 Hz, 250-500 Hz, and 500-1000 Hz. We continue like this until we have decomposed the signal to a pre-defined certain level. Then we have a bunch of signals, which actually represent the same signal, but all corresponding to different frequency bands. We know which signal corresponds to which frequency band, and if we put all of them together and plot them on a 3-D graph, we will have time in one axis, frequency in the second and amplitude in the third axis. This will show us which frequencies exist at which time. Higher frequency is better resolved in time, and lower frequency is better resolved in frequency. This means that, a certain high frequency component can be located better in time than a low frequency component. On the contrary, a low frequency component can be located better in frequency compared to high frequency component. Let's take a sinusoidal signal, which has two different frequency components at two different times. Figure 3.15 shows that the low frequency portion first, and then the high frequency. Figure 3.16 shows the result of FT and WT respectively. From the FT, figure 3.16 (a), frequency can be identified but we can’t get the order of this signal. But in WT, figure 3.16 (b), we can get time and frequency simultaneously. As mentioned earlier, higher frequency has better resolution in time and lower frequency has better
resolution in frequency. In figure 3.16 (b), you can see the first signal and second signal is different as resolution and magnitude. First signal has low resolution in time so this signal is lower frequency otherwise; second signal has a high resolution so this signal is higher frequency.

![Figure 3.15 Low Frequency is First, High Frequency is Next](image)

Figure 3.15 Low Frequency is First, High Frequency is Next

![Figure 3.16 Fourier Transform, FFT (a), Wavelet Transform (b)](image)

Figure 3.16 Fourier Transform, FFT (a), Wavelet Transform (b)

To make it clear, figure 3.17 is shown that the high frequency portion first, and then the low frequency is followed. From the FT, figure 3.17 (a), frequency can be identified but we can’t
get the order of this signal same as figure 3.16 (a). But figure 3.17 (b), you can see the first signal and second signal is different as resolution and magnitude same as figure 3.16 (b). First signal has high resolution in time so this signal is high frequency otherwise; second signal has a low resolution so this signal is lower frequency.

Figure 3.17 High Frequency is First, Low Frequency is Next

Figure 3.18 Fourier Transform, FFT (a), Wavelet Transform (b)
3.5. Wavelet Analysis of Halbach Array

The Wavelet Transform-based approach is proposed to analyze the transient oscillatory response of an EDS Maglev system that can precisely estimate the magnetic field intensity at the beginning stage of levitation. Wavelet Transform-based approach is used to analyze the harmonics of magnetization function. Then wavelet coefficients both vertical and horizontal are compared to find an enhanced and cancelled magnetic field region. The field analyses based on the Fourier series and the FEM are provided in order to illustrate the advantage of the proposed scheme in the analysis of the dynamic characteristics of the levitation.

The model, four-piece Halbach array configuration with the wavelength of $d$ and the thickness $d_t$ is used in this analysis is shown as figure 3.18 and figure 3.19.

Figure 3.19 Four-piece Halbach Array Configuration with Wavelength $d$ and Thickness $d_t$
From chapter 3.2, if there is a pair magnetization of $m_x$ and $m_z$, it can be decomposed into sine and cosine pairs

$$m = im_x + km_z = i \sum_n m_{xn} + k \sum_n m_{zn}$$

$$= \sum_n [m_0 f_n (i \sin (k_n x) + k \cos (k_n x))]$$

(3.9)

where, $f_n$ is the amplitude of the $n^{th}$ harmonic.

Let $m_{z4}(x)$ and $m_{x4}(x)$ be the vertical and horizontal components of the magnetization respectively. Then, the magnetizations of the practical four piece Halbach array can be written as [26],

$$m_{z4}(x) = m_0 \Pi(4x/\lambda) \cdot \left\{ \sum_{n=0}^{\infty} [\delta(x-n\lambda)-\delta(x-(2n-1)\lambda/2]\right\}$$

(3.10)

$$m_{x4}(x) = m_0 \Pi(4x/\lambda) \cdot \left\{ \sum_{n=0}^{\infty} [\delta(x-n\lambda-\lambda/4)-\delta(x-(2n-1)\lambda/2-\lambda/4)]\right\}$$
where, \( \lambda = 4d \) is the wavelength array, \( k_1 = 2\pi/\lambda \) is the wave number and thickness of the Halbach array is \( d_t \).

The following analyses are based on the four-piece square block Halbach arrays with \( d = d_t \), as shown in Figure 3.21 that shows ideal magnetization of Halbach array both vertical and horizontal with \( 4d \) wavelength.

![Figure 3.21 Magnetization of 4-piece Halbach Array with Wavelength = 4d](image)

(a) Vertical; (b) Horizontal

Similar to Fourier analysis, wavelet analysis deals with the expansion of functions in terms of a set of basis functions. Unlike Fourier analysis, wavelet analysis expands functions not in terms of trigonometric polynomials but in terms of wavelets, which are generated in the form of translations and dilations of a fixed function called a mother wavelet. They are localized in time and frequency, permitting a closer connection between the function being represented and
their coefficients [27]. There are two types of wavelet transforms as there are Fourier transforms. One is the Continuous Wavelet Transform (CWT) and the other is the Discrete Wavelet Transform (DWT). In DWT, any continuous functional \( f(t) \) can be approximated as a sum of wavelet and scaling functions,

\[
f(t) = f_0(t) + \sum_{j=0}^{N} \Delta f_j(t)
\]

\[
= \sum_{k \in \mathbb{Z}} c_{0,k} \varphi_{0,k}(t) + \sum_{j=0}^{N} \sum_{k \in \mathbb{Z}} d_{j,k} \psi_{j,k}(t)
\]

where \( \varphi_{0,k}(t) \) and \( \psi_{j,k}(t) \) are the scaling function and wavelet function respectively with \( c_{0,k} \) and \( d_{j,k} \) as their corresponding coefficients.

To find a harmonic amplitude of \( f_n \) in (3.9), the Haar wavelet is selected as the mother wavelet due to its similarity in shape and its simple computation. The Haar wavelet is the first known wavelet and was proposed in 1909 by Alfred Haar.

The Haar Wavelet is described as a step function \( f(x) \) with

\[
f(x) = \begin{cases} 
1 & 0 \leq x < \frac{1}{2} \\
-1 & \frac{1}{2} \leq x < 1 \\
0 & \text{otherwise} 
\end{cases}
\]

Figure 3.22 Haar Wavelet
Figure 3.22 shows Haar wavelet and Figure 3.23 shows Harr wavelet transform of vertical and horizontal magnetization with level 7 decomposition that provides the coefficient of the wavelet. The calculated wavelet coefficient is applied to estimate the Halbach array magnetic field intensity given by [11].

\[
H_{\text{enhanced}} = \sum_i \left\{ -i m_0 f_n \left(1 - e^{\frac{k d}{n t}}\right) e^{\frac{k z}{n}} \sin \left(k_n x\right) \\
- k m_0 f_n \left(1 - e^{\frac{k d}{n t}}\right) e^{\frac{k z}{n}} \cos \left(k_n x\right) \right\}.
\]  

(3.13)

Figure 3.23 Haar Wavelet Level 7 Decomposition, (a) Vertical; (b) Horizontal
Let $z$ be the distance from the observation point to the surface of the array, equation 3.13 is used to observe magnetic harmonic components at a certain distance. To investigate the enhanced region, vertical and horizontal wavelet coefficients are compared and found to be either positive or negative. These harmonic pairs can be classified as one of three types.

