Optical And Physical Properties Of Ceramic Crystal Laser Materials

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OPTICAL AND PHYSICAL PROPERTIES OF CERAMIC CRYSTAL LASER MATERIALS

by

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A dissertation submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
in the Department of Physics
in the College of Sciences
at the University of Central Florida
Orlando, Florida

Summer Term
2007

Major Professor: Michael Bass
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ABSTRACT

Historically ceramic crystal laser material has had disadvantages compared to single crystal laser material. However, progress has been made in the last decade and a half to overcome the disadvantages associated with ceramic crystal. Today, because of the promise of ceramic crystal as a high power laser material, investigation into its properties, both physical and optical, is warranted and important.

Thermal expansion was measured in this thesis for Nd:YAG (yttrium aluminum garnet) ceramic crystal using an interferometric method. The interferometer employed a spatially filtered HeNe at 633 nm wavelength. Thermal expansion coefficients measured for the ceramic crystal samples were near the reported values for single crystal Nd:YAG. With a similar experimental setup as that for the thermal expansion measurements, dn/dT for ceramic crystal Nd:YAG was measured and found to be slightly higher than the reported value for single crystal.

Depolarization loss due to thermal gradient induced stresses can limit laser performance. As a result this phenomenon was modeled for ceramic crystal materials and compared to single crystals for slab and rod shaped gain media. This was accomplished using COMSOL Multiphysics, and MATLAB. Results indicate a dependence of the depolarization loss on the grain size where the loss decreases with decreased grain size even to the point where lower loss may be expected in ceramic crystals than in single crystal samples when the grain sizes in the ceramic crystal are sufficiently small.

Deformation-induced thermal lensing was modeled for a single crystal slab and its relevance to ceramic crystal is discussed. Data indicates the most notable cause of deformation-induced thermal lensing is a consequence of the deformation of the top and bottom surfaces.
Also, the strength of the lensing along the thickness is greater than the width and greater than that due to other causes of lensing along the thickness of the slab.

Emission spectra, absorption spectra, and fluorescence lifetime were measured for Nd:YAG ceramic crystal and Yb:Lu₂O₃ ceramic crystal. No apparent inhomogeneous broadening appears to exist in the Nd:YAG ceramic at low concentrations. Concentration and temperature dependence effects on emission spectra were measured and are presented.

Laser action in a thin disk of Yb:Y₂O₃ ceramic crystal was achieved. Pumping was accomplished with a fiber coupled diode laser stack at 938 nm. A slope efficiency of 34% was achieved with maximum output energy of 28.8 mJ/pulse.
Dedicated to my wife, Janelle, my daughter, Kaley, my parents, siblings and everyone who ever hoped I would one day finally complete the requirements for graduation.
ACKNOWLEDGMENTS

This work was possible because of Dr. Michael Bass who took me into his group when I lost my first advisor to another University. Also, members of Bass group past and present made this possible. Thanks to Mark Dubinskiy, for whom I worked at the Army Research Laboratory, and who was a motivating mentor during my summer internship there. Tara Corso from VLOC has been very helpful in providing samples to work with at apparently no cost but information exchange. Dr. Alexandra Rapaport, and Dr. Rita Peterson were very helpful in getting me going on some of the experiments and even doing them with me. Also, Dr. Boris Zeldovich provided very helpful discussion, and Dr. Bin Chen originally wrote some of the code used for the models presented herein. It’s nice to be surrounded by such genius.
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CHAPTER ONE
INTRODUCTION

During the last decade, doped ceramic crystals have developed into attractive materials for use in high power laser systems. Ceramic crystal has been found to have physical and optical properties on par with its single crystal counterparts. In some instances ceramic crystal materials have even been demonstrated to possess significantly improved properties. Notably, the fracture toughness is higher for ceramic crystal than for single crystal [1]. Ceramic crystal Nd:YAG lasers have even been reported to yield higher slope efficiency than comparable single crystals [2, 3]. However, it may be some time before ceramic crystal of such high quality as was used [3] become generally available. Also, because of the random orientation of each grain in the ceramic crystal, only crystals having cubic structures are suitable for use as ceramic crystal laser materials.

Since 1995 when ceramic crystals began to show promise as a laser material, there has been an increase in research focused on ceramic crystals. Much of this has been centered on Nd\textsuperscript{3+} doped yttrium aluminum garnet (YAG) ceramic crystal. Lasing has been demonstrated in a variety of host materials and from several dopants (see Table 1). A variety of physical and optical properties have been measured including birefringence, index of refraction, scattering, and hardness. In many instances the ceramic crystal materials has been shown to be comparable to their single crystal counterparts.

Scattering in ceramic crystals had to be eliminated or reduced before it could be considered comparable to single crystals. Since a ceramic crystal is made of microscopic grains,
numerous grain boundaries exist inside the material. If there is a phase difference between the material [4] of one grain and adjoining grains or there is a pore separating the grains [5], scattering can occur at the grain boundaries. Also, grain boundary thickness influences scattering [6]. Researchers have been successful in reducing scattering so that very high quality material has been fabricated resulting in ceramic crystal materials as laser hosts. [6, 5]

Table 1:

Years in which lasing was demonstrated in several ceramic crystal materials.

<table>
<thead>
<tr>
<th>Laser dopant:host</th>
<th>Year Demonstrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dysprosium:CaF₂</td>
<td>1966</td>
</tr>
<tr>
<td>Nd:Yttralox</td>
<td>1973</td>
</tr>
<tr>
<td>Nd:YAG</td>
<td>1995</td>
</tr>
<tr>
<td>Nd:Y₂O₃</td>
<td>2001</td>
</tr>
<tr>
<td>Nd:Lu₂O₃</td>
<td>2002</td>
</tr>
<tr>
<td>Yb:Y₂O₃</td>
<td>2002</td>
</tr>
<tr>
<td>Yb:YAG</td>
<td>2003</td>
</tr>
<tr>
<td>Yb:Sc₂O₃</td>
<td>2003</td>
</tr>
<tr>
<td>Yb:Lu₂O₃</td>
<td>2004</td>
</tr>
</tbody>
</table>

Sesquioxides such as yttria (Y₂O₃) and lutetia (Lu₂O₃) are of particular interest since they have attractive thermal properties for use as laser host materials. Yet they have such high melting temperatures as to make growth by the conventional Czochralski process very difficult if not impossible [7, 8, 9, 10]. Ceramic crystal fabrication techniques allow sesquioxides to be fabricated at temperatures below their melting points. For example, the melting temperature of
Y₂O₃ is ≈2410 °C while one reported sintering temperature for Y₂O₃ ceramic crystal material is close to ≈1700 °C [8]. Therefore, Y₂O₃ and other ceramic crystal sesquioxides can be prepared and studied as viable host materials.

There are other advantages to ceramic crystal materials. Dopant concentrations can be made higher than in a single crystal. For example Nd:YAG single crystal generally has a maximum dopant concentration of 1% neodymium. As a ceramic crystal, Nd:YAG can have dopant concentrations up to 9% [11]. Higher concentrations may lead to concentration quenching, so an optimum concentration is usually sought. Another potential benefit of ceramic crystal is the possibility of graded dopant concentrations which could lead eventually to uniform heat distribution during pumping [12]. Since the ceramic crystal is formed in a mold, it is conceivable that such a mold could be shaped according to one’s desire. Additionally, composite materials that require the adhesion of two or more originally separate pieces of material could be fabricated as one piece during the ceramic crystal fabrication process [13]. The Czochralski growth process is time consuming and it takes on the order of weeks to grow a large Nd doped YAG crystal, whereas the fabrication process for a ceramic crystal takes on the order of days.

As the importance of and interest in ceramic crystal laser materials grows, it seems worthwhile to invest time in understanding both their physical and optical properties. Therefore, the thermal expansion coefficient and dn/dT of ceramic crystal YAG were measured (See Chapter 3). This was accomplished by use of an interferometric technique that had been employed by Rita Peterson [14] to measure fluoride crystal properties. Computer simulations of the depolarization loss in ceramic and single crystal were also performed. These are compared to experimental results found in the literature [15] (See Chapter 4). The fluorescence lifetime of
neodymium in ceramic crystalline YAG as a function of dopant concentration and temperature was studied in the range between 40 and 300 K (See Chapter 6). Also, a test of Yb:Y₂O₃ ceramic crystal as a laser gain medium was made. The methodology, results, and data analysis related to these experiments are the subject of this thesis along with the theory and modeling of birefringence, depolarization loss, and thermal lensing.
CHAPTER TWO
BACKGROUND AND LITERATURE REVIEW

2.1 Fabrication

The Czochralski crystal growth process of yttrium aluminum garnet (YAG) requires a great deal of time. In the Czochralski process a melt is made containing the elements required for the desired crystal. A small “seed” crystal at the end of a dowel is placed in contact with the melt and rotates with respect to the melt. Slowly the dowel is pulled out of the melt with new material grown onto the seed crystal. Depending on the size of the crystal desired it may be weeks later when the boule is finished growing. Rods may then be cut from the boule. Unfortunately the center of the YAG boule is not desirable for laser rods because of its high concentration of defects.

The process for fabricating ceramic crystal materials is decidedly different than that for growing single crystals. It consists of chemical processes for generating powders that are sintered. Ref. [3] describes the process this way:

“...aqueous solutions of aluminium, yttrium and neodymium chlorides are mixed together. This mixed aqueous solution is added dropwise and then mixed with an aqueous solution of ammonium hydrogen carbonate. Then steps of filtration and washing with water are repeated several times and the resulting material is then dried for two days at ≈ 120 °C. The obtained precursor, consisting of 10-nm-size particles, is calcined at ≈ 1200 °C to produce the Nd:YAG raw oxide powder, with a particle size of 100nm. This
powder is milled with solvent, binder and dispersion medium for 24 hours. The milled slurry is put in a gypsum mold and dried to obtain the desired form. Finally, after removing the organic components by calcination, the remaining material is vacuum-sintered at 1800 °C, after which highly transparent ceramic Nd:YAG is obtained. The average grain size in this ceramic is about 3-4 μm. The sintering time varies from 5 to 20 hours depending on the size of samples” [3].

The final ceramic when viewed with an SEM will appear as shown in Fig. 1. Grain sizes in the figure reach up to 30 μm in diameter.

Figure 1: SEM image of ceramic crystal with grain sizes up to 30 μm [16].

According to Wisdom, “the dopant percentages in the precursor materials remain in the final ceramic piece, a strong advantage compared with Czochralski growth, where the larger
dopant ions are often rejected at the crystal-melt interface, resulting in a lower doping percentage in the crystal than in the melt.” [13]. Also, significant quantities of the final ceramic material do not need to be discarded as a result of high defect density. Ceramic crystal materials can be made into very large, high quality gain media and as a result they are more cost effective than gain media made from single crystals.

2.2 Optical Properties

Scattering in ceramics can be high which can limit their usefulness as laser hosts, so studies of scattering in ceramic crystal materials have been made. Refs. [6, 5] show a dependence of scattering on pore volume and grain boundary phase (see Fig. 2). As the pore volume decreases, scattering in ceramic crystals becomes closer to the scattering in single crystals, and the ceramic crystal materials then perform more like the single crystal materials during lasing. The same applies to the grain boundary phase. As the grain boundary phase decreases the ceramic crystal scatters as would a single crystal and also performs more like a single crystal while lasing.
The concentration dependence of the fluorescence lifetime of Nd:YAG single crystal and Nd:YAG ceramic crystal was found to be the same in experiments in Ref. [17] (see Fig. 3).

The index of refraction was measured and reported in Ref. [18] for ceramic crystal and single crystal. It was found that the dispersion for ceramic crystal and single crystal coincide with each other and with the dispersion curve in the figure which was calculated from the Sellmeir equation [18]. The results are shown in Fig. 4.
Figure 3: Fluorescence lifetime of the Nd\(^{3+}\) \(^4F_{3/2}\) level as a function of Nd dopant concentration for both single crystal and ceramic crystal YAG.

Figure 4: Open circles are data for ceramic crystal YAG, filled triangles are data for single crystal YAG, and the solid line is calculated from the Sellmeir formula in [18].
2.3 Mechanical Properties

In Ref. [1] experiments were performed to ascertain the fracture toughness and the hardness of ceramic crystal as compared to single crystal (see Table 2). It was found that YAG ceramic crystal’s fracture toughness is nearly five times greater than that for single crystal. In addition, it was found that neodymium doped YAG ceramic crystal has a fracture toughness nearly three times that of undoped YAG single crystal. The hardnesses were within 15 percent of one another. Apparently, the polycrystalline nature of the ceramic crystal does not allow fractures to propagate through the bulk of the material as easily as a fracture might propagate in a single crystal. Instead, the fracture will encounter a grain boundary and a different crystal orientation before it propagates very far. In a single crystal however a fracture can travel a larger distance along a crystalline plane before encountering a defect or defects that may stop it.

Table 2:
Hardness and fracture toughness of YAG and Nd:YAG ceramic and single crystal [1].

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Ceramics</th>
<th>Y₃Al₅O₁₂</th>
<th>Y₃Al₅O₁₂ : Nd³⁺</th>
<th>Y₃Al₅O₁₂ crystal</th>
</tr>
</thead>
<tbody>
<tr>
<td>H, GPa</td>
<td>16.0⁺</td>
<td>15.5⁺</td>
<td>15.0⁺</td>
<td>14.5⁺</td>
</tr>
<tr>
<td>K_{IC}, MPa</td>
<td>8.7⁺</td>
<td>8.8⁺</td>
<td>5.2⁺</td>
<td>1.8⁺</td>
</tr>
</tbody>
</table>
2.4 Thermal Properties

The thermal conductivity of Nd:YAG ceramic crystal is reported to be approximately the same as the thermal conductivity for Nd:YAG single crystal. One reference [2] reports a value of 10.7 W/K·m$^{-1}$ for the ceramic crystal compared to 10.5 W/K·m$^{-1}$ for single crystal, while another reference simply reports them as both being approximately 0.11 W/cm·K$^{-1}$ [19].

2.5 Laser Performance

Laser experiments were reported in [2] and [3] in 2002 and 2004 respectively. The two experiments compare the output power to the pump power for both ceramic crystal and single crystal lasers. For the results from 2002, the single crystal laser was a 0.6% doped Nd:YAG rod measuring φ 4 mm x 104 mm, and the ceramic crystal laser was a 0.6% doped Nd:YAG rod measuring φ 4 mm x 105 mm. For the results from 2004, the ceramic and single crystal lasers were 0.6% doped Nd:YAG rods with dimensions φ 4 mm x 105 mm. Side pumping was achieved using 32 laser diodes capable of 10 W in a manner described in [2]. The results from 2002 show that ceramic crystal has lower slope efficiency than single crystal. In 2004 when a similar experiment was performed, the slope efficiency for ceramic crystal was found to be higher than the slope efficiency for single crystal. They mention that single crystal has defects and has not been improving in quality while the ceramic crystal has. Such a result could be accounted for if the issues with scattering at grain boundaries are reduced sufficiently and the
grains are of lower defect density. Results of the two experiments are shown in Fig. 5(a) and Fig 5(b). These results may be for very high quality ceramics compared to common quality single crystal laser media but such high quality ceramic crystal material may not be generally available. How easily results like this can be repeated remains to be seen. They have not been seen in our laboratory.
Figure 5: (a) Output power versus pump power results from 2002 for Nd:YAG single crystal and ceramic crystal [2]. (b) Output power versus pump power results from 2004 for Nd:YAG single crystal and ceramic crystal [3].
3.1 Methodology

An interferometric method was employed in measuring the thermal expansion coefficient of ceramic crystal Nd:YAG. The beam from a helium neon laser (HeNe) was spatially filtered as shown in Fig. 7. The spatially filtered beam was then split. One beam was directed such that it reflected off opposing sides of the sample of interest while the other beam was directed such that it reflected off opposing sides of the reference sample. These two beams interfered, and while the sample and its reference were heated, thermal expansion was tracked by observing the change in the interference pattern.

Two ovens housed one sample each—the sample of interest and a reference sample made of fused silica (see Fig. 6). The ovens were heated using resistive heaters (RHs). Voltage was applied to the heaters via a computer-controlled power supply, and the temperature was measured with platinum resistance temperature detectors (Pt:RTDs). The Pt:RTDs were interfaced with the computer using a National Instruments PCI-6025E. It was through this board and the use of LabVIEW [20] that control and measurement was accomplished.

The samples were specially prepared with surfaces that are transmissive on one half and reflective on the other (see Table 3). The reflective surfaces of the reference sample were gold while the reflective surfaces of the samples of interest were silver and nickel. Reflective surfaces were needed so that the same beam could reflect off opposite sides of the samples for the thermal
expansion measurement. Transmissive surfaces were needed so that a beam could travel through the sample during the measurement of dn/dT. Fig. 12 illustrates this. The HeNe wavelength used for the interferometry was 633 nm.

The fused silica reference sample has a known thermal expansion coefficient [21]. Heating the ovens led to the presence of heated air inside the ovens and around the windowless entries and exits of the ovens. The air near the entries and exits likely did not have a uniform temperature, and therefore it likely did not have a uniform index of refraction. A complicated procedure could have been devised to include the changing indices of refraction into the beam path equations, but the use of a reference sample made this unnecessary. We assumed the optical path lengths through the air in the reference and sample beam paths were reasonably the same and therefore subtracted out of our analysis.
Figure 6: Optical layout of the interferometer used for measuring thermal expansion and $dn/dT$.

Not shown in the diagram was another shield with glass windows around only the ovens.
Figure 7: The experimental setup used to measure thermal expansion coefficients.
Table 3:
Specifications of the samples and reference used to measure the thermal expansion coefficients and \( dn/dT \). Sample length needed to be taken into account in order to obtain more accurate results.

<table>
<thead>
<tr>
<th>Material</th>
<th>Fused Silica</th>
<th>Nd:YAG ceramic crystal</th>
<th>Nd:YAG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dopant %</td>
<td>N/A</td>
<td>0.09%</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4%</td>
<td>1%</td>
</tr>
<tr>
<td>Width</td>
<td>10 mm</td>
<td>10 mm</td>
<td>10 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 mm</td>
<td>10 mm</td>
</tr>
<tr>
<td>Length</td>
<td>10 mm</td>
<td>3.8 mm</td>
<td>8.85 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.9 mm</td>
<td>10.5 mm</td>
</tr>
<tr>
<td>Height</td>
<td>10 mm</td>
<td>6 mm</td>
<td>10 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 mm</td>
<td>10 mm</td>
</tr>
</tbody>
</table>

During the measurement of the thermal expansion coefficients, the output of the interferometer was an interference pattern like that shown on the computer screen in Fig. 7 with bright and dark fringes. The path difference of the two interfering beams changed, and the interference pattern shifted as the samples were heated. Initially a PIN diode was used in conjunction with a slit to measure rising and falling intensities as the fringes passed the detector. This procedure was found to be shrouded in uncertainty because it was unknown if at some point the direction of the passing fringes changed. It was found that using a CCD camera eliminated this confusion. The CCD camera was computer controlled to take a picture of the fringes about every 4 seconds over the course of the experiment. A subsequent movie file generated by
LabVIEW of the pictures allowed later viewing of the pictures in rapid succession so that the fringe movement could be viewed. A drawback to using the CCD camera was the amount of memory required to record the fringe movement. The CCD camera recorded interference patterns like that shown in Fig. 9.

Observation of the changing fringes revealed that a slow fringe shifting process and a fast fringe shifting process were occurring during the heating and subsequent temperature stabilization of the sample and reference. It was hypothesized that the fast process was the result of the thermal expansion of the sample. The slow process is thought have been due to heated air and heated optical elements causing slow changes in the optical paths of the interfering beams. To better understand what was happening, the samples were removed and mirrors were placed as shown in Fig. 8. The ovens were then heated. It was observed that the interference fringe pattern began to move very slowly after approximately 8 minutes of heating. It seems that the heating of optical elements and air paths may have been having a cumulative affect which could have tainted our measurements. The effect did not consistently result in the same amount of fringe movement. Fortunately, however, the slow fringe shifting effect did not start until 8 minutes after heating started and after it did start, it moved quite slowly at first. It was therefore determined that heating the sample in a short 8 minutes could allow measurements to be made of the fast fringe shifting process before the slow fringe shifting process could significantly influence the measurement.
Figure 8: Experimental setup for determining fringe shift as a result of heating the ovens. The samples have been removed and mirrors have been inserted behind the ovens to allow beam propagation paths to be similar to the beam propagation paths in the case where the samples are present in the setup.

Approximately 26 volts were applied to the resistive heaters and the oven temperatures increased in approximately 8 minutes from ambient temperature (approximately 30 °C) to around 90 °C. Generally ambient temperature was a few degrees above room temperature because the entire system was inside an unpowered freezer for isolation. The waste heat from the laser elevated the temperature inside the freezer and consequently also elevated the initial temperature of the ovens. During heating the computer periodically recorded a CCD image of the fringes and the subsequent movie of the shifting fringes usually took between 400 and 500 megabytes of memory. After the heating took place and the temperature was stabilized, the experiment was stopped. At that time the images of the fringes could be viewed as a movie in a matter of seconds. In this way the number of fringes passing a given point on the screen could be counted.
The number of passing fringes in conjunction with the change in temperature and other known quantities made it possible to calculate the thermal expansion coefficient as described below.

Figure 9: An example of an interference pattern recorded by the CCD camera in the interferometric experiment for measuring thermal expansion and \( \frac{dn}{dT} \).

A derivation of the formula to calculate the thermal expansion coefficient starts with

\[
\alpha \equiv \frac{1}{L} \frac{dL}{dT} .
\] (1)

\( L \) is length of the sample, \( \alpha \) is the thermal expansion coefficient and \( T \) is the temperature. Eq. (1) can be represented as

\[
\frac{T}{L} \Delta L = \Delta \alpha ,
\] (2)

and can be rewritten as

\[
\Delta L = \alpha L \Delta T .
\] (3)

As there was a sample and a reference, two equations with appropriate subscripts can be
employed in representing the thermal expansion in each material and can be written in the form of Eq. (3):

\[ \Delta L_{\text{YAG}} = \alpha_{\text{YAG}} L_{\text{YAG}} \Delta T_{\text{YAG}} \]

and

\[ \Delta L_{\text{FS}} = \alpha_{\text{FS}} L_{\text{FS}} \Delta T_{\text{FS}} . \]

As mentioned, the samples were not all of the same thickness. This means that the beam paths for the thinner samples included longer air paths. The index of refraction of the air decreased as it was heated and this changed the optical path length. Therefore, part of the beam path through air could not be subtracted out of the analysis. In the case where the sample was thinner than the reference, the path between the gray lines and the surfaces of the thermally expanded sample as shown in Fig. 10, needed to be included in the analysis.
Figure 10: The changes in the beam path as a result of heating. Red lines indicate beams whose direction is indicated by adjacent black arrows. Dotted lines represent geometries previous to heating. The thickness difference between the expanded top and bottom samples is $d$.

