Comparing Assessment Methods As Predictors Of Student Learning In Undergraduate Mathematics

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COMPARING ASSESSMENT METHODS AS PREDICTORS OF STUDENT LEARNING IN UNDERGRADUATE MATHEMATICS

by

NICHOLE APRIL SHORTER
B.S. University of Central Florida, 2007

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the Department of Mathematics in the College of Sciences at the University of Central Florida Orlando, Florida

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ABSTRACT

This experiment was designed to determine which assessment method: continuous assessment (in the form of daily in-class quizzes), cumulative assessment (in the form of online homework), or project-based learning, best predicts student learning (dependent upon posttest grades) in an undergraduate mathematics course. Participants included 117 university-level undergraduate freshmen enrolled in a course titled “Mathematics for Calculus”.

Initially, a multiple regression model was formulated to model the relationship between the predictor variables (the continuous assessment, cumulative assessment, and project scores) versus the outcome variable (the posttest scores). However, due to the possibility of multicollinearity present between the cumulative assessment predictor variable and the continuous assessment predictor variable, a stepwise regression model was implemented and caused the cumulative assessment predictor variable to be forced out of the resulting model, based on the results of statistical significance and hypothesis testing. The finalized stepwise regression model included continuous assessment scores and project scores as predictor variables of students’ posttest scores with a 99% confidence level. Results indicated that ultimately the continuous assessment scores best predicted students’ posttest scores.
This work is dedicated to my parents

Sven and Kathy Shorter

And my brother

Nicholas Shorter
ACKNOWLEDGEMENTS

The following people, Dr. Cynthia Young and Dr. Tace Crouse, are major contributors to the completion of this thesis. I would like to thank my advisor Dr. Cynthia Young for all of her guidance and support. She helped tremendously with designing and implementing my experiment. Without her, this research would not have been possible. I also thank Dr. Tace Crouse for the help she provided me with designing my experiment as well as choosing an appropriate research question.

I also thank my family, Sven, Kathy, and Nicholas, and my friends for their daily motivation. Their continuous encouragement and prayers were the underlying source of my perseverance.
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# LIST OF ACRONYMS/ABBREVIATIONS

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<th>Acronym</th>
<th>Description</th>
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<tr>
<td>AAHE</td>
<td>American Association for Higher Education</td>
</tr>
<tr>
<td>NSF</td>
<td>National Science Foundation</td>
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<tr>
<td>RMSE</td>
<td>Root Mean Square Error</td>
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<td>SSreg</td>
<td>Sum of Squares due to Regression</td>
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<tr>
<td>SSres</td>
<td>Sum of Squares Residual</td>
</tr>
<tr>
<td>SStotal</td>
<td>Sum of Squares Total</td>
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<tr>
<td>STEM</td>
<td>Science, Technology, Engineering, and Mathematics</td>
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CHAPTER ONE: INTRODUCTION

The emphasis on student learning and retention at the college level is slowly beginning to flourish. Researchers are discovering the need for educational reform, especially in undergraduate mathematics and particularly in the freshman level courses. Students who are majoring in STEM (Science, Technology, Engineering and Mathematics) disciplines are struggling with the transition from high school to college mathematics. As a result, studies on various classroom techniques aimed to improve student learning and retention in undergraduate mathematics are currently being conducted. This study analyzes the use of several different assessment methods as predictors of student learning in undergraduate mathematics, specifically at the freshman level where students are transitioning from high school to college. Its purpose is to find which of the three assessment methods examined: continuous assessment, cumulative assessment, and project-based learning, best predicted the students’ performance in a freshman level, undergraduate mathematics course. The ultimate goal is to use these results to better design undergraduate mathematics curriculum.

First, a literature review on various assessment methods is provided. It opens with a brief history on educational reform at the college level. Then there is an introduction to assessment and to student learning and retention. A discussion on pre- and post-testing follows. Next the assessment methods in which this experiment focuses upon are presented: continuous assessment, cumulative assessment, and project-based learning. Rubrics will also be examined in this section. The section is concluded with an explanation of predictors of student learning.
Second, the design of the experiment is discussed. The section begins with the purpose of this experiment followed by the hypothesis. It then describes the population, the mathematics course the experiment was performed on, the classroom environment in which the experiment took place, the professor’s teaching style of the course, and how the assessment methods in the literature review section were implemented and graded.

Third, the results of the experiment are presented. The section begins by examining the relationships between the variables using the Pearson correlation and scatter plots. A multiple regression model of the data is then formulated and discussed. Due to the possibility of multicollinearity present in the multiple regression model, a stepwise regression model was then implemented and analyzed. The section concludes with hypothesis and statistical significance tests.

Fourth, the conclusions of the experiment are described.

Lastly, the possible implementation of the results is discussed, as well as potential future experiments and research.
CHAPTER TWO: LITERATURE REVIEW

Postsecondary Educational Reform

Research on educational reform at the postsecondary level has focused primarily on students: their learning styles, which student characteristics enhance or hinder their success, what preconceived notions they bring to different subjects, as well as gender and ethnicity issues (Hart, 1999). As a result, collegiate research on assessment has been minimal. This is unfortunate since assessment is an integral part of any undergraduate course and is essential to monitoring and improving student learning. Assessment is often tied in with studies that analyze various instructional approaches. A study by Keeler and Voxman (1994) consisted of examining classes that use cooperative groups in lecture-style classes. Packard (1993) studied student learning in a statistics course where the material was taught using three different computer-presentation methods. In addition, Frid (1994) focused on the differences in teaching calculus traditionally, by concepts before practice, and by emphasizing infinitesimals. Bookman and Friedman (1994) analyzed how well students in lab-based versus traditional calculus courses problem-solved, while Park and Travers (1996) analyzed how well students succeeded in computer-based versus traditional calculus courses. Although assessment is present in these studies, it is not the central focus.

So why is educational reform important and why should assessment be its primary focus? According to “The Status of Research on Mathematics Education at the Postsecondary Level,” Hart (1999) states that “the most compelling reason to do research on postsecondary mathematics teaching is because what we are doing in mathematics
classrooms now is not working for the vast majority of students”. Furthermore, in *Assessment in practice: Putting principles to work on college campuses*, Banta, Lund, Black, and Oblander (1996) noted that the current emphasis placed on collegiate educational reform is detailed in publications such as *Involvement in Learning* (Study Group on the Conditions of Excellence in American Higher Education, 1984), *College: The Undergraduate Experience in America* (Boyer, 1987), *Education counts* (Special Study Panel on Education Indicators for the National Center for Education Statistics, 1991), and most recently, *An American Imperative: Higher Expectations for Higher Education* (Wingspread Group on Higher Education, 1993). All of these studies promote the use of assessment and feedback as tools to improve postsecondary education.

**Introduction to Assessment**

We examined the importance of educational reform at the postsecondary level and why assessment should be its quintessence. Next, we need to specify exactly what we mean by assessment. Wentzell, Richlin, and Cox (2007) define assessment as “the evaluation of educational methods and outcomes with the goal of improving practice”. In his analysis of accountability and the goals of postsecondary education, Peters (1994) defines assessment as the “systematic inquiry into learning in order to improve it” (p. 1). Assessment is a process, not just the administration and grading of tests. It is imperative to note that assessment tools are not limited by exams, but extend to the use of quizzes, projects, reports, essays, journals, portfolios, interviews, asking questions in class, speeches, skits, cooperative and collaborative group work, poster presentations, etc. The assessment tool chosen should properly test whether or not the student achieved the goals
and objectives of the lesson. It is then graded, often by using a scoring rubric (these will be discussed later) and feedback is provided to the student detailing his/her performance. Finally, faculty must use their assessment results; Ratcliff (1997) points out that “too few campuses regularly use assessment information at the departmental level to improve and enhance undergraduate teaching. Assessment can guide us not only in the improvement of student learning but also in how we teach” (p. 29). Assessment results inform faculty as to whether or not curriculum modifications are needed. For example, if a class of calculus students fails the assessment on limits, the professor may wish to spend additional time on limits before moving ahead to derivatives.

Assessments can be broken down into formative and summative assessments. O’Connor (2002) states that the purpose of formative assessment is to monitor and guide an ongoing assessment while it is still in progress, and the purpose of summative assessment is to evaluate the success of an assessment when it has been completed. O’Connor lists the following assessment types as formative assessments: observations, quizzes, homework, in-class questions, and worksheets. He lists the following assessment types as summative assessments: tests, projects, and term papers.

As we have already noted, assessment in postsecondary education has many benefits. Wlodkowski and Ginsberg (1995) argue that assessment improves learning and produces competence. Furthermore, Suskie (2004) addresses several benefits in her book Student Learning: A Common Sense Guide: “…assessment…provides the feedback essential to helping faculty understand what is and isn’t working and how to improve their curricula and teaching/learning strategies to bring about even greater learning” (p. 11). She adds that students benefit from assessment because it enables them to discover
their strengths and weaknesses resulting in school and career improvement. Suskie feels that faculty benefit as well since assessment permits them to concentrate on teaching objectives and outcomes and to obtain solid evidence of the quality of their teaching. She concludes, “In short, assessment provides the feedback faculty and institutions need to improve teaching and learning.”

**Student Learning**

Since assessment provides faculty with important student data which allows them to make appropriate curricular decisions, it results in improved student learning (“Beyond Crossroads,” 2006). The ultimate goal of the study of this thesis is to investigate assessment techniques that can demonstrate improved student learning. But what exactly is student learning? Ratcliff (1997), in his article “Improving Postsecondary Teaching, Learning and Assessment,” defines student learning as “the learning of basic knowledge in science, mathematics, and the social sciences; cognitive abilities, such as oral and written communication skills, critical thinking, and problem solving” (p. 14).

We will now examine student learning at the institutional level. Ratcliff (1997) argues that recently, colleges and universities have attempted to improve student learning through the implementation of assessment. The focus on assessment by institutions and faculty has lead to an expansion of activity. He feels that assessment can not only improve learning but also accountability. However, these benefits have yet to be documented in the literature.

There is a need for collaboration between faculty and institutional administration to get students actively involved in classroom assessment (Angelo & Cross, 1993). In
addition, that faculty and administration collaboration should also place emphasis on varying assessments and utilizing assessment results. The National Association of State Universities and Land Grant Colleges (1988) proposed that colleges and universities should use multiple methods of assessment to improve both teaching and learning. The Association claims that assessment programs within the institutions should have a strategy plan or program that implements curricular change based on assessment results. Huba and Freed (2000) build on the importance of utilizing assessment results. They believe that not only should assessment data be collected, but the assessment process must be completed by using the data results to enforce changes that will lead to better learning. However, such an activity is rare in postsecondary education (Ratcliff, 1997). This is unfortunate since the quality of student learning depends on student assessment data.

Moreover, there should be a comparison of program and classroom assessment data in order to improve student learning (Huba & Freed, 2000). This can be done at the department level, in which faculty can collaboratively construct a plan detailing the learning outcomes for mathematics courses and programs (“Beyond Crossroads,” 2000). Then, the data collected on student learning will be analyzed and based on the results, appropriate modifications to curriculum, materials, and teaching methods can be made. According to the Study on the Conditions of Excellence in American Higher Education (1984):

Very simply, institutions need to value the data they have and collection needs to be purposeful as they transform the data into useful information; they should
make a conscientious effort to acquire and use better information about student learning, the effects of courses, and the impact of programs.

The assessment cycle starts with the declaration of desired student learning outcomes and ends by providing results that instill change which leads to improved student learning ("Beyond Crossroads," 2006). Drawing our attention towards the ending behavior of assessment in student learning, assessment ultimately gives faculty and students feedback concerning each student’s progress in achieving learning outcomes. Feedback is very beneficial because it informs faculty on exactly what, how much, and how well students are learning. Overall, the assessment process, if implemented appropriately, has the potential to greatly impact undergraduate mathematics and improve student learning.

**Pre- and Post-Testing**

Pretests are often used to gauge students’ prior knowledge before implementing an assessment. Hartley and Davies (1976) define a pretest as “any set of related questions, given before instruction, that is directly relevant to the knowledge, attitude, or skill domain to be acquired.” The questions of a pretest can be identical to, selected from, or closely represent those of a posttest to be conducted after the teaching has ended. They can be written in terms of multiple-choice, matching, true-false, short-answer, essays, or long-answer questions. The purpose of a pretest is to discover any prior knowledge (regardless of accuracy) of the material to be taught, and with the aid of a posttest, to gain information about the success of the learner and the teacher.
Pretests can be used as guides to teaching, course design, student progress analysis, and assessment evaluation (Hartley & Davies, 1976). Pretests can direct students’ attention towards what they do not know; this enables them to pay special attention to these topics during instruction in an effort to learn the unfamiliar material. Hartley (1973) proclaims, “Pretests increased students’ awareness of what was expected of them, helped students to organize related material, and thus made the material easier for students to remember” (p. 2). All in all, pretests are an assessment tool that not only gauge students’ prior knowledge of a subject, but can potentially improve student learning when that subject is taught.

Continuous Assessment

We will now discuss one of the assessment methods implemented in this study: continuous assessment. In this context, continuous assessment is an assessment method that is implemented frequently in an effort to get students to study and review their course materials more often. The American Association for Higher Education (AAHE) (1992) constructed a list of the “9 Principles of Good Practice for Assessing Student Learning”. Continuous assessment was listed as the fifth principle. The AAHE argues that assessment is most effective when it is ongoing. However, one time assessments are better than none. Wargo (2006) adds that research has shown that not administering any assessment does indeed have adverse effects on students, and not merely because the assessments enforce good study habits. Two experiments were conducted in which the results showed that when students are tested, even if they do not study, long-term retention is improved. Furthermore, the AAHE points out that continuous assessment,
since it develops a collection of student performance, allows faculty to track the progress of students over time. Banta et al. (1996) agrees with the AAHE by asserting that successful assessment is ongoing, iterative, continuous, and an effective monitoring tool.

Several studies have been conducted regarding the effectiveness of continuous assessment. Some very early studies indicate that students who undergo frequent testing earn relatively higher final examination scores that those who are not tested frequently (Gable, 1936; Hertzberg, Heilman, & Leuenberger, 1932; Jones, 1923). Frequent testing, or “the testing effect” as Wargo (2006) denotes it, was originally studied using the memorization of word lists. Recently, the testing effect began incorporating other assessment methods. In one experiment, researchers had a class study science passages where half took essay tests and the other half did not. For five minutes, two days, or one week later, the groups were evaluated on their retention. The half of the class who took the tests but did not study retained the material better than those who did not take the tests. “Clearly, testing enhances long term retention through some mechanism different from restudying the material” (Wargo, 2006). Wargo also examined a study in which repeated testing of material (without studying) was compared to repeated studying of material. Those who were tested retained 61 percent of the material a week later, while those who only studied the material retained only 40 percent a week later.

However, Johnson (1984) analyzed a similar experiment where a group of students were separated into two groups and taught at different times by the same teacher. The smaller of the two groups was given short tests every two weeks. Both groups took a final examination in which most of the questions came from the short tests. The goal of the study was to see if small class size and frequent, short tests effected exam results.
Surprisingly, the results for both groups were very similar and no significant difference was found. Johnson then proposed that perhaps the study could be replicated in large classes.

Does continuous assessment even promote studying? In a study by Mawhinney, Bostow, Laws, Blumenfeld, and Hopkins (1971) where students were tested daily and biweekly, the average number of minutes studied for the students who were tested daily was higher than the number of those who were tested biweekly. Mawhinney et al. stated that

The use of these schedules of testing is likely a result of instructor convenience or the assumption that this form of testing is sufficient to motivate the student to study in a consistent fashion for the duration of the course.

Based on the results of these studies, evidence exists that continuous assessment may improve retention and promote studying.

The continuous assessment in this study was in the form of daily quizzes. Thus, it is necessary to discuss the effects of daily quizzes on student learning. Azorlosa and Renner (2006) describe a study conducted by Marchant (2002) in which announced and unannounced quizzes were administered to a class. The class scored nearly 20 percent higher on the quizzes that were announced and reported to have read the assigned materially more closely if a quiz was anticipated. In addition, Ruscio (2001) asserts that daily announced quizzes can promote reading ahead. Standlee and Popham (1960) attribute the increase in achievement from the use of quizzes to extrinsic motivation: “students will work harder throughout the course, because they want to get good grades on the quizzes, and this yields higher achievement.” Standlee and Popham further suggest
that daily quizzes will improve student achievement faster in lecture style courses because it combines the subject matter with an activity, structures the course, and provides student results and extrinsic motivation.

