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Abstract

The package can be used to analyze the performance of step-up and step-down procedures. It can be used to compare powers, calculate the "false discovery rate", to study the effects of reduced step procedures, and to calculate $P[U \leq k]$, where U is the number of rejected true hypotheses. It can be used to determine the maximum number of steps that can be made and still guarantee (with a given probability) that the number of false rejections will not exceed some specified number. The test statistics are assumed to have a multivariate-t distribution. Examples are included.

Keywords: step-up and step-down procedures, performance analysis, reduced step procedures, maximum steps to guarantee true rejection probabilities.

1. Introduction

There are many situations where a researcher is interested in the outcome of a family of tests. A classical example is the case where m experimental drugs are compared with a standard with respect to an outcome. Analysts are increasingly confronting large-mining environments, due to advances in computing facilities and data collection strategies. Multiple testing has increased in importance and micro-array experiments are common in such diverse areas as neuro-imaging, genomics and astronomy.

In multiple testing, it is important to control the probability of rejecting true hypotheses. A standard procedure has been to control the family-wise error rate (FWER), the probability of rejecting at least one true null hypotheses. For large numbers of hypotheses, using FWER can result in very low power for testing single hypotheses. Recently powerful multiple step methods have been developed for controlling the false discovery rate (FDR), that is, the expected proportion of type I errors. More recently Van der Laan, Dudoit, and Pollard (2004) proposed controlling a generalized family error rate k-FWER (also called gFWER(k)), defined as the probability of at least $(k+1)$ type I errors ($k = 0$ for the usual FWER). Lehmann and Romano (2005) suggested both a single step and a step-down procedure for controlling k-FWER. Somerville and Hemmelmann (2008) proposed limiting the number of steps in step-up or step-down procedures (reduced step procedures) to control k-FWER (and the proportion of false positives).

2. Step-down and step-up procedures

Step-down test procedures may be described as follows. Let $t_1, t_2, \ldots, t_m$ be the test statistics corresponding to the null hypotheses $H_1, H_2, \ldots, H_m$. Denote by T_i the random variable associated with t_i.

Let $T_{(1)} \leq T_{(2)} \leq \cdots \leq T_{(m)}$ be the ordered values for the test statistics and denote the corresponding hypotheses $H_{(1)}, H_{(2)}, \ldots, H_{(m)}$. If $T_{(m)} \geq d_m$, reject $H_{(m)}$ and continue for $i = m-1, m-2, \ldots$, comparing $T_{(i)}$ with d_i, rejecting $H_{(i)}$ if $T_{(i)} \geq d_i$ and continuing until for the first time $T_{(i)} < d_i$. If $T_{(m)} < d_m$, reject no hypotheses. If $T_{(i)} \geq d_i$, for all values of i, reject all the hypotheses. The values $d_1 \leq d_2 < \cdots \leq d_m$ are constants (called critical values).

For step-up procedures $T_{(i)}$ is compared with d_i, beginning with $i = 1$, continuing for $i = 2, 3, \ldots$ until, for the first time, $T_{(i)} > d_i$. $H_{(i)}, H_{(i+1)}, \ldots, H_{(m)}$ are then rejected.

3. Reduced step procedures

An s step, step-down procedure, is defined as follows: Compare $T_{(i)}$ with d_i for $i = m, m-1, \ldots, m-s+1$ until for the first time $T_{(i)} < d_i$, in which case reject $H_{(m)}, \ldots, H_{(i+1)}$. If $T_{(m)} < d_m$, reject no hypotheses. If $i = m-s+1$, reject all hypotheses for which $T_i \geq d_i$.

For the s step, step-up procedure, begin with the comparison of $T_{(m-s+1)}$ with d_{m-s+1}. If $T_{(m-s+1)} \geq d_{m-s+1}$, reject all hypotheses H_i for which $T_{(i)} \geq d_{m-s+1}$, and otherwise compare $T_{(m-s+2)}$ with d_{m-s+2}, $T_{(m-s+3)}$ with $d_{m-s+3}, \ldots$, until for the first time $T_{(i)} \geq d_i$. $H_{(i)}, H_{(i+1)}, \ldots, H_{(m)}$ are then rejected.

An s step procedure is equivalent to an m step procedure where the $m-s$ smallest values are replaced with the value d_{m-s+1}. The critical value d_{m-s+1} is the minimum critical value (MCV). The concept of an MCV was introduced by Somerville (2004).

4. Defining the problem

Assume there are m multivariately distributed test statistics. The n_T test statistics corresponding to the true hypotheses have zero means, and the n_F test statistics corresponding to the false hypotheses have means equal to δ. The test statistics corresponding to the true and false hypotheses have common correlations ϱ_T and ϱ_F respectively, with the common correlation between the true and false test statistics being ϱ_C. Using Monte Carlo methods the program `FDR` generates n m-dimensional random vectors from the multivariate distribution, and for each uses the step-up or step-down procedure to determine the number of true and false hypotheses rejected using the designated test procedure. Powers, probabilities, etc., are thus estimated.

The following parameters are needed as input:

m: Number of null hypotheses.

nT: Number of true hypotheses.

nF: Number of false hypotheses.

delta: Standardized mean of test statistics corresponding to false hypotheses.

df: Degrees of freedom for the multivariate distribution.

d: Vector of the m critical values.

rhoT, rhoF, rhoC: Correlation among true, among false and between true and false test statistics, respectively.

nFbeg, nFend, nFint: For mode 1, for a series of nF value, first, last and interval between.

nsBEG, nsEND, nsINT: For mode 2, for a series of # of steps, first, last and interval between.

kBEG, kEND, kINT: For tables of $P[U \leq k] \geq$ GAMMA first k, last k and interval between k values.

GAMMA: Given by the user. Requirement is that $P[U \leq k] \geq$ GAMMA.

5. Using the package

The package is designed to study the performance of arbitrary step-up and step-down procedures. There are three modes for the Fortran program FDR. Mode 1 was designed to calculate three kinds of power (per pair, all pairs and any pair). See Horn and Dunnett (2004). In addition FDR is calculated. Using mode 1, the user specifies the number of steps to be used, and one or more series of ranges for nF. The output file PWR.out contains, for all the values for nF specified, the three kinds of power, and the false discovery rate (FDR).