To make a precise calculation of the thermal expansion coefficient in the case where the sample thickness was different than the reference thickness, each piece of the beam path needs to be considered. In the case where the sample and reference are of equal thickness we start with:

$$2(\Delta L_{YAG} - \Delta L_{FS}) = m \lambda$$

where

$$\Delta L_{YAG} = L_{YAGf} - L_{YAGi},$$

and

$$\Delta L_{FS} = L_{FSf} - L_{FSi}.$$ 

The subscripts $f$ and $i$ are attached to final and initial lengths, respectively. In words, Eq. (6)
demonstrates that twice the difference of the thickness differences of the sample and reference is equivalent to the number of fringes multiplied by the wavelength. This is a clue to how we might incorporate more beam path segments into our calculation. Considering the equation in terms of the change in the beam path (see Fig. 11), rather than in terms of the change in the thicknesses of the samples, each of $\Delta L_{\text{YAG}}$ and $\Delta L_{\text{FS}}$ represent a decrease in the beam path.

In the case where the sample was thinner than the reference, the optical path length in the air needed to be taken into account. Assuming the index of refraction of the air in question was uniform, the optical path length is given by

$$OPL = n \cdot d ,$$  \hspace{1cm} (9)

where $n$ is the index of refraction and $d$ is defined in Fig. 10. Since the index of refraction of air decreases with increasing temperature, the heating of the air was another mechanism by which there was a decrease in the optical path length. Since one can represent the optical path length by which each beam was shortened and then take the difference between those values, the relative change in phase between the two beams’ initial state can be obtained and a relationship similar to Eq. (6) found.
Figure 11: The red lines represent the beam path lengths of the two interfering beams. When the samples are heated the beam path lengths are shortened. The lengths by which the beam paths are shortened are $2\Delta L_{FS}$ for the beam associated with the fused silica reference sample and $2\Delta L_{YAG}$ for the beam associated with the YAG sample. Heated air leads to beam path length reduction of the value $2\Delta n \cdot d$

Therefore in a manner similar to Eq. (6) where the heated air path is included it follows that

$$2(\Delta L_{YAG} + (n_y d - n_H d) - \Delta L_{FS}) = m\lambda$$  

(10)

where

$$d = (L_{FS} + \Delta L_{FS}) - (L_{YAG} + \Delta L_{YAG}),$$  

(11)

$n_R$ is the room temperature index of refraction of air and $n_H$ is index of refraction of the heated
Substituting Eqs. (7) and (8) into Eqs. (10) and (11), and substituting Eq. (11) into Eq. (10) yields

\[2(\alpha_{\text{YAG}}L_{\text{YAG}}\Delta T_{\text{YAG}} + (n_R - n_H)((L_{FS} + \alpha_{FS}L_{FS}\Delta T_{FS}) - (L_{YAG} + \alpha_{\text{YAG}}L_{YAG}\Delta T_{\text{YAG}})) - \alpha_{FS}L_{FS}\Delta T_{FS}) = m\lambda.\] (12)

Solving for \(\alpha_{\text{YAG}}\) yields

\[\alpha_{\text{YAG}} = \frac{m\lambda}{2L_{\text{YAG}}\Delta T_{\text{YAG}}(1 - (n_R - n_H))} - \frac{(n_R - n_H)(L_{FS} - L_{YAG} + \alpha_{FS}L_{FS}\Delta T_{FS})}{L_{YAG}\Delta T_{\text{YAG}}(1 - (n_R - n_H))} + \frac{\alpha_{FS}L_{FS}\Delta T_{FS}}{L_{\text{YAG}}\Delta T_{\text{YAG}}(1 - (n_R - n_H))}.\] (13)

Before applying Eq. (13) to the data one must know what the index of refraction of air is, and how it changes with temperature. A relationship is provided by Bengt Edlen [22]:

\[(n - 1)_p = (n - 1)_s \frac{0.00138823p}{(1 + 0.003671t)}\] (14)

where \(p\) is the pressure in torr, \(t\) is the temperature in °C, and \((n - 1)_s\) is the dispersion of standard air which is also given by Bengt Edlen as

\[(n - 1)_s \times 10^8 = 8342.13 + 2406030(130 - \sigma^2)^{-1} + 15997(38.9 - \sigma^2)^{-1}\] (15)

where \(\sigma\) is said to be the vacuum wave number in \(\mu\text{m}^{-1}\), but in practice is simply the inverse of \(\lambda\) in \(\mu\text{m}^{-1}\). It was assumed that \(p\) is the standard pressure of 1 atm or 760 torr.

Because \(n_R - n_H\) is very small compared to 1, and \(\alpha_{FS}L_{FS}\Delta T_{FS}\) is small compared to \(L_{FS}\) - \(L_{YAG}\), Eq. (13) can be rewritten without any loss in accuracy as:

\[\alpha_{\text{YAG}} = \frac{m\lambda}{2L_{\text{YAG}}\Delta T_{\text{YAG}}} - \frac{(n_R - n_H)(L_{FS} - L_{YAG})}{L_{\text{YAG}}\Delta T_{\text{YAG}}} + \frac{\alpha_{FS}L_{FS}\Delta T_{FS}}{L_{\text{YAG}}\Delta T_{\text{YAG}}}.\] (16)
The experimental setup for measuring the $dn/dT$ was similar to the experimental setup for measuring the thermal expansion coefficient (see Fig. 12). The primary difference was that no beam was directed toward the reference sample. Yet, two interfering beams were still required for the $dn/dT$ experiment. One of the interfering beams was reflected from the front surface of the sample while the second beam traveled through the sample and was reflected from the back surface so that it traveled through the material twice. During heating the index of refraction changed, and this change was measured by a shift in the interference pattern.

Figure 12: Optical layout of the interferometer used for measuring $dn/dT$.  

Figure 12: Optical layout of the interferometer used for measuring $dn/dT$. 
To derive the necessary equation for calculating the \( \frac{dn}{dT} \) with a known thermal expansion coefficient and a measurement of the fringe movement, we start with the change in the optical path length as a function of temperature which is given by

\[
2 \frac{d(nL)}{dT} = 2\left(n \frac{dL}{dT} + L \frac{dn}{dT}\right). \tag{17}
\]

The factor of two is a result of one of the interfering beams propagating through the medium twice. \( T \) is the temperature, \( L \) is the length of the sample, and \( n \) is the index of refraction. The change in the optical path length was measured by the number of passing fringes, so we also have the relationship

\[
2 \frac{d(nL)}{dT} = \frac{m\lambda}{\Delta T}. \tag{18}
\]

Substituting Eq. (18) into Eq. (17) and solving for \( \frac{dn}{dT} \) yields

\[
\frac{dn}{dT} = \frac{m\lambda}{2L\Delta T} - \frac{n}{L} \frac{dL}{dT}. \tag{19}
\]

Substituting Eq. (1) into Eq. (19) yields

\[
\frac{dn}{dT} = \frac{m\lambda}{2L\Delta T} - n\alpha, \tag{20}
\]

where \( \alpha \) is the thermal expansion coefficient of the sample—in this case Nd:YAG ceramic crystal.
3.2 Thermal Expansion Coefficient and dn/dT Findings

When the correction for the index of refraction of air was applied to the data, the set of results obtained for the thermal expansion coefficient of the ceramic crystals studied are as shown in Fig. 13.

![Graph showing thermal expansion coefficient vs % Nd dopant concentration]

Figure 13: The index of refraction of air was included in the calculation of the thermal expansion coefficient in this figure. The dotted line at 7.8x10^{-6} °C^{-1} is the value reported by [23], and the dotted line at 6.9 x10^{-6} °C^{-1} is the value reported on the Northrop Grumman website [24] for 1% single crystal Nd:YAG.

The data in Fig. 13 indicates a thermal expansion coefficient for Nd:YAG ceramic crystal near 6.5x10^{-6} °C^{-1}. A reason for our measured values to differ from reported values shown in Fig. 13 may be the temperature range in which the thermal expansion coefficient was measured.
In [23] the temperature range was from 0-250 °C. Perhaps over the smaller temperature range 30-90 °C, the approximate temperature range for our measurements, the thermal expansion is less. The data suggests a concentration dependence of the thermal expansion coefficient, but no dependence can be certainly born out by this data. Therefore, more refined measurements are necessary to discern a concentration dependence of the thermal expansion coefficient of Nd:YAG ceramic crystal. If there is a dependence of the thermal expansion coefficient on dopant concentration, then a sample with graded dopant concentration will have stress profiles that will be more complicated than one might otherwise expect. The concentration dependence of the thermal expansion coefficient would then need to be taken into account.

To calculate dn/dT a value of the thermal expansion coefficient was needed for each sample. An average of the measured values of the thermal expansion coefficient was used. For example the two measurements of the thermal expansion coefficient for 1% single crystal Nd:YAG were averaged to 5.56x10^{-6} °C^{-1}. This value was used in Eq. (20) to calculate the dn/dT. Also, the appropriate values for temperature change and fringe shift were used. Fig. 14 shows the experimental results of the measurements of dn/dT. The errors from the thermal expansion coefficients carried over and make a significant contribution to the error of the dn/dT values.
Figure 14: $dn/dT$ for ceramic and single crystal Nd:YAG. The ceramic crystal samples had dopant concentrations ranging from 0.09% to 4%. The dotted line represents the value of 1% Nd:YAG single crystal as found on the website of Northrop Grumman [24].

The experimental technique for the measurement of $dn/dT$ was less prone to error. The reason for this was that the two beams which interfered were collinear as they propagated through heated air and beam splitters and so fluctuations in optical path length due to heated air or optical elements automatically subtracted out. Also, many more fringes passed the detector in the measurement of the $dn/dT$ which led to less error. It was not difficult to repeat measurements as the data shows in Fig. 14. Again, the data suggests a concentration dependence of the $dn/dT$ but is not conclusive in showing one. If further experiments were to show conclusively that there is concentration dependence, the data in Fig. 14 suggests it would be slight. These measurements show the $dn/dT$ value for the 1% doped single crystal Nd:YAG to be $1.3 \times 10^{-5} \, \text{oC}^{-1}$ and the $dn/dT$ value for the 1% doped ceramic crystal Nd:YAG to be $9.6 \times 10^{-6} \, \text{oC}^{-1}$. More precise measurements of the thermal expansion coefficient would lead to more precise values of $dn/dT$. 
An improved measurement of the thermal expansion coefficient could be accomplished by using a vacuum chamber. Placing the sample and the beam path in a vacuum chamber would decrease the possibility of error from heated air. Also, it would eliminate the need for a reference sample which was necessary to subtract out the contribution of heated air paths in the analysis. A vacuum chamber would also reduce heating of beam directing optical elements such as beam splitters and mirrors. Because there would be no need for a reference sample, fewer optical elements would be needed for the experiment.
CHAPTER FOUR
DEPOLARIZATION LOSS

4.1 Introduction

4.1.1 In General

To understand how to model birefringence and depolarization loss in ceramic crystal, one needs to review the models which have been made for single crystals. Several papers have been published with calculations for the birefringence in YAG. Our group has performed such calculations and published results [25, 26]. To start, the relationships of importance need to be identified. Though the birefringence is defined as a difference between two principal indices of refraction, the quantity that relates to the index of refraction called the dielectric permeability tensor is the one of interest for performing the calculations. Because it is a second rank tensor, the dielectric permeability can be used in tensor mathematics involving physical properties of the crystal. The index of refraction which is not a tensor quantity should not be treated as one [27]. Stress in the medium is the physical quantity that gives rise to a change in the dielectric permeability and thus the principal indices of refraction. The relationship between stress and the dielectric permeability is:

\[ \Delta B_y = \pi_{ijkl} \sigma_{kl} \]  \hspace{1cm} (21)

Where B is the dielectric permeability, \( \pi \) is the piezo-optic tensor, and \( \sigma \) is the stress tensor. Since the values of \( \pi \) are given in the crystal coordinate system and the values for stress are in...
the laboratory coordinate system, a transformation must be performed so that the two quantities are in the same basis before multiplying them. To simplify the calculations a reduced suffix notation found in Nye’s book *Physical Properties of Crystals* [27] is often employed. Using Nye’s reduction methods, the number of suffixes can be reduced such that Eq. (21) can be written as

$$\Delta B_m = \pi_{mn} \sigma_n.$$  (22)

When performing the transformation, the reduced piezo-optic tensor should be expanded into its full-suffix form, transformed, and then reduced again. It is important to follow Nye’s convention closely when applying the reduced suffix notation. One source was found to have left out several factors of $\frac{1}{2}$ in the piezo-optic tensor which are necessary in order to bring the notation into agreement with Nye’s notation [28]. Ref. [25] mentions this problem as well as the problem of another paper treating the index of refraction as a tensor quantity which is only applicable in a limiting circumstance. Another equivalent expression for the dielectric impermeability is

$$\Delta B_m = p_{mn} \varepsilon_n,$$  (23)

where $p$ is the elasto-optic tensor and $\varepsilon$ is the strain tensor. It is presented here in the reduced-suffix form. Results obtained by either calculation should be equivalent. Therefore after calculating the dielectric impermeability using Eq. (22), the calculation can be compared with the result of Eq. (23).
An issue arising in [25] concerns the values for the compliance tensor used therein. Since Dixon measured the values of the elasto-optic coefficients in 1967 [29], it is possible to calculate the piezo-optic coefficients using the relationship from [27]:

\[ \pi_{mn} = P_{mr} s_{rm} \] (24)

where \( s \) is the compliance tensor in reduced-suffix form. The values for the compliance tensor are found in A. A. Kaminskii’s *Laser Crystals* [30]. Also the values for the stiffness tensor can be found there. Stiffness and compliance are inversely related such that

\[ c \cdot s = I \] (25)

where \( I \) is identity. Given that one knows the stiffness tensor, the compliance can be calculated by taking the inverse of the stiffness tensor. Using the values for the stiffness tensor given in [30], one will find that the stiffness tensor is not invertible unless one assumes symmetry in the strain tensor which is what has been assumed by [27]. Ref. [27] explicitly represents the strain as the symmetric part of the deformation. Under this assumption one can invert the stiffness tensor, but a modified version of the identity matrix must be used. The fourth-rank identity tensor with assumed symmetry for the strain tensor can be found in Walpole’s paper “Fourth-Rank Tensors of the Thirty-Two Crystal Classes: Multiplication Tables” [31]. It is given as

\[ I_{ijkl} = \frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \] (26)

where \( \delta \) is the Kronecker delta and equals 0 when the subscripts are not equal and one when they are. It can be said that the stiffness tensor is not invertible under the identity \( I_{ijkl} = \delta_{ik} \delta_{jl} \) but is invertible under the identity given in Eq. (26). In [25] the stiffness tensor from *Laser Crystals* [30] was incorrectly inverted and a factor of four appeared in the value for the \( s_{44} \) component of
the compliance tensor. The correct values for the compliance tensor, as found in [30], are

\[ s_{11} = 3.59 \cdot 10^{-12} \quad \text{Pa}^{-1}, \]

\[ s_{12} = -0.90 \cdot 10^{-12} \quad \text{Pa}^{-1}, \text{ and} \]

\[ s_{44} = 8.69 \cdot 10^{-12} \quad \text{Pa}^{-1}. \]

With the correct values of the compliance tensor one can solve for the piezo-optic tensor using Eq. (24). After transforming from the crystal coordinate system into the laboratory coordinate system, the values for the piezo-optic tensor in units of m^2N^{-1} are found to be:

\[ \pi_{11} = -3.0 \cdot 10^{-13} \]

\[ \pi_{12} = 1.1 \cdot 10^{-13} \]

\[ \pi_{13} = 1.7 \cdot 10^{-13} \]

\[ \pi_{14} = -1.7 \cdot 10^{-13} \cdot \cos(3\Phi) \]

\[ \pi_{15} = -1.7 \cdot 10^{-13} \cdot \sin(3\Phi) \]

\[ \pi_{33} = -3.6 \cdot 10^{-13} \]

\[ \pi_{44} = -2.9 \cdot 10^{-13} \]

\[ \pi_{66} = -4.1 \cdot 10^{-13} \]

In the laboratory coordinate system the z-axis is along the [111] direction and \( \Phi \) is the cut angle of the slab (see Fig. 15). After the piezo-optic tensor is found, the calculation in Eq. (22) can be performed. This yields the dielectric impermeability tensor for the crystal in the laboratory coordinate system. A submatrix is then extracted from the dielectric impermeability tensor representing the components that will be perpendicular to the direction of propagation. This submatrix is

\[ \begin{pmatrix}
B_{11} & B_{12} \\
B_{12} & B_{22}
\end{pmatrix}, \]

where \( B_{12} \) is equivalent to \( B_{21} \), so both are represented by \( B_{12} \). Again, this is in the laboratory
coordinate system, but in order to calculate the birefringence, the dielectric impermeability in its principal system is needed. We therefore find the eigenvalues of the submatrix which are

\[
B_{\pm} = \frac{1}{2} [B_{11} + B_{22} \pm ((B_{11} - B_{22})^2 + 4B_{12}^2)^{1/2}].
\] (30)

The eigenvalues are the nonzero elements of the diagonalized matrix. The index of refraction is related to the dielectric impermeability according to:

\[
B_{\pm} = \frac{1}{n_{\pm}^2}.
\] (31)

Because there is a transformation from the laboratory system to the system in which the dielectric impermeability is diagonalized, a rotational transformation through an angle \(\theta\) must be performed. That is

\[
B_{y} = a_{ik}a_{jl}B_{kl}
\] (32)

where \(a\) is a rotation matrix for the transformation. As a result

\[
\tan(2\theta) = \frac{2B_{12}}{B_{11} - B_{22}}.
\] (33)

The depolarization loss is given in Ref. [25] by

\[
Loss = \sin^2(2\theta)\sin^2\left(\frac{\delta}{2}\right),
\] (34)

where

\[
\delta = \frac{2\pi}{\lambda}L(n_+ - n_-)
\] (35)

is the phase delay. \(L\) is the length of the sample in the direction of propagation and \(\lambda\) is the wavelength.
Figure 15: Sketch showing the cut angle $\Phi$, where the $x$ primed axis is the [10$\bar{1}$] axis and the $y$ primed axis is the [12$\bar{1}$] axis [25].
A polarized beam oriented along one of the axes in the diagonalized system of the dielectric impermeability, \( B \), would not experience depolarization because for instance

\[
\begin{pmatrix}
\frac{-i\delta}{2} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \frac{i\delta}{2}
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix} =
\begin{pmatrix}
0 \\
0 \\
\frac{i\delta}{2}
\end{pmatrix}.
\]

(36)

This is not generally the case however because the phase delay matrix [9] in Eq. (36) is transformed into the laboratory coordinate system before being multiplied by the electric field vector. Prior to entering the slab, the incoming rays are assumed to be polarized along the x-axis of the laboratory coordinate system.

4.1.2 For Ceramics

In Efim A. Khazanov’s theoretical work on ceramic depolarization loss for rod shaped ceramic crystal lasers, he states that, “from the viewpoint of polarization, a ceramic is a sequence of phase plates,” [32]. As a result of analysis, [32] goes on to state that the “depolarization is proportional to \( p^2N^{-1} \)”, where

\[
N = \frac{L}{\langle l \rangle},
\]

(37)

where \( \langle l \rangle \) is the mean grain size and \( L \) is the length of the active element, and \( p \) is a function of the heat release power, \( P_h \). \( \Gamma(r,\phi) \) as defined in [32] is ,”a part of power…in the depolarized component”. In other words, \( \Gamma(r,\phi) \) is the functional form of the depolarized power of a beam that has propagated through a rod. In Figure 16, \( \sigma_I(r,\phi) \) is the spread in the depolarized
component of the power for 100 realizations. A realization is one calculation for one set of grains.

Figure 16: “Maximum cross-section (dashed curves) and average cross-section (solid curves) values $\sigma_T(r,\phi)$ at 1, $N=300$; 2, $N=100$; and 3, $N=30$. Curve 4 shows depolarization $\langle \gamma \rangle$ calculated from Eqs. (1)” of Ref. [32]. This is for an Nd:YAG rod ceramic crystal (curves 1-3) and Nd:YAG rod single crystal (curve 4) presumably.

It appears Khazanov is saying that the integrated depolarization of the output for a single crystal (curve 4 in Fig. 16) is not so different from the fluctuations in the depolarization that one sees of the output from ceramic crystal. It is unclear from [32] how the integrated depolarization compares for single and ceramic crystal but would apparently depend on the grain size.
4.2 Depolarization Loss Methodology

Preliminary models made in our group on the depolarization loss in ceramic crystal slab lasers suggested that the shear stresses in ceramic crystal are less than those in single crystal. Otherwise, a convergent value for the depolarization loss could not be achieved. Discussions with Dr. Yongho Sohn of the UCF Engineering department, suggested that there could be some relaxation in the ceramic crystal when it is heated. Also, some literature [33] indicated there might be shifting grain boundaries in stressed ceramic crystal materials. We theorized that shear stresses could be less in ceramic crystal and that the mechanism could be slipping or shifting of the crystallites at the grain boundaries. Yet, because the temperatures being reached in the slabs were not high, the stresses that were being induced in the ceramic crystal as a result of pumping were not large. Our original theory of stress relief at grain boundaries as the explanation for our quandary was on shaky footings because it appears greater stresses are required for shear stress relaxation by that mechanism.