Overall, research on daily announced quizzes and exam results has produced inconsistent conclusions. Azorlosa and Renner found a few studies in which exam performance was improved based on announced quizzes (Geiger & Bostow, 1975; Noll, 1939), and others in which the quizzes had no effect on exam performance (Beaulieu & Utecht, 1987; Lumsden, 1976). Hovell, Williams, and Semb (1979), Sporer (2001), and Wilder, Flood, and Stromsnes (2001) showed that quizzes increase attendance and self-reported studying, despite no effect on exam performance. Azorlosa and Renner propose that the value of the quizzes being too low, dropping quiz grades, or the quiz format may have caused the lack of exam performance. Thus, no clear conclusions regarding the use of daily announced quizzes and exam performance can be drawn.

**Cumulative Assessment**

We now discuss the second assessment method that was implemented in this study: cumulative assessment. Cumulative assessment is an assessment method which assesses student learning on the material from the first day of class to the present at a consistent rate during the course. Since it is implemented rather frequently, it can be considered as another form of continuous assessment.

It is important to space your cumulative assessments and administer them often to achieve better student performance (Donovan & Radosevich, 1999). Cumulative assessment allows students to space their learning and to study and absorb the material a
little bit at a time. These benefits have had the greatest effect on lecture-style classes, suggesting that students who take cumulative assessments would outperform roughly 67 percent of the students who do not take the assessments (Myers & Myers, 2007). In addition, Willingham (2002) advocates cumulative assessments because studies have found that they benefit student learning and aid in retention for several years.

Myers and Myers (2007) studied an experiment in which cumulative assessments were administered biweekly for six weeks to one class (who also took a midterm and final exam), while the other class took only the midterm and final exams. The students who took the biweekly tests scored approximately 20 percentage points higher on the midterm exam, 10 percentage points higher on the final exam, and 15 percentage points higher on their course grade compared to the students who took the midterm and final exams only.

Furthermore, the biweekly test class seemed to favor the class format more compared to the class that took the midterm and final exams only. None of the students in the biweekly test class dropped the course.

Myers and Myers (2007) feel that the students in the biweekly testing class may have performed better because the students had less material to learn for each exam, were less likely to cram for exams, received frequent feedback on their performance, and were more confident and motivated. Willingham (2002) concludes that faculty should include more assessment into courses to reduce cramming and increase student learning and performance.
Project-Based Learning

One of the assessment methods used in this study was project-based learning. Project-based learning is an assessment method in which students’ understanding of a mathematics lesson is assessed by completing a project. Appropriate implementation of project-based learning consists of projects in which students demonstrate a basic understanding of the problem at hand; simplify their methods to arriving at a solution; consider all possible cases to a solution; gather data and observations; record results; design and test hypotheses; provide evidence to accept or reject their hypotheses; utilize counter examples if necessary; make mathematical conclusions; connect, explain, and justify results; suggest future investigations; provide any formal proof (Glaister & Glaister, 2000). Glaister and Glaister further argue that project-based learning can enhance mathematics teaching and learning through teaching students how to problem-solve and mathematically communicate and reason. Projects should be designed so that they “use and apply mathematics in a practical or real-life situation and a pure mathematical context” (Glaister & Glaister, 2000).

Furthermore, Glaister and Glaister discuss guidelines regarding how much help a teacher should provide when students are being assessed through projects. Teachers should intervene just enough so that students are still able to work on their project over an extended period of time without continual teaching supervision, break down large projects into small manageable stages on their own, figure out how to experiment with different problem-solving techniques, demonstrate an understanding of the mathematical concepts that the project is assessing, and communicate the ideas and conclusions of the
work on their own. Lastly, it is essential to have students in group projects list what each group member contributed to the project to better gauge their understanding.

There has been a couple of research studies conducted on the impact that project-based learning has had on student learning. Boaler (1998) conducted a three year study in which two schools’ (high school level) teaching methods were evaluated: one school, Amber Hill, used the traditional textbook approach of teaching mathematics while the other school, Phoenix Park, used an open-ended projects method. Boaler found that students who attended Amber Hill developed more of a procedural understanding of mathematics, which was ultimately of little to no use to the students when they were presented with new problems. Unfortunately, this disconnection has been a problem among mathematics educators for quite some time. However, the students from Phoenix Park developed a conceptual understanding of mathematics that enabled them to apply their skills to problems outside of the classroom.

In the article, “Open and Closed Mathematics: Student Experiences and Understandings” Boaler (1998) mentions several studies that detail the benefits of project-based learning. Students benefit by developing decision making skills, planning skills, and the ability to apply their mathematical knowledge. Furthermore, Boaler observed that the students at Amber Hill found mathematics to be boring and tedious; these students demonstrated a lack of involvement in their work and felt that the exercises were repetitious. In contrast, the students at Phoenix Park had mixed feelings about project-based learning: some really enjoyed it and felt that it enhanced their learning and understanding of mathematics applications, while others did not like the openness of the approach nor the freedom it granted.
To better gauge the effects that project-based learning had on the students at Phoenix Park, the top 53 students from Amber Hill and 51 mixed-ability students from Phoenix Park were required to complete a mathematics project focusing on the modeling and planning of a proposed house that required them to solve two problems relating to local authority design rules. The project required students to seek information from various sources, implement their own methods, plan their problem solving strategies, combine different areas of mathematical content, and communicate their results. Keeping in mind that the students representing Amber Hill were top ability students and the students representing Phoenix Park were mixed-ability students: the students from Phoenix Park scored significantly higher than the students from Amber Hill on the applied project. The Amber Hill students had difficulty solving the applied project due to their choice of methods. Furthermore, the students were administered a regular mathematics exam in which both schools performed similarly.

Conclusively, the Amber Hill students’ performance on the project suggested that their traditional textbook learning made them develop a procedural understanding of mathematics that was of limited use to them. On the other hand, the Phoenix Park students had learned to use mathematics in new situations and realized that mathematics involves active and innovative thinking. Boaler (1998) states:

The students were able to use mathematics because of three important characteristics: a willingness and ability to perceive and interpret different situations and develop meaning from them and in relation to them; a sufficient understanding of the procedures to allow appropriate procedures to be selected;
and a mathematical confidence that enabled students to adapt and change procedures to fit new situations.

Swafford and Kepner (1980) also researched the impact that project-based learning has on student learning. They examined the First-Year Algebra via Applications Development Project developed by the National Science Foundation (NSF). The project teaches mathematics through applications and models. In the study, 20 schools in the United States administered the applications development project. The goals were to analyze the materials in a classroom setting, evaluate the students’ understanding of the mathematics concepts compared to students in traditional algebra classrooms, determine students’ attitudes towards the use of projects, gauge the students ability to solve real world problems, and judge the difficulties of implementing the projects.

Swafford and Kepner (1980) found that the schools who taught using project-based learning performed comparable to the schools that used the traditional teaching approach. The project-based group showed a little bit more of an improvement when compared to the traditional group. Six schools of the 20 studied that used the project-based materials performed significantly better on the posttest. Seventeen of the 20 schools involved favored the project-based learning. Ultimately, the project-based materials were found to be successfully useable in a variety of school settings.
Rubrics

Since rubrics were used when designing and grading the pre- and post-tests, a short discussion of what a rubric is, the qualities of a good rubric, and the benefits of using rubrics is provided.


A scoring guide: a simple list, chart, or guide that describes the criteria that you and perhaps your colleagues will use to score or grade an assignment. At a minimum, a rubric lists the things you’re looking for when you evaluate a student assignment. The list is often accompanied by guidelines for evaluating each of those things.

As an application of assessment, a rubric shows the rules of scoring and explains the criteria for which student work will be judged (Huba & Freed, 2000). Huba and Freed feel that the purpose of a rubric is to systematically create a final score for an assessment. The rubric should contain a numerical scale where the grader can allocate a specific value for each part of a question (in a math setting), then add these values to obtain a final score. Some parts of a question may be more pertinent than others, and these parts may be weighted more than others. Ultimately, rubrics provide a commentary that describes each level of mastery of an assessment.

Huba and Freed (2000) outline six questions to ask when constructing a rubric:

1. What criteria or essential elements must be present in the student’s work to ensure that it is high in quality?” Applying to mathematics rubrics, it is important to identify which steps must be presented in the student’s work, which steps may not be as
necessary, and how much weight should be allocated per step. Mathematics faculty must also consider cases in which there are multiple ways of solving a problem.

“2. How many levels of achievement do I wish to illustrate for students?” When creating a mathematics rubric, faculty must determine the breakdown of steps necessary, how valuable each step is, and the number of steps required when solving a problem.

“3. For each criterion or essential element of quality, what is a clear description of performance at each achievement level?” In mathematics assessments, a perfect score for a question may require more than just a correct numerical answer, but also perhaps an explanation, graph, verification, etc. Thus, a rubric needs to clearly specify the expectation of performance at each level.

“4. What are the consequences of performing at each achievement level?” If points are being deducted for certain mistakes, then these also need to be specified in the rubric.

“5. What rating scheme will I use in the rubric?” Mathematics faculty should consider the weight of each part of a question as well as which questions are more important than others.

“6. When I use the rubric, what aspects work well and what aspects need improvement?” Of course after implementing your rubric, it is always important to take into account what you have found to work well and not so well before creating your next assessment rubric.

Lastly, Suskie (2004) outlines several benefits to using a rubric: rubrics make expectations clear; inspire better performance; make scoring simple, quick, accurate, unbiased, and consistent; improve student communication; reduce arguments.
Predictors of Student Learning

One of the aims of this study is to determine which assessment method serves as the best predictor of student learning. More specifically, we wish to find which assessment method reflects the students’ posttest grades the closest. Predictors of student learning in the form of assessments are very useful to faculty because they gauge the level of student understanding. However, most of the studies that have been conducted on predictors of student learning are not on assessments, but on student attributes such as age, socioeconomic status, gender, ethnicity, or attitude toward the material to be learned.

Dahlke (1974) conducted a study aimed to determine predictors of student success and time of course completion or course dropout in a self-paced course. The predictors that Dahlke analyzed were students’ age, sex, years of school prior to college, number of semesters in basic mathematics, total number of semesters of high school mathematics, total number of hours at the community college, reasons for enrolling in their current mathematics course, arithmetic achievement, reading comprehension, study techniques, and attitudes towards mathematics. Another study used course performance to predict student satisfaction and self-efficacy in an online undergraduate class (Puzziferro, 2008). Similarly, Klomedge (2007) discovered that among self-efficacy, self-set goals, assigned goals, and ability, self-efficacy was the best predictor of academic performance in an undergraduate sample. In addition, Chamorro-Premuzic and Furnham (2008) completed a study on the extent to which personality, ability, and learning methods predict academic performance. Chamorro-Premuzic and Furnham believe that the two most evident predictors of student success are personality and cognitive ability. In the article, “The

Stephens and Konvalina (2001) studied how short weekly quizzes, computer algebra software projects, and a practice comprehensive final exam influence student success in an intermediate college algebra course. After implementing a stepwise regression model on the data, Stephens and Konvalina concluded that all three factors influenced student success (determined by a final exam) significantly.

It can be concluded that the majority of studies conducted on predictors of student learning focus on affective student attributes as predictors as opposed to cognitive attributes. It may be more useful, informative, and easier to implement assessment predictors than to analyze students’ affective behaviors when gauging student success.
CHAPTER THREE: EXPERIMENT

Purpose

The purpose of this study is to determine which assessment method: continuous assessment (daily in-class quizzes), cumulative assessment, or project-based learning, best predicts student learning (dependent upon posttest grades) in a freshman level, undergraduate mathematics course.

Hypothesis

Among continuous assessment, cumulative assessment, and project-based learning, continuous assessment will best predict students’ posttest scores in an undergraduate mathematics course.

Sample

The sample included 117 university-level undergraduate freshmen enrolled in a course entitled “Mathematics for Calculus”. Eighty-five students were in the university’s EXCEL program while 32 students were not. The EXCEL program is designed for students with average standardized test scores and aims to increase their success in the first two years of college. These students are STEM majors and have access to a free math and science tutoring lab. The university also has a regular free math tutoring lab available for all students to attend.
Course Design

Mathematics for Calculus is a college entry-level pre-calculus course that covers college algebra and trigonometry content, and also prepares students for calculus. The course is a traditional lecture-style course in which the professor teaches straight from the textbook. The class met three times a week for one hour and 50 minutes in a large lecture hall. The course lasted for one semester, or approximately 15 weeks. The professor used a document camera to display the textbook to the students. The lessons were taught by switching the document camera from the textbook to paper in which problems from the textbook were visually worked out. On the first day of class, students took a pretest. Students took quizzes almost every class day when there was not a test; there were 34 quizzes in total. The quizzes contained one problem from the suggested homework that was not collected. There were 10 online cumulative assessments as well as a group project. The students also took four exams and a 2 hour and 50 minute cumulative final exam that had the pretest questions verbatim embedded in it. Students were allowed non programmable, non graphing calculators on all assessments. We will now examine the descriptions and grading methods of the course assessments (see Appendix A).

Pre- and Post-Tests

The pretests (see Appendix B) were given on the first day of class. The pretest contained 12 questions that were imbedded verbatim into the final exam. The pretest did not count as a grade. The posttest (see Appendix B) is the part of the final exam that consisted of the 12 pretest questions. The final exam counted for 25 percent of their grade. The purpose of the pretest was to gauge any prior student understanding of the
topics to be taught in the course. The pre- and post-tests were used as summative assessments. Both tests were graded by two graders separately using a rubric (see Appendix B).

**Continuous Assessments (Quizzes)**

Students took a short 10 minute quiz almost every class meeting when a test was not scheduled. There were 34 quizzes in total (see Appendix C). Every quiz contained only two questions: the first question was a two-point problem-solving question from the previous nights’ suggested homework, while the second question was a one-point multiple-choice question on a concept from the section that was to be taught in class that day (thus requiring students to read ahead). After the quizzes were administered and collected, the professor displayed the solved problem and answers on the document camera. Since the quizzes were administered so frequently, they fall under the category of continuous assessment. The quizzes were used as formative assessments and counted for 15 percent of their grade.

**Cumulative Assessments**

Students were required to complete 10 cumulative assessments (see Appendix D). The cumulative assessments were completed online using the textbook’s online course management system, Wiley Plus. Wiley Plus provides the entire textbook online, the entire solutions manual online (with the problems worked out), and videos of the instructor working problems. Each assessment contained 10 questions that covered the
current material as well as material from as early as the first class. After each question, students are informed whether or not they answered it correctly. The professor created the assessments so that for the first four assessments students were allowed unlimited tries per question, as well as hints. The remaining six assessments allowed up to three tries per question, and no hints. Students were not required to complete the full assessment in one sitting. The assessments contain a score review page that displays which questions are answered and which are not, the score per each question, the points possible per each question, and the number of attempts per each question. The cumulative assessments were used as formative assessments and counted for 10 percent of their grade.

**Projects**

The students were required to complete a mathematics project on global warming (see Appendix E). The students put themselves into groups of four. The purpose of the project is for students to identify function models based on carbon dioxide levels for one of the 10 largest cities in the U.S. Students needed to gather data, research global warming, create charts and plots, linear models, nonlinear models, identify which function type best fits the data, make predictions, and draw conclusions. Students were required to write up a written report of their findings and graphs. Lastly, they needed to include a page of the division of work among members of the group. The project was used as a summative assessment and counted for 10 percent of their grade.
CHAPTER FOUR: RESULTS

Pearson Correlation

The underlying mathematical goal of this experiment is not only to determine which assessment method (among the cumulative assessments, continuous assessments, and project) best predicts students’ posttest scores, but also to create a mathematical model that can approximate students’ posttest scores given their scores on the three aforementioned assessment methods.

We begin our analysis by defining and examining the relationships among our variables. The cumulative assessment, continuous assessment, and project scores make up our three predictor variables, or independent variables. The variable that we wish to predict the outcome of, the posttest scores, is our dependent variable. Each student’s average of the 10 cumulative assessment scores make up the cumulative assessment variable, and each student’s average of the 34 daily in-class quizzes make up the continuous assessment variable.

When determining the dependence that one variable has on another, we use the Pearson product-moment correlation coefficient; or more simply, the Pearson correlation. The Pearson correlation, (denoted by a lower-case r) acts as a scale as to how dependent one variable is on another. The correlation has values between -1 and 1, where a Pearson correlation of -1 corresponds to the increase (decrease) in value of one variable depending entirely upon the decrease (increase) in value of another respectively, a Pearson correlation of 0 corresponds to no dependence at all, and a Pearson correlation of
1 corresponds to the increase (decrease) in value of one variable depending entirely upon the increase (decrease) in value of another respectively (Cohen & Cohen, 1983).