Somerville and Hemmelmann (2008) showed that, under fairly general conditions, when the means of test statistics corresponding to false hypotheses increase without limit, $P[U \leq k]$ is minimized. In Appendix A, it is shown that $P[U \leq k]$, as a function of nF, is minimized when nF is less than or equal to the number of steps minus 1. Extensive calculations, using a Fortran program BHmax with extended precision, strongly suggest, at least for the Benjamini and Hochberg (1995, BH for short) procedure, and most practical cases, that the minimum occurs when nF is exactly equal to the number of steps minus $(k + 1)$. Mode 1 can be used to verify this for particular sets of values of m, nF, d, etc.

While mode 1 uses a fixed value for the number of steps, and one or more series of ranges of values for nF, mode 2 uses nF equal to the number of steps minus $(k + 1)$, and one or more series of ranges for the number of steps.

A file CV.in must exist before executing FDR. The file must contain in non-decreasing order of magnitude, the m critical values for the procedure. A Fortran program makeCV, which calculates the ctitical values for the BH procedure, and for the step-down procedure of Lehmann and Romano (2005) is given in the supplementary materials. The user should be able to follow the instructions given in `FDR`.

The Fortran program `FDR` has been compiled using Lahey/Fujitsu Fortran 95 on Microsoft platforms XP, Vista and Windows 7, and using `g95` on Microsoft platform Windows 7.

6. Selecting the value of `kSTAR`

If the object is to calculate the maximum number of steps such that $P[U \leq k] \geq$ `GAMMA`, then the proper value for `kSTAR` is k. However the user may be interested in several values of k for several values of `GAMMA`. The minimum value of `s` is usually overestimated if `kSTAR` is chosen otherwise. The error is less serious if a lesser value is chosen for `kSTAR`. Especially for large values of `m`, `kSTAR` may be chosen slightly smaller with minimal error. Choosing `kSTAR` larger is usually more serious, with the error increasing much more rapidly than the choice of choosing the value slightly smaller. Example 3 in Appendix C gives a more detailed analysis.

7. Accuracy of estimated probabilities

The number of random m-dimensional vectors generated is given by `n`, a user input. Values of `n` of 100, 1000, 10000 and 100000 for the estimation of a probability of 0.95 result in standard errors of 0.0218, 0.0069, 0.0022 and 0.0007 respectively.

8. Miscellaneous

The following procedure is suggested when both one-sided and two-sided hypotheses are included. Calculate the p values corresponding to each test statistic, and then use the test statistic value which corresponds to that p value as a one-sided hypotheses.

Clearly the more steps that are used in a procedure, the more powerful the procedure will be. Mode 2 was added to the predecessor package to assist in determining the maximum number of steps which will still guarantee $P[U \leq k] \geq$ `GAMMA`.

The value of `n`, the number of generations of m-dimensional random vectors from a multivariate-t distribution, determines the accuracy of the estimates and the results. A useful technique is to begin a study with small values of `n`, but very large numbers of steps or numbers of false hypotheses. These preliminary runs can determine the values for the number of steps or false hypotheses in future runs where increased accuracies are needed.

The package has been used with the number of hypotheses as large as 1,000,000. With such large values for `m`, the value of `n`, the number of random generations of the m-dimensional vectors must be reduced so as to result in reasonable running times.

The original version of the program `FDR` was used to produce tables for two papers published in “Recent Developments in Multiple Comparison Procedures”, Institute of Mathematical Statistics, one by Somerville (2004) and one by Horn and Dunnett (2004).

9. Summary and conclusions

The Fortran program FDR can be a very useful tool in the evaluation of step-up or step-down procedures. It can be used to compare competing procedures.

It is worth noting that, since single step procedures are special cases of multi-step procedures, all of the capabilities of the program apply to single step procedures. See Example 5 in Appendix C.

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A. Corollary to Theorem 6.1, Somerville and Hemmelman

Somerville and Hemmelmann (2008) proved the following:

Theorem: Let $d_1 \leq d_2 \leq \cdots \leq d_m$ be the critical values corresponding to a step-up or step-down procedure used to simultaneously test m hypotheses. Let the random variable T_i, associated with the test statistic t_i be absolutely continuous or discrete, have mean μ_i, and a distribution such that

$$P[T_i \geq a|\mu_i = \mu] = P[T_i \geq (a+\delta)\mu_i = (\mu+\delta)],$$

where a, μ and δ are arbitrary constants.

Let $m = n_T + n_F$, where n_T and n_F, respectively are the number of true and false hypotheses. Suppose $T_1, T_2, \ldots, T_m$ have means $\mu_1, \mu_2, \ldots, \mu_{nF}, 0, \ldots, 0$, respectively. If U is the number of true hypotheses which are rejected, then, for the 1-sided case, $P[U \leq k]$ is minimized for $k \in [0, n_F]$, when $\mu_i \to \infty$, for $i = 1, \ldots, n_F$.

We show that the following corollary is true.

Corollary: Let $P^*(n_F, k, m)$ be the minimum value of $P[U \leq k]$ when there are `m` hypotheses, of which n_F are false. Under the conditions of the above theorem, the minimum value of P^*, as a function of n_F, occurs for a value of n_F which is never greater than $s-1$.

Proof: Assume $\mu_i \to \infty$, for $i = 1, \ldots, n_F$. Then $T_{(m)}, \ldots, T_{(m-n_F+1)} \to \infty$. If $n_F \geq s$,each of the `s` steps of an s-step procedure will result in the rejection of a false hypothesis and no true hypotheses can be rejected, i.e. $P[U \leq k] = 1$ for all possible k values. Thus a minimum value of $P^*(n_F, k, m)$, if one exists, cannot occur for a value of n_F greater than $s-1$.

Comments: The program `FDR` (mode 2) uses the value of n_F as $s-k-1$ in any calculations requiring $P^*(n_F, k, m)$. The program can also be used (mode 1) to check that $n_F = s-k-1$ gives the smallest value of $P[U \leq k]$ for arbitrary k, m, s and critical values. (See example 2 in Appendix C.)

Extensive calculations (primarily using critical values for the BH procedure) strongly suggest that, for most practical applications, $P^*(n_F, k, m)$, as a function of n_F, is minimized when $n_F = s-k-1$. Using a `Fortran` program `BHmax` with extended precision, no cases were encountered where a value of n_F resulted in $P[U \leq k]$ less than its value when $n_F = s-k-1$.