As has already been mentioned more investigation revealed an incorrect factor of four in the compliance tensor of previous calculations. When this is taken into account, the problem with non-converging depolarization in ceramics no longer exists. The depolarization loss is noisy but the noise is not so large that depolarization loss trends cannot be clearly seen. The basic model that we use for calculating depolarization loss in ceramic crystal is shown in Fig. 17 and the MATLAB [34] code can be found in Appendix A.
Figure 17: Each of the above diagrams represents a realization or grain sequence through which a ray in our model propagates. The grains of the ceramic crystal are randomly oriented and have random widths.

Our simulations were carried out for a slab where the rays enter through Brewster cut faces and then propagate in a zigzag manner through the slab. The size of the grains was generally on the order of tens of microns but was made as small as 0.625 μm. In 1995, [10] reported ceramic crystal grains to be “about 50 μm”. Other reported values have been 10 μm [11]-[13], 3-4 μm [14], and 1-3 μm [15]. Our model can be applied to all of these grain sizes.

4.3 Depolarization Loss Findings

We applied the analyses in Refs. [25] and [26] with the corrected peizo optic tensor components to the single crystal Yb:YAG slab shown in Fig. 18(a) since this is the same single crystal evaluated in Refs. [25, 26]. Through it the rays propagate in a zigzag fashion. The slab has Brewster ends angled at 30.6 degrees and is pumped with 9.8 kW of diode power. It was
found that the new results for depolarization loss as a function of cut angle are a factor of ten smaller than in Ref. [35], but that the functional form of the cut angle dependence is the same. For comparison, the cut angle dependence of depolarization loss from Ref. [35] is reproduced in Fig. 19. The new results obtained with the corrected compliance tensor are shown in Fig. 20.
Figure 18: (a) Geometry of the slab used in the simulation of depolarization loss in Ref. [35] and for the results in Fig. 20. (b) Geometry of the slab used for depolarization loss results shown in Fig. 21.
Figure 19: Cut angle dependence of the depolarization loss in a zigzag single crystal slab that was 80 mm in length [35].
Figure 20: Cut angle dependence of the depolarization loss for 80 mm long single crystal slab with corrected compliance tensor.

It has been shown theoretically in Ref [36] that certain geometries will result in all rays experiencing an integer number of bounces. Phase fronts exiting the slab will not therefore have steps resulting from some rays experiencing a different number of bounces. As a result, the length of our slab was extended as shown in Fig. 18 (b) to apply the theory in Ref [36]. The other dimensions remain unchanged. The pump power is also increased to 10.88 kW. When such a geometry is used to calculate the cut angle dependence of the depolarization loss, the
The general shape of the dependence remains unchanged as shown in Fig. 21. The magnitude of the loss has changed slightly as a result of propagating through more material.

**Figure 21**: Cut angle dependence of the depolarization loss in an 89.6 mm long single crystal with the corrected compliance tensor.

Modeling of the depolarization loss in a Yb:YAG ceramic crystal slab was also performed. Because the ceramic crystal is made up of multiple particles oriented randomly, no graph such as that in Fig. 21 for ceramics can be produced. Instead the effect of the grain size on the depolarization was investigated.
The model used was written in MATLAB. It traced one ray at a time through the ceramic crystal slab. Each ray was traced in small steps. For most of the grain sizes the step size was 0.6 \( \mu \text{m} \), but for some smaller grain sizes it was 0.06 \( \mu \text{m} \). At each step along the zigzag path, the program checked to see if it had crossed a grain boundary or crossed a boundary where the stress might be different (see Fig. 22). It also checked to see if a slab surface had been reached. The program ended when it had propagated through the slab. Because the step size is not infinitesimally small, the ray may be traced for a distance not longer than a step size where incorrect stress, grain orientation, or propagation direction may be applied. Therefore, error in the model may arise from the step size.
Figure 22: Sketch illustrating grains (in black) and boundaries where stress data may change (in green) (ceramic grains traced from Ref. [5]).
Figure 23: Graph of transmission through a single crystal slab with the dimensions shown in Fig. 18(b) and with a cut angle of 75 degrees. Losses are due to depolarization resulting from thermally induced stress gradients. It is placed here for comparison to the ceramic transmission in Fig. 26.
Figure 24: Graph of transmission through a single crystal slab with the dimensions shown in Fig. 18(b) and with a cut angle of 60 degrees. Losses are due to depolarization resulting from thermally induced stress gradients. It is placed here for comparison to the ceramic transmission in Fig. 25.
Figure 25: Transmission through a ceramic crystal slab with the dimensions shown in Fig. 18(b) and having grain sizes with a normal distribution centered at 10um and standard deviation 2.5um. Losses are due to depolarization resulting from thermally induced stress gradients.
Figure 26: Transmission through a ceramic crystal slab with the dimensions shown in Fig. 18(b) and having grain sizes with a normal distribution centered at 80 μm and standard deviation 20 μm. Losses are due to depolarization resulting from thermally induced stress gradients. 

From Figs. 25 and 26, the noise in the depolarization loss in a ceramic crystal is apparent and clearly contributes to the calculated depolarization. The distribution of the depolarization loss within the slab is similar to that in the single crystal. By evaluating ceramic crystals with different distributions of grain size, the dependence of the depolarization loss on grain size shown in Fig. 27 was found. To save time the data were calculated for a subset of rays that spanned the width of the slab. The data with a line through them represents a data set resulting
from a full set of rays spanning the width and height of the slab which have propagated through the slab.

![Grain Size Dependence of Depolarization Loss](image)

**Figure 27:** Depolarization loss in ceramic crystal as a function of the grain size. Each solid diamond is a calculation of the depolarization loss using a subset of rays spanning the width of the entrance. The open dotted curve represents the depolarization loss as calculated using a full set of rays spanning the thickness and width of the entrance.

It is important to note the limited applicability of the model. As the grain size increases significantly the model will become less reliable. This is because each ray is assumed to pass through a set of randomly oriented crystal grains. As ratio of grain size to ray spacing increases, neighboring rays may in reality see nearly the same set of randomly oriented grains (see Figs. 28 and 29), but at present our model doesn’t take that into account. Therefore our model is most accurate when the mean grain size is less than 100 μm. As previously noted, it is common for ceramic crystal laser materials to have mean grain sizes lower than 100 μm.
Figure 28: Sketch illustrating the case where the ray spacing is larger than the average grain size (ceramic grains traced from Ref. [5])
The factor of four correction to the compliance tensor component for YAG in our calculation changed the magnitude of the predicted depolarization loss in the single crystal but not the general shape of its cut angle dependence. A dependence of the depolarization loss in a ceramic crystal on the grain size was demonstrated. The grain sizes for which our model is most accurate are those grain sizes which are commonly found in ceramic crystal laser material.

In addition to performing the model for zigzag propagation through the slab, it was also carried out for straight through propagation. Again, for the single crystal, a cut angle dependence was discovered. As can be seen in Fig. 30, the overall depolarization is greater for the straight through case than the zigzag case in Fig. 21. This is to be expected since depolarization accrued in a zig is cancelled out at least in part by a zag.
Figure 30: Cut angle dependence of the depolarization loss for propagation straight through a Yb:YAG single crystal slab.

Again, the model was run for a ceramic slab as described previously, but for the case of straight through propagation. The model was run several times with a full set of rays spanning the width and thickness of the slab, and that data is shown in Fig. 31. Because the ceramic crystalline grains are randomly generated while the model is being carried out, the ceramic slab for each calculation is different. The stress distribution remains the same however. For this reason the data appear somewhat random. There does appear to be a trend to increasing depolarization loss with increasing grain size. The same limits to the model of zigzag
propagation through the ceramic slab apply to the straight through case making this model more
applicable to slabs with grain sizes smaller than 100 μm.

For the straight through case the depolarization loss in the ceramic is around 2%, while
the depolarization loss for the single crystal can be seen to be around 3-3.5%. This is explained
by examining Figs. 23-26 which demonstrate that the maximum depolarization loss of the
ceramic crystal (Figs. 25-26) is lower than the maximum depolarization loss in the single crystal
(Figs. 23-24). When the maximum depolarization loss is small, the depolarization loss
fluctuations generated in the ceramic crystal slab are a more significant contributing factor to the
depolarization loss. When the maximum depolarization loss is high, however, one can expect
the overall depolarization loss to be dominated by the region of maximum depolarization loss
which for the ceramic crystal, as seen in the zigzag case, is smaller than for the single crystal.
Figure 31: Depolarization loss for straight through propagation as a function of ceramic crystalline grain size in a Yb:YAG ceramic crystal slab. Because only two significant figures can be claimed, the dependence on grain size is not significant.

Rods are often used as laser gain media and so the model was applied to a 60 mm long Nd:YAG rod. It was 5 mm in diameter and 4 donuts pumped it with a total of 25 W of average power. Straight through propagation through the single crystal was performed to determine any cut angle dependence of the depolarization loss. The depolarization loss was found to be 2.07% and did not vary with cut angle. Rods are known to experience depolarization loss that is in the form of a Maltese cross [37] as shown in Fig. 32. The Maltese cross pattern occurred in rods of both single crystal and ceramic crystal in our models.
Figure 32: Depolarization loss in a single crystal Nd:YAG rod, illustrating the Maltese cross common to both single and ceramic crystal rods.

Propagation straight through a 60 mm long 5mm diameter Nd:YAG ceramic crystal rod was performed. The depolarization loss was 1.84% over the range 5 to 100 μm. Since input values in the model are limited to 2 and 3 significant figures, the depolarization loss values for all grain sizes are indistinguishable.
CHAPTER FIVE
THERMAL LENSING

5.1 Methodology

COMSOL Multiphysics, a finite element analysis program formerly known as FEMLAB [38], has been used to model the pump-induced thermal and stress effects in slab lasers [26, 35]. Deformation data was extracted from such models and used to calculate the deformation-induced lensing in laser gain media. As the thermal expansion coefficient of the ceramic crystal was shown to be near the thermal expansion coefficient of single crystal, these models which have been developed with the single crystal in mind can be applied to the ceramic crystal.

Thermal lensing analysis was carried out for a slab with an 89.59 mm long rectangular region, a thickness of 1 mm, and a width of 16 mm. Its Brewster ends had an apex angle of 30.61 degrees. This geometry can be calculated from [36]. Taking the index of refraction of Yb:YAG to be 1.82 and the index of refraction of air to be 1, one can solve for Brewster’s angle. The apex angle can be found according to Eq. (19) of [36]. This value is 30.61°. Next, using Eqs. (14) and (20) in [36] one can arrive at the length to thickness ration equation

\[ \frac{L}{t} = N \tan(90 - \alpha), \]  

(38)

where \( L \) is the length of the rectangular region, \( N \) is the number of bounces the rays make, \( t \) is the thickness of the slab and \( \alpha \) is the apex angle. Depending on how one chooses to calculate the length, different results may be obtained as shown in Table 4. The small variations in the
geometry yield different results as discussed below. Unless otherwise stated, the geometry used in the MATLAB and ASAP [39] portions of the foregoing analysis was 30.61° for the apex angle and 89.59 mm for the length of the rectangular region of the slab.

Table 4:
How small changes in the apex angle affect the calculated length of the slab’s rectangular region where N=53.

<table>
<thead>
<tr>
<th>α</th>
<th>L (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.6°</td>
<td>89.618…</td>
</tr>
<tr>
<td>30.606…°</td>
<td>89.594…</td>
</tr>
<tr>
<td>30.61°</td>
<td>89.582…</td>
</tr>
</tbody>
</table>

COMSOL Multiphysics was used to calculate the deformation of the slab. Data points representing the deformation were exported from COMSOL Multiphysics for further analysis and calculation. The data came in the form x, y, z, and u (deformation in the x-direction); x, y, z, and v (deformation in the y-direction); and x, y, z, and w (deformation in the z-direction). In MATLAB, the top surface data points were extracted from the three data sets. These data points are represented by the blue dots in Fig. 33. The mesh, generated by MATLAB, is an interpolation of the data. The ripples in the top surface deformation are a result of the diode bars being spaced 1 mm from each other [40]. Because of symmetry Fig. 33 also represents the bottom surface deformation with a change of axes. Also, Fig. 34 represents the entrance and exit faces of the slab. The bowing is a result of the heat at and thereby expansion of the edges where
the pump light enters. Fig. 35 which represents the deformation of the edge surfaces also has a
rippling effect which is also due to the pumping diodes being spaced.

![Deformation of Slab Top Surface](image)

**Figure 33:** Thermally-induced deformation of a slab surface. The ripples result from the spaces
between the diode bars pumping the slab edges [40]. Bending of the surface at the ends is a
result of the Brewster cut ends where the peak bows up.
Figure 34: Entrance and exit face deformation of the Yb:YAG slab.
Figure 35: Deformation of the edge surfaces is rippled as a result of the spacing of the pump diodes.

In ASAP, rays were propagated through the deformed slab. ASAP tracked the optical path length of each ray where the optical path length (OPL) is the index of refraction multiplied by the distance traveled. Optical path length difference (OPLD) is the optical path length traveled by the rays through a deformed slab minus the optical path length traveled through the same non-deformed slab. As noted above, the considered slab had a rectangular region 89.59 mm long and an apex angle of the Brewster end of 30.61°. If the geometry was made 89.6 mm in rectangular region length and 30.6° apex angle of the Brewster end, then we saw that the optical path length of the rays traced through the slab traveled equal distances when the slab was
not deformed (see Fig. 36). If, however, the geometry was made to be the slab that is now under consideration, 89.59 mm rectangular region length, and 30.61 apex angle, the rays did not all travel equal distances (see Fig. 37). One row of rays experienced one less bounce. As mentioned, both of these geometries may reasonably be calculated from [36] depending on how many digits are kept in the calculations. While the calculations done in MATLAB and ASAP were done using 30.61° apex angle and 89.59 mm rectangular region length, the calculations in COMSOL appear to have been done with an 89.58 mm rectangular region length. When 89.58 mm was used for the rectangular region and 30.61° was used for the apex angle in the MATLAB and ASAP calculations, there was still a row of rays that experienced one less bounce in the non-deformed case. Initially there were 95 rows of rays, so 1 row was approximately 1% of the total rays. This emphasizes how slight changes in the geometry can cause a loss of a flat phase front for non-deformed slabs. An algorithm in MATLAB was used to eliminate the data experiencing one less bounce from the data set and Fig. 38 shows that without the row of rays which experienced one less bounce, the phase front is flat.
Figure 36: Optical path length for rays traveling through a non-deformed slab of 89.6 mm rectangular region length, 16 mm width, 1 mm thickness, and 30.6° apex angle. Optical path length is 0.1957297 m.
Figure 37: Optical path length of rays traveling through a non-deformed slab with length 89.59 mm, 30.61° apex angle, 16 mm width, and 1 mm thickness.

Optical path length through non-deformed slab

- Optical path length (m) range: 0.1945 to 0.1955
- Width (m) range: -5 x 10^-2 to 5 x 10^-2
- Thickness (m) range: -4 x 10^-4 to 4 x 10^-4
Without the errant row of rays shown in Fig. 37, it can be seen that the optical path length indicates a flat phase front. Optical path length is 0.19571179 m.

Because the flat phase front across the width and thickness of the slab is so sensitive to the geometry of the slab, it may not be surprising that once the slab has deformed as a result of thermal expansion that the phase front will no longer be flat as shown in Fig. 39. This deviation is made more significant by the fact that without deformation there was a row of rays near the bottom of the slab that experienced one less bounce, but with the deformations there were several rows of rays at the top of the slab that experience one more bounce than the rest of the rays propagated through the slab.
Figure 39: The step in the phase front of rays traced through the slab resulting from the deformations of the slab.

The optical path length of the flat phase front in Fig. 38 was found to be 0.19571179 m which was subtracted from subsequent measurements of the optical path length of rays propagated through the deformed slab. In this way optical path length difference was obtained and comparison could be made to Ref. [40] values for thermal lensing resulting from stress and thermal gradients. Fig. 40 shows the optical path length difference where the rays experiencing an extra step shown in Fig. 39 have been expurgated. The reported optical path length difference in Ref. [40] resulting from thermal gradients is between $3 \times 10^{-6}$ m and $10 \times 10^{-6}$ m whereas the optical path length difference resulting from thermal stress is between $-5 \times 10^{-7}$ m and $15 \times 10^{-7}$ m.
Since the optical path length difference resulting from deformation as reported here is between $1.5 \times 10^{-5}$ m and $2 \times 10^{-5}$ m, the deformation-induced thermal lensing effect appears to be a significant cause of thermal lensing in the considered slab.

![Optical path length difference resulting from slab deformation](image)

**Figure 40:** Optical path length difference for rays which have propagated through the slab.

Further analysis revealed which surfaces of the slab contribute the most to the deformation-induced thermal lensing. It was possible to keep the edges and the entrance and exit surfaces flat while the top and bottom surfaces were deformed. This was done, and rays were propagated through the slab. The resultant optical path length difference is shown in Fig. 41 and can be seen to have curvature similar to the optical path length difference for all surfaces being
deformed as shown in Fig. 40. The magnitude of the optical path length difference in Fig. 41, however, is less than that in Fig. 40. This is to be expected since all surfaces were not deformed.

Figure 41: Optical path length difference of rays propagated through the slab when all surfaces are kept flat except the top and bottom surfaces.

All surfaces were kept flat except for the entrance and exit faces to ascertain the contribution to the overall optical path length difference that they make (see Fig. 42). It was found that the optical path length difference was between $7.2 \times 10^{-6}$ m and $7.6 \times 10^{-6}$ m which is noticeably smaller than for the case where just the top and bottom surfaces were deformed.
Figure 42: Optical path length difference of rays propagated through the slab when all surfaces are kept flat except for the entrance and exit surfaces.

In the case where the edges were deformed but the remainder of the slab surfaces were kept flat, there appeared to be no change from the totally non-deformed case. This is to be expected because the rays were initially directed parallel to the non-deformed edges. For the rays to interact with the edges there would have needed to be a surface that refracts them in the direction of the edges, but as the other surfaces were kept flat, this did not happen. Because the optical path length difference resulting from the deformation of the top and bottom surfaces in addition to the optical path length difference resulting from the deformation of the entrance and exit surfaces appeared to account for the overall deformation, we can therefore conclude that the
top and bottom surfaces contribute the most to the overall optical path length difference. If the
deformation of the top and bottom surfaces could be removed then a significant source of lensing
could eliminated. This could be accomplished by constraining the surfaces with a rigid material
which has high thermal conductivity.

5.2 Thermal Lensing Findings

In Ref. [40] calculations of the power were made using the following equations:

\[
\frac{1}{F_x} = \frac{-2 \cdot (OPLD(x) - OPLD(x_0))}{(x - x_0)^2} \quad \text{and} \quad \frac{1}{F_y} = \frac{-2 \cdot (OPLD(y) - OPLD(y_0))}{(y - y_0)^2}.
\]

The data was meshed in MATLAB and then cross sections of the data were extracted and
fit to a quadratic equation. It was found that \(1/F_i\) where \(i\) is either \(x\) or \(y\), is equivalent to \(-2 \cdot a\)
where \(a\) is the coefficient of \(x^2\) in the quadratic fit of a cross section of the data in Fig. 40. In this
way we found that \(1/F_y = -0.12 \, \text{m}^{-1}\). This is slightly less than the value for \(1/F_w\) calculated in
[40] as a result of thermal gradients which is over an order of magnitude larger than \(1/F_w\)
calculated in [40] as a result of thermal stress. By comparison to \(1/F_w\) as reported here, the value
for \(1/F_t\), - 0.42 \, \text{m}^{-1}, is 3.5 times larger in magnitude. This may at first seem surprising when
looking at Fig. 40, but it is more reasonable when one considers that the obvious curvature in the
width direction would appear more slight if viewed on a scale similar to the thickness of the slab.
Also, if the curvature in the thickness direction were extended to a range on par with the width it
would be more dramatic. Another important comparison between these results and the results of [40] is that $1/F_t$ is negative for the thermal lensing due to deformation and positive for thermal lensing due to stress and thermal gradients. However, the magnitude of the power of the thermal lens resulting from deformation is about two orders larger than that resulting from stress and thermal gradients and so there will be little cancellation.
6.1 Absorption Spectra

The absorption spectra were measured using a Cary 500 Dual Beam Spectrophotometer. The Cary 500 compares the transmission of two beams. One beam acts as a reference while the other beam passes through the sample. A comparison of the beams reveals the level of absorption at a given wavelength. The instrument scans through many wavelengths to generate the absorption spectrum of the sample being tested.
Figure 43: Schematic of the Cary 500 spectrophotometer used for taking absorption spectra [41].
Figure 44: Average of two absorbance spectra as a function of wavelength for 1% doped Nd:YAG ceramic crystal.

Because the Cary 500 measures absorbance, the data needs to be analyzed to obtain the absorption coefficient. The measured transmission $T_{\text{meas}}$ is a function of both the bulk transmission, $T_{\text{bulk}}$, and the Fresnel transmission, $T_{\text{fresnel}}$, or the transmission expected after Fresnel reflection has occurred.

$$T_{\text{meas}} = T_{\text{fresnel}} \cdot T_{\text{bulk}} = T_{\text{fresnel}}^2 \cdot T_{\text{bulk}}$$

Next, the absorption in a material is commonly represented by Beer’s law of attenuation,

$$I = I_o e^{-\alpha z}$$

where $I_o$ is the incident intensity into the bulk material, and $I$ is the intensity after the probing beam has passed through the bulk material, $\alpha$ is the absorption coefficient, and $z$ is the sample
thickness. Eq. (42) is in reference to the absorption in the sample without regard to the loss due to Fresnel reflection. Therefore, Eq. (42) can be written as

\[ T_{\text{bulk}} = e^{-\alpha z}. \] (43)

The quantity measured by the Cary 500 is absorbance or optical density which is defined as

\[ A = \log_{10}(\frac{1}{T_{\text{meas}}}) = -\log_{10}(T_{\text{meas}}). \] (44)

Substituting Eq. (43) into Eq. (41) and then Eq. (41) into Eq. (44) leads eventually to

\[ \alpha = \frac{A + 2\log_{10} T_{\text{fresnel}}}{z \log_{10} e}. \] (45)

Another equivalent expression for the absorption coefficient which can also be derived from Eqs. (41), (43), and (44) is

\[ \alpha = \frac{2 \ln(T_{\text{fresnel}}) + A \ln(10)}{z}, \] (46)

where in both cases

\[ T_{\text{fresnel}} = \frac{4n}{(n+1)^2}, \] (47)

where \( n \) is the index of refraction. The data for absorbance can therefore be used along with the values for sample thickness, \( z \), and Fresnel transmission, \( T_{\text{fresnel}} \), to obtain the absorption coefficient (see Fig. 45). In calculating the Fresnel transmission, one may include the dependence of the index of refraction on the wavelength (dispersion). In this case though, the range of wavelengths considered didn’t necessitate the use of dispersion in the calculation because there is very little change in the index of refraction over the wavelength range considered.
Figure 45: The absorption coefficient as a function of wavelength calculated from the data in Fig. 44. The value used for thickness, z, was 0.15 cm, and the value used for index of refraction, n, was 1.82.