We will first use the Pearson correlation to examine the reliability between two graders. We aim to have the Pearson correlation between two graders grading the same assessment to be as close to 1 as possible. In our experiment, we separated the two graders and they graded the pretest and posttest without collaborating with one another (to prevent grader bias and ensure that the rubric was well defined). The grader reliability between the two graders for the pretest and posttest were calculated using the Pearson correlation, and are presented in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grader A and B</td>
<td>0.8319</td>
<td>0.9343</td>
</tr>
</tbody>
</table>

As hoped, a high correlation between the graders is present for the pretest and the posttest. The higher correlation between the graders for the posttest may be attributed to more “practice” using the rubric, since both graders graded the pretest before the posttest. They did not collaborate at any time during, between, or after grading these assessments.

Now we will take a look at the Pearson correlation between our dependent and independent variables. When analyzing the Pearson correlation between two independent variables, we desire the correlation to be as close to 0 as possible, otherwise there is a dependency between independent variables. It is common to have a low dependency between two independent variables; however, when a higher dependency between two
independent variables is present, we run into a problem called multicollinearity, which will be explained later. The Pearson correlation among the independent and dependent variables is shown in Table 2.

Table 2: Pearson Correlation among Independent and Dependent Variables

<table>
<thead>
<tr>
<th></th>
<th>Pretest</th>
<th>Cumulative Assessment</th>
<th>Continuous Assessment</th>
<th>Project</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>1</td>
<td>0.0087</td>
<td>0.1297</td>
<td>-0.0484</td>
<td>0.1685</td>
</tr>
<tr>
<td>Cumulative Assessment</td>
<td>-</td>
<td>1</td>
<td>0.5227</td>
<td>0.098</td>
<td>0.3099</td>
</tr>
<tr>
<td>Continuous Assessment</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>0.0591</td>
<td>0.6339</td>
</tr>
<tr>
<td>Project</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>0.2137</td>
</tr>
<tr>
<td>Posttest</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

The Pearson correlation performed on the same variable is 1, because it fully depends on itself. The dashes are placed in cells where the Pearson correlation was already calculated. In other words, the Pearson correlation between the pretest and the cumulative assessment is the same as that of the cumulative assessment and the pretest, so a dash is placed to avoid redundancy. As expected, the Pearson correlation between the pretest and the independent and dependent variables is rather low (close to 0), since the pretest was given prior to any other assessment methods.

As hoped, the Pearson correlation between the cumulative assessment and the project (0.098) as well as the Pearson correlation between the continuous assessment and the project (0.0591) is close to 0, showing independence among independent variables. The problem, however, arises in the Pearson correlation between the cumulative
assessment and the continuous assessment (0.5227). This is rather high, possibly indicating multicollinearity. The details of treating multicollinearity will be discussed when we create our mathematical model.

Before a mathematical model can be developed, it is necessary to further examine the relationship between the independent variables and the dependent variable through the use of goodness of fit measurements and scatter plots.

The scatter plots shown below (Figure 1, Figure 2, and Figure 3) will help determine what type of relationship exists between each of the independent variables versus the dependent variable.

![Scatter plot of Cumulative Assessment Scores vs. Posttest Scores](image)

*Figure 1: Scatter plot of Cumulative Assessment Scores vs. Posttest Scores*
Figure 2: Scatter plot of Continuous Assessment Scores vs. Posttest Scores
The goal is to determine the simplest type of fit (for example, linear, quadratic, exponential, etc.) that best captures the relationship of the data. When analyzing goodness of fit in curve-fitting, there are two important quantities to consider: $R^2$ and the RMSE (root mean square error). $R^2$ is a statistic that measures how well the approximation curve fits the real data points. The closer $R^2$ is to 1, the better the goodness of fit. It is essentially the percentage of the variance in the dependent variable that is captured by the predictors (independent variables). The RMSE measures the difference between the values predicted using a model to the real data points. The closer the RMSE is to 0, the better the model fits the data (Cohen & Cohen, 1983).
Using the curve-fitting tool in MATLAB, several models (see Table 6) were created for each independent variable versus dependent variable. The goodness of fit for these models is summarized in Table 3, Table 4, and Table 5.

**Table 3: Goodness of Fit for Various Models of Cumulative Assessment Scores vs. Postest Scores**

<table>
<thead>
<tr>
<th>Model Type</th>
<th>R²</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.0960</td>
<td>13.25</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.1082</td>
<td>13.22</td>
</tr>
<tr>
<td>Cubic</td>
<td>0.1085</td>
<td>13.28</td>
</tr>
<tr>
<td>4th Degree</td>
<td>0.1229</td>
<td>13.23</td>
</tr>
<tr>
<td>Power</td>
<td>0.0799</td>
<td>13.43</td>
</tr>
<tr>
<td>Exponential</td>
<td>0.0986</td>
<td>13.23</td>
</tr>
<tr>
<td>Sum of Exponentials</td>
<td>0.1072</td>
<td>13.29</td>
</tr>
</tbody>
</table>

**Table 4: Goodness of Fit for Various Models of Continuous Assessment Scores vs. Postest Scores**

<table>
<thead>
<tr>
<th>Model Type</th>
<th>R²</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.4018</td>
<td>10.78</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.4090</td>
<td>10.76</td>
</tr>
<tr>
<td>Cubic</td>
<td>0.4090</td>
<td>10.81</td>
</tr>
<tr>
<td>4th Degree</td>
<td>-</td>
<td>10.76</td>
</tr>
<tr>
<td>Power</td>
<td>0.4088</td>
<td>10.86</td>
</tr>
<tr>
<td>Exponential</td>
<td>0.3926</td>
<td>10.81</td>
</tr>
<tr>
<td>Sum of Exponentials</td>
<td>0.4090</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 5: Goodness of Fit for Various Models of Project Scores vs. Posttest Scores**

<table>
<thead>
<tr>
<th>Model Type</th>
<th>R²</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.0457</td>
<td>13.62</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.0889</td>
<td>13.36</td>
</tr>
<tr>
<td>Cubic</td>
<td>0.0898</td>
<td>13.42</td>
</tr>
<tr>
<td>4th Degree</td>
<td>-</td>
<td>13.69</td>
</tr>
<tr>
<td>Power</td>
<td>0.0437</td>
<td>13.63</td>
</tr>
<tr>
<td>Exponential</td>
<td>0.0443</td>
<td>-</td>
</tr>
<tr>
<td>Sum of Exponentials</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 6: Respective Expressions for each Model Type

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Polynomial</td>
<td>$ax + b$</td>
</tr>
<tr>
<td>Quadratic Polynomial</td>
<td>$ax^2 + bx + c$</td>
</tr>
<tr>
<td>Cubic Polynomial</td>
<td>$ax^3 + bx^2 + cx + d$</td>
</tr>
<tr>
<td>4th Degree Polynomial</td>
<td>$ax^4 + bx^3 + cx^2 + dx + e$</td>
</tr>
<tr>
<td>Power</td>
<td>$ax^b + c$</td>
</tr>
<tr>
<td>Exponential</td>
<td>$ae^{bx}$</td>
</tr>
<tr>
<td>Sum of Exponentials</td>
<td>$ae^{bx} + ce^{dx}$</td>
</tr>
</tbody>
</table>

The cells that contain dashes indicated the inability for the model to converge to the data points.

To determine the best model, we want to choose the simplest model in which the $R^2$ is closest to 1 and the RMSE is relatively small. If possible, it is most efficient to choose the same model type for all of the independent versus dependent variable relationships (Cohen & Cohen, 1983).

Since the difference in $R^2$ and RMSE between the quadratic models and linear models is small, and those two models seem to be the simplest in which the $R^2$ was highest and the RMSE was lowest, we will represent the relationships between the independent variables and the dependent variables in a linear manner.

Using the curve-fitting tool in MATLAB, the following linear models were created between each independent variable versus the dependent variable.
Figure 4: Cumulative Assessment Scores vs. Posttest Scores Scatter Plot

Table 7: Coefficients and Possible Coefficient Interval with 95% Confidence Bounds for the Linear Model: Posttest Scores = p1*(Cumulative Assessment Scores) + p2

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Coefficient Interval (95% Confidence)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1 = 0.2765</td>
<td>(0.1198, 0.4331)</td>
</tr>
<tr>
<td>p2 = 57.45</td>
<td>(44.36, 70.55)</td>
</tr>
</tbody>
</table>
Figure 5: Continuous Assessment Scores vs. Posttest Scores Scatter Plot

Table 8: Coefficients and Possible Coefficient Interval with 95% Confidence Bounds for the Linear Model: Posttest Scores = p1*(Continuous Assessment Scores) + p2

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Coefficient Interval (95% Confidence)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1 = 0.6856</td>
<td>(0.531, 0.8401)</td>
</tr>
<tr>
<td>p2 = 32.12</td>
<td>(21.12, 43.13)</td>
</tr>
</tbody>
</table>
Project Scores vs. Posttest Scores Scatter Plot

Figure 6: Project Scores vs. Posttest Scores Scatter Plot

Table 9: Coefficients and Possible Coefficient Interval with 95% Confidence Bounds for the Linear Model: Posttest Scores = p1*(Project Scores) + p2

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Coefficient Interval (95% Confidence)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1 = 0.3921</td>
<td>(0.0611, 0.7232)</td>
</tr>
<tr>
<td>p2 = 45.33</td>
<td>(15.81, 74.84)</td>
</tr>
</tbody>
</table>

Multiple Regression

Choosing a linear relationship between the independent and dependent variables allows us to try modeling our data through the use of multiple regression. Multiple
regression is commonly used when researchers try to determine which predictor variable best predicts a certain outcome. It takes the following form:

\[ y = a + b_1x_1 + b_2x_2 + \ldots + b_nx_n \]

where the constant \( a \) represents the predicted criterion (\( y \)) when all the predictors \((x_1, x_2, \ldots, x_n)\) equal zero. The coefficients \( b_1, b_2, \ldots, b_n \) are known as the multiple regression coefficients or weights. They represent the average change in \( y \) for each unit increase in \( x_i \) (for \( i = 1, 2, \ldots, n \)) when the values of each of the independent variables are held constant (Cohen & Cohen, 1983).

Our multiple regression equation takes the following form:

\[ y = a + b_1x_1 + b_2x_2 + b_3x_3 \]

where \( y \) = posttest scores, \( x_1 \) = cumulative assessment scores, \( x_2 \) = continuous assessment scores, and \( x_3 \) = project scores. In other words,

\[
\text{Posttest} = a + b_1 \times \text{(Cumulative Assessment)} + b_2 \times \text{(Continuous Assessment)} + b_3 \times \text{(Project)}.
\]

Using MATLAB to create a multiple regression fit on our data, we came up with the following multiple regression equation:

\[
\text{Posttest} = 5.1432 - 0.0412 \times \text{(Cumulative Assessment)} + 0.7001 \times \text{(Continuous Assessment)} + 0.3303 \times \text{(Project)}.
\]
The maximum error is the maximum of the absolute value of the deviation of the data from the model. This equation had a maximum error of approximately 27.8325. In an attempt to reduce this error, we tried several nonlinear regression models in MATLAB to see if perhaps our linear model was the cause. However, these models (we tested quadratic, cubic, 4th degree polynomials, exponential, and power models) produced an even greater maximum error ranging from approximately 28 to 32. The RMSE for the multiple regression model is 10.39, and the $R^2$ is 0.4345.

We will now address the possibility of multicollinearity. When performing multiple regression analysis, multicollinearity occurs when one (or more) independent variable is highly correlated with another independent variable, suggesting dependence. It is expected that the independent variables are highly correlated with the dependent variable, but not with one another. Essentially, independent variables that are highly correlated with one another contribute nothing further to the ability of the regression line to predict the dependent variable. Multicollinearity causes small changes in the model to create large changes in the coefficients. When multicollinearity is present, it causes the model to provide incorrect results regarding how each individual predictor helps predict the dependent variable, but it does not affect how the multiple regression equation predicts the dependent variable when considering all of the predictor variables as a group. Multicollinearity tends to produce large standard errors (Cohen & Cohen, 1983).

**Stepwise Regression**

Multicollinearity can be treated through stepwise regression. Stepwise regression enters independent variables into a regression equation one at a time based on t-tests of
statistical significance. The predictor variable that has the highest correlation with the dependent variable is entered into the equation first. The rest of the variables are entered into the equation depending on the contribution of each predictor. Independent variables are no longer entered into the equation when they no longer make a statistically significant contribution. Therefore, the disadvantage of using stepwise regression is that one or more independent variables may be eliminated from the regression equation (Cohen & Cohen, 1983).

We used stepwise regression on our data using MATLAB and produced the following results shown in Table 10 and Table 11.

Posttest = 4.08135 + 0.67425*(Continuous Assessment) + 0.324578*(Project)

<table>
<thead>
<tr>
<th>Table 10: Goodness of Fit in Stepwise Regression Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>R²</td>
</tr>
<tr>
<td>0.432981</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 11: Coefficients, t-statistics, and p-values of the Stepwise Regression Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
</tr>
<tr>
<td>Continuous Assessment</td>
</tr>
<tr>
<td>Project</td>
</tr>
</tbody>
</table>

Hypothesis Testing and Statistical Significance

We now must determine through hypothesis testing and statistical significance tests what it is that our models are stating and with what confidence level. We begin by
providing the central measures of tendency for our independent and dependent variables (see Table 12).

\[
\text{Table 12: Central Measures of Tendency for Independent and Dependent Variables}
\]

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Range</th>
<th>Standard Deviation</th>
<th>Variance</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>9.0271</td>
<td>7.78</td>
<td>0</td>
<td>31.67</td>
<td>31.67</td>
<td>5.2396</td>
<td>27.45</td>
<td>0.4844</td>
</tr>
<tr>
<td>Cumulative Assessment</td>
<td>82.1526</td>
<td>87.5</td>
<td>21.67</td>
<td>100</td>
<td>78.33</td>
<td>15.5548</td>
<td>241.95</td>
<td>1.438</td>
</tr>
<tr>
<td>Continuous Assessment</td>
<td>70.0779</td>
<td>71.96</td>
<td>34.58</td>
<td>93.46</td>
<td>58.88</td>
<td>12.8318</td>
<td>164.66</td>
<td>1.1863</td>
</tr>
<tr>
<td>Project</td>
<td>88.8376</td>
<td>88</td>
<td>75</td>
<td>100</td>
<td>25</td>
<td>7.5646</td>
<td>57.22</td>
<td>0.6993</td>
</tr>
<tr>
<td>Posttest</td>
<td>80.1661</td>
<td>81.11</td>
<td>27.78</td>
<td>100</td>
<td>72.22</td>
<td>13.8781</td>
<td>192.60</td>
<td>1.283</td>
</tr>
</tbody>
</table>

We need these measures to examine the sum of squares due to regression (SSreg). The SSreg represents the part in which the independent (predictor) variables share with the dependent variable. The formula is

\[
SSreg = \sum (y' - \overline{y})^2
\]

where \( y' \) is the approximated dependent variable obtained from the regression, and \( \overline{y} \) is the mean of the dependent variable.

In addition to the SSreg, we also need to calculate the sum of squares residual (SSres). This is the part of the dependent variable that is not shared by any of the predictors. The formula is

\[
SSres = \sum (y - y')^2
\]
where $y$ is the dependent variable, and $y'$ is the approximated dependent variable obtained from the regression.

Lastly, the sum of squares total (SStotal) is obtained by adding the sum of squares due to regression (SSreg) to the sum of squares residual (SSres). The formula is

$$SS_{total} = \sum (y - \bar{y})^2$$

where $y$ is the dependent variable, and $\bar{y}$ is the mean of the dependent variable.

In order to gain confidence that our regression equation is able to predict the dependent variable, we must decide if SSreg is large enough in relation to SSres. This is done through hypothesis testing. We test the null hypothesis that SSreg = 0 versus the alternative hypothesis that SSreg > 0. This is done by conducting an F-test.

Table 13 corresponds to our multiple regression model:

$$\text{Posttest} = 5.1432 - 0.0412*(\text{Cumulative Assessment}) + 0.7001*(\text{Continuous Assessment}) + 0.3303*(\text{Project}).$$

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>9706.6</td>
<td>1</td>
<td>9706.6</td>
<td>88.35</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Residual</td>
<td>12634</td>
<td>115</td>
<td>109.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>22340.6</td>
<td>116</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The mean square column is calculated by dividing the SSreg and SSres by their respective degrees of freedom. The F-ratio is calculated by dividing the mean square
regression by the mean square residual. This ratio is now compared to an F-table of critical values (found in the appendices of statistics books). The critical value of the F-ratio with 1 degree of freedom in the numerator and 115 degrees of freedom in the denominator with an alpha level of .05 is 3.92. Since our F-ratio is greater than the critical F value, and our p-value is less than .05, we can conclude that the regression effect is greater than 0 and that at least one of the predictors accurately determines posttest scores.

