High resolution monopulse tracking

1988

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HIGH RESOLUTION MONOPULSE TRACKING

BY

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B.S., University of Kentucky, 1985

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Engineering
in the Graduate Studies Program
of the College of Engineering
University of Central Florida
Orlando, Florida

Fall Term
1988
ABSTRACT

High Resolution Radar techniques are applied to the problem of resolving a multiple target array and locating its geometric center without the usual biasing toward the brightest targets. Monopulse radar techniques are combined with high resolution stepped frequency pulse train signal processing in an amplitude tracking radar.

A single pulse monopulse system's aimpoint is biased toward the brightest point targets in an array. However, by using a stepped frequency pulse monopulse radar, the cross range distance to each individual scatterer may be found.

Unlike the single pulse monopulse system, the aimpoint is independent of the reflectivity of the targets. The geometric center of a multiple scatterer array is found by averaging the cross range components along both axes. For the stepped frequency high resolution monopulse system, the center of each uniquely separated pair of point targets is calculated by examining the cross-correlation function of the sum and difference channels. The autocorrelation of the sum channel is used to normalize the cross-correlation
data thereby eliminating the effects of the different targets radar cross sections (RCS). The zero separation term of the error function (DC term) remains biased toward the bigger scatterer, even after normalization. The nonzero terms (AC terms) are the cross range distances from the antenna's boresight to each scatterer and are independent of their RCS. By simply dropping this zero separation term and averaging the remaining ones together, the aimpoint becomes the unbiased geometric center of the array.

The special cases of one, two and three resolvable point scatterers are examined in detail. Analysis of a nondiscrete complex scattering array is not presented, since the requirement of separation pair uniqueness cannot be assumed.

The monopulse tracking simulation work was done on an IBM AT using Microsoft Fortran-77.
ACKNOWLEDGEMENTS

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1 INTRODUCTION

Since the 1960's the modernized military powers have relied heavily on infra-red and laser designated guidance systems for their smaller missiles. These systems are not effective in bad weather or smoke because of the increased atmospheric attenuation. Radar systems work better in these adverse conditions than infra-red or laser systems, but radar systems were dismissed because of the technical difficulties in building radar components so small. Recently there have been major advances in millimeter wave technology. The jump occurred in usable frequencies from the range of 1 to 20 GHz to the realm of 35 to 200 GHz. These higher frequencies lead to smaller waveguides, antennas and RF components. Currently it is feasible to consider making miniature radar seekers for all guided missiles.

This type of autonomous seeker is commonly called a "Fire and Forget" system since no sort of guidance needs to be provided for after firing. For ground to ground encounters, this eliminates the need for a forward observer. For air to ground or air to air encounters, the launch platform can leave after firing,
so it is not exposed to return fire as it would if it illuminates the target.

When more than one scaterer on a target is illuminated at the same sampling time, it is not possible for a simple monopulse system to resolve the two. However, a high resolution monopulse system can resolve two or more scaterers on a target and will allow for tracking of individual scaterers on a target or the center of a group of scaterers on a target. The simple monopulse system can only track the brightest point of a set of scaterers that make up the target. This means that when only two scaterers are present, the aimpoint of the simple monopulse system will be biased toward the scaterer with the largest radar cross section.

In Einstein (1984) the use of a stepped frequency pulse train is examined in detail for the purpose of target detection in clutter. However, it makes no mention of the use of this technique in a tracking sense. A high resolution noncoherent monopulse radar will yield data that allows for calculation of the cross-correlation function of the sum and difference channels as well as the autocorrelation function described in Einstein (1984).

For a simple monopulse system the difference signal is normalized by the sum signal, but in a high
resolution noncoherent monopulse system the cross-correlation function is normalized by the autocorrelation function of the sum channel. Since this is done on a term by term basis, the biasing toward the brightest scatterer is removed by ignoring the zero separation term of the normalized cross-correlation function in cases where more than one point scatterer is target is resolved. The only limitation is that each scattering pair be uniquely separated in down range distance. This requirement comes from Einstein (1984). It is necessary since the range profile is not known; only the convolution of the range profile with itself is known. Since the range profile is not known, all examples will be for point targets, which have a range profile that closely approximates an impulse function. This makes the interpretation of the convolutions much easier to recognize.

No uniquely separated pairs exist in uniform clutter. If the signal power from the target is stronger than the power return from the clutter, then the geometric tracking process may still be useful. However, if the clutter return is strong or if it is drastically non-uniform then the geometric tracking routine will give erroneous results. This means that the routine must include a threshold test of the signal
to clutter power ratio and of the signal to noise ratio before its results may be considered valid.

This thesis contains four chapters. The first is a review of radar theory and terms. Some of the topics covered are measuring range and clutter elimination by use of polarization information. To find the area of clutter illuminated by the radar system, pulsewidth and beamwidth limiting flight geometries are compared. Also, the advantages of low pulse repetition frequency waveforms and the elimination of range aliasing are discussed.

Chapter two deals with high resolution radars which use inverse digital Fourier transforms of stepped frequency pulse trains to accomplish pulse compression. Then the high resolution theory is applied to a multilobbing or monopulse system.

Chapter three gives examples of a four horn multilobbing radar. The receiver is set to yield cross range (angular) information. The first test is for an azimuth scan and the second is for an elevation scan. The tests show only one polarization at a time.

The fourth chapter states the conclusions that may be drawn from the simulations and experimental data.
1.1. The Range Equation

For a radar system the range to a target is calculated simply by measuring the time it takes a single pulse to travel to a target and reflect back. Since the pulse traveled the distance to the target twice, there and back, the total delay time from transmission to reception is divided by 2. Using 300,000 km/sec as the speed of light, the range is,

\[
\frac{1}{2} \text{ Delay Time} \times 300,000 \text{ km/sec.}
\]

When the radar is trained on a target, the power received from the target for one sampling pulse is

\[
P_{\text{rec}} = \frac{P \cdot G \cdot \sigma \cdot A_e \cdot \tau_p}{(4\pi)^2 \cdot R^4}
\]

where,

- \( P \) = average transmitted power
- \( G \) = antenna gain and losses, for transmit and receive
- \( \sigma \) = radar cross section of target
- \( A_e \) = effective antenna area
- \( \tau_p \) = pulse duration time (See Figure 4)
- \( R \) = range.

To detect targets this energy and the noise energy must exceed a threshold level that is high enough above the noise level so that noise alone will not cause false alarms.
Clutter may also be present along with the target and noise. Clutter returns come from the ground, precipitation, or chaff.

The RCS of a target, $\sigma$, is a function of the cross sectional area of the target as viewed by the radar, the target's reflectivity and its directivity. A sphere with a cross sectional area of 1 square meter has a maximum RCS of one square meter. (See Table 1, Passive Reflectors)

1.2. Searching for Targets

Before a radar can track a target, it must first detect the target in the presence of noise and clutter and then classify it. In order to accomplish this search, the antenna should scan in some fixed pattern. For this discussion, I will assume that the antenna is scanning in the azimuth plane. Figure 1 shows a typical scan pattern for an inner gimballed antenna, while Figure 2 gives an example of a scan pattern for an outer gimballed antenna. In the inner gimbal system the antenna scans about an axis that is perpendicular to the look down angle. For zero forward velocity and a fixed look down angle, the pattern for the inner gimballed antenna would be a straight line. An outer gimballed antenna scans about an axis that is vertical to the ground. When not moving and with a
<table>
<thead>
<tr>
<th>TYPE</th>
<th>Size</th>
<th>MAXIMUM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$A_T$</td>
</tr>
<tr>
<td>SPHERE</td>
<td>$a$</td>
<td>$\frac{a\lambda}{2}$</td>
</tr>
<tr>
<td>CYLINDER</td>
<td>$a$</td>
<td>$b\sqrt{\frac{a\lambda}{2}}$</td>
</tr>
<tr>
<td>FLAT PLATE</td>
<td>$a$</td>
<td>$ab$</td>
</tr>
<tr>
<td>DIHEDRAL CORNER</td>
<td>$\frac{a}{b}$</td>
<td>$\sqrt{2ab}$</td>
</tr>
<tr>
<td>TRIANGULAR</td>
<td>$\frac{a}{\sqrt{3}}$</td>
<td>$\frac{\pi a}{3\lambda^2}$</td>
</tr>
<tr>
<td>TRIHEDRAL SQUARE</td>
<td>$\frac{a}{\sqrt{3}}$</td>
<td>$\sqrt{3a^2}$</td>
</tr>
</tbody>
</table>

$A_T =$ Equivalent Flat Plate Area  
RCS= Radar Cross Section  
$\lambda =$ Wavelength  
Objects Assumed Large in $\lambda$.  

\[
\begin{align*}
A_T &= \frac{a\lambda}{2} \\
RCS &= \frac{2}{\pi a} \\
\lambda &= \text{Wavelength} \\
\text{Objects Assumed Large in } \lambda.
\end{align*}
\]
Figure 1. Antenna Footprints for an Inner Yaw Gimbal.

Figure 2. Antenna Footprints for an Outer Yaw Gimbal.
fixed look down angle, the pattern scanned is an arc of a circle on the ground.

1.2.1. Clutter Power

The clutter power, as in Nathanson (1969), at the receiver is

\[ P_c = \frac{P G A_e \tau_p (\sigma_o A_c)}{(4\pi)^2 R^4} \]

where,

- \( A_c \) = Area of clutter cell illuminated
- \( \sigma_o \) = Backscatter cross section per unit area (\( m^2/m^2 \)) of a reflecting surface, (assume the mean value).

There are two cases for calculating the value of \( A_c \), the clutter area (See Figure 3 a,b). They depend on whether the pulse length is larger or smaller than the projected radar footprint in the radial direction of the beam. The footprint is the ellipse on the ground that is under illumination. For both cases the earth is assumed to be flat. As illustrated, the clutter area may be approximated by the clutter cell limited by beamwidth

\[ A_c = \pi R^2 \tan(\theta_{az}/2) \tan(\theta_{el}/2) \csc(\phi) \]

when \( \tan(\phi) > R \tan(\theta_{el}) / (c \tau_p/2) \)
Radar $\Theta_{el} = -3\text{dB Beam Width in Elevation}$

$\Theta_{az} = -3\text{dB Beam Width in Azimuth}$

Figure 3a. Beamwidth Limiting Case.

Flat Earth

$2R \tan (\Theta_{el}/2)$

$2R \tan (\Theta_{az}/2)$

Figure 3b. Pulsewidth Limiting Case.
or by the clutter cell limited by pulse length,

\[ Ac = 2R \tan(\theta_{az}/2) \times \left( \frac{c \tau_p}{2} \right) \sec(\phi) \]

where \[ \tan(\phi) < \frac{R \tan(\theta_{el})}{(c \tau_p/2)} \]

where

- \( \theta_{az} \) = Azimuth beam width (3dB, Two Way)
- \( \theta_{el} \) = Elevation beam width (3dB, Two Way)
- \( \phi \) = Look down angle

The choice of either equation depends on the look down angle. For pencil beam radars with beamwidths less than 10 degrees, the following approximation holds since \( \tan(x) = x \) for small values of \( x \):

\[ Ac = \left( \frac{\pi R^2}{4} \right) \times \theta_{el} \times \theta_{az} \times \csc(\phi), \]

where \[ \tan(\phi) > \theta_{el} \times R / (c \tau_p/2) \]

and

\[ Ac = R \theta_{el} (c \tau_p/2) \sec(\phi), \]

when \[ \tan(\phi) < \theta_{el} \times R / (c \tau_p/2) \].