The absorption coefficient was measured for a range of concentrations in the ceramic crystal from 0.09% to 1.8%. The data is shown in Fig. 46. At the maximum absorption peak for the 1.8% sample, the percent transmission went to less than 0.02% making the data unreliable because at such low transmission levels the detector noise plays a more significant role in the data. However, the ratio between the peak at 808.6 nm and 795.4 nm was measured for the other samples represented in Fig. 46 and that data is shown in Table 5. Using the expected ratio for the two peaks, the value of the absorption coefficient at 808.6 nm for 1.8% Nd:YAG ceramic crystal was calculated from the value of the peak at 795.4 nm. With this data graphs were made of the absorption coefficients of the two peaks as they vary with dopant concentration. It can be seen in Figs. 47 (a)-(b) that the data is linear and can be fit as such. Error may have arisen for
the 1.8% sample from the fact that the transmission through the 1.8% sample at 795.4 nm went below 0.2%. Again, this may be low enough to reduce the signal to noise to an undesirable level. Also there may be error in the value of the percent dopant concentration.

Figure 46: Absorption coefficient of ceramic crystal Nd:YAG for concentrations from 0.09% to 1.8%. The dot near the top of the graph represents the expected maximum of the absorption coefficient for 1.8% Nd<sup>3+</sup> dopant concentration in ceramic crystal YAG.
Table 5:
The dependence of the peak absorption on the dopant concentration, and the ratio of $\alpha_{p1}=\alpha(808.6) \text{ to } \alpha_{p2}=\alpha(795.4)$. The ratio for 1.8\% is the average of the other ratios. It was used to obtain $\alpha_{p1}$ for 1.8\% from $\alpha_{p2}$.

<table>
<thead>
<tr>
<th>Dopant Concentration</th>
<th>$\alpha_{p1}/\alpha_{p2}$</th>
<th>$\alpha_{p2}$</th>
<th>$\alpha_{p1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09%</td>
<td>1.503</td>
<td>0.6191</td>
<td>0.9305</td>
</tr>
<tr>
<td>0.32%</td>
<td>1.516</td>
<td>2.117</td>
<td>3.211</td>
</tr>
<tr>
<td>1%</td>
<td>1.508</td>
<td>6.522</td>
<td>9.835</td>
</tr>
<tr>
<td>1.8%</td>
<td><strong>1.509</strong></td>
<td><strong>10.765</strong></td>
<td><strong>16.244</strong></td>
</tr>
</tbody>
</table>
Figure 47: Absorption coefficient, $\alpha$, as a function of dopant concentration, $\varepsilon$, for Nd:YAG ceramic crystal. Fitted data is for dopant concentrations from 0.09\% to 1.8\% for (a) the 808.6 nm peak and (b) the 795.4 nm peak.

In Silfvast’s book *Laser Fundamentals* [42 Eq. 7.32] it is found that

$$\alpha(\nu) = \sigma_{ul}(\nu) \frac{g_u}{g_l} N_l$$  \hspace{1cm} (48)

where $\nu$ is the excitation frequency, and the subscripts $u$ and $l$ represent upper and lower energy levels respectively. $N_l$ is the density of atoms in the lower level and $g_u$ and $g_l$ are statistical weights relating to their respective energy levels. If we assume the frequency (or wavelength) is constant and that $N_l$ is proportional to the total dopant concentration, it can be seen that the absorption coefficient, $\alpha$, is proportional to the dopant concentration. Error may arise from inexact determination of dopant concentration. Without using a method such as Rutherford backscattering to confirm the concentrations, the precise dopant concentration values remain in question [43].
The absorption coefficient was also measured for a Yb:Lu₂O₃ ceramic crystal. When doped in Lu₂O₃ ceramic crystal, Yb³⁺ has three characteristic absorption peaks at 903, 948.6, and 976 nm. Its maximum absorption peak is at 976 nm which corresponds to [44] for ceramic crystal and [45] for single crystal. Absorption spectra show at what wavelength a sample may be pumped in order to achieve lasing with good slope efficiency. For Yb:Lu₂O₃, pumping at 976 nm appears to be the best option, but because that peak is so narrow, it may be easier to pump at 948.6 nm.

![Yb:Lu₂O₃ Absorption Coefficient](image)

**Figure 48:** Absorption coefficient as a function of wavelength for Yb:Lu₂O₃ ceramic crystal taken with the Cary 500 spectrophotometer.

### 6.2 Emission Spectra

Emission spectra were measured for 0.32%, 1%, 1.8%, and 4% Nd doped YAG ceramic crystals from Baikowski with dimensions 3 x 3 x 6 mm and a 1% Nd:YAG ceramic crystal from
VLOC with dimensions 1.6 x 4.05 x 39.8 mm. A fiber-coupled diode pumped the sample at 808 nm. The fiber was positioned above the sample which can be seen in Fig. 49, and the pump light from the fiber was focused into the sample. The sample was attached to the end of the cold finger of a cryostat cooled by compressed helium. The vacuum chamber was pumped by a roughing pump. A silicon sensor was mounted at the tip of the cold finger to measure the temperature and was part of a temperature control system that included a heating element which was a ribbon heater but was then replaced by a cartridge heater also mounted near the tip of the cold finger. The sensor detected the temperature and reported it to the temperature controller which determined whether or not to heat the cold finger. When the controller determined that heating was necessary, current was passed through the heater to heat the tip of the cold finger where the sample was mounted. In this way the temperature dependence of the emission spectra was measured. Once the temperature of the sample was sufficiently stabilized the sample was pumped with 808 nm radiation. The fluorescence was collected by a lens or lens system and imaged onto an entrance slit of a SpectraPro-300i monochromator from Acton Research Corporation or a Thermo Jarrell Ash monochromator. Through the use of mirrors and a grating, the monochromator selected a wavelength which exited at the exit slit where a detector was placed. A schematic of the experimental setup is shown in Fig. 49.
Figure 49: Experimental setup for measuring emission spectra. The dotted red lines represent the laser beam that is either in a higher plane or directed down into the vacuum chamber where the sample is mounted.

Either a 600 or a 1200 groove/mm grating could be used in the monochromator. Though the 1200 groove/mm grating offered higher resolution, generally the 600 groove/mm grating was used because the resulting intensity to the 280 United Detector Technology (UDT) sensor was
greater and made for a better signal to noise ratio. With higher resolution comes a decreased signal because the bandwidth and consequently the number of photons to the detector is reduced.

Using the compressed-helium cryostat, the emission spectrum was measured at different temperatures starting as low as 37 K. Light from the fiber-coupled diode laser was directed into the top window of the vacuum chamber. Prior to entering the chamber a glass slide acting as a beam splitter reflected a small percentage of the beam to a 221 UDT detector for reference. The beam entering the chamber was focused onto the sample via two lenses. Fluorescence emitted from the sample was imaged onto the entrance slit of the monochromator. The power of a narrow band of fluorescence passed by the monochromator was detected by means of the UDT instruments 280 sensor and displayed on the UDT Instruments S380 Dual Channel Optometer. The monochromator slowly scanned through the emission spectrum and the optometer read the power. The information was passed to the computer where a LabVIEW program recorded the information and produced a plot of emission as a function of wavelength. The data was saved to be examined in Origin [46], a spreadsheet program.

To achieve accurate measurements with our experimental setup, a calibration needed to be employed. A standard source with a known emission spectrum such as an iodine tungsten lamp was used as our black body radiator at 3350K peaking in its emission near 866 nm. The emission of the lamp was measured using the monochromator and a correction for our spectroscopic system was obtained (see Fig. 50). The noisy look to the measured emission between 1350nm and 1450nm is most likely a result of absorption by water in the air or on an optical surface.
Figure 50: A graph showing in red the measured unpolarized emission from an iodine tungsten lamp as a function of wavelength and in black the real emission from the tungsten lamp at 8.3 amps.

$I_{\text{standard}}$ is defined here as the actual intensity out from the tungsten lamp, and $I_{\text{measured}}$ is the intensity that was measured through our spectroscopic system. With the data shown in Fig. 51, the following operation was performed in Origin:

$$
\frac{I_{\text{standard}}(\lambda)}{I_{\text{measured}}(\lambda)} = \zeta(\lambda). 
$$  \hspace{1cm} (49)

Multiplying whatever emission intensity we measure by $\zeta(\lambda)$ corrects for the changes to the spectrum caused by our system. The calibration function, $\zeta(\lambda)$, was used to obtain an adjusted emission spectrum for subsequent data such as that shown in Fig. 52.
Figure 51: The emission spectrum as measured for a 1% Nd:YAG ceramic crystal from VLOC.

The calibration function, $\zeta(\lambda)$, was multiplied by the signal data shown in Fig. 51 in the following manner:

$$I_{\text{adjusted}}(\lambda) = \zeta(\lambda)I_{\text{measured}}(\lambda). \quad (50)$$

Here, $I_{\text{adjusted}}(\lambda)$ is the intensity data adjusted for the unwanted changes made to the spectrum by our system including monochromator and detector. A plot of $I_{\text{adjusted}}(\lambda)$ is shown in Fig. 52.
Figure 52: Emission signal of 1% Nd:YAG crystalline ceramic from VLOC adjusted according to the calibration function of the system.

The emission cross section as a function of wavelength, \( \lambda \), is given by [47] as

\[
\sigma(\lambda) = \frac{\eta \lambda^5}{\tau_f \left( \int \lambda I_{\text{adjusted}}(\lambda) d\lambda \right) f 8 \pi n^2 c} I_{\text{adjusted}}(\lambda)
\]  

(51)

where \( \eta \) is the radiative quantum efficiency of the upper states, \( f \) is the fraction of the pumped population in the upper state, \( c \) is the speed of light, \( \tau_f \) is the fluorescent lifetime, \( n \) is the index of refraction, and \( \lambda \) is the wavelength. The following values were used: \( \eta=1 \), \( \tau_f=212 \, \mu s \), \( f=1 \), \( n=1.815 \), \( c=3 \times 10^8 \, \text{m/s} \), and the integral of the measured and corrected function \( \lambda I(\lambda) \) was found numerically in Origin to be near \( 2.55 \times 10^{-5} \). Also the final values of cross section were written in units of \( \text{cm}^2 \) and are shown in Fig. 53.
In the literature [23] the stimulated emission cross section for Nd:YAG single crystal is reported as $6.5 \times 10^{-19}$ cm$^2$. But because of thermalization at 295 K only 42.7% of the $^4F_{3/2}$ manifold is in the upper Stark level, the upper laser level. Therefore, to obtain the effective emission cross section the spectroscopic cross section was multiplied by 0.427 to obtain the effective emission cross section. Therefore, the effective emission cross section of single crystal Nd:YAG is $2.8 \times 10^{-19}$ cm$^2$. The peak cross section which occurs at 1064 nm in our data is $3.98 \times 10^{-19}$ cm$^2$. Multiplying this value also by 0.427 yields an effective cross section of $1.7 \times 10^{-19}$ cm$^2$ for ceramic crystal Nd:YAG.

A similar process as above was repeated for 0.32%, 1%, 1.8% and 4% ceramic crystal Nd:YAG samples from Baikowski, a United States distributor of material that is made by Konoshima Chemical Company in Japan. Also, for comparison 0.32% and 1% single crystal Nd:YAG samples from VLOC were measured. The calibration source for the system was an
Oriel tungsten lamp with a spectral range of 250-2400 nm peaking at 913 nm whose black body temperature is 3174 K. The relevant curves associated with the calibration are shown in Fig. 54.

Figure 54: In black, the measured unpolarized emission from a tungsten lamp as a function of wavelength, and in green, the real emission from the tungsten lamp, and in red, the correction function, $\zeta(\lambda)$.

Fig. 55 shows data taken for Nd:YAG ceramic crystal at room temperature that has been adjusted using the calibration function, $\zeta(\lambda)$, from Fig. 54. This data suggests that as the dopant concentration increases up to 4% there is a fall off in intensity. However, between each measurement the sample had to be removed and another sample placed in the sample holder. During this process a lens had to be moved and then replaced, and sometimes small adjustments were made to the position of the incoming diode laser light. These factors may have influenced the absolute value of the intensity. Even so, Fig. 55 is suggestive of concentration quenching effects in higher dopant concentrations of Nd:YAG ceramic crystal.
Figure 55: The emission spectra as measured for 0.32%, 1%, 1.8% and 4% Nd:YAG ceramic crystals from Baikowski. Resolution was better than 0.4 nm.

It may be thought that in ceramic crystal materials, dopant ions may be lodged at grain boundaries and thus experience different potentials than ions located in the bulk of ceramic crystal grains. If this were the case then one might expect to see inhomogeneous broadening in the emission for ceramic crystal samples as compared to single crystal samples. Yet Fig. 56 and Fig. 57 illustrate the uniformity in emission cross section peak widths between ceramic and single crystal for concentrations of 0.32% and 1% dopant concentrations. This suggests no inhomogeneous broadening in ceramic crystal. In amorphous materials such as glass, inhomogeneous broadening occurs which is a broadening of the emission peaks of the dopant ions. This is a result of the dopant ions sitting in different energy potentials from one another so that their emissions vary slightly. Cumulatively this leads to broadening of the emission peaks. Dopant ions in a crystal such as YAG will emit more uniformly leading to sharper peaks in the
emission. Because there is no apparent broadening (see Figs. 56-57) of the emission peaks of the ceramic crystal emission in comparison to the single crystal emission it is inferred that there is no significant ion accumulation at the grain boundaries. Still, one might wonder if inhomogeneous broadening would show up at higher concentrations. Fig. 58 shows a comparison between the emissions of different concentrations of ceramic crystal Nd:YAG measured at room temperature illustrating the lack of inhomogeneous broadening as a function of dopant concentration.

![Comparison of emission cross sections](image.png)

Figure 56: Comparison of the emission cross sections of 1% single crystal (red) and 1% ceramic crystal (black) at room temperature. Resolution was better than 0.4 nm.
Figure 57: Comparison of the emission cross sections of 0.32% single crystal (red) and 0.32% ceramic crystal (black) at room temperature. Resolution was better than 0.4 nm.
Figure 58: Concentration dependence of emission cross section of Nd:YAG ceramic crystal near the peak emission near 1064 nm at room temperature. Resolution was better than 0.4 nm.

Investigations of the temperature dependence of the emission cross section were also made. It was found that the peak in the emission cross section shifts from the 1064 nm to the 1061 nm peak as the sample material is cooled. This appears to be a common feature for these materials regardless of dopant concentration or of whether the host material is single crystal or ceramic crystal. Figs. 59 to 61 are illustrative.
Figure 59: Emission cross section of 4% ceramic crystal Nd:YAG in the region of the lasing wavelength for temperatures ranging from 50K to room temperature (~300K). Resolution was better than 0.4 nm.
Figure 60: Emission cross section of 1% ceramic crystal Nd:YAG in the region of the lasing wavelength for temperatures ranging from 50K to room temperature (~300K). Resolution was better than 0.4 nm.
Figure 61: Emission cross section of 1% single crystal Nd:YAG in the region of the lasing wavelength for temperatures ranging from 50K to room temperature (~300K). Resolution was better than 0.4 nm.

Fig. 62 is provided to show more clearly the change in the two cross section emission peaks near 1061 nm and 1064 nm for the 1% Nd:YAG single crystal and the 1% Nd:YAG ceramic crystal samples. As can be seen in Figs. 59-61, the center wavelength of the peaks also shifts with temperature.
Figure 62: Emission cross section summary for 1% ceramic and single crystal Nd:YAG showing the rapid increase of the emission cross section for the 1061 nm emission peak as temperature decreases.

The emission cross section has been measured for Yb:Lu$_2$O$_3$ ceramic crystal. The calibration source for the system was an Oriel tungsten lamp with a spectral range of 250-2400 nm peaking at 913 nm whose black body temperature is 3174 K. The emission was measured for several temperatures from 50 K to room temperature (Fig. 64). Also, in place of the diode pump source, a MOPO (master oscillator and power oscillator) system was used which could be set to emit various wavelengths of laser radiation.

The lasing wavelength of Yb$^{3+}$ doped laser gain media is usually taken to be the peak near 1030 nm, but it can also lase near 1070 nm. It can be seen in Fig. 64 that the emission at 968 nm decreases as temperature decreases. The emission peaks at 1025 and 1070 however increase with decreasing temperature. A possible explanation of this behavior is that the lower
Stark sublevel to which the 968 emission decays in the $^2F_{7/2}$ manifold is less populated at higher temperatures than at lower temperatures making for more probable transitions. On the other hand the Stark levels to which the 1025 nm and 1070 nm emissions decay is less populated at lower temperatures making for more probable transitions. See Fig. 63.

Figure 63: Energy level diagram for Yb:Lu$_2$O$_3$ showing emission at 968 nm, 1025 nm, and 1070 nm. Taken in part from [45].

In Ref. [23] we find that for Yb:YAG at room temperature 5.5% of the dopant is at the lower laser energy level. When cooling the sample, the thermalized population of the lower laser level of the 1025 nm transition may change more substantially than the lower laser level of the
1070 nm transition resulting in a more dramatic increase in the 1025 nm emission as seen in Fig. 64.

Figure 64: Emission cross section of 10% doped Yb:Lu$_2$O$_3$ ceramic crystal as it varies with temperature. Temperatures range from 50 K to room temperature (~300 K).

In calculating the emission spectra in Fig. 64 the index of refraction used was $n=1.93$ [48]. Dispersion was assumed to be small in the region of interest from 900 to 1200 nm. The other constants used to solve for the emission cross section according to Eq. (51) were $\eta=1$, $\tau_f=1.38$ ms, $f=1$, and $c=3 \times 10^8$. A comparison of the peak values from Fig. 64 is shown in Table 6.
Table 6:
Comparison of the peak emission cross section values from Fig. 64. The units for the cross section values are cm$^2$.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Peak wavelength</th>
<th>968nm</th>
<th>1025nm</th>
<th>1070nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room Temp</td>
<td>2.47</td>
<td>5.82</td>
<td>2.84</td>
<td></td>
</tr>
<tr>
<td>250 K</td>
<td>1.49</td>
<td>5.70</td>
<td>3.75</td>
<td></td>
</tr>
<tr>
<td>200 K</td>
<td>1.37</td>
<td>6.80</td>
<td>4.12</td>
<td></td>
</tr>
<tr>
<td>150 K</td>
<td>1.19</td>
<td>8.70</td>
<td>4.40</td>
<td></td>
</tr>
<tr>
<td>100 K</td>
<td>0.831</td>
<td>11.02</td>
<td>4.48</td>
<td></td>
</tr>
<tr>
<td>50 K</td>
<td>0.392</td>
<td>11.96</td>
<td>4.41</td>
<td></td>
</tr>
</tbody>
</table>

Another interesting feature of the data in Fig. 64 is the points where there appears to be no temperature dependence of the emission. One such point is at 1032 nm, and another is at 999 nm. It is conceivable that this material could be used to build a temperature independent laser albeit one that would not lase at a peak of the material’s emission.

6.3 Lifetime

Another fundamental and important laser material property is fluorescence lifetime. Although it has been measured before, we show here that fluorescence lifetime varies little from position to position inside the same sample. A comparison is made with the lifetime values for
our samples and the lifetime values found in the literature. Two experimental setups for measuring fluorescent lifetime are shown in Fig. 65. A pulse of light excited the ions in the sample into the upper laser level. As these ions relaxed into lower energy levels they fluoresced. A lens imaged the emission region onto the entrance slit of the monochromator. Next, the monochromator selected the emission wavelength which was detected by the photomultiplier and displayed by the oscilloscope. Alternatively, a lens focused the emission onto an Analog Modules Inc. detector model 710-120 having a 10 ns response time. Prior to entering the detector the light was filtered to significantly reduce the pump light. The detector was also connected to the oscilloscope from which the decaying signal was recorded by the computer. The data was fit to an equation of the form

\[ I(t) = I_0 e^{-\frac{t}{\tau}} + C, \quad (52) \]

where \( I_0 \) and \( C \) are constants and \( \tau \) is the fluorescence lifetime.

After it was recorded by the computer the data was plotted on a logarithmic or natural logarithmic scale as shown in Fig. 66. The decay time was extracted by fitting the measured data to Eq. (52). This means we are assuming the decay is governed by a single exponential so that the fit in this log-linear plot should be a straight line. The value for lifetime reported in Table 7 corresponds to the data shown in Fig. 66. Eq. (52) can be fit to the data in Fig. 66 using software such as Origin. Origin gives an \( R^2 \) value when fitting data. \( R^2 \) represents the degree to which the fit represents the data. The closer \( R^2 \) is to 1, the better the fit. In Fig. 66 the \( R^2 \) value for the fit is 0.99859, and \( \tau \) is 211\( \mu \)s.
Our samples included an Nd:YAG ceramic crystal from VLOC which had a Neodymium concentration of 1.0%. Other samples were from Baikowski and had Nd dopant concentrations of 0.32%, 1.0%, 1.8%, and 4.0%.
Table 7:
Fluorescence lifetime data for Nd:YAG ceramic and single crystal.