To determine which predictors accurately determine posttest scores, we must now test each regression coefficient \( (b_1, b_2, b_3) \) for statistical significance. The null hypothesis is \( b_i = 0 \) and the alternative hypothesis is that \( b_i \neq 0 \) (for \( i = 1, 2, 3 \)). We now calculate the t-statistic for each using the following formula:

\[
t = \frac{b_i}{\left( \sqrt{\frac{\sum(y-y')^2}{n-k-1}} \right) \left( \sqrt{\sum(x-\bar{x})^2} \right)}
\]

where \( b_i \) is the regression coefficient, \( y \) is the posttest scores, \( y' \) is the approximated posttest scores obtained from the regression, \( n \) is equal to the sample size, \( k \) is the number of predictor variables, \( x \) is the predictor that corresponds to the regression weight \( b_i \) that is being examined, and \( \bar{x} \) is the mean of that predictor. The t-statistics for each predictor variable are summarized in Table 14.

\[
\begin{array}{|c|c|c|}
\hline
\text{t-statistic} & \text{Cumulative Assessment} & \text{Continuous Assessment} & \text{Project} \\
\hline
-0.6528 & 9.1506 & 2.5451 \\
\hline
\end{array}
\]

\textbf{Table 14: The t-statistics for each Predictor Variable (Multiple Regression Model)}
The t-statistic for the cumulative assessment predictor does not exceed the critical t-statistic at 0.1 (which is 1.289); therefore we fail to reject the null hypothesis that the cumulative assessment regression coefficient is nonzero. However, the t-statistic for the continuous assessment predictor does exceed the critical t-statistic at 0.1. Moreover, it exceeds the critical t-statistic at 0.0005 (which is 3.373), creating a 99.95% confidence level. Therefore, we reject the null hypothesis that the continuous assessment regression coefficient is equal to zero. In addition, the t-statistic for the project predictor does exceed the critical t-statistic at 0.1. Moreover, it exceeds the critical t-statistic at 0.01 (which is 2.358), creating a 99% confidence level. Thus, again we reject the null hypothesis that the project regression coefficient is equal to zero. This allows us to conclude that only the continuous assessment predictor and the project predictor are significantly related to the posttest. Simply, only the continuous assessment scores and project scores are accurate predictors of students’ posttest performance.

Since we only performed these statistical significance tests on our multiple regression model, we now perform them on our stepwise regression model:

Posttest = 4.08135 + 0.67425*(Continuous Assessment) + 0.324578*(Project)

<table>
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<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
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<td>Total</td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 15: Source Table for Stepwise Regression Model
The t-statistic for the continuous assessment predictor does exceed the critical t-statistic at 0.1. Moreover, it exceeds the critical t-statistic at 0.0005 (which is 3.373), creating a 99.95% confidence level. Therefore, we reject the null hypothesis that the continuous assessment regression coefficient is equal to zero. In addition, the t-statistic for the project predictor does exceed the critical t-statistic at 0.1. Moreover, it exceeds the critical t-statistic at 0.01 (which is 2.358), creating a 99% confidence level. Thus, again we reject the null hypothesis that the project regression coefficient is equal to zero. This allows us again to conclude that the continuous assessment scores and project scores are accurate predictors of students’ posttest performance.

Recall that the closer the RMSE is to 0, and the closer R² is to 1, the better the goodness of fit. The RMSE of the multiple regression model is 10.39 which is closer to 0 than that of the stepwise regression model (10.5416), and the R² of the multiple regression model is 0.4345 which is closer to 1 than that of the stepwise regression model (0.4329). This indicates that the model that best fits the data is the multiple regression model. This makes sense since our data contains the cumulative assessment scores which made an impact on posttest scores (just not a statistically significant one). However, our main priority is not to find a model that best fits our data, but to find a model that best predicts students’ posttest scores. Therefore, based on the results of the statistical significance t-tests, the best predictor model for our data is the stepwise regression model:
Posttest = 4.08135 + 0.67425*(Continuous Assessment) + 0.324578*(Project)

Lastly, recall that the regression coefficients indicate the average change in posttest scores for each unit increase in one of the predictor variables when the value of each of the other predictor variables is held constant. Since the regression coefficient of the continuous assessment predictor is larger than that of the project predictor, we may conclude that students’ continuous assessment scores best predict their posttest scores.
CHAPTER FIVE: CONCLUSIONS

The main finding of this experiment was that the continuous assessments, in the form of daily in-class quizzes containing one homework question and one concept question that required students to read ahead before class, were the primary predictors of students’ posttest scores. This affirms our hypothesis to be true. Moreover, the combination of the project scores and the continuous assessment scores served as a stronger predictor of students’ posttest scores. Lastly, we were able to construct a mathematical model (by modifying a multiple regression model to a more accurate and statistically significant stepwise regression model) to model the relationship between the two predictor variables, continuous assessment scores and project scores, to the outcome variable, posttest scores.

Reflecting upon the implementation of this experiment, it is evident that the results were logical. The students completed the cumulative assessments online and at home. This allowed them to work the problems relatively at their own pace (the assignments were, however, only available for a short period of time), and they had an unlimited amount of tries per problem on some of the assignments. Perhaps the lower-achieving students may have merely copied other students’ answers on the cumulative assignments, without attempting to learn and understand the material. Or perhaps students took a lot of time to complete the cumulative assessments, and thus struggled with time on the posttest. This may explain why the cumulative assessment variable was not statistically significant, and thus, was eliminated from the stepwise regression model.

The reason that the project grades are good predictors of students’ posttest grades is most likely due to the fact that not only do projects require students to understand the
mathematical material, but also to apply what they have learned. This is consistent with Boaler’s (1998) findings that project-based learning increases students’ abilities to apply their mathematical knowledge. The projects were done in groups, permitting the students to collaborate with their peers and allowing an exchange of mathematical thinking. If one group member did not understand the mathematics, other group members may have found a better way to connect and teach the concepts to the misunderstood peer.

Furthermore, it is logical that the continuous assessments were the best predictors of students’ posttest performance. In essence, the continuous assessments were the best monitor of student performance throughout the course. To perform well on the daily in-class quizzes, students needed to have had their homework completed, or at least have understood the material assessed in the homework, and they needed to have been prepared for class by having read the section to be covered that day. The students who had completed these tasks performed well on the quizzes, and ultimately performed well on the posttest. This is consistent with Mawhinney, Bostow, Laws, Blumenfeld, and Hopkins’ (1971) findings that continuous assessment promotes studying; with Marchant’s (2002) findings that announced quizzes cause students to read the material more closely; with Ruscio’s (2001) findings that announced quizzes can promote reading ahead; and with Stephens and Konvalina’s (2001) findings that short quizzes influence student success. All in all, these conclusions follow closely to any teacher’s ideals: teachers want students to come to class having completed the homework and read the material. They are then ready to ask questions and have any misconceptions corrected.

This experiment may not produce the same results for other classes; every teacher has a different teaching style, various groups of students have different learning styles
and ability levels, perhaps this experiment worked well with a large class size as opposed to a small class size, or maybe it works better for pre-calculus courses than for calculus or trigonometry courses. Only further experimentation would be able to confirm or falsify these claims. However, the stepwise regression model that was formulated was proven to be statistically significant with a 99% confidence level. Thus, if professors implement daily in-class quizzes and projects they will be able to monitor student progress and better predict how students will perform on final exams. This allows professors to detect early on (based on quiz and/or project performance) whether or not students are struggling with the material. Thus, they are given feedback, as Suskie (2004) mentioned, on what is and is not working and how to improve teaching and learning. Moreover, professors should use their data results to enforce changes that will lead to better learning, as Huba and Freed (2000) suggested. Furthermore, the projects teach students how to apply their mathematical skills and learn to work in collaborative groups. The in-class quizzes will promote students to complete their homework and read ahead, better preparing them for class. On a grander scale, perhaps students will maintain such a habit and apply it to other courses, and more professors will adopt these assessment types in their courses, ultimately leading to greater student success in college mathematics.
CHAPTER SIX: FUTURE WORK

One possible extension of this experiment would be to repeat the experiment on either larger or smaller class sizes. This may provide more insight on how class size affects the stepwise regression model. In addition, the experiment should also be implemented in other various undergraduate mathematics classes, and to different academic standings (i.e., sophomores, juniors, and seniors). Then, a comparison can be made among the stepwise regression models of this experiment and the experiments tested on other mathematics courses and academic levels. It would also be informative to know how the model changes with respect to different instructors.

An ideal follow-up experiment would be to test each independent variable separately against a control group in a series of three courses. For example, implementing the cumulative assessments to one course and not implementing them in another, then comparing student performance. Then repeat the experiment for continuous assessments and projects. This may allow researchers to conclude which assessment method improves student learning the most. Lastly, it would be interesting to analyze the model if it included SAT scores and math placement scores as predictor variables as well.
APPENDIX A: SYLLABUS
MAC 2147

Math for Calculus: Algebra & Trigonometry  
M/W/F 2:30pm-4:20pm  
HPA 112

Professor: Dr. Cynthia Y. Young  
Office: MAP 231G  
Office Phone: 823-5987  
Email: cyyoung@mail.ucf.edu  
Office Hours: Mondays 1:30-2:30pm  
Wednesdays 11:00am-12:00pm  
Fridays 1:30pm -2:30pm

Math Lab: MAP 113 offers free tutoring for UCF students  
EXCEL Lab: CCII room 223 offers free tutoring for EXCEL students


Classroom Rules:
No hats are to be worn in class.  
Cell phones must be on SILENT (not vibrate or ringing).  
Quizzes will start at 2:30pm and end at 2:35pm. If you are late then you will receive a 0 for that quiz.

Study Expectations: This class is a five-credit course and it is expected that you spend a minimum of 10 hours outside of class reading, working homework problems, completing assignments and studying. Many students will require more than 10 hours.

Grades:

<table>
<thead>
<tr>
<th>Assessment</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tests</td>
<td>40%</td>
</tr>
<tr>
<td>Final Exam</td>
<td>25%</td>
</tr>
<tr>
<td>Quizzes</td>
<td>15%</td>
</tr>
<tr>
<td>Cumulative Assignments (Wiley Plus)</td>
<td>10%</td>
</tr>
<tr>
<td>Team Project: Climate Change</td>
<td>10%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
</tr>
</tbody>
</table>

A: 90%-100%  B: 80%-89%  C: 70%-79%  F: 0%-69%

D’s cannot be assigned in this course. A grade of NC (No Credit) will be assigned if the following criteria are met: Course grade is less than 70%, and you complete the course.

ASSESSMENT DESCRIPTIONS
Tests: There will be four tests given (see schedule). Make up tests will only be given to students representing the university during a test with proper documentation. If a test is missed for any other reason it will be replaced by the final exam grade. Your test average is worth 40% of your grade.

Final Exam: The final exam is cumulative and is worth 25% of your grade. It will be given at 1:00pm – 3:50pm on Wednesday December 5, 2007.

Quizzes: There will be 34 scheduled quizzes (see schedule). Each scheduled quiz is worth 3 points and will have two problems. One problem (worth 2 points is a problem directly from the suggested homework due that day) and one problem (worth 1 point) is a pretest knowledge question to ensure that you have read the material to be covered that class period.

Homework: A list of homework problems for each section is given in this syllabus. It is suggested homework to do after we have covered the material. The two-point problem on every quiz comes directly from these assigned problems.

Wiley Plus: Wiley Plus is an online course management system with many valuable features. The entire book is there electronically as well as the entire student solutions manual (with full solutions). In addition there are videos. Anywhere you see a video icon next to an example in the book; you can click in Wiley plus and watch a video of me working that problem. I encourage you to use this system as a supplement. There are 10 scheduled cumulative assignments (see schedule) that you are required to complete. Your Wiley plus average is worth 10% of your grade.

Course Logins: http://edugen.wiley.com/edugen/class/cls40433/
First Day (Registering) http://wiley.breezecentral.com/firstday
Tech Support: http://hesupport.wiley.com/wileyplus

Project: The GEP Unifying Theme is *Global Climate Change*. When UCF asked students what they care about, undergraduates responded with a clear, unified voice: the environment, specifically Global Climate Change. There will be a project (see schedule) that will be completed in assigned teams. The project will be 10% of your grade.

Calculators: Only non programmable, non graphing calculators allowed on tests.

*The withdrawal deadline is Friday October 12, 2007.*

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<th>Date</th>
<th>Class Material</th>
<th>Assessment</th>
<th>Wiley Plus</th>
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<td>Pretests</td>
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### Homework Problems

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RUBRIC/ANSWER KEY

1. Find $BC$ if $B = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 2 & -1 \\ 3 & 1 \end{bmatrix}$.

Step 1: Knowing that it is $BC$ not $CB$:

Step 2: Demonstration of Matrix Multiplication Process:

$$BC = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 + 4 + 3 & 0 - 2 + 1 \\ 0 - 4 + 12 & 3 + 2 + 4 \end{bmatrix}_{2 \times 2}$$

Step 3: Simplification:

$$\begin{bmatrix} 7 & -1 \\ 8 & 9 \end{bmatrix}$$

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TOTAL VALUE: 5 points

2. Find the sum of the infinite series if possible: $\sum_{n=1}^{\infty} \left(-\frac{1}{3}\right)^n$

Step 1: Write in the form $\sum_{k=1}^{\infty} a_k r^{k-1}$.

$$\sum_{n=1}^{\infty} \left(-\frac{1}{3}\right)^n = \sum_{n=1}^{\infty} \left(-\frac{1}{3}\right)^{n-1} = \sum_{n=1}^{\infty} \left(a_n \left(-\frac{1}{3}\right)^n\right)$$

Step 2: Demonstrate knowledge of formula: $S_\infty = a_1 \frac{1}{1-r}$, $r < |1|$

Let $a_1 = -\frac{1}{3}$ and $r = -\frac{1}{3}$

$$S_\infty = -\frac{1}{3} \frac{1}{1 - \left(-\frac{1}{3}\right)} = \left(-\frac{1}{3}\right) \left(\frac{1}{1 + \frac{1}{3}}\right)$$
Step 3: Simplify

\[
\left(-\frac{1}{3}\right)\left(\frac{1}{1+\frac{1}{3}}\right) = \left(-\frac{1}{3}\right)\left(\frac{1}{\frac{4}{3}}\right) = \left(-\frac{1}{3}\right)\left(\frac{3}{4}\right) = -\frac{1}{4}
\]

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Total Value: 6 Points

3. The structure of the \((\text{FeCl}_4\text{Br}_2)^3\) ion (dibromatetrachlorideferrate III) is shown in the figure. Determine the angle, \(\theta\) (the angle between the axis containing the apical atom bromide (Br) and the segment connecting Br to Cl).

Let \(\alpha\) be the supplementary angle to \(\theta\).

Step 1: Set up right triangle trigonometric ratio

\[
\tan \alpha = \frac{2.249}{2.354}
\]

\[
\alpha = \tan^{-1}\left(\frac{2.249}{2.354}\right)
\]

\(\alpha \approx 43.7^\circ\)

\[\theta = 180^\circ - 43.7^\circ = 136.3^\circ\]

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Total Value: 9 points
4. If \( f(x) = \frac{2x + 5}{7 + x} \)

A. Find its inverse, \( f^{-1} \).
B. State the domain of \( f^{-1} \).
C. State the range of \( f^{-1} \).

Step 1: Let \( f(x) = y \)

\[
y = \frac{2x + 5}{7 + x}
\]

interchange \( x \) and \( y \).

Step 2: Solve for \( y \).

\[
x(7 + y) = 2y + 5
\]
\[
7x + xy = 2y + 5
\]
\[
2y - xy = 7x - 5
\]
\[
y(2 - x) = 7x - 5
\]
\[
y = \frac{7x - 5}{2 - x}
\]

Step 3: Let \( y = f^{-1}(x) \)

\[
f^{-1}(x) = \frac{7x - 5}{2 - x}
\]

Step 4: State domain of \( f^{-1} \).

\((\infty, 2) \cup (2, \infty) \) or \( x \neq 2 \)

Step 5: Find the range of \( f^{-1} \).