1. Coefficients with the same position and sign generate an enhanced region.
2. Coefficients with the same position but different sign generate a cancelled region.
3. Coefficients with the different position generate neither an enhanced nor cancelled region.

Figure 3.24 shows that enhanced harmonic components at 0.1$d$ levitation height for one period both horizontal and vertical from wavelet transform is applied to equation 3.13 $B_{H}$ denotes horizontal components and $B_{V}$ denotes vertical components. The three types of classification decide the number of harmonic components. For enhanced region classification condition 1 is applied. Figure 3.25 shows that Fourier transform is applied to same condition. From those figures, different harmonic results are shown and these illustrate the advantage of the proposed approach.
Figure 3.24 Enhanced Harmonic Components at Distance 0.1$d$ for One Period, Wavelet Transform, Vertical (top) and Horizontal (bottom)

Figure 3.25 Enhanced Harmonic Components at Distance 0.1$d$ for One Period, Fourier Transform, Vertical (top) and Horizontal (bottom)
Figure 3.26 shows the result of wavelet analysis for transient-state at 0.25$d$, 0.5$d$ and steady-state at 0.9$d$, 1$d$. Then, in order to investigate and represent this phenomenon, proposed wavelet transform analysis, Fourier series analysis and FEM analysis are compared at 0.25$d$, 0.5$d$, 0.9$d$ and 1$d$. Figure 3.27 and figure 3.28 show the results of three methods at 0.25$d$ and 0.5$d$ for transient-state and figure 3.29 and figure 3.30 show at 0.9$d$ and 1$d$ for steady-state. The solid line shows the Wavelet analysis result, the dashed line shows Fourier series analysis and the dotted line shows FEM analysis. In these figures, below 0.5$d$ represents the beginning stage of levitation. Unlike the other two methods, proposed wavelet analysis reveals an irregular distribution of the magnetic field below 0.5$d$ in figure 3.27 and figure 3.28. But in figure 3.29 and figure 3.30, the distance between observation point and surface array approaches equilibrium levitation height, 1$d$; the magnetic field intensity becomes sinusoidal shape as the same results in Fourier series analysis and FEM analysis.

The proposed scheme explains the under-damped dynamics that results from the cradle’s dynamic response to the irregular distribution of the magnetic field. This keeps the cradle oscillating in order to balance between the levitation force and gravity of the cradle at the beginning stage of levitation. This suggests that this EDS Maglev system that responds to a vertical repulsive force could be subject to such instability at the beginning stage of a low levitation height.
Figure 3.26 Wavelet Analysis of Enhanced Horizontal Components at Different Distances for One Period

Figure 3.27 Enhanced Horizontal Components at Different Distances for One Period when Levitation Height Approaches $0.25d$
Figure 3.28 Enhanced Horizontal Components at Different Distances for One Period when Levitation Height Approaches 0.5d

Figure 3.29 Enhanced Horizontal Components at Different Distances for One Period when Levitation Height Approaches 0.9d

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Figure 3.30 Enhanced Horizontal Components at Different Distances for One Period when Levitation Height Approaches 1\textit{d}
4. CHAPTER FOUR: EDS MAGLEV SYSTEM MODELING

4.1. Modeling Approach

In Maglev system, forces are produced by the interaction of the magnet arrays as they travel across the coils in the track. These forces are used to levitate and position the carriage through its travel. Due to the design of the carriage with magnets acting on both the upper and lower surfaces of the track, the system has a nominally designed levitation height. It is assumed from the design that when the carriage is at its nominal position on the track, gravitational forces and the forces from the upper array and the lower left and lower right arrays will be in equilibrium, holding the carriage at or near its nominal position.

To model the suggested EDS maglev system, two different stages are proposed. Let define nominal point for the system modeling, then it is classified two stages:

- Until the levitation height reaches a nominal point
- After the levitation height reaches a nominal point

At the stage I, also we can say initial stage of the system; the objective is the height control. System based design and analysis is suggested. And for the stage II, the objective is the system stability. Damping and optimal control is suggested.
4.2. Dynamics Analysis Theory

Let define first the coordination for body fixed and inertial coordinate system.

Figure 4.1 Classified Stages for the Controller Design

Figure 4.2 Body-fixed and Inertial Coordinate Systems
\[\eta_1 = \{x, y, z\}^T \text{ Inertial Position}\]

\[\eta_2 = \{\phi, \theta, \psi\}^T \text{ Inertial Orientation}\]

\[v_1 = \{u, v, w\}^T \text{ Body-fixed Linear Velocity}\]

\[v_2 = \{p, q, \tau\}^T \text{ Body-fixed Angular Velocity}\]

\[\tau_1 = \{F_x, F_y, F_z\}^T \text{ External Forces}\]

\[\tau_2 = \{M_k, M_M, M_N\}^T \text{ External Moments}\]

The equations of motion for rigid body, expressed in the body-fixed reference coordinate system, can be written as [67-69]

\[M_{RB} \ddot{v} + C_{RB}(v)v = \tau_{RB}\] (4.1)

where \(M_{RB}\) is a matrix of inertial and mass terms and \(C_{RB}\) is a matrix of centrifugal and Coriolis terms. They are given by

\[
M_{RB} = \\
\begin{bmatrix}
    m & 0 & 0 & 0 & mz_G & -my_G \\
    0 & m & 0 & -mz_G & 0 & mx_G \\
    0 & 0 & m & my_G & -mx_G & 0 \\
    0 & -mz_G & my_G & I_{xx} & -I_{sy} & -I_{xz} \\
    mz_G & 0 & -mx_G & -I_{sy} & I_{yy} & -I_{yz} \\
    -my_G & mx_G & 0 & -I_{xz} & -I_{yz} & I_{zz}
\end{bmatrix}
\]
\[ C_{RB}(v) = \begin{bmatrix}
0 & 0 & 0 & m_{zG}r + m_{yG}q & mw - mxGq & -mv - mxGr \\
0 & 0 & 0 & m_{wG} - m_{yG}p & mzGp + mxGr & m_{uG} - m_{yG}r \\
0 & 0 & 0 & m_{vG} - m_{zG}p & mu - m_{zG}q & mxGp + m_{yG}q \\
-m_{zG}r - m_{yG}q & m_{yG}p & mzGp & -I_{xG}q + I_{yG}r & -I_{zG}q - I_{yG}r & I_{zG}q + I_{yG}r \\
m_{xG}q & -m_{zG}r - m_{xG}p & mzGq & I_{xG}r + I_{zG}p & I_{zG}q - I_{yG}r & -I_{zG}q - I_{xG}r \\
m_{xG}r & m_{yG}r & -m_{xG}q - m_{yG}q & -I_{yG}q - I_{xG}p & I_{zG}q - I_{yG}r & I_{zG}q - I_{yG}r
\end{bmatrix} \]

(4.2)

\( \mathbf{v} = [u \ v \ w \ p \ q \ r]^T \) is the vector of translational and rotational velocities of the vehicle with respect to the vehicle body-fixed reference frame, \( \mathbf{\tau}_{RB} = \begin{bmatrix} F_x & F_y & F_z & M_x & M_y & M_z \end{bmatrix}^T \) is the vector that represents all external forces and moments applied to the carriage.