B. Random multivariate normal vector generator

This appendix outlines the methodology used to construct an m-dimensional vector with the first n_T components having mutual correlation ϱ_T, and the last $n_F = m - n_T$ components having mutual correlation ϱ_F, and with the correlation between elements of the two sets of components having mutual correlation ϱ_C.

For the i-th m-dimensional random vector, generate z_T, z_F and z_C as follows:

$$\begin{aligned} z_T &= \text{rnor}(nz, nw) \cdot \sqrt{\varrho_T - \varrho_C} \\ z_F &= \text{rnor}(nz, nw) \cdot \sqrt{\varrho_F - \varrho_C} \\ z_C &= \text{rnor}(nz, nw) \cdot \sqrt{\varrho_C} \end{aligned}$$

where $\text{rnor}(nz, nw)$ are randomly generated $N(0,1)$ variates. (Note that the generation requires that ϱ_C can never be larger than the smaller of ϱ_T and ϱ_F.)

Generate n_T random normal variates:

$$z(i,j) \quad = \quad \text{rnor}(nz, nw) \cdot \sqrt{1-\varrho_T} + z_T + z_C \quad (j = 1, \ldots, n_T).$$

Generate $n_F = m - n_T$ random normal variates:

$$z(i,j) \quad = \quad \text{rnor}(nz, nw) \cdot \sqrt{1-\varrho_F} + z_F + z_C \quad (j = n_T + 1, \ldots, m).$$

It is not difficult to show that $\text{Var}(z(i,j)) = 1$ Also

$$\begin{aligned}
\text{Cov}(z(i,j), z(i,j')) &= \varrho_C \quad (j = 1, \ldots, n_T; j' = n_T + 1, \ldots, m) \\
\text{Cov}(z(i,j), z(i,j')) &= \varrho_T \quad (j, j' = 1, \ldots, n_T; j, j' \text{unequal}) \\
\text{Cov}(z(i,j), z(i,j')) &= \varrho_F \quad (j, j' = n_T + 1, \ldots, m; j, j' \text{unequal})
\end{aligned}$$

The i-th m-dimensional vector $(z(i,1), z(i,2), \ldots, z(i,m))$ has the required characteristics.

C. Worked examples (tutorial)

This appendix illustrates how to use the Fortran program `FDR` for 6 different objectives. (A seventh possible objectivemfor the user is listed but not illustrated.

The program is interactive and its use begins with the user typing `FDR` and pressing `ENTER` for each example. All the program responses and the user actions are replicated. Upon completeion of the calculations, `FDR` notifies the user of the output files which contain the results of the computations (e.g. `The output file is AT1.out.`) The appropriate output file is explicitly presented for the user.

The seven different objectives are:

1. Calculate powers and false discovery rate (FDR) for multiple step procedures.
2. Check that minimum of $P[U \leq k]$ occurs when `nF` $= s - k - 1$.
3. Determine the maximum number of steps so that $P[U \leq k] \geq$ `GAMMA`.
4. Check accuracies when `kSTAR` is not chosen to equal k.
5. Calculate powers and false discovery rate (FDR) for single step procedures.
6. Examine effects of number of steps on powers and probabilities.
7. Compare two step-up or step-down procedures (not included).

In all cases, before executing the program `FDR`, the file `CV.in` must exist and contain the critical values for the step-up or step-down procedure to be analysed by `FDR`. The critical values must be arranged in non-descending order from smallest to largest.

A Fortran program `makeCV`, which can be used to give the critical values (in `CV.in`) for both the BH procedure and the step-down procedure of Lehmann and Romano (2005), is provided in the supplementary materials. Also included are the `CV.in` files needed for the examples in the appendix.

For all of the examples we assume the conditions for Theorem 6.1 of Somerville and Hemmelmann (2008), see Appendix A. Assume also that the means of the test statistics corresponding to a null hypothesis equal zero.

Example 1

Calculate powers and false discovery rate (FDR) for multiple step procedures.

(a) There are 50 null one-sided hypotheses to be simultaneously tested. Using the BH procedure with `q = 0.05` $= \alpha$, determine the per pair, all pairs and any pair power, and the actual false discovery rate (FDR) when there are 1, 11, 21, 31 or 41 false hypotheses, each of which are 3 standard errors greater than those of the true hypotheses (`delta = 3`). Use `n = 100000` and `seed = 757`. Assume the test statistics have a multivariate-t distribution with degrees of freedom equal to infinity and `rhoT = rhoF = rhoC = 0.0`. We will need a file `CV.in`, containing the critical values (ascending order) for the BH procedure.

(b) Would the powers be different with `rhoT = 0.2`, and `rhoF = rhoC = 0.0`?

(a) Executing `FDR`, we obtain:

```
TYPE 1 and press ENTER for MODE 1
TYPE 2 and press ENTER for MODE 2
TYPE 3 and press ENTER for MODE 3

1

In the next step user will need to give values for the following parame-
ters.
n        number of random vectors used
iseed    seed used for generation, e.g. 757, 1234998
m        number of hypotheses
df       degrees of freedom for test statistics
         use "-1" if infinity
nbrsided use "1" for 1-sided tests
         use "2" for two-sided tests
upordown use pos. integer or zero for stepup
         use neg. integer for down

GIVE the following PARAMETERS needed for FDRpwr5 (as integers)
and TYPE ENTER
  n iseed m df upordown nbrsided

100000 757 50 -1 1 1

Type a value for delta, the common expected value of the
test statistics corresponding to the false hypotheses.
If you wish to have all false hypotheses rejected, TYPE "10"
Press ENTER
```

```
3

rhoT is the correlation among TRUE test statistics
rhoF is the correlation among FALSE test statistics
rhoC is the correlation between TRUE and FALSE
Give rhoT,rhoF and rhoC

0.0 0.0 0.0

Give number of steps for procedure

50

GIVE values for nFbeg, nFend, nFint.
TYPE ENTER
P[U <= k] will be calculated for values of nF
from nFbeg to nFend, in intervals of nFint
U is the number of false positives

1 41 10

Do you want to repeat with new numbers of false Hypotheses?
If YES, type any positive number, press ENTER
If NO, type 0 or any negative number, press ENTER

0

The file PWR.IN is complete.
PRE completed
Program calculates P[U <= k] for all k <= m
and all  the values used in FDR
For which values of k would you like tables?
The program will make tables of P[U <= k] for values
of k from kBEG to kEND in intervals of kINT
Maximum number of tabulated k values in a run is 110!
Give kBEG, kEND and press "ENTER"
Warning:kEND must not be larger than m, the number of hypotheses to be
tested.

1 11 1

instructPOST completed
date and time 20110811 153257.199
Finished calculating for nF = 1
...
Finished calculating for nF = 41
Number of steps is      50
date and time 201110811 15336.543
```

```
BODY completed
POST has been successfully executed
Outout file is AT1.out
Ouput file PWR.out was completed in BODY
date and time 20110811 053306.559

FDR completed
```