The range of \( f^{-1} \) is equal to the domain of \( f \). \((\infty, -7) \cup (-7, \infty) \) or \( x \neq -7 \)

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Total Value: 9 points

5. Solve the equation \( 6t^{-2/3} - t^{-1/3} - 1 = 0 \).

Step 1: Let \( u = t^{-1/3} \).

\[
6u^2 - u - 1 = 0
\]

Step 2: Solve for \( u \).

\[
(3u + 1)(2u - 1) = 0
\]

\( 3u + 1 = 0 \) or \( 2u - 1 = 0 \)

\( u = -\frac{1}{3} \) or \( u = \frac{1}{2} \)

Step 3: Let \( u = t^{-1/3} \).

\[
t^{-1/3} = -\frac{1}{3} \text{ or } t^{-1/3} = \frac{1}{2}
\]

Step 4: Solve for \( t \).

\[
t = \left(-\frac{1}{3}\right)^{-3} \text{ or } t = \left(\frac{1}{2}\right)^{-3}
\]
\[ t = (-3)^3 \quad \text{or} \quad t = 2^3 \]
\[ t = -27 \quad \text{or} \quad t = 8 \]

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Total Value: 9 points

6. Solve the equation \[ \log_2(x + 1) + \log_2(4 - x) = \log_2(6x) \]

Step 1: Write both sides as a single log.
\[ \log_2[(x + 1)(4 - x)] = \log_2(6x) \]

Step 2: Use one-to-one function or inverse function to eliminate logs.
Note: They can say \( [x] = 6x \) or exponentiate both sides to get same result.
\[ (x + 1)(4 - x) = 6x \]

Step 3: Solve quadratic for \( x \).
\[ -x^2 + 3x + 4 = 6x \]
\[ x^2 + 3x - 4 = 0 \]
\[ (x + 4)(x - 1) = 0 \]
\[ x = -4 \quad \text{or} \quad x = 1 \]

Step 4: Eliminate any extraneous solutions.
\( x = -4 \) is not in domain of equation, answer is only \( x = 1 \).

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Total Value: 8 points

7. The concentration \( C \) of a particular drug in a person’s bloodstream \( t \) minutes after injection is given by

\[ C(t) = \frac{2t}{t^2 + 100} \]

What is the concentration in the bloodstream after 1 minute?
What is the concentration in the bloodstream after 1 hour?
What is the concentration in the bloodstream after 1 day?

Step 1: Find \( C(1) \).
\[ C(1) = \frac{2(1)}{1^2 + 100} = \frac{2}{101} \approx 1.98\% \]
Step 2: Find $C(60)$. 

$$C(60) = \frac{2(60)}{60^2 + 100} = \frac{120}{3700} \approx 3.2\%$$

Step 3: Find $C(1440)$. 

$$C(1440) = \frac{2(1440)}{1440^2 + 100} = \frac{2880}{2073700} \approx 0.14\%$$

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Total Value: 6 points

8. In 2003, there were an estimated 25 million people who had been infected with HIV in sub-Saharan Africa. If the infection rate increases at an annual rate of 9% a year compounding continuously, how many Africans will be infected with HIV by 2010?

Step 1: Write the population model. 

$$N = N_0 e^{rt}$$

Step 2: Let $N_0 = 25$, $r = 0.09$, and $t = 7$. 

$$N = 25e^{(0.09)(7)}$$

Step 3: Use a calculator and approximate. 

$$N \approx 46.9 \text{ million people}$$

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Total Value: 5 points

9. Graph the piecewise defined function and state its domain and range.

$$f(x) = \begin{cases} 
x & x \leq -1 \\
x^3 & -1 < x < 1 \\
x^2 & x > 1 
\end{cases}$$

Step 1: Graph the three functions 
(1 point for structure and 1 point for open hole at $x=1$).
Step 2: State the Domain. \((-\infty,1) \cup (1,\infty)\) or \(x \neq 1\)
Step 3: State the Range. \((-\infty,1) \cup (1,\infty)\) or \(y \neq 1\)

Total Value: 7 points

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10. Find the partial fraction decomposition of \(\frac{3x+1}{x^2+4x+4}\).

Step 1: Split into a sum of a linear and repeated term.
\[
\frac{3x+1}{x^2+4x+4} = \frac{3x+1}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}
\]

Step 2: Equate Numerators.
\[A(x+2) + B = 3x+1\]
Step 3: Solve for A and for B.
\[Ax + 2A + B = 3x+1\]

Match linear terms: \(A = 3\)
Match constant terms: \(2A + B = 1\)
\[A = 3\] and \(B = -5\)

Total Value: 6 points

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<td>Step 1</td>
<td>1</td>
</tr>
<tr>
<td>Step 2</td>
<td>1</td>
</tr>
<tr>
<td>Step 3</td>
<td>1</td>
</tr>
</tbody>
</table>

11. State the center and radius of the circle given by the equation:
\[x^2+y^2-10x+6y+22=0\]

Step 1: Complete the square:
\[(x^2-10x)+(y^2+6y)=-22\]
\[(x^2-10x+25)+(y^2+6y+9)=-22+25+9\]
\[(x-5)^2+(y+3)^2=12\]
Step 2: Identify the center. \((5,-3)\)
Step 3: Identify the radius. \(r = \sqrt{12}\)

Total Value: 7 points

<table>
<thead>
<tr>
<th>Process</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>2</td>
</tr>
<tr>
<td>Step 2</td>
<td>1</td>
</tr>
<tr>
<td>Step 3</td>
<td>1</td>
</tr>
</tbody>
</table>
12. A plane has a compass heading of $60^\circ$ (east of north) and an airspeed of 300 mph. The wind is blowing at 40 mph with a heading of $30^\circ$ (west of north). What is the plane’s actual heading an airspeed?

Step 1: Draw the picture.

Add the vectors (Tail to Tip) and draw resultant vector.

Step 2: Find $x$.

\[ 300^2 + 40^2 = x^2 \]
\[ x^2 = 91,600 \]
\[ x = \sqrt{91,600} \approx 302.65 \]

Step 3: Find $\theta$.

\[ \tan \theta = \frac{40}{300} \]
\[ \theta = \tan^{-1}\left(\frac{40}{300}\right) \approx 7.6^\circ \]

Step 4: Find $\beta$.

$\beta = 60^\circ - \theta \approx 52.4^\circ$

Step 5: Give answer in words.

Heading 302.54 mph at $52.4^\circ$ East of North

<table>
<thead>
<tr>
<th>Total Value: 13 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
</tr>
<tr>
<td>Step 1</td>
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<td>Step 3</td>
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<td>Step 4</td>
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<td>Step 5</td>
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</tbody>
</table>
APPENDIX C: CONTINUOUS ASSESSMENTS (QUIZZES)
Quiz 1

1. (Section 0.5, #29) Factor into a product of three polynomials:

\[ x^3 - 9x \]

2. The following:

\[ x + 5 = x - 2 \]

is an example of

(a) a conditional equation

(b) an extraneous solution

(c) a quadratic equation

(d) an inconsistent equation or contradiction
Quiz 2

1. For a certain chemistry experiment, a student requires 100 ml of a solution that is 8% HCl (hydrochloric acid). The storeroom has only solutions that are 5% HCl and 15% HCl. How many milliliters of each available solution should be mixed to get 100 ml of 8% solution?

2. In the quadratic formula:

\[
ax^2 + bx + c = 0, \quad a \neq 0,
\]

then

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

the term inside the radical, \(b^2 - 4ac\), is called the discriminant. The solutions or roots of a quadratic equation with a \textit{negative} discriminant are:

(a) One double or repeated root

(b) Two distinct real roots

(c) Two complex roots, complex conjugates

(d) Nonexistent; there is no solution
1. (Section 1.4, #47) Solve the equation by introducing a substitution that transforms the equation to quadratic form.

\[ 6t^{2/3} - t^{1/3} - 1 = 0 \]

2. Which of the following is the graph of \(|x| > a\)?

(a) ![Graph A]

(b) ![Graph B]

(c) ![Graph C]

(d) ![Graph D]
1. (Section 1.6, #33) Solve the rational inequality and express the solution in interval notation.

\[ \frac{3p - 2p^2}{4 - p^2} < \frac{3 + p}{2 - p} \]

2. The graph shown above is symmetric with respect to

(a) the origin
(b) the \(x\)-axis
(c) the \(y\)-axis
(d) the line \(y = x + 1\)
1. (Section 2.2, #45) Calculate the length and the midpoint for the segment joining the points \((a, b)\) and \((b, a)\).

2. The lines:

\[
y = \frac{1}{3} x - 1
\]

\[
y = \frac{1}{3} x - 6
\]

are

(a) perpendicular

(b) vertical

(c) parallel

(d) horizontal
1. (Section 2.4, #33) State the center and radius of the circle given by the equation:

\[ x^2 + y^2 - 10x + 6y + 22 = 0 \]

2. \( h(x) = x^2 - x \)

The function \( h(x) \) (described above) is

(a) even

(b) odd

(c) neither even nor odd

(d) symmetric with respect to the origin
Quiz 7

1. (Section 3.2, #65) A famous author negotiates with her publisher the monies she will receive for her next suspense novel. She will receive $50,000 up front and a 15% royalty rate on the first 100,000 books sold, and 20% on any books sold beyond that. If the book sells for $20 and royalties are based on the selling price, write a royalties function, \( R(x) \), as a function of the total number of books sold, \( x \).

2. The graph of \( f(x-a)+b \) is the graph of \( f(x) \) shifted:

   (a) \( a \) units right, \( b \) units up
   (b) \( b \) units right, \( a \) units down
   (c) \( a \) units left, \( b \) units up
   (d) \( b \) units left, \( a \) units down
1. (Section 3.4, #31) Evaluate $f(g(1))$ and $g(f(2))$, if possible.

$$f(x) = \sqrt{1-x} \quad g(x) = x^2 + 2$$

2. The inverse of the function $f(x) = |x|$ shown below is

(a) $f^{-1}(x) = -|x|$
(b) $f^{-1}(x) = |y|$
(c) $f^{-1}(x) = \frac{1}{|x|}$
(d) $f^{-1}(x)$ does not exist
1. (Section 3.5, #52) The function $f$ is one-to-one. Find its inverse, and check your answer. State the domain and range of both $f$ and $f^{-1}$.

$$f(x) = -\frac{3}{x}$$

2. The quadratic function $f(x) = (x-3)^2 - 1$

(a) opens up and has a vertex of $(3,-1)$

(b) opens up and has a vertex of $(3,-1)$

(c) opens down and has a vertex of $(-3,1)$

(d) opens down and has a vertex of $(3,1)$
Name: ________________________________

Quiz 10

1. (Section 4.2, #30) Find all the real zeros (and state their multiplicity) of the polynomial function.

\[ f(x) = 5x^3(x+1)^4(x-6) \]

2. What can be concluded based on the correctly solved division problem shown below?

\[
\begin{array}{c|ccccc}
\multicolumn{2}{c}{2x^3 + 3x^2 - 1x - 5} \\
\hline
x - 2 & 2x^4 & -1x^3 & -7x^2 & -3x & +10 \\
\hline
& 2x^4 & -4x^2 & & & \\
& 0 & 3x^3 & -7x^2 & -3x & +10 \\
& & 3x^3 & -6x^2 & & \\
& & 0 & -x^2 & -3x & +10 \\
& & & -x^2 & +2x & \\
& & & 0 & -5x & +10 \\
& & & & -5x & +10 \\
& & & & 0 & \\
\end{array}
\]

(a) \(x - 2\) is a factor of the polynomial \(2x^4 -1x^3 -7x^2 -3x +10\)

(b) \(x - 2\) is a factor of the polynomial \(2x^3 +3x^2 -1x -5\)

(c) 2 is a zero of the polynomial \(2x^3 +3x^2 -1x -5\)

(d) \(-2\) is a zero of the polynomial \(2x^4 -1x^3 -7x^2 -3x +10\)
Quiz 11

1. Given the polynomial \( P(x) = x^3 - 7x^2 - x + 7 \) write the polynomial in terms of a product of linear factors.

2. Rational Functions have vertical asymptotes that correspond to
   (a) The values that make the numerator equal to zero.
   (b) The values that make the denominator equal to zero.
   (c) The degree of the numerator is less than the degree of the denominator.
   (d) The degree of the numerator is equal to the degree of the denominator.
1. (Section 4.5, #43) Graph the following rational function. Label the intercepts and asymptotes (if there are any).

\[ f(x) = \frac{2x^3 - x^2 - x}{x^2 - 4} \]

2. Which of the following statements is true regarding the exponential function:

\[ f(x) = b^x \quad b > 0 \quad b \neq 1 \]

(a) The domain of \( f(x) \) is \((-\infty, \infty)\) and the range of \( f(x) \) is \((0, \infty)\)

(b) \( f(x) \) has no x-intercepts, but it has a y-intercept at \((0,1)\)

(c) The x-axis is a horizontal asymptote in the graph of \( f(x) \)

(d) All of the above
Quiz 13

3. (Section 5.2, #26) A college student placed an ad on www.match.com on a
   Monday, and by Wednesday she had received 60 e-mail messages from
   potential suitors. She found that every day the number of e-mails from new
   potential suitors decreased 10 percent from the day before. How many new e-
   mails would she expect to receive on the following Sunday?

4. Which of the following equations is equivalent to $y = \log_b x$?
   
   (a) $y = b^x$
   
   (b) $x = b^y$
   
   (c) $b = x^y$
   
   (d) $b = y^x$
Quiz 14

1. (Section 5.4, #41) Write the expression as a single logarithm.

\[ \ln(x+1) + \ln(x-1) - 2\ln(x^2 + 3) \]

2. What would be the first step in solving the following equation:

\[ \ln(3 - x^2) = 7 \]

a) Subtract 7 from both sides of the equation to make the equation equal zero.

b) Divide both sides of the equation by ln.

c) Exponentiate (base e) both sides of the equation.

d) Take the natural logarithm (ln) of both sides of the equation.
Quiz 15

1. (Section 5.5, #24) Solve the logarithmic equation exactly.

   \[ \log_2 (3x - 1) = 3 \]

2. Using the triangle below, which of the following equations is true?

   \[ \begin{align*}
   \text{(a)} \quad \cos \theta &= \frac{a}{b} \\
   \text{(b)} \quad \sec \theta &= \frac{c}{a} \\
   \text{(c)} \quad \cot \theta &= \frac{b}{a} \\
   \text{(d)} \quad \sin \theta &= \frac{c}{a}
   \end{align*} \]
Quiz 16

1. (Section 6.3, #41) Use the illustration below that shows a search and rescue helicopter with a 30° field of view with a search light to answer the following question: If the search and rescue helicopter is flying at an altitude of 150 feet above sea level, what is the diameter of the circle that is illuminated on the surface of the water?

2. Which of the following is the reference angle for 600°?

   (a) 420°
   (b) 240°
   (c) 60°
   (d) 30°
1. (Section 6.5, #13) If $\sec \theta = -2$, and the terminal side of $\theta$ lies in quadrant III, find $\tan \theta$.

2. Graphs of the form $y = A \sin Bx$ and $y = A \cos Bx$ have

   (a) amplitude $|A|$ and period $\frac{B}{2\pi}$

   (b) amplitude $B$ and period $2\pi |A|$  

   (c) amplitude $|A|$ and period $\frac{2\pi}{B}$

   (d) amplitude $2\pi |A|$ and period $B$
Quiz 18

1. (Section 6.8, #32) Graph the given function over one period.

\[ y = -4 \sin \left( \frac{\pi}{2} x \right) \]

2. The graphs of

\[ y = \tan x \]
\[ y = \cot x \]
\[ y = \sec x \]
\[ y = \sec x \]

all have

(a) vertical asymptotes

(b) an amplitude of 1

(c) a range of \(( -\infty, \infty )\)

(d) a period of \( \pi \)
Quiz 19

1. (Section 6.9, #33) Graph the function over at least one period.

\[ 3 - 2 \sec \left( x - \frac{\pi}{2} \right) \]

2. Which of the following statements is false?

(a) \( \tan \theta = \frac{\sin \theta}{\cos \theta} \)

(b) \( \cot \theta = \frac{1}{\tan \theta} \)

(c) \( \sec \theta = \frac{1}{\sin \theta} \)

(d) \( \cot \theta = \frac{\cos \theta}{\sin \theta} \)
Quiz 20

1. (Section 7.2, #28) Verify the trigonometric identity.

\[
\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} = 2 \csc^2 x
\]

2. Which of the following statements is always true?

(a) \(\sin\left(\frac{\pi}{2} - \theta\right) = \sin \theta\)

(b) \(\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta\)

(c) \(\sin\left(\frac{\pi}{2} - \theta\right) = 1 - \sin \theta\)

(d) \(\cos\left(\frac{\pi}{2} - \theta\right) = -\cos \theta\)
Name: ________________________________

**Quiz 21**

1. (Section 7.3, #27) Find the exact value of \( \sin(\alpha - \beta) \) if \( \sin \alpha = -\frac{3}{5} \) and \( \sin \beta = \frac{1}{5} \) and the terminal side of \( \alpha \) lies in QIII and the terminal side of \( \beta \) lies in QI.

2. Which of the following statements is always true?

   (a) \( \cos A \cos B = \cos(AB) \)

   (b) \( \sin A \sin B = \sin^{2}(AB) \)

   (c) \( \sin A \cos B = \sin B \cos A \)

   (d) none of the above
1. (Section 7.5, #3) Use the half-angle identity to find the exact value of the trigonometric expression \( \cos \left( \frac{11\pi}{12} \right) \).