Equation 4.1 consists of two parts, translational and rotational, which can be rewritten as

\[ F = m \left( \mathbf{\dot{v}}_1 + \mathbf{v}_2 \times \mathbf{v}_1 + \mathbf{v}_2 \times \mathbf{r}_G + \mathbf{\dot{v}}_2 \times (\mathbf{\dot{v}}_2 \times \mathbf{r}_G) \right) \]

(4.3)

\[ M = \mathbf{I}_o \mathbf{\dot{v}}_2 + \mathbf{v}_2 \times (\mathbf{I}_o \mathbf{v}_2) + m \mathbf{r}_G \times (\mathbf{\dot{v}}_1 + \mathbf{v}_2 \times \mathbf{v}_1) \]

(4.4)

where \( \mathbf{I}_o \) is the inertial tensor as defined at the origin of the body-fixed coordinate system, and \( \mathbf{r}_G \) is the vector from origin of the body-fixed frame to the body center of gravity, and is defined as:

\[ \mathbf{r}_G = [x_G \ y_G \ z_G]^T \]

(4.5)

The Equation 4.3 and Equation 4.4 can be expanded as:

\[ F_x = m[\mathbf{\dot{u}} - v \mathbf{r} + wq - x_G (q^2 + r^2) + y_G (pq - p\dot{r}) + z_G (pr + q\dot{r})] \]

\[ F_y = m[\mathbf{\dot{v}} - wp + ur - y_G (r^2 + p^2) + z_G (qr - p\dot{r}) + x_G (qp + p\dot{r})] \]

(4.6)

\[ F_z = m[\mathbf{\dot{w}} - uq + vp - z_G (p^2 + q^2) + x_G (rp - q\dot{r}) + y_G (rq + p\dot{r})] \]
If the vehicle is symmetric around both the \(x-z\) and \(y-z\) planes, it is implied that \(I_{xy} = I_{xz} = I_{yz} = 0\). This reduces the rigid body inertia tensor to:

\[
I_o = \begin{bmatrix}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{bmatrix}
\] (4.8)

The equation 4.7 can be rewritten as

\[
M_K = I_{xx} \dot{p} + (I_{zz} - I_{xy}) qr - (\dot{r} + pq) I_{xx} + (r^2 - q^2) I_{yz} + (pr - \dot{q}) I_{xy} + m[y_G (\dot{w} - uq + vp) - z_G (\dot{v} - wp + ur)]
\]

\[
M_M = I_{yy} \dot{q} + (I_{xx} - I_{zz}) rp - (\dot{p} + qr) I_{xy} + (p^2 - r^2) I_{xz} + (qp - \dot{r}) I_{yz} + m[z_G (\dot{u} - vr + wq) - x_G (\dot{w} - uq + vp)]
\] (4.7)

\[
M_N = I_{zz} \dot{r} + (I_{yy} - I_{xx}) pq + m[x_G (\dot{v} - wp + ur) - y_G (\dot{u} - vr + wq)]
\]

If the center of mass of the vehicle is at the origin of the body-fixed reference frame, \(x_G = y_G = z_G = 0\), and the vehicle fixed coordinate frame and track reference coordinate have same origin.

The equations 4.7 and 4.9 are further simplified as
\[ F_x = m[\dot{u} - vr + wq] \]
\[ F_y = m[\dot{v} - wp + ur] \quad (4.10) \]
\[ F_z = m[\dot{w} - uq + vp] \]
\[ M_K = I_{xx} \dot{p} + (I_{zz} - I_{yy}) qr \]
\[ M_M = I_{yy} \dot{q} + (I_{xx} - I_{zz}) rp \quad (4.11) \]
\[ M_N = I_{zz} \dot{r} + (I_{yy} - I_{xx}) pq \]

With these six equations, we can simulate the translational and rotational dynamics. The results are \( \mathbf{v}_1 = [u \ v \ w]^T \) (Body-fixed linear velocity) and \( \mathbf{v}_2 = [p \ q \ r]^T \) (Body-fixed angular velocity). The following coordinate transform relates translational velocities between body-fixed and inertial coordinates:

\[
\begin{bmatrix}
  x \\
  y \\
  z 
\end{bmatrix} = 
\begin{bmatrix}
  u \\
  v \\
  w 
\end{bmatrix} \mathbf{J}_1(\mathbf{\eta}_2), \quad \mathbf{\eta}_1 = \mathbf{J}_1(\mathbf{\eta}_2) \mathbf{v}_1 \quad (4.12)
\]

where \( \mathbf{\eta}_1 = [x \ y \ z]^T \) (Inertial Position) and \( \mathbf{\eta}_2 = [\phi \ \theta \ \psi]^T \) (Inertial Orientation) are the final dynamic simulation results.

\[
\mathbf{J}_1(\mathbf{\eta}_2) = 
\begin{bmatrix}
  \cos \theta \cos \phi - \cos \phi \sin \phi + \cos \phi \sin \theta \sin \phi & \sin \phi \sin \phi + \cos \phi \sin \theta \cos \phi \\
  \cos \theta \sin \phi - \cos \phi \cos \phi + \sin \phi \sin \theta \sin \phi & -\sin \phi \cos \phi + \sin \phi \sin \theta \cos \phi \\
  -\sin \theta & \cos \phi \cos \phi & \cos \theta \sin \phi 
\end{bmatrix} \quad (4.13)
\]

The coordinate transform relates rotational velocities between body-fixed and inertial reference coordinates by:
\[ \dot{\eta}_2 = J_2(\eta_2) v_2 \]  

(4.14)

and

\[
J_2(\eta_2) = \begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi / \cos \theta & \cos \phi / \cos \theta
\end{bmatrix}
\]

(4.15)

\[
\begin{bmatrix}
\phi \\
\theta \\
\varphi
\end{bmatrix} = J_2(\eta_2) \begin{bmatrix}
p \\
q \\
r
\end{bmatrix} = \begin{bmatrix}
p + q \tan \theta \sin \phi + r \tan \theta \cos \phi \\
q \cos \phi \tan \theta - r \sin \phi \\
\cos \phi \sec \theta + q \sin \phi \cos \theta
\end{bmatrix}
\]

(4.16)

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} = J_2^{-1}(\eta_2) \begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\varphi}
\end{bmatrix} = \begin{bmatrix}
\dot{\phi} \sec \theta \\
\dot{\theta} \cos \phi + \dot{\phi} \cos \theta \sin \phi \\
-\dot{\theta} \sin \phi + \dot{\phi} \cos \theta \cos \phi
\end{bmatrix}
\]