Note that the clutter area is a function of the range so that unlike a target signal the clutter signal will decay only on the order of \( 1/R^2 \) for the beamwidth limited case and as \( 1/R^3 \) for the pulsewidth limited case.

In the absence of clutter, the return signal is due only to the target and thermal noise. As such, locating a target is accomplished by looking for the
return to exceed a fixed level. It is assumed that there is an automatic adjustment in the IF gain made to cancel the effect of the differences in range so that different range cell returns may be compared on the same scale. In the presence of clutter a return signal is composed of a target, ground clutter and thermal noise. The ground, trees, and buildings are all sources of clutter.

1.2.2. Clutter Filtering

To find a target in clutter, the return of the range cell under test is compared to the average of all of the cells surrounding it. If it exceeds this value by some certain amount, then a potential target is indicated. This type of processing leads to a constant false alarm rate (CFAR).

1.3. Pulse Compression

The basic limit of a radar system is that the resolution in range is inversely proportional to the bandwidth of the transmitted radar pulse. If we were to transmit a perfect impulse, it would result in complete resolution in range, since a perfect impulse would have infinite bandwidth. However, it is not possible to transmit a perfect impulse. There are methods to increase the bandwidth and transmit enough power to get a return signal. These are called pulse
compression techniques. Analog pulse compression type systems have a transmit waveform that is a continuous sweep of frequencies and is commonly referred to as a "chirp waveform."

In this thesis, digital post detection processing is presented. Digital pulse compression uses a series of pulses at different frequencies. Digital pulse compression uses radar frequency pulses which are composed of stepped RF modulation from a tunable local oscillator. The pulse is generated simply by switching the RF modulation on then off. While the pulse is off, the antenna is switched to receive the reflected energy from the targets.

Each one of these pulses is sent out at N different carrier frequencies. They are spaced apart in time so that a low pulse repetition frequency (low PRF) is maintained. The radar has no way of knowing which transmit pulse gave rise to a signal return, so the pulses must be separated by sufficient time so that the assumption can be made that all of the return power is from the last pulse.

Low PRF insures that there is no ambiguity in range, since the return had to be due to the last pulse that was transmitted. This is because the power from the earlier pulses is below the noise level of the radar since it decays at $1/R^4$ as the range to the
target increases, which is the same as saying as the range delay in time increases.

For pulse width limiting radars, the illumination patch or footprint needs to be subdivided. I will call the interval of the footprint resolved by a single pulse a range cell. The length of a range cell is directly proportional to the pulse width in time (See Figure 4a and 4b).

\[
\text{Range Cell Length (m)} = \tau_P \text{ (sec)} \times 3 \times 10^8 \text{ (m/sec)}
\]

Without the use of high resolution methods this would be the limit of the resolving ability of a simple radar system. For example, if a pulse is on for 100 nano-seconds and the speed of light is assumed to be \(3 \times 10^8\) meters per second, then such a radar system will have a resolution of 30 meters.

1.4. Tracking in Range

After a target is detected by the CFAR method and it has been classified as a real target, then it must be tracked in down range distance as well as angle. As in Skolnik (1980), range tracking is done by using two adjacent range cells or by sampling a low resolution range cell at twice the normal rate.

The return signal from one gate is subtracted from the other and this difference is normalized by the
\[ \tau_p = \text{Time Duration of One Low Resolution Range Cell.} \]

Figure 4a. Pulse Shape Function.

Modulation Not to Scale

Figure 4b. Pulse Shape with Modulation.
sum of the two to generate a range error signal. The result is used in the range tracking feedback control loop to reposition the timing of the range gates. Shifting the timing of the range gates keeps the A/D sampling time synchronized with the returning signal from the target. The magnitude of the range error signal is directly proportional to the amount of time the gates must be moved and its sign points in the direction the gates must move. The accuracy of the range tracking is limited to half of the length of the range gates used.

Figures 5 and 6 show the A/D output level for 16 of the low resolution range cells. Figure 5 is the return from two bounce scatterers or dihedral targets, and Figure 6 is the return for the odd number of bounces or trihedral targets.

1.5. Polarization

To get more information from the radar system, the signal polarization is utilized. For example, the signals could be circularly polarized. If the transmitted pulse was polarized right hand circular (RHC) and the received signal is also RHC, then the pulse had been reflected an even number of times. If the transmitted pulse was polarized right hand circular (RHC) and the received signal is now left hand circular
(LHC), then the pulse had been reflected an odd number of times. If the return signal had an elliptical polarization, then that ellipse may be treated as the sum of a LHC component and a RHC component.

The polarization information can be used to improve the constant false alarm rate method of clutter canceling. For example, a smooth metal building returns a large signal when the beam hits both the ground and wall. This causes the return signal to have the same polarization as the transmitted signal, so the building wall is a source of even clutter.

**TABLE 2. RECEIVER POLARIZATIONS**

<table>
<thead>
<tr>
<th>TRANSMITTER POLARITY</th>
<th>RECEIVER LEFT HAND CIRCULAR</th>
<th>RECEIVER RIGHT HAND CIRCULAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEFT HAND CIRCULAR</td>
<td>EVEN RETURN</td>
<td>ODD RETURN</td>
</tr>
<tr>
<td>RIGHT HAND CIRCULAR</td>
<td>ODD RETURN</td>
<td>EVEN RETURN</td>
</tr>
</tbody>
</table>
Figure 5. Dihedral Return vs. Low Resolution Range Cells.
Figure 6. Trihedral Return vs. Low Resolution Range Cells.
2. MONOPULSE AND HIGH RESOLUTION RADARS

2.1. Monopulse Radar Systems

A monopulse radar system has two receiver horns in the antenna. These are placed parallel to each other and balanced so that they receive the same wave front when the antenna is pointing on target (See Figure 7). More horns may be used to apply the tracking to two axes. For simplicity I will consider a system that calculates an error function for a scanning sample along only one axis, then a two axis system will be examined.

Monopulse systems are generally broken down into two groups, the phase and amplitude tracking systems (See Figures 8a and 8b). The amplitude tracking system is the one that is examined in detail.

2.1.1. Phase Tracking

A phase tracking monopulse radar has a phase detector to compare the relative phase difference between the two horns. This phase difference is proportional to the sine of the phase shift $\theta$ between the two receiver horns.

\[ \text{Phase Track Error} = k \sin(\theta) \]
Figure 7. Single Coordinate Amplitude Monopulse Antenna.
A) AMPLITUDE MONOPULSE

Figure 8a. Amplitude Monopulse Antenna Configuration.

B) PHASE MONOPULSE

Figure 8b. Phase Monopulse Antenna Configuration.
This type of system is very sensitive to phase shifts caused by the relative motion of the target and to any imbalances in the two receiver channels. Because of this limitation the rest of the thesis will focus on the amplitude tracking monopulse method.

2.1.2. Amplitude Tracking

An amplitude tracking monopulse radar is composed of two horns which I will call left and right. These are connected through a hybrid-T so that the outputs are the sum and the difference of the left and right horns. The antenna patterns (See Figures 9 and 10) illustrate the properties of the sum and difference patterns. Figure 9 shows the return from a pair of trihedrals averaged over all of the frequencies. Figure 10 is the return from a pair of dihedrals, and it too was averaged over the entire bandwidth.

The sum channel data will reach a maximum when the target is on bore sight, while the magnitude of the difference (delta) channel data goes to a minimum and its phase changes sign. It is this delta channel data that is so useful as an error function. Since the sum channel is nearly flat when on target, it does not work well as an error function to close the tracking loop. The delta channel has a nearly linear behavior near the zero error angle. The ratio of the difference
Figure 9. Sum and Difference Antenna Pattern from a Pair of Trihedrals.
Figure 10. Sum and Difference Antenna Pattern from a Pair of Dihedrals.
over the sum is nearly linear between the 3 dB down points that define the beam's width.

\[
\text{Amplitude Track Error} = \frac{L - R}{L + R} = \frac{\Delta(f,t)}{\Sigma(f,t)}
\]

The amplitude tracking error function is simply the difference between the left and right horns normalized by the sum of the two horns. This is the result when using a low resolution radar with no pulse compression. For this type of system, the tracking is biased toward the brightest reflector when many are illuminated simultaneously on the same target.

2.2. High Resolution Radar

A stepped frequency pulse train as described by Einstein (1984) is utilized to increase the bandwidth of the receiver to yield high resolution range information. For a coherent system, the range profile can be solved for explicitly. In a non-coherent system the phase angle information is not preserved between transmission and reception, so only the autocorrelation of the range profile may be found. This function gives the target's down range separations, but not the exact range to the target. This separation profile is useful in identifying a target, but it alone will
not locate a target any more precisely than a simple low resolution system.

The magnitude of the return signal from a single point scatterer with amplitude $A$, at a sampling time $t_s$ will be

$$|E(f, t_s)| = A \left[ Ps(t) \cdot \frac{r}{c} \right]$$

where $Ps(t)$ is the pulse shape in time, $r$ is the range to the target and $c$ is the speed of light. Let the return $E(t_s)$ be defined to be the center of the range cell, so for the best return signal, the A/D sampling should be timed with the maximum of the pulse shape function $Ps(t)$, (See Figure 4).

It is important to note that the return signal is frequency dependent, and that the high resolution radar is sampling in frequency domain.

2.2.1. Coherent Processing

For coherent processing each pulse is coherently detected with the transmitted carrier for that pulse. The complex return from a distributed target after coherent mixing is:

$$V(f_k, t) = e^{-j \psi_k(t)} \left[ e^{\frac{-j 2\pi f_k x}{c}} \int_{-\infty}^{+\infty} \rho(x) \cdot Ps(t-x)e^{j \frac{2\pi f_k x}{c}} dx \right]$$

Where $\rho(x)$ is the distributed target's profile in down range delay time, $x$, and an ideal local oscillator is
assumed (no frequency drift while waiting for the return). From the form of the previous equation; it follows that the return is the Fourier transform of the down range target profile of the distributed target \( \rho(x) \), weighted by the pulse shape function \( P_s(x) \).

\[
\text{Frequency Domain} \quad \longleftrightarrow \quad \text{Time Domain}
\]

\[
V(f_k, t) \quad \longleftrightarrow \quad \rho(x) P_s(t-x)
\]

Since this is so, the range profile can be found by taking the inverse Fourier transform, \( \mathcal{F}^{-1} \), of the return signal, sampled in the frequency domain. When \( N \) different stepped frequency pulses are transmitted and their returns are sampled, an inverse discrete Fourier transform may be used to find the range profile. In order to apply the inverse discrete Fourier transform, the frequency steps must all be equal.

\[
f_k = f_0 + k x \Delta f \quad \text{for} \quad k = 0, \ldots, N-1
\]

where,

- \( f_k \) is the transmitted frequency for the \( k \)th pulse.
- \( \Delta f \) is the frequency step size.
- \( N \) is the number of pulses in the train.