<table>
<thead>
<tr>
<th>Material Type</th>
<th>Dopant Concentration</th>
<th>Fluorescence lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our measurement</td>
<td>CC 1.0%</td>
<td>211 μs</td>
</tr>
<tr>
<td>Ref. [49]</td>
<td>CC 1.0%</td>
<td>219 μs</td>
</tr>
<tr>
<td>Ref. [50]</td>
<td>CC 1.0%</td>
<td>237.6 μs</td>
</tr>
<tr>
<td>Ref. [51]</td>
<td>CC 1.1%</td>
<td>210 μs</td>
</tr>
<tr>
<td>Ref. [51]</td>
<td>SC 0.9%</td>
<td>217 μs</td>
</tr>
<tr>
<td>Ref. [23]</td>
<td>SC 1.0%</td>
<td>230 μs</td>
</tr>
</tbody>
</table>

The commonly accepted value for the fluorescence lifetime of 1.0% Nd:YAG single crystal is 230 μs as found in Koechner’s book *Solid-State Laser Engineering* [23]. The measurement of single crystal lifetime in [51] falls below the generally accepted value suggesting that their measured value of fluorescence lifetime for ceramic crystal may also be low. Our measurement would then also be low. Our measurements along with other measurements from the literature as seen in Table 7 indicate a shorter lifetime from the ceramic crystal than for the single crystal.

We did not know how uniformly the dopant was distributed in the ceramic crystal or if there was an accumulation of the dopant at the grain boundaries. If that were so, the measured lifetime could have fluctuated from position to position in the ceramic crystal if in one position there happened to be more grain boundaries and therefore higher effective local dopant
concentration. Exciting different regions of the same sample would then result in different lifetimes due to these non-uniformities in the dopant distribution.

Figure 66: Plot of the irradiance on a logarithmic scale as a function of time for pulse-pumped 1% Nd:YAG ceramic crystal from VLOC.

Some investigation into the variability of the fluorescence lifetime measurement was made. We attempted to ascertain what changes might occur depending on what part of the sample was being excited and whether the pump was focused or unfocused. A summary of the data is shown in Table 8
Table 8:

Summary of lifetime data for changing focus and pump spot position.

<table>
<thead>
<tr>
<th>Focused pump beam</th>
<th>Unfocused pump beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot</td>
<td>Lifetime (μs)</td>
</tr>
<tr>
<td>1</td>
<td>211 ±2</td>
</tr>
<tr>
<td>2</td>
<td>211 ±2</td>
</tr>
<tr>
<td>3</td>
<td>214 ±2</td>
</tr>
<tr>
<td>4</td>
<td>213 ±2</td>
</tr>
</tbody>
</table>

Because data was taken every 2 μs, the error bars in Table 8 are given as ±2 μs. However more error may be introduced as a result of the fitting. With an error of ±2 μs six of the seven measured lifetimes are essentially the same. Those six data points vary by only 1.4%. The odd data point is spot 2 with an unfocused pump beam. It should be noted that the R² value for that data point happens to be the smallest, meaning the fit of the data is the worst. In order to determine if there is a significant difference between unfocused pumping and focused pumping, more data is needed.

Lifetime measurements were made for samples from Baikowski with four different dopant concentrations. Results of these measurements are shown in Figs. 67-71. The linearity of the natural logarithmic plots, Figs. 67-69 indicate the single exponential nature of the decay.
Figure 67: The normalized emission signal for 0.32%-4% Nd:YAG ceramic on a natural logarithmic plot. The data was taken at 77 K.
Figure 68: The normalized emission signal for 0.32%-4% Nd:YAG ceramic on a natural logarithmic plot. This data was taken at room temperature.

Because the lifetime changed from one concentration to another, the data ranges also changed. This is illustrated in Figs. 69 and 70. This change in data range affected the error in the data. In shorter data ranges there were the same number of points taken as for longer ranges, so the digitization error for the shorter ranges is less. The error bars in Fig. 71 reflect the digitization error.
Figure 69: Normalized emission signals for 0.32%-4% Nd:YAG ceramic crystal on a logarithmic plot. The data ranges were not all the same leading to different values for digitization error. This data was taken at 77 K.
Figure 70: Normalized emission signals at 77K for 0.32%-4% Nd:YAG ceramic. This data is not on a logarithmic plot. This also illustrates the difference in the data ranges.

Since the sample was mounted to the cold finger of a cryostat, it was possible to make many measurements of the lifetime at different temperatures to ascertain the dependence of fluorescence lifetime on temperature. Or, in other words, to reduce the effect of the nonradiative lifetime on our measured results, the sample was cooled. We could therefore get a better idea of the radiative lifetime value. The results of these measurements are summarized in Fig. 71. The fact that the curves for the different dopant concentrations are at different levels in Fig. 71 is likely due to ion-to-ion interaction which changes with dopant concentration. When the dopant ion concentration increases there is more probability that the ions will interact. The interaction may be in the form of energy exchange which effectively gives the excited ions more paths by which they can decay, making the decay faster. It can be seen in Fig. 71 that the lifetime of 1%
Nd:YAG ceramic crystal from Baikowski is near the value 230 $\mu$s found in [23] for 1% Nd:YAG single crystal.

Figure 71: Dependence of lifetime on temperature and concentration for Nd:YAG ceramic crystal.

Fluorescence lifetime has also been measured for 10% Yb:Lu$_2$O$_3$ ceramic crystal. To measure the lifetime of the 10% Yb:Lu$_2$O$_3$ the setup in Fig. 65(b) was used. The lifetime was measured 5 times and recorded in the computer. Each set of lifetime data was fit to a single exponential and from the fit equation the lifetime was ascertained. The lifetime data for the five sets of data was averaged and found to be $1.38\pm0.05$ms where 0.04 ms is the standard deviation of the five data points. Data for the lifetime was taken every 0.01 ms which may be added to the uncertainty making the value for the fluorescence lifetime $1.38\pm0.06$ms. The lifetime was measured at the 1025 nm peak in the emission. The lifetime was also measured using the same
process as for the 1025 nm peak near the 968 nm peak and found to be 1.30±0.04 ms. Also near 1030 nm and with only four lifetime measurements, the lifetime was found to be 1.37±0.06 ms. The value used in the calculation of the emission spectra in an earlier section 6.2 was 1.38 ms.
7.1 Yb:Y₂O₃ Ceramic Crystal Laser

7.1.1 Background and Methodology

At the time of our study the best results in the literature for a Yb:Y₂O₃ ceramic laser were found in [7], and are summarized in Table 9.

Table 9:
Recent literature results for lasing Yb:Y₂O₃ [7].

<table>
<thead>
<tr>
<th>Publication Year</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorption Power</td>
<td>27 W</td>
</tr>
<tr>
<td>Output Power</td>
<td>9.22 W</td>
</tr>
<tr>
<td>Output Type</td>
<td>CW</td>
</tr>
<tr>
<td>Pump $\lambda$</td>
<td>937 nm</td>
</tr>
<tr>
<td>Laser $\lambda$</td>
<td>1078 nm</td>
</tr>
<tr>
<td>Output Reflectivity</td>
<td>96 % @ 1064 nm</td>
</tr>
<tr>
<td>Spot size</td>
<td>450 um</td>
</tr>
<tr>
<td>Threshold</td>
<td>3.1 W</td>
</tr>
<tr>
<td>Slope Efficiency</td>
<td>41 %</td>
</tr>
<tr>
<td>Dopant Concentration</td>
<td>8 %</td>
</tr>
<tr>
<td>Sample Size</td>
<td>3 mm cube</td>
</tr>
<tr>
<td>Emission Cross-section</td>
<td>$0.5 \times 10^{-20} \text{ cm}^2$</td>
</tr>
</tbody>
</table>
In the experimental setup shown in Fig. 72, the 180 W laser used for pumping was a fiber-couple diode laser by Laserline in Mülheim-Kärlich, Germany. It coupled stacks of diode bar emission into a 400 μm fiber and had a numerical aperture of 0.22. Since the emission wavelength of the diodes depended on temperature it could be controlled by selecting the coolant temperature and/or the power supplied to the diodes. The peak emission wavelength was near 938 nm. To eliminate the need for cooling the thin ceramic disk, we operated the pump source and hence the laser in a pulsed manner. The pulses were approximately 1 ms long and the repetition rate was 10 Hz. Since the fluorescence lifetime of Yb$^{3+}$ in Y$_2$O$_3$ is 0.82 ms [4] the length of the pulse was appropriate to achieve lasing. This regime of operation is referred to as quasi-CW. Because the pulses were only coming at a repetition rate of 10 Hz, the gain medium was cooled enough by convection into the air between pulses that a coolant system for the Yb:Y$_2$O$_3$ ceramic crystal was not necessary.

Figure 72: Experimental configuration for the 10% doped Yb:Y$_2$O$_3$ ceramic crystal laser.
The lens system for focusing the pump consisted of a 50 mm focal length anti-reflective coated plano-convex lens that was placed about 50 mm from the tip of the fiber and a 75 mm focal length anti-reflective coated plano-convex lens. Another 75 mm to the right of the lenses the HR coated surface of the ceramic crystal Yb:Y₂O₃ sample was located. 75 mm further to the right of the sample was the 300 mm radius of curvature 95% reflective output coupler.

To aid in the determination of output coupler placement, the spot size of the pump light after the lenses was measured. An approximately 9 μm core diameter fiber was mounted to a translation stage and pointed directly into the beam and moved to the focal plane of the focused pump radiation. As shown in Fig. 73, the fiber terminated at a detector that was connected to an oscilloscope to record the maximum energy for the pulse as shown in the figure. That value was recorded for many positions as the fiber moved through the focal plane of the pump radiation. From the data we could determine the approximate positions that represented 1/e² of the maximum voltage read on the oscilloscope and thereby determine the approximate width of the beam at the focus or in other words the spot size of the pump. The pump spot size was found to be approximately 600 microns in diameter.
Figure 73: The experiment used to determine the spot size of the pump radiation. The fiber tip passed through the beam while intensities were monitored on the oscilloscope.

Because Yb$^{3+}$ is essentially a two level system where $^{2}F_{5/2}$ is the upper level and $^{2}F_{7/2}$ is the lower level [45], fluorescence is readily reabsorbed. We therefore wanted to match pump spot size to the mode size of the cavity as shown in Fig. 74 where the pump spot size is slightly larger than the mode size. It would of course be most efficient if they matched exactly because in that case more of the pumped volume would result in laser output, but it is better to err with a larger pump spot size than a smaller one. In this way the laser emission won’t be reabsorbed. If the mode size were larger than the pump size, the laser light could be absorbed in the region of the mode that wasn’t pumped. For illustration purposes Fig. 74 shows an exaggerated difference between the mode size and pump size.
Figure 74: The mode size should be smaller than the pump spot size at the focal plane as illustrated here. This is to avoid absorption losses.

The mode size can be calculated using equations (5.11-5.12) from Koechner’s book *Solid-State Laser Engineering* [23]. Equivalent expressions are reproduced here as Eqs. (53-55). These were entered into Mathcad [52] and graphs were produced showing the mode size as a function of mirror position. Fig. 75 shows a diagram of the cavity and Fig. 76 is a graph of Eqs. (53-55) produced in Mathcad.

\[
\begin{align*}
    w_o(L) &= \sqrt{\left(\frac{\lambda}{\pi}\right)^2 \cdot L \cdot (R_2 - L) \cdot (R_1 - L) \cdot \frac{R_1 + R_2 - L}{(R_1 + R_2 + 2 \cdot L)^2}} \\
    w_1(L) &= \sqrt{\left(\frac{\lambda \cdot R_1}{\pi}\right)^2 \cdot \frac{L \cdot (R_2 - L)}{(R_1 - L) \cdot (R_1 + R_2 - L)}} \\
    w_2(L) &= \sqrt{\left(\frac{\lambda \cdot R_2}{\pi}\right)^2 \cdot \frac{L \cdot (R_1 - L)}{(R_2 - L) \cdot (R_1 + R_2 - L)}}
\end{align*}
\]

$L$ is the distance between the mirrors, $w_1$ is the radius of the beam at mirror 1, $R_1$ is the radius of curvature of mirror 1, $w_2$ is the radius of the beam at mirror 2, $R_2$ is the radius of curvature of mirror 2, and $w_o$ is the minimum beam waste.
As the spot size we measured was around 600 microns in diameter the mode size needed to be slightly smaller than that. From Fig. 76 it can be seen that a mode size less than 600 microns was achievable, but mode size very close to 600 microns appears to have been unachievable. It was found by trial that placing the output coupler at 75 mm to the right of the sample, and thus the HR mirror, as in Fig. 72 the best results were achieved. It would seem from Fig. 76 that better efficiency should have resulted from placing the output coupler further from the sample, but the thermal lensing of the sample may have altered the cavity in such a way as to require that the output coupler be closer to the sample than expected. An analysis of thermal effects in this system has not been carried out. The analysis would be complicated by the fact that the thermal lensing not only introduces a focal length from the intra-cavity sample, but if the sample thermally distorts, the dielectric HR coating on the sample that was assumed flat may curve slightly and become a focusing element. In any case, the best results obtained were for the output coupler positioned 75 mm from the sample.
Figure 76: Graph of the spot sizes as a function of the cavity length. It can be seen that \( w_0 \) and \( w_1 \) track one another because the flat HR cavity mirror is the position of the beam waist. Note that the y-axis represents radii.

7.1.2 Laser Findings

Output energy from the Yb:Y\(_2\)O\(_3\) ceramic crystal laser is shown in Fig. 77. Fitting the data to a trendline reveals a slope efficiency of approximately 34%. Commonly the data below is reported in units of watts, but the pulse generator we used generated pulses whose length
increased with increasing amplitude. Some work was done to characterize this effect as shown in Fig. 78.

Figure 77: Output energy, $E_0$, as a function of absorbed energy, $E_a$, for the Yb:Y$_2$O$_3$ ceramic crystal laser.

In the quasi-CW regime, the width of a laser pulse also changes with increasing power but in a different way than does the pump pulse. The reason for this is that as the sample is pumped it begins to lase when enough energy has been absorbed. If the pump pulse width stays the same, but the energy of the pump increases, then the energy required for lasing is absorbed sooner and lasing results sooner. An oscilloscope image illustrates the difference between pump pulse width and lasing pulse width in Fig. 79.
Figure 78: Pulse width vs. absorbed energy per pulse.
Figure 79: An example of the oscilloscope display when pumping and lasing. The pulse indicated by a 1 is the pump pulse while the pulse indicated by a 2 is the laser pulse. As is evident, the widths of the two pulses are not identical [53].

As shown in Fig. 79, a quasi-CW laser starts pulsing sometime during the pump pulse. Increasing the pump power causes the starting time of the laser pulse or pulses to be closer to the starting time of the pump pulse. At the start of the laser pulse, relaxation oscillations in the emission occur which settle down to a more constant value as seen in Fig. 79. Therefore after the initial relaxation oscillations in the laser pulse a time follows in which the laser can be thought of as being in a CW regime. The focus of the measurements being on output power it is regrettable that the widths of the changing laser pulses were not measured.

The maximum output energy achieved was 28.8 mJ/pulse. The pulse width when 44 amps were applied to the pulse driver was 1400 \( \mu \)s and the pulse width when 46 amps were
applied to the pulse driver was 1410 μs. Extrapolating between these values gives the pulse width at 45 amps as 1405 μs. At this current the highest energy reported here was obtained. It may be assumed that the lasing action at this power starts almost directly after pumping begins and so the maximum power would be 20.5 W. This conversion is made for the purpose of more direct comparison with other reports. Another possible boost to our results could have occurred if we had been able to pump at 1 Hz instead of 10 Hz. Previously we had measured an increase in output of 4.5% when pumping at 1 Hz instead of 10 Hz (though this was when 50 amps were applied by the pulse driver). If this were true with the final maximum data point, then we could expect the output to increase to 21.3 W.

If the index of refraction of the sample is taken to be 1.915 [30], then the percent transmission through the sample, taking Fresnel reflection into account on the two surfaces of the sample, should have been near 80%. This ceramic crystal may have performed less well than it should have because the sample transmission was measured, and the baseline was approximately 70% [54]. There were more absorption losses in the sample than expected. A subsequent sample was measured (with the same Shimadzu spectrophotometer that was used to obtain the baseline of 70 percent) and found to have a baseline of 80 percent as expected. The sample we used therefore was of lower quality.

Future work could include using another sample of Yb:Y₂O₃ ceramic crystal with the expected transmission losses. Also, to obtain more reliable results a pulse driver that doesn’t change output pulse widths could be used or the pulse widths could be measured along with the laser power. If the pump pulses were to be kept at or below 1 ms then the Pyro-Electric detectors which don’t accurately measure pulses whose widths exceed 1ms could be used to detect the
pulses. If instead the pulses were to be allowed to vary, then the thermal detectors would need to be used. To achieve better results, the repetition rate could be reduced to 1 Hz and the sample actively cooled.
CHAPTER EIGHT
CONCLUSION

The thermal expansion coefficient has been measured for Nd:YAG ceramic crystal and found to be near the value 6.5·10^{-6} \, \text{oC}^{-1}. When compared to the literature value 7.8·10^{-6} \, \text{oC}^{-1} for single crystal as found in [23], the value 6.9·10^{-6} \, \text{oC}^{-1} for single crystal as found in [24], and the value for single crystal, 5.5·10^{-6} \, \text{oC}^{-1}, measured here, our value for the thermal expansion coefficient for Nd:YAG ceramic crystal agrees best with the single crystal value found in [24]. This study does not confirm a dopant concentration dependence of the thermal expansion coefficient but the data suggests there may be a slight dependence. This calls for further investigation. Also, dn/dT was measured for Nd:YAG ceramic crystal and found to be near the value 1.1x10^{-5} \, \text{oC}^{-1} compared to our measured value for single crystal Nd:YAG 1.3x10^{-5} \, \text{oC}^{-1} and the value for Nd:YAG single crystal in the literature of 7.3x10^{-6} \, \text{oC}^{-1} [23]. Errors in our values for dn/dT were influenced by our errors in thermal expansion coefficient values. Discrepancies between reported values may be due to different temperature ranges in which the thermal expansion coefficient and dn/dT were measured.

Depolarization loss in a ceramic crystal slab where the rays propagate in a zigzag fashion was found to depend on the grain size. It was found that for grain sizes smaller than 8.5 \, \mu m the depolarization loss can be less than for single crystal laser material. It was also found that for propagation straight through an Nd:YAG ceramic crystal rod, the depolarization loss does not depend on the crystalline grain size. We also found no significant dependence of the depolarization loss on grain size for propagation straight through a slab. This lack of any
significant dependence on grain size for straight through propagation can be attributed to the fact that the overall depolarization loss is high. Consequently, varying the grain size and thus the small fluctuations in the depolarization loss does not significantly influence the overall depolarization loss in the considered range of grain sizes.

It has been found that thermal lensing as a result of deformations is the most significant cause of thermal lensing in slab lasers. In a slab where the rays zigzag, the power of the deformation-induced lensing in the thickness direction is greater than for any other cause of thermal lensing. In the width direction the power of thermal lensing resulting from deformations is the same order of magnitude as that resulting from thermal gradients. The optical path length difference resulting from deformations is greater than the optical path length difference resulting from thermal gradients and thermal stress.

Spectroscopically, Nd:YAG ceramic crystal is nearly identical to Nd:YAG single crystal. Both of their emission spectra change with temperature in the same manner. Yb:Lu$_2$O$_3$ ceramic crystal contains points of potential temperature independence for laser operation as indicated by its emission spectra at different temperatures.

Laser operation was achieved in Yb:Y$_2$O$_3$ ceramic crystal with a slope efficiency of 34%. The maximum output energy achieved was 28.8 mJ in a pulse having duration 1.405 ms.

Perhaps one of the most surprising things about ceramic crystal laser materials is how similar they have become to their single crystal counterparts. Besides their ability to withstand higher stresses, ceramic crystal laser materials behave essentially the same as single crystal laser materials. As a result, ceramic crystal laser materials are sought after because they are easier and less expensive to fabricate. They can also be customized to have unique geometries, exotic
dopant concentrations, and much larger size than single crystals. A simultaneously positive and negative feature of the ceramic crystal materials is that they are still being developed. It may be difficult to obtain the very high quality material fabricated in Japan where the process has matured over the last decade and a half.