The file `PWR.out` is as follows:

```
 nF  Per Pair Power   All Pairs Power  Any Pair Power  Actual FDR
   1       0.473            0.473            0.473            0.049
  11       0.730            0.053            1.000            0.039
  21       0.815            0.023            1.000            0.029
  31       0.859            0.015            1.000            0.019
  41       0.887            0.011            1.000            0.009
Number of steps is  50
m =  50 df= -1
rhoT=  0.00000000 rhoF=  0.00000000 rhoC=  0.00000000
seed= 757 n=  100000 1 -sided test
STEP UP testing    delta=  3.00000000        mode = 1
```

(b) Executing `FDR` with `rhoT = 0.2`, `rhoF = 0.0`, `rhoC = 0.0`, `PWR.out` is as follows:

```
 nF  Per Pair Power   All Pairs Power  Any Pair Power  Actual FDR
   1       0.474            0.474            0.474            0.046
  11       0.730            0.054            1.000            0.038
  21       0.815            0.023            1.000            0.029
  31       0.859            0.015            1.000            0.019
  41       0.887            0.011            1.000            0.009
Number of steps is  50
m =  50 df= -1
rhoT=  0.200000003 rhoF=  0.00000000E+00 rhoC=  0.00000000E+00
seed= 757 n=  100000 1 -sided test
STEP UP testing    delta=  3.00000000    mode = 1
```

Example 2

Check that minimum of $P[U \le k]$ occurs when `nF` $= s - k - 1$

Check that $P[U \le k]$ is smallest when `nF` $= s - k - 1$ for the BH procedure with `m = 1000`, `q` $= \alpha = 0.05$ and `s = 143`. (Assume that means of test statistics increase without limit.) Using mode 1, `delta = 10`, and `nF` first ranging from 0 to 900 in intervals of 100, and second, ranging from 949 to 999 in intervals of 50, k values from 1 to 11, execute FDR and obtain $P[U \le k]$ from AT1.out. Observe that $P[U \le k]$ is ∪-shaped as a function of `nF`.

Execute again, as previous, but this time with nF ranging from 135 to 150, again obtaining $P[U \le k]$ from AT1.out. Observe that the smallest value of P occurs when `nF` $= s - k - 1$.

(Using `n` = 100,000, an estimate of a value with a true probability of 0.95 would have a standard error of 0.00069.)

Execute `FDR` (first execution for Example 2) results in:

```
TYPE 1 and press ENTER for MODE 1
TYPE 2 and press ENTER for MODE 2
TYPE 3 and press ENTER for MODE 3

1

In the next step user will need to give values for the following parameters.
n        number of random vectors used
iseed    seed used for generation, e.g. 757, 1234998
m        number of hypotheses
df       degrees of freedom for test statistics
         use "-1" if infinity
nbrsided use "1" for 1-sided tests
         use "2" for two-sided tests
upordown use pos. integer or zero for stepup
         use neg. integer for down

GIVE the following PARAMETERS needed for FDR (as integers)
and TYPE ENTER
  n iseed m df upordown nbrsided

100000 757 1000 -1 1 1

Type a value for delta, the common expected value of the
test statistics corresponding to the false hypotheses.
If you wish to have all false hypotheses rejected, TYPE "10"
Press ENTER

10

rhoT is the correlation among TRUE test statistics
rhoF is the correlation among FALSE test statistics
rhoC is the correlation between TRUE and FALSE
Give rhoT,rhoF and rhoC

0.0  0.0  0.0

Give number of steps for procedure

143
```

```
GIVE values for nFbeg, nFend, nFint.
TYPE ENTER
P[U <= k] will be calculated for values of nF
from nFbeg to nFend, in intervals of nFint
U is the number of false positives

0 900 100

Do you want to repeat with new numbers of false Hypotheses?
If YES, type any positive number, press ENTER
If NO, type 0 or any negative number, press ENTER

9

GIVE values for nFbeg, nFend, nFint.
TYPE ENTER
P[U <= k] will be calculated for values of nF
from nFbeg to nFend, in intervals of nFint
U is the number of false positives

949 999 50

Do you want to repeat with new numbers of false Hypotheses?
If YES, type any positive number, press ENTER
If NO, type 0 or any negative number, press ENTER

0

The file PWR.IN is complete.
PRE completed
Program calculates P[U <= k] for all k <= m
and all  the values used in FDR
For which values of k would you like tables?
The program will make tables of P[U <= k] for values
of k from kBEG to kEND in intervals of kINT
Maximum number of tabulated k values in a run is 110!
Give kBEG, kEND and press "ENTER"
Warning:kEND must not be larger than m, the number of hypotheses to be tested.

        1  11  1

instructPOST completed
date and time 20110817 104014.322
Finished calculating for nF = 0
...
Finished calculating for nF = 999
```

```
Number of steps is  143
date and time 20110817 104904.729
BODY completed
POST has been successfully executed
Output file is AT1.out
Output file PWR.out was completed in BODY
date and time 20110817 104905.634
FDR completed
```