The bandwidth is simply \( N x \Delta f \), so the resolution in range is,

\[
\text{Resolution} = \frac{C}{(N x \Delta f)}
\]
In order to avoid folding the returns from scatterers within the same low resolution range cell, the range of the profile, $C / \Delta f$, must be made greater than two times the width of a single pulse,

$$C / \Delta f > 2C x \tau_p$$

so that returns from scatterers located more than one pulse width away from the sampling time (the center of the pulse) will be strongly attenuated. By solving for $\Delta f$ the requirement is expressed as,

$$\Delta f < \frac{1}{2\tau_p}$$

For an example consider the simplest cases of a single point (Figure 11a) and of two point targets in the same range cell separated by a distance $d$ (Figure 11b). For a single scatterer no additional information is gained by the high resolution system. For the two target case the simple monopulse radar (See Figure 12a) will track the brightest point of the target array, it is commonly called an RCS tracking radar. The high resolution monopulse system will track on the geometric center of the array of targets, hence, it is referred to as a geometric tracker.

If the radar is to find a vehicle in clutter, the range cell will be generally larger than the
Figure 11a. Single Point Reflector and Aimpoint Displacement.

Figure 11b. Two Reflector Target and Its Aimpoint Displacement.
Figure 12. Simple Monopulse Aimpoint vs. High Resolution Monopulse Aimpoint.
vehicle, because available hardware cannot generate a powerful pulse with a large bandwidth. By using the high resolution radar, the resolution can be improved to the point where the range bins are smaller than the vehicle. Now the target is resolved into a set of reflectors on the vehicle's surface.

Assuming that the longest length of the target is $L$ meters, then the returns from a pair of reflectors on the surface of the target cannot be more than $L$ meters apart. However, the target might resonate with the clutter and the clutter may resonate with itself at any distance within the range cell.

While tracking on a distributed target, only separations that are less than $L$ need to be considered. Those greater than $L$ are used as a clutter reference. This may help in determining if the return is due to a target in clutter or simply a high amount of clutter. By using the inverse Fourier transform, each range cell is magnified into $N$ high resolution range bins.

2.2.2. Non-Coherent Processing

Relative motion of the targets and the antenna is a difficult problem to overcome in a coherent radar system. It requires the use of three accelerometers on the antenna and assumes that the target has no motion of its own. The accelerometer outputs are
integrated twice to give the spatial displacement of the antenna so that the phase shift of the radar frequency signal that this displacement caused may be removed.

In a non-coherent radar the phase information is not preserved. This makes the non-coherent system more immune to relative motion of the target and the antenna. Since there is no fixed reference for phase, the sum channel is assumed to be real and positive. This will be the phase reference for both channels. Figures 13 and 14 show how the inphase and quadrature delta channels are formed by splitting the delta channel and multiplying one side by a cosine and the other side by a sine function.

The reference cosine function is generated by hard limiting the sum channel. The reference sine function is formed by phase shifting the limited sum channel by 90 degrees. The following Figure 15 is a flow chart of the process that uses the outputs of the A/Ds to compute the autocorrelation function of the sum channel and the crosscorrelation of the sum and difference channels. After the threshold test, the normalization is done. Finally, the remaining values are averaged to arrive at the cross range error angle.
Figure 13. RF to IF and IF to A/D’s Block Diagrams.
Figure 14. Phase Detector Detail.
Set Constants and rotation factors for IFFT[]

Get Number of Samples and Threshold fraction

Call Source Routine For Raw Radar Data

Square the Sum Channel then Compute the Autocorrelation or 'Separation Profile'

\[ \Phi^{-1}\{ \text{Sum} \times \text{Difference} \} \]

Form the product (Sum x Difference), then Compute the Crosscorrelation

\[ \Phi^{-1}\{ \text{Sum} \times \text{Difference} \} \]

Find the Magnitude of each Autocorrelation Term

Set threshold as a fraction of the zero separation term and apply the threshold to each autocorrelation term

\[ \frac{-1}{\phi \times \frac{\Delta \Sigma}{\Phi \times [\Sigma \Sigma]}} \]

Ratio =

Average the Error Values

More Data?

Figure 15. Flow Chart for High Resolution Monopulse Programs.
3. SIMULATIONS AND EXPERIMENTS WITH POINT TARGETS

3.1. Example System

All of the following examples are limited to the use of point scattering targets. These are simple to simulate and are easily constructed. The results may be considered valid for any target since a complex target may be modeled as a set of point reflectors. The only restraint is that each pair of point targets be separated by a unique number of high resolution range bins.

For this example, let us assume the system to be designed will be mounted on an earth satellite. The system is to track another earth satellite. Since this is to be a space based system, the operating frequency may be chosen to be in a range where the atmospheric losses are high. The purpose here is to prevent a terrestrial source from interfering.

For simplicity, assume the system is operating at a base frequency of 100 GHz. Let the pulse length be 32 m, so the pulse duration is 106.6 nanoseconds. Also, a square pulse shape is assumed. To divide the 32 m range cell into 1.0 m long, high resolution
cells, the step in frequency must be 4.6875 MHz if 64 frequencies are used.

\[ \text{Resolution} = \frac{c}{\text{Bandwidth}} = \frac{c}{N \times \Delta f} \]

so,

\[ \Delta f = \frac{300,000 \text{ km/s}}{N \times \text{Resolution}} \]

In the examples N is 64 and the resolution is 1 meter. For 1 meter resolution the total bandwidth is 300 MHz, and the frequency step size should be 4.6875 MHz.

All radar systems are limited by the range equation,

\[ P_{\text{rec}} = \frac{P \cdot G \cdot \sigma \cdot A_e \cdot \tau_p}{(4\pi)^2 \cdot r^4} \]

where,

- \( P \) = average transmitted power
- \( G \) = antenna gain and losses, for transmit and receive
- \( \sigma \) = radar cross section of target
- \( A_e \) = effective antenna area
- \( \tau_p \) = pulse duration time (See Figure 4)
- \( r \) = range.

The \( 1/R^4 \) losses limit the range to the point where the received power is lost in the thermal noise. In this example, zero clutter is assumed for space. Assume that the maximum range is 3200 Km; then a pulse repetition frequency of 93.75 pulses/second will insure against range ambiguity. After each pulse the antenna is switched to receive, with the A/D converter sampling at intervals equal to the pulse duration, 106.6 nanoseconds or a sampling rate of 9.375 MHz.
3.1.1. Single Target Simulation

For one target with a reflectivity of $r^a e^{j\theta a}$, the limited channel output $E(f)$ is given by the following equation assuming $A_s(\alpha)$ is the gain of the sum channel as a function of scan angle,

$$E(f) = A_s(\alpha) \sigma_a$$

where $\alpha$ is the angle from the bore sight of the antenna to the line of sight to the target.

As in Barton (1977), the simplest case of uniform far field illumination, $S(\alpha)$, is

$$S(\alpha) = d \left[ \sin(\alpha/2) \right] / (\alpha/2)$$

where $d$ is the distance to the target, and the following near field approximations are assumed.

$$A_s(\alpha) = 1 \quad \text{and} \quad A_\Delta(\alpha) = x$$

When looking at only one scatterer in the far field, $E(f)$ is constant in magnitude for all frequencies. The difference channel, $\Delta(f)$ is also constant for all frequencies at a given $\alpha$. It is passed through the phase detector circuit (Figures 13 and 14). The phase detector has two outputs. These are the inphase component $\Delta I(f)$ and the quadrature component $\Delta Q(f)$ of the nonlimited channel, which is the difference channel for tracking. The sum channel is passed through a limiter to be used as a phase reference for
the phase detector. Since \( E(f) \) is defined to be real,

\[ E(f) = E^*(f) \]

This means that the inphase difference channel is the component of the difference channel that is inphase with the sum channel vector. The quadrature difference channel is the component of the difference channel that is \( \pi/2 \) radians out of phase with the sum channel vector. Where \( \phi \) is the angle between the sum and difference channels,

\[ \Delta(f) = A_\Delta(\alpha) \times \sqrt{\sigma_a} \]

\[ \Delta I(f) = \Delta(f) \cos(\phi) \]

\[ \Delta Q(f) = \Delta(f) \sin(\phi) \]

Then the inphase and quadrature difference channels are as follows,

\[ \Delta I(t) = \Delta(f) \cos(\omega t + \phi) \times \cos(\omega t) \]

\[ \Delta I(t) = \Delta(f) \cos(2\omega t + \phi) + \Delta(f) \cos(\phi) \]

\[ \Delta Q(t) = \Delta(t) \cos(\omega t + \phi) \times \cos(\omega t - \pi/2) \]

\[ \Delta Q(t) = \Delta(t) \cos(2\omega t - \pi/2 + \phi) + \Delta(t) \cos(\phi + \pi/2) \]

After low pass filtering,

\[ \Delta I(t) = \Delta(t) \cos(\phi) / 2 \]

\[ \Delta Q(t) = -\Delta(t) \sin(\phi) / 2 \]

The negative sign in the quadrature channel can be canceled by reversing the connection of the envelope detector to the A/D converter. The inphase and quadrature A/D outputs are bipolar since they can be either
positive or negative. The sum channel A/D output is always positive and therefore unipolar. If 8 bit A/D's are used on both the unipolar and bipolar channels, the need for a sign bit in the bipolar channel will reduce the dynamic range by 6 dB compared to the unipolar channels.

The dynamic range of an 8 bit A/D is 48 dB if the noise floor is one quantum level. If the noise floor is two quantum levels, then the dynamic range is cut to 42 dB and for a noise floor of four quantum levels the dynamic range is reduced to only 36 dB.

The amplitude monopulse tracking error is

\[
\text{Monopulse Error}(nr) = \frac{-1}{\phi \left[ (\Delta I(f) + j \Delta Q(f)) \Sigma^*(f) \right]}
\]

\[
\frac{-1}{\phi \left[ \Sigma(f) \times \Sigma^*(f) \right]}
\]

\[
\frac{R_{\Delta \Sigma}(nr)}{R_{\Sigma \Sigma}(nr)}
\]

The monopulse error is defined to exist only for those values of \((nr)\) where the autocorrelation of the sum channel exceeds an adaptive threshold of a selected fraction of the zero separation term of the autocorrelation function. In the examples given 1/20 of the zero separation term was arbitrarily chosen as the threshold. This is required to prevent a division by
zero and/or to prevent a division by a small number. This also helps eliminate the effects of clutter. In terms of the targets this means that the monopulse error function is defined only at range separations where there are relatively strong scattering centers.