For several reasons, ceramic crystal laser materials are very attractive as high power laser gain media. Since they can be made arbitrarily large and the cost to make them is less than single crystal laser materials, their power output per cost is potentially high. Also since their physical and optical properties are so similar to their single crystal counterparts, there is not a significant trade off for choosing lower cost ceramic laser material.
APPENDIX A
THERMAL LENSING CODE
MATLAB code for extracting slab surface data from COMSOL multiphysics

%% The Data Selecting Program

\textbf{global} \ L_{\text{rect}} \ L_{\text{angle}} \ \text{wid} \ t \ \text{angl}

\begin{verbatim}
angl1 = 30.61*pi/180; %Angle of brewster slab ends
t = .001; %slab thickness
w_dp = 16e-3; %slab width
w_undp = 2*0; %total width of two undoped slab edges
wid = w_dp+w_undp; %total width of slab including undoped regions
L_dp = 89.58e-3; %the total length of rectangular part of the doped slab
L_undp = 2*0; %the total length of rectangular parts of the two undoped slab ends
L_rect = L_dp+L_undp; %the total length of the rectangular region...doped and undoped
L_angle = t/tan(angl1); %the length of one brewster angle end
L = L_dp+L_undp+L_angle; %full slab length along center line.
ZL_top = -0.5*(L_dp+L_undp); %the coordinate component of the left-most point of top face
ZR_top = 0.5*(L_dp+L_undp)+L_angle; %the coordinate component of the right-most point of top face
ZL_btm = -0.5*(L_dp+L_undp)-L_angle; %the coordinate component of the left-most point of bottom face
ZR_btm = 0.5*(L_dp+L_undp); %the coordinate component of the right-most point of bottom face
Zstep = 120; %the number of divisions along the length of the top and bottom surfaces
Estep = 50; %the number of divisions along the rotated x-direction of the ent and ext surfaces
Wstep = 65; %the number of division along the width direction (currently for ent and ext surfaces)

%---------------- CREATE A FILE WITH DATA THAT CAN BE PASSED TO ASAP ---------

fid=fopen('Dimensions.TXT','wt');
fprintf(fid,'%g
',t);
fprintf(fid,'%g
',wid);
fprintf(fid,'%g
',L_rect+L_undp);
fprintf(fid,'%g
',L_angle);
fclose(fid);
\end{verbatim}
x = u_data(:,1);  
y = u_data(:,2);  
z = u_data(:,3);  
u = u_data(:,4);  
v = v_data(:,4);  
w = w_data(:,4);  
clear u_data v_data w_data;

x = [x;x]; %x=[x;-x]; % the second column represents a reflection of data along the length
y = [y;-y]; %y=[y;y]; % the first column represents a reflection of data along the width
z = [z;z]; %z=[z;-z]; % this leaves us with the data for all slab surfaces
u = [u;u]; %u=[u;-u]; %if the data coming in is for half the slab, the second column can be commented out
v = [v;-v]; %v=[v;v];
w = [w;-w];

u_data = [x y z u];
v_data = [x y z v];
w_data = [x y z w];

% [ystep, zstep] = steps(u_data); % This tries to discover the number of points in the y-direction and the number of points in the z-direction.

index = size(u_data);
%----------------select the data and all displacements for the data   %rowt stands for row top
s = [t/2 -t/2 wid/2 0];
c = [1 1 2 3];
for j = 1:4
  rows = 1;
  for row = 1:index(1)
    s(4) = -((L_rect/2) - (t/2) - u_data(row,1))/tan(ang1)); % distance from the center of the slab to the end surface
    tolerance(row,j) = abs(u_data(row,c(j)) - s(j));
    if tolerance(row,j) < 5*10^-5
      surf(rows,1:4) = u_data(row,1:4);
      surf(rows,5) = v_data(row,4);
      surf(rows,6) = w_data(row,4);
      rows = rows + 1;
    end
  end
  dsurf(:,1) = surf(:,1) + surf(:,4);
  dsurf(:,2) = surf(:,2) + surf(:,5);
  dsurf(:,3) = surf(:,3) + surf(:,6);
  if j == 1
    top = dsurf;
  end
end
if j==2
    bot=dsurf;
end
if j==3
    edg=dsurf;
end
if j==4
    ent=dsurf;
end
clear rows surf dsurf
end

%--------TOP SURFACE-----TOP SURFACE-----TOP SURFACE-----TOP SURFACE-----
Xi=top(:,1); index=size(Xi);
for m=1:index(1)
    %brings the data down to the z-y plane
    Xi(m,1)=(Xi(m,1)-(t/2));
end
Yi=top(:,2);
Zi=top(:,3);

%X1=Y1.*0;                                   %initialization
%X1=GRIDDATA(Xi,Yi,Zi,Xi,Y1,Z1,'cubic');        %the format of X1 is Y1 by Z1
X1=mycubicinterp(Xi, Yi, Zi, Y1, Z1);
figure('NumberTitle','off','Name','top surface mesh');
mesh(Y1,Z1,X1)
axis tight; hold on
plot3(Yi,Zi,Xi,'.')%,'MarkerSize',15) %nonuniform

index=size(X1);
fid=fopen('TOP_X.TXT','wt');
fprintf(fid,'DISPLAY -9\n');                %file header
fprintf(fid,'Z 0 SPOTS X ');
fprintf(fid,'%g ', ZL_top);
fprintf(fid,'%g ', ZR_top);
fprintf(fid,'%g', index(1));
fprintf(fid,' Y');
fprintf(fid, ' %g ', -wid/2);
fprintf(fid, ' %g ', wid/2);
fprintf(fid,'\n', index(2));
fprintf(fid,'%FAST ');
fprintf(fid,'%g', index(2));
fprintf(fid,' %g', index(1));

for jj=1:index(1)
    %number of rows
    fprintf(fid,'\n');
    for kk=1:index(2)
        fprintf(fid,'%13.9e ',(X1(jj,kk))); %increase accuracy by removing
        %common factor

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end
end
fclose(fid);

%----ENTRANCE-----ENTRANCE-------ENTRANCE------ENTRANCE------ENTRANCE-----
clear Xi Yi Zi X1 Y1 Z1 m index
Xi=ent(:,1); index=size(Xi);
Yi=ent(:,2);
Zi=ent(:,3);

gangle = (pi/180)*((180-30.61)/2);  %reconstituting angle
for m=1:index(1)            %brings the data over to the x-y plane
    Zi(m,1)=(Zi(m,1)+L/2);
end
rotation2=[cos(ang1),0,-sin(ang1);  0,1,0;  sin(ang1),0,cos(ang1)];
for n=1:index(1)   %rotates the data
    ent(n,1:3) = rotation2*[Xi(n,1);Yi(n,1);Zi(n,1)];
end

[X1, Y1, Z1]= entrancefit(ent);   %entrancefit is a function I(Jed) wrote to
fit the data

index=size(X1);
fid=fopen('BACK_Z.TXT','wt');
fprintf(fid,'DISPLAY -9
'); %file header
fprintf(fid,'%g ', -wid/2);  %the center of the end face is first put at 0 in
my ASAP simulation codes
fprintf(fid,'%g ', wid/2);
for jj=1:index(1)       %
    fprintf(fid,'%6.3e ',(X1(jj,kk)));
end
fclose(fid);
clear X1 Y1 Z1 Xi Yi Zi m index; %release memory
Xi=edg(:,1); index=size(Xi);
Yi=edg(:,2);
Zi=edg(:,3);

for m=1:index(1)
    Yi(m,1)=Yi(m,1)-wid/2;
end

Ui=Xi; %Make a copy
Vi=Yi; %Make a copy
Wi=Zi;

Xi=Vi; %Rotate the surface
Yi=-Ui; %Rotate the surface

clear Vi Wi Ui
[Y1,Z1]=MESHGRID(-(t/2):t/Estep:(t/2),ZL_btm:((ZR_top-ZL_btm)/Zstep):ZR_top); %watch memory! X1=Y1.*0;
X1=mycubicinterp(Xi, Yi, Zi, Y1, Z1);
figure('NumberTitle','off','Name','edge surface mesh');
mesh(Y1,Z1,X1)
axis tight; hold on
plot3(Yi,Zi,Xi,'.')U1=X1;W1=Z1;
X1=-W1;
Z1=U1;
index=size(Z1)
clear U1 W1
%
index=size(X1);
fid=fopen('EDG_Y.TXT','wt');
fprintf(fid,'DISPLAY -9\n'); %file header
fprintf(fid,'Z 0 SPOTS X ');
fprintf(fid,'%g ', ZL_btm);
fprintf(fid,'%g ', ZR_top);
fprintf(fid,'%g', index(1));
fprintf(fid,' Y');
fprintf(fid,' %g ', -t/2);
fprintf(fid,' %g ', t/2);
fprintf(fid,'%g\n', index(2));
fprintf(fid,'$FAST ')fprintf(fid,'%g', index(2));
fprintf(fid,'%g', index(1));

for jj=1:index(1) %number of rows
    fprintf(fid,'\n');
    for kk=1:index(2)
        fprintf(fid,'%13.9e ',(Z1(jj,kk))); %increase accuracy by removing common factor
    end

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function [X1]=mycubicinterp(Xi, Yi, Zi, Y1, Z1)
Xm=Xi;
Ym=Yi;
Zm=Zi;
ordery=5;
orderz=11;

indexi=size(Yi);
indexm=size(Ym);
index1=size(Y1);
counter=1;
for j=1:index1(1)
    for i=1:index1(2)
        toly=.00005;
        tolz=.00005;
        numpoints=0;
        while numpoints<(ordery^2)
            toly=toly+.00005;
            tolz=tolz+.00005;
            counter=1;
            for k=1:indexm(1)
                if (Ym(k,1)>=(Y1(j,i)-toly)) & (Ym(k,1)<=(Y1(j,i)+toly))
                    if (Zm(k,1)>=(Z1(j,i)-tolz)) & (Zm(k,1)<=Z1(j,i)+tolz))
                        relevantdata(counter)=k;
                        counter=counter+1;
                    end
            end
        end
    end
end
end
A=exist('relevantdata','var');
if A==1
rdindex=size(relevantdata);
numpoints=rdindex(2);
if numpoints<ordery
    clear relevantdata k
end
end

%%
x=Xm(relevantdata,1);
y=Ym(relevantdata,1);
z=Zm(relevantdata,1);
index=size(y);
for jj=1:ordery
    ys(1:index(1),jj)=y.^jj;
end
for ii=1:orderz
    zs(1:index(1),ii)=z.^ii;
end
clear jj ii

col=1;
for jj=1:ordery
    for ii=1:orderz
        crstrms(1:index(1),col)=ys(1:index(1),jj).*zs(1:index(1),ii);
        col=col+1;
    end
end
clear jj ii col

D=[ones(size(y)) ys zs crstrms ]; % cwavtrms swavtrms cwavtrmsb swavtrmsb]; % make a matrix with (1 x^2 x y^2)
a=D\x; % solve for the constants a1, a2, a3, and a4 from an
equation that looks like this:
% [1 x(1)^2 x(1) y(1)^2][a(1)]  [opl(1)]
% [1 x(2)^2 x(2) y(2)^2][a(2)]= [opl(2)]
% [1 ... . ] [a(3)]  [opl(3)]
% [. . ] [a(4)]  [opl(4)]
% [. . ] [ . ]
% [. . ] [ . ]
for jj=1:ordery
    ysp(jj)=Y1(j,i).^jj;
end
for ii=1:orderz
    zsp(ii)=Z1(j,i).^ii;
end
clear jj ii

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col=1;
for jj=1:ordery
    for ii=1:orderz
        crstrmsp(col)=ysp(jj).*zsp(ii);
        col=col+1;
    end
end

Dfit=[1 ysp zsp crstrmsp];
fit=Dfit*a;
X1(j,i)=fit;
clear D MaxErr Z a crstrms index zs ys zsp ysp z x y relevantdata counter k A;
end
end

%unwrap the data and generate new data file that includes original data.

MATLAB code for interpolating slab entrance or exit surface data

function [X1, Y1, Z1]= entrancefit(ent)
global L_rect L_angle wid t ang1
%------------------FITTING---------------------------------------------
x=ent(:,1);  %the x-coordinate data
y=ent(:,2);  %the y-coordinate data
z=ent(:,3); %the z-coordinate data
index=size(y);
ordery=4;
orderz=4;
% wavs=order^2;
% for m=1:index(1)
%     x(m,1)=(x(m,1)-.001);
% end

for j=1:ordery
    ysp(1:index(1),j)=y.^j;
end

for i=1:orderz
    zsp(1:index(1),i)=z.^i;
end
clear j i
col=1;
for j=1:ordery
    for i=1:orderz
        crstrms(1:index(1),col)=ys(1:index(1),j).*zs(1:index(1),i);
        col=col+1;
    end
end
clear j i col

D=[ones(size(y)) ys zs crstrms ]; %cwavtrms swavtrms cwavtrmsb swavtrmsb]; %
make a matrix with (1 x^2 x y^2)
a=D\x;  %solve for the constants a1, a2, a3, and a4 from an equation that
looks like this:
    %   [1 x(1)^2 x(1) y(1)^2][a(1)]  [opl(1)]
    %   [1 x(2)^2 x(2) y(2)^2][a(2)]  [opl(2)]
    %   [1 ... . ][a(3)]  [opl(3)]
    %   [. . ][a(4)]  [opl(4)]
    % [. . ][ . ]
    % [. . ][ . ]
X= D*a;  %calculate the opl values with the given constants
clear D
MaxErr = max(abs(X - x)); %find the error?

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  plotting %%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[Y1,Z1]=MESHGRID(-wid/2:wid/80:wid/2,-
    (t/2)/sin(ang1):(t/sin(ang1))/40:(t/2)/sin(ang1));%watch memory!% change
range for different thicknessOPD=Y1.*0; %initialization
X1=GRIDDATA(y,z,X,Y1,Z1,'v4');%the format of X1 is Y1 by Z1
figure('NumberTitle','off','Name','entrance surface mesh');
mesh(Y1,Z1,X1)
axis tight; hold on
plot3(y,z,x,'.')%,'MarkerSize',15) %nonuniform
%release memory
clear x y OPD3;

ASAP code for tracing rays through a slab

SYSTEM NEW !!Initializes ASAP at the start of execution.  Reinitializes the
data storage.
RESET     !!Reinitializes all control settings to those at program startup.
Also resets all ray data with the equivalent of the RAYS 0 command.
&TIC      !!Starts timer
$CASE BOTH  !! Instructs the parser to recognize both upper and lower case
letters.
-- Define System Units and Wavelengths --

UNITS

METERS 'WATTS' ! Units of length and flux
WAVELENGTH 1030 NM ! Value of wavelength in specified units
SF=0!!!5E5 ! Scale Factor

READ Dimensions.TXT ! Contains the values of thickness, T, width, W, Length

    part, LA

SDA=1.83 ! Source and Detector Angle
D_BOT=(T/2)*(COS((SDA))+SIN((SDA))) ! FOR the CASE When T/2 is the
distance away from

    !! the end of the slab that the source

is placed

    !!, D_bot is the length of source

below the z-y

    !! plane
D_FULL=(T)*(SIN[30.61-(SDA)]/SIN[30.61]) ! The Full length of the source.
ZPOS=-(LR/2+(LA)+(T)/2) ! Z-Dimension POSITION of source
DXB=-(D_BOT) ! Source X-dimension Bottom
DXT=(D_FULL)-(D_BOT) ! Source X-dimension Top

-- Define Coatings --

COATING PROPERTIES

0 1 'TRANSMIT' ! TRANSMIT coating is 100% transmissive
1 0 'REFLECT' ! REFLECT coating is 100% reflective
0 0 'ABSORB' ! ABSORB coating is 100% absorptive

-- Define Media --

MEDIA 1.82 'YAG' ! Index of Refraction and name of media

-- Create Top Surface --

READ TOP_X.TXT ! Start reading future records from the beginning

    of the given

    file "name" (with default extension .INR). The

    file Top_X.TXT

    contains ASAP code: DISPLAY -9 (Reads

    BRO009.DAT File)

    !! Z 0 SPOTS X -0.04 0.04 33.3

    !! (Sets the plane (z=0), step

    size and max

    !! and min values for the

    other two dimensions)

    !! $FAST 33 41 (Performs fast

    reading of numeric arrays.)

RETURN ! Returns to ASAP top command level (that is, the

    ASAP> prompt).

! Alternatively, RETURN can be used to terminate
the graphics mode

! (that is, OVERLAY) and return you to text
command level.
DISPLAY  !!Reads previously created distribution data file into ASAP where
!!it may be modified and/or examined by DISPLAY subcommands.
ISOMETRIC 'TOP face1'  !!Creates an isometric view of the current distribution data file.
!!Title of plot is in single quotes.
MODIFY 1 66 1 121 0 (SF)  !!MODIFY [ m m' n n' [ a b ] ] [ 'flabel' ].
!!Modifies the data
!!region specified by the two integer pixel ranges, m to m' (across)
!!and n to n' (down). The data in that region is replaced by a a
!!plus b times the data value, that is:
!!f'(i,j)=a+b(f(i,j)) i=m,m' j=n,n'
WRITE TOP_X DIS  !!Writes the current data to the binary file
!!TOP_X.dis
RETURN  !!Returns to ASAP top command level (that is, the ASAP> prompt).
!!Alternatively, RETURN can be used to terminate the graphics mode
SURFACE  !!Signals ASAP that surface definition commands follow
SAMPLED TOP_X  !!Creates an explicit surface interpolated from sampled data
!!defined by reading deformation samples found in the binary
!!distribution file name.dis (default BRO009.DAT) external file.
OBJECT 'SLAB_TOP1'  !!Defines an object (top surface) based upon formerly defined entities.
INTERFACE 1 0 YAG AIR  !!Defines a 100% reflective surface at the YAG/AIR boundary
ROTATE Y 90  !!Rotates object -90 degrees about the y-axis
SHIFT X (T)/2  !!Shifts the entity in the x-direction by specified amount
SURFACE
SAMPLED TOP X
OBJECT 'SLAB_BOTTOM1'
INTERFACE 1 0 YAG AIR
ROTATE Y -90
ROTATE x 180
SHIFT X -(T)/2

!!-----------------Create Rear (Entrance) End Surface------------------!!

$READ BACK_Z.TXT
RETURN
DISPLAY
ISOMETRIC 'ENTRANCE face1'
MODIFY 1 41 1 81 0 (0)
WRITE BACK_Z DIS
RETURN

SURFACE
  SAMPLED BACK Z
  OBJECT 'SLAB_BACK1' !!RIGHT ENTRANCE FACE
  INTERFACE 0 1 AIR YAG
  ROTATE Z 90
  ROTATE Y 120.61
  SHIFT Z -((LR)/2+(LA)/2)

SURFACE
  SAMPLED BACK_Z
  OBJECT 'SLAB_BACK2' !!RIGHT ENTRANCE FACE
  INTERFACE 0 1 AIR YAG
  ROTATE Z 90
  ROTATE Y -59.39
  SHIFT Z ((LR)/2+(LA)/2)

!!-----------------Create edge Surface------------------!!
$READ EDG_Y.TXT
RETURN
DISPLAY
  ISOMETRIC 'EDGE1 face1'
MODIFY 1 51 1 121 0 -(0)
WRITE EDG_Y DIS
RETURN

SURFACE
  SAMPLED EDG_Y
  OBJECT 'SLAB_EDG1' !!EDGE
  INTERFACE 0 1 AIR YAG
  ROTATE Y -90
  ROTATE Z 90
  SHIFT Y ((W)/2)
  BOUNDS +3 +4

$READ EDG_Y.TXT
RETURN
DISPLAY
  ISOMETRIC 'EDGE2 face1'
MODIFY 1 51 1 121 0 (0)
WRITE EDG2_Y DIS
RETURN

SURFACE
  SAMPLED EDG2_Y
  OBJECT 'SLAB_EDG2' !!EDGE
  INTERFACE 0 1 AIR YAG
  ROTATE Y 90
  ROTATE Z -90
  SHIFT Y -((W)/2) BOUNDS +3 +4
!!-------------------------------------DETECTORS---------------------------------!!
SURFACE
   PLANE Z -(ZPOS) RECT (D_BOT) (W)/2  !!Generates a plane at 
z=L+1E-3 perp. to the z-axis  !!and bound by a rectangle
with the given dimensions
   OBJECT 'DETECTOR01'  !!Makes the plane into an
object
   INTERFACE 0 0 AIR AIR  !!Gives the detector plane
air/air interface
   ROTATE Y 1.83 -(ZPOS) 0
   PLOT FACETS  !!Plots parametric mesh
representation of objects in the
   Graphics window.

!!--------------------------------CREATE SOURCES---------------------------------!!
GRID RECT Z (ZPOS) (DXB) (DXT) -(W)/2 (W)/2 95 145  !!Creates rectangular
source @ Z=-(L+1E-3)
   SOURCE DIR 0 0 1  !!grid of rays
   FLUX TOTAL 63  !!Rays go in +Z
   ROTATE Y (SDA) (ZPOS) 0
   !-$GO
.Skip5
!-$GO
.Skip5
!!3D view
!-$GO
.Skip5
!!WINDOW X Z

!!--------------------------------ANALYSIS---------------------------------!!
CONSIDER ONLY DETECTOR01  !! Considers the detector for
STATS  !! the stats command.

!!--------------------------------Output Data---------------------------------!!
$ECHO NONE
$IO OUTPUT OPL3_ZIG ONLY FILE(11)
   LIST POSITION
$IO OUTPUT CLOSE
$ECHO

!!3D view
!!WINDOW X Z
MATLAB Code for plotting optical path length difference

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Thermal lens plotting
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% part I: load ASAP data for OPL due to end effect