The file `AT1.out` is as follows:

```
P[U <= k]   U is # of false rejections
     nF\k   1     2     3     4     5     6     7     8     9    10    11
       0 0.995 0.999 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000
     100 0.055 0.156 0.310 0.488 0.654 0.787 0.880 0.937 0.969 0.985 0.994
     200 0.022 0.075 0.174 0.320 0.491 0.649 0.782 0.875 0.933 0.968 0.985
     300 0.040 0.121 0.261 0.437 0.616 0.761 0.867 0.932 0.968 0.986 0.994
     400 0.071 0.195 0.376 0.571 0.738 0.858 0.930 0.968 0.988 0.995 0.998
     500 0.126 0.305 0.518 0.712 0.848 0.929 0.970 0.989 0.996 0.999 1.000
     600 0.218 0.452 0.677 0.839 0.930 0.974 0.991 0.997 0.999 1.000 1.000
     700 0.365 0.635 0.830 0.934 0.978 0.994 0.998 1.000 1.000 1.000 1.000
     800 0.579 0.827 0.944 0.984 0.997 0.999 1.000 1.000 1.000 1.000 1.000
     900 0.839 0.964 0.994 0.999 1.000 1.000 1.000 1.000 1.000 1.000 1.000
     950 0.949 0.994 0.999 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000
     999 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000
```

Observe from `AT1.out` that $P[U \leq k]$ as a function of `nF` is ∪-shaped.

Executing the second time with `nF` ranging from 130 to 145, the file `AT1.out` is as follows:

```
   P[U <= k]   U is # of false rejections
     nF\k   1     2     3     4     5     6     7     8     9    10    11
     130 0.022 0.071 0.164 0.299 0.457 0.611 0.743 0.842 0.910 0.952 0.976
     131 0.021 0.069 0.160 0.294 0.451 0.605 0.737 0.839 0.908 0.950 0.975
     132 0.020 0.067 0.157 0.289 0.445 0.600 0.732 0.835 0.905 0.948 0.975
     133 0.020 0.065 0.154 0.284 0.439 0.594 0.727 0.831 0.902 0.949 0.975
     134 0.019 0.064 0.150 0.279 0.434 0.588 0.721 0.827 0.902 0.949 0.975
     135 0.018 0.062 0.147 0.274 0.428 0.582 0.717 0.828 0.903 0.950 0.976
     136 0.018 0.060 0.145 0.269 0.421 0.576 0.718 0.829 0.904 0.950 0.976
     137 0.017 0.059 0.141 0.264 0.416 0.577 0.719 0.830 0.904 0.950 0.976
     138 0.017 0.057 0.138 0.260 0.417 0.578 0.720 0.830 0.905 0.951 0.976
     139 0.016 0.056 0.135 0.261 0.418 0.580 0.721 0.831 0.905 0.951 0.976
     140 0.015 0.054 0.136 0.262 0.419 0.581 0.722 0.832 0.906 0.951 0.977
     141 0.015 0.054 0.137 0.263 0.421 0.582 0.723 0.832 0.906 0.951 0.977
     142 0.015 0.055 0.137 0.263 0.422 0.583 0.724 0.833 0.907 0.952 0.977
     143 0.015 0.055 0.138 0.264 0.423 0.584 0.725 0.834 0.907 0.952 0.977
     144 0.015 0.055 0.138 0.265 0.424 0.585 0.726 0.835 0.908 0.952 0.977
     145 0.015 0.056 0.139 0.266 0.426 0.586 0.727 0.836 0.908 0.952 0.977
```

From this `AT1.out` file, we observe that the value of $P[U \leq k]$ is never less than its value at `nF` $= s - k - 1$. (The values are mined from the file `MISC.out` where $P[U \leq k]$ is given to 4 decimal places. An estimate of a probability of 0.95 would have a standard error of 0.00069.)

Example 3

Determine the maximum number of steps so that $P[U \leq k] \geq$ `GAMMA`.

(a) Given 1000 hypotheses, make tables for the largest number of steps so that $P[U \leq k] \geq$ `GAMMA`. Use the BH procedure (one-sided) with `GAMMA` values of 0.95 and 0.99. Assume all correlations are zero and degrees of freedom infinity. Use `kSTAR` value of 1.

(b) Having completed 3a, and having the external file `MISC.out`, we can now use mode 3 to determine the maximum number of steps such that $P[U \leq k] \geq$ `GAMMA` for a different set of k values, and, or different values of `GAMMA` for the same number of hypotheses and the same values of α and degrees of freedom. Find the maximum number of steps such that $P[U \leq k] \geq$ `GAMMA`, for $k < 12$ when `GAMMA` equals 0.50 and 0.90.

(a) Two approaches can be used. We can make a preliminary study using numbers of steps from 3 to 400 in intervals of 10, (for example) using a small value of `n` (say 1000). The subroutine `POST` will use linear interpolation to give preliminary values, and then we can make a second study using one or more series of values for the number of steps and a larger value of `n`, and all numbers of steps from 3 to 400 in intervals of 1, and use `n = 100000`. This is much more time consuming.

Be sure that `CV.in` has the critical values (non-decreasing order) for the desired procedure. The required file for the BH procedure can be found in the supplementary materials. The first step is to execute `FDR`, using mode 2, and `delta = 10`.

Executing `FDR`, using `n=100000`, we have:

FDR

```
TYPE 1  and PRESS ENTER for MODE 1
TYPE 2  and PRESS ENTER for MODE 2
TYPE 3  and PRESS ENTER for MODE 3
```