(4.17)

\[
\begin{bmatrix}
\ddot{p} \\
\ddot{q} \\
\ddot{r}
\end{bmatrix} = \begin{bmatrix}
\ddot{\phi} \phi \sec \theta - \dot{\phi} \dot{\phi} \cos \theta \\
\ddot{\theta} \cos \phi - \dot{\theta} \dot{\phi} \cos \phi + \dot{\phi} \cos \theta \sin \phi + \dot{\phi} \cos \theta \cos \phi - \dot{\varphi} \dot{\varphi} \cos \phi - \dot{\phi} \dot{\theta} \sin \theta \sin \phi \\
-\ddot{\theta} \sin \phi + \dot{\theta} \dot{\phi} \cos \phi + \dot{\phi} \cos \theta \cos \phi - \dot{\varphi} \dot{\varphi} \cos \phi - \dot{\phi} \dot{\theta} \sin \theta \cos \phi
\end{bmatrix}
\]

(4.18)

4.2.1. System Based Design

The coordinate system for the inertial reference frame is defined the horizontal direction of travel as the \( x \)-axis, the lateral across the track as \( y \)-axis and the vertical direction as \( z \)-axis. The orientation of the three linear velocities \( u \), \( v \) and \( w \) and three rotation velocities \( p \), \( q \) and \( r \) are
shown in figure 4.2. Each labeling is a Halbach magnet array of the reference system [57].

Figure 4.3 Carriage Axis Arrangements

Levitation system equations are [58]

\[ \dot{z} = \gamma \]

\[ \dot{\gamma} = g \cos \varphi \cos \theta - pv + qu + \frac{\mu_0}{4m\pi} \frac{u_0^2}{u_0^2 + \lambda^2} \frac{I_i^2}{z} - \mu_0 \frac{u^2}{4m\pi (u^2 + \lambda^2)^2} \frac{I_i^2}{z_0} \frac{d}{dt}(\Delta z) - \frac{\mu_0}{4m\pi (u^2 + \lambda^2)^2} \frac{I_i^2}{z_0} \frac{d}{dt}(\Delta z) \quad (4.19) \]

\[ \dot{I}_i = -\frac{R}{L} I_i + \frac{v_l}{L} \]

\[ \lambda = \frac{2}{\mu_0 \sigma \theta} \quad \mu_0 = 4\pi \times 10^{-7} [H/m] \]

where \( g \) [m/s²] is gravity, \( z \) [m] is vertical position, \( \gamma \) [m/s] is the vertical velocity, \( I \) [A] is the current for levitation, \( R[\Omega] \) is the resistance, \( L[H] \) is the inductance, \( v_l \) [V] is the levitation...
voltage controller, $\lambda$[m/s] is the characteristic velocity, $\sigma$ is the conductivity, $\delta$[m] is the thickness of the rails, $u_0$ is the desired linear speed, m[Kg] is the vehicle mass, $\mu_0$ is the permeability, and $\Delta z$ [m] is the perturbation.

Rotational motion equations are

$$\dot{p} = \frac{1}{I_{xx}} \left[ (I_{yy} - I_{zz}) qr + (q^2 - r^2) I_{yz} \right]$$

$$\dot{q} = \frac{1}{I_{yy}} \left[ (I_{zz} - I_{xx}) pr - pq I_{yz} \right] + \frac{I_{yz}}{I_{yy} I_{zz}} \left[ (I_{xx} - I_{yy}) pq - pr I_{yz} \right]$$

$$\dot{r} = \frac{1}{I_{zz}} \left[ (I_{xx} - I_{yy}) pq + pr I_{yz} \right] + \frac{I_{yz}}{I_{yy} I_{zz}} \left[ (I_{zz} - I_{xx}) pr - pq I_{yz} \right]$$

(refer to figure 4.3) where $[p, q, r]$ are the body-fixed angular velocity (rad/s), $I_{ij}$ are the inertial terms, and $[\phi, \theta, \psi]$ are the inertial orientation (Euler Angles) of the vehicle (rad) and are governed by the following set of differential equations:

$$\dot{\phi} = p + \tan \theta (q \sin \phi + r \cos \phi)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\psi} = \sec \theta (q \sin \phi + r \cos \phi)$$

(4.21)
4.3. Point Mass Based Design

For the stage II, Point Mass Based Design is proposed. First, the levitation force produced per circuit by the magnet array is given by [6-9],

\[
F = \frac{B_0^2 w^2}{2kL} \frac{1}{1 + (R/\omega L)^2} \exp(-2kd) \text{ [Newton]} \tag{4.22}
\]

where \( R \) is the resistance of the circuit, \( w \) is the transverse width of each array, \( d \) is the distance from the array to the coils, \( \omega \) is the excitation frequency of the circuit and is given by the relationship \( \omega = kv \), \( v \) is the array velocity as it passes over the coils (m/s), \( k \) is given by the relationship \( k = \frac{2\pi}{\lambda} \), where \( \lambda \) is the array wavelength (0.1m), \( L \) is the inductance of the circuit and \( B_0 \) is the peak field at the surface of the array

\[
B_0 = B_r [1 - \exp(-kd_r)] \sin(\frac{\pi}{M}) \frac{\sin(\pi/M)}{(\pi/M)} \text{ [Tesla]} \tag{4.23}
\]

where, \( d_r \) is the thickness in the vertical direction of the Halbach array, \( B_r \) is the permanent magnet remanent field and \( M \) is the number of magnet elements per wavelength.

For the vehicle levitated above the track coils, the magnetic levitation force \( F \) is a function of the distance between the magnetic array and levitation coils, \( d \). The force can be expanded in a Taylor series in the perturbed small displacement variables about the equilibrium-nominal position.
\[ F(d) = F(d_0) + \left. \frac{\partial F}{\partial d} \right|_{d=d_0} (d - d_0) \]  \hspace{1cm} (4.24)

where, \( F(d_0) = mg \), \( m \) is the mass of the vehicle, and \( g \) is the gravity acceleration rate of earth.

\[ F_L = \frac{B_0^2 w^2}{2kL} \frac{1}{1 + (R/\omega L)^2} \exp(-2k\delta d) \]
\[ = F(d_0)(1 - 2k\delta d) \]  \hspace{1cm} (4.25)

For the vehicle at the equilibrium nominal position, the magnetic guidance force \( F \) is balanced by the force generated by both sides of the arrays on the vehicle. These forces are a function of the distance \( d \) between the magnetic array and track coils at a giving traveling speed. Similarly to the levitation dynamics,

\[ F(d) = F_{14}(d_0) - F_{25}(d_0) + \left. \frac{\partial(F_{14}(d) - F_{25}(d))}{\partial d} \right|_{d=d_0} (d - d_0) \]  \hspace{1cm} (4.26)

Substitute equation 4.25 into equation 4.26, then we get

\[ F(d) = -4kF_{14}(d_0)\delta d \]  \hspace{1cm} (4.27)

where, \( F_{14} \) and \( F_{25} \) are the lateral forces generated by guidance array 1, 4 and array 2, 5 respectively from figure 4.3.