%Firstly, from data file to load data: dataOPL3
%the format of the loaded data: (Ray X Y Z Flux Object OPL)
% x=dataOPL3(:,1); % 2 for ASAP output file
% y=dataOPL3(:,2);% 3 for ASAP output file
% OPL3=dataOPL3(:,3);% 7 for ASAP output file
% x=dataOPL3(:,2); % 2 for ASAP output file
% y=dataOPL3(:,3);% 3 for ASAP output file.
% OPL3=dataOPL3(:,7);% 7 for ASAP output file.
%clear u_data v_data w_data;%release memory

indxopl=size(OPL3);
row=1;
% for j=1:indxopl(1)
% if  OPL3(j,1)>0.1975 | OPL3(j,1)<0.195
%    deleter(row)=j;
%    row=row+1
% end
% end
% x(deleter)=[];
% y(deleter)=[];
% OPL3(deleter)=[];

ymin=min(y);
ymax=max(y);
ystep=((ymax-ymin)/200);

xmin=min(x);
xmax=max(x);
```
[X1,Y1]=MESHGRID(xmin:((xmax-xmin)/100):xmax,ymin:((ymax-ymin)/200):ymax); % watch memory! OPL= Y1.*0; % initialization
OPL=GRIDDATA(x,y,OPL3,X1,Y1,'nearest') % the format of OPL is X1 by Y1

to remove the dip feature at y=0
% % Num_c=1+floor(abs(0-ymin)/ystep)
% % rows_to_fix=2*2; % number of rows to be fix, try from 0, 1...% % for
kk=0:0.5*rows_to_fix
% % OPL(Num_c-kk,:)=OPL(Num_c-0.5*rows_to_fix,:);
% % OPL(Num_c+kk,:)=OPL(Num_c-0.5*rows_to_fix,:);
% % end
% % % indexmeshopl=size(OPL)
% % % for ii=1:indexmeshopl(1)
% % % for jj=1:indexmeshopl(2)
% % % OPL(ii,jj)=OPL(ii,jj)-0.19571179;
% % % end
% % % end

figure(1);
mesh(X1,Y1,OPL)
axis tight; hold on
plot3(x,y,OPL3,'.','color','red') %,'MarkerSize',15) % nonuniform%
% release memory
figure(2);
mesh(X1,Y1,OPL)
clear ystep ymin ymax Num_c rows_to_fix kk xmin xmax xstep % x y OPL3;
MATLAB code for modeling depolarization loss for zigzag propagation through a ceramic crystal slab

function[] = ceramicZXXZ3(tip_angle, prop_angle, slab_t, slab_recL, wavelength, no, step,...
   L_triangle, slab_Flength, slab_Clength, k, del_Lx, del_Lz, ii_end, Bo, deltaz, x,...
   sigma11, sigma12, sigma13, sigma22, sigma23, sigma33)
Iout=0;

prompt={'Standard deviation in mm:';'Central grain size in mm:'};
name='Grain size & distribution';
umlines=1;
defaultanswer={'5e-3';'20e-3'};
answer=inputdlg(prompt,name,numlines,defaultanswer);
mu= str2num( answer{2,1});
sigma= str2num( answer{1,1});

for ii=1:step(1)    %%%% start point of ray goes in (assuming ii=1 is the bottom of the slab)
   for jj=1:step(2)
      %jj=step(2);
      E1=[1 ; 0]; %%Iin_node=abs(E1(1,1))^2;
      kk0=ii*del_Lx/(del_Lz*tan(tip_angle));  %the number of steps in the z-direction that correspond to ii on the ray path
      entH=ii*del_Lx;
      slabsurface(1)=entH/sin(prop_angle);%from the entrance to the first slab surface
      nextslabsurface=slab_t/sin(prop_angle);%distance to next slab surface.
      numbounces=round(tan(prop_angle)*slab_Clength/slab_t);%how many bounces
      nextslabsurface=slab_Clength/(numbounces*cos(prop_angle));
      for dv=2:numbounces
         slabsurface(dv)=slabsurface(dv-1)+nextslabsurface;
      end
      slabsurface(numbounces+1)=slabsurface(numbounces)+(nextslabsurface-slabsurface(1));

%%%starting values
bbstp=0.06*10^-3 ;  %baby step in millimeters
for i=1:step(1)
   for j=1:step(2)
      d=-1;
      while (d<0) | (d>2*mu)
         d=mu+sqrt(sigma)*randn(1);
      end
      gs=d;  %grain size
      dl=0;   %total path length
      gsc=0;  %initialize grain size counter
      reflect=1;DC=0;
endzag = length(slabsurface);
po = redpiezoCeramic;

iizzold = ceil(dL*sin(prop_angle)/del_Lx + .00001);  % ii steps defined by the
distance travelled.
kkkold = ceil(dL*cos(prop_angle)/del_Lz + kk0);  % true z-steps from ent. tip

iiold = round(tooth(((66-ii)+iizzold-1)));  % the absolute x step position

while (dL < slabsurface(endzag))
    dL = dL + bbsstp;
gsc = gsc + bbsstp;
    iizz = ceil(dL*sin(prop_angle)/del_Lx + .0000001);  % ii steps defined by the
distance travelled.
kkk = ceil(dL*cos(prop_angle)/del_Lz + kk0);  % true z-steps from ent. tip

    iii = round(tooth(((66-ii)+iizz-1)));  % the absolute x step position  66 is
    2*step(1). if step(1) does not equal 32 then modify tooth.m

    DC = DC + bbsstp;
    if gsc >= gsc | iizz > iizzold | kkk > kkkold | dL >= slabsurface(reflect)
         B = po * [sigma11(iiold,jj,kkkold); sigma22(iiold,jj,kkkold);
            sigma33(iiold,jj,kkkold); sigma23(iiold,jj,kkkold);
            sigma13(iiold,jj,kkkold); sigma12(iiold,jj,kkkold)] + ...
            [Bo; Bo; Bo; 0; 0; 0];  % make the indices correct
         zigorzag = (-1)^reflect;  % create a number that will tell me if I'm zigging
            or zagging
            if zigorzag == -1  % zig down
                BB11 = B(1,1)*(cos(prop_angle))^2 + B(5,1)*sin(2*prop_angle) + B(3,1)*sin(prop_angle))^2;
                BB22 = B(2,1);
                BB12 = B(6,1)*cos(prop_angle) + B(4,1)*sin(prop_angle);
            end
            if zigorzag == 1  % zag up
                BB11 = B(1,1)*(cos(prop_angle))^2 - B(5,1)*sin(2*prop_angle) + B(3,1)*sin(prop_angle))^2;
                BB22 = B(2,1);
                BB12 = B(6,1)*cos(prop_angle) - B(4,1)*sin(prop_angle);
            end

    BBplus = 0.5*(BB11 + BB22) + 0.5*sqrt((BB11 - BB22)^2 + 4*(BB12)^2);
    BBminus = 0.5*(BB11 + BB22) - 0.5*sqrt((BB11 - BB22)^2 + 4*(BB12)^2);
    nzplus = 1/sqrt(BBplus);
    nzminus = 1/sqrt(BBminus);

    deltan = (nzplus - nzminus);
    if (BB11 == BB22)
        theta = 0;
    else
        theta = ((1/2)*atan(2*BB12/(BB11 - BB22)));
    end

    beta = theta;
phase=k*deltan*DC; % DC is Distance since last Change
a=[cos(beta),-sin(beta);sin(beta),cos(beta)];
b=[exp(-i*phase/2),0;0,exp(i*phase/2)];
E1=(inv(a)*b*a)*E1; DC=0;
if gsc>=gs
    gsc=0;
d=-1;
    while (d<=0) | (d>=2*mu)
        d=mu+sqrt(sigma)*randn(1);
    end
    gs=d; % grain size
po=redpiezoCeramic;
end
if dL>=slabsurface(reflect)
    reflect=reflect+1;
end
if iizz>iizzold
    iizzold=iizz;
    iiiiold=iii;
end
if kkk>kkkold
    kkkold=kkk;
    % howcls=dL*cos(prop_angle)/del_Lz+kk0;
end
if kkkold>step(3), break, end
end
end

end

Iout_node(ii,jj) = abs(E1(1,1))^2;
Iout=Iout+Iout_node(ii,jj);
end

mu sigma
figure(1);
mesh(Iout_node)
% dd=15:0.5:25; plot(dd, Pzigzag_loss);
% hold on; hold off;
MATLAB code for modeling depolarization loss for straight through propagation
in a ceramic slab

% function[]=ceramic2XXZ3(tip_angle, prop_angle, slab_t, slab_recL,
wavelength, no, step,...
% L_triangle, slab_Flength, slab_Clength, k, del_Lx, del_Lz, ii_end, Bo,
deltaz, x,...
% sigma11, sigma12, sigma13, sigma22, sigma23, sigma33)
tic

% tip angle of Brewster end
tip_angle= 30.6*pi/180;

% ray propagation angle, the angle between ray path and top or bottom surface
prop_angle= 30.6*pi/180;

% full thickness of the slab
slab_t= 1;

% full length of rectangular region of the slab
slab_recL= 89.6;

% lasing wavelength
wavelength= 1.030e-3;

% refractive index
no= 1.82;

% obtain the number of slices along thickness, half width and full length. step(1)=32, step(2)=64, step(3)=798
step= size(sigma11);

% bottom length of Brewster triangle
L_triangle= slab_t/tan(tip_angle);

% full length of the slab, the length from left tip to right tip along length of the slab
slab_Flength= slab_recL+2*L_triangle;

% length of top or bottom slab surface
slab_Clength= slab_recL+L_triangle;

% length along thickness of the slab
k= 2*pi/wavelength;

% step length along thickness of the slab
del_Lx= slab_t/(step(1));

% step length along full length of the slab
del_Lz= slab_Flength/(step(3));

% total number of steps along the thickness direction for a passing ray rounded up
ii_end= ceil(tan(prop_angle)*slab_Clength/del_Lx);

% 1/no^2
Bo= 1/no^2;
deltaz = del_Lx / sin(prop_angle); \% length the ray travels in an x-step. (confusingly, this is along the direction of propagation and NOT along the z-direction necessarily)

x = 1;

Iout = 0;

prompt = {'Standard deviation in mm:'; 'Central grain size in mm:'};
name = 'Grain size & distribution';
numlines = 1;
defaultanswer = {'5e-3'; '20e-3'};
answer = inputdlg(prompt, name, numlines, defaultanswer);
mu = str2num(answer{2, 1});
sigma = str2num(answer{1, 1});

for ii = 1:step(1) \% start point of ray goes in (assuming ii=1 is the bottom of the slab)
    \% ii=step(1)/2
    for jj = 1:step(2)
        \% jj=step(2)
        E1 = [1; 0]; \% Iin_node = abs(E1(1,1))^2;
        kk0 = ii * del_Lx / (del_Lz * tan(tip_angle)); \% the number of steps in the z-direction that correspond to ii on the ray path
        \%--create a column vector with data representing path distance travelled
        \%--at the time of each reflection
        \%--starting values
        bbstp = 0.6 * 10^-3; \% baby step in millimeters
        d = -1;
        while (d <= 0) | (d >= 2 * mu)
            d = mu + sqrt(sigma) * randn(1);
        end
        gs = d; \% grain size
        dL = 0; \% total path length
        gsc = 0; \% initialize grain size counter
        reflect = 1;
        DC = 0;
        endthrough = slab_Clength;
        po = redpiezoCeramic;

        kkkold = ceil(dL / del_Lz + kk0); \% true z-steps from ent. tip
        iiiold = ii; \% the absolute x step position
        \%
        while (dL < endthrough)
            dL = dL + bbstp;
            gsc = gsc + bbstp;
            kkk = ceil(dL / del_Lz + kk0); \% true z-steps from ent. tip
iii=ii; % the absolute x step position 66 is 2*step(1). if step(1) does not equal 32 then modify tooth.m
DC=DC+bbstp;
if gsc>=gs|kkk>kkkold|dL=endthrough
    B=po* [sigma11(ii,jj,kkkold); sigma22(ii,jj,kkkold);
sigma33(ii,jj,kkkold); sigma23(ii,jj,kkkold); sigma13(ii,jj,kkkold);
sigma12(ii,jj,kkkold)]+...  
    [Bo; Bo; Bo; 0; 0; 0];  % make the indices correct
end

BB11=B(1,1);
BB22=B(2,1);
BB12=B(6,1);

BBpluse= 0.5*(BB11 + BB22) + 0.5*sqrt((BB11-BB22)^2 + 4*(BB12)^2);
BBminus= 0.5*(BB11 + BB22) - 0.5*sqrt((BB11-BB22)^2 + 4*(BB12)^2);
nzpluse=1/sqrt(BBpluse);
nzminus=1/sqrt(BBminus);

deltan=(nzpluse-nzminus);
if (BB11==BB22)
    theta=0;
else
    theta=((1/2)*atan(2*BB12/(BB11-BB22)));  
end

beta=theta;
phase=k*deltan*DC;  % DC is Distance since last Change
a=[cos(beta),-sin(beta); sin(beta),cos(beta)];
b=[exp(-i*phase/2),0; 0, exp(i*phase/2)];
E1=(inv(a)*b*a)*E1;
DC=0;
if gsc>=gs
    gsc=0;
d=-1;
    while (d<=0) | (d>=2*mu)
        d=mu+sqrt(sigma)*randn(1);
    end
    gs=d;  % grain size
    po=redpiezoCeramic;
end
if kkk>kkkold
    kkkold=kkk;
    % howcls=dL*cos(prop_angle)/del_Lz+kk0;
end
% if kkkold>step(3), break, end
end
MATLAB code for modeling depolarization loss for straight through propagation through a single crystal slab

```matlab
function []=ceramicZXXZ3(tip_angle, prop_angle, slab_t, slab_recL, wavelength, no, step,...
    L_triangle, slab_Flength, slab_Clength, k, del_Lx, del_Lz, ii_end, Bo, delta_Lx, x,...
    sig1, sig2, sig3, sig4, sig5, sig6, sig7, sig8)
tic

    tip_angle= 30.6*pi/180;%% tip angle of Brewster end
    prop_angle= 30.6*pi/180;%% ray propagation angle, the angle between ray path and top or bottom surface
    slab_t= 1;% full thickness of the slab
    slab_recL= 89.6;% full length of rectangular region of the slab
    wavelength= 1.030e-3;% lasing wavelength
    no= 1.82;% refractive index
    step= size(sig1);%% obtain the number of slices along thickness, half width and full length. step(1)=32, step(2)=64, step(3)=798
    
    % function
    %    Iout_node(ii,jj) = abs(E1(1,1))^2;
    %    Iout=Iout+Iout_node(ii,jj);
    % end % match jj
    % end %ii

    Pzigzag_loss(x)=(1-Iout/(step(1)*step(2)))*100
    % Pzigzag_loss(x)=(1-Iout)*100
    % Pzigzag_loss(x)=(1-Iout/(step(2)))*100
    % x=x+1;
    % end % match dd
    %figure(1); plot(Iout_node(16,:))
    mu
    sigma
    figure(1);
    plot(Iout_node)
    mesh(Iout_node)
    % dd=15:0.5:25 ; plot(dd, Pzigzag_loss);
    % hold on; hold off;
toc
```
L_triangle = \frac{slab_t}{\tan(tip\_angle)}; \% \text{ bottom length of Brewster triangle}

slab_Flength = slab_recL + 2*L_triangle; \% \text{ full length of the slab, the length from left tip to right tip along length of the slab}

slab_Clength = slab_recL + L_triangle; \% \text{ length of top or bottom slab surface}

k = \frac{2\pi}{\text{wavelength}};

del_Lx = \frac{slab_t}{\text{step}(1)}; \% \text{ step length along thickness of the slab}

del_Lz = \frac{slab_Flength}{\text{step}(3)}; \% \text{ step length along full length of the slab}

ii_end = \frac{\text{ceil}(\tan(prop\_angle) \times \text{slab\_Clength} / \text{del}_Lx)}{\% \text{total number of steps along the thickness direction for a passing ray rounded up}

Bo = \frac{1}{\text{no}^2};

deltaz = \frac{\text{del}_Lx}{\sin(prop\_angle)}; \% \text{ length the ray travels in an x-step. (confusingly, this is along the direction of propagation and NOT along the z-direction necessarily)}

x = 1;

\text{for cgl=0:120}
\%cgl=60;
Cangle = cgl*\pi/180;
Iout = 0;

\text{for ii=1:step(1)} \% \% \% \text{ start point of ray goes in (assuming ii=1 is the bottom of the slab)}
\% ii=1
\text{for jj=1:step(2)} \% jj=1
\text{El} = [1; 0]; \% \text{In\_node} = \text{abs}((\text{El}(1,1))^2);
\text{kk0} = ii \times \text{del}_Lx / (\text{del}_Lz \times \tan(tip\_angle)); \% \text{the number of steps in the z-direction that correspond to ii on the ray path}

\% \% \% \text{--create a column vector with data representing path distance travelled at the time of each reflection} \% \% \% \% \text{starting values}
bbstp = \text{del}_Lz / 2; \% \text{baby step in millimeters}
gs = \text{slab\_Clength}; \% \text{grain size}
dL = 0; \% \text{total path length}
gsc=0;       %initialize grain size counter
DC=0;
endthrough=slab_Clength;
po=redpiezoSingle(Cangle);

kkkold=ceil(dL/del_Lz+kk0);  %true z-steps from ent. tip
iiiold=ii; %the absolute x step position

% while (dL<endthrough)
dL=dL+bbstp;
gsc=gsc+bbstp;
kkk=ceil(dL/del_Lz+kk0);  %true z-steps from ent. tip
iii=ii; %the absolute x step position

DC=DC+bbstp;
if gsc>=gs|kkk>kkkold|dL>=endthrough
   B=po* [sigma11(ii,jj,kkkold); sigma22(ii,jj,kkkold);
          sigma33(ii,jj,kkkold); sigma23(ii,jj,kkkold);
          sigma13(ii,jj,kkkold)];
   [Bo; Bo; Bo; 0; 0; 0];  %make the indices correct
      BB11=B(1,1);
      BB22=B(2,1);
      BB12=B(6,1);

      BBpluse= 0.5*(BB11 + BB22) + 0.5*sqrt((BB11-BB22)^2 + 4*(BB12)^2);
      BBminus= 0.5*(BB11 + BB22) - 0.5*sqrt((BB11-BB22)^2 + 4*(BB12)^2);
      nzpluse=1/sqrt(BBpluse);
      nzminus=1/sqrt(BBminus);
      deltan=(nzpluse-nzminus);

      if (BB11==BB22)
         theta=0;  %this is because when BB11=BB22, B12 is zero and according
to matlab
            %atan(0)=0, atan(+Inf)=+-1.57 but atan (0/0)=NaN and
            %0/0=NaN and 1/0=Inf and -1/0=-Inf. atan(0/0) occurs when
            %there has been no change in the stress profile from the
            %last calculation
         else
            theta=((1/2)*atan(2*BB12/(BB11-BB22))); %
         end

      beta=theta;
      phase=k*deltan*DC;  %DC is Distance since last Change
      a=[cos(beta),-sin(beta); sin(beta),cos(beta)];
      b=[exp(-i*phase/2),0; 0, exp(i*phase/2)];
      El=(inv(a)*b*a)*El;
DC=0;

if kkk>kkkold
  kkkold=kkk;
  %howcls=dL*cos(prop_angle)/del_Lz+kk0;
end
  %if kkkold>step(3), break, end
end
end

%end

Iout_node(ii,jj) = abs(E1(1,1))^2;
Iout=Iout+Iout_node(ii,jj);

end
end

Pzigzag_loss(x)=(1-Iout/(step(1)*step(2)))*100
  % Pzigzag_loss(x)=(1-Iout)*100
  %Pzigzag_loss(x)=(1-Iout/(step(2)))*100
  x=x+1;

end

% end
% figure(1);  plot(Iout_node(16,:))
% figure(1);
% mesh(Iout_node)
% dd=15:0.5:25 ; plot(dd, Pzigzag_loss);
% hold on;  hold off;
toc

MATLAB Code for modeling depolarization loss for zigzag propagation through a single crystal slab

function[Iout_node, Pzigzag_loss]=singleZXXZ(tip_angle, prop_angle, slab_t, slab_recL, wavelength, no, step, L_triangle, slab_Flength, slab_Clength, k, del_Lx, del_Lz, ii_end, Bo, deltaz, x, sigma11, sigma12, sigma13, sigma22, sigma23, sigma33)
for cgl=0:120
  Cangle=cgl*pi/180;
  %for Cangle=0:pi/180:2*pi/3
    po=redpiezoSingle(Cangle);
    Iout=0;
  end
for ii=1:step(1)    %%% start point of ray goes in (assuming ii=1 is
the bottom of the slab)

    kk0=ii*del_Lx/(del_Lz*tan(tip_angle));  %the number of steps in the
    z-direction that correspond to ii steps along x for the ray path

    for jj=1:step(2)            E1=[1 ; 0];
        path=0;
        n=1;
        dL=0; %represents the total distance travelled.
        for iii=1:ii_end
            kkk=round(iii*del_Lx/(del_Lz*tan(prop_angle))+kk0);  %true z-
            switch path
                case {0}
                    ii_zig=ii-iii+1; %first zig part of path
                    while dL<n*deltaz
                        %po=redpiezoSingle(Cangle);
                        d=deltaz;     %single crystal has all the same d
                        B=po* [sigm11(ii_zig,jj,kkk);
                        sigma22(ii_zig,jj,kkk); sigma33(ii_zig,jj,kkk); sigma23(ii_zig,jj,kkk);
                        sigma13(ii_zig,jj,kkk); sigma12(ii_zig,jj,kkk)]+...
                        [Bo; Bo; Bo; 0; 0; 0];

                        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% components corresponding to zig
                        propagation %%%
                        BB11_down(ii_zig,jj,kkk)=B(1,1)*(cos(prop_angle))^2+B(5,1)*sin(2*prop_angle)+
                        B(3,1)*(sin(prop_angle))^2;
                        BB22_down(ii_zig,jj,kkk)=B(2,1);
                        BB12_down(ii_zig,jj,kkk)=B(6,1)*cos(prop_angle)+B(4,1)*sin(prop_angle);
                        BBpluse_down(ii_zig,jj,kkk)=
                        0.5*(BB11_down(ii_zig,jj,kkk) + BB22_down(ii_zig,jj,kkk)) +
                        0.5*sqrt((BB11_down(ii_zig,jj,kkk)-BB22_down(ii_zig,jj,kkk))^2 +
                        4*(BB12_down(ii_zig,jj,kkk))^2);
                        BBminus_down(ii_zig,jj,kkk)=
                        0.5*(BB11_down(ii_zig,jj,kkk) + BB22_down(ii_zig,jj,kkk)) -
                        0.5*sqrt((BB11_down(ii_zig,jj,kkk)-BB22_down(ii_zig,jj,kkk))^2 +
                        4*(BB12_down(ii_zig,jj,kkk))^2);

                        nzpluse_down(ii_zig,jj,kkk)=1/sqrt(BBpluse_down(ii_zig,jj,kkk));
                        nzminus_down(ii_zig,jj,kkk)=1/sqrt(BBminus_down(ii_zig,jj,kkk));
                        deltanz_down(ii_zig,jj,kkk)=nzpluse_down(ii_zig,jj,kkk)-
                        nzminus_down(ii_zig,jj,kkk);
if 
(BB11_down(ii_zig,jj,kkk)==BB22_down(ii_zig,jj,kkk))
    theta_down(ii_zig,jj,kkk)=0;
else

theta_down(ii_zig,jj,kkk)=((1/2)*atan(2*BB12_down(ii_zig,jj,kkk)/(BB11_down(ii_zig,jj,kkk)-BB22_down(ii_zig,jj,kkk))));
end

beta=theta_down(ii_zig,jj,kkk);
phase=k*deltan_down(ii_zig,jj,kkk)*d;

a=[cos(beta),-sin(beta); sin(beta),cos(beta)];
b=[exp(-i*phase/2),0; 0, exp(i*phase/2)];
E1=(inv(a)*b*a)*E1;
dL=d+dL;
end

if dL>=n*deltaz  %% n*0.2
    n=n+1;
end

if ii_zig<=1
    ii_start=iii+1; %reset for next upward zig
    path=1; % 1 represents upward zig
end

case {1}
    ii_zag=iii-ii_start+1; %upward part of path

while dL<n*deltaz

    % po=redpiezoSingle(Cangle);
    d=deltaz;
    B=po* [sigma11(ii_zag,jj,kkk); sigma22(ii_zag,jj,kkk); sigma33(ii_zag,jj,kkk); sigma23(ii_zag,jj,kkk); sigma13(ii_zag,jj,kkk); sigma12(ii_zag,jj,kkk)];...
    [Bo; Bo; Bo; 0; 0; 0];

    BB11_up(ii_zag,jj,kkk)=B(1,1)*(cos(prop_angle))^2-
    B(5,1)*sin(2*prop_angle)+B(3,1)*(sin(prop_angle))^2;
    BB22_up(ii_zag,jj,kkk)=B(2,1);
    BB12_up(ii_zag,jj,kkk)=B(6,1)*cos(prop_angle)-
    B(4,1)*sin(prop_angle);
    BBpluse_up(ii_zag,jj,kkk)=
    0.5*(BB11_up(ii_zag,jj,kkk) + BB22_up(ii_zag,jj,kkk)) +
    0.5*sqrt(((BB11_up(ii_zag,jj,kkk)-BB22_up(ii_zag,jj,kkk))^2 +
    4*(BB12_up(ii_zag,jj,kkk))^2);
BBminus_up(ii_zag,jj,kkk)=
0.5*(BB11_up(ii_zag,jj,kkk) + BB22_up(ii_zag,jj,kkk)) -
0.5*sqrt((BB11_up(ii_zag,jj,kkk)-BB22_up(ii_zag,jj,kkk))^2 +
4*(BB12_up(ii_zag,jj,kkk))^2);

nzpluse_up(ii_zag,jj,kkk)=1/sqrt(BBpluse_up(ii_zag,jj,kkk));
nzminus_up(ii_zag,jj,kkk)=1/sqrt(BBminus_up(ii_zag,jj,kkk));
deltan_up(ii_zag,jj,kkk)=nzpluse_up(ii_zag,jj,kkk)-nzminus_up(ii_zag,jj,kkk);

if
(BB11_up(ii_zag,jj,kkk)==BB22_up(ii_zag,jj,kkk))
theta_up(ii_zag,jj,kkk)=0;
else
theta_up(ii_zag,jj,kkk)=((1/2)*atan(2*BB12_up(ii_zag,jj,kkk)/(BB11_up(ii_zag,jj,kkk)-BB22_up(ii_zag,jj,kkk))));
end

beta=theta_up(ii_zag,jj,kkk);
phase=k*deltan_up(ii_zag,jj,kkk)*deltaz;
a=[cos(beta),-sin(beta); sin(beta),cos(beta)];
b=[exp(-i*phase/2),0; 0, exp(i*phase/2)];
E1=(inv(a)*b*a)*E1;
dL=d+dL;
end

if dL>=n*deltaz  %% n*0.2
n=n+1;
end
if ii_zag==step(1);
ii_start=iii+1; %reset for next downward zig
path=2;  % 2 represents downward zig
end

case {2}
ii_zig=step(1)-(iii-ii_start);  %downward zig part of
path

while dL<n*deltaz

% po=redpiezoSingle(Cangle);
d=deltaz;
B=po* [sigma11(ii_zig,jj,kkk);
sigma22(ii_zig,jj,kkk); sigma33(ii_zig,jj,kkk); sigma23(ii_zig,jj,kkk);
sigma13(ii_zig,jj,kkk); sigma12(ii_zig,jj,kkk)]+...
[Bo; Bo; Bo; 0; 0; 0];

%%%%%%%%%%%%%%%%%%% components corresponding to zig

propagation %%%%
BB11_down(ii_zig,jj,kkk)=B(1,1)*(cos(prop_angle))^2+B(5,1)*sin(2*prop_angle)+B(3,1)*(sin(prop_angle))^2;
BB22_down(ii_zig,jj,kkk)=B(2,1);
BB12_down(ii_zig,jj,kkk)=B(6,1)*cos(prop_angle)+B(4,1)*sin(prop_angle);
BBpluse_down(ii_zig,jj,kkk)=
0.5*(BB11_down(ii_zig,jj,kkk) + BB22_down(ii_zig,jj,kkk)) +
0.5*sqrt((BB11_down(ii_zig,jj,kkk)-BB22_down(ii_zig,jj,kkk))^2 +
4*(BB12_down(ii_zig,jj,kkk))^2);
BBminus_down(ii_zig,jj,kkk)=
0.5*(BB11_down(ii_zig,jj,kkk) + BB22_down(ii_zig,jj,kkk)) -
0.5*sqrt((BB11_down(ii_zig,jj,kkk)-BB22_down(ii_zig,jj,kkk))^2 +
4*(BB12_down(ii_zig,jj,kkk))^2);
nzpluse_down(ii_zig,jj,kkk)=1/sqrt(BBpluse_down(ii_zig,jj,kkk));
nzminus_down(ii_zig,jj,kkk)=1/sqrt(BBminus_down(ii_zig,jj,kkk));
deltan_down(ii_zig,jj,kkk)=nzpluse_down(ii_zig,jj,kkk)-
nzminus_down(ii_zig,jj,kkk);

if (BB11_down(ii_zig,jj,kkk)==BB22_down(ii_zig,jj,kkk))
    theta_down(ii_zig,jj,kkk)=0;
else
    theta_down(ii_zig,jj,kkk)=(1/2)*atan(2*BB12_down(ii_zig,jj,kkk)/(BB11_down(ii_zig,jj,kkk)-BB22_down(ii_zig,jj,kkk)));
end
beta=theta_down(ii_zig,jj,kkk);
phase=k*deltan_down(ii_zig,jj,kkk)*deltaz;
a=[cos(beta),-sin(beta); sin(beta),cos(beta)];
b=[exp(-i*phase/2),0; 0, exp(i*phase/2)];
El=(inv(a)*b*a)*E1;
dL=d+dL;
if dL>=n*deltaz  %% n*0.2
    n=n+1;
end
if ii_zig==1;
    ii_start=iii+1; %reset for next upward zig
    path=1; % 2 represents upward zig
end

end % match "switch"
end  %% match iii
Iout_node(ii,jj) = abs(E1(1,1))^2;
Iout=Iout+Iout_node(ii,jj);
end  %% match jj
end  %% match ii
Pzigzag_loss(cgl+1)=(1-Iout/(step(1)*step(2)))*100;
Pzigzag_loss(x)=(1-Iout/(step(1)*step(2)))*100;
x=x+1;
end

figure(1);  mesh(Iout_node)% dd=15:0.5:25 ; plot(dd, Pzigzag_loss);  hold on;
hold off;

MATLAB code for calculating the reduced piezo-optic tensor for ceramic crystal

function [pzr]=redpiezoCeramic

%this file should give the piezo-optic tensor for a randomly oriented
%crystalling grain.

%First define some values

%These are the piezo-optical coefficients usually represented by the greek
symbol for pi, but here I represent them as pz. These were calculated
%from data found in the literature.
pz11=-.12049*10^-12;
pz12=0.050579*10^-12;
pz44=-2*0.2672175*10^-12;

%to perform the calculation deltaB=pz*sigma, pz must be transformed into
%the laboratory coordinate system where the data for sigma will be.

%for ceramics the laboratory coordinate system is randomly oriented for
%each crystalline grain, so I define three random rotation angles that will
%be used to generate a transformation matrix.

 epsilon=pi/3;
    trform_1=[1/sqrt(2) 0 -1/sqrt(2);...
              -1/sqrt(6) sqrt(2)/sqrt(3) -1/sqrt(6);...
              1/sqrt(3) 1/sqrt(3) 1/sqrt(3)];
    trform_2=[cos(epsilon) sin(epsilon) 0;...
               -sin(epsilon) cos(epsilon) 0;...
               0 0 1];
    trform=trform_2*trform_1;

alpha=2*pi*rand(1);
beta=2*pi*rand(1);
gamma=2*pi*rand(1);

%now I make my transformation matrices holding z and rotating left, then
%holding y and rotating right then holding z again and rotating left again.
%This is in accordance with the scheme used by Efim A. Khazanov

trform_1=[cos(alpha) sin(alpha) 0; -sin(alpha) cos(alpha) 0; 0 0 1];
trform_2=[cos(beta) 0 sin(beta); 0 1 0; -sin(beta) 0 cos(beta)];
trform_3=[cos(gamma) sin(gamma) 0; -sin(gamma) cos(gamma) 0; 0 0 1];

trform=trform_3*trform_2*trform_1;

%I'd like to also have another rotation scheme more similar to the one used
% in the paper by Chen so that I can check results.
% theta=pi/6;
%
% a=cos(theta)/sqrt(2);  b=sin(theta)/sqrt(6);  c=1/sqrt(3);
d=sin(theta)/sqrt(2); e=cos(theta)/sqrt(6);
%
% trform=[a-b 2*b -(a+b);-(d+e) 2*e d-e; c c c];

%expand the piezo-optical tensor to full-suffix

pz=zeros(3,3,3,3);
pz(1,1,1,1)=pz11;
pz(2,2,2,2)=pz11;
pz(3,3,3,3)=pz11;
pz(1,2,1,2)=0.5*pz44;
pz(1,2,2,1)=0.5*pz44;
pz(1,3,1,3)=0.5*pz44;
pz(1,3,3,1)=0.5*pz44;
pz(2,1,1,2)=0.5*pz44;
pz(2,1,2,1)=0.5*pz44;
pz(2,3,2,3)=0.5*pz44;
pz(2,3,3,2)=0.5*pz44;
pz(3,1,1,3)=0.5*pz44;
pz(3,1,3,1)=0.5*pz44;
pz(3,2,2,3)=0.5*pz44;
pz(3,2,3,2)=0.5*pz44;

%transform the pz-tensor into the laboratory coordinate system

for i=1:3
    for j=1:3
        for k=1:3
            for l=1:3
                sum=0;
                for m=1:3
                    for n=1:3
                        for o=1:3
                            for p=1:3
                                pzp(i,j,k,l)=trform(i,m)*trform(j,n)*trform(k,o)*trform(l,p)*pz(m,n,o,p);
                                sum=sum+pzp(i,j,k,l);
                            end
                        end
                    end
                end
            end
        end
    end
end
MATLAB code for calculating the reduced piezo-optic tensor for single crystal