2. The range of \( y = \sin^{-1}(x) \) is

   (a) \([-1,1]\]

   (b) \([0,\pi]\]

   (c) \(\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \)

   (d) \((-\infty, \infty)\)
Quiz 23

1. (Section 7.8, #92) If a person breathes in and out every 3 seconds, the volume of air in lungs can be modeled by \( A = 2\sin\left(\frac{\pi}{3}x\right) + 3 \), where \( A \) is in liters of air and \( x \) is in seconds. How many seconds into the cycle is the volume of air equal to 2 liters?

2. A little girl is standing at a known distance away from the bottom of a slide. The angle of elevation from her feet to the top of the slide and the angle formed between the slide and the ground are also known. Is it possible to figure out how long the slide is?

   (a) No, because we also need to know the distance from the girl’s feet to the top of the slide.

   (b) No, because we also need to know the height of the slide’s ladder.

   (c) Yes, because we can solve an Angle-Angle-Side triangle case.

   (d) Yes, because we can solve an Angle-Side-Angle triangle case.
1. (Section 8.3, #26) Calculate the area of the so called Bermuda Triangle if, as other people define it, its vertices are located in Miami, Bermuda, and San Juan:

<table>
<thead>
<tr>
<th>Location</th>
<th>Location</th>
<th>Distance (nautical miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miami</td>
<td>Bermuda</td>
<td>898</td>
</tr>
<tr>
<td>Bermuda</td>
<td>San Juan</td>
<td>831</td>
</tr>
<tr>
<td>Miami</td>
<td>San Juan</td>
<td>890</td>
</tr>
</tbody>
</table>

2. If the dot product of two vectors is zero, then
   
   (a) The two vectors are parallel.
   
   (b) The two vectors are perpendicular.
   
   (c) The magnitude of one of the vectors must be zero.
   
   (d) The angle between the two vectors is 0° or 180°.
1. (Section 8.4, #63) A plane has a compass heading of 60° (east of due north) and an airspeed of 300 mph. The wind is blowing at 40 mph with a heading of 30° (west of due north). What is the plane’s actual heading and airspeed?

2. The point plotted in the graph above can be written as:

(a) \( b + ai \)

(b) \( b - ai \)

(c) \( (a + b)i \)

(d) \(- a - bi \)
Quiz 26

1. (Section 8.7, #15) Find the quotient, $\frac{z_1}{z_2}$, and express it in rectangular form.

$$z_1 = \sqrt{12} \left[ \cos 350^\circ + i \sin 350^\circ \right] \text{ and } z_2 = \sqrt{3} \left[ \cos 80^\circ + i \sin 80^\circ \right]$$

2. How many solutions does the following system of linear equations have?

$$y = \frac{1}{3} x - 1$$
$$y = \frac{1}{3} x + 1$$

a. One
b. Two
c. Infinitely many
d. None
1. (Section 8.8, #37) Graph \( r = 2 \cos \theta \).

2. Match the rational expression with the form of the partial fraction decomposition.

\[
\frac{3x + 2}{x(x^2 - 25)}
\]

(a) \( \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 - 25} \)

(b) \( \frac{A}{x} + \frac{B}{x + 5} + \frac{C}{x - 5} \)

(c) \( \frac{A}{x} + \frac{Bx + C}{x^2 - 25} \)

(d) \( \frac{A}{x} + \frac{Bx + C}{x^2 + 25} \)
Quiz 28

1. (Section 9.3, #22) Find the partial fraction decomposition of

\[
\frac{3x + 1}{x^2 + 4x + 4}
\]

2. Which of the following statements is true about the linear inequality below?

\[3x + y < 2\]

(a) The origin is not a solution to the inequality.

(b) The graph will contain a solid line.

(c) Quadrant III in the graph will be shaded.

(d) When solving the inequality for y, you will need to flip the inequality sign.
1. (Section 9.5, #49) Find the area enclosed by the system of inequalities:

\[ y > |x| \]
\[ y < 2 \]

2. Which of the following represents the system of linear equations shown below as an augmented matrix?

\[
\begin{align*}
2x - y &= 5 \\
-x + 2y &= 3
\end{align*}
\]

(a) \[
\begin{bmatrix}
2 & -1 & 1 \\
2 & 5 & 3
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
2 & -1 & 5 \\
-1 & 2 & 3
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
2 & -1 & 3 \\
-1 & 2 & 5
\end{bmatrix}
\]

(d) \[
\begin{bmatrix}
2 & -1 & 3 \\
2 & -1 & 5
\end{bmatrix}
\]
1. (Section 10.2, #45) Solve the system of linear equations using augmented matrices.

\[
\begin{align*}
x - z - y &= 10 \\
2x - 3y + z &= -11 \\
y - x + z &= -10
\end{align*}
\]

2. Which of the following statements is true regarding the two matrices shown below?

\[
A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}
\]

(a) A = B
(b) A + B and A – B are acceptable operations
(c) A\times B is an acceptable operation
(d) A\text{ }^{-1} \text{ exists}
1. (Section 10.4, #62) For the system of equations:

\[ 3x + 2y = 5 \]
\[ ax - 4y = 1 \]

Find \( a \) that guarantees no unique solution.

2. \[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

The equation described above is the equation of

(a) a circle
(b) a hyperbola
(c) a parabola
(d) an ellipse
Name: ________________________________

Quiz 32

1. (Section 11.4, #35) Graph the hyperbola.

\[-9x^2 - 18x + 4y^2 - 8y - 41 = 0\]

2. Which of the following statements is true about parametric equations?

(a) They are a way of describing the path an object takes along a curve.
(b) They have equivalent rectangular equations.
(c) Two important applications are cycloids and projectiles, whose paths can be traced using parametric equations.
(d) All of the above.
1. (Section 11.7, #36) A gun is fired from the ground at an angle of 60°, and the bullet has an initial speed of 2000 ft/sec. How high does the bullet go? What is the horizontal (ground) distance between where the gun was fired and where the bullet hit the ground?

2. The notation

\[ \sum_{n=0}^{k} n \]

means to

(a) Multiply the integers from 0 to n.

(b) Add 0, n, and k.

(c) Add the integers from 0 to n.

(d) Add the integers from 0 to k.
1. (Section 12.1, #71) Use sigma notation to write the sum.

\[ 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120} + \ldots \]

2. What is the first step to using mathematical induction?

(a) Show the statement is true for \( n = 1 \).

(b) Assume the statement is true for \( n = k \).

(c) Show the statement is true for \( k + 1 \).

(d) None of the above.
APPENDIX D: CUMULATIVE ASSESSMENTS
Chapter 1, Section 1.1, Question 35
Specify any values that must be excluded from the solution set and then solve the equation.

\[ \frac{10}{x} - 40 = \frac{40}{5} \]

\[ x = \boxed{5} \] must be excluded from the solution set.

Give exact answer in form of fraction.

The solution is \[ x = \boxed{20} \].

Chapter 1, Section 1.2, Question 31
When landscaping their yard, a couple budgeted 4,200 dollars. The irrigation system is 2,400 dollars and the sod is 1,500 dollars. The rest they will spend on trees and shrubs. Trees each cost 28 dollars and shrubs each cost 4 dollars. Combined they plant a total of 37 trees and shrubs. How many of each did they plant in their yard?

They plant in their yard \[ \boxed{12} \] trees and \[ \boxed{25} \] shrubs.

Answer *1: significant digits are disabled, the tolerance is +/-2%
Answer *2: significant digits are disabled, the tolerance is +/-2%

Chapter 1, Section 1.3, Question 07
If a person drops a water balloon off the rooftop of a 64 foot building the height of the water balloon is given by the equation

\[ h = -16t^2 + 64 \]

where \( t \) is in seconds. When will the water balloon hit the ground?

The water balloon will hit the ground in \[ \boxed{2} \] sec.

Significant digits are disabled, the tolerance is +/-2%

Chapter 1, Section 1.4, Question 19
Solve the radical equation for the given variable.

\[ \sqrt{1 - 5x} = x + 2 \]

\[ x = \boxed{-1} \]

Significant digits are disabled, the tolerance is +/-2%

Chapter 1, Section 1.4, Question 47
Solve the equation by introducing a substitution that transforms this equation to quadratic form.

\[ 20t^{2/3} - t^{1/3} - 1 = 0 \]

\[ t = \boxed{-1} \] or \[ t = \boxed{1} \] (Write your answers in ascending order.)

Answer *1: significant digits are disabled, the tolerance is +/-2%
Answer *2: significant digits are disabled, the tolerance is +/-2%

Chapter 1, Section 1.7, Question 33
Solve the inequality and express the solution in interval notation.

\[ |t - 13| > 6 \]

Choose the correct answer.

\[ (7, 19) \]

\[ (7, 19] \]

\[ (-\infty, 7) \cup (19, \infty) \]

\[ (-19, -7) \]

\[ (-\infty, -19) \cup (-7, \infty) \]

GO Problem, Section 1.5, Question 58
Solve and express solution in interval notation.
Cumulative Assessment #3

Start Date: 1 Feb 2008 at 03:00 PM
Due Date: 12 Feb 2008 at 03:00 PM
Student Access after Due Date: Yes. View Only
Graded: Yes

Chapter 1, Section 1.4, Question 47

Solve the equation by introducing a substitution that transforms this equation to quadratic form.

\[ 56t^{-2/3} - t^{-1/3} - 1 = 0 \]

\[ t = \underline{\text{---}}^1 \text{ or } t = \underline{\text{---}}^2 \] (Write your answers in ascending order.)

Answer *1: significant digits are disabled; the tolerance is +/-2%
Answer *2: significant digits are disabled; the tolerance is +/-2%

Chapter 1, Section 1.6, Question 27

Solve the rational inequality and express the solution set in interval notation.

\[ \frac{5 + 1}{81 - s^2} \geq 0 \]

\[ (-\infty, -9) \cup (-1, 9) \]
\[ (-\infty, -9] \cup [-1, 9] \]
\[ (-\infty, -9] \cup [-1, 9] \]
\[ (-9, -1] \cup (9, \infty) \]
\[ [-9, -1] \cup [9, \infty) \]

GO Problem, Section 2.3, Question 48

Write the equation of the line that passes through the given points. Express the equation in slope-intercept form.

\((-8, -7) \text{ and } (9, 1)\)

Give exact answer in form of fraction.
Chapter 2, Section 2.4, Question 35

Transform the equation into standard form by completing the square. Then state the center and radius of the circle.

\[ x^2 + y^2 - x + y + \frac{17}{36} = 0 \]

Center is at \((\underline{\underline{\text{}}}, \underline{\underline{\text{}}})\)

Radius = \underline{\underline{\text{}}}

**Answer 1:** significant digits are disabled; the tolerance is +/-2%

**Answer 2:** significant digits are disabled; the tolerance is +/-2%

Chapter 3, Section 3.4, Question 33

Evaluate \(f(g(1))\) and \(g(f(2))\), if possible.

\[ f(x) = \frac{1}{|x - 1|} \quad g(x) = x + 1/7 \]

If it is impossible to evaluate, use a Capital X. Give exact answers in fraction form.

\[ f(g(1)) = \underline{\underline{\text{}}} \]

\[ g(f(2)) = \underline{\underline{\text{}}} \]

**Significant digits are disabled; the tolerance is +/-2%**

Chapter 3, Section 3.5, Question 53

The function \(f\) is one-to-one. Find its inverse.

\[ f(x) = \frac{20}{15 - x} \]

\[ f^{-1}(x) = \underline{\underline{\text{}}} \]
Chapter 3, Section 3.5, Question 55

The function $f$ is one-to-one. Find its inverse.

$$f(x) = \frac{3x + 9}{6-x}$$

$f^{-1}(x) =$

Chapter 3, Section 3.2, Question 27

Find the difference quotient for the function $f(x) = x^2 - x$.

$$\frac{f(x+h) - f(x)}{h} =$$

Chapter 4, Section 4.5, Question 5

Find the domain of the rational function:

$$f(x) = \frac{x^2 + 8}{x^2 + x - 6}$$

- All $\mathbb{R}$ except $x = 6$
- All $\mathbb{R}$ except $x = 3$, $x = 2$
Chapter 4, Review Problems, Question 101

Determine the vertical and horizontal (or oblique) asymptotes (if they exist) for the following rational functions.

\[ f(x) = \frac{11 - x}{x + 2} \]

**Vertical Asymptote:** \( x = \) \[\text{ }^1\]

**Horizontal Asymptote:** \( y = \) \[\text{ }^2\]

**Answer \#1:** significant digits are disabled; the tolerance is +/-2%

**Answer \#2:** significant digits are disabled; the tolerance is +/-2%
Cumulative Homework #4
Start Date: 12 Feb 2008 at 03:00 PM
Due Date: 21 Feb 2008 at 03:00 PM
Student Access after Due Date: Yes, View Only
Graded: Yes

Chapter 1, Section 1.4, Question 47
Solve the equation by introducing a substitution that transforms this equation to quadratic form.

\[ 20t^{-2/3} - t^{-1/3} - 1 = 0 \]

\[ t = \boxed{[\text{expression}]}^1 \text{ or } t = \boxed{[\text{expression}]}^2 \] (Write your answers in ascending order.)

**Answer** *1*: significant digits are disabled; the tolerance is +/-2%
**Answer** *2*: significant digits are disabled; the tolerance is +/-2%

GO Problem, Section 1.1, Question 17
Solve the equation.

\[ 28 - [7 + 4x - 3(x + 7)] = 4(3x + 8) - [8(3x - 8) + 2 - 10x] \]

\[ x = \boxed{[\text{expression}]}^1 \]

Significant digits are disabled; the tolerance is +/-2%

Chapter 2, Section 2.4, Question 35
Transform the equation into standard form by completing the square. Then state the center and radius of the circle.

\[ x^2 + y^2 - x + y + \frac{23}{50} = 0 \]

\[ \text{Center is at } (\boxed{[\text{expression}]}^1, \boxed{[\text{expression}]}^2) \]

\[ \text{Radius } = \boxed{[\text{expression}]}^1 \]

**Answer** *1*: significant digits are disabled; the tolerance is +/-2%
**Answer** *2*: significant digits are disabled; the tolerance is +/-2%

Chapter 4, Section 4.1, Question 23
Rewrite the quadratic function \( f(x) = x^2 + 6x - 3 \) in standard form by completing the square.

\[ f(x) = (x + a)^2 + b \]

where \( a = \boxed{[\text{expression}]}^1 \) and \( b = \boxed{[\text{expression}]}^2 \).
GO Problem, Section 1.6, Question 45

A .22 caliber gun fires a bullet at a speed of 1,960 feet per second. If a .22 caliber is fired straight upward into the sky the height of the bullet in feet is given by the equation

\[ h = -16t^2 + 1960t \]

where \( t \) is the time in seconds with \( t = 0 \) corresponding to the instant the gun is fired. How long is the bullet in the air?

Bullet is in the air for \( \tfrac{\text{Answer}}{\text{Answer}} \) sec.

Significant digits are disabled; the tolerance is +/-2%

Chapter 2, Section 2.1, Question 15

Calculate the distance and midpoint between the segment joining the given points.

\((-2, 20)\) and \((-7, -17)\)

Round the distance to 2 decimal places.

Distance = \( \tfrac{\text{Answer}}{\text{Answer}} \) units

Midpoint is \( \tfrac{\text{Answer}}{\text{Answer}} \)

Answer *1: significant digits are disabled; the tolerance is +/-2%
Answer *3: significant digits are disabled; the tolerance is +/-3%

Chapter 2, Section 2.2, Question 43

Use symmetry to help you choosing the correct plot of the given equation.

\[ y = \tfrac{x^3}{3} \]

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Cumulative Assessment #2

Start Date: 18 Jan 2008 at 04:00 PM
Due Date: 31 Jan 2008 at 09:00 PM
Student Access after Due Date: Yes, View Only
Graded: Yes

Chapter 1, Section 1.1, Question 45

Specify any values that must be excluded from the solution set and then solve the equation.

\[
\frac{t - 1}{1 - t} = \frac{6}{5}
\]

\[t = \underline{\phantom{1}}\] must be excluded from the solution set.

The solution is \[t = \underline{\phantom{1}}\]. (If no solution, write \(t\).