The six degree of freedom equations for the motion of a rigid body are from figure 4.3:

\[ f_o = m\left(\frac{\partial V_o}{\partial t}\right)_{rel} + \omega \times v_o + \dot{\omega} \times r_G + \omega \times (\omega \times r_G) \]  \hspace{1cm} (4.28)
\[ \mathbf{m}_0 = \mathbf{I}_o \dot{\omega} + \omega \times (\mathbf{I}_o \omega) + m \mathbf{r}_G \times \left( \dot{\mathbf{v}_0} \right)_{\text{rel}} + \omega \times \mathbf{v}_0 \]  

(4.29)

where, \( f_0 \) is force, \( m \) is mass, \( v_0 \) is velocity, \( \omega_0 \) is the total angular velocity, \( \mathbf{m}_0 \) is moment, \( \mathbf{I}_o \) is the inertial tensor as defined at the origin of the carriage-fixed coordinate system, and \( \mathbf{r}_G \) is the vector from the inertial frame to the carriage center of gravity.

The designed nominal levitation height for this track is 0.01 m. At this height the forces from the three magnet arrays will be in equilibrium. Perturbations from the design height will increase the force between the array and the track on the surfaces that are closer, and reduce the force on the arrays that are farther away.

For the equilibrium state,

**Vertical translation:**

\[
\sum F_z = F_{z_3} - F_{z_1} - F_{z_2} + F_{z_6} - F_{z_4} - F_{z_5} 
\]

(4.30)

**Lateral translation:**

\[
\sum F_y = F_{y_1} - F_{y_2} + F_{y_4} - F_{y_5} 
\]

(4.31)

The force and moment equations are given by

\[
F_y = mg \cos \theta \sin \phi - (\sum F_z)(\cos \theta \sin \phi) \\
+ (\sum F_y)(\cos \phi \cos \varphi + \sin \varphi \sin \theta \sin \phi) 
\]

(4.32)

\[
F_z = mg \cos \theta \cos \phi - (\sum F_z)(\cos \theta \cos \phi) \\
+ (\sum F_z)(- \sin \phi \cos \varphi + \sin \varphi \sin \theta \cos \phi) 
\]

(4.33)
\[
M_K = (F_{z_3} - F_{z_1} + F_{z_2} + F_{z_6} - F_{z_4} + F_{z_5})(\cos \theta \cos \phi) \cdot L_y \\
- (F_{y_1} - F_{y_2} + F_{y_4} - F_{y_5})(- \sin \phi \cos \phi + \sin \phi \sin \theta \cos \phi) \cdot L_{ylt} \\
+ (F_{z_3} - F_{z_1} + F_{z_2} + F_{z_6} - F_{z_4} + F_{z_5})(\cos \theta \sin \phi) \cdot L_z \\
+ (F_{y_1} + F_{y_2} - F_{y_4} - F_{y_5})(\cos \phi \cos \phi + \sin \phi \sin \theta \sin \phi) \cdot L_{zlt}
\] (4.34)

\[
M_M = (F_{z_3} - F_{z_1} - F_{z_2} - F_{z_6} + F_{z_4} + F_{z_5})(\cos \theta \cos \phi) \cdot L_x \\
+ (F_{y_1} + F_{y_2} - F_{y_4} - F_{y_5})(- \sin \phi \cos \phi + \sin \phi \sin \theta \cos \phi) \cdot L_{ylt} \\
+ (-F_{z_3} + F_{z_1} + F_{z_2} + F_{z_6} - F_{z_4} - F_{z_5})(\sin \theta) \cdot L_x \\
- (F_{y_1} + F_{y_2} - F_{y_4} - F_{y_5})(\cos \theta \cos \phi) \cdot L_{zlt}
\] (4.35)

\[
M_N = (F_{z_3} - F_{z_1} - F_{z_2} - F_{z_6} + F_{z_4} + F_{z_5})(\cos \theta \sin \phi) \cdot L_x \\
+ (-F_{y_1} + F_{y_2} - F_{y_4} + F_{y_5})(\cos \theta \sin \phi) \cdot L_{ylt} \\
+ (-F_{z_3} + F_{z_1} + F_{z_2} + F_{z_6} - F_{z_4} - F_{z_5})(\sin \theta) \cdot L_y \\
+ (F_{y_1} - F_{y_2} + F_{y_4} - F_{y_5})(\cos \phi \cos \phi + \sin \phi \sin \theta \sin \phi) \cdot L_{zlt}
\] (4.36)

From equation 4.34 to equation 4.36, \( L_x, L_y, \) and \( L_z \) are the distances between center of each levitation array and the center of the vehicle in x, y, and z directions. \( L_{xlt}, L_{ylt}, \) and \( L_{zlt} \) are the distances between the center of the lateral array and the center of the vehicle in the x, y, and z directions accordingly.
4.4. Controller Design

4.4.1. Approach and Simulation Result for System Based Design

In this section, for levitation height control design, first define, the nonlinear state transformation from equation 4.19 [58],

\[ x_1 = z, \quad x_2 = \gamma, \quad x_3 = \frac{I_i^2}{|x_1|}. \]  

(4.37)

\[ \dot{x}_1 = x_2, \]

\[ \dot{x}_2 = g \cos \phi \cos \theta - pv + qu + \frac{\mu_0}{4m\pi} \frac{u_0^2}{(u_0^2 + \lambda^2)} x_3 - \frac{\mu_0}{4m\pi} \frac{u^2}{(u^2 + \lambda^2)} z_0^2 \Delta z(\cdot) \]

\[ - \frac{\mu_0}{4m\pi} \frac{\lambda^2 - u^2}{(u^2 + \lambda^2)^2} x_1 x_3 \frac{d}{dt} \Delta z(\cdot) \]

\[ \dot{x}_3 = -\frac{2R}{L} x_3 + \frac{2}{L} \frac{x_1^2 v_1}{x_1} - \frac{x_2 x_3}{x_1} \]  

(4.38)

To use a fictitious controller design, define error system \( e_i = x_i - x_i^d \) where \( x_i^d \) are fictitious controllers. What we want is

\[ \dot{e}_i = -k_i e_i \]

\[ e_1 = x_1 - x_1^d \]

\[ \dot{e}_1 = \dot{x}_1 - \dot{x}_1^d \]

\[ = x_2 - \dot{x}_1^d \]

\[ = e_2 + x_2^d - \dot{x}_1^d \]  

(4.39)

If we define, \( x_2^d = -k_i e_1 + \dot{x}_1^d \),

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Then, $\dot{e}_1 = -k_1e_1 + e_2$, this means next job is to make $e_2$ makes 0.

\[
\dot{e}_2 = \dot{x}_2 - \dot{x}_2^d
\]

\[
= g \cos \phi \cos \theta - pv + qu + \frac{\mu_0}{4m \pi u_0^2 + \lambda^2} (e_3 + x_3^d) - \frac{\mu_0}{4m \pi u^2 + \lambda^2} x_1 x_3 \frac{\Delta z(.)}{z_0}
\]

\[
= -\frac{\mu_0}{4m \pi (u^2 + \lambda^2)^{\frac{1}{2}}} \frac{\lambda}{z_0} \frac{d}{dt} (\Delta z(.) - \dot{x}_2^d)
\]