In the presence of very heavy clutter the assumption that each resonating scattering pair is unique falls apart. This can lead to angle error calculations that point completely off of the target or in some cases even outside the illuminated area. Those that are obviously too large can be tested for and eliminated. At this time I can not see a simple solution to filtering out those components that point off the target, but not out of the beam.

3.1.2. Two Target Array

For two targets separated by $r_{bc}$ meters in down range distance, the time delay between them is the time for light to travel past one to the other and back again. If the two targets are fixed so their separation does not change, the time delay is constant.

$$t_{bc} = \frac{2 r_{bc}}{300,000} \text{ (km/sec)}$$

Assume the two targets have reflectivity vectors $\bar{B}$ and $\bar{C}$ defined as follows.

$$\bar{B} = B e^{jb} \quad \text{and} \quad \bar{C} = C e^{jc}$$
the sum channel is given by,
\[
\Xi(f) = A_{\xi}(\alpha) \left[ (B + C) \left( \frac{B}{2} + C \right) \right]^{1/2}
\]
and the difference channel is given by,
\[
\Delta I(f) = A_{\Delta}(\alpha) \left[ \left( \frac{B}{2} + C \right) + 2BC \cos(\omega t_{bc}) \right]^{1/2}
\]
\[
\Delta Q(f) = A_{\Delta}(\alpha) \left[ \left( \frac{B}{2} + C \right) + 2BC \sin(\omega u_{bc}) \right]^{1/2}
\]
where the following definitions hold,
\[
\alpha = \text{angle between the Target and Bore sight}
\]
\[
t_{bc} = \text{time delay due to down range separation of B and C.}
\]
\[
u_{bc} = \text{time delay due to cross range separation of B and C.}
\]

The 3-D plot for the two target case is shown in Figure 16. The error component at the -16 m down range separation bin passes through zero when the nearer of the two targets is on bore sight. The +16 meter separation term passes through zero when the further of the two targets is on boresight. For zero meters separation the error function goes through zero when the peak of the return is on boresight. Remember that this point is equal to the geometric center only when the
Figure 16. 3-D Plot of the Monopulse Error Function While Scanning a Pair of Point Targets.
two scatterers have the same RCS and are aimed the same way.

The geometric center of the two scatterers is found by averaging the -16 m and +16 m separation terms without the zero separation term.

3.1.3. Three Target Array

For three targets separated by \( r_{ab} \), \( r_{bc} \) and \( r_{ca} \) meters in down range distance, where it is assumed the separations are each distinct and not equal, the sum channel is as follows,

\[
\Sigma(f) = A^2(\alpha) \left[ \frac{1}{2} \sqrt{A^2 + B^2 + C^2} \right]
\]

\[
\Sigma(f) = A^2(\alpha) \left[ \frac{1}{2} \sqrt{(A + B + C)^2 + 2AB \cos(\omega t_{ab}) + 2BC \cos(\omega t_{bc}) + 2CA \cos(\omega t_{ca})} \right]
\]

and the difference channel is given by,

\[
\Delta I(f) = A^2(\alpha) \left[ \frac{1}{2} \sqrt{(A + B + C)^2 + 2AB \sin(\omega t_{ab}) + 2BC \sin(\omega t_{bc}) + 2CA \sin(\omega t_{ca})} \right]
\]

\[
\Delta Q(f) = A^2(\alpha) \left[ \frac{1}{2} \sqrt{(A + B + C)^2 + 2AB \sin(\omega t_{ab}) + 2BC \sin(\omega t_{bc}) + 2CA \sin(\omega t_{ca})} \right]
\]
where,
\[ \alpha \] = angle between the target and boresight
\[ t_{ab} \] = time delay due to down range separation of A and B
\[ t_{bc} \] = time delay due to down range separation of B and C
\[ t_{ca} \] = time delay due to down range separation of C and A
\[ u_{ab} \] = time delay due to cross range separation of A and B
\[ u_{bc} \] = time delay due to cross range separation of B and C
\[ u_{ca} \] = time delay due to cross range separation of C and A.

For the simulation of three targets assume the following parameters are given:

- RCS Amplitude of Target A: \( 50.0 \text{ m}^2 \)
- RCS Amplitude of Target B: \( 25.0 \text{ m}^2 \)
- RCS Amplitude of Target C: \( 15.0 \text{ m}^2 \)
- Cross Range Distance of Target A: \(-8.0 \text{ m}\)
- Cross Range Distance of Target B: \(4.0 \text{ m}\)
- Cross Range Distance of Target C: \(12.0 \text{ m}\)
- Down Range Distance to Target A: \(300.0 \text{ m}\)
- Down Range Distance to Target B: \(307.0 \text{ m}\)
- Down Range Distance to Target C: \(319.0 \text{ m}\)

Figure 17 is a 3-D plot and shows the error function for the three target case. The +7 m and +19 m terms go to zero when target A is on boresight. The -7 m and +12 m error terms pass through zero when target B is on boresight. Similarly, the +7 m and the +19 m terms are zero when target C is on boresight. As before, the zero meter separation term is biased toward the peak reflective center and is not included in calculating the geometric center of the array of scatterers.
Figure 17. 3-D Plot of the Monopulse Error Function While Scanning a Three Point Target Array.
3.2. High Resolution Monopulse Tests

Three sets of test data were collected. They differ only by the receive polarization and the axis of scanning. The target array for both tests consists of two dihedrals and two trihedrals arranged as follows where the distances are in the units of high resolution bins (1 meter for this example). Figures 18 and 19 show the top and side views of the radar range, and Figure 20 is a detail of the target array.

**TABLE 3**

**TARGET POLARIZATIONS AND COORDINATES**

<table>
<thead>
<tr>
<th>TARGET</th>
<th>TYPE</th>
<th>LOCATION RELATIVE TO TARGET A (CROSS RANGE, DOWN RANGE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Dihedral</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>B</td>
<td>Dihedral</td>
<td>(2, -10)</td>
</tr>
<tr>
<td>C</td>
<td>Trihedral</td>
<td>(-4, 13)</td>
</tr>
<tr>
<td>D</td>
<td>Trihedral</td>
<td>(-2, -7)</td>
</tr>
</tbody>
</table>

For all the tests the transmitter sends out a right hand circular (RHC) polarized wave.

3.2.1. Polarmetric Analysis of High Resolution Data

In the first test the receiver passes both of the sum channels, left and right hand circular polarization.
Figure 18. Side View of the Radar Test Range.

Figure 19. Top View of the Radar Test Range.
Figure 20. Top View of Experimental Array.
The antenna is slowly scanning along the azimuth axis. For simplicity, I will call the right hand circularly polarized receiver channel "even" and the left hand circularly polarized receiver channel "odd." The range plots (Figures 5 and 6) show the sum channel even and odd low resolution response for 16 low resolution range cells. Note how the return is smeared across three adjacent low resolution cells.

The Time Series I.F. plots (Figures 21 and 22) show the even and odd sum channel returns vs. the 64 sampling frequencies. Since both the unipolar A/D channels are real, their transforms may be computed simultaneously by placing the even polarization component in the real part and the odd polarization component in the imaginary part of a complex variable. Then after taking the inverse Fourier transform, the even and odd properties of the transform makes possible the separation of the two autocorrelation functions. Figure 23 shows the output from the real separator routine, where the left side of the graph is the odd return autocorrelation and the right side is the even return autocorrelation. For comparison these are computed separately as in Figures 24 and 25.
Figure 21. Even Return vs. N Frequency Steps.
Figure 22. Odd Return vs. N Frequency Steps.
Figure 23. Real Separator Output from Simultaneous DFT's.
Figure 24. Magnitude of the Inverse Transform of the Odd Channel Data Squared.
Figure 25. Magnitude of the Inverse Transform of the Even Channel Data Squared.
Two useful transforms can be generated from this data. These are the autocorrelation functions of each of the polarized sum channels mentioned above. The autocorrelation peaks of the sum odd data show the resonation of trihedral targets with other trihedrals. Likewise the autocorrelation of the sum even data shows the resonations of dihedrals with dihedrals. These functions are used to apply the thresholding and also are used for normalizing the difference channel's inverse Fourier transform, on a term by term basis.

3.2.2. Monopulse Tests

For both the second and third tests, the receiver is set to pass one sum channel and the corresponding difference channel of the same polarization. One is the azimuth scan of the same test array as before, and the later is an elevation scan of the same array.

3.2.2.1. Monopulse Test, Azimuth Scan, Even Return

For the second test the receiver is set to pass the sum even signal to the limited channel, and the delta even signal is passed through the phase detector and broken into its inphase and quadrature components. Since only the even channels are being used, only the dihedral targets can be seen. Due to the lack of isolation, the trihedrals will appear as faint dihedrals. The thresholding will eliminate them.
3.2.2.2. Monopulse Test, Elevation Scan, Odd Return

For the third test the receiver is set to pass the sum odd signal to the limited channel, and the delta odd signal is passed through the phase detector and broken into its inphase and quadrature components. Since only the odd channels are being used only the trihedral targets can be seen. Due to imperfect isolation the dihedrals will appear as faint trihedrals.

3.2.3. Comments on the Graphs Included

For each of these tests the following graphs are included. The Sum Delta Patterns for both the azimuth and elevation scans are shown for 250 dwells in Figures 9 and 10. A dwell is the time for one set of \( N \) frequencies to be sampled. The 3-D plots (Figures 26 and 27) showing the monopulse error vs. down range separation vs. cross range distance are the output of the fortran program Slope.exe, and they represent the normalized cross-correlation of the sum and difference channels vs. the dwell. Note that the angle scan rates of the antenna were scaled so that at this down range distance each dwell sample is proportional to one meter in cross range distance.
Figure 27. 3-D Plot of Monopulse Error vs. Down Range Separation vs. Cross Range Distance for Even Channel Data, Elevation Scan.
Figure 26. 3-D Plot of Monopulse Error vs. Down Range Separation vs. Cross Range Distance for Odd Channel Data, Azimuth Scan.
The two-dimensional Monopulse Error vs. Cross Range Distance plots (Figures 28 and 29) represent the same data as the three-dimensional graphs. Each line on these plots represents a plus or minus separation term. When one of these lines passes through zero, one of the targets is on boresight.

Unfortunately the targets were right at the limit of being resolvable in the cross range direction, so their geometric center and brightest point are very close together. Another problem is the unexpected flat region near the zero crossing. A lack of dynamic range in the inphase and quadrature channels caused this. The magnitude of the difference channel from the unipolar A/D has a dynamic range that is 6 dB larger. The last pair of plots (Figures 30 and 31) are the same as the Monopulse Error vs. Cross Range plots, except the inphase and quadrature channels were used only for their phase information, while the magnitude component of the difference channel came from the unipolar A/D.
Figure 28. Monopulse Error vs. Cross Range Distance for Odd Channel Data, Azimuth Scan.
Figure 29. Monopulse Error vs. Cross Range Distance for Even Channel Data, Elevation Scan.
Figure 30. Monopulse Error vs. Cross Range Distance for Odd Channel Data, Azimuth Scan. Note: Unipolar A/D used for magnitude and Bipolar A/D used for phase only.
Figure 31. Monopulse Error vs. Cross Range Distance for Even Channel Data, Elevation Scan.
Note: Unipolar A/D used for magnitude and Bipolar A/D used for phase only.
4. CONCLUSIONS

The improvement in tracking realized by using a high resolution monopulse radar vs. a low resolution monopulse system is very data dependent. For single point target or when a pair of targets have nearly the same RCS, both systems will track on the same point. However, when more than one scatterer is present and they have RCS values that are not equal, then the brightest point in the array is no longer at the geometric center of the array.