2

```
In the next step user will need to give values for the following parame-
ters.
n        number of random vectors used
iseed    seed used for generation, e.g. 757, 1234998
m        number of hypotheses
df       degrees of freedom for test statistics
         use "-1" if infinity
nbrsided use "1" for 1-sided tests
         use "2" for two-sided tests
upordown use pos. integer or zero for stepup
         use neg. integer for down
```

```
GIVE the following PARAMETERS needed for FDRpwr (as integers)
and TYPE ENTER
  n iseed m df upordown nbrsided

100000 757 1000 -1 1 1

Type a value for delta, the common expected value of the
test statistics corresponding to the false hypotheses.
If you wish to have all false hypotheses rejected, TYPE "10"
Press ENTER

10

rhoT is the correlation among TRUE test statistics
rhoF is the correlation among FALSE test statistics
rhoC is the correlation between TRUE and FALSE
Give rhoT,rhoF and rhoC

0.0 0.0 0.0

Type a value for kSTAR
To give the most accurate computation of P[U <= k] use kSTAR=k.
However, especially for large values of m, the value
of P is only very slightly overestimated using values
of k which are moderately larger than kSTAR.
Using values for k smaller than kSTAR usually results in large
overestimates of P.
Thus a value of kSTAR greater than contemplated values of k is
 not recommended
A value of kSTAR equal to 1 is often sufficient.

1

Type nsBEG nsEND nsINT, press ENTER

3 400 1

Do you want to repeat the process with a different value
for the number of steps?
If YES, type any positive number, press ENTER
If NO, type 0 or any negative number, press ENTER

0

File PWR.in is complete.
PRE completed
Program calculates P[U <= k] for all k <= m
and all the values of nF used in FDR
```

```
For which values of k would you like tables?
The program will make tables of P[U <= k] for values
of k from kBEG to kEND in intervals of kINT
Maximum number of tabulated k values in a run is 110!
Give kBEG, kEND, kINT and press "ENTER"
WARNING: kEND must not be larger than m, the number of hypotheses tested

1 22 1

The program can calculate the largest number of steps
such that P[U <= k] >= GAMMA.
How many Gammas will you be using?

2

Give the  2   values and press ENTER.
For Example .99 .95 .90

.95 .99

instructPOST completed
date and time 20110813 125256.970
Finished calculating for steps =  3
...
Finished calculating for steps =  400
date and time 20110813  150842 .856
Finished forMAXsteps
POST has been successfully executed
Output files are AT1.out,AT2.out
Output file PWR.out was completed in BODY
See also file MAXsteps.out
date and time 20110813 150844.369
FDR completed
```

The file `MAXsteps.out` is as follows:

```
 m =  1000 df= -1
 rhoT=  0.00000000 rhoF=  0.00000000 rhoC=  0.00000000
 seed= 757 n=  100000 1 -sided test
 STEP UP testing  delta=  10.0000000   mode=  2
 NOTE: A negative number (or zero) for the max number of steps (eg -xx)
 indicates an undetermined max less than xx.
 A number equal to nsEND usually indicates an undetermined max number of
 steps exceeding nsEND. However too small a value for n and or nsEND
 could be the cause.  Examination of the appropriate AT file, or a rerun
 with a larger nsINT (quickest) or a larger value of n is suggested.
 nsEND= 400.
```

```
 Maximum number of steps to guarantee P[U <= k] >= GAMMA.   kSTAR=  1
GAMMA  0.950
     k    1     2     3     4     5     6     7     8     9    10    11
          7    16    28    41    54    70    86   104   123   144   166

     k   12    13    14    15    16    17    18    19    20    21    22
        189   215   245   279   318   368   400   400   400   400   400

GAMMA  0.990
     k    1     2     3     4     5     6     7     8     9    10    11
         -3     8    16    26    36    48    61    75    89   106   123

     k   12    13    14    15    16    17    18    19    20    21    20
        142   160   182   206   233   261   295   336   400   400   400
```

We now use an alternate (quicker and less exact) procedure. Before proceeding, save the file `MISC.out` (use another name as `MISC.out` will be overwritten in the following). We shall again execute `FDR`, but this time use `n = 1000`. We will again obtain the exterior files `MISC.out`, `AT1.out`,`AT2.out` and `MAXsteps.out`. The resulting `MAXsteps.out` exterior file is given below:

```
 Maximum number of steps to guarantee P[U <= k] >= GAMMA.   kSTAR=  1
GAMMA  0.950
     k    1     2     3     4     5     6     7     8     9    10    11
          7    17    28    42   400    71   400   400   400   400   400

     k   12    13    14    15    16    17    18    19    20    21    22
        400   400   400   400   400   400   400   400   400   400   400

GAMMA  0.990
     k    1     2     3     4     5     6     7     8     9    10    11
         -3     8    18    24    34    40    56   400    87   400   400

     k   12    13    14    15    16    17    18    19    20    21    22
        400   400   400   400   400   400   400   400   400   400   400
```

It is obvious that the maximum number of steps such that $P[U \leq k] \geq$ `GAMMA` is a non-decreasing function of `k`. However the file suggests otherwise. The problem is that we are using `n` $= 1000$, and the standard error of an estimate of a probability which is 0.95 is 0.007. An examination of the `AT1.out` file has probability estimates with obvious 'errors'. (See `GAMMA = 0.95`, and the sequence of probabilities for `nsteps` $= 45, 46, 47$ for example).

We shall try to circumvent the problem by repeating the execution of `FDR` using `nsINT` $= 10$ instead of 1. The output of the file `MAXsteps.out` is now:

```
 Maximum number of steps to guarantee P[U <= k] >= GAMMA.   kSTAR=  1
GAMMA  0.950
     k    1     2     3     4     5     6     7     8     9    10    11
```

```
             5     16     28     42     56     70     86    101    122    143    163

      k     12     13     14     15     16     17     18     19     20     21     22
           184    209    237    393    393    393    393    393    393    393    393

 GAMMA   0.990
      k      1      2      3      4      5      6      7      8      9     10     11
            -3      7     16     23     33     41     55     71     88    103    119

      k     12     13     14     15     16     17     18     19     20     21     22
           137    149    171    191    216    393    393    303    393    393    393
nsEND= 393
```

This has removed the inconsistency, but reducing `n` from 100000 to 1000 has decreased the accuracy materially.

(b) Use mode 3 (and existing external file `MISC.out`) to find the maximum number of steps such that $P[U \leq k] \geq$ `GAMMA` for `k` less than or equal to 11 when `GAMMA` equals 0.50 and 0.90. Execution of `FDR` results in the following exterior file `MAXsteps.out`:

```
 Maximum number of steps to guarantee P[U <= k] >= GAMMA.   kSTAR=  1
GAMMA  0.500
     k      1      2      3      4      5      6      7      8      9     10     11
           34     56     79    103    129    157    188    222    261    307    368

GAMMA  0.900
     k      1      2      3      4      5      6      7      8      9     10     11
           10     22     36     51     67     84    103    123    145    168    194
```

Example 4

Check accuracies when `kSTAR` is not chosen to equal `k`.

We have demonstrated that $P[U \leq k]$ as a function of `nF` is ∪-shaped, and have proved that, under the conditions of Theorem 6.1 of Somerville and Hemmelmann (2008), the minimum occurs when `nF` is not greater than $s-1$, and assert that, in most practical cases, the minimum occurs when `nF` equals $s - k - 1$.