Let,

\[ A = g \cos \phi \cos \theta - pv + qu \]

\[ B = \frac{\mu_0}{4m \pi u_0^2 + \lambda^2} \]

\[ C = -\frac{\mu_0}{4m \pi u^2 + \lambda^2} \frac{x_1 x_3}{z_0} \]

\[ D = -\frac{\mu_0}{4m \pi (u^2 + \lambda^2)^{\frac{1}{2}}} \frac{\lambda}{z_0} x_1 x_3 \]

\[
\dot{e}_2 = A + B\left( e_3 + x_3^d \right) + C \Delta z(.) + D \frac{d}{dt}(\Delta z(.)) - \dot{x}_2^d
\]  

(4.41)

So, $-k_2 e_2 = A + B\left( e_3 + x_3^d \right) + C \Delta z(.) + D \frac{d}{dt}(\Delta z(.)) - \dot{x}_2^d$

and

\[
x_3^d = \frac{1}{B} \left( \dot{x}_2^d - k_2 e_2 - A \right) - \frac{1}{B} (C \Delta z(.) + D \frac{d}{dt}(\Delta z(.)))
\]  

(4.42)

Finally,
\[ \dot{e}_3 = \dot{x}_3 - \dot{x}_3^d \]
\[ = - \frac{2R}{L} x_3 - \frac{x_2 x_3}{x_4} \dot{x}_3^d + \frac{2}{L} \sqrt{\frac{x_3}{x_4}} v_i \]

Again, what we want is
\[ \dot{e}_3 = - k_3 e_3 \]
\[
v_i = \frac{L}{2} \frac{1}{\sqrt{x_3}} \left( \dot{x}_3^d + \frac{2R}{L} x_3 - k_3 e_3 + \frac{x_2 x_3}{x_1} \right), \tag{4.43}\]

From equation 4.38, this system is simulated by matlab/simulink. This simulation shows the performance of the levitation trajectory and controllers. The parameters used in this simulation are shown table 4.1 [57].

Table 4.1 Simulation Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Cradle</td>
<td>65 cm</td>
</tr>
<tr>
<td>Cradle Mass</td>
<td>9.3 Kg</td>
</tr>
<tr>
<td>Volume of Magnet</td>
<td>0.01 m × 0.13 m × 0.12 m</td>
</tr>
<tr>
<td>Wavelength (λ)</td>
<td>0.1 m</td>
</tr>
<tr>
<td>Total length of Halbach Array (w)</td>
<td>0.12 m</td>
</tr>
<tr>
<td>Resistance / loop (R)</td>
<td>1.5 × 10⁻³ Ohm</td>
</tr>
<tr>
<td>Inductance / loop (L)</td>
<td>2.6 × 10⁻⁶ Henry</td>
</tr>
<tr>
<td>Theoretical Surface Field (B0)</td>
<td>0.9 Tesla</td>
</tr>
</tbody>
</table>
The following figures show the results of height and voltage response, left side figure is height response and the right side is voltage response. Figure 4.4 shows Ideal system response, it illustrates no disturbance. Figure 4.5 shows with a disturbance at 5[sec] and figure 4.6 are used to see the response in detail. Figure 4.7 and figure 4.8 show a trajectory of height and voltage response. The Dotted line is the desired response and the solid line is the achieved response.

Figure 4.4 Ideal System, without Disturbance

Figure 4.5 with Disturbance at 5[sec]
Figure 4.6 Magnification Response with Disturbance at 5[sec]

Figure 4.7 Trajectory of Height and Magnification of Left Figure
4.4.2. Approach and Simulation Result for Point Mass Based Design

Because the vehicle dynamic damping is not large enough to damp the oscillations in lateral $y$, levitation $z$, roll $\phi$, pitch $\theta$, and yaw $\phi$, a control mechanism is desired to damp these oscillations. The rigid body dynamics with a small displacement around the nominal position are given by equation 4.44 [55, 56]. For small displacements around the nominal position, the high order terms are negligible compared to the principal terms. The simplified vehicle dynamics are given by

$$\mathbf{M}_{RB} \ddot{\mathbf{a}}_0 = \tau_{RB}$$  

(4.44)

where,

$$\tau_{RB} = [F_y \; F_z \; M_k \; M_M \; M_N]^T, \; \mathbf{a}_0 = [\delta y \; \delta z \; \phi \; \theta \; \phi]^T$$
\[
M_{RB} = \begin{bmatrix}
m & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & 0 \\
0 & 0 & I_{xx} & 0 & 0 \\
0 & 0 & 0 & I_{yy} & 0 \\
0 & 0 & 0 & 0 & I_{zz} \\
\end{bmatrix}
\]

\(\delta z\) is the vehicle displacement from equilibrium position in levitation direction, and \(\delta y\) is the vehicle displacement laterally from equilibrium position.

The system equation with control is

\[
M_{RB}\ddot{a}_0 = -K_{sl0}a_0 = \tau_0
\]

(4.45)

where,

\[
K_{sl0} = \begin{bmatrix}
8F_yk & 0 & 0 & 0 & 0 \\
0 & 2mgk & 0 & 0 & 0 \\
0 & 0 & 8F_ykL_z^2 & 0 & 0 \\
0 & 0 & 0 & 0.12F_ykL_x^2 & 0 \\
0 & 0 & 0 & 0 & 8F_ykL_{slt}^2 k \\
\end{bmatrix}
\]

From equation 4.45, it is clear that the dynamic modeling has no damping terms. The system equation with control is given by

\[
M_{RB0}\ddot{a}_0 + K_{sl0}a_0 + u = 0
\]

(4.46)

where, \(u\) is the control input with are give as \(u = [u_y, u_z, u_\phi, u_\theta, u_\varphi]\)

Then, control force equations are given as
\[ u_y = \sum F_y = K_y \delta \dot{y}, \quad u_z = \sum F_z = K_z \delta \dot{z} \]

\[ u_\phi = L_y \cdot \sum M_x = K_\phi \dot{\phi}, \quad u_\theta = L_x \cdot \sum M_z = K_\theta \dot{\theta} \]

(4.47)

\[ u_\psi = L_{x_{lat}} \cdot \sum M_y = K_\psi \dot{\psi} \]

Let define \( K_d = [K_y \ K_z \ K_\phi \ K_\theta \ K_\psi]^T \) is the damping control factors. To solve these equations some constraints can be given to get optimized solutions. To minimize the mean square of control forces, it can be solved with Lagrange multiplier optimization method. For a levitation,

\[
L = F_{z_1}^2 + F_{z_2}^2 + F_{z_6}^2 + F_{z_4}^2 + F_{z_5}^2 + \lambda_1 (F_{z_3} - F_{z_1} - F_{z_2} + F_{z_6} - F_{z_4} - F_{z_5} - u_z) \\
+ \lambda_2 (F_{z_3} - F_{z_1} - F_{z_2} + F_{z_6} + F_{z_4} + F_{z_5} - \frac{u_\theta}{L_x})
\]

(4.48)