In the case of two scatterers, if one is twice as reflective as the second. The RCS center of the pair is 1/3 of the distance between the two, from the brighter reflector. So, the low resolution system will track this point, which is off of the geometric center by 1/6 of the distance separating them. The general expression for the improvement in tracking by a high resolution monopulse radar and a single pulse radar follows:

\[
\% \text{ Improvement} = \frac{d}{2} \left( \frac{(A-B)}{(A+B)} \right)
\]

where \( A \) = RCS of target A
\( B \) = RCS of target B
\( d \) = Cross range distance separating A and B.
When three scatterers are illuminated, by a single pulse in the same low resolution range cell, the track aimpoint improvement is expressed as follows:

\[
\%I = \frac{\text{Geometric Center} - \text{Average Brightest Point}}{\frac{A_c r_a + B_c r_b + C_c r_c}{3} - \frac{A_c r_a + B_c r_b + C_c r_c}{A + B + C}}
\]

where

- \(A\) = RCS of target A
- \(B\) = RCS of target B
- \(C\) = RCS of target C
- \(c_{ra}\) = Cross range distance of target A
- \(c_{rb}\) = Cross range distance of target B
- \(c_{rc}\) = Cross range distance of target C

The assumption that always applies to the high resolution system is the requirement that the point scattering targets be separated by unique distances and that the return from the clutter not invalidate that assumption.
Software Listings

***************************************************************
***************************************************************

SLOPE: Mono Pulse Error Slope Program Version 1.0
Copyright (C) 1987 Martin Marietta Corporation, U.S.A.
Written by Larry Alan Young of Martin Marietta Missile Systems Radar Systems Analysis Dept.

This program simulates a High Resolution Mono-pulse antenna scanning a fixed target array. The error angle is the average of the following ratio for each IFFT bin where IFFT(Sum x Sum ) exceeds a threshold.

\[
\frac{\text{IFFT}(\Delta(f) \times \text{Sum}(f))}{\text{IFFT}(\text{Sum}(f) \times \text{Sum}(f))}
\]

This program calculates:

\[
\text{Track Error} = \frac{\text{MAG}[ \text{IFFT}(\Delta(f) \times \text{Sum}(f)) ]}{\text{MAG}[ \text{IFFT}(\text{Sum}(f) \times \text{Sum}(f)) ]}
\]

where the denominator exceeds a threshold. The points thus calculated are placed in array RATIO(ND,II) where II indexes the separation bins (-31 to 32) and ND indexes the number of frequency templates sampled (dwell). This program uses the following subroutines, SOURCE2 SOURCE3, LYFFT, AXIS, PLOTTER and CURVE all developed by Larry Young.

Contact Larry Alan Young (305) 356-5422 for technical assistance.

***************************************************************
***************************************************************
PROGRAM SLOPE

$INCLUDE: 'MONBLK.FOR'

COMPLEX*8 SMCHNL(64),SUMSQ(64),DELSUM(64),PHASER
COMPLEX*8 FFTSS(64),FFTDSS(64)
REAL *4 PKHSUM(33),PKHDEL(33)
REAL *4 PKLSUM(33),PKLDEL(33),THOLD,THRESH
CHARACTER*1 ANS
CHARACTER*12 DATINP

C Open an output file to dump the results.
OPEN(3,FILE='MONO.OUT',STATUS='NEW',FORM=*
   'FORMATTED')
OPEN(5,FILE='MONO.DAT',STATUS='NEW',FORM=*
   'FORMATTED')
WRITE(3,*) 'Sep. Ratio Mag[IFFT(DS)]
           Mag[IFFT(SS)]'

C Set some constants.
PI = 3.1415927
J  = (0.0,1.0)
PHASER = 1.0
SCREEN = 3
THRESH = 0
ND = 1

C Define I and W for use in the LLYFFT Routine.
CALL LYTWID(I,W)

10 CALL CLEAR(SCREEN)
WRITE(*,*)
WRITE(*,900) 'How many Dwell of data should be
   processed (2-33)'
900 FORMAT(1X,A)
990 FORMAT(1X,F19.4,A,I3,A)
910 FORMAT(1X,I4,F15.4,2F15.3)
911 FORMAT(1X,2I8,F15.5)
READ(*,*) NDWELL
IF ((NDWELL.LT.2).OR.(33.LT.NDWELL)) GOTO 10

C Use a threshold.
OPEN(4,FILE='THRESH',STATUS='OLD',FORM=*
   'FORMATTED')
READ(4,*) THRESH,XK,CRSCALE
CLOSE(4)
9 FORMAT (1X,A,F5.4,A)
C Get the data. One call to SORCE produces 64 pulses of radar data for the Sum and Delta Channels

DO 5000 ND = 1, NDWELL
CALL SORCE

C Move the Sum Data into the SMCHNL (Sum Channel).
DO 1010 II = 1, 64
SMCHNL(II) = SON(II)
1010 CONTINUE

C Calculate Sum Squared.
DO 1020 II = 1, 64
SUMSQ(II) = SMCHNL(II) * SMCHNL(II)
1020 CONTINUE

C Calculate the FFT [Sum x Sum].
CALL LYFFT (SUMSQ, FFTSS, I, W)

C Make the Delta * Sum Product.
DO 1090 II = 1, 64
DELSUM(II) = (SPRDI(II) + J* SPRDQ(II)) * SMCHNL(II)
1090 CONTINUE

C Calculate the FFT [Delta Sum].
CALL LYFFT (DELSUM, FFTDS, I, W)

C Find the Peak values of the Mag[IFFT(Sum Sum)] and Mag[IFFT(Delta Sum)].
PKHSUM(ND) = 0.
DO 1130 II = 1, 64
FFTSS(II) = FFTSS(II)/64.
MAGFSS(ND, II) = SQRT((REAL(FFTSS(II)))**2 &
+ (AIMAG(FFTSS(II)))**2)
IF (MAGFSS(ND, II) .GT. PKHSUM(ND)) THEN
PKHSUM(ND) = MAGFSS(ND, II)
END IF
IF (II .EQ. 1) PKLSUM(ND) = PKHSUM(ND)
IF ((MAGFSS(ND, II) .LT. PKLSUM(ND)).AND.
(MAGFSS(ND, II) .NE. 0)) THEN
PKLSUM(ND) = MAGFSS(ND, II)
END IF
1130 CONTINUE

PKHDEL(ND) = 0.
DO 1135 II = 1, 64
    FFTDS(II) = FFTDS(II)/64.
    MAGFDS(ND, II) = SQRT((REAL(FFTDS(II)))**2 & +(AIMAG(FFTDS(II)))**2)
IF (MAGFDS(ND, II) .GT. PKHDEL(ND)) THEN
    PKHDEL(ND) = MAGFDS(ND, II)
END IF
IF (II .EQ. 1) PKLDEL(ND) = PKHDEL(ND)
IF ((MAGFDS(ND, II).LT.PKLDEL(ND)).AND. & (MAGFDS(ND, II).NE.0)) THEN
    PKLDEL(ND) = MAGFDS(ND, II)
END IF
CONTINUE

C Threshholding is required, set up the YES/NO array in JT array. Move the Mag [FFT(Sum **) ] into MSUMSQ
CALL CLEAR(SCREEN)
WRITE(*,*)' For Dwell Number ', ND ', '
WRITE(*,*)' The peak value of the IFFT [Sum * Sum] ' & ,PKHSUM(ND)
WRITE(*,*)' The peak value of the IFFT [Delta x Sum] ' & ,PKHDEL(ND)
WRITE(*,*)

C Use a threshold .
THOLD = PKHSUM(ND) * THRESH
WRITE(*,*)
WRITE(*,*)' The following Range Separation Bins passed ',THOLD
WRITE(*,*)' Monopulse Error for each separation bin ... '
DETECT = 0
DO 1060 II= 1, 64
    JT(ND, II) = 0
    IF (MAGFSS(ND, II) .GT. THOLD) THEN
        JT(ND, II) = 1
        DETECT = DETECT + 1
    END IF
1060 CONTINUE
C Apply the YES/NO array to the both the numerator and the denominator before dividing.

DO 1140 II=1,64
  FFTDS(II) = FFTDS(II) * JT(ND,II)
  FFTSS(II) = FFTSS(II) * JT(ND,II)
  MAGFSS(ND,II) = MAGFSS(ND,II) * JT(ND,II)
  MAGFDS(ND,II) = MAGFDS(ND,II) * JT(ND,II)
1140 CONTINUE

C Check for Zero Numerator or for Zero Denominator.
DO 1500 II = 1,64
  IF ((MAGFDS(ND,II).EQ.0.0).OR.
     (MAGFSS(ND,II).EQ.0.0))THEN
    RATIO(ND,II)=0.
  ELSE
    FFTDS(II) = FFTDS(II) * PHASER
    RATIO(ND,II) = REAL( FFTDS(II)/FFTSS(II) )
    RATIO(ND,II) = XK * CRSCAL * RATIO(ND,II)
    IF (RATIO(ND,II) .LT. 0) THEN
      MAGFDS(ND,II)= -MAGFDS(ND,II)
    IF (II .EQ. 1) THEN
      THE zero seperation term ....
      TERMO = RATIO(ND,II)
    ELSE
      TOTAL = TOTAL + RATIO(ND,II)
    END IF
  END IF
  IF(JT(ND,II).EQ.1)THEN
    IF(II.LT.33) IS = II - 1
    IF(II.GE.33) IS = II - 65
    WRITE(*,990) RATIO(ND,II) , ' at ' , IS, & ' m seperation'
    WRITE(3,910) IS,RATIO(ND,II),MAGFDS(ND,II),MAGFSS(ND,II)
  END IF
1500 CONTINUE
Dump the data to a file for Surfer software.