Significant digits are disabled; the tolerance is +/-2%

Chapter 1, Section 1.4, Question 21

Solve the radical equation for the given variable.

\[\sqrt{x^2 - 20} = x - 2\]

\[x = \underline{\phantom{1}}\]

Significant digits are disabled; the tolerance is +/-2%

Chapter 1, Section 1.4, Question 45

Solve the equation by introducing a substitution that transforms this equation to quadratic form.

\[z^{2/7} - 6z^{1/7} + 9 = 0\]

\[z = \underline{\phantom{1}}\]

Significant digits are disabled; the tolerance is +/-2%

Chapter 2, Section 2.1, Question 29

Determine if the triangle with the given vertices is a right triangle, isosceles triangle, neither, or both.

(Recall that a right triangle satisfies the Pythagorean theorem and an isosceles triangle has two sides of equal length.)

(1, 1), (2, 0), and (-3, -4)

\(\triangle\) The triangle is isosceles and right.
Chapter 2, Section 2.3, Question 57
Write the equation of the line that passes through the given points. Express the equation in slope-intercept form.

(0, 25) and (-24, 0)

\[ y = \]

GO Problem, Section 2.4, Question 60
Find the equation of a circle that has a diameter with endpoints (15, 0) and (-1, -16).

- \((x - 7)^2 + (y + 8)^2 = 128\)
- \((x + 7)^2 + (y - 8)^2 = 128\)
- \((x - 7)^2 + (y + 8)^2 = \sqrt{128}\)

Chapter 3, Section 3.1, Question 51
Find the domain of the given function. Express the domain in interval notation.

\[ g(t) = t^3 + 11t \]

- \((-\infty, 11)\)
- \((11, \infty)\)
- \((-\infty, 0)\)
- \((-\infty, \infty)\)
- \((0, \infty)\)

Chapter 3, Section 3.2, Question 27
Find the difference quotient for the function \( f(x) = x^2 - x \).

\[
\frac{f(x+h) - f(x)}{h} =
\]

Chapter 3, Section 3.4, Question 29

Evaluate \( f(g(1)) \) and \( g(f(2)) \), if possible.

\[
f(x) = \frac{1}{x} \quad g(x) = 19x + 1
\]

\( f(g(1)) = \)

\( g(f(2)) = \)

Chapter 3, Section 3.5, Question 55

The function \( f \) is one-to-one. Find its inverse.

\[
f(x) = \frac{4x + 10}{17 - x}
\]

\( f^{-1}(x) = \)
Chapter 4, Section 4.1, Question 43

Find the vertex, \((h, k)\), of the parabola associated with the quadratic function.

\[ f(x) = \frac{1}{6}x^2 - 6x + 17 \]

\((h, k) = (\_\_\_, \_\_\_)^2\)

Answer *1: significant digits are disabled; the tolerance is +/-2%
Answer *2: significant digits are disabled; the tolerance is +/-2%

Chapter 5, Section 5.1, Question 41

If $3500 is put in a savings account that earns 2.5% interest per year compounded quarterly, how much is expected to be in that account in 11 years?

Round the answer to 2 decimal places.

The amount in the account after 11 years is expected to be $\_\_\_\_\_\_\_\_^1$

Significant digits are disabled; the tolerance is +/-2%

Chapter 5, Section 5.5, Question 27

Solve the logarithmic equation exactly

\[ \log_4(x - 3) + \log_4x - 1 \]

\[ x = \_\_\_\_\_\_\_\_^1 \]

Significant digits are disabled; the tolerance is +/-2%

Chapter 3, Section 3.1, Question 37

Evaluate the given quantity using the following function:

\[ f(x) = 78x - 69 \]

\[ f(x + 3.4) - f(x - 3.4) = \_\_\_\_\_\_\_\_^1 \]

Significant digits are disabled; the tolerance is +/-2%

Chapter 3, Section 3.1, Question 43

Evaluate the given quantity using the following function:

\[ g(t) = 33 + t \]

\[ \frac{g(t + h) - g(t)}{h} = \_\_\_\_\_\_\_\_^1 \]
Chapter 3, Section 3.5, Question 45

The function $f$ is one-to-one. Find its inverse.

$$f(x) = \sqrt{x - 7}$$

$$f^{-1}(x) =$$
Cumulative Homework #5

Start Date: 21 Feb 2008 at 04:00 PM
Due Date: 6 Mar 2008 at 10:00 PM
Student Access after Due Date: Yes. View Only
Graded: Yes

Chapter 1, Section 1.4, Question 47

Solve the equation by introducing a substitution that transforms this equation to quadratic form.

\[ 42t^{-2/3} - t^{-1/3} - 1 = 0 \]

\[ t = \underline{\text{ }} \text{ or } t = \underline{\text{ }} \] (Write your answers in ascending order.)

Answer *1: significant digits are disabled; the tolerance is +/-2%
Answer *2: significant digits are disabled; the tolerance is +/-2%

Chapter 2, Section 2.4, Question 57

Describe the graph (if it exists) of

\[ x^2 + y^2 + 18x - 18y + 162 = 0 \]

- The graph is a circle of radius \( \sqrt{162} \) centered at (9, 9).
- The graph consists of the single point (-9, 9).
- The graph consists of the single point (9, -9).
- The graph does not exist.
- The graph is a circle of radius \( \sqrt{162} \) centered at (-9, 9).

GO Problem, Section 2.3, Question 89

Find an equation of a line that passes through the point \((-78, 10.4 + 1)\) and parallel to the line \(Ax + By = C\). Express your answer in slope-intercept form.

\[ y = \underline{\text{ }} \]
Chapter 4, Review Problems, Question 17

Find the vertex of the parabola associated with the given quadratic function:

\[ f(x) = 19x^2 - 23x + 29 \]

Do not round off, use exact values (fractions).

\[ x_{\text{vertex}} = \]

\[ y_{\text{vertex}} = \]

Chapter 4, Review Problems, Question 93

List the possible rational zeros, and test to determine all rational zeros.

\[ p(x) = x^4 - 5x^3 + 20x - 16 \]

- Possible rational zeros: \( 1, 2, 4 \).

No rational zeros.

- Possible rational zeros: \( 1, 2, 4, 8, 16 \).

No rational zeros.

- Possible rational zeros: \( 1, 2, 4, 8, 16 \).

The rational zeros are \( 1, 2, 4 \) and \( -2 \).

- Possible rational zeros: \( 1, 2, 4, 8 \).

The rational zeros are \( 1, 2, 4 \) and \( -2 \).

- Possible rational zeros: \( 1, 2, 4, 8, 16 \).

The rational zeros are \( 1, 2, 4 \).

GO Problem, Section 4.5, Question 71

Write a rational function (in simplest form) that has vertical asymptotes at \( x = -3 \) and \( x = 1 \) and a horizontal asymptote at \( y = 16 \).

\[ f(x) = \frac{16x}{x^2 + 2x - 3} \]
Chapter 5, Section 5.5, Question 41

If money is invested in a savings account earning 3.7% interest compounded yearly, how many years until the money triples?

Round the answer to 2 decimal places.

\[ t = \quad \text{years} \]

*Significant digits are disabled; the tolerance is +/-2%*

Chapter 6, Section 6.3, Question 7

Use the right triangle diagram below and \( \beta = 25^\circ \), \( a = 14 \text{ km} \) to find \( c \).

Round your answer to two significant digits.

\[ c \approx \quad \text{km} \]

*Significant digits are disabled; the tolerance is +/-2%*

Chapter 6, Section 6.4, Question 91

A right triangle is drawn in QI with one leg on the \( x \)-axis and its hypotenuse on the terminal side of \( \theta \) drawn in standard position.
If \( \sin \theta = \frac{9}{41} \), then what is \( \tan \theta \)?

Give exact answer in form of fraction.

\( \tan \theta = \) 

---

Chapter 3, Section 3.4, Question 25

Evaluate the functions for the specified values, if possible.

\[ f(x) = x^2 + 7 \quad g(x) = \sqrt{x} - 1 \]

\[ f(g(2)) = \] 

*Significant digits are disabled; the tolerance is +/-2%*
GO Problem, Section 1.4, Question 48

Solve the equation by introducing a substitution that transforms the equation to quadratic form.

\[ t^{-2/3} - t^{-1/3} - 20 = 0 \]

Give exact answers in form of fraction. Write the answers in ascending order.

The solutions are

\[ t = \boxed{\text{expression}} \]

or

\[ t = \boxed{\text{expression}} \]

**Answer** *1*: significant digits are disabled; the tolerance is +/-2%

**Answer** *2*: significant digits are disabled; the tolerance is +/-2%

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Chapter 3, Section 3.5, Question 53

The function \( f \) is one-to-one. Find its inverse.

\[ f(x) = \frac{12}{14 - x} \]

\[ f^{-1}(x) = \]

---

GO Problem, Section 3.3, Question 37

For the given graph of a function \( f(x) \), draw the indicated function.

\[ y = 2f(x + 4) - 5 \]
Choose the correct answer.
Chapter 4, Section 4.1, Question 49

Find the quadratic function that has the given vertex and goes through the given point.

vertex: \((-1, 5)\) point: \((0, 3)\)

Write your answer in standard form.

\[ f(x) = \]

Chapter 4, Section 4.4, Question 11

Given a zero of the polynomial, determine all other zeros and write the polynomial in terms of a product of linear factors:

\[ P(x) = x^3 - 13x + 12 \]

zero: \(x = 1\)

- The zeros are 1, 4 and -13, \(P(x) = (x - 1)(x + 4)(x + 13)\).
- The zeros are 1, 3 and -4, \(P(x) = (x - 1)(x + 4)(x - 3)\).
- The zeros are 1, 4 and -3, \(P(x) = (x - 1)(x - 4)(x + 3)\).
- The zeros are 1, -1 and 12, \(P(x) = (x - 1)(x + 1)(x - 12)\).
The zeros are 0 and -1,  \( P(x) = x(x - 1) \).

---

**GO Problem, Section 5.5, Question 30**

Solve the logarithmic equation exactly.

\[
\log_3(17 - x) - \log_3(x + 3) = 1
\]

\[x = \boxed{10}^1\]

*Significant digits are disabled; the tolerance is +/-2%*

---

**Chapter 6, Section 6.2, Question 57**

Calculate the index of refraction, \( n_r \), given the following assumptions:

- Incident medium is air
- Air has an index of refraction value of \( n_i = 1.00 \)
- Incidence angle \( \theta_i = 30^\circ \)
- Refractive medium is plastic
- Refractive angle \( \theta_r = 20^\circ \)

Round to three decimal places.

\[n_r = \boxed{1.331}\]

*Significant digits are disabled; the tolerance is +/-2%*

---

**Chapter 6, Section 6.5, Question 49**

Find all values of \( \theta \), where \( 0^\circ \leq \theta \leq 360^\circ \), when \( \cos \theta = 0 \).

Write your answers in the ascending order.

\[\theta = \boxed{90^\circ} \text{ or } \boxed{270^\circ}\]

*Answer 1: significant digits are disabled; the tolerance is +/-2%*

*Answer 2: significant digits are disabled; the tolerance is +/-2%*

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**Chapter 7, Section 7.1, Question 33**

Use a Pythagorean identity to find \( \csc \theta \) if \( \cos \theta = -\frac{7}{15} \) and the terminal side of \( \theta \) lies in quadrant III. Give exact answer.

\[
\csc \theta = \boxed{\text{expression}}\]
GO Problem, Section 7.2, Question 13

Simplify the trigonometric expression.

\[
1 - \frac{\sin^2 x}{1 - \cos x}
\]

Choose the correct answer.

- \cos x

\(- \sin x\)

\(\cos x\)
Chapter 1, Section 1.4, Question 53

Solve the equation by introducing a substitution that transforms this equation to quadratic form.

\[ u^{4/3} - 65u^{2/3} = -64 \]

\[ u = \pm \phantom{-} \]  
\[ *1 \text{ or } u = \pm \phantom{-} \]  
\[ *2 \]

Write the answers in ascending order; give positive answers.

**Answer *1:** significant digits are disabled; the tolerance is +/-2%

**Answer *2:** significant digits are disabled; the tolerance is +/-2%

GO Problem, Section 3.5, Question 55

Find the inverse of the one-to-one function \( f \). Check your answer. State the domain and range of both \( f \) and \( f^{-1} \).

\[ f(x) = \frac{2x + 1}{3 - x} \]

\[ f^{-1}(x) = \]

Domain of \( f \) is _______

Range of \( f \) is _______

- Domain of \( f \) is \((-\infty, -3) \cup (-3, \infty)\)

- Range of \( f \) is \((-\infty, 2) \cup (2, \infty)\)

- Domain of \( f \) is \((-\infty, -2) \cup (-2, \infty)\)
Range of \( f \) is \(( -\infty, 3) \cup (3, \infty)\)

\( C \) Domain of \( f \) is \(( -\infty, 3) \cup (3, \infty)\)

Range of \( f \) is \(( -\infty, -2) \cup (-2, \infty)\)

Domain of \( f^{-1} \) is ________

Range of \( f^{-1} \) is ________

\( C \) Domain of \( f^{-1} \) is \(( -\infty, 2) \cup (2, \infty)\)

Range of \( f^{-1} \) is \(( -\infty, -3) \cup (-3, \infty)\)

\( C \) Domain of \( f^{-1} \) is \(( -\infty, -2) \cup (-2, \infty)\)

Range of \( f^{-1} \) is \(( -\infty, 3) \cup (3, \infty)\)

\( C \) Domain of \( f^{-1} \) is \(( -\infty, -3) \cup (-3, \infty)\)

Range of \( f^{-1} \) is \(( -\infty, 2) \cup (2, \infty)\)

---

**GO Problem, Section 3.5, Question 62**

The equation used to convert from degrees Fahrenheit, \( x \), to degrees Celsius is \( C(x) = \frac{5}{9}(x - 32) \).

Determine inverse function, \( C^{-1}(x) \).

\[ C^{-1}(x) = \]

What does the inverse represent?

The inverse represents converting from degrees ________
Chapter 4, Section 4.1, Question 55

Find the quadratic function that has the given vertex and goes through the given point.

vertex: \((\frac{1}{19}, -\frac{3}{38})\) point: \((\frac{3}{38}, 0)\)

Write your answer in standard form.

\[
f(x) =
\]

GO Problem, Section 4.4, Question 52

(a) Use Descartes' rule of signs to determine the possible combination of positive real zeros, negative real zeros, and imaginary zeros; (b) use the rational zero test to determine possible rational zeros; (c) test for rational zeros; and (d) factor as a product of linear factors.

\[P(x) = x^4 - 5x^3 + 5x^2 + 25x - 26\]

(d) Write \(P(x)\) as a product of linear factors.

\[\begin{align*}
\mathbb{C} \quad P(x) &= (x + 1)(x - 2)(x - 3 - 2i)(x - 3 + 2i) \\
\mathbb{C} \quad P(x) &= (x + 2)(x - 1)(x - 3 - 2i)(x - 3 + 2i) \\
\mathbb{C} \quad P(x) &= (x + 2)(x - 1)(x + 3 - 2i)(x + 3 + 2i)
\end{align*}\]

GO Problem, Section 4.5, Question 42

Use the graphing strategy to graph the rational function.

Match the graph of \(f(x) = \frac{x^2 - 9}{x + 2}\) with one of the following choices:
Print Assignment:
GO Problem, Section 5.5, Question 30

Solve the logarithmic equation exactly.

\[ \log_3(33 - x) - \log_3(x + 7) = 1 \]

\[ x = 1 \]

Significant digits are disabled; the tolerance is +/-2%

Chapter 6, Section 6.8, Question 43

Find an equation of the graph.

![Graph Image]

Chapter 7, Section 7.8, Question 25

Solve \( 6\cos^2 \theta - 3\cos \theta = 0 \) exactly on \( 0 \leq \theta \leq 2\pi \). [Choose the correct answer.]

- \( \theta = \frac{\pi}{3}, \frac{5\pi}{3} \)
- \( \theta = 0, \pi, 2\pi, \frac{5\pi}{3} \)
- \( \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3} \)
Chapter 8, Section 8.3, Question 17

Find the area of the described triangle:

\[ a = 7, \quad b = 13, \quad \text{and} \quad c = 10. \]

Round your answer to three significant digits.

\[ A \approx \quad \text{**1**} \]

*Significant digits are disabled; the tolerance is +/-2%*
GO Problem, Section 10.3, Question 62

For the system of equations

\[ 5x + 8y = 13 \]
\[ ax - 32y = 1 \]

find \( a \) that guarantees no unique solution.

\[ a = \boxed{} \]

Significant digits are disabled; the tolerance is +/-2%

GO Problem, Section 9.3, Question 24

Find the partial fraction decomposition for the rational function.

\[ \frac{42x^2 - 82x - 8}{(x + 7)(x - 1)^2} \]

C

\[ \frac{41}{x + 7} + \frac{1}{x - 1} + \frac{6}{(x - 1)^2} \]

C

\[ \frac{41}{x + 7} - \frac{6}{(x - 1)^2} \]

C

\[ \frac{41}{x + 7} + \frac{1}{x - 1} - \frac{6}{(x - 1)^2} \]

C

\[ \frac{41}{x + 7} - \frac{1}{x - 1} - \frac{6}{(x - 1)^2} \]

C

\[ \frac{41}{x + 7} - \frac{1}{x - 1} + \frac{6}{(x - 1)^2} \]

Chapter 8, Section 8.5, Question 1

Find the dot product: \( (7, -3) \cdot (3, 4) \).

\[ (7, -3) \cdot (3, 4) = \boxed{} \]

Significant digits are disabled; the tolerance is +/-2%
Chapter 7, Section 7.8, Question 71

Solve $2\sin^2 x + 3\cos x = 0$ exactly on $0 \leq x \leq 2\pi$.