The solutions are given as:

\[
F_{z_1} = -\frac{\lambda_1 - \lambda_2}{2} = \frac{u_z}{4} + \frac{u_\theta}{4L_x}, \quad F_{z_2} = -\frac{\lambda_1 - \lambda_2}{2} = \frac{u_z}{4} + \frac{u_\theta}{4L_x} \\
F_{z_3} = -\frac{\lambda_1 + \lambda_2}{2} = -\frac{u_z}{4} - \frac{u_\theta}{4L_x}, \quad F_{z_4} = -\frac{\lambda_1 + \lambda_2}{2} = -\frac{u_z}{4} - \frac{u_\theta}{4L_x} \\
F_{z_5} = -\frac{\lambda_1 + \lambda_2}{2} = \frac{u_z}{4} - \frac{u_\theta}{4L_x}, \quad F_{z_6} = -\frac{\lambda_1 - \lambda_2}{2} = -\frac{u_z}{4} + \frac{u_\theta}{4L_x}
\]

For a lateral,

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\[
L = F^2_{y_1} + F^2_{y_2} + F^2_{y_4} + F^2_{y_5} + \lambda_1(F_{y_1} - F_{y_2} + F_{y_4} - F_{y_5} - u_y) \\
+ \lambda_2(F_{y_1} - F_{y_2} + F_{y_4} - F_{y_5} - \frac{u_\phi}{L_y}) + \lambda_3(F_{y_1} - F_{y_2} + F_{y_4} - F_{y_5} - \frac{u_\phi}{L_{slt}})
\]  

(4.49)

The solutions are given as:

\[
F_{y_1} = -\frac{\lambda_1 + \lambda_2 + \lambda_3}{2} = -\frac{u_y}{4} - \frac{u_\phi}{4 L_y} - \frac{u_\phi}{4 L_{slt}}
\]

\[
F_{y_2} = -\frac{\lambda_1 - \lambda_2 - \lambda_3}{2} = \frac{u_y}{4} + \frac{u_\phi}{4 L_y} + \frac{u_\phi}{4 L_{slt}}
\]

\[
F_{y_4} = -\frac{\lambda_1 + \lambda_2 + \lambda_3}{2} = -\frac{u_y}{4} - \frac{u_\phi}{4 L_y} - \frac{u_\phi}{4 L_{slt}}
\]

\[
F_{y_5} = -\frac{\lambda_1 - \lambda_2 - \lambda_3}{2} = \frac{u_y}{4} + \frac{u_\phi}{4 L_y} + \frac{u_\phi}{4 L_{slt}}
\]

As an alternative, linear quadratic regulator (LQR) control is investigated. For a system

\[
\dot{x} = Ax + Bu
\]

(4.50)

The performance index is given by

\[
J = \int_0^\infty (x^* Q x + u^* R u) dt
\]

(4.51)

where \( x^* \) is the complex conjugate of the transpose of matrix \( x \) and the control vector is given by

\[
u(t) = -Kx(t)
\]

(4.52)
where, $Q$ is the weighting matrix on the states,

$R$ is a positive scalar and yields a matrix of optimal gains $K$ for the state feedback.

![LQR Optimal Control Block Diagram](image)

Figure 4.9 LQR Optimal Control Block Diagram

$$K = R^{-1}B^*P$$ (4.53)

where $P = P^*$, is the unique positive definite solution of the algebraic Riccati equation

$$A^*P + PA - PBP^{-1}B^*P + Q = 0$$ (4.54)

The weighting matrix $Q = 1$ and $R = 1$ are used in this paper to demonstrate the effectiveness of the design. The controller can be designed separately according to the LQR optimized control theory. Let define, $x_y = \delta Y, x_{2y} = \delta \dot{Y}, K_{u0y} = 8F_bk$. The lateral system equation is given by
\[ \dot{x}_y = A_y x_y + B_y u_y \]  
\[ \text{(4.55)} \]

where

\[ \dot{x}_y = [x_{1y}, x_{2y}], \quad A_y = \begin{bmatrix} 0 & 1 \\ -K_{sr0y} / m & 0 \end{bmatrix}, \quad B_y = \begin{bmatrix} 0 \\ -1 / m \end{bmatrix} \]  
\[ \text{(4.56)} \]

substituting equation 4.56 into equation 4.54, we get equation

\[ \begin{bmatrix} -K_{sr0y} / m & -K_{sr0y} / m \\ p_{12} & p_{22} \end{bmatrix} \begin{bmatrix} p_{12} \\ p_{11} \end{bmatrix} + \begin{bmatrix} -K_{sr0y} / m & p_{12} \\ -K_{sr0y} / m & p_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0 \]  
\[ \text{(4.57)} \]

solving it and select the positive values, the results are given

\[ p_{12} = m(\sqrt{K_{sr0y}^2 + 1} - 1) \]
\[ p_{11} = \sqrt{1 + 2m(\sqrt{K_{sr0y}^2 + 1} - 1)((\sqrt{K_{sr0y}^2 + 1} - 1) + K_{sr0y})} \]  
\[ p_{22} = m(\sqrt{1 + 2m(\sqrt{K_{sr0y}^2 + 1} - 1)}) \]  
\[ \text{(4.58)} \]

substituting equation 4.58 into equation 4.53, the optimized control gain matrix K is,
\[ K = R^{-1}B^*P = \begin{bmatrix} 0 & \frac{1}{m} \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \\ \end{bmatrix} = [(\sqrt{K_{st0y}^2 + 1 - 1}) \sqrt{1 + 2m(\sqrt{K_{st0y}^2 + 1 - 1})}]
\]

and the optimized control is

\[ u_y = Kx = \begin{bmatrix} \sqrt{K_{st0y}^2 + 1 - 1} + 1 + 2m(\sqrt{K_{st0y}^2 + 1 - 1}) \end{bmatrix} \begin{bmatrix} \delta Y \\ \delta Y \end{bmatrix}^T \]

\[ = [(\sqrt{K_{st0y}^2 + 1 - 1})\delta Y + \sqrt{1 + 2m(\sqrt{K_{st0y}^2 + 1 - 1})}\delta \dot{Y}]
\]

Similarly, the optimized control can be derived for \( u_z, u_\phi, u_\theta, \) and \( u_\phi \).

\[ u_z = [\sqrt{K_{st0z}^2 + 1 - 1}]\delta Z + \sqrt{1 + 2m(\sqrt{K_{st0z}^2 + 1 - 1})}\delta \dot{Z}
\]

\[ u_\phi = [\sqrt{K_{st0\phi}^2 + 1 - 1}]\phi + \sqrt{1 + 2I_{zz}(\sqrt{K_{st0\phi}^2 + 1 - 1})}\phi
\]

\[ u_\theta = [\sqrt{K_{st0\theta}^2 + 1 - 1}]\theta + \sqrt{1 + 2I_{zz}(\sqrt{K_{st0\theta}^2 + 1 - 1})}\phi
\]

\[ u_\phi = [\sqrt{K_{st0\phi}^2 + 1 - 1}]\phi + \sqrt{1 + 2I_{zz}(\sqrt{K_{st0\phi}^2 + 1 - 1})}\phi
\]
where \( K_{st0z} = 2mgk \), \( K_{st0\phi} = 8F_bkL_y^2 \),

\( K_{st0\theta} = 0.12F_bkL_x^2 \), and \( K_{st0\varphi} = 8F_bkL_{slt}^2 \).