We have observed that the slope of $P[U \leq k]$ is steepest when `nF` is less than $s - k$, and is nearly flat for a large proportion of the range from `nF` $= s - k$ to `m`.

Table 1 illustrates the differences between the maximum number of steps such that $P[U \leq k] \geq$ `GAMMA` when `kSTAR` = `k`, and when `kSTAR` is 5.

Note that, as previously indicated, using `kSTAR` larger or smaller, in `FDR` results in an overestimate of the number of steps which can be used, and still maintain $P[U \leq k] \geq$ `GAMMA`. If `kSTAR` is less than `k`, the overestimate is usually slight, particularly if `m` or `k` or both are large. However, when `kSTAR` is greater than `k` the overestimate is much more serious, even when the difference is small.

k	1	2	3	4	5	6	7	8	9	10	15	20
kSTAR = 5	11	19	30	41	54	69	86	103	123	143	276	1000
kSTAR = k	7	16	28	40	54	69	85	103	122	142	270	1000

Table 1: Table comparing $P[U \leq k]$ when `kSTAR = 5`, and `kSTAR = k`, `m = 1000`, `q = 0.05`, BH procedure, `GAMMA = 0.95`, `n = 100000`.

Example 5

Calculate powers and false discovery rate (FDR) for single step procedures.

A single step procedure can be regarded as a multi-step procedure with $\mathtt{s} = 1$. The following example uses `FDR` to determine the powers and false discovery rate (FDR) for the Bonferroni procedure. Note that, using $\mathtt{s} = 1$, the file `CV.in` can use the critical values for the Holm procedure, the Lehman-Romano procedure with $\mathtt{k} = 1$, or the BH procedure.

Problem: For `m = 1000`, `q = 0.05` and `delta = 3`, find the powers and false discovery rate (FDR) of the Bonferroni procedure.

Use $\mathtt{s} = 1$, and assume all correlations equal to 0.

Executing `FDR` we obtain the followimg:

```
TYPE 1 and press ENTER for MODE 1
TYPE 2 and press ENTER for MODE 2
TYPE 3 and press ENTER for MODE 3

1

In the next step user will need to give values for the following parameters.
n         number of random vectors used
iseed     seed used for generation, e.g. 757, 1234998
m         number of hypotheses
df        degrees of freedom for test statistics
          use "-1" if infinity
nbrsided use "1" for 1-sided tests
          use "2" for two-sided tests
upordown use pos. integer or zero for stepup
          use neg. integer for down

GIVE the following PARAMETERS needed for FDRpwr (as integers)
and TYPE ENTER
  n iseed m df upordown nbrsided

100000 757 1000 -1 1 1

Type a value for delta, the common expected value of the
test statistics corresponding to the false hypotheses.
If you wish to have all false hypotheses rejected, TYPE "10"
Press ENTER
```

3

```
rhoT is the correlation among TRUE test statistics
rhoF is the correlation among FALSE test statistics
rhoC is the correlation between TRUE and FALSE
Give rhoT,rhoF and rhoC
```

0.0 0.0 0.0

```
Give number of steps for procedure
```

1

```
GIVE values for nFbeg, nFend, nFint.
TYPE ENTER
```

1 20 1

```
P[U <= k] will be calculated for values of nF
from nFbeg to nFend, in intervals of nFint
U is the number of false positives

Do you want to repeat with new numbers of false Hypotheses?
If YES, type any positive number, press ENTER
If NO, type 0 or any negative number, press ENTER
```

0

```
File PWR.IN is complete.
PRE completed
Program calculates P[U <= k] for all k <= m
and all the values of nF used in FDR
For which values of k would you like tables?
The program will make tables pf P[U <= k] for values
of k from kBEG to kEND in intervals of kINT.
Maximum number of tabulated k values in a run is 110!
Give kBEG, kEND, kINT and press "ENTER"
WARNING: kEND must not be larger than m, the number of hypotheses.
```

1 22 1

```
InstructPOST completed
date and time 20111012 113006.187
Finished calculating for nF = 1
...
Finished calculating for nF = 20
```

```
Number of steps is   1
date and time 20111012 113621.062
BODY completed

POST has been successfully executed
Output files are AT1.out,  AT2.out
Output file PWR.out was completed in BODY
date and time 20111012 113621.234
FDR completed
```

The following is the output in `PWR.out`:

```
  nF  Per Pair Power   All Pairs Power  Any Pair Power  Actual FDR
   1       0.189            0.189            0.189            0.044
   2       0.188            0.036            0.339            0.040
   3       0.187            0.007            0.462            0.037
   4       0.187            0.001            0.564            0.033
   5       0.188            0.000            0.645            0.031
   6       0.187            0.000            0.711            0.028
   7       0.187            0.000            0.766            0.026
   8       0.187            0.000            0.811            0.024
   9       0.187            0.000            0.844            0.023
  10       0.187            0.000            0.874            0.021
  11       0.187            0.000            0.898            0.020
  12       0.187            0.000            0.916            0.019
  13       0.187            0.000            0.932            0.017
  14       0.187            0.000            0.945            0.017
  15       0.187            0.000            0.955            0.016
  16       0.187            0.000            0.964            0.015
  17       0.187            0.000            0.970            0.014
  18       0.187            0.000            0.976            0.013
  19       0.187            0.000            0.981            0.013
  20       0.187            0.000            0.984            0.012
 Number of steps is  1
 m =  1000 df= -1
 rhoT=  0.00000000 rhoF=  0.00000000 rhoC=  0.00000000
 seed= 757 n=  100000 1 -sided test
 STEP UP testing    delta=  3.00000000   mode=  1
```

Example 6

Examine effects of number of steps on powers and probabilities.

Compare the per pair power and $P[U \leq 1]$ for s $= 1, 11, ..., 110$ and 800 for values of nF $=$ 1, 11, ..., 110, and 800. Use `q = 0.05`, `m = 1000`, `n = 100000`, `delta = 3`, `rhoT = rhoF = rhoC = 0.0` for the BH procedure.