NICH = 33
DO 2740 NFB = 1, 64
   IF(NICH.GE.33) IS = NICH - 65
   IF(NICH.LT.33) IS = NICH - 1
   WRITE(5, 911) IS, ND, RATIO(ND, NICH)
   NICH = NICH + 1
   IF (NICH .GT. 64) THEN
      NICH = 1
   END IF
2740 CONTINUE

The 65th point is the same as the 1st point.
NICH = 33
IF(NICH.GT.33) IS = NICH - 65
IF(NICH.LE.33) IS = NICH - 1
WRITE(5, 911) IS, ND, RATIO(ND, NICH)
CRNG = CRSCALE * REAL(ND - 17)

Now the total error calculation.
IF (DETECT .EQ. 1) THEN
   Only one point target was detected.
   TOTAL = TERM0
ELSE
   Note: the number of point targets detected:
      ( AC and DC terms - 1 ) / 2
      TOTAL = TOTAL / (DETECT-1.0)
END IF
WRITE(*,*) 'Monopulse error : ', TOTAL
WRITE(3,*) 'Monopulse error : ', TOTAL

Plot the results.
CALL AXIS
STOP 'Leaving the Monopulse Error Slope Routine.'
END

SLOPE: Monopulse Error Slope Program Version 1.0
Copyright (C)1987 Martin Marietta Corporation,
U.S.A.
SOURCE2: Monopulse Error Slope Data Generating Subroutine Version 1.0

Copyright (C)1987 Martin Marietta Corporation, U.S.A.

Written by
Larry Alan Young
of
Martin Marietta Missile Systems
Radar Systems Analysis Dept.

This subroutine will generate Sum and Difference channel data to simulate the antenna scanning past the target array. The inputs parameters are in a file and arranged as follows,

Amplitude of target 1
Amplitude of target 2
Cross range distance to target 1
Cross range distance to target 2
Down range distance to target 1
Down range distance to target 2

Contact Larry Alan Young (305) 356-5422 for technical assistance.

SUBROUTINE SORCE
$INCLUDE: 'MONBLK.FOR'
CHARACTER *14 DATIN

C Get the file name of the test parameters ...
IF ( ND .EQ. 1 ) THEN
WRITE(*,*)
&'Enter a file name for the data parameters'
READ(*, '(A)') DATIN
OPEN(6, FILE=DATIN, STATUS='OLD', FORM='FORMATTED')
915 FORMAT(F6.1)
C Get the target's reflectivity ...
READ(6,915) A
READ(6,915) B

C Get the target Cross range Distances...
READ(6,915) CRA
READ(6,915) CRB

C Get the target Down Range Distances ...
READ(6,915) DRA
READ(6,915) DRB
CLOSE(6)
END IF

PI = 3.1415927

C Calculate the Down range separations ...
RNGAB = ABS( DRA - DRB)
RNGBC = ABS( DRB - DRC)

C Where ND goes from 1 to NDWELL ,say 1 to 33 ,
C Let XSCANG ( scan angle ) go from -NDWELL/2 to
C NDWELL/2 .

XSCANG = REAL(ND-1) - REAL(NDWELL-1)/2.

C SumPat is a linear approximation of the sum channel
C as the radar scans past the target array.

C Generate the 64 Frequencies of Sum Channel Data ... 
PeakSum = ( A * CRA + B * CRB ) / 2.0
SumPat = 1.0
DC = (A*A + B*B )
AC1 = ( 2*A*B )
DO 710 IFREQ = 1 , 64
SON(IFREQ)=DC+AC1* COS(2*PI * RNGAB * IFREQ/64.)
SON(IFREQ) = SumPat * SQRT( SON(IFREQ) )
710 CONTINUE

C Make the 64 Frequencies of Inphase Difference Channel
C Data. ACI and ACQ are linear approximations of the
C delta channel as the radar scans past the target
C array in cross range, as are DCI and DCQ.
C The even part of A resonating with B is symmetric
C about the point half way between the two targets in
C cross range. This even component is approximately
C linear in magnitude, with respect to scan angle, and
C goes through zero as the scan angle passes the
C geometric center of the pair.
The odd part of A resonating with B is symmetric about the point half way between the two targets in cross range. It is a fixed value, independent of the scan angle.

\[
DC = (A^2 + B^2) \\
DC10 = (DC * XSCANG - ((A^2 * CRA + B^2 * CRB) / 2.0)) \\
DCQ0 = DC10 \\
AC11 = AC1 * (XSCANG - (CRA + CRB)/2.) \\
ACQ1 = AC1 * (CRA - CRB)/2.
\]

Make the 64 Frequencies of Inphase Difference channel data.

```
DO 720 IFREQ= 1, 64
   SPRDI(IFREQ) = DC10 + AC11 * COS( 2 * PI * RNGAB * IFREQ/64.)
   SPRDI(IFREQ) = SPRDI(IFREQ) / SON(IFREQ)
720 CONTINUE
```

Make the 64 Frequencies of Quadrature Difference channel data.

```
DO 730 IFREQ= 1, 64
   SPRDQ(IFREQ) = DCQ0 + ACQ1 * SIN( 2 * PI * RNGAB * IFREQ/64.)
   SPRDQ(IFREQ) = SPRDQ(IFREQ) / SON(IFREQ)
730 CONTINUE
```

RETURN
END

SOURCE2: Monopulse Error Slope Data Generating Subroutine Version 1.0

Copyright (C) 1987 Martin Marietta Corporation, U.S.A.
SOURCE3: Monopulse Error Slope Data Generating
Subroutine Version 1.0

Copyright (C) 1987 Martin Marietta Corporation,
U.S.A.

Written by
Larry Alan Young
of
Martin Marietta Missile Systems
Radar Systems Analysis Dept.

This subroutine will generate Sum and Difference
cchannel data to simulate the antenna scanning
past the target array. The inputs parameters are
in a file and arranged as follows,

Amplitude of target 1
Amplitude of target 2
Amplitude of target 3
Cross range distance to target 1
Cross range distance to target 2
Cross range distance to target 3
Down range distance to target 1
Down range distance to target 2
Down range distance to target 3

Contact Larry Alan Young (305) 356-5422 for
technical assistance.

SUBROUTINE SORCE
$INCLUDE: 'MONBLK.FOR'
CHARACTER *14 DATIN

Get the file name of the test parameters ...
IF ( ND .EQ. 1 ) THEN
WRITE(*,*)
&' Enter a file name for the data parameters'
READ(*,'(A)') DATIN
OPEN(6,FILE=DATIN,STATUS='OLD',FORM='FORMATTED')
915 FORMAT(F6.1)
C Get the target's reflectivity ...
READ(6,915) A
READ(6,915) B
READ(6,915) C

C Get the target Cross range Distances...
READ(6,915) CRA
READ(6,915) CRB
READ(6,915) CRC

C Get the target Down Range Distances ... 
READ(6,915) DRA
READ(6,915) DRB
READ(6,915) DRC
CLOSE(6)
END IF
PI = 3.1415927

C Calculate the Down range separations ...
RNGAB = (DRA - DRB)
RNGBC = (DRB - DRC)
RNGCA = (DRC - DRA)

C Where ND goes from 1 to NDWELL ,say 1 to 33 ,
C XSCANG (The scan Angle) goes from -NDWELL/2 to
C NDWELL/2 .
C
XSCANG = REAL(ND-1) - REAL(NDWELL-1)/2.

C SumPat is a Cosinusoidal approximation of the sum
C channel as the radar scans past the target array.

C Generate the 64 Frequencies of Sum Channel Data.
C In the near field the Sum Pattern is a constant
C function while the Difference Pattern is a linear
C function.
SumPat = 1
DC = (A*A + B*B + C*C)
AC1 = (2*A*B)
AC2 = (2*B*C)
AC3 = (2*C*A)
DO 710 IFREQ= 1, 64
   SON(IFREQ) = DC +
   AC1 * COS(2 * PI * RNGAB * IFREQ/64.)
   * + AC2 * COS(2 * PI * RNGBC * IFREQ/64.)
   * + AC3 * COS(2 * PI * RNGCA * IFREQ/64.)
   SON(IFREQ) = SumPat * SQRT(SON(IFREQ))
710 CONTINUE
Make the 64 Frequencies of Inphase Difference Channel Data.

I AND Q are linear approximations of the delta channel as the radar scans past the array in cross range.

The even part of A resonating with B is symmetric about the point half way between the two targets in cross range. This even component is approximately linear in magnitude, with respect to scan angle, and goes through zero as the scan angle passes the geometric center of the pair.

The odd part of A resonating with B is symmetric about the point half way between the two targets in cross range. It is a fixed value, independent of the scan angle.

\[
\begin{align*}
DCI &= DC \times \left( XSCANG - (A \times CRA + B \times CRB + C \times CRC) / (3 \times (A+B+C)) \right) \\
DCQ &= DCI \\
ACI1 &= AC1 \times \left( XSCANG - (CRA + CRB) / 2 \right) \\
ACQ1 &= AC1 \times (CRA - CRB) / 2 \\
ACI2 &= AC2 \times \left( XSCANG - (CRB + CRC) / 2 \right) \\
ACQ2 &= AC2 \times (CRB - CRC) / 2 \\
ACI3 &= AC3 \times \left( XSCANG - (CRC + CRA) / 2 \right) \\
ACQ3 &= AC3 \times (CRC - CRA) / 2
\end{align*}
\]

Make the 64 Frequencies of Inphase Difference channel data.

DO 720 IFREQ = 1 , 64

\[
\begin{align*}
SPRDI(IFREQ) &= DCI \\
&+ ACI1 \times \cos\left( 2 \times \pi \times RNGAB \times IFREQ / 64 \right) \\
&+ ACI2 \times \cos\left( 2 \times \pi \times RNGBC \times IFREQ / 64 \right) \\
&+ ACI3 \times \cos\left( 2 \times \pi \times RNGCA \times IFREQ / 64 \right) \\
SPRDI(IFREQ) &= SPRDI(IFREQ) / \text{SON(IFREQ)}
\end{align*}
\]

720 CONTINUE
C Make the 64 Frequencies of Quadrature Difference

C channel data.
DO 730 IFREQ= 1 , 64
  SPRDQ(IFREQ) = DCQ
  & + ACQ1 * SIN( 2 * PI * RNGAB * IFREQ/64.)
  & + ACQ2 * SIN( 2 * PI * RNGBC * IFREQ/64.)
  & + ACQ3 * SIN( 2 * PI * RNGCA * IFREQ/64.)
SPRDQ(IFREQ) = SPRDQ(IFREQ) / SON(IFREQ)
730 CONTINUE
RETURN
END

************************************************
C SOURCE3: Monopulse Error Slope Data Generating
Subroutine Version 1.0
C
C Copyright (C) 1987 Martin Marietta Corporation,
U.S.A.
C
************************************************
**LYFFT: Monopulse Error Slope Subroutine to calculate IFFT{X} Version 1.0**

**Copyright (C) 1987 Martin Marietta Corporation, U.S.A.**

Written by
Larry Alan Young
of
Martin Marietta Missile Systems
Radar Systems Analysis Dept.

This subroutine will compute the Inverse Fourier Transform of the data in the array X(n), by using a 64 point radix 4 method.

Contact Larry Alan Young (305) 356-5422 for technical assistance.

**SUBROUTINE LYFFT(X,X3,I,W)**

COMPLEX X(64),X1(64),X2(64),X3(64),J,W(64)
INTEGER*4 I(64)
J=(0.,1.)