From figure 4.10 to figure 4.12, the 6DOF vehicle dynamics with a small disturbance is shown under no control and a steady acceleration force. The simulation shows that small disturbance affects system dynamics and the oscillation will not be decayed. In view of height, maximum oscillation region is 0.4cm.

Figure 4.10 System Response Under No Control, Levitation Height
From figure 4.13 to figure 4.16, the system dynamics under optimized damping control are showed, which has the advantage of simple and the requirement for implementation. The results show that oscillation of levitation height, pitch, roll and lateral position are decreased. In view of height, maximum oscillation region is 0.25cm. Compared to condition under no control performance is enhanced 37.5%.
Figure 4.13 System Response Under Optimized Damping, Levitation Height

Figure 4.14 System Response Under Optimized Damping, Pitch
From figure 4.17 to figure 4.19 show the system dynamics under the optimized LQR control and compared it to the damping control response, better results are shown in levitation height, pitch and lateral position. The maximum height oscillation region is 0.06cm, which is a 76% performance enhancement. Optimized damping and LQR controls reduce instability of system dynamics.
Figure 4.17 System Response Under Optimized LQR, Levitation Height

Figure 4.18 System Response Under Optimized LQR, Pitch
Figure 4.19 System Response Under Optimized LQR, Lateral Position
5. CHAPTER FIVE: CONCLUSION

In this dissertation, the Wavelet Transform-based approach is studied to analyze the transient oscillatory response of an EDS Maglev system. The magnetic field intensity is mainly a function of the levitation height between the cradle and the track. The Wavelet Transform-based approach is used to analyze the harmonics of the magnetization function. Haar Wavelet is selected as the mother wavelet to decompose the coefficients of the Wavelet Transform. An irregular magnetic field distribution at the beginning stage of levitation has not been characterized by Fourier series and Finite Element Method. Field analyses based on the Fourier series and the Finite Element Method are provided in order to illustrate the advantage of the proposed scheme in the analysis of dynamic characteristics in the beginning stage of levitation.

The proposed scheme explains the under-damped dynamics that results from the cradle’s dynamic response to the irregular distribution of the magnetic field. This keeps the cradle oscillating in order to balance between the levitation force and gravity of the cradle at the beginning stage of levitation. This suggests that the EDS Maglev system that responds to a vertical repulsive force could be subjected to such instability at the beginning stage of a low levitation height. The proposed method can be useful in analyzing instability at the beginning stage of levitation height.

In order to model and analyze the EDS Maglev system, two different stages are proposed. Stage I is defined until the levitation height reaches a nominal point, and stage II is defined after
the levitation height reached that nominal point.

   During the stage I, the initial stage of the system, the objective is the height control. For this purpose, system based design and analysis is performed. Using backstepping approach, nonlinear robust control design method is used and simulated via Matlab/Simulink. For stage II, the objective is the system stability. An optimized damping and LQR control were designed and verified through the reference system, Inductrack. As shown in the simulation results, the proposed control schemes provide operational stability for Inductrack. The analysis and simulation results will be used as guidance for theoretical and experimental research.

   Based on the work done in this dissertation, there are several challenges for the Halbach array Maglev system. Future researches include:

   The active Halbach magnetic array design and build in practical way.

   The Halbach array geometry optimization based on the nominal levitation height has been studied, but the optimized geometry may need some modification.

   The control design has been investigated for ideal cases based on the states being measurable and without considering noise. There are many research opportunities in Maglev studies.
APPENDIX: HALBACH ARRAY LEVITATION FORCE
One of the most important equations in this thesis is the levitation force equation. The derivation was given in [8].

As the Halbach array moves above the coils, the magnetic field cuts through the upper conductors of the coil, the time-variation in magnetic field acts as a voltage source in each closed loop of wire. The effective circuit of this wire is an inductor $L$ and resistor $R$ in series. The standard circuit theory applies.

\[
V = L \frac{dI}{dt} + RI = (\phi_0 \sin(\omega t))' = \omega \phi_0 \cos(\omega t)
\]  

(A.1)

where $V$ is the induced voltage, $I$ is the induced current, $L$ is the inductance (self plus mutual) of a circuit, and $R$ is its resistance, and $\phi_0$ is the peak flux linked by the circuit. Equation A.1 can be rewritten as

\[
\frac{dI}{dt} + \frac{R}{L} I = \frac{\omega \phi_0}{L} \cos(\omega t)
\]  

(A.2)

The steady-state solution of equation A.2 is

\[
I(t) = \frac{\phi_0}{L} \left[ \frac{1}{1 + (R/\omega L)^2} \right] \sin(\omega t) + (R/\omega L) \cos(\omega t)
\]  

(A.3)
where excitation frequency $\omega$ of the circuit is $\omega = k \nu$ and $k = 2 \pi / \lambda$, $\nu$ is the array velocity, and $\lambda$ is the array wavelength.

The approximation Halbach array magnetic field flux densities are given as

\[
B_x = B_r \left(1 - \exp(-kd)\right) \frac{\sin(\pi/M)}{\pi/M} \sin(kx) \exp(-k\Delta z) \tag{A.4}
\]

\[
B_z = B_r \left(1 - \exp(-kd)\right) \frac{\sin(\pi/M)}{\pi/M} \cos(kx) \exp(-k\Delta z) \tag{A.5}
\]

where $\Delta z$ is the distance between the magnet array to the coil, $d$ is the thickness of the Halbach array, $M$ is the number of magnet bars per wavelength in the array, and $B_r$ is the remanence of the permanent magnet material.

The approximation induced flux is given by

\[
\phi = B_r \left(1 - \exp(-kd)\right) \sin(\pi/M) \frac{w \exp(-2k\Delta z) \sin(kx)}{k} \frac{1 - \exp(-kh)}{k} \tag{A.6}
\]

\[
\approx B_r \left(1 - \exp(-kd)\right) \sin(\pi/M) \frac{w \exp(-2k\Delta z) \sin(kx)}{k}
\]

where $h$ is the distance between the lower and upper legs of the coil.

\[
\phi_0 \approx \frac{wB_r}{k} \left(1 - \exp(-kd)\right) \frac{\sin(\pi/M)}{\pi/M} \exp(-2k\Delta z) \tag{A.7}
\]
Inserting equation A.7 into equation A.3, the induced current is given as

\[ I(t) = \frac{wB_r}{kL} (1 - \exp(-kd)) \frac{\sin(\pi/M)}{\pi/M} \exp(-2k\Delta z) \]

\[ \ast \left[ \frac{1}{1 + (R/\omega L)^2} \right] \left[ \sin(\omega t) + (R/\omega L) \cos(\omega t) \right] \]  \hspace{1cm} (A.8)

The levitation force is given by

\[ F_z = I(t) \ast B_x \ast w \]  \hspace{1cm} (A.9)

Averaging equation A.9 over the wavelength the average levitation force is given by:

\[ <F_z> = K_f \frac{B_r^2 \left[1 - \exp(-kd)\right] \frac{\sin(\pi/M)}{\pi/M}^2 \omega^2}{2kL} \]

\[ \ast \frac{1}{1 + (R/\omega L)^2} \exp(-2k\Delta z) \]  \hspace{1cm} (A.10)
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