We will need a run for each value of `s`. We illustrate for s $= 1$. Executing FDR, we obtain:

```
TYPE 1 and press ENTER for MODE 1
TYPE 2 and press ENTER for MODE 2
TYPE 3 and press ENTER for MODE 3

1

In the next step user will need to give values for the following parameters.
n        number of random vectors used
iseed    seed used for generation, e.g. 757, 1234998
m        number of hypotheses
df       degrees of freedom for test statistics
         use "-1" if infinity
nbrsided use "1" for 1-sided tests
         use "2" for two-sided tests
upordown use pos. integer or zero for stepup
         use neg. integer for down

GIVE the following PARAMETERS needed for FDR (as integers)
and TYPE ENTER
  n iseed m df upordown nbrsided

100000 757 1000 -1 1 1

Type a value for delta, the common expected value of the
test statistics corresponding to the false hypotheses.
If you wish to have all false hypotheses rejected, TYPE "10"
Press ENTER

3

rhoT is the correlation among TRUE test statistics
rhoF is the correlation among FALSE test statistics
rhoC is the correlation between TRUE and FALSE
Give rhoT,rhoF and rhoC

0.0  0.0  0.0

Give number of steps for procedure

1

GIVE values for nFbeg, nFend, nFint.
TYPE ENTER
P[U.le.k] will be calculated for values of nF
from nFbeg to nFend, in intervals of nFint
U is the number of false positives
```

```
1 101 10

Do you want to repeat with new numbers of false Hypotheses?
If YES, type any positive number, press ENTER
If NO, type 0 or any negative number, press ENTER

9

GIVE values for nFbeg, nFend, nFint.
TYPE ENTER
P[U.le.k] will be calculated for values of nF
from nFbeg to nFend, in intervals of nFint
U is the number of false positives

800 800 1

Do you want to repeat with new numbers of false Hypotheses?
If YES, type any positive number, press ENTER
If NO, type 0 or any negative number, press ENTER

0

The file PWR.IN is complete.
PRE completed
Program calculates P[U <= k] for all k <= m
and allthe values of nF used in FDR.
For which values of k would you like tables?
The program will make tables of P[U <= k] for values
of k from kBEG to kEND in intervals of kINT
Maximum number of tabulated k values in a run is 110!
Give kBEG, kEND, kINT and press "ENTER"
Warning: kEND must not be larger than m, the number of hypotenuses

1 22 1

INSTRUCTpost COMPLETED
Date and time 20111012 130407.2811
Finished calculating for nF = 1
...
Finished calculating for nF = 101
Finished calculating for nF = 800
Number of steps is  1
Date and time 20111012 130407.281
BODY completed

POST has been successfully executed
```

```
Output files are AT1.out, AT2.out
Output file PWR.out was completed in BODY
Date and time 20111012 130409.562
FDR completed
```

The output for $P[U \leq k]$ and per pair power can be found in `AT1.out` and `PWR.out` and are given below:

P[U <= k]   U is # of false rejections

nF\k	1	2	3	4	5	6	7	8	9	10	11
1	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
11	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
21	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
31	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
41	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
51	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
61	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
71	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
81	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
91	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
101	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
800	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

nF	Per Pair Power	All Pairs Power	Any Pair Power	Actual FDR
1	0.189	0.189	0.189	0.044
11	0.187	0.000	0.898	0.020
21	0.187	0.000	0.987	0.012
31	0.187	0.000	0.998	0.008
41	0.187	0.000	1.000	0.006
51	0.186	0.000	1.000	0.005
61	0.186	0.000	1.000	0.004
71	0.186	0.000	1.000	0.003
81	0.186	0.000	1.000	0.003
91	0.186	0.000	1.000	0.003
101	0.186	0.000	1.000	0.002
800	0.187	0.000	1.000	0.000

We use column 2 of each table to produce the line for `s` $= 1$ for the tables of $P[U \leq k]$, Table 2 and per pair power, Table 3. Additional runs for `s` $= 11$, 21, ... are needed for the respective lines in Tables 2 and 3 given below.

s\nF	1	11	21	31	41	51	61	71	81	91	101	800
1	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	1.000
11	.994	.965	.919	.902	.902	.903	.905	.907	.909	.910	.912	.994
21	.994	.965	.905	.824	.757	.739	.741	.745	.749	.752	.756	.980
31	.994	.965	.905	.822	.724	.632	.585	.578	.582	.587	.593	.960
41	.994	.965	.905	.822	.724	.623	.526	.461	.442	.444	.449	.935
51	.994	.965	.905	.822	.724	.623	.524	.433	.364	.334	.332	.905
61	.994	.965	.905	.822	.724	.623	.524	.432	.353	.288	.253	.872
71	.994	.965	.905	.822	.724	.623	.524	.432	.353	.284	.228	.839
81	.994	.965	.905	.822	.724	.623	.524	.432	.353	.284	.227	.804
91	.994	.965	.905	.822	.724	.623	.524	.432	.353	.284	.227	.768
101	.994	.965	.905	.822	.724	.623	.524	.432	.353	.284	.227	.730
800	.994	.965	.905	.822	.724	.623	.524	.432	.353	.284	.227	.007

Table 2: Table of `P[U.le.1]`.

s\nF	1	11	21	31	41	51	61	71	81	91	101	800
1	.189	.187	.187	.187	.187	.186	.186	.186	.186	.186	.186	.187
11	.194	.296	.366	.392	.396	.396	.396	.396	.396	.396	.396	.396
21	.194	.296	.371	.424	.457	.468	.469	.469	.469	.469	.469	.470
31	.194	.296	.371	.425	.466	.497	.513	.516	.517	.517	.517	.517
41	.194	.296	.371	.425	.466	.500	.527	.545	.550	.551	.551	.552
51	.194	.296	.371	.425	.466	.500	.528	.552	.570	.577	.579	.579
61	.194	.296	.371	.425	.466	.500	.528	.552	.573	.590	.599	.602
71	.194	.296	.371	.425	.466	.500	.528	.552	.574	.592	.608	.621
81	.194	.296	.371	.425	.466	.500	.528	.552	.574	.592	.609	.638
91	.194	.296	.371	.425	.466	.500	.528	.552	.574	.592	.609	.652
101	.194	.296	.371	.425	.466	.500	.528	.552	.574	.592	.609	.666
800	.194	.296	.371	.425	.466	.500	.528	.552	.574	.592	.609	.884

Table 3: Table of per pair power.

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