```matlab
function [pzr]=redpiezoSingle(Cangle)

%multiply the pz-tensor by the stress tensor
```

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%this file should give the piezo-optic tensor for a randomly oriented %crystalling grain.

%First define some values

%These are the piezo-optical coefficients usually represented by the greek %symbol for pi, but here I represent them as pz. These were calculated %from data found in the literature.
pz11=-.12049*10^-12;
pz12=0.050579*10^-12;
pz44=-2*0.2672175*10^-12;

%to perform the calculation deltaB=pz*sigma, pz must be transformed into %the laboratory coordinate system where the data for sigma will be.

%for ceramics the laboratory coordinate system is randomly oriented for %each crystalline grain, so I define three random rotation angles that will %be used to generate a transformation matrix.

external\input{rotation_angles}

%alpha=2*pi*rand(1);
%beta=2*pi*rand(1);
%gamma=2*pi*rand(1);

% %now I make my transformation matrices holding z and rotating left, then %holding y and rotating right then holding z again and rotating left again. %This is in accordance with the scheme used by Efim A. Khazanov %
% trform_1=[cos(alpha) sin(alpha) 0; -sin(alpha) cos(alpha) 0; 0 0 1];
% trform_2=[cos(beta) 0 sin(beta);0 1 0; -sin(beta) 0 cos(beta)];
% trform_3=[cos(gamma) sin(gamma) 0; -sin(gamma) cos(gamma) 0; 0 0 1];
% trform=trform_3*trform_2*trform_1;

%I'd like to also have another rotation scheme more similar to the one used %in the paper by Chen so that I can check results.
%theta=pi/6;
% a=cos(theta)/sqrt(2); b=sin(theta)/sqrt(6); c=1/sqrt(3);
d=sin(theta)/sqrt(2); e=cos(theta)/sqrt(6);
% trform=[a-b 2*b -(a+b);-(d+e) 2*e d-e; c c c];
%expand the piezo-optical tensor to full-suffix

```matlab
pz=zeros(3,3,3,3);
pz(1,1,1,1)=pz11;       pz(1,2,1,2)=0.5*pz44;        pz(1,1,2,2)=pz12;
pz(2,2,2,2)=pz11;       pz(1,2,2,1)=0.5*pz44;        pz(1,1,3,3)=pz12;
pz(3,3,3,3)=pz11;       pz(1,3,1,3)=0.5*pz44;        pz(2,2,1,1)=pz12;
pz(1,3,3,1)=0.5*pz44;        pz(2,3,2,3)=0.5*pz44;        pz(3,3,2,2)=pz12;
pz(2,1,1,2)=0.5*pz44;        pz(3,1,3,1)=0.5*pz44;        pz(3,3,1,1)=pz12;
pz(2,1,2,1)=0.5*pz44;        pz(3,1,1,3)=0.5*pz44;        pz(3,3,2,2)=pz12;
pz(3,1,3,2)=0.5*pz44;        pz(3,2,3,2)=0.5*pz44;

%transform the pz-tensor into the laboratory coordinate system

for i=1:3
    for j=1:3
        for k=1:3
            for l=1:3
                sum=0;
                for m=1:3
                    for n=1:3
                        for o=1:3
                            for p=1:3
                                pzp(i,j,k,l)=trform(i,m)*trform(j,n)*trform(k,o)*trform(l,p)*pz(m,n,o,p);
                                sum=sum+pzp(i,j,k,l);
                            end
                        end
                    end
                end
            end
        end
    end
end
pzr(1,1)=pzp(1,1,1,1);
pzp(1,2)=pzp(1,1,2,2);
pzp(1,3)=pzp(1,1,3,3);
pzp(1,4)=2*pzp(1,1,2,3);
pzp(1,5)=2*pzp(1,1,1,3);
pzp(1,6)=2*pzp(1,1,1,2);
pzp(2,1)=pzp(2,2,1,1);
```

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% multiply the pz-tensor by the stress tensor

MATLAB code for modeling depolarization loss for zigzag propagation through a single crystal rod

% function[]=ceramic2XXZ3(tip_angle, prop_angle, slab_t, slab_recL, wavelength, no, step,...
% L_triangle, slab_Flength, slab_Clength, k, del_Lx, del_Lz, ii_end, Bo, deltaz, x,...
% sigma11, sigma12, sigma13, sigma22, sigma23, sigma33)
tic

rod_L= 60;%% full length of rectangular region of the slab
wavelength=1.064e-3;%% lasing wavelength
no=1.82;%% refractive index
step=size(sigma11);%% obtain the number of slices along thickness, half width and full length. step(1)=32, step(2)=64, step(3)=798
\[ k = \frac{2\pi}{\text{wavelength}}; \]
\[ \text{del}_L = \frac{\text{rod}_L}{\text{step}(3)}; \]  \text{% step length along full length of the slab}
\[ \text{Bo} = \frac{1}{\text{no}^2}; \]
\[ x = 1; \]

%for cgl=0:120
cgl=60;
Cangle=cgl*pi/180;
Iout=0;

for ii=1:step(1)  \text{% start point of ray goes in (assuming ii=1 is the bottom of the slab)}
    \% ii=step(1)/2
    for jj=1:step(2)
        \% jj=step(2);
            E1=[-1;0]; \text{% Iin_node=abs(E1(1,1))^2;}
            kk=1;
    \%--create a column vector with data representing path distance travelled
    \%--at the time of each reflection
    \%\%\%starting values
    bbstp=del_L; \% baby step in millimeters
    gs=60; \% grain size
    dL=0; \% total path length
    gsc=0; \% initialize grain size counter
    reflect=1;
    Dc=0;
endthrough=rod_L;
po=redpiezoSingle(Cangle);

kkkold=kk; \% true z-steps from ent. tip
iiiold=ii; \% the absolute x step position

while (dL<endthrough)
    dL=dL+bbstp;
    gsc=gsc+bbstp;
    kkk=ceil(dL/del_L); \% true z-steps from ent. tip
    iii=ii; \% the absolute x step position 66 is 2*step(1). if step(1) does not equal 32 then modify tooth.m
    DC=DC+bbstp;
    if gsc>=gs | kkk>=kkkold | dL>=endthrough
        B=po* [sigma11(ii,jj,kkkold); sigma22(ii,jj,kkkold); sigma33(ii,jj,kkkold); sigma23(ii,jj,kkkold); sigma13(ii,jj,kkkold); sigma12(ii,jj,kkkold)]+...
        [Bo; Bo; Bo; 0; 0; 0]; \% make the indices correct
    end
end

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BB11 = B(1,1);
BB22 = B(2,1);
BB12 = B(6,1);

BBplus = 0.5*(BB11 + BB22) + 0.5*sqrt((BB11-BB22)^2 + 4*(BB12)^2);
BBminus = 0.5*(BB11 + BB22) - 0.5*sqrt((BB11-BB22)^2 + 4*(BB12)^2);
nzplus = 1/sqrt(BBplus);
nzminus = 1/sqrt(BBminus);

deltan = (nzplus - nzminus);

if (BB11 == BB22)
  theta = 0;
else
  theta = ((1/2)*atan(2*BB12/((BB11-BB22))));  
end

beta = theta;
phase = k*deltan*DC;  % DC is Distance since last Change
a = [cos(beta), -sin(beta); sin(beta), cos(beta)];
b = [exp(-i*phase/2), 0; 0, exp(i*phase/2)];
E1 = (inv(a)*b*a)*E1;
DC = 0;

if kkk > kkkold
  kkkold = kkk;
  % howcls = dL*cos(prop_angle)/del_Lz+kk0;
  end
  % if kkkold > step(3), break, end
end
end

if kkk > kkkold
  kkkold = kkk;
  % howcls = dL*cos(prop_angle)/del_Lz+kk0;
  end
  % if kkkold > step(3), break, end
end

Pzigzag_loss(x) = (1 - Iout/(step(1)*step(2)))*100
% Pzigzag_loss(x) = (1 - Iout)*100
% Pzigzag_loss(x) = (1 - Iout/(step(2)))*100
MATLAB code for modeling depolarization loss for zigzag propagation through a ceramic crystal rod

```matlab
x=x+1;
% end  % match dd
%figure(1);  plot(Iout_node(16,:))
%end
figure(1);
mesh(Iout_node)
% dd=15:0.5:25 ; plot(dd, Pzigzag_loss);
% hold on;  hold off;
toc
```

```matlab
% function[]=ceramicZXXZ3(tip_angle, prop_angle, slab_t, slab_recL, wavelength, no, step,...
%   L_triangle, slab_Flength, slab_Clength, k, del_Lx, del_Lz, ii_end, Bo, deltaz, x,...
%   sigma11, sigma12, sigma13, sigma22, sigma23, sigma33)
tic
rod_L= 60;%% full length of rectangular region of the slab
wavelength= 1.064e-3;%% lasing wavelength
no= 1.82;%% refractive index
step= size(sigma11);%% obtain the number of slices along thickness, half width and full length. step(1)=32, step(2)=64, step(3)=798
k= 2*pi/wavelength;

del_Lz= rod_L/(step(3));%% step length along full length of the slab
Bo= 1/no^2;
x= 1;
Iout=0;

prompt={'Standard deviation in mm:';'Central grain size in mm:'};
name='Grain size & distribution';
umlines=1;
defaultanswer={'5e-3';'20e-3'};
answer=inputdlg(prompt,name,numlines,defaultanswer);
mu=str2num(answer{2,1});
sigma=str2num(answer{1,1});

for ii=1:step(1)  %%%% start point of ray goes in (assuming ii=1 is the bottom of the slab)
    % ii=step(1)/2
    
    
end
```

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for jj=1:step(2)
%jj=step(2);
  E1=[1 ; 0]; %E1=abs(E1(1,1))^2;
  kk=1;
%%--create a column vector with data representing path distance travelled
%%--at the time of each reflection
%%%starting values
bbstp=0.6*10^-3 ; %baby step in millimeters
d=-1;
while (d<=0) | (d>=2*mu)
  d=mu+sqrt(sigma)*randn(1);
end
%grain size
%total path length
%i initialize grain size counter
reflect=1;
%end through = rod_L;
po=redpiezoCeramic;

kkkold=kk; %true z-steps from ent. tip
iiold=ii; %the absolute x step position
%
while (dL<endthrough)
  dL=dL+bbstp;
  gsc=gsc+bbstp;
  kkk=ceil(dL/del_Lz); %true z-steps from ent. tip
  iii=ii; %the absolute x step position 66 is 2*step(1). if step(1) does not
  %equal 32 then modify tooth.m
  DC=DC+bbstp;
  if gsc>=gs|kkk>kkkold|dL>=endthrough
    B=po*[sigma11(ii,jj,kkkold); sigma22(ii,jj,kkkold);
          sigma33(ii,jj,kkkold); sigma23(ii,jj,kkkold); sigma13(ii,jj,kkkold);
          sigma12(ii,jj,kkkold)]+...
    [Bo; Bo; Bo; 0; 0; 0]; %make the indices correct
  BB11=B(1,1);
  BB22=B(2,1);
  BB12=B(6,1);
  BBpuse= 0.5*(BB11 + BB22) + 0.5*sqrt((BB11-BB22)^2 + 4*(BB12)^2);
  BBminus= 0.5*(BB11 + BB22) - 0.5*sqrt((BB11-BB22)^2 + 4*(BB12)^2);
  nzpuse=1/sqrt(BBpuse);
  nzminus=1/sqrt(BBminus);
  deltan=(nzpuse-nzminus);
  if (BB11==BB22)
    theta=0;
  else
    theta=((1/2)*atan(2*BB12/(BB11-BB22)));
if theta<0
    theta=theta+pi/2;
end

beta=theta;
phase=k*deltan*DC;  %DC is Distance since last Change
a=[cos(beta),-sin(beta); sin(beta),cos(beta)];
b=[exp(-i*phase/2),0; 0, exp(i*phase/2)];
E1=(inv(a)*b*a)*E1;
DC=0;

if gsc>=gs
    gsc=0;
d=-1;
    while (d<=0) | (d>=2*mu)
        d=mu+sqrt(sigma)*randn(1);
    end
    gs=d;  %grain size
    po=redpiezoCeramic;
end
if kkk>kkkold
    kkkold=kkk;
    %howcls=dL*cos(prop_angle)/del_Lz+kk0;
end
if kkkold>step(3), break, end
end
end

end  %%
Iout_node(ii,jj) = abs(E1(1,1))^2;
Iout=Iout+Iout_node(ii,jj);
huhttheta(ii,jj)=theta;
end  %% match jj
end  %ii

Pzigzag_loss(x)=(1-Iout/(step(1)*step(2)))*100
  % Pzigzag_loss(x)=(1-Iout)*100
  %Pzigzag_loss(x)=(1-Iout/(step(2)))*100
  % x=x+1;
  % end  %% match dd
figure(1);  plot(Iout_node(16,:))
mu
sigma
figure(1);
mesh(Iout_node)
% dd=15:0.5:25 ; plot(dd, Pzigzag_loss);
% hold on; hold off;
toc
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