**COMPUTE X1 THE FIRST RECURSION**

DO 30 N=1,16
   X1(N) = X(N) + X(N+16) + X(N+32) + X(N+48)
   X1(N+16) = (X(N) - X(N+32) + J*(-X(N+16) + X(N+48))) * W(N)
   X1(N+32) = (X(N) - X(N+16) + X(N+32) - X(N+48)) * W(2*N-1)
   X1(N+48) = (X(N) - X(N+16) + X(N+32) - X(N+48)) * W(3*N-2)
30 CONTINUE
COMPUTE X2 THE SECOND RECURSION

N=1
NK=1
DO 50 LC2=1,4
  DO 40 LC1=1,4
    X2(N) = X1(N) + X1(N+4) + X1(N+8) + X1(N+12)
    X2(N+4) = (X1(N) - X1(N+8) + J*(-X1(N+4) + X1(N+12))) * W(4*NK-3)
    X2(N+8) = (X1(N) - X1(N+4) + X1(N+8) - X1(N+12)) * W(8*NK-7)
    X2(N+12) = (X1(N) - X1(N+4) + J*(X1(N+4) - X1(N+12))) * W(12*NK-11)
    N=N+1
    NK=NK+1
  40 CONTINUE
N=N+12
NK=1
50 CONTINUE

COMPUTE X3 THE THIRD RECURSION

DO 60 N=1,61,4
  X3(I(N)) = X2(N) + X2(N+1) + X2(N+2) + X2(N+3)
  X3(I(N+1)) = X2(N) - X2(N+2) + J*((-X2(N+1)) + X2(N+3))
  X3(I(N+2)) = X2(N) - X2(N+1) + X2(N+2) - X2(N+3)
  X3(I(N+3)) = X2(N) - X2(N+2) + J*(X2(N+1) - X2(N+3))
60 CONTINUE

The FFT2D is now in the X3 array.
RETURN
END

******************************************************************************************

LYFFT: Mono Pulse Error Slope Subroutine to calculate IFFT{X} Version 1.0

Copyright (C) 1987 Martin Marietta Corporation, U.S.A.

******************************************************************************************
SUBROUTINE LYTWID(I,W)

COMPLEX J,W(64)
INTEGER*4 I(64)
J=(0,1)
DO 400 NK=1,64
C The value THETA is (2*pi/64)*NK
THETA= 0.0981747704*(NK-1)
W(NK)=COS(THETA)-J*SIN(THETA)
400 CONTINUE

C ARRAY I IS FOR REARRANGING THE OUTPUT INTO SEQUENTIAL C ORDER.
I( 1)= 1 
I( 2)=17 
I( 3)=33 
I( 4)=49
<table>
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<tr>
<th>i</th>
<th>value</th>
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<td>48</td>
<td>63</td>
</tr>
</tbody>
</table>
I(49) = 4
I(50) = 20
I(51) = 36
I(52) = 52
I(53) = 8
I(54) = 24
I(55) = 40
I(56) = 56
I(57) = 12
I(58) = 28
I(59) = 44
I(60) = 60
I(61) = 16
I(62) = 32
I(63) = 48
I(64) = 64
RETURN
END

*****************************************************************************
LYFFT: Monopulse Error Slope Subroutine to calculate IFFT(x) Version 1.0
Copyright (C) 1987 Martin Marietta Corporation, U.S.A.
*****************************************************************************
**AXIS: Mono Pulse Error Slope Axis Plotting**  
**Subroutine Version 1.0**

Copyright (C) 1987 Martin Marietta Corporation,  
U.S.A.

Written by  
Larry Alan Young  
of  
Martin Marietta Missile Systems  
Radar Systems Analysis Dept.

This subroutine will draw the coordinate axis on  
the Vectrix High Resolution Monitor.

Contact Larry Alan Young (305) 356-5422 for  
technical assistance.

```
SUBROUTINE AXIS

$INCLUDE: 'MONBLK.FOR'

INTEGER *4 IX4
INTEGER *2 BC, FC, SCLC, IX, IX1
C IX1 IS FILLER FOR THE INTEGER *4 FIELD ...

INTEGER *2 IYEAR, IMON, IDAY, I HOUR, I MIN, IS EC, IHUN
INTEGER *2 GETTIM, GETDAY, GETKEY, ICR
CHARACTER *19 TITLE
CHARACTER *2 CR, SCAN
CHARACTER *30 EXTRA
CHARACTER *1 CM(2)
CHARACTER *14 RNAME

EQUIVALENCE (TNAME, RNAME)
EQUIVALENCE (ICR, CR)
EQUIVALENCE (IX4, IX)
EQUIVALENCE (M, CM(1))
```
OPEN(7, FILE='PLOT.INP', STATUS='OLD', FORM=
 & 'FORMATTED')
910 FORMAT(F8.3)
911 FORMAT(3x,A)
READ(7,910) APITCH
READ(7,910) AYAW
READ(7,910) PXSCAL
READ(7,910) DFSCAL
READ(7,910) FG
READ(7,911) Scan
CLOSE(7)
ICR=13+256*13
IXI = 0
C Define Background Color and foreground Color
OPEN(1, FILE='VECTRIX')
BC = 192
FC = 511
SCLC = 2
PC = 63
C Erase the vectrix screen and set the background & foreground colors.
WRITE(1,'(A,I3,A,I3)') 'SV G E',BC,' C ',FC
C Set the view port for the PRINTER WINDOW.
WRITE(1,'(A)') 'V 000 616 047 479'
C Vertical Axis.
WRITE(1,'(A)') 'RE M 100 95 L 100 255'
C Vertical scale's tick marks.
DO 7100 IY = 95,255,20
WRITE(1,'(A,I4,A,I4)') 'RE M 95 ',IY,' L 99 ',IY
7100 CONTINUE
C Label the Data Graph.
YMAX = DFSCAL
YMIN = 0
WRITE(1,'(A)') 'RE M 66 300 $Monopulse'$
WRITE(1,'(A)') 'RE M 66 285 $ Error''
IF(ABS(DFSCAL).GE.100 ) THEN
  LABLE = INT( YMAX-YMIN )
  LLYMAX = INT( YMAX )
  LLYMIN = INT( YMIN )
  WRITE(1,'(A,I4)') 'REM 66 255 $',LYMAX
  WRITE(1,'(A,I4)') 'REM 66 235 $',3*LABLE/4+LYMIN
  WRITE(1,'(A,I4)') 'REM 66 215 $',LABLE/2 +LYMIN
  WRITE(1,'(A,I4)') 'REM 66 195 $',LABLE/4 +LYMIN
  WRITE(1,'(A,I4)') 'REM 66 175 $',LYMIN
  WRITE(1,'(A,I4)') 'REM 66 155 $',-LABLE/4 +LYMIN
  WRITE(1,'(A,I4)') 'REM 66 135 $',-LABLE/2 +LYMIN
  WRITE(1,'(A,I4)') 'REM 66 115 $',-3*LABLE/4+LYMIN
  WRITE(1,'(A,I4)') 'REM 66 95 $',-LYMAX
ELSE
  WRITE(1,'(A,F5.0)') 'REM 66 255 $',YMAX
  WRITE(1,'(A,F5.0)') 'REM 66 235 $',3*(YMAX-YMIN)/4.+YMIN
  WRITE(1,'(A,F5.0)') 'REM 66 215 $',1*(YMAX-YMIN)/2.+YMIN
  WRITE(1,'(A,F5.0)') 'REM 66 195 $',1*(YMAX-YMIN)/4.+YMIN
  WRITE(1,'(A,F5.0)') 'REM 66 175 $',YMIN
  WRITE(1,'(A,F5.0)') 'REM 66 155 $',-1*(YMAX-YMIN)/4.+YMIN
  WRITE(1,'(A,F5.0)') 'REM 66 135 $',-1*(YMAX-YMIN)/2.+YMIN
  WRITE(1,'(A,F5.0)') 'REM 66 115 $',-3*(YMAX-YMIN)/4.+YMIN
  WRITE(1,'(A,F5.0)') 'REM 66 95 $',-YMAX
END IF

C Horizontal Axis.
WRITE(1,'(A)') 'REM 100 175 L 356 175 '
WRITE(1,'(A)') 'REM 100 95 L 356 95 '
WRITE(1,'(A)') 'REM 155 70 $Down Range Separations'
WRITE(1,'(A)') 'REM 435 210 $Cross Range Distance'

C Horizontal scale's tick marks.
DO 7201 IX =100,348,16
  WRITE(1,'(A,I4,A,I4,A)') 'RE M ',IX, ' 85 L ',IX,' 95'
& WRITE(1,'(A,I4,A,I4,A)') 'RE M ',IX+4, ' 92 L ',IX+4,' 95'
& WRITE(1,'(A,I4,A,I4,A)') 'RE M ',IX+8, ' 90 L ',IX+8,' 95'
& WRITE(1,'(A,I4,A,I4,A)') 'RE M ',IX+12, ' 92 L ',IX+12,' 95'
7201 CONTINUE
Scale for IFFT outputs (-32 - +32).

WRITE(1,'(A)') 'RE M 88 83 $-32'
WRITE(1,'(A)') 'RE M 152 83 $-16'
WRITE(1,'(A)') 'RE M 226 83 $0'
WRITE(1,'(A)') 'RE M 287 83 $16'
WRITE(1,'(A)') 'RE M 352 83 $32'

WRITE(1,'(A,I4,A,I4,A)') 'RE M ',356, ', 165 L ',
& ,356, ', 175 '
WRITE(1,'(A,I4,A,I4,A)') 'RE M ',356, ', 85 L ',
& ,356, ', 95 '
WRITE(1,930)'Press'
WRITE(1,931)'SPACE'
WRITE(1,932)'To Cont'
WRITE(1,940)'PrtSc'
WRITE(1,941)'Print'
WRITE(1,942)'Screen.'
CLOSE(1)

Plot the data.
CALL PLOTER
Sound the Bell ...
WRITE(*,*)''
CALL BIGGER

930 FORMAT('RE M 624 45 $',A)
931 FORMAT('RE M 624 33 $',A)
932 FORMAT('RE M 624 21 $',A)
940 FORMAT('RE M 000 45 $',A)
941 FORMAT('RE M 000 33 $',A)
942 FORMAT('RE M 000 21 $',A)
RETURN
END

AXIS: Monopulse Error Slope Axis Plotting
       Subroutine Version 1.0

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U.S.A.

*****************************************************************
PLOTTER: Monopulse Error Slope Data Plotting Subroutine Version 1.0

Copyright (C) 1987 Martin Marietta Corporation, U.S.A.
Written by Larry Alan Young of Martin Marietta Missile Systems Radar Systems Analysis Dept.

This subroutine will plot the results of the ratio function in 3 - Dimensions.

Contact Larry Alan Young (305) 356-5422 for technical assistance.