Write your answers in ascending order in terms of $\pi$.

$x =$

Chapter 6, Section 6.3, Question 35

This exercise illustrates a mid-air refueling scenario that our military aircraft often use. Assume the elevation angle that the hose makes with the plane being fueled is $\theta = 36^\circ$.

If the hose is 150 feet long, what should the altitude difference, $\Delta h$, be between the two planes?

Round your answer to two significant digits.

Your answer: $\Delta h = 1$ ft.

*Significant digits are disabled; the tolerance is +/-2%*

GO Problem, Section 5.5, Question 30

Solve the logarithmic equation exactly.

$$\log_3(22 - x) - \log_3(x + 2) = 1$$

$x =$
GO Problem, Section 4.1, Question 29
Rewrite the quadratic function in standard form by completing the square.

\[ f(x) = -4x^2 + 16x - 4 \]

\[ f(x) = \quad \quad \quad \]

Answer: significant digits are disabled; the tolerance is +/-2%

Chapter 3, Section 3.5, Question 55
The function \( f \) is one-to-one. Find its inverse.

\[ f(x) = \frac{3x + 22}{7 - x} \]

\[ f^{-1}(x) = \quad \quad \quad \]

GO Problem, Section 2.3, Question 48
Write the equation of the line that passes through the given points. Express the equation in slope-intercept form.

\((-4, -3)\) and \((5, 1)\)

Give exact answer in form of fraction.

\[ y = \quad \quad \quad \]

GO Problem, Section 1.4, Question 48
Solve the equation by introducing a substitution that transforms the equation to quadratic form.
\( t^{-2/3} - t^{-1/3} - 6 = 0 \)

Give exact answers in form of fraction. Write the answers in ascending order.

The solutions are

\[ t = \quad \]  

or

\[ t = \quad \]  

**Answer 1:** significant digits are disabled; the tolerance is +/-2%  
**Answer 2:** significant digits are disabled; the tolerance is +/-2%
Cumulative Homework #9

Start Date: 26 Mar 2008 at 04:00 PM
Due Date: 13 Apr 2008 at 11:00 PM
Student Access after Due Date: Yes. View Only
Graded: Yes

GO Problem, Section 1.4, Question 72

Solve the equation

\[ 5x^{5/6} - x^{7/12} - 42x^{1/3} = 0 \]

Write the answers in ascending order. The solutions are \( x = \boxed{\text{\textsuperscript{1}x}}, \ x = \boxed{\text{\textsuperscript{2}x}} \).

**Answer *1:** significant digits are disabled; the tolerance is +/-2%
**Answer *2:** significant digits are disabled; the tolerance is +/-2%
**Answer *3:** significant digits are disabled; the tolerance is +/-2%
**Answer *4:** significant digits are disabled; the tolerance is +/-2%
**Answer *5:** significant digits are disabled; the tolerance is +/-2%

GO Problem, Section 2.4, Question 40

Find the equation of the circle centered at \((8, 6)\) and passing through the point \((2, 3)\). Write the equation of the circle in general form.

\[ x^2 + y^2 + (\boxed{\text{\textsuperscript{1}x}})x + (\boxed{\text{\textsuperscript{2}y}})y + (\boxed{\text{\textsuperscript{3}c}}) = 0 \]

**Answer *1:** significant digits are disabled; the tolerance is +/-2%
**Answer *2:** significant digits are disabled; the tolerance is +/-2%
**Answer *3:** significant digits are disabled; the tolerance is +/-2%
**Answer *4:** significant digits are disabled; the tolerance is +/-2%

Chapter 3, Section 3.5, Question 65

A student is working at Target making $13 per hour and the weekly number of hours worked per week, \( x \), varies. If Target withholds 25% of his earnings for taxes and social security, write a function, \( E(x) \), that expresses the student’s take-home pay each week. Find the inverse function, \( E^{-1}(x) \).

\[ E(x) = 9.75x \]
\[ E^{-1}(x) = \frac{x}{9.75} \]

\[ E(x) = 9.75x \]
\[ E^{-1}(x) = \frac{x}{13} \]
$E(x) = 13x$
$E^{-1}(x) = \frac{x}{13}$

$\therefore E(x) = 9.75x$
$E^{-1}(x) = 9.75x$

$\therefore E(x) = 13x$
$E^{-1}(x) = \frac{x}{9.75}$

GO Problem, Section 4.1, Question 53

Find the quadratic function in standard form with vertex \((-2, -7)\) and goes through the point \((-8, 65)\).

$$f(x) =$$

Chapter 5, Section 5.5, Question 27

Solve the logarithmic equation exactly

$$\log_3(x - 5) + \log_3 x = 1$$

$$x =$$

Significant digits are disabled; the tolerance is +/-2%

Chapter 6, Section 6.3, Question 33

If the flagpole that a golfer aims at on a green measures 5 feet from the ground to the top of the flag, and a golfer measures a 3 degree angle from the ground at the golfer's feet to the top of the flag, how far (horizontal distance) is the golfer from the flag?

Round your answer to three significant digits.

Your answer: \(x^{\text{1st}}\) ft.
Chapter 7, Section 7.3, Question 1

Find the exact value for the trigonometric expression.

\[ \sin \left( \frac{n}{12} \right) \]

C \[ \frac{1}{4} \]

C \[ \frac{1 - \sqrt{2}}{6} \]

C \[ \frac{1 + \sqrt{2}}{4} \]

C \[ \frac{\sqrt{6} - \sqrt{2}}{2} \]

C \[ \frac{\sqrt{6} - \sqrt{2}}{4} \]

Chapter 8, Section 8.6, Question 51

**Resultant Force.** Force A, at 90 pounds, and force B, at 150 pounds, make an angle of \( 20^\circ \) with each other. Represent their respective vectors as complex numbers written in trigonometric form, and solve for the resultant angle (between A and the resultant force) expressed as a degree. Carry out all calculations exactly and round to one decimal place the final answer only.

Angle of the resultant force is \( \theta \).

**Significant digits are disabled; the tolerance is +/-2%**

Chapter 9, Section 9.3, Question 17

Find the partial decomposition for the rational function.

\[ \frac{5x}{x(x - 5)} \]

C \[ \frac{5x}{x - 5} \]

C \[ \frac{5}{x - 5} + \frac{5}{x + 5} \]

C \[ \frac{5}{x(x - 5)} + \frac{5}{x - 5} \]
Chapter 10, Section 10.3, Question 31

Evaluate $3 \times 3$ determinant.

\[
\begin{vmatrix}
-1 & -1 & 3 \\
-4 & 7 & 8 \\
-2 & -3 & 8 \\
\end{vmatrix}
\]

*1

Significant digits are disabled; the tolerance is +/-2%
Cumulative Homework #10

Start Date: 26 Mar 2008 at 04:00 PM
Due Date: 17 Apr 2008 at 11:00 PM
Student Access after Due Date: Yes, View Only
Graded: Yes

GO Problem, Section 1.1, Question 17

Solve the equation.

\[35 - [2 + 6x - 5(x + 2)] = 2(2x + 3) - [4(2x - 3) + 6 - 2x]\]

\[x = \boxed{1}\]

Significant digits are disabled; the tolerance is +/-2%

Chapter 3, Section 3.1, Question 41

Evaluate the given quantity using the following function:

\[f(x) = 80x - 95\]

\[\frac{f(x + h) - f(x)}{h} = \boxed{1}\]

Significant digits are disabled; the tolerance is +/-2%

Chapter 4, Section 4.4, Question 13

Given a zero of the polynomial, determine all other zeros and write the polynomial in terms of a product of linear factors:

\[P(x) = 2x^3 + x^2 - 25x + 12\]

Zero: \(x = \frac{1}{2}\)

- The zeros are \(\frac{1}{2}, -\frac{1}{2}\) and 12, \(P(x) = (x - \frac{1}{2})(x + \frac{1}{2})(x - 12)\).
- The zeros are 0 and \(\frac{1}{2}\), \(P(x) = x(x - \frac{1}{2})\).
- The zeros are \(\frac{1}{2}, 4\) and -25, \(P(x) = 2\left(x - \frac{1}{2}\right)(x + 4)(x + 25)\).
- The zeros are \(\frac{1}{2}, 3\) and 4, \(P(x) = 2\left(x - \frac{1}{2}\right)(x + 4)(x - 3)\).
- The zeros are \(\frac{1}{2}, 4\) and -3, \(P(x) = 2\left(x - \frac{1}{2}\right)(x - 4)(x + 3)\).
Chapter 5, Section 5.5, Question 49

In 2003, there were an estimated 1 million people who had been infected with HIV in the United States. If the infection rate increases at an annual rate of 2.5% a year compounding continuously, how many years, $t$, until there are 2.0 million Americans infected with HIV?

Round the answer to 2 decimal places.

$t = \underline{\hspace{2cm}} \text{years}$

*Significant digits are disabled; the tolerance is +/-2%*

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Chapter 6, Section 6.6, Question 19

Convert $50^\circ$ from degrees to radians. Leave answer in terms of $\pi$.

---

Chapter 7, Section 7.7, Question 49

Evaluate the given expression exactly. Give your answer in terms of $\pi$.

$\sin^{-1}\left(\sin\left(-\frac{6\pi}{5}\right)\right) =$

---

Chapter 8, Section 8.7, Question 21

Find the result of the expression using De Moivre's theorem. Write the answer in rectangular form.

$(-6 + 6i)^5 = \underline{\hspace{2cm}} \text{cis}^{\underline{\hspace{2cm}}} + (\underline{\hspace{2cm}})^i$

*Answer *1: significant digits are disabled; the tolerance is +/-2%

*Answer *2: significant digits are disabled; the tolerance is +/-2%

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Chapter 9, Section 9.3, Question 21

Find the partial decomposition for the rational function.
\[
\frac{24x - 18}{x^2 + 6x + 9}
\]

\[
\frac{24}{x + 3} + \frac{90}{(x + 3)^2}
\]

\[
\frac{24}{x + 3} + \frac{-90}{(x + 3)^2}
\]

\[
\frac{1}{x + 3} + \frac{90}{(x + 3)^2}
\]

\[
\frac{24}{x + 24} + \frac{90}{(x + 90)^2}
\]

\[
\frac{24}{x + 3} + \frac{-90}{(x + 90)^2}
\]

\[
\frac{24}{x + 3} + \frac{-90}{(x + 3)^3}
\]

Chapter 10, Section 10.4, Question 13

\[
A = \begin{bmatrix} -7 & 4 & 7 \\ -6 & 9 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 0 & 1 \\ -8 & -9 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 4 \\ -5 & 7 \\ -7 & -6 \end{bmatrix}
\]

Calculate the given expression.

\[(A + B)C\]

Chapter 12, Section 12.5, Question 45

In a state lottery in which six numbers are drawn from a possible 35 numbers, the number of possible six-number combinations is equal to \(\binom{35}{6}\). How many possible combinations are there?

Your answer: \(\framebox{1,085,970}\) combinations are possible.

Significant digits are disabled; the tolerance is +/-2%
Global Warming Project

Part 1 (40 points)
Are the levels rising? Are U.S. Cities Getting Warmer?

Purpose: Create function models based on carbon dioxide levels for one of the 10 largest cities in the US.

Gather Data:
From the latest US Census Bureau data available, select the $x$th largest city in the US, where $x$ is the last digit of the PID of the oldest student in your group (note 0 will be the $x = 10$). This information is available from the US Census Bureau at http://www.census.gov.

Find the historical carbon dioxide levels for your city for every 5 years starting at 1960 and ending at 2005.
Find the historical temperatures for your city for every 10 years starting at 1960 and ending at 2005.

Project (Part 1) Instructions:

(2 points) 0. Explain the term “global warming” and “greenhouse effect”. What are the effects of high carbon dioxide levels? How are they harmful?

(7 points) 1. Create a chart with the carbon dioxide level data that the group has found. Chart should include the following: Year, years past 1960, and Carbon Dioxide level. Plot your data for the carbon dioxide levels on a graph for each year; x-axis is years past 1960 and y-axis is carbon dioxide level (parts per million, ppm). Make sure your graph is properly labeled.

(4 points) 2. A linear model of the carbon dioxide data was created in the plot. Find the equation for the model. (i.e. The equation should be in the form $f(x) = ax + b$).

(2 points) 3. Predict the Carbon Dioxide level for year 2020 and 2040.
4. Predict the year that the Carbon Dioxide level will reach 500 ppm.

5. Create a chart with the temperatures that the group has found for the years. Chart should include the following: Year, years past 1960, and temperature. Plot your data for the carbon dioxide levels on a graph for each year; x-axis is years past 1960 and y-axis is the temperature (Fahrenheit)

6. A linear model of the temperature was created in the plot. Find the equation for the model. (i.e. The equation should be in the form f(x) = ax + b).

7. Predict the temperature for year 2020 and 2040.

8. Plot the graph of carbon dioxide levels versus temperature for the years between 1960 and 2005. What can you conclude about the graph?

Conclusion:

9. Do your models provide evidence to support the theory of global warming? Why or why not? What are your conclusions on global warming? What are things the society could do to help global warming in the future? Use your models to justify response.

Part 2 (50 points)
Where are we headed with current trends? What can be done?

Below are some graphs of different quantities often associated with pollution.
Investigate a particular “cause” of global warming or a particular result of global warming. Find data that represents the particular phenomenon (10 points). Then use one of the functions we have discussed in class (linear, piecewise, quadratic, higher order polynomial, exponential, logarithmic, or rational) to model the phenomenon that interests you. Determine the specific functions (10 points) and label particular points on the graph of the function. Label the axes (5 points) correctly. What does that model predict for behavior of the function in 2050 (5 points)? 20200 (5 points)? Use your models developed in this project to develop a plan that will reduce global warming (15 points).

Project Specifications

Paper will be 8-10 pages (including graphs and charts).
All graphs and tables will be labeled
Title page will have a title and all authors’ names
Last page will be a division of work explanation (who did what on this project)
APPENDIX F: IRB
Notice of Exempt Review Status

From: UCF Institutional Review Board
FWA0000351, Exp. 6/24/11,
IRB00001138

To: Nichole A Shorter and Co-PI: Cynthia Young, Ph.D.

Date: August 25, 2008

IRB Number: SBE-08-05779

Study Title: Comparing Assessment Methods as Predictors of Student Learning in Undergraduate Mathematics

Dear Researcher:

Your research protocol was reviewed by the IRB Chair on 8/25/2008. Per federal regulations, 45 CFR 46.101, your study has been determined to be minimal risk for human subjects and exempt from 45 CFR 46 federal regulations and further IRB review or renewal unless you later wish to add the use of identifiers or change the protocol procedures in a way that might increase risk to participants. Before making any changes to your study, call the IRB office to discuss the changes. A change which incorporates the use of identifiers may mean the study is no longer exempt, thus requiring the submission of a new application to change the classification to expedited if the risk is still minimal. Please submit the Termination/Final Report form when the study has been completed. All forms may be completed and submitted online at https://iris.research.ucf.edu.

The category for which exempt status has been determined for this protocol is as follows:

4. Research involving the collection or study of existing data, documents, records, pathological specimens or diagnostic specimens, if these sources are publicly available or if the information is recorded by the investigator in such a manner that subjects cannot be identified, directly or through identifiers linked to the subjects. ("Existing" means already collected and/or stored before your study starts, not that collection will occur as part of routine care.)

All data, which may include signed consent form documents, must be retained in a locked file cabinet for a minimum of three years (six if HIPAA applies) past the completion of this research. Any links to the identification of participants should be maintained on a password-protected computer if electronic information is used. Additional requirements may be imposed by your funding agency, your department, or other entities. Access to data is limited to authorized individuals listed as key study personnel.

On behalf of Tracy Dietz, Ph.D., UCF IRB Chair, this letter is signed by:

IRB Coordinator
REFERENCES


tests as teaching devices in educational psychology classes. *Journal of
educational Psychology, 23*, 371-380.

attendance at class meetings with and without grade-related contingencies: A