SUBROUTINE PLOTER

$INCLUDE: 'MONBLK.FOR'

FFT output for 64 frequencies for up to 33 evenly spaced dwells.
XCON = 0.017453292
IF ( AYAW .GE. 0 ) THEN
IX0  = 96
ELSE
IX0  = 352
END IF
IF ( APITCH .GE. 0 ) THEN
IY0  = 175
ELSE
IY0  = 325
END IF
IXZERO = -4
IYZERO = 0
DELTA = 4.
HMAX  = 65
NICH  = 33
NPLANE = 0
C Move the data into a buffer.
DO 2750 ND = 1, NDWELL
C 3D projection of the data ...
  IXZERO = IX0 + NPLANE * 17.066667
  SIN(XCON*AYAW) &
  IYZERO = IY0 + NPLANE * 7.071068
  SIN(XCON*APITCH) &
ARABUF(65) = RATIO(ND,NICH)
DO 2740 NFB = 1, 64
ARABUF(NFB) = RATIO(ND,NICH)
NICH = NICH + 1
IF (NICH .GT. 64) THEN
  NICH = 1
END IF
2740 CONTINUE
CALL CURVE
NPLANE = NPLANE + 1
2750 CONTINUE
900 FORMAT(1X,A\)
RETURN
END

*****************************************************************************

C C PLOTTER: Monopulse Error  Slope Data Plotting
C C Subroutine Version 1.0
C C Copyright (C) 1987 Martin Marietta Corporation,
C C U.S.A.
C C*****************************************************************************
CURVE: Monopulse Error Slope Line Drawing Subroutine Version 1.0

Copyright (C) 1987 Martin Marietta Corporation, U.S.A.

Written by Larry Alan Young of Martin Marietta Missile Systems Radar Systems Analysis Dept.

This subroutine will draw a curve on the Vectrix screen.

Contact Larry Alan Young (305) 356-5422 for technical assistance.

SUBROUTINE CURVE

$INCLUDE: 'MONBLK.FOR'

INPUT TO THIS ROUTINE .................

IXZERO (0,0) POINT ON THE SCREEN
IYZERO
DELTA # PIXELS / UNIT X INCREMENT
HMAX # OF INCREMENTS
DFSCAL DATA FULL SCALE VALUE
PXSCAL PIXELS FOR FULL SCALE VALUE
PC PLOT COLOR

INTEGER *2 BC,FC

Initialize everything to defaults.
OPEN(1,FILE='VECTRIX')

Set the view port for the Data Graph.
WRITE(1,'(A)') V 000 671 000 479'

Set the color for the line to plot.
WRITE(1,'(A,I3)')'C',PC
C Set to the (0,0) point of the screen.
IX = IXZERO
IY = IYZERO
WRITE(1,910) IX,IY

C Move to the first point of graph.
IX = INT(DELTA) + IXZERO
IY = INT(PXSCAL * (ARABUF(1)/DFSCAL)) + IYZERO
WRITE(1,915) IX,IY

C Draw the middle of the graph.
IF ( HMAX .NE. 1 ) THEN

DO 100 INC = 2,HMAX
   IY = INT(PXSCAL * (ARABUF(INC)/DFSCAL)) + IYZERO
C Increment to next point to plot.
   IX = IXZERO + INC * DELTA
C Draw a line to this point.
   WRITE(1,920) IX,IY
100 CONTINUE
END IF

CLOSE(1)
910 FORMAT( 'RE M ',2I4)
915 FORMAT( 'RE D ',2I4)
920 FORMAT( 'RE L ',2I4)

RETURN
END

C**************************************************************
C CURVE:     Monopulse Error  Slope Line Drawing
            Subroutine  Version 1.0
C
C Copyright (C) 1987 Martin Marietta Corporation,
C              U.S.A.
C**************************************************************
INTEGER*2 SCREEN

INTEGER*4 JT(33,64), I(64)
INTEGER*4 PC, IXZERO, IYZERO, HMAX, ND, NDWELL, RNGBIN, & NFB
REAL*4 SON(64), SPRDQ(64), SPRDI(64)
REAL*4 RATIO(33, 64), MAGFSS(33, 64), MAGFDS(33, 64)
REAL*4 ARRAY(250, 64), ARABUF(250)
REAL*4 DFSCAL, PXSCAL, DELTA

COMPLEX*8 X(64), X3(64)
COMPLEX*8 J, W(64)

CHARACTER*1 ANSWER
CHARACTER*2 UNITS

COMMON /GOT1/ SCREEN
COMMON /GOT2/ JT, I
COMMON /GOT3/ PC, IXZERO, IYZERO, HMAX, ND, NDWELL, & RNGBIN, NFB
COMMON /GOT5/ SON, SPRDQ, SPRDI, ARRAY, ARABUF
COMMON /GOT6/ RATIO, MAGFSS, MAGFDS
COMMON /GOT7/ DFSCAL, PXSCAL, DELTA
COMMON /GOT8/ X, X3
COMMON /GOT9/ J, W
COMMON /GOT10/ ANSWER
COMMON /GOT12/ UNITS
APPENDIX B

DEVELOPMENT OF AUTOCORRELATION FUNCTION
Development of Autocorrelation Function

The development of the autocorrelation function as in Einstein (1984). Assume an agile frequency exciter output,

\[ u(t) = e^{j(2\pi f_k t + \theta_k + \psi_k(t))} \]

where \( \theta_k \) is some constant phase angle and \( \psi_k(t) \) is the phase drift from the stable frequency \( f_k \). Let \( t = 0 \) be the time delay that corresponds to zero range.

The exciter serves also as the local oscillator so \( r(t) \), the reference is given as follows,

\[ r(t) = u(t) = e^{j(2\pi f_k t + \theta_k + \psi_k(t))} \]

If a near perfectly stable frequency source is used the term for the drift in frequency is zero, \( \psi_k(t) = 0 \).

The reflected signal from a single point target is the targets reflective coefficient modulated by the transmitted signal, delayed in time because of the distance to the target. For a single horn antenna, the received signal before mixing is,

\[ E(t) = A_t e^{j\phi_t} u(t - \frac{r}{c}) \]

The received signal is mixed with the local oscillator. The received signal is multiplied by the conjugate
of the reference $r(t)$, the local oscillator. So after filtering, the result $v(t)$ is,

$$v(t) = A_t e^{j \phi_t} u(t - 2\tau) \ast r(t) = A_t e^{j(2\pi f_k t + \Theta_k + \psi_k(t))}$$

The single point target has a range profile $\rho(x)$ defined as,

$$\rho(x) = A_t \delta(x)e^{j \phi_t}$$

where, the range delay in time is proportional to the range,

$$r = \frac{x}{c}$$

Consider a distributed target the range profile $\rho(x)$, if illuminated by a pulsed radar, with a pulse shape of $Ps(x)$ the received signal for a differential element of the distributed target is,

$$dE(f,t) = \rho(x) Ps(t-x) e^{j(2\pi f_k (t-x) + \Theta_k + \psi_k(t))} dx.$$  

The term $e^{j(2\pi f_k t + \Theta_k + \psi_k(t))}$ represents the phase of the return from $\rho(x)dx$.

The receiver voltage for a given frequency is the coherent sum of all of the differential scattering elements in the target. So in general the receiver voltage is the convolution of the range profile $\rho(x)$.
and the pulse shape \( P_s(x) \).

\[
E(f_k, t) = \int_{-\infty}^{+\infty} \rho(x) \, P_s(t-x) \, e^{j(2\pi f_k (t-x) + \Theta_k + \psi_k(t))} \, dx
\]

Assuming there is no frequency drift ,

\[ f_k(t-x) = f_k(x) \text{ and } \psi_k(t-x) = 0. \]

Using these facts and rearranging terms yields,

\[
E(f_k, t) = e^{-j(2\pi f_k x)} \int_{-\infty}^{+\infty} \rho(x) \, P_s(t-x) \, e^{-j2\pi f_k x} \, dx.
\]

The received signal is then mixed with the local oscillator, the voltage output from the detector is:

\[
V(f_k, t) = e^{-j\psi_k(t)} \int_{-\infty}^{+\infty} \rho(x) \, P_s(t-x) \, e^{-j2\pi f_k x} \, dx
\]

Mixing the signals has removed the frequency \( f_k \) and the phase \( \Theta_k \), but it introduces again the phase drift \( \psi(t) \). By regrouping terms the received signal is seen to have the form of a Fourier transform.

\[
Q(f_k, t) = V(f_k, t) e^{j\psi_k(t)} = \int_{-\infty}^{+\infty} \rho(x) \, P_s(t-x) \, e^{-j2\pi f_k x} \, dx
\]
Let $\Phi[g]$ represent the Fourier transform of $g(x)$ then,

$$q(f_k, t) = \rho(x) \ast Ps(t-x)$$

$$Q(f_k, t) = \Phi[\rho(x) \ast Ps(t-x)]$$

By applying the inverse Fourier transform to the receiver output voltage, the product of the range profile and pulse shape is found.

$$\rho(x) Ps(t-x) = \int_{-\infty}^{+\infty} V(f_k, t) e^{j\psi_k(t)} e^{j2\pi f_k x} df$$

or,

$$\rho(x) Ps(t-x) = \Phi^{-1}[Q(f,t)]$$

Since the return is sampled at discrete frequencies the discrete form of the transform is

$$\rho(x) Ps(t-x) = \sum_{k=0}^{N-1} e^{j\psi_k(t)} V(f_k, t) e^{j2\pi f_k x} \Delta f$$

where $\psi_k(t) = \psi(f_k, t)$

If there is no drift in the local oscillator or if it is negligible coherent processing is simplified. But when the phase drift is too large then they must be measured and compensated for in the above equation. The other alternative is to use noncoherent radar. The range profile, weighted by the pulse shape $\rho(x) Ps(t-x)$ is not obtained from a noncoherent system. But it is
possible to find the autocorrelation $R_q(z,t)$.

$$R_q(z,t) = \int_{-\infty}^{+\infty} q^*(x,t) q(x+z,t) \, dx$$

In reality, $|Q(f_k,t)|^2$ is the power spectral density of the received signal sampled at $N$ frequencies $f_k$. The spectral density $S(f)$ is a real even nonnegative function of frequency which gives the total average power per ohm when integrated.

$$P = \int_{-\infty}^{+\infty} S(f) \, df = <q(t)>^2$$

The receiver power spectral density is the square of the receiver voltage.

$$S(f) = |Q(f,t)|^2$$

$$P = \int_{-\infty}^{+\infty} |Q(f,t)|^2 \, df = <q(t)>^2$$

From Ziemer / 1976, the Wiener-Khinchine theorem says that the autocorrelation function $R(x)$ of a signal and its power spectral density $S(f)$ are Fourier transform pairs.

$$R_q(z,t) = \Phi^{-1} [S(f)] = \Phi^{-1} [|Q(f,t)|^2]$$

$$R_q(z,t) = \int_{-\infty}^{+\infty} |V(f,t)|^2 \left| e^{j2\pi fz} \right| df$$
In the discrete form,

\[ Rq(z,t) = \sum_{k=0}^{N-1} |V(f,t)|^2 e^{j\frac{2\pi f_k z}{\Delta f}} \]

The important point to be made is that finding \( Rq(z,t) \) does not depend on the phase of the receiver voltage, it depends only on the magnitude of the received signal.
LIST OF REFERENCES


